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The Optimum Quantity of Central Bank Reserves

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Abstract

This paper analyzes the optimal quantity of central bank reserves in an economy where reserves and other financial assets provide liquidity benefits. Using a static model, I derive a constrained Friedman rule that characterizes the socially optimal level of reserves, demonstrating that this quantity is neither necessarily large nor small but depends on the marginal benefits of reserves relative to alternative safe assets. The model highlights how the supply of government and private-sector liquid assets influences demand for reserves and the size of a central bank balance sheet. I calibrate and estimate the model to determine the optimal amount of central bank holdings in the United States. I extend the analysis to account for shadow banking, where non-bank intermediaries create short-term liquid assets but generate monitoring costs and externalities. The presence of shadow banking alters the optimal balance of reserves and other assets, potentially constraining optimal balance sheet policy. The results offer new insights into the debate over the size of central bank balance sheets and the interaction between public and private liquidity provision.

Topics: Monetary policy implementation; Financial institutions; Financial markets; Financial system regulation and policies

JEL codes: E41, E42, E58, G21, G28

Résumé

Dans la présente étude, j'analyse la quantité optimale de réserves que la banque centrale d'un pays devrait détenir dans une économie où les réserves et les autres actifs financiers offrent des avantages sur le plan de la liquidité. À l'aide d'un modèle statique, j'obtiens une règle de Friedman modifiée, à laquelle j'ajoute des contraintes. Ce modèle permet de caractériser la quantité socialement optimale des réserves et de démontrer que cette quantité n'est pas nécessairement grande ni petite, mais qu'elle dépend des avantages marginaux des réserves par rapport à d'autres actifs sûrs. Le modèle illustre la façon dont l'offre d'actifs liquides du secteur public et du secteur privé influe sur la demande de réserves et sur la taille du bilan d'une banque centrale. J'étalonne et estime le modèle afin de déterminer le montant optimal des avoirs que devrait détenir la banque centrale des États-Unis. J'élargis l'analyse pour tenir compte du secteur bancaire parallèle, dans lequel des intermédiaires financiers non bancaires créent des actifs liquides à court terme, mais génèrent des coûts de surveillance et des externalités. La présence du secteur bancaire parallèle modifie l'équilibre optimal entre les réserves et les autres actifs, ce qui peut constituer une contrainte pour la conduite d'une politique concernant le bilan optimal. Les résultats de l'analyse apportent de nouvelles perspectives dans le débat sur la taille du bilan des banques centrales et sur l'interaction entre l'offre de liquidités du secteur public et celle du secteur privé.

Sujets : Mise en œuvre de la politique monétaire; Institutions financières; Marchés financiers; Réglementation et politiques relatives au système financier
Codes JEL : E41, E42, E58, G21, G28

1. Introduction

The aftermath of the global financial crisis and the COVID-19 pandemic saw central banks worldwide significantly expand their balance sheets. This expansion has not only reshaped the implementation of monetary policy but also altered the composition of financial assets held by the public. In particular, it has led to an increased share of short-term, central bank-issued liabilities (reserves) that can only be held by banks. As several policymakers signal a preference for maintaining larger balance sheets in normal times (e.g., Logan (2019)), fundamental questions arise: What constitutes an optimal central bank balance sheet? More specifically, under what conditions is it optimal to expand reserve balances that are exclusively held by banks? What economic forces determine this optimal balance sheet size?

The classic irrelevance proposition of Wallace (1981) suggests that the composition of the central bank's balance sheet does not matter under certain conditions. This neutrality result relies on two key assumptions: (i) financial assets provide no non-pecuniary benefits, and (ii) all investors have unrestricted access to all asset markets on identical terms (Cúrdia and Woodford (2010)). In practice, both assumptions fail to hold. Some financial instruments, including bank deposits and central bank reserves, provide liquidity or transaction services, while others serve as collateral or hedging instruments. Additionally, central bank reserves are held only by banks, introducing segmentation into financial markets. In this paper, I develop a static general equilibrium model in which both households and banks derive liquidity benefits from financial assets, but only banks can hold central bank reserves. This framework allows me to characterize the optimal quantity of reserves and identify the key determinants of an optimal central bank balance sheet.

The model yields a modified version of the Friedman rule, where the return on reserves should equal the returns on both short-term and long-term government bonds. While the Friedman rule has been a cornerstone of monetary theory for decades, I extend its implications along three dimensions. First, I incorporate constraints on the central bank's asset holdings, leading to a constrained Friedman rule. Specifically, if there is an insufficient supply of short-term government bonds, households may face a shortage of these bonds even when the central bank minimizes its own holdings. In such cases, short-term bond yields can be lower than the return on reserves and long-term bonds. I demonstrate that issuing central bank bills as substitutes for short-term government bonds can alleviate this inefficiency and improve welfare.

Second, I provide a quantitative assessment of the optimal central bank balance sheet by calibrating my model to U.S. data. While much of the existing literature on optimal reserves frames the discussion in qualitative terms—whether the balance sheet should be large or small, or whether reserves should be scarce, ample, or abundant—I estimate an explicit quantity of reserves that optimizes welfare. Prior research argues for a smaller balance sheet to mitigate risks to central bank net worth and monetary policy independence (e.g., Sims (2013); Hall and Reis (2013)) or to reduce the costs imposed on the balance sheets of commercial banks (e.g., Williamson (2019)). Conversely, a larger balance sheet may be desirable if it reduces reliance on private sector maturity transformation (Greenwood et al. (2016)) or enhances liquidity management (Gagnon and Sack (2014)). My model is most closely related to Martin et al. (2019), in which setting the optimal quantity of reserves balances liquidity benefits against the costs to the banking sector's balance sheet. Consistent with Vissing-Jorgensen (2023), my framework suggests that the central bank should aim to equalize the convenience yields of different financial assets.

By calibrating the model, I estimate demand curves for various financial instruments and derive an optimal level of reserves. This approach provides a structural method to determine whether reserves should be scarce, ample, or abundant, thereby contributing to the literature on the optimal policy for a central bank's balance sheet. The results highlight that the optimal reserve quantity depends not only on liquidity preferences of investors and banks but also on the supply of government debt and regulatory constraints. In environments where the stock of government bonds is limited, a small central bank balance sheet is optimal because government bonds offer a higher marginal liquidity benefit than reserves. Conversely, when the supply of government bonds is large, an ample or abundant reserve regime is preferable.

Finally, I extend the model to examine cases where deviations from the Friedman rule may be optimal. In particular, I analyze externalities associated with shadow banking, where lenders do not fully internalize the costs of monitoring loans and create over-collateralized securities that mimic the liquidity properties of government bonds. When such externalities are present, increasing the supply of short-term government bills raises short-term yields and reduces shadow bank leverage, thereby mitigating financial instability. This insight contributes to the broader literature on deviations from the Friedman rule, which includes cases related to foreign seigniorage (Schmitt-Grohé and Uribe (2012)), market power in goods markets (Schmitt-Grohé and Uribe (2004)), and distortionary taxation (Chari et al. (1996); Kimbrough (1986); Phelps (1973)).

The model is a non-stochastic, perfect foresight equilibrium framework in which households allocate wealth across short-term and long-term government bonds, bank deposits, reverse repos, and bank equity. Banks invest in government bonds, corporate loans, and reserves, funding themselves through deposits, equity, and repos. I incorporate bank capital constraints and calibrate the model to match observed money market rates. Equilibrium asset prices reflect convenience yields arising from liquidity benefits (see Krishnamurthy and Vissing-Jorgensen (2012)). When banks are capital-constrained, a wedge arises between returns on bank assets and liabilities, analogous to intermediary spreads in Gorton et al. (2016). The Friedman rule implies that the return on bank reverse repos should equal the interest rate on reserves, with bank repo rates slightly below. The calibrated model generates demand curves for U.S. financial assets, complementing empirical studies on reserve demand (e.g., Smith (2019); Smith and Valcarcel (2023); Afonso et al. (2022); Lopez-Salido and Vissing-Jorgensen (2024)). Unlike econometric approaches that estimate local slopes of demand curves (e.g., Lagos and Navarro (2019); Armenter and Lester (2017); Afonso et al. (2019)), my calibration-based approach provides a global estimate and allows for counterfactual analysis under changing regulatory and fiscal conditions. The model also captures interactions between secured funding markets and the broader financial system, including constraints on repo market intermediation (e.g., Anbil et al. (2022)). Overall, this paper provides a theoretical and quantitative framework for assessing the optimal central bank balance sheet. The findings offer policy-relevant insights into the trade-offs between balance sheet expansion, financial stability, and liquidity provision, shedding light on the evolving role of central bank liabilities in modern financial systems.

