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# Uncovering Subjective Models from Survey Expectations

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## Abstract

Households may perceive that macroeconomic variables move together in a different way from that implied by their actual realizations and sophisticated models. We use a structural test derived from a multivariate noisy-information framework and additional evidence from survey data and newspaper narratives to show that information friction alone cannot explain households' tendency to associate higher future inflation with a worse labor market outlook. We also show that the subjective model empirically uncovered from survey data implies amplified output responses to supply shocks, but dampened output and price responses to demand shocks.

*Topics: Business fluctuations and cycles; Monetary policy; Inflation and prices; Labour Markets*

*JEL codes: E21, E30, E32, E71, D84*



## Résumé

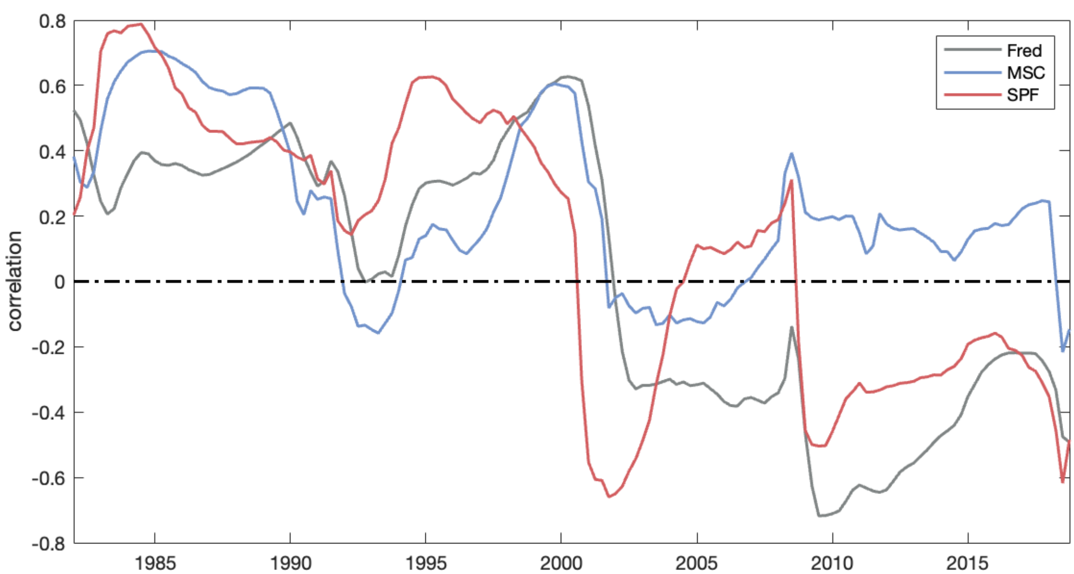
Les ménages peuvent avoir l'impression que les variables macroéconomiques évoluent ensemble d'une manière différente de ce qu'impliquent les données réelles et les modèles sophistiqués. Nous utilisons un test structurel dérivé d'un cadre multivarié avec information bruitée ainsi que des données d'enquête et des articles de journaux pour montrer que les frictions informationnelles ne peuvent pas à elles seules expliquer la tendance des ménages à associer une hausse future de l'inflation à un assombrissement des perspectives sur le marché du travail. Nous montrons également que le modèle subjectif empiriquement dégagé des données d'enquête implique une réponse amplifiée de la production répondaux chocs d'offre, mais une réponse modérée de la production et des prix aux chocs de demande.

*Sujets : Cycles et fluctuations économiques; Politique monétaire; Inflation et prix; Marchés du travail*
*Codes JEL : E21, E30, E32, E71, D84*

# 1 Introduction

When households expect a higher inflation rate, they also anticipate higher unemployment rates and an underperforming economy.[1] Figure 1 depicts such a pattern using the rolling-window time-series correlation between average households' inflation and unemployment expectations in the *Michigan Survey of Consumer Expectations* (MSC), that of professionals in the *Survey of Professional Forecasters* (SPF), and those of the realization of the two series.[2] Although the realized correlation between the two variables is positive before the 1990s and turns negative after 2000, as reflected more or less by professionals' forecasts in SPF, the correlation of the two expectations in MSC remains mostly positive throughout the entire sample period.[3]

Figure 1: Time Varying Correlation between Inflation and Unemployment Change




Correlation using 10-year rolling window, 1982–2018. Gray line: realized data from FRED. Blue line: expectations from the MSC. Red line: expectations from the SPF.

[1]Several contemporaneous studies, such as (Bhandari *et al.*, 2025; Kamdar, 2019; Andre *et al.*, 2022; Candia *et al.*, 2020; Han, 2023), also document a similar pattern.

[2]Obtained from the Federal Reserve Bank of St. Louis (FRED). A detailed data description is included in Appendix A.1.

[3]Additional results in Appendix A.3 and A.4 confirm that such a positive correlation is seen across time and not driven by a certain group of consumers.

Such a data pattern naturally calls for an examination of how agents jointly form expectations about different macroeconomic variables. We extend the commonly used test on information rigidity as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013) from a single-variable to a multi-variable environment, allowing for potentially subjective perceptions of correlation between variables. In the presence of both information rigidity and a subjective model, we characterize exactly how expectations are jointly formed rather than independently formed, and investigate the causes of such a correlation in expectations.

Two possibilities arise when expectations regarding different variables are correlated with each other. First, the agent may hold a subjective belief about the correlation between variables (i.e., the transition matrix in the noisy-information model). Second, they may simply receive signals that provide information about both variables. One example of such correlated information is a non-sophisticated newspaper article commenting on both inflation and general macroeconomic conditions. Another example involves pessimistic/optimistic heuristics, where an agent may get information about both variables that is biased in the same direction. We derive differentiating predictions from models with only information friction and those that are subjective.

The essence of the test is a joint sign restriction on the contemporaneous correlations of expectations and their between-variable serial correlation of forecast errors. We show that under very general conditions, a subjective model perceiving a positive relationship between today's inflation and tomorrow's unemployment is necessary for generating the coexistence of positively correlated expectations in the survey data and a positive between-variable serial correlation in forecast errors. Not only do households expect the two variables to move in the same direction, overforecasting inflation today leads to overforecasting the unemployment rate tomorrow. In contrast, such a joint pattern is not evident in an alternative environment that only features incomplete information about the state of the economy, where correlated signals could also drive expectation co-movements.

With the test results, we proceed with a structural estimation of a vector-autoregression (VAR) model to uncover the perceived law of motion of the macroeconomy by households and professionals. We report on various statistical tests with estimates to determine the existence of the wedge between the two. Our estimation unambiguously confirms that households, as opposed to professionals, associate current inflation with worsening future labor markets. In addition, the direction of the subjective association between inflation and the labor market goes from the former to the latter.

Such an expectational pattern has an important macroeconomic implication. Once the uncovered subjective model is used to calibrate the dynamics of the expectations in a modified textbook New Keynesian model (Galí, 2015), the economy's output and price responses to a standard supply shock are amplified while the response to demand shocks is dampened. When a persistent negative supply shock leads to the initial rise in prices and drop in output, the upward change in inflation expectations induces an additional pessimistic shift in future output expectations, reducing demand and output. In contrast, when a negative demand shock hits the economy, pushing down inflation expectations, an associated improvement in the economic outlook counterbalances the negative output impact.[4]

Recognizing that survey expectations reflect both their perceived laws of motion and changes in households' information, we supplement the structural estimation with micro survey evidence using the self-reported news exposures in the MSC as a direct control for their information set changes. We show that different types of news have domain-specific impacts on consumers' expectations. For example, consumers who hear news about inflation are likely to expect a higher inflation rate, and those exposed to labor market news revise their unemployment expectations accordingly. Consumers can distinguish between different types of news. However, among all types of news, inflation news predominantly leads to expectations of worse future economic conditions across domains, including higher unemployment expectations. The association in expectations between inflation and unem-

[4] Adams and Barrett (2024) show that empirically identified sentiment shocks to inflation expectations have subsequent deflationary impacts.

ployment is particularly strong among households that have heard unfavorable news about inflation.

Lastly, we investigate what is special about inflation that triggers an inflation-unemployment association. We use directly measured news coverage on macroeconomic topics in a sample of 250,000 economic news articles published in *The Wall Street Journal* between 1984 and 2022.[5] We first confirm that newspaper coverage of inflation and unemployment is indeed highly correlated with self-reported and topic-specific exposures in the MSC. Then, we show that, central to the asymmetric impacts of inflation news, more intense news coverage of inflation is perceived to be particularly unfavorable,[6] while households perceive no such directional implications for unemployment news. Meanwhile, relying upon the identified topics of each news article, we show that newspaper articles are particularly likely to draw an inflation-unemployment association during episodes of high realized inflation rather than with high unemployment rates. Altogether, the evidence suggests that the negativity associated with inflation news might be one possible explanation for why the perceived correlation between inflation and unemployment goes from the former to the latter.

## Related Literature

This paper is based on the literature on information rigidity, which uses the implications of forecasting errors and forecasting revisions from the noisy-information model (Woodford, 2001; Sims, 2003) or sticky-expectation model (Mankiw *et al.*, 2004) to understand expectation formation. The seminal work of Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013) considers tests using current and lag forecast errors. Coibion and Gorodnichenko (2015) and Bordalo *et al.* (2018) use forecast errors and revisions obtained from survey data. We extend the insight from these papers that the serial correlation of forecasting

[5] As a robustness check, we also find similar patterns with a sample of 250,000 articles published in *The New York Times* between 1989 and 2022.

[6] This is related to the argument by Chahrour *et al.* (2024) that unfavorable inflation news is more frequently reported than favorable news, which causes a stronger expectational response to unfavorable inflation news.

errors of single variables reveals information rigidity. We show that between-variable correlations in forecasting errors reveal a correlation in the information or a perceived correlation in subjective models. In our framework, the forecasters may have a subjective understanding of the law of motion of states that differs from the actual one. This is similar to the single-variable case in Ryngaert (2018).[7]

We are among the few contemporaneous papers that study the positive correlation between inflation and unemployment rate in household expectations, which include Bhandari *et al.* (2025); Kamdar (2019); Candia *et al.* (2020); Andre *et al.* (2022); Han (2023); Stantcheva (2024). Our additional finding regarding this empirical pattern is that the direction of such a perceived correlation in subjective models particularly goes from inflation to unemployment rather than the other way around. Closely related is the expanding empirical evidence that most households hold negative views toward inflation, despite the potential macroeconomic benefits of mild inflation (Shiller, 1997). Various hypotheses have been put forward to explain this pattern, such as the supply-over-demand view (Kamdar, 2019; Andre *et al.*, 2022; Han, 2023); ambiguity aversion (Bhandari *et al.*, 2025); neglect of macroeconomic trade-offs (Stantcheva, 2024); partisan biases (Gillitzer *et al.*, 2021); personal finance (Bolhuis *et al.*, 2024); and the erosion of real income (Hajdini *et al.*, 2022; Jain *et al.*, 2022; Stantcheva, 2024). Compared with these studies, this paper is agnostic about the relative importance of these channels. Instead, we show that the well-documented negative views of inflation held by households, and reported in newspapers, make inflation more likely to be the trigger of the inflation-unemployment association.

More generally, this paper contributes to the literature on subjective models in macroeconomic expectation formation, particularly how expectations of different macroeconomic variables are related to each other. Andre *et al.* (2022) use survey vignettes to show that both households and experts hold heterogeneous views about how the same hypothetically exogenous macroeconomic shocks affect inflation and unemployment rates. Complementary

[7] Andrade *et al.* (2016) represents one exception of studying expectations with a multivariable environment, with a focus on the term structure of disagreement.

to their paper, we adopt a different approach to detect the subjective perceptions of how macroeconomic variables are correlated with each other, relying on cross-variable restrictions in observational data. Similar to their finding, we find that households have a strong tendency to predict the same directions of the changes in the unemployment rate and inflation, regardless of the nature of the macroeconomic shocks.

The rest of the paper proceeds as follows. Section 2 derives testable implications and performs a test of joint expectation formation under the noisy information model. Section 3 structurally estimates the subjective model and shows its macroeconomic implications in a modified textbook New Keynesian model. Section 4 documents independent evidence on the connection between cross-correlation and joint learning using perceived news data in the MSC. Section 5 provides further supporting evidence for subjective models using newspaper-based narratives. Finally, Section 6 concludes.

# 2 Test of Joint Expectation Formation

In this section, we first examine different possible sources of the positive correlation between expected inflation and unemployment documented in the introduction. We do so through the lens of the noisy-information model, as in Woodford (2001) and Sims (2003). We show that in this simple framework, different hypotheses can lead to the same correlation between expectations. Consequently, we cannot distinguish between these different hypotheses using the correlation between expectations alone. To solve this problem, we show that these various explanations have different testable implications on the *serial correlations of forecast errors* for inflation and unemployment. Furthermore, the serial correlations of forecast errors are informative about whether the agent jointly or independently forms expectations.

## 2.1 Model Environment

The testable implications on forecast errors that we consider are in the spirit of those from Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). In our model, there are multiple macroeconomic states that are not directly observable to the agent. The agent may have subjective beliefs about how these states evolve. They observe multiple noisy signals about the states that can be arbitrary combinations of these states that they try to form beliefs about. Consider the states $\boldsymbol{L}_{t+1,t}$, which are macroeconomic variables that follow the state-space representation (1). The agent observes noisy signals on these variables, with the observational equation given by (2).

$$\boldsymbol{L}_{t+1,t} = A\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{1}$$

$$\boldsymbol{s}_t^i = G\boldsymbol{L}_{t,t-1} + v_t^i + \eta_t \tag{2}$$

In contrast to the Actual Law of Motion (ALM) summarized in Equation (1), the agent may have a subjective model, the Perceived Law of Motion (PLM), about how states evolve.[8]

$$\boldsymbol{L}_{t+1,t} = \hat{A}\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{3}$$

$\hat{A}$, namely the subjective model, may or may not be the same as $A$. We show later that whether $\hat{A}$ is diagonal or not has testable implications on the serial correlations of the agent's forecast errors as well as on the correlations between expectational variables.

The signals observed contain an individual-specific noise $v_t^i$ and a time-specific one $\eta_t$, both of which follow a normal distribution with mean zero. The individual noise is independent across agent and time, and the time-specific noise is not autocorrelated and independent

[8]We do not consider the case where $G$ is also subjective, as in the rational inattention literature where $G$ can usually be chosen by the agents themselves. See Mafákowiak *et al.* (2018) as an example. For this reason, we assume the agents always use the correct $G$.

with the structural shock $w_{t+1,t}$. Adding a time-specific noise does not change the nature of the individual's signal extraction problem. The only difference is that it allows for an imprecise signal after aggregation at each time point. To ease notations, we define $\epsilon_{i,t} := v_t^i + \eta_t$. The distribution of shocks and noises is

$$w_{t+1,t} \sim N(0,Q) \quad \epsilon_{i,t} := v_t^i + \eta_t \sim N(0,R),$$

where $Q$ and $R$ are the corresponding variance-covariance matrices.

The agent then updates their beliefs upon observing $\boldsymbol{s}_t^i$ and form expectations according to a linear Kalman Filter as described in (4), where $K$ is the Kalman Gain.[9]

$$\begin{aligned}\boldsymbol{L}^i_{t+1,t|t} &= \hat{A}\boldsymbol{L}^i_{t,t-1|t} \\ &= \hat{A}\big(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}_t^i - G\boldsymbol{L}^i_{t,t-1|t-1})\big)\end{aligned} \tag{4}$$

From equation (4), it is immediately clear that the beliefs about different macroeconomic states in $\boldsymbol{L}^i_{t+1,t|t}$ are correlated for different reasons, even if the actual states are not correlated (i.e., $A$ and $Q$ are diagonal). First, consider the case where the agent learns about different states independently (i.e., $\hat{A}$ is diagonal).[10] We call this scenario "independent learning." The beliefs are correlated if either the signals are combinations of the states (i.e., $G$ is non-diagonal) or the noises in signals are correlated (i.e., $R$ is non-diagonal). These two cases mainly consider the information frictions that can lead to correlations in expectation variables. They can also be thought of as two different formulations of pessimistic/optimistic heuristics. In the first case, the agent confuses multiple states in one signal and adjusts their beliefs on all the states while observing this signal. In the second case, if the noises are positively correlated, the agent is more likely to observe signals about states biased toward the same direction.

[9]For derivations of the standard Kalman Filter, please see Appendix B.1.

[10]This case includes Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Ryngaert (2018), and many others.

Another possibility for observing correlated beliefs is that the agent has a subjective model $\hat{A}$ that is non-diagonal. The form of $\hat{A}$ represents the agent's understanding of the joint dynamics of the macroeconomic states in $\boldsymbol{L}_{t+1,t}$. We call this scenario "joint learning," as the agent believes the underlying macroeconomic states are correlated, and this is incorporated into their belief formation process. As a result, the agent adjusts their beliefs on multiple states even if they observe uncorrelated noisy signals about only one of the states.

As all of the aforementioned possibilities can give rise to the same correlation between beliefs, it is important to consider other moments from the belief data that can distinguish between these possibilities. To achieve this, we propose a test using the serial correlations of forecasting errors because they give distinct testable implications for independent learning and joint-learning models. We call this the *"joint learning test."* To derive this test, consider the forecasting error for one period ahead:

$$\begin{aligned} FE^i_{t+1,t|t} &\equiv \boldsymbol{L}_{t+1,t} - \boldsymbol{L}^i_{t+1,t|t} \\ &= \hat{A}(I - KG)FE^i_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\epsilon_{i,t} \end{aligned} \tag{5}$$

where $M = (A - \hat{A}KG - \hat{A}(I - KG))$. Averaging across agents $i$ at each time $t$, we get an aggregate test on forecasting errors:

$$FE_{t+1,t|t} = \hat{A}(I - KG)FE_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\eta_t \tag{6}$$

Equation (5) and Equation (6) are the basis of our *joint learning test* at the individual and consensus level, respectively.

The key parameters we focus on are the elements in $\hat{A}(I - KG)$. Considering the state vector $\boldsymbol{L}$ contains unemployment rate change and inflation, both equations can be estimated from survey data using OLS as $w_{t+1,t}$ and $\eta_t$ are independent with $FE_{t,t-1|t-1}$ and $\boldsymbol{L}_{t,t-1}$. Before we show the results from actual survey data, we discuss what the joint learning tests tell us about the different possibilities that can result in correlated expectation variables.

