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Youming Liu
Banking and Payments Department
Bank of Canada
youmingliu@bank-banque-canada.ca

Francisco Rivadeneyra
Banking and Payments Department
Bank of Canada
frivadeneyra@bank-banque-canada.ca

Edona Reshidi
International Monetary Fund
ereshidi@imf.org

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Public vs. Private Payment Platforms: Market Impacts and Optimal Policy*

Youming Liu† Edona Reshidi‡ Francisco Rivadeneyra§

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Abstract

We study competition between a welfare-maximizing public platform and a profit-maximizing private platform in a two-sided payment market. We characterize the public platform's optimal pricing and show that it balances the benefits of increased competition against the welfare costs of network fragmentation. While introducing a public platform generally raises aggregate welfare and financial inclusion, the competing private platform may respond by raising its fees, disadvantaging merchants that continue to accept payments from the private platform. Finally, we show that cost-recovery and zero-fee mandates constrain public pricing, making welfare improvements uncertain and conditional on network effects, user switching behavior, and the degree of platform differentiation.

JEL Classification: E58, E42, D4.

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†Bank of Canada, Banking and Payments Department; youmingliu@bank-banque-canada.ca.

‡IMF, Payments, Currencies and Infrastructure Division, MCM; ereshidi@imf.org

§Bank of Canada, Banking and Payments Department; frivadeneyra@bank-banque-canada.ca.

1 Introduction

Competition in payment markets is increasingly characterized by a mix of public and private platforms. Traditionally, retail payments were dominated by privately operated networks, such as card schemes and bank-based payment rails, overseen by public authorities primarily in their regulatory capacity. Today, however, public institutions are entering the market as active platform providers. Central banks, for example, are introducing public payment infrastructures, such as fast payment systems (including Pix in Brazil, UPI in India, FedNow in the US, TIPS in EU countries, and RTR in Canada), and are issuing or exploring central bank digital currencies (such as eNaira, Digital Euro, and e-CNY) that operate alongside and in competition with private platforms. At the same time, innovation continues within private payment platforms, ranging from established card networks to newer digital wallets and stablecoin-based payment rails that compete based on price, speed, and convenience while relying on strong network effects. Taken together, these developments imply that competition in payments is no longer purely private but increasingly reflects mixed public–private competition in a two-sided market.

This evolution raises a fundamental policy question: how should a welfare-maximizing public payment platform optimally price its services when competing directly with a profit-maximizing private platform? This question is central to current policy debates regarding FPS and CBDC pricing, access conditions, and cost recovery, as well as to regulatory discussions on the interaction between public payment rails and private stablecoin-based payment systems. Yet existing models provide limited guidance. Most analyses either abstract from payment competition or treat the public platform as a monopolist, while the platform-competition literature typically assumes that all platforms maximize profits.

This paper integrates the mixed-oligopoly and platform-competition literature to address these questions. We develop a two-sided payments framework in which platforms compete for consumers and merchants, but differ in their objectives. We compare two competitive regimes. Under private competition, both platforms are privately operated and maximize profits. Under benevolent competition, one platform is publicly operated and maximizes social welfare, while the other remains profit-maximizing. This comparison enables us to isolate the interaction among public objectives, two-sided pricing, and network effects. It also characterizes how the presence of a welfare-maximizing platform reshapes pricing incentives, participation, and surplus allocation in payment markets.

Our framework is designed to capture two essential features of payment markets. First, we model an oligopolistic market with asymmetric platforms. Oligopolistic competition reflects the fact that payment markets are typically dominated by a small number of large players, while platform asymmetry is intrinsic to public–private competition, wherein public and private platforms differ in objectives, user perceptions, and the scope, coverage, or quality of the payment services they offer. Second, we allow platforms to implement flexible pricing schemes that combine a fixed fee with a per-transaction fee. This two-part tariff structure reflects common industry practice and enables platforms to balance participation incentives with usage incentives.

When platforms compete using such flexible pricing schemes, equilibrium outcomes are generally not unique and may involve a continuum of equilibria (Armstrong, 2006). To address this indeterminacy, we follow White and Weyl (2016) and Ekmekci et al. (2025) and assume that platforms set per-transaction fees equal to the marginal network benefits and compete only on fixed fees. While much of the payments literature emphasizes per-transaction fees, particularly in the context of interchange fees, our solution concept instead treats fixed fees as the primary strategic variable. In practice, payment platforms charge both types of fees, and focusing exclusively on either is an abstraction. Our approach preserves a realistic fee structure while remaining analytically tractable and allows for a transparent characterization of equilibrium pricing and welfare, especially in the presence of a welfare-maximizing public platform.

We begin our analysis by solving the social planner benchmark problem in which a planner directly sets the prices of both payment platforms to maximize total welfare. The resulting first-best pricing rule is intuitive: prices are set equal to the marginal cost of facilitating transactions for an additional user, net of the cross-side network benefits generated for participants on the other side of the market.

Our main analysis focuses on the competitive equilibrium. It yields four primary results. First, we characterize optimal public pricing under public–private competition. Unlike a social planner who dictates prices across the entire market, the public platform acts strategically within a mixed duopoly, taking the private platform's prices as given. We find that the optimal pricing of the public platform deviates from the first-best benchmark via an upward 'benevolent adjustment'. While intuition might suggest that a welfare-maximizing entity should aggressively undercut its private rival to intensify market competition, doing so inefficiently diverts users away from the private platform. This diversion undermines the private platform's network effects, thereby diminishing the surplus for users who continue to use it for payments. By internalizing this externality, the public platform optimally refrains from aggressive undercutting. The finding that a public firm may optimally restrain its competitiveness is consistent with results from the one-sided mixed-oligopoly literature, which typically relies on assumptions such as convex cost structures (De Fraja and Delbono, 1989) and increasing returns to private investment (Ma, 2004). In contrast, our results highlight a novel channel through which two-sided network interactions lead to a similar conclusion.

Our second result concerns the distributional effects of public competition. We find that the private platform sets fees differently when competing with a public platform than when competing with another private platform: it sets lower consumer fees, including both fixed and total fees, while raising the fixed fee charged to merchants. As a result, merchants using the private platform are worse off in public-private competition compared to purely private competition. This outcome, where increased competition leads to a higher merchant fee, is consistent with findings in the literature, yet our result emerges without relying on the assumption of price coherence (Edelman and Wright, 2015) or differential price sensitivity between consumers and merchants (Wang, 2022). Instead, the underlying mechanism aligns with the theory of competitive bottlenecks (Armstrong and Wright, 2007): platforms compete more intensely for single-homing users (consumers) and therefore attempt to recoup losses on the multi-homing side (merchants).

Third, we analyze how common public policy mandates may affect welfare. We show that commonly proposed rules, such as zero pricing for consumers, marginal-cost pricing, or strict cost recovery, are generally not welfare-optimal in competitive two-sided markets. Such constraints are increasingly common in practice: for example, FedNow must recover operational costs under the Monetary Control Act, Brazil's Pix offers zero fees to consumers, and the Digital Euro is expected to be free of charge for consumers as well. We show that while these policies aim to promote financial inclusion and fiscal sustainability, their welfare effects critically depend on how flexibly they are implemented. Forcing the public platform to price too aggressively can exacerbate inefficient diversion from the private platform, while overly restrictive pricing rules reduce the welfare gains from public entry. These results imply that optimal public pricing is inherently contextdependent and must account for the structure of competition and network effects rather than relying on simplistic pricing heuristics. Overall, efficient implementation of zero-fee or cost-recovery policies requires policymakers to have a clear understanding of market conditions, including the strength of network effects across platforms, user switching behavior in response to relative prices, and the degree of platform differentiation. Absent such information, rigid application of these mandates may fail to deliver the intended welfare gains from public platform provision.

Finally, we show that public–private competition enhances overall market participation. Compared to an environment with only private platforms, benevolent competition results in increased participation on both sides of the market, with a greater number of consumers utilizing digital payment instruments and a higher proportion of merchants accepting them.

2 Related Literature:

This paper contributes to three distinct bodies of literature: two-sided platform competition, public payment platforms (including FPS and CBDC) and mixed oligopolies.

First, within the domain of two-sided platforms, an extensive body of research has delved into the intricate dynamics of competition in these markets, with a specific emphasis on payment platform industries.[1] A central question in this literature is whether competition can enhance market efficiency. Seminal works have explored the effects of competition on interchange fees, with Guthrie and Wright (2007) noting that outcomes vary by case, while Chakravorti and Roson (2006) found that competition generally lowers prices, albeit with potential distortions. Armstrong (2006) explored oligopolistic markets with differentiated platforms, uncovering how network effects and pricing strategies shape competitive outcomes. They investigated scenarios involving flat pricing and more flexible models with two-part tariffs, illustrating how different pricing structures can either amplify or mitigate the pro-competitive nature of network effects. More recently, Jain and Townsend (2021) showed that competition among platforms can lead to Pareto efficiency but assumed a perfectly competitive market, which may not hold in payments markets.

Our paper extends this literature by introducing a public platform into the compet-

[1]See Rysman and Wright (2014) for a comprehensive review of this literature

itive landscape and examines how its presence changes equilibrium outcomes. We focus on a market where public and private platforms coexist and compete, which remains underexplored in the existing literature. This unique perspective allows our paper to offer new and valuable insights into the competitive implications of public payment platforms offering valuable guidance for policymakers and practitioners alike.

Second, we contribute to the emerging literature on public payment platforms, which has focused mostly on FPS and CBDC. On the CBDC front, most research has primarily focused on evaluating CBDC's function as a store of value and exploring the potential implications for monetary and financial stability.[2] In contrast, our paper pivots its attention to the means of payment aspect of CBDC and, following an industrial organization approach, scrutinizes its potential impacts on the payment industry. More specifically, in terms of modeling, we incorporate a two-sided platform setup into the literature on CBDC that is inherent to payment systems and account for the competitive landscape within this industry. Such two-sidedness issues have not been studied so far from a CBDC perspective.

On the FPS front, recent studies have started to empirically quantify the benefits brought by introducing an FPS into the market. Those benefits are not only speeding up transactions but also producing broader economic impacts such as financial inclusion and increased efficiency (Cornelli et al., 2024; Ouyang, 2021; Sampaio and Ornelas, 2024). Our paper contributes to this literature by highlighting the public nature of FPS, a feature present in many countries, where systems are developed and operated by central banks or other public entities. Studying this public aspect is important because it shapes the system's objectives, access, and pricing structure, potentially amplifying its role as a digital public good that promotes inclusion, competition and equitable access to financial infrastructure. More recently, (Frost et al., 2025) study payment platform competition in the presence of public payment infrastructures, such as CBDC or FPS. They analyze how these public infrastructures affect market outcomes under different interoperability regimes. In contrast, we focus on the design of optimal fee structures for public platforms.

Finally, this paper contributes to the mixed oligopoly literature, particularly where privately-owned, profit-maximizing firms compete against welfare-maximizing public firms (De Fraja and Delbono, 1989; Matsumura, 1998). This study extends this analysis to a

[2] For extensive surveys of recent research in this context see Ahnert et al. (2022) and Chapman et al. (2023).

two-sided market context, like digital platforms, where the interactions between different user groups, such as consumers and merchants, are critical. As in standard one-sided mixed oligopoly, public entry can raise aggregate welfare but may also induce inefficient switching when products are differentiated. We show that in a two-sided platform setting, network effects amplify this trade-off: by reallocating users across platforms, public pricing changes not only rival profits but also the strength of rival network effects. As a result, unlike in one-sided mixed oligopoly where higher public prices typically reflect cost inefficiencies or constraints, the welfare-maximizing public platform may optimally set higher prices even with equal costs and full benevolence, in order to preserve efficient network structure.

The rest of the paper is organized as follows. Section 3 lays out the theoretical framework. Section 4.1 provides the social planner case, while in Section 4.2 we analyse the case of competition and provide comparative static analysis. Section 5 examines the impact of policy practices such as mandating zero consumer fees and ensuring cost recovery and discusses how greater flexibility in public platform policy design can lead to improved welfare outcomes. Finally, Section 6 concludes.

3 The Model

The model considers a two-sided market with two types of agents: merchants (m) and consumers (c), each with a mass of one. Each consumer transacts with every merchant using one of two payment platforms, indexed by $\{1, 2\}$, or cash, indexed by $\{0\}$.

The two payment platforms compete by setting prices, while the payment in cash is always free.[3] Each platform sets a two-part tariff consisting of a fixed fee denoted as t_i^x, and a per-transaction fee denoted as r_i^x. This two-part tariff structure reflects the realistic pricing strategy employed by most card networks.

