BANK OF CANADA
BANQUE DU CANADA

Staff working paper / Document de travail du personnel—2026-14

Last updated: May 7, 2026

To Tokenize, or Not to Tokenize: The Design Question for a Central Bank Digital Currency

Jonathan Chiu
Banking and Payments Department
Bank of Canada
jchiu@bankofcanada.ca

Cyril Monnet
Study Center Gerzensee
University of Bern
cyril.monnet@unibe.ch

Oliver Xu
Banking and Payments Department
Bank of Canada
jxu@bankofcanada.ca

DOI: https://doi.org/10.34989/swp-2026-14 | ISSN 1701-9397

To Tokenize, or Not to Tokenize: The Design Question for a Central Bank Digital Currency*

Jonathan Chiu
Bank of Canada

Cyril Monnet
University of Bern

Oliver Xu
Bank of Canada

February 2026

Abstract

This paper develops a general equilibrium model to assess central bank digital currency (CBDC) design in a monetary system where traditional banks and "crypto banks" (i.e., banks that issue stablecoins) coexist. We compare tokenized and non-tokenized CBDC, showing that their desirability depends on the reliability of private money provision, the availability of collateral assets and the features of the crypto sector. Crucially, we show that the tokenization decision of CBDC matters for the equilubrium outcomes only when collateral use differs across sectors, identifying conditions under which tokenization is necessary to improve welfare. Tokenized CBDC can crowd out stablecoins and improve efficiency when crypto banks are not that trustworthy and crypto assets are scarce. Non-tokenized CBDC may be preferred when crypto transactions are less desirable or when reallocating reserves from traditional to crypto banks is beneficial. Our results highlight a trade-off between gains in payment efficiency and potential reductions in bank lending. These findings offer new policy insights on CBDC design under evolving financial conditions.

JEL Codes: E50, E58.
Keywords: Central Bank Digital Currencies, Crypto Banks, Stablecoins, Tokenization

*We thank many friends and colleagues for many useful conversations. We thank the Editor and an anonymous referee for their helpful comments. The views expressed here are those of the authors and do not necessarily reflect the position of the Bank of Canada.

1 Introduction

A central question in the design of a central bank digital currency (CBDC) is whether it should be *tokenized* or not.[1] A non-tokenized CBDC competes with bank deposits as a means of payment in traditional (off-chain) markets; a tokenized CBDC can be used on programmable ledgers and competes with stablecoins in the crypto sector. The choice affects payment efficiency in the traditional sector, the level of activities in the crypto sector, the allocation of collateral across traditional and crypto banks, and the supply of bank credit to firms. This paper develops a general equilibrium model to assess CBDC design when traditional and crypto banks coexist and to identify conditions under which tokenization is necessary to improve welfare.

Central bankers and policymakers are actively considering the optimal design of the future monetary system.[2] New technologies such as distributed ledger technology (DLT), smart contracts, and stablecoins are reshaping payments and monetary systems. At the same time, transactions in crypto sector are pseudonymous and border-less, posing challenges for regulation. Given these developments, it is crucial to investigate whether and how central banks should redesign the monetary system to accommodate new technologies. Central banks play a pivotal role in ensuring payment efficiency and stability. As such, they must consider the potential benefits and risks of integrating new technologies into the monetary system. In particular, to harness the benefits of technological advancements while mitigating the associated risks, the central bank needs to prepare a blueprint for the future monetary system as a function of future developments. Some central bankers argue for crowding stablecoins out of existence by issuing central bank digital currencies (CBDC).[3] Against this context, this paper develops a general equilibrium monetary model to study the following questions and derive the optimal policy in a calibrated model under different scenarios:

[1]By tokenization, we mean representing a claim (e.g., a payment balance or an underlying asset) as a digital token recorded on a programmable ledger, such that ownership and transfer of the claim are executed by ledger updates (and, when relevant, smart-contract rules), rather than through updates on a conventional account-based system.

[2]For example, the U.S. Department of the Treasury published a report on "The Future of Money and Payments" in September 2022 (https://home.treasury.gov/system/files/136/Future-of-Money-and-Payments.pdf). Also, the 2023 Bank for International Settlements Annual Report discussed the "Blueprint for the future monetary system" (https://www.bis.org/publ/arpdf/ar2023e.pdf).

[3]Federal Reserve Chair Jerome Powell argued that the introduction of a CBDC would eliminate the use case for cryptocurrencies. He stated, "You wouldn't need stablecoins; you wouldn't need cryptocurrencies if you had a digital U.S. currency. I think that's one of the stronger arguments in its favor." Also, Reserve Bank of India Deputy Governor T. Rabi Sankar also asserted that a CBDC would "kill whatever little case there could be" for cryptocurrencies. Academics like Gary Gorton suggest that the government could "introduce a central bank digital currency and tax private stablecoins out of existence" (Gorton, 2023).

- What roles should traditional banks, crypto banks and central banks play in the future monetary system?[4]
- Should a central bank issue a CBDC? Should it be tokenized or not?
- What are the implications for traditional and tokenized monetary systems?

Our model extends the framework in Chiu and Monnet (2025) to consider an economy consisting of two sectors: a traditional sector (for conventional "off-chain" transactions) and a crypto sector (for "on-chain" transactions). In the traditional sector, sector, households use cash and bank deposits, which are issued respectively by the central bank and traditional banks, as a means of payment. In the crypto sector, traditional, non-tokenized deposits are not feasible. Therefore households use stablecoins issued by crypto banks as a means of payment. Banks, in both traditional and crypto sectors, are susceptible to an agency problem, giving rise to an incentive constraint according to which they must secure their issuance of means of payments with collateral.

The crypto sector differs from the traditional sector in three aspects. First, some crypto transactions may be socially undesirable (e.g., involving criminal activities). Second, traditional banks are more tightly regulated and monitored than crypto banks. Third, traditional banks specialize in credit creation but are not allowed to invest in crypto assets (e.g., Bitcoin, Ether), which crypto banks can hold as reserve assets. Overall, the regulation over traditional and crypto banks and the characteristics of their collateral holdings determine the tightness of their incentive constraints.

After characterizing the basic model in Sections 2 and 3, we examine how the features of the crypto sector (e.g., size of the sector and the abundance of crypto assets) affect the whole economy, including the traditional sector in Section 4. We then use the model to discuss two potential future monetary systems. Section 5 considers the central bank issuing a non-tokenized CBDC used exclusively in the traditional sector. The exercise can be interpreted as an extension of Keister and Sanches (2023) by examining the implications for the crypto sector. We found that a CBDC can have complex effects on the crypto sector, crowding in or crowding out stablecoins depending on economic conditions.

Section 6 allows the central bank to issue a tokenized CBDC for use in the crypto sector. The exercise can be interpreted as an extension of Chiu and Monnet (2025) by considering a generalized environment

[4] Here, we use the term "crypto banks" to refer to banks that issue stablecoins.

with cash, bank credit creation and endogenous crypto asset prices. While issuing a tokenized CBDC generally lowers crypto asset prices, it can crowd in or crowd out traditional bank deposit and credit creations depending on the role of bonds and CBDC in the crypto sector. We also consider the case where the central bank issues both a tokenized and a non-tokenized CBDC and provide a condition under which issuing a tokenized CBDC is equivalent to an issuance of a non-tokenized CBDC.

Section 7 examines the quantitative implications of these two CBDC systems and compares computationally the relative desirability of introducing these two new money balances in a parameterized model. We find that it tends to be optimal to issue a tokenized CBDC when the crypto transactions are socially desirable, crypto banks are not trustworthy and crypto assets are scarce.

Literature Our paper contributes to several lines of research in the literature. First of all, papers such as Ahnert et al. (2025), Andolfatto (2020), Chiu and Davoodalhosseini (2021), Chiu et al. (2023a), Keister and Sanches (2023), and Williamson (2022a) study the effects of CBDC issuance on traditional bank intermediation in normal times. In addition, Fernandez-Villaverde et al. (2020), Keister and Monnet (2022), Monnet et al. (2020), Schilling et al. (2020), and Williamson (2022b) examines the effects on bank stability in crisis times. Wang (2023) studies the implications of money laundering for the optimal design of a CBDC and Tinn (2024) considers a CBDC with asymmetric privacy. None of these papers examine the role of tokenized CBDC on blockchains in serving the crypto sector.[5] The most related paper is Chiu and Monnet (2025). This paper generalizes their model by incorporating cash, bank loans and endogenous crypto asset prices, which are important for computational work. Also, this paper considers not only tokenized money for the crypto sector but also the potential impacts of a non-tokenized CBDC on the crypto sector. Another related paper is Huang and Keister (forthcoming) which studies the narrow banking debate between stablecoins and tokenized deposits.

Our work is also related to the emerging literature on stablecoin and decentralized finance. Theoretical papers by D'Avernas, Bourany, and Vandeweyer (2021), Li and Mayer (2021), Kozhan and Viswanath-Natraj (2021)) study decentralized stablecoins such as Dai issued by the MakerDAO. Bertsch (2023) analyses stablecoins adoption and fragility and Carapella (2024) studies regulatory arrangements. Other DeFi platforms are also actively studied. For example, Aoyagi and Itoy (2021), Capponi and Jia

[5]See Auer et al. (2022) for a review of some policy issues and the academic literature.

(2021), Lehar and Parlour (2021) study decentralized exchanges in the form of automated market makers (e.g., Uniswap), while Chiu, et al (2022) and Lehar and Parlour (2022) focus on lending platforms. Chiu et al. (2023b) use a network model to investigate the relationships among different crypto tokens and DeFi activities. Cong and Mayer (2022) study the competition among national fiat currencies, cryptocurrencies, and central bank digital currencies. None of these papers evaluate the general-equilibrium impact of a CBDC on crypto activities, which is the main contribution of our analysis.

2 Benchmark Economy

The model is an extension of Chiu and Monnet (2025). Time is discrete and continues forever: $t = 0, 1, 2, ...$ Each period consists of two sub-periods with two alternating markets, à la Lagos and Wright (2005). In the first sub-period, segmented frictional markets open, and we call them AM. In the second sub-period, a Walrasian centralized market opens, and we call it PM. The discount factor between the PM and the next AM is $\beta < 1$.

The economy consists of a measure 2 of infinitely lived buyers and sellers, as well as large measures of short-lived bankers, and entrepreneurs, who enter the economy in each PM and exit in the following PM. The bankers can run a "traditional" or a "crypto" bank. Traditional banks offer loans to entrepreneurs while crypto banks do not offer loans. All agents trade a common numeraire good in the PM. Buyers and sellers derive utility $V(y)$ from consuming y units of the numeraire and suffer a linear cost $-h$ from producing h units of the numeraire. We assume $V'(y) > 0$ and $V''(y) \leq 0$. Bankers and entrepreneurs have a linear utility from consuming the numeraire. There is also a government and a central bank.

Young bankers enter the economy in the PM and exit in the following PM. In the PM, bankers can produce and consume the numeraire good with linear utility. Each banker chooses to operate either a traditional bank or a crypto bank and issues deposits or stablecoins as payment instruments. Traditional banks also extend loans to entrepreneurs.

The AM consists of two segmented sectors: traditional sector and crypto sectors. In the traditional sector, buyers and sellers interact directly to trade physical goods. In the crypto sector, they interact on a digital ledger called a "blockchain" to trade crypto goods e.g., digital collectibles in the form of NFTs, decentralized finance services, decentralized services like data indexing and cloud storage. We add a "tilde" ($\tilde{X}$) to denote all "on-chain" variables X in the crypto sector. As explained below, buyers can use three different means of payments: cash, z, issued by the central bank, deposits, d, issued by traditional bankers, and stablecoins, $\tilde{s}$, issued by crypto bankers.[6] We start with an economy where cash and deposits are used in the traditional sector as they can only be transferred off-chain, while stablecoins can only be used in the crypto sector as they are tokens recorded on the blockchain. The interpretation is that, due to technological barriers, regulatory restrictions, or transaction costs, deposits and cash are off-chain payment instruments and thus cannot be directly used to settle on-chain transactions. Conversely, stablecoins, recorded and transacted exclusively on blockchain ledgers, cannot be used in off-chain traditional markets.

Later, we will consider a design where central bank money can also be "tokenized" (i.e., have a digital representation on the blockchain) to be used on-chain. The three means of payment are created to trade three goods. In the traditional sector, there are cash goods, x_z, and deposit goods, x, that buyers can purchase off-chain from sellers using cash and deposits respectively; In the crypto sector, there are crypto goods, $\tilde{x}$, that buyers can purchase on-chain using stablecoins. Buyers derive utility $U(x_z, x, \tilde{x})$ from consuming these goods in the AM. Sellers incur a marginal cost of 1 when producing the three goods. In the benchmark model, we assume

$$U(x_z, x, \tilde{x}) = u_z(x_z) + u(x) + \tilde{u}(\tilde{x})$$

Each period in the PM, the government issues a fixed supply, B, of illiquid one-period bonds trading at a price q_b (in terms of the numeraire good), which is endogenously determined. The rate of return of bonds is $R_b = 1/q_b$. Each unit of bonds pays one unit of numeraire good in the following PM, financed by lump-sum taxation in the PM. In addition, in each PM, buyers are endowed with one unit of a 1-period crypto asset that pays E units of numeraire goods in the next PM.[7] These assets, denoted by $\tilde{e}$, are traded at the equilibrium price q_e in the PM.

In this sense, the "crypto asset" in the model is best interpreted as a representative on-chain asset that serves as a collateral/reserve instrument for the crypto sector. E summarizes the (possibly indirect)

[6] The existence of cash goods and cash is not critical for our analytical results. They are introduced just to match empirical data in the calibration exercise.

[7] Here, the return to holding the crypto asset can arise either from an explicit cash-flow-like payout or from expected price appreciation generated by protocol mechanisms: (i) staking or similar protocol rewards (e.g., staked ETH or liquid-staking tokens), (ii) buybacks/burning financed by protocol revenues that reduce token supply (e.g., MKR burns or fee burning under EIP-1559), or (iii) interest earned when the asset is supplied to decentralized lending markets (e.g., Aave/Compound). Also, E should be interpreted as the expected value as the market value of crypto assets can be random. As long as the random payoff is realized after AM transactions, uncertainty does not matter for our analysis due to risk neutrality in the PM.

flow of value associated with holding such an asset in practice.

The role of the central bank and bankers, both traditional and crypto, is to provide buyers with a means of payment. We use z, d and $\tilde{s}$ to denote the real payment balances of cash, deposits and stablecoins. The central bank issues cash with a fixed growth rate τ. Traditional bankers issue deposits d in the PM and promise to redeem them in the following PM. However, they cannot commit to repaying their outstanding claims, and they need to back them by holding government bonds b_T and loans ℓ_T as reserve assets. Banks can abscond with a fraction $1-\rho$ of reserve assets, so we call ρ the pledgeability parameter of traditional banks. Loans are borrowed by n_ℓ competitive entrepreneurs at rate R_ℓ to finance a project that can hire sellers in the AM to incur ℓ units of disutility to produce $F(\ell)$ units of numeraire goods in the next PM. The price of each unit of loans is $q_\ell = 1/R_\ell$.