2. Model

2.1. Households

The economy features a representative household that encapsulates the behavior of all non-bank investors. In period 1, the household is endowed with goods G and has the ability to produce additional goods H at a disutility cost $c(H) > 0$, where $c'(H) > 0$. The household derives utility from consumption in period 2 and from the liquidity benefits associated with holding assets other than equity, denoted as $\lambda Q > 0$. The marginal utility from holding liquid assets is diminishing, satisfying $Q'() > 0$ and $Q''() < 0$. A more specific functional form for liquidity benefits will be introduced in the calibration exercise. The household's utility function is assumed to be linear in period-2 consumption, such that:

$$U = C + \lambda Q - c(H). \tag{1}$$

Households do not consume in period 1 but own a technology that allows them to transform period-1 goods into capital goods at a one-for-one rate. The capital goods are subsequently sold to firms. The price of period-1 goods is normalized to $P_1 = 1$, and the price of period-2 consumption is denoted by P_2. The household's nominal wealth in period 1 is therefore given by $W = P_1(G+H)$. Households allocate their wealth across a portfolio consisting of short-term bonds B_{ST}^{HH}, long-term bonds B_{LT}^{HH}, deposits D^{HH}, equity E^{HH}, reverse repos $REVERSE^{HH}$, and central bank liabilities M^{HH}. Central bank liabilities—which could represent central bank bills, digital currency, or expanded access to central bank deposits—are assumed to be perfect substitutes for short-term government bonds. While this assumption is not critical to the results, it allows exploration of counterfactuals where central banks issue securities functionally equivalent to short-term bonds. The household's budget constraint in period 1 is given by:

$$B_{LT}^{HH} + B_{ST}^{HH} + D^{HH} + E^{HH} + REVERSE^{HH} + M^{HH} \leq P_1(G+H). \tag{2}$$

In period 2, households consume out of their accumulated wealth, which comprises the returns on their investments, profits from firm and bank ownership (π_F and π_B), and a lump-sum tax T. Specifically,

$$\begin{aligned} P_2C = r_E E^{HH} + r_{B_{LT}} B_{LT}^{HH} + r_{B_{ST}} B_{ST}^{HH} + r_D D^{HH} + r_{REVERSE} REVERSE^{HH} \\ + r_{M^{HH}} M^{HH} + \pi_F + \pi_B - T. \end{aligned} \tag{3}$$

where the gross returns on various assets are denoted by r_E, $r_{B_{LT}}$, $r_{B_{ST}}$, r_D, $r_{REVERSE}$, and $r_{M^{HH}}$.

Households may hold long-term bonds directly or obtain them through reverse repo agreements, with banks providing the bonds as collateral. The liquidity benefits derived from direct bond holdings versus reverse repos may differ due to frictions such as regulatory constraints (e.g., Duffie (1996)).

2.2. Banks

A representative bank optimizes over reserves M^B, loans L^B, short-term government bonds B_{ST}^B, long-term government bonds B_{LT}^B, repo transactions $REPO^B$, and deposits D^B. Equity E^B is determined by the balance sheet identity. The bank receives liquidity benefits, $b(\frac{M^B}{P_1}, \frac{D^B}{P_1})$, which are decreasing in deposits and are realized in real goods. For simplicity, liquidity benefits from other assets are omitted. [1]

The bank maximizes profits subject to three key constraints: (i) a repo constraint ensuring that repo transactions do not exceed bond holdings, (ii) a leverage constraint imposing a minimum equity ratio k, and (iii) a non-negativity constraint on short-term bond holdings. The bank's Lagrangian is expressed as:

$$\begin{aligned}\mathcal{L}^B &= r_{M^B} M^B + r_L L^B + r_{B_{LT}} B_{LT}^B + r_{B_{ST}} B_{ST}^B + P_2 b(\frac{M^B}{P_1}, \frac{D^B}{P_1}) \\ &- r_E E^B - r_D D^B - r_{REPO_B} REPO_B^B \\ &+ \lambda_{REPO}(B_{LT}^B - REPO_B^B) \\ &+ \lambda_{CAP}((1-k)(B_{ST}^B + B_{LT}^B + M^B + L^B) - D^B - REPO^B) \\ &+ \lambda_{BST}(B_{ST}^B).\end{aligned} \tag{4}$$

The bank's optimality conditions yield standard equilibrium relationships where constraints create wedges between asset returns (e.g., Ennis (2018)):

$$r_L = r_{M_B} + P_2 \frac{\partial b}{\partial M^B} \tag{5a}$$

$$r_{B_{LT}} = r_{M_B} + P_2 \frac{\partial b}{\partial M^B} - \lambda_{REPO} \tag{5b}$$

$$r_{B_{ST}} = r_{M_B} + P_2 \frac{\partial b}{\partial M^B} - \lambda_{BST} \tag{5c}$$

$$r_E = r_{M_B} + P_2 \frac{\partial b}{\partial M^B} + (1-k)\lambda_{CAP} \tag{5d}$$

$$r_D = r_{M_B} + P_2 \frac{\partial b}{\partial M^B} + P_2 \frac{\partial b}{\partial D^{HH}} - k\lambda_{CAP} \tag{5e}$$

$$r_{REPO_B} = r_{B_{LT}} - k\lambda_{CAP}. \tag{5f}$$

2.3. Firms

A representative firm borrows L^F at rate r_L to acquire G^F goods from households. Production follows a decreasing returns technology $F(G^F)$, with output sold to households at

[1] There are reasons why reserves may be preferable to other assets such as t-bills for liquidity. For example, the liquidity benefit of reserves could be due to their utility in meeting intraday regulatory liquidity requirements (e.g., d'Avernas and Vandeweyer (2020)) or in making intraday payments (Ashcraft et al. (2011)). Other benefits may be due to the difficulty in rapidly converting other assets such as long-term Treasury securities into cash (Bush et al. (2019)), or because of bank or regulatory preferences for reserves (Covas and Nelson (2019)).

price P_2. The firm's capital goods are fully depreciated in production. Firms solve:

$$\max_{G^F} \Pi_F = P_2 F(G^F) - r_L L^F, \tag{6}$$

subject to $P_1 G^F \leq L^F$. The optimality condition equates the marginal revenue product of capital to the borrowing rate:

$$P_2 F'(G^F) = r_L. \tag{7}$$

2.4. Central bank and government

Government bond issuance, B_{ST} and B_{LT}, is exogenously given and finances government expenditures G^G. The central bank conducts monetary policy by acquiring bonds B_{ST}^{CB} and B_{LT}^{CB}, funded by reserves and central bank bills. Central bank profits,

$$\Pi_{CB} = r_{B_{LT}} B_{LT}^{CB} + r_{B_{ST}} B_{ST}^{CB} - r_{M^B} M^B - r_{M^{HH}} M^{HH}, \tag{8}$$

are remitted to the government, which levies a lump-sum tax to cover net liabilities:

$$T = r_{BLT}(B_{LT} - B_{LT}^{CB}) + r_{BST}(B_{ST} - B_{ST}^{CB}) + r_{M^B} M_B + r_{M^{HH}} M_{HH}. \tag{9}$$

Following Martin et al. (2019), monetary policy determines the price level via an active monetary and passive fiscal policy regime, where the central bank sets r_{M^B} to target inflation $\bar{\pi} = P_2/P_1$.

3. Equilibrium

In this section, I formally define the equilibrium conditions of the model, incorporating the insights developed in previous sections. The equilibrium characterization depends on the set of binding constraints faced by banks, with the general solution allowing for cases where all constraints are binding (i.e., $\lambda_j > 0$ for $j \in CAP, REPO, BST$).

