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Forecasting Risks to the Canadian Economic Outlook at a Daily Frequency

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Abstract

In this paper, we estimate the distribution of future inflation and growth in real gross domestic product (GDP) for the Canadian economy at a daily frequency. To do this, we model the conditional moments (mean, variance, skewness and kurtosis) of inflation and GDP growth as moving averages of economic and financial conditions. Then, we translate the conditional moments into conditional distributions using a flexible parametric distribution known as the skewed generalized error distribution. We show that the probabilities of inflation and GDP growth derived from the conditional distributions accurately reflect realized outcomes during the sample period from 2002 to 2022. Our methodology offers daily-frequency forecasts with flexible forecasting horizons. This is highly useful in an environment of elevated uncertainty surrounding the inflation and growth outlook.

Topics: Econometric and statistical methods; Business fluctuations and cycles
JEL codes: C32, C58, E44, G17

Résumé

Dans cette étude, nous estimons la distribution quotidienne de l'inflation et de la croissance du produit intérieur brut (PIB) réel futures au sein de l'économie canadienne. Pour ce faire, nous modélisons les moments conditionnels (moyenne, variance, asymétrie et aplatissement) de l'inflation et de la croissance du PIB comme des moyennes mobiles des conditions économiques et financières. Ensuite, nous traduisons les moments conditionnels en distributions conditionnelles à l'aide d'une distribution paramétrique souple connue sous le nom de distribution d'erreurs généralisée asymétrique. Nous montrons que les probabilités d'inflation et de croissance du PIB calculées à partir des distributions conditionnelles reflètent fidèlement les résultats obtenus au cours de la période de 2002 à 2022. Notre méthodologie procure des prévisions quotidiennes sur des horizons variables, ce qui est très utile dans un contexte de forte incertitude entourant les perspectives d'inflation et de croissance.

Sujets : Méthodes économétriques et statistiques, Cycles et fluctuations économiques
Codes JEL : C32, C58, E44, G17

Introduction

Economic forecasts such as for inflation and growth in gross domestic product (GDP) are typically provided in point estimates. This is in part because point estimates are simple and provide a useful reference point for communicating how macroeconomic risks have evolved. However, they start to lose their value when the outlook becomes highly uncertain or the potential for tail risks becomes significant. This was the main reason why the Bank of Canada decided not to provide a projection at the onset of the COVID-19 pandemic but instead focused on assessing the overall economic impact of the pandemic for Canada and identifying the channels through which it is likely to affect the economy. Any forecasts provided at the time would have been meaningless given the unprecedented nature of the shock and the wide range of possible macroeconomic outcomes.

Policy-makers and financial market participants have long been concerned about tail risks in macroeconomic outcomes. Over the past two decades, two notable tail risk events have challenged our understanding of inflation. Following the 2008–09 global financial crisis (GFC), macroeconomic models tended to overestimate the recovery in inflation, whereas they underestimated the persistence in inflation following the pandemic. This is because the conditional mean of any model may fail to adequately capture the outlook in the presence of tail risks, particularly those of significant magnitude. Indeed, this "inflation puzzle" is better explained when models take into accounts tail risks, or higher moments of the distribution (Lopez-Salido and Loria 2020).

Evidently, tail risks can have a profound impact on the economy and asset prices across sectors, either directly or indirectly, through shifts in interest rate expectations. Therefore, better understanding and managing tail risks have become imperative for policy-makers and investors alike. Some surveys (e.g., the Blue Chip Economic Indicators and market participant surveys of several central banks, including the Bank of Canada) provide a useful way to quantify tail risks by collecting respondents' beliefs regarding the probability distribution around the point forecast. However, the low frequency of these surveys constrains their usefulness, particularly in fast-changing environments.

In this paper, we estimate the full distribution of future inflation and real GDP growth for the Canadian economy as a function of economic and financial conditions at a daily frequency. Our methodology includes three steps. First, we individually model the four moments (mean, variance, skewness and kurtosis) of inflation and GDP growth as moving averages of economic and financial news and expectations. This step is rooted in the principle that economic agents update their expectations only when new information becomes available. Examples of economic and financial news include key macroeconomic data releases such as inflation and GDP growth and well-known financial variables such as the yield curve and stock market index, all of which have a natural linkage to future inflation and GDP growth. Second, we construct conditional distributions using a flexible parametric distribution known as the skewed generalized error distribution (SGED) with four parameters that correspond to the four moments obtained from the first step. Finally, we estimate the marginal probability of risks based on the probability density function obtained from the second step. We also estimate the copula and examine the joint probability of inflation and GDP growth outcomes, a highly pertinent aspect in the post-pandemic era.

Our estimates of tail risks in inflation and GDP growth accurately capture key events in our sample period from 2002. During both the GFC and the pandemic, the 12-month-ahead probability of inflation falling in the Bank of Canada target range of 1% to 3% drops to near zero while the recession probability soars to near 100%. The model also correctly estimates the low probability of a recession during the euro debt crisis and the oil price shock when Canada narrowly avoided a recession. Our findings also show that stagflation risk (i.e., the risk of both high inflation and a recession), which was negligible throughout most of the sample period, rose significantly in 2022 due to a combination of tighter monetary policy and financial conditions.

Our methodology is, in principle, similar to quantile regressions, which are more commonly used in the literature to model conditional distributions (Adrian, Boyarchenko and Giannone 2019 and Lopez-Salido and Loria 2020). Whereas quantile regressions model the relationship to percentiles of a target variable, we model the relationship to moments of a target variable. Otherwise, both methods should lead to similar findings on quantifying the balance of risks around macroeconomic forecasts. We cannot use quantile regressions, however, because the sample period in our analysis is limited by the lack of available data in our covariates.

