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Fire Sales and Liquidity Requirements

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Abstract

We study liquidity requirements in a framework with fire sales. The framework nests three common pricing mechanisms—cash-in-the-market, second-best-use, and adverse selection—and can produce the same observables under different pricing mechanisms. We identify three forces that shape the optimal policy. Absent risk-sharing considerations, the equilibrium is efficient with cash-in-the-market pricing; a liquidity requirement is optimal with second-best-use pricing; and a liquidity ceiling (i.e., a cap on liquid assets) is optimal with adverse selection. Accounting for risk-sharing considerations, we find the optimal level of liquidity remains higher with second-best-use pricing relative to cash-in-the-market pricing, and a liquidity ceiling remains optimal with adverse selection.

Topics: Asset pricing; Financial markets; Financial system regulation and policies

JEL codes: G12, G23, G28

Résumé

Nous étudions les exigences de liquidité dans un cadre de ventes en catastrophe (ou « liquidations ») d'actifs. Ce cadre intègre trois mécanismes communs d'établissement des prix – fondés sur la liquidité disponible sur le marché, sur la vente à des acheteurs qui feront une utilisation sous-efficace des actifs ou sur l'antisélection – et peut produire les mêmes variables observables avec chacun de ces mécanismes. Nous dégageons trois facteurs qui déterminent la politique optimale. Si nous ne tenons pas compte des considérations liées au partage des risques, l'équilibre est efficace (au sens de Pareto) quand les prix sont déterminés par la liquidité disponible sur le marché; une exigence de liquidité est optimale lorsque les actifs sont vendus à des acheteurs qui en feront une utilisation sous-efficace; et un plafond de liquidité (c'est-à-dire un plafond sur les actifs liquides) est optimal en présence d'antisélection. En revanche, si nous tenons compte des considérations liées au partage des risques, nous constatons que le niveau de liquidité optimal demeure plus élevé lorsque les actifs sont vendus à des acheteurs qui en feront une utilisation sous-efficace que lorsque les prix sont déterminés par la liquidité disponible sur le marché, et qu'un plafond de liquidité reste optimal en présence d'antisélection.

Sujets : Évaluation des actifs; Marchés financiers; Réglementation et politiques relatives au système financier
Codes JEL : G12, G23, G28

1 Introduction

Fire sales are common phenomena in periods of financial distress. These episodes are characterized by large sales of financial assets and a reduction in their prices, despite little to no change in the fundamentals, and they occur when investors are forced to sell their assets for various reasons. Examples abound across markets and asset classes, ranging from assets held by distressed banks (Granja, Matvos, and Seru, 2017) to asset-backed securities (Merrill et al., 2021) and highly rated corporate bonds (Falato, Goldstein, and Hortaçsu, 2021; Ma, Xiao, and Zeng, 2022).

The literature provides several theories to explain fire sales that are based on very different mechanisms that lead to low asset prices. Some theories are based on the assumption that buyers have limited cash available to purchase assets (Allen and Gale, 1998). Others assume that buyers have a low willingness to pay because they can collect lower cash flows than sellers (i.e., the so-called second-best-use assumption; Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Lorenzoni, 2008; Dávila and Korinek, 2018). A third set of theories is based on asymmetric information and adverse selection (Guerrieri and Shimer, 2014; Kurlat, 2016; Chang, 2018; Dow and Han, 2018).

Liquidity requirements are one of the policies introduced to mitigate fire sales. For instance, banks and money market mutual funds (MMMFs) are subject to such rules. In addition, the Securities and Exchange Commission (SEC) is considering liquidity requirements for open-end mutual funds, motivated by the "dash for cash" in March 2020 that resulted in mutual funds selling large quantities of high-quality corporate bonds at low prices (Haddad, Moreira, and Muir, 2021).[1] However, little is known about whether the different pricing mechanisms proposed by the literature call for different policy stances regarding liquidity regulation and whether other features of a fire sale interact with such mechanisms to further affect the optimal policy.

This paper analyzes liquidity requirements, using a unifying framework that nests three main pricing mechanisms used in the fire-sale literature; namely, cash-in-the-market pricing, second-best-use pricing, and adverse-selection pricing. Our contribution is to offer a single unifying framework to highlight the forces that generate fire-sale externalities associated with market incompleteness and how different fire-sale pricing mechanisms interact with such forces to shape the optimal policy. Some of our results build on the insights of Dávila and Schaab (2023) regarding

[1]The proposal is available at Securities and Exchange Commission, 17 CFR Parts 270 and 274 [Release Nos. 33-11130; IC-34746; File No. S7-26-22], RIN 3235-AM98, "Open-End Fund Liquidity Risk Management Programs and Swing Pricing; Form N-PORT Reporting," https://www.sec.gov/rules/proposed/2022/33-11130.pdf.

the general effects of policy interventions in incomplete-markets heterogeneous agent models.

We show that the optimality of the interventions that regulate investors' liquidity holdings and whether regulators should force investors to hold more or less liquidity depend on three main elements: (i) the difference between the sellers and the buyers' ability to collect cash flow from the *marginal* unit traded, (ii) the sensitivity of the fire-sale price to the sellers' liquidity holdings, and (iii) how market incompleteness affects the investors' ability to share risk. Importantly, even if investors are able to collect the same cash flow from any given asset (Kurlat 2021), they might trade assets of different quality on the margin, opening up a role for policy interventions.

The optimal policy can vary significantly, depending on the pricing mechanism—ranging from a liquidity requirement to a liquidity ceiling (i.e., an upper limit on liquidity holdings), and include no interventions. In addition, risk-sharing considerations could tilt the direction of the optimal policy (i.e., liquidity requirement or liquidity ceiling) even for a given pricing mechanism.

We first use a simple baseline framework that allows us to abstract from risk-sharing considerations to deliver stark results. The model has two assets (i.e., a short-term liquid asset and a long-term asset) and, similar to the fire-sale literature, two sets of agents (which we label the sellers and the buyers, as in Dávila and Korinek, 2018). An exogenous shock that increases the sellers' liquidity needs triggers a fire sale, forcing them to sell long-term assets to the buyers. In the efficiency and policy analysis, we focus on the composition of the sellers' portfolios in terms of their liquid and long-term assets before the possible realization of fire sales, aiming to determine whether and how the mix of the two assets should be regulated. We design our framework such that different pricing mechanisms, under appropriate restrictions, give rise to observationally equivalent models of fire sales. That is, our framework can produce the same portfolio choices, trading volumes, prices, and buyers' demand under cash-in-the-market, second-best-use, and adverse-selection pricing. We are thus able to highlight the role played by the pricing mechanism in shaping the optimal policy after "controlling" for easily observable variables.

With cash-in-the-market pricing and no risk-sharing considerations, the equilibrium is (Pareto) efficient and, thus, no liquidity regulation is necessary. Fire sales simply redistribute resources from the sellers to the buyers—as they allow the buyers to buy at a low price—and generate no aggregate welfare losses.

With second-best-use pricing, the buyers collect less cash flow from their long-term assets than the sellers do. Thus, the fire sales that transfer long-term assets from the sellers to the buyers result in a reduction in aggregate efficiency. Abstracting from the risk-sharing considerations, the optimal policy is a liquidity requirement, which reduces the depth of a fire sale.

With adverse-selection pricing, the equilibrium is again inefficient. Even if all of the investors can collect the same cash flow from any given asset, the cash flow collected from the *marginal* unit traded is different between the buyers and the sellers. In the version of the model with adverse selection, there are high- and low-quality long-term assets, and the sellers have private information about the quality of each unit they hold. On the margin, the sellers sell high-quality assets because all of the low-quality ones are sold as infra-marginal units. But for the buyers, the marginal unit traded is the average asset in the market, which includes both high- and low-quality ones. Thus, similar to the second-best-use version of the model, the sellers collect more cash flow from the marginal unit traded than the buyers do, giving rise to an inefficiency that can be corrected with a regulation that avoids a too-low price during a fire sale. Crucially, however, this objective is achieved with a liquidity ceiling in the adverse-selection model, as opposed to a liquidity requirement as in the second-best-use model. The logic of this result is similar to that in Malherbe (2014). A ceiling is required because if the sellers enter a fire-sale episode with less liquidity, a larger fraction of the sales will be due to fundamental reasons and a smaller fraction to private information, reducing the extent of the adverse-information problem. Imposing a liquidity requirement when fire sales are driven by adverse selection would *amplify* a fire sale and reduce welfare. Our optimal policy analysis under adverse-selection pricing represents a contribution in itself. While the literature provides deep microfoundations to understand how adverse selection affects fire sales and the response to some policy interventions, our simpler framework allows us to undertake a full *optimal* policy analysis that speaks about liquidity requirements.

We then extend our model to include risk-sharing considerations. The forces that arise in the baseline model continue to operate, but the optimal policy is also affected by the investors' inability to efficiently share risk, due to market incompleteness. The equilibrium can be inefficient even with cash-in-the-market pricing, similar to the cash-in-the-market banking model of Allen and Gale (2004). We establish that if a liquidity requirement is optimal with cash-in-the-market pricing, then the optimal requirement is stricter in an observationally equivalent model that is based on second-best-use pricing. And if a liquidity ceiling is optimal with cash-in-the-market pricing, then in an equivalent model with second-best-use pricing the optimal regulation is a lower ceiling or even a liquidity requirement. With adverse-selection pricing, the optimal regulation remains a liquidity ceiling under regularity conditions that are likely to hold in practice.

Our analysis focuses on the inefficiencies driven by market incompleteness, which the literature has labeled *distributive externalities* (Dávila and Korinek, 2018; Lanteri and Rampini, 2023). The effects of these externalities are often difficult to sign. Our results make progress by distinguishing the difference between the cash flow that the sellers and buyers are able to collect from the marginal units they trade and the role of imperfect risk sharing. While the first force typically points in one direction—in nearly all models in the literature, the sellers can collect the same or more cash flow than the buyers can—the second one is ambiguous and depends on whether imperfect risk sharing has higher impacts on the buyers or the sellers. We abstract from the so-called *collateral externalities* that are driven by binding collateral constraints, which are typically easier to sign and have been studied in greater detail (Bianchi, 2011; Dávila and Korinek, 2018).

Our results provide guidance for policymakers, as they identify key forces that shape optimal liquidity policies under various pricing mechanisms. While we conduct our analysis using a simple framework, these forces are likely to remain valid even in richer environments.

A direct policy implication is related to the debate about the introduction of liquidity requirements for open-ended mutual funds, proposed by the SEC and motivated by the March 2020 "dash for cash." Recall that this event was a fire sale of high-quality corporate bonds and, thus, was likely unrelated to second-best-use considerations as the investors should easily collect cash flow from corporate bonds. There is also no evidence that this event was driven by adverse selection (Haddad, Moreira, and Muir, 2021). If the fire-sale prices in this event are driven by cash-in-the-market pricing, our analysis suggests that the impact of market incompleteness on risk sharing should have first-order importance in determining the optimal policy.

Additional comparisons with the literature. Among the papers that study optimal policies to mitigate fire sales of financial assets, several focus on regulating ex-ante borrowing and total investments (e.g., Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021). Our paper complements these studies, as we focus on the composition of investors' portfolios and the share invested in liquid assets, abstracting from the size of investors' borrowing and investments.

Our work is closely related to Dávila and Korinek (2018). They identify, in a general second-best-use model, collateral externalities driven by collateral constraints and distributive externalities driven by incomplete markets. They also provide sufficient statistics to guide policy interventions. While our policy analysis builds on their approach, there are important distinctions. First, Dávila and Korinek (2018) focus on the size of investors' borrowing and investments, whereas we focus on the composition of their portfolios, in terms of liquid and illiquid assets, to study liquidity requirements. Second, we show that the sufficient statistics identified by Dávila and Korinek (2018) can be used not only with second-best-use pricing but also with cash-in-the-market and asymmetric-information pricing—overturning the conjecture of Kurlat (2021) about the inability to use the approach of Dávila and Korinek (2018) with asymmetric information. Third, we use the insights of Dávila and Schaab (2023) to further distinguish two forces that affect distributive externalities (i.e., the cash flow collected from the marginal unit traded, and imperfect risk sharing), allowing us to make progress in understanding the effects of distributive externalities. While Dávila and Korinek (2018) show that distributive externalities can lead to choices that are either too high or too low relative to those preferred by the regulator, we establish that the inability to unambiguously sign these effects to study liquidity requirements is due to imperfect risk sharing.

Another closely related paper is Kurlat (2021), which compares the optimal size of ex-ante investments, using second-best-use and adverse-selection pricing. While the spirit of our exercise is similar, there are again important differences besides the fact that we also consider cash-in-the-market pricing. First, Kurlat (2021) focuses on the size of ex-ante investments, whereas we focus on the composition in terms of liquid and illiquid assets. Second, in Kurlat (2021), investors have linear utility, whereas we extend our analysis to a setting with general utility to study the impact of risk-sharing considerations. Third, Kurlat (2021) states that despite the fact that "[t]he result of Dávila and Korinek (2018) that there are measurable statistics that suffice to determine the direction of the externality [...] does not extend to the asymmetric-information model," we show that the sufficient statistics identified in Dávila and Korinek (2018) can actually be used with asymmetric-information pricing to perform policy analysis. We also show that these statistics can be used with cash-in-the-market pricing—a mechanism not analyzed in Kurlat (2021). Third, we show that the optimal regulatory stance with asymmetric-information pricing is related to asset-price responses to the sellers' liquidity holdings (consistent with Malherbe, 2014) and the cash flow collected by the buyers and sellers from the marginal unit traded. Whether or not the buyers and sellers can collect the same cash flow from any given assets—a point the literature has often focused on (Dow and Han, 2018; Kurlat, 2021)—matters only insofar as this affects the cash flow collected from the marginal unit traded.

Several other papers study liquidity regulation for financial intermediaries but focus on other aspects. Farhi, Golosov, and Tsyvinski (2009) show that liquidity requirements can mitigate the problem of hidden trades in a Diamond-Dybvig framework. Calomiris, Heider, and Hoerova (2015) show that regulating banks' liquidity holdings is beneficial because such assets are easily observable and do not suffer from asymmetric-information problems. Kara and Ozsoy (2020) and Kashyap, Tsomocos, and Vardoulakis (2024) show that capital requirements and liquidity requirements should be used together to improve welfare. Hachem and Song (2021) show that liquidity regulation can trigger credit booms, focusing on China from 2007 to 2014. Robatto (2023) studies the interaction between liquidity requirements and central bank liquidity injections.[2]

Our paper is also related to a large empirical literature on fire sales.[3] Coval and Stafford (2007) and Jotikasthira, Lundblad, and Ramadorai (2012) document fire sales in equity securities. Ellul, Jotikasthira, and Lundblad (2011), Falato et al. (2021), Falato, Goldstein, and Hortaçsu (2021), and Manconi, Massa, and Yasuda (2012) provide evidence of fire sales in corporate bond markets both in normal times and during crises (2007-2008 and COVID-19 crises), although the evidence about normal times is challenged by Ambrose, Cai, and Helwege (2012) and Choi et al. (2020). Li and Schürhoff (2019) focus on fire sales in municipal bond markets.

2 The intuition for our results

To provide a clear intuition for the forces that affect the results and the optimal policy, we build on the insights of Dávila and Schaab (2023).[4] Policy interventions are beneficial in our framework if they improve aggregate efficiency (i.e., the total cash flow collected from financial assets) or if they improve risk sharing among agents by offsetting the inefficiencies driven by incomplete markets. To clarify this idea, consider the marginal value of a dollar that is available to an investor. Our three-period model ($t = 0, 1, 2$) includes a long-term asset that is traded at time $t = 1$ and produces some cash flow at time $t = 2$. Given the time-1 price of the long-term asset, q_1, one dollar allows an investor to purchase ${}^{1}\!/_{q_1}$ units of such an asset. The gains from holding the additional ${}^{1}\!/_{q_1}$ units of the long-term asset depend on the cash flow collected at $t = 2$ and the value of such cash flow

[2] A closely related strand of the banking literature uses fire-sale pricing mechanisms in models of runs and banking crises; see, for instance, Diamond and Dybvig (1983); Acharya and Yorulmazer (2008); Gale and Yorulmazer (2013); Gertler and Kiyotaki (2015); Robatto (2019); Goldstein et al. (2022).

