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Modelling the Sovereign Debt Strategy: A Practical Primer

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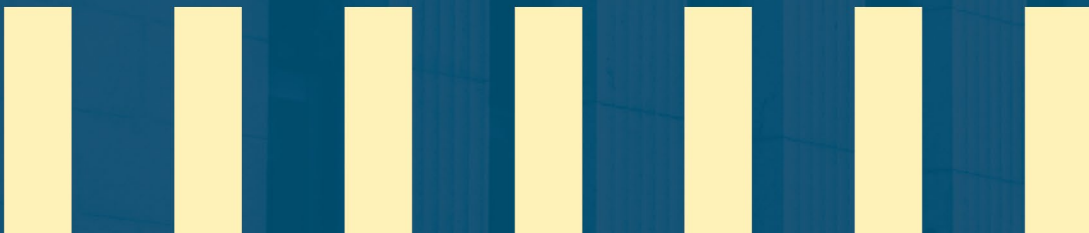
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Abstract

This paper provides a primer on the role of debt modelling in developing a sovereign debt issuance strategy, and how the policy objectives of a sovereign debt manager influence design decisions within their models. The insights provided here are supported by current and past uses of the Canadian Debt Strategy Model, which is a key component of Canada's process to set its annual Debt Management Strategy and Medium-Term Debt Strategy. We address specific challenges that issuers of public debt often face. Those challenges include defining an appropriate objective function, specifying a strategy adaptable to an uncertain economic environment, operating within computational limitations and integrating qualitative considerations about liquidity and the needs of the investor base with quantitative assessments of costs and risks.

Topics: Debt management; Econometric and statistical methods; Financial markets; Fiscal policy

JEL codes: G11, G17, H63, H68

Résumé

Cette étude explique le rôle de la modélisation de la dette dans l'élaboration d'une stratégie d'émission de titres souverains, et la façon dont les objectifs de politique d'un gestionnaire de dette souveraine influencent les décisions concernant la conception de ses modèles. Les observations présentées ici sont étayées par l'utilisation actuelle et passée du modèle canadien de gestion de la dette – un élément clé du processus mis en place par le Canada pour établir sa stratégie annuelle de gestion de la dette et sa stratégie de gestion de la dette à moyen terme. Nous abordons des défis particuliers auxquels les émetteurs de titres de dette publique sont souvent confrontés. Ces défis sont les suivants : définir une fonction objectif appropriée, déterminer une stratégie pouvant être adaptée à un environnement économique incertain, fonctionner dans les limites des capacités de calcul, et intégrer des considérations qualitatives sur la liquidité et les besoins des investisseurs avec des évaluations quantitatives des coûts et des risques.

Sujets : Gestion de la dette; Méthodes économétriques et statistiques; Marchés financiers; Politique budgétaire

Codes JEL : G11, G17, H63, H68

1 Introduction

Managing a sovereign's domestic debt portfolio is a critical and complex task that requires careful planning over both short and long horizons. In Canada, this task is guided by the creation of an annual Debt Management Strategy (DMS) and a five-year Medium-Term Debt Strategy (MTDS). Together, the DMS and MTDS help achieve the dual objectives of raising stable, low-cost funding for the federal government and maintaining well-functioning markets.

The core of the annual DMS is deciding how that year's total sovereign issuance will be allocated between different debt instruments. For that purpose, debt modelling plays a crucial role by quantifying the trade-offs between issuing each instrument. In doing so, modelling offers debt managers a set of efficient strategies to choose from based on their preferences.

The original steady-state Canadian Debt Strategy Model (CDSM) was developed more than a decade ago (Bolder and Deeley 2011). Since then, Canada's stock of debt has systematically increased, macro-financial uncertainty has risen and computing technology has advanced. Together, these issues motivated the creation of the enhanced Dynamic CDSM (Audet et al. 2025). It considers the current and projected environment and allows the model's portfolios to be state-dependent, which is now a key component of the process to set Canada's DMS and MTDS.

Unlike Bolder and Deeley (2011) and Audet et al. (2025) who provide more comprehensive, technical primers intended for modelling specialists, we aim to help debt policy-makers understand how the modelling is integrated into the debt strategy process. Specifically, we explain how the policy objectives of the DMS and MTDS, along with current economic, market and technological realities, drive how the CDSM is designed and applied.

The outline of the paper is as follows. [Section two](#) discusses the key debt management questions and how modelling can help on both a strategic and tactical basis. [Section three](#) discusses the CDSM design choices around quantifying the debt manager's objective function. [Section four](#) discusses design choices around simulating an uncertain environment and leveraging those simulated results with state-dependent strategies. [Section five](#) discusses design choices to adapt to computational limitations. [Section six](#) discusses design choices around qualitative inputs for a debt strategy. [Section seven](#) concludes.

2 Overview of the Canadian model and process to set a debt strategy

2.1 Role of modelling in the debt strategy

A sovereign's debt strategy is defined by how it allocates debt issuance across different instruments. While those instruments vary solely by term in Canada, in some countries they include foreign currencies and inflation-linked bonds.^{1, 2} The core sectors in Canada are 3-, 6- and 12-month treasury bills, and 2-, 5-, 10- and 30-year bonds.

The DMS helps the federal government achieve its first objective of stable, low-cost funding through “a balanced debt structure that contributes to maintaining the stability of debt costs and to reducing the risk of the debt portfolio” (Department of Finance Canada 2025).

Based on this, Canada's debt strategy model in practice takes the form of a cost-risk portfolio optimization from the issuer's standpoint.

Canada's commitment to fostering well-functioning markets and ensuring stable, predictable issuance underpins the government's stable, low-cost framework. Garbade (2007) finds that moving from tactical issuance to regular and predictable issuance patterns in the United States reduced market uncertainty, thereby helping to minimize the overall cost of borrowing. This principle is reflected in Canada's portfolio constraints, such as minimum issuance requirements.

This micro-portfolio approach is a simplification in that it treats debt management decisions as separate from the government's broader fiscal policy decisions, which is consistent with the trend since the 1990s of most countries establishing independent offices to manage debt (Blommestein and Hubig 2012). For Canada, it also reflects the specific role of the Bank of Canada—which is responsible for developing the model and interpreting its outputs—in the decision-making process. As the government's fiscal agent, the Bank's mandate is mostly limited to advising on the micro approach.

This approach also doesn't consider the welfare implications of debt issuance on end investors—such as their returns from holding the bonds and how well those align with their own portfolio objectives. Thus, consulting directly with market participants in Canada (Bank of Canada 2025) is an important part of the setting the debt strategy so market demand factors are well-integrated with model results in the final DMS.

¹ Canada does issue debt in foreign currencies. However, the proceeds are used to maintain liquid foreign reserves. This type of issuance is excluded from the CDSM and is not covered in the DMS or the MTDS.

² Canada stopped issuing real return bonds in 2022. At the time of writing, about \$70 billion was outstanding.

2.2 The baseline Canadian model

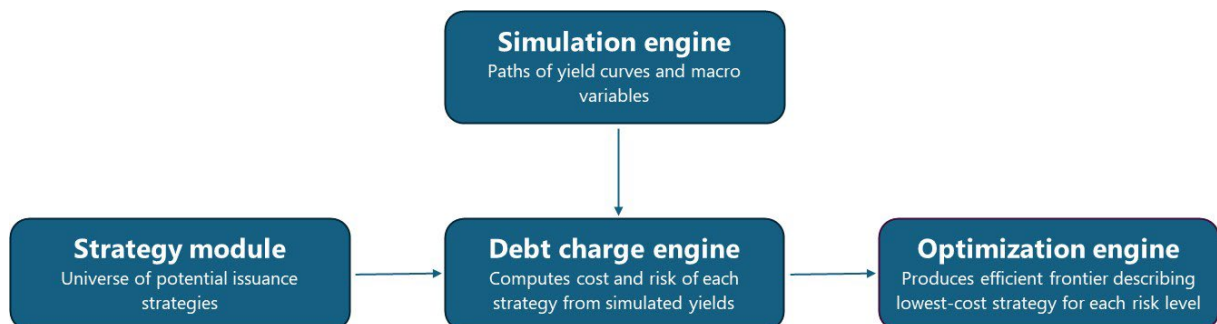
Bolder and Deeley (2011) provide a foundational overview of the baseline CDSM, a simulation-based framework to formulate issuance strategies that balance minimizing costs with managing risk.