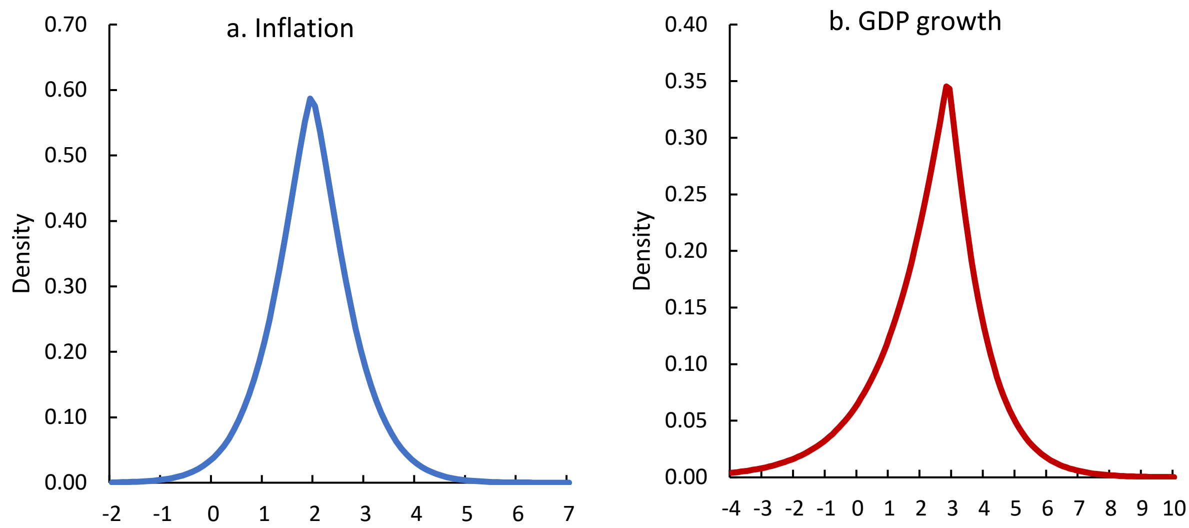
Our paper makes important contributions to the literature in two key ways. First, our methodology stands out as one of the first to generate conditional distributions at a daily frequency. By contrast, most if not all existing methods in the literature are restricted to quarterly, or at best, monthly frequency. The second contribution of our paper lies in its ability to link conditional distributions across various forecasting horizons. To our knowledge, this area of research has remained relatively unexplored due to its complexities. We anticipate that these two features will prove valuable to policy-makers and market participants alike.

The rest of this paper is organized as follows. Section 1 presents the descriptions of Canadian inflation and GDP growth and provides the motivation for a flexible parametric distribution. Section 2 describes this distribution and our modelling framework to forecast future distributions. Section 3 discusses the choice of covariates and treatment of outliers. The empirical results are presented in section 4, where we show our parameter estimates and examine the goodness of fit. Section 5 concludes.

1. Overview of Canadian inflation and GDP growth

In this section, we analyze the distributions of realized year-over-year (y-o-y) inflation and y-o-y real GDP growth in Canada. We focus on the sample period since 2000 due to the availability of data for market expectations of economic variables, which are needed to calculate the unexpected component of data releases (i.e., economic news). **Chart 1** shows the distributions of inflation and GDP growth from 2000 to 2019. The distribution of realized inflation in Canada is a near-perfect symmetric distribution around 2% during this sample period. This is not surprising given that the Bank of Canada has been operating under an inflation-targeting framework since 1991. The framework aims to keep inflation at the 2% midpoint of a target range of 1% to 3%. The distribution of realized GDP growth in Canada is also symmetric but exhibits mild skewness to the left (i.e., negatively skewed). This means that the mass of the GDP distribution is more concentrated on growth above the mean of 3% with relatively less occurrence of growth below the mean of 3% (e.g., during the GFC).

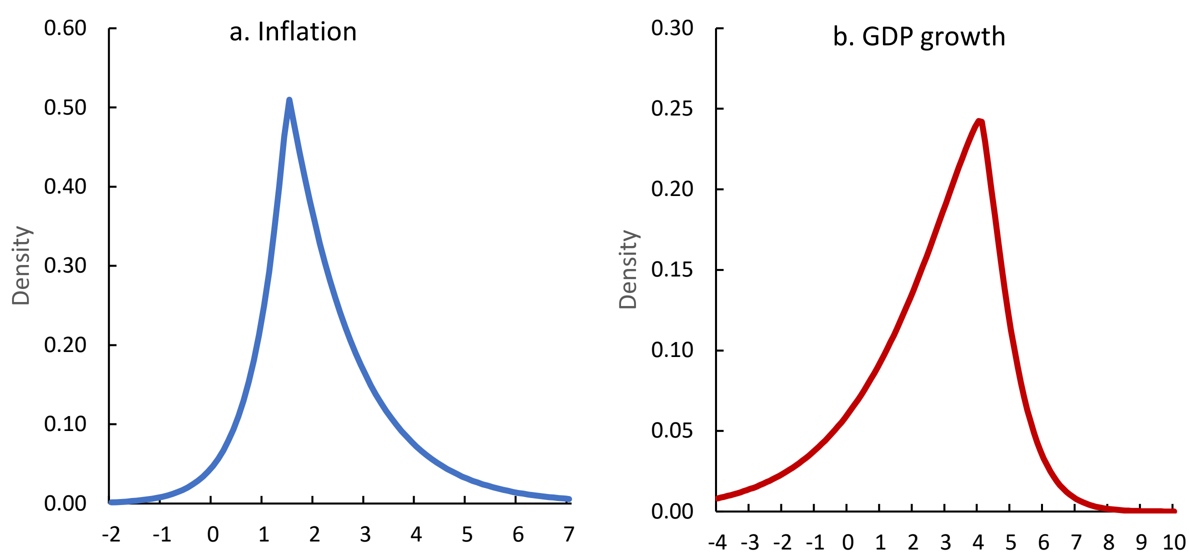
Chart 1: Distribution of inflation and GDP growth before COVID-19, year-over-year (2000–19)



Sources: Bloomberg, Haver Analytics and Bank of Canada calculations — Last observation: December 2019

Chart 2 shows how the higher moments of the distribution change when we expand the period by just three years, from 2000 to 2022, reflecting the significance of the pandemic. The distribution of inflation becomes more skewed to the right (i.e., positively skewed) due to the historically high inflation seen since the start of the pandemic. In contrast, the distribution of GDP growth becomes more skewed to the left due to extreme negative growth episodes seen during the pandemic. The kurtosis, which measures the thickness of tails of the distribution, increases for both inflation and GDP growth given the extreme realizations during the pandemic.