Similar to traditional banks, crypto bankers can issue stablecoins $\tilde{s}$ backed by crypto assets $\tilde{e}$ and government bonds b_C. Crypto bankers cannot commit, and they can abscond with a fraction $1-\kappa$ of their off-chain assets and $1-\tilde{\kappa}$ of their on-chain assets. The case of $\kappa < \rho$ captures the idea that, unlike traditional banks, crypto banks are unregulated.[8] The case of $\kappa < \tilde{\kappa}$ captures the idea that on-chain assets can be monitored more easily than off-chain assets.

The timeline is the following: In the PM, traditional bankers acquire government bonds, make loans to entrepreneurs and issue deposits to buyers while crypto bankers acquire crypto assets and government bonds and issue stablecoins to buyers. The central bank also buys bonds to back the issuance of cash. In the next AM, buyers trade x and x_z in the traditional sector and $\tilde{x}$ in the crypto sector using the corresponding means of payment. Entrepreneurs use bank loans to hire sellers for their projects. In the following PM, entrepreneurs repay loans and holders of deposits and stablecoins redeem their tokens with the respective issuer. The government repays bonds. Figure 2 is a succinct representation of our economy. The balance sheets of the traditional, crypto and central banks are given by the following table:

Trad.	bank
Assets	Liabilities
ℓ_T	d
b_T	Equity

Crypto	bank
Assets	Liabilities
$\tilde{e}$	$\tilde{s}$
b_C	Equity

Central	bank
Assets	Liabilities
b_{CB}	z

Balance Sheets of Traditional, Crypto and Central Banks

[8] In practice, regulation typically precludes, or discourages, traditional banks from holding crypto assets.

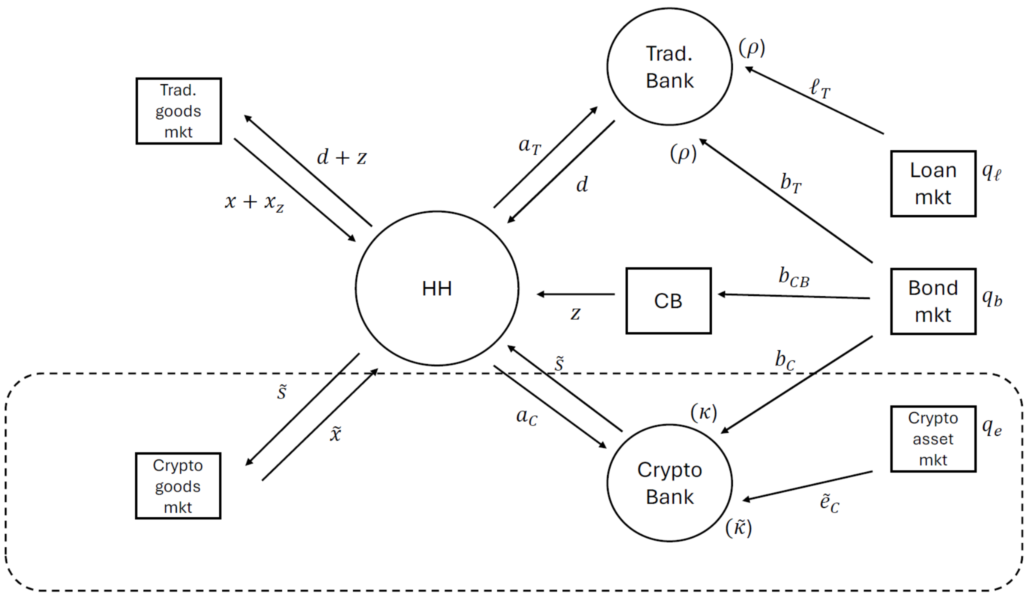

Figure 1: Model Setup

3 Equilibrium Characterization

3.1 Buyers' problems

In the PM, each buyer makes an offer (a_T, d, ℓ_T, b_T) to a competitive traditional banker where a_T is an initial payment to the banker in the PM, d is the quantity of deposits issued by the banker to be used off-chain by the buyer in the following AM, ℓ_T and b_T are loans and bonds that the banker will hold to back its issuance of deposits. The offer must satisfy the participation constraint (PC) and the incentive constraint (IC), respectively:

$$(a_T - q_\ell \ell_T - q_b b_T) + \beta(\ell_T + b_T - d) \geq 0 \quad , \tag{1}$$

$$\rho \ell_T + \rho b_T \geq d. \tag{2}$$

The terms on the left-hand-side of the PC capture the traditional banker's payoffs in the first and the next PMs. Specifically, the PC requires that the discounted payoff be non-negative. The IC requires that, to incentivize the bankers to redeem the deposits, the value of the pledgeable reserve assets on the left-hand-side needs to be higher than the redemption value of the deposits on the right-hand-side.

Similarly, the buyer makes an offer $(a_C, \tilde{s}, \tilde{e}, b_C)$ to a competitive crypto banker where a_C is an initial payment to the banker in the PM, $\tilde{s}$ is the quantity of stablecoins issued by the banker to be used on-chain by the buyer in the following AM, $\tilde{e}$ and b_T are crypto assets and bonds that the banker will hold to back its issuance of stablecoins. The offer needs to satisfy respectively the participation constraint (PC) and the incentive constraint (IC) of the crypto banker:

$$(a_C - q_e\tilde{e} - q_b b_C) + \beta(\tilde{e} + b_C - \tilde{s}) \geq 0, \tag{3}$$

$$\tilde{\kappa}\tilde{e} + \kappa b_C \geq \tilde{s}. \tag{4}$$

The two constraints (3) and (4) have similar interpretations as those discussed above. In addition, the buyer needs to carry cash z to buy cash goods subject to an inflation rate of τ. Given monetary policy τ, asset prices q_e, q_ℓ, q_e, the buyer chooses $(z, a_C, \tilde{s}, \tilde{e}, b_C, a_T, d, b_T, \ell_T)$ to maximize

$$\underbrace{-a_C - a_T - (1+\tau)z}_{\text{PM payoff}} + \underbrace{\beta U(x_z, x, \tilde{x})}_{\text{AM payoff}}$$

subject to (1), (2), (3) and (4), and liquidity constraints with respect to cash, deposits and stablecoins

$$x_z \leq z, \qquad x \leq d, \qquad \tilde{x} \leq \tilde{s}.$$

Clearly, constraints (1) and (3) will bind. Otherwise, it is optimal to reduce a_T and a_C. In addition, it is obvious that all the liquidity constraints will bind when

$$U_1(x_z, x, \tilde{x}) = \frac{\partial}{\partial x_z} U(x_z, x, \tilde{x}) > 1,$$
$$U_2(x_z, x, \tilde{x}) = \frac{\partial}{\partial x} U(x_z, x, \tilde{x}) > 1,$$
$$U_3(x_z, x, \tilde{x}) = \frac{\partial}{\partial \tilde{x}} U(x_z, x, \tilde{x}) > 1.$$

Otherwise, it is optimal to increase the consumption of one or more goods. We assume these inequalities hold and provide a condition below to justify them. With this understanding and replacing a_T and a_C

using the PC at equality, we can rewrite the problem as

$$\max_{z,\tilde{e},b_C,\tilde{s},\ell_T,b_T,d} -(1+\tau)z + \beta U(z,d,\tilde{s})$$
$$-q_e\tilde{e} - q_b b_C + \beta(\tilde{e} + b_C - \tilde{s}) + \beta\lambda_C(\tilde{\kappa}\tilde{e} + \kappa b_C - \tilde{s})$$
$$-q_\ell \ell_T - q_b b_T + \beta(\ell_T + b_T - d) + \beta\lambda_T(\rho\ell_T + \rho b_T - d)$$

where $\beta\lambda_T$ and $\beta\lambda_C$ are respectively the Lagrange multipliers on the traditional and crypto bank's IC. The first-order conditions are given in the online appendix. Note that the liquidity constraints are binding when $\min\{q_b, q_e, q_\ell, 1+\tau\} > \beta$. When this condition is satisfied, it is straightforward to obtain the following result from the FOC.

Lemma 1. *All the liquidity constraints bind. The optimal reserve decision of a traditional bank is*

$$(b_T, \ell_T) = \begin{cases} (d/\rho, 0), & \text{if } q_\ell > q_b, \\ (0, d/\rho), & \text{if } q_\ell < q_b, \\ \{b_T, \ell | \rho(b_T + \ell_T) = d\}, & \text{if } q_\ell = q_b, \end{cases} \tag{5}$$

and the optimal reserve decision of a crypto bank is

$$(b_C, \tilde{e}) = \begin{cases} (\tilde{s}/\kappa, 0), & \text{if } \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}} > \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa}, \\ (0, \tilde{s}/\tilde{\kappa}), & \text{if } \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}} < \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa}, \\ \{b_C, \tilde{e} | b_C\kappa + \tilde{e}\tilde{\kappa} = \tilde{s}\}, & \text{if } \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}} = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa}. \end{cases} \tag{6}$$

The optimal portfolio of the buyer satisfies

$$\tilde{u}'(\tilde{s}) = 1 + \min\left\{\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right), \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right)\right\}, \tag{7}$$

$$u'(d) = 1 + \frac{1}{\rho}\left(\frac{\min\{q_\ell, q_b\}}{\beta} - 1\right), \tag{8}$$

$$u_z'(z) = \frac{1+\tau}{\beta}. \tag{9}$$

3.2 Steady State equilibrium

We now derive the equilibrium conditions for the rest of the economy.

(i) Loan market clearing

Entrepreneurs take q_ℓ as given and choose the size of loan repayment, ℓ, to solve

$$\max_{\ell} F(L) - \ell,$$

where $L \equiv q_\ell \ell$ is the size of the bank loan. Hence, the FOC implies that $F'(L)q_\ell = 1$. The loan market clears when entrepreneurs' demand for loans equals banks' supply of loans

$$q_\ell \ell_T = n_\ell L,$$

which implies:

$$F'(\frac{q_\ell \ell_T}{n_\ell})q_\ell = 1. \tag{10}$$

(ii) Bond market clearing

The central bank needs to satisfy a balance-sheet constraint which requires that its cash issuance is fully backed by government bonds:

$$b_{CB} = z. \tag{11}$$

Note that feasibility requires $z \leq B$. Otherwise the implied private-sector bond holdings would be negative. Here, we focus on parameterizations in which the inflation (money growth) policy is feasible given the available bond supply. In general, bond supply places an upper bound on real cash balances under full backing, which in turn implies a lower bound on the inflation rate required to support equilibrium when bonds are scarce.

Hence, the bond market clears when total demand for bonds (from central, traditional, and crypto banks) equals the fixed government bond supply:

$$B = b_T + b_C + b_{CB}. \tag{12}$$

Through out the paper, we impose parameter values so that the private sector holds some bonds, or $B - b_{CB} = b_T + b_C > 0$.

(iii) Crypto asset market clearing

The crypto market clears when

$$\tilde{e} = E. \tag{13}$$

Definition 2. A steady state monetary equilibrium consists of prices (q_ℓ, q_e, q_b), quantities of means of payments $(d, \tilde{s}, z)$, quantities of reserve assets $(b_T, b_C, b_{CB}, \ell, \tilde{e})$ satisfying conditions (5)-(13).

Finally, we define the social welfare by allowing the social planner to assign a weight on the surplus from consuming crypto goods to capture the fact that some activities conducted in the crypto sector can be less desirable. With a weight $\omega \in (0,1]$ on crypto goods, the social welfare function is

$$\mathcal{W} = [u_z(x_z) - x_z] + [u(x) - x] + \omega[\tilde{u}(\tilde{x}) - \tilde{x}] + n_\ell[F(L) - L].$$

Hence, the regulator may want to discourage crypto goods transactions when $\omega < 1$. We denote by $(x_z^*, x^*, \tilde{x}^*, L^*)$ the efficient level of consumption in each sector that equates the marginal utility of consumption to the marginal cost of production.

4 Effects of introducing the Crypto Sector

To further characterize the equilibrium outcome, we specify the forms of the utility and production functions. Suppose the utility in the AM is

$$U(x_z, x, \tilde{x}) = u_z(x_z) + u(x) + \tilde{u}(\tilde{x}),$$

where

$$u_z(x_z) = \frac{x_z^{1-\sigma}}{1-\sigma},\ u(x) = \alpha\frac{x^{1-\sigma}}{1-\sigma},\text{and } \tilde{u}(\tilde{x}) = \tilde{\alpha}\frac{\tilde{x}^{1-\sigma}}{1-\sigma}.$$

And the utility function in the PM is given by

$$V(y) = A \log y.$$

The production function of entrepreneurs is

$$F(L) = \frac{L^{1-\chi}}{1-\chi},$$

with $\chi < 1$. [9] The optimal loan repayment size for the firm is given by

$$\ell = q_\ell^{\frac{1-\chi}{\chi}},$$

and the loan market clears when

$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}}.$$

Consider first an economy where the crypto sector is absent ($\tilde{\alpha} = 0$). Proposition 3 summarizes the key comparative statics. The following proposition is proved in the appendix:

[9] $\chi < 1$ also implies that ℓ_T decreases with the loan rate $1/q_\ell$ (i.e., $d\ell_T/dq_\ell > 0$).

Proposition 3. *Comparative statics in an economy without a crypto sector.*

(1) A higher B reduces q_b and ℓ_T , but raises d and b_T.

(2) A higher n_ℓ reduces q_b, but raises d and ℓ_T.

(3) A high τ reduces q_b, z and ℓ_T, but raises d and b_T.

Note that results (1) and (3) are similar. In this simplified setup, cash issuance is fully backed by government bonds, so the central bank holds $b_{CB} = z$, where z denotes real cash balances demanded by households. Bond-market clearing then implies that the quantity of bonds available to back private payment instruments is the net supply $B - b_{CB} = B - z$. Since deposit issuance (and the associated balance-sheet allocations) depends on the availability of backing collateral only through this net quantity, an increase in bond supply B and a decrease in cash demand z have identical effects on deposits through the same channel: both relax the collateral constraint faced by banks.

In addition, results (1) and (2) highlight that the composition and supply of backing assets matter for both the scale of deposit issuance and the portfolio mix used to back deposits: increasing the availability of bonds versus entrepreneurs' loan claims shifts not only the quantity of deposits that banks can supply, but also whether deposits are predominantly backed by public bonds or by private lending assets.

4.1 Equilibrium Outcomes with the Crypto Sector

As shown in the appendix, there are three possible equilibrium outcomes depends on whether $b_T \geq 0$ and $b_C \geq 0$ bind or not:[10]

		b_T	b_C
Eq. A	Bonds are held by all banks	+	+
Eq. B	Bonds are not held by crypto banks	+	0
Eq. C	Bonds are not held by traditional banks	0	+

For example, in Eq. A , both traditional and crypto banks hold bonds. The equilibrium q_b satisfies

$$B = [\frac{d}{\rho} - n_\ell q_b{}^{\frac{1-\chi}{\chi}}] + [\frac{\tilde{s} - E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

[10]Under our assumption that $B > b_{CB}$, we rule out Eq. D where bonds are not held by any private banks.

where

$$\tilde{s} = \tilde{\alpha}^{\frac{1}{\sigma}} \left[1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$
$$d = \alpha^{\frac{1}{\sigma}} \left[1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}.$$

4.2 Effects of increasing the Crypto Sector

Figure 2 reports the distribution of the equilibrium outcomes over the $(\tilde{\alpha}, E)$ space, when n_ℓ is not too high so that the case with $b_C = b_T = 0$ is ruled out. Eq. B is uniquely supported when the size of the crypto sector is small relative to the endowment of crypto assets. Crypto banks do not rely on bonds to issue stablecoins. Eq. A arises when the size of the crypto sector increases, with both traditional and crypto banks holding bonds. Eq. C emerges when the crypto sector becomes even larger, with traditional bonds holding only loans. The result is summarized by the following proposition. Depending on the values of the remaining parameters, some regions may be empty.