An equilibrium consists of a set of bank allocations (M^B, L^B, B_{LT}^B, B_{ST}^B, E^B, D^B, $REPO^B$), household allocations ($B_{LT}^{HH}, B_{ST}^{HH}, D^{HH}, E^{HH}, REVERSE^{HH}, M^{HH}, H$), firm allocations (L^F, G^F), market-determined interest rates $(r_{M^B}, r_{M^{HH}}, r_L, r_{B_{LT}}, r_{B_{ST}}, r_{REPO})$, and the aggregate price level (P_2). These variables must satisfy three equilibrium conditions: (i) profit maximization by banks (4), (ii) profit maximization by firms (25), and (iii) utility maximization by the representative household (1), subject to market clearing constraints:

$$G + H = G^F + G^G \tag{10a}$$

$$C = F(G^F) \tag{10b}$$

$$B_{LT} = B_{LT}^{CB} + B_{LT}^{HH} + B_{LT}^{B} \tag{10c}$$

$$B_{ST} = B_{ST}^{CB} + B_{ST}^{HH} + B_{ST}^{B} \tag{10d}$$

$$D^{HH} = D^B \tag{10e}$$

$$E^{HH} = E^B \tag{10f}$$

$$REVERSE^{HH} = REPO^B. \tag{10g}$$

From the household's optimization problem, the equilibrium spread between equity returns and the interest rate on liquid assets corresponds to the representative household's marginal liquidity benefit of those assets. Consequently, by imposing structure on the Lagrange multipliers in the bank's first-order conditions (5a) - (5f), the equilibrium conditions for the Lagrange multipliers can be expressed as:

$$\lambda_{CAP} = P_2(\lambda \frac{\partial Q}{\partial D^{HH}} + \frac{\partial b}{\partial D^{HH}}) = \frac{P_2}{1-k}(\frac{\partial c}{\partial H} - \frac{\partial F}{\partial G^F}), \tag{11a}$$

$$\lambda_{REPO} = P_2 \lambda \frac{\partial Q}{\partial B_{LT}^{HH}} - (1-k)\lambda_{CAP}, \tag{11b}$$

$$\lambda_{BST} = P_2 \lambda \frac{\partial Q}{\partial B_{ST}^{HH}} - (1-k)\lambda_{CAP}. \tag{11c}$$

By substituting these Lagrange multipliers back into the bank's first-order conditions (5a) - (5f), I obtain expressions for equilibrium interest rates. In equilibrium, all rates are linearly related to the interest rate on reserves, r_{M^B}. The spreads between r_{M^B} and other rates are determined by the liquidity benefits conferred to households and banks, the capital constraint k, and the cost of deposits. These liquidity benefits, in turn, are influenced by the supply of liquid assets available in the economy. Notably, equation (11a) implies that the capital constraint binds when $\frac{\partial \ell}{\partial D^{HH}} > \frac{\partial b}{\partial D^{HH}}$.

The equilibrium inflation rate is determined by combining equations (5a), (7), and (11a):

$$\pi = \frac{r_{M_B}}{F'(G^F) - \frac{\partial b}{\partial M^B} + (k^L - k^B)(\lambda \frac{\partial Q}{\partial D^{HH}} + \frac{\partial b}{\partial D^{HH}})}. \tag{12}$$

For any given level of central bank assets and liabilities, the central bank can achieve its target inflation rate, $\bar{\pi}$, by adjusting the interest rate on reserves. This result implies that the central bank retains the flexibility to optimize the level of reserves while still meeting its inflation objective through appropriate adjustments to r_{M^B}.

4. Model Calibration

This section calibrates the model to align with prevailing rates and quantities in U.S. money markets. To achieve this, I impose a specific functional form on household and bank liquidity preferences. Household liquidity preferences are incorporated into the utility function (equation 1) using a constant elasticity of substitution (CES) aggregator to capture preferences over liquid assets.[2] This formulation extends the approaches of Drechsler et al. (2017), Krishnamurthy and Li (2023), and Nagel (2016) by incorporating a broader set of assets:

$$\lambda Q() = \lambda(\alpha_D D^{HH\rho} + \alpha_{BST}(B_{ST}^{HH} + M^{HH})^{\rho} + \alpha_{BLT} B_{LT}^{HH\rho} + (1 - \alpha_D - \alpha_{BST} - \alpha_{BLT}) REVERSE^{HH\rho})^{\frac{\nu}{\rho}}, \tag{13}$$

[2]The arguments for the liquidity preferences of households and banks are expressed in real terms. For clarity of exposition, I omit the P_1 denominators, given that $P_1 = 1$.

where $0 < \rho < 1$ governs substitutability across liquid assets, λ reflects the relative weight of liquidity benefits against portfolio returns, and $0 < \nu < 1$ captures diminishing marginal returns to liquidity. In this setup, households treat short-term bonds and central bank bills as perfect substitutes.[3]

For banks, I specify the liquidity benefit function of reserves and deposits as follows:

$$b(M^B, D^B) = -\gamma((M^B - T)\ln(M^B - T) - M^B) + \phi M^B - cD^B. \tag{14}$$

This structure ensures that the marginal benefit of reserves declines logarithmically, similar to the functional form in Lopez-Salido and Vissing-Jorgensen (2024):[4]

$$\frac{\partial b}{\partial M^B} = \phi - \gamma ln(M^B - T), \tag{15}$$

where ϕ represents a reserve demand shock, $\gamma > 0$ captures the declining marginal utility of reserves, and T represents a threshold level of reserves demanded by banks. Deposits impose a linear marginal cost due to their liquidity demands and potential regulatory constraints Acharya and Rajan (2024):

$$\frac{\partial b}{\partial D^B} = -c. \tag{16}$$

4.1. Empirical Rate Counterparts

The empirical calibration focuses on the period from 2013 to mid-2024. This time frame is chosen to capture the post-crisis regulatory landscape, including Basel III liquidity regulations. To ensure consistency with the model's deterministic structure, I focus on short-term rates with minimal credit risk. I use end-of-quarter rates sourced primarily from the Federal Reserve Bank of St. Louis' FRED database. Unless otherwise noted, all rates are expressed relative to the interest on reserves. Short-term bond yields are converted into overnight rates by adjusting the one-month Treasury bill rate using the spread of one-month overnight index swaps over the effective federal funds rate. This adjustment removes expected policy rate changes from the measure of r_{BST}. I use the Secured Overnight Financing Rate (SOFR) as a proxy for the expected returns on long-term government bonds, consistent with the model's treatment of banks viewing reverse repos and bonds as perfect substitutes. Bank deposit rates, r_D, are equated to the effective federal funds rate, while the bank lending rate, r_L, is computed as the average of the overnight-equivalent, 30-day commercial paper rate and the expected return on long-term government bonds plus the option-adjusted spread of the ICE BofA AAA US Corporate Index.

Figure 2 shows that rates (on an overnight basis) were mostly below the interest on reserves, except for the bank lending rate. The bank lending rate remained above the interest on reserves throughout the period, reflecting the need for a higher return to compensate for

[3] If the central bank were to introduce a central bank digital currency (CBDC), the CBDC's representation in the utility function might differ. See Bordo and Levin (2017) for a discussion on CBDC characteristics.

[4] Unlike Lopez-Salido and Vissing-Jorgensen (2024), the specified function includes a threshold to account for a desired minimum quantity of reserves.

less liquidity relative to reserves. This rate spiked in March 2020, reflecting market stress at the onset of the pandemic. The only period when these yields were above the interest on reserves occurred in 2019, with the largest spike above the interest on reserves coinciding with the repo pressures in September 2019.

4.2. Empirical Quantity Counterparts

Because the U.S. economy comprises more sectors than the model, mapping observed quantities to the four financial agents in the model—households, banks, the central bank, and the government—requires simplifications. These adjustments are made while ensuring that the balance sheet identities of the agents remain consistent. The following section outlines the key modeling choices underlying this mapping.

Balance sheet quantities are sourced from FRED, with most data originating from the Federal Reserve's Flow of Funds. Bank reserves M^B are measured as Depository Institution Reserves (MADIRL). Since households do not currently hold central bank bills, their reserve holdings M^{HH} are set to zero, allowing for counterfactual analysis in later sections.

Household holdings of short-term bonds B_{ST}^{HH} are estimated as net Treasury bills outstanding (BOGZ1FL313161110Q - BOGZ1FL713061113Q), while holdings of long-term bonds B_{LT}^{HH} are calculated as net Treasury bonds outstanding (BOGZ1FL313161275Q - BOGZ1FL713061125Q) plus overnight reverse repos at the Federal Reserve (BOGZ1FL712151103Q), less long-term bond holdings of banks.

To maintain the central bank's balance sheet identity, I measure the central bank's holding of long-term bonds as reserves less its t-bill holdings (BOGZ1FL713061113Q). That is, $B_{LT}^{CB} = M^B - B_{ST}^{CB}$.[5]

Banks are assumed not to hold any t-bills, which is consistent with the model when t-bill yields are below the interest on reserves, as in the data. Banks' long-term bond holdings, B_{LT}^{B}, include banks' treasury holdings and reverse repo positions with the private sector (BOGZ1FL762051005Q + BOGZ1FL752051005Q + BOGZ1FL662051003Q), since these assets are identical for banks within our overnight model. Bank deposits, $D^B = D^{HH}$, consist of checkable deposits plus time and saving deposits (BOGZ1FL703127005Q + BOGZ1FL703130005Q). Bank repos are measured as the sum of chartered bank repos, foreign bank repos, and dealer repos (BOGZ1FL762151005Q + BOGZ1FL752151005Q + BOGZ1FL662151003Q). From the model, bank repos represent reverse repos from the household's perspective. To facilitate cross-period comparisons, all assets are normalized by household wealth ($B_{LT}^{HH}+B_{ST}^{HH}+M^{HH}+E^{HH}+D^{HH}$). Figure 1 illustrates stable relative quantities across most of the sample period, except during the early 2020 pandemic shock, when reserves and short-term bonds increased sharply while long-term bond holdings declined due to the Federal Reserve's quantitative easing programs.