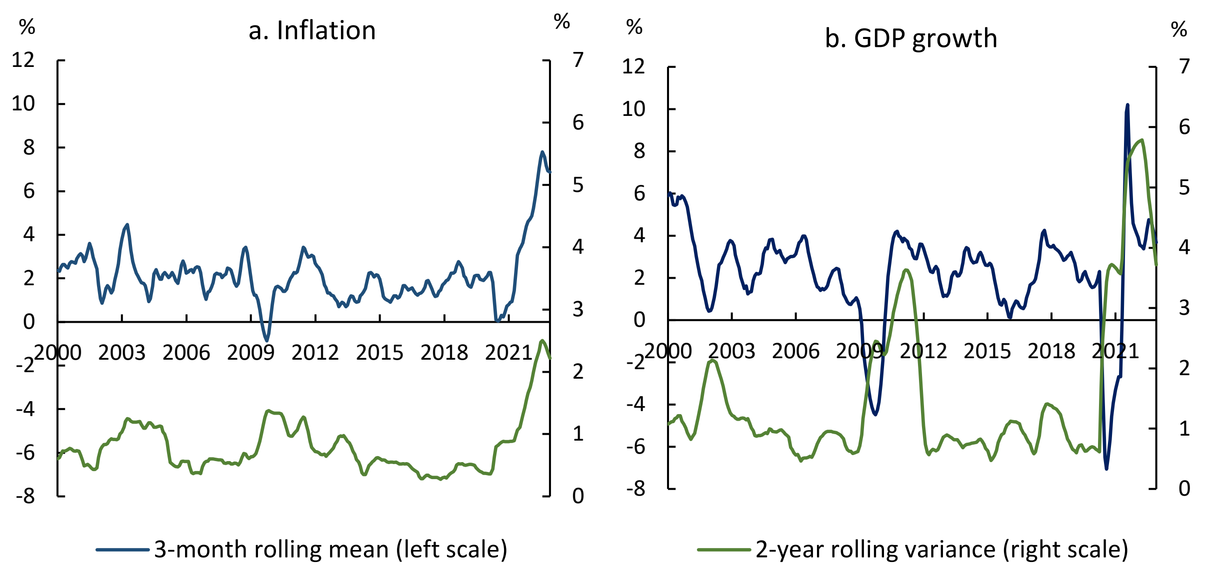
Chart 2: Realized distribution of inflation and GDP growth, year-over-year, full sample (2000–22)



Sources: Haver Analytics and Bank of Canada calculations — Last observation: December 2022

To better visualize the changing properties of distributions, we show the rolling-window moments of inflation and GDP growth. We use different rolling windows suitable for each moment. For instance, a shorter rolling window (e.g., three months) is more suitable for the mean of the distribution, while a longer rolling window (e.g., 20 years) is required for the third and fourth moments to better capture the asymmetry and the tails of the distribution. **Chart 3** shows the three-month and the two-year rolling windows for the mean and variance, respectively, of the distributions of inflation and GDP growth. For most of the sample period, the rolling mean for inflation fluctuates around 2%, consistent with the Bank's inflation target. Moreover, its rolling standard deviation stays close to 0 except for during crisis episodes (e.g., the burst in the dot-com bubble in 2000, the GFC and the pandemic). After March 2020, both the rolling mean and variance spiked to their highest values in the sample period, reflecting the initial drop and the subsequent sharp increase in inflation. The three-month rolling mean of GDP growth is mostly positive, diving below zero only during the GFC and the pandemic.[1] The two-year rolling variance is relatively stable except for substantial spikes around the GFC and the pandemic.

Chart 3: Rolling windows for the mean and variance of the distributions of inflation and GDP growth



Sources: Haver Analytics and Bank of Canada calculations

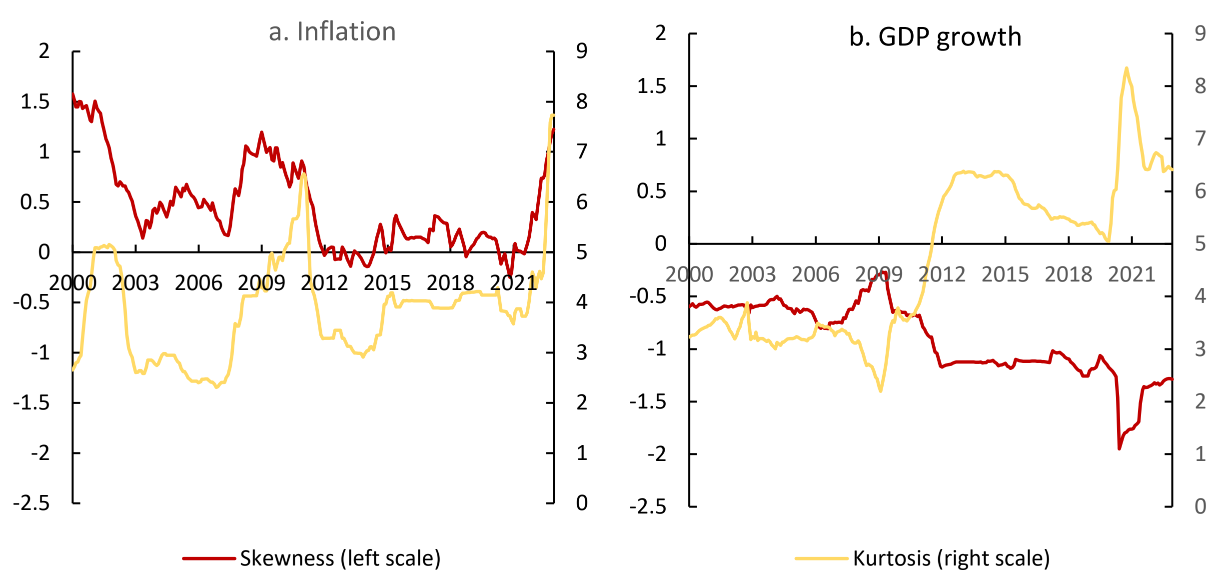
Last observation: December 2022

Chart 4 shows the 20-year rolling window for skewness and kurtosis of the distributions of inflation and GDP growth. The rolling skewness of the inflation distribution is slightly positive and stable for the majority of the sample, with a few leaps around 2008 and 2020. The value of the rolling kurtosis is stable for most of the sample, with a couple of significant spikes (the GFC and the pandemic). These two moments show that the distribution of inflation was close to a normal distribution before the pandemic, with a skewness value close to 0 and a kurtosis value close to 3. Historically high inflation after the start of the pandemic led to a surge in the rolling skewness. Similarly, the kurtosis spiked because more of the variance is caused by these high values of inflation.

[1] While negative growth rates occurred in quarter-over-quarter terms in the first half of 2015, this did not occur in the three-month rolling mean of growth in year-over-year terms.