At its core, the CDSM is structured around four interdependent components (**Figure 1**):

- a simulation engine
- a strategy module
- a debt charge engine
- an optimization engine

The CDSM's simulation engine jointly simulates the evolution of Nelson-Siegel yield curve factors (level, slope and curvature) and macroeconomic variables (primary deficit, inflation, gross domestic product) in a vector autoregression (VAR) framework. This is done across thousands of simulations, capturing the inherent uncertainty in each of those variables. These scenarios serve as the backdrop against which various issuance strategies—defined by weights across sectors—are tested.

Figure 1: The Canadian Debt Strategy Model's components interact to provide debt managers with efficient strategies



The strategy module constructs these issuance profiles, which are constrained to ensure they align with broader policy objectives (e.g., maintaining market liquidity). These strategies are deterministic (fixed over time) in this baseline model. The debt charge engine then evaluates each strategy across those thousands of simulated paths to calculate the associated cost and risk metrics. Finally, the optimization engine finds the lowest-cost strategy for each risk level to produce the efficient frontier.

By providing an efficient frontier of potential issuance strategies instead of prescribing a single optimal solution, the CDSM allows policy-makers to incorporate their preferences and qualitative considerations into the final strategy.

Later sections of this paper will dive into specific important decisions that were made while designing these components to address common challenges and ensure the model's realism and policy relevance.

2.3 The MTDS and DMS and how they are integrated

The World Bank and International Monetary Fund (2019) recommend that all sovereign debt managers have a medium-term debt strategy—which aims to achieve a desired debt composition over multiple years—to guide their decisions. The medium-term framework provides a structured approach to evaluate different debt strategies and their cost and risk implications in a future-focused way that acknowledges the inherent uncertainty in the macro-financial environment.

In Canada, the MTDS uses a 15-year period to evaluate the costs and risks of different debt strategies. The first five years are dedicated to transitioning from the current year's issuance to the medium-term target. This transition phase is crucial because it allows debt managers to gradually adjust their issuance strategies, thus maintaining the necessary stability and predictability of sovereign debt issuance.

More generally, because sovereign debt cannot be easily bought back (unlike corporate debt), the decisions made today have long-lasting implications for future debt management strategies. The forward-looking approach in the MTDS ensures that the current year's debt strategy, the DMS, is sustainable and aligned with long-term objectives.

Canada's annual debt strategy process starts by running the MTDS analysis with updated fiscal and macroeconomic projections. From there, policy-makers select one MTDS target from a set of cost-risk efficient portfolios within pre-defined ranges of risk tolerance. The first year of the MTDS is then used as an analytical grounding for the issuance amounts in the annual DMS (**Table 1**). The final DMS builds in any further adjustments to reflect feedback from consultations.

Table 1: Hypothetical issuance path under a high-risk efficient frontier point

	t-bill stock	2-year	5-year	10-year	30-year
Year 1 (DMS)	50.1%	19.2%	13.4%	13.4%	3.8%
Year 2	54.9%	18.0%	12.0%	11.3%	3.9%
Year 3	56.8%	17.7%	11.5%	10.2%	3.8%
Year 4	60.7%	17.1%	10.2%	8.3%	3.7%
Year 5 (MTDS)	64.1%	17.1%	8.8%	6.6%	3.4%

Note: Debt Management Strategy (DMS) is for Year 1. The Medium-Term Debt Strategy (MTDS) is for Year 5 and beyond. The 30-year bond issuance is set to a fixed amount in this example. The issuance values are at the fifth year of the horizon and are as a share of the total stock of treasury bills (t-bills) and bond issuance.

Preliminary analysis for the MTDS is typically conducted several months before the release of the federal budget using current financial requirements published in the latest non-budgetary economic update. The MTDS is finalized before release of the budget. The DMS is then fine-tuned in the months leading up to its publication in the federal budget (**Table 2**).

Table 2: Canada's Debt Management Strategy over three fiscal years

Can\$ billions

	FY2024-25	FY2025-26	FY2026-27
Stock of treasury bills at fiscal year-end	285	293	291
2-year	94	120	110
5-year	63	84	80
10-year	63	84	80
30-year	17	24	24
10-year, green bond	4	4	4
Total bonds	241	316	298
Total issuance	526	609	589
Long bonds (10-year +) as a share of total bonds	33%	34%	35%
Treasury bills as a share of total issuance	54%	48%	49%

Note: FY is fiscal year. For simplicity, green bonds are generally modelled as part of the 10-year sector.

Source: Department of Finance Canada

By jointly managing the MTDS and DMS, debt managers can balance:

- strategic considerations, which involve long-term planning and alignment with policy objectives and cost-risk preferences
- tactical decisions, which require agility and responsiveness to the short-term environment

Balancing these two issues also helps reduce biases in the DMS process because it forces policy-makers to provide a reason for any deviations from the MTDS. Ultimately, this joint MTDS-DMS approach allows debt managers to optimize their debt strategy to achieve a sustainable long-term path of issuance that is both stable and flexible. This ensures that the government's issuance can remain predictable while still being adaptable to changing circumstances.

2.4 CDSM enhancements to improve the MTDS–DMS process

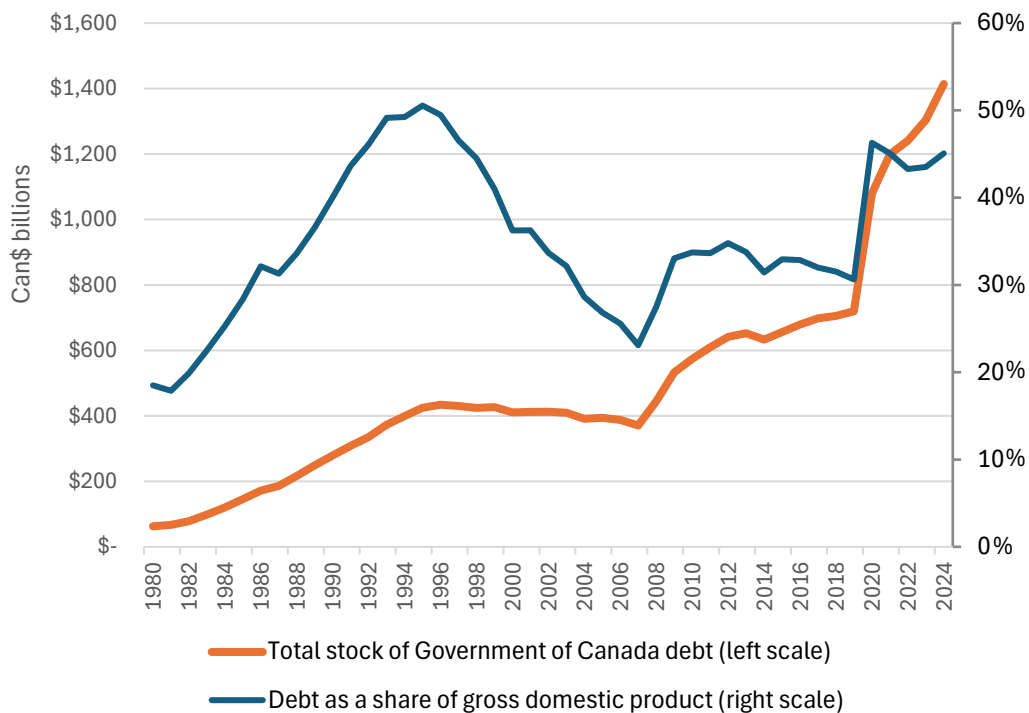
The nature of the model means that it should always be changing to best represent the current reality. In the last 15 years, the CDSM had undergone several changes from the baseline model (see [section 2.2](#)) to adapt to changing fiscal and computation conditions and to better assist with helping achieve policy objectives. Broadly, these changes improved the CDSM’s realism and supported the joint MTDS-DMS process outlined in [section 2.3](#).