Consumers and merchants receive idiosyncratic membership values from adopting each platform i, denoted by v_i^c for consumers and v_i^m for merchants. The membership value is independent of transaction volumes and represents the additional benefit that an agent

[3] In practice, payment platforms also set an interchange fee, which is the amount paid by merchant side intermediaries to consumer side intermediaries. This fee is a crucial component in the payment ecosystem, influencing the incentives for business acquirers on the merchant side and payment service providers on the consumer side. However, in this paper, we have chosen to abstract away from the complexities associated with end-user intermediaries. In Section 6, we discuss the intermediary market in more detail and outline how future work could extend the model to incorporate intermediary dynamics.

gains from adopting a payment platform compared to using only cash, which provides no membership benefit. Examples of the membership value include transaction security, privacy protection, access to business lounges or exclusive club memberships. Agents have heterogeneous preferences or beliefs regarding these features; for instance, individuals may have varying perceptions of the privacy protection offered by a particular payment method. To account for this heterogeneity, we assume that the membership values v_1^x and v_2^x are randomly drawn from the joint distribution F_x. Similar to White and Weyl (2016), Tan and Zhou (2021) and Teh et al. (2022), we keep the distribution function F_x general, with the only assumptions being that F_x is continuously differentiable and log-concave.

During transactions, agents also receive a constant marginal benefit per transaction from using platform i, denoted as b_i^m for merchants and b_i^c for consumers. In contrast, the transaction benefit from using cash is normalized to zero. Thus, these per-transaction benefits represent the additional utility of using a digital payment platform relative to cash, reflecting features such as convenience, speed, and fraud protection. Unlike membership value, which is fixed, the total transaction benefit depends on the volume of transactions, which in turn is determined by the number of users adopting the payment method on each side of the market. Therefore, network effects are central to adoption: the value of a payment method for a merchant (or consumer) increases with the number of consumers (or merchants) using it.

We also assume that consumers singlehome, meaning they adopt at most one platform, while merchants are allowed to multihome, meaning they can adopt both platforms. This assumption is supported by empirical findings from Rysman and Wright (2014), which show that although consumers may hold payment cards from more than one platform, the majority of cardholders prefer to use only one card. Modeling consumer single-homing also implies stronger competitive pressure on the consumer side of the market than on the merchant side, reflecting the empirical relevance of consumer acquisition as a key margin of competition among payment platforms.

An important distinction of our model from those in the literature that focus on platform competition (Guthrie and Wright, 2007; Rochet and Tirole, 2002) is that we incorporate differentiated payment schemes along with preference heterogeneity among consumers and merchants. In our model, the membership value differs between payment platforms, reflecting the differentiation between them. Additionally, the membership value can differ both between consumers and merchants and within the same agent type, capturing the heterogeneity in user preferences. Differentiation is especially crucial when one of the platforms is public because public and private platforms may not be able to offer the same services, and they are often perceived differently by users.

Note that although our model assumes that the per-transaction benefits b_i^c and b_i^m are exogenous and abstracts from merchant surcharging, in the Appendix B we show that the results go through even if we allow for endogenous benefits and merchant surcharging.

The timing of the game is as follows:

Stage 1. Platforms simultaneously set their fixed fees $\{t_i^c, t_i^m\}_{i\in\{1,2\}}$ and per-transaction fees $\{r_i^c, r_i^m\}_{i\in\{1,2\}}$.

Stage 2. After observing the fixed fees, consumers and merchants choose whether and which payment platform to adopt. We denote the number of merchants or consumers of each platform with N_i^x, where $i \in \{1, 2\}$ and $x \in \{c, m\}$.

Stage 3. A consumer (merchant) who has adopted platform i chooses the number of merchants (consumers), denoted by $\tilde{N}_i^m$ ($\tilde{N}_i^c$), with whom to interact. As a result the total fee paid by each user, denoted by P_i^c and P_i^m, respectively, is given by

$$P_i^c = t_i^c + r_i^c \tilde{N}_i^m, \tag{1}$$

$$P_i^m = t_i^m + r_i^m \tilde{N}_i^c. \tag{2}$$

We now formally define the decision problem of agents. A merchant can choose to singlehome, multihome, or not adopt any payment platform. After choosing the platforms, the merchant can further choose the number of consumers to interact with on that platform based on the per-transaction fee. Formally, a merchant's utility maximization problem can be expressed as follows:

$$u_m = \sum_{i\in 1,2} \max\{0, v_i^m + (b_i^m - r_i^m)\tilde{N}_i^c - t_i^m\}, \tag{3}$$

where $\tilde{N}_i^c$ represents the number of consumers that merchant chooses to transact with on platform i and $\tilde{N}_i^c < N_i^c$.

A consumer can adopt at most one payment platform. Therefore, a consumer will choose the platform that provides the highest utility. Formally, a consumer's utility can be expressed as follows:

$$u_c = \max\{0, v_1^c + (b_1^c - r_1^c)\tilde{N}_1^m - t_1^c, v_2^c + (b_2^c - r_2^c)\tilde{N}_2^m - t_2^c\}, \tag{4}$$

where $\tilde{N}_i^m$ is the number of merchants that a consumer chooses to transact with on the platform i and $\tilde{N}_i^m < N_i^m$.

The transaction volumes on each platform are given by $\tilde{N}_i^m \tilde{N}_i^c$. The marginal (social) cost per transaction on each platform is denoted by c_i, which reflects the resources required to facilitate a transaction. This cost is assumed to be fully borne by the platform. To ensure that transactions on the platform generate positive externalities, we assume that $c_i < b_i^c + b_i^m$.

Platforms also incur costs associated with providing membership benefits to users. Similarly to the membership value, these costs are independent of transaction volumes but depend on the number of users on the platform. We assume that the marginal cost of accepting users is constant, but differs across platforms and agent types, although it is uniform within the same agent type. We denote the costs per merchant and per consumer for platform i as k_i^m and k_i^c, respectively.

The profit of platform i, denoted by Π_i, is determined by the difference between the revenue from the fees and the costs of facilitating transactions and providing membership benefits. Formally, the profit function is given by:

$$\Pi_i = (t_i^c + r_i^c \tilde{N}_i^m - k_i^c)\tilde{N}_i^c + (t_i^m + r_i^m \tilde{N}_i^c - k_i^m)\tilde{N}_i^m - c_i \tilde{N}_i^m \tilde{N}_i^c. \tag{5}$$

4 Equilibrium Analysis

In this section, we first derive the socially optimal pricing in the payments market. We then analyze platform competition under two scenarios: one with two profit-maximizing private platforms, and another with a benevolent public platform competing against a private one. Finally, we compare the market outcomes under these two settings.

4.1 Planner's Choice

The social planner's objective is to set the membership prices for each platform in a way that maximizes the total welfare. Total welfare is defined as the sum of consumer surplus, merchant surplus and platform profits. Platform profits were defined in the previous section; in what follows, we define the user surplus. The merchant side surplus, denoted as V_m, is the sum of the individual merchant net utilities given the number of consumers adopting each platform:

$$V_m = \sum_{i \in \{1,2\}} \left[\int_v max\{0, v_i^m + (b_i^m - r_i^m)\tilde{N}_i^c - t_i^m\} dF_m(v) \right] \tag{6}$$

where $F_m(v)$ is the cumulative distribution function of the merchant membership values.

Similarly, the consumer side surplus, denoted as V_c, is defined as the maximum value that consumers can obtain from the platform, given the number of merchants accepting each platform:

$$V_c = \int_v \max\{0, v_1^c + (b_1^c - r_1^c)\tilde{N}_1^m - t_1^c, v_2^c + (b_2^c - r_2^c)\tilde{N}_2^m - t_2^c\} dF_c(v) \tag{7}$$

where $F_c(v)$ is the cumulative distribution function of the consumer membership values.

Overall, total welfare is defined as:

$$Wel = V_m + V_c + \sum_{i \in \{1,2\}} \Pi_i \tag{8}$$

The planner's objective is to maximize the social welfare as defined above. By solving the social planner's problem we obtain the socially optimal prices (denoted by the subscript FB) as stated in the following proposition:

Proposition 1. *A social planner maximizes social welfare in the payments market via setting following prices for each platform $i \in \{1, 2\}$:*

$$t_{i,FB}^c = k_i^c + (c_i - b_i^c - b_i^m)N_{i,FB}^m; \quad r_{i,FB}^c = b_i^c. \tag{9}$$

$$t_{i,FB}^m = k_i^m + (c_i - b_i^c - b_i^m)N_{i,FB}^c; \quad r_{i,FB}^m = b_i^m. \tag{10}$$

where $N_{i,FB}^m$ and $N_{i,FB}^c$ represent user allocations that maximize the social welfare.

Proof. Because transactions via platforms have positive externalities, the social planner would set the per-transaction at the level of marginal transaction benefit to maximize transaction volumes on a platform, that is $r_i^m = b_i^m$ and $r_i^c = b_i^c$, under which each consumer will transact with all merchants who join the same platforms. The first-order conditions of the social welfare function (8) with respect to N_i^m and N_i^c imply that the efficient total fees satisfy:

$$P_i^c = t_i^c + r_i^c N_i^m = k_i^c + (c_i - b_i^m) N_i^m \tag{11}$$

$$P_i^m = t_i^m + r_i^m N_i^c = k_i^m + (c_i - b_i^c) N_i^c \tag{12}$$

which can be achieved by setting the variable fee equal to b_i^c and the fixed fee equal to $k_i^c + (c_i - b_i^c - b_i^m) N_i^m$. □

The first-best total fees as shown in Equation 11 highlight a key distinction between two-sided payment markets and traditional one-sided markets. In one-sided markets, economic efficiency is typically achieved when prices equal marginal costs. In two-sided markets, in contrast, efficient pricing requires setting prices below marginal costs due to the presence of network externalities: the participation of an additional user increases the platform's value for users on the other side.

This socially optimal pricing approach provides a valuable framework for regulatory design. If a well-informed regulator acts as a social planner, setting fees that reflect marginal costs and accounts for network benefits, regulation could theoretically reach the first-best outcome, aligning prices with social value and maximizing welfare through balanced participation. However, in practice, regulators may only be able to impose fee caps rather than mandate exact prices, especially when the platform is privately operated. This flexibility would enable private platforms to set prices below the cap, leading to varied pricing strategies that may dilute the intended welfare benefits of regulation. Furthermore, platforms could add new fees or find ways to bypass these caps, complicating efforts to reach a socially optimal pricing structure and potentially undermining regulatory goals.

4.2 Platform Competition

In this section, we characterize competitive equilibrium under two distinct scenarios. The first scenario analyses private competition (PC), where both platforms aim to maximize profit. The second scenario examines benevolent competition (BC), where Platform 1 takes a benevolent approach, maximizing the social welfare, while Platform 2 remains profit maximizing. We then present the equilibrium prices for both scenarios and analyze the differences between them.

In the presence of network effects and strategic pricing by platforms, competition can give rise to multiple equilibria. This multiplicity arises from two key factors. First, there is a coordination problem between users on both sides of the market. Consumers' willingness to join a platform depends on their expectations regarding merchant participation, and vice versa (Katz and Shapiro, 1985). Second, platforms compete using two-part tariffs, which consist of a fixed membership fee and a per-transaction fee. This form of pricing introduces additional complexity, allowing for a continuum of equilibria, each associated with different levels of profits for platforms and surplus for users on both sides (Armstrong, 2006).

To resolve the coordination problem and the issue of continuum of equilibria, we adopt the insulated equilibrium concept developed by White and Weyl (2016)[4]. This framework relies on two-part insulating tariffs, in which a platform's pricing adjusts according to the number of users who join. The intuition behind this mechanism is that users on each side facing this two-part tariff will have a dominant strategy for platform participation. When both platforms commit to this pricing strategy, the decisions of merchants and consumers about which platform to join become independent of participation decisions on the other side. By employing this solution concept, we can identify a unique equilibrium and study its properties.

Following White and Weyl (2016), we define an insulated equilibrium as follows:

Definition 1. An insulated equilibrium is a Nash Equilibrium in which both platforms commit to set per-transaction fees such that after seeing the fees, users' decisions of which platform to join are independent of participation on the other side, i.e., $r_i^m = b_m$ and $r_i^c = b_c$, $\forall i \in \{1, 2\}$.

Under insulated pricing, all consumers on platform i transact with all merchants on the same platform, and likewise all merchants transact with all consumers on that platform, therefore, $\tilde{N}_i^m = N_i^m$ and $\tilde{N}_i^c = N_i^c$.