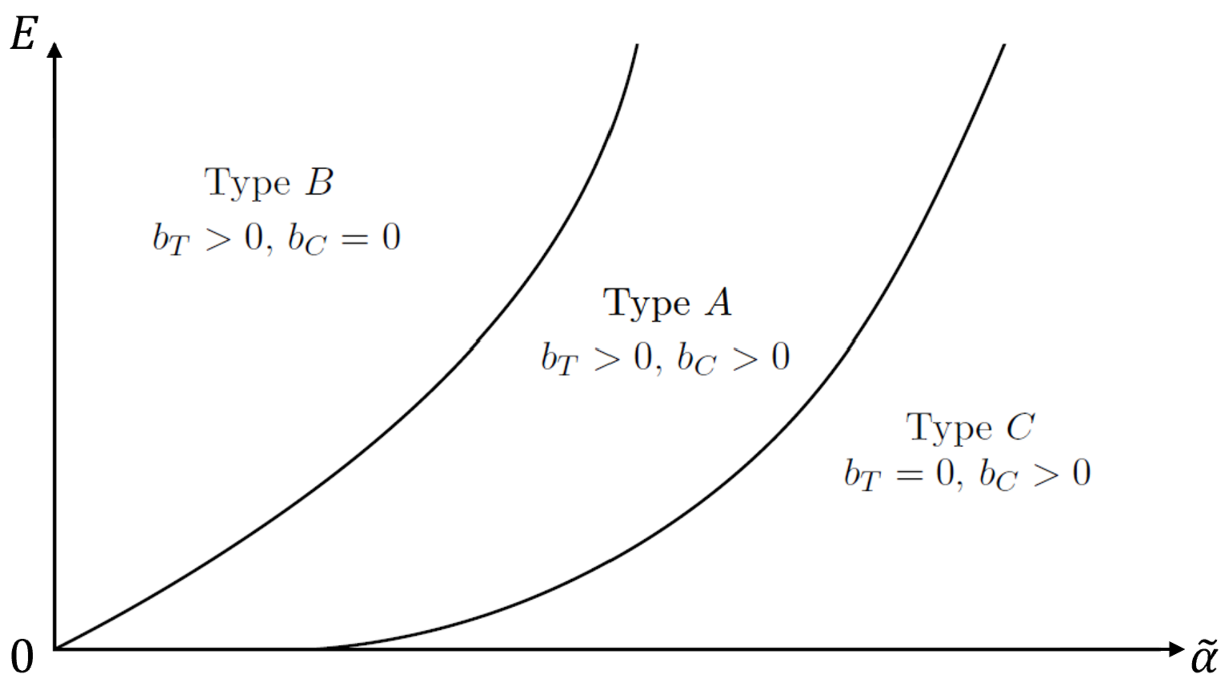

Figure 2: Equilibrium Outcomes with a Crypto Sector

Proposition 4. *Given our assumption on parameter values, for each* $(\tilde{\alpha}, E)$ *there exists a* unique *steady-state equilibrium, and it belongs to* exactly one *of equilibria A–C. The regions in the* $(\tilde{\alpha}, E)$*-plane associated with equilibria A–C are characterized by the complementary-slackness conditions for bond holdings* $b_T \geq 0$ *and* $b_C \geq 0$*. Depending on the values of the remaining parameters, some regions may be empty.*

5 Non-tokenized CBDC

This section considers the case where the central bank issues a non-tokenized CBDC with a (gross) return rate R_m (see Figure 3). After setting the CBDC return rate, the central supplies CBDC to satisfy the market demand. Like deposits, CBDC can be used in the traditional sector to buy good x. Given R_m, buyers endogenously choose the amount of CBDC balance denoted by m. Note that both crypto and traditional banks have no incentives to hold CBDC as reserves as $R_m \leq R_b = 1/q_b$. The buyer's problem is modified: a buyer chooses $(z, a_C, s, \tilde{s}, \tilde{e}, b_C, a_T, d, b_T, \ell_T, m)$ to maximize

$$-a_C - a_T - (1+\tau)z - m + \beta u_z(x_z) + \beta u(x) + \beta \tilde{u}(\tilde{x})$$

subject to

$$x_z = z, \qquad \text{(LC of cash consumption)}$$

$$\tilde{x} = \tilde{s}, \qquad \text{(LC of crypto consumption)}$$

$$x = mR_m + d, \qquad \text{(LC of trad. consumption)}$$

$$a_C \geq q_e\tilde{e} + q_b b_C - \beta\left[\tilde{e} + b_C - \tilde{s}\right], \qquad \text{(PC of crypto banks)}$$

$$\tilde{\kappa}\tilde{e} + \kappa b_C \geq \tilde{s}. \qquad \text{(IC of crypto banks)}$$

$$a_T \geq q_\ell \ell_T + q_b b_T - \beta\left[\ell_T + b_T - d\right], \qquad \text{(PC of trad. banks)}$$

$$\rho\ell_T + \rho b_T \geq d. \qquad \text{(IC of trad. banks)}$$

The central bank's demand of bonds becomes

$$b_{CB} = R_m m + z.$$

The following tables capture the balance sheets of banks:

Trad. bank

Assets	Liabilities
ℓ_T	d
b_T	Equity

Crypto bank

Assets	Liabilities
$\tilde{e}$	$\tilde{s}$
b_C	Equity

Central bank

Assets	Liabilities
b_{CB}	z
	m

Balance Sheets of Traditional, Crypto and Central Banks

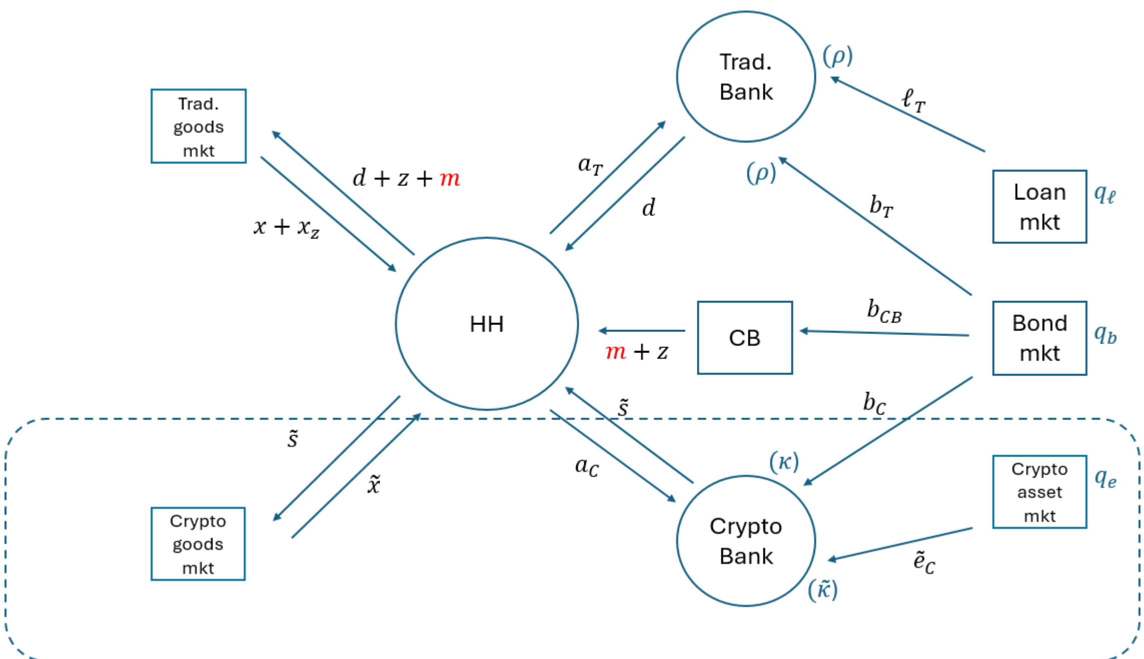

Figure 3: Non-Tokenized CBDC

The buyer's problem becomes

$$
\begin{aligned}
&\max_{z}[-(1+\tau)z + \beta u_z(z)] \\
&+ \max_{\tilde{e}, b_C, \tilde{s}} [-q_e\tilde{e} - q_b b_C + \beta(\tilde{e} + b_C - \tilde{s}) + \beta\tilde{u}(\tilde{s}) + \beta\lambda_C(\tilde{\kappa}\tilde{e} + \kappa b_C - \tilde{s})] \\
&+ \max_{\ell_T, b_T, d, m} [-m - q_\ell \ell_T - q_b b_T + \beta(\ell_T + b_T - d) + \beta u(d + mR_m) + \beta\lambda_T(\rho\ell_T + \rho b_T - d)] .
\end{aligned}
$$

The first-order conditions and the other equilibrium conditions are given in the online appendix.

Given the policy rate R_m set by the central bank, we need to consider different cases depending on whether the following inequalities are binding or not: $m \geq 0, b_T \geq 0, b_C \geq 0$. As shown in the appendix, there are six possible equilibrium outcomes:[11]

	m	b_T	b_C
Eq. A	0	+	+
Eq. B	0	0	+
Eq. C	0	+	0
Eq. E	+	+	+
Eq. F	+	0	+
Eq. G	+	+	0

In the first three cases, $m = 0$. Hence the allocations are just equivalent to those in Section 4.1, with an additional condition on R_m to ensure zero demand for CBDC. In contrast, the remaining three cases are characterized by positive CBDC demand ($m > 0$).

Effects of non-tokenized CBDC

Proposition 5. *When there is a positive demand for non-tokenized CBDC, increasing the interest rate R_m always crowds out bank loans. In addition,*

(1) when $b_T > 0, b_C > 0$, it crowds in stablecoins and lowers both q_b and q_e;

(2) when $b_T > 0, b_C = 0$, it has no impacts on stablecoins and q_e but lowers q_b;

(3) when $b_T = 0, b_C > 0$, it crowds out stablecoins, and raises both q_b and q_e.

	L	q_b	$\tilde{s}$	q_e
(1) $b_T > 0, b_C > 0$	−	−	+	−
(2) $b_T > 0, b_C = 0$	−	−	0	0
(3) $b_T = 0, b_C > 0$	−	+	−	+

Note that the CBDC competes with bank deposits as a means of payment. This will relax buyers' liquidity constraints in the traditional sector, lowering traditional banks' demand for reserve assets, including loans. Moreover, when the CBDC is issued, the central bank needs to acquire bonds from the two sectors to back the CBDC creation. If the central bank acquires these bonds from traditional banks to issue the CBDC, these reallocated collateral assets are now used more efficiently by the central bank who faces no incentive constraints. With more efficient usage of bonds, the bond market becomes less tight and the bond price drops. If the crypto sector does not use bonds (case 2), there are

[11] Under our assumption that $B > b_{CB}$, we rule out Eq. D and Eq. H where bonds are not held by any private banks.

no spillover effects. If the crypto sector uses bonds (case 1), this induces more stablecoin creation and hence lowers the demand for crypto asset as a collateral. When the new CBDC is backed completely by bonds acquired from the crypto sector (case 3), the bond market becomes tighter and the bond price rises, driving up the crypto asset price as well. As a result, stablecoins are crowded out.

6 Tokenized CBDC

Suppose the central bank issues tokenized CBDC with a return rate $\tilde{R}_m$ (see Figure 4). Notice that CBDC can be used for crypto goods transactions or used by crypto banks as a reserve asset (subject to pledgeability $\tilde{\kappa}$). We denote them as $\tilde{m}$ and $\tilde{M}$ respectively. Crypto banks may have an incentive to hold $\tilde{M}$ as reserve asset when $\tilde{\kappa} > \kappa$. By managing the ledger, the central bank has an option to impose surveillance on transactions facilitated by CBDC. This, however, will lead to a privacy discount so that the value of CBDC balances is discounted by $\mu \in (0,1)$. Hence, the degree of surveillance is captured by $1-\mu$. The central bank should set a lower μ if it intends to discourage the use of CBDC for crypto transactions. Note that traditional banks have no incentive to hold CBDC as reserves as these balances are always dominated by bonds. A buyer chooses $(a_C, z, \tilde{s}, \tilde{e}, b_C, a_T, d, b_T, \ell_T, \tilde{m}, \tilde{M})$ to maximize

$$-a_C - a_T - (1+\tau)z - \tilde{m} + \beta u_z(x_z) + \beta u(x) + \beta \tilde{u}(\tilde{x})$$

subject to

$$x_z = z, \qquad \text{(LC of cash consumption)}$$

$$\tilde{x} = \tilde{s} + \mu \tilde{m} \tilde{R}_m, \qquad \text{(LC of crypto consumption)}$$

$$x = d, \qquad \text{(LC of trad. consumption)}$$

$$a_C \geq \tilde{M} + q_e \tilde{e} + q_b b_C - \beta \left[\tilde{M} \tilde{R}_m + \tilde{e} + b_C - \tilde{s} \right], \qquad \text{(PC of crypto banks)}$$

$$\tilde{\kappa} \tilde{e} + \kappa b_C + \tilde{\kappa} \tilde{M} \tilde{R}_m \geq \tilde{s}. \qquad \text{(IC of crypto banks)}$$

$$a_T \geq q_\ell \ell_T + q_b b_T - \beta \left[\ell_T + b_T - d \right], \qquad \text{(PC of trad. banks)}$$

$$\rho \ell_T + \rho b_T \geq d. \qquad \text{(IC of trad. banks)}$$

The central bank's demand of bonds becomes

$$b_{CB} = z + \tilde{R}_m(\tilde{m} + \tilde{M}).$$

The balance sheets of different banks are:

Trad.	bank	Crypto	bank	Central	bank
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
ℓ_T	d	$\tilde{e}$	$\tilde{s}$	b_{CB}	z
b_T	Equity	b_C	Equity		$\tilde{m}$
		$\tilde{M}$			