2.4.1 From hypothetical equilibrium to transition model

The baseline CDSM was developed in an environment where the Government of Canada (GoC) was running surpluses and the federal budget forecasted balanced budgets in future years. Therefore, the model assumed that, in the steady state, budgets would be balanced. Given the stable fiscal outlook and computational constraints at the time, the model made a reasonable simplification where it assumed a hypothetical equilibrium where the target portfolio was fully implemented.

This design suited the context of the time, when the model’s sole objective was to define an MTDS under the conditions of a balanced budget. Therefore, the model focused on steady-state allocations where the allocation had already reached its hypothetical equilibrium. Specifying a transition path from the current issuance pattern to the MTDS, as described in [section 2.3](#), was left for future work. Cost and risk metrics were calculated as if the portfolio had always been in equilibrium without accounting for the refinancing of the existing stock of debt. Since the model’s release, the stock of GoC debt has risen steadily, making the assumption of a steady-state allocation less realistic (**Chart 1**).

Chart 1: Stock of Government of Canada debt rose steadily starting in 2010, and sharply since 2020



Sources: Statistics Canada and Bank of Canada

Last observation: December 2024

Changes in the fiscal environment created more challenges to interpreting outputs from the steady-state model in a way relevant for designing the MTDS, particularly for senior decision-makers. The absence of a transition path meant that bridging the MTDS and DMS required a manual process. These evolving requirements have created the need for a framework that incorporates a transition path and accounts for current portfolio dynamics.

The updated transition model addresses these limitations. It does so by using the outstanding debt portfolio as the starting point and building in a transition path to the MTDS target (with transition speed as a choice variable in the model). The transition reflects any shifts in total allocation to reach the target and any increases in outstanding debt.

By jointly optimizing the medium-term target strategy and the path to reach it, the model can evaluate the longer-term impact of issuance during the transition period—specifically, the first year of transition that informs the DMS. The CDSM can therefore solve for the MTDS and DMS issuance allocations at the same time.

2.4.2 From deterministic to state-dependent model

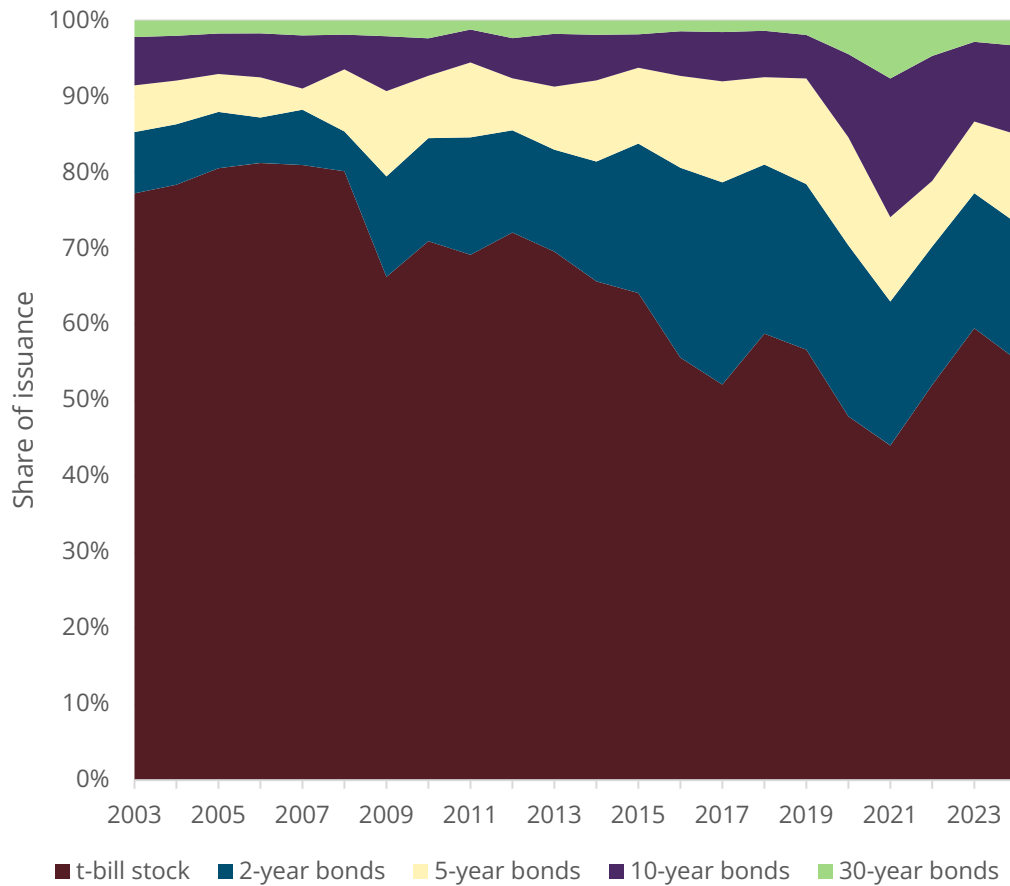
Even with a transition model (see [section 2.4.1](#)), debt managers still rely on a fixed path of allocations to guide the issuance for both the MTDS and the DMS. And committing to a set transition to the MTDS is not always feasible because unknown short-term factors could arise in the future that the debt manager will want to react to.

The solution is to enhance the definition of a debt strategy to include a state-dependent component. Instead of treating a strategy as just a deterministic allocation in each sector (over multiple years), the strategy should also include parameters on how to adjust to various prevailing macro-financial variables. This allows the model to set a long-term strategy while adjusting to short-term changes.

Audet et al. (2025) summarize the mechanics of this dynamic enhancement. These state-dependent strategies are more cost-risk efficient and provide better guidance for the DMS on whether and how to deviate from the MTDS path based on short-term market conditions. They also make the MTDS itself more realistic because it now builds in an accurate expectation that the debt manager will deviate from the path in the future depending on the realized state or states. **Chart 2** shows a considerable lack of smoothness in the historical issuance mix, reflecting frequent deviations.³

³ We observe a lack of historical year-to-year smoothness in the issuance allocations published in the annual DMS.

Chart 2: Annual Government of Canada issuance mix has not followed a smooth path historically



Note: For t-bills, the number refers to year-end t-bill stock.

Source: Bank of Canada calculations

Last observation: June 2025

3 Quantifying the debt manager's objective

Under the portfolio optimization set-up of the debt manager's problem, the objective function is to minimize the cost and risk of the debt portfolio—reflecting the goal for stable, low-cost funding. The trade-off between cost and risk is not strictly defined and depends on the debt manager's risk preferences.

The specific cost and risk metrics used in the CDSM have not changed significantly since Bolder and Deeley (2011). This section will focus on the intuition of these metrics and how they impact portfolio outcomes.

3.1 Defining cost

The core assumption behind a cost metric for a sovereign debt portfolio is that the issuer doesn't typically buy back their debt—similar to an investor's buy-and-hold

portfolio.⁴ This is in line with the debt manager's responsibility to maintain a supply of outstanding debt at each maturity point that is predictable, stable and sufficiently large at all times to ensure well-functioning markets. Thus, any valuation changes of the bonds between issuance and maturity dates—known as mark-to-market value—is much less relevant.

Therefore, cost is primarily measured by *debt-service charges*, which include interest costs on treasury bills and bonds, plus inflation adjustments for inflation-linked bonds.⁵ Because all instruments are assumed to be issued at par, the interest costs for nominal debt are captured entirely through quarterly coupon payments.^{6, 7} The par rate of each bond is obtained from the simulated yield curve at the time of issue, which is constructed from the prevailing values for the Nelson-Siegel factors (see [section 2.2](#)). This definition of cost focuses on the actual cash flows required to service the debt rather than changes in the market value.

This cost measure is a simplification in that it only captures the tangible financial costs to the federal government. It does not consider the cost to society nor the dead weight loss of a strategy. Those other factors of liquidity, yield curve maintenance, minimizing market distortion, supporting other markets (e.g., repo) and investor preferences, among others, are instead reflected through:

- the model's constraints (see [section 6](#))
- adjustments to the model results in the final DMS

Defining cost this way delineates the quantitative considerations from the qualitative and allows a simple, objective and universally understood basis for debt managers' discussions on cost-risk preferences and trade-offs.