4.2.1 Private Competition (PC)

In this scenario, both platforms are private and they choose their prices to maximize their own profits. In this model, uniform network effects lead to insulated pricing, with

[4]A closely related approach is the "Net Fee" model by Ekmekci et al. (2025), in our model setting the "Net Fee" approach and the insulated equilibrium are equivalent.

$r_i^m = b_m$ and $r_i^c = b_c$. To find the equilibrium in this scenario, one must determine the pair of values $\{t_i^m, t_i^c\}$ such that no platform will deviate from this pricing. Specifically, we can express the profit function of platform i as follows:

$$\Pi_i = (t_i^m + b_i^m N_i^c - k_i^m)N_i^m + (t_i^c + b_i^c N_i^m - k_i^c)N_i^c - c_i N_i^m N_i^c, \tag{13}$$

where N_i^c and N_i^m represent the share of consumers and merchants, respectively, participating in platform i. Given insulated pricing and letting j denote the competing platform, the participation shares on each side can be expressed as follows:

$$N_i^m = \int_{v_i^m \geq t_i^m} dF_m(v), \tag{14}$$

$$N_i^c = \int_{v_i^c \geq \max t_i^c, v_j^c t_i^c - t_j^c} dF_c(v). \tag{15}$$

By setting the per-transaction fee equal to the marginal network benefit, insulated pricing eliminates the problem of a continuum of equilibria. Furthermore, under insulated pricing, the benefit of transacting with users on the other side of the platform is fully extracted by the platform, making users' participation decisions independent of their expectations regarding the participation on the other side. As a result, the multiplicity that arises from coordination issues disappears. The only conditions necessary are those that ensure the concavity of the profit function. The following proposition characterizes the unique equilibrium of the private competition, given the sufficient conditions:

Proposition 2. *Under private platform competition, the insulated equilibrium implies platform i sets prices according to the following pricing functions:*

$$t_{i,PC}^m = k_i^c + (c_i - b_i^m - b_i^c)N_{i,PC}^c + \underbrace{\mu_i^m}_{Markup}, \quad r_{i,PC}^m = b_i^m. \tag{16}$$

$$t_{i,PC}^c = k_i^m + (c_i - b_i^m - b_i^c)N_{i,PC}^m + \underbrace{\mu_i^c}_{Markup}, \quad r_{i,PC}^c = b_i^c. \tag{17}$$

where μ_i^m and μ_i^c are defined as

$$\mu_i^m = -\frac{N_i^m}{\partial N_i^m / \partial t_i^m}; \quad \mu_i^c = -\frac{N_i^c}{\partial N_i^c / \partial t_i^c}. \tag{18}$$

The equilibrium is unique if the following sufficient conditions hold:

$$b_i^m + b_i^c - c_i \leq \min\{\lim_{\substack{t_i^c \to \infty \\ t_j^c \to 0}} \mu_i^m, \lim_{\substack{t_i^m \to \infty \\ t_j^m \to 0}} \mu_i^c\},$$

where $i, j \in \{1, 2\}$ *and* $i \neq j$.

The terms μ_i^m and μ_i^c represent the markups that platform i applies on the merchant and consumer sides, respectively. These markups are positive and capture the degree of market power the platform holds on each side of the market. Since the markups are positive, they indicate that the platform sets prices above the socially efficient levels, reflecting distortions due to market power.

The sufficient conditions that ensure the concavity of the profit function require that the marginal network benefits are dominated by the markups at any price level. Since the markup arises from platform differentiation, these conditions imply that platforms must be sufficiently differentiated relative to the marginal network benefit. To illustrate this, we consider a specific example where the idiosyncratic membership values v_x^i are independently and identically distributed across platforms according to the Type I extreme value distribution with dispersion parameter λ_x. This implies that the markup of the platforms are

$$\mu_i^m = \frac{\lambda_m}{1 - N_i^m}$$

and

$$\mu_i^c = \frac{\lambda_c}{1 - N_i^c}.$$

As $\lim_{\substack{t_i^m \to \infty \\ t_m^j \to 0}} N_i^m = 0$ and $\lim_{\substack{t_i^c \to \infty \\ t_c^j \to 0}} N_i^c = 0$, we have $\lim_{\substack{t_i^m \to \infty \\ t_m^j \to 0}} \mu_i^m = \lambda_m$ and $\lim_{\substack{t_i^c \to \infty \\ t_c^j \to 0}} \mu_i^c = \lambda_c$. Therefore, the sufficient conditions imply that:

$$b_i^m + b_i^c - c_i \leq \min\{\lambda_m, \lambda_c\}.$$

The uniqueness of equilibrium requires sufficient horizontal differentiation between platforms relative to network benefits, because the dispersion parameter λ_x captures the level of horizontal differentiation of platforms. This condition is in the same spirit as Armstrong (2006), which assumes sufficient differentiation between platforms to achieve market-sharing equilibrium.

4.2.2 Benevolent Competition (BC)

Now suppose that platform 1 is benevolent and seeks to maximize either social welfare or end-user surplus instead of maximizing profit. To accommodate a general class of such objectives that the platform may have, we set the objective function as a weighted sum of user surplus and platform profits:

$$\mathcal{W}_1 = \Pi_1 + \omega_p \Pi_2 + \omega_u \left(V_m + V_c\right), \tag{19}$$

where ω_u represents the weight given to the merchants' and consumers' surplus, and ω_p represents the weight assigned to the private platform's profit. The weight of the benevolent platform's profit is normalized to 1. The only constraint on weight parameters ω_u and ω_p is that they must be non-negative. Both weights may exceed 1, allowing for cases where the benevolent platform may prioritize end-users' surplus or the rival's profit more than its own profit.

Allowing the public platform to place positive weight on private platform profits captures realistic policy considerations faced by central banks. Public payment infrastructures are typically introduced alongside, rather than as substitutes for, private platforms, and policymakers may value the continued viability of private providers to preserve competition, resilience, and innovation in the payments ecosystem. In this sense, private profits proxy for platform sustainability and rival network strength, and the weights in the objective function provide a reduced-form way to capture these broader policy trade-offs, rather than reflecting an intrinsic concern for private returns.

As in the previous section, we apply the Insulated Equilibrium solution concept. In equilibrium, each platform chooses its fixed fees (t_i^c and t_i^m) while setting per-transaction fees equal to the marginal network benefits, i.e., $r_i^c = b_i^c$ and $r_i^m = b_i^m$. This leads to the following proposition:

Proposition 3. *Let the diversion ratio be defined as* $\eta_1^c \equiv -\frac{\partial N_2^c/\partial t_1^c}{\partial N_1^c/\partial t_1^c}$. *Given the sufficient condition in Proposition 2, there is a unique equilibrium in the setting where Platform 1 is benevolent and Platform 2 is profit-maximizing. In this equilibrium, the benevolent*

Platform 1 sets fixed prices according to the following pricing functions:

$$t^m_{1,BC} = k^m_1 + (c_1 - b^m_1 - b^c_1)N^c_{1,BC} + (1-\omega_u)\mu^m_1, \tag{20}$$

$$t^c_{1,BC} = k^c_1 + (c_1 - b^m_1 - b^c_1)N^m_{1,BC} + \omega_p\mu^c_2\eta^c_1 + (1-\omega_u)\mu^c_1. \tag{21}$$

The private platform sets prices using the same pricing functions as in the private competition as shown in equation (16) *and* (17)*, except that* $N^c_{2,PC}$ *and* $N^m_{2,PC}$ *are replaced by* $N^c_{2,BC}$ *and* $N^m_{2,BC}$ *respectively.*

Diversion ratio η^c_1 captures the share of consumers diverted from Platform 1 to Platform 2 in response to a marginal price increase by Platform 1.

4.2.3 Comparison of Outcomes: Private vs. Benevolent Competition

Having derived the equilibrium prices under both private competition (PC) and benevolent competition (BC), we now turn to a comparison of outcomes across the two cases. This comparison illustrates how the presence of a benevolent public platform, which prioritizes user welfare and potentially rival profits over its own, affects platform pricing and market structure.

When comparing the pricing behavior of a benevolent platform to that of a profit-maximizing one, it is clear that the benevolent platform tends to set lower prices overall, in order to enhance market competition and welfare. This benevolent platform places a smaller weight, specifically $1 - \omega_u$, on the markup terms μ^m_1 and μ^c_1, which measure market power distortions on the merchant and consumer sides, respectively. By doing so, the platform mitigates the extent to which it exploits market power.

However, a key distinction in benevolent pricing for consumers is the presence of the *benevolent adjustment term*, $\omega_p\mu^c_2\eta^c_1$, which does not appear in private platform pricing. This term captures an additional trade-off faced by the benevolent platform: while it aims to increase user surplus through lower pricing, it must also consider the adverse effect of user diversion from the private platform. Specifically, by attracting consumers away from the rival platform, the benevolent platform may contribute to inefficient user allocation and a reduction in the rival's network effects, which can diminish overall welfare. The adjustment term thus captures the benevolent platform's internalization of the competitive externality, particularly when it places positive weight on the rival's profits.

To understand this trade-off more clearly, consider a simplified environment where network effects are absent on the consumer side but remain on the merchant side (i.e., $b_i^c = 0$ and $b_i^m > 0$).

Lowering the benevolent platform's price attracts two types of users:

1. New consumers who would otherwise use cash (extensive margin),

2. Consumers who switch from the private platform (intensive margin).

While the first channel unequivocally improves aggregate welfare, the second yields ambiguous welfare effects for two reasons. First, the migration of consumers from platform 2 to platform 1 reduces the cross-group network externalities for merchants who continue to accept the private platform. Second, if consumers switch despite possessing a stronger intrinsic preference (i.e., a higher membership benefit) for the private platform, this misallocation reduces total social surplus.

In other words, the benevolent platform would set a higher price when the private platform has a greater ability to generate network benefits for its users compared to when the private platform's network benefits are relatively weak. Similarly, it sets a higher price when the private platform provides greater membership value or processes payments more cost-effectively. These results are formalized in the following proposition:

Proposition 4. *Given t_2^c and t_2^m, the benevolent adjustment set by the benevolent platform increases with b_2^c, b_2^m while decreases with c_2 and k_2^c. Furthermore, the adjustment increases following a first-order stochastic dominance shift in the distribution of the private platform's consumer membership value, v_2^c.*

Interestingly, this seemingly anti-competitive pricing behavior becomes less pronounced when the private platform possesses greater market power. To demonstrate this, we consider the special case of logit demand, where market power is fully captured by the dispersion parameter λ_c of the consumer membership value distribution. We then establish the following:

Proposition 5. *If the idiosyncratic membership values $v_x^i, x \in \{c, m\}$, are independently and identically distributed across platforms according to the Type I extreme value distribution with the dispersion parameter $\lambda_x, x \in \{c, m\}$. Then, the benevolent adjustment decreases with λ_c.*

In other words, as the private platform's market power increases (i.e., demand becomes less elastic), the benevolent platform faces less incentive to accommodate it by raising its own prices. In such cases, the marginal welfare loss from diverting users away is smaller, weakening the rationale for pricing conservatively to protect the rival platform.

Our result aligns with the mixed-oligopoly literature, which also show that a welfare-maximizing public firm may optimally restrain its competitiveness. In particular, De Fraja and Delbono (1989) show that when production costs are convex, efficiency requires output to be distributed as evenly as possible across firms. As a result, the public firm optimally restricts its production capacity and sets prices above marginal cost so that part of production is shifted to private firms. Similarly, Ma (2004) consider an environment in which private firms can incur a fixed cost to invest in technologies that reduce marginal costs. In their setting, rationing public supply improves welfare by inducing greater reliance on private firms, thereby strengthening incentives for cost-reducing investment and lowering average production costs. In contrast, our analysis identifies a novel channel specific to two-sided markets: by internalizing the effect of participation on rival network size and the surplus generated on the private platform, the public platform optimally softens price competition and reallocates participation across platforms.

The difference between equation (20) and equation (16) shows that the public platform charges a lower fixed price to merchants compared to the same prices charged by the same but profit-maximizing platform in PC. However, due to the benevolent adjustment, it is unclear whether the public platform sets lower prices on the consumer side as well. Nevertheless, the following proposition shows that the public platform unambiguously charges lower consumer fees than in the counterfactual where it is replaced by a profit-maximizing private platform. As a result, consumer surplus is strictly higher under benevolent competition than under private competition.