## 2.2 Properties of Joint Learning Test

To ease the exposition, we make the following assumptions:

**Assumption 1.** *The subjective transition matrix $\hat{A}$ has positive eigenvalues within the unit circle.*

**Assumption 2.** *The diagonal elements of the $G$ matrix are positive.*

**Assumption 3.** *The variance-covariance matrix of prior $\boldsymbol{L}^i_{t,t-1|t-1}$ is a diagonal matrix and common to each individual:*

$$\Sigma := diag(\{\sigma_i^2\}_{i=1}^n)$$

Assumption 1 suggests that the agent considers a stationary process for the unobservable states. Assumption 2 guarantees that each signal increases as the corresponding state increases.[11] Finally, Assumption 3 assumes that the agent uses priors where the two variables are not correlated with each other.[12]

Under these assumptions, expectations formed by independent learning and joint learning will lead to different properties of the coefficient matrix $\hat{A}(I-KG)$. Following the convention from the literature, we first consider the case of FIRE.

**Proposition 1.** *Under FIRE, e.g., $A = \hat{A}$, $G = I$ and $R \to \mathbf{0}$, the coefficient matrix $\hat{A}(I-KG) = \mathbf{0}$.*

*Proof.* See Appendix C.1. □

This proposition makes clear that lag forecast errors do not predict current forecast errors under FIRE. Note that this is true even under joint expectation formation (i.e., $A$ is non-diagonal). This is consistent with the standard results from the single variable noisy-information model.

[11] This is a regularity assumption, which helps anchor our discussions about the sign restrictions regarding $\hat{A}(I-KG)$. Oppositely moved signals relative to the underlying states imply similar predictions.

[12] We do not separately consider another scenario where the *prior* beliefs of the agent perceives non-zero correlations (i.e., a non-diagonal $\Sigma$), as it is inherently similar to the case of the subjective model perceiving such a correlation (i.e., a non-diagonal $\hat{A}$.)

Next, we turn to the cases with imperfect information where $R \neq \mathbf{0}$. The matrix $\hat{A}(I - KG)$ has different patterns under joint or independent learning. First, we consider the case of independent learning where $\hat{A}$ is diagonal.

**Proposition 2.** ***(Independent Learning)*** *If $\hat{A} = diag(\{a_i\}_{i=1}^n)$, denote the off-diagonal elements of $\hat{A}(I - KG)$ as $w_{ij}$ with $i \neq j$. We have:*

*(1) $w_{ij} = 0$ if $G$ and $R$ are diagonal.*

*(2) $w_{ij} = w_{ji} = 0$ or $w_{ij}w_{ji} > 0$ if $G$ or $R$ is non-diagonal.*

*Proof.* See Appendix C.2. □

Proposition 2 makes two distinct points. First, if the agent does not consider the macroeconomic states to be correlated ($\hat{A}$ is diagonal) and they observe uncorrelated, separate signals regarding each state, the expectation formation process collapses to the single-variable noisy-information model as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). The forecast errors of one variable predict its future forecast errors due to information rigidity, but the forecast errors of other variables can not. Second, under independent learning, if signals on different states are mixed, the forecast errors of one state can predict future forecast errors of the other *symmetrically*. In particular, the directions of such predictability are related to how the signals are generated. For simplicity, we formalize these properties in the case with two states.

**Corollary 1.** ***(Non-diagonal $R$: correlated noises)*** *If $\hat{A}$ and $G$ are diagonal and $R = \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I - KG)$ have the same signs as $\rho$ if $\hat{A}$ has positive entries on the diagonal.*

*Proof.* See Appendix C.3. □

Corollary 1 shows that the forecast error of one state positively predicts the future forecast error of the other if the noises are positively correlated. The intuition is as follows. Without loss of generality, suppose the agent wants to infer the first state. When they see both signals, as they recognize the noises are positively correlated, the agent puts positive weight on the signal about the first state and negative weight on the signal about the other state to correct for the correlation in noises.[13] As a result, a positive shock to state 1 leads to positive forecast errors in both states. The forecast errors of both states are persistent due to information rigidity, so a positive forecast error in the first state predicts a positive forecast error in the second state.

Another possibility is that the signal observed combines information about both states (i.e., $G$ is non-diagonal). In this case, we consider only *triangular* $G$. This configuration is without loss of generality, as any signals with general 2 by 2 $\hat{G}$ can be reformulated into signals with triangular $G$ and they lead to the same posterior beliefs.[14]

**Corollary 2.** ***(Non-diagonal** $G$**: correlated signals)*** *If $\hat{A}$ is diagonal, $R = \begin{pmatrix} \sigma^2_{1,s} & 0 \\ 0 & \sigma^2_{2,s} \end{pmatrix}$ is diagonal and $G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I - KG)$ have the opposite signs as $g_1 g_2$.*

*Proof.* See Appendix C.4. □

To understand the intuition behind Corollary 2, consider the case where $g_1$ and $g_2$ are both positive. When the first state increases and the second state stays the same, the agent sees a positive signal 1. As they are not sure which state increases, the agent adjusts beliefs upwards on both signals. As a result, they have a positive forecast error in the first state and a negative forecast error in the second. Due to information rigidity, a positive forecast error in one state now predicts a negative forecast error in the other state in the future.

Now we move to the case of joint learning. Note that the counter-positive argument of Proposition 2 leads to the testable implications under models of *joint expectation formation.*

[13] We can see this from the fact that the off-diagonal elements in the Kalman Gain are both negative in this case.

[14] The formal proof is included in Appendix C.

**Proposition 3.** ***(Joint Learning)*** *If off-diagonal elements of $\hat{A}(I-KG)$ are not both zeros and of different signs, then $\hat{A}$ is non-diagonal, regardless of whether $G$ and $R$ are diagonal or not.*

*Proof.* This is the counter-positive of Proposition 2. □

Moreover, if we consider the case where signals are separate and not correlated, we can get more informative results about $\hat{A}$ by looking at the off-diagonal elements of $\hat{A}(I-KG)$.

**Proposition 4.** ***(Joint Learning with separate signals)*** *If both $G$ and $R$ are diagonal and $\hat{A} = \left(a_{ij}\right)_{n\times n}$ is non-diagonal, denote $\hat{A}(I-KG) = \left(w_{ij}\right)_{n\times n}$. The signs of these off-diagonal elements are the same as their counterparts in $\hat{A}$ (i.e., $w_{ij}a_{ij} > 0$).*

*Proof.* See Appendix C.5. □

This proposition shows that when the signals on multiple states are separate and noises are uncorrelated, the coefficient matrix $\hat{A}(I-KG)$ has non-zero off-diagonal elements if and only if the agent believes in a non-diagonal $\hat{A}$. The signs on the off-diagonal elements in $\hat{A}(I-KG)$ are the same as those in $\hat{A}$.

The intuition behind this proposition is also straightforward. Suppose that the first element in $\boldsymbol{L}_{t,t-1}$ is the change in the unemployment rate, and the second element is inflation. If the agent over-predicted inflation yesterday due to a noise shock to the inflation signal, they will also over-predict current inflation due to information rigidity. Such an over-prediction will create an over-prediction of unemployment today if the agent believes that higher inflation leads to a higher unemployment rate. In contrast, it will create an under-prediction of unemployment today if they believe that inflation lowers unemployment.

Finally, it is important to note that the properties of the joint learning test we describe in this section do not depend on the actual $A$ matrix at all. In other words, the joint learning test is useful to uncover the agent's subjective model $\hat{A}$ no matter what the true model ($A$) is.

### 2.3 Taking Stock

In Section 2.2, we show that the coefficient matrix $\hat{A}(I-KG)$ in the proposed joint learning test has different properties when beliefs are formed under FIRE, single-variable learning, or joint learning. It is now useful to link the results from such tests with implied correlations between belief variables under these different scenarios. We focus on the case where the hidden macroeconomic states $\boldsymbol{L}_{t+1,t}$ are inflation and change in unemployment rate. Recall the consensus mean forecast is given by the average of (4) across individuals. Define $Y_t = \begin{pmatrix} L_{t,t-1|t-1} \\ L_{t,t-1} \end{pmatrix}$ and we can write (4) and ALM (1) as the following vector autoregression (VAR) model:

$$Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I-KG) & \hat{A}KG \\ \mathbf{0}_{2\times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & I_{2\times 2} \end{pmatrix}}_{F} \cdot \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix} \tag{7}$$

Then we know the stationary variance-covariance matrix is given by:

$$vec(\Sigma_L) = (I_{16} - \Phi \otimes \Phi)^{-1} vec\left(F(R+Q)F'\right) \tag{8}$$

The correlation between belief variables implied by the this covariance matrix differs depending on whether expectations are formed independently, jointly, or under FIRE. Guided by the results from the previous section, we can simply separate these different frictions into the following formulations w.l.o.g.:

$$\hat{A} = \begin{pmatrix} \cdot & m_1 \\ 0 & \cdot \end{pmatrix}, \quad G = \begin{pmatrix} \cdot & g_2 \\ 0 & \cdot \end{pmatrix}, \quad R = \begin{pmatrix} \cdot & \rho \\ \rho & \cdot \end{pmatrix}$$

Table 1 summarizes the testable implications of these different frictions in the noisy-information model.

Table 1: Summary of Joint Learning Test

| | Assuming actual $A$ being diagonal | | | | |
|---|---|---|---|---|---|
| Cases: | $\hat{A}$ | $G$ | $R$ | Off-diagonal elements of $\hat{A}(I-KG)$ | $Corr(E\pi, Eun)$ |
| FIRE | $=A$ | N/A | $=\mathbf{0}$ | both $=0$ | $=Corr(\pi, un)$ |
| $\Sigma$ is Diagonal | Diag | Diag | Diag | both $=0$ | $=0$ |
| | Diag | Diag | $\rho>0$ | both $>0$ | $\gtrless 0$ |
| | Diag | Diag | $\rho<0$ | both $<0$ | $\gtrless 0$ |
| | Diag | $g_2>0$ | Diag | both $<0$ | $\gtrless 0$ |
| | Diag | $g_2<0$ | Diag | both $>0$ | $\gtrless 0$ |
| | $m_1>0$ | Diag | Diag | $>0$ at (1,2), $=0$ at (2,1) | $>0$ |
| | $m_1<0$ | Diag | Diag | $<0$ at (1,2), $=0$ at (2,1) | $<0$ |

Notes: The implied signs of the cross-terms in the forecast error test we proposed before, and the correlation between two macroeconomic states, for different configurations of $\hat{A}$, $G$, and $R$: $R = \begin{pmatrix} \cdot & \rho \\ \rho & \cdot \end{pmatrix}$, $G = \begin{pmatrix} \cdot & g_2 \\ 0 & \cdot \end{pmatrix}$, $\hat{A} = \begin{pmatrix} \cdot & m_1 \\ 0 & \cdot \end{pmatrix}$ We maintain the assumption as in Section 2.2 and 2.3 that $A$ and $\Sigma$ are both diagonal.

Unlike the properties of the joint learning test, the correlation between belief variables clearly depends on the form of $A$. We focus on the most clear-cut case where $A$ is diagonal.[15]

In Table 1, first note that under FIRE or independent learning with separate signals ($\hat{A}$, $G$, and $R$ are all diagonal), they have the same implications on the off-diagonal elements of $\hat{A}(I-KG)$ and $Corr(E\pi, Eun)$. However, under FIRE the diagonal elements of $\hat{A}(I-KG)$ would be zeroes, whereas under independent learning they would be between zero and one. More importantly, Table 1 shows that the positive correlation between expected inflation and unemployment status can come from a correlation in noises, a mix of states in the signals observed, or the agent's subjective model. The off-diagonal elements of $\hat{A}(I-KG)$ from the joint learning test offer additional moments that can help distinguish between these possible explanations. In particular, if the off-diagonal elements are estimated to have different signs, it suggests the agent has a non-diagonal subjective model $\hat{A}$ and correlated or mixed signals *cannot* be the only reasons that lead to the correlation between expectation variables.

[15] When $A$ is non-diagonal, the correlation between inflation and unemployment will be non-zero. In that case, the properties of off-diagonal elements in $\hat{A}(I-KG)$ remain the same as in Table 1. The correlation $corr(E\pi, Eun)$ will be bigger (smaller) than $corr(\pi, un)$ if the off-diagonal elements in $\hat{A}$ are bigger (smaller) than the corresponding elements in $A$.

## 2.4 Empirical Tests on Joint Learning

Guided by Table 1, we perform the joint learning test using survey data from the MSC and the SPF. To do this, we follow (6) and simply estimate the following regressions:

$$\begin{pmatrix} fe^{\pi}_{t+1,t|t} \\ fe^{un}_{t+1,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t,t-1|t-1} \\ fe^{un}_{t,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t,t-1} + e_t \tag{9}$$

where $fe^{x}_{t+h,t|t}$ stands for the h-period ahead forecasting errors of variable $x$.

However, with the MSC, we do not observe $fe^{x}_{t+1,t|t}$ directly; rather, we have data on year-ahead forecast errors $fe^{x}_{t+4,t|t}$. We can then use the four-period-ahead version of equation (6):

$$\begin{aligned} FE_{t+4,t|t} &= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I-\hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\ &\quad - (\hat{W}\hat{A}KG+I)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} + w_{t+4,t+3} - \hat{W}\hat{A}K\eta_t \end{aligned} \tag{10}$$

where $\hat{W} = I + \hat{A} + \hat{A}^2 + \hat{A}^3$, and the fact that $\hat{A}$ is stationary guarantees that $\hat{W}$ is invertible. The derivation that extends (6) to (10) is in **Appendix B.2**. More importantly, the properties of $\beta$s derived in the last section hold true for the year-ahead specification as well. To illustrate the similar performance of the proposed quarter-ahead test (6) and year-ahead test (10), we perform the proposed tests with simulated data and include these results in **Appendix D**. We can then estimate:

$$\begin{pmatrix} fe^{\pi}_{t+4,t|t} \\ fe^{un}_{t+4,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t+3,t-1|t-1} \\ fe^{un}_{t+3,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t+3,t-1} + e_t \tag{11}$$

The parameters of interest are $\beta_{11}, \beta_{12}, \beta_{21}$, and $\beta_{22}$. They can be estimated using OLS because, in equation (10), the two components of the error term are uncorrelated with all the regressors. The $w_{t+4,t+3}$ is an unpredictable error happening after $t+3$, thus uncorrelated with forecasting errors up to $t+3$ as well as any variable realized before $t+4$. The noise attached to public signal $\eta_t$ is realized at time $t$ and thus does not correlate with the forecast error with the information set at time $t-1$. Here we have to assume there is no feedback effect of $\eta_t$ on realized macroeconomic variables after time $t$ through general equilibrium so that $\eta_t$ is uncorrelated with any variable (except for expectational ones) realized beyond time $t$.[16]

Another complication to performing the test is that it requires the unemployment rate change to be comparable to the realized data to create forecast errors. The data in the MSC on unemployment expectation is categorical. We follow Bhandari *et al.* (2025) and Mankiw *et al.* (2004) to impute the expectation series.[17]

It is worth noting that the assumption essential to recovering unemployment expectation is that the predicted unemployment change follows a normal distribution with a constant variance across time. This assumption is particularly plausible in the framework of a noisy-information model with a stationary Kalman Filter, as the posterior distributions of forecasted variables are normally distributed and stationarity guarantees a time-invariant posterior variance.

We then estimate (11) with year-ahead forecast errors on expected inflation and expected unemployment rate change with OLS, controlling for corresponding realized variables according to (10).[18] Four coefficients in (11) are estimated. Among these, $\beta_{11}$ and $\beta_{22}$ are the typical indicators for the presence of information rigidity as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). Higher values of these terms imply higher degrees of information rigidity (noisier signals). The key coefficients related to joint learning are $\beta_{12}$ and $\beta_{21}$. We call them *the cross-terms* of coefficients on forecast errors, the properties of which are summarized in Table 1. The goal of this exercise is to assess which model of expectation

[16]Notice $v_t^i$ disappears as we derive the consensus expectation. This is because the idiosyncratic noise has a zero mean at each time point.

[17]We discuss the imputation approach in Appendix A.5.

[18]The imputation method involves the use of SPF and uses the consensus expectation on unemployment status. Such an approach does not apply to panel data. For this reason, in the baseline analysis for the SPF and the MSC, we consider the aggregate version of the joint-learning test (10).

formation can be reconciled with the estimates of these four coefficients from survey data. Table 2 presents the key results with the MSC and the SPF.

Table 2: Aggregate Test on Joint Learning, MSC vs SPF

| | MSC | | | SPF | | |
|---|---|---|---|---|---|---|
| | 1984-2023 | 1981-2018 | 1990-2018 | 1984-2023 | 1981-2018 | 1990-2018 |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\beta_{11}$ | 0.64*** | 0.61*** | 0.65*** | 0.79*** | 0.74*** | 0.76*** |
| | (0.080) | (0.066) | (0.085) | (0.064) | (0.061) | (0.093) |
| $\beta_{12}$ | −0.11 | −0.14 | −0.02 | 0.19 | −0.28 | −0.08 |
| | (0.076) | (0.087) | (0.095) | (0.117) | (0.200) | (0.199) |
| $\beta_{21}$ | 0.13*** | 0.11*** | 0.21*** | 0.05 | 0.04* | 0.06 |
| | (0.033) | (0.039) | (0.063) | (0.034) | (0.024) | (0.049) |
| $\beta_{22}$ | 0.71*** | 0.60*** | 0.50*** | 0.63*** | 0.55*** | 0.51*** |
| | (0.044) | (0.079) | (0.091) | (0.060) | (0.072) | (0.097) |
| Observations | 152 | 149 | 116 | 152 | 149 | 116 |

* ***,**,*: Significance at 1%, 5%, and 10% level. Estimation results for joint-learning test (11). Columns (1)–(3) are results from the MSC and (4)–(6) are results from SPF. Columns (1) and (4) use a sample of 1984–2023, excluding the outlier of the year 2019, where the change in unemployment is around 10%. Columns (2) and (5) use samples 1981q3–2018q4 to avoid the COVID-19 period. Columns (3) and (6) use a sample from 1990–2018 to stay away from the Volker and COVID-19 periods. Newey-West standard errors are reported in brackets.

The first column of Table 2 contains estimation results using the baseline sample for 1984–2023. The estimates on $\beta_{11}$ and $\beta_{22}$ are significantly positive, meaning that the consumers form expectations with limited information. The significant estimates on $\beta_{21}$ suggest that consumers *do not* form expectations on unemployment and inflation independently, with separate signals. Moreover, the fact that $\beta_{12}$ and $\beta_{21}$ have different signs suggests that consumers are forming expectations jointly with subjective beliefs about the structural relationship between inflation and unemployment, $\hat{A}$.[19] According to Proposition 4, the agent's subjective model indicates that past inflation will lead to an increase in the unemployment rate. From Table 1, such a belief structure $\hat{A}$ can induce a positive correlation between the two expectations.

