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Credit Conditions, Inflation, and Unemployment

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Abstract

We construct a New Monetarist model with labor market search and identify two channels that affect the long-term relationship between inflation and unemployment. First, inflation lowers wages through bargaining because unemployed workers rely more heavily on cash transactions and suffer more from inflation than employed workers: this wage-bargaining channel generates a downward Phillips curve without assuming nominal rigidity. Second, inflation increases the firm's financing costs, which discourages job creation and increases unemployment; this cash-financing channel leads to an upward-sloping Phillips curve. We calibrate our model to the U.S. economy. The improvement in firm financing conditions can explain the observation that the slope of the long-run Phillips curve has switched from positive to negative post-2000.

Topics: Business fluctuations and cycles; Credit and credit aggregates; Inflation and prices; Labour markets

JEL codes: E24, E31, E44, E51

Résumé

Nous élaborons un modèle néo-monétariste de l'appariement sur le marché du travail et recensons deux canaux ayant une incidence sur la relation à long terme entre l'inflation et le chômage. Premièrement, l'inflation entraîne une diminution des salaires par son effet sur les négociations, puisque les personnes sans emploi ont davantage recours aux transactions au comptant et souffrent plus de l'inflation que celles qui ont un emploi : ce canal de négociation des salaires donne une courbe de Phillips à pente descendante sans supposer la rigidité nominale. Deuxièmement, l'inflation entraîne une hausse des coûts de financement des entreprises, ce qui les dissuade de créer des emplois et fait monter le chômage : ce canal de financement par trésorerie donne une courbe de Phillips à pente ascendante. Nous étalonnons le modèle en fonction de l'économie américaine. L'amélioration des conditions de financement des entreprises peut expliquer l'inversion de la courbe de Phillips à long terme, qui est passée de positive à négative après 2000.

Sujets : Cycles et fluctuations économiques; Crédit et agrégats du crédit; Inflation et prix; Marchés du travail

Codes JEL : E24, E31, E44, E51

1 Introduction

Empirical studies have found significant temporal and spatial variations in the slope of the long-run Phillips curve. King and Watson (1994) find a negative slope in U.S. data from 1954 to 1969, no consistent pattern from 1970 to 1987, and a positive slope from 1954 to 1987. Berentsen et al. (2011) apply the HP filter to U.S. data from 1955 to 2005 and find a positive relationship between inflation and unemployment at low frequencies. Similarly, Haug and King (2014), using a band-pass filter approach, support Berentsen et al.'s (2011) findings. In Figure 1 we follow the approach of Berentsen et al. (2011) and Haug and King (2014) and find that the slope of the Phillips curve is positive before 2000 and negative afterward. Cross-country variations in the Phillips curve slope have also been documented. Karanassou et al. (2010) report a negative relationship between inflation and unemployment in a panel of European countries. Dolado and Jimeno (1997) find that Spain exhibited a positive Phillips curve slope from 1970 to 1977, which turned negative from 1977 to 1994. For Germany, Franz (2005) and Schreiber and Wolters (2007) both identify a negative Phillips curve slope.

Given the conflicting empirical findings, it is useful to construct a theoretical model that accommodates the mixed evidence on the slope of the long-run Phillips curve. The dominant theoretical thinking is that the long-run Phillips curve is vertical: from the new Keynesian point of view, inflation and monetary policy do not affect unemployment in the long run when nominal variables have time to adjust. The new monetarist literature emphasizes the liquidity role of money and tends to prescribe an upward-sloping Phillips curve. As shown in Berentsen et al. (2011), inflation increases the liquidity cost of holding money, taxes monetary transactions, and dampens economic activity. Little work has been done on channels that lead to a negatively sloped long-run Phillips curve. Without such channels, it is impossible to explain the observed variation in the slope of the long-run Phillips curve.

We develop a model that holds the promise of explaining the mixed evidence on the slope of the long-run Phillips curve. The labor market features search frictions, as in Mortensen and Pissarides (1994). The goods market is competitive but lacks record-keeping, requiring households to use some means of payment to transact, as in Lagos and Wright (2005) and Rocheteau and Wright (2005). Firms face credit frictions and must use cash or credit backed by pledged capital to finance wage payments, as in Kiyotaki and Moore (1997).

In our baseline model, inflation affects unemployment through two channels. The first is the *firm cash-financing channel*. If a firm uses cash to finance wage payments, inflation raises the firm's financing costs, reduces its profits, discourages job creation, and increases unemployment. The implication of this channel is consistent with the new monetarist view

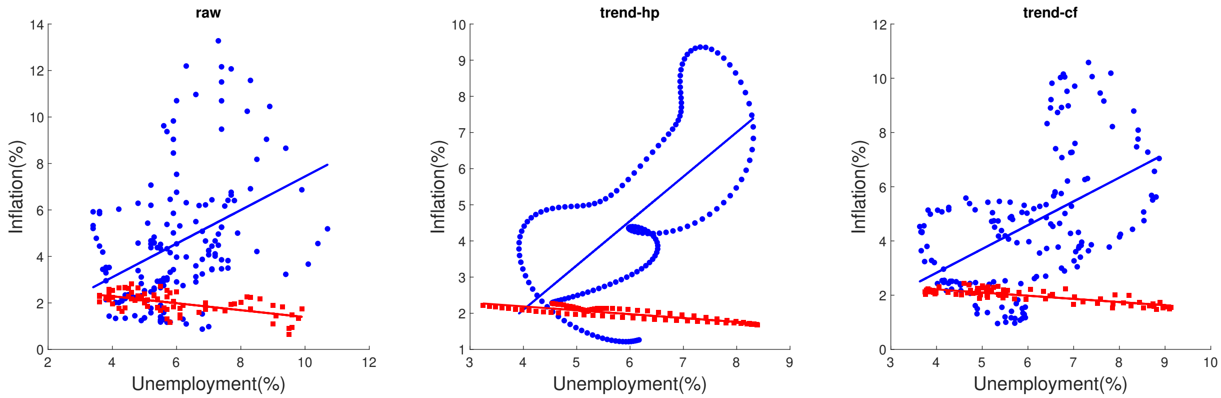


Figure 1: Long-Run Phillips Curve in the U.S.

Notes. Unemployment: FRED series UNRATE; inflation: FRED series CPILFESL. Both series are in quarter frequency. The first panel plots the raw data, the second panel plots the trend components from applying the 1600 HP filter, and the last panel plots the trend components from applying the band-pass filter in Christiano and Fitzgerald (2003). The red (blue) dots and curves represent data for the period 2000–2019 (1960–1999).

that inflation acts as a tax on cash holdings and hurts the economy. The cash-financing channel implies an upward-sloping Phillips curve.

The second channel, the *wage-bargaining channel*, is more novel and implies a downward-sloping Phillips curve. Employed workers receive wage income to meet some of their liquidity needs and therefore carry lower real balances than the unemployed. As a result, inflation hurts unemployed workers more. When inflation rises, workers become more willing to accept lower wages to get employed, which increases firms' profits, encourages job creation, and reduces unemployment. This wage-bargaining channel aligns with empirical evidence. Consumer payment surveys show that the unemployed engage in a significantly larger share of cash transactions than the employed (see, for example, Burdett et al. 2016; Greene et al. 2024).[1]

Given these two opposing channels, the overall impact of inflation on unemployment can be either positive or negative. In particular, this relationship depends on credit conditions measured by capital pledgeability. When credit is scarce, firms rely more heavily on cash to finance wage payments, so the cash-financing channel tends to dominate and the Phillips curve is more likely to exhibit a positive slope. As credit conditions improve, the cash-financing channel weakens, the wage-bargaining channel becomes dominant, and the Phillips curve slopes downward.

[1] See also our calculation of cash transaction shares using the data from the Federal Reserve Bank of Atlanta's Survey and Diary of Consumer Payment Choice in Section 5.1. Related findings are that lower-income individuals, who are more likely to be unemployed, are disproportionately affected by inflation (Easterly and Fischer 2001).

To clarify our theoretical contribution, it is useful to compare our model to Berentsen et al. (2011). There are two major differences. First, in Berentsen et al. (2011), search frictions and bargaining in the goods market create strategic complementarity between the firm's entry decision and the worker's liquidity choice, which amplifies the positive relationship between inflation and unemployment. In contrast, we purposefully shut down the Berentsen et al. (2011) channel by assuming a competitive goods market with no search frictions, where firms make no profit. Instead, we emphasize the liquidity choices of firms, which, in reality, hold a significant share of total money demand.[2] The cash-financing channel we propose is also consistent with empirical evidence. For example, Coibion et al. (2020) use Italian firm survey data to show that firms tend to reduce hiring and increase their use of credit when they expect inflation to rise.[3]

Another key difference in our model is that firms pay wages before workers consume, so wage income is a source of liquidity. As a result, employed workers hold less cash and bear a lower liquidity cost than the unemployed. This differential liquidity cost is key to the wage-bargaining channel. In contrast, in Berentsen et al. (2011), firms pay wages after workers purchase goods, so all workers hold the same amount of cash, and the wage-bargaining channel is absent. In the extended model, we let the firm and the worker jointly decide the timing of wage payments. In equilibrium, the firm pays all or part of the wages before the goods market and the remainder afterward. In the latter case, the firm's cash-financing costs decrease and the total surplus increases. With the endogenous timing of wage payments, the cash-financing channel is dominated by the wage-bargaining channel and the Phillips curve is unambiguously downward sloping. However, adding goods market search frictions reintroduces ambiguity to the slope of the Phillips curve.

We calibrate our model to the U.S. economy from 2000 to 2019. For calibration, besides the two channels featured in our baseline model, we incorporate the Berentsen et al. (2011) channel by having a frictional goods market. We target household money demand, as is standard in the literature, and set the capital pledgeability parameter to match firm money demand.

We use the calibrated model to quantify the contributions of inflation and firm credit conditions to changes in unemployment and the slope of the long-run Phillips curve between

[2]We calculate firm and household money demand (currency and checkable deposits holdings over GDP) in the U.S. using FRED data. From 2000 to 2019, firm money demand is 0.0459 and household money demand is 0.0374. See Section 5.1 for more details.

[3]See also Christiano et al. (2015), who show that the bulk of movements in aggregate economic activity during the Great Recession can be attributed to shocks to financial frictions that raised the cost of working capital.

1960–1999 and 2000–2019. Over these two periods, unemployment decreases by 0.37%, from 6.26% to 5.89%. The improvement in firm credit conditions reduces unemployment by 0.48%, while the decline in inflation increases unemployment by 0.11%. Regarding the slope of the Phillips curve, the increase in χ causes the slope to switch from positive to negative, capturing the observed change in the Phillips curve, as shown in Figure 1. Meanwhile, the decrease in inflation flattens the negatively sloped Phillips curve slightly by 12%.

Our short-run analysis finds that credit and monetary shocks play an important role in explaining the volatility of business debt, the share of labor compensation in GDP, and the business and household components of money demand. However, these shocks have little impact on the volatility of unemployment. The two channels that affect the relationship between inflation and unemployment in the long run also operate in the short run. However, their effects tend to be obscured by technology shocks, which are the main driving force of the cyclical fluctuations in unemployment.

Related Literature. Our paper is most closely related to studies on the long-run relationship between inflation and unemployment following Berentsen et al. (2011). Most models predict an upward-sloping Phillips curve, with a few exceptions, which we discuss in more detail.[4]

Bethune and Rocheteau (2023) suggest an interest-rate channel: higher inflation reduces the supply of public liquid assets and thus lowers the real interest rate, or the required real return on assets. As a result, the present value of firm profits increases, encouraging job creation. According to their calibration, the interest-rate channel is dominated by the Berentsen et al. (2011) channel, leading to an upward-sloping Phillips curve. They show that two factors can strengthen the interest-rate channel: (i) if money creation is distributed as a lump-sum transfer and (ii) if non-monetary wealth becomes more liquid.

Bethune et al. (2024) introduce an outside option for consumers to search for another seller in the frictional goods market. Higher inflation reduces this outside option, allowing firms to charge higher prices, earn greater profits, and open more vacancies. They find that this new effect dominates the Berentsen et al. (2011) effect when inflation is low, while the reverse is true when inflation is high.

[4]Lehmann (2022) uses a cash-in-advance model with a mechanism similar to Berentsen et al. (2011): agents must hold cash to finance consumption, inflation reduces the surplus of the worker-firm pair, and thus job openings decline. Rocheteau et al. (2007) and Dong (2011) show that the relationship between inflation and unemployment can be negative or positive depending on the utility function, in particular, whether the CM and DM consumption goods are complements or substitutes. They do not have a search labor market; unemployment arises from indivisible labor, as in Rogerson (1988).

An unpublished working paper by Liu (2008) proposes a mechanism similar to our wage-bargaining channel. In her model, the unemployed have a higher probability of being matched in the goods market and, therefore, carry more cash than the employed. Compared to her model, our wage-bargaining channel is more robust in that it operates even if all workers have the same opportunity to consume in the goods market. Her work does not include formal calibration or analysis of the reversal of the sign of the slope of the Phillips curve.

Other works that study liquidity choice and labor market search in the New Monetarist framework include Gomis-Porqueras et al. (2020), who identify a hump-shaped relationship between inflation and capital stock; He and Zhang (2020), who find that incorporating firm money demand amplifies the effect of inflation on unemployment and that inflation reduces the firm's money share; Bethune et al. (2015), who examine the effect of unsecured consumer credit on unemployment; and Gabrovski et al. (2025), who show that the impact of financial disruptions on macroeconomic variables is modest due to the substitutability among different means of payment.

Finally, our paper is related to studies on the effects of liquidity shocks on business cycles. In a cash-in-advance model, Cooley and Quadrini (1999, 2004) show that the slope of the short-run Phillips curve depends on the types of shocks and the central bank's policy response. Monetary shocks induce a negative relationship between inflation and unemployment, while technological shocks induce a positive one. Cooley and Quadrini (2004) further show that the optimal monetary policy is pro-cyclical: the central bank should increase the money supply to counteract the high nominal interest rate induced by a positive technology shock. Ait Lahcen et al. (2022) study a stochastic version of Berentsen et al. (2011) to account for the nonlinear relationship between unemployment and inflation and to explain the increasing volatility of unemployment at higher inflation rates. Cui et al. (2025) examine the steady-state and business-cycle implications of liquidity on capital reallocation. Herkenhoff (2019) analyzes how cyclical fluctuations and long-term increases in consumer credit access impact the business cycle. Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2013), and Petrosky-Nadeau (2013) explore how frictions in the market where firms finance the vacancy-posting costs affect the cyclical volatility of the labor market.

The rest of the paper is organized as follows. Section 2 introduces the baseline model with a competitive goods market and characterizes the equilibrium. Section 3 examines the model's implications for the effects of inflation and firm credit conditions on unemployment and the Phillips curve. Section 4 extends the model by incorporating search frictions in the goods market and endogenizing the timing of wage payments. Section 5 calibrates the model and conducts both long-run and short-run quantitative analyses. Section 6 concludes.

2 The Baseline Model

The model environment builds on Berentsen et al. (2011) with three innovations. First, we include capital, which can be combined with labor to produce output and used as collateral for loans. Second, firms must finance their working capital for wage payments using either cash or bank loans secured by capital. Third, workers can use both cash and wage income to purchase goods, meaning that unemployed workers bear higher inflation costs than employed workers.

Time is discrete and infinite. Agents discount between periods by $0 < \beta < 1$. In each period, three markets open sequentially: a labor market (LM), followed by a decentralized goods market (DM), and finally a centralized goods market (CM). The CM is frictionless. LM and DM have frictions, as detailed below.

There are two types of infinitely lived agents, workers and firms. In the LM, workers are endowed with one unit of labor. Firms are endowed with a production technology that transforms capital and one unit of labor into consumption goods valued in the CM according to the production function $f(k)$, where $f' > 0$, $f'' < 0$, and $f(0) = 0$. Labor is indivisible, and each firm can hire at most one worker. There is a unit measure of workers. The measure of firms is arbitrarily large, but not all firms are active at any given point in time. Firms are owned by workers, and in each firm, each worker has an equal share. In addition to these two types of agents, there are a large number of one-period lived bankers. Bankers are not endowed with anything but the ability to commit to debt redemption. They value consumption in the CM.

In the LM, firms with workers produce output. After production, an employed worker separates from their job with probability s. Firms without workers can meet unemployed workers bilaterally after paying a vacancy-posting cost. An unemployed worker matches with a job-posting firm according to a matching function, $\mathcal{M}(u, v)$, where u is the measure of unemployed workers and v is the measure of vacancy-posting firms. The matching function is increasing, concave, twice differentiable, and homogeneous of degree one. The labor market tightness is defined as $\tau = v/u$. Newly matched firm-worker pairs start producing in the next LM.

In the DM, firms can transform goods produced in the LM by a linear cost function into DM goods. The marginal cost is $c > 0$. Workers value DM good according to $v(q)$, with $v' > 0$,

$\upsilon'' < 0$, and $\upsilon(0) = 0$. For now, assume that the DM is competitive and that there is no search friction. We will relax this assumption in Section 4.1.

In the CM, firms sell unsold goods and repay bank loans, and all agents adjust their money and capital holdings. Agents have linear utility in CM consumption, which can also be produced one-for-one with labor. Negative consumption indicates that the agent produces in the CM.

Two payment instruments, money and bank IOUs, are used for LM and DM transactions.[5] As is well-known in the literature, money has no role in the economy if credit is perfect (Kocherlakota 1998). We assume that workers lack commitment and there is no record-keeping device for worker's borrowing, so consumer credit is infeasible.[6] Firms cannot commit to wage payment after they sell the output, so they must pay the workers in the LM.[7] They can secure intra-period loans from banks in the form of bank IOUs by pledging capital at the beginning of the LM. Bank IOUs are recognizable by all agents, so they can be used to pay wages in the LM and goods in the DM. Firms repay loans in the CM to redeem collateral, and banks redeem IOUs from the IOU holders. The money supply grows at a rate of $\pi > \beta - 1$; there are no monetary equilibria if $\pi < \beta - 1$. Changes in M are accomplished by lump-sum transfers if $\pi > 0$ and lump-sum taxes if $\pi < 0$.