[3] Some other closely related papers use structural models to study issues similar to ours. Kargar, Passadore, and Silva (2023) use a search model with aggregate risk to study transaction costs, trading volumes, and asset prices, with a focus on the 2020 "dash for cash." Geromichalos and Herrenbrueck (2016) focus on assets with different maturities, as we do, but abstract from fire sales.

[4] Dávila and Schaab (2023) dissect the way through which policy interventions affect welfare in general heterogeneous agents models with incomplete markets.

for the investor (i.e., the marginal utility of the investor at $t = 2$). That is,

$$\begin{array}{c}\text{marginal value}\\ \text{of a dollar}\end{array} = \frac{1}{q_1} \times \left\{ \begin{array}{c}\text{cash flow collected from}\\ \text{marginal unit traded at } t = 1\end{array} \right\} \times \left\{ \begin{array}{c}\text{marginal utility at}\\ t = 2,\ \text{discounted}\end{array} \right\}. \quad (1)$$

The gap between the sellers and buyers' marginal utility of wealth (i.e., the marginal value of a dollar) in (1) is a key element that affects the optimal liquidity policy. Specifically, in a fire sale, any gap that opens up between the cash flow collected from the marginal unit traded by the buyers and the sellers affects aggregate efficiency. And any gap in the discounted time-2 marginal utility affects risk sharing. The third element that affects the optimal policy stance does not appear in (1) and is given by the sensitivity of the asset price, q_1, to changes in the liquidity that the investors carry into the market at time $t = 1$. The role of this element is discussed below when comparing the optimal policy under adverse selection with the optimal one under second-best-use pricing.[5]

Our baseline framework (sections 3-5) assumes that all of the investors have linear utility at $t = 2$. This assumption eliminates any difference in the time-2 marginal utility of the sellers and buyers (i.e., the cash flow extracted from the long-term asset has the same value for both) and, with it, any consideration about risk sharing. Thus, the only element that affects the efficiency of the equilibrium and the optimal policy is whether the sellers and buyers are able to extract the same cash flow from the assets they trade.

With cash-in-the-market pricing (Section 3), all agents collect the same cash flow from the long-term assets. This assumption, combined with the lack of risk-sharing considerations, implies that the equilibrium is efficient.

With second-best-use pricing (Section 4), in which the buyers are able to collect less cash flow than the sellers, the equilibrium is inefficient even if we shut down the risk-sharing considerations. In a fire sale, long-term assets are transferred to agents that can extract less cash flow than the sellers can, reducing the total cash flow that will be extracted from such assets, relative to a situation with no fire sales. A liquidity requirement imposed on the sellers reduces the supply of assets in a fire sale, increasing the price. The higher price means that the sellers need to sell even fewer long-term assets to meet their liquidity needs, reducing the trading volume even more. Ultimately, many more assets remain with the sellers, increasing the economy-wide cash flow collected from the long-term

[5] Another element that quantitatively affects the optimal policy stance is the amount of assets sold by the sellers, similar to Dávila and Korinek (2018); see Section 3.6. However, because we focus on a fire *sale*, we design our model so that it always produces positive sales of assets and, thus, the sign of this element is unambiguous in our analysis.

assets.

With adverse-selection pricing (Section 5), in which the sellers have private information about the quality of the assets they sell, the buyers and sellers have the same ability to collect cash flow from any given asset. However, as we highlighted in equation (1), the marginal value of a dollar depends on the cash flow collected from the *marginal* unit of the asset that is traded and this marginal unit is different when comparing sellers and buyers. The seller's marginal unit is a high-quality asset because they liquidate all of their low-quality assets before selling their high-quality ones—thanks to their informational advantage. But the buyer's marginal unit traded is represented by the average asset in the market, which includes both low- and high-quality assets. The gap in the marginal cash flow collected by the buyers and the sellers creates an inefficiency because the sellers reduce their ex-ante investments, knowing that they will have to liquidate, on the margin, their high-quality assets at a low price. Regulation that reduces the depth of a fire sale is again beneficial.

Crucially, the optimal policy in the model with adverse selection is a liquidity ceiling (or a tax on liquidity). The logic is similar to Malherbe (2014), in which the adverse-selection problem is mitigated when the sellers hold less cash; this is because more sales reflect cash needs and the fraction of high-quality assets that are traded is higher. This result highlights the other element—besides the marginal value in (1)—that affects the optimal policy stance; that is, the sensitivity of the fire-sale price to the sellers' ex-ante liquidity holdings, which is positive with cash-in-the-market and second-best-use pricing but negative with adverse-selection pricing.

In Section 6, we extend the model to account for risk-sharing considerations (i.e., the discounted value of the time-2 cash flow is not the same across agents), using a more general utility at $t = 2$. The forces that affect the optimal policy in the baseline model remain the same but the additional risk-sharing considerations might tilt the optimal regulatory stance with cash-in-the-market and second-best-use pricing. In the asymmetric-information model, under some regularity conditions, the optimal policy remains a liquidity ceiling.

3 Model with cash-in-the-market pricing

We begin by describing the model in which a low price in a fire sale is driven by a cash-in-the-market mechanism, similar to Allen and Gale (1998). Section 4 describes the model in which the low price is driven by a second-best-use assumption, and Section 5 describes the model in which the low price is driven by an adverse-selection problem. For all of these models, we use a simple formulation of agents' utility that allows us to abstract from risk-sharing considerations and derive stark results. Section 6 extends the analysis of all of the models to a general utility function to account for risk-sharing considerations.

Following a standard approach in the fire-sale literature (e.g., Dávila and Korinek, 2018), we consider an economy populated by two sets of investors—the sellers (s) and the buyers (b)—and the economy lasts for three periods, $t = 0, 1, 2$. At $t = 0$, the sellers make their portfolio choices by choosing their investments in a liquid asset and a long-term asset. The buyers are born at $t = 1$ (similar to the patient investors in Stein, 2012, as discussed further in Section 3.1). At $t = 1$, a fire sale can occur, depending on the realization of an exogenous shock that forces the sellers to sell some of their holdings of the long-term asset. At $t = 2$, the payoff of the long-term asset is realized.

We begin by describing the choices and equilibrium at $t = 1$ and $t = 2$, taking as given the portfolio choices the sellers made at $t = 0$. In Section 3.5, we look at the decisions the sellers made at $t = 0$; and in Section 3.6, we turn to the policy analysis.

3.1 Environment at $t = 1$ and $t = 2$

The sellers have linear utility from consumption, c_2^s, at $t = 2$. The buyers also consume at $t = 2$ and derive utility from consumption at $t = 1$. Their utility is $u\left(c_1^b\right) + c_2^b$. We assume that $u\left(c\right) = \log c$, to simplify the exposition. But the model can be extended to allow a more general strictly increasing and strictly concave function. The linearity of the buyers and sellers' utility functions allows us to keep the exposition simple and derive stark results. We extend the analysis in Section 6.1 to a framework with a general utility function at $t = 2$.

There are two assets: a short-term (liquid) asset, which can be interpreted as a storage technology, and a long-term asset. The liquid asset is standard; for each unit invested at time t, there is one unit available at $t + 1$. The long-term asset has payoff $R > 1$ at $t = 2$.

The sellers begin $t = 1$ with an amount l_0^s of liquidity and k_0^s of the long-term asset, both of which are determined at $t = 0$ (see Section 3.5 for the analysis of the sellers' time-0 portfolio choices). They also have liabilities, d_0^s, which we assume are to external agents, with d_0^s representing the face value; more discussion about these liabilities is provided in Section 3.1.1. The buyers, who are born at $t = 1$ (as noted at the beginning of Section 3), are endowed with only one unit of liquidity.[6]

At $t = 1$, there is a centralized market in which the investors can trade the liquid and long-term assets. We denote q_1 as the price of the long-term asset and normalize the price of the liquid asset to one. We assume that short selling is not allowed, although the analysis can be extended without altering the logic of the results. For instance, we can allow for short selling that is subject to some costs or limits.

We note that at $t = 1$, the buyers and sellers are able to adjust their portfolio holdings of the liquid and long-term assets by trading in the centralized market. However, at the economy-wide level, it is not possible to change the overall supply of the two assets at $t = 1$; the overall supply is given by $1 + l_0^s$ and k_0^s, respectively (i.e., the amounts that the buyers and sellers have at the beginning of $t = 1$). The choices that lead to the economy-wide supply of these two assets will be analyzed in Section 3.5, when studying the sellers' ex ante investment decisions.

Some elements of this environment are very similar to Stein (2012). In particular, the buyers are isomorphic to the patient investors in Stein (2012). The buyers derive utility from time-1 consumption, and the patient investors in Stein (2012) have access to a productive project. But both the consumption here and the investment in Stein (2012) represent possible uses of the buyers' time-1 endowments, and both generate benefits according to a strictly concave function (i.e., the time-1 utility function here and the production function in Stein (2012).

3.1.1 Sellers' liabilities and aggregate-withdrawal shock at $t = 1$

At $t = 1$, the sellers have to repay a fraction γ of debt d_0^s, while the remaining fraction $1 - \gamma$ will be due at $t = 2$. We assume that γ is an aggregate shock that can take values $\gamma \in \{0, \bar{\gamma}\}$, with $\bar{\gamma} \in (0, 1)$. The shock is realized at the beginning of $t = 1$; that is, before the time-1 market opens. The realization of γ is common knowledge. In the state in which $\gamma = \overline{\gamma}$, the equilibrium will display fire sales; that is, the sellers will sell their long-term assets at a "low" price.

We interpret the sellers as banks, MMMFs, or mutual funds that experience withdrawals or outflows or, more generally, acute liquidity needs. We refer to $\gamma = 0$ as the low-withdrawal state and $\gamma = \overline{\gamma}$ as the high-withdrawal state.

[6]To prove some of our results, we require the endowment of the buyers' liquid asset to be $1 + \varepsilon$ for an arbitrarily (small) $\varepsilon > 0$. However, that is just a technical assumption and, for simplicity, we focus the exposition on the limiting case $\varepsilon \to 0$.

3.2 Sellers' choices at $t = 1$

The sellers consume only at $t = 2$ and, thus, they choose their time-1 portfolio, given their withdrawals, γd_0^s, and their initial holdings, l_0^s and k_0^s, of the liquid and long-term assets. Formally, the sellers choose their non-negative holdings of liquid and long-term assets at $t = 1$, $l_1^s \geq 0$ and $k_1^s \geq 0$ to maximize consumption, c_2^s, subject to the time-1 budget constraint

$$l_1^s + q_1 k_1^s \leq q_1 k_0^s + l_0^s - \gamma d_0^s, \tag{2}$$

where consumption, c_2^s, is given by the payoff of their time-1 investments, net of the repayments, $(1 - \gamma)\, d_0^s$, that are due to their debt holders

$$c_2^s = R k_1^s + l_1^s - (1 - \gamma)\, d_0^s. \tag{3}$$

We restrict our attention to the relevant equilibrium cases in which $q_1 \leq R$.[7] If $q_1 = R$, which will be the case in the low-withdrawal state $\gamma = 0$ (i.e., when no fire sales occur), the liquid and long-term assets have the same returns, so the sellers are indifferent between the two. Without loss of generality, we focus on the case in which the sellers do not engage in any trade, so that their holdings are $l_1^s = l_0^s$ and $k_1^s = k_0^s$. When $q_1 < R$, which will be the case in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when a fire sale occurs), the long-term asset has a higher return than the liquid asset and, thus, the sellers invest all of their wealth in the long-term asset. That is, $l_1^s = 0$, and k_1^s is residually determined by the budget constraint

$$k_1^s = \frac{q_1 k_0^s - (\gamma d_0^s - l_0^s)}{q_1}. \tag{4}$$

Thus, we can summarize the sellers' choices as

$$\{l_1^s, k_1^s\} = \begin{cases} \{l_0^s, k_0^s\} & \text{if } q_1 = R \\ \left\{0, \frac{q_1 k_0^s - \left(\gamma d_0^s - l_0^s\right)}{q_1}\right\} & \text{if } q_1 < R. \end{cases} \tag{5}$$

[7]If $q_1 > R$, then the expected return of the long-term asset is negative, but because the return of the liquid asset is zero, no agent would invest in the long-term asset. This cannot be an equilibrium because the market-clearing condition for the long-term asset would not hold.

3.3 Buyers' choices at $t = 1$

The buyers choose their holdings of the liquid and long-term assets at $t = 1$ and their consumption at $t = 1$ and $t = 2$, to solve the following problem:

$$\max_{l_1^b,\, k_1^b, c_1^b, c_2^b} \; u\left(c_1^b\right) + c_2^b, \tag{6}$$

where time-2 consumption is given by

$$c_b^2 = l_1^b + Rk_1^b, \tag{7}$$

and where the choices are subject to non-negativity constraints and to the budget constraint

$$c_1^b + l_1^b + q_1 k_1^b \leq 1. \tag{8}$$

Note that the resources available to the buyers (i.e., the right-hand side of (8)) are equal to one because the buyers enter $t = 1$ with a unit of the liquid asset and no holdings of the long-term asset; see Section 3.1.

The maximization in (6) implies the standard asset pricing condition

$$q_1 = \frac{1}{u'\left(c_1^b\right)} \times R, \tag{9}$$

where $1/u'(c_1^b)$ is the ratio of the marginal utility at $t = 2$ (i.e., one) and the marginal utility at $t = 1$ (i.e., $u'\left(c_1^b\right)$). Note that the time-1 consumption choice satisfies $u'\left(c_1^b\right) \geq 1$ because the buyers have linear utility from their time-2 consumption, so they can save on the liquid asset, and $u'(1) = 1$, since $u(c) = \log c$. Hence, the buyers will never choose to consume more than one unit at $t = 1$.

Focusing again on the relevant case in which $q_1 < R$, and using $u(c) = \log c$, the buyers' optimal choices are

$$\left\{c_1^b, l_1^b, k_1^b\right\} = \begin{cases} \{1, 0, 0\} & \text{if } q_1 = R \\ \left\{\frac{q_1}{R}, 0, \frac{1}{q_1} - \frac{1}{R}\right\} & \text{if } q_1 < R \end{cases} \tag{10}$$

To preview some of the results, we note that in the low-withdrawal state $\gamma = 0$ (i.e., when no fire sales occur), the buyers consume $c_1^b = 1$ so that their marginal utility is $u'(c_1^b) = 1$, resulting in a time-1 price of $q_1 = R$ for the long-term asset. Hence, q_1 is equal to the cash flow that the asset produces at $t = 2$. In contrast, in the high-withdrawal state, $\gamma = \bar{\gamma}$ (i.e., when a fire sale occurs), the buyers consume $c_1^b < 1$, so that their marginal utility is $u'\left(c_1^b\right) > 1$. Hence, the time-1 price of the long-term asset is $q_1 < R$; that is, it is lower than the cash flow, R.

3.4 Equilibrium at $t = 1, 2$

We solve for the equilibrium at $t = 1, 2$ under two parameter restrictions. First, we assume that the buyers' initial holdings of liquidity are at an intermediate level; that is,

$$l_0^s < \bar{\gamma} d_0^s < 1 + l_0^s. \tag{11}$$

The inequality $l_0^s < \bar{\gamma} d_0^s$ implies that in the high-withdrawal state, $\bar{\gamma}$, the sellers' initial liquidity holdings, l_0^s, are not sufficient to cover all of their withdrawals. Thus, the sellers are forced to sell some of their long-term asset holdings. The inequality $\bar{\gamma} d_0^s < 1 + l_0^s$ guarantees that the withdrawals the sellers need to make do not exceed the total amount of their liquidity, $1 + l_0^s$, available in the economy; recall that the buyers start $t = 1$ with one unit of liquidity and the sellers start with l_0^s units. Regarding the second parameter restriction, we assume that k_0^s is large enough so that the sellers' time-2 consumption is always non-negative. This second assumption guarantees that the sellers are always solvent, which allows us to sidestep the potential issue of the sellers' default. As a result, we can focus on the key forces that drive the results of the policy analysis. In Section 3.5, we show that these parameter restrictions arise endogenously from the sellers' time-0 portfolio choices.