4.3. Calibrated Parameters

Table 1 summarizes the calibrated parameters. Bank regulation primarily determines these values. The capital constraint ($k = 0.06$) aligns with regulatory leverage ratio require-

[5]Since we are ignoring money holdings, our measure of central bank long-term bond holdings is likely understated.

ments, reflecting observed banking practices Hintze (2022).[6] The cost of deposits is set at $c = 0.1$, consistent with the average FDIC fees documented by Banegas and Tase (2020).

Using first-order conditions from equations 5a and 5e, I estimate γ and T via equation 15, restricting data to pre-September 2019 to mitigate repo market distortions. This yields an estimate of $\gamma = 0.0063$ and $T = 0.046$:

$$\frac{\partial b}{\partial R^B} = -\underset{(0.005)}{0.063} ln(R^B - \underset{(0.003)}{0.046}). \tag{17}$$

Household's liquidity preferences are restricted to $0 < \nu, \rho < 1$ so that there are diminishing returns to liquidity. For parsimony and ease of estimation, I assume that $\nu = \rho$. Taking the derivative of equation 13, and then taking logs, I can represent the marginal liquidity benefit of a given household asset A by the following equation:

$$ln(\frac{\partial \ell()}{\partial A}) = ln(\rho \lambda \alpha_A) + (\rho - 1) ln(A). \tag{18}$$

Given the assumed parameter estimates so far, I can measure the household's liquidity benefits of the different assets using equations 5a through 5f. For households, I estimate liquidity preferences by regressing the marginal liquidity benefit of asset A on its log quantity. Pooled OLS implicitly assumes that the αs for each asset class are the same (I get parameter estimates of these αs later and show that they are similar):

$$ln(\frac{\partial \ell()}{\partial A}) = 1.21 - 0.03 ln(A). \tag{19}$$

The coefficient estimate implies $\rho = 0.97$, which is consistent with Nagel (2016)'s estimate of $\rho = 1$ but exceeds Krishnamurthy and Li (2023)'s estimate of $\rho = 0.6$.

Table 1: Calibrated Model Parameters

Household Parameter	Value	Bank Parameter	Value
ρ	0.97	k	0.06
ν	0.97	c	0.10
		γ	0.063
		T	0.046

Remaining parameters (λ, α_D, α_{BST}, α_{BLT}, ϕ) are calibrated period-by-period to match observed yields using equations 5a-5f, ensuring perfect yield alignment. Figure 3 shows

[6] Banks are required to maintain a leverage ratio that exceeds 5% including a G-SIB add-on, but in practice have a leverage ratio close to 6%. This higher leverage ratio could reflect additional, confidential leverage requirements that regulators impose on individual institutions as discussed in Allen and Wittwer (2022).

that ϕ, capturing reserve demand shocks, spiked in March 2020 and subsequently declined, mirroring fluctuations in λ. The relative stability of α parameters supports the assumption that liquidity preferences remain consistent over time. These calibrated parameters provide a foundation for analyzing the optimum quantity of reserves in the subsequent section.

5. The Optimum Quantity of Central Bank Reserves

In the model, overall welfare is a function of several factors. Households derive benefit from holding liquid assets instead of equity. However, when banks are constrained by equity requirements, more equity allows them to issue more deposits, which provides liquidity benefits to households but also entails a cost for banks. By issuing equity, banks can also increase the asset side of their balance sheet. This allows banks to hold more reserves, which provides liquidity benefits.

A central planner, subject to the same regulatory and technological constraints as market participants, optimally determines household and bank portfolios, firm borrowing, and production, as well as the composition of the central bank's balance sheet. By the first welfare theorem, the planner's allocation coincides with the competitive equilibrium under the appropriate central bank balance sheet policy.

$$\begin{aligned} SW_{baseline} =& \ell(D^{HH}, B^{HH}_{LT}, REVERSE^{HH}, B^{HH}_{ST} + M^{HH}) \\ &+ b(M^B, D^B) + F(G^F) - c(H) \\ &+ \lambda_{REPO}(B^B_{LT} - REPO^B_B) \\ &+ \lambda_{CAP}((1-k)(B^B_{ST} + B^B_{LT} + M^B + L^B) - D^B - REPO^B) \\ &+ \lambda_{B_{ST}} B^B_{ST} \\ &+ \lambda_{CB_{BLT}} B^{CB}_{LT} \\ &+ \lambda_{CB_{BST}} B^{CB}_{ST} \\ &+ \lambda_{CB_{MB}} M^B \\ &+ \lambda_{CB_{MHH}}(B^{CB}_{LT} + B^{CB}_{ST} - M^B). \end{aligned} \tag{20}$$

The first term on the right-hand side of equation (20) represents the liquidity benefits that households derive from holding deposits, government bonds, reverse repos and central bank bills. The second term on the right-hand side represents the social benefits from banks holding higher reserves as well as the costs of bank deposits. The third term on the right-hand side represents production. The fourth term represents disutility of working. The first set of constraints in the Lagrangian corresponds to the representative bank's regulatory and non-negativity constraints. Meanwhile, the remaining four constraints in the Lagrangian represent the central bank's non-negativity constraints. The central bank's holdings of short-term and long-term bonds are not constrained to be less than the amount issued by the government since the central bank will never find it optimal to exceed government issuance. Notably, the household would derive infinite utility from the liquidity benefits of its short-term and long-term bond holdings as those holdings approach 0.

Let us first consider the case where the central bank cannot issue central bank bills ($M^{HH} = 0$). In this case, the social planning central bank will select B^{CB}_{ST}, B^{CB}_{LT}, and R^B to maximize social welfare. Since government bond issuance is fixed, this implies that $\frac{\partial L}{\partial M^B} = \frac{\partial H}{\partial M^B}$. The two first-order conditions associated with the choice of central bank balance sheet positions are:

$$\frac{\partial \ell}{\partial B^{HH}_{LT}} - \lambda_{CB_{BLT}} = \frac{\partial b}{\partial M^B} + (1-k^B)(\frac{\partial \ell}{\partial D^{HH}} + \frac{\partial b}{\partial D^{HH}}) \tag{21}$$

$$\frac{\partial \ell}{\partial B^{HH}_{ST}} - \lambda_{CB_{BST}} = \frac{\partial b}{\partial M^B} + (1-k^B)(\frac{\partial \ell}{\partial D^{HH}} + \frac{\partial b}{\partial D^{HH}}). \tag{22}$$

These two conditions respectively show the social optimum central bank holdings of long-term and short-term bonds. They also determine the optimum quantity of reserves through the identity of the central bank balance sheet (given $M^{HH} = 0$). Ignoring the Lagrangian coefficient, the left-hand side of equation 21, which shows the marginal social cost of an increase in central bank holdings of long-term bonds, is equal to the marginal liquidity benefit of household holdings of government long-term bonds. The right-hand side of equation 21 shows the marginal social benefit. Banks gain an extra dollar of reserves, and the first term represents the marginal direct benefit that banks receive from holding more reserves. The remaining term represents the marginal liquidity benefit of household deposits and the marginal cost to banks of deposits after accounting for capital constraints. As long as the central bank is not constrained by its non-negativity constraint (i.e., as long as $\lambda_{CB_{BLT}} = 0$), it will equate this marginal social cost and benefit. A similar result holds for short-term bonds because of the similarity of equations 21 and 22.

Proposition 1 summarizes the socially optimal relationship between the return on bonds and the return on central bank reserves in the model.

Proposition 1. *The socially optimal relationship between the return on bonds and the return on reserves is a constrained Friedman rule:*

$$r_{B_{LT}} + \lambda_{CB_{BLT}} = r_{B_{ST}} + \lambda_{CB_{BST}} = r_{M^B}. \tag{23}$$

Proof. From the household's first order conditions, $r_{B_{LT}} = r_E - P_2 \frac{\partial \ell}{\partial B^{HH}_{LT}}$, $r_{B_{ST}} = r_E - P_2 \frac{\partial \ell}{\partial B^{HH}_{ST}}$ and $r_D = r_E - P_2 \frac{\partial \ell}{\partial D^{HH}}$. Then, substitute these three equations along with equations (5e) and (11a) into (21) and (22). □

This Friedman rule, together with the equilibrium bank repo rate in equation 5f, implies that $r_{M_B} \geq r_{B_{LT}} > r_{REPO}$ is the social optimum when banks are subject to non-zero capital costs. This suggests that targeting the bank repo borrowing rate to equal the return on reserves would result in a central bank balance sheet that would be smaller than the social optimum (i.e., the return on long-term bonds is above the return on reserves so the central bank should remove more bonds from the market). Left unclear is whether this small wedge due to the capital costs between these two rates is economically important. It is much smaller than the wedge between the return on bonds and the return on currency implied by the traditional Friedman rule. Further, Curdia and Woodford (2011) consider a deviation of

25 basis points (bps) from their optimum policy rule to be quite small, and capital costs in the model are smaller than 25 bps.