2.1 Worker's Problem

Let U_j, V_j, and W_j, $(j = 0, 1)$, denote the worker's value function in the LM, DM, and CM, respectively, where j represents employment status, with 0 indicating unemployed and 1 indicating employed. For simplicity, assume that workers do not carry capital.[8] A worker of type j entering the CM with real balance z and unspent labor income ω solves the following

[5]Rocheteau and Rodriguez-Lopez (2014) consider a broader scope of liquidity, including government bonds and claims on a firm's profit as possible components of an agent's portfolio. Here we focus on traditional monetary policy instead and model money and secured credit backed by the firm's asset.

[6]We abstract from consumer credit and focus on firm credit, as the effects of consumer credit on labor market performance are studied extensively in the literature. For example, Bethune et al. (2015) study the household unsecured credit in a model based on Berentsen et al. (2011) and endogenize the credit limit. They show that the availability of consumer credit reduces long-run unemployment. Herkenhoff (2019) shows that access to consumer credit prolongs recessions but enhances welfare by reducing consumption volatility and improving the quality of job matches.

[7]Although there is no monitoring device, the worker can punish the firm by quitting. Therefore, some unsecured credit, in the spirit of Kehoe and Levin (1993), is feasible between the worker and the firm. However, modeling both unsecured and secured credit is challenging. Thus, we focus on secured credit.

[8]The equilibrium capital price includes its liquidity value and is above (or at least equal to) its marginal product. We assume that only firms can pledge capital to acquire credit, while workers cannot. Following Aruoba et al. (2011), the underlying friction is that workers cannot bring physical capital to the goods market and they can counterfeit certificates of the capital costlessly.

problem at the beginning of the CM:

$$W_j(z,\omega) = \max_{x,\hat{z}_j} \left[x + \beta U_j(\hat{z}_j)\right]$$
$$\text{st } x + (1+\pi)\hat{z}_j + T = z + \omega + \Delta,$$

where x is the consumption of the CM good, T is a lump-sum tax (a transfer if negative), Δ is dividend income, π is the inflation rate, and $\hat{z}_j$ is the real balance in terms of good x in the next period. Workers may choose different amounts of money according to their employment status because employed workers anticipate labor income in the LM. The first-order condition with respect to $\hat{z}_j$ is

$$-(1+\pi) + \beta U_j'(\hat{z}_j) \leq 0,$$

where the equality is strict if and only if $\hat{z}_j > 0$. As standard in this framework, $\hat{z}_j$ does not depend on z or ω. The envelope conditions are $\partial W_j/\partial z = \partial W_j/\partial \omega = 1$, which implies that W_j is linear in $z + \omega$.

In the LM, an employed worker works and receives wage w, which was determined bilaterally between the firm and the worker upon hiring. After production, the worker-firm pair separates with probability s. The value function of an employed worker is

$$U_1(z_1) = (1-s)V_1(z_1,w) + sV_0(z_1,w).$$

It follows that $U_1'(z_1) = (1-s)\partial V_1/\partial z_1 + s\partial V_0/\partial z_1$. An unemployed worker does not have wage income. They search for a job and match with a firm with probability λ_h. If matched, they will start working in the next LM. The unemployed worker's LM value is given by

$$U_0(z_0) = \lambda_h V_1(z_0) + (1-\lambda_h)V_0(z_0),$$

and $U_0'(z_0) = \lambda_h \partial V_1/z_0 + (1-\lambda_h)\partial V_0/\partial z_0$. Note that the subscript of V represents the employment status at the end of the LM, after the separation shocks are realized and the job search is completed.

In the DM, workers can use cash earned in the previous CM and wage income from the previous LM to pay for good q. The DM is a Walrasian market, and due to competition, the price of the DM good is equal to the marginal cost c. The worker chooses the quantity of the DM good to buy and how to pay for the good given their liquidity. The DM problem of an employed worker is

$$\begin{aligned} V_j(z_1, w) &= \max_{q_1, z', w'} \left[\upsilon(q_1) + W_j(z_1 - z', w - w') \right] \\ \text{st } cq_1 &= z' + w',\ z' \leq z_1,\ w' \leq w, \end{aligned}$$

where z' and w' are the transfers of real balance and wage income to firms, respectively. Consolidating the liquidity constraints and by the linearity of W_j, we can write the DM problem as

$$V_j(z_1, w) = \max_{q_1} \left[\upsilon(q_1) - cq_1 + W_j(z_1, w) \right]$$
$$\text{st } cq_1 \leq z_1 + w.$$

Let q^* solve $\upsilon'(q^*) = c$. The first-order condition with respect to q results in $q_1 = q^*$ if $z_1 + w \geq cq^*$ and $q_1 = (z_1 + w)/c$ otherwise. Taking derivative to get $\partial V_j / \partial z_1 = 1$ if $z_1 + w \geq cq^*$ and $\partial V_j / \partial z_1 = \upsilon'(q_1)/c$ otherwise. Notice that the DM surplus, $\upsilon(q) - cq$, strictly increases in liquidity if the liquidity constraint binds.

Combining the first-order conditions and the envelope conditions, we get the following solution to q_1 and z_1:

$$\begin{array}{lll} q_1 = q_i & \text{and } z_1 = cq_i - w, & \text{if } w < cq_i, \\ q_1 = w/c & \text{and } z_1 = 0, & \text{if } cq_i \leq w < cq^*, \\ q_1 = q^* & \text{and } z_1 = 0, & \text{if } w \geq cq^*, \end{array} \tag{1}$$

where q_i solves $\upsilon'(q_i) = (1 + i)c$ and $i = (1 + \pi)/\beta - 1$ is the nominal interest rate according to the Fisher equation. Note that q_i decreases in i and is independent of w. If w is higher than cq^*, employed workers are not liquidity constrained, as w is sufficient to pay for optimal q. If w is insufficient to buy q^* but is sufficient to purchase q_i, workers are liquidity constrained. However, since holding money is too costly relative to the marginal benefit, they choose not to hold money. Lastly, if w is not sufficient to pay q_i, workers supplement their wages with cash to buy q_i.

Next, we solve the DM problem of an unemployed worker:

$$V_j(z_0) = \max_{q_0} \left[\upsilon(q_0) - cq_0 + W_j(z_0) \right]$$
$$\text{st } cq_0 \leq z_0.$$

The solution to q_0 and z_0 is

$$q_0 = q_i \text{ and } z_0 = cq_i. \tag{2}$$

Since unemployed workers are more liquidity-constrained, their consumption and DM trade surplus are (weakly) lower than that of the employed.

We consolidate the value functions and simplify them to get the following in the steady state:

$$W_1(0,0) = -T + \beta\left[w - iz_1 + \upsilon(q_1) - cq_1 + (1-s)W_1(0,0) + sW_0(0,0)\right] \tag{3}$$

and

$$W_0(0,0) = -T + \beta\left[-iz_i + \upsilon(q_i) - cq_i + \lambda_h W_1(0,0) + (1-\lambda_h)W_0(0,0)\right]. \tag{4}$$

Subtract (4) from (3) to get the surplus of a worker matched with a firm. Let $S_h \equiv W_1(0,0) - W_0(0,0)$ and $r \equiv 1/\beta - 1$:

$$S_h = \frac{w + [\upsilon(q_1) - cq_1 - iz_1] - [\upsilon(q_i) - (1+i)cq_i]}{r + s + \lambda_h}. \tag{5}$$

From (5), being employed has three advantages: First, employed workers earn wage income w. Second, they get more trade surplus in the DM because they are less liquidity-constrained, that is, $\upsilon(q_1) - cq_1$ is (weakly) higher than $\upsilon(q_0) - cq_0$. Third, they save on inflation costs by holding less cash: z_1 is strictly lower than z_0, which just covers cq_i. These advantages also imply that S_h increases with w. Another result is that given w, S_h increases with i because the outside option, captured by $\upsilon(q_i) - (1+i)cq_i$, decreases in i.

2.2 Firm's Problem

We now turn to the firm's problem. Let $\tilde{W}_j$, $\tilde{U}_j$, and $\tilde{V}_j$ denote the value of the firm of type j at the beginning of the CM, LM, and DM, respectively, where firms with workers are type 1 and those without workers are type 0. A firm enters the CM with real balance z_f, capital k, bank loan l, and output y. It adjusts money and capital holdings to bring to the following LM. The CM value of a firm with a worker is

$$\tilde{W}_1(z_f, k, l, y) = z_f + (1-\delta)k + y - (1+i_l)l + \max_{\hat{z}_f, \hat{k}}\left[-(1+\pi)\hat{z}_f - \hat{k} + \beta\tilde{U}_1\left(\hat{z}_f, \hat{k}\right)\right],$$

where δ is the capital depreciation rate, i_l is the interest rate on the bank loan, and $\hat{z}_f$ and $\hat{k}$ are the real balance and capital carried to the LM, respectively. The first-order conditions are

$$\hat{k} \;:\; -1+\beta\partial\tilde{U}_j\left(\hat{z}_f,\hat{k}\right)/\partial\hat{k}=0,$$
$$\hat{z}_f \;:\; -(1+\pi)+\beta\partial\tilde{U}_j\left(\hat{z}_f,\hat{k}\right)/\partial\hat{z}_f\leq 0,$$

where the equality is strict if and only if $\hat{z}_f>0$. The envelope conditions are $\partial\tilde{W}_j/\partial z_f = \partial\tilde{W}_j/\partial y=1$, $\partial\tilde{W}_j/\partial l=-(1+i_l)$, and $\partial\tilde{W}_j/\partial k=1-\delta$.

In the LM, the firm can borrow secured loans from the bank by pledging a fraction χ of capital, as in Kiyotaki and Moore (1997). The loans are repaid in the following CM. The pledegeability of capital, measured by χ, indicates the firm's credit conditions. Upon completion of production in the LM, the firm retains the worker with probability $1-s$ and loses the worker with probability s. Let z' be the real balance paid to the worker. The expected value of the firm in the LM is

$$\tilde{U}_1(z_f,k)=(1-s)\tilde{V}_1(z_f-z',k,l,y)+s\tilde{V}_0(z_f-z',k,l,y),$$

where $w=z'+l$, $z'\leq z_f$, $l\leq\chi(1-\delta)k$, and $y=f(k)$. The envelope conditions are $\partial\tilde{U}_1/\partial z_f=(1-s)\partial\tilde{V}_1/\partial z_f+s\partial\tilde{V}_0/\partial z_f$ and $\partial\tilde{U}_1/\partial k=(1-s)\partial\tilde{V}_1/\partial k+s\partial\tilde{V}_0/\partial k$.

For a firm without a worker, money and capital are not needed but costly to obtain. Therefore, its money and capital holdings are zero. A firm is matched with an unemployed worker with probability λ_f after paying cost κ to post the vacancy. A matched firm does not produce in the current period or participate in the following DM but will become active in the LM and DM next period. A vacancy-posting firm's expected value is

$$\tilde{U}_0=-\kappa+\lambda_f\tilde{W}_1(0,0,0,0)+(1-\lambda_f)\tilde{W}_0(0,0,0,0).$$

In the DM, producing firms transform some of their LM output into DM goods using a linear cost function. As the DM is competitive, firms do not make profits in that market. By the linearity of $\tilde{W}$, a producing firm's DM value is

$$\tilde{V}_j(z_f,k,l,y)=\tilde{W}_j(z_f,k,l,y).$$

Before we solve the firm's decisions on real balance and capital, we turn to the bank's problem to solve the loan rate. Banks extend secured loans to firms in the LM and receive repayment in full in the CM within the same period. Competitive banking implies that the intra-period loan rate, i_l, is 0. This implies that firms first use their capital to get free loans and then use cash to top up the wage payments. We combine this result with the first-order conditions and the envelope conditions to solve the firm's choice of k and z_f, given w. The solution is in one of the three regimes:

$$\begin{aligned}
&\text{Regime 1:} \quad k = k_i \text{ and } z_f = w - \chi(1-\delta)k_i, \text{ if } w \geq \chi(1-\delta)k_i, \\
&\text{Regime 2:} \quad k = w/\chi(1-\delta) \text{ and } z_f = 0, \text{ if } \chi(1-\delta)k^* \leq w < \chi(1-\delta)k_i, \\
&\text{Regime 3:} \quad k = k^* \text{ and } z_f = 0, \text{ if } w < \chi(1-\delta)k^*,
\end{aligned} \tag{6}$$

where k^* solves $f'(k^*) = r+\delta$ and k_i solves $f'(k_i) = r+\delta-i\chi(1-\delta)$. Notice that k_i increases in i and χ. By the concavity of f, $k_i > k^*$. In Regime 1, firms are liquidity-constrained. They use both secured loans and cash to pay workers. Capital is over-accumulated as in Lagos and Rocheteau (2008). Note that k_i increases in i: A higher nominal interest rate induces firms to substitute more capital for cash to meet their liquidity needs, which is the Tobin effect. In Regime 2, firms are still liquidity constrained but they do not hold money and rely solely on capital to finance wages: the cost of over-accumulated capital is smaller than the cost of cash. In Regime 3, the liquidity constraint does not bind. Capital accumulation is at its first best, and firms secure enough free liquidity to pay wages and do not hold cash.

Note that given w, k is not monotone in χ. As χ increases from 0, the capital per firm first increases from k^* in Regime 1, then decreases in Regime 2, and finally becomes constant at k^* in Regime 3. In other words, as χ increases, firms initially over-accumulate capital for its liquidity function to save on cash. With a further increase in χ, the economy transitions to Regime 2, where firms no longer require cash to finance wage payments. As χ increases, firms need less capital to finance wage payments, so they shed some of their capital. As χ continues to increase, the economy enters Regime 3, where the efficient level of capital alone is sufficient to pay workers, and firms do not over-accumulate capital.[9]

Combining the value functions in the three markets, the value of producing firms can be written as

$$\begin{aligned}
\tilde{W}_1(0,0,0,0) \;=\; & \beta[-iz_f + f(k) - (r+\delta)k - w + \\
& + (1-s)\tilde{W}_1(0,0,0,0) + s\tilde{W}_0(0,0,0,0)],
\end{aligned} \tag{7}$$

[9]Without loss of generality, we assume there is no capital market. The capital market would be inactive even if we allow for it. Since workers or inactive firms do not use capital, they lend only if the real interest rate on the loan is $r+\delta$. As producing firms can pay at most $r+\delta$, there is no active lending and borrowing in the capital market. If the real loan rate is $r+\delta$ (it occurs when $\chi = 0$ or when the firms are in Regime 3), the firms are indifferent between borrowing capital from others and self-financing.

where z_f and k are given by (6). For a firm without a worker, the expected value is

$$\tilde{W}_0(0,0,0,0) = \beta \left[-\kappa + \lambda_f \tilde{W}_1(0,0,0,0) + (1-\lambda_f)\tilde{W}_0(0,0,0,0) \right]. \tag{8}$$

Subtract (8) from (7) to get the surplus of a firm in a match in the LM. Let $S_f = \tilde{W}_1(0,0,0,0) - \tilde{W}_0(0,0,0,0)$. Then

$$S_f = \frac{f(k) - (r+\delta)k - w + \kappa - iz_f}{r + s + \lambda_f}. \tag{9}$$

Compared to a non-producing firm, the producing firm generates output $f(k)$, pays wages w to the worker, incurs $(r+\delta)k$ as the opportunity cost of holding k, bears an inflation cost iz_f by holding cash, and saves on job-posting cost κ. Taking the derivative of S_f with respect to i in Regime 1, given w, we get $\partial S_f/\partial i = -z_f$.[10] Intuitively, inflation raises the cost of wage financing, so firms are worse off. In Regimes 2 and 3, since firms do not use cash, inflation does not affect their surplus.

2.3 Wage Bargaining

Let us turn to the determination of wages. We assume that once a worker and a firm agree upon a contract, they cannot renegotiate the wage.[11] Wage is determined by splitting the matching surplus according to the Kalai bargaining solution. Let $\rho \in (0,1)$ be the worker's bargaining power. The worker's and firm's surpluses must satisfy

$$\frac{S_h}{S_f} = \frac{\rho}{1-\rho}. \tag{10}$$

To solve for λ_f and λ_h, we use the zero-profit condition for entering firms and the law of motion for unemployment. The free-entry condition is $\tilde{W}_0(0,0) = 0$, which yields

$$\lambda_f \left[f(k) - (r+\delta)k - w - iz_f \right] = \kappa (r+s). \tag{11}$$

[10] To see this, note that in Regime 1, $k = k_i$, so $\partial S_f/\partial i = \{f'(k_i) - [r+\delta-i\chi(1-\delta)]\}dk_i/di + \chi(1-\delta)k_i - w$. By definition of k_i, $f'(k_i) - [r+\delta-i\chi(1-\delta)] = 0$, so $\partial S_f/\partial i = -z_f$.

[11] The assumption of no renegotiation is innocuous in a stationary economy. In Section 5.3, where we introduce shocks in the CM for short-run analysis, we allow renegotiation to occur in the CM simultaneously with portfolio choice. We do not allow renegotiation in the LM after the worker and the firm choose their portfolios because that will induce a two-sided hold-up problem.

The law of motion for unemployment is

$$u_{t+1} = u_t (1 - \lambda_h) + (1 - u_t)s.$$

In the steady state, the measure of unemployed workers remains constant, which results in

$$u = \frac{s}{\lambda_h + s}. \tag{12}$$

Finally, we can derive the matching probabilities for vacant firms and unemployed workers from the matching function

$$\lambda_f = \mathcal{M}(1/\tau, 1) \tag{13}$$

and

$$\lambda_h = \mathcal{M}(1, \tau). \tag{14}$$

2.4 Government Policy

The government spends G, levies tax T, and receives seigniorage πz, where $z \equiv uz_0 + (1 - u)(z_1 + z_f)$ is the total demand for real balances. The government maintains a balanced budget, so $G = T + \pi z$ in each period. In the steady state, targeting the nominal interest rate is equivalent to targeting the money growth rate.