The original CDSM did not discount debt costs because it was not feasible or relevant in the hypothetical steady-state context. With the updated CDSM being based on the current time and projected future, an argument can be made to take costs at present value. Still, undiscounted cost remains the baseline because it is simpler and the

⁴ Note that Canada conducts Cash Management Bond Buybacks. These operations target bonds close to maturity to help smooth out the cash requirement and minimize the need for large on-time cash outflows.

⁵ Inflation-linked bonds are no longer part of Canada's strategy, but the outstanding amounts are still accounted for.

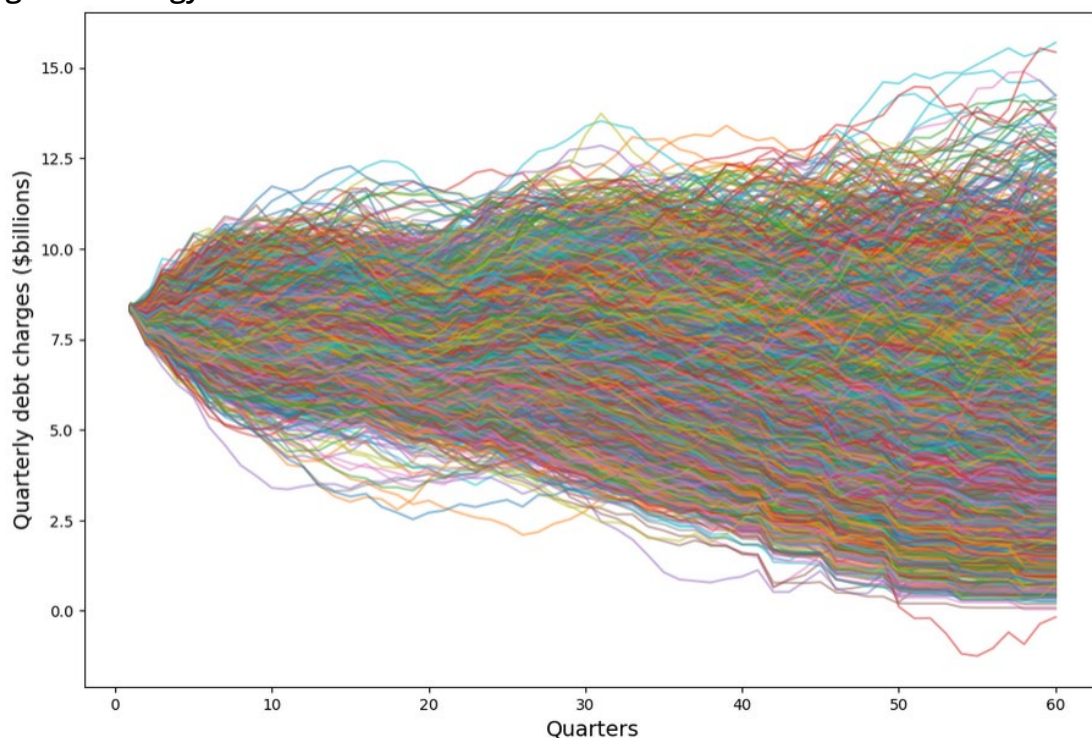
⁶ Defining coupon payments as quarterly (i.e., each period) instead of their actual semi-annual frequency in Canada simplifies the model's computations. And since the CDSM reports cost as an annual figure, simplifying to quarterly payments does not impact our analysis.

⁷ The first coupon payment is always a full one because a bond is always issued exactly one quarter before its first quarterly coupon in the model. This is inherent to the model's quarterly granularity and is a simplification from reality (see [section 5.3](#)).

government's investment activities (which would otherwise give relevance to the time value of money) don't enter the model's equation.

The single cost metric for a strategy is obtained by averaging the annual debt costs over the horizon and across simulation paths (**Figure 2**). The more granular distribution of these costs informs several risk metrics outlined in [section 3.2](#).

Figure 2: Debt costs have a wide distribution across time and simulations for a given strategy



3.2 Different measures of risk

Bolder and Deeley (2011) explain how the CDSM defines risk to help achieve the government's stability goal and visualize it in the stochastic framework.

The natural trade-off of risk with cost in debt management comes from the fact that:

- yield curves tend to slope upward (i.e., longer debt is more expensive)
- more frequent refinancing (from maturities of shorter debt) exposes the issuer to greater interest rate volatility and risks of relying on market access in an uncertain environment

Therefore, the simplest proxy of risk for a debt portfolio is the *average term to maturity* (ATM). The ATM is widely reported and considered by rating agencies. However, it is severely limited because it ignores the actual distribution of maturities and the costs of refinancing. For example, a debt portfolio that has 50% issuance at 3-month treasury

bills and 50% at 30-year bonds would have a similar ATM to a portfolio that is 75% 10-year bonds and 25% 30-year bonds. These two portfolios would have very different risk and cost characteristics and would have noticeably different refinancing needs in the medium term.

The measure of *rollover* captures the average amount of debt that is maturing—and hence needs to be refinanced at an uncertain future rate—each year relative to the size of the economy. Senior policy-makers can easily interpret this measure, which is why it is used as a guidepost for setting risk preferences. Also, like ATM, rollover is easy to compare with other sovereigns. However, while rollover effectively captures a source of risk, it still doesn't reflect costs and hence the actual level of risk the government is facing.

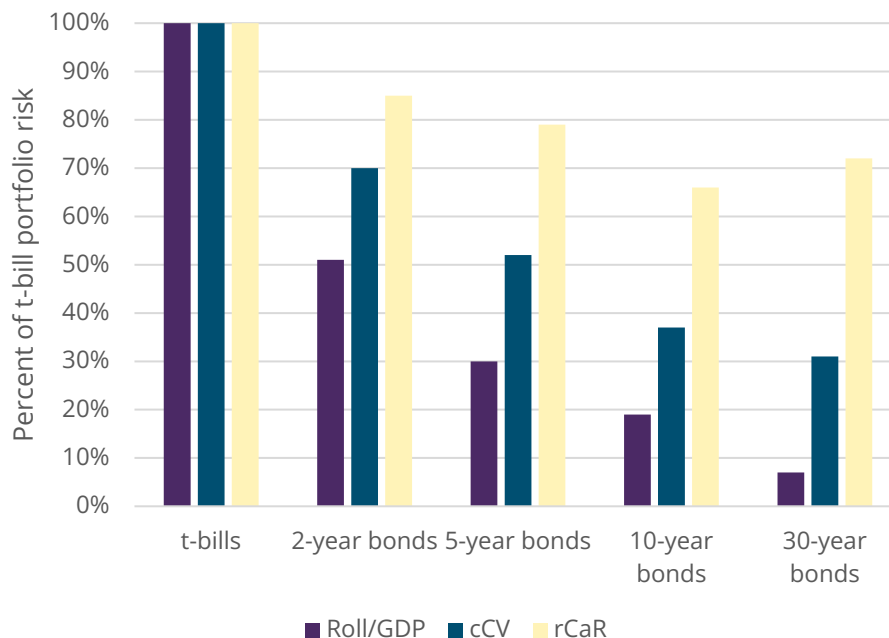
The primary risk measure used in Canada is the *conditional cost volatility* (cCV). It reflects the uncertainty in debt-service costs for any given year, conditional on costs in the previous year. Unlike unconditional volatility that measures overall variability over the full horizon, cCV specifically captures the year-to-year surprises in debt costs. Capturing this is important because the federal budget is especially sensitive to sudden changes in debt costs that would leave the government with little time to adjust fiscal policy.

An alternative risk measure is the relative cost-at-risk (rCaR). It is the difference between the debt-service cost in the worst-case simulated path (95th percentile for Canada) and the average cost, which is similar to the value-at-risk measure.

Both the rollover and cCV risk metrics are focused on the refinancing volumes and exposure to debt-cost shocks over the horizon of a single year. Hence, the greatest marginal reduction in risk occurs between treasury bills (1 year or less) and 2-year bonds—meaning treasury bills are the primary driver of risk. For rCaR, which is based on the entire horizon, the reduction in risk by shifting from treasury bills to bonds is smaller (**Chart 3**).