The equilibrium definition is standard. Given a realization of the shock $\gamma \in \{0, \bar{\gamma}\}$, an equilibrium at $t = 1, 2$ is a collection of the sellers and buyers' portfolio choices at $t = 1$ (i.e., $\{l_1^s, k_1^s\}$ and $\{l_1^b, k_1^b\}$), the buyers' consumption choices at $t = 1$ and $t = 2$ (i.e., c_1^b and c_2^b), the sellers' consumption choices at $t = 2$ (i.e., c_2^s), and a time-1 price for the long-term asset (i.e., q_1), such that the buyers and sellers maximize their utilities and the time-1 market clears.[8] Specifically, the market-clearing condition for liquidity at $t = 1$ is

$$c_1^b + l_1^b + l_1^s + \gamma d_0^s = 1 + l_0^s, \tag{12}$$

[8] Formally, the equilibrium satisfies equations (3) and (7), the two expressions in (5), the three expressions in (10), and the market-clearing condition (12).

where the right-hand side uses the assumption that the buyers are endowed with one unit of liquidity (see Section 3.1). That is, the liquid asset available in the economy, $1 + l_0^s$, is allocated between the buyers' consumption, c_1^b, their liquidity holdings, l_1^b, and the sellers' liquidity holdings, l_1^s, carried to $t = 2$, and the resources, γd_0^s, that are used to repay the sellers' debt holders at $t = 1$. The other market-clearing condition—for the long-term asset—holds by Walras' law, but we also state it for completeness:

$$k_1^b + k_1^s = k_0^s, \tag{13}$$

where the right-hand side uses the assumption that the buyers have no endowment of the long-term assets (see Section 3.1).

We first solve for the equilibrium at $t = 1, 2$ given the realization of the low-withdrawal state $\gamma = 0$. In this case, the sellers do not need to sell any debt and the market-clearing condition in (12) implies that all of the liquidity remains in the hands of the buyers. Because we have normalized the buyers' initial liquidity holdings to one (see Section 3.1), they have enough liquidity to achieve the level of consumption, $c_1^b = 1$, that equalizes their marginal utility of consumption at $t = 1$ to that at $t = 2$. No fire sales arise when $\gamma = 0$. The next proposition formalizes the result and describes the equilibrium; all of the proofs are provided in Appendix A.

Proposition 3.1. ***(Cash-in-the-market pricing, equilibrium at $t = 1, 2$ with low withdrawals: no fire sales)*** *Given $\gamma = 0$, there exists an equilibrium in which the time-1 price of the long-term asset is $q_1 = R$, the buyers' time-1 consumption is $c_1^b = 1$, and the buyers and sellers engage in no trade at $t = 1$ (i.e., $l_1^s = l_0^s$, $k_1^s = k_0^s$ for the sellers, and $l_1^b = 0$, $k_1^b = 0$ for the buyers). Consumption at $t = 2$ is $c_2^s = Rk_0^s + l_0^s - d_0^s$ for the sellers and $c_2^b = 0$ for the buyers.*

Next, we solve for the equilibrium, given the realization of the high-withdrawal shock $\gamma = \bar{\gamma}$. When this shock is realized, the sellers do not have enough liquidity to pay for all of the withdrawals (see equation (11)) and, thus, sell some of their holdings of the long-term assets to the buyers, in exchange for liquidity. As a result, the buyers' remaining liquidity with which to finance their time-1 consumption decreases relative to the low-withdrawal state and the buyers' time-1 consumption, c_1^b, also decreases. This lack of liquidity introduces a wedge, between the buyers' $t = 1$ and $t = 2$ marginal utilities of consumption, that lowers the price, q_1, of the long-term asset. Thus, a fire sale arises. That is, the assets the sellers sold to pay for their withdrawals are traded at a price that is lower than the expected payoff, R, as formalized by the next proposition. The proof in Appendix A states the equilibrium values of all of the endogenous variables as a function of the parameters.

Proposition 3.2. ***(Cash-in-the-market pricing, equilibrium at*** $t = 1, 2$ ***with large withdrawals: fire sales)*** *Given* $\gamma = \overline{\gamma}$*, there exists an equilibrium with fire sales. The time-1 price of the long-term asset is* $q_1 = R\left[1 - (\bar{\gamma} d_0^s - l_0^s)\right] < R$*, the sellers sell part of their initial holdings of the long-term asset and hold no liquidity at* $t = 1$ *(i.e.,* $k_1^s < k_0^s$ *and* $l_1^s = 0$*), the buyers reduce their liquidity holdings and increase their long-term asset holdings at* $t = 1$ *(i.e,* $l_1^b = 0$ *and* $k_1^b > 0$*), and the buyers' time-1 consumption is* $c_1^b = 1 - (\bar{\gamma} d_0^s - l_0^s) < 1$*.*

3.5 Sellers' choices at $t = 0$

We now turn to the analysis at $t = 0$, when the sellers decide how to allocate their resources between the liquid and long-term assets. Then, in Section 3.6, we ask whether the sellers' choices at $t = 0$ are efficient and whether regulatory interventions can improve the equilibrium outcome.

At $t = 0$, the sellers have an endowment, e^s, and issue debt, d_0^s, and allocate their resources, $e^s + d_0^s$, to liquid and long-term assets, subject to the budget constraint

$$l_0^s + k_0^s \le e^s + d_0^s. \tag{14}$$

We assume that the debt, d_0^s, is exogenously given by $d_0^s = d^s$, and we focus on the choices of $\{l_0^s,\ k_0^s\}$.[9] This allows us to take the size of the sellers' portfolio as given (i.e., $e^s + d^s$) and focus on whether the allocation of these resources to long-term and liquid assets is efficient or the sellers' liquidity holdings should be regulated. Our analysis complements that of several other fire-sales papers, which often focus on the inefficiencies that lead to overborrowing (Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021).

When making their time-0 choices, the sellers know the probability distribution over the withdrawal shock γ, which is given by

$$\gamma = \begin{cases} 0 & \text{with probability } 1 - \pi \\ \bar{\gamma} & \text{with probability } \pi. \end{cases} \tag{15}$$

[9] Regarding d_0^s, one can assume that there is a mass of external agents that may deposit their endowments with the sellers. Assuming the external agents are risk neutral and that they can only deposit with the sellers or use storage technology, and that the sellers can make a take-it-or-leave-it offer, the sellers will offer a zero return on deposits, and d_0^s will be equal to the external agents' total endowment.'

We assume that π is sufficiently large,

$$\pi > \frac{(R-1)(1-\bar{\gamma}d^s)}{\bar{\gamma}d^s}, \tag{16}$$

which guarantees that the possibility of fire sales at $t = 1$ is not negligible and, thus, the sellers want to have positive holdings of the liquid assets at $t = 0$.

To determine the sellers' portfolio choices at $t = 0$, we proceed along the lines of Dávila and Korinek (2018) and derive the sellers' time-0 choices that maximize their time-1 indirect utility function at $t = 1$. This approach is very convenient because it will make the comparison with the regulator's problem and solutions very transparent. In addition, the equations that describe both the sellers and the regulator's choices in the second-best-use and asymmetric-information models of sections 4 and 5 will have an identical structure, thereby allowing us to build on the results of this section to study liquidity regulation in those models.

The sellers' indirect utility function at $t = 1$ is

$$V_1^s\left(l_0^s, k_0^s\right) = c_2^s + \lambda_1^s\left[l_0^s + q_1 k_0^s - \left(l_1^s + q_1 k_1^s + \gamma\, d^s\right)\right] + \mu_1^s l_1^s. \tag{17}$$

The first term on the right-hand side is the sellers' time-2 utility, which is linear in consumption, c_2^s. The second term is the Lagrange multiplier λ_1^s of the sellers' time-1 budget constraint (equation (2)) times the budget constraint itself. The last term is the Lagrange multiplier μ_1^s of the non-negative constraint on liquidity holdings, times such holdings, l_1^s.

The sellers choose liquidity l_0^s and long-term asset holdings k_0^s to maximize their expected indirect utility function

$$\max_{l_0^s, k_0^s} \mathbb{E}_0\left\{V_1^s\left(l_0^s, k_0^s\right)\right\}, \tag{18}$$

subject to the budget constraint (14). The problem in (18) is easy to analyze because we can exploit the envelope theorem to obtain

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}. \tag{19}$$

Recall that λ_1^s is the Lagrange multiplier of the sellers' budget constraint at $t = 1$ and, thus, it represents the sellers' marginal value of wealth. Equation (19) states that the sellers choose their time-0 portfolio so that the time-1 marginal value of holding one additional unit of the long-term asset, represented by the left-hand side, is equal to the time-1 marginal value of holding one additional unit of liquidity, on the right-hand side. That is, a marginal dollar of investments at $t = 0$ could be used to invest in the long-term asset or in liquidity, which have market values of q_1 and one at $t = 1$ and which the sellers value according to their time-1 marginal utility of wealth λ_1^s.

The marginal utility of the sellers' wealth, λ_1^s, (and the equivalent object for the buyers, λ_1^b) is a crucial object for our analysis. Because λ_1^s is formally defined as the Lagrange multiplier of (2), the analysis in Section 3.2 implies

$$\lambda_1^s = \frac{R}{q_1}. \tag{20}$$

That is, a marginal unit of wealth available to sellers at $t = 1$ can be used to purchase $1/q_1$ units of the long-term assets. Each unit of the asset will then produce a payoff R, which is evaluated according to the linear marginal utility of wealth. Note that λ_1^s corresponds to the "marginal value of a dollar" in (1), as the cash flow collected from the marginal unit traded is given by R and the marginal utility of the sellers' consumption at $t = 2$ is one.

Given $q_1 = R$ in the low-withdrawal state (i.e., when there are no fire sales), we can use equations (19) and (20) to pin down the value of q_1 in the high-withdrawal state (i.e., in the fire-sale state), which we denote by $q_1(\bar{\gamma})$ (see Appendix E for the derivation):

$$q_1(\bar{\gamma}) = R\,\frac{\pi}{(R-1)+\pi} < 1, \tag{21}$$

where the inequality follows from $R > 1$. We can then combine (21) with the expression for q_1 derived in Proposition 3.2 to solve for the sellers' choice of liquidity l_0^s, given $d_0^s = d^s$:

$$l_0^s = \frac{\pi}{(R-1)+\pi} + \bar{\gamma} d^s - 1, \tag{22}$$

which is strictly positive because of the restriction in (16). Appendix B summarizes the equilibrium at $t = 0, 1, 2$ as a function of the parameters and the time-0 endowments.

Finally, note that the sellers' choice of liquidity in (22) implies that the restrictions in (11) used to derive the equilibrium at $t = 1, 2$ hold. In addition, because the sellers' choice of the long-term asset, k_0^s, is residually determined by the budget constraint (14), the assumption in Section 3.4, that k_0^s is sufficiently large, holds if the sellers' endowment, e^s, is sufficiently large.

3.6 Efficiency and policy analysis

We now study whether the equilibrium is efficient; that is, whether the equilibrium allocation—and, in particular, the sellers' time-0 portfolio choice—corresponds to that of a planner or regulator (hereinafter simply referred to as the "regulator"). Under the assumption that the buyers and sellers have linear utility at $t = 2$, we show that the equilibrium is efficient and, thus, no liquidity regulation should be imposed on the sellers' time-0 choices. In sections 4 and 5, we show that the equilibrium is, instead, inefficient in the second-best-use and the asymmetric-information models, requiring liquidity regulation in those cases.

We use a standard approach employed in the fire-sale literature. Various papers, such as Lorenzoni (2008), Dávila and Korinek (2018), and Kurlat (2021), consider a regulator that makes the initial portfolio choices at $t = 0$ but has no influence on the trading and choices that occur in the subsequent time periods (i.e., at $t = 1$ and $t = 2$). Crucially, the regulator internalizes the effects of the time-0 portfolio choices on the time-1 price, q_1, which is different from the individual sellers that take the time-1 price, q_1, as given. By following the same approach, our results are easily comparable with the literature. In addition, this approach has a good fit with the analysis of the actual liquidity requirements that are imposed, in practice, before the possible realization of fire sales.

To define efficiency, we rely on the concept of Pareto optimality because our model—like several others in the fire-sale literature—has two sets of agents (i.e., buyers and sellers). Thus, an equilibrium is constrained efficient if no regulatory intervention at $t = 0$ can improve the welfare of the buyers, the sellers, or both.

We consider the problem of a regulator aiming to maximize the sellers' welfare while ensuring that the buyers' welfare is at least as high as it would be in the unregulated equilibrium. At $t = 0$, the regulator chooses thir investments in the sellers' liquidity and long-term assets, l_0^s and k_0^s, that will maximize the sellers' utility. In addition, the regulator chooses a transfer, T, from the seller to the buyers to make sure that the buyers achieve the same level of utility as that in the unregulated equilibrium. Because the buyers are born at $t = 1$, we assume that the transfer from the sellers to the buyers involves an amount T of the liquid asset[10] Thus, the sellers will enter $t = 1$ with liquidity $l_0^s - T$ and the buyers with liquidity $1 + T$. The regulator's problem is

$$\max_{l_0^s, k_0^s, T} \ \mathbb{E}_0 \left\{ V_1^s \left(l_0^s - T, k_0^s;\, q_1 \right) \right\} \tag{23}$$

where $V_1^s(\cdot)$ is the sellers' indirect utility functions, defined in (17), in which we have highlighted the sellers' dependence on the price, q_1. The maximization is subject to the sellers' budget con-

[10] As in the literature, the transfer cannot be contingent on the state of the economy at $t = 1$, otherwise it would violate the assumption that the regulator can affect only the time-0 choices.

straint, (14) evaluated at $d_0^s = d^s$,

$$l_0^s + k_0^s \leq e^s + d^s \tag{24}$$

and to the constraint that the buyers' time-1 indirect utility $V_1^b\left(T;\, q_1\right)$ should be no less than the level $\overline{V}$ they achieved in the unregulated equilibrium:

$$V_1^b(T; q_1) \geq \overline{V}. \tag{25}$$

Specifically, the buyers' time-1 indirect utility is defined analogously to that of the sellers:[11]

$$V_1^b\left(T; q_1\right) = u(c_1^b) + c_2^b + \lambda_1^b\left[1 + T - \left(l_1^b + q_1 k_1^b + c_1^b\right)\right] + \mu_1^b l_1^b. \tag{26}$$

The term λ_1^b is the Lagrange multiplier of the buyers' time-1 budget constraint and, thus, represents the buyers' marginal utility of wealth. The term μ_1^b is the Lagrange multiplier on the non-negativity constraint $l_1^b \geq 0$.

The key difference, compared to the sellers' individual problem, is that the regulator accounts for the effects of its choices on the long-term asset's time-1 price, q_1. Thus, denoting ξ as the Lagrange multiplier of the buyers' utility constraint (25), the regulator's first-order conditions for the choice of the sellers' holdings of liquidity, l_0^s, and long-term assets, k_0^s, imply[12]

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)\right\}. \tag{27}$$

The first-order condition for the choice of transfers, T, is

$$\mathbb{E}_0\left\{\frac{\partial q_1}{\partial T}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right) + \xi\lambda_1^b\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}. \tag{28}$$

The regulator's optimality condition (27) that pins down the optimal choice of the liquidity and long-term assets differs from that of the individual sellers in (19) because the regulator internalizes the effects of its choices on the time-1 price, q_1, as noted before. The difference between (19) and (27) can introduce a wedge between the regulator's choices and those of the private agents, which is captured by the second term on the right-hand side of (27). This wedge is affected by three elements, which are similar to those identified by Dávila and Korinek (2018) in regard to what they

[11]Recall from Section 3.1 that the buyers have no holdings of the long-term asset at the beginning of $t = 1$.

[12]See Appendix E for more details on how we derive the first-order conditions.

refer to as distributive externalities (i.e., externalities due to incomplete markets):

1. The sensitivity of the time-1 price q_1 with respect to the sellers' $t = 0$ choice of liquidity holdings, l_0^s, that is, $\partial q_1 / \partial l_0^s$;

2. The sellers' purchases of the long-term assets at $t = 1$, $k_1^s - k_0^s$ (or sales, if negative);

3. The difference between the buyers' marginal utility of wealth, λ_1^b, and that of the sellers, λ_1^s, adjusted by the Lagrange multiplier ξ: $\xi\lambda_1^b - \lambda_1^s$.