2.5 Equilibrium

A stationary monetary equilibrium is a list of $(w, k, z_f, z_1, z_0, q_1, q_0, \lambda_f, \lambda_h, \tau)$ that solves (1), (2), (5), (6), and (9)–(14), with $z_0 + z_1 + z_f > 0$. Plug (13) and (14) into (10) and (11), and the steady state can be characterized by a pair of (w, τ) that solves the wage-bargaining equation and the firm free-entry equation:

$$\frac{w + \upsilon(q_1) - cq_1 - iz_1 - \upsilon(q_i) + (1+i)\, cq_i}{f(k) - (r+\delta)\, k - w - iz_f + \kappa} \frac{r + s + \mathcal{M}\left(\frac{1}{\tau}, 1\right)}{r + s + \mathcal{M}(1, \tau)} = \frac{\rho}{1 - \rho} \tag{15}$$

and

$$\mathcal{M}(1/\tau, 1)\left[f(k) - (r+\delta)\, k - w - iz_f\right] = \kappa(r + s), \tag{16}$$

where k, q_1, z_1, and z_f are functions of w, as described in (1) and (6). We first show that there exists an equilibrium with active firm entry if the entry cost is not too big. The proofs of all lemmas and propositions are in Appendix C.

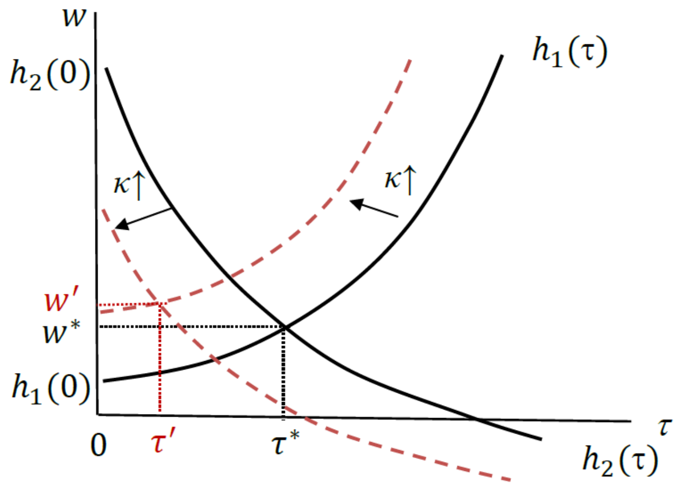


Figure 2: Equilibrium

Notes: The figure characterizes the equilibrium (τ, w), where $h_1(\tau)$ is defined by the wage-bargaining equation (15) and $h_2(\tau)$ by the firm's free-entry condition (16).

Lemma 1 *Equation (15) can be written as* $w = h_1(\tau)$, *where* $h_1' > 0$ *and* $\underline{w} \equiv h_1(0) > 0$. *In the* (τ, w) *space,* h_1 *shifts to the left if* κ *increases.*

The wage-bargaining equation is upward sloping in the (τ, w) space. The LHS of (15) represents the ratio of the tenure-long matching surplus. When w increases, the employed worker receives higher pay, has more liquidity for DM purchases, and saves on the cost of holding cash. Consequently, the worker's surplus strictly increases in w. As τ increases, the worker discounts the value of tenure more because it becomes easier to find a new job. Hence, the worker's surplus strictly decreases in τ. The reverse holds for the firm. Therefore, w must increase with τ to maintain a constant surplus ratio. Intuitively, if there are more firms in the market, workers are in a better position in that they are more likely to find new jobs, so wages must increase to attract workers. Holding τ fixed, if κ rises, the incumbent firm's surplus increases, which requires a higher w to rebalance the surplus share. As a result, h_1 shifts to the left.

Lemma 2 *Equation (16) can be written as* $w = h_2(\tau)$, *where* $h_2' < 0$. *Suppose* $f(k^*) - (r+\delta)k^* > (r+s)\kappa$. *Then* $\bar{w} \equiv h_2 > 0$ *and* $\bar{\tau} \equiv h_2^{-1}(0) \in (0, \infty)$. *In* (τ, w) *space,* h_2 *shifts to the left if* κ *increases.*

Equation (16) is downward sloping in the (τ, w) space. This follows from the firm's free-entry condition. If w increases, an active firm's profit decreases. Then τ must fall to increase the firm's matching probability, thereby maintaining a constant expected profit. Intuitively, higher wages reduce the firm's profitability, so fewer firms post vacancies and the labor market becomes less tight. When κ increases, an entering firm must expect higher profits. Holding τ constant, w must decrease to satisfy this condition. Therefore, h_2 shifts to the left.

Figure 2 shows h_1 and h_2 and how they shift as κ increases. By Lemmas 1 and 2, the following proposition establishes the existence of a unique monetary stationary equilibrium.

Proposition 1 *There exists a positive $\hat{\kappa}$ such that if $\kappa < \hat{\kappa}$, there exists a unique stationary monetary equilibrium with active production.*

The existence result is common in the literature. In Berentsen et al. (2011), multiple steady states can arise due to the strategic complementarity between the worker's choice of money holdings and the firm's job-opening decisions. However, in our baseline model, the DM is Walrasian, meaning that firms earn zero profit in the DM regardless of the workers' money holdings, and a worker's probability of DM trade is always 1 regardless of firm entry. Because strategic complementarity is absent, the steady state is unique.

3 Model Implications

In this section, we analyze the model's implications for how economic variables respond to firm credit conditions, captured by capital plegdeability χ, and monetary policy, captured by inflation or nominal interest rate i. We show the analytical results in the main text. Numerical examples are provided in Appendix D.1.

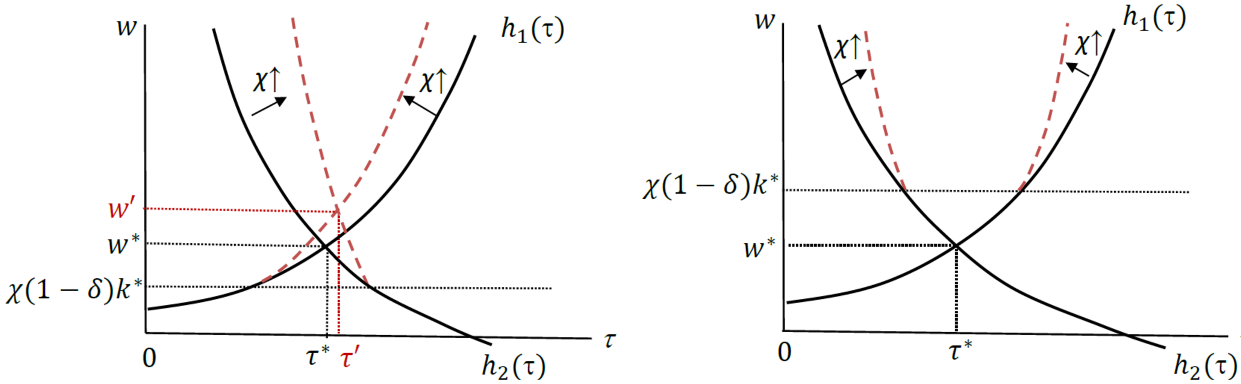
3.1 Effects of Capital Pledgeability

Higher pledgeability may result from advancements in technology that enhance asset verification or from stronger enforcement of debt repayment. An increase in pledgeability alleviates liquidity frictions for firms and leads to an improvement in resource allocation. Proposition 2 summarizes the effects of a higher χ.

Proposition 2 *An increase in χ raises w and τ and lowers u in Regimes 1 and 2. It does not change the equilibrium in Regime 3.*

Given w, a change in χ does not affect the worker's surplus. However, a higher χ raises the firm's surplus in Regimes 1 and 2. In Regime 1, higher pledgeability allows firms to save on cash. Regime 2 allows firms to shed over-accumulated capital to lower costs. As a result, the total surplus increases, and w rises to rebalance the surplus shares. With higher profits, firms are more willing to enter the labor market. Therefore, τ increases and u declines. In Regime 3, where the firm's liquidity constraint does not bind, an increase in pledgeability does not affect the firm's surplus, and the equilibrium remains unchanged.

Figure 3 depicts the effects of an increase in χ on w and τ. Given τ, the LHS of (15) decreases with χ in Regimes 1 and 2, which requires a higher w to restore the equation. The LHS of (16) increases with χ, which also requires a higher w to restore the equation. Hence, h_1 rotates counterclockwise around $\chi\,(1-\delta)\,k^*$, while h_2 rotates clockwise. Panel (a) illustrates the steady state in Regime 1 or 2, where the intersection is above $\chi\,(1-\delta)\,k^*$. Here, an increase in χ results in a new intersection with higher w and τ. Panel (b) shows the steady state in Regime 3, where the intersection of h_1 and h_2 is below $\chi\,(1-\delta)\,k^*$. In this case, changes in χ do not affect the intersection point, leaving w unchanged.



(a) $w > \chi\,(1-\delta)\,k^*$, Regimes 1 and 2

(b) $w < \chi\,(1-\delta)\,k^*$, Regime 3

Figure 3: The Effects of an Increase in χ

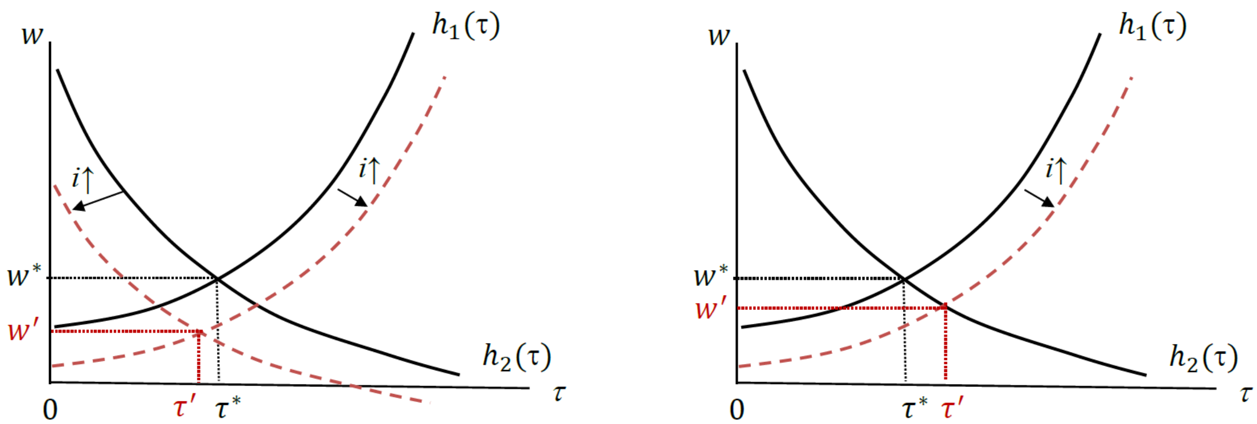
It should be noted that unique cutoff χ's do not necessarily exist between regimes. As χ increases, k_i rises, as do $\chi(1-\delta)k_i$ and $\chi(1-\delta)k^*$. By Proposition 2, w also increases. It is not clear whether w grows faster than $\chi(1-\delta)k_i$ and $\chi(1-\delta)k^*$. However, in all numerical examples we tried, the economy moves from lower to higher regimes as χ increases.

Figure D.1 in Appendix D.1 presents a numerical example of the effects of χ. As χ increases from 0 to 1, the economy transitions from Regime 1 to Regime 2 and then to Regime 3, with regime switches occurring at the kinks. Wages, market tightness, and unemployment rate evolve, as described in Proposition 2. Capital increases in Regime 1 as k_i rises with χ, then decreases in Regime 2, as firms can use less capital to meet their wage payment obligations, and stays constant at k^* in Regime 3. In this example, since more firms are producing in Regimes 1 and 2, DM output increases with χ. LM output follows the trajectory of k as output per firm is an increasing function of k. The firm's cash holdings decline because less cash is needed when capital can be pledged for more credit.

3.2 Effects of Inflation

Next, we turn to the effects of i. Consider an increase in i. Given w and τ, an increase in i reduces the unemployed worker's trade surplus in all regimes and decreases the firm's surplus in Regime 1 due to higher liquidity costs. Consequently, the LHS of (15) increases in all regimes, while the LHS of (16) decreases only in Regime 1. Therefore, h_1 shifts downward, as illustrated in Figure 4, and h_2 shifts downward only in Regime 1. In all cases, w falls.[12] Unemployment decreases in Regimes 2 and 3 but is ambiguous in Regime 1. Therefore,

Proposition 3 *As i increases, w decreases in all regimes, and u decreases in Regimes 2 and 3.*



(a) $w > \chi(1-\delta)k_i$, Regime 1

(b) $w < \chi(1-\delta)k_i$, Regimes 2 and 3

Figure 4: The Effects of an Increase in i

The intuition behind Proposition 3 is as follows. Given w, being unemployed becomes more painful with a higher i because unemployed workers must acquire cash in the CM and bear higher inflation costs in order to consume in the DM. The LM surplus of an employed worker increases as the threat point of being unemployed is lowered in wage bargaining. On the other hand, given w, a producing firm is worse off in Regime 1 with a higher i because it must pay a higher inflation tax by acquiring cash to pay the workers. Taken together, w must fall to maintain a constant share of the total surplus.

[12]Both channels suggest that inflation reduces real wages, which is consistent with the empirical result in Cardoso (1992), Braumann (2004), Sanchez (2015), and Sanchez and Wilkinson (2022), showing that higher inflation is associated with lower (or even negative) real wage growth.

Following the above explanation, inflation affects unemployment through two opposing channels. The first is the wage-bargaining channel. Inflation hurts the unemployed more as they must finance all their DM purchases with cash. So workers are willing to accept a lower wage, which increases the firm's profit and vacancy posting. This results in lower unemployment. Therefore, the wage-bargaining channel implies a downward-sloping Phillips curve. This channel is active in all regimes.

The second channel is the cash-financing channel. If a firm uses cash to pay wages, then higher inflation increases a firm's financing costs, reducing its profitability and incentive to post vacancies. The cash-financing channel implies that inflation leads to higher unemployment and an upward-sloping Phillips curve. This channel is only active in Regime 1 where firms use cash.

In Regime 1, due to the coexistence of both channels, the effect of inflation on unemployment is ambiguous, depending on which channel dominates. When the economy is in Regime 2 or 3, only the wage-bargaining channel is active, so inflation reduces unemployment. Again, as firms do not make profits in the DM in our model, worker's cash holding does not affect a firm's job-posting decision. Hence, the channel in Berentsen et al. (2011) is muted. Inflation impacts the economy through two channels that differ from those in Berentsen et al. (2011) and other studies focusing on consumers' liquidity choices.

The aggregate output depends on the measure of producing firms and the output per producing firm. In Regime 3, output per producing firm remains constant at $f(k^*)$. However, as i increases, the wage-bargaining channel induces higher employment and a higher measure of producing firms. As a result, the aggregate output increases. In Regimes 1 and 2, the effect of inflation on output is less clear. In Regime 1, firms accumulate more capital as i increases, so output per producing firm increases. The measure of producing firms can change in either direction. In Regime 2, output per producing firm decreases as k decreases, but more firms are producing.

Because w decreases with i and k_i increases in i, the economy transitions between regimes as i changes. It is straightforward to show that

Corollary 1 *As i increases, the economy moves from lower regimes to higher regimes.*

It is difficult to analytically establish how the Phillips curve slope changes with χ when the economy is in Regime 1. Therefore, we resort to numerical exercises. Figures D.2–D.5 in Appendix D.1 illustrate how i affects steady-state allocation. The slope of the Phillips curve is sensitive to credit conditions. Unemployment increases with i when χ is low, becomes

U-shaped as χ increases, and decreases with i when χ is sufficiently high.

4 Extensions

In this section, we explore two extensions. First, we add DM search and bargaining, as well as unemployment benefit and leisure; these features will be incorporated into our calibration in Section 5 to better match the data. Second, we relax the assumption that firms must pay the full wage in the LM and allow firms and workers to negotiate the timing of wage payments.

4.1 Unemployment Benefits, Leisure, and DM Search

We modify the baseline model as follows. Unemployed workers receive unemployment benefits, denoted by b. Similar to wage income w, b is paid in the LM and can be used for DM purchases. Unemployed workers enjoy leisure, denoted by ℓ, in the LM. Assume utility is linear in ℓ. Let $\mathcal{N}(B,S)$ be the matching function in the DM, where B and S denote the measures of buyers and sellers, respectively. The matching function is strictly increasing, strictly concave, and homogeneous of degree 1. As all workers participate in the DM and only producing firms can sell, the measure of buyers is 1 and the measure of sellers is $1-u$. A buyer meets a seller with probability $\sigma_h = \mathcal{N}(1, 1-u)$ and a seller meets a buyer with probability $\sigma_f = \mathcal{N}(1/1-u, 1)$. The DM terms of trade are determined by Kalai bargaining. The buyer's payment for the DM good is $g(q) = \psi cq + (1-\psi)\upsilon(q)$, where $\psi \in (0,1)$ is the buyer's bargaining power.

The worker and the firm's CM and LM problems stay the same as in the baseline model. The DM problem of workers with wage income is

$$V_j(z_1, w) = \max_{q_1} \{\sigma_h[\upsilon(q_1) - cq_1] + W_j(z_1, w)\}$$
$$\text{st } g(q_1) \le z_1 + w.$$

Combined with the worker's CM and LM value functions, the worker's choices of q_1 and z_1 are as follows: If $w < g(q_i)$, $q_1 = q_i$ and $z_1 = g(q_i) - w$, where q_i solves $\upsilon'(q_i)/g'(q_i) = 1 + i/\sigma_h$. If $g(q_i) \le w < g(q^*)$, $q_1 = g^{-1}(w)$ and $z_1 = 0$. If $w \ge g(q^*)$, then $q_1 = q^*$ and $z_1 = 0$.

Similarly, workers without wage income solve

$$V_j(z_0, b) = \max_{q_0} \{\sigma_h[\upsilon(q_0) - cq_0] + W_j(z_0, b)\}$$
$$\text{st } g(q_0) \leq z_0 + b.$$

Their choices of q_0 and z_0 are: If $b < g(q_i)$, $q_0 = q_i$ and $z_0 = g(q_i) - b$. If $g(q_i) \leq b < g(q^*)$, $q_0 = g^{-1}(b)$ and $z_0 = 0$. If $b \geq g(q^*)$, $q_0 = q^*$ and $z_0 = 0$.