Chart 3: Longer-term debt provides more risk reduction under rollover and cCV metrics

Based on a hypothetical 100% issuance in each instrument



Note: *Roll/GDP* is how much debt on average is maturing each year relative to gross domestic product (GDP). *cCV* is conditional cost volatility, and *rCaR* is relative cost-at-risk.

One limitation of this risk framework is that it assumes returns follow normal Gaussian distributions (based on the classic Markowitz efficient frontier), which doesn't necessarily represent reality where, for instance, fatter tails are common. Also, it would be useful to consider a wider set of metrics, beyond the existing *rCaR*, that place greater emphasis on tail risks—instead of penalizing upside and downside volatility equally. Such metrics would be useful because sovereign debt managers are particularly sensitive to crisis outcomes and the potential political and reputational consequences.

3.3 Selecting portfolios based on cost-risk trade-off

Based on the chosen cost and risk metrics, the optimization engine will find the lowest cost portfolio for each point of risk. The collection of these results forms the efficient frontier (**Chart 4**).

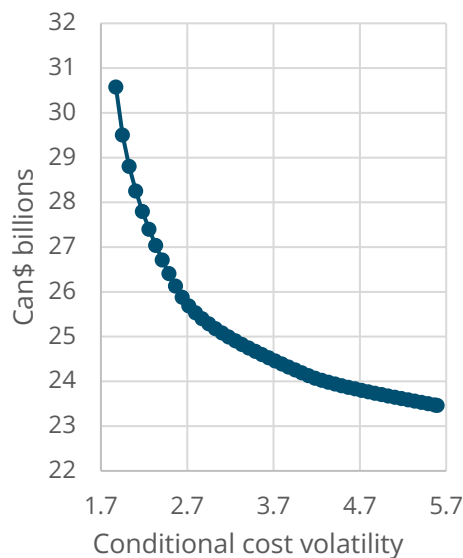
To select a portfolio on the efficient frontier that matches their cost-risk preferences, senior policy-makers can consider many possible criteria. Canada's current MTDS approach is to have a pre-defined desired range of risk (see [section 3.2](#)) and then comprehensively evaluate the candidate portfolios within that range based on the full set of policy considerations (e.g., supporting liquidity, market demand, rate forecasts and issuance consistency with the previous year).

Another approach is to set a target slope value or range on the efficient frontier, representing how much in annual costs the policy-maker is willing to pay to reduce one unit of risk. (Slope becomes greater, or steeper, as you move left on the frontier to less risky portfolios.) Whichever criteria are used, it's important that they be consistent from year to year (or to carefully justify any changes) so that the underlying preferences they reflect are well-defined.

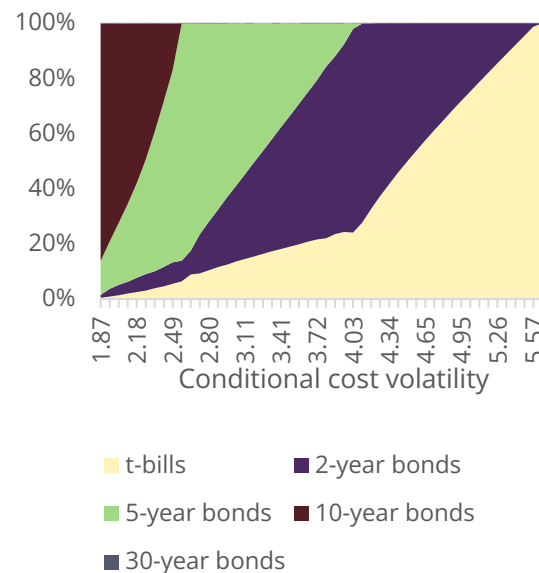
Chart 4: How each point on an efficient frontier, based on 15-year horizon, corresponds to an allocation

Conditional cost volatility metric

a. Average annual costs



b. Outstanding allocation as a share of total allocation



Sources: Department of Finance Canada and Bank of Canada Calculations

3.3.1 Results depend on risk metric

The risk metrics are generally correlated since each risk metric is based on the same distribution of simulated shocks. However, the choice of metric does have some effect on the optimized portfolio because each metric captures a somewhat different aspect of risk. Since cCV puts higher weight than rCaR on the risk of treasury bills (see [section 3.2](#)), portfolios optimized using cCV tend to allocate more to the bonds—within Canada's typical risk preference ranges. This can be shown by comparing two issuance strategies five years out with the same weighted ATM ([Table 3](#)).

Table 3: Hypothetical issuance strategies when optimizing under different risk metrics

	Relative cost-at-risk (rCaR)	Conditional cost volatility (cCV)
t-bill stock	62.6%	52.9%
2-year bond (annual)	8.7%	11.9%
5-year bond (annual)	5.7%	13.3%
10-year bond (annual)	19.3%	17.6%
30-year bond (annual)*	3.7%	4.3%
Average annual cost	65.64	66.06
Average term to maturity	6.98	6.98
cCV	4.63	4.36
rCaR	21.82	21.86
Roll/GDP [†]	14.49	13.32

Note: The issuance values are at the fifth year of the horizon and are as a share of the total stock of treasury bills (t-bills) and bond issuance.

*The 30-year bond issuance is set to a fixed dollar amount in this example, though its amount in percentage terms would vary.

[†] Roll/GDP is how much debt on average is maturing each year relative to gross domestic product (GDP).

3.3.2 Results depend on time horizon

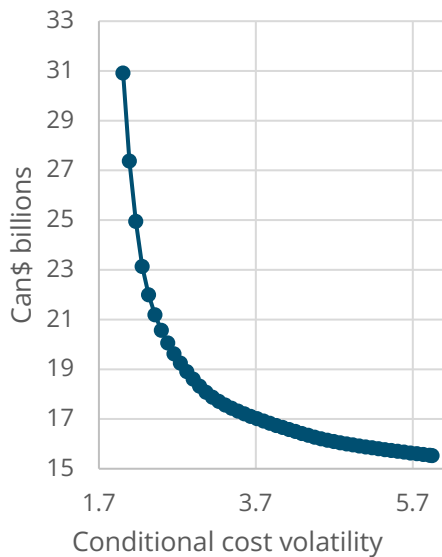
Note that the CDSM doesn't favour 30-year bonds. This is because the risk reduction benefits from these bonds are not fully realized under the CDSM's 15-year horizon.

Chart 5 shows how these bonds become more relevant under a longer horizon.

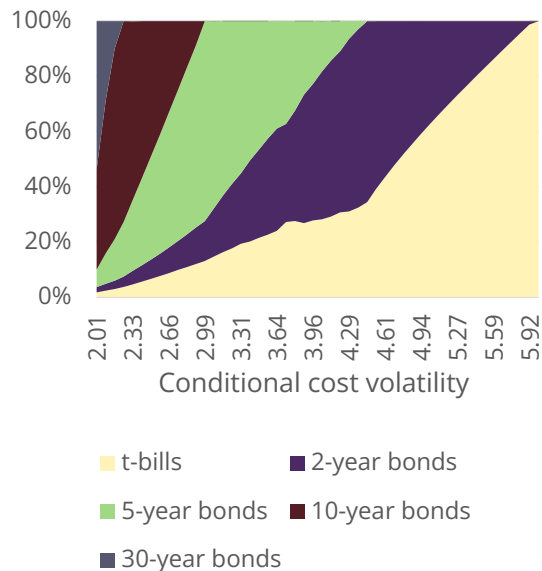
However, extending the horizon also makes the model's results less meaningful (i.e., overemphasizing the uncertain far future). So, a balance must be struck. For Canada, the shorter horizon is preferred since the 30-year allocation in the DMS has typically been set based on factors outside the model—namely, demand from investors, such as insurance companies and pension funds, who need long-duration assets to match their long-term liabilities.

Chart 5: 30-year bonds become more relevant under horizon longer than 15 years

a. Average annual costs



b. Outstanding allocation as a share of total allocation



Sources: Department of Finance Canada and Bank of Canada Calculations

4 Modelling issuance decisions under uncertainty

The debt manager is tasked with selecting a financing strategy that meets policy objectives, doing so under substantial uncertainty over future interest rates and macroeconomic outcomes. A debt model therefore should be able to:

- simulate this uncertain environment in a way that is consistent with policy expectations and observed economic dynamics
- incorporate as much meaningful information as possible from these simulations into the process to select a strategy

4.1 Combining expectations with uncertainty in simulations

In the prior versions of the CDSM, the simulations were estimated from historical data using a simple stationary VAR model (see [section 2.2](#)). However, in the context of the new transition model that starts from the current state and aims to simulate the near-term future, this procedure had two drawbacks.