Proposition 6. *Suppose $\omega_u = \omega_p = 1$:*

- *Users of platform 1 are better off under BC than under PC as the platform charges lower fixed prices to both consumers and merchants than its profit-maximizing counterpart would under PC, i.e., $t^c_{1,BC} < t^c_{1,PC}$ and $t^m_{1,BC} < t^m_{1,PC}$*
- *The total fee P^m_1 paid by merchants of platform 1 is unambiguously lower in BC than that in PC, i.e., $P^m_{1,BC} < P^m_{1,PC}$; but whether the total fee P^c_1 paid by consumers is lower is ambiguous.*

We next examine how the private platform responds to the entry of a benevolent competitor.

Proposition 7. *Suppose $\omega_u = \omega_p = 1$. Then,*

- *Consumers of platform 2 are better off under BC as the platform charges a lower fixed fee, i.e., $t^c_{2,BC} \leq t^c_{2,PC}$, and they face a lower total fee under BC. i.e., $P^c_{2,BC} \leq P^c_{2,PC}$.*
- *Merchants of platform 2 are worse off under BC as the platform charges a higher fixed fee, i.e., $t^m_{2,BC} \geq t^m_{2,PC}$.*

Because consumers are single-homing, the competitive pressure from the benevolent platform compels the private platform to lower its consumer prices in order to retain users. As a result, consumers on the private platform are better off under benevolent competition (BC) than under private competition (PC). In contrast, merchants are multi-homing, meaning that platforms do not compete directly for them. Consequently, the introduction of a benevolent platform does not exert the same downward pressure on merchant fees. In fact, the private platform's pricing strategy results in lower merchant surplus for two key reasons: First, fewer consumers participate on the private platform under BC than under PC, i.e., $N^2_{c,BC} \leq N^2_{c,PC}$, reducing the network benefits passed on to merchants. Second, in line with the competitive bottleneck equilibrium proposed by Armstrong and Wright (2007), platforms face stronger competition on the single-homing (consumer) side, prompting them to recover revenue from the multi-homing (merchant) side.

These pricing and surplus effects translate directly into the role of public platforms in improving financial inclusion. Proposition 6 and 7 jointly show that consumers are better off under the BC than PC, implying that there are more consumers participating under the BC equilibrium. On the merchant side, multi-homing and the greater surplus offered by the benevolent platform imply that more merchants choose to participate in at least one platform under benevolent competition.

5 Other policy practices of the benevolent platform

Public payment platforms are often subject to policy mandates that are different from the purely benevolent objectives assumed in the benchmark model. These mandates reflect a trade-off between promoting social goals, such as financial inclusion and broad market access, and adhering to practical constraints, including fiscal sustainability and limited market intervention. As a result, public platforms must design their pricing strategies not only to enhance welfare, but also to achieve budgetary balance.

These policy-driven practices may impose additional constraints that limit the public platform's ability to fully optimize market outcomes. For example, cost-recovery requirements or restrictions on subsidizing specific user groups can reduce pricing flexibility, preventing the platform from offering low or zero fees, even when such pricing would be welfare-enhancing. In addition, political considerations or the need for public accountability may discourage the platform from engaging in aggressive competition with private providers, even if such behavior could improve efficiency and inclusion.

In this section, we examine two common practices of public platforms: (i) mandating zero consumer prices, and (ii) ensuring cost recovery. For each case, we will discuss the conditions under which a benevolent platform can still enhance the efficiency in the payments market. We focus on the setup where the benevolent puts equal weights on the end-user surplus and on the private platform profit, i.e., $\omega_u = \omega_p = 1$.

5.1 Zero consumer fees

The concept of zero consumer fees has gained significant attention in the context of payment systems, particularly with the introduction of PIX in Brazil and the proposed Digital Euro by the European Central Bank (ECB). The rationale behind this zero-consumer fee structure is to promote financial inclusion, increase competition among payment providers, and reduce reliance on cash transactions, while minimizing the financial burden on the public budget.

Against this backdrop, we extend our model to study the optimal fee structure and its welfare implications when a benevolent public platform sets the consumer fee to zero. The results are summarized in the following proposition:

Proposition 8. *If the public platform fixes its consumer price at zero in an insulated* *equilibrium such that:* $P_1^c = t_1^c + b_1^c N_1^m = 0$ *the public platform maximizes the social welfare by setting the merchant price at* $P_1^m = k_1^m + (c_1 - b_1^c)N_1^c$. *Introducing the public platform is welfare-improving compared to private competition if the following condition is satisfied:* $k_1^c + (c_1 - b_1^c)N_1^m + \mu_2^c \eta_1^c \leq 0$.

The proposition above shows that the benevolent platform can still improve payment efficiency even when constrained to charge a zero consumer fee. This is particularly the case when, absent the constraint, the platform would have chosen to set a negative consumer fee - that is, to subsidize consumer participation. In such settings, both the zero consumer fee and the optimally chosen merchant fee are lower than what a private platform would charge under private competition (PC), leading to stronger competitive pressure and greater user participation. The sufficient condition ensures that this competitive pressure is not excessively strong, such that it would harm the network effects of the private platform. When the condition is satisfied, the introduction of the benevolent platform results in a net welfare gain, even under the zero-fee constraint.

However, if the unconstrained optimal strategy for the benevolent platform would involve charging a positive consumer fee, then imposing a zero-fee constraint may lead to excessively strong competitive pressure on the private platform. In this case, the forced reduction in consumer pricing could result in a distortion of user allocation and a loss of network effects on the private platform. As a result, the overall welfare impact becomes ambiguous.

5.2 Cost-recovery approach

Cost-recovery is another common practice requiring public payment platforms to set fees that cover their operational costs. For example, the U.S. FedNow FPS is required to set prices for its payment services in order to recover costs under the Monetary Control Act of 1980.[5]

Therefore, we extend our model to study the optimal pricing of the benevolent platform under a cost-recovery approach. The results are summarized in the following proposition:

Proposition 9. *If the public platform is subject to a cost-recovery constraint in an insulated equilibrium, such that* $(P_1^c - k_1^c)N_1^c + (P_1^m - k_1^m)N_1^m - c_1 N_1^m N_1^c = 0$, *the public*

[5] The Federal Reserve must establish fees for “priced services” to recover, over the long run, all the direct and indirect costs associated with its payment and settlement system services.

platform maximizes the social welfare by setting

$$P_1^m = k_1^m + (c_1 - b_1^c)N_1^c + (1 - \varphi)\mu_1^m,$$

and

$$P_1^c = k_1^c + (c_1 - b_1^m)N_1^m + (1 - \varphi)\mu_1^c + \varphi\mu_2^c\eta_1^c,$$

where

$$\varphi = \frac{\mu_1^m N_1^m + \mu_1^c N_1^c - (b_1^c + b_1^m - c_1)N_1^c N_1^m}{\mu_1^m N_1^m + \mu_1^c N_1^c - \mu_2^c \eta_1^c N_1^c}.$$

With the cost-recovery constraint, the market power terms μ_1^m and μ_1^c reappear in the benevolent platform's pricing strategy. However, whether the reappearance of the market power terms will increase or decrease the benevolent prices depends on whether the Lagrangian parameter φ is smaller or greater than 1. Based on the formulation of the Lagrangian term, it is clear that the parameter is smaller than 1 when

$$(b_1^c + b_1^m - c_1)N_1^c N_1^m \geq \mu_2^c \eta_1^c N_1^c \implies (c_1 - b_1^c - b_1^m)N_1^c N_1^m + \mu_2^c \eta_1^c N_1^c \leq 0. \tag{22}$$

The equation on the right is the profit of a benevolent platform under the BC equilibrium without the cost-recovery constraint. This condition implies that the benevolent platform must increase its price to achieve a balanced budget if it would otherwise incur a loss in the BC equilibrium. As the benevolent platform raises its price, the total number of transactions it facilitates decreases, along with the network benefits received by users on both sides. In other words, the benevolent platform diminishes its network effects in order to meet the cost-recovery constraint.

Implementing the cost-recovery objective comes at the expense of reducing social welfare. Equation 22 suggests that the condition for a cost-recovery benevolent platform to raise prices occurs when its network benefits, $b_1^c + b_1^m$, exceed a certain threshold. Although the platform could provide more transactional benefits to users under this condition, it increases prices to reduce the total transactions on its platform. However, the following proposition shows that the benevolent platform with cost-recovery constraint can still improve welfare if $\varphi < 1$.

Proposition 10. *Introducing a cost-recovery benevolent platform is welfare-improving compared to a private platform in the PC equilibrium if the optimal pricing of the benev-*

olent platform without the cost-recovery constraint would result in a loss in profit.

This proposition highlights that even with a cost-recovery constraint, the public platform improves welfare as long as the platform's socially optimal pricing strategy, without the constraint, would require setting prices so low that they generate a loss. This indicates the platform's intention to prioritize welfare over profit by enhancing competition and consumer access.

5.3 Optimal public policy design

Determining which policy, enforcing a zero-consumer fee constraint or a cost-recovery constraint, yields better welfare outcomes is challenging, as either constraint can become more binding depending on market conditions. However, a key normative lesson from the analysis is that adopting a more flexible policy framework can enhance welfare while minimizing unnecessary interventions or fiscal burdens.

Proposition 8 implies that ambiguity in welfare outcomes arises when the public platform, in the absence of the zero-consumer fee constraint, would find it optimal to charge a positive consumer fee. Similarly, Proposition 10 suggests that ambiguity also occurs when the public platform would generate non-negative profit without a cost-recovery constraint. These insights suggest that a more effective policy design involves relaxing both the zero-consumer fee and cost-recovery constraints.

By allowing the platform to charge non-negative consumer prices or pursue non-negative profit, the platform gains the flexibility to set fees optimally based on market conditions. Specifically, the platform will charge a positive consumer fee or generate positive profit only if these strategies improve welfare. Conversely, if the socially optimal price for consumers would be negative, or if operating at a loss would maximize welfare, the platform can still improve social welfare by setting a zero fee or aiming for cost recovery.

This flexibility ensures that the public platform avoids the inefficiencies associated with rigidly enforced zero-fee or cost-recovery policies. A more relaxed policy framework will also allow the platform to respond dynamically to changes in market conditions, promoting welfare through competitive pricing while maintaining least market intervention or sustainable public budget.

In practice, public platforms often implement these policy constraints based on limited internal information. For example, US Federal Reserve sets fees for its payment services based on its own operating costs, imputed financial costs, and projected transaction demand that is assumed to be invariant across different fee schedules. However, an important policy implication based on Proposition 8 and 10 suggest effective implementation of these policy constraints requires the public platform to set the fees based on broader knowledge on market conditions, which includes the network effects of all platforms, user behavior in response to price changes, and the extent of platform differentiation. Without accounting for these factors, rigidly applied policy rules may fail to deliver the welfare improvements that motivate the introduction of a public payment platform in the first place.

6 Conclusion and Discussion

This paper investigates the role of public payment platforms in a two-sided market where a public payment platform, such as a CBDC or a public FPS, competes against an incumbent private payment platform. We model the public platform as a benevolent agent that maximizes total surplus rather than its own profits and analyze how its entry differs in terms of equilibrium prices, user participation, and welfare from the one with private entry.

Our analysis shows that optimal public platform pricing is not equivalent to that of a social planner. Because the public platform can only influence the private platform prices and user participation via its own pricing strategy, it must strategically navigate a trade-off. On the one hand, lowering prices promotes participation and competitive pressure. On the other hand, setting prices too low may inefficiently reallocate users who would otherwise benefit more from the private platform due to heterogeneous preferences. It can also reduce the network benefits associated with the competing private platform, thus harming total welfare.

We further examine two common policy constraints commonly imposed on public platforms: zero consumer fees and cost-recovery mandates. Our findings indicate that a public platform can still outperform a private entrant in terms of welfare under these constraints, provided they are applied with sufficient flexibility. In particular, the public platform should charge a positive fee or generate a positive profit only if, in the absence of these constraints, doing so improves welfare. Conversely, when the unconstrained optimal consumer price is negative, or when operating at a loss would maximize welfare, a public entry with a zero-fee or cost-recovery can still be welfare-enhancing.

There are several important features in the payments market that are not explicitly modeled. While our analysis abstracts from these factors, we offer some preliminary intuitions on how they may affect the robustness and applicability of our results.

Intermediaries: This paper abstracts away from intermediaries to focus on the direct dynamics between the public payment platform and end-users. For example, private networks such as Visa and MasterCard, rely heavily on issuing and acquiring banks for end-user engagement. On the other hand, public payment systems either already incorporate or are exploring partnerships with intermediaries like banks or payment service providers (PSPs). Our results would be robust if we extended the model to include a perfectly competitive intermediary market, where the public platform sets fees to intermediaries and they are passed on to end-users without additional markups. It would be interesting to explore how our work could be extended to markets where the perfectly competitive intermediary market assumption may not be suitable. In markets with intermediary market power, the analysis would be more complex and the welfare benefits could be diminished due to double marginalization and other conflicting intermediary incentives.

