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Anastasiia Antonova
Canadian Economic Analysis Department
Bank of Canada
AAntonova@bank-banque-canada.ca

Mykhailo Matvieiev
Canadian Economic Analysis Department
Organization
MMatvieiev@bank-banque-canada.ca

Céline Poilly
Aix-Marseille University
celine.poilly@univ-amu.fr

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Supply Shocks in the Fog: The Role of Endogenous Uncertainty*

Anastasiia Antonova[†] Mykhailo Matvieiev[‡] Céline Poilly[§]

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Abstract

Recessions are often accompanied by heightened uncertainty. We build an imperfect-information New Keynesian model in which procyclical information quality generates endogenous countercyclical uncertainty, and the nonlinear structure allows for a precautionary saving motive. We show theoretically that endogenous uncertainty operates entirely through aggregate demand. For negative supply shocks, the induced rise in uncertainty can depress demand enough to dominate the shock's inflationary force, turning the shock deflationary. Monetary policy can fully eliminate the adverse effect of endogenous uncertainty by stabilizing the output gap. We quantify the endogenous uncertainty channel in the US data and find it to be strong enough to generate deflation in response to negative supply shocks.

JEL classification: E32, D81, E52, D83, E21.

Keywords: endogenous uncertainty, precautionary saving, aggregate demand, imperfect information.

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[†]Bank of Canada - Banque du Canada. Email: aantonova@bank-banque-canada.ca

[‡]Bank of Canada - Banque du Canada. Email: mmatvieiev@bank-banque-canada.ca

[§]Aix-Marseille Univ., CNRS, AMSE. Email: celine.poilly@univ-amu.fr

1 Introduction

Bad times are uncertain times: recessions are marked not only by low output but also by heightened uncertainty. Empirical evidence suggests that this uncertainty is, at least in part, an endogenous response to downturns (Ludvigson et al., 2021). In flexible-price models, endogenous uncertainty arising from procyclical information quality has been shown to amplify recessions by reducing productive capacity (Fajgelbaum et al., 2017). Yet in monetary economies, where fluctuations operate through aggregate demand and inflation, the role of this feature remains largely unexplored. We therefore ask: how does procyclical information quality shape aggregate demand, inflation, and the conduct of monetary policy?

We address this question within a noisy-information New Keynesian framework of Woodford (2001) and Lorenzoni (2009), extended along two dimensions. First, we allow information quality to improve with economic activity, generating endogenous countercyclical uncertainty. Second, we introduce nonlinearities that capture the precautionary saving motive, so consumption and saving respond to changes in uncertainty. In this environment, activity shapes uncertainty because production generates information, while uncertainty feeds back into activity through consumption choices. This interaction gives rise to a novel mechanism of shock propagation in the New Keynesian model—the endogenous uncertainty channel.

We show theoretically that the endogenous-uncertainty channel operates entirely through aggregate demand. In the absence of price adjustment, higher uncertainty reduces output via precautionary saving rather than by affecting productive capacity. The mechanism therefore arises only in sticky-price economies, where output is demand-determined, giving monetary policy a powerful role in offsetting cyclical informational imperfections. In this environment, negative supply shocks can generate a contraction in demand rather than the overheating predicted by the standard New Keynesian model. As a result, the output decline is amplified, inflation turns into deflation, and supply disturbances trigger demand recessions. The central bank can fully neutralize these effects by closing the output gap.

To quantify the endogenous uncertainty channel, we use forecast errors from the Michigan Survey of Consumers. We find this channel to be sizable: after a negative productivity shock, the induced demand feedback generates deflation and substantially amplifies the output decline.

The conventional view that negative supply shocks are inflationary was challenged during the Covid crisis, when apparent supply disruptions coincided with deflation (Guerrieri et al., 2022). In the context of recent tariff episodes, Barnichon and Singh (2025) document that tariffs have historically been associated with deflation rather than inflation, partly because they occur during downturns with heightened uncertainty. We provide a theoretical rationale for how negative supply shocks can become deflationary when they endogenously raise uncertainty. The mechanism carries important policy implications: procyclical information quality calls for a more accommodative monetary stance in supply-driven recessions to offset the endogenous-uncertainty distortion. Consistent with the demand nature of the mechanism, monetary policy can fully correct the inefficiency of such recessions.

The noisy-information New Keynesian model with procyclical signal precision provides a natural framework to study countercyclical endogenous uncertainty. Households are imperfectly informed about aggregate productivity and update beliefs using signals whose precision varies with economic activity, consistent with a learning-by-doing interpretation, while firms set prices subject to nominal rigidities. Within this environment, fluctuations in information quality translate into movements in uncertainty that shape consumption and output.

We provide an analytical characterization of the endogenous-uncertainty mechanism within a tractable version of our model. For this, we assume prices are rigid only in the first period, which allows beliefs to be expressed purely in terms of exogenous variables. We also employ a risk-adjusted log-linearization that retains higher-order terms in the household optimality conditions, since precautionary saving hinges on these risk components.

We analytically decompose the effect of a productivity (supply) shock on aggregate demand — the output gap — into a standard New Keynesian component and an endogenous-uncertainty channel. Under the standard New Keynesian mechanism, a negative productivity shock raises the output gap because potential output falls while sticky prices prevent actual output from adjusting. By contrast, the endogenous-uncertainty channel moves the gap in the opposite direction by contracting demand. The aggregate-demand response therefore depends on the strength of the endogenous-uncertainty channel. When the endogenous-uncertainty channel dominates, a negative supply shock becomes a Keynesian supply shock—it contracts aggregate demand.

Further, we analyze how endogenous uncertainty shapes the effect of government spending on private consumption. While the mechanism amplifies the standard crowding-out effect, it also generates an additional crowding-in force. This crowding-in arises because higher activity induced by government spending reduces uncertainty. The relative importance of crowding-in depends on the persistence of spending: when the increase is sufficiently transitory, crowding-in can dominate and private consumption rises.

We quantify the amplification generated by the endogenous-uncertainty channel using the fully fledged nonlinear version of the model, parameterized for the US. We estimate the parameters governing information precision and its cyclicality by matching the time-series distribution of income-growth forecast errors in the model and in the data. To do so, we construct aggregate expectation errors from the Michigan Survey of Consumers by first computing individual income-growth errors and then aggregating them to isolate the aggregate component. The estimates imply a statistically significant degree of procyclicality in signal precision.

Finally, we simulate the responses to productivity and government spending shocks in the nonlinear model with procyclical signal precision and compare them to an economy where this mechanism is shut down. For productivity shocks, the estimated cyclicality substantially amplifies the output decline and generates deflation relative to the constant-precision benchmark. For persistent government spending shocks, the decline in private consumption is likewise amplified. Moreover, strict inflation targeting eliminates the distortions generated by information imperfections.

The rest of the paper is organized as follows. The remainder of the introduction discusses the related literature. Section 2 presents the model. Section 3 derives analytical results on the role of endogenous uncertainty for aggregate demand and shock propagation. Section 4 quantifies the endogenous-uncertainty channel for the US. Section 5 concludes.

Related Literature. In addition to the papers mentioned above, we relate our work to the broader literature on endogenous uncertainty generated by procyclical signal precision over the business cycle. Existing work emphasizes propagation through investment (Saijo, 2017; Schaal and Taschereau-Dumouchel, 2023), Knightian uncertainty (Ilut and Saijo, 2021), financial frictions (Benhabib et al., 2019; Straub and Ulbricht, 2024), or labor-market frictions (Bernstein et al., 2024). In contrast, we study the transmission of endogenous consumer uncertainty through precautionary saving and its effects on aggregate demand.

Our demand amplification through precautionary saving makes the paper conceptually related to the HANK/SAM frameworks of Ravn and Sterk (2017, 2021) and Challe (2020). The crucial difference is the origin of risk: in our framework uncertainty arises from informational frictions rather than labor-market imperfections. Consequently, it is aggregate rather than idiosyncratic and triggers precautionary saving even under full risk sharing among households, rendering redistributive policies such as insurance or transfers ineffective.

We also relate to the Keynesian supply shock literature, which studies how supply disturbances can endogenously generate strong aggregate demand responses. Bilbiie and Melitz (2023) obtain demand amplification from firm entry and exit, Cesa-Bianchi and Ferrero (2021) from sectoral linkages, Fornaro and Wolf (2023) from productivity-enhancing investment, and L'Huillier et al. (2024) from diagnostic expectations. Our mechanism instead relies on a feedback between activity and endogenous household uncertainty operating through precautionary saving, transmitting supply shocks to aggregate demand without relying on production-side features.

Finally, we relate to the literature on learning under noisy and imperfect information (Veldkamp, 2005; Van Nieuwerburgh and Veldkamp, 2006; Ordonez, 2009; Mäkinen and Ohl, 2015) and to empirical literature rejecting the Full Information Rational Expectations hypothesis (Mankiw et al., 2003; Andrade and Le Bihan, 2013; Coibion and Gorodnichenko, 2012, 2015). We contribute by structurally quantifying the cyclicality of consumers' information frictions in the US We also connect to work on aggregate uncertainty shocks and precautionary saving demand (Fernández-Villaverde et al., 2011; Leduc and Liu, 2016; Basu and Bundick, 2017; Fernández-Villaverde and

Guerrón-Quintana, 2020), but differ by making uncertainty endogenous.

2 A New Keynesian Model with Endogenous Uncertainty

We build on the noisy-information New Keynesian framework of Woodford (2001); Lorenzoni (2009), and extend it along two dimensions. First, starting from the premise that economic activity generates information, we introduce procyclical signal precision, which gives rise to endogenous, time-varying uncertainty. Second, we account for household precautionary saving by departing from a linear framework. Together, these two extensions give rise to a novel endogenous-uncertainty channel of aggregate shock propagation, driven by feedback between consumer uncertainty and economic activity.

We present the model in three steps. First, we lay out the standard components of a New Keynesian environment. Second, we describe the information structure and belief updating. Third, we define the equilibrium and characterize the joint determination of economic activity and agents' beliefs. Appendix A lists all model equations.

2.1 Environment

The economy consists of a representative household that consumes, saves, and supplies labor, and a continuum of monopolistically competitive firms — owned by the household — that produce differentiated goods using labor and set prices subject to Rotemberg adjustment costs. The differentiated goods are aggregated into a final good, which is purchased by the household and the government. Monetary policy is conducted by a central bank according to a Taylor rule, and a fiscal authority finances government spending through lump-sum taxes. Aggregate productivity shocks drive economic fluctuations. All expectation operators, $\mathbb{E}(.)$, denote expectations conditional on the (imperfect) information available to agents at the time decisions are made.

2.1.1 Households

A representative household maximizes its expected lifetime utility

$$\max_{C_t, L_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \xi \frac{L_t^{1+\omega}}{1+\omega} \right), \tag{1}$$

where C_t denotes the household's consumption, and L_t is the labor supply (hours worked); η denotes the household's relative aversion to risk, ω the inverse Frisch elasticity of labor supply, ξ labor disutility weight, and β the discount factor. The household lifetime utility in Eq. (1) is subject to the following sequence of budget constraints

$$P_tC_t + Q_tB_t = B_{t-1} + P_tW_tL_t + D_t, \tag{2}$$

where P_t is the price of the final consumption good, W_t is the real wage, B_t denotes holdings of riskless bonds, Q_t is the bond price, D_t denotes lump-sum transfers from firm ownership and government. The household's optimization yields the Euler equation and the labor supply equation, given respectively by

$$Q_t = \beta \cdot \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \cdot \frac{1}{\pi_{t+1}} \right\}, \tag{3}$$

$$\xi L_t^{\omega} = C_t^{-\eta} W_t, \tag{4}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the gross inflation rate.

2.1.2 Firms

A continuum of monopolistically competitive firms, indexed by $i \in [0,1]$, produces differentiated goods $Y_t(i)$ which are aggregated into the final output via the CES aggregator $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}$, where ϵ is the elasticity of substitution between differentiated goods. The demand for good i is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{5}$$

where $P_t(i)$ is the price of good i, and the aggregate price index is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}$.

Firm i produces its output using the following technology

$$Y_t(i) = \tilde{A}_t L_t(i)^{1-\alpha}, \tag{6}$$

where $\tilde{A}_t$ is the aggregate productivity (described in detail below) and α governs the returns to scale.

Each firm i sets the nominal price of its good, $P_t(i)$, subject to quadratic adjustment costs a la Rotemberg (1982). The firm maximizes its expected discounted stream of profits

$$\max_{P_t(i)} \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \left\{ P_t(i)Y_t(i) - (1-\bar{\tau})P_tW_tL_t(i) - \frac{\Phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_tY_t \right\}, \tag{7}$$

subject to the firm-specific demand Eq. (5). Here Φ denotes the strength of price adjustment costs and thus governs the degree of price rigidity, $\bar{\tau} = \epsilon^{-1}$ is a standard labor subsidy that ensures the efficiency of the flexible-price equilibrium, and $Q_{t,t+s}$ is the stochastic discount factor, defined as $Q_{t,t+s} \equiv \beta^s \, \mathbb{E}_t \big[(C_{t+s}/C_t)^{-\eta} / \prod_{j=1}^{s} \pi_{t+j} \big]$.