Liquidity requirements are optimal when the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s}(k_1^s - k_0^s)(\xi\lambda_1^b - \lambda_1^s)\}$ in (27), evaluated at the unregulated equilibrium, is positive. This is because the right-hand side of (27) represents the regulator's marginal value of investing in liquidity at $t = 0$. Hence, a positive value for the term $\mathbb{E}\frac{\partial q_1}{\partial l_0^s}(k_1^s - k_0^s)(\xi\lambda_1^b - \lambda_1^s)\}$ means that, at the unregulated equilibrium, the regulator's value of investing in liquidity exceeds that of the private agents.

However, the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s}(k_1^s - k_0^s)(\xi\lambda_1^b - \lambda_1^s)\}$ in (27) is zero in this model with cash-in-the-market pricing and linear utility of time-2 consumption, as shown formally by Proposition 3.3 below. Thus, the planner's first-order condition (27) coincides with that of the private agents in (19), and the equilibrium is efficient. As a result, no liquidity regulation is required with cash-in-the-market pricing when all of the investors have linear utility at $t = 2$.[13]

Proposition 3.3. ***(Efficiency in the cash-in-the-market model)*** *The unregulated equilibrium is constrained efficient.*

To clarify this result, we appeal to the discussion in Section 2. The marginal value of a dollar in equation (1) is formally represented by the Lagrange multipliers λ_1^s and λ_1^b for the sellers and the buyers, respectively. In this model with cash-in-the-market pricing and linear utility of consumption at $t = 2$, λ_1^s and λ_1^b are equalized,

$$\lambda_1^s = \lambda_1^b = \frac{R}{q_1}, \tag{29}$$

because the buyers and sellers both collect the same cash flow, R, from any unit traded—including the marginal units—and they both have constant linear utility at $t = 1$. The linear utility at $t = 2$ also prevents any wealth effect that could arise from the planner's transfers, T. This implies that

[13] Dávila and Korinek (2018) show that when markets between $t = 0$ and $t = 1$ are complete, the equilibrium is efficient. Our model has two assets at $t = 0$ (i.e., long-term asset and liquidity) and two states at $t = 1$ (i.e., two possible realizations of γ), but the markets are not complete here because the buyers cannot invest in the long-term asset at $t = 0$. Hence, the efficiency result in Proposition 3.3 arises *despite* market incompleteness.

the time-1 price, q_1, is unresponsive to the transfers, T, and, thus, the term $\partial q_1/\partial T$ in the regulator's first-order condition (28) is zero. All of these results together imply that the Lagrange multiplier ξ of the regulator's constraint (25) is equal to one. That is, the sellers and buyers are effectively "symmetric"—not just at the unregulated equilibrium but also as we change the sellers and buyers' wealth, using the transfers, T. In other words, fire sales just entail a redistribution from the sellers to the buyers and create no inefficiencies. Formally, under $\xi = 1$ and $\lambda_1^s = \lambda_1^b$, the first-order condition (27) simplifies to $\mathbb{E}_0 \{\lambda_1^s q_1\} = \mathbb{E}_0 \{\lambda_1^s\}$ and is, thus, identical to that of the individual sellers, that is, to equation (19).

4 Model with second-best-use pricing

We now present a version of the model where the decline in the price of long-term assets during a fire sale is due to the buyers' reduced ability to collect cash flow from these assets, compared to the sellers. This assumption is common in the literature (Shleifer and Vishny 1992) and is relevant in various contexts, such as in the sale of failed banks (Acharya and Yorulmazer, 2008; Granja, Matvos, and Seru, 2017). We design the second-best-use model to be fully comparable with the baseline model of Section 3. Specifically, the two models produce the same sellers' portfolio choice at $t = 0$, the same buyers' demand at $t = 1$, and the same price and quantity traded in a fire sale, given the appropriate parameter restrictions.

The key result is that the equilibrium with second-best-use pricing is inefficient and a liquidity requirement is necessary to achieve an efficient outcome. As in Section 3, we assume linear utility at $t = 2$ to isolate the inefficiencies arising from the buyers' reduced ability to collect cash flow, thereby abstracting from inefficient risk sharing. The case with more general utility functions is analyzed in Section 6.

4.1 Second-best-use model: environment

As in the baseline model, the framework consists of three periods indexed by $t = 0, 1, 2$. At $t = 1$, fire sales may occur and, at $t = 2$, the long-term asset generates its output. The endowments are identical to those in the baseline model. At $t = 0$, the sellers have endowment e^s and issue debt $d_0^s = d^s$. The buyers enter the model at $t = 1$ with one unit of liquidity and no holdings of the long-term asset. The sellers' preferences are the same as in the baseline model, meaning they have linear utility from consumption c_2^s at $t = 2$. They are also subject to the same withdrawal shock, $\gamma \in \{0, \bar{\gamma}\}$, at $t = 1$.

The second-best-use model differs from the cash-in-the-market model of Section 3 in terms of the buyers' preferences and ability to manage long-term assets. First, the buyers derive utility only at $t = 2$ and their utility is linear in consumption, c_2^b. Thus, unlike in Section 3, the buyers do not obtain utility from consumption at $t = 1$. Second, while the sellers can collect a cash flow of $R > 1$ per unit of the long-term asset at $t = 2$, as in Section 3, the buyers receive a lower cash flow. Specifically, if a buyer purchases $k_1^b \geq 0$ units of the long-term asset at $t = 1$, they can collect a cash flow of $f(k_1^b)$ at $t = 2$, where $f(\cdot)$ is a strictly increasing and concave function that satisfies $f(0) = 0$ and $f'(0) = R$. Thus, for $k_1^b > 0$, we have $f(k_1^b) < Rk_1^b$.

Note that the sellers' problem is identical to that in Section 3, both at $t = 0$ and $t = 1$. Thus, their objective functions, budget constraints, and first-order conditions are unchanged. However, the buyers' problem is different, as we discuss in the next section.

4.2 Buyers' choices in the second-best-use model

At $t = 1$, the buyers choose their time-1 holdings of liquidity l_1^b and long-term asset k_1^b to maximize their time-2 consumption, c_2^b, which is given by

$$c_2^b = l_1^b + f\left(k_1^b\right). \tag{30}$$

The maximization is subject to the budget constraint, which is similar to (8) but does not include the time-1 consumption:

$$l_1^b + q_1 k_1^b \leq 1. \tag{31}$$

The first-order conditions now imply

$$q_1 = f'(k_1^b) \leq R, \tag{32}$$

where the inequality follows from the assumption about $f(\cdot)$, introduced in Section 4.1. In particular, $q_1 < R$ when $k_1^b > 0$. That is, the buyers are willing to purchase long-term assets at a low price because they are able to collect a lower cash flow than the sellers.

4.3 Equilibrium in the second-best-use model

The equilibrium is similar to that of the cash-in-the-market model.[14] In the low-withdrawal state $\gamma = 0$, the price, q_1, of the long-term asset is equal to its payoff, R, and the trading volume is zero. In the high-withdrawal state $\gamma = \bar{\gamma}$, the price, q_1, of the long-term asset is lower than its payoff, R, and the sellers sell some of their holdings of the long-term assets. The next proposition formalizes these results, and Appendix C provides a complete characterization of the equilibrium.

Proposition 4.1. *(Equilibrium in the second-best-use model)*

- *At $t = 0$, the sellers invest amounts $l_0^s = \bar{\gamma} d^s - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ in liquidity and $k_0^s = e^s + d^s - \frac{\pi R}{R-1+\pi}\left[\bar{\gamma} d^s - (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right]$ in the long-term asset.*
- *At $t = 1$,*
  - *If $\gamma = 0$, the price of the long-term asset is $q_1 = R$, and the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 1$ for the buyers);*
  - *If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = R\frac{\pi}{R-1+\pi} < 1$ and the trading volume is positive (i.e., $k_1^s < k_0^s$ and $l_1^s = 0$ for the sellers and $k_1^b > 0$ and $l_1^b < 1$ for the buyers), with $k_1^b = (f')^{-1}\left(R\frac{\pi}{R-1+\pi}\right)$.*

To ensure that the second-best-use model is fully comparable to the cash-in-the-market one, we can impose some restriction on the function, f, that describes the buyers' ability to extract cash flow from the long-term asset.[15] Specifically, if $f(k) = \log(1+Rk)$ (which satisfies the assumption that f is strictly increasing, f is strictly concave, $f(0) = 0$, and $f'(0) = R$ as introduced in Section 4.1), the second-best-use model produces the same time-0 portfolio for the sellers as in the cash-in-the-market model of Section 3, the same time-1 price q_1 for any realization of γ, the same trading volume at $t = 1$, and the same sensitivity of the price, q_1, to the trading volume, k_1^b.[16] The result about the sensitivity of q_1 with respect to k_1^b can be derived by noting that, in the second-best-use model, the buyers' first-order condition (32), evaluated using $f(k) = \log(1 + Rk)$, implies that

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[14] The equilibrium in the second-best-use model solves the following equations: the sellers' time-0 budget constraint (14), their first-order condition at $t = 0$, the two expressions in (5) that pin down their portfolio choices at $t = 1$ in each of the two states $\{0, \bar{\gamma}\}$, and their time-2 consumption (3) in each of the two states $\{0, \bar{\gamma}\}$; and the buyers' time-1 portfolio choices and time-2 consumption in (30)-(32) in each of the two states $\{0, \bar{\gamma}\}$, and the market-clearing condition (12) evaluated at $c_1^b = 0$ in each of the two states $\{0, \bar{\gamma}\}$.

[15] We also assume that the sellers' endowment, e^s, and debt, d^s, are the same.

[16] The two models also have the same price sensitivity, q_1, to the sellers' time-0 holdings of liquidity, l_0^s, and time-2 consumption.

$q_1 = \frac{R}{1+Rk_1^b}$, which is identical to the corresponding first-order condition (9) for the cash-in-the-market model that is evaluated using the functional form $u(\cdot) = \log(\cdot)$ (as assumed in Section 3.1), the sellers' budget constraint (in equation (8)), and $l_1^b = 0$ (as discussed in Section 3.3). The other results follow from evaluating the equilibrium in the second-best-use model in Proposition 4.1, using $f(k) = \log(1 + Rk)$ and comparing it with the equilibrium in the cash-in-the-market model stated in Appendix B.

4.4 Inefficiency and liquidity requirements in the second-best-use model

We now analyze efficiency and policy interventions in the second-best-use model. The regulator's problem in the second-best-use model can be formulated in the same way as for the baseline cash-in-the-market model. That is, equations (23)-(25) are still valid for the second-best-use model to describe the planner's problem, their budget constraint, and the constraint that the buyers' utility has to be at least as large as in the unregulated equilibrium, respectively. As a result, the regulator's first-order conditions (19) and (28) are also the same.[17]

The key difference with the cash-in-the-market model of Section 3 is in the buyers' marginal utilities of wealth. In all states at $t = 1$, the buyers' marginal utility of wealth λ_1^b is now given by

$$\lambda_1^b = 1 \tag{33}$$

and, thus, is independent of the price, q_1, of the long-term asset—compare (33) with the expression for the cash-in-the-market model in (29). Equation (33) follows from the fact that λ_1^b is the Lagrange multiplier of the buyers' time-1 budget constraint (31). In contrast, the sellers' marginal utility of wealth, λ_1^s, is the same as in the baseline model because their problem is the same. That is, λ_1^s is still given by (20), namely, $\lambda_1^s = R/q_1$.

Comparing the sellers and buyers' marginal utilities in (20) and (33) shows that the two sets of agents have the same marginal utility of wealth in the low-withdrawal state $\gamma = 0$ (i.e., when $q_1 = R$) but different marginal utilities in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when $q_1 < R$ and a fire sale occurs). That is, a gap between the two marginal utilities opens up when a fire sale occurs;

[17] The sellers' time-1 indirect utility function, V_1^s, is also unchanged and, thus, is given by (17), whereas the buyers' time-1 indirect utility function in (26) is replaced by $V_1^b(T; q_1) = c_2^b + \lambda_1^b \left[1 + T - \left(l_1^b + q_1 k_1^b\right)\right]$ because they do not derive utility from the time-1 consumption in the second-best-use model and we can ignore the non-negativity constraint on l_1^b, as it does not bind). However, this difference does not affect the regulator's first-order conditions because, due to the envelope theorem, such optimality conditions do not depend on the time-1 or time-2 choices.

specifically, $\lambda_1^s > \lambda_1^b$.

The gap that opens up between the buyers' and sellers' marginal utilities of wealth is due to the buyers' lower ability to extract cash flow and, as denoted in equation (1), because the cash flow collected is from the marginal unit traded. Because of this gap, when assets are transferred to buyers in fire sales, the total output available in the economy at $t = 2$ is lower, compared to the non-fire-sales state. Similar to Section 3, the linearity of the time-2 utility of both the buyers and the sellers implies that the considerations about risk sharing discussed in Section 2 do not affect the policy analysis in this baseline version of the model. These considerations will play a role when considering a more general time-2 utility in Section 6.

The planner can improve welfare by forcing the sellers to invest more in liquidity at $t = 0$. With more liquidity available at $t = 1$, each seller needs to sell fewer assets, resulting in a higher price, q_1, during a fire sale. The higher price, in turn, implies that the sellers need to sell even less, increasing the quantity of the long-term assets that remain in their hands and, thus, increasing the total output available in the economy at $t = 2$. The next proposition formalizes this result.

Proposition 4.2. ***(Inefficiency and liquidity requirements in the second-best-use model)*** *In the second-best-use model, the unregulated equilibrium is not constrained efficient and the sellers' time-0 liquidity holdings are lower than the socially optimal level.*

5 Model with asymmetric information and adverse selection

Our third model is based on the classic assumption that long-term assets might vary in quality (i.e., some might generate more cash flow than others) and sellers have better information than buyers about the quality of the assets they are selling (Akerlof, 1970). This gives rise to the type of adverse selection that reduces the price of the assets that are sold.

To make the asymmetric-information model fully comparable with those in sections 3 and 4, we address a common issue in asymmetric-information models of fire sales. In these models, increasing the sellers' liquidity needs (which leads to more asset sales) typically results in a rise in asset prices. This occurs because higher liquidity needs force sellers to offload high-quality assets, increasing the proportion of such high-quality assets in the market and, thus, raising prices (Eisfeldt, 2004; Uhlig, 2010). This contrasts with the cash-in-the-market and second-best-use models from sections 3 and 4, where prices and trading volumes have a negative relationship—a pattern observed in actual fire sales (Uhlig, 2010).

To align the asymmetric-information model with the other models from sections 3 and 4, we use a simple approach. We assume that an increase in the sellers' liquidity needs is correlated with a greater fraction of low-quality assets. This assumption allows us to generate a negative link between prices and trading volume. The increase in the fraction of low-quality assets leads to lower prices and higher trading volumes, offsetting the positive link that would otherwise arise from higher liquidity needs. Kurlat (2016, 2021) and Chang (2018) develop deeper microfoundations for the negative price-volume link, but we highlight in Section 5.5 that the intuition for the inefficient composition of investments between the long-term and liquid assets in our model is similar to that of Kurlat (2016, 2021), where they explain the inefficiencies in the total sizes of investments. And despite the simple structure we use, our model captures the narrative about periods of financial distress characterized by fire sales and worsening adverse selection, as described, for instance, in Gorton and Ordonez (2014).

5.1 Asymmetric-information model: environment

There are, again, three periods indexed by $t = 0, 1, 2$. As in the models discussed in sections 3 and 4, the sellers make investments in liquidity and long-term assets at $t = 0$. The buyers are born at $t = 1$, and trading occurs in a centralized market at $t = 1$, when a fire sale can happen. The payoffs of the long-term assets are realized at $t = 2$.