The rolling skewness of the GDP distribution is negative throughout the sample period. Before the GFC, the skewness was only slightly negative. However, the distribution became more negatively skewed following the GFC, reflecting large negative growth rates followed by relatively small positive growth rates during the recovery phase. A similar pattern can be observed around the pandemic, with the distribution displaying increased negative skewness. The rolling kurtosis of the GDP distribution is also characterized by three distinct periods that switched during the GFC and the pandemic. Before the GFC, the kurtosis was relatively low and stable. Then, the level of rolling kurtosis shifted up twice, once after the GFC and again after the pandemic.

Chart 4: Rolling windows for the skewness and kurtosis of the distributions of inflation and GDP growth



Sources: Haver Analytics and Bank of Canada calculations

Last observation: December 2022

These observed variations in skewness and kurtosis highlight the presence of pronounced asymmetry in the distributions of inflation and GDP growth. As such, modelling them as normal distributions would introduce significant biases, which can lead to inaccurate risk assessments. This warrants a more sophisticated modelling technique that can account for time-varying asymmetry in the distributions of inflation and GDP growth. By doing so, we can obtain a more comprehensive understanding of the potential risks and uncertainties in the economy.

2. Modelling framework

We rely on a flexible parametric distribution known as the skewed generalized error distribution (SGED). The SGED density function $f_t^{(\tau)}$ has four parameters that conveniently correspond to the first four moments: mean $\mu_t^{(\tau)}$, variance $\sigma_t^{(\tau)}$, skewness $s_t^{(\tau)}$and kurtosis $k_t^{(\tau)}$.

Formally, we have:

$$z_{t+\tau} = \frac{\pi_{t+\tau} - \mu_t^{(\tau)}}{\sigma_t^{(\tau)}}, \quad \text{where the density of } z_{t+\tau} \text{ is given by:}$$

$$f_{z,t}^{(\tau)}(z) = C_t^{(\tau)} \exp\left(-\left|z + \delta_t^{(\tau)}\right|^{\eta_t^{(\tau)}} / \left[1 + sign\left(z + \delta_t^{(\tau)}\right)\lambda_t^{(\tau)}\right]\left(\theta_t^{(\tau)}\right)^{\eta_t^{(\tau)}}\right)$$

$$C_t^{(\tau)} = \left(\eta_t^{(\tau)}/2\,\theta_t^{(\tau)}\right)\Gamma\left(1/\eta_t^{(\tau)}\right)^{-1}$$

$$\theta_t^{(\tau)} = \Gamma\left(1/\eta_t^{(\tau)}\right)^{1/2}\Gamma\left(3/\eta_t^{(\tau)}\right)^{-1/2}\left(B_t^{(\tau)}\right)^{-1}$$

$$\delta_t^{(\tau)} = 2\lambda_t^{(\tau)} A_t^{(\tau)}\left(B_t^{(\tau)}\right)^{-1}$$

$$B_t^{(\tau)} = \sqrt{1 + 3\left(\lambda_t^{(\tau)}\right)^2 - 4\left(A_t^{(\tau)}\right)^2\left(\lambda_t^{(\tau)}\right)^2}$$

$$A_t^{(\tau)} = \Gamma\left(2/\eta_t^{(\tau)}\right)\Gamma\left(1/\eta_t^{(\tau)}\right)^{-1/2}\Gamma\left(3/\eta_t^{(\tau)}\right)^{-1/2},$$

where $\Gamma(.)$ denotes the gamma function. The scaling parameters $\eta_t^{(\tau)}$ and $\lambda_t^{(\tau)}$ are subject to $\eta_t^{(\tau)} > 0$ and $-1 < \lambda_t^{(\tau)} < 1$.

This density function nests a large set of conventional densities. For example, when $\lambda_t^{(\tau)} = 0$, we have the generalized error distribution, as in Nelson (1991); when $\lambda_t^{(\tau)} = 0$ and $\eta_t^{(\tau)} = 2$, we have the standard normal distribution; when $\lambda_t^{(\tau)} = 0$ and $\eta_t^{(\tau)} = 1$, we have the double-exponential distribution; and when $\lambda_t^{(\tau)} = 0$ and $\eta_t^{(\tau)} = \infty$, we have the uniform distribution on the interval $\left[-\sqrt{3}, \sqrt{3}\right]$. The parameter $\eta_t^{(\tau)}$ controls the height and the tails of the density function, and the skewness parameter $\lambda_t^{(\tau)}$ controls the rate of descent of the density around the mode ($-\delta_t^{(\tau)}$).

The third and fourth moments are defined as:

$$s_t^{(\tau)} = A_{3t}^{(\tau)} - 3\delta_t^{(\tau)} - \left(\delta_t^{(\tau)}\right)^3$$

$$k_t^{(\tau)} = A_{4t}^{(\tau)} - 4A_{3t}^{(\tau)}\delta_t^{(\tau)} + 6\left(\delta_t^{(\tau)}\right)^2 + 3\left(\delta_t^{(\tau)}\right)^4,$$

where

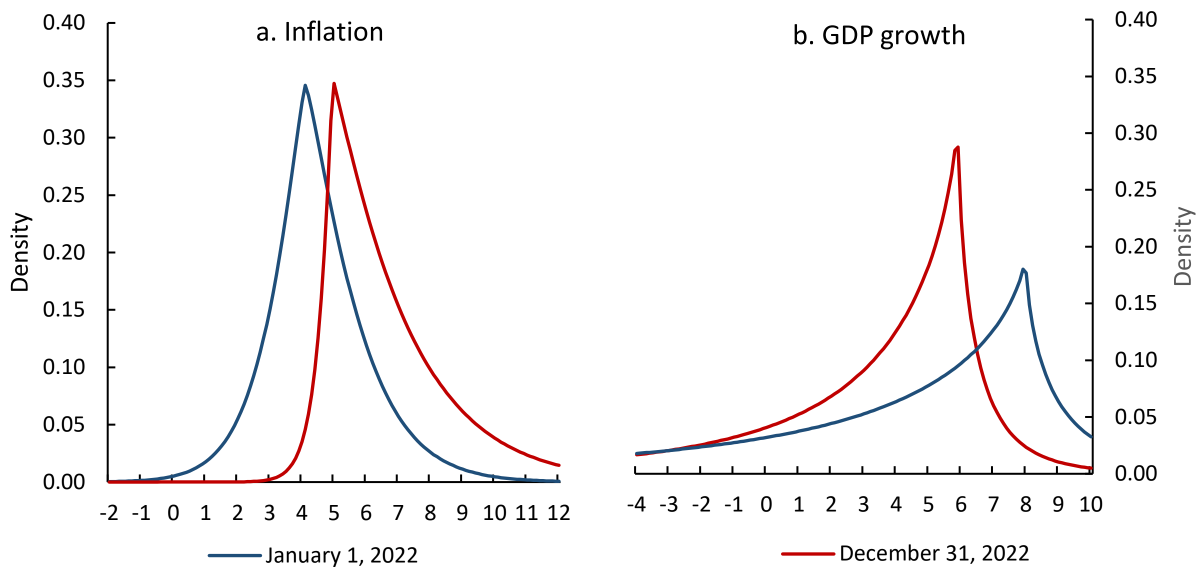
$$A_{3t}^{(\tau)} = 4\lambda_t^{(\tau)}\left(1 + \left(\lambda_t^{(\tau)}\right)^2\right)\Gamma\left(4/\eta_t^{(\tau)}\right)\Gamma\left(1/\eta_t^{(\tau)}\right)^{-1}\left(\theta_t^{(\tau)}\right)^3 \text{ and}$$