The first-order condition for profit maximization, evaluated in a symmetric equilibrium, yields the conventional New Keynesian Phillips curve:

$$\epsilon \left(1 - (1-\bar{\tau})MC_t\right) = 1 - \Phi \left(\pi_t - 1\right) \pi_t + \Phi \mathrm{E}_t \left\{ Q_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\}, \tag{8}$$

where MC_t denotes real marginal cost. The labor demand equation, derived from firms' cost-minimization problem and evaluated in symmetric equilibrium, is given by

$$W_t = (1-\alpha) \, \tilde{A}_t MC_t L_t^{-\alpha}. \tag{9}$$

2.1.3 Policy and resource constraint

The central bank sets the nominal interest rate R_t according to the Taylor (1993) rule

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}, \tag{10}$$

where $\bar{R} = 1/\beta$ is the steady-state nominal interest rate (with $\bar{\pi} = 1$). Parameter ϕ_π determines the sensitivity of the interest rate to inflation. The nominal rate and the bond prices are linked as $R_t = 1/Q_t$. We also assume an exogenous path of government spending G_t, financed through lump-sum taxes.

Output is either consumed by the household and the government or spent as price adjustment costs. Hence, the aggregate resource constraint is

$$Y_t = C_t + G_t + \frac{\Phi}{2}(\pi_t - 1)^2 Y_t. \tag{11}$$

2.1.4 Productivity

Log-productivity, $\tilde{a}_t \equiv \log(\tilde{A}_t)$, consists of two components: a persistent component a_t and a transitory component f_t:

$$\tilde{a}_t = a_t + f_t. \tag{12}$$

The transitory component follows an i.i.d. process, $f_t \sim \mathcal{N}(0, \sigma_f^2)$ while the persistent component evolves according to

$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \epsilon_t^a. \tag{13}$$

where ρ_a governs the persistence and $\epsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$. The shocks f_t and ϵ_t^a are mutually independent. We normalize steady-state productivity by setting $\bar{a} = 0$.

2.2 Information structure

We adopt a noisy-information framework in which agents observe aggregate productivity, $\tilde{a}_t$, but *cannot* disentangle its transitory component f_t from its persistent component a_t (e.g. Lorenzoni, 2009). The persistent component represents the true state of the economy and is inferred from noisy signals. At the beginning of period t, agents hold a prior belief over a_t and update it using two signals: (*i*) observed productivity $\tilde{a}_t$, and (*ii*) an additional learning-by-doing signal whose precision varies over time. Crucially, the precision of the learning-by-doing signal increases with output, making information quality procyclical. This endogenous cyclicality of information quality is a novel feature of our model and is the source of the endogenous uncertainty channel studied in the paper.

2.2.1 Priors

Let Ω_t denote the information set available to households at the beginning of period t, before they receive any signals pertaining to the *current* period. Given this information, the prior belief about the persistent productivity component is

$$a_t|\Omega_t \sim \mathcal{N}(\theta_t, \gamma_t^{-1}), \tag{14}$$

where θ_t is the perceived mean, and γ_t^{-1} is the perceived variance (γ_t is information precision).

2.2.2 Signals

Within the period t, agents receive two noisy signals. The first signal, z_t, is an observation of realized productivity, $z_t = \tilde{a}_t$. Conditional on z_t, agents' belief about the persistent component is $a_t|z_t \sim \mathcal{N}(\tilde{a}_t, [\gamma^z]^{-1})$, where $\gamma^z = \sigma_f^{-2}$ denotes signal precision. Using Eq. (12), the signal can be written as

$$z_t = a_t + \epsilon_t^z \tag{15}$$

where $\epsilon_t^z \sim \mathcal{N}(0, [\gamma^z]^{-1})$ is the a noise term.

The second signal is a learning-by-doing signal, denoted by s_t. It is defined as

$$s_t = a_t + \epsilon_t^s, \tag{16}$$

where $\epsilon_t^s \sim \mathcal{N}(0, [\gamma_t^s]^{-1})$ is noise term. The precision of this signal is time-varying and increases with economic activity, $\gamma_t^s = \gamma(Y_t/\bar{Y})$, where $\gamma(\cdot)$ governs the strength of cyclicality and satisfies $\gamma'(Y_t/\bar{Y}) > 0$, and $\bar{Y}$ denotes the steady-state level of output. Appendix B provides learning-by-doing microfoundations for procyclical precision—higher activity increases agents' exposure to production technology, improving inference about the economy's state—and discusses alternative learning protocols.

Information precision is procyclical in our model, unlike in standard New Keynesian noisy-information frameworks that typically assume constant signal precision (e.g., Lorenzoni, 2009). This procyclicality captures the idea that economic activity generates information, allowing agents to form more precise beliefs in expansion. While related in spirit to learning-from-activity models (e.g., Van Nieuwerburgh and Veldkamp, 2006; Fajgelbaum et al., 2017; Ilut and Saijo, 2021), our mechanism operates through aggregate demand rather than the production side of the economy, so its macroeconomic effects are tightly disciplined by monetary policy, as we show below.

2.2.3 Beliefs formation

Bayesian updating implies that agents' posterior beliefs about a_t, conditional on prior information Ω_t and two signals z_t and s_t satisfy

$$\mathrm{E}(a_t|\Omega_t, s_t, z_t) = \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{17}$$

$$\mathrm{Var}(a_t|\Omega_t, s_t, z_t) = \frac{1}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{18}$$

The information set at the end of the period t is $\Omega_{t+1} = \{\Omega_t, z_t, s_t\}$. Combining the AR(1) process in Eq. (13) with the posterior mean in Eq. (17) yields agents' mean beliefs about a_{t+1} at the beginning of period $t+1$, denoted by $\theta_{t+1} \equiv \mathrm{E}(a_{t+1}|\Omega_{t+1})$:

$$\begin{aligned} \theta_{t+1} &= (1-\rho_a)\bar{a} + \rho_a \mathrm{E}(a_t \mid \Omega_t, s_t, z_t) + \mathrm{E}(\epsilon_{t+1}^a \mid \Omega_{t+1}), \\ &= (1-\rho_a)\bar{a} + \rho_a \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}. \end{aligned} \tag{19}$$

The corresponding belief precision, $\gamma_{t+1} \equiv [\mathrm{Var}(a_{t+1}|\Omega_{t+1})]^{-1}$, follows from Eq. (13) and Eq. (18):

$$\begin{aligned} \gamma_{t+1} &= \left[\rho_a^2 \mathrm{Var}(a_t \mid \Omega_t, s_t, z_t) + (1-\rho_a)^2 \mathrm{Var}(\bar{a}) + \mathrm{Var}(\epsilon_t^a)\right]^{-1}, \\ &= \left[\frac{\rho_a^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2\right]^{-1}. \end{aligned} \tag{20}$$

Eqs. (19) and (20) jointly define the recursive law of motion for beliefs about a_t.

2.3 Equilibrium

Definition (Equilibrium). *A dynamic symmetric equilibrium in this economy is a sequence of quantities* $\{C_t, L_t, Y_t, G_t, D_t\}_{t=0}^{\infty}$*, prices* $\{P_t, W_t, Q_t, R_t\}_{t=0}^{\infty}$*, signals* $\{z_t, s_t\}_{t=0}^{\infty}$*, and belief states* $\{\theta_t, \gamma_t, \gamma_t^s\}_{t=0}^{\infty}$*, such* *that: (i) households' and firms' optimality conditions hold; (ii) all firms choose the same price, P_t; (iii) beliefs evolve according to their recursive Bayesian updating rule; (iv) quantity and price paths are consistent with beliefs; (v) learning-by-doing precision satisfies $\gamma_t^s = \gamma(Y_t/\bar{Y})$; (vi) the nominal interest rate satisfies the monetary-policy rule; and (vii) all markets clear.*

Note that information precision depends on output through condition (*v*), while output depends on information precision through beliefs in condition (*iv*). This two-way feedback is central to our endogenous-uncertainty amplification mechanism: under procyclical signal precision, uncertainty and economic activity are jointly determined in equilibrium.

3 The Uncertainty Channel: a Stylized Approach

In this section, we analyze the role of procyclical information quality in the transmission of aggregate shocks within a tractable version of our model. We show analytically that the interaction between endogenous uncertainty – arising from procyclical information quality – and the precautionary-saving motive generates a novel transmission mechanism absent from the standard New Keynesian model, which we refer to as the endogenous-uncertainty channel.

Capturing the precautionary-saving motive requires moving beyond the first-order approximation. We therefore adopt a risk-adjusted approximation that preserves tractability while allowing uncertainty to affect equilibrium outcomes. In particular, following the approach of Skinner (1988), we augment the linearized Euler equation with a second-order term that captures precautionary-saving.

In what follows, lowercase letters denote log variables $x_t = log(X_t)$, unless stated otherwise. Log-deviations from the steady state are denoted by $\hat{x}_t$. All derivations are available in Appendix C.

3.1 Tractability assumptions

We impose the following tractability assumptions: (i) nominal price rigidities apply only in the current period t, so that the inefficient wedge between the wage and the marginal product of labor is purely contemporaneous and does not persist into the future; (ii) households save in real bonds, allowing us to abstract from the inflation-risk premium induced by inflation expectations; (iii) production is linear ($\alpha = 0$), and (iv) steady-state government spending is zero ($\bar{G} = 0$); (v) agents have no prior information about productivity, implying zero prior precision, $\gamma_t = 0$. Assumptions (i) and (ii) are required for tractability, while (iii)-(v) are introduced to simplify derivations.[1]

[1] In Section 4 we relax these assumptions, and confirm that the quantitative results are consistent with the model's theoretical mechanism.

3.2 Inefficiency wedge and output path

Given our assumptions, log-linearizing the labor demand Eq. (9) yields a simple relationship between wages and productivity

$$w_{t+j} = mc_{t+j} + \tilde{a}_{t+j} \quad \text{with} \begin{cases} mc_{t+j} \neq 0 & \text{if} \quad j = 0 \\ mc_{t+j} = 0 & \text{if} \quad j > 0 \end{cases} . \tag{21}$$

Here, mc_{t+j} is the log real marginal cost and captures the inefficient wedge between wages and productivity. This wedge is present only in period t, when nominal price rigidity generates inefficiency; for $j > 0$ the economy is efficient and real marginal cost is zero, i.e., nominal marginal cost equals the price.

Combining labor supply Eq. (4), labor demand Eq. (9), the production function Eq. (6), and the resource constraint Eq. (11) yields the following expressions for output

$$y_t = \left(\frac{1}{\omega+\eta}\right) mc_t + \left(\frac{1+\omega}{\omega+\eta}\right) \tilde{a}_t + \left(\frac{\eta}{\omega+\eta}\right) g_t, \tag{22}$$

$$y_{t+j} = \left(\frac{1+\omega}{\omega+\eta}\right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega+\eta}\right) g_{t+j}, \quad \text{for } j > 0. \tag{23}$$

where $g_t = \frac{G_t}{Y}$ denotes government spending relative to steady-state output.

Eqs. (22)-(23) show that output in period t depends on the endogenous wedge mc_t, while for $j > 0$ output is fully determined by exogenous variables – technology and government spending.

3.3 IS curve and law of motion of beliefs

We derive a risk-augmented IS curve by combining a risk-adjusted approximation of the Euler equation Eq. (3), which captures precautionary saving, with the resource constraint Eq. (11). This yields

$$y_t = (1-\rho_g)g_t - \frac{1}{\eta}(r_t - \rho) + \mathrm{E}_t\{y_{t+1}\} - \frac{1}{2}(1+\eta)\mathrm{Var}_t\{y_{t+1}\}, \tag{24}$$

where $\rho = -log(\beta)$, r_t denotes the real interest rate, and ρ_g is the persistence parameter of government spending, such that $g_{t+1} = \rho_g g_t$.

Eq. (24) establishes the demand determinants of output. The first three terms form a standard New Keynesian IS curve (Galí, 2008): higher government spending g_t and higher expected future output $E_t y_{t+1}$ increase current output, while a higher real interest rate r_t lowers it. Relative to the standard New Keynesian IS curve, Eq. (24) features an additional fourth term capturing uncertainty about future output, measured by the conditional variance $\mathrm{Var}_t\{y_{t+1}\}$. It implies that greater uncertainty about future output depresses current output, as households cut consumption and increase precautionary saving.