The endowments are the same as in the baseline model. Specifically, the sellers have an endowment e^s and issue debt $d_0^s = d^s$ at $t = 0$. When the buyers are born at $t = 1$, they are endowed with one unit of liquidity and no holdings of the long-term asset.

The sellers' preferences are the same as in the cash-in-the-market and second-best-use models of sections 3 and 4. Specifically, they have linear utility from consumption c_2^s at $t = 2$ and are subject to the same withdrawal shock $\gamma \in \{0, \bar{\gamma}\}$ at $t = 1$.

The buyers have the same preferences as in the second-best-use model of Section 4; that is, they enjoy linear utility from consumption c_2^b at $t = 2$. Thus, the buyers do not consume at $t = 1$; and this differs from the cash-in-the-market model of Section 3.

The key difference in this version of the model is in the quality of the long-term assets and the information the agents have about it. Recall that only the sellers make the initial investment in the long-term assets. For each unit invested at $t = 0$, a fraction, θ, becomes a "lemon" at $t = 1$ (i.e., it will produce nothing at $t = 2$), whereas the remaining fraction $1 - \theta$ produces R as in the baseline model. We assume that θ is a random variable taking values $\theta \in \{0, \bar{\theta}\}$, with $\bar{\theta} \in (0, 1)$

and $(1-\theta)R > 1$, ensuring that the long-term assets have higher productivity than the liquid assets. Crucially, each seller has private information about the quality of the assets they own. That is, each seller can distinguish between the low-quality assets (that produce no cash flow at $t = 2$) and the high-quality ones (that produce $R > 1/(1-\theta)$ at $t = 2$). Note that the buyers can collect the same time-2 cash flow, from their long-term assets, as the sellers (i.e., zero from the low-quality assets and R from the high-quality assets), similar to the cash-in-the-market model of Section 3. However, the buyers cannot recognize the quality of each individual asset traded at $t = 1$.

The adverse-selection problem arises only when $\{\gamma, \theta\} = \{\bar{\gamma}, \bar{\theta}\}$ (i.e., when some of the long-term assets are lemons). In the other state, $\theta = 0$ and all of the long-term assets are high quality. This keeps the analysis simple, allowing us to focus on illustrating the core results.

We assume that the realization of γ and θ are correlated. This generates a link between the sellers' liquidity needs and the degree of adverse selection, as discussed at the beginning of Section 5. At $t = 1$, there are two possible states: $\{\gamma, \theta\} = \{0, 0\}$ (with probability $1 - \pi$), and $\{\gamma, \theta\} = \{\bar{\gamma}, \bar{\theta}\}$ (with probability π). In equilibrium, a fire sale occurs in the latter state. Although not essential for our results, we also assume that the link between γ and θ is given by $\theta = \nu\gamma^{\chi}$, with $\nu > 0$ and χ sufficiently large. This assumption ensures that a higher trading volume is associated with a lower price in a fire sale, as discussed at the beginning of Section 5, thereby making the model with asymmetric information observationally equivalent to those of sections 3 and 4.[18]

When $\gamma, \theta = \{\bar{\gamma}, \bar{\theta}\}$, we consider a pooling equilibrium. Unlike the classic lemon problem (Akerlof, 1970), where only lemons are traded and the equilibrium price is zero, here the sellers will also sell some high-quality assets to meet their liquidity needs, resulting in a positive price for such assets.[19]

Finally, we impose the following parameter restriction:

$$\frac{(R-1)\bar{\gamma}d^s - R\pi e^s}{(R-1) - R\pi\bar{\theta}} > 0. \tag{34}$$

We also assume that e^s is sufficiently large, as in Section 3. These assumptions ensure that the sellers' investments in liquidity and long-term assets at $t = 0$ are both strictly positive and their time-2 consumption is also strictly positive.

[18]The assumption $\theta = \nu\gamma^{\chi}$ can be relaxed, for example, by assuming that $\theta = \nu\gamma^{\chi} + \varepsilon$, where $\varepsilon \in \mathbb{R}$ is either negative or sufficiently small.

[19]Note that, in equilibrium, the sellers cannot sell only lemons if their liquidity holdings, l_0^s, are less than the withdrawals $\bar{\gamma}d^s$. To see why, assume for contradiction that only lemons were traded. In that case, the price, q_1, would be zero and the sellers would not raise enough resources to meet their withdrawals.

5.2 Sellers' choices in the asymmetric-information model

We start by solving the sellers' problem at $t = 1$ and then proceed to the analysis at $t = 0$. At the beginning of $t = 1$, the sellers hold k_0^s units of the long-term asset and l_0^s units of liquidity. A fraction θ of their long-term asset holdings is of low quality (i.e., "lemons"), while the remaining fraction $1 - \theta$ is high quality.

In the low-withdrawal state $\gamma = 0$, we also have $\theta = 0$, meaning that all of the long-term assets are high quality. Thus, the sellers' problem and choices are the same as in the models with cash-in-the-market and second-best-use pricing; see Section 3.2. That is, in the relevant case in which $q_1 = R$, the choices $k_1^s = k_0^s$ and $l_1^s = l_0^s$ are optimal (i.e., the sellers do not engage in any trades).

In the high-withdrawal state $\gamma = \bar{\gamma}$, we also have $\theta = \bar{\theta}$ and, thus, a fraction of the sellers' long-term assets are low quality. The sellers sell all of their holdings of low-quality assets plus some of their high-quality assets, as discussed at the end of Section 5.1. Let k_1^s denote the holdings of the high-quality assets that the sellers retain. Their problem is to maximize consumption c_2^s at $t = 2$, which is the sum of the cash flow, R, from each retained high-quality long-term asset, k_1^s, and from their liquidity, l_1^s, minus the repayment, $(1 - \bar{\gamma})d^s$, owed to the debt holders:

$$\max_{k_1^s, l_1^s} Rk_1^s + l_1^s - (1 - \bar{\gamma})d^s \tag{35}$$

subject to $l_1^s \geq 0$ and the budget constraint $l_1^s + \bar{\gamma}d^s \leq l_0^s + q_1\bar{\theta}k_0^s + q_1 \left[(1 - \bar{\theta})k_0^s - k_1^s\right]$. That is, the sellers finance their holdings l_1^s of liquidity and their withdrawals $\bar{\gamma}d^s$ by using the liquidity l_0^s carried from $t = 0$, selling their holdings of low-quality long-term assets $\bar{\theta}k_0^s$ at price q_1, and selling an amount $(1 - \bar{\theta})k_0^s - k_1^s$ of their high-quality long-term assets, also at price q_1.

The sellers' problem in (35) is, thus, identical to that in the cash-in-the-market model, despite the added asymmetric information friction. Hence, as discussed in Section 3.2, the sellers choose $l_1^s = 0$, and the amount of high-quality assets they retain, k_1^s, is determined residually by the budget constraint evaluated at $l_1^s = 0$. This amount is given by (4).

Moving to the choices at $t = 0$, the problem (18) is unchanged and, thus, their first-order condition is again given by (19).

5.3 Buyers' choices in the asymmetric-information model

The buyers' problem is to maximize their time-2 consumption, c_2^b, given their linear utility at $t = 2$. To formalize the buyers' problem, let $\mathbb{E}_1^b \{R_2\}$ denote the buyers' belief about the payoff of a long-term asset purchased at $t = 1$, based on their information set. We assume that the buyers' beliefs are rational; that is, that the buyers form their expectation $\mathbb{E}_1^b \{R\}$ based on the amount of high- and low-quality assets the sellers sold:

$$\mathbb{E}_1^b \{R_2\} = \frac{\theta k_0^s \times 0 + R\left[(1-\theta) k_0^s - k_1^s\right]}{k_0^s - k_1^s}. \tag{36}$$

That is, the sellers sell all of their holdings of low-quality long-term assets, θk_0^s, which produce zero at $t = 2$, and an amount, $(1-\theta) k_0^s - k_1^s$, of high-quality long-term assets that produces R at $t = 2$. The total sales, given by the denominator in (36), are $k_0^s - k_1^s$.

We can thus express the buyers' expected time-2 consumption as $\mathbb{E}_1^b\{c_2^b\} = l_1^b + \mathbb{E}_1^b \{R_2\} k_1^b$. The buyers' optimization problem is

$$\max_{l_1^b, k_1^b} l_1^b + \mathbb{E}_1^b \{R_2\} k_1^b, \tag{37}$$

subject to their budget constraint $l_1^b + q_1 k_1^b \leq 1$. Their optimality condition is

$$q_1 = \mathbb{E}_1^b \{R_2\}. \tag{38}$$

That is, the buyers are willing to trade as long as the price, q_1, is equal to the expected payoff, $\mathbb{E}_1^b \{R_2\}$.

5.4 Equilibrium in the asymmetric-information model

This section describes the main features of the equilibrium. The full characterization is provided in Appendix D.

In the low-withdrawal state $\{\gamma, \theta\} = \{0, 0\}$, all of the long-term assets are of high quality. Thus, the buyers' belief about the productivity of the long-term assets that are traded is $\mathbb{E}_1^b \{R_2\} = R$. As a result, (38) implies that the price, q_1, is also equal to R.

In the high-withdrawal state $\{\gamma, \theta\} = \{\bar{\gamma}, \bar{\theta}\}$, some of the assets the sellers sold are of low quality. Consequently, the buyers' belief about the productivity of the long-term assets that are traded is $\mathbb{E}_1^b\left\{R_2\right\} < R$. Specifically, using the sellers' optimal choice for k_1^s, which is given by (4) as discussed in Section 5.2, the buyers' belief in (36) can be rewritten as

$$\mathbb{E}_1^b\left\{R_2\right\} = R\left(1 - \frac{q_1\bar{\theta}k_0^s}{\bar{\gamma}d^s - l_0^s}\right). \tag{39}$$

Combining (38) and (39) evaluated at $q_1 = q_1\left(\bar{\gamma}, \bar{\theta}\right)$ and rearranging, we obtain

$$q_1\left(\bar{\gamma}, \bar{\theta}\right) = \frac{R}{\left(1 + R\frac{\bar{\theta}k_0^s}{\bar{\gamma}d^s - l_0^s}\right)}. \tag{40}$$

We can then solve for the price, $q_1\left(\bar{\gamma}, \bar{\theta}\right)$, in the high-withdrawal state and for the sellers' portfolio choices at $t = 0$ as a function of the parameters. To this end, we combine (40) with the sellers' time-0 first-order condition and time-0 budget constraint, which are given by (14) and (19), as discussed in Section 5.2. The price, $q_1\left(\bar{\gamma}, \bar{\theta}\right)$, is the same as in the cash-in-the-market and second-best-use models and, thus, is given by (21). The sellers' time-0 liquidity holdings are then given by

$$l_0^s = \frac{(R-1)\bar{\gamma}d^s - R\pi\theta e^s}{(R-1) - R\pi e^s}, \tag{41}$$

and the time-0 investments in long-term assets, k_0^s, are residually determined by the budget constraint (14). These quantities are both positive, given the assumption in (34).

Finally, we compare the equilibrium in this model with those in the cash-in-the-market and second-best-use models. As noted before, the equilibrium price at $t = 1$ is the same and because the sellers' problem is also identical—as discussed in Section 5.2—the trading volume is also the same. Using (22) and (41), we can also derive the same portfolio choices at $t = 0$, given the appropriate restrictions on θ and on other parameters. We also examine how the price, q_1, responds to changes in the trading volume during a fire sale. In this model with asymmetric information, the trading volume is related to both the withdrawal pressure, γ, and the share of low-quality assets, θ. As assumed in Section 5.1, γ and θ are positively correlated (specifically, $\theta = \nu\gamma^{\chi}$ with $\nu > 0$ and χ sufficiently large) and, under this assumption, (40) implies $\partial q_1\left(\bar{\gamma}, \bar{\theta}\right)/\partial\bar{\gamma} < 0$. Thus, as γ (and θ) increases, the buyers sell more long-term assets (i.e., the trading volume increases) and the price, $q_1\left(\bar{\gamma}, \bar{\theta}\right)$, decreases. Therefore, the link between the trading volume and the price in a fire sale is negative, similar to the cash-in-the-market and second-best-use models of sections 3 and 4.

5.5 Inefficiency and liquidity ceiling in the asymmetric-information model

We now study the efficiency of the equilibrium and policy interventions in the version of the model with asymmetric information. The regulator's problem remains unchanged and, thus, is as described by equations (23)-(25).[20] The first-order conditions are (27) and (28).

As discussed in sections 2, 3.6, and 4.4, the efficiency is crucially affected by the sellers' and buyers' marginal utilities of wealth; that is, λ_1^s and λ_1^b. The sellers' marginal utility, λ_1^s, is again given by $\lambda_1^s = R/q_1$, as in the cash-in-the-market and second-best-use models (see (20)). Here, the logic is that with an additional dollar available, the sellers can *avoid* selling an amount, $1/q_1$, of *high-quality assets*, which are the assets with payoff R at $t = 2$. For the buyers, however, the marginal utility of wealth is $\lambda_s^b = 1$, independently of the realization of $\{\gamma, \theta\}$; this result follows from the buyers' problem in (37). Why are the buyers and sellers' marginal utilities of wealth different, even if they have the same ability to collect cash flow at $t = 2$? As emphasized in equation (1), it is important to understand the cash flow produced by the *marginal* unit that is traded. For the sellers, this marginal unit consists of high-quality assets, because all of the low-quality holdings are sold first as inframarginal units. But for the buyers, the marginal unit traded is the average asset in the market, whose cash flow is the average of high- and low-quality assets.

Thus, similar to the second-best-use model of Section 4, a gap between the buyers and sellers' marginal utilities of wealth opens up during a fire sale, though for different reasons. In the second-best-use model, the gap arises from the buyers' lower ability to collect cash flow from long-term assets, at $t = 2$, which reduces aggregate efficiency (in the sense discussed in Section 2). This reduction occurs because the total cash flow collected at $t = 2$ in a fire sale is lower compared to the state with no fire sales. With asymmetric information, total consumption at $t = 2$ is not directly affected by the trading volume because the buyers and sellers are able to collect the same cash flow. Instead, the gap arises because at $t = 0$, the sellers invest too little in the (highly productive) long-term asset, anticipating that they will have to sell the marginal high-quality asset at an adverse-selection discount during a fire sale. This intuition is similar to the one discussed by Kurlat (2021) but, here, it affects the composition of the investments between long-term and liquid assets, as opposed to the total size of the portfolio, as in Kurlat (2021).

The optimal policy intervention supports the time-1 asset price to induce higher investments in

[20]Similar to the planner's problem in the second-best-use model, the buyers' indirect utility function includes only time-2 consumption. However, this does not affect the regulator's first-order conditions because of the envelope theorem.

the long-term asset at $t = 0$. While optimal regulation aims to also generate higher asset prices in the second-best-use model, it is achieved here by forcing the sellers to invest *less* in liquidity. This regulation is the opposite to that in the second-best-use model. With asymmetric information, reducing liquidity results in a higher time-1 price, q_1, during a fire sale, due to the same logic discussed in Malherbe (2014). That is, if the sellers hold less liquidity, a larger fraction of the assets traded are sold to meet their liquidity needs and, thus, consists of high-quality assets. Consequently, the share of lemons in the market is lower, mitigating the adverse-selection problem. Formally, in the adverse-selection model, the term $\partial q_1 / \partial l_0^s$ in the regulator's first-order condition (19) is negative, in contrast to the positive term found in the cash-in-the-market and second-best-use models of sections 3 and 4.

Proposition 5.1. ***(Inefficiency and liquidity ceiling in the asymmetric-information model)*** *In the asymmetric-information model, the unregulated equilibrium is not constrained efficient. Specifically, the sellers' time-0 liquidity holdings exceed the socially optimal level.*

6 General time-2 utility and risk-sharing considerations

The models of sections 3-5 yield a simple and stark result: Three observationally equivalent models of fire sales that abstract from risk-sharing considerations have very different implications regarding the liquidity requirements that, when imposed on financial players, might force them to sell assets in a fire sale. With cash-in-the-market pricing, the equilibrium is efficient and no regulation is needed. With a second-best-use assumption, a liquidity requirement is optimal. And with asymmetric information, the opposite regulation (i.e., a ceiling on liquidity) is optimal.