$$A_{4t}^{(\tau)} = \left(1 + 10\left(\lambda_t^{(\tau)}\right)^2 + 5\left(\lambda_t^{(\tau)}\right)^4\right)\Gamma\left(5/\eta_t^{(\tau)}\right)\Gamma\left(1/\eta_t^{(\tau)}\right)^{-1}\left(\theta_t^{(\tau)}\right)^4.$$

In practice, after estimating $s_t^{(\tau)}$ and $k_t^{(\tau)}$, we can solve the above two equations numerically with two unknown parameters to find $\lambda_t^{(\tau)}$ and $\eta_t^{(\tau)}$. Finally, we can plug the two parameters and the first two moments into the SGED formula to derive a density function at a monthly (data release) frequency.

For example, using the rolling window moments of inflation and GDP growth from **Chart 3** and **Chart 4**, **Chart 5** shows the density functions for inflation $f_t^{(\tau)}(\pi)$ and GDP growth $f_t^{(\tau)}(g)$ at the beginning and end of 2022. The inflation distribution shifted to the right with a fatter tail, reflecting increased upside risks to inflation over the year. In contrast, the GDP growth distribution shifted to the left, indicating an increase in downside risks to growth.

Chart 5: Change in inflation and GDP growth distributions over 2022



Sources: Haver Analytics and Bank of Canada calculations

Last observation: December 31, 2022

The density functions shown so far are backward-looking and thus are not informative of future economic outcomes. In order to measure and quantify risks to the economic outlook, we specify a parametric framework to model future moments of macroeconomic variables ($t + \tau$) conditional on the information available at time t. Since the SGED is characterized by its moments, the conditional density of future macroeconomic variables can be obtained from estimating their conditional moments. Then, we can use the conditional density to quantify the risks to the outlook.

2.1. Dynamics of the conditional moments

Our modelling framework for conditional moments is rooted in the principle that economic agents' perceptions of future economic outcomes are updated only with new information. For instance, if there is no new information today, the expectation of future moments of macroeconomic variables is simply the expectation from yesterday. If there is new information today, the expectation of future moments would build on yesterday's to form a new expectation. We apply this framework to all four moments of inflation and GDP growth.

We denote the four moments of inflation ($\pi_{t+\tau}$) as follows:

- First moment $\mu_t^{(\tau)} = E_t[\pi_{t+\tau}]$; time t expectation of future $\pi_{t+\tau}$.
- Second moment $\sigma_t^{(\tau)} = \sqrt{E_t\left[\left(\pi_{t+\tau} - \mu_t^{(\tau)}\right)^2\right]}$; time t expectation of volatility in future $\pi_{t+\tau}$.
- Third moment by $s_t^{(\tau)} = \frac{E_t\left[\left(\pi_{t+\tau} - \mu_t^{(\tau)}\right)^3\right]}{\left(\sigma_t^{(\tau)}\right)^3}$; time t expectation of asymmetry in the distribution of future $\pi_{t+\tau}$.
- Fourth moment by $k_t^{(\tau)} = \frac{E_t\left[\left(\pi_{t+\tau} - \mu_t^{(\tau)}\right)^4\right]}{\left(\sigma_t^{(\tau)}\right)^4}$; time t expectation of tails in the distribution of future $\pi_{t+\tau}$.

We model each moment as functions of macroeconomic and financial variables. More specifically, we use their market expectations, E_t, and their news component, $News_t$. More details on how they are measured are discussed in section 3. For illustrative purposes, we show only the dynamic for the first moment.

The first moment $\mu_t^{(\tau)}$ can be expressed as the sum of two components: (1) a long-run component denoted by μ_t^* and (2) a mean-reverting short-run component $\tilde{\mu}_t^{(\tau)}$:

$$\mu_t^{(\tau)} = \mu_t^* + \tilde{\mu}_t^{(\tau)} \tag{1}$$

The long-run component is driven by expectations E_t:

$$\mu_t^* = \mu_{t-1}^* + \gamma_e'(\mathrm{E}_t - E_{t-1}[\mathrm{E}_t]),$$

where

$$E_{t-1}[\gamma_e'\mathrm{E}_t] = (1 - \beta_e)\mu_{t-1}^* - \omega$$

$$\text{and } 0 < \beta_e < 1.$$

Then, combining the last two equations implies that:

$$\mu_t^* = \omega + \beta_e\mu_{t-1}^* + \gamma_e'\mathrm{E}_t.$$

The short-run component is driven by $News_t$:

$$\tilde{\mu}_t^{(\tau)} = \beta_n\tilde{\mu}_{t-1}^{(\tau)} + \gamma_{n,\tau}'News_t,$$

$$\text{where } 0 < \beta_n < 1.$$

Finally, we can rewrite both components of $\mu_t^{(\tau)}$ as follows:

$$\mu_t^* = \sum_{j=0}^{t-1} \beta_e^j (\omega + \gamma_e' \mathrm{E}_{t-j}) \tag{2}$$

$$\tilde{\mu}_t^{(\tau)} = \sum_{j=0}^{t-1} \beta_n^j (\gamma_{n,\tau}' \, News_{t-j}) \tag{3}$$

Therefore, μ_t^* is simply a moving average of current and past expectations and $\tilde{\mu}_t^{(\tau)}$ is a moving average of current and past news. It is clear from these expressions that the expectation of the first moment converges to its long-run component after a sufficiently long period of no (or offsetting) news.