In Eq. (24) current output depends on *beliefs* about future output, in particular on the conditional mean $\mathrm{E_t}\{y_{t+1}\}$ and variance $\mathrm{Var_t}\{y_{t+1}\}$. Our tractable setup allows us to compute these beliefs analytically as functions of the exogenous variables $\tilde{a}_t$ and g_t. Using Eq. (23), we obtain

$$\mathrm{E_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)\mathrm{E_t}\{\tilde{a}_{t+1}\} + \rho_g\left(\frac{\eta}{\omega+\eta}\right)g_t, \tag{25}$$

$$\mathrm{Var_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)^2 \mathrm{Var_t}\{\tilde{a}_{t+1}\}. \tag{26}$$

That is, beliefs about next-period output y_{t+1} can be constructed from beliefs about next-period productivity. Productivity beliefs, in turn, are governed by beliefs about the persistent component a_t, summarized by the mean θ_t and precision γ_t in Eq. (14). In particular, $\mathrm{E_t}\{\tilde{a}_{t+1}\} = \mathrm{E_t}\{a_{t+1}\} = \theta_{t+1}$ and $\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \mathrm{Var_t}\{a_{t+1}\} + [\gamma^z]^{-1} = \gamma_{t+1}^{-1} + [\gamma^z]^{-1}$. Using the belief dynamics for the persistent component a_t in Eqs. (19) and (20), we obtain the following linear approximation for beliefs about next-period productivity $\tilde{a}_{t+1}$

$$\mathrm{E_t}\{\tilde{a}_{t+1}\} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t], \tag{27}$$

$$\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \sigma_a^2 + [\gamma^z]^{-1} + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma}v^2 \cdot y_t, \tag{28}$$

where $v \equiv \frac{\gamma}{\gamma+\gamma^z}$ denotes the steady-state weight given to the learning-by-doing signal s_t when forming beliefs, and $\gamma \equiv \gamma(1)$ is the steady-state precision of signal s_t.[2] Eq. (28) shows that uncertainty about future productivity decreases with output, reflecting the procyclical precision of the learning-by-doing signal; $\bar{y}$ denotes the steady-state output.

Substituting the belief expressions in Eqs. (27)–(28) into Eqs. (25)-(26) and plugging the result into the IS curve Eq. (24), yields the following steady-state deviation for output as a function of the real interest rate deviation and exogenous variables

$$\hat{y}_t = f \cdot \left[-\frac{1}{\eta}\hat{r}_t + \frac{1+\omega}{\omega+\eta}\rho_a\big(vs_t + (1-v)z_t\big) + \Big(1 - \rho_g \cdot \frac{\omega}{\omega+\eta}\Big)g_t \right], \tag{29}$$

$$\text{where } f \equiv \left[1 - \frac{1}{2} \cdot \frac{\rho_a^2(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\gamma}{(\gamma^z+\gamma)^2}\right]^{-1}. \tag{30}$$

In Eq. (29), output fluctuations are governed by the composite parameter f. For reasonable parameter values – such that positive signals s_t and z_t raise output – f is positive and satisfies $f \geq 1$. Eq. (30) shows that f depends on the precision of the learning-by-doing signal, γ, and therefore determines the strength of amplification generated by endogenous uncertainty. When the signal is absent ($\gamma = 0$), we obtain $f_{|\gamma=0} = 1$; we refer to this benchmark as the *constant-uncertainty* case. By contrast, when the signal is operative ($\gamma > 0$), fluctuations are amplified,

[2] Under our assumption of no prior information ($\gamma_t = 0$), beliefs are formed using the two signals s_t and z_t, with weights v and $1 - v$, respectively.

with $f_{|\gamma>0} > 1$, corresponding to the *endogenous-uncertainty* case.

Note that f is non-monotonic in γ. As $\gamma \to 0$, the signal s_t becomes uninformative and $f \to 1$, since output has little effect on information precision. As $\gamma \to \infty$, the economy approaches full information and again $f \to 1$, because additional output generates negligible new information. For intermediate values $\gamma \in (0, \infty)$, f rises with γ, peaks at $\gamma = \gamma^z$, and then declines. Hence, amplification is strongest when γ is neither too low nor too high. In Section 4, we discipline γ using empirical evidence on consumers' forecast errors.

3.4 Aggregate demand effect of productivity shock

We now turn to the analysis of aggregate-demand amplification through the endogenous-uncertainty channel. Following the New Keynesian tradition, we measure aggregate demand by the output gap.

We begin by defining flexible-price output using Eq. (22)

$$\hat{y}_t^f = \left(\frac{1+\omega}{\omega+\eta}\right)\tilde{a}_t + \left(\frac{\eta}{\omega+\eta}\right)g_t, \tag{31}$$

which corresponds to the level of output that would obtain under fully flexible prices. The output gap is defined as the deviation of actual output from its flexible-price benchmark:

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^f = \left(\frac{1}{\omega+\eta}\right)mc_t. \tag{32}$$

Log-linearizing the Phillips curve Eq. (8) yields a positive link between aggregate demand and inflation

$$\pi_t = \frac{\epsilon}{\Phi}\cdot mc_t = \frac{\epsilon(\omega+\eta)}{\Phi}\tilde{y}_t. \tag{33}$$

Note that, in this economy, inflation is fully determined by aggregate demand. Moreover, assumption (i) implies a static Phillips curve.

We assume that the central bank controls the real interest rate and targets the output gap according to the rule $\hat{r}_t = \phi\tilde{y}_t$. By Eq. (33), this policy is equivalent to inflation targeting, since inflation is proportional to the output gap. The implied inflation targeting rule is therefore $\hat{r}_t = \phi'\pi_t$, where $\phi' = \phi\frac{\Phi}{\epsilon(\omega+\eta)}$.

Combining Eqs. (29) and (31), we obtain the following expression characterizing the output gap

$$\left[1+\frac{\phi}{\eta}\cdot f\right]\cdot\tilde{y}_t = \frac{1+\omega}{\omega+\eta}\cdot\left[-\tilde{a}_t + \rho_a f(vs_t + (1-v)z_t)\right] + f\cdot\left(1-\frac{\omega\rho_g}{\omega+\eta}-\frac{\eta}{\omega+\eta}\right)g_t. \tag{34}$$

We predominantly focus on the economy's response to a productivity shock– an innovation to the persistent component a_t. We do so because it is a supply shock: a decline in productive capacity which, in a standard New Keynesian model, generates an overheated economy by raising the output gap and inflationary pressures. To construct the response of the output gap to a productivity shock, we shut down government spending and noise shocks by setting $g_t = 0$, and $\epsilon_t^z = \epsilon_t^s = 0$. Under these restrictions, $z_t = s_t = \tilde{a}_t = a_t$. The relationship between productivity and the output gap is summarized in Proposition 1.

Proposition 1 (Aggregate demand effect of a productivity shock)**.** *The effect of a productivity shock a_t on the output gap $\tilde{y}_t$ is*

$$\tilde{y}_t = -\psi(1-\rho_a)\, a_t + \psi\rho_a(f-1)\, a_t, \tag{35}$$

where $\psi = \left[1 + \frac{\phi}{\eta} \cdot f\right]^{-1} \frac{1+\omega}{\omega+\eta}$. The first term is the standard New Keynesian effect; the second term reflects amplification through endogenous uncertainty.

The first term in Eq. (35) captures the standard New Keynesian mechanism. When $\rho_a < 1$, a *negative* productivity shock generates *positive output gap*: actual output falls by less than flexible-price output. This occurs because price rigidities prevent firms from adjusting prices sufficiently to replicate the flexible-price allocation, leaving prices inefficiently low and generating excess demand.

The second term in Eq. (35) operates when $f > 1$ and captures transmission through endogenous-uncertainty channel. Through this channel, a negative productivity shock raises precautionary saving and depresses aggregate demand, thereby reducing the output gap. When the endogenous-uncertainty channel dominates the standard New Keynesian channel – i.e., when $f > \rho_a^{-1}$ – a negative productivity shock generates a *negative output gap*. In this case, a negative productivity shock becomes a *Keynesian supply shock*: a supply disturbance for which the endogenous demand contraction dominates the direct supply effect.

Note that the endogenous-uncertainty channel operates entirely through aggregate demand. The implications for monetary policy are summarized in the following Corollary.

Corollary 1 (Aggregate-demand nature of the endogenous-uncertainty channel)**.** *The endogenous-uncertainty channel operates solely through the output gap. Accordingly, by stabilizing the output gap, monetary policy can fully neutralize its effects.*

To understand Corollary 1, note that flexible-price output is unaffected by information imperfections. From Eq. (31), the response of flexible-price output to a productivity shock is $\hat{y}_t^f = \frac{1+\omega}{\omega+\eta} a_t$, so the flexible-price allocation coincides with the full-information benchmark. Intuitively, under flexible prices, output is supply-determined, and the demand-side frictions are neutralized through price adjustment. Informational frictions on the demand side matter only when prices are sticky, in which case they operate through aggregate demand. In particular, stronger information frictions imply a larger demand contraction following a negative productivity shock. Hence, by stabilizing the output gap, monetary policy can fully neutralize the endogenous-uncertainty channel and restore the flexible-price, full-information allocation.

We also examine how the endogenous-uncertainty channel affects the responses of output, inflation, and hours worked to a technology shock. The responses of output and inflation are

$$y_t = f \cdot \psi \cdot \left[\frac{\phi}{\eta} + \rho_a \right] a_t, \tag{36}$$

$$\pi_t = \frac{\epsilon \rho_a (\omega + \eta)}{\Phi} \left[f - \frac{1}{\rho_a} \right] a_t. \tag{37}$$

Eq. (36) shows that the endogenous-uncertainty channel ($f > 1$) amplifies the output response to a productivity shock (through its effect on aggregate demand). The sign of the inflation response in Eq. (37) mirrors the sign of the output gap: inflation is negative whenever the output gap is negative, i.e., when ($f > \rho_a^{-1}$).

The response of hours worked is

$$l_t = \tilde{l}_t + l_t^f = \tilde{y}_t - \frac{\eta - 1}{\omega + \eta} a_t. \tag{38}$$

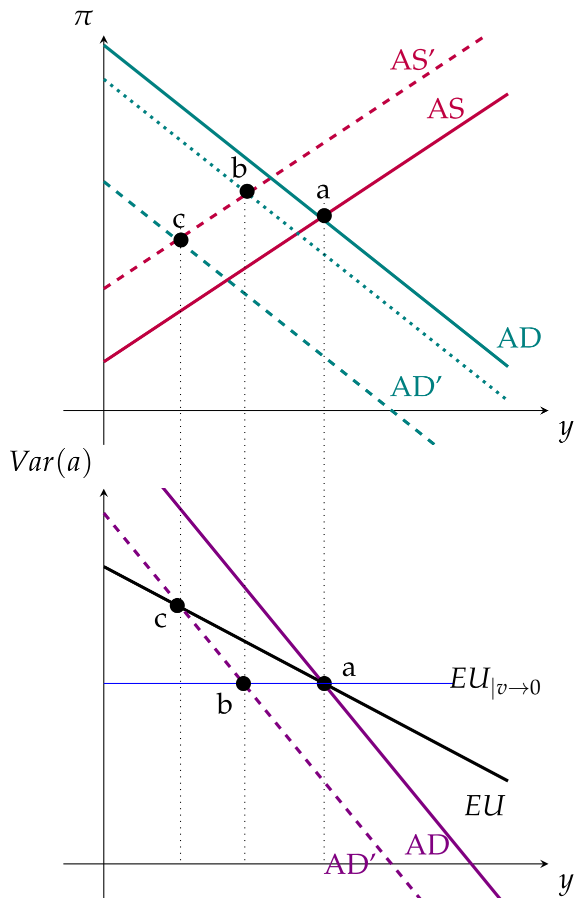
Here, efficient hours in a flexible-price economy are $l_t^f = -\frac{\eta-1}{\omega+\eta} a_t$, and the inefficient hours gap satisfies $\tilde{l}_t = \tilde{y}_t$. Efficient hours fall with productivity (and thus rise after a negative productivity shock) as long as $\eta \geq 1$. The hours gap moves one-for-one with the output gap. Without endogenous uncertainty, negative productivity shock raises both efficient hours and the hours gap, implying higher total hours worked, as in the textbook New Keynesian model (Galí, 2008). With the endogenous uncertainty, the response becomes ambiguous: efficient hours still rise, but the hours gap may fall after a negative productivity shock.

3.5 Graphical illustration

Now we graphically illustrate the joint equilibrium adjustment of economic activity and uncertainty to a productivity shock via the interaction of the aggregate demand (AD), aggregate supply (AS), and endogenous uncertainty (EU) curves.