An employed worker's surplus in the LM is

$$S_h = \frac{w - b - \ell + \sigma_h[\upsilon(q_1) - g(q_1)] - iz_1 - \sigma_h[\upsilon(q_i) - g(q_i)] + iz_0}{r + s + \lambda_h}. \quad (17)$$

The firm's DM value is

$$\tilde{V}_j(z_f, k, y) = A + \tilde{W}_j(z_f, k, y),$$

where

$$A \equiv \sigma_f\{(1-u)[g(q_1) - cq_1)] + u[g(q_0) - cq_0)]\}$$

is the firm's expected profit in the DM. Notice that the firm's profit depends on two factors, as in Berentsen et al. (2011). The first is the worker's real balances, z_0 and z_1. Lower inflation reduces the worker's cost of acquiring cash, which increases real balances and leads to higher profit for the firm. Second, the firm's matching probability with each type of worker. If a firm is more likely to match with an employed worker, who holds more liquidity, then its expected profit will be higher.

As in the baseline model, there are three regimes regarding the firm's choice of cash and capital, and conditions for the regimes remain the same. A matched firm's surplus in the LM is

$$S_f = \frac{f(k) - (r+\delta)k - w + A + \kappa - iz_f}{r + s + \lambda_f}.$$

The equilibrium w solves $S_h/S_f = \rho/(1-\rho)$. The free-entry condition is

$$\lambda_f[f(k) - (r+\delta)k - w + A - iz_f] = \kappa(r+s)$$

and λ_h and u are determined by (12)–(14), as in the baseline model. It can be shown that if κ is sufficiently small, firms will enter.

As in Berentsen et al. (2011), there is strategic complementarity between firms and workers: When more firms post vacancies, it becomes easier for workers to find jobs. This leads to more firms actively producing, which increases the likelihood of workers finding sellers in the DM. As a result, workers acquire more cash for DM transactions, increasing the DM trade surplus. This, in turn, encourages more firms to enter the market. Strategic complementarity may result in multiple monetary steady states. Appendix D.2 provides an example of three steady states. Following Berentsen et al. (2011), we focus on the one with the highest τ or the lowest unemployment rate if there are multiple steady states.[13]

DM search and bargaining is an important channel in Berentsen et al. (2011) that amplifies the positive effect of inflation on unemployment. However, our novel wage-bargaining channel is robust and the overall effect of inflation on unemployment can still be ambiguous with the presence of the Berentsen et al. (2011) channel. Numerical examples show that the Phillips curve can be downward sloping, U-shaped, or upward sloping, depending on capital pledgeability. In Section 5, we will do a quantitative analysis to determine the slope of the Phillips curve using calibrated parameters.

4.2 Endogenous Timing of Wage Payments

The baseline model assumes that firms must pay wages immediately after LM production, making wage income a source of liquidity for workers in the DM. This creates different liquidity costs for the employed and unemployed, which is central to the wage-bargaining channel. In this section, we extend the baseline model by allowing the timing of wage payments to be an endogenous choice. While firms commit to wage payments, their promises cannot serve as a means of payment in the DM. Due to banking regulations, firms can only obtain secured loans. Matched firms and workers negotiate the wage in two parts: w_1, paid immediately after LM production, and w_2, paid in the CM after all output is sold. The firm prefers to delay payment until the CM to save on cash-financing costs, while the worker prefers to receive wages in the LM to reduce cash holdings.

The worker's CM problem remains unchanged except that they may receive wage income w_2 in the CM. The LM value function of an employed worker is

$$U_1(z_1) = (1-s) V_1(z_1, w_1, w_2) + sV_0(z_1, w_1, w_2),$$

[13] Kehoe (1985) points out the difficulty in conducting comparative statics in models with multiple equilibria. For discussions on how to use government policies to select the desired equilibrium, see Schreft and Smith (1998), Ennis and Keister (2005), and Antinolfi et al. (2007).

and the LM value function of an unemployed worker is

$$U_0(z_0) = \lambda_h V_1(z_0, 0, 0) + (1 - \lambda_h) V_0(z_0, 0, 0).$$

Workers can use cash and wage income w_1 to pay for q. The DM problem of workers with wage income is

$$V_j(z_1, w_1, w_2) = \max_{q_1} [v(q_1) - cq_1 + W_j(z_1, w_1 + w_2)]$$
$$\text{st } cq_1 \leq z_1 + w_1.$$

The first-order condition with respect to q_1 results in $q_1 = q^*$ if $z_1 + w_1 \geq cq^*$ and $q_1 = (z_1 + w_1)/c$ otherwise.

Consolidating the value functions, we get the following solution to the employed worker's problem:

$$\begin{aligned}
&q_1 = q_i && \text{and } z_1 = cq_i - w_1, && \text{if } w_1 < cq_i, \\
&q_1 = w_1/c && \text{and } z_1 = 0, && \text{if } cq_i \leq w_1 < cq^*, \\
&q_1 = q^* && \text{and } z_1 = 0, && \text{if } w_1 \geq cq^*.
\end{aligned}$$

The unemployed worker's choices of q_0 and z_0 are given by (2), as in the baseline model. Using these, we get the surplus of a worker employed at wages w_1 and w_2 as

$$S_h = \frac{w_1 + w_2 + v(q_1) - cq_1 - iz_1 - [v(q_i) - (1+i)cq_i]}{r + s + \lambda_h}.$$

The firm's CM value now includes its payment of w_2:

$$\tilde{W}_j(z_f, k, l, y - w_2) = y - w_2 + z_f + (1 - \delta)k - l + \max_{\hat{z}_f, \hat{k}} \left[-(1+\pi)\hat{z}_f - \hat{k} + \beta \tilde{U}_j\left(\hat{z}_f, \hat{k}\right)\right],$$

where we apply the result that $i_l = 0$ for the intra-period loan.

The expected value of producing firms in the LM is

$$\tilde{U}_1(z_f, k) = (1 - s)\tilde{V}_1(z_f - z', k, l, y - w_2) + s\tilde{V}_0(z_f - z', k, l, y - w_2),$$

where $w_1 = z' + l \leq z_f + \chi(1 - \delta)k$ and $y = f(k)$. Again, since firms do not make profits in the DM, their DM value is

$$\tilde{V}_j(z_f, k, l, y - w_2) = \tilde{W}_j(z_f, k, l, y - w_2).$$

The solution to the producing firm's problem is in one of the three regimes:

$$\begin{array}{lll}\text{Regime 1:} & k = k_i \text{ and } z_f = w_1 - \chi(1-\delta)k_i, & \text{if } \chi(1-\delta)k_i \le w_1, \\ \text{Regime 2:} & k = w_1/\chi(1-\delta) \text{ and } z_f = 0, & \text{if } \chi(1-\delta)k^* \le w_1 < \chi(1-\delta)k_i, \\ \text{Regime 3:} & k = k^* \text{ and } z_f = 0, & \text{if } \chi(1-\delta)k^* > w_1.\end{array}$$

A matched firm's surplus is

$$S_f = \frac{f(k) - (r+\delta)k - w_1 - w_2 + \kappa - iz_f}{r + s + \lambda_f}.$$

Wages are negotiated according to Kalai bargaining. That is,

$$\max_{w_1, w_2} S_h + S_f, \tag{18}$$

$$\text{st } (1-\rho)S_h - \rho S_f = 0, \tag{19}$$

$$w_2 \ge 0, \tag{20}$$

where the non-negativity constraint (20) means that workers are not allowed to borrow from firms and pay back in the CM. The objective function is concave, and the constraint set is convex. If (20) is slack, the FOC is

$$[f'(k) - (r+\delta)]\frac{\partial k}{\partial w_1} - i\frac{\partial z_f}{\partial w_1} + [\upsilon'(q_1) - c]\frac{\partial q_1}{\partial w_1} - i\frac{\partial z_1}{\partial w_1} = 0. \tag{21}$$

The first two terms capture the marginal cost for the firm of increasing w_1, and the last two terms capture the marginal gain of increasing w_1 for the worker. Under Kalai bargaining, the firm's marginal cost and the worker's marginal gain sum to zero. If w_2, derived from (19) and (21), is negative, then the solution lies at the corner where $w_2 = 0$ and w_1 solves (19), which is the same as in the baseline model.

There are three cases for an interior solution of w_1 and w_2, depending on the ordering of cq_i, cq^*, $\chi(1-\delta)k_i$, and $\chi(1-\delta)k^*$.

1. $\chi(1-\delta)k^* < \chi(1-\delta)k_i < cq_i < cq^*$. The objective function is strictly concave for

$w_1 < \chi(1-\delta) k_i$ and $w_1 > cq_i$, and flat for $w_1 \in [\chi(1-\delta) k_i, cq_i]$. Any $w_1 \in [\chi(1-\delta) k_i, cq_i]$ and $w_2 > 0$ satisfying

$$(1+i) w_1 + w_2 = \frac{D_1}{D_1 + D_2} \left[f(k_i) - (r+\delta) k_i + \kappa + i\chi(1-\delta) k_i \right], \tag{22}$$

where $D_1 = \rho(r + s + \lambda_h)$ and $D_2 = (1-\rho)(r + s + \lambda_f)$, is the solution. In this case, $q_1 = q_i$, $k = k_i$, $z_1 = cq_i - w_1$, and $z_f = w_1 - \chi(1-\delta) k_i$. The marginal cost of holding cash is constant at i for both parties. Because both parties hold cash, when to pay becomes irrelevant as the worker-firm pair can always transfer payments between themselves at zero net cost. As w_2 decreases in w_1, w_2 reaches its highest value at $w_1 = \chi(1-\delta) k_i$. At $w_1 = \chi(1-\delta) k_i$, if (22) yields $w_2 < 0$, then an interior solution does not exist. In that case, the solution is $w_2 = 0$ and w_1 solves (19).

2. $\max\{\chi(1-\delta) k^*, cq_i\} < \min\{\chi(1-\delta) k_i, cq^*\}$. In this case, the objective function is strictly concave. The first-order condition solves w_1 as follows:

$$\left[f'\left(\frac{w_1}{\chi(1-\delta)} \right) - (r+\delta) \right] \frac{1}{\chi(1-\delta)} + \left[\upsilon'\left(\frac{w_1}{c} \right) - c \right] \frac{1}{c} = 0, \tag{23}$$

and

$$\begin{aligned} w_2 \;\; = \;\; & \frac{D_1}{D_1 + D_2} \left[f\left(\frac{w_1}{\chi(1-\delta)} \right) - \frac{(r+\delta) w_1}{\chi(1-\delta)} - w_1 + \kappa \right] \\ & - \frac{D_2}{D_1 + D_2} \left[\upsilon\left(\frac{w_1}{c} \right) - \upsilon(q_i) + (1+i) cq_i \right] \end{aligned} \tag{24}$$

needs to be positive. Here, $q_1 = w_1/c$, $k = w_1/[\chi(1-\delta)]$, and $z_1 = z_f = 0$. Neither party holds cash, yet both are liquidity-constrained. The selection of w_1 equates the firm's marginal cost of obtaining extra capital with the worker's marginal gain from receiving extra wage payment in the LM. If (24) is negative, then (20) binds, and the solution is $w_2 = 0$ with w_1 determined by solving (19).

3. $cq_i < cq^* \leq \chi(1-\delta) k^* < \chi(1-\delta) k_i$. The objective function is strictly concave for $w_1 < cq^*$ and $w_1 > \chi(1-\delta) k^*$ and is flat for $w_1 \in [cq^*, \chi(1-\delta) k^*]$. Any $w_1 \in [cq^*, \chi(1-\delta) k^*]$ and $w_2 > 0$ that satisfy

$$w_1 + w_2 = \frac{D_1 \left[f(k^*) - (r+\delta) k^* + \kappa \right] - D_2 \left[\upsilon(q^*) - cq^* - \upsilon(q_i) + (1+i) cq_i \right]}{D_1 + D_2} \tag{25}$$

is the interior solution. In this case, $q_1 = q^*$, $k = k^*$, and $z_1 = z_f = 0$. Intuitively, as long as the wage paid in the LM achieves the first-best for both parties without incurring any

Table 1: Comparative Statics

	$\frac{dw_1}{d\chi}$	$\frac{dw_2}{d\chi}$	$\frac{dw}{d\chi}$	$\frac{d\tau}{d\chi}$	$\frac{dw_1}{di}$	$\frac{dw_2}{di}$	$\frac{dw}{di}$	$\frac{d\tau}{di}$
Regime 1 interior	$+^*$	$-^*$	$-^*$	$+$	$+^*$	$-^*$	$-^*$	$+$
Regime 2 interior	$+$	$\pm$	$\pm$	$+$	0	$-$	$-$	$+$
Regime 2 corner	$+$	0	$+$	$+$	$-$	0	$-$	$+$
Regime 3 interior	0	0	0	0	0^*	$-^*$	$-$	$+$
Regime 3 corner	0	0	0	0	$-$	0	$-$	$+$

liquidity cost to either party, only the amount needed to cover DM spending must be paid early. The firm has plenty of liquidity, so it can pay more than cq^* in the LM as long as (20) holds. At $w_1 = cq^*$, if (25) yields $w_2 < 0$, then the solution is $w_2 = 0$ and w_1 is determined by solving (19).

In all cases, the corner solution with $w_2 = 0$ occurs at $w_1 < \max\{\chi(1-\delta)k_i, cq^*\}$. There are two implications. First, as $w_1 < \chi(1-\delta)k_i$, the corner solution does not occur in Regime 1. Second, as $w_1 < cq^*$, workers are liquidity constrained in the DM under the corner solution.

The model is closed by the free-entry condition $\tilde{W}_0(0,0,0,0) = 0$ or

$$\lambda_f [f(k) - (r+\delta)k - w_1 - w_2 - iz_f] = \kappa(r+s) \tag{26}$$

and (12)–(14). Propositions 1–3 and Corollary 1 have their modified counterparties. The proofs are in the Appendix.

Proposition 4 *With endogenous timing of wage payments, there exists a $\hat{\kappa} > 0$ such that if $\kappa < \hat{\kappa}$, there exists a unique monetary stationary equilibrium with active production.*

We do comparative statics in each regime for interior and corner solutions. Let $w = w_1 + w_2$ be the total wage payments. For multiple interior solutions in Regime 1 or 3, we pick the lowest w_1. That is, $w_1 = \chi(1-\delta)k_i$ is chosen in Regime 1 and $w_1 = cq^*$ in Regime 3. The results are in Table 1.

The asterisk indicates that the result depends on the choice of the solution, since there are multiple solutions in the regime. An important result is that how τ varies with χ or i does not depend on the choice of the solution. In the entire range of i, τ increases with i, implying that u decreases with i, resulting in an unambiguously downward-sloping Phillips curve. As in the baseline model, only the wage-bargaining channel operates in Regimes 2 and 3. In Regime 1, the worker and the firm share the inflation cost by endogizing the timing of wage payments. Compared to the baseline model, the firm's financing cost is lower, which weakens the cash-financing channel and allows the wage-bargaining channel to dominate. Summarizing the comparative statics with respect to χ and i, we have the following propositions.

Proposition 5 *With endogenous timing of wage payments, an increase in χ does not affect the equilibrium in Regime 3. It raises τ and lowers u in Regimes 1 and 2. Pick $w_1 = \chi(1-\delta)k_i$ if the economy is in Regime 1. An increase in χ raises w_1 in Regimes 1 and 2.*

Proposition 6 *With endogenous timing of wage payments, an increase in i raises τ and lowers u. Pick $w_1 = \chi(1-\delta)k_i$ if the economy is in Regime 1. As i increases, w decreases.*

Corollary 2 *With endogenous timing of wage payments, as i increases, the economy moves from the lower regimes to the higher regimes.*

We derive the results with DM search frictions and endogenous timing of wage payments in Appendix A. Search frictions amplify the positive effect of inflation on unemployment, making the wage-bargaining channel less dominant. Therefore, the slope of the Phillips curve becomes ambiguous again. Appendix D.3 provides a series of numerical examples. The Phillips curve can be upward-sloping, U-shaped, or downward-sloping, depending on χ.

5 Quantitative Analysis

We calibrate our extended model in Section 4.1 to quantify the effects of firm credit conditions and inflation on labor market outcomes, as well as the impact of credit conditions on the Phillips curve. Our main focus is on the long run, but we also explore some short-run analysis.

5.1 Calibration

We use $f(k) = A_f k^\theta$ for the production function in the LM. As in Berentsen et al. (2011), the matching function in this market has the Cobb-Douglas form,

$$\mathcal{M}(u,v) = u^\iota v^{1-\iota}.$$

It follows that the job-finding rate and the vacancy-filling rate are, respectively,

$$\lambda_h = \min\{(v/u)^{1-\iota}, 1\} \text{ and } \lambda_f = \min\{(v/u)^{-\iota}, 1\}.$$

The worker's DM utility function is $\upsilon(q) = A_\upsilon q^\alpha$, and the production function is $q = y$, where y is the amount of the LM good transformed into the DM good. Following Kiyotaki and Wright (1993) and Berentsen et al. (2011), the DM matching function is

$$\mathcal{N}(B, S) = \frac{BS}{B+S}.$$

We calibrate our model to the U.S. economy from 2000 to 2019. Each period in our model corresponds to a quarter. Refer to Appendix F for the data used for calibration.

We use the average AAA corporate bond yield of 5% to proxy the annual nominal interest rate, which implies a quarterly nominal interest of 1.23%. The average annual inflation rate is 2.01%. The implied annual discount factor is 0.9714, and the quarterly discount factor is 0.9928.