The news beta (β_n) gives the relative weight of news between two consecutive days. Our estimates of β_n are all between 0 and 1. This implies that the model gives more weight to the most recent information, and the relative importance of past news decays exponentially with the gap between the date at which the forecast is done and the date of the release of that past news. The higher the beta, the more persistent the news information content in shaping the inflation outlook. Similarly, the expectations beta (β_e) gives the relative weight of expectations between two consecutive days.

Gamma helps us understand and interpret the dynamics of the estimated conditional moments. The news gamma (γ_n) is a vector of parameters that has the same size as the sets of news X. The news gamma serves two purposes. First, it enables us to collapse the sets of multivariate news $News_t$ to only one scalar $\gamma_{n,\tau}' News_t$, which can be interpreted as the best combination of news that is relevant for forecasting a particular characteristic of the distribution. Second, as the sets of news are standardized, different components of γ convey useful information about the relative importance of each set of news. The expectations gamma (γ_e) plays a similar role as γ_n. It allows us to weight different expectations components and form the expectations indicator ($\gamma_e' \mathrm{E}_t$) that is the most relevant for forecasting a particular characteristic of the distribution.

2.2. Linking moments across horizons

A common problem faced in quantile regressions, or forecasting exercises in general, is a fixed forecasting window, which poses a challenge for policy-makers. This is because updated forecasts of the model are not directly comparable with their previous forecasts because each data point covers a different forecasting period. For example, if a three-month forecasting model is updated in January and February, the January data point will cover the sample period from January to April and the February data point will cover the sample period from February to May. Therefore, a change in estimates from January to February cannot be solely attributed to a change in the underlying macroeconomic conditions. One possible solution to this problem is estimating multiple regressions with different forecasting horizons. Using the earlier example, one can estimate a three-month model in January and subsequently a two-month model in February to have both models provide a forecast for the month of April. However, the results are again not directly comparable since each forecasting horizon will have a unique set of parameters.

Fortunately, our specification allows us to link moments across different forecasting horizons and share one common set of parameters. Given that news typically averages out to 0 ($E_{t-1}[News_t] = 0$), we can easily establish that:

$$\mu_t^{(\tau)} = \mu_t^* + \beta_n^{\tau-\tau_0} \left(\mu_t^{(\tau_0)} - \mu_t^* \right). \tag{4}$$

This implies that the same set of parameters allows us to compute the moments (and hence the distribution) of the variable of interest at any given horizon. To see that, let us assume that our dynamic is true for horizon 1, that is:

$$\mu_t^{(1)} = \mu_t^* + \tilde{\mu}_t^{(1)}.$$

We have $E_{t-1}\left[\tilde{\mu}_t^{(1)}\right] = \beta_n \tilde{\mu}_{t-1}^{(1)}$ and $E_{t-1}[\mu_t^*] = \mu_{t-1}^*$, implying that:

$$\mu_t^{(\tau)} = E_t[\pi_{t+\tau}] = E_t\left[E_{t+\tau-1}[\pi_{t+\tau}]\right] = E_t\left[\mu_{t+\tau-1}^{(1)}\right] = E_t\left[\mu_{t+\tau-1}^* + \tilde{\mu}_{t+\tau-1}^{(1)}\right] = \mu_t^* + \beta_n^{\tau-1}\tilde{\mu}_t^{(1)}.$$

Hence $\mu_t^{(\tau)} = \mu_t^* + \beta_n^{\tau-1}\left(\mu_t^{(1)} - \mu_t^*\right)$, which is equivalent to:

$$\mu_t^{(\tau)} = \mu_t^* + \beta_n^{\tau-\tau_0}\left(\mu_t^{(\tau_0)} - \mu_t^*\right). \qquad (5)$$

The same dynamic applies to all three other moments, as shown below:

$$\mu_t^{(\tau_0)} = \mu_t^* + \tilde{\mu}_t^{(\tau_0)};\ \tilde{\mu}_t^{(\tau_0)} = \beta_{\mu,n}\tilde{\mu}_{t-1}^{(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^{\mu};\ \mu_t^* = \ \omega_\mu + \beta_{\mu,e}\mu_{t-1}^* + \mathrm{E}_t{}'\gamma_e^{\mu};\ \ \mu_t^{(\tau)} = \mu_t^* + \beta_{\mu,n}^{\tau-\tau_0}(\mu_t^{(\tau_0)} - \mu_t^*)$$

$$\hat{\sigma}_t^{(\tau_0)} = \sigma_t^* + \tilde{\sigma}_t^{(\tau_0)};\ \tilde{\sigma}_t^{(\tau_0)} = \beta_{\sigma,n}\tilde{\sigma}_{t-1}^{(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^{\sigma};\ \sigma_t^* = \omega_\sigma + \beta_{\sigma,e}\sigma_{t-1}^* + \mathrm{E}_t{}'\gamma_e^{\sigma};\ \ \hat{\sigma}_t^{(\tau)} = \sigma_t^* + \beta_{\sigma,n}^{\tau-\tau_0}(\hat{\sigma}_t^{(\tau_0)} - \sigma_t^*)$$

$$s_t^{(\tau_0)} = s_t^* + \tilde{s}_t^{(\tau_0)};\ \tilde{s}_t^{(\tau_0)} = \beta_{s,n}\tilde{s}_{t-1}^{(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^{s}\ ;\ s_t^* = \ \omega_s + \beta_{s,e}s_{t-1}^* + \mathrm{E}_t{}'\gamma_e^{s};\ \ s_t^{(\tau)} = s_t^* + \beta_{s,n}^{\tau-\tau_0}(s_t^{(\tau_0)} - s_t^*)$$

$$\hat{k}_t^{(\tau_0)} = k_t^* + \tilde{k}_t^{(\tau_0)};\ \tilde{k}_t^{(\tau_0)} = \beta_{k,n}\tilde{k}_{t-1}^{(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^{k}\ ;\ k_t^* = \ \omega_k + \beta_{k,e}k_{t-1}^* + \mathrm{E}_t{}'\gamma_e^{k};\ \ \hat{k}_t^{(\tau)} = k_t^* + \beta_{k,n}^{\tau-\tau_0}(\hat{k}_t^{(\tau_0)} - k_t^*)$$

where we set $\sigma_t^{(\tau)} = \left(\hat{\sigma}_t^{(\tau)}\right)^2$ and $k_t^{(\tau)} = \left(\hat{k}_t^{(\tau)}\right)^2$ to ensure the positivity of the second and fourth moments.