Columns (1)–(3) in Table 2 also suggest that the pessimistic heuristics in the form of

[19]This follows from Proposition 3. To be clear, the test results in Table 2 suggest that $\hat{A}$ is non-diagonal, but they DO NOT exclude the possibility that $G$ and $R$ may also be non-diagonal.

non-diagonal $R$ or $G$ cannot be the *only* reason for the positive correlation between expected inflation and unemployment status. If pessimistic heuristics are the only frictions in expectation formation, $\beta_{21}$ and $\beta_{12}$ would both be negative. These are inconsistent with the estimates in Table 2.

On the other hand, the results from columns (4)–(6) show that the professionals seem to have a different $\hat{A}$ from consumers. The significant $\beta_{11}$ and $\beta_{22}$ suggest again the presence of information rigidity. The estimates are comparable with previous studies imposing independent learning.[20] Contrary to the results with the MSC, the small and insignificant $\beta_{12}$ and $\beta_{21}$ imply that they do not believe lagged inflation will raise the future unemployment rate. These results are consistent with the finding that there is a positive correlation between expected unemployment and inflation in the MSC, whereas such a correlation does not appear in SPF. All in all, the estimates from SPF suggest that professionals are closer to independent expectation formation, or at least use a different structure $\hat{A}$ from consumers, when forming expectations.

Moreover, all the above results hold for different cuts of samples. In columns (2) and (5), we omit the COVID-19 episode and the results for both the MSC and the SPF are consistent with those in the baseline results. Recall in Figure 1 that the correlations between realized inflation and unemployment fall below zero after the 1990s.[21] Meanwhile, the correlation between expected variables in MSC stays positive. It is in this episode that the two correlations have the starkest disconnection. In columns (3) and (6), we include the estimates using a subsample from 1990–2018 for both the MSC and the SPF. The results are qualitatively in line with those using the baseline sample. Moreover, the estimated $\beta_{21}$ is twice as large, suggesting that consumers believe in a stronger response of the future unemployment rate to current inflation.

[20] For example, in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013).

[21] In Figure 1, we used a 10-year rolling window and plotted the correlation against the ending date of that window. The figure suggests using realized data after the 1990s, inflation and unemployment became negatively correlated.

# 3 Uncovering the Subjective Model

The previous sections focus on the joint learning test about forecast errors under sign restrictions according to Table 1. The test results suggest that the households use a subjective model $\hat{A}$ to jointly form expectations on inflation and unemployment. In particular, they perceive that past inflation increases unemployment but not vice versa. Expectations based on such a model $\hat{A}$ generate a positive association between expected inflation and unemployment, consistent with survey data. However, there are two caveats for the previous test scheme: (1) it relies on the assumption of an uncorrelated prior, and (2) it does not assess whether the agent's perceived law of motion (PLM) aligns with the actual law of motion (ALM) (i.e., whether $\hat{A} = A$). To address these concerns, this section directly estimates the joint dynamics of inflation, changes in the unemployment rate, and their respective expectations, as specified in (7):

$$Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I-KG) & \hat{A}KG \\ \mathbf{0}_{2\times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & I_{2\times 2} \end{pmatrix}}_{F} \cdot \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix}$$

Note that this VAR representation follows directly from the general noisy information framework and the ALM, thus it does not depend on the assumption of uncorrelated priors. The estimation of (7) yields estimates of $A$, $\hat{A}KG$, and $\hat{A}(I-KG)$. Summing the estimated $\hat{A}(I-KG) \equiv B$ and $\hat{A}G \equiv C$ yields estimates of $\hat{A}$. This is convenient in that we can directly test if $A = \hat{A}$, and the estimation of the PLM $\hat{A}$ does not rely on uncovering the exact degree of information frictions governed by $K$ and $G$.

We estimate (7) with the same dataset as in our joint learning tests, using the iterative generalized method of moments (GMM) with an efficient weighting matrix. Note that to be consistent with the baseline model assumption where there is no feedback loop from expectations to realized values, the estimation also includes restrictions that the bottom-right 2-by-2 submatrix in $\Phi$ contains all zeros.[22] Lastly, because quarterly observations of annualized changes are used, we use Newey-West standard errors up to four quarters when calculating the variance and covariance matrix of moment conditions. Table 3 reports the estimation results with our baseline sample 1984q1–2023q4.[23]

Table 3: Estimates of Joint Learning Model (7)

| **MSC, quarterly, Q1 1984–Q4 2023** | | |
|---|---|---|
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.836 & -0.058 \\ 0.034 & 0.617 \end{bmatrix}$ | $\begin{bmatrix} 0.053 & 0.057 \\ 0.042 & 0.095 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.741 & -0.149 \\ 0.137 & 0.831 \end{bmatrix}$ | $\begin{bmatrix} 0.050 & 0.082 \\ 0.044 & 0.048 \end{bmatrix}$ |
| T-test: | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | 1.581 | 0.057 |
| **SPF, quarterly, Q1 1984–Q4 2023** | | |
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.837 & -0.056 \\ 0.014 & 0.751 \end{bmatrix}$ | $\begin{bmatrix} 0.061 & 0.074 \\ 0.041 & 0.093 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.955 & -0.038 \\ 0.040 & 0.495 \end{bmatrix}$ | $\begin{bmatrix} 0.019 & 0.016 \\ 0.035 & 0.239 \end{bmatrix}$ |
| T-Test | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | 0.546 | 0.293 |

The table reports the estimates and their Newey-West standard errors from the GMM estimation of the four-variable VAR model. An iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ are used to calculate the standard errors of $\hat{A}$ estimates.

We primarily focus on the estimates of the off-diagonal terms of $\hat{A}$, which reveal the perceived between-variable serial correlation between inflation and unemployment rate changes. Although according to the ALM, $A_{2,1}$, the realized unemployment rate change's response

[22] In the Appendix, we report the estimates from an unrestricted VAR allowing for feedback effects from expectations to realized values in Table 15; our major findings remain intact.

[23] Estimation results based on alternative samples, 1984-2019 or 1990-2018, yield identical conclusions (Table 14).

to the lagged inflation rate, is 0.034 and is insignificantly different from 0, the perceived response is as big as 0.14 with a standard error of 0.04. This result is consistent with the evidence thus far that current inflation leads to more pessimistic labor market expectations. To decide if the difference is statistically different from zero, we perform a statistical test of GMM estimates under a null hypothesis of $A_{2,1} < B_{2,1} + C_{2,1} \equiv \hat{A}_{2,1}$. The null hypothesis is easily rejected at the 5% significance level.

In contrast, the estimation of professional forecasts in the same sample confirms different PLM patterns from those of households. In particular, the professionals' subjective model perceives little impact of current inflation on future unemployment rate changes ($\hat{A}_{2,1} = 0.04, s.e. = 0.035$). This is not significantly different from the actual impacts of inflation on unemployment ($A_{2,1} = 0.014, s.e. = 0.041$). It is also worth noting that in both MSC and SPF cases, estimates of $A_{2,1}$ are not significantly different from zero.

Besides the between-variable correlation, PLM also differs from ALM in terms of the persistence of the inflation rate. Households, on average, underperceive the persistence of inflation. In contrast, the professionals' subjective model overly perceives the persistence of inflation. This is consistent with Ryngaert (2018), who finds overperception of the persistence in SPF inflation forecasts.

Up to this point, our analysis remains agnostic about the reasons behind the subjective model as captured in $\hat{A}$. One important consideration is the role of monetary policy. One may argue that the positive subjective association of future unemployment rate and current inflation reflects a sensible expectation that a tightening response of monetary policy to current inflation may lead to a weakening of labor markets. However, this alone cannot explain the gap between PLM and ALM in terms of the between-variable correlation. The gap could be because households overperceive the central bank's responsiveness to inflation or the impacts of monetary policy on labor markets. This is reminiscent of the findings from Carvalho and Nechio (2014); Dräger *et al.* (2016); Bauer *et al.* (2024), which show the time-varying and subjective patterns of monetary policy perceptions. Our findings do not directly include monetary policy as an additional variable in the VAR. Instead, we treat the uncovered $\hat{A}$ as a sufficient statistic for the perceived correlation that may stem from all kinds of naive to sophisticated reasons, including misperceptions about monetary policy rules.

## 3.1 Shock Propagation Under a Subjective Model

How does the uncovered subjective model affect the dynamics of realized inflation and unemployment rate in general equilibrium? To answer this, we consider a simple three-equation New Keynesian Model as in Galí (2015):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + s_t \tag{12}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + d_t \tag{13}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t, \tag{14}$$

where $y_t$ denotes the output gap, and $s_t$ and $d_t$ are supply and demand shocks that follow persistent AR(1) processes. To incorporate our expectation formation model, we first invoke Okun's Law and assume $u_t = -\chi y_t$, and then assume expectations are formed according to the consensus version of (4):[24]

$$L_{t+1,t|t} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t, \tag{15}$$

where $L_{t+1,t|t} \equiv \begin{pmatrix} E_t \pi_{t+1} \\ E_t u_{t+1} \end{pmatrix}$ and $L_{t,t-1} \equiv \begin{pmatrix} \pi_t \\ u_t \end{pmatrix}$. Equation (15) restates how expectation is formed using the subjective model $\hat{A}$ and (signals of) realized economic variables, and equations (12)–(14) illustrate how these expectations, in turn, influence the actual evolution of economic outcomes.

[24] Here we assume both $E_t \pi_{t+1}$ and $E_t u_{t+1}$ are formed by households using the subjective model $\hat{A}$.

**Calibration:** The micro-founded model that gives (12)–(14) features households with intertemporal elasticity $\sigma$ and a Frisch elasticity of labor supply $\psi$. There is a continuum of monopolistic competitive firms with $1-\theta$ probability to adjust prices every period. As a result, the slope of the Phillips curve is $\kappa = \lambda(\sigma + \frac{\psi+\alpha}{1-\alpha})$, where $1-\alpha$ is the share of labor input in the firm's production function and $\lambda$ is the coefficient on marginal cost. It follows that $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, with $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

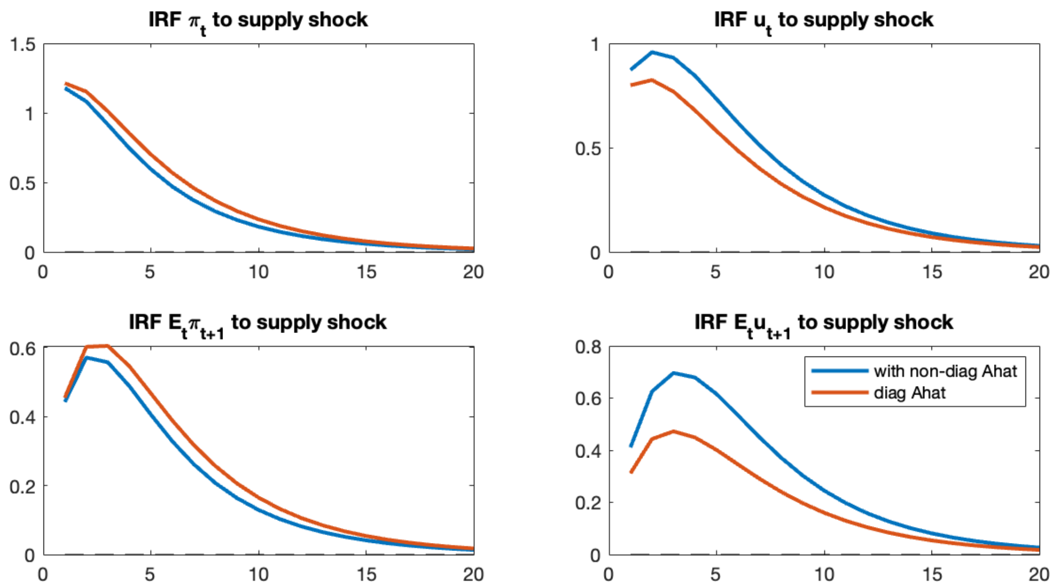
We follow the baseline calibration in Galí (2015) by setting $\beta = 0.99$, $\sigma = 1$, $\psi = 1$, $\alpha = 1/3$, $\epsilon = 6$, and $\theta = 2/3$, which implies an average price duration of three quarters. The policy parameters in the Taylor rule are $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, which guarantee the determinacy of the equilibrium. Finally, we use $\chi = 0.43$ from Ball *et al.* (2017). The persistence of supply and demand shocks are set to be 0.8 with unit standard deviations.

The purpose of this exercise is to illustrate how the subjective belief that inflation leads to unemployment influences the dynamics of realized inflation and unemployment. To this end, we first examine the impulse response functions (IRFs) to supply and demand shocks in our baseline model, where $\hat{A} = \begin{pmatrix} 0.74 & 0 \\ 0.14 & 0.83 \end{pmatrix}$—a structure chosen to align with the lower triangular matrix estimated in Section 3. We then compare these IRFs to those obtained when $\hat{A}$ has zero off-diagonal elements.

Figure 2 shows the responses of inflation, the unemployment rate, and their corresponding expectations to a 1% increase in the supply shock. The blue lines represent the IRFs under the baseline model, where the agent holds a subjective belief that past inflation leads to higher unemployment. The red lines correspond to the case where $\hat{A}$ is diagonal, implying no such perceived link between inflation and unemployment.

Consistent with the standard New Keynesian (NK) model, both inflation and unemployment rise following a positive supply shock. However, the responses in our framework are more persistent, reflecting the role of noisy information in expectation formation. Comparing the two models, the key difference lies in the behavior of unemployment: It rises more

Figure 2: IRF in Response to Supply Shock

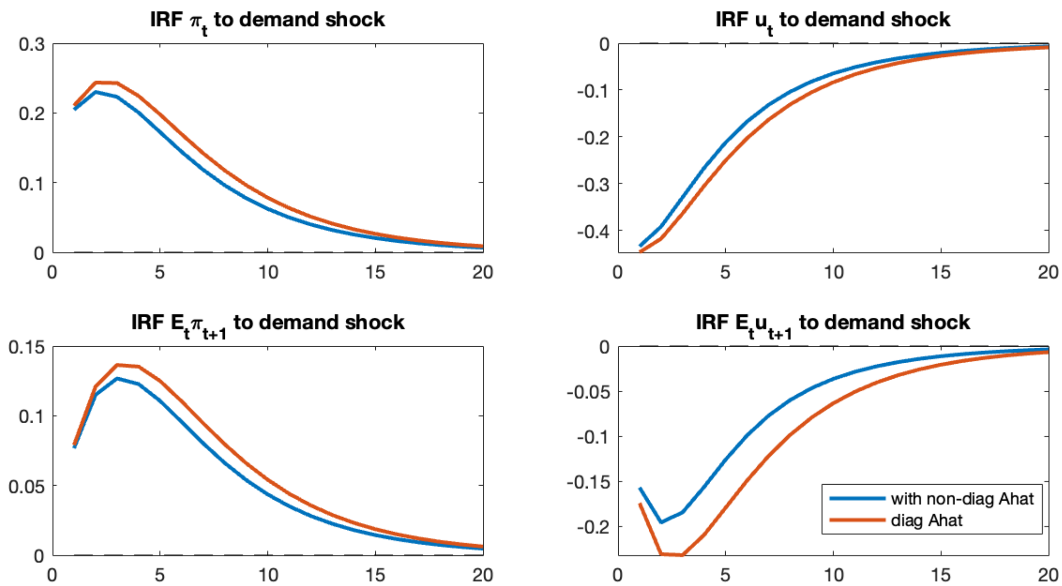



sharply in the baseline model. This is because households, believing that higher inflation signals deteriorating economic conditions, reduce their demand more aggressively than in the diagonal $\hat{A}$ case. This additional contractionary force also slightly dampens inflation through the New Keynesian Phillips Curve (NKPC) channel.

In contrast, responses to a positive demand shock reveal a different pattern, as depicted in Figure 3. In the baseline model, higher inflation leads households to perceive the economy as less overheated compared to the diagonal $\hat{A}$ case. This perception induces a moderating effect on both expectations of inflation and unemployment, which dampens the responses of these expected and realized variables both on impact and in subsequent periods.

Overall, when households form expectations based on a subjective model in which past inflation is believed to lead to higher unemployment, policymakers face a more challenging trade-off between inflation and unemployment following a supply shock. In contrast, this belief can help stabilize both inflation and unemployment in response to demand shocks. During episodes characterized by significant supply-side disruptions, policymakers may incur greater welfare losses if they fail to account for the subjective models underpinning household

Figure 3: IRF in Response to Demand Shock




expectations.

# 4 Empirical Evidence I: Inflation News and Expectations

The goal of this section is to supplement structural evidence presented thus far with micro survey evidence regarding the pattern of $\hat{A}$, by directly controlling the information set of individuals (i.e., $s_t^i$ in the model environment).

In particular, we utilize the self-reported exposure to macroeconomic news in the MSC.[25] The survey question asks what kind of news the respondent has heard in the last few months. The answers are categorized into different types of news reported by the survey respondents, and we further summarize these types of news into 10 categories.[26]

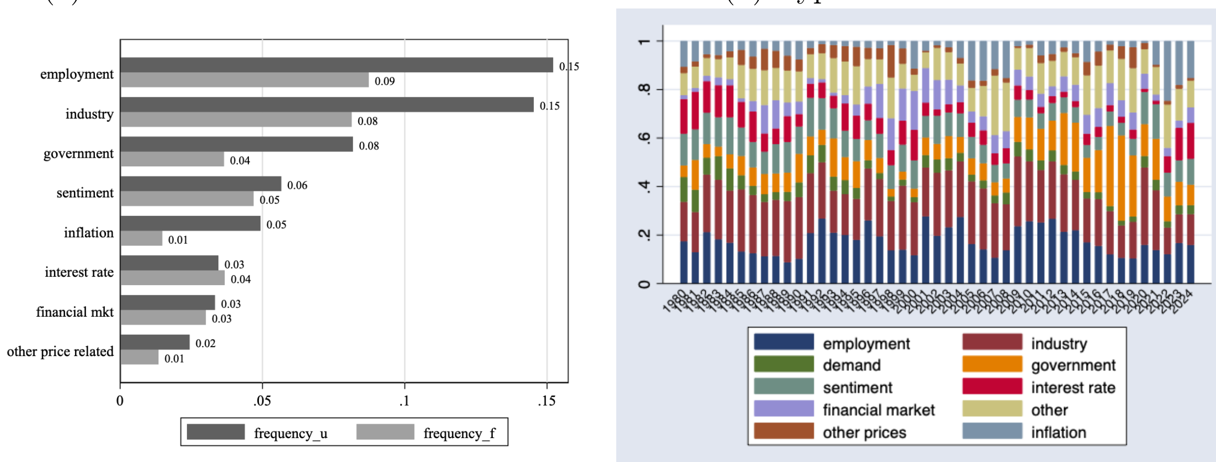
The MSC labels the reported news as "favorable" or "unfavorable" according to the description of the news. For all the panel respondents in the MSC, around 37% report that

[25]The idea of using the MSC's news exposure as a proxy of the respondents' information set dates back to Doms and Morin (2004), Pfajfar and Santoro (2013), and Lamla and Maag (2012).