The AD curve is obtained by substituting the monetary policy rule $r_t = \bar{r} + \phi' \pi_t$ and output beliefs, Eqs. (25)-(26), into the IS curve, Eq. (24). The AS curve is derived by substituting the output gap definition $\tilde{y}_t = y_t - y_t^f$ and flexible output, Eq. (31), into the Phillips curve, Eq. (33). The EU curve is given by Eq. (28). The resulting system is

Figure 1: Adjustments after negative productivity shock: illustration



Notes: The figure plots the equilibrium adjustment in economic activity (upper figure) and uncertainty (lower figure) to a negative productivity shock. Point (a) is the initial equilibrium, point (b) is the equilibrium without endogenous uncertainty, and point (c) is the equilibrium with endogenous uncertainty.

$$\textbf{AD:}\quad y_t = -\frac{1}{\eta}(\phi'\pi_t + \bar{r} - \rho) + \frac{1+\omega}{\omega+\eta}\mathrm{E_t}\{\tilde{a}_{t+1}\} - \frac{1}{2}(1+\eta)\left(\frac{1+\omega}{\omega+\eta}\right)^2 \mathrm{Var_t}\{\tilde{a}_{t+1}\},$$

$$\textbf{AS:}\quad \pi_t = \frac{\epsilon(\omega+\eta)}{\Phi}\left(y_t - \frac{1+\omega}{\omega+\eta}\tilde{a}_t\right),$$

$$\textbf{EU:}\quad \mathrm{Var_t}\{\tilde{a}_{t+1}\} = \sigma_a^2 + \frac{1}{\gamma^z} + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma}v^2 y_t.$$

Figure 1 plots the AD and AS curves on the output-inflation plane (y, π), and AD and EU curves on the output-uncertainty plane $(y, Var(a))$. The EU curve is shown for two cases: it is downward-sloping when the endogenous uncertainty channel is active, and horizontal in the absence of endogenous uncertainty ($EU_{|v\to 0}$). General equilibrium is given by the intersection of

AD with AS in the top panel and AD and EU in the bottom panel.

Suppose the economy starts at point (a). A negative productivity shock shifts both AS and AD leftward: AS shifts because efficient output falls, and AD shifts because expected future productivity declines. Without endogenous uncertainty, the economy moves to point (b), characterized by lower output and higher inflation.

With endogenous uncertainty, equilibrium in the bottom panel is pinned down by the intersection of AD′ and the downward-sloping EU curve, yielding point (c) with lower output and higher uncertainty. Higher uncertainty then feeds back into precautionary saving, shifting AD further left in the top panel and moving the economy to point (c), where both output and inflation fall.

3.6 Crowding out/in effect of public spending shocks

We conclude the theoretical section by analyzing the propagation of a government spending shock. In this government spending shock exercise, we assume that $z_t = s_t = \tilde{a}_t = 0$ and that $g_t > 0$, while monetary policy continues to respond to the output gap. The relationship between private consumption and government spending is given by the following proposition.

Proposition 2 (Crowding out/in of private consumption)**.** *The link between private consumption and government spending is given by*

$$c_t = -f\tilde{\psi}\left[\rho_g + \frac{\phi}{\eta}\right]g_t + (f-1)\tilde{\psi}\left[1 + \frac{\eta}{\omega}\right]g_t \tag{39}$$

where $\tilde{\psi} = \frac{\omega}{1+\omega}\psi$.

When uncertainty is endogenous ($f > 1$), two opposing forces arise. On the one hand, endogenous uncertainty amplifies the traditional crowding-out channel, since the first term in Eq. (39) is scaled by f. This is because higher government spending acts as a negative demand shift for private households by lowering their expected income, and this contraction is amplified through precautionary saving.[3] In other words, if uncertainty reacted to private consumption rather than total output, only the first term would operate: uncertainty would rise and the crowding-out effect would be amplified.

On the other hand, endogenous uncertainty introduces an additional term (the second term in Eq. (39)) capturing a crowding-in effect: higher government spending raises total output, reduces uncertainty, lowers precautionary saving, and thus increases private consumption for a given expected income. Since the crowding-out component strengthens with shock persistence, while the crowding-in component is unaffected by it, consumption falls for sufficiently persistent shocks and rises for sufficiently transitory shocks.

[3] Note that the magnitude of this crowding-out effect increases with the persistence of government spending, ρ_g, and with the strength of the monetary policy response, ϕ, as emphasized by Leeper et al. (2017).

4 Quantification of Endogenous Uncertainty Channel

We now turn to the quantitative evaluation of the endogenous-uncertainty channel within the nonlinear New Keynesian model laid out in Section 2. To this end, we parametrize the model using US data. We then examine the quantitative role of endogenous uncertainty in the transmission of productivity and public spending shocks, and analyze how this mechanism interacts with monetary policy. Throughout this section we continue to assume a linear dependence of signal precision on output: $\gamma\left(\frac{Y_t}{\bar{Y}}\right) = \gamma \cdot \frac{Y_t}{\bar{Y}}$. We solve the model taking into account that the model economy is driven by agents' beliefs and expectation errors about the unobserved state variable. To account for nonlinearities, we use third-order perturbation. Further details on the solution method and simulation procedures are provided in Appendix E.

4.1 Parametrization

We parametrize the model at a quarterly frequency. The parameters fall into two categories. The first category contains standard parameters, which we calibrate to match conventional data moments or take directly from the literature. The second category consists of parameters specific to our endogenous-uncertainty mechanism. These include the signal-precision parameters γ and γ^z from the noisy-information block of the model, as well as the productivity shock standard deviation σ_a; together, these parameters govern the law of motion for households' beliefs. We estimate this second set of parameters by matching moments of consumer income forecast errors constructed from the Michigan Survey of Consumers. Next, we describe (*i*) the calibration of the standard parameters, (*ii*) the construction of the survey-based forecast errors, and (*iii*) the estimation procedure for the information parameters.

4.1.1 Calibrated parameters

Table 1 reports the calibrated parameters. The time discount factor is set to $\beta = 0.99$, implying an annual interest rate of approximately 4% in the steady state. The coefficient of relative risk aversion is set to $\eta = 3$. We assume a unitary Frisch elasticity of labor supply ($\omega = 1$), a standard value in the business-cycle literature. The scale parameter ξ is set to normalize the steady-state labor to $\bar{L} = 0.3$. The elasticity of substitution across varieties is set to $\epsilon = 8$, which delivers a steady-state markup of roughly 14.5%, consistent with the estimates of Farhi and Gourio (2018). We set the returns-to-scale such that $\alpha = 0.3$, implying a labor share in output of about 60%, in line with the empirical evidence in Autor et al. (2020). The price adjustment cost parameter is set to $\Phi = 100$, which maps – under our assumed elasticity of substitution – to a Calvo probability of price non-adjustment of approximately 0.75 (to a first-order approximation), implying an average price duration of about a year. This is in line with the findings of Nakamura and Steinsson

(2013). The monetary policy parameter governing the response of the interest rate to inflation is set to the conventional value $\phi_\pi = 1.5$. Finally, the government spending-to-output ratio is set to 18%, matching its average level in US data. We set the persistence of productivity to $\rho_a = 0.97$, in line with standard estimates from AR(1) regressions of utilization-adjusted TFP on its lag (see, e.g., Fajgelbaum et al., 2017; Antonova and Matvieiev, 2025).[4]

Table 1: Calibrated parameters

Parameter	Description	Value	Source/Target
β	Discount factor	0.99	Annual interest rate, 4%
η	Degree of risk aversion	3	Standard value
ω^{-1}	Frisch elasticity of labor supply	1	Standard value
ϵ	Elast. of substitution btw goods	8	Price markup, 14.5%
α	Returns-to-scale complement	0.3	Labor share in output, 60%
Φ	Price adjustment cost parameter	100	Frequency of price change, 1 year
ϕ_π	Interest rate rule parameter	1.5	Standard value
g/y	Government spending share of GDP	0.18	BEA, 18%
ρ_a	Persistence of productivity	0.97	Correlation of TFP with its lag

4.1.2 Forecast errors construction

To calibrate the imperfect-information block in our model (parameters γ and γ^z), we exploit variation in households' forecast errors over time. We construct the aggregate income growth forecast errors by aggregating individual forecast errors computed from the Michigan Survey of Consumers (MSC). The MSC is a rotating monthly panel survey in which respondents are eligible to be re-interviewed six months after the initial interview.[5] In each interview, respondents are asked to report their current household income (in dollars) as well as their expected income growth over the next 12 months.[6]

For each month, we consider only respondents who have been re-interviewed at least once. Let $\mathbb{E}_{t-12}[\Delta inc_{j,t}]$ denote the expected income growth reported by the j-th respondent during her first interview, referring to the income growth she expects to receive over the upcoming year. Using the data from two subsequent interviews, we also calculate the realized income growth

[4]We calibrate ρ_a rather than estimate it from forecast-error dynamics because, in our setting, belief dynamics depend only on composite objects that combine ρ_a with signal precision (γ, γ^z), making these parameters separately unidentified.

[5]The Michigan Survey of Consumers employs a rotating sample design: each month combines a newly drawn cross-sectional sample with follow-up interviews of prior respondents, typically conducted about six and twelve months after the initial interview (see Surveys of Consumers Technical Report, 2024).

[6]The current income corresponds to 'INCOME: total household income - current dollars' (the asked question is "*Now, thinking about your total income from all sources (including your job), how much did you receive in the previous year?*") and the expected income is 'FAMILY INCOME % u/d next year' (the asked question is "*By about what percent do you expect your income to (increase/decrease) during the next 12 months?*")

of household j over the same year, $\Delta inc_{j,t}$. For each month, we compute the individual forecast errors as the difference between expected and realized income growth: $e^{j}_{t,t-12} = \mathbb{E}_{t-12}[\Delta inc_{j,t}] - \Delta inc_{j,t}$.

We then aggregate errors across individuals to obtain a monthly series of the aggregate income growth forecast errors, $e^{f}_{t,t-12}$, calculated as $e^{f}_{t,t-12} = \Sigma^{J}_{j=1}\omega_{j,t}e^{j}_{t,t-12}$; here $\omega_{j,t}$ is the weight assigned to the j-th respondent's forecast error at time t ($\Sigma^{J}_{j=1}\omega_{j,t} = 1$) and provided by the MSC, where J is the number of respondents. Since income growth has both aggregate and idiosyncratic components, aggregating across households averages out idiosyncratic dispersion, leaving only the aggregate component and its associated forecast error. The resulting series spans the period 1981M1–2019M12. Appendix D provides details on the aggregation procedure and plots the distribution of the aggregate income forecast error.

4.1.3 Estimation

Next, we outline our strategy for calibrating the remaining parameters γ, γ^z, and σ_a, which govern the evolution of households' beliefs. To identify the parameters related to signal precision (γ^z and γ), we draw on the approach of Fajgelbaum et al. (2017), using selected moments of the forecast error distribution. In particular, we target the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentiles of the aggregate forecast error distribution of income growth over time. The idea is that the 5$^{\text{th}}$ percentile corresponds to periods of recession, when the precision of the cyclical signal is low and uncertainty is largely driven by the time-invariant signal z, which helps identify γ^z. In contrast, the 95$^{\text{th}}$ percentile corresponds to periods of expansion, when output increases and the cyclical signal s_t receives greater weight in the posterior belief, which is informative for pinning down γ. We pin down the standard deviation σ_a by matching the empirical unconditional volatility of real private consumption in US data over the period 1981Q1–2019Q4.[7]

We minimize the weighted squared distance between a vector of three empirical moments, $m^{\text{data}} \in \mathbb{R}^3$, and the corresponding model-simulated moments, $m^{\text{sim}}(\psi) \in \mathbb{R}^3$. Our simulated method of moments estimator is defined as

$$\hat{\psi} = \arg\min_{\psi} \left(m^{\text{data}} - m^{\text{sim}}(\psi)\right)' W \left(m^{\text{data}} - m^{\text{sim}}(\psi)\right),$$

where $\psi = [\gamma \; \gamma_z \; \sigma_a]'$ is the parameter vector and W is a symmetric, positive semi-definite 3×3 weighting matrix constructed from the variance-covariance matrix of the empirical moments. The standard errors are constructed using stationary block bootstraps Politis and Romano (1994), which allows resampling without destroying the time dependence of the data. We use 1,000 bootstrap replications.

[7] The series is expressed in logs and one-sided HP-filtered before computing the standard deviation. The same transformation is applied to the model-generated data during the moment-matching procedure.

Table 2: Model's parameters and matched moments

Panel A: Parameters Value		
Parameter	Description	Value
γ	Sensitivity of signal s_t precision wrt output	38.10 (3.63)
γ^z	Precision of signal z_t	5.27 (1.67)
σ_a	TFP level shock: s.d	0.018 (0.002)

Panel B: Matched Moments		
	Data, %	Model, %
Forecast error, 5^{th} percentile	−6.91	−6.87
Forecast error, 95^{th} percentile	6.11	6.14
Real consumption, s.d	1.04	1.04

Note: The values under brackets are the bootstrapped standard errors of the estimates.