Banks will be constrained in their long-term bond holdings in the social optimum ($B^B_{LT} = REPO^B$). To see this, the social optimum condition for long-term bonds implies that the Lagrangian coefficient for holding long-term bonds (λ_{REPO}) is greater than the marginal benefit banks get for holding reserves. Since the marginal benefit for holding reserves is greater than zero $\frac{\partial b}{\partial M_B} > 0$, the Lagrangian coefficient must also be greater then zero, meaning banks are constrained. Martin et al. (2019) find that it is socially optimal to have a floor system with an ample amount of reserves that is not quite abundant.

The model's constrained Friedman rule would suggest that the optimal monetary policy operating system depends on the environment. I define a zero-reserves system as one in which $M_B \to 0$. Technically, it will always be socially optimal to hold a non-zero amount of reserves since equation 15 implies that $\frac{\partial b}{\partial R^B} = \infty$ when $M^B = 0$.

- If there is small gross supply of short-term and long-term bonds such that the central bank is constrained, then a zero-reserves system would be socially optimal (i.e., $B^{CB}_{ST} = B^{CB}_{LT} \approx 0$). In this case, banks are also constrained and do not hold short-term bonds. Expanding the central bank balance sheet beyond a zero-reserves regime would remove assets with a high marginal liquidity benefit from the market and replace them with assets with a lower marginal liquidity benefit.

- If there is a large gross supply of long-term bonds but a small gross supply of short-term bonds where commercial banks are constrained in their short-term bond holdings (i.e., $B^B_{ST} = 0$), then a partially constrained Friedman rule is optimal ($r_{B_{ST}} < r_{B_{LT}} = r_{M_B}$). In this environment, the central bank is not constrained in its long-term bond holdings ($\lambda_{CB_{B_{LT}}} = 0$) but is constrained in its short-term bond holdings ($\lambda_{CB_{B_{ST}}} > 0$) and can implement a partially constrained Friedman rule.

- If there is a small enough gross supply of long-term bonds but a large gross supply of short-term bonds where commercial banks are not constrained in their short-term bond holdings (i.e., $B^B_{ST} > 0$), then a partially constrained Friedman rule is also optimal ($r_{B_{LT}} < r_{B_{ST}} = r_{M_B}$). In this environment, the central bank is not constrained in its short-term bond holdings ($\lambda_{CB_{B_{ST}}} = 0$) but is constrained in its long-term bond holdings ($\lambda_{CB_{B_{LT}}} > 0$).

- If there is a large gross supply of short-term and long-term bonds, then a central bank can implement an unconstrained Friedman rule ($r_{B_{LT}} = r_{B_{ST}} = r_{M^B}$). By equations 5b and 5c, the unconstrained Friedman rule implies that $\frac{\partial b}{\partial M^B} = \lambda_{REPO} = \lambda_{BST}$.

I illustrate the effect of the environment on the socially optimum level of reserves in the calibration section below.

5.1. The optimum quantity of reserves in the calibrated model

I conduct two exercises to illustrate the social optimum level of reserves in the calibrated model. In the first exercise, I use the actual gross supply of bonds during the sample period along with the calibrated parameters to estimate the optimum level of central bank bond holdings in each sample period. In the second exercise, I use calibrated parameters from June 2013 and June 2021 and look at how equilibrium yields and the social optimum are impacted by central bank bond holdings and the gross outstanding supply of bonds.

Figure 4a compares the optimal amount of reserves from 2013–2024 with the actual amount of the Fed's reserves. The optimum quantity of reserves has increased over time. This is mainly a reflection of growth in the economy. When reserves are scaled by household wealth, there is no trend (Figure 4b). Apart from the repo market stress in 2019, actual reserves have been above the optimum quantity of reserves. This behavior is consistent with the intuition of the optimum rule. In Figure 2, bond and bill yields are only above the interest on reserves when actual reserves are below optimal reserves. The model rule suggests it is optimal to increase reserves when bond and bill yields are higher than the interest on reserves.

The quantity of reserves fell below the optimal level in December 2018, approximately nine months before the repo market disruptions of September 2019. Reserves continued to decline leading up to the stress episode. However, the model indicates that the optimal quantity of reserves increased sharply in September 2019, implying that reserves would have needed to rise by several trillion dollars in that quarter to reach the optimal level. This result arises because the model shifts the reserve demand curve to align with prevailing yields in September 2019, requiring a substantial movement along the demand curve to implement the constrained Friedman rule. The pronounced increase in the model-implied optimal reserves suggests the presence of nonlinearities in reserve demand that the framework does not explicitly capture. Potential drivers of these nonlinearities include banks strategically hoarding reserves by delaying interbank payments (Yang (2020)) and the uneven distribution of reserves across more and less active intermediaries (Copeland et al. (2021)).

To illustrate these two effects more directly, Figure 5 shows how yields vary with the quantity of central bank reserves. To produce this figure, I fix the central bank holdings of short-term bonds at 0 in the model, so all changes in reserves come from central bank holdings of long-term bonds. The model uses the calibrated parameters from June 2013 and June 2021 to illustrate how changes in preferences and the environment impact the optimum quantity. The figure illustrates two demand curves. The first demand curve uses parameters from June 2013 and illustrates how the reverse repo yield (i.e., SOFR) changes when the Fed expands its balance sheet while the second demand curve uses parameters from June 2021. To put these curves in context, the figure also includes plots of SOFR at the end of each quarter (above interest on reserves) versus the scaled quantity of reserves. By construction, the respective demand curves will pass through the dots representing the observations for June 2013 and June 2021. The social optimum value is where the SOFR demand curve intersects the x-axis, given this is the point where SOFR equals interest on reserves, which is approximately 5% in June 2013 and over 6% in June 2021.

Despite being calibrated to June 2013 and June 2021 data, the demand curves fit the observations relatively well. The decent fit is due to the ρ parameter, which determines the slope of the demand curve. If we were to increase the parameter towards 1 from its current value of $\rho = 0.97$, the demand curve would be flatter. This would imply a lower social optimum quantity of reserves (again, since by construction the curve contains the observation for June 2013 and June 2021). On the other hand, if the ρ parameter was much lower, the demand curve would become steeper and the social optimum would become closer to the actual amount of reserves in June 2013 and June 2021.

5.2. The optimum quantity of reserves with central bank bills

I have modelled central bank bills (also interpreted as CBDC or expansion of the central bank balance sheet to non-bank counterparties) as a perfect substitute for short-term bonds in household portfolios. In this section, I investigate the impact on the constrained Friedman rule from introducing central bank bills by allowing $M_{HH} \geq 0$ instead of $M_{HH} = 0$. In effect, this is the same as relaxing the non-negativity constraint on central bank holdings of short-term bills.

Proposition 2. *With the ability to issue central bank bills, the constrained Friedman rule becomes:*

$$r_{B_{LT}} + \lambda_{CB_{BLT}} = r_{B_{ST}} = r_{M^B}. \tag{24}$$

Proof. Since $\frac{\partial b}{\partial M^B} = \infty$ when $M^B = 0$, the central bank reserves constraint will not bind except in the degenerate case when the government does not issue any bonds. The rest of the proof follows the proof for the optimum without central bank bills. □

By construction, the social optimum in Proposition 2 does not differ from the social optimum in Proposition 1 when $B_{ST}^{CB} > 0$ is optimal in both.[7] When it is optimal to issue central bank bills ($M_{HH} > 0$ and $B_{ST}^{CB} = 0$), two possibilities emerge for the social optimum.

First, the central bank could implement an unconstrained Friedman rule ($r_{B_{LT}} = r_{B_{ST}} = r_{M_B}$). In this case, which is shown in Figure 6, there is a sufficient gross supply of long-term bonds. The central bank is not constrained in that it can issue the required amount of central bank bills and central bank reserves by removing long-term bonds from the market without its long-term bond non-negativity constraint becoming binding.

Second, it is possible that the gross supply of long-term bonds may be scarce such that the central bank cannot leave enough long-term bonds in the market to implement the unconstrained optimum. In this case, market rates would be such that $r_{B_{LT}} < r_{B_{ST}} = r_{M^B}$.