Balance Sheets of Traditional, Crypto and Central Banks

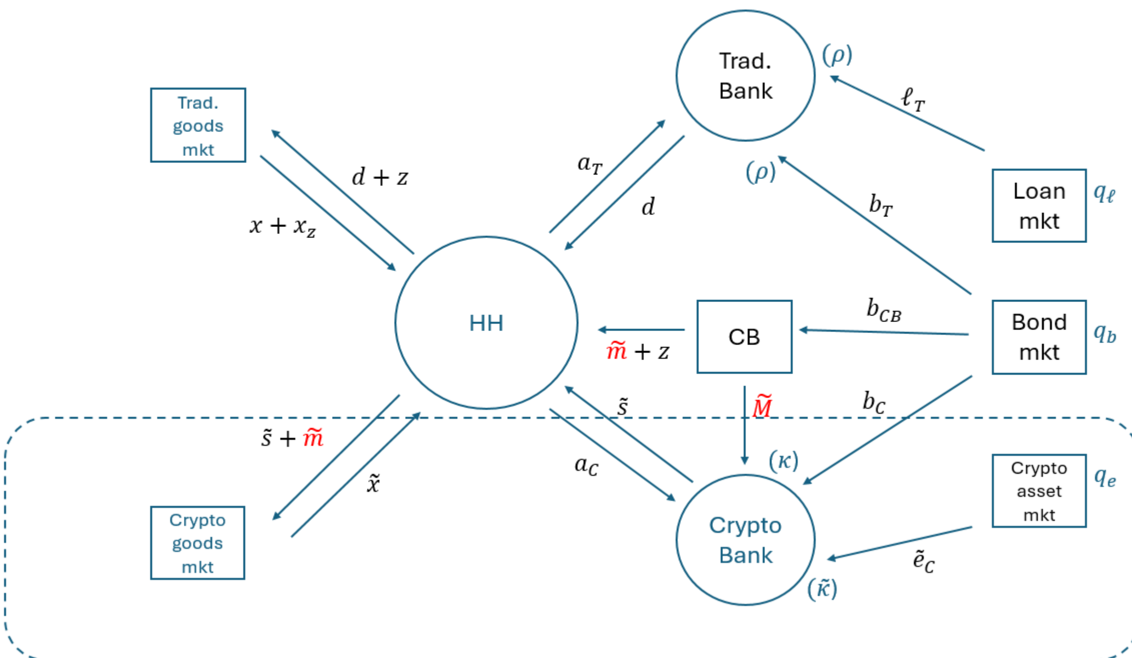

Figure 4: Tokenized CBDC

The buyer's problem can be simplified as the following problem:

$$\max_z[-(1+\tau)z + \beta u_z(z)]$$
$$+ \max_{\tilde{e}, b_C, \tilde{M}, \tilde{s}} \left[-\tilde{m} - \tilde{M} - q_e\tilde{e} - q_b b_C + \beta\left[\tilde{M}\tilde{R}_m + \tilde{e} + b_C - \tilde{s}\right] + \beta\tilde{u}(\tilde{s} + \mu\tilde{m}\tilde{R}_m) + \beta\lambda_C(\tilde{\kappa}\tilde{e} + \tilde{\kappa}\tilde{M}\tilde{R}_m + \kappa b_C - \tilde{s})\right]$$
$$+ \max_{\ell_T, b_T, d}\left[-q_\ell\ell_T - q_b b_T + \beta(\ell_T + b_T - d) + \beta u(d) + \beta\lambda_T(\rho\ell_T + \rho b_T - d)\right].$$

The first-order conditions and the other equilibrium conditions are given in the online appendix. Given the policy rate R_m set by the central bank, we need to consider different cases depending on whether the following inequalities are binding or not: $\tilde{m}_t \geq 0, \tilde{M}_t \geq 0, b_T \geq 0, b_C \geq 0$. As shown in the appendix, there are twelve possible equilibrium outcomes:[12]

	$\tilde{m}$	$\tilde{M}$	b_T	b_C		$\tilde{m}$	$\tilde{M}$	b_T	b_C
Eq. A	0	0	+	+	Eq. I	0	+	+	+
Eq. B	0	0	0	+	Eq. J	0	+	0	+
Eq. C	0	0	+	0	Eq. K	0	+	+	0
Eq. E	+	0	+	+	Eq. M	+	+	+	+
Eq. F	+	0	0	+	Eq. N	+	+	0	+
Eq. G	+	0	+	0	Eq. O	+	+	+	0

First, we show in the next lemma that, in equilibrium, the CBDC can be used either as a collateral or as a means of payment, but not both. Hence, equilibria M-O are not feasible under generic parameter values.

Lemma 6. *$\tilde{m}, \tilde{M} > 0$ is a non-generic (measure zero) outcome in the parameter space.*

Proof. When $\tilde{m}, \tilde{M} > 0$,

$$\tilde{M}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) = \left(\frac{1}{\beta\tilde{R}_m} - 1\right)\frac{1}{\tilde{\kappa}} + 1,$$
$$\tilde{m}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) = \frac{1}{\beta\mu\tilde{R}_m}$$

implying

$$(\frac{1}{\mu} + \beta\tilde{R}_m)\tilde{\kappa} = 1 + \beta\tilde{R}_m$$

which is a knife-edge case. □

[12] Under our assumption that $B > b_{CB}$, we rule out Eq. D, H, L P where bonds are not held by any private banks.

Next, we characterize when a CBDC is used as a means of payment and not a collateral. A necessary condition for $\tilde{m} > 0$ and $\tilde{M} = 0$ is

$$\tilde{M}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) < \left(\frac{1}{\beta\tilde{R}_m} - 1\right)\frac{1}{\tilde{\kappa}} + 1,$$
$$\tilde{m}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) = \frac{1}{\beta\mu\tilde{R}_m}$$

implying

$$(\frac{1}{\mu} + \beta\tilde{R}_m)\tilde{\kappa} < 1 + \beta\tilde{R}_m$$

which requires $\tilde{\kappa}$ small and μ large. Finally, the following proposition characterizes the effects of raising the CBDC interest rate.

Effects of tokenized CBDC

Proposition 7. *When there is a positive demand for a tokenized CBDC ($\tilde{m} + \tilde{M} > 0$), an increase in the interest rate always lowers crypto asset price. In addition,*

(1) when $b_T > 0, b_C > 0$, it crowds in d, crowds out L and lowers q_b;

(2) when $b_T = 0, b_C > 0$, it crowds out d, but has no impacts on L and d;

(3) when $b_T > 0, b_C = 0$, it crowds in d, crowds in L and lowers q_b.

	q_e	q_b	L	d
(1) $b_T > 0, b_C > 0$	−	−	−	+
(2) $b_T = 0, b_C > 0$	−	−	0	0
(3) $b_T > 0, b_C = 0$	−	+	+	−

The intuition is similar to those of Proposition 5. The tokenized CBDC relaxes buyers' liquidity constraint in the crypto sector, lowering crypto banks' demand for crypto assets. Again, the central bank needs to acquire bonds from the two sectors to back the CBDC creation. If the central bank acquires these bonds from crypto banks, the overall allocation of bonds becomes more efficient, the bond market becomes less tight and the bond price drops. There will be a spillover effect to the traditional sector when traditional banks use bonds (case 3), and no spillover effects if traditional banks do not use bonds (case 2). When the bonds are acquired completely from the traditional sector (case 3), the bond market becomes tighter as a result, bank deposits are crowded out.

The equivalence and coexistence of tokenized and non-tokenized CBDC

After characterizing the effects of a non-tokenized and a tokenized CBDC, we now examine when they are similar and when they are different. The following proposition establishes a condition under which the two systems are equivalent.

Proposition 8. *When $\rho = \kappa$, any equilibrium allocation with a positive supply of CBDC and $b_T, b_C > 0$ can be supported by issuing only a tokenized CBDC with $\mu = 1$, only a non-tokenized CBDC, or a mix of both.*

When $\rho = \kappa$, traditional and crypto banks are equally reliable. The only difference between them is the set of collateral assets they can use—yet this distinction becomes irrelevant when both banks hold bonds. According to Propositions 5 and 7, CBDC introduced in one sector generates spillover effects to the other sector whenever $b_T, b_C > 0$. In such cases, the entry point of the CBDC—whether it is introduced via traditional or crypto sectors—does not affect the set of implementable allocations. However, when bonds are not used in both sectors, the entry point becomes crucial. In particular, when traditional banks do not hold bonds, only a tokenized CBDC can correct inefficiencies in the crypto sector. The following proposition characterizes the condition under which it is socially optimal to introduce a tokenized CBDC to crowd out stablecoins.

Proposition 9. *When $b_C > 0, b_T = 0$, it is welfare-improving to issue a tokenized CBDC with $\mu = 1$ to crowd out stablecoins.*

More generally, Propositions 5 and 7 imply that, when $b_T, b_C > 0$, issuing a tokenized or a non-tokenized CBDC lowers q_b, driving both x and $\tilde{x}$ closer to their first-best levels. Loans, however, will decline. When there was originally under-investment in the economy, issuing a CBDC will lead to a trade-off between consumption and investment efficiencies, related to that studied by Keister and Sanches (2023).

Another natural question is whether to add another type of CBDC or not. This is equivalent to asking whether it is optimal to issue both a tokenized and a non-tokenized CBDC. In general, the answer to this question depends on the complex trade-offs between the impacts on x, $\tilde{x}$ and L.

However, one can establish simple sufficient conditions under which issuing an additional CBDC can improve welfare:

Proposition 10. *1. If $b_T = 0, b_C > 0$ when there is only a non-tokenized CBDC, then issuing a tokenized CBDC can improve welfare when*

$$\omega[\tilde{u}'(\tilde{x}) - 1] > 0.$$

2. Suppose $n_\ell = 0$. If $b_T > 0, b_C = 0$ when there is only a tokenized CBDC, then issuing a non-tokenized CBDC can improve welfare when

$$[u'(x) - 1] > 0.$$

3. Suppose $n_\ell = 0$. If $b_T > 0, b_C = 0$ when there is only a non-tokenized CBDC, then issuing a tokenized CBDC can improve welfare when

$$\rho[u'(x) - 1] < \omega[\tilde{u}'(\tilde{x}) - 1].$$

This proposition characterizes the conditions under which introducing an additional CBDC instrument raises welfare by improving x and/or $\tilde{x}$ locally, depending on which sector holds bonds. The intuition is that a second CBDC type relaxes a binding liquidity constraint in the sector that the existing CBDC does not serve. In (1), only non-tokenized CBDC is present and bonds sit with crypto banks; adding tokenized CBDC lets the central bank supply liquidity on-chain and improves welfare when crypto consumption is valued ($\omega[\tilde{u}' - 1] > 0$). In (2) and (3), with no bank loans ($n_\ell = 0$), one sector holds all bonds and the other is liquidity-constrained; introducing the missing CBDC type eases that constraint. The inequality in (3) says that adding tokenized CBDC is welfare-improving when the weighted marginal gain in crypto consumption exceeds the marginal loss in traditional consumption, given the pledgeability ρ of traditional banks.

Note that there are practical considerations that may limit the central bank's ability to set different interest rates for the two types of CBDC. For example, if both are available to households and the two rates are constrained to be equal, then the household's intratemporal optimality conditions imply an equalization of marginal utilities across the two consumption margins (informally, $u'(x) = \tilde{u}'(\tilde{x})$)). But the optimality generally does not call for that equalization for a given ω. Hence, if the goal is to implement the socially desired tradeoff between traditional and crypto activity, differential effective remuneration across the two CBDC is a natural instrument, allowing the central bank to tilt the relative attractiveness of liquidity across sectors. In practice, offering two “forms” of CBDC is conceptually feasible (e.g., a retail, account-based/off-chain CBDC and a tokenized/on-chain CBDC for specialized settlement use cases). One natural implementation is that users maintain an account-based CBDC balance and “withdraw” into a tokenized wallet when on-ledger use is required, then redeem back. Arbitrage/sweeping if conversion is frictionless: If the two forms are freely convertible 1:1 at all times, and conversion can be done costlessly between two PM markets, then a rate differential tends to induce “sweeping” into the higher-remunerated form when interest accrues, with the lower-rate form held only transiently for payments. This is the practical analogue of why some wedge is typically needed to sustain two coexisting forms with different returns (e.g., time delays, quantity limits, fees, or compliance checks on conversion/withdrawals).

7 Quantitative Exercises

This section conducts a quantitative exercise to study the effects of introducing a CBDC in a calibrated model.

7.1 Calibration: Economy without the Crypto Sector

We calibrate the model to the U.S. economy prior to the introduction of crypto assets (i.e., $\tilde{\alpha} = E = 0$). Hence, we have

$$\tilde{x} = \tilde{e} = b_C = 0.$$

We focus on an equilibrium where traditional banks hold both loans and bonds on their balance sheets:

$$\ell_T > 0, b_T > 0,$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, d, b_T, \ell_T, z, b_{CB})$ satisfy the following seven conditions:

$$q_b = q_\ell,$$
$$\rho(b_T + \ell_T) = d,$$
$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right),$$
$$z^{-\sigma} = \frac{1+\tau}{\beta},$$
$$\ell_T = n_\ell {q_\ell}^{\frac{1-\chi}{\chi}},$$
$$b_{CB} = z,$$
$$B = b_T + b_{CB}.$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{d}{\rho} - n_\ell {q_b}^{\frac{1-\chi}{\chi}}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$d = \alpha^{\frac{1}{\sigma}}\left[1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}.$$

The table below summarizes all the parameter values along with their calibration targets. Three parameters (i.e., β, τ and ρ) are set directly. The rest are calibrated internally. The calibration of production and preference parameters (χ, σ) is based on money and loan time series data from 2000 to 2015. The rest, (A, α, B, n_ℓ), are calibrated to match targets of money velocity, the share of deposits in money and the asset composition of banks.

List of Parameters

Parameter	Notation	Value	Notes
Discount factor	β	0.96	Standard in literature
Money growth rate	τ	0.02	Average inflation is 2%
Traditional bank pledgeability	ρ	0.9	Asset-liability ratio of banks in 2015 is 1.12
Curvature of production	χ	0.81	Interest elasticity of loans (2000-2015) is -0.236
Curvature of preference	σ	3.17	Interest elasticity of money demand (2000-2015) is -0.087
Scale of the PM market	A	98	Income velocity of money in 2015 is 25.74
Preference for x	α	23.17	Deposits-money ratio 2015 is 0.66
Bond and loan supplies	B	2.83	Real interest rate is 0.5%
	n_ℓ	1.1	Loan-bond ratio of banks in 2015 is 85%

To parameterize the economy with the crypto sector, we set $E = 0.12$, $\tilde{\alpha} = 0.0015$, $\kappa = 0.8$ so that the market capitalization of stablecoins is about 4.5% of that of bank deposits, that the fraction of bonds held by stablecoin issuers is less than 1%, and that the pledgeability parameters for stablecoin issuers are such that $\kappa = 0.8 < \tilde{\kappa} = \rho = 0.9$, so that bonds are more pledgeable for banks than for stablecoin issuers while tokenized assets have the same pledgeability parameter as that of bank reserves. Also we assume that $\mu = 1$, so that the new money design preserves privacy.

Non-tokenized CBDC

Figure 5 reports the effects of increasing R_m on asset prices (q_b, q_ℓ, q_e), money balances $(d, \tilde{s}, m)$,and reserve assets (b_T, b_C, ℓ_T). The last panel reports the impacts on the social welfare in terms of consumption equivalence Δ measured in percentage of consumption. In our example, $b_T, b_C > 0$ for the relevant region of parameter values. An increase of R_m lowers all asset prices, crowds out bank loans and deposits but crowds in stablecoin creation. Also, the FOC with respect to m implies that, as R_m increases, the total real balances used to purchase traditional good, $d + mR_m$, must go up when $m > 0$. Welfare goes up for R_m not too high as consumption in both traditional and crypto sectors improves, despite a smaller banking sector.