First, the resulting simulated distributions won't likely match the debt manager's expectations of the evolution of the economic environment. However, this discrepancy is

unavoidable. Statistical models can replicate only the statistical features—such as mean, covariances and autocovariances—of the underlying (historical) dataset, while the debt manager’s expert judgment incorporates their assessment of the specifics of the current economic conditions.

Second, the procedure does not allow for a statistically well-grounded method to incorporate scenarios when generating the simulated data. These scenarios can be used to assess how much model risk resides in the simulation model by estimating how different the chosen strategy would be if the process to generate real data was markedly different from the one implied by the model.

The updated CDSM improves on this method by incorporating conditioning information to the simulation output. Specifically, this information is the forecasts of private sector economists that are typically updated quarterly for key economic inputs and the projected federal deficit, both of which are published by the Department of Finance Canada.⁸ The method generates simulated paths where:

- the expected values of the simulations match the forecasts
- the distribution of paths around that expectation matches the history

This method is based on Jarociński (2010) and its application is summarized in Audet et al. (2025).

This conditioning allows the debt manager to integrate their expectations while still maintaining the stochastic nature of the model. This provides a more complete idea of future uncertainty and is necessary for calculating risk metrics.

Note that beyond interest rates and their direct relevance to cost (see [section 3.1](#)), each macro variable also serves a distinct functional purpose (outside of their VAR interactions with rates). The primary deficit gives each year’s financing needs in addition to what’s maturing. Gross domestic product (GDP) is important as an index for several model parameters (i.e., well-functioning market constraints) and denominator for some risk metrics. Inflation is needed to price outstanding inflation-linked bonds.

⁸ For more about forecasts, see Department of Finance Canada, “[Department of Finance Survey of Private Sector Economic Forecasts](#)” (last updated November 4, 2025). Deficit projections are published in the federal budget document. For an example of these projections, see Department of Finance Canada, “[Annex 1: Details of Economic and Fiscal Projections](#)” in *Budget 2025* (November 2025).

4.1.1 Interest rate expectations impact optimal issuance strategies

We present results under two hypothetical settings for interest rate forecasts where throughout the forecast horizon:

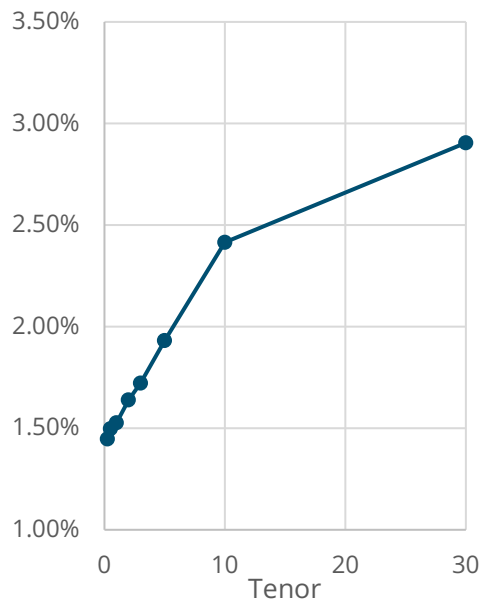
- a low, relatively flat yield curve is expected
- a higher, steeper yield curve is expected

Chart 6 summarizes the resulting issuance strategies under both low- and high-yield forecasts, highlighting how variations in the forecasted yield curve influence decisions about the optimal issuance.

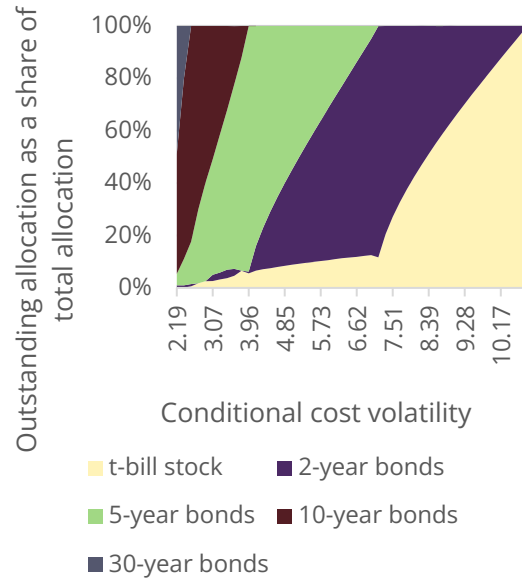
This exemplifies the added value of combining conditioning with stochastic paths. The conditioning allows the debt manager to produce an efficient frontier of strategies that respects their own interest rate expectations, while the stochastic paths allow risk metrics to be calculated (cCV in this case) and an efficient frontier to be produced.

Chart 6: Issuance strategies differ under low- and high-yield forecasts

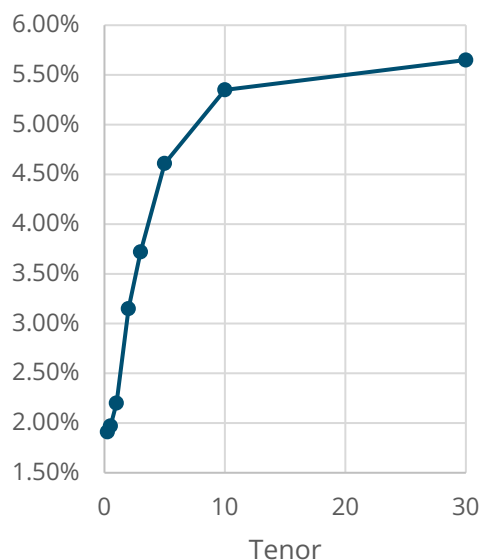
a. Yield curve under low-yield forecast



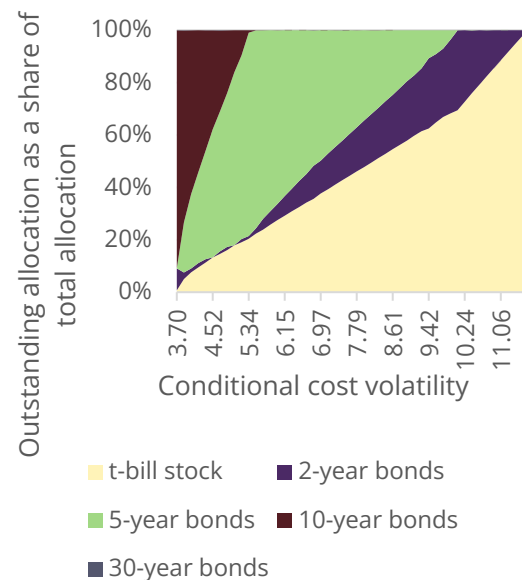
b. Efficient frontier allocations under low-yield forecast



c. Yield curve under high-yield forecast



d. Efficient frontier allocations under high-yield forecast



Sources: Department of Finance Canada and Bank of Canada Calculations

4.2 Extending the DMS to state-dependent strategies

As mentioned in [section 2.4.2](#), the newest version of the CDSM has enhanced the very definition of a debt strategy to include not only the target issuance path but also parameters on how to deviate temporarily from that path based on prevailing macro-financial conditions.

Audet et al. (2025) show that dynamic strategies offer a strict improvement in cost and risk over deterministic strategies under all risk preferences and specifications of common state variables. They also find that most of this improvement can be achieved by reacting to just one state variable: the interest rate level.

This level most directly captures the savings from:

- issuing more long-term debt to lock in rates when they are low
- issuing more treasury bills to limit exposure to rates when they are high

The level factor explains a large percentage of changes in the yield curve over time. Also, the interest rate level is at least moderately correlated with each of the five other core state variables. This means that the marginal information value gained from adding a second variable to the reaction function is limited.