More platforms: In this analysis, we consider a model with only two platforms for two primary reasons. First, by focusing on the competition between a single public and a single private platform, we can more clearly observe how the public platform's welfare-maximizing approach influences market outcomes, separately from any competitive effects that would arise from simply increasing the number of platforms. Second, one can think of this analysis as reflecting a policymaker's decision framework: whether to enter the market with a public platform or to promote the entry of additional private platforms. This setup clarifies the unique competitive and welfare impacts of a public platform, providing insights into the potential benefits of public sector involvement over simply increasing private sector competition. Future research could examine scenarios with multiple platforms, including the coexistence of public platforms like FPS and CBDC systems alongside private ones, which many jurisdictions are actively considering.[6]

Holding Limits: An additional factor relevant to public platforms, especially CBDCs,

[6]The direction of welfare change becomes ambiguous when an additional platform is introduced in these markets due to network effects. Without any differentiation, the optimal scenario is to have a single platform.

is the potential implementation of holding limits or transaction limits that many central banks are considering. While this paper does not explore holding limits directly, it may be interesting for future research on public platforms to consider how such limits could shape competitive dynamics. For instance, low holding limits on CBDC could mitigate the risk of bank disintermediation while potentially encouraging different pricing strategies to enhance their appeal compared to private platforms. These limits may not necessarily restrict the competitive effect of CBDC, as they could instead lead to more aggressive pricing strategies to attract users. This aspect could be contrasted with FPS systems, which typically lack such restrictions, possibly leading to differing competitive effects. Further exploration of these elements could deepen our understanding of the varied impacts of CBDC and FPS in the payments ecosystem.

Consumer multihoming: It is important to discuss the case where consumers are allowed to multihome, as consumers are indeed holding multiple payment methods nowadays. In such setting, consumers adopt both payment methods to transact with more merchants. When meeting with a single-homing merchant, a multihoming consumer can still transact with the payment method accepted by the merchant. Merchants, therefore, can strategically choose to singlehome, thereby steering consumer toward their preferred payment method. As such, merchants' adoption decisions become interdependent: the decision to adopt one platform depends on the fees set by the other platform.

The optimal pricing of the public platform would be different under consumer multihoming; however, the main trade-off faced by a benevolent public platform still remains. Intuitively, when the public platform sets lower fees, merchants have stronger incentive to singlehome, as they can steer transactions toward the lower-cost option. However, if the fees are set too low, this may attract merchant who are intrinsically better off using the private platform - leading to inefficient allocation of users. Moreover, more merchants migrating to the public platform further dampens the network effects of the private platform. To mitigate these distortions, the public platform should adjust to a higher price in order to mitigate distortions of merchant allocations and weakening rival network effects.

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A Proofs

A.1 Lemmas

Lemma 1. *For $i, j \in \{1, 2\}$ and $i \neq j$, the following statements hold*

1. N_i^c, $1 - N_i^c$, N_i^m *and* $1 - N_i^m$ *are log concave functions of* t_i^c, t_j^c, t_i^m *and* t_j^m*;*
2. *For* $x \in \{c, m\}$, $\frac{\partial N_i^x}{\partial t_i^x} \leq 0$, $\frac{\partial N_i^x}{\partial t_j^x} \geq 0$, *and* $\frac{\partial N_i^x}{\partial t_i^x} + \frac{\partial N_i^x}{\partial t_j^x} \leq 0$*;*
3. $-\frac{1}{1-N_i^c}(\frac{\partial N_i^c}{\partial t_i^c})^2 \leq \frac{\partial^2 N_i^c}{(\partial t_i^c)^2} \leq \frac{1}{N_i^c}(\frac{\partial N_i^c}{\partial t_i^c})^2$ *and* $-\frac{1}{1-N_i^c}(\frac{\partial N_i^c}{\partial t_j^c})^2 \leq \frac{\partial^2 N_i^c}{(\partial t_j^c)^2} \leq \frac{1}{N_i^c}(\frac{\partial N_i^c}{\partial t_j^c})^2$*;*
4. $\mu_i^c \equiv -\frac{N_i^c}{\frac{\partial N_i^c}{\partial t_i^c}}$ *is decreasing in* t_i^c *and increasing in* t_j^c*, similarly* $\mu_i^m \equiv -\frac{N_i^m}{\frac{\partial N_i^m}{\partial t_i^m}}$ *is decreasing in* t_i^m *and increasing in* t_j^m*;*
5. $\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_j^c}{(\partial t_i^c)^2} \geq 0$, $\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_j^c}{(\partial t_i^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{(\partial t_i^c)^2} \geq 0$, *and* $\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \leq 0$*;*
6. *Define the diversion ratio as* $\eta_i^c \equiv -\frac{\partial N_i^c/\partial t_j^c}{\partial N_i^c/\partial t_i^c}$*, we have* $\frac{\partial \eta_i^c}{\partial t_i^c} \leq 0$.

Proof. 1. Implied by Theorem 1 in Caplin and Nalebuff (1991) when $\rho = 0$.

2. Implied by taking derivative of equation (15) and equation (14) with respect to t_i^c, t_j^c, t_i^m, and t_j^m.

3. Because N_i^c is a log-concave function of t_i^c, the second-order derivative of $log(N_i^c)$ is negative. That is

$$\frac{\partial^2 log(N_i^c)}{\partial (t_i^c)^2} = \frac{N_i^c \frac{\partial^2 N_i^c}{(\partial t_i^c)^2} - (\frac{\partial N_i^c}{\partial t_i^c})^2}{(N_i^c)^2} < 0$$

Because $1 - N_i^c$ is also a log-concave function of t_i^c, the second-order derivative of $log(1 - N_i^c)$ is negative. That is

$$\frac{\partial^2 log(1 - N_i^c)}{\partial (t_i^c)^2} = \frac{-(1 - N_i^c)\frac{\partial^2 N_i^c}{(\partial t_i^c)^2} - (\frac{\partial N_i^c}{\partial t_i^c})^2}{(1 - N_i^c)^2} < 0$$

Similar proof applies to N_i^m.

4. Because N_i^c is a log-concave function of t_i^c, the first-order derivative of $log(N_i^c)$ with respect to t_i^c is decreasing. That is

$$\frac{\partial log(N_i^c)}{\partial t_i^c} = \frac{\frac{\partial N_i^c}{\partial t_i^c}}{N_i^c}$$

is decreasing. Because $\frac{\partial N_i^c}{\partial t_i^c} < 0$, $\frac{N_i^c}{-\frac{\partial N_i^c}{\partial t_i^c}}$ is decreasing in t_i^c as well.

By taking derivative of μ_i^c with respect to t_j^c, one gets

$$\frac{\partial \mu_i^c}{\partial t_j^c} = \frac{-\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial N_i^c}{\partial t_j^c} + N_i^c \frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}}{(\partial N_i^c/\partial t_i^c)^2}$$

From previous lemma, we have

$$-\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial N_i^c}{\partial t_j^c} + N_i^c \frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \geq (\frac{\partial N_i^c}{\partial t_j^c})^2 - \frac{N_i^c}{1-N_j^c}(\frac{\partial N_j^c}{\partial t_i^c})^2 = \frac{1-N_i^c-N_j^c}{1-N_j^c}(\frac{\partial N_j^c}{\partial t_i^c})^2 \geq 0.$$

Therefore, μ_i^c is increasing in t_j^c. Similar proof applies to μ_i^m.

5. Because a log-concave function is also a quasi-concave function, the following second-order bordered hessian matrix of N_i^c has a non-negative determinant:

$$\begin{aligned} & det \begin{bmatrix} 0 & \frac{\partial N_i^c}{\partial t_i^c} & \frac{\partial N_i^c}{\partial t_j^c} \\ \frac{\partial N_i^c}{\partial t_i^c} & \frac{\partial^2 N_i^c}{\partial (t_i^c)^2} & \frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \\ \frac{\partial N_i^c}{\partial t_j^c} & \frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} & \frac{\partial^2 N_i^c}{(\partial t_j^c)^2} \end{bmatrix} \\ = & -\frac{\partial N_i^c}{\partial t_i^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}\right] + \frac{\partial N_i^c}{\partial t_j^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}\right] \geq 0. \end{aligned}$$

Suppose

$$-\frac{\partial N_i^c}{\partial t_i^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}\right] < 0$$

because the non-negative determinant we have

$$\frac{\partial N_i^c}{\partial t_j^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}\right] \geq 0.$$

The first equation implies

$$\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} > \frac{\frac{\partial N_i^c}{\partial t_i^c}}{\frac{\partial N_i^c}{\partial t_j^c}}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2}$$

which further implies

$$\begin{aligned} & -\frac{\partial N_i^c}{\partial t_i^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}\right] + \frac{\partial N_i^c}{\partial t_j^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}\right] \\ < & \quad (\frac{\partial N_i^c}{\partial t_j^c})^2\frac{\partial^2 N_i^c}{(\partial t_i^c)^2} - (\frac{\partial N_i^c}{\partial t_i^c})^2\frac{\partial^2 N_i^c}{(\partial t_j^c)^2}. \end{aligned}$$

However, the second equation implies

$$\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \leq \frac{\frac{\partial N_i^c}{\partial t_j^c}}{\frac{\partial N_i^c}{\partial t_i^c}} \frac{\partial^2 N_i^c}{(\partial t_i^c)^2}$$

which further implies

$$\begin{aligned} & -\frac{\partial N_i^c}{\partial t_i^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}\right] + \frac{\partial N_i^c}{\partial t_j^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}\right] \\ \geq \; & (\frac{\partial N_i^c}{\partial t_j^c})^2 \frac{\partial^2 N_i^c}{(\partial t_i^c)^2} - (\frac{\partial N_i^c}{\partial t_i^c})^2 \frac{\partial^2 N_i^c}{(\partial t_j^c)^2}, \end{aligned}$$

a contradiction. Therefore,

$$-\frac{\partial N_i^c}{\partial t_i^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c}\right] \geq 0,$$

which implies

$$\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \geq 0$$

because $-\frac{\partial N_i^c}{\partial t_i^c} > 0$ Following the similar proof we can show that

$$\frac{\partial N_i^c}{\partial t_j^c}\left[\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}\right] \geq 0,$$

because $\frac{\partial N_i^c}{\partial t_j^c} > 0$, we have

$$\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2} \geq 0.$$

Now we show that $\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} \leq 0$: Note that $\frac{\partial^2 N_i^c}{\partial t_j^c \partial t_i^c} = \frac{\partial^2 N_j^c}{(\partial t_i^c)^2}$, and with

$$\frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_i^c}{(\partial t_j^c)^2} - \frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \geq 0$$

and

$$\frac{\partial N_j^c}{\partial t_j^c}\frac{\partial^2 N_j^c}{(\partial t_i^c)^2} - \frac{\partial N_j^c}{\partial t_i^c}\frac{\partial^2 N_j^c}{\partial t_j^c \partial t_i^c} \geq 0$$

It is clear that both inequalities hold when $\frac{\partial^2 N_i^c}{\partial t_i^c \partial t_j^c} \leq 0$ and $\frac{\partial^2 N_j^c}{\partial t_j^c \partial t_i^c} \leq 0$.

6. Take derivative of diversion ratio η_i^c with respect to t_i^c, one get

$$\frac{\partial \eta_i^c}{\partial t_i^c} = \frac{\frac{\partial N_i^c}{\partial t_j^c}\frac{\partial^2 N_i^c}{(\partial t_i^c)^2} - \frac{\partial N_i^c}{\partial t_i^c}\frac{\partial^2 N_j^c}{(\partial t_i^c)^2}}{(\frac{\partial N_i^c}{\partial t_i^c})^2}$$

which is non-positive based on the pervious lemma.

□

A.2 Proof of Proposition 2

Given both platforms commit to the insulated pricing:

$$P_i^m = t_i^m + b_i^m N_i^c,$$

and

$$P_i^c = t_i^c + b_i^c N_i^m.$$

The profit function for platform i becomes

$$\pi_i(N_c, N_m) = (t_i^c + b_i^c N_i^m - k_i^c)N_i^c + (t_i^m + b_i^m N_i^c - k_i^m)N_i^m - c_i N_i^m N_i^c.$$

Under insulated prices, both N_i^m is a function of $\overrightarrow{t_m}$ and N_i^c is a function of $\overrightarrow{t_c}$ only, where $\overrightarrow{t_x} = \{t_x^1, t_x^2\}$. This is the case because under the insulated pricing, consumer and merchant optimization problems become

$$\max\{0, v_1^c - t_1^c, v_2^c - t_2^c\}$$

and

$$\max\{0, v_i^m - t_i^m\}$$

respectively.