This section extends the framework used in sections 3-5 by incorporating the impact of market incompleteness on the investors' inability to fully share risk in financial markets. To this end, we relax the assumption that the investors have linear utility at $t = 2$ and instead allow for an arbitrary concave utility function. Under this assumption, the forces discussed in sections 3-5 continue to operate. However, the optimal policy stance is also influenced by the possibility that the unregulated equilibrium may not achieve full risk sharing.

The equilibrium in the general-utility model is generically inefficient in the cash-in-the-market model, and the optimal policy could be either a liquidity requirement or a liquidity ceiling, depending on the exact parameterization. However, if a liquidity requirement is optimal in the cash-in-the-market model, we show that it must be tighter in an observationally equivalent second-best-use model. And if a liquidity ceiling is optimal in the cash-in-the-market model, we show that the optimal regulation in the second-best-use model is either a lower ceiling or a liquidity requirement. The optimal regulation in the asymmetric-information model remains a liquidity ceiling, under regularity conditions.

6.1 Model with general time-2 utility

In each of the three models (i.e., cash-in-the-market, second-best-use, and asymmetric-information), the assumptions regarding technology, endowments, information, and preferences are identical, except for the functional form of the sellers and buyers' time-2 utility. We assume that the sellers' time-2 utility from consuming c_2^s is $u_2^s(c_2^s)$ and the buyers' time-2 utility from consuming c_2^b is $u_2^b(c_2^b)$, where $u_2^s(\cdot)$ and $u_2^b(\cdot)$ are strictly increasing and weakly concave functions and at least one of them is strictly concave. For the cash-in-the-market model, we relabel the time-1 utility function as $u_1^b(\cdot)$ to avoid confusion.

To facilitate the comparison among the three models, we impose the same sellers' utility function, $u_2^s(\cdot)$, in all models. For the buyers, however, the utility function needs to be different to generate the same observationally equivalent equilibrium. This is because the buyers consume at $t = 1$ and $t = 2$ in the cash-in-the-market model but only at $t = 2$ in the other two models. We normalize $u_2^b(\cdot)$ *in each version of the model* so that the buyers' marginal utility is one at the equilibrium with low withdrawals (i.e., with no fire sales). That is, ${}^{\partial u_2^b(0)}/_{\partial c_b^2} = 1$ in the cash-in-the-market model and ${}^{\partial u_2^b(1)}/_{\partial c_b^2} = 1$ in the second-best-use and asymmetric-information models.

We derive the policy analysis (in the next section) under the assumption that there exists an equilibrium with the same features as in the baseline: an interior portfolio choice for liquidity and long-term asset holdings at $t = 0$, no trading at $t = 1$ in the low-withdrawal state $\gamma = 0$, a positive trading volume and a fire sale at $t = 1$ in the high-withdrawal state $\gamma = \bar{\gamma}$, and a buyers' demand that is downward sloping in the trading volume.[21] The remainder of this section provides some remarks to show that the environment can generate an equilibrium with these features. Without loss of generality, we normalize the sellers' time-2 utility function so that their time-1 marginal utility of wealth in the low-withdrawal state, $\gamma = 0$ (i.e., when no fire sales occur), is $\lambda_1^s(0) = 1$.

[21] As in Dávila and Korinek (2018), we sidestep the issue of the existence of the equilibrium, given the generality of the utility functions we consider.

Remark #1: Sellers and buyers' choices. Because the sellers' utility depends only on their time-2 consumption and because there is no uncertainty between $t = 1$ and $t = 2$, their objective at $t = 1$ is unchanged (i.e., maximizing time-2 consumption) and their choices are, thus, the same as in the baseline analyses of sections 3-5. For the buyers, the time-1 first-order condition (9) in the cash-in-the-market model is replaced by

$$q_1 = \frac{(u_2^b)'(c_2^b)}{(u_1^b)'(c_1^b)} R. \tag{42}$$

In the second-best-use and asymmetric-information models, because the buyers' utility depends only on time-2 consumption, the first-order conditions (32) and (38) are unchanged—the logic is identical to that discussed for the sellers' problem.

Remark #2: Sellers and regulator's problem at $t = 0$. The formulation of the sellers and regulator's problem in (18) and (23), respectively, is unchanged. While the indirect utility functions are slightly different because of the general time-2 utility, this difference does not affect the time-0 first-order conditions because of the envelope theorem (see discussion in Section 3.6). Thus, the sellers' time-0 first-order condition is still given by (19) and the regulator's first-order conditions are given by (27) and (28).

Remark #3: Equilibrium in the low-withdrawal state $\gamma = 0$. Under the normalizations regarding the buyers' marginal utility, which we introduced before, and taking as given time-0 choices, the price in the low-withdrawal state, $\gamma = 0$, is $q_1 = R$ in all three models and no trade takes place, as in the baseline.

Remark #4: Equilibrium in the high-withdrawal state $\gamma = \bar{\gamma}$. Similar to the baseline, we can use (19) to pin down the price, $q_1(\bar{\gamma})$, in the fire-sale state.[22] To do so, we note that the time-1 marginal utility of the sellers' wealth, λ_1^s, which is given by (20) in the baseline, is now given by

$$\lambda_1^s = \frac{R}{q_1} \frac{\partial u_2^s\left(c_2^s\right)}{\partial c_2^s} \tag{43}$$

[22]In the version of the model with asymmetric information, the state is fully described by the tuple $\{\gamma, \theta\}$. However, as assumed in Section 5, θ is a function of γ and, thus, we simply use the notation $q_1(\gamma)$ to refer to the price in state γ in all three versions of the model, including the version with asymmetric information.

in all three versions of the model. That is, an additional unit of wealth at $t = 1$ allows the sellers to reduce their sales by $1/q_1$ units of the long-term asset, obtaining a payoff, R, per unit of asset, which is then valued according to their time-2 marginal utility of consumption, $\partial u_2^s(c_2^s)/\partial c_2^s$. Note that (43) formalizes (1) for the case in which the sellers have a general marginal utility, $\partial u_2^s(c_2^s)/\partial c_2^s$. Combining (15), (19), (43) and the assumption that the sellers' marginal utility of wealth, λ_1^s, is normalized to one in the low-withdrawal state, we can solve for the price, $q_1(\bar{\gamma})$, of the long-term asset in the fire-sale state:

$$q_1(\bar{\gamma}) = \frac{\pi R \frac{\partial u_2^s\left(c_2^s(\bar{\gamma})\right)}{\partial c_2^s}}{(1-\pi)(R-1) + \pi R \frac{\partial u_2^s\left(c_2^s(\bar{\gamma})\right)}{\partial c_2^s}} < 1. \tag{44}$$

While (44) expresses $q_1(\bar{\gamma})$ as a function of the endogenous level of time-2 consumption, c_2^s, it shows that $q_1(\bar{\gamma}) < 1$ because $R > 1$ and $\pi < 1$. Thus, an equilibrium with high withdrawals, $\bar{\gamma}$, is characterized by a drop in the long-term asset price. In addition, because the sellers make the same choices as in the baseline (see Remark #1), the high-withdrawal state is again characterized by a higher trading volume relative to normal times, similar to the baseline of sections 3-5.

6.2 Inefficiency and liquidity regulation with general utility

We are now ready to state our main results in the model with general utility. We begin by comparing efficiency and regulation in the cash-in-the-market and second-best-use models, and then we turn to the model with asymmetric information.

Proposition 6.1. ***(General utility: cash-in-the-market and second-best-use models)*** *Consider a cash-in-the-market model and a second-best-use model that are observationally equivalent (i.e., the sellers make the same time-0 choices in the two models and for any γ, the time-1 price, q_1, the time-1 trading volume, k_1^b, and the sensitivity of the price, q_1, to the trading volume, k_1^b, are the same in the two models). Assume also that the sensitivity of the price, q_1, to the sellers' time-0 liquidity holdings, $\partial q_1/\partial l_0^s$, is the same. Then,*

(i) If the sellers' time-0 liquidity holdings are higher than the socially optimal level in the cash-in-the-market model (i.e., the optimal policy is a liquidity ceiling), the optimal policy in the second-best-use model is either a lower liquidity ceiling or a liquidity requirement.

(ii) If the sellers' time-0 liquidity holdings are lower than the socially optimal level in the cash-in-the-market model (i.e., the optimal policy is a liquidity requirement), then the optimal policy in the second-best-use model is a tighter liquidity requirement.

In the cash-in-the-market model with general utility, the equilibrium is generically inefficient and it is not possible to establish the general direction of the inefficiency, similar to Dávila and Korinek (2018). However, we can compare the efficiency and regulation in a cash-in-the-market model with that of an equivalent second-best-use model.

Proposition 6.1 states that the socially optimal level of liquidity is always higher through the lenses of a second-best-use model, compared to a cash-in-the-market model. To understand this result, note that in the model with general utility, during a fire sale a gap can open up between the sellers and the buyers' marginal utilities of wealth (i.e., between λ_1^s and λ_1^b), so that the regulator's first-order condition (19) does not coincide with those of the individual sellers in (19). Importantly, with cash-in-the-market pricing, this gap is small—or even negative—because both the sellers and the buyers' marginal utilities, λ_1^s and λ_1^b, increase in a fire sale, relative to normal times. The sellers' marginal utility of wealth increases because they lose some wealth in a fire sale. The buyers' marginal utility of wealth increases because the increase in the demand for liquidity (due to the shock $\bar{\gamma}$) drains liquidity from the market and reduces the buyers' ability to consume at $t = 1$. In the second-best-use model, the sellers' marginal utility of wealth also increases, but the buyers' marginal utility of wealth decreases because these buyers gain on the inframarginal units they purchase. That is, for such inframarginal units, the cash flow collected is greater than the price paid—the cash flow is equal to the price only for the marginal unit.

Next, we analyze efficiency and regulation with asymmetric information. The forces that operate in the baseline model with linear utility continue to operate. The additional element that arises with general utility is related to the term $\partial q_1/\partial T$ in the planner's first-order condition (28). In the baseline model, the linearity of the sellers' time-2 utility implies that a transfer, T, that reduces the sellers' wealth (and transfers such wealth to the buyers) has no effect on the equilibrium price, q_1, and, thus, $\partial q_1/\partial T = 0$. With general utility, a reduction in the sellers' wealth generates a "wealth effect," resulting in a change in the sellers time-0 portfolio and, with it, the equilibrium price.[23] Hence, the term $\partial q_1/\partial T$ is not necessarily zero with general utility. However, the next

[23]Formally, in the baseline model, the linearity of the time-2 sellers' utility implies that these sellers' time-0 liquidity choices are infinitely elastic and, thus, their time-0 first-order condition pins down the price, $q_1(\bar{\gamma})$. Because the tax, T, is a lump sum, it does not affect the time-0 first-order or the price, $q_1(\bar{\gamma})$. In the version of the model with general utility, the time-0 first-order condition depends on the time-2 marginal utility of consumption and, thus, $\partial q_1(\bar{\gamma})/\partial T$ is

proposition provides a sufficient condition under which the optimality of the liquidity ceiling in the asymmetric-information model—derived in the baseline with linear utility—extends to the case with general utility.

Proposition 6.2. ***(General utility: asymmetric-information model)*** *Consider the asymmetric-information model augmented with a transfer, T, of liquidity from the sellers to the buyers, at the beginning of $t = 1$, with the transfer, T, announced at the beginning of $t = 0$. Define the sellers' $fundamental\ liquidity\ needs$ as*

$$(fundamental\ liquidity\ needs) = \frac{\bar{\gamma}d^s - (l_0^s - T)}{\theta k_0^s}, \tag{45}$$

which is the ratio of the liquidity the sellers raise in the market to finance their time-1 withdrawals in the high-withdrawal state, $\bar{\gamma}d^s - (l_0^s - T)$, to the quantity of lemons in that state, θk_0^s. If

$$\left.\frac{\partial\,(fundamental\ liquidity\ needs)}{\partial T}\right|_{T=0} \leq 0, \tag{46}$$

then the sellers' time-0 liquidity holdings are higher than the socially optimal level, so that the optimal policy is a liquidity ceiling.

We argue that the sufficient condition (46) has a natural interpretation and, thus, we expect it to be satisfied. To understand (46), fix the price $q_1(\bar{\gamma})$ in a fire sale. Condition (46) says that when the sellers face a higher tax, T, that reduces their wealth, they tilt their time-0 portfolio in a way that reduces their *fundamental liquidity needs*; that is, the amount of liquidity they need to raise through selling long-term assets in a fire sale, relative to the stock of such assets. Because, on the margin, the sellers sell high-quality long-term assets to finance their liquidity needs (i.e., the assets with payoff $R > 1$), there is a cost to finance this liquidity need, given by the fire-sale discount. Under (46), an increase in the tax, T, that makes the sellers poorer creates an incentive for them to reduce the losses related to the fire-sale discount. Thus, (46) seems natural in the sense that the loss of wealth due to the tax creates an incentive for the seller to hedge against further losses they would face in a fire sale.

not necessarily zero.

7 Conclusions

This paper analyzes liquidity requirements—a policy that has attracted growing attention over time from policymakers and academics—in a model in which financial intermediaries are forced to sell some assets to meet high liquidity needs. The model nests three mechanisms commonly employed in the literature to generate low fire-sale prices: cash-in-the-market pricing, second-best-use pricing, and adverse-selection pricing.

The optimal liquidity policy involves a liquidity requirement, or a liquidity ceiling, or no intervention, depending on the pricing mechanism and the effects of market incompleteness on investors' ability to share risk. More generally, we identified three main forces that determine the direction of the optimal policy and that are common to all three pricing mechanisms: (i) the cash flow that the buyers and sellers collect from the marginal unit traded, (ii) the sensitivity of the fire-sale price to the investors' liquidity holdings, and (iii) how market incompleteness affects the investors' ability to share risk.

For policymakers that are considering liquidity requirements—such as the SEC, which is considering this policy for mutual funds—our results suggest that fire sales alone, such as those that took place during the March 2020 dash for cash, do not justify liquidity requirements. Instead, our results suggest that policymakers should consider whether liquidity requirements are warranted by the pricing mechanism that applies to the situation they are considering, whether market incompleteness gives rise to imperfect risk sharing, and whether the buyers or the sellers would suffer more, in a fire sale, from such imperfect risk sharing.

We derived our results using a standard fire-sale framework in which trades take place in centralized markets. In practice, some assets that have experienced fire sales—such as asset-backed securities and corporate bonds—are traded in decentralized over-the-counter (OTC) markets. While the forces we identified are likely to be important even in OTC markets, future research could study whether such forces interact with other possible distortions that are driven by the lack of centralized trading venues.

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ONLINE APPENDIX

A Proofs

Proof of Proposition 3.1. Given $\gamma = 0$, we can use the equilibrium equations (i.e., namely, equations (3) and (7), the two expressions in (5), the three expressions in (10), and the market-clearing condition (12)) to solve for the eight equilibrium objects stated in the proposition.

Proof of Proposition 3.2. When $\gamma = \bar{\gamma}$, we can use the equilibrium equations (i.e., namely, equations (3) and (7), the two expressions in (5), the three expressions in (10), and the market-clearing condition (12)) to solve for the price, q_1, and the buyers' time-1 consumption, c_1^b, as stated in the proposition, the sellers' time-1 portfolio,

$$\{l_1^s, k_1^s\} = \left\{0, k_0^s - \frac{\bar{\gamma} d_0^s - l_0^s}{R\left(1 - (\bar{\gamma} d_0^s - l_0^s)\right)}\right\},$$

the buyers' time-1 portfolio,

$$\{l_1^b, k_1^b\} = \left\{0, \frac{\bar{\gamma} d_0^s - l_0^s}{R\left(1 - (\bar{\gamma} d_0^s - l_0^s)\right)}\right\},$$

the sellers' time-2 consumption,

$$c_2^s = \frac{(1-\bar{\gamma})\bar{\gamma}\left(d_0^s\right)^2 - d_0^s(\bar{\gamma} k_0^s R + (1-\bar{\gamma}) l_0^s + 1) + k_0^s l_0^s R + k_0^s R + l_0^s}{1 + l_0^s - \bar{\gamma} d_0^s} > 0$$

(where the inequality follows from (11) and the assumption that k_0^s is sufficiently large), and the buyers' time-2 consumption, $c_b^2 = \frac{\bar{\gamma} d_0^s - l_0^s}{1 + l_0^s - \bar{\gamma} d_0^s} > 0$, where the inequality follows from (11). The inequalities stated in the proposition follow from (11).