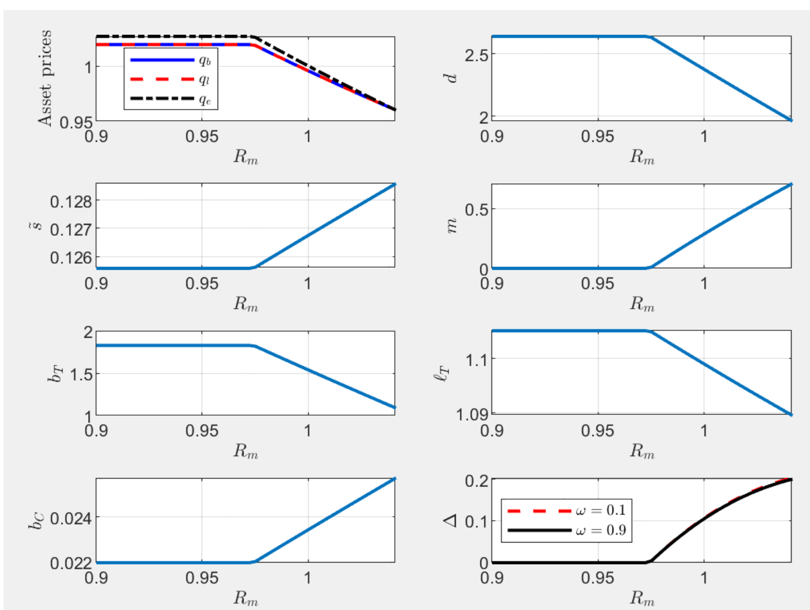

Figure 5: Comparative statics with a non-Tokenized CBDC

Tokenized CBDC

Figure 6 reports the effects of increasing $\tilde{R}_m$. There are three regions. When $\tilde{R}_m$ is sufficiently low, there are no effects as $\tilde{m} = \tilde{M} = 0$. For an intermediate range of $\tilde{R}_m$, tokenized CBDC is held as a means of payment. When b_C stays positive, all asset prices drop, crowding out stablecoins and bank loans but crowds in deposits. Welfare goes up as the scarce bond supply is allocated more efficiently from the crypto sector to the central bank and traditional banks. Also, the FOC with respect to $\tilde{m}$ implies that, as $\tilde{R}_m$ increases, the total real balances used to purchase crypto good, $\tilde{s} + \tilde{m}\tilde{R}_m$, must go up when $\tilde{m} > 0$. For a sufficiently higher $\tilde{R}_m$ however, further increase in the CBDC rate starts to move bond holdings from traditional banks to the central bank, crowding out bank deposits and crowding in bank loans, while stablecoins are not affected. As a result, welfare declines.

To understand the results reported in Figures 5 and 6, note that in the calibrated economy without CBDC, $F'(L) = 1/q_\ell < 1$, implying that the supply of bank loans is inefficiently high. Hence, when a non-tokenized CBDC is issued, it is optimal to set a high interest rate R_m to support an efficient level of x consumption, while crowding out bank deposits and bank loans. When a tokenized CBDC is issued, it is optimal to increase $\tilde{R}_m$ to crowd out stablecoins, driving b_C to zero. This is welfare-improving as both x_T and x_C will increase, and bank loans are optimally crowded out. Once $x_C = 0$, however, any further increase in $\tilde{R}_m$ will start to reduce b_T, crowding out d and crowding in ℓ_T. Since the supply of loans is already inefficiently high, further increase in $\tilde{R}_m$ is not optimal even when ω is high. This explains why the optimal interest rate on CBDC in Figure 6 is so much lower than in Figure 5, and why the optimal interest rate not seem to depend on ω.

Design choice between tokenized and non-tokenized CBDC

Figure 7 reports the welfare effects of introducing different types of CBDC (when it is sub-optimal to introduce a CBDC, the corresponding curve is not shown). When the pledgeability parameter of crypto banks, κ, is relatively large as shown in panels (1) and (2), it is optimal to introduce a non-tokenized CBDC to crowd out bank deposits and loans, reallocating bonds from the traditional to the crypto sector and crowding in stablecoins. The welfare benefit drops as crypto transactions becomes more desirable. Overall, the welfare gain is around 0.3% to 0.6% of consumption, depending on the availability of crypto collateral.

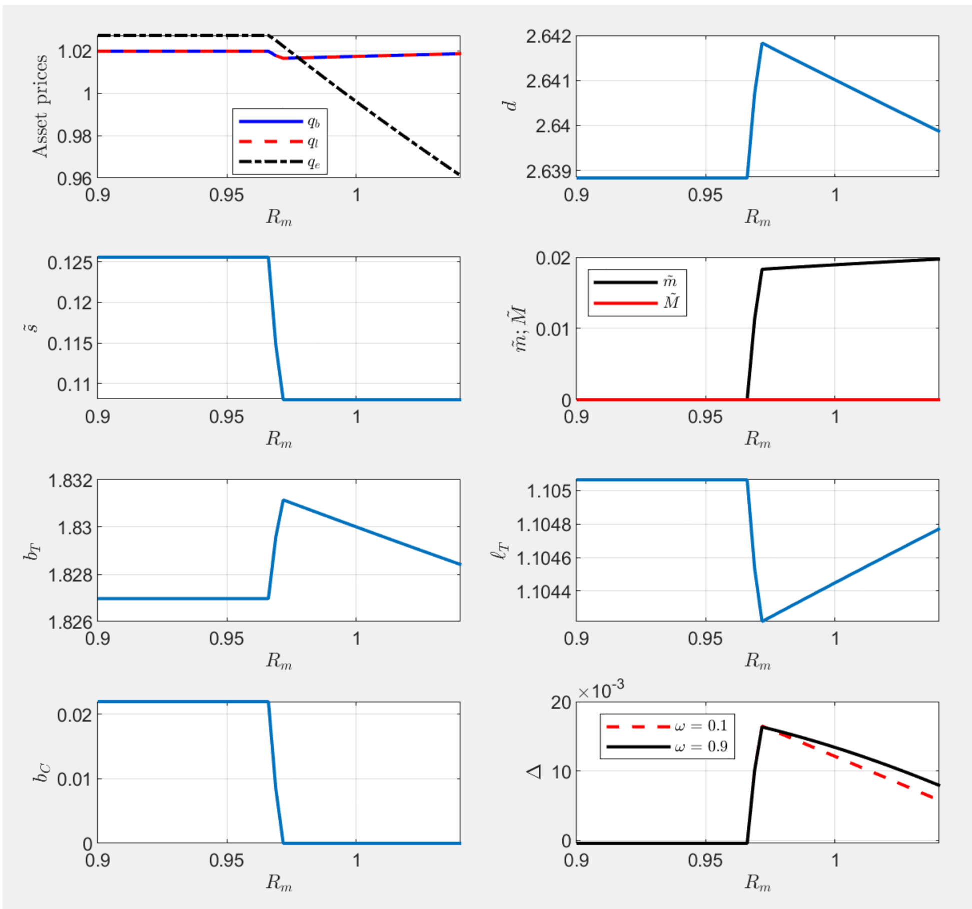

Figure 6: Comparative statics with a Tokenized CBDC

When the pledgeability parameter of crypto banks, κ, is much smaller as shown in panels (3) and (4), introducing non-tokenized CBDC becomes less desirable as the bonds reallocated to the crypto sector cannot be used efficiently to create stablecoins. When crypto transactions are socially more desirable (i.e., a large ω), it becomes optimal to issue a tokenized CBDC to support crypto consumption. Issuing a tokenized CBDC is especially desirable when E is low. In the extreme and somewhat unrealistic case, the welfare gain of introducing a tokenized CBDC can be as high as 9% of consumption when the crypto transactions are considered as desirable as traditional ones but the crypto banks are very unreliable and the supply of crypto collateral is very limited.

8 Conclusion

This paper develops a general equilibrium framework to evaluate the optimal design of central bank digital currencies in an economy where traditional finance and decentralized finance coexist. We show that the issuance of a CBDC—whether tokenized or not—can have complex effects on financial intermediation, asset prices, and the allocation of collateral across sectors. Tokenized CBDC crowd out stablecoins and improve welfare when crypto banks are less trustworthy and crypto assets are scarce. In contrast, non-tokenized may be more effective when crypto activity is less socially desirable or when reallocating collateral to the crypto sector is welfare-improving. Our quantitative analysis highlights key trade-offs: CBDC can enhance payment efficiency, but may reduce bank lending depending on how reserve assets are redistributed. These findings provide timely insights for central banks weighing how to incorporate new forms of public money into evolving financial ecosystems.

A key policy implication of our analysis is that central banks must account for the general equilibrium effects that link the balance sheets of money issuers—traditional banks, crypto banks, and the central bank itself. The introduction of a CBDC affects not only payment efficiency but also the allocation of reserve assets and the viability of private money creation across sectors. The optimal design of the future monetary system thus depends critically on three structural factors: the supply of reserve assets (such as bonds, private investment projects, and crypto collateral), the reliability of crypto banks (as measured by pledgeability κ and $\tilde{\kappa}$), and the social desirability of crypto transactions (ω).
Policymakers should recognize that decisions about whether and how to tokenize CBDCs will reverberate through both traditional and decentralized financial sectors, influencing the stability,

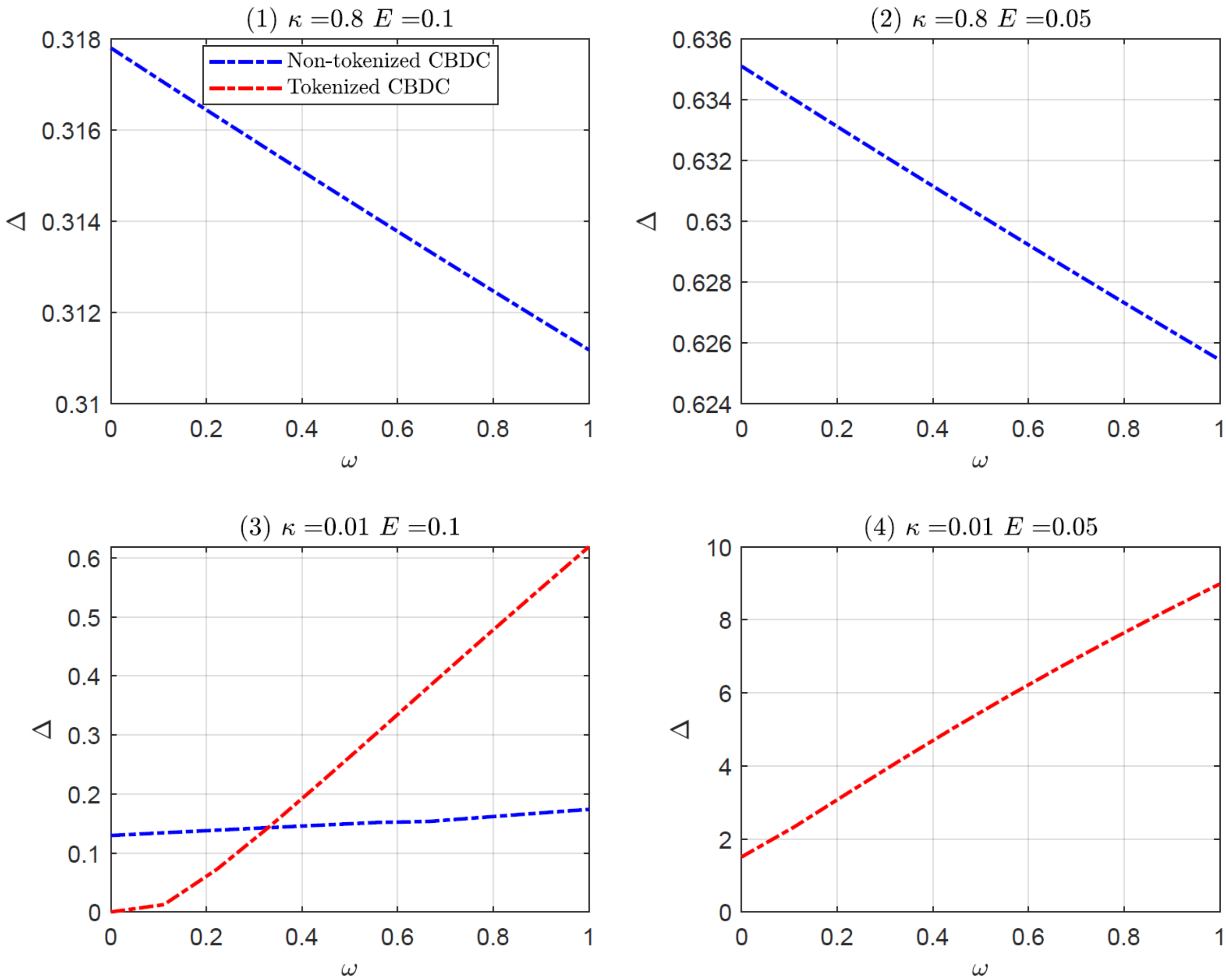

Figure 7: Comparative statics

efficiency, and composition of the overall monetary architecture.

References

[1] Ahnert, T., Hoffmann, P., and Monnet, C. (2025). Payments and privacy in the digital economy. Journal of Financial Economics, Volume 169, 104050 https://doi.org/10.1016/j.jfineco.2025.104050.

[2] Andolfatto, D. (2021). Assessing the impact of central bank digital currency on private banks. The Economic Journal, 131(634), 525–540.

[3] Aoyagi, J., and Itoy, Y. (2021). Coexisting exchange platforms: Limit order books and automated market makers. SSRN Working Paper. url: https://ssrn.com/abstract=3808755.

[4] Auer, R., Frost, J., Gambacorta, L., Monnet, C., Rice, T., and Shin, H. S. (2022). Central bank digital currencies: Motives, economic implications, and the research frontier. Annual Review of Economics, 14, 697–721.

[5] Bertsch, C. (2023). Stablecoins: Adoption and fragility. Mimeo.

[6] Capponi, A., and Jia, R. (2021). Decentralized stablecoins and collateral risk. arXiv preprint. url: https: //arxiv.org/pdf/2103.08842.pdf.

[7] Carapella, F. (2024). Stablecoin self-regulation. Federal Reserve Board of Governors, manuscript.

[8] Chiu, J., and Davoodalhosseini, M. (2021). Central bank digital currency and banking: Macroeconomic benefits of a cash-like design. Bank of Canada Staff Working Paper, 2021-63.

[9] Chiu, J., Davoodalhosseini, S. M., Jiang, J., and Zhu, Y. (2023a). Bank market power and central bank digital currency: Theory and quantitative assessment. Journal of Political Economy, 131(5), 1213–1248.

[10] Chiu, J., Koeppl, T., Yu, H., and Zhang, S. (2023b). Understanding DeFi through the lens of a production-network model. Bank of Canada Staff Working Paper, 2023-42.

[11] Chiu, J., and C. Monnet. (2025) Money creation for distributed ledgers: Stablecoins, tokenized deposits, or central bank digital currencies? Mimeo.

[12] Chiu, J., Ozdenoren, E., Yuan, K., and Zhang, S. (2022). On the fragility of DeFi lending. SSRN Working Paper. https://ssrn.com/abstract=4328481

[13] Cong, L. W., and Mayer, S. (2022). The coming battle of digital currencies. SC Johnson College of Business Applied Economics and Policy Working Paper Series, 2022-04.

[14] Cong, L. W., Harvey, C. R., Rabetti, D., and Wu, Z. Y. (2024). An anatomy of crypto-enabled cybercrimes. SSRN Working Paper. https://ssrn.com/abstract=4188661

[15] D'Avernas, A., Bourany, T., and Vandeweyer, Q. (2021). Are stablecoins stable? Working paper.

[16] Fernandez-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2021). Central bank digital currency: Central banking for all? Review of Economic Dynamics, 41, 225–242.

[17] Financial Stability Board. (2020). Regulation, supervision and oversight of "global stablecoin" arrangements. FSB Report.