The first order condition of π_i w.r.t

$$t_i^m : g_i^m \equiv b_i^c \frac{\partial N_i^m}{\partial t_i^m} N_i^c + N_i^m + (t_i^m + b_i^m N_i^c - k_i^m)\frac{\partial N_i^m}{\partial t_i^m} - c_i N_i^c \frac{\partial N_i^m}{\partial t_i^m} = 0$$

$$t_i^c : g_i^c \equiv b_i^m \frac{\partial N_i^c}{\partial t_i^c} N_i^m + N_i^c + (t_i^c + b_i^c N_i^m - k_i^c)\frac{\partial N_i^c}{\partial t_i^c} - c_i N_i^m \frac{\partial N_i^c}{\partial t_i^c} = 0$$

by solving equations above we get

$$\tilde{P}_i^m = k_i^m + (c_i - b_i^c)\tilde{N}_i^c - \frac{N_i^m}{\frac{\partial N_i^m}{\partial t_i^m}},$$

$$\tilde{P}_i^c = k_i^c + (c_i - b_i^m)\tilde{N}_i^m - \frac{N_i^c}{\frac{\partial N_i^c}{\partial t_i^c}}.$$

where $\tilde{N}_i^c$ and $\tilde{N}_i^m$ are jointly determined by the FOCs of platforms 1 and 2 in equilibrium.

Next we check the second-order conditions. The Hessian matrix is

$$H_i(t_i^m, t_i^c) = \begin{bmatrix} \frac{\partial g_i^m}{\partial t_i^m} & \frac{\partial g_i^m}{\partial t_i^c} \\ \frac{\partial g_i^c}{\partial t_i^m} & \frac{\partial g_i^c}{\partial t_i^c} \end{bmatrix}$$

We check if $\frac{\partial g_i^m}{\partial t_i^m}, \frac{\partial g_i^c}{\partial t_i^c} < 0$ and $\frac{\partial g_i^m}{\partial t_i^m}\frac{\partial g_i^c}{\partial t_i^c} - \frac{\partial g_i^c}{\partial t_i^m}\frac{\partial g_i^m}{\partial t_i^c} > 0$

$$\begin{aligned} \frac{\partial g_i^m}{\partial t_i^m} &= 2\frac{\partial N_i^m}{\partial t_i^m} + [t_i^m + (b_i^m + b_i^c - c_i)N_i^c - k_i^m]\frac{\partial^2 N_i^m}{\partial (t_i^m)^2} \\ &= \frac{\partial N_i^m}{\partial t_i^m}\left[2 - \frac{N_i^m}{(\partial N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2}\right] < 0 \end{aligned}$$

The last two inequalities come from the facts that (i) $\frac{\partial N_i^m}{\partial t_i^m} < 0$ and (ii) f_m is log-concave, so the N_i^m is log-concave, and then $-\frac{N_i^m}{\partial (N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2} \geq -1$. Similar proofs apply to $\frac{\partial g_i^c}{\partial t_i^c}$.

Now we look at

$$\begin{aligned} &\frac{\partial g_i^m}{\partial t_i^m}\frac{\partial g_i^c}{\partial t_i^c} - \frac{\partial g_i^c}{\partial t_i^m}\frac{\partial g_i^m}{\partial t_i^c} \\ = &\frac{\partial N_i^m}{\partial t_i^m}\frac{\partial N_i^c}{\partial t_i^c}\left[(2 - \frac{N_i^m}{(\partial N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2})(2 - \frac{N_i^c}{(\partial N_i^c/\partial t_i^c)^2}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2}) - \frac{\partial N_i^m}{\partial t_i^m}\frac{\partial N_i^c}{\partial t_i^c}(b_i^c + b_i^m - c_i)^2\right] \end{aligned}$$

The above equation is positive iff

$$(b_i^c + b_i^m - c_i)^2 \leq \frac{(2 - \frac{N_i^m}{(\partial N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2})(2 - \frac{N_i^c}{(\partial N_i^c/\partial t_i^c)^2}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2})}{\frac{\partial N_i^m}{\partial t_i^m}\frac{\partial N_i^c}{\partial t_i^c}}$$

It is sufficient to show that given $2 - \frac{N_i^m}{(\partial N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2} \geq 1 \geq N_i^m$ and $2 - \frac{N_i^c}{(\partial N_i^c/\partial t_i^c)^2}\frac{\partial^2 N_i^c}{\partial (t_i^c)^2} \geq$

$1 \geq N_i^c$

$$(b_i^c + b_i^m - c_i)^2 \leq \frac{N_i^m N_i^c}{\frac{\partial N_i^m}{\partial t_i^m}\frac{\partial N_i^c}{\partial t_i^c}}$$

which is the case because $b_i^c + b_i^m - c_i \leq \min\{\bar{\mu}_i^c, \bar{\mu}_i^m\}$ for all $t_i^c, t_c^j \in R^2$ and $t_i^m, t_m^j \in R^2$, μ_i^c is a decreasing function of t_i^c and increasing function of t_c^j, and μ_i^m is a decreasing function of t_i^m.

A.3 Proof of Proposition 3

When platform 1 becomes benevolent, the maximization problem of platform 1 is, by restricting to the insulated pricing:

$$\begin{aligned}\max_{t_1^m, t_1^c} \mathcal{W}(\overrightarrow{t_m}, \overrightarrow{t_c}) \;=\; & \omega_u \left[\sum_i \int_v \max\{0, v_i^m - t_i^m\} dF_m(v) + \int_v \max\{0, v_1^c - t_1^c, v_2^c - t_2^c\} dF_c(v)\right] \\ & + \left[(t_1^c - k_1^c)N_1^c + (b_1^c + b_1^m - c_1)N_1^m N_1^c + (t_1^m - k_1^m)N_1^m\right] \\ & + \omega_p \left[(t_2^c - k_2^c)N_2^c + (t_2^m - k_2^m)N_2^m + (b_2^c + b_2^m - c_2)N_2^m N_2^c\right].\end{aligned}$$

By taking the FOC of above w.r.t

$$t_1^m : g_1^m \equiv (b_1^c + b_1^m - c_1)N_1^c \frac{\partial N_1^m}{\partial t_1^m} + (t_1^m - k_1^m)\frac{\partial N_1^m}{\partial t_1^m} + (1 - \omega_u)N_1^m = 0$$

$$t_1^c : g_1^c \equiv (t_1^c - k_1^c)\frac{\partial N_1^c}{\partial t_1^c} + (b_1^c + b_1^m - c_1)N_1^m \frac{\partial N_1^c}{\partial t_1^c} + \omega_p (t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m)\frac{\partial N_2^c}{\partial t_1^c} + (1 - \omega_u)N_1^c = 0$$

solving above equations we get

$$\hat{t}_1^m = k_1^m + (c_1 - b_1^c - b_1^m)\hat{N}_1^c + (1 - \omega_u)\mu_1^m,$$

$$\hat{t}_1^c = k_1^c + (c_1 - b_1^c - b_1^m)\hat{N}_1^m + \omega_p \mu_2^c \eta_1^c + (1 - \omega_u)\mu_1^c.$$

where

$$\eta_1^c = -\frac{\partial N_2^c / \partial P_1^c}{\partial N_1^c / \partial P_1^c}$$

Now we check the second-order conditions for the benevolent platform:

$$\begin{aligned}\frac{\partial g_1^m}{\partial t_1^m} &= \frac{\partial N_1^m}{\partial t_1^m} + (t_1^m - k_1^m + (b_1^c + b_1^m - c_1)N_1^c)\frac{\partial^2 N_1^m}{(\partial t_1^m)^2} + (1-\omega_u)\frac{\partial N_1^m}{\partial t_1^m} \\ &= \frac{\partial N_1^m}{\partial t_1^m}(2 - \omega_u - \frac{N_i^m}{(\partial N_i^m/\partial t_i^m)^2}\frac{\partial^2 N_i^m}{\partial (t_i^m)^2}) \leq 0\end{aligned}$$

Next, we check if $\frac{\partial g_1^c}{\partial t_1^c} < 0$

$$\begin{aligned}\frac{\partial g_1^c}{\partial t_1^c} &= (2-\omega_u)\frac{\partial N_1^c}{\partial t_1^c} + [t_1^c - k_1^c + (b_1^c + b_1^m - c_1)N_1^m]\frac{\partial^2 N_1^c}{(\partial t_1^c)^2} + \omega_p(t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m)\frac{\partial^2 N_2^c}{(\partial t_1^c)^2} \\ &= (2-\omega_u)\frac{\partial N_1^c}{\partial t_1^c} + \omega_p \mu_2^c \eta_1^c \frac{\partial^2 N_1^c}{(\partial t_1^c)^2} + \omega_p \mu_2^c \frac{\partial^2 N_2^c}{(\partial t_1^c)^2} \\ &= \frac{\partial N_1^c}{\partial t_1^c}\left[2 - \omega_u - \omega_p \mu_2^c \frac{\frac{\partial N_1^c}{\partial t_2^c}\frac{\partial^2 N_1^c}{(\partial t_1^c)^2} - \frac{\partial N_1^c}{\partial t_1^c}\frac{\partial^2 N_2^c}{(\partial t_1^c)^2}}{(\frac{\partial N_1^c}{\partial t_1^c})^2}\right] = \frac{\partial N_1^c}{\partial t_1^c}(2 - \omega_u - \omega_p \mu_2^c \frac{\partial \eta_1^c}{\partial t_1^c}) \leq 0\end{aligned}$$

The last inequality holds because $\frac{\partial N_1^c}{\partial t_1^c} \leq 0$ and $\frac{\partial \eta_1^c}{\partial t_1^c} \leq 0$.

Next, we check if

$$\frac{\partial g_i^m}{\partial t_i^m}\frac{\partial g_i^c}{\partial t_i^c} - \frac{\partial g_i^c}{\partial t_i^m}\frac{\partial g_i^m}{\partial t_i^c} > 0$$

Note that

$$\frac{\partial g_i^c}{\partial t_i^m} = \frac{\partial g_i^m}{\partial t_i^c} = (b_1^c + b_1^m - c_1)\frac{\partial N_1^m}{\partial t_1^m}\frac{\partial N_1^c}{\partial t_1^c}$$

therefore,

$$\frac{\partial g_i^m}{\partial t_i^m}\frac{\partial g_i^c}{\partial t_i^c} - \frac{\partial g_i^c}{\partial t_i^m}\frac{\partial g_i^m}{\partial t_i^c} \geq \frac{\partial N_1^m}{\partial t_1^m}\frac{\partial N_1^c}{\partial t_1^c}(1 - (b_1^c + b_1^m - c_1)^2\frac{\partial N_1^m}{\partial t_1^m}\frac{\partial N_1^c}{\partial t_1^c})$$

With the sufficient condition of Proposition 1, we have the RHS greater than 0.

A.4 Proof of Proposition 4

Given the first-order condition of the benevolent platform, the benevolent adjustment is $d_2^c\eta_1^c$, where $d_2^c = t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m$. Given insulated pricing and a fixed t_2, the diversion ratio η_1^c does not change with b_2^c, b_2^m, k_2^c, or c_2, but it increases following a first-order stochastic dominance shift in the distribution of the private platform's consumer membership value, v_2^c. Conversely, the term d_2^c increases as b_2^c or b_2^m increases, decreases as c_2 or k_2^c increases, and remains invariant to changes in the distribution of v_2^c. Because both d_2^c and η_1^c are non-negative, their product $d_2^c\eta_1^c$ unambiguously increases with b_2^c and b_2^m, decreases with c_2 and k_2^c, and increases following a first-order stochastic dominance shift in the distribution of v_2^c.

A.5 Proof of Proposition 5

Given the first order condition of the benevolent platform, the logit demand suggests that the benevolent adjustment is $d_2^c\frac{N_2^c}{1-N_1^c}$, where $d_2^c = t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m$.

The benevolent adjustment decreases with λ_c given the fact that t_2^m and t_2^c are fixed so that d_2^c is fixed, and $\frac{N_2^c}{1-N_1^c} = \frac{exp(\frac{v_2^c - t_2^c}{\lambda_c})}{1+exp(\frac{v_2^c - t_2^c}{\lambda_c})}$ does not vary in t_1^c and is decreasing with λ_c.