An important policy insight from a state-dependent model comes from the guidance it gives the debt manager on different future optimal issuances across different values of a future state variable. For contingency planning, this approach enables policy-makers to examine issuance under the average, low (25th) or high (75th) percentiles of next year's possible values for the level variable (**Table 4**). Such analysis provides a framework for planning around the inherent uncertainty of interest rate forecasts and ensures that issuance strategies remain robust under a range of plausible market conditions.

Table 4: Change in issuance under the low and high percentiles of the interest rate level of the yield curve simulation

Interest rate level (percentile)	Stock of t-bills*	2-year-bond (annual)	5-year bond (annual)	10-year bond (annual)	30-year bond (annual) [†]
Low (25 th)	-1.21%	-2.04%	4.19%	7.88%	0%
High (75 th)	1.30%	1.87%	3.65%	-11.65%	0%

Note: Projection is for fiscal year 2027–28.

*t-bills is treasury bills.

[†]The 30-year bond issuance is set to a fixed amount in this example.

While state-dependent strategies are a new idea in the modelling space, conceptually they are consistent with how debt managers make decisions. Prevailing market conditions are inherently reflected in the DMS because consultations with market participants are a key input (see [section 2.3](#)). Even without a formal dynamic model, Canada has previously incorporated the current rate environment into its debt strategies. For example, the 2021–22 DMS noted that the government issued significantly more long-term bonds when interest rates were historically low to support debt sustainability (Department of Finance Canada 2021).

That said, having a state-dependent model offers a new and robust tool to provide more systematic, quantitative guidance on how debt managers could respond to rate levels. This can inform a DMS approach that is overall both more time-consistent and efficient.

4.2.1 Practically integrating state-dependent and deterministic strategies

While state-dependent models are shown to align with debt managers' practices and offer cost-risk efficiency benefits, they should not be thought of as complete or immediate replacements for the existing deterministic models.

First, enhancing the debt strategy to state-dependent issuances is a new idea that requires a conceptual leap for the general policy-maker, especially since the reaction function itself isn't as tangible a value as issuance allocation. Related to this, the reaction function must be examined to ensure that deviations in year-to-year issuance lie within what policy-makers consider stable, predictable issuance.

This, combined with the existing anchoring of debt issuance to the output of a deterministic model (for reasons of predictability), means that debt managers introducing a state-dependent model to their policy are best served by starting off with a hybrid approach to form their own versions of the MTDS and DMS. For example, managers could use a deterministic strategy as the baseline MTDS and deviate from that in the DMS based on directional insights obtained from the state-dependent strategy under the prevailing interest rate environment.

Note that the state-dependent CDSM fully incorporates deterministic strategies into its functionality. Since the model's users can specify any number of state variables to react to, they can simply specify zero variables to reflect the properties of a deterministic model.

For contingency analysis, having a state-dependent reaction function to unexpectedly high or low rates, or more complex defined macroeconomic scenarios, can be valuable for debt managers (**Table 4**). This is true even if the issuance numbers themselves are not followed. Communicating such a plan to senior policy-makers in the DMS can help them better prepare to take more substantive action in a future DMS if the corresponding scenario is realized.

5 Implementing the debt model computationally

The CDSM doesn't map debt strategies to cost and risk through a functional form. Instead, the CDSM relies on making a large set of calculations (i.e., running the debt

charge engine across all simulation paths for each strategy) that requires substantial computing capacity to generate robust results.

The approach for the CDSM is to first evaluate an initial space of a large number of debt strategies. For each risk point on the frontier, the lowest-cost strategy at or below that risk level is chosen as the starting value for the optimization. The next step is determining the efficient frontier of optimal strategies using the iterative random search algorithm from Audet et al. (2025) at each point. Running the model can take several hours. The reason is that evaluating the cost and risk of a strategy at each step of the algorithm takes a nontrivial amount of time, and optimization tolerance is set extremely low to ensure no corner solutions.

Introducing state-dependent strategies intensifies the computational challenge because it at least doubles the number of choice variables. Beyond the fixed allocation in each tenor, the algorithm must now also solve for every reaction parameter for each tenor. This not only expands the space of strategies that must be searched but also increases the likelihood that two very different strategies have very similar cost-risk properties and the optimization gets trapped at a local minimum. Both of these factors could lead to allocations fluctuating along the frontier.

Therefore, a strong preference exists for parsimonious strategies that use only one state variable, which is typically the interest rate level because it is by far the most meaningful.

5.1 Computing technology and its limitations

For many years, this tension between smooth results and fast running time that gets exacerbated from higher dimensionality was a major reason why developmental work on dynamic models was limited. However, major advancements in recent years have made computational power more affordable.

The current CDSM leverages the high-performance computing cluster at the Bank of Canada. It does so by separating the most computationally-intensive part of the model—optimizing each point along the frontier—into individual jobs (one for each point) that are run in parallel. This drastic improvement in efficiency is necessary just to feasibly run the model at the scale it is currently defined.

To facilitate this parallelization, the model code (currently in MATLAB, but adaptable to other languages) is structured to be highly modular, with the components cleanly separated from one another. This is supported by the object-oriented nature of the coding.

For policy purposes, the CDSM aims for a run time of a few hours at most, which allows for generating results under multiple settings within a day. Generating multiple results is

especially valuable closer to the decision-making deadline for the DMS when turnaround times are short and the environment and exact policy preferences are still in flux.

Work is under way to embed machine learning techniques in the optimization module, which could offer new ways to explore the optimal debt strategies.

5.2 Simplifying the strategy via kernels

Including each new instrument in the debt strategy imposes significant computational complexity through the addition of two choice variables. One way to alleviate this challenge is by limiting the number of instruments in the strategy to only those that are necessary.

An obvious way the CDSM alleviates this challenge is by collapsing the 3-, 6- and 12-month treasury bill sectors into one instrument. Canada's three bill sectors are issued simultaneously at the same biweekly auctions and at relatively consistent proportions. Thus, it is reasonable to set up the model to solve for a simple aggregate allocation of treasury bills with a fixed rule defining the split between the 3-, 6- and 12-month sectors.

Also, the unfavourable cost-risk trade-off of 30-year bonds means that debt managers rely heavily on market intelligence to inform their final allocations. This makes these bonds much less meaningful as a choice variable in the model, which is why they have been excluded as a choice variable in the strategy in recent years. Instead, all debt charge calculations use a fixed issuance amount for the sector.

Finally, while the deterministic model has a choice variable for transition speed, it was found that much of the variation in allocations affected by that could be achieved through varying the choice variables for the reaction function itself. Therefore, that parameter could also be removed without meaningfully impacting the debt manager's true choice set.

Note that the mechanism by Audet et al. (2025) to translate the reaction function output to allocations precludes the need to solve for one of the instruments. This is because the solution is simply the remainder once the total allocation of the other instruments is subtracted from 1.

This approach reduces the number of choice variables in a strategy from 15 to 6 while still accurately recognizing the full set of seven treasury bills and bonds as instruments in Canada's regular debt program.

To make the most efficient use of computing time, certain policy decisions are excluded from the model. The reason is that accurately characterizing these decisions would be immaterial. An ongoing example is Canada's green bond issuance. These bonds are issued annually in small amounts and in a potentially irregular term. Decisions on their size and term are made at the time of issuance instead of before the start of the fiscal

year like regular nominal bonds. The treatment takes the amount as a fixed issuance (outside the strategy itself) in the most common term.⁹

5.3 Set time granularity to only what is necessary

At each step in the model's debt charge engine, a series of calculations are done on maturities, fiscal needs, issuances and debt costs. With that engine being the key computational bottleneck, the number of such steps should be minimized to the extent reasonable.

A quarterly frequency was determined to be sufficient for the CDSM because it captures:

- most of the intra-year macroeconomic dynamics (e.g., GDP releases are quarterly)
- the necessary timing of semi-annual coupon payments
- the rough timelines for each bond's issuance cycles (e.g., a new 10-year bond typically starts being issued at around 10.5 years to maturity, so the first issuance will occur 42 quarters before)

A quarterly frequency is also sufficient for defining the cost measure annually and reporting DMS numbers as annual totals.