Following Berentsen et al. (2011), we set $\iota = 0.72$ to match the elasticity of the job-finding rate with respect to labor market tightness. Using the method in Shimer (2005), we calculate the monthly job-finding rate to be 0.3092, which implies a quarterly job-finding rate of 0.6703. The separation rate, s, is set to match the average unemployment rate from 2000 to 2019 of 5.87%, resulting in $s = 4.18\%$ per quarter.[14] The unemployment benefit is set to $b = 0.5w$ to target a replacement ratio of 0.5. The value of leisure is set to $\ell = 0.45w$ so that $(b + \ell)/w = 0.95$, as in Hagedorn and Manovskii (2008). The worker's DM bargaining power, $\rho = 0.06$, is calibrated to match the share of labor compensation in output from FRED, which averaged 0.6048 from 2000 to 2019.

We normalize A_f to 1 in the production function and set $\theta = 0.3905$ to match the average capital-to-output ratio in the same time period.[15] Following Aruoba et al. (2011) and Aruoba (2011), we choose $\delta = 0.018$ (corresponding to 0.07 annually) to match the investment-to-capital ratio. The capital pledgeability parameter χ is set to match firm money demand, defined as $(1-u)z_f/Y$ in the model, where Y is the annualized total output.[16] To measure firm money demand in the data, we use the sum of the quarterly series of checkable deposits and currency for nonfinancial noncorporate businesses and nonfinancial corporate businesses to divide by the nominal GDP.[17] We set $\chi = 0.0309$ to target the average firm money demand

[14] We derive s from Equation (12), given the targets of the job-finding rate and unemployment.

[15] Two data series from FRED, Capital Stock at Constant National Prices for the United States and real output, are used to generate the capital-to-output ratio. Both series are measured in 2017 dollars and the average ratio is 3.5227 in the period 2000–2019.

[16] $Y = (1-\psi)\mathcal{N}(1, 1-u)\{(1-u)[\upsilon(q_1) - cq_1] + u[\upsilon(q_0) - cq_0]\} + (1-u)f(k)$.

[17] As is standard for quantitative analysis with new monetarist models, we use M1 or the sum of currency and checkable deposits as the measure of money demand.

of 0.0449 at the average annualized interest rate of 5%.

We interpret the DM as the retail market, as is standard in the literature. We set the worker's bargaining power, ψ, to target a retail markup of 39%, as in Stroebel and Vavra (2019), Bethune et al. (2020), and Wang et al. (2020), derived from data in the U.S. Census Bureau Annual Retail Trade Report. We set $\alpha = 0.5$ and choose A_v to match household demand for money, defined in the model as $[(1-u)z_1 + uz_0]/Y$. To measure household money demand in the data, we divide the quarterly series of household's checkable deposits and currency by nominal GDP. We find that $A_v = 3.85$ matches the average household money demand of 0.0373 at an average annualized interest rate of 5%.

The parameters β, i, ι, s, b, ℓ, δ, and κ can be taken directly from the data or set to match individual targets, while χ, ρ, θ, A_v, and ψ are jointly calibrated to match firm money demand, labor share, capital-to-output ratio, household money demand, and retail markup (see Appendix E for formulae of these targets). The values of these five parameters are chosen to minimize the squared percentage distance between the targets in the data and model with equal weights on each target.

Table 2 summarizes the calibrated parameter values and their corresponding targets. At the calibrated parameters, firms hold a positive amount of cash, so the firm cash-financing channel is operative. All workers hold a positive amount of cash, but the employed hold less. In all exercises in this section based on the calibrated values, the steady state is unique.

As shown in Table 3, our calibrated model hits the targets for labor share, firm money demand, household money demand, markup, and capital-to-output ratio very well (the hit of unemployment rate and job-finding rate is exact). The model also captures some non-targeted moments well. On the firm side, we calibrate χ to target firm money demand. The calibrated model implies the ratio of bank loans acquired by the firm to GDP is 0.11, which aligns with the data.[18]

On the worker side, given that the differential reliance on cash payments is the key to the wage-bargaining channel, it is useful to see whether the model's prediction is consistent with the data. We calculate three metrics: the share of cash transactions for the employed, the share of cash transactions for the unemployed, and their ratio. We derive these statistics from the Survey and Diary of Consumer Payment Choice (SDCP) from the Atlanta Fed.[19]

[18]The data series of business bank loans is the sum of the depository institution loans for nonfinancial corporate business (FRED series BLNECLBSNNCB) and for nonfinancial noncorporate business (FRED series NNBDILNECL).

[19]In the model, the share of cash transactions is $z_0/[\psi c q_0 + (1-\psi)v(q_0)]$ for unemployed and $(z_1 +$

Table 2: Parameters in the Calibrated Model

Parameter	Target	Value
Calibrated externally		
β discount factor (quarterly)	real interest rate	0.9928
i nominal interest rate (quarterly)	AAA corporate bond yield	0.0123
ι LM matching elasticity	Berentsen et al. (2011)	0.7200
δ depreciation rate (quarterly)	Aruoba et al. (2011)	0.0180
Calibrated internally to hit individual targets exactly		
s job separation (quarterly)	unemployment 5.87%	0.0418
b EI benefit	replacement ratio 0.5	1.9784
ℓ leisure	Hagedorn and Manovskii (2008)	1.7806
κ entry cost	quarterly job-finding rate 0.6703	4.8513
Jointly calibrated internally with grid search		
χ capital pledgeability	firm money demand	0.0309
ρ worker bargaining power	labor share	0.0600
θ LM production	capital-to-output ratio	0.3905
A_v DM utility	household money demand	3.8500
ψ buyer bargaining power	retail mark up	0.6400

The survey collects demographic characteristics, including employment status. In the Diary component, consumers report their daily transactions, including the payment instrument used and the dollar value of each transaction. From Table 3, we see that our model's prediction is closely aligned with the data. The cash share for the employed is 0.41 from the model and 0.48 from the data; the cash share for the unemployed is 0.58 from the model and 0.66 from the data. The cash-share ratio is very close between the model (0.73) and the data (0.71).

5.2 The Long Run

Figure 5 illustrates the impact of changing capital pledgeability as χ increases from 0 to 0.1, while the other parameters are kept at their calibrated values. As χ increases, the economy transitions from Regime 1 to 2 and then to 3. During the transition, firms create more vacancies in Regimes 1 and 2, and the unemployment rate falls. If χ increases from 0 to 0.05, the long-run unemployment declines from 6.83% to 5.54%. As firms save on wage-financing costs, productivity rises, and wages increase.

$z_f)/[\psi c q_1 + (1-\psi)v(q_1)]$ for employed. In the data, it is calculated as the share of currency and debit transactions, averaged across 2015–2019.

Table 3: Targets and Model Predicted Values

	Data	Model
Targeted moments		
labor share	0.6048	0.6066
firm money demand	0.0449	0.0449
hhd money demand	0.0373	0.0373
retail markup	1.3900	1.3889
capital-GDP ratio	3.5227	3.5116
Non-targeted moments		
firm bank loans-output ratio	0.1082	0.1067
share of cash purchases of employed	0.4798	0.4146
share of cash purchases of unemployed	0.6609	0.5841
cash-share ratio (employed/unemployed)	0.7259	0.7098

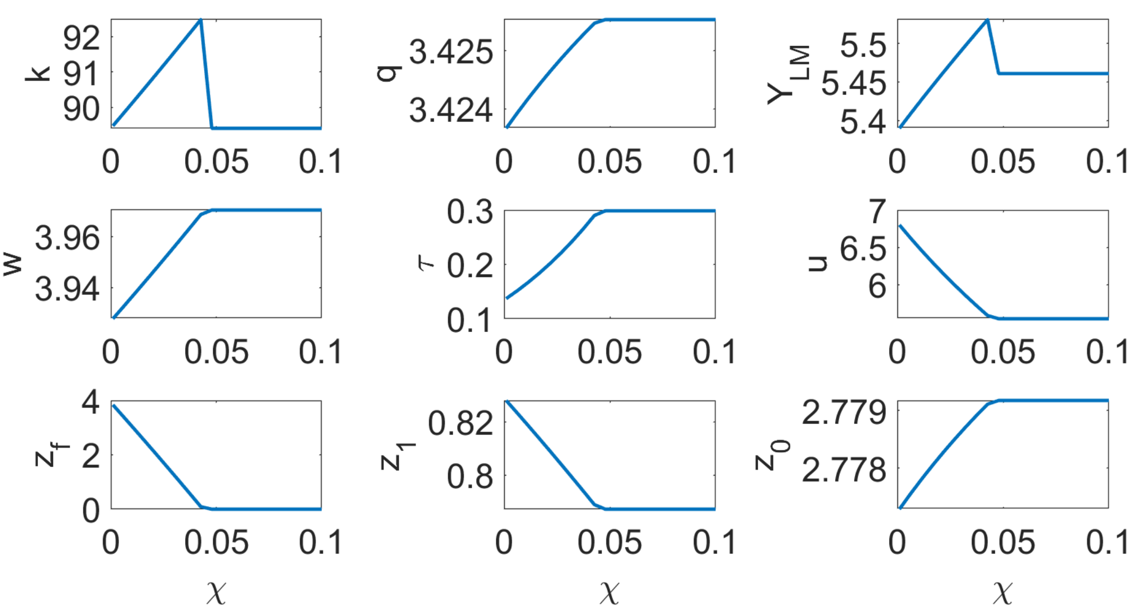


Figure 5: Calibrated Effects of χ

Notes: The figure shows the effects of χ on the capital of producing firms (k), total trading volume in the DM ($q = (1-u)q_1 + uq_i$), output in the LM ($Y_{LM} = (1-u)f(k)$), wage (w), labor market tightness (τ), unemployment (u), and the demand for real cash balances by firms (z_f), employed workers (z_1), and unemployed workers (z_0).

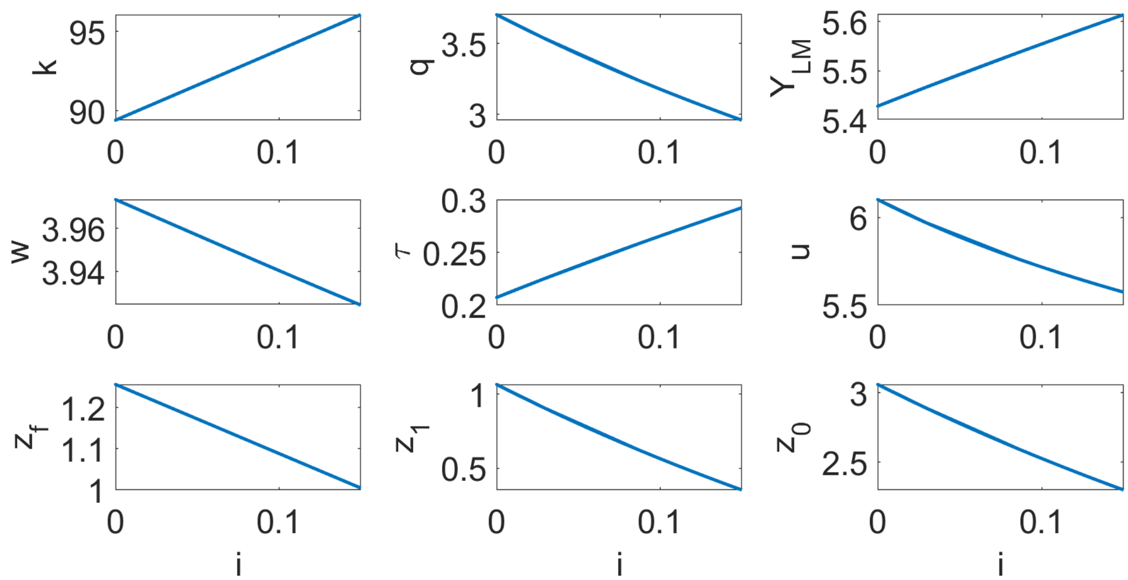


Figure 6: Calibrated Effects of i

Notes: The figure shows the effects of χ on the capital of producing firms (k), total trading volume in the DM ($q = (1-u)q_1 + uq_i$), output in the LM ($Y_{LM} = (1-u)f(k)$), wage (w), labor market tightness (τ), unemployment (u), and the demand for real cash balances by firms (z_f), employed workers (z_1), and unemployed workers (z_0).

Figure 6 shows the effects of inflation as the nominal interest rate increases from 0 to 0.15 while holding all other parameters at their calibrated values. The economy stays in Regime 1 throughout given the changes in i. As inflation rises, firms increasingly rely on capital to finance wage payments. Their cash holding decreases, capital investment increases, and LM output rises. Wages fall and workers carry less real cash balance. As liquidity becomes more expensive, DM trade declines. At the same time, more job openings lead to higher employment. The Phillips curve is downward sloping. As the nominal interest rate increases from 0 to 10%, the unemployment rate decreases from 6.10% to 5.72%, while the real wage decreases by 1%.

Figure 7 plots two Phillips curves with different χ values: the blue curve uses $\chi = 0.0204$, corresponding to the period 1960–1999, and the red curve uses $\chi = 0.0309$, corresponding to the period 2000–2019.[20] We can use Figure 7 to quantify how much χ and inflation, respectively, contribute to the change in unemployment and the change in the slope of the Phillips curve from A, where χ and inflation are at their 1960–1999 values, to C, where χ

[20] We impute the value of χ by assuming that the increase in the ratio of business debt to capital is attributed to the rise in χ. Data suggest that the average ratio of business debt to capital increased by 51.8% from 1960–1999 to 2000–2019. Thus, we impute $\chi_{1960-1999} = \chi_{2000-2019}/1.518$.

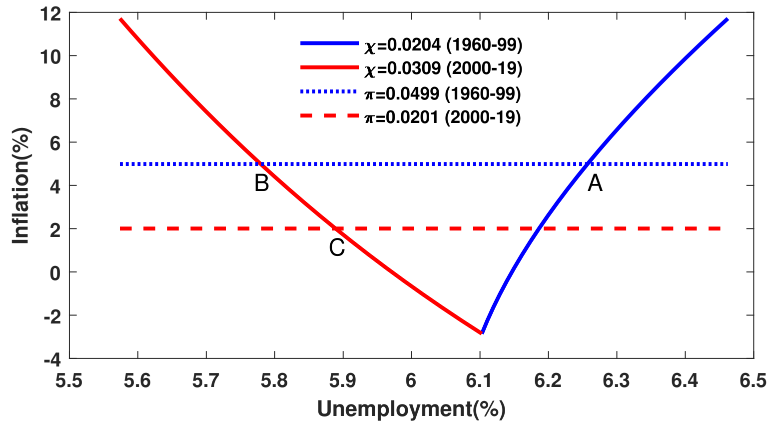


Figure 7: Effects of χ on the Long-Run Phillips Curve

Notes: The red (blue) solid curve is generated with χ for the period 2000–2019 (1960–1999). The red (blue) dotted line marks the average inflation rate for the period 2000–2019 (1960–1999).

and inflation are at their 2000–2019 values. The effect of the improvement in firm credit conditions, with χ increasing from 0.0204 to 0.0309, is captured by the movement from A on the blue curve to B on the red curve. The effect of a decrease in inflation from 4.99% to 2.01% is represented by the movement from B to C along the red curve.

We first check the effect on unemployment. From A to C, unemployment decreases by 0.37%, from 6.26% to 5.89%. The improvement in firm credit conditions reduces unemployment by 0.48% (from A to B), while the decrease in inflation brings unemployment back by 0.11% (from B to C).

Regarding the slope of the Phillips curve, the increase in χ causes the slope to switch from positive to negative (A to B), and the decrease in inflation flattens the negatively sloped Phillips curve slightly by 12% (from B to C). When $\chi = 0.0204$, firms rely more heavily on cash to finance their wage payments and the cash-financing channel is stronger, resulting in a positively sloped Phillips curve. As χ increases to 0.0309, the cash-financing channel becomes weaker and the wage-bargaining channel dominates. Consequently, the Phillips curve takes on a negative slope.

5.3 The Short Run

Following Aruoba (2011), Berentsen et al. (2011), and Cui et al. (2025), we analyze the model's business-cycle implications in this section. We introduce three types of shocks: a technology shock to A_f, a credit shock to χ, and a monetary shock to gross inflation $\Pi = 1 + \pi$. Four specifications are considered: (1) technology shock only, (2) technology shock plus credit shock, (3) technology shock plus monetary shock, and (4) all three shocks combined. The stochastic version of the model is presented in Appendix B.

The processes of the shocks are described by

$$\begin{aligned}
\ln A_{ft+1} &= \rho_A \ln A_{ft} + \varepsilon_{A,t}, \varepsilon_{A,t} \sim N(0, \sigma_A^2), \\
\ln \Pi_{t+1} - \ln \bar{\Pi} &= \rho_\Pi (\ln \Pi_t - \ln \bar{\Pi}) + \varepsilon_{\Pi,t}, \varepsilon_{\Pi,t} \sim N(0, \sigma_\Pi^2), \\
\ln \chi_{t+1} - \ln \bar{\chi} &= \rho_\chi (\ln \chi_t - \ln \bar{\chi}) + \varepsilon_{\chi,t}, \varepsilon_\chi \sim N(0, \sigma_\chi^2),
\end{aligned}$$

where ε_A, ε_Π, and ε_χ are i.i.d. and orthogonal. We set $\rho_A = 0.95$, as is standard in quarterly models. The values of ρ_Π and ρ_χ are chosen to match the autocorrelations of total money demand and the ratio of business debt to output, respectively. Total money demand is defined as the sum of firm and household money demand, and business debt is defined as collateralized bank loans. We set σ_A, σ_Π, and σ_χ to match the standard deviations of GDP, total money demand, and the ratio of business debt to GDP, respectively.

Table 4 shows cyclical statistics for GDP, unemployment, labor share, wage rate, inflation, the ratio of business debt to GDP, total money demand, household money demand, and firm money demand, calculated from the data and derived from the model. The first five columns show the standard deviations, and the last five columns show the correlations with GDP. All statistics are computed after taking logs and HP filtering.

Our model captures the monetary side of the economy well. Credit and monetary shocks help to explain the volatility of the ratio of business debt to GDP and total money demand, both of which are directly targeted by the shock processes. The model can also account for non-targeted moments of business and household components of money demand. The credit and monetary shock help to explain the countercyclicality of household money demand, as well as the procyclicality of firm money demand.