Proof of Proposition 3.3. To prove the results, we evaluate the first-order condition of the regulator at the unregulated equilibrium. Specifically, we rederive the equilibrium in a version of the model in which the regulator announces a transfer, T, close to zero before the sellers make their time-0 decisions, so that we can compute the expression $\partial q_1 / \partial T$ that appears in the regulator's first-order condition (28). We then evaluate this equilibrium at $T = 0$.

In the version of the model with the transfer, T, close to zero, the sellers' time-0 problem (18) becomes

$$\max_{l_0^s, k_0^s} \mathbb{E}_0 \left\{ V_1^s(l_0^s - T, k_0^s) \right\},$$

subject to the budget constraint (14) evaluated at $d_0^s = d^s$. The first-order condition (19), however, is unchanged. The expression for q_1 in a fire sale in Proposition 3.2 is also unchanged because the time-1 market-clearing condition for liquidity (12) is independent of T.This is the case because the buyers enter $t = 1$ with $1 + T$ units of liquidity and the sellers enter $l_0^s - T$ and, thus, the total liquidity available in the economy is unchanged at $1 + l_0^s$. Hence, the price $q_1(\overline{\gamma})$ in (21) that follows from (19) and (20) is also unchanged. Note that $q_1(\overline{\gamma})$ in (21) does not depend on T and, thus, ${}^{\partial q_1(\overline{\gamma})}/_{\partial T} = 0$. Turning to the price $q_1(0)$ (i.e., in the low-withdrawal state $\gamma = 0$), and under the assumption that the buyers' endowment is $1 + \varepsilon$ (see footnote 6), a marginal change in T away from $T = 0$ does not affect the buyers' optimal choice $c_1^b = 1$, and because (9) implies the price q_1 depends only on c_1^b, we have $\partial q_1(0)/\partial T = 0$.

In all states at $t = 1$, the expression $\partial q_1/\partial T$ is zero when evaluated at $T = 0$, therefore, the first-order condition (28) of the regulator simplifies to

$$\xi = \frac{\mathbb{E}\left\{\lambda_1^s\right\}}{\mathbb{E}\left\{\lambda_1^b\right\}}. \tag{47}$$

Then, using (29), we obtain $\xi = 1$. Thus, using $\xi = 1$ and (29), the regulator's optimality condition (27) becomes identical to that of the sellers in (19). In other words, the regulator's first-order condition holds when evaluated at the unregulated equilibrium and, thus, such an equilibrium is constrained efficient.

Proof of Proposition 4.1. The equilibrium objects solve the list of equations in footnote 14. The price in the high-withdrawal state ($\gamma = \bar{\gamma}$) comes from (19), as in the cash-in-the-market model. The sellers' time-0 liquidity investment, l_0^s, comes from (32) the buyers' budget constraint at $t = 1$ (31) and the market-clearing condition (12). The time-0 choice of k_0^s follows from the sellers' time-0 budget constraint (14).

Proof of Proposition 4.2. We proceed as in the proof of Proposition 3.3 by rederiving the equilibrium with a transfer, T, that is close to zero, evaluating such an equilibrium at $T = 0$, and showing that ${}^{\partial q_1}/_{\partial T} = 0$ in all states at $t = 1$, when evaluating this derivative at $T = 0$.

In the high-withdrawal state, $\gamma = \bar{\gamma}$, the result ${}^{\partial q_1(\overline{\gamma})}/_{\partial T} = 0$ can be shown as in the proof of

Proposition 3.3. In the low-withdrawal state, we have $k_1^b = 0$ for any T close to zero. Thus, because q_1 is pinned down by (32) evaluated at $k_1^b = 0$, we have ${\partial q_1}/{\partial T} = 0$ when evaluated at $T = 0$.

Thus, as in the proof of Proposition 3.3, the value of ξ is given by (47). However, the expression for λ_1^b is different here, relative to the proof of Proposition 3.3, and in particular, relative to $\lambda_1^b = 1$ in both states in the second-best-use model. Thus, $\xi = \mathbb{E}\left\{\lambda_1^s\right\}$.

Next, again using $\lambda_1^b = 1$ and $\lambda_1^s = R/q_1$ together with the last result $\xi = \mathbb{E}\left\{\lambda_1^s\right\}$, the regulator's first-order condition (27) becomes

$$\begin{aligned} \mathbb{E}\left\{\lambda_1^s\left(q_1-1\right)\right\} &= \mathbb{E}\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\mathbb{E}\left\{\lambda_1^s\right\}-\lambda_1^s\right)\right\} \\ &= (1-\pi)\times 0+\pi(1-\pi)\frac{\partial q_1(\bar{\gamma})}{\partial l_0^s}\left(k_1^s(\bar{\gamma})-k_0^s\right)\left(1-\frac{R}{q_1(\bar{\gamma})}\right), \end{aligned} \tag{48}$$

where $q_1(\bar{\gamma})$ and $k_1^s(\bar{\gamma})$ denote the price and the sellers' end-of-period holdings of the long-term assets, respectively. The second line uses $\frac{\partial q_1}{\partial l_0^s} = 0$ where there are no fire sales (which holds because the sellers enter $t = 1$ with $l_0^s > 0$, where the inequality follows from (16)) and fire sales happen with probability π (according to (15)). As a last step, we show that the right-hand side of (48) is not zero and, thus, the equilibrium is not efficient. Specifically, the right-hand side is positive and, thus, the right-hand side of the regulator's first-order condition (27) evaluated at the unregulated equilibrium is higher than the marginal value of the sellers' wealth λ_1^s. Because the right-hand side of (27) is the marginal social value of investing in liquidity, such a value is higher for the regulator than for the individual agents and, thus, the sellers' time-0 liquidity holdings are lower than the socially optimal level.

To establish that $\frac{\partial q_1(\bar{\gamma})}{\partial l_0^s}\left(k_1^s(\bar{\gamma})-k_0^s\right)\left(1-\frac{R}{q_1(\bar{\gamma})}\right) > 0$, we begin by noting that $k_1^s(\bar{\gamma}) - k_0^s < 0$ because the sellers sell some of their long-term asset holdings in a fire sale and that $1 - {R}/{q_1(\bar{\gamma})} < 0$ because $q_1(\bar{\gamma}) < R$ in a fire sale. Thus, we need to show that ${\partial q_1(\bar{\gamma})}/{\partial l_0^s} > 0$. To establish this result, Figure 1 plots the left- and right-hand sides of the buyers' first-order condition (32) evaluated at the equilibrium value of k_0^b; that is, using the time-1 market-clearing condition for capital, (13), and the budget constraint of the sellers in times of fire sales (4):

$$q_1(\bar{\gamma}) = f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right) \tag{49}$$

as a function of $q_1(\bar{\gamma})$. The left-hand side is given by $q_1(\bar{\gamma})$ and, thus, is represented by the 45-degree line (solid line). The right-hand side $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$ is represented by the dotted line and

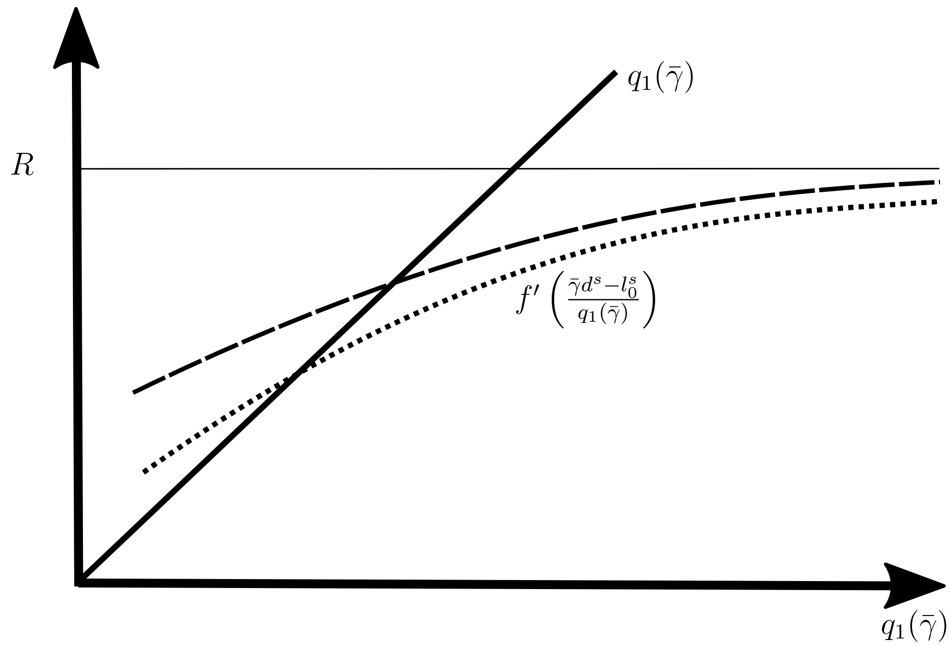

Figure 1: Establishing the sign of $\partial q_1(\bar{\gamma})/\partial l_0^s$ in the second-best-use model. The figure plots $q_1(\bar{\gamma})$ (solid line) and $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$ (dotted and dashed lines). An increase in l_0^s causes an increase in $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$, represented by the shift from the dotted to the dashed line, and thus, an increase in the equilibrium value of $q_1(\bar{\gamma})$.

depends on $q_1(\bar{\gamma})$ through $f'(\cdot)$. As $q_1(\bar{\gamma}) \to \infty$, the argument of $f'(\cdot)$ goes to zero and, thus, $f'(\cdot)$ converges to R, given the assumptions in Section 4.1. And as $q_1(\bar{\gamma})$ decreases, the argument of $f'(\cdot)$ increases, and $f'(\cdot)$ decreases because $f'' < 0$. In addition, the intersection with the 45-degree line is somewhere at a point where $q_1(\bar{\gamma}) < R$. If l_0^s increases, the argument of $f'(\cdot)$ decreases and, again, because $f'' < 0$, the value of $f'(\cdot)$ increases for any $q_1(\bar{\gamma})$. Thus, an increase in l_0^s causes an upward shift in Figure 1, the shift from the dotted line to the dashed line. In other words, an increase in l_0^s generates an increase in $q_1(\bar{\gamma})$, and vice versa, establishing $\partial q_1(\bar{\gamma})/\partial l_0^s > 0$.

Proof of Proposition 5.1. When we allow the regulator to make transfers, T, from the sellers' to the buyers, the equilibrium price in the high-withdrawal state $\gamma, \theta = \left\{\bar{\gamma}, \bar{\theta}\right\}$ is still given by (21). This can be shown as in the proof of Proposition 3.3. In the low-withdrawal stat,e $\{\gamma, \theta\} = \{0, 0\}$, the price is unchanged at $q_1 = R$, using (38). Hence, $\partial q_1/\partial T = 0$ in all states.

We can thus follow the same steps as in the proof of Proposition 4.2 to derive (48). While the signs of $k_1^s(\bar{\gamma}) - k_0^s$ and $(1 - R/q_1(\bar{\gamma}))$ are the same as in the proof of Proposition 4.2, because of the same logic (i.e., $k_1^s(\bar{\gamma}) - k_0^s < 0$ and $(1 - R/q_1(\bar{\gamma}) < 0)$, the sign of $\partial q_1(\bar{\gamma})/\partial l_0^s$ is negative here (and thus different from the positive sign in the proof of Proposition 4.2). The result, $\partial q_1(\bar{\gamma})/\partial l_0^s < 0$, follows directly from (40).[24] As a result, the right-hand side of (48) is negative in the asymmetric-information model, rather than positive as in the second-best-use model. The result that the sellers' holdings of the liquidity assets at $t = 0$ in the unregulated equilibrium are higher than the socially optimal level follows the same logic as the result of Proposition 4.2, which states that such holdings are too low—discussed in the proof of Proposition 4.2—with the sign flipped because ${}^{\partial q_1(\bar{\gamma})}/_{\partial l_0^s} < 0$ in the asymmetric-information model.

Proof of Proposition 6.1. We begin by establishing the intermediate results that ${}^{\partial q_1}/_{\partial T} = 0$ in the low-withdrawal state (i.e., when $\gamma = 0$) both in the cash-in-the-market and second-best-use models, similar to the baseline. This follows from the same logic used in the baseline. That is, for the cash-in-the-market model, the buyers' first order conditions when $\gamma = 0$ are

$$(u_1^b)'(c_1^b) = \frac{R}{q_1}(u_2^b)'(c_2^b), \tag{50}$$
$$(u_1^b)'(c_1^b) = (u_2^b)'(c_2^b), \tag{51}$$

using the assumption that the buyers are endowed with $1 + \varepsilon$ units of liquidity (see footnote 6) and that the trading volume is zero in equilibrium (so that $k_1^b = 0$). These equations imply that $q_1 = R$, independently of the level of the buyers' consumption and, thus, independently of T. In the second-best-use model, the result can be shown as in the proof of Proposition 4.2.

The term ${}^{\partial q_1}/_{\partial T}$ in the high-withdrawal state $\gamma = \bar{\gamma}$, however, is not zero, in general, because the price, $q_1(\bar{\gamma})$, depends on T; see (44) and note that the time-2 consumption c_2^s is, in general, a function of T. However, because $q_1(\bar{\gamma})$ and c_2^s are the same in the cash-in-the-market and second-best-use models (see Remarks #1 and #4 in Section 6.1), ${}^{\partial q_1}/_{\partial T}$ will also be the same in both models.

Next, we establish another intermediate result. That is, we show that

$$\lambda_1^b = 1 \tag{52}$$

in both models in the low-withdrawal state (i.e., when $\gamma = 0$). In the cash-in-the-market model, the buyers' marginal utility of wealth is $\lambda_1^b = (u_1^b)'(c_1^b)$. This can be obtained by differentiating the buyers' time-1 Lagrangian with respect to c_1^b. The result, $\lambda_1^b = 1$, in the cash-in-the-market model

[24]More precisely, when allowing for transfers, T, the price $q_1(\bar{\gamma}$ is given by $q_1\left(\bar{\gamma}, \bar{\theta}\right) = {}^{R}/\left(1 + R\frac{\bar{\theta} k_0^s}{\bar{\gamma} d^s - (l_0^s - T)}\right)$. However, the dependence on T does not affect the sign of ${}^{\partial q_1(\bar{\gamma})}/_{\partial l_0^s}$.

follows from the fact that, in the low-withdrawal state (in which $\gamma = 0$, and $q_1 = R$) and given that the buyers have one unit of endowments, the normalization $(u_2^b)'(0) = 1$ and the first-order conditions (50) and (51) imply that $c_1^b = 1$ and, thus, $\lambda_1^b = (u_1^b)'(1) = 1$, given the functional form $(u_1^b)'(c) = \log c$ (see Section 3). In the second-best-use model, the buyers' marginal utility of wealth is $\lambda_1^b = (u_2^b)'(c_2^b)$; this can be obtained by differentiating the buyers' time-1 Lagrangian with respect to l_1^b. The result, $\lambda_1^b = 1$, in the second-best-use model follows from the fact that, in the low-withdrawal state (in which $\gamma = 0$, and $q_1 = R$), no trading takes place and the buyers' time-2 consumption is equal to their endowment of liquidity, $c_2^b = 1$, so that $(u_2^b)'(1) = 1$ because of the normalization introduced in Section 6.1.