A quarterly frequency does lose some precision for the timing of debt operations. With the GoC doing multiple treasury bill and bond auctions each quarter, the model aggregates these auctions and doesn't consider the exact date (month) or size of each.¹⁰ Auction-level granularity is important for debt managers to set quarterly auction schedules or decide the size of each auction (vs. the number of auctions).¹¹ But that level of detail is mostly immaterial for the more strategic decisions about the DMS and MTDS that the model seeks to inform.

Therefore, a quarterly frequency is ideal. Moving to a monthly frequency would triple computation time while the increased precision would have minimal impact on results.

⁹ Canada's discontinued 1-month treasury bill was not incorporated into the model because its amount was small and its cost-risk properties very similar to the 3-month bill. As well, any extensions to the model's quarterly granularity would have had an exponential computing cost.

¹⁰ Besides the biweekly treasury bill auctions, there are five 2-year auctions, four 5-year auctions, four 10-year auctions, and two 30-year auctions projected to take place every quarter in the 2025–26 fiscal year.

¹¹ The GoC's [quarterly bond auction schedule](#) is published for market participants, typically a few weeks before the start of the quarter.

6 Incorporating non-model considerations

Debt managers have a critical responsibility for maintaining the health of the sovereign debt, fixed-income and wider financial markets domestically. Their role goes beyond designing the DMS and underpins several other debt management functions, including:

- auction design and scheduling (see [section 5.3](#))
- the Debt Distribution Framework
- public and bilateral messaging to market participants
- facilities to support market functioning
- domestic liquidity management
- liaising with fiscal and other political authorities

To the extent these overarching objectives can be reflected in the DMS itself, it is done so through setting constraints and making qualitative adjustments to the model results to better reflect the current market conditions.

These constraints and adjustments can be a significant—even predominant—driver of the final DMS decision. Even without any quantitative model, a responsible debt manager should already have a basic idea of how much they should be issuing in each tenor based on the needs of their investor base, the structure and liquidity properties of their fixed-income market and the known priorities of politicians, among others. Ultimately, a debt model only provides improved guidance within those carefully defined parameters.

6.1 Minimum issuance constraints

Since the CDSM does not natively account for market functioning in its optimization, well-functioning market (WFM) minimums for bonds and treasury bills are imposed in the model. This ensures that the government maintains a sufficient level of issuance in each sector to:

- support liquidity at the relevant points along the yield curve
- maintain predictability in its year-to-year issuance

The minimums are imposed externally based on prevailing market intelligence and data analysis. They are scaled into the future based on projected GDP since the need for safe debt instruments increases as the economy grows.

For treasury bills, analysis by Bank staff finds that a minimum volume of cash-like instruments should be in the economy (as a share of its size). Otherwise, stress indicators such as the OIS-bill spread will become disconnected. Hence, to guide the

WFM minimum for treasury bills, a model estimating the OIS-bill spread as a product of the stock of treasury bills is also used as an input.

For bonds, market input is obtained in the form of minimum benchmark sizes, meaning how much of a particular maturity investors and dealers believe must be maintained for adequate liquidity and price discovery. These minimum benchmark sizes are then scaled accordingly. Previous years' benchmarks are often used as a reference point in these discussions, which take place during annual consultations on the debt strategy and on an ad hoc basis.

These minimum constraints tend to take up a large percentage of the total issuance and can be binding for some sectors that have highly unfavourable cost-risk trade-offs (namely 30-year bonds when it was a choice variable). This somewhat limits allocation possibilities for the cost-risk optimization to find efficient strategies.¹² It leaves even less room for policy-makers to choose between strategies. This is done by design to maintain predictability and respect the larger considerations of debt management discussed earlier.

For the state-dependent model, the method of implementing WFM minimums was adjusted to address computational concerns and the difficulty of translating strategies into issuances over time. Before the constraints simply reduced the search space of eligible strategies at each risk point (see [section 3.3](#)). Now the allocations being optimized in each strategy specifically refer to the allocations above the WFM minimums (with a subsequent scaling and addition to get the final numbers).

6.2 Accounting for the impact of over-issuance

Conversely, issuing too much in one sector can cause market demand to be saturated, leading to market participants having more difficulty absorbing new supply. As a result, participants may demand higher yields as a form of compensation, which raises issuance costs for the government.

One way the CDSM accounts for this is by including penalty functions through increases in the coupon rates (beyond the simulated level of the yield curve level) for issuances whose size exceeds a certain threshold. The penalty function is based on historical multi-bid auction data. For each sector, the penalty function calculates the slope of the bidders' aggregate demand curve using the method described in Chang (2023). It then uses that slope to estimate how auction yields change with issuance size.

The penalty function complements the minimum issuance constraints in steering the model to more balanced and realistic issuance strategies that are not unduly

¹² With the growth in debt issuance, this space has increased as minimums have become less binding. Therefore, assessing over-issuance has become more important (see [section 6.2](#)).

concentrated in the sectors that are most purely cost-risk efficient. It also helps keep issuance within historical norms because the function is based on actual issuance numbers in the past. All this helps support the broader objectives of stability and predictability.

However, the penalty function has the downside of being based only on actual auctions in the past and thus does not fully quantify the impact of hypothetical issuance well above market capacity. Therefore, when evaluating the final DMS, it is important to also impose an upper bound for each sector. Based on consultations and other market intelligence, this limit estimates how much market participants could absorb before demanding materially higher yields. This concept of maximum issuance will continue to have a greater role in driving the final DMS results as debt issuance worldwide rises—with no parallel to recent history—and dealers' balance sheets remain constrained.

While the above mechanisms assume sovereign debt demand is purely segmented by sectors, it can also be thought of as partly fungible on a duration basis. In the Diez de los Rios (2024) portfolio balance model, issuing more debt, especially long-term debt, increases the supply of duration risk to market participants and leads investors to demand more compensation for this new supply of duration (more so for the duration from long-term debt). Taking this perspective in the DMS, consideration could be given to limiting issuance of long-term bonds specifically in an environment where fiscal spending and concerns about absorbing debt are both high.

6.3 Other considerations

In this context of maximum issuance, the debt manager could find themselves in an uncertain environment where fiscal needs could change dramatically during the year. Examples of this include the emergency funding needed in 2020 during the COVID-19 crisis and the 2008–09 global financial crisis. For an annual DMS, there is also value in retaining contingency room in shorter sectors whose issuance can be ramped up more quickly and cost-effectively on short notice.

While the issuance path in the MTDS is smooth by definition, it does not guarantee the maturity structure will be smooth. This is especially true if there were spikes in total annual issuance in the recent or semi-recent history. For example, rollover will be much higher in 2025 and 2030 as large amounts of 5-year and 10-year bonds, respectively, mature because of Canada's increased issuance during the COVID-19 crisis of 2020. Manual adjustments to the MTDS should be made to reduce these year-to-year fluctuations.

The Government of Canada holds debt assets, such as Canada Mortgage Bonds, which are of comparable credit quality to GoC bonds.¹³ A future addition to the model may incorporate these holdings in an asset-liability netting framework to allow for more representative cost-risk calculations. This potential adjustment would ensure the CDSM's analysis reflects the government's net position rather than treating liabilities in isolation.

7 Conclusion

Debt modelling may be one component in the broader process to set a sovereign debt strategy and manage debt, but it is a very well-defined and important component. Debt models quantify potential solutions to the complex, high-dimensional problem of allocating between several debt instruments (and adjusting those allocations based on varying states) where precision is critical. The potential solutions that models provide are not only robust but can be effectively integrated with qualitative analysis to speak directly to the preferences of policy-makers.

Drawing from practical experiences in Canada, this paper illustrates exactly how these specialized and flexible modelling tools can be designed and applied to best inform a country's short-term (DMS) and medium-term (MTDS) debt issuance policies. In particular, we explain the intuition behind how the CDSM:

- encodes the debt manager's objective function
- more effectively achieves policy objectives by moving to a state-dependent framework
- adapts to computational limitations
- operates collaboratively with fundamental qualitative considerations

Through this paper we hope to promote the continued development of accessible, rigorous and highly relevant debt issuance models by sovereign debt managers worldwide.

¹³ For more details, see Bank of Canada, "[Canada Mortgage Bonds: Government purchases and holdings.](#)"

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