The model does a decent job of capturing the cyclical behaviors of labor compensation. The monetary shock is particularly effective in explaining the volatility and co-movement of the labor share. Our model suggests that the wage rate is less volatile than output, as in the data. Adding credit and monetary shocks weakens the strong pro-cyclicality of the wage rate implied by the model without these shocks. The wage-bargaining channel plays an important role: inflation reduces wage rates but increases firms' profitability and output. Credit and monetary shocks play a limited role in accounting for the high volatility of unemployment.

Before the pandemic, a widely held view was that the statistical short-run Phillips curve, derived from aggregate inflation time series, had become flatter since the 1990s. Explana-

Table 4: Business-Cycle Statistics: Data and Model

	Standard deviation					Correlation with output				
	Data	A_f	A_f, χ	A_f, Π	All 3	Data	A_f	A_f, χ	A_f, Π	All 3
GDP	1.05	1.05	1.05	1.05	1.05	1.00	1.00	1.00	1.00	1.00
unemployment	10.96	8.13	8.16	8.40	8.04	-0.90	-1.00	-1.00	-0.97	-0.97
labor share	0.67	0.02	0.02	0.44	0.47	-0.06	-0.94	-0.95	-0.03	-0.28
wage rate	0.83	0.46	0.46	0.57	0.50	-0.10	1.00	1.00	0.82	0.75
inflation	0.33	0.00	0.00	0.44	0.46	0.36	-	-	-0.02	0.24
firm debt/GDP	2.73	0.08	2.73	0.57	2.73	-0.35	1.00	-0.47	0.13	-0.70
total money demand	6.00	1.90	3.14	6.00	6.00	-0.34	-1.00	-0.04	-0.30	-0.09
hhd money demand	22.24	3.84	3.84	9.89	11.21	-0.60	-1.00	-1.00	-0.37	-0.55
business money demand	7.03	0.28	6.46	2.84	6.52	0.52	-1.00	0.46	-0.08	0.63

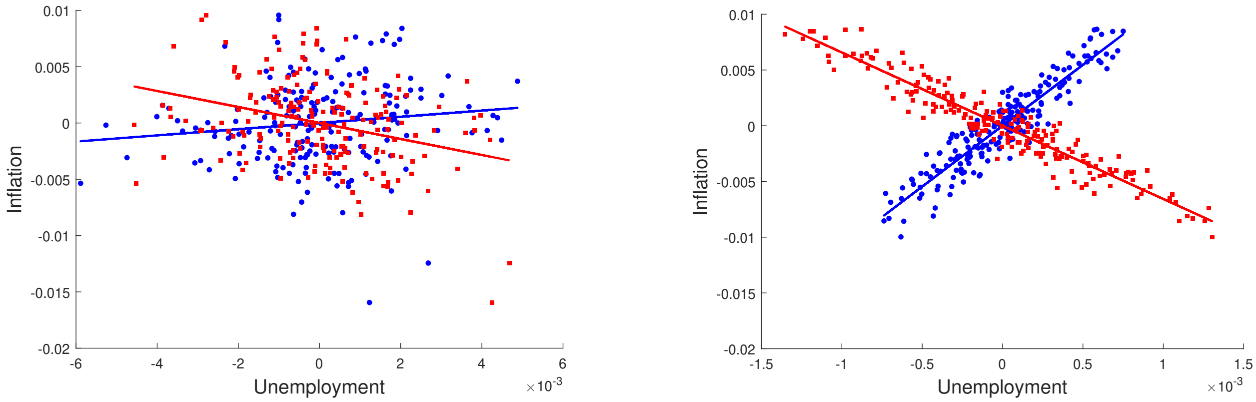
Notes: The standard deviations are relative to output except for output itself.

tions for this include inflation targeting, more firmly anchored inflation expectations, and structural changes driven by globalization (Kohlscheen and Moessner 2022).[21] The mechanisms that we identify in our paper might help explain the flattening of the short-run Phillips curve: starting with a negatively sloped Phillips curve, an improvement in credit conditions weakens the cash-financing channel and makes the wage-bargaining channel more dominant. Therefore, unemployment is more responsive to inflation.

Figure 8 shows the effects of χ on the slope of the short-run Phillips curve.[22] The left panel is derived from the model with all three shocks, while the right panel shuts down the technology shock to isolate our proposed mechanisms. By themselves, the mechanisms work as described earlier in the long-run analysis. The presence of the technology shock, however, dominates and obscures the effects of improved credit conditions.

[21]The view that the Phillips curve has become flatter has been scrutinized. Hooper et al. (2020), using state and MSA-level data, suggest that the flattening of the Phillips curve has been greatly exaggerated. Hazell et al. (2022), constructing state-level price indices for nontradable goods, estimate that the slope of the Phillips curve was small even during the 1980s. Braun (2024), adjusting the unemployment rate by accounting for job-seeking activities, finds no sign of a flattening Phillips curve in the post-2008 recession. Jorgensen and Lansing (2024) suggest that while the relationship between changes in inflation and unemployment (the accelerationist Phillips curve) has become flatter, the relationship between the level of inflation and unemployment (the original Phillips curve) has become steeper.

[22]We also explored how a change in χ affects the Beveridge curve and how a change in inflation affects the slope of the Beveridge curve and found almost no effect.



(a) All 3 shocks (b) No A_f shock

Figure 8: Effect of χ on the Short-Run Phillips Curve

Notes: The red dots and fitted line represent the simulated unemployment and inflation with $\chi = 0.0307$ (the value for the period 2000–2019), and the blue dots and fitted line represent the simulation with $\chi = 0.0201$ (the value for the period 1960–1999). The left panel is derived from the model with all three shocks, corresponding to the "All 3" column in Table 4. The right panel shuts down the technology shock.

6 Conclusions and Future Research

We present a search-theoretical model to study the channels through which inflation impacts the economy in the presence of search frictions and imperfect credit. In addition to the common understanding that inflation acts as a tax on holding money, thereby harming economies where money is essential, we identify a novel channel called the wage-bargaining channel, through which inflation can enhance labor market performance. Since being employed provides wage income and reduces reliance on cash holdings, workers are willing to accept lower wages to be hired, leading to increased job creation.

A general takeaway from our study is that changes in the features of credit, labor, and goods markets can induce structural changes in the Phillips curve. Our study highlights one such source: firm credit conditions. Our calibrated model suggests that improvements in firm credit conditions can account for the observed shift in the slope of the long-run Phillips curve from positive to negative around 2000.

We view our study as a solid starting point for future research aimed at better understanding the Phillips curve across different historical periods and countries. The two channels we propose can serve as a foundation for this line of research. Our work can be extended to incorporate other forms of firm credit, such as credit secured by future revenue, as in Holmström and Tirole (1998). One could also follow Wasmer and Weil (2004) to model credit frictions in the financing of job postings. Consumer credit, whether secured or unsecured, has ambiguous implications for the wage-bargaining channel. Other factors affecting liquidity demand, such as unemployment insurance and financial inclusion, are also interesting topics for further exploration.

Finally, the focus of this paper is on the long-run relationship between inflation and unemployment. Our model incorporates labor market search, goods market search, capital accumulation, and liquidity considerations. In future work, We can explore modifying the model for comprehensive business-cycle analysis. For example, one could follow Arouba (2011) and Cui et al. (2025) to combine our model with neoclassical RBC models to better capture cyclical observations on consumption and investment. In our model the effects of credit and monetary shocks tend to be much weaker than those of technology shocks. Introducing credit that can be used directly for opening vacancies might have a stronger effect on the labor market.

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Appendix

A Endogenous Timing of Wage Payments with DM Frictions

We modify the model in Section 4.2 in two dimensions. First, the DM is subject to search frictions. Let σ_h be the probability that a buyer matches a seller and σ_f be the probability that a seller matches a buyer. Second, once a buyer and a seller are matched, they decide the terms of trade in the DM according to a generic trading mechanism, as in Gu and Wright (2016). The mechanism specifies the payment and trading quantity given the buyer's liquidity. Let $g(q)$ denote the payment for q units of DM good, where $g(0) = 0$ and $g' > 0$, and let L denote the buyer's liquidity. According to the mechanism, $g(q) = L$ and $q = g^{-1}(L)$ if $L < g(q^*)$, and $g(q) = g(q^*)$ and $q = q^*$ if $L \geq g(q^*)$. The CM and LM Bellman equations for workers and firms are the same as in Section 4.2. The worker's DM value is

$$\begin{aligned} V_j(z_j, w_1, w_2) &= \max_{q,z',\omega'} \sigma_h [\upsilon(q) - g(q)] + W_j(z_j, w_1 + w_2) \\ \text{st } g(q) &\leq z + w_1. \end{aligned}$$

The firm's DM value is

$$\tilde{V}_j(z_f, k, l, y - w_2) = \sigma_f [g(q) - cq] + \tilde{W}_j(z_j, w_1 + w_2).$$

The LM surplus of a worker employed with wages w_1 and w_2 is

$$S_h = \frac{w_1 + w_2 + \sigma_h [\upsilon(q_1) - g(q_1) - \upsilon(q_i) + g(q_i)] + i(z_0 - z_1)}{r + s + \lambda_h}$$

and of the firm is

$$S_f = \frac{f(k) - (r+\delta)k - w_1 - w_2 + \kappa - iz_f + \sigma_f \{(1-u)[g(q_1) - c(q_1)] + u[g(q_i) - c(q_i)]\}}{r + s + \lambda_f}.$$

Wage bargaining results in the following FOC wrt w_1:

$$[f'(k) - (r+\delta)] \frac{\partial k}{\partial w_1} - i\frac{\partial z_f}{\partial w_1} + \sigma_h [\upsilon'(q_1) - g'(q_1)] \frac{\partial q_1}{\partial w_1} - i\frac{\partial z_1}{\partial w_1} = 0.$$

The interior solution $(w_1, w_2) > (0,0)$, if it exists, is in one of three cases.

1. $\chi(1-\delta)k^* < \chi(1-\delta)k_i < g(q_i) < g(q^*)$. Any $w_1 \in [\chi(1-\delta)k_i, g(q_i)]$ and $w_2 > 0$ satisfying

$$(1+i)w_1 + w_2 = \frac{D_1}{D_1 + D_2}\left\{f(k_i) - [r + \delta - i\chi(1-\delta)]k_i + \kappa + A\right\},$$

where $A \equiv \sigma_f \{(1-u)[g(q_1) - c(q_1)] + u[g(q_i) - c(q_i)]\}$, is the solution. The solution implies $q_1 = q_i$, $k = k_i$, $z_1 = g(q_i) - w_1$, and $z_f = w_1 - \chi(1-\delta)k_i$.

2. $\max\{\chi(1-\delta)k^*, g(q_i)\} < \min\{\chi(1-\delta)k_i, g(q^*)\}$. The interior solution $(w_1, w_2) > (0,0)$ solves

$$\left[f'\left(\frac{w_1}{\chi(1-\delta)}\right) - (r+\delta)\right]\frac{1}{\chi(1-\delta)} + \sigma_h\left[\frac{\upsilon' \circ g^{-1}(w_1)}{g' \circ g^{-1}(w_1)} - 1\right] = 0$$

and

$$\begin{aligned} w_2 \quad = \quad & \frac{D_1}{D_1 + D_2}\left[f\left(\frac{w_1}{\chi(1-\delta)}\right) - \frac{w_1(r+\delta)}{\chi(1-\delta)} - w_1 + \kappa + A\right] \\ & - \frac{D_2}{D_1 + D_2}\left\{w_1 + ig(q_i) + \sigma_h[\upsilon(q_1) - g(q_1) - \upsilon(q_i) + g(q_i)]\right\}. \end{aligned}$$

In this solution, $\max\{g(q_i), \chi(1-\delta)k^*\} < w_1 \leq \min\{g(q^*), \chi(1-\delta)k_i\}$, $g(q_1) = w_1$, $k = w_1/\chi(1-\delta)$, and $z_1 = z_f = 0$.

3. $g(q_i) < g(q^*) < \chi(1-\delta)k^* < \chi(1-\delta)k_i$. Any $w_1 \in [g(q^*), \chi(1-\delta)k^*]$ and $w_2 > 0$ satisfying

$$\begin{aligned} w_1 + w_2 \quad = \quad & \frac{D_1}{D_1 + D_2}\left[f(k^*) - (r+\delta)k^* + \kappa + A\right] \\ & - \frac{D_2}{D_1 + D_2}\left\{ig(q_i) + \sigma_h[\upsilon(q^*) - g(q^*) - \upsilon(q_i) + g(q_i)]\right\}, \end{aligned}$$

is the solution. The solution implies $q_1 = q^*$, $k = k^*$, and $z_1 = z_f = 0$.

With the free-entry condition

$$f(k) - (r+\delta)k - w_1 - w_2 - iz_f + A = \frac{\kappa(r+s)}{\lambda_f}$$

and the law of motion of unemployment, we can solve for the steady state (w_1, w_2). We continue Example 1 with $\psi = 0.5$ and DM matching function $\mathcal{N}(B,S) = BS/(B+S)$. The plots are in Appendix D.3. The Phillips curve can be upward-sloping, downward-sloping, or U-shaped.

B The Dynamic-Stochastic Model

Let A_f, Π_t, and χ_t be random variables realized in the CM at $t-1$. To avoid the double holdup problem, we assume that matched firms and workers can renegotiate contracts in the CM when shocks are realized. Productivity, inflation rate, and capital pledgeability follow this exogenous process:

$$\begin{aligned} \ln A_{ft+1} &= \rho_A \ln A_{ft} + \varepsilon_{A,t},\ \varepsilon_{A,t} \sim N(0, \sigma_A), \\ \ln \Pi_{t+1} - \ln \bar{\Pi} &= \rho_\Pi (\ln \Pi_t - \ln \bar{\Pi}) + \varepsilon_{\Pi,t},\ \varepsilon_{\Pi,t} \sim N(0, \sigma_i), \\ \ln \chi_{t+1} - \ln \bar{\chi} &= \rho_\chi (\ln \chi_t - \ln \bar{\chi}) + \varepsilon_{\chi,t},\ \varepsilon_{\chi,t} \sim N(0, \sigma_\chi). \end{aligned}$$

By the Fisher equation, $1 + i_t = (1+r)(1+\Pi_t)$.

The unemployment rate at the beginning of t is determined at the end of the LM in $t-1$ when job finding and separation occur. So the DM matching probabilities in t are also known at the end of $t-1$. Specifically,

$$\begin{aligned} u_t &= u_{t-1}(1 - \lambda_{h,t-1}) + (1 - u_{t-1})s, \\ \sigma_{f,t} &= \mathcal{N}(1/1 - u_t, 1), \\ \sigma_{h,t} &= \mathcal{N}(1, 1 - u_t). \end{aligned}$$

Given the recursivity and linearity of the problem, an agent's choices of $z_{0,t}$, $z_{1,t}$, z_f, and k_t do not depend on future variables. As A_f, i_t, and χ_t are realized at $t-1$, $\{z_{0,t}, z_{1,t}, z_f, k_t\}$ is determined as follows:

$$\begin{aligned} z_{0,t} &= \max\{g(q_{i,t}) - b, 0\}, \\ z_{1,t} &= \max\{g(q_{i,t}) - w_t, 0\}, \\ z_{f,t} &= \max\{w_t - \chi_t(1-\delta)k_{i,t}, 0\}, \end{aligned}$$

where $g(q) = \psi c(q) + (1-\psi)\upsilon(q)$, $q_{i,t}$ is solved from $i_t = \sigma_{h,t}[\upsilon'(q_{i,t})/g'(q_{i,t}) - 1]$, and $k_{i,t}$

is solved from $f'(k_{i,t}) = r + \delta - i_t \chi_t (1-\delta)$. The choice of $\{q_{0,t}, q_{1,t}, k_t\}$ is given by

$$q_{0,t} = \begin{cases} q_{i,t}, & \text{if } b \leq g(q_{i,t}) \\ g^{-1}(b), & \text{if } g(q_{i,t}) < b \leq g(q^*) \\ q^*, & \text{if } b > g(q^*), \end{cases}$$

$$q_{1,t} = \begin{cases} q_{i,t}, & \text{if } w_t \leq g(q_{i,t}) \\ g^{-1}(w_t), & \text{if } g(q_{i,t}) < w_t \leq g(q^*) \\ q^*, & \text{if } w_t > g(q^*), \end{cases}$$

$$k_t = \begin{cases} k^* & \text{if } w_t \leq \chi_t (1-\delta) k^* \\ w_t/\chi_t (1-\delta) & \text{if } \chi_t (1-\delta) k^* < w_t \leq \chi_t (1-\delta) k_{i,t} \\ k_{i,t} & \text{if } w_t > \chi_t (1-\delta) k_{i,t}. \end{cases}$$

Given w_t, the surplus of the matched worker and firm in the CM at $t-1$ is

$$\begin{aligned} S_{h,t-1} &= \beta\{w_t - b + i_t (z_{0,t} - z_{1,t}) + \sigma_{h,t}[\upsilon(q_{1,t}) - g(q_{1,t}) - \upsilon(q_{0,t}) + g(q_{0,t})]\} \\ &\quad + \beta (1 - s - \lambda_{h,t}) \mathbb{E} S_{h,t}, \\ S_{f,t-1} &= \beta [-i_t z_{f,t} + f(k_t) - (r+\delta) k_t - w_t] \\ &\quad + \beta \sigma_{f,t} \{(1-u_t)[g(q_{1,t}) - c(q_{1,t})] + u_t [g(q_{0,t}) - c(q_{0,t})]\} \\ &\quad + \beta\kappa + \beta (1 - s - \lambda_{f,t}) \mathbb{E} S_{f,t}, \end{aligned}$$

where the expectation is taken over A_f, i_t, and χ_t. Wage bargaining for w_t in the CM solves

$$\rho S_{f,t-1} = (1-\rho) S_{h,t-1}.$$

The free-entry condition is

$$\kappa = \lambda_{f,t} S_{f,t}.$$

Finally, LM matching follows:

$$\begin{aligned} \lambda_{f,t} &= \mathcal{M}(1/\tau_t, 1), \\ \lambda_{h,t} &= \mathcal{M}(1, \tau_t). \end{aligned}$$

C Proofs of Lemmas, Propositions, and Corollaries

Proof of Lemma 1: The numerator of the first term on the LHS of Equation (15) is a one-period surplus of an employed worker. If $w > cq^*$, $q = q^*$, $z_1 = 0$, and the numerator is $w + \upsilon(q^*) - cq^* - \upsilon(q_i) + (1+i)\,cq_i$, which increases in w. If $w \in (cq_i, cq^*]$, $q = w/c$, $z_1 = 0$, and the numerator is $\upsilon(w/c) - \upsilon(q_i) + (1+i)\,cq_i$, which increases in w. If $w \leq cq_i$, $q_1 = q_i$, $z_1 = cq_i - w$, and the numerator is $(1+i)\,w$, which again increases in w. In all three cases, the numerator continuously increases in w.