[26]The descriptions of the question and the variable are included in Appendix F.1. Table 16 in Appendix F.1 summarizes what types of news are included in each category.

Figure 4: Type of News

(a) Fraction of fav and unfav news (b) Type of news across time




*Notes: Panel (a): fractions of favorable and unfavorable news reported by individuals with news in the MSC. Panel (b): shares of different types of news out of total news reported each year.*

they have heard no news in the past few months. For the remaining 63% of respondents who have heard news, Figure 4 summarizes the types of news they report.

In Figure 4, panel (a) shows the fraction of favorable and unfavorable news reported by the survey respondents who have heard news. News on industry, employment, government, and inflation account for 60% of the news reported. Among these types, the respondents report much more unfavorable than favorable news. News on industry, employment, and demand are major categories related to real activities in the economy. Panel (b) plots the shares of different types of news out of the total news reported in each year. From Figure 4, we see that most of the news is clearly labeled to be related to some specific economic aspect. The news with unclear labels is categorized as "sentiment"[27] and only accounts for around 11% of news reported.

We perform a panel regression of expected inflation and unemployment status[28] on indicators of receiving different types of news, controlling for individual and time-fixed effects. Table 4 suggests that hearing news on high (low) inflation increases (decreases) reported

[27]See Table 16.

[28]The expected unemployment variable takes the value 1/0/-1 if the survey respondent says unemployment rates will increase/stay the same/decrease.

expected inflation by about 0.43% (0.21%) and increases the probability of believing the unemployment rate will rise (fall) by 2.5%. However, employment news only has a significant impact on unemployment expectations, not on inflation expectations. This pattern remains in columns (3) and (4), where we include all types of news in the regression. Another pattern worth mentioning is that, unlike unemployment expectations, inflation expectations rarely react to news from other domains.

We also examine how news exposure affects the positive association between expected inflation and unemployment at the individual level. We run a panel regression of expected inflation on the measure of expected unemployment, indicators of different news reported, and the interactions between expected unemployment and news indicators. We are interested in whether the correlation between expected inflation and unemployment changes depends on what news the individuals hear about.

In column (1) of Table 5, we include only indicators for inflation, employment, and interest rate news. The correlation between expected inflation and unemployment for individuals without this news is around 0.36. This number doubles for the individuals who report hearing news about inflation going up, and it is significantly smaller for individuals who hear unfavorable news about employment or favorable news about interest rates. In column (2), we further include more indicators of all types of news in Table 16. The correlation for individuals hearing no news is 0.38. This correlation is significantly lower for those who hear news about real activities like employment, specific industries, and demand, a pattern consistent with our explanation through a subjective model. On the other hand, such a correlation is much higher for individuals who hear inflation news or an unfavorable sentiment. This latter result aligns with the finding in Bhandari *et al.* (2025).

How much does the individual-level correlation, conditional on news heard, help explain the correlation in consensus expectations? In Figure 5, we plot the mean of each year for consensus expectations on inflation and unemployment for 1984–2023, conditional on

Table 4: FE Panel Regression with Self-reported News

| Expectation on: | Inflation | Likelihood Unemployment Increase | Inflation | Likelihood Unemployment Increase |
|---|---|---|---|---|
| news on: | (1) | (2) | (3) | (4) |
| Inflation fav | -0.21* | -0.06*** | -0.21* | -0.05*** |
| | (0.117) | (0.017) | (0.118) | (0.017) |
| Inflation unfav | 0.43*** | 0.06*** | 0.42*** | 0.05*** |
| | (0.085) | (0.010) | (0.085) | (0.010) |
| Employment fav | -0.03 | -0.14*** | -0.01 | -0.13*** |
| | (0.056) | (0.009) | (0.057) | (0.009) |
| Employment unfav | 0.05 | 0.10*** | 0.04 | 0.09*** |
| | (0.054) | (0.007) | (0.054) | (0.007) |
| Interest rate fav | -0.03 | -0.06*** | -0.01 | -0.04*** |
| | (0.071) | (0.012) | (0.072) | (0.012) |
| Interest rate unfav | 0.02 | 0.11*** | 0.02 | 0.10*** |
| | (0.081) | (0.012) | (0.081) | (0.012) |
| Industry fav | | | -0.20*** | -0.10*** |
| | | | (0.059) | (0.008) |
| Industry unfav | | | 0.11** | 0.08*** |
| | | | (0.053) | (0.006) |
| Demand fav | | | -0.16 | -0.09*** |
| | | | (0.104) | (0.014) |
| Demand unfav | | | -0.04 | 0.07*** |
| | | | (0.111) | (0.013) |
| Gov fav | | | -0.12 | -0.09*** |
| | | | (0.077) | (0.012) |
| Gov unfav | | | 0.21*** | 0.10*** |
| | | | (0.058) | (0.008) |
| Sentiment fav | | | -0.12* | -0.12*** |
| | | | (0.069) | (0.010) |
| Sentiment unfav | | | 0.09 | 0.07*** |
| | | | (0.078) | (0.009) |
| Stock fav | | | -0.07 | -0.07*** |
| | | | (0.059) | (0.011) |
| Stock unfav | | | 0.05 | 0.07*** |
| | | | (0.077) | (0.011) |
| Other prices fav | | | -0.22** | -0.04*** |
| | | | (0.102) | (0.016) |
| Other prices unfav | | | 0.04 | 0.04*** |
| | | | (0.087) | (0.013) |
| Other real fav | | | -0.02 | -0.07*** |
| | | | (0.108) | (0.019) |
| Other real unfav | | | 0.22* | 0.04*** |
| | | | (0.117) | (0.013) |
| Wage fav | | | 0.03 | -0.03 |
| | | | (0.158) | (0.024) |
| Wage unfav | | | -0.09 | 0.07*** |
| | | | (0.149) | (0.016) |
| Observations | 169304 | 189158 | 169304 | 189158 |
| $R^2$ | 0.673 | 0.677 | 0.673 | 0.681 |
| Time F.E. | Y | Y | Y | Y |
| Individual F.E. | Y | Y | Y | Y |

[*] ***,**,*: Significance at 1%, 5%, and 10% level. Results come from fixed-effect panel regressions of expectations on different dummies of self-reported news. Columns (1) and (3) use expected inflation as the dependent variable; columns (2) and (4) use the categorical variable of the expected unemployment rate to increase/stay the same/decrease as the dependent variable. The results control for individual fixed effects and time-fixed effects. Standard errors are adjusted for heteroscedasticity and autocorrelation.

Table 5: Correlation Conditional on News Heard

| Dependent var: | $E\pi$ | |
|---|---|---|
| | (1) | (2) |
| $Eun$ | 0.36*** | 0.38*** |
| | (0.034) | (0.047) |
| Inflation fav $\times Eun$ | 0.17 | 0.16 |
| | (0.164) | (0.164) |
| Inflation unfav $\times Eun$ | 0.36*** | 0.36*** |
| | (0.117) | (0.118) |
| Employment fav $\times Eun$ | 0.03 | 0.03 |
| | (0.089) | (0.090) |
| Employment unfav $\times Eun$ | -0.20*** | -0.16** |
| | (0.073) | (0.074) |
| Interest rate fav $\times Eun$ | -0.23** | -0.24** |
| | (0.104) | (0.104) |
| Interest rate unfav $\times Eun$ | -0.16 | -0.16 |
| | (0.114) | (0.115) |
| Industry fav $\times Eun$ | | 0.06 |
| | | (0.092) |
| Industry unfav $\times Eun$ | | -0.23*** |
| | | (0.073) |
| Demand fav $\times Eun$ | | -0.14 |
| | | (0.145) |
| Demand unfav $\times Eun$ | | -0.57*** |
| | | (0.155) |
| Gov fav $\times Eun$ | | 0.08 |
| | | (0.107) |
| Gov unfav $\times Eun$ | | 0.01 |
| | | (0.079) |
| Sentiment fav $\times Eun$ | | 0.01 |
| | | (0.112) |
| Sentiment unfav $\times Eun$ | | 0.24** |
| | | (0.113) |
| Stock fav $\times Eun$ | | -0.11 |
| | | (0.085) |
| Stock unfav $\times Eun$ | | 0.06 |
| | | (0.115) |
| Other prices fav $\times Eun$ | | -0.01 |
| | | (0.152) |
| Other prices unfav $\times Eun$ | | -0.16 |
| | | (0.130) |
| Other real fav $\times Eun$ | | -0.11 |
| | | (0.168) |
| Other real unfav $\times Eun$ | | -0.21 |
| | | (0.157) |
| Wage fav $\times Eun$ | | -0.17 |
| | | (0.235) |
| Wage unfav $\times Eun$ | | 0.00 |
| | | (0.224) |
| Observations | 167346 | 167346 |
| $R^2$ | 0.674 | 0.675 |
| Time F.E. | Y | Y |
| Individual F.E. | Y | Y |

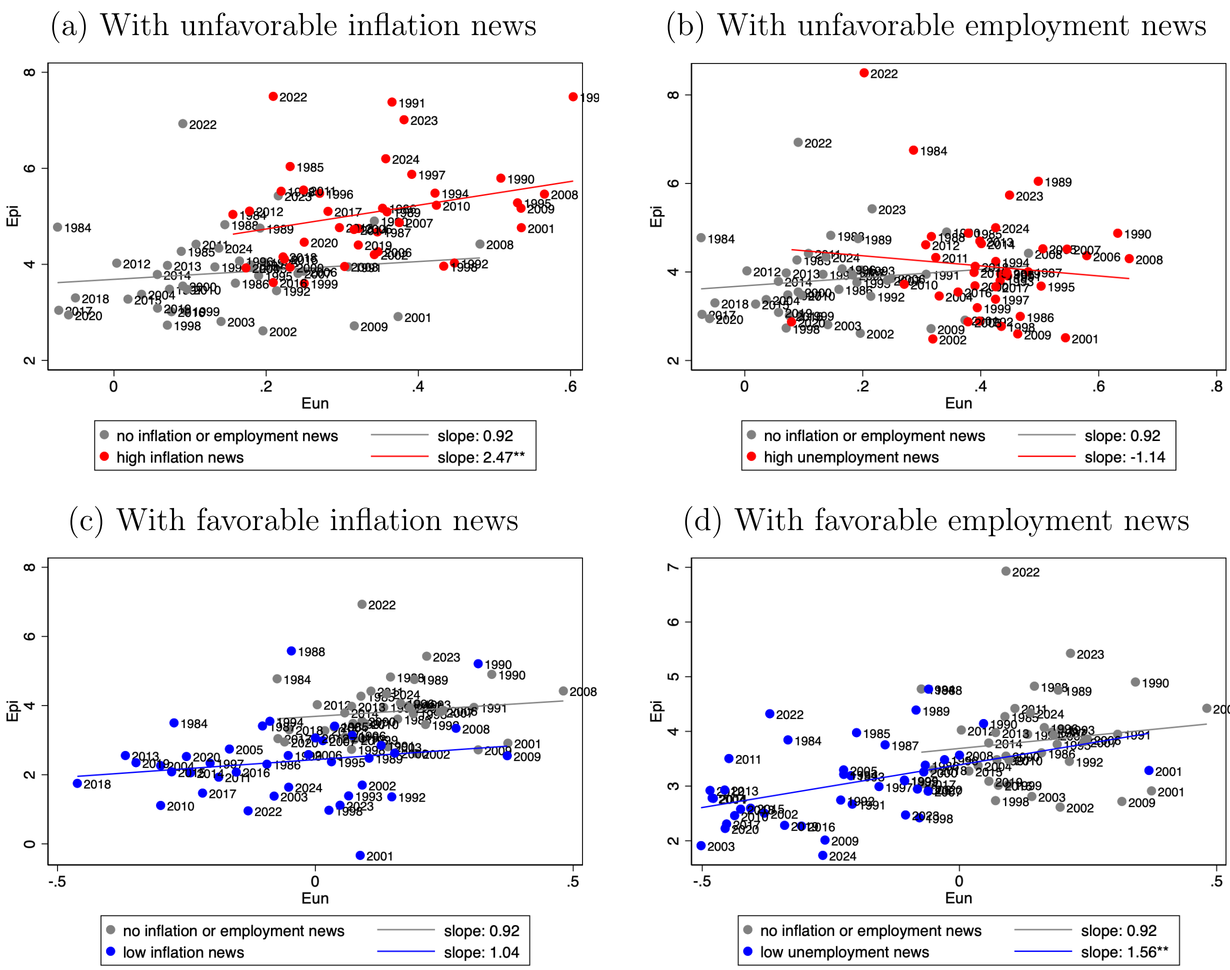
* ***,**,*: Significance at 1%, 5%, and 10% level. The results control for individual fixed effects and time-fixed effects. Standard errors are adjusted for heteroscedasticity and autocorrelation.

hearing inflation news or unemployment news.[29] In Figure 5 panel (a) and (b), the red dots are consensus expectations in each year, conditional on hearing unfavorable inflation or unemployment news. The gray dots represent people without employment or inflation news. Several patterns emerge from this figure. First of all, from panel (a), individuals who hear high inflation news on average expect higher inflation and unemployment changes than those who do not report hearing inflation or employment news. This contrasts with individuals who hear about news on high unemployment rates. These individuals only adjust their unemployment expectations upwards, but not their inflation expectations (panel (b)). Second, we see a positive correlation between the two expectations across time for individuals who hear high inflation news. The correlation of consensus expectations among people who do not hear inflation or employment news is low and insignificant. Moreover, the correlation becomes negative for people who hear unfavorable employment news. Finally, hearing favorable inflation news lowers both expected inflation and unemployment, similar to hearing favorable employment news. The correlation between consensus expectations among individuals who hear favorable news is not significantly different from those among people who do not hear this news. These results are in line with the findings from Table 5. Altogether, they suggest that types of news play a crucial role in explaining the correlation between consensus expectations. The positive correlations are mostly among individuals who hear news about inflation being high. Such a correlation disappears among those who hear about a bad employment status.

We consider these empirical patterns to support the importance of subjective model friction in explaining the correlation between expected inflation and unemployment, both at the aggregate and the individual level. The households from the MSC distinguish the types of information they hear about and adjust their expectations in different ways depending on the content of the information. In particular, news about high inflation leads them to adjust both inflation and unemployment expectations upwards, contributing to the positive

[29] We exclude the COVID-19 pandemic years, 2020 and 2021, as the unemployment rate numbers are extremely high, making them outliers for the sample.

Figure 5: Consensus Expectations Conditional on News Heard




*Notes: Scatterplot for consensus expected inflation and unemployment each year for 1984–2023. Gray dots in all panels are expectations for individuals who do not hear employment or inflation news. Top left panel: red dots are expectations conditional on hearing high inflation news. Top right panel: red dots are expectations conditional on hearing high unemployment news. Bottom left panel: blue dots are expectations conditional on hearing low inflation news. Bottom right panel: blue dots are expectations conditional on hearing low unemployment news.*

correlation between these two expectations. On the contrary, bad employment news only moves unemployment expectations, thus lowering the correlation.

# 5 Empirical Evidence II: Inflation-Unemployment Narratives in Newspapers

The previous section shows that self-reported news exposure changes households' domain-specific expectations, but only inflation news has an impact on the expectations across domains. What is special about inflation?

Recognizing the mass news media as one of the important sources of information for households to learn about the macroeconomy,[30] we further corroborate these findings by directly measuring news coverage on inflation, unemployment, and other related macroeconomic topics from a historical news archive. We confirm that measured news coverage is indeed correlated with self-reported news exposure and is also domain-specific. Second, inflation news coverage is often associated with unfavorable perceptions, while unemployment news coverage has a relatively neutral connotation. Third, news articles are more likely to simultaneously discuss inflation and unemployment when inflation is high, while there is no such pattern with unemployment rates.

In practice, we use a selected sample of 150,000 news articles published in *The Wall Street Journal*[31] (WSJ) between January 1984 and June 2022. These are filtered based on several criteria from a repeated random sample of 25,0000 articles in the database, around 25% of the total number of articles published in the WSJ in this period. In particular, we exclude articles directly covering the news in non-U.S. countries/regions, and those that are not directly related to macroeconomic and financial markets (e.g., sports and culture, and so

[30] See evidence from Carroll (2003), Doms and Morin (2004), Larsen *et al.* (2021), and Chahrour *et al.* (2024).

[31] We choose the WSJ as its main focus is economic and financial news targeted at the U.S. audience. Our results are confirmed by the same analysis of another major news outlet, *The New York Times*.

on). In the main body of the paper, we primarily rely on simple keyword counts to determine if a news article is related to a particular topic.[32] Then we construct article-specific news coverage of each topic using the frequency of keywords or average topic weights.

We define the news coverage of a particular topic (e.g., inflation) as the sum of the frequencies of the term "inflation" mentioned as a share of the total number of words within each article. Over the sample period, the time series of the news coverage of inflation and unemployment are highly correlated with their respective self-reported news exposure in the MSC. In particular, the correlation coefficient between the news measure and the share of MSC households that report having heard *any* news about prices is 0.6. The correlation regarding unemployment news is around 0.37 (see Table 6). Note that here *any* news is measured by gross exposure: the total fraction who have heard some either good or bad news (see Figure 6).

The news coverage is often domain-specific. Over the sample period, the time variations of news coverage of inflation and unemployment exhibit patterns of their own and do not simultaneously move. The correlation coefficients between two measures of news coverage are close to zero across various measures. This suggests that at least the joint news coverage of unemployment and inflation cannot be the common factor that drives the correlations between unemployment and inflation expectations. This is consistent with the finding in the previous section that households can distinguish between news on inflation and news on unemployment.