Next, we turn to the regulator's first-order conditions, which are the same as in the baseline, as noted in Remark #2 in Section 6.1. For both models, we can rewrite the regulator's first-order condition (28) using (i) the result ${}^{\partial q_1}/_{\partial T} = 0$ in the low-withdrawal state (i.e., when $\gamma = 0$), (ii) the normalization $\lambda_1^s = 1$ in the low-withdrawal state (i.e., $\gamma = 0$), and (iii) the fact that $\lambda_1^b = 1$ in the low-withdrawal state (i.e., $\gamma = 0$), from (52). Thus, using "$(\bar{\gamma})$" to denote the variables in the high-withdrawal state, $\gamma = \bar{\gamma}$, (28) becomes

$$\pi \frac{\partial q_1(\bar{\gamma})}{\partial T} \left(k_1^s(\bar{\gamma}) - k_0^s\right) \left[\xi \lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma})\right] + \mathbb{E}_0 \left\{\xi \lambda_1^b - \lambda_1^s\right\} = 0$$

and rearranging

$$\xi = \frac{1 - \pi + \lambda_1^s(\bar{\gamma})\pi \frac{\partial q_1(\bar{\gamma})}{\partial T} \left[k_1^s(\bar{\gamma}) - k_0^s(\bar{\gamma})\right] + \pi \lambda_1^s(\bar{\gamma})}{1 - \pi + \lambda_1^b(\bar{\gamma})\pi \left(1 + \frac{\partial q_1(\bar{\gamma})}{\partial T} \left[k_1^s(\bar{\gamma}) - k_0^s(\bar{\gamma})\right]\right)}. \tag{53}$$

Note that (53) holds in both the cash-in-the-market and second-best-use models.

The last step is to compare the expression $\mathbb{E}_0 \left\{\frac{\partial q_1}{\partial l_0^s} \left(k_1^s - k_0^s\right) \left(\xi \lambda_1^b - \lambda_1^s\right)\right\}$ in the other regulator's first-order condition, (27), in the cash-in-the-market and second-best-use models. In both models, ${}^{\partial q_1}/_{\partial l_0^s} = 0$ in the low-withdrawal state $\gamma = 0$, which can be established similarly to the result ${}^{\partial q_1}/_{\partial T} = 0$ for that state derived before. Thus, because the two models are comparable (i.e., the price sensitivity, ${}^{\partial q_1}/_{\partial l_0^s}$, and the trading volume, $k_1^s - k_0^s$, are the same in the high-withdrawal state $\gamma = \bar{\gamma}$), we only need to show

$$\left[\xi \lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma})\right]_{\text{cash-in-the-market model}} > \left[\xi \lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma})\right]_{\text{second-best-use model}}. \tag{54}$$

To see why this is the case, note that if $\xi \lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma}) > 0$ in the cash-in-the-market model, the expression ${}^{\partial q_1}/_{\partial l_0^s} \left(k_1^s - k_0^s\right) \left(\xi \lambda_1^b - \lambda_1^s\right) < 0$ in the high-withdrawal state $\gamma = \bar{\gamma}$, using ${}^{\partial q_1}/_{\partial l_0^s} > 0$

and $k_1^s - k_0^s < 0$ in that state. Hence, the sellers' liquidity holdings are higher than the socially optimal level (and the optimal policy is a liquidity ceiling), as discussed in the proof of Proposition 5.1. Therefore, if the term $\xi\lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma})$ is smaller in the second-best-use model, the expression ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)$ is closer to zero or positive, implying that the optimal policy in the second-best-use model is a lower ceiling (if ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right) < 0$) or a liquidity requirement (if ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right) > 0$).

As $\lambda_1^s(\bar{\gamma})$ is the same in both models, establishing (54) is equivalent to showing

$$\left[\xi\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market model}} > \left[\xi\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use model}} \tag{55}$$

or, using (53),

$$\left[\frac{1-\pi+\lambda_1^s(\bar{\gamma})\pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]+\pi\lambda_1^s(\bar{\gamma})}{1-\pi+\lambda_1^b(\bar{\gamma})\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)}\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market model}}$$
$$> \left[\frac{1-\pi+\lambda_1^s(\bar{\gamma})\pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]+\pi\lambda_1^s(\bar{\gamma})}{1-\pi+\lambda_1^b(\bar{\gamma})\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)}\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use model}}.$$

The numerator is the same in both models and, thus, we need to show that

$$\left[(1-\pi)\frac{1}{\lambda_1^b(\bar{\gamma})}+\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)\right]_{\text{cash-in-the-market model}}$$
$$< \left[(1-\pi)\frac{1}{\lambda_1^b(\bar{\gamma})}+\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)\right]_{\text{second-best-use model}}.$$

The only term that is different in both models is the buyers' marginal utility of wealth $\lambda_1^b(\bar{\gamma})$. Thus, we need to show that

$$\left[\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market model}} > \left[\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use model}},$$

and we do so by showing that

$$\left[\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market model}} > 1 > \left[\lambda_1^b(\bar{\gamma})\right]_{\text{second-best- use model}}.$$

To establish this result, we show that, in the high-withdrawal state, $\gamma = \bar{\gamma}$ (i.e., when fire sales occur), the buyers' marginal utility increases in the cash-in-the-market model, relative to the low-withdrawal state, whereas it decreases in the second-best-use model.

In the cash-in-the-market model, the equilibrium in the low-withdrawal state is the same as in the baseline; that is, $c_1^b = 1$ and $c_2^b = 0$. With this allocation, the buyers' first-order condition (42) holds, given the normalization $(u_2^b)'(0) = 1$ and the fact that sellers behave as in the baseline, and the market-clearing condition for liquidity, which is still given by (12), holds as well. Then, as in the baseline, the market-clearing condition evaluated at $l_1^b = 0$ and $l_1^s = 0$ implies that $c_1^b < 1$ in the high-withdrawal state $\gamma = \bar{\gamma}$ and, thus, $\lambda_1^b(\bar{\gamma}) > 1$.

In the second-best-use model, the buyers' time-2 consumption is

$$\begin{aligned} c_2^b &= l_1^b + f\left(k_1^b\right) \\ &= l_0^b - q_1 k_1^b + f\left(k_1^b\right), \end{aligned}$$

where the last line uses the time-1 budget constraint. Differentiating with respect to k_1^b,

$$\begin{aligned} \frac{\partial c_2^b}{\partial k_1^b} &= -\frac{\partial q_1}{\partial k_1^b} k_1^b - q_1 + f'\left(k_1^b\right) \\ &= -f''\left(k_1^b\right) k_1^b > 0, \end{aligned}$$

where the second line uses $q_1 = f'(k_1^b)$ in (32), which continues to hold in the model with general utility, as noted in Remark #1 in Section 6.1. Because k_1^b increases in the high-withdrawal state $\gamma = \bar{\gamma}$ relative to the low-withdrawal state $\gamma = 0$ (i.e., trading increases in a fire sale and, thus, buyers acquire more assets in a fire sale), the buyers' time-2 consumptionin the second-best-use model also increases. As a result, the marginal utility of wealth $\lambda_1^b = (u_2^b)'(c_2^b)(\bar{\gamma})$ is lower than in the low-withdrawal state $\gamma = 0$; that is, it is less than one.

Proof of Proposition 6.2. We can proceed as in the proof of Proposition 6.1 to derive (53); note that with asymmetric information, the result $\partial q_1 / \partial l_0^s < 0$ can be derived as in baseline model; see the proof of Proposition 5.1.

Next, we show that $\lambda_1^b = 1$ in all states at $t = 1$. Because on the margin, the buyers can purchase one unit of liquidity at $t = 1$,which allows them to increase consumption by one unit at $t = 2$, we can express their marginal utility of wealth as $\lambda_1^b = (u_2^b)'(c_2^b)$. The buyers' consumption is

$$\begin{aligned} c_2^b &= 1 - q_1 k_0^b + \mathbb{E}_1^b \{R_2\} k_0^b \\ &= 1, \end{aligned}$$

where the first list follows the derivation in Section 5.3 and the budget constraint stated in Section 5.3 that replaces l_1^b, and the second line rearranges using the buyers' first-order condition (38), which is unchanged in the model with general utility. Given the normalization $(u_2^b)'(1) = 1$ in Section 6.1, the result that $\lambda_1^b = 1$ in all states follows.

We can thus use (53) and the fact that $\lambda_1^b = 1$ (as established above) to obtain

$$\begin{aligned} \xi\lambda_1^b - \lambda_1^s &= \frac{1 - \pi + \lambda_1^s(\bar{\gamma})\pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma}) - k_0^s\right] + \pi\lambda_1^s(\bar{\gamma})}{1 + \pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma}) - k_0^s\right]} - \lambda_1^s(\bar{\gamma}) \\ &= -\frac{\left(\lambda_1^s(\bar{\gamma}) - 1\right)(1 - \pi)}{1 + \pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma}) - k_0^s\right]} < 0, \end{aligned}$$

where the second line rearranges and uses (i) the fact that the sellers' marginal utility of wealth in the high-withdrawal state, $\lambda_1^s(\bar{\gamma})$, is greater than one (this result holds because the marginal utility in the low-withdrawal state, $\lambda_1^s(0)$, is normalized to one, the sellers' consumption decreases in the high-withdrawal state, relative to the low-withdrawal one, because total output drops due to the fact that some long-term assets become lemons in that state, and the buyers' consumption is unchanged as previously established); (ii) the trading volume is positive in the high-withdrawal state and, thus, $k_1^s(\bar{\gamma}) - k_0^s < 0$ because the sellers sell some of their holdings of k_0^s; and (iii) the term $\partial q_1(\bar{\gamma})/\partial T < 0$, which holds because we are using (40), we have

$$\frac{\partial q_1(\bar{\gamma})}{\partial T} = -\frac{R}{\left(1 + R\frac{\bar{\theta}k_0^s}{\bar{\gamma}d^s - (l_0^s - T)}\right)} \frac{\partial\left(\frac{R\bar{\theta}k_0^s}{\bar{\gamma}d^s - (l_0^s - T)}\right)}{\partial T}\Bigg|_{T=0} \leq 0,$$

where the inequality uses the assumption in (46).

B Equilibrium in the cash-in-the-market model

The equilibrium at $t = 0, 1, 2$ in the cash-in-the-market model of Section 3 is the following:

- At $t = 0$, the sellers invest an amount $l_0^s = \frac{\pi}{R-1+\pi} + \bar{\gamma}d^s - 1$ in liquidity and $k_0^s = e^s + d^s(1-$

$\bar{\gamma}) - \frac{\pi R}{R-1+\pi} + 1$ in the long-term asset.

- At $t = 1$
  - If $\gamma = 0$, the price of the long-term asset is $q_1 = R$, the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 1$ for the buyers), and the buyers' consumption is $c_1^b = 1$;
  - If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = \frac{\pi R}{R-1+\pi} < 1$, the sellers' portfolio choices are

$$k_1^s = \frac{\pi^2 R\left(d^s(1-\bar{\gamma}) + e^s\right) + \pi(R-1)\left[R\left(d^s(1-\bar{\gamma}) + e^s\right) + R - 1\right] - (R-1)^2}{\pi R\left(R-1+\pi\right)}$$

    and $l_1^s = 0$, the buyers' portfolio choices are $k_1^b = \frac{R-1}{\pi R}$ and $l_1^b = 0$ and their consumption is $c_1^b = \frac{\pi}{R-1+\pi}$.

- At $t = 2$
  - If $\gamma = 0$, the sellers consume $c_2^s = Re^s + d^s\left(R-1\right) - \left(R-1\right)\left[\frac{\pi}{R-1+\pi} + \bar{\gamma}d^s - 1\right]$ and buyers consume $c_2^b = 1$;
  - If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = \frac{\pi(R-1+\pi)[d(R-1)(1-\bar{\gamma})+e^s R]-(1-\pi)(R-1)^2}{\pi(R-1+\pi)}$ and the buyers consume $c_2^b = \frac{R-1}{\pi}$.

C Equilibrium in the second-best-use model

The equilibrium at $t = 0, 1, 2$ in the asymmetric-information model of Section 4 is the following:

- At $t = 0$, the sellers invest an amount $l_0^s = \bar{\gamma}d^s - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ in liquidity and $k_0^s = e^s + d^s - \frac{\pi R}{R-1+\pi}\left[\bar{\gamma}d^s - (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right]$ in the long-term asset.

- At $t = 1$
  - If $\gamma = 0$, the price of the long-term asset is $q_1 = R$ and the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 1$ for the buyers);

– If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = R\frac{\pi}{R-1+\pi} < 1$ and the sellers' portfolio choices are

$$k_1^s = e^s + d^s(1-\bar{\gamma}) + (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right) \times \left(\frac{\pi R}{R-1+\pi} - 1\right)$$

and $l_1^s = 0$, and buyers' portfolio choices are $k_1^b = (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ and $l_1^b = 1 - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(R\frac{\pi}{R-1+\pi}\right)$.

• At $t = 2$

– If $\gamma = 0$, the sellers consume $c_2^s = Re^s + (R-1)\left[d^s(1-\bar{\gamma}) + \left(\frac{\pi R}{(R-1)+\pi}\right)(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right]$ and the buyers consume $c_2^b = 1$;

– If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = Re^s + (R-1)d^s(1-\bar{\gamma}) + R\left(\frac{\pi R}{(R-1)+\pi} - 1\right)(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ and the buyers consume $c_2^b = f\left[(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right] + 1 - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(R\frac{\pi}{R-1+\pi}\right)$.

D Equilibrium in the asymmetric-information model

The equilibrium at $t = 0, 1, 2$ in the asymmetric-information model of Section 5 is the following:

• At $t = 0$, the sellers invest an amount $l_0^s = \frac{(R-1)\bar{\gamma}d^s - R\pi\theta e^s}{(R-1) - R\pi e^s}$ in liquidity and

$$k_0^s = \frac{(R-1+R\pi\theta)e^s - R\pi(e^s)^2 + (R-1)(1-\gamma)d_0^s - R\pi e^s d_0^s}{(R-1) - R\pi e^s}$$

in the long-term asset.

• At $t = 1$

– If $\gamma = 0$, the price of the long-term asset is $q_1 = R$ and the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers);

– If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = \frac{\pi R}{R-1+\pi} < 1$ and the sellers' portfolio choices are $k_1^s = \frac{d^s((R-1)(-1+\gamma)+e^s(\pi R+\gamma-(\pi+R)\gamma))+e^s(e^s\pi R-(R-1)(1+(\pi-1)\theta))}{1+(e^s\pi-1)R}$ and $l_1^s = 0$.

• At $t = 2$

– If $\gamma = 0$, the sellers consume $c_2^s = Re^s + d^s(R-1) - (R-1)\left[\frac{\pi}{R-1+\pi} + \bar{\gamma}d^s - 1\right]$ and the buyers consume $c_2^b = 1$;

- If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = \frac{d^s((R-1)(-1+\gamma)+e^s(\pi R+\gamma-(\pi+R)\gamma))+e^s(e^s\pi R-(R-1)(1+(\pi-1)\theta))R}{1+(e^s\pi-1)R}$ and the buyers consume $c_2^b = 1$.

E Additional derivations

Price q_1 and time-0 liquidity choices of sellers. To derive (21), we first rewrite (19) using the process for γ in (15), denoting $\lambda_1^s(0)$ and $\lambda_1^s(\bar{\gamma})$ to be the seller's marginal utilities of wealth conditional on $\gamma = 0$ and $\gamma = \bar{\gamma}$, respectively, and similarly for $q_1(0)$ and $q_1(\bar{\gamma})$:

$$(1-\pi)\lambda_1^s(0)q_1(0) + \pi\lambda_1^s(\bar{\gamma})q_1(\bar{\gamma}) = (1-\pi)\lambda_1^s(0) + \pi\lambda_1^s(\bar{\gamma}).$$

Then, using (20) and $q_1(0) = R$, we rearrange to obtain (21).

Regulator's first-order condition. To derive (27), we take the derivative of the regulator's objective function (23), with respect to l_0^s, using ξ as the Lagrange multiplier of (25) and using the budget constraint (24) to obtain $\frac{\partial k_0^s}{\partial l_0^s} = -1$:

$$\mathbb{E}_0 \left\{ \lambda_1^s(1-q_1) + \frac{\partial q_1}{\partial l_0^s} \left[\lambda_1^s \left(k_0^s - k_1^s \right) - \xi \lambda_1^b k_1^b \right] \right\} = 0$$

We then rearrange using the time-1 market-clearing condition for the long-term asset, (13), which implies that $k_1^b = k_0^s - k_1^s$ when evaluated at $k_0^b = 0$, to obtain (27). Equation (28) is derived similarly.