To link the moments across horizons, we need to specify the number of horizons we want to link them with. By increasing the number of horizons, the model can better address the issue of over-parametrization, but this comes at the cost of a lower goodness of fit. Although there is no objective criterion to determine the optimal number of horizons, we choose to include four horizons (i.e., 3, 6, 9 and 12 months) in our analysis because the trade-off appears significant beyond that threshold.

2.3. Parameter estimation

We estimate the parameters (beta, gamma and omega) for each conditional moment dynamic independently. This is a robust approach because it prevents a given moment misspecification from affecting the estimation of other moments. The alternative is the maximum likelihood estimation (MLE) method where the parameters of all four moments dynamics are estimated jointly. While the MLE is more efficient in principle, it entails greater computational burden, which will likely lead to a local optimum. Our methodology mitigates this burden by estimating one moment at a time.

First, we compute the realized moments simply as rolling sample mean, variance, skewness and kurtosis.

- The realized mean is $RM_t = \left(\sum_{u=t-T_m+1}^{t}\pi_u\right)/T_m$.

- The realized variance is $RV_t = \sqrt{\left(\sum_{u=t-T_v+1}^{t}(\pi_u - RM_t)^2\right)/T_v}$.
- The realized skewness is $RS_t = \frac{\left(\sum_{u=t-T_s+1}^{t}(\pi_u - RM_t)^3\right)/T_s}{RV_t^3}$.
- The realized kurtosis is $RK_t = \frac{\left(\sum_{u=t-T_k+1}^{t}(\pi_u - RM_t)^4\right)/T_k}{RV_t^4}$.

Next, we denote that the conditional moments $\mu_t^{(\tau)}$, $\sigma_t^{(\tau)}$, $s_t^{(\tau)}\, and\ k_t^{(\tau)}$ are time *t* forecasts of future realized moments, or formally:

$$\mu_t^{(\tau)} = E_t[RM_{t+\tau}];\ \ \sigma_t^{(\tau)} = E_t[RV_{t+\tau}];\ \ s_t^{(\tau)} = E_t[RS_{t+\tau}];\ k_t^{(\tau)} = E_t[RK_{t+\tau}].$$

To estimate the parameters, we minimize the gap between the conditional moments $\mu_t^{(\tau)}$, $\sigma_t^{(\tau)}$, $s_t^{(\tau)}\, and$ $k_t^{(\tau)}$ and the associated realized moments $RM_{t+\tau}, RV_{t+\tau}, RS_{t+\tau}\ and\ RK_{t+\tau}$:

$$\left\{\hat{\beta}_{\mu,n}; \hat{\gamma}_{n,\tau_0}^{\mu}; \widehat{\omega}_{\mu}; \hat{\beta}_{\mu,e}; \hat{\gamma}_e^{\mu}\right\}$$

$$= arg \min_{\beta_{\mu,n};\gamma_{n,\tau_0}^{\mu};\omega_{\mu};\beta_{\mu,e};\gamma_e^{\mu}} \left[\sum_{j=0}^{J-1}\left\{\sum_{t=1}^{T-\tau_j}\left(RM_{t+\tau_j} - \mu_t^{(\tau_j)}\right)^2\right\}\right]$$

$$+ \sum_{t=1}^{T}\left(\mathrm{E}_t{}'\gamma_e^{\mu} - \left((1-\beta_{\mu,e})\mu_{t-1}^{*} - \omega_{\mu}\right)\right)^2$$

$$\left\{\hat{\beta}_{\sigma,n}; \hat{\gamma}_{n,\tau_0}^{\sigma}; \widehat{\omega}_{\sigma}; \hat{\beta}_{\sigma,e}; \hat{\gamma}_e^{\sigma}\right\}$$

$$= arg \min_{\beta_{\sigma,n};\gamma_{n,\tau_0}^{\sigma};\omega_{\sigma};\beta_{\sigma,e};\gamma_e^{\sigma}} \sum_{j=0}^{J-1}\left\{\sum_{t=1}^{T-\tau_j}\left(RV_{t+\tau_j} - \sigma_t^{(\tau_j)}\right)^2\right\}$$

$$+ \sum_{t=1}^{T}\left(\mathrm{E}_t{}'\gamma_e^{\sigma} - \left((1-\beta_{\sigma,e})\sigma_{t-1}^{*} - \omega_{\sigma}\right)\right)^2$$

$$\left\{\hat{\beta}_{s,n}; \hat{\gamma}_{n,\tau_0}^{s}; \widehat{\omega}_{s}; \hat{\beta}_{s,e}; \hat{\gamma}_e^{s}\right\}$$

$$= arg \min_{\beta_{s,n};\gamma_{n,\tau_0}^{s};\omega_{s};\beta_{s,e};\gamma_e^{s}} \sum_{j=0}^{J-1}\left\{\sum_{t=1}^{T-\tau_j}\left(RS_{t+\tau_j} - s_t^{(\tau_j)}\right)^2\right\}$$

$$+ \sum_{t=1}^{T}\left(\mathrm{E}_t{}'\gamma_e^{s} - \left((1-\beta_{s,e})s_{t-1}^{*} - \omega_{s}\right)\right)^2$$

$$\{\hat{\beta}_{k,n}; \hat{\gamma}^k_{n,\tau_0}; \widehat{\omega}_k; \hat{\beta}_{k,e}; \hat{\gamma}^k_e\}$$

$$= arg \min_{\beta_{k,n};\gamma^k_{n,\tau_0};\omega_k;\beta_{k,e};\gamma^k_e} \sum_{j=0}^{J-1} \left\{ \sum_{t=1}^{T-\tau_j} \left(RK_{t+\tau_j} - k_t^{(\tau_j)} \right)^2 \right\}$$

$$+ \sum_{t=1}^{T} \left(\mathrm{E}_t{}'\gamma^k_e - \left((1-\beta_{k,e}) k^*_{t-1} - \omega_k \right) \right)^2$$

3. Data

We estimate the model for Canadian inflation and real GDP growth, both in y-o-y terms, using a combination of monthly macroeconomic data and daily financial market returns. Incorporating financial market data enables us to leverage signals from forward-looking data at a daily frequency, which complements the important yet lagged signals from monthly macroeconomic data. This can prove particularly valuable to policy-makers during periods of uncertainty and rapidly changing market conditions. To merge two datasets with different frequencies, we transform the monthly data into a daily frequency by assuming a step function behaviour for inflation and GDP growth data. That is, inflation and GDP growth rates between monthly data releases are assigned the value of the latest release until the next release. In doing so, we can collapse the set of multivariate news to one scalar. All news variables, both macroeconomic and financial, are standardized to compare the importance of each news item for different moments.