But there are differences between the two types of news. Unlike unemployment news, inflation news coverage is mostly labeled as unfavorable. This can be seen from the fact that the high correlation between news coverage and self-reported exposure to any news on inflation is entirely driven by the share of agents who "have heard about *unfavorable* news about prices." The correlation between self-reported negative exposure and news coverage

[32]In the Appendix, we report results with topic modeling tools based on Latent Dirichlet Allocation (LDA) as applied by Bybee *et al.* (2020). Compared to the simple metric of frequency counts, LDA admits a topic to be represented by not only one keyword but by a cluster of commonly used words that differ across topics. See Bybee *et al.* (2020) and Macaulay and Song (2022) for similar applications.

is almost equal to that of the gross measure. In contrast, news coverage of unemployment is less correlated with exposure to either positive or negative news alone than gross exposure (see Table 6). This suggests that although labor market news coverage is likely to be either favorable or unfavorable from households' point of view, inflation news coverage is more likely to be associated with a negative connotation.

Table 6: News Coverage and Self-Reported News Exposure

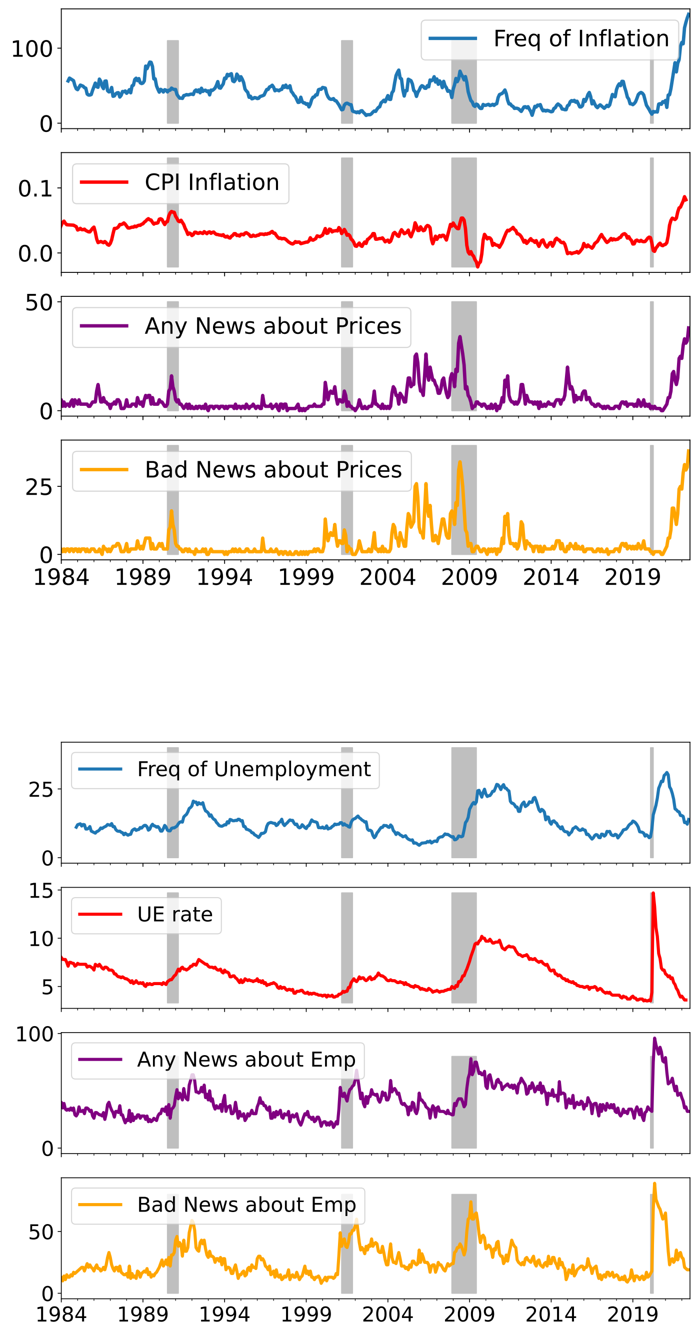
| Topic | Any News | Bad News | Good News |
|---|---|---|---|
| Inflation | 0.605 | 0.627 | -0.048 |
| Unemployment | 0.373 | 0.295 | 0.153 |

To more systematically assess what drives newspaper articles' association between inflation and unemployment, we run a Probit regression to explore the factors correlated with an article's tendency to draw an association between inflation and unemployment. The regressors include a range of article-specific topic dummies and the realized inflation rates $\pi_t$ and unemployment rates $u_t$. Columns 1–3 in Table 7 report the results.

The association between unemployment and inflation is more likely to be seen in a news article that is also about "Fed," "growth," "economy," "recession," and "uncertainty." In addition, columns (1)–(3) include only realized unemployment rates, inflation rates, and both, respectively. Together, they show that a higher inflation rate $\pi_t$ is associated with a higher probability of an article mentioning both inflation and unemployment, while the level of unemployment rate does not have any effect. Higher inflation rates not only lead to more coverage of inflation, but also result in more associations made between inflation and unemployment in news articles.

To summarize, this section shows that inflation news coverage is not only directionally negative as perceived by households, but also more likely to lead to news coverage across domains on topics such as unemployment. One hypothesis regarding this asymmetric pattern might be that inflation news serves as a more salient memory cue for selective recall of subjective models in the minds of households. Andre *et al.* (2022) provide suggestive evidence

Figure 6: News Coverage, Self-reported News Exposure, and Macroeconomic Realizations




This plots news coverage measured in the WSJ sample, realized inflation and unemployment rates, and two self-reported news exposures in the MSC.

Table 7: Drivers of Inflation-Unemployment Association

| | (1) | (2) | (3) |
|---|---|---|---|
| economy | 1.07*** | 1.07*** | 1.07*** |
| | (0.03) | (0.03) | (0.03) |
| fed | 0.22*** | 0.21*** | 0.21*** |
| | (0.03) | (0.03) | (0.03) |
| growth | 0.60*** | 0.61*** | 0.61*** |
| | (0.03) | (0.03) | (0.03) |
| oil price | 0.24*** | 0.24*** | 0.24*** |
| | (0.05) | (0.05) | (0.05) |
| recession | 0.48*** | 0.47*** | 0.47*** |
| | (0.03) | (0.03) | (0.03) |
| uncertainty | 0.14*** | 0.15*** | 0.15*** |
| | (0.05) | (0.05) | (0.05) |
| $\pi_t$ | | 3.73*** | 3.62*** |
| | | (0.93) | (0.96) |
| $u_t$ | -0.01 | | -0.00 |
| | (0.01) | | (0.01) |
| N | 150465 | 150465 | 150465 |

** p<0.001, ** p<0.01 and * p<0.05.
The table reports results from Probit regressions with the dependent variable being the dummy indicating if an article mentions both "inflation" and "unemployment" in the texts. Regressors are dummy variables to indicate if the particular keyword, e.g., "growth", is mentioned in the article. $\pi_t$ and $u_t$ are the inflation and unemployment rates at time $t$, the date of publication of the article.

for such mechanisms, which we leave for further exploration in future research.

# 6 Conclusion

Several studies have documented that households tend to unconditionally associate current and future inflation with a worse economic outlook and labor market, the so-called "stagflation view" or "supply view" of the economy.[33] We study the theoretically relevant mecha-

[33] See Bhandari *et al.* (2025); Kamdar (2019); Andre *et al.* (2022); Candia *et al.* (2020); Han (2023).

nisms that can generate such belief patterns and conclude that information friction alone is insufficient. Rather, it reflects households' subjective views regarding how macroeconomic variables move together.

By extending the single-variable noisy information model to a multivariable setting, we derive a pair of sign restrictions on the correlation of expectations and the serial correlation between forecast errors of different macroeconomic variables. This restriction informs a test against data that helps differentiate expectation patterns due to only information friction versus those due to subjective models. Our claim is further supported by self-reported news exposure in the survey and narratives in newspapers.

We also illustrate that the presence of the uncovered subjective model of "inflation means bad economy" alters the propagation mechanisms of macroeconomic shocks. In particular, it amplifies the output and price responses to a supply shock but dampens those to a demand shock. This poses a more stark trade-off faced by central banks in response to adverse supply shocks. It also questions the effectiveness of demand management policies, especially through increasing inflation expectations. Our findings speak to the macro implications of the emerging micro causal evidence that suggests household consumption responses to inflation expectations are often negative, due to expected real-income erosion or the precautionary responses to uncertainty.[34]

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[34]See Candia *et al.* (2020); Jain *et al.* (2022); Stantcheva (2024); Georgarakos *et al.* (2024).

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# A Data Appendix

## A.1 Data Description

**SCE:** The Survey of Consumer Expectations is run by the New York Fed, starting in June 2013 and available monthly.[35] We use the median year-ahead inflation expectation as proxy for expected inflation and the expected chance that unemployment rate will increase in 12 months as proxy for expected unemployment rate change.

**MSC:** The monthly component of the Michigan Survey of Consumers is available starting in 1978.[36] We use the expected price change in one year as proxy for expected inflation and the question about whether the unemployment rate will go up, go down, or stay the same as proxy for expected unemployment rate change.

**FRED:** We use year-to-year Headline CPI (CPIAUCSL) as a measure of realized inflation and year-to-year change of unemployment rate (UNRATE) as a measure of changes in unemployment status.

**SPF:** We use the series on CPI inflation rate (CPI) from the Survey of Professional Forecasters as a measure of expected inflation. We use the series on civilian unemployment rate as a measure of expected unemployment rate. To make it comparable with consumer surveys, we compute the expected year-ahead change in unemployment rate from this series.

## A.2 Aggregate Survey Forecast and Real-time Data

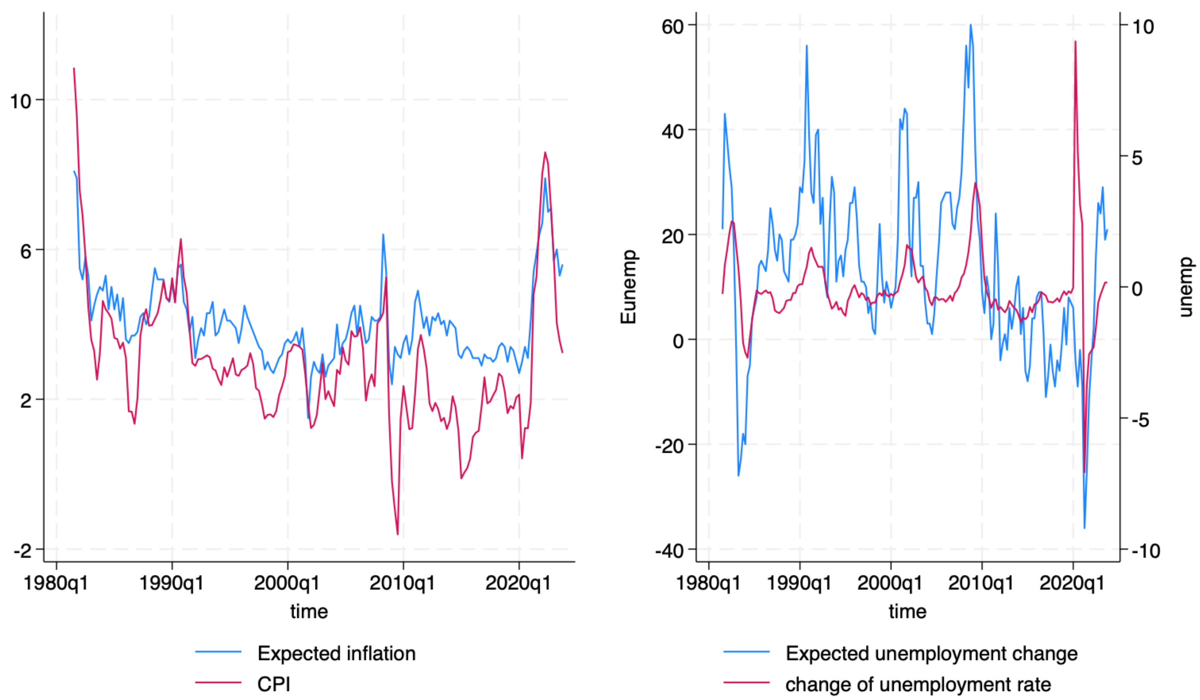
To illustrate the difference between the survey expectation and realized data, Figure 7 plots raw data on average expectation from the MSC with realized data for inflation and unemployment rate change. All real-time series are change from a year ago, as the corresponding

[35] For details of SCE, see Armantier *et al.* (2016).

[36] Quarterly data starts earlier, from 1960, but many dimensions are missing.

expectation series are one-year-forward forecasts. The abnormal spikes in unemployment rate changes correspond to the COVID-19 pandemic.

Figure 7: Actual and Expected Inflation and Unemployment




Survey expectation from the MSC against the realized data. All macro data are changes from a year ago, survey expectations are one-year-forward forecasts. Unemployment expectation is aggregated from categorical data. Positive number means more people believe unemployment will increase in the future.

## A.3 Time Series Evidence

We first report the simultaneous correlation between consensus expectations on inflation and unemployment from the MSC, the SPF, and realized data. All the expectation variables are the average of individual expectations within the quarter.[37]

[37]In the MSC, expectation data is available monthly. We use quarterly data to keep the MSC at the same frequency as the SPF. The use of monthly data does not change our results qualitatively.

Table 8: Correlations between Expected/Actual Inflation and Unemployment

| Sample | MSC | SPF | FRED |
|---|---|---|---|
| 1984-2023 | $0.14^{*}$ | $-0.03$ | $-0.32^{***}$ |
| 1981-2018 | $0.16^{**}$ | $0.03$ | $0.00$ |
| 1990-2018 | $0.27^{***}$ | $0.05$ | $-0.08$ |

* **** means significant at 1%,** means 5% and * means 10%, indicating significance level of Pearson Correlation. In sample 1984–2023, we exclude the COVID year 2021.*

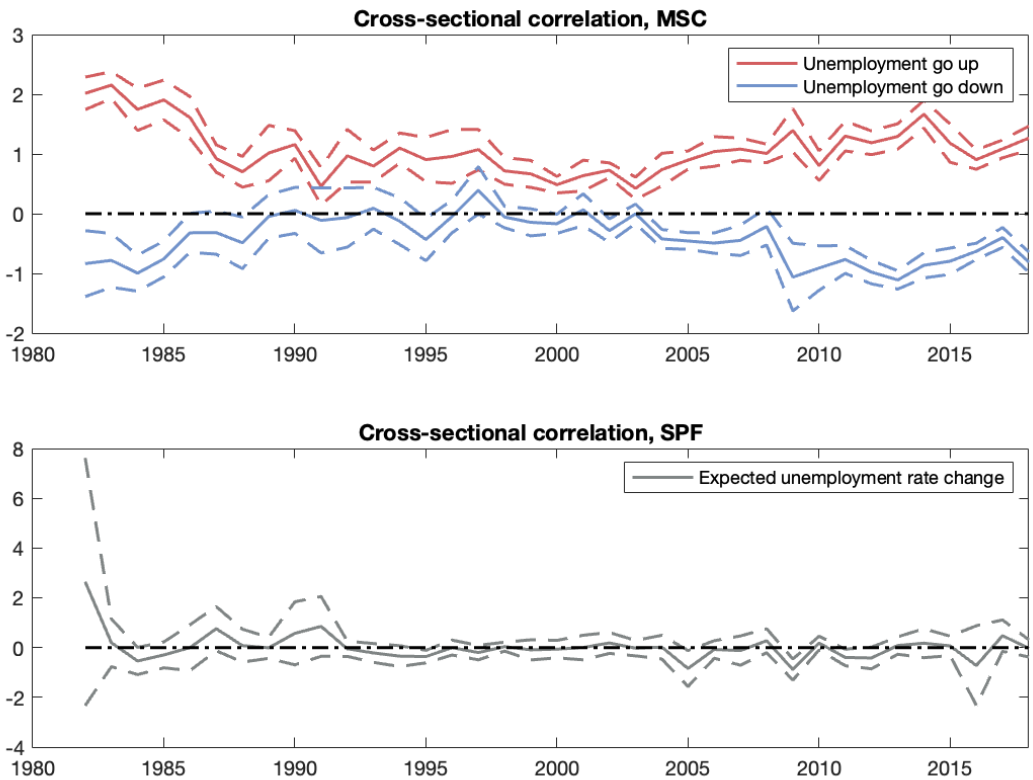
Table 8 summarizes the Pearson correlation between (expected) inflation and unemployment change in different samples that we considered in our empirical analysis. Throughout the different samples, the correlation between these expected variables in household surveys are significantly positive, different from those in the SPF and actual data.

## A.4 Evidence from Individual-level Cross-correlation

There are potentially many possible explanations for the observed positive correlation between consensus expectations. One possibility is that waves of pessimism and optimism move the average unemployment and inflation beliefs in the same direction. Furthermore, as seen in Figure 1, the time-series correlation heavily depends on the presence of aggregate shocks.

To rule out these possibilities, we examine whether individual respondents in household surveys make a similar association. This will help us understand whether the patterns in aggregate-level data have a micro-level foundation or are mainly due to the aggregation process. Previous research suggests that the properties of consensus expectations may differ from those of individual expectations.[38] Figure 8 shows the estimated correlation from the cross-sectional regression in each year.

Figure 8: Time-varying Correlation between Inflation and Unemployment Change




The top panel reports estimates $\beta_1$ from: $E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 U_{t+12,t} + \theta\mu_i + D_t + \epsilon_{i,t}$, where $U_{t+12,t}$ stands for two dummy variables. This indicates the MSC consumer believes the unemployment rate will go up or down in the next 12 months. The bottom panel reports estimates $\beta_1$ from: $E_{i,t}\pi_{t+4,t} = \beta_0 + \beta_1 E_{i,t}un_{t+4,t} + \theta\mu_i + D_t + \epsilon_{i,t}$, where $E_{i,t}un_{t+4,t}$ stands for the expected change of unemployment rate from SPF. The MSC data is monthly and the SPF data is quarterly. The dashed lines show the 10% confidence interval.

The top panel of Figure 8 uses data from the MSC. In this survey, respondents are asked whether they think unemployment will go up, stay the same, or go down a year from now. The two lines are the differences in inflation expectations relative to consumers who believe unemployment will stay the same for each year. The figure suggests that households' beliefs about inflation are again positively associated with their beliefs about unemployment change.

[38]For instance, Coibion and Gorodnichenko (2015) suggest the predictability of forecast errors from forecast revisions is an emerging property of aggregation across individuals and may not be seen at the individual level; Bordalo *et al.* (2018) document over-reaction of inflation expectation to new information on the individual level, contrary to under-reaction typically found with consensus expectations.

Such a positive relation is significant and relatively stable across time. This finding is the same as in Kamdar (2019).

The bottom panel of Figure 8 is the cross-sectional correlation between expected inflation and unemployment rate change in the SPF. Contrary to consumers, professionals do not associate inflation with unemployment rate when forming their beliefs.