A.6 Proof of Proposition 6

Denote $t^1 = \{t_1^c, t_1^m\}$ as the solution of platform 1's FOCs in the benevolent competition, and denote $\tilde{t}^1 = \{\tilde{t}_1^c, \tilde{t}_1^m\}$ as the solution in the private competition given t_2. Based on g_2^m, the best response of the benevolent platform is

$$t_1^m = k_1^m + (c_1 - b_1^c - b_1^m)N_1^c,$$

while the best response of the private platform is

$$\tilde{t}_1^m = k_1^m + (c_1 - b_1^c - b_1^m)\tilde{N}_1^c + \mu_i^m.$$

First, we show that $t_1^m \leq \tilde{t}_1^m$ and $P_1^m \leq \tilde{P}_1^m$:

$$\begin{aligned} \tilde{t}_1^m - t_1^m &= (c_1 - b_1^c - b_1^m)(\tilde{N}_1^c - N_1^c) + \mu_1^m \\ &\geq (c_1 - b_1^c - b_1^m) + \mu_1^m \geq 0 \end{aligned}$$

The last inequality holds because $b_1^c + b_1^m - c_1 \leq \bar{\mu}_1^m$.

Similarly

$$\begin{aligned} \tilde{P}_1^m - P_1^m &= (c_1 - b_1^c)(\tilde{N}_1^c - N_1^c) + \mu_1^m \\ &\geq (c_1 - b_1^c) + \mu_1^m \geq 0. \end{aligned}$$

We now show that $t_1^c \le \tilde{t}_1^c$. To show that, we use the proof by contradiction - assuming that $t_1^c > \tilde{t}_1^c$

By Taylor expansion of g_1^c and g_1^m around $\tilde{t}_1$, one gets

$$\begin{bmatrix} N_1^m(\tilde{t}_1, t_2) \\ \\ N_1^c(\tilde{t}_1, t_2) - d_2^c \frac{\partial N_2^c(\tilde{t}_1, t_2)}{\partial t_1^c} \end{bmatrix} = G \begin{bmatrix} t_1^m - \tilde{t}_1^m \\ \\ t_1^c - \tilde{t}_1^c \end{bmatrix}, \tag{23}$$

where $G = \begin{bmatrix} \frac{\partial g_1^m}{\partial t_1^m}(\hat{\tilde{t}}_1, t_2) & \frac{\partial g_1^m}{\partial t_1^c}(\hat{\tilde{t}}_1, t_2) \\ \\ \frac{\partial g_1^c}{\partial t_1^m}(\hat{\tilde{t}}_1, t_2) & \frac{\partial g_1^c}{\partial t_1^c}(\hat{\tilde{t}}_1, t_2) \end{bmatrix}$,$\hat{\tilde{t}}_1 \in \left[\tilde{t}_1, t_1\right]$, and $d_2^c = t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m$.

We now show that the left hand side of equation (23) are all positive: Note that N_1^m is non-negative, and thus it is enough to show that

$$N_1^c(t_1, t_2) - d_2^c \frac{\partial N_2^c(t_1, t_2)}{\partial t_1^c} \ge 0$$

for any t_1 and t_2.

By taking derivative of the left-hand-side of above equation with respect to t_2^c, one gets

$$\frac{\partial \left[N_1^c(t_1, t_2) - d_2^c \frac{\partial N_2^c(t_1, t_2)}{\partial t_1^c}\right]}{\partial t_2^c} = -d_2^c \frac{\partial^2 N_2^c}{\partial t_1^c \partial t_2^c} \ge 0,$$

with Lemma 5

$$\frac{\partial^2 N_2^c}{\partial t_1^c \partial t_2^c} \le 0.$$

Therefore, $N_1^c(t_1, t_2) - d_2^c \frac{\partial N_2^c(t_1, t_2)}{\partial t_1^c}$ is increasing in t_2^c.

Note that d_2^c is monotonously increasing w.r.t. t_2^c, hence when t_2^c reaches to the lowest value such that $d_2^c = 0$, we have

$$N_1^c(t_1, t_2) - d_2^c \frac{\partial N_2^c(t_1, t_2)}{\partial t_1^c} = N_1^c(t_1, t_2) > 0$$

Next, we show that $\frac{\partial g_1^c}{\partial t_1^c}(\hat{\tilde{t}}_1, t_2) < 0$.

$$\frac{\partial g_1^c}{\partial t_1^c}(\hat{\tilde{t}}_1, t_2) = \hat{\tilde{d}}_1^c \frac{\partial^2 N_1^c}{(\partial t_1^c)^2} + \frac{\partial N_1^c}{\partial t_1^c} + d_2^c \frac{\partial^2 N_2^c}{(\partial t_1^c)^2} \tag{24}$$

where $\hat{\hat{d}}_1^c = \hat{\hat{t}}_1^c - k_1^c + (b_1^c + b_1^m - c_1)N_1^m$.

Note that $\frac{\partial N_1^c}{\partial t_1^c} \leq 0$ and by Lemma 5, $\frac{\partial^2 N_2^c}{(\partial t_1^c)^2} = \frac{\partial^2 N_1^c}{\partial t_2^c \partial t_1^c} \leq 0$.

Therefore, if $\frac{\partial^2 N_1^c}{(\partial t_1^c)^2} \leq 0$, then $\frac{\partial g_1^c}{\partial t_1^c} \leq 0$ trivially.

Now we discuss the case that $\frac{\partial^2 N_1^c}{(\partial t_1^c)^2} \geq 0$, under which equation (24) is increasing w.r.t $\hat{\hat{d}}_1^c$. Because $\hat{\hat{t}}_1^c < t_1^c$, one has $\hat{\hat{d}}_1^c \leq -d_2^c \frac{\partial N_2^c/\partial t_1^c}{\partial N_1^c/\partial t_1^c}$, therefore, with the assumption that $\frac{\partial \eta_1^c}{\partial t_1^c} \leq 0$

$$d_1^c \frac{\partial^2 N_1^c}{(\partial t_1^c)^2} + d_2^c \frac{\partial^2 N_2^c}{(\partial t_1^c)^2} \leq d_2^c \frac{\frac{\partial N_2^c}{\partial t_1^c}\frac{\partial^2 N_1^c}{(\partial t_1^c)^2} - \frac{\partial^2 N_2^c}{(\partial t_1^c)^2}\frac{\partial N_1^c}{\partial t_1^c}}{-\frac{\partial N_1^c}{\partial t_1^c}} = -\frac{\partial N_1^c}{\partial t_1^c} d_2^c \frac{\partial \eta_1^c}{\partial t_1^c} \leq 0.$$

Based on the sign of derivatives, we show the contradiction.

Because of the assumption that $t_1^c > \tilde{t}_1^c$, we have

$$\frac{\partial g_1^c}{\partial t_1^m}(t_1^m - \tilde{t}_1^m) + \frac{\partial g_1^c}{\partial t_1^c}(t_1^c - \tilde{t}_1^c) \leq 0.$$

Because $t_1^m \leq \tilde{t}_1^m$ and $\frac{\partial g_1^c}{\partial t_1^m} \geq 0$, the first term is the non-positive. Similarly, because $\frac{\partial g_1^m}{\partial t_1^m} < 0$ and $t_1^c > \tilde{t}_1^c$, the second term is non-positive too.

But from the implicit function theorem above, we have

$$\frac{\partial g_1^c}{\partial t_1^m}(t_1^m - \tilde{t}_1^m) + \frac{\partial g_1^c}{\partial t_1^c}(t_1^c - \tilde{t}_1^c) = N_1^c(t^1, t^2) - d_2^c \frac{\partial N_2^c(t^1, t^2)}{\partial t_1^c} > 0.$$

A contradiction!

Now we show that given $c_1 < b_1^m$, $P_1^c < \tilde{P}_1^c$. Because $t_1^m < \tilde{t}_1^m$, there are more merchants joining platform 1 in BC than PC, therefore $N_1^m > \tilde{N}_1^m$.

However this means

$$P_1^c - \tilde{P}_1^c = t_1^c - \tilde{t}_1^c + b_1^c(N_1^m - \tilde{N}_1^m)$$

while $t_1^c - \tilde{t}_1^c < 0$ but $b_1^c(N_1^m - \tilde{N}_1^m) > 0$ which leads to ambiguity of whether P_1^c is greater or smaller than $\tilde{P}_1^c$.

A.7 Proof of Proposition 7

Now we show that given $t_1^c \leq \tilde{t}_1^c$ and $t_1^m \leq \tilde{t}_1^m$, the private platform will respond with lower t_2^c but higher t_2^m.

The prices of the private platform 2 are determined by the following two equations

$$g_2^m \equiv t_2^m - (c_2 - b_2^c - b_2^m)N_2^c - \mu_2^m = 0,$$

$$g_2^c \equiv t_2^c - (c_2 - b_2^c - b_2^m)N_2^m - \mu_2^c = 0.$$

and $t_1 = t_1^m, t_1^c$. Therefore, by the implicit function theorem, one gets the following.

$$\begin{bmatrix} \frac{\partial t_2^m}{\partial t_1^m} & \frac{\partial t_2^m}{\partial t_1^c} \\ \\ \frac{\partial t_2^c}{\partial t_1^m} & \frac{\partial t_2^c}{\partial t_1^c} \end{bmatrix} = - \begin{bmatrix} \frac{\partial g_2^m}{\partial t_2^m} & \frac{\partial g_2^m}{\partial t_2^c} \\ \\ \frac{\partial g_2^c}{\partial t_2^m} & \frac{\partial g_2^c}{\partial t_2^c} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial g_2^m}{\partial t_1^m} & \frac{\partial g_2^m}{\partial t_1^c} \\ \\ \frac{\partial g_2^c}{\partial t_1^m} & \frac{\partial g_2^c}{\partial t_1^c} \end{bmatrix}$$

$$= - \begin{bmatrix} 1 - \frac{\partial \mu_2^m}{\partial t_2^m} & -(c_2 - b_2^c - b_2^m)\frac{\partial N_2^c}{\partial t_2^c} \\ \\ -(c_2 - b_2^c - b_2^m)\frac{\partial N_2^m}{\partial t_2^m} & 1 - \frac{\partial \mu_2^c}{\partial t_2^c} \end{bmatrix}^{-1} \begin{bmatrix} 0 & -(c_2 - b_2^m - b_2^c)\frac{\partial N_2^c}{\partial t_1^c} \\ \\ 0 & -\frac{\partial \mu_2^c}{\partial t_1^c} \end{bmatrix}$$

$$= \Lambda^{-1} \begin{bmatrix} 1 - \frac{\partial \mu_2^c}{\partial t_2^c} & (c_2 - b_2^c - b_2^m)\frac{\partial N_2^c}{\partial t_2^c} \\ \\ (c_2 - b_2^c - b_2^m)\frac{\partial N_2^m}{\partial t_2^m} & 1 - \frac{\partial \mu_2^m}{\partial t_2^m} \end{bmatrix} \begin{bmatrix} 0 & (c_2 - b_2^m - b_2^c)\frac{\partial N_2^c}{\partial t_1^c} \\ \\ 0 & \frac{\partial \mu_2^c}{\partial t_1^c} \end{bmatrix}$$

$$= \Lambda^{-1} \begin{bmatrix} 0 & (c_2 - b_2^c - b_2^m)\left[(1 - \frac{\partial \mu_2^c}{\partial t_2^c})\frac{\partial N_2^c}{\partial t_1^c} + \frac{\partial N_2^c}{\partial t_2^c}\frac{\partial \mu_2^c}{\partial t_1^c}\right] \\ \\ 0 & (c_2 - b_2^c - b_2^m)^2\frac{\partial N_2^m}{\partial t_2^m}\frac{\partial N_2^c}{\partial t_1^c} + (1 - \frac{\partial \mu_2^m}{\partial t_2^m})\frac{\partial \mu_2^c}{\partial t_1^c} \end{bmatrix}$$

where Λ is the determinant of the matrix $\begin{bmatrix} \frac{\partial g_2^m}{\partial t_2^m} & \frac{\partial g_2^m}{\partial t_2^c} \\ \\ \frac{\partial g_2^c}{\partial t_2^m} & \frac{\partial g_2^c}{\partial t_2^c} \end{bmatrix}$ that is positive given the sufficient conditions that the profit function is concave.