Figure 7 illustrates what happens to the social optimum level of reserves when the central bank has the ability to issue central bank bills. In this case, the model produces a social optimum level of reserves that is slightly lower. Assuming the central bank is not constrained in its long-term bond holdings, its holdings of long-term bonds must increase to accommodate the increase in central bank bills. This follows from equation 23 when $\lambda_{CB_{BLT}} = 0$, since central bank bills must increase and household holdings of long-term bonds must decrease to equalize their yields. Since long-term bond yields must fall, the left hand side of equation 21 must increase. Reserves must decrease for the right hand side of equation 21 to equal the increase on the left hand side.

6. Extension: Shadow banking production of liquid assets

The baseline model has thus far considered banks as the sole financial intermediaries. In practice, various other institutions perform similar intermediation functions. For lack

[7] We assume that the central bank either holds short-term bonds or issues central bank bills, but does not do both.

of a better term, we refer to these institutions as shadow banks. These entities compete with traditional banks in the lending market, offering loans to firms at the same rate, r_L. However, shadow banks may operate with less efficiency, incurring a monitoring cost, $m(X)$, which is an increasing function of the amount of lending, $X \geq 0$. Specifically, $m'(X) \geq 0$, $m'(0) = 0$, and $m''(X) > 0$, reflecting that as shadow banks extend more credit, they either lend to riskier borrowers or face higher marginal monitoring costs. Alternatively, $m(X)$ may represent an insurance cost, increasing at an accelerating rate with shadow bank lending. For simplicity, assume that shadow banks fund themselves by issuing securitized assets—such as asset-backed commercial paper (ABCP)—that provide liquidity benefits similar to those of short-term government bonds. While ABCP is not perfectly liquid, it serves as a short-term funding instrument commonly used by corporations for liquidity management. This assumption captures the fundamental incentive of shadow banks to supply short-term assets when short-term government bond yields are relatively low. To ensure solvency, shadow banks must over-collateralize, requiring a fraction z, $0 \leq z \leq 1$, of lending to be financed with equity, with the remainder, $(1 - z)$, funded through the securitized asset. The profit function for shadow banks is thus:

$$\Pi_{SB} = (r_L - zr_E - (1 - z)r_{BST})X - m(X). \tag{25}$$

At an interior optimum, shadow banks choose X such that:

$$r_{BST} = r_L - \frac{\partial m}{\partial X} - z\frac{\partial \ell}{\partial B_{ST}}, \tag{26}$$

where z is assumed to be sufficiently small to ensure an interior solution.[8]

Next, we examine how shadow banking influences the socially optimal level of reserves. In addition to monitoring costs, shadow banking activity may generate negative externalities, $\omega(X)$, which are increasing in X ($\omega''(X) > 0$). These externalities could represent costs associated with financial instability, such as bankruptcy deadweight losses or inefficient runs on shadow banks. While competitive agents do not internalize these costs, a social planner accounts for them in welfare maximization. This framework aligns with Williamson (2016), who models negative externalities in the production of drugs. Incorporating $m(X)$ and $\omega(X)$ into the baseline social welfare function yields the following optimality condition:

$$r_{B_{LT}} - P_2\frac{\partial \omega}{\partial X}\frac{\partial X}{\partial B_{LT}} + \lambda_{CB_{BLT}} = r_{B_{ST}} - P_2\frac{\partial \omega}{\partial X}\frac{\partial X}{\partial B_{ST}} + \lambda_{CB_{BST}} = r_{M_B}. \tag{27}$$

In the absence of externalities ($\omega = 0$ for all X), the social optimum maintains the constrained Friedman rule. However, with shadow banking externalities, the optimal policy implies a higher yield on both long-term and short-term bonds relative to reserves. This result suggests that the central bank's balance sheet should be smaller than in a setting without externalities, contrasting with Greenwood et al. (2016). In their model, bank reserves crowd out private intermediation, whereas in this framework, government bills displace private

[8] This holds if $z < 1 - \frac{(1-k^B)\frac{\partial \ell}{\partial D_{HH}}}{\frac{\partial \ell}{\partial B_{ST}}}$ at $X = 0$.

sector intermediation, and reserves indirectly reduce the supply of government bills.

As in the baseline case, the central bank may be constrained by its non-negativity condition, preventing it from fully implementing the social optimum. When this constraint binds, shadow banking expands beyond the socially optimal level because shadow banks capitalize on the low yields of short-term debt to issue liquid securities. The introduction of central bank bills can mitigate these distortions in two ways: first, by dampening shadow banking externalities when the central bank is otherwise constrained; and second, by increasing the supply of liquid assets available to households, thereby improving social welfare.

7. Conclusion

This paper examines the optimal quantity of reserves within a static framework, deriving a constrained Friedman rule as the optimal policy outcome. The model is agnostic ex ante regarding whether a large central bank balance sheet is optimal, demonstrating that the optimal level of reserves depends critically on the broader financial environment. In particular, because the optimal reserve quantity is shaped by the marginal benefits not only of reserves but also of other financial assets, the model highlights how shifts in the supply of alternative safe assets influence the equilibrium quantity of reserves.

Several avenues for future research remain. First, the analysis assumes a representative bank, but heterogeneity in bank balance sheets, liquidity preferences, and funding constraints could alter the optimal reserve level. Introducing such heterogeneity, as emphasized by Kashyap and Stein (2000) and Copeland et al. (2021), could provide a more nuanced understanding of reserve demand and financial stability implications. Second, the social planner's objective function could be expanded to account for additional considerations, such as the potential impact of a large central bank balance sheet on inflation risk, central bank profitability, or institutional independence. Third, relaxing the assumption of perfect competition in the deposit market—allowing for bank market power—could generate additional deviations from the Friedman rule and alter policy prescriptions. These extensions present fruitful directions for future research.

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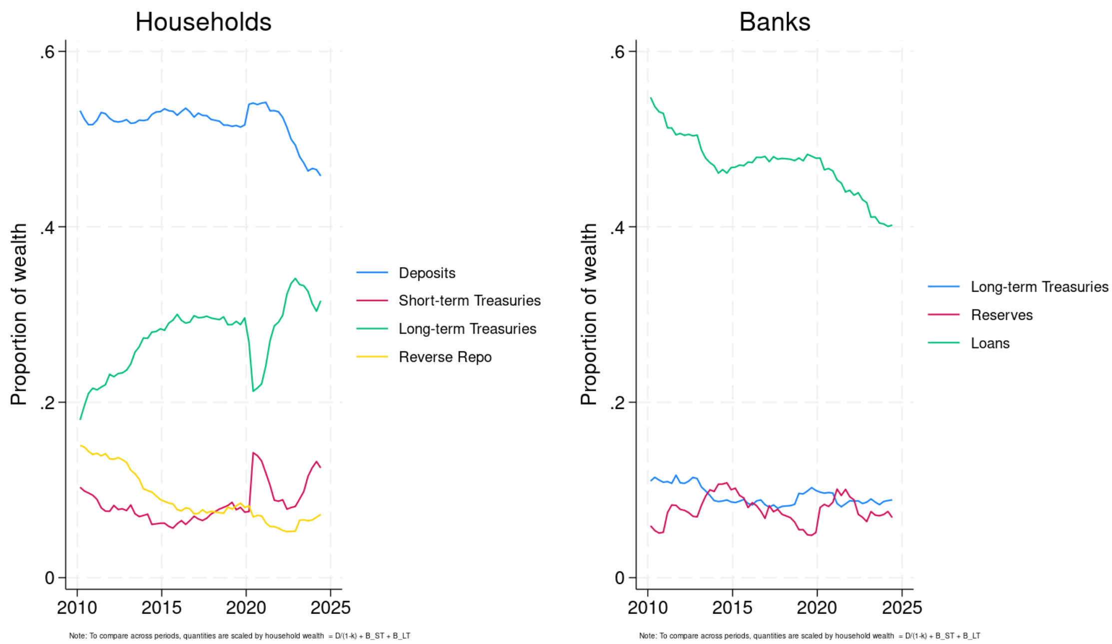
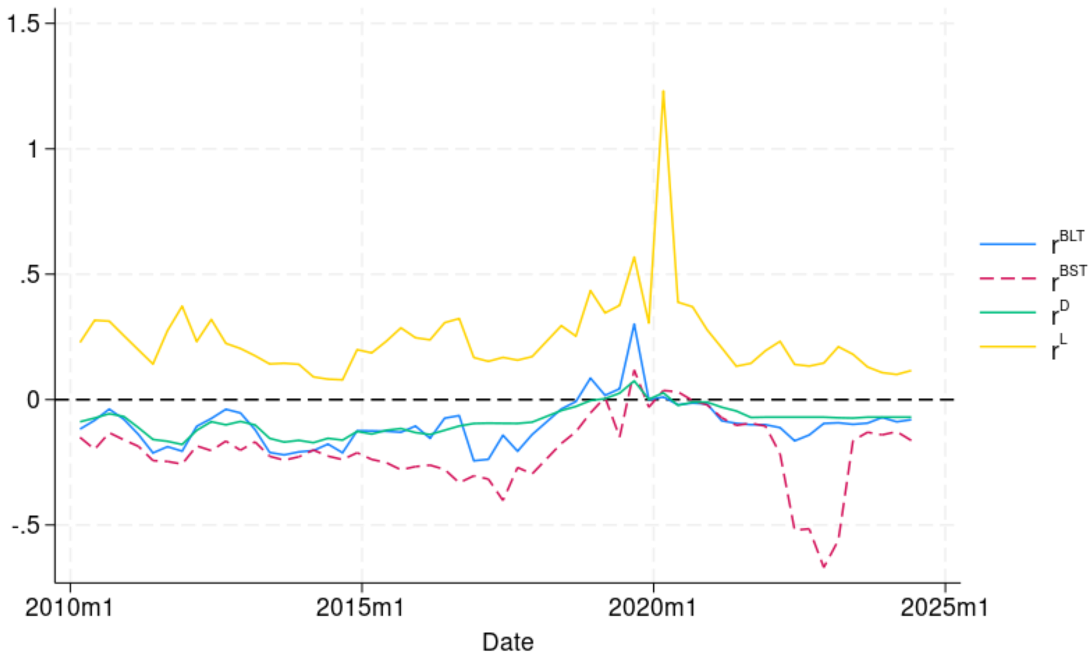
8. Figures