The denominator of the first term on the LHS of (15) is the one-period surplus of a producing firm. If $w > \chi(1-\delta)\,k_i$, $k = k_i$, $z_f = w - \chi(1-\delta)\,k_i$, and the denominator is $f(k_i) - [r + \delta - i\chi(1-\delta)]\,k_i - (1+i)\,w + \kappa$, which decreases in w. If $w \in (\chi(1-\delta)\,k^*, \chi(1-\delta)\,k_i]$, $k = w/\chi(1-\delta)$, $z_f = 0$, and the denominator is $f(w/\chi(1-\delta)) - (r+\delta)\,w/\chi(1-\delta) - w + \kappa$. Take the derivative of w to get $[f' - (r+\delta)]/\chi(1-\delta) - 1$. Because $k > k^*$, $f' - (r+\delta) < 0$. So the denominator decreases in w. If $w \leq \chi(1-\delta)\,k^*$, $k = k^*$, $z_f = 0$, and the denominator is $f(k^*) - (r+\delta)\,k^* - w + \kappa$, which decreases in w. In all cases, the denominator continuously decreases in w. It follows that the first term strictly increases in w.

The second term on the LHS strictly decreases in τ because $\mathcal{M}$ strictly increases in both arguments. Altogether, when τ increases, w must increase so the RHS stays at $\rho/(1-\rho)$, which means w is a strictly increasing function of τ. So (15) can be written as $w = h_1(\tau)$, where $h_1' > 0$.

At $\tau = 0$, $\mathcal{M}(1/\tau, 1) = 1$, $\mathcal{M}(1, \tau) = 0$, and $\underline{w}$ solves

$$\frac{\underline{w} + \upsilon(q_1) - cq_1 - iz_1 - \upsilon(q_i) + (1+i)\,cq_i}{f(k) - (r+\delta)\,k - \underline{w} - iz_f + \kappa}\frac{r+s+1}{r+s} = \frac{\rho}{1-\rho}.$$

If $\underline{w} = 0$, then $q_1 = q_i$ and $z_1 = z_i$. The LHS is 0. Hence, $\underline{w}$ must be strictly positive if $\rho > 0$, so that the above equation holds. ■

Proof of Proposition 1. A stationary monetary equilibrium exists if h_1 and h_2 cross at $(w, \tau) > (0, 0)$. By Lemma 1, h_1 increases with $h_1(0) = \underline{w} > 0$, and h_2 decreases with $h_2(0) = \bar{w} > 0$ and $h_2(\bar{\tau}) = 0$. So h_1 and h_2 cross if $\underline{w} > \bar{w}$. Notice that $\underline{w}$ increases in κ and $\bar{w}$ decreases in κ. If $\bar{w} > \underline{w}$ at $\kappa = 0$, then there is a stationary monetary equilibrium at $\kappa = 0$. When κ increases, h_1 shifts up and h_2 shifts down. For a large κ, $\bar{w}$ becomes negative and there is no entry and no monetary equilibrium. Therefore, there exists a $\hat{\kappa}$ below which there is a unique stationary monetary equilibrium and above which there is no stationary monetary equilibrium.

Next, we show that $\bar{w} > \underline{w}$ when $\kappa \to 0$. Let $\underline{w}_0 = h_1(0)$ and $\bar{w}_0 = h_2(0)$ when $\kappa \to 0$. Then $\bar{w}_0$ solves

$$f(k) - (r+\delta)k - \bar{w}_0 - iz_f = 0.$$

Plug $\bar{w}_0$ into (15) with $\tau = \kappa = 0$. The LHS of (15) is ∞. As the LHS of (15) increases in w, $\underline{w}_0$ must be less than $\bar{w}_0$. ■

Proof of Proposition 2. If the economy is in Regime 3, $k = k^*$, increasing χ does not change equations (15) and (16). Therefore, the equilibrium allocation is not affected. Holding τ constant, if the economy is in Regime 2, the LHS of (16) decreases in χ, and the LHS of (15) increases. The same argument applies to Regime 1. So h_1 rotates counterclockwise and h_2 rotates clockwise at $w = \chi(1-\delta)k^*$. Therefore, the equilibrium w is higher. To seeτ is higher, plug (16) into (15) to get

$$\frac{w + v(q_1) - cq_1 - iz_1 - v(q_i) + (1+i)cq_i}{\kappa} \frac{\mathcal{M}\left(\frac{1}{\tau}, 1\right)}{r + s + \mathcal{M}(1,\tau)} = \frac{\rho}{1-\rho}.$$

The first term increases in w and the second decreases in τ. It follows that an increase in w results in a higher τ. By (12), u falls. ■

Proof of Proposition 3. Given τ, by (15), w decreases in i in all regimes. So h_1 shifts down as i increases. Given τ, by (16), w decreases in i in Regime 1. So as i increases, h_2 shifts down if the economy is in Regime 1, as shown in Figure 4 (a). In the new equilibrium, w is lower, but the change in τ is ambiguous, as is u. If the economy is in Regime 2 or 3, as i does not appear in (16) in these two regimes, h_2 does not move when i changes. In Figure 4 (b), with an increase in i, h_1 shifts down and h_2 stays the same. In the new equilibrium, w is lower and τ is higher. By (12), u is lower. ■

Proof of Proposition 4. For case 1, the interior solution occurs at $w_1 \in [\chi(1-\delta)k_i, cq_i)$. Pick $w_1 = \chi(1-\delta)k_i$. By (22), w_2 strictly increases in τ. Let $w_2 = H_1(\tau)$, where H_1 is from (22). By (26), w_2 strictly decreases in τ. Let $w_2 = H_2(\tau)$, where H_2 is from (26). Note that for w_2 to be nonnegative in (26), we need

$$f(k_i) - [r + \delta + \chi(1-\delta)]k_i > 0. \tag{C.1}$$

Consider $\kappa \to 0$. By (22), $w_2 = 0$ implies $\tau = \infty$. For H_1 and H_2 to intersect at positive w_2

and τ, H_1 must be positive at $\tau = \infty$. That is,

$$\frac{r+s+1}{r+s} > \frac{1-\rho}{\rho} \frac{(1+i)\chi(1-\delta)k_i}{f(k_i) - [r+\delta+\chi(1-\delta)]k_i}. \tag{C.2}$$

For a higher κ, H_1 shifts up and H_2 shifts down. At $\hat{\kappa}$, H_1 and H_2 intersect at $w_2 = 0$ and $\hat{\tau} \in (0, \infty)$, where $\hat{\tau}$ solves

$$\lambda_h(\hat{\tau}) = \frac{1-\rho}{\rho} \frac{(1+i)\chi(1-\delta)k_i}{f(k_i) - [r+\delta+\chi(1-\delta)]k_i}(r+s) - (r+s)$$

and $\hat{\kappa} = \lambda_f(\hat{\tau})\{f(k_i) - [r+\delta+\chi(1-\delta)]k_i\}/(r+s)$. If (C.1) or (C.2) does not hold, then there is no interior solution of (w_1, w_2) for case 1. The cutoff $\hat{\kappa}$ is given by Proposition 1.

In case 2, for w_2 to be nonnegative in (26), we need

$$f\left(\frac{w_1}{\chi(1-\delta)}\right) - \frac{r+\delta+\chi(1-\delta)}{\chi(1-\delta)}w_1 > 0, \tag{C.3}$$

where w_1 is given by (23). Consider $\kappa \to 0$. For H_1 and H_2 to intersect at a positive w_2, we need

$$\frac{r+s+1}{r+s} > \frac{1-\rho}{\rho} \frac{\upsilon(w_1/c) - \upsilon(q_i) + (1+i)cq_i}{f\left(\frac{w_1}{\chi(1-\delta)}\right) - \frac{r+\delta+\chi(1-\delta)}{\chi(1-\delta)}w_1}. \tag{C.4}$$

At $\hat{\kappa}$, H_1 and H_2 intersect at $w_2 = 0$ and $\hat{\tau} \in (0, \infty)$, where $\hat{\tau}$ solves

$$\lambda_h(\hat{\tau}) = \frac{1-\rho}{\rho} \frac{\upsilon\left(\frac{w_1}{c}\right) - \upsilon(q_i) + (1+i)cq_i}{f\left(\frac{w_1}{\chi(1-\delta)}\right) - \frac{r+\delta+\chi(1-\delta)}{\chi(1-\delta)}w_1}(r+s) - (r+s)$$

and $\hat{\kappa} = \lambda_f(\hat{\tau})\left[f\left(\frac{w_1}{\chi(1-\delta)}\right) - \frac{(r+\delta)+\chi(1-\delta)}{\chi(1-\delta)}w_1\right]/(r+s)$. If (C.3) or (C.4) does not hold, then there is no interior solution of (w_1, w_2) for case 2. The cutoff $\hat{\kappa}$ is given by Proposition 1.

In case 3, pick $w_1 = cq^*$. For w_2 to be nonnegative in (26), we need

$$f(k^*) - (r+\delta)k^* - cq^* > 0. \tag{C.5}$$

Consider $\kappa \to 0$. For H_1 and H_2 to intersect at a positive w_2, we need

$$\frac{r+s+1}{r+s} > \frac{1-\rho}{\rho} \frac{\upsilon(q^*) - \upsilon(q_i) + (1+i)cq_i}{f(k^*) - (r+\delta)k^* - cq^*}. \tag{C.6}$$

At $\hat{\kappa}$, H_1 and H_2 intersect at $w_2 = 0$ and $\hat{\tau} \in (0, \infty)$, where $\hat{\tau}$ solves

$$\lambda_h(\hat{\tau}) = \frac{1-\rho}{\rho} \frac{\upsilon(q^*) - \upsilon(q_i) + (1+i) cq_i}{f(k^*) - (r+\delta) k^* - cq^*} (r+s) - (r+s)$$

and $\hat{\kappa} = \lambda_f(\hat{\tau}) [f(k^*) - (r+\delta) - cq^*] / (r+s)$. If (C.5) or (C.6) does not hold, then there is no interior solution of (w_1, w_2) for case 3. The cutoff $\hat{\kappa}$ is given by Proposition 1. ■

Proof of Proposition 5: In Regime 3, it is clear that χ does not show up in (18), (19), or (26), so w_1, w_2, and τ do not depend on χ. The comparative statics shows $dw_1/d\chi > 0$ in Regimes 1 and 2. If the interior solution becomes corner or corner becomes interior as χ increases, w_1 increases because it is continuous in χ. The same argument applies to τ. By (12), u decreases with χ in Regimes 1 and 2. ■

Proof of Proposition 6. It is clear from the comparative statics that $d(w_1 + w_2)/di < 0$ in each regime and $d\tau/di > 0$ in Regimes 2 and 3. Because $w_1 + w_2$ is continuous in i, it continuously decreases if the solution switches from corner to interior or interior to corner or from one regime to another. The same argument applies to τ. By (12), u decreases with i in Regimes 2 and 3. ■

Proof of Corollary 2. Suppose that the economy is in Regime 1 with an interior solution. Increasing i has two effects. First, it lowers w_2 (by the result in Table 1), which will eventually violate the non-negativity condition of (20). Suppose $w_1 = \chi(1-\delta) k_{\hat{\imath}}$ and $w_2 = 0$ at $\hat{\imath}$, but $\chi(1-\delta) k_{\hat{\imath}} < cq_{\hat{\imath}}$ still holds. Increasing i to $i + \varepsilon$ results in a corner solution in which $w_2 = 0$ and $w_1 \in (\chi(1-\delta) k^*, \chi(1-\delta) k_i)$, as the objective function strictly increases in $w_1 \in [0, \chi(1-\delta) k_i]$. Therefore, the solution becomes a corner solution in Regime 2. Second, increasing i raises k_i and lowers q_i, which will eventually violate the condition that $\chi(1-\delta) k_i < cq_i$ as long as υ or f satisfies the Inada condition. Suppose $\chi(1-\delta) k_{\hat{\imath}} = cq_{\hat{\imath}}$ at $\hat{\imath}$, but $w_2 > 0$. Then (23) is satisfied at $\hat{\imath}$. The economy moves to Regime 2 with an interior solution. Further increases in i reduce w_2 but do not change w_1.

Suppose the economy is in Regime 2 with an interior solution. Increasing i again has two effects. First, it violates (20). In that case, the solution becomes corner in Regime 2 if $\chi(1-\delta) k^* < cq_i < \min\{\chi(1-\delta) k_i, cq^*\}$ or Regime 3 if $cq_i < \chi(1-\delta) k^* < \min\{\chi(1-\delta) k_i, cq^*\}$, since the objective function is strictly concave. Second, increasing i raises k_i and lowers q_i. However, the order of cq_i and $\chi(1-\delta) k_i$ does not change the necessary condition that $\max\{cq_i, \chi(1-\delta) k^*\} < \min\{\chi(1-\delta) k_i, cq^*\}$. So, the interior solution still holds.

Suppose the economy is in Regime 2 with a corner solution that $w_2 = 0$. By Table 1, as i

increases, w_1 decreases and (23) is not satisfied. So the solution will not go back to interior. With w_1 getting lower, it will eventually be less than $\chi(1-\delta)k^*$, and the economy moves to Regime 3.

Suppose the equilibrium is interior in Regime 3. A higher i does not affect the interval $[cq^*, \chi(1-\delta)k^*]$ in which w_1 resides. As w_2 becomes smaller, it will violate (20) and the solution will be corner at $w_1 < \chi(1-\delta)k^*$, which means the equilibrium is still in Regime 3. Further reductions in w_1 by raising i reinforce $w_1 < \chi(1-\delta)k^*$, and the economy stays in Regime 3. ■

D Numerical Examples

In this Appendix, we show numerical examples to illustrate the effect of χ (Figure D.1) and i (Figures D.2–D.5). The relationship between i and u can be positive when χ is small (Figure D.2), U-shaped when χ is medium (Figure D.3), and downward sloping when χ is large (Figures D.4 and D.5).

In all examples, we use the following functions:

$$\begin{aligned} \mathcal{M}(u,v) &= A_m u^{\iota} v^{1-\iota}, \\ F(K,L) &= A_f K^{\theta} L^{1-\theta}, \\ \upsilon(q) &= A_{\upsilon} q^{\alpha}. \end{aligned}$$

If there is DM matching, the matching function is

$$\mathcal{N}(B,S) = \frac{BS}{B+S}.$$

D.1 Baseline

Example 1 Let $A_{\upsilon} = 1.5$, $\alpha = 0.6$, $A_f = 1$, $\theta = 0.3$, $A_m = 0.35$, and $\iota = 0.7$. Other parameters are $\beta = 0.96$, $i = 0.05$, $\delta = 0.15$, $\kappa = 0.05$, $s = 0.05$, and $\rho = 0.7$. Figure D.1 plots the equilibrium capital per producing firm, DM output q, LM output, wage, labor-market tightness, unemployment, real balances for firms, and employed and unemployed workers, as χ varies in $[0,1]$.

Example 2 Continue with Example 1. Figures D.2–D.5 plot the equilibrium values for $\chi = 0.02$, 0.05, 0.4, and 0.5, respectively, as i varies in $[0, 0.2]$.