The above equation implies that platform 2's prices do not respond to platform 1's price on merchants, because

$$\frac{\partial t_2^m}{\partial t_1^m} = 0, \frac{\partial t_2^c}{\partial t_1^m} = 0.$$

The platform 2's merchant price responds to the platform 1's consumer price in the following way

$$\frac{\partial t_2^m}{\partial t_1^c} = \Lambda^{-1}(c_2 - b_2^c - b_2^m)\left[(1 - \frac{\partial \mu_2^c}{\partial t_2^c})\frac{\partial N_2^c}{\partial t_1^c} + \frac{\partial N_2^c}{\partial t_2^c}\frac{\partial \mu_2^c}{\partial t_1^c}\right] \leq 0$$

The inequality above holds because $c_2 - b_2^c - b_2^m < 0$ and

$$(1 - \frac{\partial \mu_2^c}{\partial t_2^c})\frac{\partial N_2^c}{\partial t_1^c} + \frac{\partial N_2^c}{\partial t_2^c}\frac{\partial \mu_2^c}{\partial t_1^c} = -\frac{\partial \mu_2^c}{\partial t_2^c}\frac{\partial N_2^c}{\partial t_1^c} + N_2^c \frac{\frac{\partial^2 N_2^c}{\partial t_1^c \partial t_2^c}}{\frac{\partial N_2^c}{\partial t_2^c}} \geq 0$$

Finally, the platform 2's consumer price responds to the platform 1's consumer price in the following way

$$\frac{\partial t_2^c}{\partial t_1^c} = \Lambda^{-1}\left[(c_2 - b_2^c - b_2^m)^2 \frac{\partial N_2^m}{\partial t_2^m}\frac{\partial N_2^c}{\partial t_1^c} + (1 - \frac{\partial \mu_2^m}{\partial t_2^m})\frac{\partial \mu_2^c}{\partial t_1^c}\right] > 0$$

The above inequality holds because

$$\begin{aligned}
\frac{\partial \mu_2^c}{\partial t_1^c} &= \frac{-\frac{\partial N_2^c}{\partial t_2^c}\frac{\partial N_2^c}{\partial t_1^c} + N_2^c \frac{\partial^2 N_2^c}{\partial t_1^c \partial t_2^c}}{(\frac{\partial N_2^c}{\partial t_2^c})^2} \\
&\geq -\frac{\frac{\partial N_2^c}{\partial t_1^c}}{\frac{\partial N_2^c}{\partial t_2^c}}\frac{N_2^c}{1 - N_2^c} > -\frac{\frac{\partial N_2^c}{\partial t_1^c}}{\frac{\partial N_2^c}{\partial t_2^c}} N_2^c \\
&= \mu_2^c \frac{\partial N_2^c}{\partial t_1^c} \geq (b_2^c + b_2^m - c_2)\frac{\partial N_2^c}{\partial t_1^c}
\end{aligned}$$

Given that $t_1^c \leq \tilde{t}_1^c$, we have $t_2^m \geq \tilde{t}_2^m$ and $t_2^c \leq \tilde{t}_2^c$.

A.8 Proof of Proposition 8

When the benevolent platform sets zero price on the consumer side, that is

$$P_1^c = t_1^c + b_1^c N_1^m = 0$$

which implies that

$$t_1^c = -b_1^c N_1^m$$

The maximization problem of the benevolent platform is

$$\max_{t_1^m} Wel(\overrightarrow{t_m}, \overrightarrow{t_c}) = \left[\sum_i \int_v \max\{0, v_i^m - t_i^m\} dF_m(v) + \int_v \max\{0, v_1^c + b_1^c N_1^m, v_2^c - t_2^c\} dF_c(v)\right]$$
$$+ [-k_1^c N_1^c + (b_1^m - c_1) N_1^m N_1^c + (t_1^m - k_1^m) N_1^m]$$
$$+ [(t_2^c - k_2^c) N_2^c + (t_2^m - k_2^m) N_2^m + (b_2^c + b_2^m - c_2) N_2^m N_2^c]$$

The FOC implies the same pricing equation on the merchant side as in the case without the consumer price cap

$$t_1^m = k_1^m + (c_1 - b_1^c) N_1^c.$$

To find sufficient conditions for the benevolent platform to improve the market welfare from the PC, we look into the concavity of the social welfare function.

Fix the platform prices at (t_2^m, t_2^c), the welfare function is concave in (t_1^m, t_1^c).Denote $(\hat{t}_1^m, \hat{t}_1^c)$ and $(\tilde{t}_1^m, \tilde{t}_1^c)$ as two prices set by the platform 1, we have the following equation on the difference in welfare under two prices:

$$Wel(\hat{t}_1^m, \hat{t}_1^c) - Wel(\tilde{t}_1^m, \tilde{t}_1^c) \leq \nabla Wel(\tilde{t}_1^m, \tilde{t}_1^c)(\hat{t}_1 - \tilde{t}_1)$$

where $\nabla Wel(\tilde{t}_1^m, \tilde{t}_1^c)$ is the gradient of the welfare function.

Suppose that $\hat{t}_1^m$ and $\hat{t}_1^c$ are prices of platform 1 in PC, and $\tilde{t}_1^m$ and $\tilde{t}_1^c$ are prices set by the benevolent platform with the constraint, to prove that

$$Wel(\hat{t}_1^m, \hat{t}_1^c) - Wel(\tilde{t}_1^m, \tilde{t}_1^c) < 0$$

it is sufficient to show that

$$\nabla Wel(\tilde{t}_1^m, \tilde{t}_1^c)(\hat{t}_1 - \tilde{t}_1) < 0.$$

Because $\hat{t}_1^c > \tilde{t}_1^c$,

$$\frac{\partial Wel}{\partial t_1^c}(\tilde{t}_1^m, \tilde{t}_1^c) = [(b_1^m - c_1) N_1^m - k_1^c] \frac{\partial N_1^c}{\partial t_1^c} + \mu_2^c \frac{\partial N_2^c}{\partial t_1^c},$$

and

$$\frac{\partial Wel}{\partial t_1^m}(\tilde{t}_1^m, \tilde{t}_1^c) = 0.$$

It is clear that the condition holds when

$$k_1^c + (c_1 - b_1^m)N_1^m + \mu_2^c \eta_1^c < 0.$$

A.9 Proof of Proposition 9

When the benevolent platform is subject to cost-recovery objective, that is

$$(P_1^c - k_1^c)N_1^c + (P_1^m - k_1^m)N_1^m - c_1 N_1^c N_1^m = 0.$$

The cost-recovery condition implies, under the insulated equilibrium, that the platform sets t_1^c and t_1^m such that

$$(t_1^c - k_1^c)N_1^c + (t_1^m - k_1^m)N_1^m + (b_1^c + b_1^m - c_1)N_1^c N_1^m = 0.$$

The benevolent platform's maximization problem becomes

$$\begin{aligned}\max_{t_1^m, t_1^c} Wel(\overrightarrow{t_m}, \overrightarrow{t_c}) \quad = \quad & \left[\sum_i \int_v \max\{0, v_i^m - t_i^m\} dF_m(v) + \int_v \max\{0, v_1^c - t_1^c, v_2^c - t_2^c\} dF_c(v)\right] \\ & + \left[(t_1^c - k_1^c)N_1^c + (b_1^c + b_1^m - c_1)N_1^m N_1^c + (t_1^m - k_1^m)N_1^m\right] \\ & + \left[(t_2^c - k_2^c)N_2^c + (t_2^m - k_2^m)N_2^m + (b_2^c + b_2^m - c_2)N_2^m N_2^c\right],\end{aligned}$$

$$s.t. \quad (t_1^c - k_1^c)N_1^c + (t_1^m - k_1^m)N_1^m + (b_1^c + b_1^m - c_1)N_1^c N_1^m = 0$$

By taking the FOC of above w.r.t

$$t_1^m : g_1^m \equiv (1-\varphi)N_1^m + (b_1^c + b_1^m - c_1)N_1^c \frac{\partial N_1^m}{\partial t_1^m} + (t_1^m - k_1^m)\frac{\partial N_1^m}{\partial t_1^m} = 0$$

$$t_1^c : g_1^c \equiv (1-\varphi)N_1^c + (t_1^c - k_1^c)\frac{\partial N_1^c}{\partial t_1^c} + (b_1^c + b_1^m - c_1)N_1^m \frac{\partial N_1^c}{\partial t_1^c} + \varphi(t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m)\frac{\partial N_2^c}{\partial t_1^c} = 0$$

where φ is the Lagrangian multiplier.

The FOCs imply that

$$P_1^m = k_1^m + (c_1 - b_1^c)N_1^c + (1-\varphi)\mu_1^m$$

and

$$P_1^c = k_1^c + (c_1 - b_1^m)N_1^m + (1-\varphi)\mu_1^c + \varphi\mu_2^c\eta_1^c$$

Substituting the FOCs into the cost recovery condition yields:

$$\Pi_1 = (1-\varphi)(\mu_1^m N_1^m + \mu_1^c N_1^c) + (c_1 - b_1^c - b_1^m)N_1^c N_1^m + \varphi(t_2^c - k_2^c + (b_2^c + b_2^m - c_2)N_2^m)\eta_1^c N_1^c = 0$$

which implies

$$\varphi = \frac{\mu_1^m N_1^m + \mu_1^c N_1^c - (b_1^c + b_1^m - c_1)N_1^c N_1^m}{\mu_1^m N_1^m + \mu_1^c N_1^c - \mu_2^c \eta_1^c N_1^c}$$

A.10 Proof of Proposition 10

Similar to the proof of Proposition 8, it is enough to show that

$$\nabla Wel(\tilde{t}_1^m, \tilde{t}_1^c)(\hat{t}_1 - \tilde{t}_1) < 0.$$

Given the assumptions, the benevolent platform would set a lower prices in BC compare to the private platform in PC. Therefore

$$\hat{t}_1^c - \tilde{t}_1^c > 0, \hat{t}_1^m - \tilde{t}_1^m > 0$$

At the benevolent pricing ($\tilde{t}_1$), the gradient of the welfare function is

$$\frac{\partial Wel}{\partial t_1^c}(\tilde{t}_1^m, \tilde{t}_1^c) = -(1-\varphi)N_1^c - \varphi\mu_2^c \frac{\partial N_2^c}{\partial t_1^c} < 0,$$

and

$$\frac{\partial Wel}{\partial t_1^m}(\tilde{t}_1^m, \tilde{t}_1^c) = -N_1^m < 0.$$

Above all, we have

$$Wel(\hat{t}_1^m, \hat{t}_1^c) < Wel(\tilde{t}_1^m, \tilde{t}_1^c).$$

B Endogenous network benefits and surcharging

This section shows a flexible model of payments market that include two important features: i) platforms can endogenously choose their per-transaction benefits b_i^c and b_i^m; ii) whether merchants are required to set equal prices across platforms and the outside option. We show that the main conclusions derived from the baseline model do not depend on whether these features are included.

To include the first feature, it is natural to assume that platforms increasing the per-transaction benefit also face a higher marginal cost per transaction. Therefore, the per-transaction function c_i is a function of b_i^c and b_i^m and we assume that the function is convex. To include the second feature, we assume that each merchant produces a product/service that is valued at ν to all consumers, and in each transaction, a consumer who purchases the final product / service is charged for a price f and a surcharge τ_i. The surcharge can be different across platforms if merchants are allowed to set different prices across platforms, while if merchants are asked to charge the same price across platforms (price coherence), the surcharge is fixed at 0.

Therefore, each consumer choosing platform i receives following utility:

$$u_i^c = v_i^c + (b_i^c - r_i^c - f - \tau_i)N_i^m - t_i^c \tag{25}$$

and each merchant choosing platform i receives following utility:

$$u_i^m = v_i^m + (b_i^m - r_i^m + f + \tau_i)N_i^c - t_i^m \tag{26}$$

Platforms set prices by committing to insulated pricing, that is, the platform sets per-transaction fees that make users face a dominant decision of participation regardless of how many users participating on the other side. Given the insulated pricing commitment, we have

$$r_i^m = b_i^m + \tau_i \tag{27}$$

$$r_i^c = b_i^c - \tau_i \tag{28}$$

The platform profit then becomes

$$\pi_i = t_i^c N_i^c + t_i^m N_i^m + (b_i^m + b_i^c - c_i) N_i^m N_i^c. \tag{29}$$

Due to the property of insulated pricing, N_i^m and N_i^c are not functions of b_i^m and b_i^c. Therefore, the platform maximizes its profit by choosing b_i^m and b_i^c such that $b_i^m + b_i^c - c_i$ is maximized, and because the cost function is convex, the maximum exists. It is then clear that the platform sets its network benefits separately from its decisions on fixed fees t_i^c and t_i^m. Given the benefits of the fixed network, the platform has the same profit function as the baseline model. Hence, the same analysis of the baseline model still applies to this more flexible model.