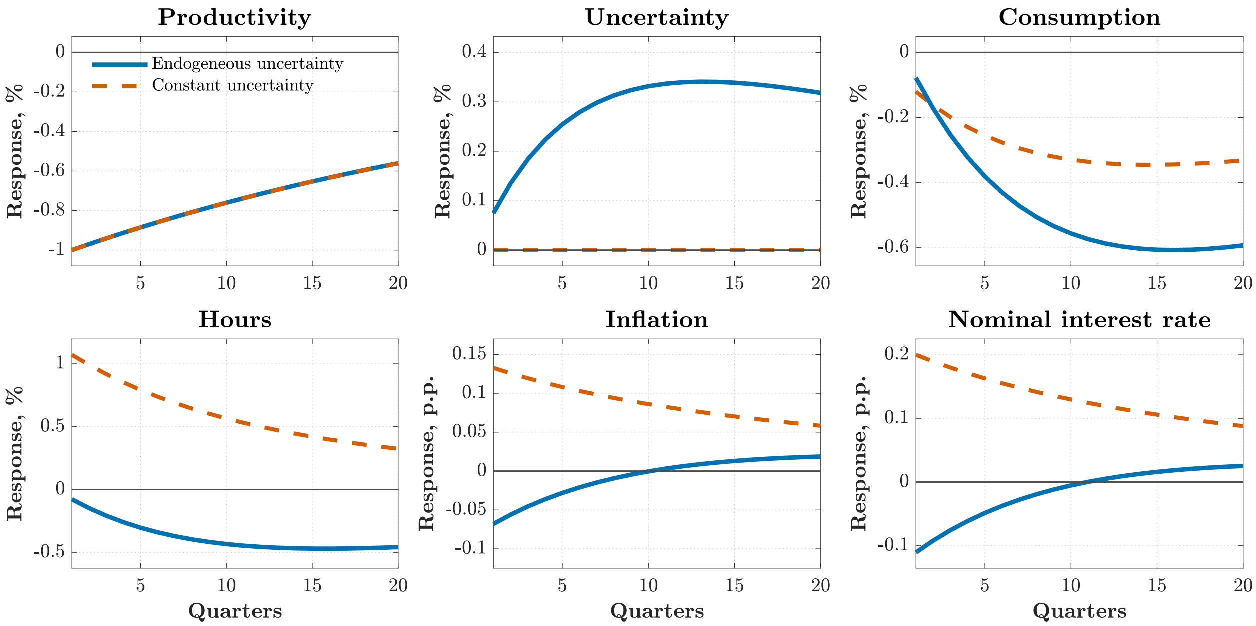
Table 2 presents the estimated parameter values along with the empirical and model moments. The model successfully reproduces the magnitude of the 5^{th} and 95^{th} percentiles of household forecast income errors as well as consumption volatility. The parameter governing the sensitivity of the cyclical signal to output, $\gamma = 38.1$, is positive and statistically significant, indicating that a procyclical signal precision in our model aligns with the household expectations data. The estimated precision of the time-invariant signal, $\gamma^z = 5.27$, is also significant, suggesting that a sizable share of informational imperfections comes from factors unrelated to cyclical economic conditions. Finally, the parameter σ_a, which shapes dynamics through both the speed of information updating and the volatility of the fundamental, is estimated at 0.018, consistent with empirical TFP volatility.

To interpret these estimation results, note that the signal-to-noise ratio of z_t is $\sigma_a/(\gamma^z)^{-\frac{1}{2}} = 0.041$. The signal-to-noise ratio of s_t depends on output level; evaluated at steady state output it equals $\sigma_a/(\gamma)^{-\frac{1}{2}} = 0.11$. These values are broadly in line with, though somewhat lower than, those reported by Melosi (2014). While our estimates pertain to household information, Melosi (2014) studies a dispersed-information model in which firms are imperfectly informed and finds signal-to-noise ratios of 0.09 for monetary policy shocks and 0.6 for technology shocks.

4.2 Impulse response analysis

We assess the role of endogenous uncertainty in shock propagation by comparing impulse responses to a negative productivity shock in the baseline model with those from a counterfactual economy in which the endogenous-uncertainty channel is shut down. In the counterfactual, information remains imperfect but uncertainty is fixed at its long-run baseline level. Addition-

Figure 2: Impulse response to a negative productivity shock



Note: Blue lines: baseline with endogenous uncertainty. Orange dashed lines: counterfactual with uncertainty fixed at its long-run level. Uncertainty is the inverse of precision, γ_t.

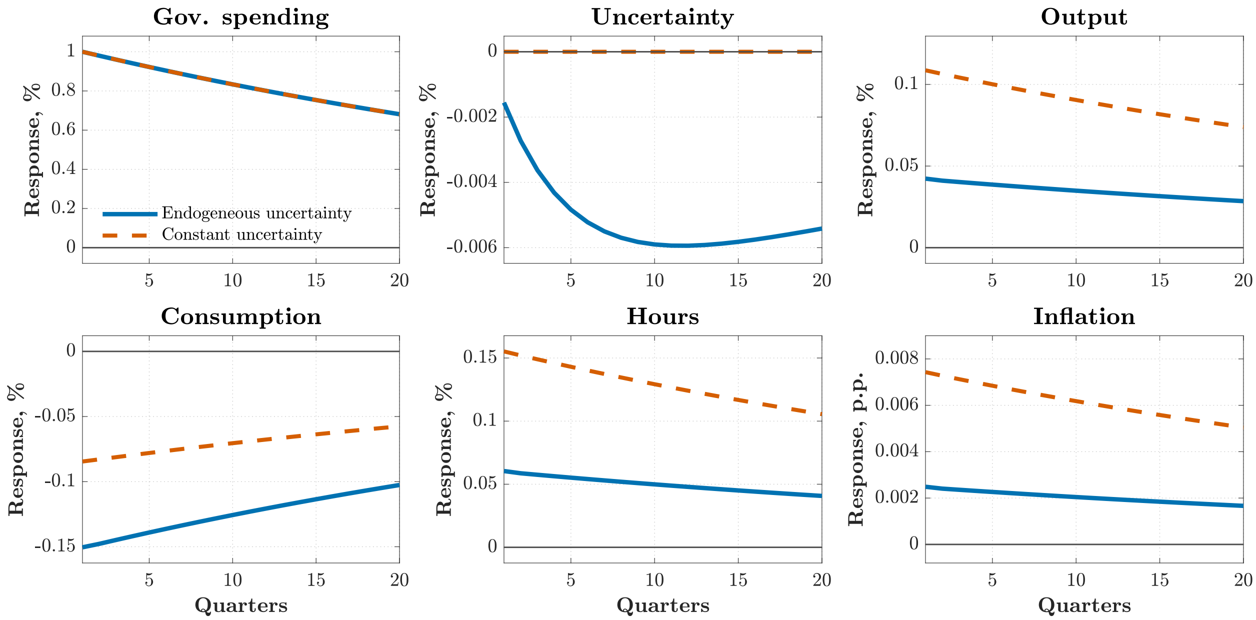
ally, we examine how endogenous uncertainty affects the economy's response to an increase in government spending and assess the effectiveness of monetary policy in offsetting the effects of endogenous uncertainty. Finally, we discuss the model features that distinguish our framework from models with exogenous uncertainty.

4.2.1 Negative productivity shock

Figure 2 reports impulse responses to a negative productivity shock in our baseline model, where the endogenous-uncertainty channel is active (blue lines), and in a counterfactual economy in which the endogenous-uncertainty channel is shut down (red lines with markers). In the baseline model, a negative productivity shock raises uncertainty, reinforcing the precautionary saving motive and thereby depressing aggregate demand and prices. As a result, the economy experiences deflation over the first several quarters and a sizable contraction in consumption, reaching 0.6% at its trough. Weak demand puts downward pressure on nominal interest rates and generates slack in the labor market, reflected in falling employment. As a result, consumption, employment, and prices end up co-moving, and our productivity shock acquires Keynesian features, namely a supply shock that is observationally similar to a demand shock.

In the counterfactual scenario with constant uncertainty, the impulse responses are markedly different. A decline in productivity generates inflation, which is in line with the traditional view that supply shocks cause output and prices to move in opposite directions. Moreover,

Figure 3: Impulse response to a fiscal expansion



Note: Blue lines: baseline with endogenous uncertainty. Orange dashed lines: counterfactual with uncertainty fixed at its long-run level. Uncertainty is the inverse of precision, γ_t.

employment now respond positively, and consumption falls by only 0.3% – about half the decline in the baseline model. This indicates that the endogenous uncertainty generates strong demand effects, making it a powerful demand-side amplification channel in the model.

4.2.2 Increase in public spending

We now turn to the quantitative impact of the endogenous-uncertainty channel on the effects of fiscal expansion. We model an increase in public spending as an initial 1% rise followed by an AR(1) path with persistence $\rho_g = 0.98$.[8] We choose high persistence in line with the evidence that the government spending shocks are found to be very persistent, notably in the context of defense spending Galí et al. (2007); Fisher and Peters (2010); Ramey (2011).

Figure 3 reports the results. A persistent increase in government spending raises output and inflation while crowding out private consumption. Endogenous uncertainty amplifies this crowding-out effect: private consumption falls substantially more in the model with endogenous uncertainty than in the constant-uncertainty counterfactual. As a result, the output and inflation responses are weaker under endogenous uncertainty. The consumption response is consistent with our theoretical results in Section 3: although uncertainty declines following the fiscal expansion, the large persistence of the fiscal shock causes the crowding-out channel to dominate

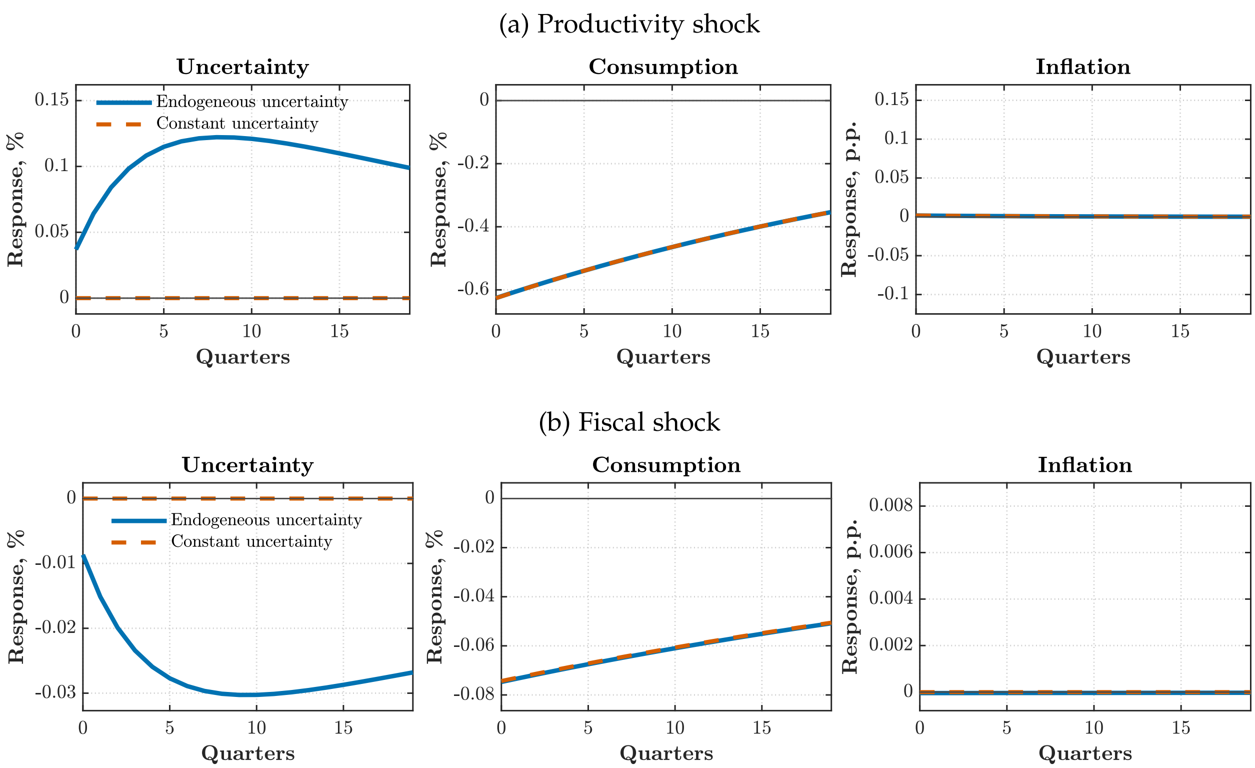
[8]The increase in government spending is a fully unexpected, zero-probability ("MIT") shock, so households do not assign any ex ante probability to a government spending innovation when forming beliefs.

the crowding-in channel. Section 4.4 discusses the persistence for which crowding in dominates and private consumption rises in response to fiscal expansion.

4.3 Implication for Monetary Policy

Section 3, we have shown that the endogenous-uncertainty channel has no impact on flexible-price allocation, since cyclical fluctuations in desired saving affect only aggregate demand. This implies that, in the absence of cost-push disturbances, monetary policy can simultaneously stabilize prices and close the output gap; that is, "divine coincidence" (Blanchard and Gali, 2007) holds, as in the canonical New Keynesian model.

Figure 4: Impulse responses under strong monetary policy



Note: The figure reports the impulse responses under a strict inflation targeting rule ($\phi_\pi = 1000$). Blue lines denote the baseline with endogenous uncertainty. Orange lines denote the counterfactual with uncertainty fixed at its long-run level. Uncertainty is the inverse of precision, γ_t.