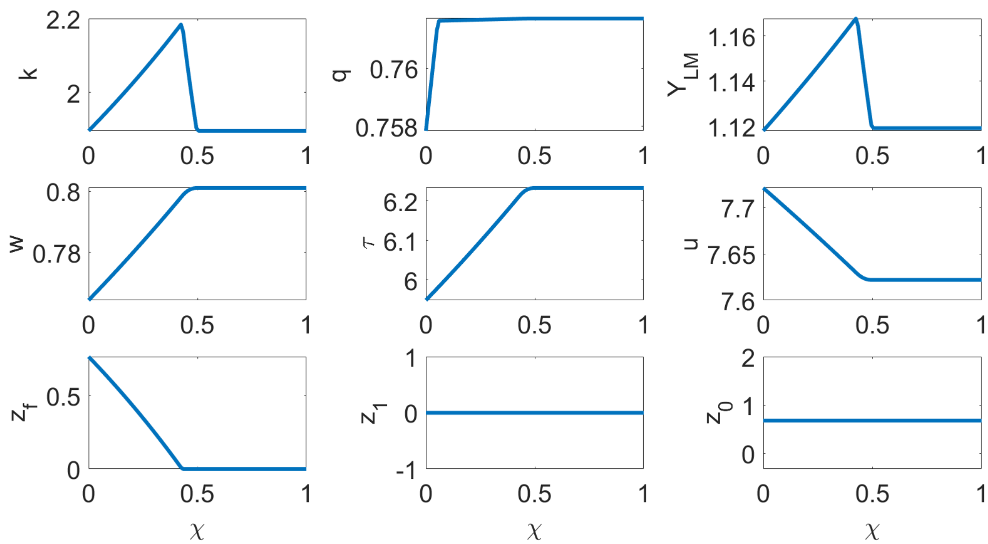

Figure D.1: Effects of χ

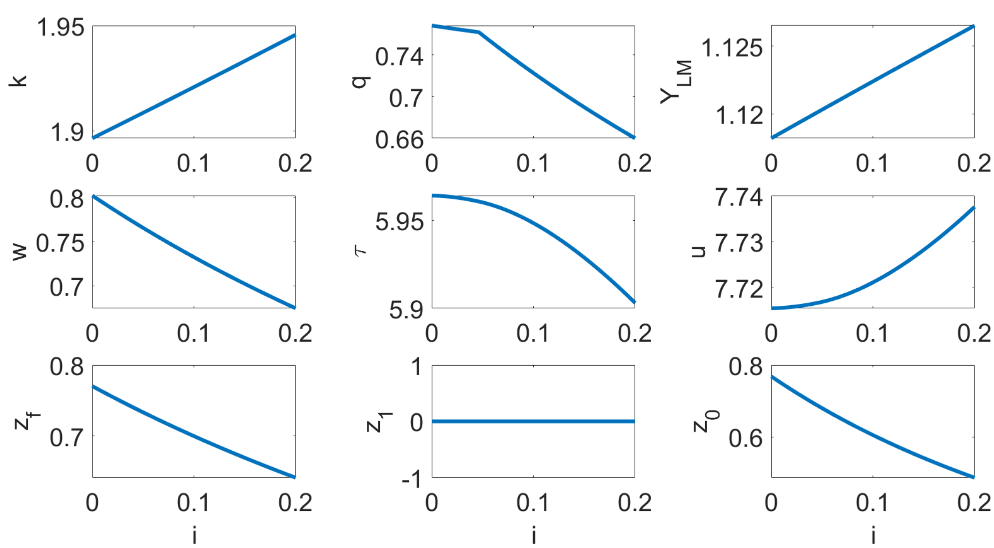

Figure D.2: Effects of i ($\chi = 0.02$, Regime 1)

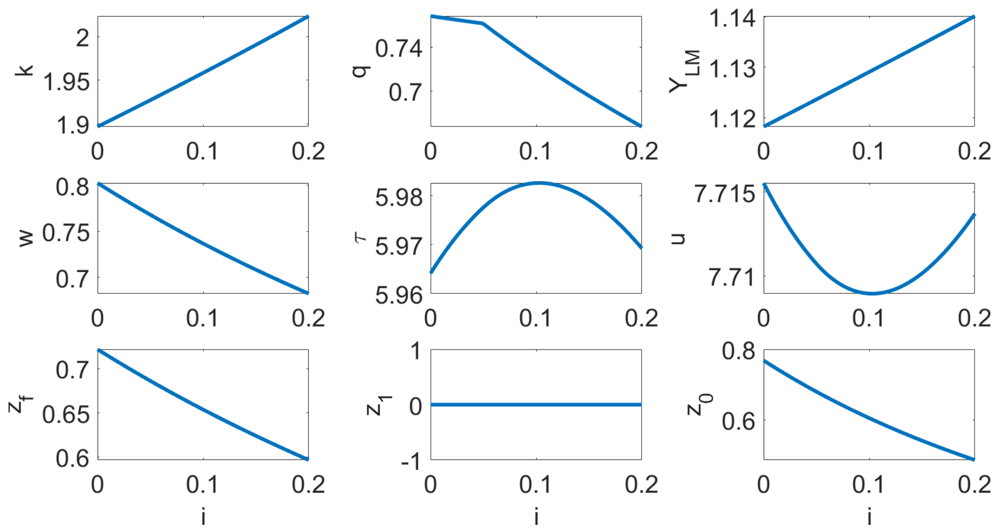

Figure D.3: Effects of i ($\chi = 0.05$, Regime 1)

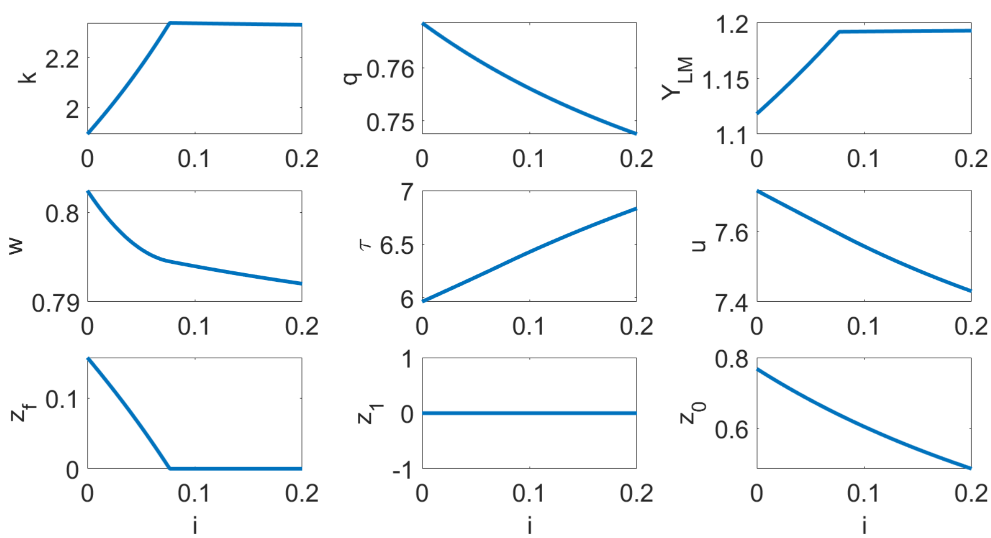

Figure D.4: Effects of i ($\chi = 0.4$, Regimes 1 and 2)

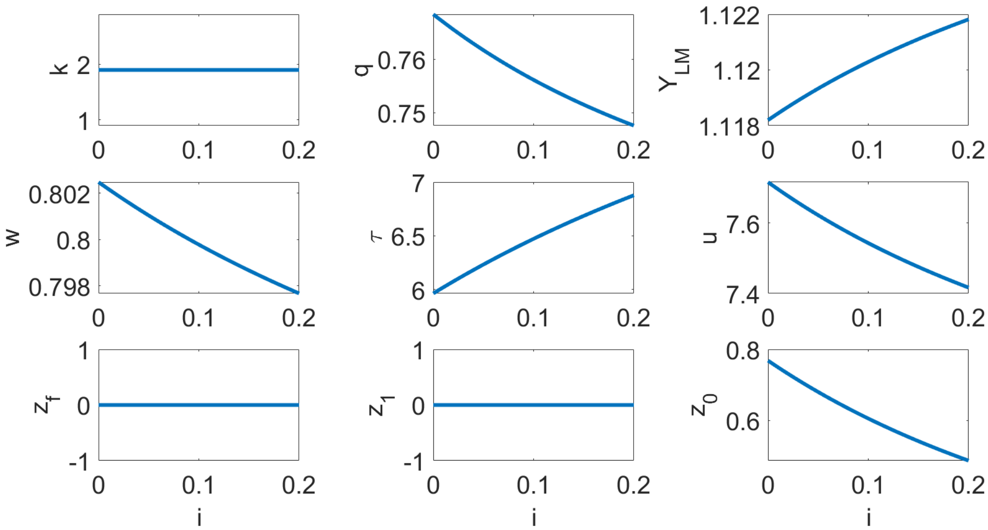

Figure D.5: Effects of i ($\chi = 0.5$, Regime 3)

D.2 Unemployment Benefits, Leisure and DM Frictions

Example 3 Let $A_v = 1.5$, $\alpha = 0.6$, $A_f = 1$, $\theta = 0.1$, $A_m = 0.35$, and $\iota = 0.4$. Other parameters are $\beta = 0.96$, $i = 0.05$, $\chi = 0.14$, $b = 0.1$, $\ell = 0.05$, $c = 1$, $\delta = 0.2$, $\kappa = 0.76$, $s = 0.05$, $\rho = 0.9$, and $\psi = 0.5$. There are three steady states: $(\tau_1, w_1) = (0.0003.7868)$, $(\tau_2, w_2) = (0.0010, 0.7990)$, and $(\tau_3, w_3) = (0.0070, 0.8709)$.

Example 4 Continue with the parameter values in Example 1, and let $\psi = 0.5$. The Phillips curve is upward sloping when $\chi = 0.02$, U-shaped when $\chi = 0.25$, and downward sloping when $\chi = 0.4$. The graphs are similar to Figures D.2–D.5.

D.3 Endogenous Timing of Wage Payments with DM Frictions

Example 5 Continue with the parameter values in Example 1. Figures D.6–D.8 plot the equilibrium values for $\chi = 0.02$, 0.15, and 0.24, respectively, as i varies in $[0, 0.1]$. The steady state is unique, and the Phillips curve is downward sloping at $\chi = 0.02$, U-shaped at $\chi = 0.15$, and upward sloping at $\chi = 0.24$.

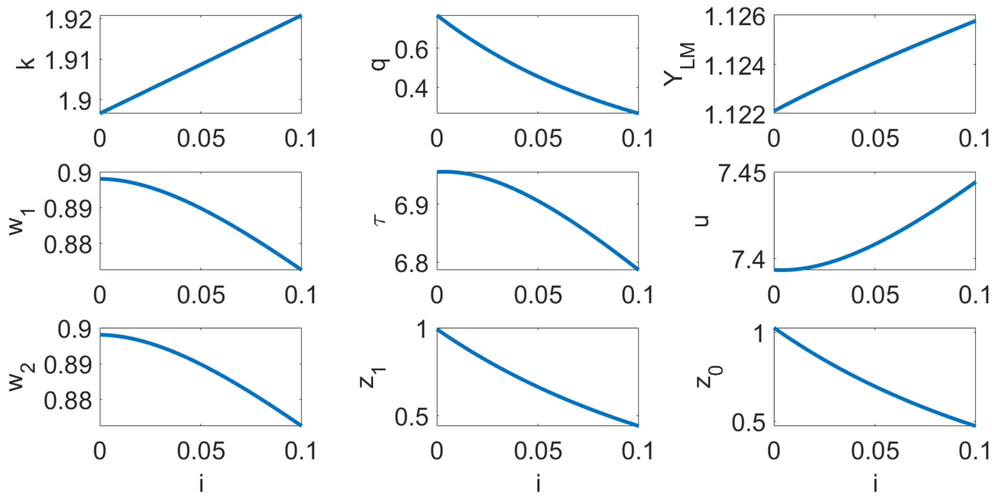

Figure D.6: Effects of i ($\chi = 0.02$)

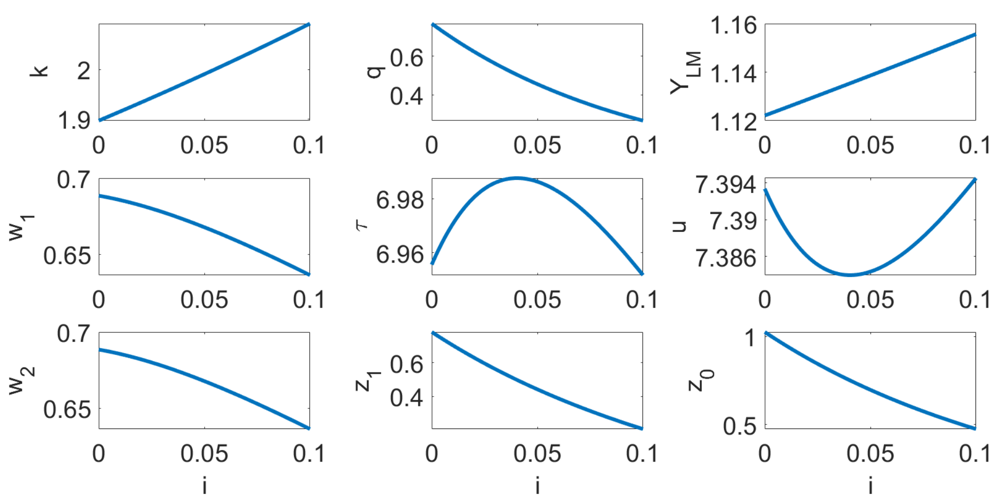

Figure D.7: Effects of i ($\chi = 0.15$)

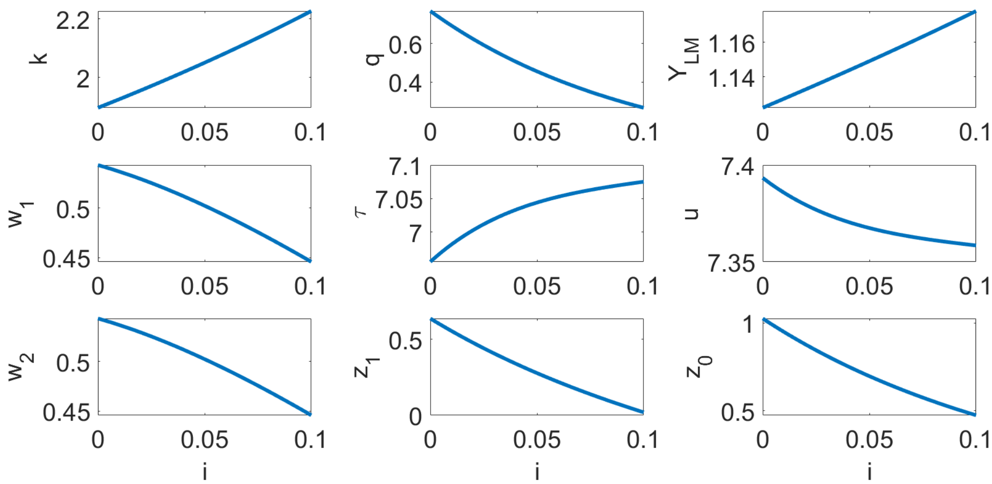

Figure D.8: Effects of i ($\chi = 0.24$)

E Calibration Formulae

In this Appendix we define the formulae to calculate the model's targeted moments.

The real output, Y, is given by

$$Y = (1-\psi)\,\mathcal{N}\,(1, 1-u)\,\{(1-u)\,[\upsilon(q_1) - cq_1] + u\,[\upsilon(q_0) - cq_0]\} + (1-u)\,f(k).$$

The capital-to-output ratio is defined as

$$\frac{(1-u)\,k}{Y}.$$

The share of labor compensation in output is

$$\frac{(1-u)\,w}{Y}.$$

The average markup in the DM is

$$(1-u)\,\frac{\psi cq_1 + (1-\psi)\,\upsilon\,(q_1)}{cq_1} + u\frac{\psi cq_0 + (1-\psi)\,\upsilon\,(q_0)}{cq_0}.$$

The total money demand is

$$\frac{uz_0 + (1-u)\,z_1 + (1-u)\,z_f}{Y}.$$

The firm money demand is

$$\frac{(1-u)\,z_f}{Y}.$$

The household money demand is

$$\frac{uz_0+(1-u)\,z_1}{Y}.$$

F Data

F.1 Figure 1

The data are from FRED, and the time period is 1960–2019.

- UNRATE: Unemployment Rate
- CPILFESL: Consumer Price Index for All Urban Consumers: All Items Less Food and Energy

F.2 Table 2 and Table 3

The following data are used to calculate the targeted moments and non-targeted moments in Table 2 and Table 3. Data are from FRED, and the time period is 2000–2019 unless marked otherwise.

1. Target for nominal interest rate i:
   - AAA (yield for AAA corporate bonds)
2. Target for real interest rate to set β:
   - CPILFESL: Consumer Price Index for All Urban Consumers: All Items Less Food and Energy
   - AAA (Yield for AAA Corporate Bonds)
3. Target for unemployment rate to set s:
   - UNRATE: Unemployment Rate
4. Target for job-finding rate to set κ:
   - FRED UEMPLT5: Number Unemployed for Less Than 5 Weeks

- FRED UEMPLOY: Unemployment Level

5. Target for firm money demand to set χ:
   - NNBCDCA: Nonfinancial Noncorporate Business; Checkable Checkable Deposits and Currency
   - NCBCDCA: Nonfinancial Corporate Business; Checkable Deposits and Currency
6. Target for labor share to set ρ:
   - FRED LABSHPUSA156NRUG: Share of Labor Compensation in GDP at Current National Prices (Annual)
7. Target for capital-GDP ratio to set θ:
   - RKNANPUSA666NRUG: Capital Stock at Constant National Prices
   - GDPC1: Real Gross Domestic Product
8. Target for household money demand to set A_v:
   - FRED BOGZ1FL193020005Q: Households; Checkable Deposits and Currency
9. Firm bank loans-output ratio
10. Share of cash transactions:
    - Survey and Diary of Consumer Payment Choice 2015–2019. The data are available from the website of the Federal Reserve Bank of Atlanta (2019) and the Federal Reserve Bank of St. Louis (2015–2018).

F.3 Figure 7

In Figure 7, we compare the PC for the period 2000–2019 to that for the period 1960–1999. To derive the χ value for the pre-2000 curve, we compare the value of business debt/capital value for the two periods. We use the following series:

- GDPC1: Real Gross Domestic Product
- RKNANPUSA666NRUG: Capital Stock at Constant National Prices
- TBSDODNS: Nonfinancial Business; Debt Securities and Loans; Liability, Level

- (Nominal) GDP: Gross Domestic Product (GDP)

F.4 Table 4

The following FRED data series are used to calculate the standard deviation and correlation in Table 4; the time period is 2000–2019.

1. GDP:
    - GDPC1: Real Gross Domestic Product
2. Consumption:
    - PCECC96: Real Personal Consumetion Expenditures
3. Investment:
    - GPDIC1: Real Gross Private Domestic Investment
4. Money demand:
    - NNBCDCA: Nonfinancial Noncorporate Business; Checkable Checkable Deposits and Currency
    - NCBCDCA: Nonfinancial Corporate Business; Checkable Deposits and Currency
    - FRED BOGZ1FL193020005Q: Households; Checkable Deposits and Currency
    - (Nominal) GDP: Gross Domestic Product (GDP)
5. Unemployment:
    - UNRATE: Unemployment Rate
6. Inflation:
    - CPILFESL: Consumer Price Index for All Urban Consumers: All Items Less Food and Energy
7. Business debt-GDP ratio:
    - NNBDILNECL: Nonfinancial Noncorporate Business; Depository Institution Loans N.E.C.

- BLNECLBSNNCB: Nonfinancial Corporate Business; Depository Institution Loans NEC
- (Nominal) GDP: Gross Domestic Product (GDP)

8. Labor share:
   - COE: National Income: Compensation of Employees, Paid
   - (Nominal) GDP: Gross Domestic Product (GDP)
9. Wage rate:
   - COMPRNFB: Nonfarm Business Sector: Real Hourly Compensation for All Workers