Figure 1: Quantities



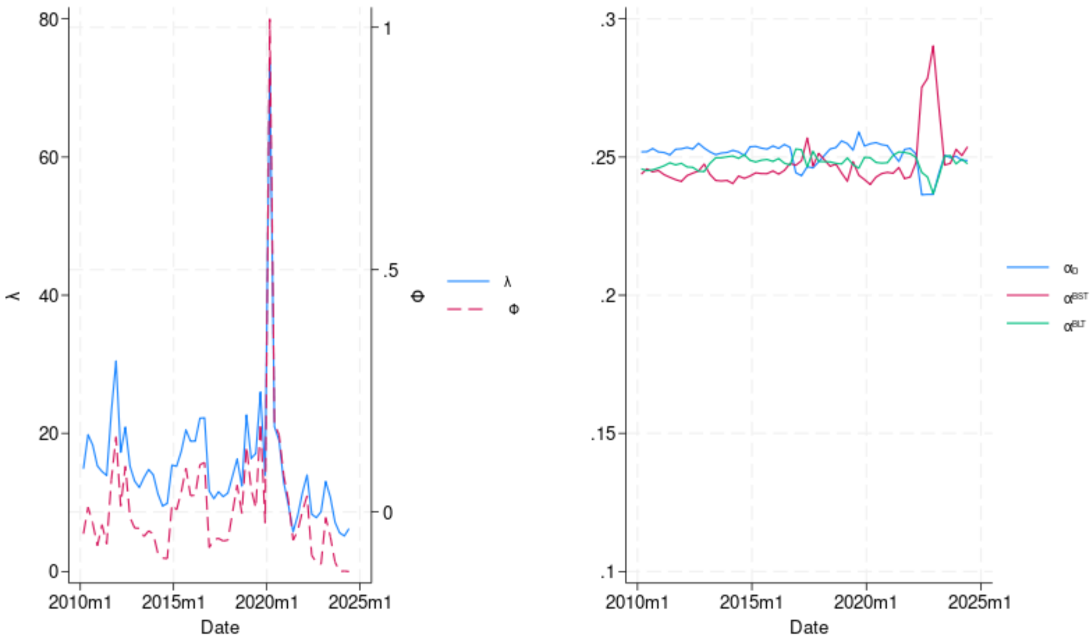
The left side of the above figure displays the quantities of deposits, government t-bills, government bonds, and reverse repos held by households, as a proportion of household wealth. The right side of the above figure displays the quantities of reserves, government t-bills, and government bonds held by banks, as a proportion of household wealth.

Figure 2: Yields used for calibration



This figure displays the monthly yields used in the analysis, relative to the interest on reserves. r^{BLT} represents the daily expected return on long-term government bonds and is equal to SOFR. r^{BST} represents the short-term t-bill yield and is calculated as the 1-month t-bill yield converted back to an overnight rate by subtracting the difference between 1-month overnight index swap rates and the effective federal funds rate. r^{D} represents the yield on deposits and is calculated as the effective federal funds rate. r^{L} represents the yield on bank loans and is equal to the average of: the 30- day commercial paper rate converted to an overnight figure and the expected return on long-term government bonds plus the option-adjusted spread of the ICE BofA AAA US Corporate Index.

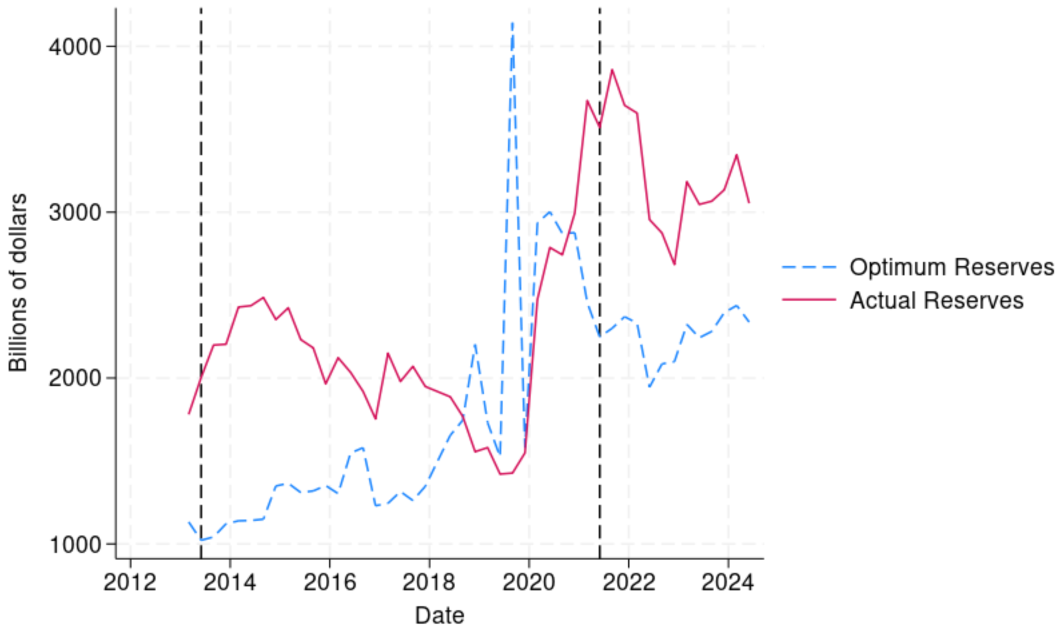
Figure 3: Parameter values



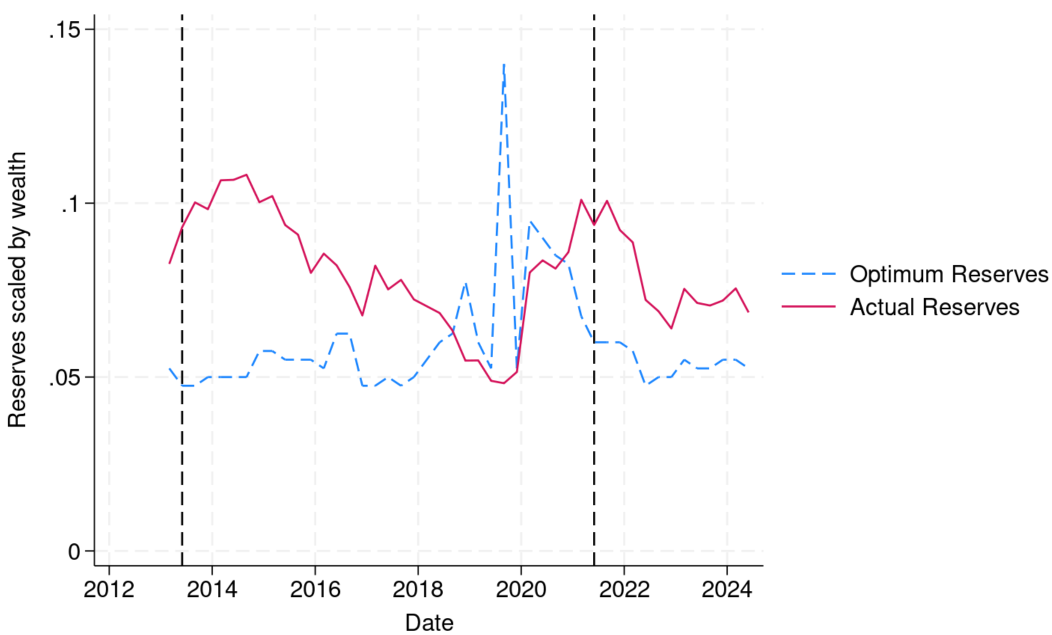
The above figure presents the parameter values used to reproduce actual yields in each period. The left-hand side chart display the parameters λ and ϕ from equations 13 and 15, respectively. The right-hand side chart displays α^{D}, α^{BST}, and α^{REV} from equation 13.