We now assess the role of monetary policy in stabilizing aggregate volatility in the quantitative version of our model. We conduct a quantitative experiment in which the central bank follows a strict inflation-targeting rule. Specifically, the policy rate responds aggressively to deviations of inflation from its target, with a large Taylor-rule coefficient; we set $\rho_\pi = 1000$ in contrast to the baseline value of 1.5. Figure 4 compares the response to shocks when uncertainty is allowed to evolve endogenously with a counterfactual scenario in which uncertainty is held constant.

Following a negative productivity shock, the precision of information declines, which increases incentives to save in order to insure against uncertain future aggregate states. This increase in desired saving can potentially set off a self-reinforcing precautionary spiral, as in our baseline model. The central bank, however, now almost fully stabilizes inflation and therefore reacts aggressively to aggregate demand. In this way, monetary policy neutralizes inefficient precautionary demand and breaks the feedback loop between uncertainty and economic activity. In the case of constant uncertainty, monetary policy also eliminates cyclical fluctuations in aggregate demand, which in this case arise solely from intertemporal substitution. In both models, monetary policy effectively restores the flexible-price equilibrium, rendering differences in consumption and inflation responses between the baseline and counterfactual negligible, as shown in Panel (a) of Figure 4.

In the same way, strong monetary policy eliminates the aggregate demand effects of fiscal expansion. The Panel (b) of Figure 4 shows that, although expansionary government spending can induce an endogenous decline in uncertainty, this decline has little to no effect on real variables when the central bank fully stabilizes inflation.

4.4 Discussion

Now we discuss the role of persistence of shocks for the quantitative results. Then we relate our results to the frameworks in which uncertainty fluctuations are exogenous.

Persistence and demand effects. Our theoretical analysis in Section 3 shows that the intensity of the endogenous uncertainty channel largely depends on the persistence of the underlying economic shocks. In the context of productivity shocks, we have shown in Proposition 1 that the responses of aggregate demand and inflation can be expressed as a convex combination of the standard New Keynesian channel and the endogenous uncertainty channel, with persistence acting as the weight between the two forces. Figure F.1 in Appendix F illustrates this result within the quantitative model: when productivity persistence is lower, the economy exhibits substantially weaker deflationary pressures due to reduced demand-side amplification.

In the case of government spending shocks, we have shown in Proposition 2 that its persistence governs the relative strength of crowding out versus crowding in in private consumption: when persistence is low crowding-in dominates. In practice, however, for crowding-in to dominate, the shock should be highly transitory: Figure F.2 in Appendix F demonstrates this result in the quantitative model.

Exogenous vs endogenous uncertainty. We now discuss whether the endogenous-uncertainty amplification mechanism can be mimicked within an exogenous-uncertainty model through a combination of a level shock and an exogenous uncertainty shock. To this end, in Appendix F we conduct several quantitative exercises to assess whether a negative productivity shock accompanied by a positive uncertainty shock can replicate the effects generated by our model.

We first consider a standard full-information New Keynesian model (which corresponds to our model under $\theta_t = a_t$; $\gamma_t = \infty$) and assume that the productivity process features stochastic volatility, following the specification used by Born and Pfeifer (2014); Basu and Bundick (2017), among others. We find that, even when the size of stochastic volatility shock of substantially larger magnitude than those typically found in the literature, the combination of productivity and stochastic volatility shock is not able to generate a sizable amplification over a sustained horizon. We also consider a constant-uncertainty version of our model, perturbed simultaneously by a productivity shock and an exogenous shock to information precision γ_t. Again, we find that even very large increases in uncertainty generate only a modest difference compared to the constant-uncertainty model.

This highlights a key distinction between environments in which uncertainty and economic activity evolve exogenously and those in which they are endogenously intertwined. In the latter case, feedback mechanisms can generate destabilizing spirals and strong amplification, whereas such mechanisms are absent in the former.

5 Conclusion

Uncertainty accompanies many recessions. In this paper, we develop an imperfect-information New Keynesian model with procyclical information precision, which generates endogenous countercyclical uncertainty.

We show theoretically that procyclical information precision creates a feedback loop between economic activity and uncertainty operating through precautionary saving. This mechanism fundamentally alters the transmission of economic shocks to aggregate demand. In particular, negative supply shocks that generate excess demand in a standard New Keynesian model instead depress aggregate demand when uncertainty responds endogenously. We further show that monetary policy can neutralize this feedback loop by closing the output gap.

Quantitatively, we document that this endogenous-uncertainty channel is economically significant for the US economy. The feedback between information and activity generates strong demand-driven effects and is sufficiently powerful to reverse the sign of the aggregate-demand response to productivity shocks.

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Supply Shocks in the Fog
Appendix

Anastasiia Antonova
Mykhailo Matvieiev
Céline Poilly

A Model equations

Household:

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \frac{R_t}{\pi_{t+1}} \right\}, \tag{A.1}$$

$$W_t = L_t^{\omega} C_t^{\eta}. \tag{A.2}$$

Firm:

$$MC_t = \frac{1}{1-\alpha} \frac{W_t}{\tilde{A}_t} L_t^{\alpha}, \tag{A.3}$$

$$Y_t = \tilde{A}_t L_t^{\alpha}. \tag{A.4}$$

Price-setting:

$$\epsilon \left(1 - MC_t\right) = 1 - \Phi \left(\pi_t - 1\right) \pi_t + \Phi \mathrm{E}_t \left\{ Q_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} \right\}, \tag{A.5}$$

where the discount factor is $Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \frac{1}{\pi_{t+1}}$.

Market clearing:

$$Y_t = C_t + G_t + \frac{\Phi}{2} \left(\pi_t - 1\right)^2 Y_t. \tag{A.6}$$

Monetary policy:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y}. \tag{A.7}$$

Evolution of beliefs about a_t:

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a \frac{\gamma_t \theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{A.8}$$

$$\gamma_{t+1}^{-1} = \frac{\rho_a^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2. \tag{A.9}$$

B Procyclical precision through learning

As a first possibility, consider a standard "learning-by-doing" assumption that each unit of production generates information. A noisy productivity signal for j-th unit of good produced

$$s_t(j) = a_t + \epsilon_t^s(j), \;\; \epsilon_t^s(j) \sim \mathcal{N}(0, \; \gamma^{-1})$$

The total amount of goods produced is $\sum j = Y_t$. Then the average of these signals generated the overall noisy signal of precision equal the sum of precisions of the underlying signals:

$$s_t = \frac{1}{Y_t} \sum s_t(j) = a_t + \epsilon_t^s, \;\; \epsilon_t^s \sim \mathcal{N}(0, \; [\gamma \cdot Y_t]^{-1}) \tag{B.1}$$

which yields pro-cyclical precision of information flow from signal s_t.

The second possibility is to consider a combination of learning by doing and social learning, see Foster and Rosenzweig (1995). Let us assume that each worker i (out of total employment L_t) gets a noisy signal about productivity for each unit j produced (learning by doing)

$$s_t(i,j) = a_t + \epsilon_t^s(i,j), \; \epsilon_t^s(i,j) \sim \mathcal{N}(0, \; \gamma^{-1}) \tag{B.2}$$

Each worker produces $y_t = \frac{Y_t}{L_t}$ goods, hence has an overall signal about productivity computed as the average of her learning by doing signals

$$s_t(i) = a_t + \epsilon_t^s(i), \; \epsilon_t^s(i) \sim \mathcal{N}(0, \; (\gamma \cdot y_t)^{-1}) \tag{B.3}$$

Finally, workers meet and exchange their information about productivity (social learning). The overall signal is the average of worker-specific signals and has precision $\gamma \cdot y_t \cdot L_t = \gamma_t \cdot Y_t$

$$s_t = a_t + \epsilon_t^s, \;\; \epsilon_t^s \sim \mathcal{N}(0, \; [\gamma \cdot Y_t]^{-1}) \tag{B.4}$$

C Risk-adjusted linear model

Market clearing. Reduced form price rigidity only in the present period: $MC_t \neq P_t$ but $MC_{t+j} = P_{t+j}$ for all $j > 0$. Combining (A.2), (A.3), and (A.6) and log-linearizing, we obtain

$$y_t = \left(\frac{1}{\omega + \eta}\right) mc_t + \left(\frac{1+\omega}{\omega + \eta}\right) \tilde{a}_t + \left(\frac{\eta}{\omega + \eta}\right) g_t \tag{C.1}$$

$$y_{t+j} = \left(\frac{1+\omega}{\omega + \eta}\right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega + \eta}\right) g_{t+j}, \quad j > 0 \tag{C.2}$$

IS equation. Assume that the household saves only in real bonds. We perform a risk-adjusted log-linearization of a corresponding Euler equation A.1. There are two stages to this linearization: first, derive the risk premium (in the spirit of Skinner (1988)) and then do a standard log-linearization. Our target is to derive the Euler equation in the form of a certainty-equivalent Euler equation adjusted for risk premium. We define certainty-equivalent consumption as consumption that households would choose if future income was certain and equal to its expected value. First, consider a nonlinear Euler equation:

$$u'(C_t) = \beta(1 + r_t)E_t u'(C_{t+1})$$

Consider the point $C^e_{t+1} = E_t C_{t+1}$. To derive the risk-premium arising due to the consumption uncertainty (a la Skinner) we take the 2nd order Taylor expansion for RHS around C^e_{t+1}

$$u'(C_t) = \beta(1 + r_t)E_t\left\{u'(C^e_{t+1}) + u''(C^e_{t+1}) \cdot (C_{t+1} - C^e_{t+1}) + \frac{1}{2}u'''(C^e_{t+1}) \cdot (C_{t+1} - C^e_{t+1})^2\right\}$$

Taking expectations form the RHS:

$$u'(C_t) = \beta(1 + r_t)\left(u'(C^e_{t+1}) + \frac{1}{2}u'''(C^e_{t+1}) \cdot E_t(C_{t+1} - C^e_{t+1})^2\right)$$

We factor out the certainty-equivalent part:

$$u'(C_t) = \beta(1 + r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})} \cdot E_t(C_{t+1} - C^e_{t+1})^2\right)$$

Multiplying and dividing by $(C^e_{t+1})^2$ we obtain the expression in terms of the relative risk aversion:

$$u'(C_t) = \beta(1 + r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2 \cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2\right)$$

This equation can be rewritten as:

$$u'(C_t) = \beta(1 + r_t)(1 + \psi_t)u'(E_t C_{t+1}) \tag{C.3}$$

where $\psi_t = \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2 \cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2$ is time-varying risk premium arising from uncertainty about future consumption.

We have CRRA utility $u(C) = \frac{C^{1-\eta}}{1-\eta}$. From now on, for the Euler equation and all equations that follow (including the low of motion of uncertainty), we work with *linear* approximations. Taking the logs from Equation C.3 we get

$$-\eta log(C_t) = log\beta + log(1 + r_t) + log(1 + \psi_t) - \eta log(E_t c_{t+1}) \tag{C.4}$$

Next we denote $c_t = log(C_t)$. Using the fact that $log(1 + x_t) \approx x_t$ and $logE_tX_{t+1} = E_tlog(X_t)$ to the first order, we get

$$c_t = -\frac{1}{\eta}\log(\beta) + E_tc_{t+1} - \frac{1}{\eta}r_t - \frac{1}{\eta}\psi_t$$

With CRRA utility we have

$$\psi_t = \frac{1}{2}\eta(1+\eta) \cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2$$

Noting that $\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}} \approx c_{t+1} - Ec_{t+1}$ and $E_t(c_{t+1} - E_tc_{t+1})^2 = Var_tc_{t+1}$, and substituting for the log-linear resource constraint around the steady state with $\bar{G} = 0$ such that $y_t = c_t + g_t$ (where $g_t = \frac{G_t}{\bar{Y}}$) we obtain:

$$y_t - g_t = E_ty_{t+1} - \rho_g g_t - \frac{1}{\eta}(r_t - \rho) - \frac{1}{2}(1+\eta)Var_ty_{t+1} \tag{C.5}$$

where $\rho = -log(\beta)$.