Could this correlation be driven by a specific group of individuals? For example, if there are groups of pessimistic individuals, they will always form worse-than-average unemployment expectations together with higher-than-average inflation expectations. This creates a positive association in the cross-sectional analysis above. We use the panel dataset in the MSC and the SPF to control for individual fixed effects as well as time-fixed effects:

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \beta_2 E_{i,t}i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t} \tag{16}$$

Again because in MSC, the expected unemployment change is a categorical variable, $\beta_1$ in (16) contains coefficients when expected unemployment goes up or down. $X_{i,t}$ includes controls such as expectations on other subjects and social-economic status, $\mu_i$ and $D_t$ stand for individual and time-fixed effects respectively. Because the panel dataset from MSC contains fewer observations and only keeps the participants for two waves of surveys six months apart, we also report the results from the same regression using panel data from SCE.[39]

[39]When using MSC, the expected unemployment and interest rate change are categorical variables, and we construct dummies that stand for increase or decrease for each of these variables. In SCE, those variables are reported as percentage points for the likelihood of the corresponding variable increasing.

Table 9: FE Panel Regression

| | MSC | | SCE | | SPF |
|---|---|---|---|---|---|
| Unemployment up | $0.30^{***}$ | $\hat{\beta}_1$ | $0.012^{***}$ | $\hat{\beta}_1$ | $-0.17^{***}$ |
| | (0.05) | | (0.002) | | (0.06) |
| Unemployment down | $-0.22^{***}$ | | | | |
| | (0.05) | | | | |
| FE | Y | | Y | | Y |
| Time dummy | Y | | Y | | Y |

* ***,**,*: Significance at 1%, 5%, and 10% level. Estimation results for specification (16) controlling for individual and time-varying characteristics, individual fixed effect, and time-fixed effect. Standard errors are adjusted for heteroscedasticity and autocorrelation.

Table 9 column 1 shows that for the MSC, an agent who expects the unemployment to go up will predict inflation to be 0.3% higher on average than one who believes unemployment to be stable, and 0.52% higher than one who believes the unemployment rate will fall. Meanwhile, the standard deviation of expected inflation across this episode is 1.17%, and the standard deviation of CPI is around 2.19%. These results are comparable to those from Kamdar (2019), where the author estimates a similar fixed-effect model but only on the correlation between expected inflation and unemployment change, without controlling for other expectational variables. The estimates shown in column 2 from the SCE are consistent with those from the MSC: If the consumer expects a 22% higher chance (which is the standard deviation of the variable) the unemployment rate will increase in 12 months, they will also expect inflation to be 0.22% higher. It is worth noting that controlling individual and time-fixed effects means the positive correlation between unemployment and inflation is not due to a common time-varying bias, which should have been captured by the time-fixed effect. It is also not due to the effect of "pessimistic individuals," which is taken out by individual fixed effects. Finally, in contrast with consumers' expectations, column 3 shows that there is a negative correlation between expected inflation and change in the unemployment rate. On average, a 1% increase in the expected unemployment rate is associated with a 0.17% fall in expected inflation for professionals. This again coincides with the message from the aggregate correlation that professionals believe in a different relationship between future inflation and unemployment movements compared with consumers.

## A.5 Recover Survey Mean from Categorical Data

From the cross-sectional dataset of the MSC, we can acquire information on the fraction of respondents with different answers. Denote $f_t^u$ as fraction of responses that are "increase" and $f_t^d$ as "decrease." Assume for each period of $t$, individuals form a cross-section of answers about the change of the asked subject (unemployment rate or business condition and price). And assume this measure follows a normal distribution with mean $\mu_t$ and variance $\sigma_t^2$.

**Assumption 4.** *At each period $t$, survey respondent $i$ forms a belief $x_{i,t}$ that indicates the change of asked variable $x$. This belief follows a normal distribution:*

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

Then suppose the agents have a common scale in answering the categorical question. If $x_{i,t}$ is close to some level $b$, then they will consider the subject will barely change; if $x_{i,t}$ is much bigger than $b$, they will answer increase. Otherwise, they will answer decrease.

$$category_{i,t} = \begin{cases} increase & x_{it} > b + a \\ decrease & x_{it} < b - a \\ same & x_{it} \in [-a + b, b + a] \end{cases}$$

Then the fraction of answer "increase," denoted as $f_t^u$, and "decrease," denoted $f_t^d$, will directly follow from normality:

$$f_t^d = \Phi\left(\frac{b-a-\mu_t}{\sigma_t}\right) \tag{17}$$

$$f_t^u = 1-\Phi\left(\frac{a+b-\mu_t}{\sigma_t}\right) \tag{18}$$

The item we want to recover is $\mu_t$, which is the corresponding average change of the asked subject a year from now. This can be computed using:

$$\sigma_t = \frac{2a}{\Phi^{-1}(1-f_t^u)-\Phi^{-1}(f_t^d)} \tag{19}$$

$$\mu_t = a+b-\sigma_t\Phi^{-1}(1-f_t^u) \tag{20}$$

From (19) and (20), computing the average across time gives us:

$$\hat{\sigma} = 1/T\sum_t^T \sigma_t = 1/T\sum_t^T \frac{2a}{\Phi^{-1}(1-f_t^u)-\Phi^{-1}(f_t^d)} \tag{21}$$

$$\hat{\mu} = 1/T\sum_t^T \mu_t = 1/T(a+b-\sigma_t\Phi^{-1}(1-f_t^u)) \tag{22}$$

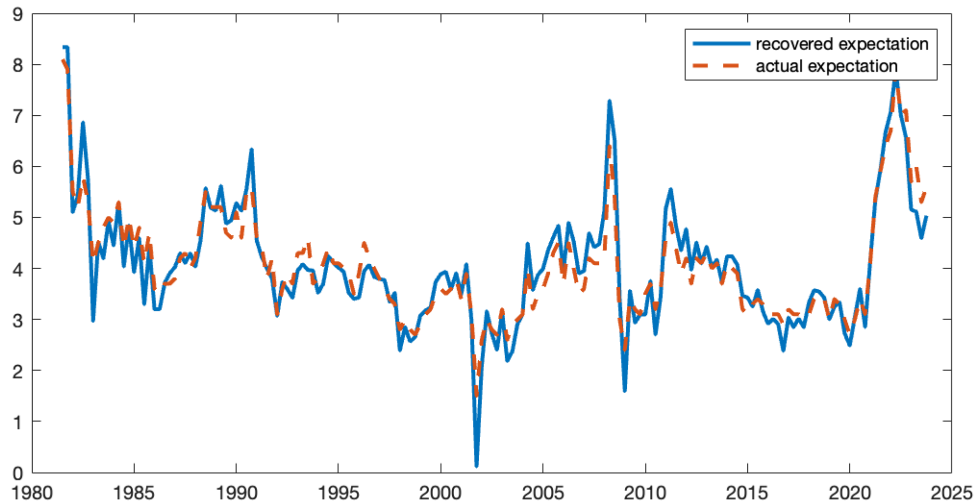
As in the MSC, there is no information on $\hat{\sigma}$ and $\hat{\mu}$, so we use the time-series mean of the data from SPF on comparable questions to approximate those from the MSC.[40] Following Bhandari *et al.* (2025), we assume the ratio of the time-series average between inflation expectation and other expectations in the MSC equals its counterpart in the SPF:

**Assumption 5.** *For the variable $x$ asked in the survey:*

$$\hat{\sigma}_x^{MCS} = \frac{1/T\sum_t^T \sigma_{E\pi,t}^{MCS}}{1/T\sum_t^T \sigma_{E\pi,t}^{SPF}} \times 1/T\sum_t^T \sigma_{x,t}^{MCS}$$

[40] For unemployment rate change, we use the average difference between projected unemployment rate at $t+3$ and the historical data at $t-1$, which is the last information available to the economist. For real GDP growth, we use the real GDP growth projection for the next four quarters after $t-1$.

Figure 9: Recovered Expected Inflation vs. Actual




*And*

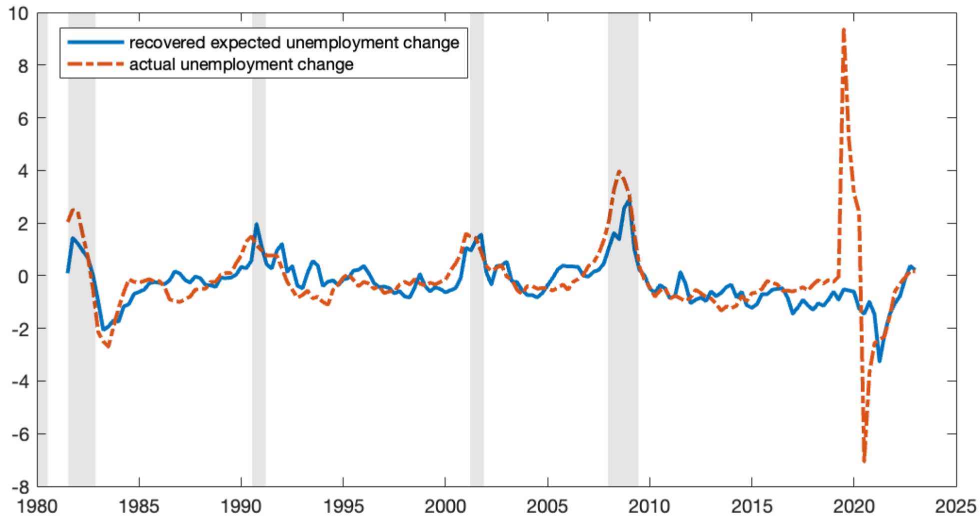
$$\hat{\mu}_x^{MCS} = \frac{1/T \sum_t^T \mu_{E\pi,t}^{MCS}}{1/T \sum_t^T \mu_{E\pi,t}^{SPF}} \times 1/T \sum_t^T \mu_{x,t}^{MCS}$$

Then from (21) and (22) and Assumption 5, we can back out $a$ and $b$, and with (20) we can recover $\mu_{x,t}$ for the expectational variable $x$.

**Recovered series:** To test whether the above method is plausible, we use the proposed method to recover $\mu_{\pi,t}$ and compare it with the actual average of expected inflation from the MSC. Figure 9 plots the recovered mean and the actual mean.

Figure 9 shows that the recovered data is actually quite close to the actual mean expectation, with a correlation of 0.95. Figure 10 shows the recovered data on expected unemployment change compared to actual data.

Figure 10: Recovered Expected Unemployment Change vs. Actual




Data from 1981q3 to 2023q4 due to availability of quarterly SPF on CPI inflation.

# B Derivation of Noisy Information Model

## B.1 Basic Stationary Kalman Filter

Consider the ALM and observational equation as in (1) and (2), where $w_{t+1,t}$, $v_t^i$, and $\eta_t$ are independent and normally distributed:

$$w_{t+1,t} \sim N(\mathbf{0}, Q) \quad v_t^i \sim N(\mathbf{0}, R_1) \quad \eta_t \sim N(\mathbf{0}, R_1)$$

Consistent with the main-text, we denote $R = R_1 + R_2$, and the perceived value of $\boldsymbol{L}_{t,t-1}$ for individual $i$ at time $t$ as $\boldsymbol{L}^i_{t,t-1|t}$. The filtering process is:

$$\boldsymbol{L}^i_{t,t-1|t} = \hat{A}\boldsymbol{L}^i_{t,t-1|t} = \boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1}) \tag{23}$$

The Kalman Filter is given by:

$$K = \Sigma G'(G\Sigma G' + R)^{-1}$$

$$\Sigma_p = \hat{A}\Sigma\hat{A}' - \hat{A}K_t G\Sigma\hat{A}' + Q,$$

where $\Sigma$ is the covariance matrix of priors as defined in assumption 2 and $\Sigma_p$ is the covariance matrix of posteriors.[41] Then the expectation is given by:

$$\boldsymbol{L}^i_{t+1,t|t} = \hat{A}\big(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1})\big)$$

## B.2 Derivation of Year-ahead Forecasting Error Rule

Consider the year-ahead consensus forecast $\boldsymbol{L}^c_{t+4,t|t}$ and year-ahead realization $\boldsymbol{L}_{t+4,t}$. Using ALM (1), we have:

$$\boldsymbol{L}_{t+4,t} \equiv \sum_{j=1}^{4} \boldsymbol{L}_{t+j,t+j-1} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} \tag{24}$$

Similar to equation (4), the year-ahead consensus expectation is:

$$\boldsymbol{L}^c_{t+4,t|t} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)[\hat{A}(I - KG)\boldsymbol{L}^c_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t] \tag{25}$$

Meanwhile from (23) and ALM we know:

$$\boldsymbol{L}^c_{t+3,t-1|t-1} = \sum_{j=0}^{3} \boldsymbol{L}^c_{t+j,t+j-1|t-1} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)\boldsymbol{L}^c_{t,t-1|t-1}$$

Denote $\hat{W} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)$ and the stationarity of $\hat{A}$ guarantees $\hat{W}$ is invertible. Plug the above equation into (25) we have:

$$\boldsymbol{L}^c_{t+4,t|t} = \hat{W}[\hat{A}(I - KG)\hat{W}^{-1}\boldsymbol{L}^c_{t+3,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t]$$

[41] Given common beliefs on $\hat{A}$ and $G$, it can be shown that prior and posterior covariance matrices converge.

Now write the forecasting error $FE_{t+4,t|t}$ as defined:

$$
\begin{aligned}
FE_{t+4,t|t} &\equiv \boldsymbol{L}_{t+4,t} - \boldsymbol{L}^c_{t+4,t|t} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} - \boldsymbol{L}^c_{t+4,t|t} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} - \hat{W}\hat{A}K\eta_t + \sum_{j=1}^{4} w_{t+j,t+j-1} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} + \boldsymbol{L}_{t+3,t} - A\boldsymbol{L}_{t+2,t-1} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- (I + \hat{W}\hat{A}KG)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \qquad (26)
\end{aligned}
$$

The last equation follows from the fact:

$$
\boldsymbol{L}_{t+3,t-1} = \boldsymbol{L}_{t+3,t+2} + \boldsymbol{L}_{t+2,t+1} + \boldsymbol{L}_{t+1,t} + \boldsymbol{L}_{t,t-1} = \boldsymbol{L}_{t+2,t-1} + \boldsymbol{L}_{t+3,t+2}
$$

# C Proofs

## C.1 Proposition 1

*Proof.* The Kalman Gain in this case:

$$
K = \Sigma G'(G\Sigma G' + R)^{-1} = I \quad \Rightarrow \quad \hat{A}(I - KG) = \mathbf{0}
$$

□

## C.2 Proposition 2

*Proof.* (1) From Kalman Filter:

$$KG = \Sigma G'(G\Sigma G' + R)^{-1}G$$

If both $G$ and $R$ are diagonal, $KG$ will be diagonal and $\hat{A}(I - KG)$ is diagonal.

(2) Define

$$V = G'(G\Sigma G' + R)^{-1}G$$

As both $\Sigma$ and $R$ are symmetric and positive semi-definite, $G$ is non-singular and it follows that $G\Sigma G' + R$ is invertible and symmetric. We can immediately see that $V$ is symmetric. Denote $V := \left(v_{ij}\right)_{n\times n}$, we have:

$$KG = \Sigma V = \left(\sigma_i^2 v_{ij}\right)_{n\times n}$$

The off-diagonal elements of the coefficient matrix, $w_{ij}$, is given by:

$$w_{ij} = -a_i \sigma_i^2 v_{ij}$$

As $v_{ij} = v_{ji}$ for any $i \neq j$, it is obvious that either $w_{ij} = w_{ji} = 0$ if $v_{ij} = 0$, or $w_{ij}w_{ji} = a_i a_j \sigma_i^2 \sigma_j^2 v_{ij}^2 > 0$ if $v_{ij} \neq 0$. $\square$

## C.3 Corollary 1

*Proof.* Denote $G = \begin{pmatrix} g_1 & \rho \\ \rho & g_4 \end{pmatrix}$ and $\Omega = (G\Sigma G' + R)$, we have:

$$KG = \Sigma G'\Omega^{-1}G = \begin{pmatrix} g_1\sigma_1^2 & 0 \\ 0 & g_4\sigma_2^2 \end{pmatrix} \frac{1}{det(\Omega)} \begin{pmatrix} \sigma_{2,s}^2 & -\rho \\ -\rho & \sigma_{1,s}^2 \end{pmatrix} \begin{pmatrix} g_1 & \rho \\ \rho & g_4 \end{pmatrix}$$

The off-diagonal elements are $-\frac{1}{det(\Omega)}\rho g_1 g_4 \sigma_1^2$ and $-\frac{1}{det(\Omega)}\rho g_1 g_4 \sigma_2^2$. As $\Omega$ is positive definite, the off-diagonal elements of $\hat{A}(I-KG)$ have the same signs as $\rho$ if $\hat{A}$ have positive entries on the diagonal. □

## C.4 Corollary 2

**Lemma 1.** *Consider 2-dimensional $\boldsymbol{L}_{t,t-1}$, 2 by 2 $G$, signals generated by $s_t = G\boldsymbol{L}_{t,t-1} + \eta_t$ with $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$, and $\eta_t$ independent normal. $\exists\, \tilde{G}$ triangular and $\tilde{\eta}_t$ independent normal such that $\tilde{s}_t = \tilde{G}\boldsymbol{L}_{t,t-1} + \tilde{\eta}_t$ and $\mathbb{E}[\boldsymbol{L}_{t,t-1}|s_t] = \mathbb{E}[\boldsymbol{L}_{t,t-1}|\tilde{s}_t]$.*

*Proof.* Denote the noise $\eta_t \sim N\left(0, \begin{pmatrix} \sigma_{s,1}^2 & 0 \\ 0 & \sigma_{s,2}^2 \end{pmatrix}\right)$. Consider $\Gamma = \begin{pmatrix} \frac{\sigma_{s,2}^2 g_1}{\sigma_{s,1}^2 g_3} & 1 \\ -\frac{g_3}{g_1} & 1 \end{pmatrix}$ and the new signals:

$$\tilde{s}_t = \Gamma G \boldsymbol{L}_{t,t-1} + \Gamma \eta_t$$

Define $\tilde{G} \equiv \Gamma\eta_t$ and $\tilde{\eta}_t \equiv \Gamma\eta_t$. It is easy to verify that $\tilde{\eta}_t$ is independent normal and $\tilde{G}$ has only one non-zero off-diagonal element. Denote the Kalman gain of the original signals as $K$ and the new signals as $\tilde{K}$. It is straightforward that:

$$\begin{aligned}
\tilde{K}\Gamma G &= \Sigma G'\Gamma'\Big(\Gamma(G\Sigma G' + R)\Gamma'\Big)^{-1}\Gamma G \\
&= \Sigma G'\Gamma'(\Gamma')^{-1}(G\Sigma G' + R)^{-1}\Gamma^{-1}\Gamma G \\
&= \Sigma G'(G\Sigma G' + R)^{-1} G = KG
\end{aligned}$$