[18] Gorton, G. B., and Zhang, J. Y. (2023). Taming wildcat stablecoins. University of Chicago Law Review, 90, 909–957.

[19] Huang, X. and T. Keister, (forthcoming). Stablecoins vs. Tokenized Deposits: The Narrow Banking Debate Revisited. Canadian Journal of Economics.

[20] Keister, T., and Monnet, C. (2022). Central bank digital currency: Stability and information. Journal of Economic Dynamics and Control, 142, 104501.

[21] Keister, T., and Sanches, D. (2023). Should central banks issue digital currency? The Review of Economic Studies, 90(1), 404–431.

[22] Kozhan, R., and Viswanath-Natraj, G. F. (2021). Decentralized stablecoins and collateral risk. SSRN Working Paper. https://dx.doi.org/10.2139/ssrn.3866975

[23] Lagos, R., and Wright, R. (2005). A unified framework of monetary theory and policy analysis. Journal of Political Economy, 113(3), 463–484.

[24] Lehar, A., and Parlour, C. A. (2021). Decentralized exchanges. SSRN Working Paper. https://dx.doi.org/10.2139/ssrn.3905316

[25] Li, Y., and Mayer, S. (2021). Money creation in decentralized finance: A dynamic model of stablecoin and crypto shadow banking. SSRN Working Paper. https://dx.doi.org/10.2139/ssrn.3757083

[26] Monnet, C., Petursdottir, A., and Rojas-Breu, M. (2020). Central bank account for all: Efficiency and stability. Mimeo.

[27] Schilling, L., Fernandez-Villaverde, J., and Uhlig, H. (2020). Central bank digital currency: When price and bank stability collide. NBER Working Paper, w28237.

[28] Tinn, K. (2024). A theoretical model of digital currency with asymmetric privacy. SSRN Working Paper. https://ssrn.com/abstract=4891933

[29] Wang, Z. (2023). Money laundering and the privacy design of central bank digital currency. Review of Economic Dynamics.

[30] Williamson, S. (2022a). Central bank digital currency: Welfare and policy implications. Journal of Political Economy, 130(11), 2829–2861.

[31] Williamson, S. (2022b). Central bank digital currency and flight to safety. Journal of Economic Dynamics and Control, 142, 104146.

Appendix

Proof of proposition 3

Proof:

(1) Effect of Higher B:

From bond market clearing, we have

$$B = b_T + z \Rightarrow \frac{db_T}{dB} = 1, \quad \frac{dz}{dB} = 0,$$

and, from loan market clearing, we obtain

$$\ell_T = n_\ell q_b^{\frac{1-\chi}{\chi}} \Rightarrow \frac{d\ell_T}{dB} = n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{dB}.$$

The deposit demand implies that

$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right) \Rightarrow -\sigma\alpha d^{-\sigma-1}\frac{dd}{dq_b} = \frac{1}{\rho\beta} \Rightarrow \frac{dd}{dq_b} = -\frac{d^{\sigma+1}}{\sigma\alpha\rho\beta} < 0,$$

implying that $\frac{dd}{dB} = \frac{dd}{dq_b} \cdot \frac{dq_b}{dB} < 0$ if $\frac{dq_b}{dB} > 0$, and vice versa. The collateral constraint implies that

$$\rho(b_T + \ell_T) = d \Rightarrow \frac{d\ell_T}{dB} = \frac{1}{\rho}\frac{dd}{dB} - \frac{db_T}{dB}.$$

Now equate the above two expressions for $\frac{d\ell_T}{dB}$, we obtain

$$\frac{1}{\rho}\frac{dd}{dB} - 1 = n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{dB}.$$

Substituting $\frac{dd}{dB}$, we yield

$$\frac{1}{\rho}\left(-\frac{d^{\sigma+1}}{\sigma\alpha\rho\beta} \cdot \frac{dq_b}{dB}\right) - 1 = n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{dB},$$

which implies

$$\left[n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} + \frac{d^{\sigma+1}}{\sigma\alpha\rho^2\beta}\right] \cdot \frac{dq_b}{dB} = -1 \Rightarrow \frac{dq_b}{dB} < 0.$$

Therefore,

$$\frac{dd}{dB} > 0, \quad \frac{d\ell_T}{dB} < 0$$

(2) Effect of Higher n_ℓ:

From the loan market equilibrium, we have

$$\ell_T = n_\ell q_b^{\frac{1-\chi}{\chi}} \Rightarrow \frac{d\ell_T}{dn_\ell} = q_b^{\frac{1-\chi}{\chi}} + n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{dn_\ell},$$

and from the collateral constraint, we obtain

$$\rho(b_T + \ell_T) = d \Rightarrow \frac{d\ell_T}{dn_\ell} = \frac{1}{\rho}\frac{dd}{dn_\ell},$$

since b_T is fixed. The deposit demand then implies that

$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right) \Rightarrow \frac{dd}{dq_b} = -\frac{d^{\sigma+1}}{\sigma\alpha\rho\beta} \Rightarrow \frac{dd}{dn_\ell} = \frac{dd}{dq_b} \cdot \frac{dq_b}{dn_\ell}.$$

Using the above expressions, we yield

$$q_b^{\frac{1-\chi}{\chi}} + n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{dn_\ell} = -\frac{d^{\sigma+1}}{\sigma\alpha\rho^2\beta} \cdot \frac{dq_b}{dn_\ell},$$

which implies

$$\left[n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} + \frac{d^{\sigma+1}}{\sigma\alpha\rho^2\beta}\right] \cdot \frac{dq_b}{dn_\ell} = -q_b^{\frac{1-\chi}{\chi}} \Rightarrow \frac{dq_b}{dn_\ell} < 0.$$

Hence, we have

$$\frac{dd}{dn_\ell} > 0, \quad \frac{d\ell_T}{dn_\ell} > 0.$$

(3) Effect of Higher Inflation τ:

From cash demand, we have

$$z^{-\sigma} = \frac{1+\tau}{\beta} \Rightarrow z = \left(\frac{\beta}{1+\tau}\right)^{1/\sigma} \Rightarrow \frac{dz}{d\tau} < 0,$$

and, from bond market clearing, we have

$$B = b_T + z \Rightarrow \frac{db_T}{d\tau} = -\frac{dz}{d\tau} > 0.$$

The deposit demand implies that

$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right) \Rightarrow \frac{dd}{dq_b} < 0 \Rightarrow \frac{dd}{d\tau} > 0,$$

while the collateral constraint implies that

$$\frac{d\ell_T}{d\tau} = \frac{1}{\rho}\frac{dd}{d\tau} - \frac{db_T}{d\tau} < 0.$$

Using the loan market condition, we then obtain

$$\frac{d\ell_T}{d\tau} = n_\ell \cdot \frac{1-\chi}{\chi} q_b^{\frac{1-\chi}{\chi}-1} \cdot \frac{dq_b}{d\tau} \Rightarrow \frac{dq_b}{d\tau} < 0,$$

or

$$\frac{dd}{d\tau} > 0, \quad \frac{db_T}{d\tau} > 0, \quad \frac{dz}{d\tau} < 0, \quad \frac{d\ell_T}{d\tau} < 0.$$

Proof of proposition 4

Proof. We use the banks' FOCs together with complementary slackness for $b_T \geq 0$ and $b_C \geq 0$.

(i) Equilibrium B. From the crypto bank's FOC, equilibrium B (in which the crypto sector does not hold bonds, $b_C = 0$) is supported when

$$\tilde{u}'(\tilde{s}) - 1 = \tilde{\alpha}(\tilde{\kappa}E)^{-\sigma} - 1 = \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right) < \frac{1}{\kappa}\left(\frac{q_b^*}{\beta} - 1\right), \tag{14}$$

where q_b^* is the equilibrium bond price under the candidate equilibrium with $b_C = 0$. Hence, equilibrium B is supported when $\tilde{\alpha}$ is low and E is high.

(ii) Equilibrium C. To support equilibrium C (in which the traditional sector does not hold bonds, $b_T = 0$), we need

$$q_\ell^* < q_b = \frac{\kappa(\tilde{\alpha}(\tilde{\kappa}E)^{-\sigma} - 1) + 1}{\beta}, \tag{15}$$

where q_ℓ^* is the equilibrium loan price under the candidate equilibrium with $b_T = 0$. This happens when $\tilde{\alpha}$ is high and E is small.

(iii) Equilibrium A. If neither (14) nor (15) holds, then both nonnegativity constraints on bond holdings are slack, so $b_T > 0$ and $b_C > 0$ and equilibrium A is supported. In other words, equilibrium A arises for intermediate values of $(\tilde{\alpha}, E)$.

Uniqueness and "exactly one case applies." Conditions (14) and (15) are precisely the complementary-slackness inequalities that deliver $b_C = 0$ (for B) and $b_T = 0$ (for C), respectively. If both were to hold, we would have $b_T = b_C = 0$, which is ruled out by the maintained restriction excluding equilibrium D. Therefore at most one of B or C can apply. If neither holds, A applies. Hence for each $(\tilde{\alpha}, E)$ exactly one of A–C obtains.

Possible empty regions. The cutoff loci implied by (14)–(15) may be degenerate for extreme configurations of the parameters (e.g., if bonds are always strictly dominated or always strictly preferred as reserves/collateral), in which case some regimes may be absent. □

Proof of proposition 5

Proof. (1) Note that $\ell_T > 0$ implies $d > 0$. Hence, when $b_T, m > 0$, we have

$$u'(d + mR_m) - 1 = 1/\beta R_m - 1 = \lambda_T = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho} = \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho}.$$

Hence an increase in R_m implies lower q_b and q_ℓ. Hence, as $\ell_T q_\ell = n_\ell q_\ell^{\frac{1}{\chi}}$ declines. When, in addition, $b_C > 0$,

$$\tilde{u}'(\tilde{s}) = \lambda_C = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa} = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}},$$

the drop in bond price implies a lower q_e and a higher $\tilde{s}$. Hence $b_C = \tilde{s}/\kappa - \tilde{\kappa}E/\kappa$ goes up.

(2) When $b_C = 0$, however, we have

$$\tilde{s} = \tilde{\kappa}E$$
$$\tilde{u}'(\tilde{s}) - 1 = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}$$

and hence there are no effects on stablecoins or crypto asset prices.

(3) When $b_T = 0, m > 0$, we have

$$u'(d + mR_m) - 1 = 1/\beta R_m - 1 = \lambda_T = \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho}.$$

Hence an increase in R_m implies a lower q_ℓ, leading to a lower $\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}}$ for $\chi < 1$. Hence, deposit d drops. This means mR_m and $b_{CB} = z + mR_m$ must rise. As a result, $b_C = B - b_{CB}$ and $\tilde{s} = \tilde{\kappa}\tilde{e} + \kappa b_C$ must decline, leading to higher q_e and q_b. □

Proof of proposition 7

Proof. (1) When $b_C, \tilde{m} > 0$, we have

$$\frac{1}{\beta\mu\tilde{R}_m} - 1 = \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) - 1 = \lambda_C = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa} = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}.$$

An increase in $\mu\tilde{R}_m$ lowers q_b, q_e. When $b_T > 0$, we have

$$\lambda_T = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho} = \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho} = u'(d),$$

implying a lower q_ℓ, $\ell_T q_\ell = n_\ell q_\ell^{\frac{1}{\chi}}$ and a higher d.

When $b_C, \tilde{M} > 0$, we have

$$\tilde{u}'(\tilde{s}) - 1 = \lambda_C = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}} = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa} = \left(\frac{1}{\beta\tilde{R}_m} - 1\right)\frac{1}{\tilde{\kappa}}.$$

An increase in $\tilde{R}_m$ lowers q_b, q_e, and raises $\tilde{s}$. When $b_T > 0$, we have

$$u'(d) - 1 = \lambda_T = \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho} = \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho},$$

implying a lower q_ℓ, $\ell_T q_\ell = n_\ell q_\ell^{\frac{1}{\chi}}$ and a higher d.

(2) When $b_C > 0, b_T = 0$, the mechanisms in (1) still apply, except that there will be no effects on deposits and loans as traditional banks do not use bonds.

(3) When $b_C = 0, \tilde{m} > 0$, we have

$$\frac{1}{\beta\mu\tilde{R}_m} - 1 = \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) - 1 = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}.$$

An increase in $\tilde{R}_m$ lowers q_e. Since $\tilde{s} = \tilde{\kappa}E$ stays unchanged while $\tilde{x} = \tilde{s} + \mu\tilde{m}\tilde{R}_m$ goes up, $\tilde{m}$ and $\tilde{m}\tilde{R}_m$ must increase. Hence more bonds will be held by the central bank $b_{CB} = z + \tilde{R}_m\tilde{m}$, and less left for traditional banks $b_T = B - b_{CB}$. To determine bank deposits, we use

$$\begin{aligned} u'(d) &= \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho} = \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho}, \\ d &= \rho\ell_T + \rho b_T, \\ \ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}. \end{aligned}$$

Notice that d cannot go up. If it were true, then $\ell_T = (d/\rho - b_T)$ must increase. When $\chi < 1$, this requires q_ℓ increase as well. But this is inconsistent with the assumption of a higher d. Hence, it must be true that d drops while q_b, q_ℓ and $q_\ell\ell_T$ both go up.