Evolution of beliefs. We linearize beliefs (A.8), (A.9) around a stationary point $\gamma_t = \bar{\gamma}$, $\gamma^s_t = \bar{\gamma}^s$, $\theta_t = s_t = \tilde{a}_t = \bar{a}$. We obtain

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a\bar{a} + \rho_a\frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\theta_t - \bar{a}) + \rho_a\bar{a} + \rho_a\frac{\gamma^z}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\tilde{a}_t - \bar{a}) + \rho_a\frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(s_t - \bar{a}) +$$
$$+ \left(\frac{\bar{\theta}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}\right)(\gamma_t - \bar{\gamma}) + \left(\frac{\bar{s}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}\right)(\gamma^s_t - \bar{\gamma}^s) =$$
$$= (1-\rho_a)\bar{a} + \rho_a\frac{\bar{\gamma}\theta_t + \gamma^z\tilde{a}_t + \bar{\gamma}^s s_t}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}$$

Now, let us denote the prior information and productivity observation as a joint signal $z_t = \frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z}\theta_t + \frac{\gamma^z}{\bar{\gamma}+\gamma^z}\tilde{a}_t$ of precision $\bar{\gamma}+\gamma^z$. Also, let us denote $v = \frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}$. Then, we can rewrite the next period belief as:

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t] \tag{C.6}$$

Assuming that $\gamma_t = \bar{\gamma}$, using the fact that $\gamma^s_t = \gamma\frac{Y_t}{\bar{Y}}$, and the definition of v we obtain

$$\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} + \sigma_a^2 - \frac{\rho_a^2}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}(\gamma_t - \bar{\gamma} + \gamma^s_t - \bar{\gamma}^s) =$$
$$= \rho_a^2\left(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}\frac{\gamma^s_t - \bar{\gamma}^s}{\bar{\gamma}^s}\right) + \sigma_a^2 = \rho_a^2\left(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}(y_t - \bar{y})\right) + \sigma_a^2$$

which equals

$$\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\gamma}v(1 - v(y_t - \bar{y})) + \sigma_a^2 \tag{C.7}$$

Finally, we are interested in $E_t\tilde{a}_{t+1}$ and $Var\tilde{a}_{t+1}$, which are obtained from the updated beliefs about a_{t+1} as

$$E_t(\tilde{a}_{t+1}) = E_t a_{t+1} = (1-\rho_a)\bar{a} + \rho_a[v s_t + (1-v)z_t] \tag{C.8}$$

$$Var(\tilde{a}_{t+1}) = \gamma_{t+1}^{-1} + \sigma_f^2 = \sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma} v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma} v^2 \cdot y_t \tag{C.9}$$

Output gap response to productivity shock. Combining IS equation (C.5) with (C.2) and beliefs (C.8), (C.9) we obtain

$$\left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right] \cdot y_t = -log(\beta) - \frac{1}{\eta} \cdot r_t + (1-\rho_g+\rho_g\frac{\eta}{\omega+\eta})g_t +$$
$$+ \frac{1+\omega}{\omega+\eta} \cdot ((1-\rho_a)\bar{a} + \rho_a[v s_t + (1-v)z_t]) - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}(\sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}))$$

Taking the log deviations from the steady state, we get

$$\left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right] \cdot \hat{y}_t = -\frac{1}{\eta}\hat{r}_t + \frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t + (1-v)\hat{z}_t] + (1-\rho_g \cdot \frac{\omega}{\omega+\eta})\hat{g}_t$$

Let us denote $f = \left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right]^{-1}$, which gives

$$\hat{y}_t = -\frac{f}{\eta}\hat{r}_t + f\frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t + (1-v)\hat{z}_t] + f(1-\rho_g \cdot \frac{\omega}{\omega+\eta})\hat{g}_t \tag{C.10}$$

From C.1 we have the natural output (in log-deviation from the steady state) equal $\hat{y}_t^n = \frac{1+\omega}{\omega+\eta}\hat{a}_t + \frac{\eta}{\omega+\eta}\hat{g}_t$. Let the output gap be denoted as $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$. Let the monetary policy response be such that $r_t = \phi \cdot \tilde{y}_t$. Then, from (C.10) we can write

$$\left[1 + \frac{\phi}{\eta} \cdot f\right] \cdot \tilde{y}_t = \frac{1+\omega}{\omega+\eta} \cdot [-\hat{a}_t + \rho_a f(v\hat{s}_t + (1-v)\hat{z}_t)] + f(1-\rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\eta}{\omega+\eta})\hat{g}_t \tag{C.11}$$

Now, consider the change in productivity Δa_t. Change in productivity is reflected in the corresponding change in $\Delta\hat{a}_t = \Delta a_t$, $\Delta\hat{s}_t = \Delta a_t$, and $\Delta\hat{z}_t = \frac{1}{1+\bar{\gamma}\sigma_f^2}\Delta a_t$ (since z_t is the combination of prior belief and observation of the current productivity); to simplify further, we assume that $\bar{\gamma} = 0$, that is, no prior information is available about the productivity. Then the response of the output gap to productivity shock is computed from

$$\frac{\omega+\eta}{1+\omega}\left[1 + \frac{\phi}{\eta} \cdot f\right]\Delta\tilde{y}_t = [\rho_a f - 1] \cdot \Delta a_t = -(1-\rho_a) \cdot \Delta a_t + \rho_a(f-1) \cdot \Delta a_t \tag{C.12}$$

It is clear that $f \geq 1$ always, as is the effect of endogenous uncertainty channel. Endogenous uncertainty channel is absent when: 1) no learning from economic activity $\gamma = 0$, then we have $v = 0$ and $f = 1$, or 2) full information model $\gamma \to \infty$ (or $\sigma_f^2 = 0$), with $v = 1$ and $f = 1$.

Crowding in private consumption. Now consider an increase in government spending Δg_t. The response of private consumption is $\Delta c_t = \Delta y_t - \Delta g_t$. The response of output to government spending shock is given from C.10 as $\Delta y_t = -\frac{1}{\eta} f \Delta r_t + f(1 - \rho_g \cdot \frac{\omega}{\omega+\eta})\Delta g_t$ and the response of interest rate is given from C.11 as $\Delta r_t = \phi \Delta \tilde{y}_t = \frac{\phi f (1-\rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\eta}{\omega+\eta})}{1+\frac{\phi}{\eta} f}$. Then, the consumption response to government spending shock is

$$\Delta c_t = \left[-1 + \frac{(1 - \rho_g \cdot \frac{\omega}{\omega+\eta} + \frac{\phi}{\omega+\eta}) f}{1 + \frac{\phi f}{\eta}} \right] \Delta g_t$$

The response of consumption depends on the persistence of government spending. Rearranging the terms we get

$$\left[1 + \frac{\phi f}{\eta}\right] \Delta c_t = (f-1)(1 - \rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\phi}{\eta} \frac{\omega}{\omega+\eta}) \Delta g_t - (\rho_g \cdot \frac{\omega}{\omega+\eta} + \frac{\phi}{\eta} \frac{\omega}{\omega+\eta}) \Delta g_t \quad \text{(C.13)}$$

The first term is the crowding in effect from the endogenous uncertainty channel ($f > 1$). The second effect is the grounding out effect from the traditional government spending channel. When government spending is persistent (large ρ_g), the standard crowding out effect is strong and the endogenous uncertainty crowding in is weak.

D Aggregating individual forecast errors

We derive the recursive process for aggregate forecast error in the Bayesian learning framework with imperfect information and procyclical precision. Let aggregate income follow the AR1 process:

$$inc_t = \rho \cdot inc_{t-1} + (1-\rho) \cdot \bar{inc} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

An individual household has income is:

$$inc_t^i = inc_t + v_t^i, \quad v_t^i \sim \mathcal{N}(0, (\gamma_t^v)^{-1})$$

v_t^i captures the individual component of income; γ_t^v is a precision parameter governing the dispersion of income distribution across individuals. The household does not observe inc_t directly and has a set of information Ω_t^i available at the beginning of period t. The corresponding prior belief about inc_t is

$$inc_t | \Omega_t^i \sim \mathcal{N}(\theta_t^i, (\gamma_t)^{-1})$$

where $E[inc_t|\Omega_t^i] = \theta_t^i$. During t household observes g_t^i and a noisy signal about inc_t

$$s_t^i = inc_t + u_t^i, \quad u_t^i \sim \mathcal{N}(0, (\gamma_t^u)^{-1})$$

Define expectation error as $e_{t-1}^i = \theta_t^i - inc_t^i$. Household updates information about inc_t so that: $E[inc_{t+1}|\Omega_{t+1}^i] = \rho E[inc_t|\Omega_{t+1}^i] + (1-\rho)\bar{inc}$ where the optimal combination of signals yields $E[inc_t|\Omega_{t+1}^i] = \frac{\gamma_t \theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u}$ and $\gamma_{t+1} = \left[\frac{\rho^2}{\gamma_t + \gamma_t^v + \gamma_t^u} + \sigma^2\right]^{-1}$. The individual expectation error is then:

$$e_t^i = \theta_{t+1}^i - inc_{t+1}^i = \rho \frac{\gamma_t \theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - \rho \cdot inc_t - v_{t+1}^i - \varepsilon_{t+1} =$$
$$= \rho \frac{\gamma_t e_t^i + (\gamma_t + \gamma_t^v) v_t^i + \gamma_t^u u_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - v_{t+1}^i - \varepsilon_{t+1}$$

Taking cross-sectional expectations (aggregating across individuals) and using that $E[v_t^i] = E[u_t^i] = E[E_{t-1}^i v_t^i] = 0$ because noise is idiosyncratic we obtain the average expectation error $e_t = E[e_t^i]$ as:

$$e_{t+1} = \rho \frac{\gamma_t}{\gamma_t + \tilde{\gamma}_t} \cdot e_t - \varepsilon_{t+1}$$

where $\tilde{\gamma}_t = \gamma_t^v + \gamma_t^u$ is the joint precision of two noisy signals. For a given prior precision γ_t, the persistence of forecast error is decreasing in signal precision $\tilde{\gamma}_t$. Finally, note that this aggregation relies on the fact that the precision of belief is the same across individuals, even though the mean belief is idiosyncratic.

E Numerical solution and simulation details

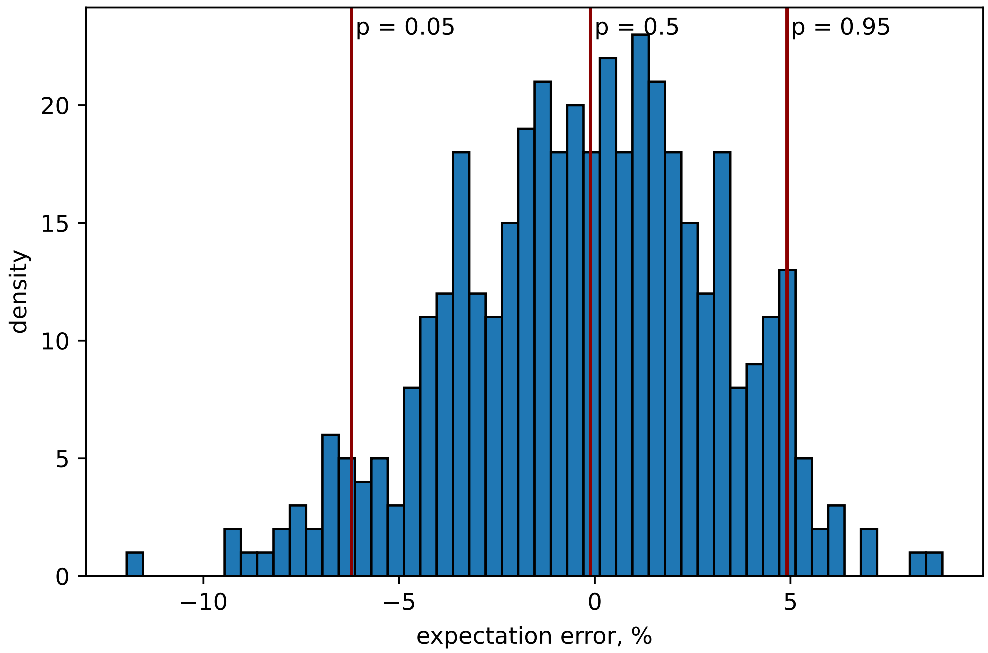
We solve the rational expectation model taking into account that agents have imperfect information about productivity. That is, beliefs θ_t and γ_t are the observable state variables but not the persistent productivity component a_t. As households compute their expectations based on these beliefs, they perceive deviation of signals from these beliefs as realizations of expectation errors. Given the prior information Ω_t, observing signals z_t and s_t consists of the expected part θ_t and the innovation part. Signal z_t is

$$z_t = \theta_t + \tilde{e}_t^z,$$

where $\tilde{e}_t^z = (a_t - \theta_t) + \epsilon_t^z$ is innovation, consisting of expectation error $a_t - \theta_t$ and realization of temporary productivity component $f_t = \epsilon_t^z$. Similarly, the public signal s_t is

$$s_t = \theta_t + \tilde{e}_t^s$$

Figure D.1: Distribution over time of the average income forecast errors



Note: The figure displays the distribution of the average forecast error, $e^f_{t,t-12}$, computed from the Michigan Survey of Consumers.

where $\tilde{e}^s_t = (a_t - \theta_t) + \epsilon^s_t$ is a sum of expectation error $(a_t - \theta_t)$ and a realization if signal noise ϵ^s_t. Hence, treating errors $\tilde{e}^z_t$ and $\tilde{e}^s_t$ as exogenous disturbances (which they are from the point of view of a household) account for the imperfect information of the household in the dynamic RE model.