Figure 4: Actual reserves vs. optimum reserves

(a) Unscaled

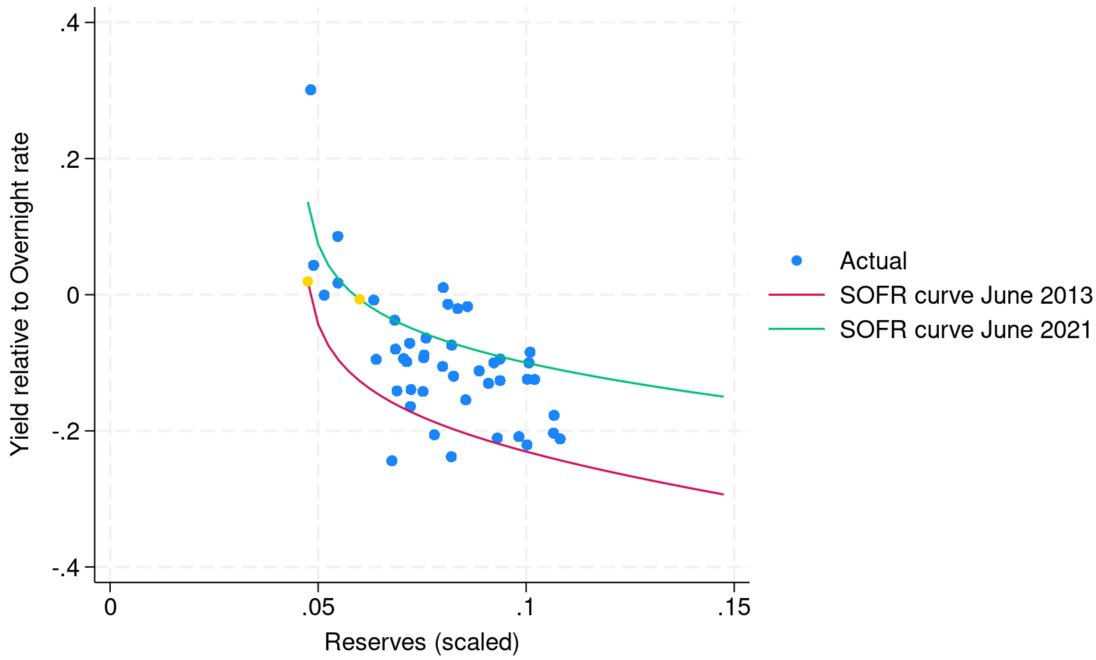


(b) Scaled by wealth



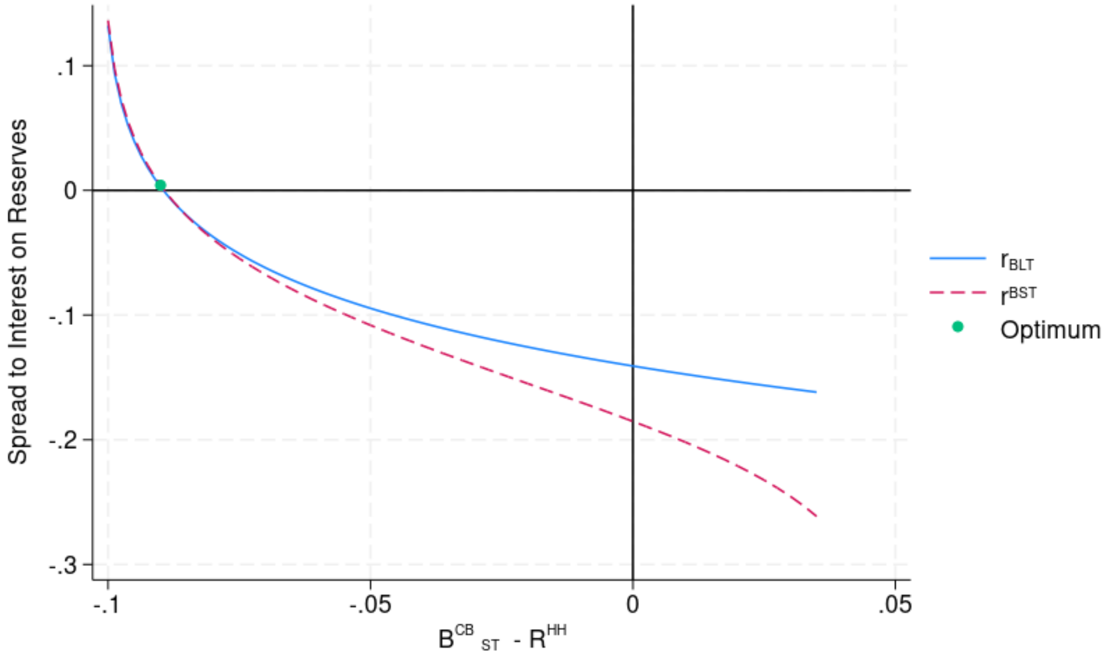
This figure displays the actual level of reserve balances over time, measured as Depository Institution Reserves from the Flow of Funds tables. It compares this actual level of reserves to the model's estimate of the social optimum level of reserves based on the calibrated parameters in each period.

Figure 5: Demand curve: June 2013 and June 2021



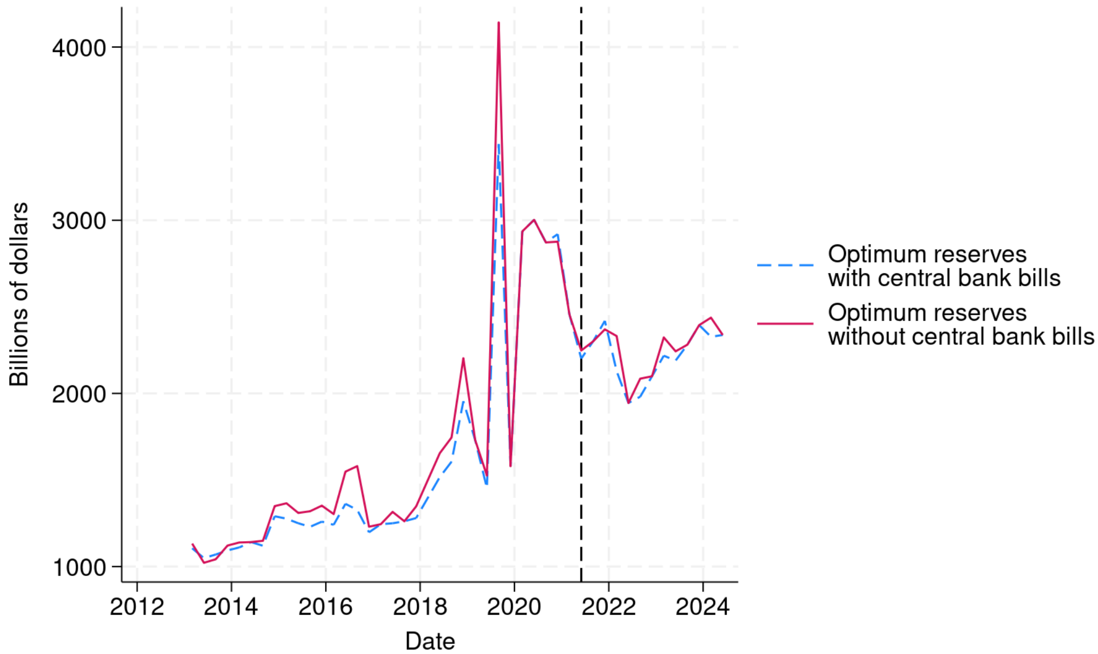
The above figure displays the model-estimated demand curve for SOFR based on calibrated parameters in June 2013 and June 2021.

Figure 6: Social optimum with central bank bills



This figure plots equilibrium yields against central bank short-term bond holdings, less central bank bills. This figure uses the calibrated parameters from June 2021 and assumes that $B_{LT} = 0.5$, $B_{ST} = 0.05$, and $B_{LT}^{CB} = 0.1475$. The social optimum displayed in the figure is at the intersection of the three lines.

Figure 7: Optimum reserves with and without central bank bills



This figure compares the model's estimate of the social optimum level of reserves with and without the ability to issue central bank bills, based on the calibrated parameters in each period.