When $b_C = 0, \tilde{M} > 0$, we have

$$\tilde{u}'(\tilde{s}) - 1 = \lambda_C = \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}} = \left(\frac{1}{\beta \tilde{R}_m} - 1\right)\frac{1}{\tilde{\kappa}}.$$

An increase in $\tilde{R}_m$ lowers q_e, and raises $\tilde{s}$. This means $\tilde{M}\tilde{R}_m = \tilde{s}/\tilde{\kappa} - E$ must go up. Hence $b_{CB} = z + \tilde{R}_m \tilde{M}$ increases as well, implying a lower $b_T = B - b_{CB}$. As shown above, it must be true that d drops while q_b, q_ℓ and $q_\ell \ell_T$ both go up. $\square$

Proof of proposition 8

Proof. When $b_T, b_C > 0$, an allocation with a positive supply of non-tokenized CBDC, m^n, satisfies

$$\rho[\frac{1}{\beta R_m} - 1] = \kappa[\tilde{u}'(\tilde{s}^n) - 1] \quad = \quad \kappa\lambda_C^n = \left(\frac{q_\ell^n}{\beta} - 1\right) = \left(\frac{q_b^n}{\beta} - 1\right) = \left(\frac{q_e^n}{\beta} - 1\right)\frac{\kappa}{\tilde{\kappa}} = \rho[u'(d^n + \Delta) - 1] = \rho\lambda_T^n$$

$$\Delta = R_m m^n,$$

$$\rho\ell_T^n + \rho b_T^n - d^n = 0,$$

$$\tilde{\kappa}\tilde{e}^n + \kappa b_C^n - \tilde{s}^n = 0,$$

$$B = b_T^n + b_C^n + b_{CB}^n$$

$$b_{CB}^n = z + \Delta,$$

where the superscript "n" is used to denote the endogenous variables when there is a non-tokenized CBDC. An allocation with a positive supply of tokenized CBDC, m^t, with $\mu = 1$ satisfies

$$\kappa[\frac{1}{\beta\tilde{R}_m} - 1 = \kappa[\tilde{u}'(\tilde{s}^t + \Delta) - 1] \quad = \quad \kappa\lambda_C^t = \left(\frac{q_e^t}{\beta} - 1\right)\frac{\kappa}{\tilde{\kappa}} = \left(\frac{q_b^t}{\beta} - 1\right) = \left(\frac{q_\ell^t}{\beta} - 1\right) = \rho\lambda_T^t = \rho[u'(d^t) - 1],$$

$$\Delta = \tilde{R}_m\tilde{m}^t,$$

$$\lambda_T^t[\rho\ell_T^t + \rho b_T^t - d^t] = 0,$$

$$\lambda_C^t[\tilde{\kappa}\tilde{e}^t + \kappa b_C^t - \tilde{s}^t] = 0,$$

$$B = b_T^t + b_C^t + b_{CB}^t,$$

$$b_{CB}^t = z + \Delta,$$

where the superscript "t" is used to denote the endogenous variables when there is a tokenized CBDC. When $\rho = \kappa$, the two allocations are equivalent by setting

$$\tilde{s}^t + \Delta = \tilde{s}^n,$$

$$d^t = d^n + \Delta,$$

$$\rho(b_T^t - b_T^n) = \rho(b_C^n - b_C^t) = \Delta.$$

□

Proof of proposition 9

Proof. Recall that the welfare is given by

$$\mathcal{W} = [u_z(x_z) - x_z] + [u(x) - x] + \omega[\tilde{u}(\tilde{x}) - \tilde{x}] + n_\ell[F(L) - L].$$

When $b_T = 0$, Proposition 7 implies a tokenized CBDC has no effects on the traditional sector. The welfare hence depends on the response of $\tilde{x}$. Note that, when $b_C > 0$, $\tilde{u}'(\tilde{x}) - 1 = \lambda_C > 0$, implying that $\tilde{x}$ is below the first-best level. Then, the FOC in Sections 4 and 5 imply that issuing a tokenized CBDC with $\mu = 1$ is most effective as the central bank can avoid the deadweight loss due to μ and $\tilde{\kappa}$. From the FOC of a tokenized CBDC, we know that when $\tilde{m} > 0$,

$$\frac{1}{\beta \tilde{R_m}} - 1 = \tilde{u}'(\tilde{x}) - 1,$$

implying that introducing a CBDC with $\tilde{R_m}$ can drive $\tilde{x}$ closer to the first-best level. Finally, the quantity of equilibrium stablecoins is

$$\tilde{s} = \tilde{\kappa}E + \kappa b_C^n = \tilde{\kappa}E + \kappa(B - z - \Delta)$$

implying that acquiring $\Delta = \tilde{m}\tilde{R}_m$ to back the tokenized CBDC crowds out stablecoins. □

Proof of proposition 10

Proof. First consider the case with $b_T = 0, b_C > 0$ when there is only a non-tokenized CBDC. By offering a tokenized CBDC with a sufficiently high $\tilde{R}_m$, the central bank can induce $\tilde{m}' = \Delta > 0$ (small enough that the equilibrium remains in the same region) backed by bonds channeled from the crypto sector, without any effects on the traditional sector. Hence, $b_C' = b_C - \Delta$. As a result, crypto consumption goes up

$$\tilde{x}' = \tilde{x} + (1 - \kappa)\Delta.$$

This is welfare improving as long as

$$\omega[\tilde{u}'(\tilde{x}) - 1] > 0.$$

Second, suppose $n_\ell = 0$, and assume $b_T > 0, b_C = 0$ when there is only a tokenized CBDC. By offering a non-tokenized CBDC with a sufficiently high R_m, the central bank can induce $m' = \Delta > 0$ backed by bonds channeled from the traditional sector, without any effects on the crypto sector. Hence, $b'_T = b_T - \Delta$. As a result, traditional consumption goes up to

$$x = x + (1-\rho)\Delta.$$

This is welfare improving as long as

$$[u'(x) - 1] > 0$$

Third, suppose $n_\ell = 0$, and assume $b_T > 0, b_C = 0$ when there is only a non-tokenized CBDC. By offering a tokenized CBDC with a sufficiently high $\tilde{R}_m$, the central bank can induce $\tilde{m}' = \Delta > 0$ backed by bonds channeled from the traditional sector. Hence, $b'_T = b_T - \Delta$. As a result, traditional consumption goes down to

$$x = x - \rho\Delta,$$

while crypto consumption goes up to

$$\tilde{x}' = \tilde{x} + \Delta.$$

This is welfare improving as long as

$$\rho[u'(x) - 1] < \omega[\tilde{u}'(\tilde{x}) - 1].$$

□

For Outline Appendix

Section 3.1: First-order conditions

The first-order conditions are

$$
\begin{aligned}
\tilde{s}: \quad U_3(z,d,\tilde{s}) &= 1+\lambda_C,\\
d: \quad U_2(z,d,\tilde{s}) &= 1+\lambda_T,\\
z: \quad \beta U_1(z,d,\tilde{s}) &= 1+\tau,\\
b_T: \quad \lambda_T &\leq \left(\frac{q_b}{\beta}-1\right)\frac{1}{\rho},\\
\ell_T: \quad \lambda_T &\leq \left(\frac{q_\ell}{\beta}-1\right)\frac{1}{\rho},\\
\tilde{e}: \quad \lambda_C &\leq \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}},\\
b_C: \quad \lambda_C &\leq \left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa},\\
\lambda_T[\rho\ell_T+\rho b_T-d] &= 0,\\
\lambda_C[\tilde{\kappa}\tilde{e}+\kappa b_C-\tilde{s}] &= 0.
\end{aligned}
$$

Section 4: Four Possible Equilibrium Outcomes with the Crypto Sector

		b_T	b_C
Eq. A	Bonds are held by all banks	+	+
Eq. B	Bonds are not held by crypto banks	+	0
Eq. C	Bonds are not held by traditional banks	0	+
Eq. D	Bonds are not held by both banks	0	0

Eq. A: Bonds are held by all banks

This is an equilibrium where traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$
\ell_T > 0, b_T > 0,
$$

$$
\tilde{e} > 0, b_C > 0.
$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following eleven conditions:

$$
\begin{aligned}
q_b &= q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right) &= \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\rho(b_T+\ell_T) &= d \\
b_C\kappa + \tilde{e}\tilde{\kappa} &= \tilde{s} \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right) \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_T + b_C + b_{CB}.
\end{aligned}
$$

It is then straightforward to show that the equilibrium q_b satisfies

$$
B = [\frac{d}{\rho} - n_\ell q_b^{\frac{1-\chi}{\chi}}] + [\frac{\tilde{s}-E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}
$$

where

$$
\begin{aligned}
\tilde{s} &= \tilde{\alpha}^{\frac{1}{\sigma}}\left[1+\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}, \\
d &= \alpha^{\frac{1}{\sigma}}\left[1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}.
\end{aligned}
$$

Eq. B: Bonds are not held by crypto banks

This is an equilibrium where traditional banks hold both loans and bonds on their balance sheets and crypto banks hold only crypto assets on their balance sheets:

$$
\begin{aligned}
\ell_T > 0, b_T > 0, \\
\tilde{e} > 0, b_C = 0.
\end{aligned}
$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following eleven conditions:

$$\begin{aligned}
q_b &= q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right) &> \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\rho(b_T+\ell_T) &= d \\
\tilde{e}\tilde{\kappa} &= \tilde{s} \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right) \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_T + b_{CB}.
\end{aligned}$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{d}{\rho} - n_\ell q_b^{\frac{1-\chi}{\chi}}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$\begin{aligned}
E\tilde{\kappa} &= \tilde{\alpha}^{\frac{1}{\sigma}}\left[1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}, \\
d &= \alpha^{\frac{1}{\sigma}}\left[1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}
\end{aligned}$$

with the following condition satisfied:

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right).$$

Eq. C: Bonds are not held by traditional banks

This is an equilibrium where traditional banks hold loans but no bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$\ell_T > 0, b_T = 0,$$
$$\tilde{e} > 0, b_C > 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following eleven conditions:

$$\begin{aligned}
q_b &> q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) &= \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right), \\
\rho \ell_T &= d \\
b_C \kappa + \tilde{e}\tilde{\kappa} &= \tilde{s} \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) \\
\alpha d^{-\sigma} &= 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_C + b_{CB}.
\end{aligned}$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{\tilde{s} - E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$\begin{aligned}
\tilde{s} &= \tilde{\alpha}^{\frac{1}{\sigma}}\left[1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}, \\
\rho n_\ell q_\ell^{\frac{1-\chi}{\chi}} &= \alpha^{\frac{1}{\sigma}}\left[1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},
\end{aligned}$$

with the following condition satisfied:

$$q_b > q_\ell.$$

Eq. D: Bonds held only by the central bank

This is an equilibrium where traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$\ell_T > 0, b_T = 0,$$

$$\tilde{e} > 0, b_C = 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following eleven conditions:

$$\begin{aligned}
q_b &> q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) &> \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right), \\
\rho \ell_T &= d \\
\tilde{e}\tilde{\kappa} &= \tilde{s} \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right) \\
\alpha d^{-\sigma} &= 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_{CB}.
\end{aligned}$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = \left(\frac{1+\tau}{\beta}\right)^{-\frac{1}{\sigma}}$$

where

$$
\begin{aligned}
E\tilde{\kappa} &= \tilde{\alpha}^{\frac{1}{\sigma}} \left[1 + \frac{1}{\tilde{\kappa}} \left(\frac{q_e}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}, \\
\rho n_\ell q_\ell^{\frac{1-\chi}{\chi}} &= \alpha^{\frac{1}{\sigma}} \left[1 + \frac{1}{\rho} \left(\frac{q_\ell}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},
\end{aligned}
$$

with the following condition satisfied:

$$
\begin{gathered}
q_b > q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right).
\end{gathered}
$$

8.1 Section 5: First-order conditions and equilibrium conditions

$$\begin{aligned}
\tilde{s}: \quad \tilde{u}'(\tilde{s}) &= 1 + \lambda_C, \\
d: \quad u'(d + mR_m) &\leq 1 + \lambda_T, \\
m: \quad u'(d + mR_m) &\leq 1/\beta R_m, \\
z: \quad u_z'(z) &= \frac{1+\tau}{\beta}, \\
b_T: \quad \lambda_T &\leq \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho}, \\
\ell_T: \quad \lambda_T &\leq \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho}, \\
\tilde{e}: \quad \lambda_C &\leq \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}, \\
b_C: \quad \lambda_C &\leq \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa}, \\
\lambda_T[\rho\ell_T + \rho b_T - d] &= 0, \\
\lambda_C[\tilde{\kappa}\tilde{e} + \kappa b_C - \tilde{s}] &= 0,
\end{aligned}$$

and the other equilibrium conditions are

$$\begin{aligned}
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z + mR_m, \\
\tilde{e} &= E, \\
B &= b_T + b_C + b_{CB}, \\
q_b(b_{CB} - z) &\leq m,
\end{aligned}$$

where the last condition ensures that the central bank raises sufficient resources to buy bonds to back non-tokenized CBDC from their issuance.

Section 5: Eight Possible Equilibrium Outcomes with Non-Tokenized CBDC

Given the policy rate R_m set by the central bank, we need to consider eight different cases depending on whether the following inequalities are binding or not.

$$m \geq 0,$$

$$b_T \geq 0,$$

$$b_C \geq 0.$$

As shown in the appendix, there are eight possible equilibrium outcomes:

	m	b_T	b_C
Eq. A	0	+	+
Eq. B	0	+	0
Eq. C	0	0	+
Eq. D	0	0	0
Eq. E	+	+	+
Eq. F	+	+	0
Eq. G	+	0	+
Eq. H	+	0	0

In the first four cases, $m = 0$. Hence they are just equivalent to those in Section 4.1, with an additional condition on R_m. In the next four cases, $m > 0$.

Eq. A: CBDC is not held and bonds are held by all banks

This is an equilibrium where no one holds CBDC, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m = 0, \ell_T > 0, b_T > 0,$$

$$\tilde{e} > 0, b_C > 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$
\begin{aligned}
\alpha d^{-\sigma} &< 1/\beta R_m, \\
q_b &= q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right) &= \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\rho(b_T+\ell_T) &= d, \\
b_C\kappa + \tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_b}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_T + b_C + b_{CB}.
\end{aligned}
$$

It is then straightforward to show that the equilibrium q_b satisfies

$$
B = [\frac{d}{\rho} - n_\ell q_b^{\frac{1-\chi}{\chi}}] + [\frac{\tilde{s}-E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}
$$

where

$$
\begin{aligned}
\tilde{s} &= \tilde{\alpha}^{\frac{1}{\sigma}}\left[1+\frac{1}{\kappa}\left(\frac{q_b}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}, \\
d &= \alpha^{\frac{1}{\sigma}}\left[1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right)\right]^{-\frac{1}{\sigma}}.
\end{aligned}
$$

Eq. B: CBDC is not held and bonds are not held by crypto banks

This is an equilibrium where no one holds CBDC, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold only crypto assets on their balance sheets:

$$m = 0, \ell_T > 0, b_T > 0,$$
$$\tilde{e} > 0, b_C = 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha d^{-\sigma} < 1/\beta R_m,$$

$$q_b = q_\ell,$$
$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$
$$\rho(b_T + \ell_T) = d,$$
$$\tilde{e}\tilde{\kappa} = \tilde{s},$$
$$\tilde{\alpha}\tilde{s}^{-\sigma} = 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$
$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right),$$
$$z^{-\sigma} = \frac{1+\tau}{\beta},$$
$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}},$$
$$b_{CB} = z,$$
$$\tilde{e} = E,$$
$$B = b_T + b_{CB}.$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{d}{\rho} - n_\ell q_b^{\frac{1-\chi}{\chi}}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$E\tilde{\kappa} = \tilde{\alpha}^{\frac{1}{\sigma}} \left[1 + \frac{1}{\tilde{\kappa}} \left(\frac{q_e}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$

$$d = \alpha^{\frac{1}{\sigma}} \left[1 + \frac{1}{\rho} \left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}$$

with the following condition satisfied:

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right).$$

Eq. C: CBDC is not held and bonds are not held by traditional banks

This is an equilibrium where no one holds CBDC, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m = 0, \ell_T > 0, b_T = 0,$$
$$\tilde{e} > 0, b_C > 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha d^{-\sigma} < 1/\beta R_m,$$

$$q_b > q_\ell,$$

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) = \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$

$$\rho \ell_T = d,$$

$$b_C \kappa + \tilde{e}\tilde{\kappa} = \tilde{s},$$

$$\tilde{\alpha}\tilde{s}^{-\sigma} = 1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right),$$

$$\alpha d^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right),$$

$$z^{-\sigma} = \frac{1+\tau}{\beta},$$

$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}},$$

$$b_{CB} = z,$$

$$\tilde{e} = E,$$

$$B = b_C + b_{CB}.$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{\tilde{s} - E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$\tilde{s} = \tilde{\alpha}^{\frac{1}{\sigma}}\left[1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$

$$\rho n_\ell q_\ell^{\frac{1-\chi}{\chi}} = \alpha^{\frac{1}{\sigma}}\left[1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$

with the following condition satisfied:

$$q_b > q_\ell.$$

Eq. D: CBDC is not held and bonds held only by the central bank

This is an equilibrium where no one holds CBDC, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m = 0, \ell_T > 0, b_T = 0,$$
$$\tilde{e} > 0, b_C = 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha d^{-\sigma} < 1/\beta R_m,$$

$$\begin{aligned}
q_b &> q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) &> \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right), \\
\rho \ell_T &= d \\
\tilde{e}\tilde{\kappa} &= \tilde{s} \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right) \\
\alpha d^{-\sigma} &= 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_{CB}.
\end{aligned}$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = \left(\frac{1+\tau}{\beta}\right)^{-\frac{1}{\sigma}}$$

where

$$E\tilde{\kappa} = \tilde{\alpha}^{\frac{1}{\sigma}} \left[1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$

$$\rho n_\ell q_\ell^{\frac{1-\chi}{\chi}} = \alpha^{\frac{1}{\sigma}} \left[1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},$$

with the following condition satisfied:

$$q_b > q_\ell,$$

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right).$$

Eq. E: CBDC is held and bonds are held by all banks

This is an equilibrium where CBDC is held, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m > 0, \ell_T > 0, b_T > 0,$$

$$\tilde{e} > 0, b_C > 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha(d + mR_m)^{-\sigma} = 1/\beta R_m,$$

$$q_b = q_\ell,$$

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) = \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$

$$\rho(b_T + \ell_T) = d,$$

$$b_C\kappa + \tilde{e}\tilde{\kappa} = \tilde{s},$$

$$\tilde{\alpha}\tilde{s}^{-\sigma} = 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_b}{\beta} - 1\right),$$

$$\alpha(d + mR_m)^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right),$$

$$z^{-\sigma} = \frac{1+\tau}{\beta},$$

$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}},$$

$$b_{CB} = z + R_m m,$$

$$\tilde{e} = E,$$

$$B = b_T + b_C + b_{CB}.$$

Eq. F: CBDC is held and bonds are not held by crypto banks

This is an equilibrium where CBDC is held, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold only crypto assets on their balance sheets:

$$m > 0, \ell_T > 0, b_T > 0,$$

$$\tilde{e} > 0, b_C = 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha(d + mR_m)^{-\sigma} = 1/\beta R_m,$$

$$q_b = q_\ell,$$

$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$

$$\rho(b_T + \ell_T) = d,$$

$$\tilde{e}\tilde{\kappa} = \tilde{s},$$

$$\tilde{\alpha}\tilde{s}^{-\sigma} = 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$

$$\alpha(d + mR_m)^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right),$$

$$z^{-\sigma} = \frac{1+\tau}{\beta},$$

$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}},$$

$$b_{CB} = z + R_m m,$$

$$\tilde{e} = E,$$

$$B = b_T + b_{CB}.$$

Eq. G: CBDC is held and bonds are not held by traditional banks

This is an equilibrium where CBDC is held, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m > 0, \ell_T > 0, b_T = 0,$$

$$\tilde{e} > 0, b_C > 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha(d + mR_m)^{-\sigma} = 1/\beta R_m,$$

$$\begin{aligned}
q_b &> q_\ell, \\
\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) &= \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right), \\
\rho \ell_T &= d, \\
b_C \kappa + \tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right), \\
\alpha(d + mR_m)^{-\sigma} &= 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z + R_m m, \\
\tilde{e} &= E, \\
B &= b_C + b_{CB}.
\end{aligned}$$

It is then straightforward to show that the equilibrium q_b satisfies

$$B = [\frac{\tilde{s} - E\tilde{\kappa}}{\kappa}] + (\frac{1+\tau}{\beta})^{-\frac{1}{\sigma}}$$

where

$$\begin{aligned}
\tilde{s} &= \tilde{\alpha}^{\frac{1}{\sigma}}\left[1 + \frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}}, \\
\rho n_\ell q_\ell^{\frac{1-\chi}{\chi}} &= \alpha^{\frac{1}{\sigma}}\left[1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right)\right]^{-\frac{1}{\sigma}},
\end{aligned}$$

with the following condition satisfied:

$$q_b > q_\ell.$$

Eq. H: CBDC is held and bonds held only by the central bank

This is an equilibrium where CBDC is held, traditional banks hold both loans and bonds on their balance sheets and crypto banks hold both bonds and crypto assets on their balance sheets:

$$m > 0, \ell_T > 0, b_T = 0,$$
$$\tilde{e} > 0, b_C = 0.$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, m, d, \tilde{s}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the following conditions:

$$\alpha(d + mR_m)^{-\sigma} = 1/\beta R_m,$$

$$q_b > q_\ell,$$
$$\frac{1}{\kappa}\left(\frac{q_b}{\beta} - 1\right) > \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right),$$
$$\rho \ell_T = d$$

$$\tilde{e}\tilde{\kappa} = \tilde{s}$$
$$\tilde{\alpha}\tilde{s}^{-\sigma} = 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right)$$
$$\alpha(d + mR_m)^{-\sigma} = 1 + \frac{1}{\rho}\left(\frac{q_\ell}{\beta} - 1\right),$$
$$z^{-\sigma} = \frac{1+\tau}{\beta},$$
$$\ell_T = n_\ell q_\ell^{\frac{1-\chi}{\chi}},$$

$$b_{CB} = z + R_m m,$$

$$\tilde{e} = E,$$

$$B = b_{CB}.$$

8.2 Section 6: First-order conditions and equilibrium conditions

The first-order conditions are

$$\begin{aligned}
\tilde{s}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) &= 1 + \lambda_C, \\
d: \quad u'(d) &\leq 1 + \lambda_T, \\
\tilde{m}: \quad \tilde{u}'(\tilde{s} + \mu\tilde{m}\tilde{R}_m) &\leq \frac{1}{\beta\mu\tilde{R}_m}, \\
z: \quad u_z'(z) &= \frac{1+\tau}{\beta}, \\
b_T: \quad \lambda_T &\leq \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\rho}, \\
\ell_T: \quad \lambda_T &\leq \left(\frac{q_\ell}{\beta} - 1\right)\frac{1}{\rho}, \\
\tilde{e}: \quad \lambda_C &\leq \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}, \\
b_C: \quad \lambda_C &\leq \left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa}, \\
\tilde{M}: \quad \lambda_C &\leq \left(\frac{1}{\beta\tilde{R}_m} - 1\right)\frac{1}{\tilde{\kappa}}.
\end{aligned}$$

The other equilibrium conditions are

$$\begin{aligned}
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z + \tilde{R}_m(\tilde{m} + \tilde{M}), \\
\tilde{e} &= E, \\
B &= b_T + b_C + b_{CB}, \\
q_b(b_{CB} - z) &\leq \tilde{m} + \tilde{M},
\end{aligned}$$

where the last condition ensures that the central bank raises sufficient resources to buy bonds to back tokenized CBDC from their issuance.

Section 6: Sixteen Possible Equilibrium Outcomes with Tokenized CBDC

Given the policy rate R_m set by the central bank, we need to consider sixteen different cases depending on whether the following inequalities are binding or not.

$$\tilde{m}_t \geq 0,$$

$$\tilde{M}_t \geq 0,$$

$$b_T \geq 0,$$

$$b_C \geq 0.$$

As shown in the appendix, there are sixteen possible equilibrium outcomes:

	$\tilde{m}$	$\tilde{M}$	b_T	b_C		$\tilde{m}$	$\tilde{M}$	b_T	b_C
Eq. A	0	0	+	+	Eq. I	0	+	+	+
Eq. B	0	0	0	+	Eq. J	0	+	0	+
Eq. C	0	0	+	0	Eq. K	0	+	+	0
Eq. D	0	0	0	0	Eq. L	0	+	0	0
Eq. E	+	0	+	+	Eq. M	+	+	+	+
Eq. F	+	0	0	+	Eq. N	+	+	0	+
Eq. G	+	0	+	0	Eq. O	+	+	+	0
Eq. H	+	0	0	0	Eq. P	+	+	0	0

Eq. A: Tokenized CBDC is not held as means of payments or collateral, and bonds are held by all banks

$$\tilde{m} = 0, \tilde{M} = 0, b_T > 0, b_C > 0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1 + \frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta} - 1\right), \\
\alpha d^{-\sigma} &= 1 + \frac{1}{\rho}\left(\frac{q_b}{\beta} - 1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta} - 1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta} - 1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_T + b_C + b_{CB}, \\
\rho(b_T + \ell_T) &= d, \\
b_C\kappa + \tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu \tilde{R}_m}, \\
q_e \tilde{R}_m &< 1.
\end{aligned}$$

with $\tilde{m}$=$\tilde{M} = 0$.

Eq. B: Tokenized CBDC is not held as means of payments or collateral, and bonds are not held by crypto banks

$$\tilde{m} = 0, \tilde{M} = 0, b_T > 0, b_C = 0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_T + b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{m}{=}\tilde{M}=0$.

Eq. C: Tokenized CBDC is not held as means of payments or collateral, and bonds are not held by traditional banks

$$\tilde{m}=0, \tilde{M}=0, b_T=0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_C + b_{CB}, \\
\rho \ell_T &= d, \\
b_C\kappa + \tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu \tilde{R}_m}, \\
q_e \tilde{R}_m &< 1.
\end{aligned}$$

with $\tilde{m}=\tilde{M}=0$.

Eq. D: Tokenized CBDC is not held as means of payments or collateral, and bonds are held only by the central bank

$$\tilde{m}=0, \tilde{M}=0, b_T=0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z, \\
\tilde{e} &= E, \\
B &= b_{CB}, \\
\rho\ell_T &= d, \\
\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{m}$=$\tilde{M}=0$.

Eq. E: Tokenized CBDC is held only as means of payments, and bonds are held by all banks

$$\tilde{m}>0, \tilde{M}=0, b_T>0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{m}, \\
\tilde{e} &= E, \\
B &= b_T+b_C+b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{M}=0$.

Eq. F: Tokenized CBDC is held only as means of payments, and bonds are not held by crypto banks

$$\tilde{m}>0, \tilde{M}=0, b_T>0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{m}, \\
\tilde{e} &= E, \\
B &= b_T+b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{M}=0$.

Eq. G: Tokenized CBDC is held only as means of payments, and bonds are not held by traditional banks

$$\tilde{m}>0, \tilde{M}=0, b_T=0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{m}, \\
\tilde{e} &= E, \\
B &= b_C+b_{CB}, \\
\rho\ell_T &= d, \\
b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{M}=0$.

Eq. H: Tokenized CBDC is held only as means of payments, and bonds are held only by the central bank

$$\tilde{m}>0, \tilde{M}=0, b_T=0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{m}, \\
\tilde{e} &= E, \\
B &= b_{CB}, \\
\rho\ell_T &= d, \\
\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &< 1.
\end{aligned}
$$

with $\tilde{M}=0$.

Eq. I: Tokenized CBDC is held only as collateral, and bonds are held by all banks

$$\tilde{m}=0, \tilde{M}>0, b_T>0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{M}, \\
\tilde{e} &= E, \\
B &= b_T+b_C+b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$

with $\tilde{m}$=0.

Eq. J: Tokenized CBDC is held only as collateral, and bonds are not held by crypto banks

$$\tilde{m}=0, \tilde{M}>0, b_T>0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{M}, \\
\tilde{e} &= E, \\
B &= b_T+b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$

with $\tilde{m}$=0.

Eq. K: Tokenized CBDC is held only as collateral, and bonds are not held by traditional banks

$$\tilde{m}=0, \tilde{M}>0, b_T=0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{M}, \\
\tilde{e} &= E, \\
B &= b_C+b_{CB}, \\
\rho\ell_T &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$

with $\tilde{m}$=0.

Eq. L: Tokenized CBDC is held only as collateral, and bonds are held only by the central bank

$$\tilde{m}=0, \tilde{M}>0, b_T=0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$\begin{aligned}
\tilde{\alpha}\tilde{s}^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m\tilde{M}, \\
\tilde{e} &= E, \\
B &= b_{CB}, \\
\rho\ell_T &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}\tilde{s}^{-\sigma} &< \frac{1}{\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}$$

with $\tilde{m}$=0.

Eq. M: Tokenized CBDC is held as both means of payments and collateral, and bonds are held by all banks

$$\tilde{m}>0, \tilde{M}>0, b_T>0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right),\\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right),\\
z^{-\sigma} &= \frac{1+\tau}{\beta},\\
q_b &= q_\ell,\\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}},\\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}},\\
b_{CB} &= z+\tilde{R}_m(\tilde{m}+\tilde{M}),\\
\tilde{e} &= E,\\
B &= b_T+b_C+b_{CB},\\
\rho(b_T+\ell_T) &= d,\\
\tilde{M}\tilde{R}_m\tilde{\kappa}+b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s},\\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m},\\
q_e\tilde{R}_m &= 1.
\end{aligned}$$

Eq. N: Tokenized CBDC is held as both means of payments and collateral, and bonds are not held by crypto banks

$$\tilde{m}>0, \tilde{M}>0, b_T>0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_b}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &= q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m(\tilde{m}+\tilde{M}), \\
\tilde{e} &= E, \\
B &= b_T+b_{CB}, \\
\rho(b_T+\ell_T) &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$

Eq. O: Tokenized CBDC is held as both means of payments and collateral, and bonds are not held by traditional banks

$$\tilde{m}>0, \tilde{M}>0, b_T=0, b_C>0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right), \\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right), \\
z^{-\sigma} &= \frac{1+\tau}{\beta}, \\
q_b &> q_\ell, \\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &= \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}}, \\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}}, \\
b_{CB} &= z+\tilde{R}_m(\tilde{m}+\tilde{M}), \\
\tilde{e} &= E, \\
B &= b_C+b_{CB}, \\
\rho\ell_T &= d, \\
\tilde{M}\tilde{R}_m\tilde{\kappa}+b_C\kappa+\tilde{e}\tilde{\kappa} &= \tilde{s}, \\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m}, \\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$

Eq. P: Tokenized CBDC is held as both means of payments and collateral, and bonds are held only by the central bank

$$\tilde{m}>0, \tilde{M}>0, b_T=0, b_C=0$$

Hence, the equilibrium quantities and prices $(q_b, q_\ell, q_e, R_m, d, \tilde{s}, \tilde{m}, \tilde{M}, b_T, b_C, \ell_T, \tilde{e}, z, b_{CB})$ satisfy the

following conditions:

$$
\begin{aligned}
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= 1+\frac{1}{\tilde{\kappa}}\left(\frac{q_e}{\beta}-1\right),\\
\alpha d^{-\sigma} &= 1+\frac{1}{\rho}\left(\frac{q_\ell}{\beta}-1\right),\\
z^{-\sigma} &= \frac{1+\tau}{\beta},\\
q_b &> q_\ell,\\
\left(\frac{q_b}{\beta}-1\right)\frac{1}{\kappa} &> \left(\frac{q_e}{\beta}-1\right)\frac{1}{\tilde{\kappa}},\\
\ell_T &= n_\ell q_\ell^{\frac{1-\chi}{\chi}},\\
b_{CB} &= z+\tilde{R}_m(\tilde{m}+\tilde{M}),\\
\tilde{e} &= E,\\
B &= b_{CB},\\
\rho\ell_T &= d,\\
\tilde{M}\tilde{R}_m\tilde{\kappa}+\tilde{e}\tilde{\kappa} &= \tilde{s},\\
\tilde{\alpha}(\tilde{s}+\mu\tilde{m}\tilde{R}_m)^{-\sigma} &= \frac{1}{\beta\mu\tilde{R}_m},\\
q_e\tilde{R}_m &= 1.
\end{aligned}
$$