Let e^a_t denote the expectation error between realized productivity component, a_t, and the expected one, $\mathrm{E}(a_t|\Omega_t)$, such that $e^a_t \equiv a_t - \theta_t$. Notice that expectation error is drawn from the distribution $e^a_t \sim \mathcal{N}(0, \gamma_t^{-1})$. We can therefore rewrite the observed signals in terms of the expected element θ_t and standard normal innovations scaled by the corresponding (time-varying) standard deviations

$$z_t = \theta_t + e^a_t + \epsilon^z_t = \theta_t + [\gamma_t]^{-1/2}\epsilon^e_t + [\gamma^z]^{-1/2}\epsilon^z_t \tag{E.1}$$

$$s_t = \theta_t + e^a_t + \epsilon^s_t = \theta_t + \gamma_t^{-1/2}\epsilon^e_t + [\gamma^s_t]^{-1/2}\epsilon^s_t \tag{E.2}$$

where ϵ^e_t, ϵ^z_t and ϵ^s_t are i.i.d. drawn from a standard normal distribution. We solve the model consisting of equations (A.1) - (A.9) and (E.1) - (E.2) using third-order perturbation method, allowing us to capture the precautionary saving channel. Simulating the model response to shocks amounts to recursive construction of expectation error e^a_t and computing the model path conditional on its realization.

F Robustness Analysis

F.1 Response to shocks with lower persistence

Figure F.1: Impulse response to a negative productivity shock: $\rho_a = 0.85$

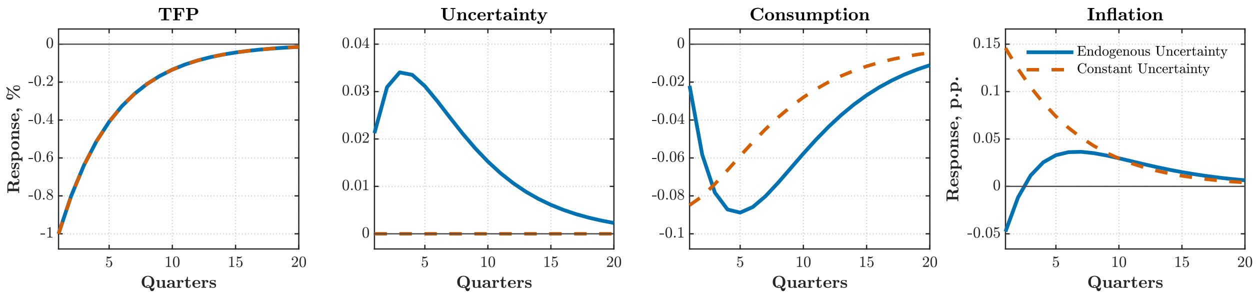
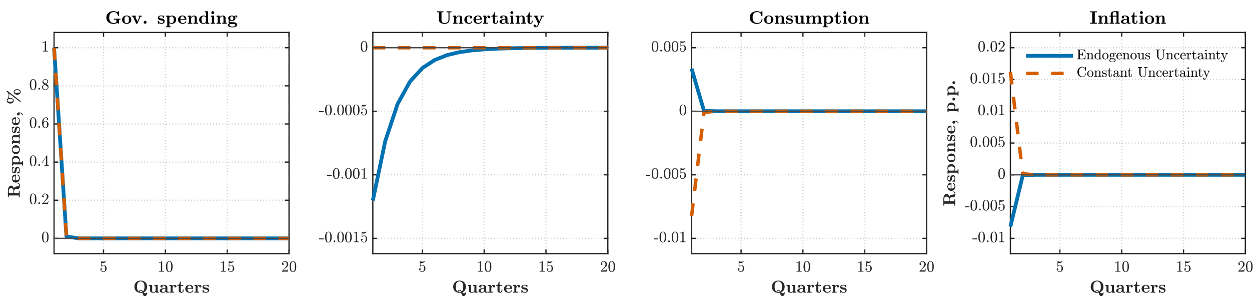


Figure F.2: Impulse response to a fiscal expansion: $\rho_g = 0.01$



F.2 Productivity shock and exogenous time-varying uncertainty

We consider a full-information New Keynesian model (which obtains from our model under $\theta_t = a_t$; $\gamma_t = \infty$) and assume that the productivity process features stochastic volatility (SV, henceforth). Following the specification used by Born and Pfeifer (2014); Basu and Bundick (2017) we model productivity as

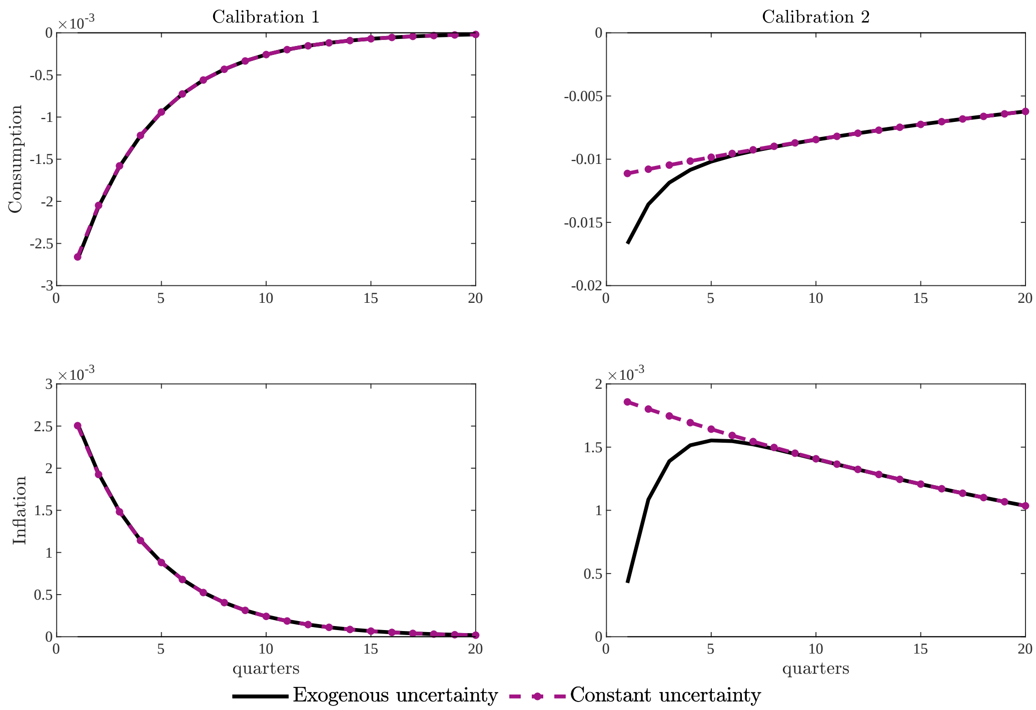
$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_{a,t}\epsilon_t^a, \tag{F.1}$$

$$\sigma_{a,t} = (1-\rho_{\sigma_a})\,\bar{\sigma}_a + \rho_{\sigma_a}\sigma_{a,t-1} + \left(1-\rho_{\sigma_a}^2\right)^{\frac{1}{2}}\eta_{\sigma_a}\epsilon_t^{\sigma_a}, \tag{F.2}$$

where $\epsilon_t^a \sim N(0,1)$ and $\epsilon_t^{\sigma_a} \sim N(0,1)$. Here, $\sigma_{a,t}$ is the time-varying volatility of productivity innovations. Parameter ρ_{σ_a} governs the persistence of the volatility (second-order) shocks, while and η_{σ_a} determines their magnitude. Compared to our endogenous-uncertainty model, any variation in uncertainty here arises entirely from changes in the fundamental volatility $\sigma_{a,t}$, rather than from cyclical informational imperfections. Hence, uncertainty in this model is exogenous and is fully driven by the process defined in Eq. (F.2).

We consider two sets of parameters for Equations (F.1)–(F.2). For the *first* set, we use the estimates of Born and Pfeifer (2021) for both the productivity and SV processes, setting $\bar{\sigma}_a = 0.007$, $\rho_a = 0.773$, $\eta_{\sigma_a} = 0.002$ and $\rho_{\sigma_a} = 0.517$. Under this parameterization, a one–standard deviation positive innovation to volatility increases the standard deviation of the TFP shock from 1.8 to 1.8040. This magnitude is broadly in line with the values reported by Fernández-Villaverde et al. (2015) for fiscal shocks.

Figure F.3: IRFs to a negative productivity shock with exogenous uncertainty



Note: The lines correspond to the response of the variables to a negative productivity shock driven by process (F.1)-(F.2) and under full information. The solid lines are when the stochastic volatility process is active ($\eta_{\sigma_a} > 0$). The dashed lines are when the stochastic volatility process is absent ($\eta_{\sigma_a} = 0$). The left panel corresponds to the responses when we calibrate the process following Born and Pfeifer (2021). The right panel corresponds to our own calibration. Consumption is expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

The left panels of Figure F.3 display the responses of consumption (top left) and inflation (bottom left) to a negative productivity shock under our first set of parameters. In each panel, the dashed line shows the IRFs to a pure negative productivity shock ($\epsilon_t^a = -1$, $\epsilon_t^{\sigma_a} = 0$), while the solid line shows the IRFs when the level shock is accompanied by a positive volatility shock ($\epsilon_t^a = -1$, $\epsilon_t^{\sigma_a} = 1$).[9] The adjustment of both consumption and inflation is driven almost entirely by the first-moment shock, while second-order effects play a negligible, though not strictly zero,

[9] In practice, we obtain the combined IRFs by adding the responses to the negative level shock and the positive volatility shock, abstracting from any covariance between the two shocks.

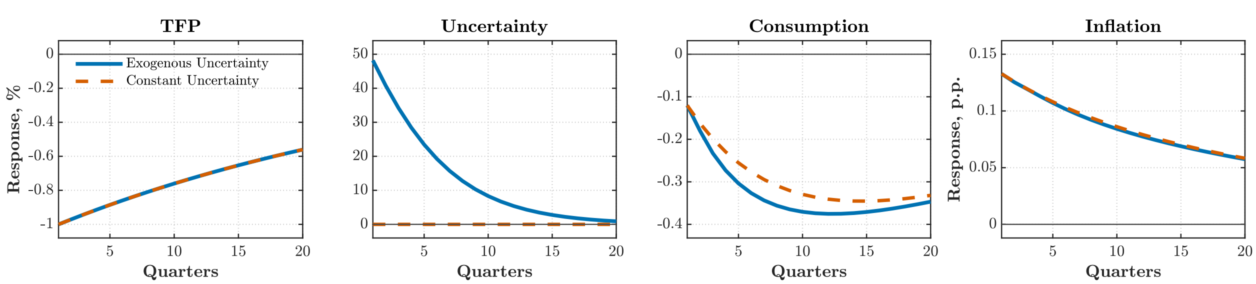
role.

In our *second* calibration, we set the parameters of the productivity process to the same values as in the baseline calibration, namely $\bar{\sigma} = 0.018$ and $\rho_a = 0.97$, while calibrating η_{σ_a} to generate an additional decline in consumption comparable in magnitude to that shown in Figure 2. In other words, we ask how large an exogenous volatility shock must be to generate the same amplification as that produced by the endogenous uncertainty channel. We, thus, set $\eta_{\sigma_a} = 0.3$. The persistence of the volatility process remains $\rho_{\sigma_a} = 0.517$. The top-right and bottom-right panels of Figure F.3 show the responses of consumption and inflation to the combination of the two shocks.

Two conclusions emerge. First, setting $\eta_{\sigma_a} = 0.3$ implies that a positive one–standard-deviation innovation to volatility raises the average standard deviation of the TFP innovation from 1.80 to 2.52. This indicates that an exceptionally large increase in uncertainty is required to generate effects of the same magnitude as those obtained in our imperfect-information model. Under this calibration, exogenous uncertainty does generate sizable demand-side effects: consumption falls more sharply following a negative productivity shock, and the inflation response is muted. However, these demand effects are not strong enough to reverse the sign of inflation, in contrast to the outcome under endogenous uncertainty (see Figure 2). Moreover, over the amplification from such shock does not hold over a sustained horizon.

We emphasize that our endogenous-uncertainty channel not only amplifies demand-driven fluctuations but also generates persistence through information frictions. While it is well established that imperfect information generates persistence in the transmission of shocks (see, for example, Melosi (2014)), we find that endogenous uncertainty further magnifies the long-lasting effects of shocks in a way that models with exogenous uncertainty cannot replicate.

Figure F.4: Impulse response to a negative productivity shock combined with exogenous shift in uncertainty



Note: The blue lines correspond to the response to a negative productivity shock accompanied by a negative shock to information precision which doubles uncertainty. The orange lines show the response in the counterfactual scenario where uncertainty is held constant.

We also consider an exercise where a negative productivity shock is accompanied by an exogenous uncertainty shock in our imperfect-information model in the form of an exogenous change in γ_t. Figure F.4 shows that even a substantial increase in uncertainty through an exogenous drop of beliefs precision leads to only a slightly smaller decline in consumption following a negative productivity shock compared to the constant-uncertainty case. This result confirms that the endogenous-uncertainty channel acts as a powerful amplification mechanism.