The second equality holds as $\Gamma$ is invertible. For the same reason, $K = \tilde{K}\Gamma$. Then we have:

$$\begin{aligned}
\mathbb{E}[\boldsymbol{L}_{t,t-1}|s_t] &= \hat{A}\big((I - KG)\boldsymbol{L}_{t,t-1|t-1} + Ks_t\big) \\
&= \hat{A}\big((I - \tilde{K}\tilde{G})\boldsymbol{L}_{t,t-1|t-1} + \tilde{K}\tilde{s}_t\big) \\
&= \mathbb{E}[\boldsymbol{L}_{t,t-1}|\tilde{s}_t]
\end{aligned}$$

$\square$

Here we prove the corollary with the general $G$:

**Corollary 3.** *(**Non-diagonal** $G$) If $\hat{A}$ is diagonal, $R = \begin{pmatrix} \sigma_{1,s}^2 & 0 \\ 0 & \sigma_{2,s}^2 \end{pmatrix}$ is diagonal, and $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I-KG)$ have signs depending on $g_1 g_2 \sigma_{2,s}^2 + g_3 g_4 \sigma_{1,s}^2$.*

*Proof.* Again, denote $\Omega = G\Sigma G' + R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where:

$$\begin{cases} a = g_1^2\sigma_1^2 + g_2^2\sigma_2^2 + \sigma_{1,s}^2 \\ b = g_1 g_3 \sigma_1^2 + g_2 g_4 \sigma_2^2 \\ c = g_1 g_3 \sigma_1^2 + g_2 g_4 \sigma_2^2 \\ d = g_3^2\sigma_1^2 + g_4^2\sigma_2^2 + \sigma_{2,s}^2 \end{cases}$$

Denote the matrix $KG := \frac{1}{det(\Omega)} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$. The off-diagonal elements of $\hat{A}(I-KG)$ depend on the signs of $x_2$ and $x_3$. It is easy to show:

$$\begin{cases} x_2 = \sigma_1^2(g_1g_2d - g_2g_3c - g_1g_4b + g_3g_4a) = \sigma_1^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \\ x_3 = \sigma_2^2(g_1g_2d - g_1g_4c - g_3g_2b + g_3g_4a) = \sigma_2^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \end{cases}$$

As $det(\Omega) > 0$, if the diagonal elements of $\hat{A}$ are both positive, the off-diagonal elements of $\hat{A}(I-KG)$ are:

$$\begin{cases} \text{negative} & \text{if } g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2 > 0 \\ \text{positive} & \text{if } g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2 < 0 \end{cases}$$

$\square$

The proof of Corollary 2 follows directly from Lemma 1 and Corollary 3.

### C.5 Proposition 4

*Proof.* If both $G$ and $R$ are diagonal, $KG = \Sigma G'(G\Sigma G' + R)^{-1}G$ is also diagonal. Denote $G = diag(\{g_i\}_{i=1}^n)$ and $R = diag(\{\sigma_{s,i}^2\})$. The matrix $KG$ is also diagonal:

$$KG = \Sigma G'(G\Sigma G' + R)^{-1}G = diag\left(\left\{\frac{g_i^2\sigma_i^2}{g_i^2\sigma_i^2 + \sigma_{s,i}^2}\right\}\right)$$

with diagonal elements $0 < \frac{g_i^2\sigma_i^2}{g_i^2\sigma_i^2+\sigma_{s,i}^2} < 1$. It follows immediately that:

$$w_{ij} = a_{ij}\frac{\sigma_{s,j}^2}{g_j^2\sigma_j^2 + \sigma_{s,j}^2}$$

Consequently, $w_{ij}$ has the same sign as $a_{ij}$. □

## D Monte Carlo Simulation

We consider the different learning structures discussed in Table 1 and simulate expectation data according to the noisy information model from (1) and (3) with sample sizes similar to the survey data used in Section 2.4. We then perform our joint learning test with year-ahead forecast as in (10), or with quarter-ahead forecast as in (6). This comparison is to show the test with year-ahead forecasts has similar performance to the one using quarter-ahead forecasts. Table 10 summarizes the parameters we use for simulation.

Table 10: Parameters for Simulation

| Variable | Value | Description |
|---|---|---|
| **Fixed Parameters** | | |
| $Q := \begin{pmatrix} \sigma_{1,t}^2 & 0 \\ 0 & \sigma_{2,t}^2 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ | Cov matrix of shocks |
| $\Sigma := \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$ | $\begin{pmatrix} 2 & 0 \\ 0 & 2.5 \end{pmatrix}$ | Cov matrix of prior |
| $A := \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$ | $\begin{pmatrix} 0.9 & 0 \\ 0 & 0.7 \end{pmatrix}$ | Structural parameters from ALM |
| $T$ | 152 | time-series sample size |
| **Model-specific Parameters** | | |
| $\hat{A} := \begin{pmatrix} \rho_1 & m_1 \\ 0 & \rho_2 \end{pmatrix}$ | $\begin{pmatrix} 0.9 & m_1 \\ 0 & 0.7 \end{pmatrix}$ | Structural parameters from PLM |
| $G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$ | $\begin{pmatrix} 1 & g_2 \\ 0 & 1 \end{pmatrix}$ | Signal generating matrix |
| $R := \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$ | $\begin{pmatrix} 3 & \rho \\ \rho & 4 \end{pmatrix}$ | Cov matrix of noises |

As in Table 1, we consider five different cases: (1) FIRE; (2) Independent Learning with noisy but uncorrelated signals; (3) Independent Learning with mixture of states (i.e., $G$ is non-diagonal); (4) Independent Learning with correlated noise (i.e., $R$ is non-diagonal); and (5) Joint Learning with $\hat{A}$ being non-diagonal. In Table 11, we show the results with the first two cases. In both cases, $\hat{A} = A$ and $G = I$. The difference is that under FIRE, $\sigma_{1,s} = \sigma_{2,s} = 0$.

Table 11: Simulation Results: FIRE or Independent Learning with Uncorrelated Signals

| FIRE or Independent Learning: $\hat{A} = A$, $g_2 = 0$, $\rho = 0$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | FIRE | | | | Independent Learning | | | |
| | Y-ahead Spec (10) | | Q-ahead Spec (6) | | Y-ahead Spec (10) | | Q-ahead Spec (6) | |
| | Truth | Test | Truth | Test | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\beta_{11}$ | 0 | $-0.01$ | 0 | 0.04 | 0.54 | $0.51^{***}$ | 0.54 | $0.47^{***}$ |
| | - | (0.03) | - | (0.09) | - | (0.09) | - | (0.09) |
| $\beta_{12}$ | 0 | 0.03 | 0 | 0.15 | 0 | $-0.14$ | 0 | $-0.14$ |
| | - | (0.04) | - | (0.11) | - | (0.010) | - | (0.10) |
| $\beta_{21}$ | 0 | 0.01 | 0 | 0.10 | 0 | $-0.03$ | 0 | $-0.09$ |
| | - | (0.02) | - | (0.09) | - | (0.04) | - | (0.11) |
| $\beta_{22}$ | 0 | $-0.00$ | 0 | 0.18 | 0.43 | $0.49^{***}$ | 0.43 | $0.61^{***}$ |
| | - | (0.05) | - | (0.12) | - | (0.07) | - | (0.11) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Columns (2) and (6) are estimation results for one-year-ahead joint-learning test (10), and columns (4) and (8) are for the quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

The results in Table 11 show the clear differences in test results under FIRE and Independent learning. For all specifications considered, if the expectation is formed under FIRE, all the $\beta$s will be insignificantly different from zero. Meanwhile, if expectations are formed independently but with information friction, only estimates on $\beta_{11}$ and $\beta_{22}$ are significantly positive. The estimates on $\beta_{21}$ and $\beta_{12}$ will be insignificant.

Table 12: Simulation Results: Independent Learning with Correlated Signals

| | Independent Learning when $G$ or $R$ are non-diagonal | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | G non-diagonal: $m_1 = 0,\ g_2 = 0.5,\ \rho = 0$ | | | | R non-diagonal: $m_1 = 0,\ g_2 = 0,\ \rho = -2$ | | | |
| | Y-ahead spec (10) | | Q-ahead spec (6) | | Y-ahead spec (10) | | Q-ahead spec (6) | |
| | Truth | Test | Truth | Test | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\beta_{11}$ | 0.57 | $0.56^{***}$ | 0.57 | $0.52^{***}$ | 0.49 | $0.43^{***}$ | 0.49 | $0.37^{***}$ |
| | – | (0.05) | – | (0.08) | – | (0.05) | – | (0.09) |
| $\beta_{12}$ | -0.14 | $-0.28^{***}$ | -0.10 | $-0.26^{***}$ | -0.17 | $-0.25^{***}$ | -0.13 | $-0.24^{***}$ |
| | – | (0.09) | – | (0.10) | – | (0.09) | – | (0.09) |
| $\beta_{21}$ | -0.07 | $-0.10^{***}$ | -0.10 | $-0.20^{**}$ | -0.09 | $-0.11^{***}$ | -0.12 | $-0.17$ |
| | – | (0.04) | – | (0.10) | – | (0.04) | – | (0.11) |
| $\beta_{22}$ | 0.40 | $0.46^{***}$ | 0.40 | $0.55^{***}$ | 0.39 | $0.49^{***}$ | 0.39 | $0.63^{***}$ |
| | – | (0.07) | – | (0.11) | – | (0.07) | – | (0.11) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Columns (2) and (6) are estimation results for year-ahead joint-learning test (10), and columns (4) and (8) are for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

Table 12 shows the results if beliefs are formed under independent learning with noisy signals that are correlated. We consider two different cases of correlated signals: either $G$ is non-diagonal or $R$ is non-diagonal. In particular, we consider either $g_2 = 0.5$ or $\rho = -2$. According to Corollary 2 and 1, in both these two scenarios $\beta_{12}$ and $\beta_{21}$ will be negative. Both regressions with (6) and (10) perform well to uncover such a pattern.

We then consider the test results under joint learning when $\hat{A}$ is non-diagonal and signals are uncorrelated. In Table 13, we report the test results from simulated data for both year-ahead specification (6) and quarter-ahead specification (10). Both test results are in line with the predictions from Proposition 4.

Table 13: Simulation Results: Joint Learning

| | Joint Learning: $m_1 = 0.5$, $G$ and $R$ are diagonal | | | |
|---|---|---|---|---|
| | Year-ahead spec (10) | | Quarter-ahead spec (6) | |
| | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) |
| $\beta_{11}$ | 0.54 | 0.48*** | 0.54 | 0.44*** |
| | - | (0.08) | - | (0.08) |
| $\beta_{12}$ | 0.32 | 0.49** | 0.31 | 0.35*** |
| | - | (0.22) | - | (0.10) |
| $\beta_{21}$ | 0 | $-0.02$ | 0 | $-0.08$ |
| | - | (0.04) | - | (0.09) |
| $\beta_{22}$ | 0.43 | 0.54*** | 0.43 | 0.70*** |
| | - | (0.12) | - | (0.14) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Column (2) contains estimation results for year-ahead joint-learning test (10), and column (4) is for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

All in all, the test results using simulated data are consistent with the theoretical predictions. The performance of tests using year-ahead forecast error or quarter-ahead forecast error is similar throughout the different scenarios we considered.

# E Estimation: Robustness

Table 14 estimates the parameters with an alternative data sample to those used in Table 3. It yields very similar estimates. Furthermore, our benchmark estimation assumes no feedback loop from expectations to realized data. For robustness, we estimate an unrestricted version of the VAR model dropping such an assumption, and report the results in Table 15.

Table 14: Estimates of Joint Learning Model (7): Alternative Sample

| | **MSC, quarterly** | | | |
|---|---|---|---|---|
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.807 & -0.070 \\ 0.062 & 0.922 \end{bmatrix}$ | $\begin{bmatrix} 0.059 & 0.114 \\ 0.022 & 0.072 \end{bmatrix}$ | $\begin{bmatrix} 0.781 & -0.060 \\ 0.059 & 0.930 \end{bmatrix}$ | $\begin{bmatrix} 0.068 & 0.145 \\ 0.031 & 0.082 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.663 & -0.096 \\ 0.189 & 0.807 \end{bmatrix}$ | $\begin{bmatrix} 0.063 & 0.089 \\ 0.057 & 0.056 \end{bmatrix}$ | $\begin{bmatrix} 0.663 & -0.081 \\ 0.271 & 0.769 \end{bmatrix}$ | $\begin{bmatrix} 0.080 & 0.094 \\ 0.064 & 0.057 \end{bmatrix}$ |
| T-test: | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | 2.094 | 0.018 | 2.999 | 0.001 |
| | **SPF, quarterly** | | | |
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.788 & -0.070 \\ 0.048 & 0.906 \end{bmatrix}$ | $\begin{bmatrix} 0.070 & 0.100 \\ 0.024 & 0.071 \end{bmatrix}$ | $\begin{bmatrix} 0.749 & -0.047 \\ 0.042 & 0.920 \end{bmatrix}$ | $\begin{bmatrix} 0.079 & 0.113 \\ 0.030 & 0.077 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.951 & 0.004 \\ 0.026 & 0.787 \end{bmatrix}$ | $\begin{bmatrix} 0.018 & 0.041 \\ 0.016 & 0.041 \end{bmatrix}$ | $\begin{bmatrix} 0.937 & -0.027 \\ 0.026 & 0.806 \end{bmatrix}$ | $\begin{bmatrix} 0.021 & 0.030 \\ 0.031 & 0.044 \end{bmatrix}$ |
| T-Test | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | -0.883 | 0.811 | -0.410 | 0.659 |

The table reports the estimates and their Newey-West standard errors from the GMM estimation of the four-variable VAR model. An iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ is used to calculate the standard errors of $\hat{A}$ estimates.

# F News Measure from MSC

## F.1 Description

In the MSC, there is a question asking about news heard recently about business conditions:

*A6. During the last few months, have you heard of any favorable or unfavorable changes in business conditions?*

Table 15: Estimates of Joint Learning Model (7): with Feedback Loop

| | **MSC, quarterly** | | | |
|---|---|---|---|---|
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.863 & 0.021 \\ -0.003 & 0.751 \end{bmatrix}$ | $\begin{bmatrix} 0.073 & 0.162 \\ 0.042 & 0.074 \end{bmatrix}$ | $\begin{bmatrix} 0.863 & 0.051 \\ -0.017 & 0.721 \end{bmatrix}$ | $\begin{bmatrix} 0.078 & 0.169 \\ 0.042 & 0.076 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.663 & -0.096 \\ 0.189 & 0.807 \end{bmatrix}$ | $\begin{bmatrix} 0.063 & 0.089 \\ 0.057 & 0.056 \end{bmatrix}$ | $\begin{bmatrix} 0.663 & -0.081 \\ 0.271 & 0.769 \end{bmatrix}$ | $\begin{bmatrix} 0.080 & 0.094 \\ 0.064 & 0.057 \end{bmatrix}$ |
| T-test: | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | 2.227 | 0.013 | 3.112 | 0.001 |
| | **SPF, quarterly** | | | |
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.696 & -0.091 \\ 0.021 & 0.792 \end{bmatrix}$ | $\begin{bmatrix} 0.078 & 0.090 \\ 0.031 & 0.072 \end{bmatrix}$ | $\begin{bmatrix} 0.678 & -0.062 \\ 0.019 & 0.785 \end{bmatrix}$ | $\begin{bmatrix} 0.086 & 0.107 \\ 0.034 & 0.089 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.951 & 0.004 \\ 0.026 & 0.787 \end{bmatrix}$ | $\begin{bmatrix} 0.018 & 0.041 \\ 0.016 & 0.041 \end{bmatrix}$ | $\begin{bmatrix} 0.937 & -0.027 \\ 0.026 & 0.806 \end{bmatrix}$ | $\begin{bmatrix} 0.021 & 0.030 \\ 0.031 & 0.044 \end{bmatrix}$ |
| T-Test | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} > A_{21}$ | 0.136 | 0.446 | 0.1534 | 0.439 |

The table reports the estimates and their standard errors from the GMM estimation of the unrestricted four-variable VAR model. Iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ is used to calculate the standard errors of $\hat{A}$ estimates.

*A6a. What did you hear?*

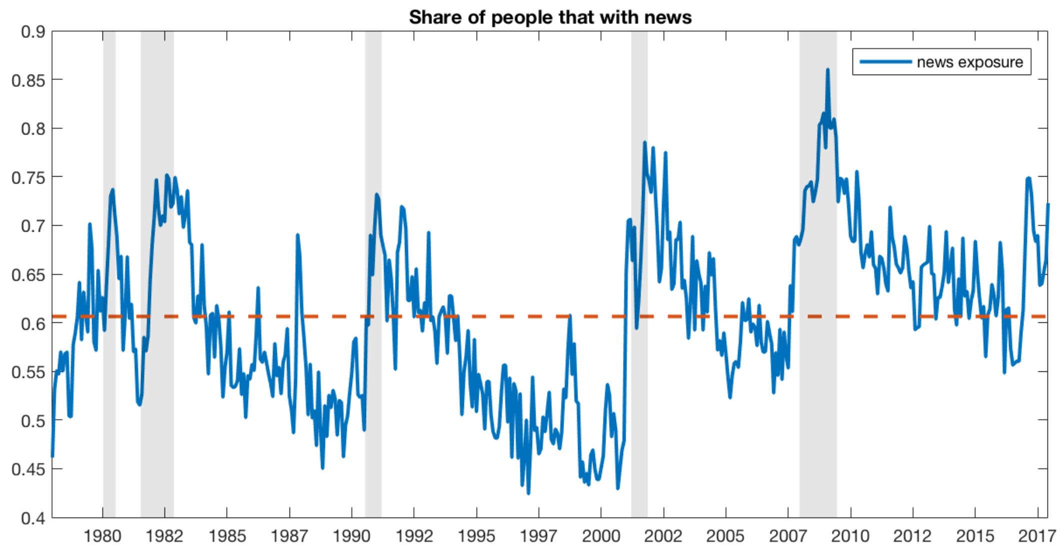
The news reported in this question should be considered as self-reported information. It may contain both public and private information heard by the survey respondents. The MSC categorizes the content of news described by the respondents is categorized into 80 different categories. We further summarize these categories into 10 different types of news, as described in Table 16. In Figure 11 we plot the share of survey respondents that report hearing any news. Figure 12 depicts the fraction of agents hearing news about unemployment and inflation conditional on hearing any news.

Table 16: Types of News Reported

| Categories Defined | News description in the MSC | |
|---|---|---|
| | Favorable | Unfavorable |
| Employment | Employ is high, plenty of jobs | Drop in employ, less overtime |
| | Other references to employ and purch power (fav) | Other references to employ and purch power (unfav) |
| Industry | Opening of plants, factories, stores | Closing of plants, factories, stores |
| | Improvements in specific industries | Decline in specific industries |
| | Farm situation good, crops good | Farm situation is bad, low farm prices, drought |
| Inflation | Lower/stable prices, less inflation | Prices falling, deflation |
| Interest rate | Easier money, credit easy to get, low int rates | Tight money, int rates high |
| Demand | Consumer/auto demand high | Consumer/auto demand low |
| | Population increase, more people to buy | Population increase, immigration |
| Government | Elections, admin, Congress, President (fav) | Elections, admin, Congress, President (unfav) |
| | More military spending, more war/tensions (fav) | More military spending, more war/tensions (unfav) |
| | Less military spending, few tensions (fav) | Less military spending, few tensions (unfav) |
| | etc. | etc. |
| Sentiment/Unclear | Better race relations, less crime | Bad race relations; more crime |
| | Times/business is good in the coming year | Times are bad now and won't change in next year |
| | Economy more stable, optimism | Economy in general less stable, lack of confidence |
| | etc. | etc. |
| Financial Market | Stock market, rise in price of stocks | Stock market decline |
| Other Real Activities | Low debts, higher savings/assets, invest up | High(er) debts, lower savings/assets |
| | Production increasing, GNP is up | Production decreasing, GNP down |
| Other Price Related | Profits high/rising | Profits high, too high |
| | Balance of payments, dollar devalue | Balance of payments, dollar devalue |
| | Price or wage controls (fav) | Price or wage controls (unfav) |
| | etc. | etc. |

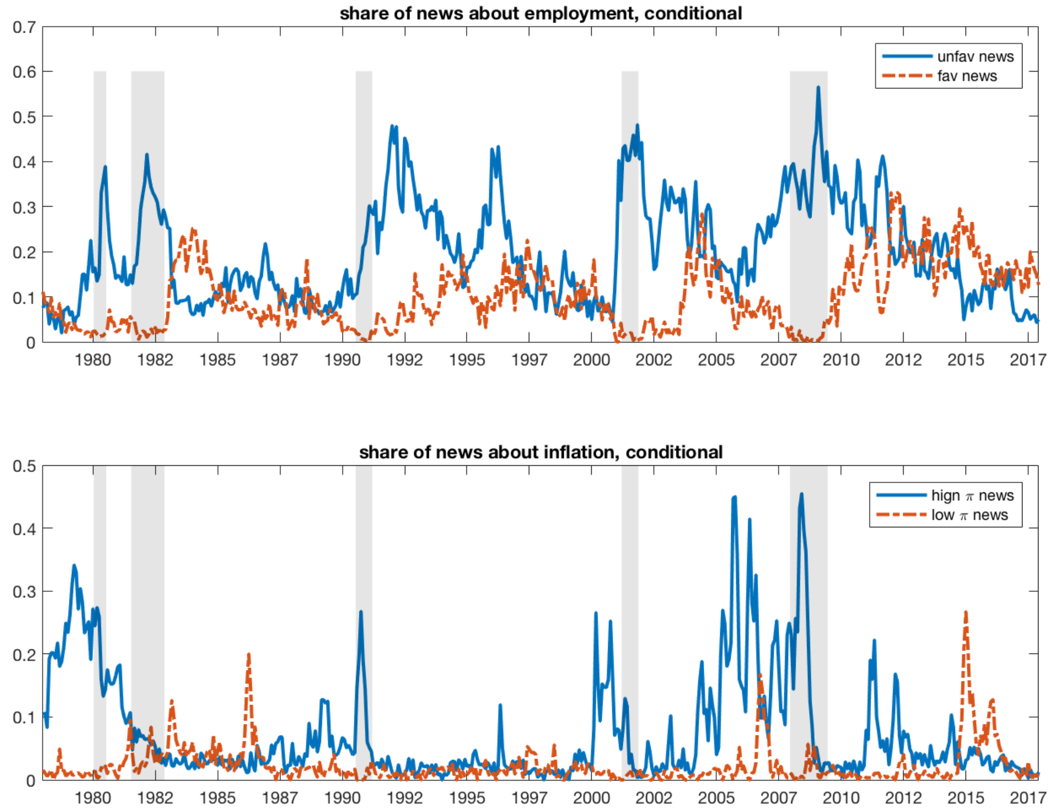
*Notes: The descriptions of news are documented by the Michigan Survey of Consumers. We reclassified them according to these descriptions.*

Figure 11: Share of People Who Report Hearing News




Share of people who report hearing any news across time. The dashed line represents on average 60% of survey participants report hearing about some news in the past few months.

Figure 12: Share of People Who Report Hearing News on Inflation and Employment




Share of people who report hearing news on employment or inflation, conditional on hearing any news. In the top panel, the blue line is the fraction with unfavorable news on employment and the red dash line is the fraction with favorable news. In the bottom panel, the blue line is the fraction with news on higher inflation.

On average, more than 60% of agents report they have heard some news about the economy, and the fraction is co-moving with the business cycle, peaking in each recession. This news about unemployment and inflation accounts for more than 40% on average, peaking at about 80% in the recent recession. There is an asymmetry in tones of news: the blue curve is almost always above the red one, which suggests agents report hearing bad news more often than good news.

# G Additional Evidence from Newspapers

### The inflation-unemployment association is seen in different narratives

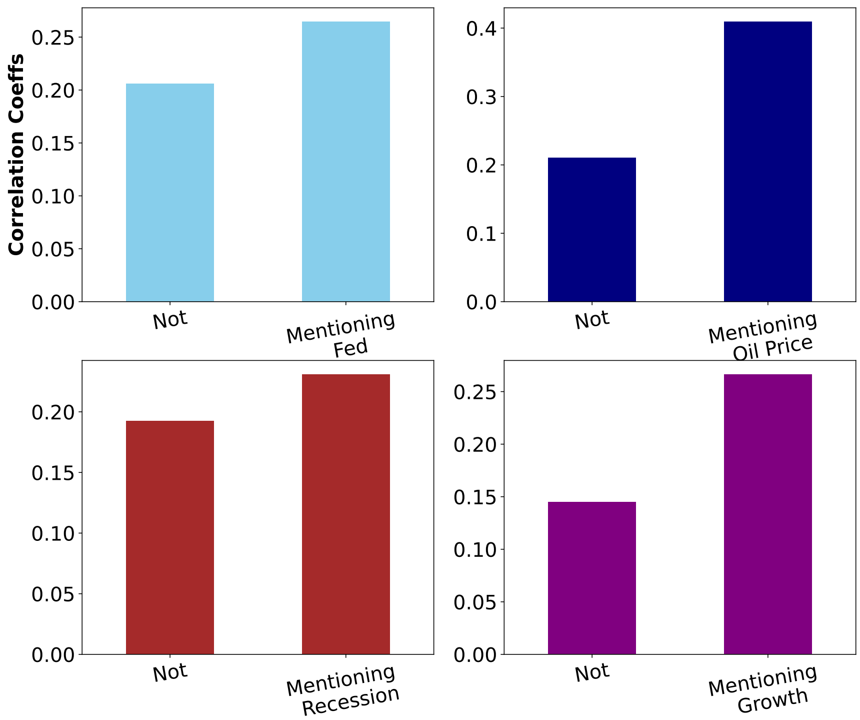
Since the association between unemployment and inflation is not driven by common signals in the newspaper, we inspect, instead, if such an association is driven by different subjective models or narratives in news discourses. We identify a narrative as a correlation between different topics that are *within* a news article.

To get some context, consider monetary policy as one example of a topic. It is indicated by an article mentioning the keyword "Fed," or by having a positive weight of a topic consisting of a list of keywords that can be interpreted as primarily related to the monetary policy (e.g.,"Fed," "Rate," "Inflation," "Economy," etc.). With these measures, we can examine if a particular article discussing monetary policy is more likely to draw connections between unemployment and inflation than other articles. We are not trying to identify causal links or directional correlations made in news articles. Instead, we treat the correlation between the frequencies of mentioning both terms as an indication of an article associating the two variables according to some model. Our goal is then to identify the topics prevailing in inflation-unemployment narratives, and if such an association is more common in certain narratives than in others.

Throughout the entire sample, the correlation between the frequencies of mentioning "inflation" and "unemployment" within each article is 0.2. This indicates that economic news articles tend to associate the two variables/concepts in economic discussions. Note that this is different from the zero correlation across time between the news coverage of unemployment and inflation.

We also find that there is a wide range of contexts in which articles make an association between inflation and unemployment. Figure 13 shows that conditional on mentioning any one of the keywords such as "Fed," "Oil price," "growth," and "recession," economic news has higher correlation coefficients between the frequencies of jointly discussing inflation and unemployment.

Figure 13: Associations between "Inflation" and "Unemployment" by Topic



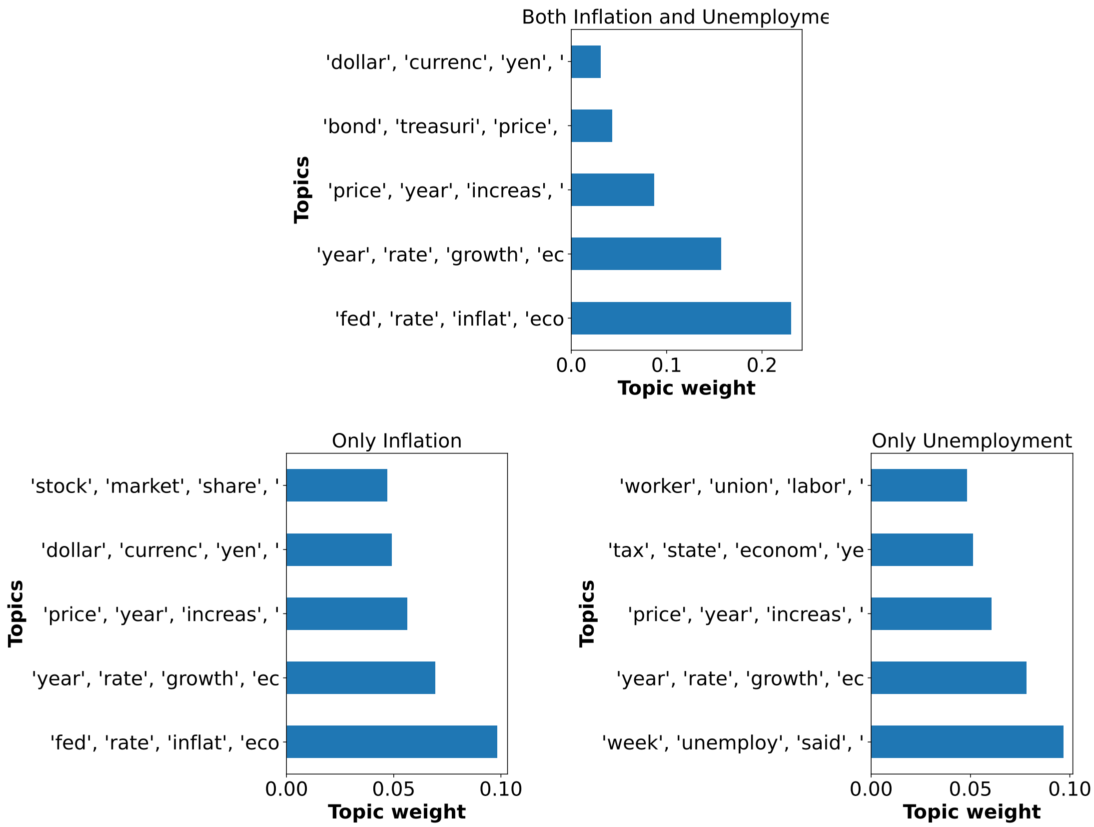

This bar chart shows the correlation coefficients between frequencies of mentioning "inflation" and "unemployment" by all articles, conditional on mentioning four other keywords.

Going beyond simple word counts, Figure 14 plots the most common LDA topics, ranked by their weights, in articles mentioning both inflation and unemployment and mentioning either topic alone. The articles that jointly mention both words and inflation-only articles largely overlap in the common topics, such as monetary policy, economic growth, prices, and exchange rates. In contrast, the most common topics in unemployment-only articles are not the same. For instance, unemployment, tax policy, and union topics are all specific to unemployment news.

### Negative sentiment cannot be the common factor, either

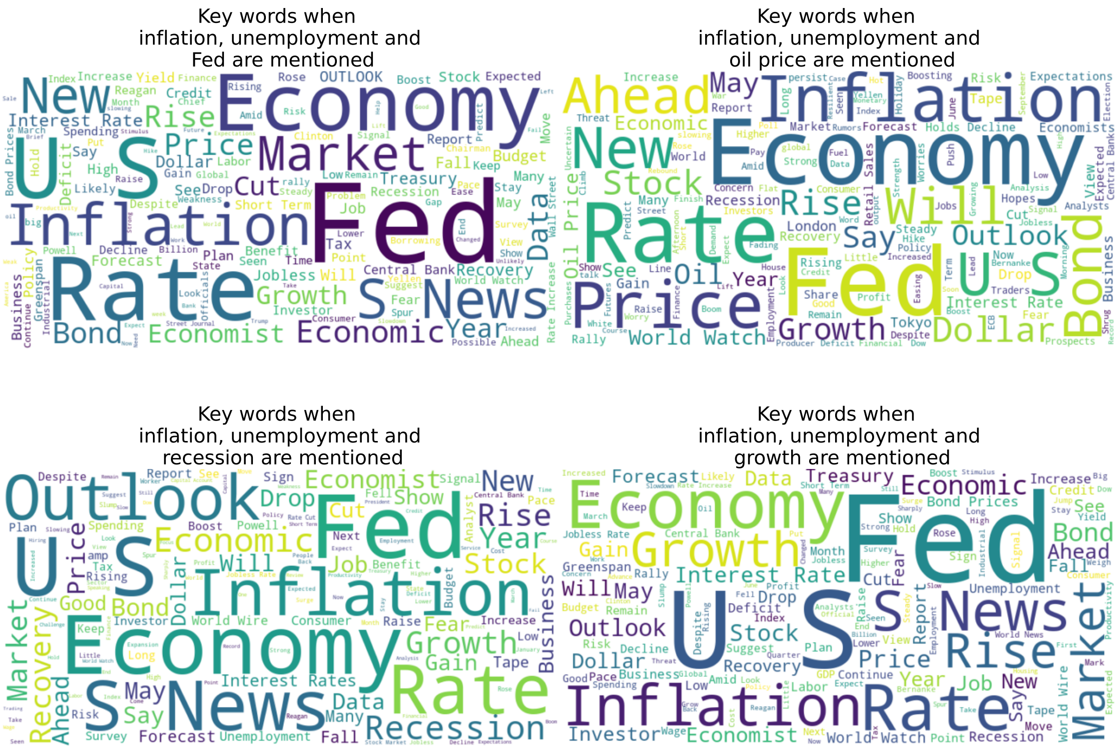
One alternative explanation for the correlated inflation and unemployment expectations is a broadly defined negative sentiment. Based on measures of overall and topic-specific sentiment

Figure 14: Topics in Inflation-Unemployment Narratives



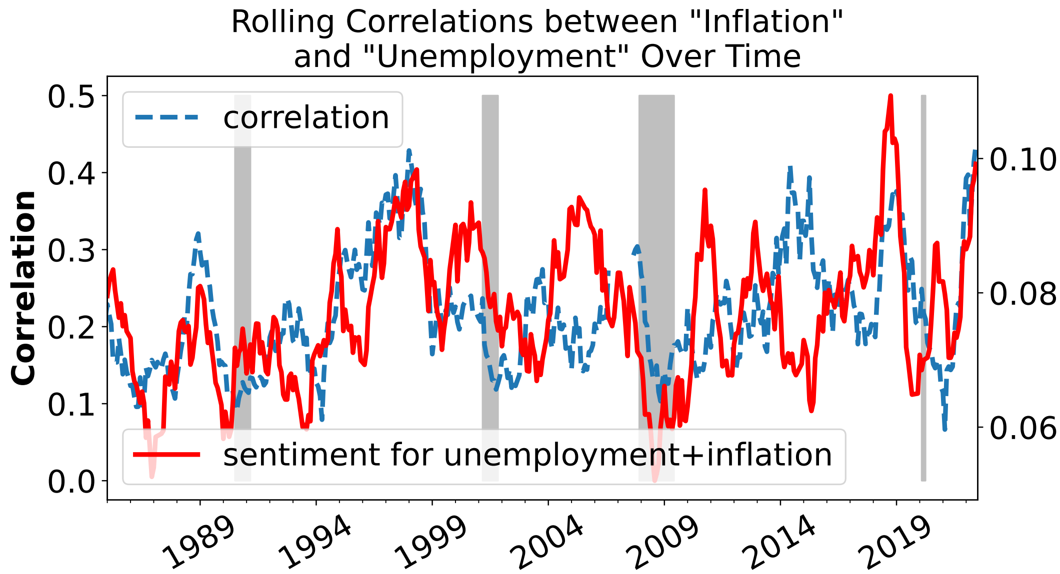

The bar charts plot the top five topics identified by the topic model in articles that mention both inflation and unemployment and those that only mention inflation or unemployment. Topic weights are between 0–1.

Figure 15: Key Words in Different Inflation-Unemployment Narratives




The figure plots the 100 most frequently used words in news articles that mention inflation, unemployment, and one of the four economic topics: Fed, oil price, recession, and growth, respectively.

Figure 16: Sentiment in Inflation-Unemployment News




On the left axis is the average within-article correlation coefficients between frequencies of "inflation" and "unemployment" for a rolling window of two years. In the right axis is the average sentiment score of articles mentioning both terms.

using newspaper texts, we find no direct support for this hypothesis. In particular, we show that the average sentiment score of articles that mention both inflation and unemployment is uncorrelated with the tendency of economic articles to associate the two within articles.

Figure 16 shows the time series of within-article correlation between coverage of unemployment and inflation in rolling windows and the measured sentiment of articles that mention both unemployment and inflation. The correlation between the two is weakly positive. It suggests that negative sentiment, as measured in inflation-unemployment news, cannot be the only driver of the inflation-unemployment association.