Although GDP growth is more commonly measured and discussed in quarter-over-quarter terms, we choose to fit y-o-y GDP growth instead, not only to be consistent with the inflation measure but also to overcome the limited sample size. From 2000 to 2022, there are only 22 unique 12-month windows. Therefore, we use overlapping 12-month windows, which increases the number of observations to 264 and mitigates seasonality issues that would result from using overlapping quarterly windows. Our measurement of GDP growth is consistent with Adrian, Boyarchenko and Giannone (2019) for a 12-month-ahead forecast.[2]

3.1. Macroeconomic and financial news

News variables are essential to our modelling framework because positive (negative) news increases upside (downside) risks to the economic outlook. For example, the distribution of near-term inflation expectations should shift or skew to the right following higher-than-expected inflation data because that may suggest some momentum for the next release. We consider two types of news variables: macroeconomic and financial.

Macroeconomic news includes surprises in y-o-y inflation, month-over-month (m-o-m) GDP growth, the unemployment rate and m-o-m retail sales, where a surprise is measured as the difference between the macroeconomic data released and the financial market’s expectations for these data from Bloomberg. Although it would be more consistent for GDP growth and retail sales news to be in y-o-y terms, this is not readily available from Bloomberg because market participants typically forecast them in m-o-m

[2] Adrian, Boyarchenko and Giannone (2019) annualize the average growth rate of GDP between *t* and *t + h*.

terms. That said, this should not materially affect the main results since the surprise in the y-o-y measure should be highly correlated with the surprise in the m-o-m measure.

We select these macroeconomic data because they have a natural linkage to future inflation and GDP growth. To reiterate, past surprises in inflation and GDP growth have a direct impact on their distribution of risks. The unemployment rate is often regarded as one of the most important indicators for inflation (the Phillips curve) and GDP growth (Okun's law). Additionally, the growth of retail sales serves as a strong indicator for inflation and GDP growth, given its close correlation with consumer spending, which represents around 60% of Canada's GDP.

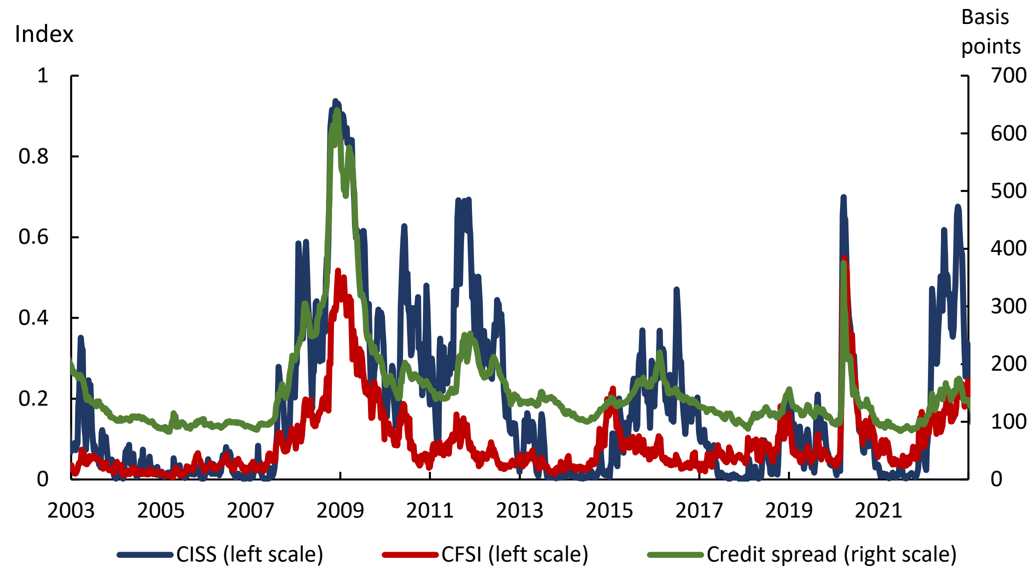
In addition to these backward-looking macroeconomic news variables, we consider forward-looking financial market news variables at a daily frequency. Since these financial asset returns have zero means and exhibit white noise–like patterns, we assume that financial news is simply daily returns in asset prices. These include daily returns in the S&P/TSX Composite Index, the USD/CAD exchange rate, three-month West Texas Intermediate (WTI) futures, the daily change in the Canadian term spread measured by the difference between Government of Canada 10-year and 2-year bond yields, the Bloomberg US investment grade spread (i.e., credit spread) and the Euro Composite Indicator of Systemic Stress (CISS), which is an aggregation of five market-specific subindexes created from financial stress measures (Holló, Kremer and Lo Duca 2012).

The credit spread and CISS are complementary proxies for financial conditions that were used in Lopez-Salido and Loria (2020) and Adams et al. (2020), respectively. In our analysis, we use the US credit spread because the Canadian corporate bond index is not as liquid and has shown delayed responsiveness to tightening financial conditions. Additionally, we include the CISS for the euro area as a proxy for the Canadian Financial Stress Index (CFSI), which is available at a weekly but not daily frequency (Duprey 2020). **Chart 6** shows that the euro CISS is a good proxy for the CFSI because it picks up all episodes of high financial market stress signalled by the CFSI.[3] This is not surprising given that they both use a similar methodology to measure systemic financial stress.

The long-term trend is similar between the credit spread and CISS in that they both tend to increase in response to shocks (**Chart 6**). The additional benefit of including the CISS is that it captures different facets of financial conditions that are particularly useful at measuring tail risks spilling over from Europe. Notably, the CISS displays higher volatility during periods such as the 2011 European debt crisis and the 2022 UK liability-driven investment crisis, both of which added tail risks to the Canadian macroeconomic outlook.

[3] A potential downside to using the CISS over the CFSI is that the CISS fails to directly capture developments in the Canadian housing market, a key feature of the CFSI.

Chart 6: Comparison of financial conditions indexes



Note: CISS is the Composite Indicator of Systemic Stress; CFSI is the Canadian Financial Stress Index
Sources: Bloomberg, European Central Bank and Bank of Canada calculations

Last observation: December 31, 2022

Similar to macroeconomic news variables, financial news variables have a well-known relationship with the outlooks for inflation and GDP growth. For instance, the S&P/TSX Composite Index typically moves higher (lower) on a better (worse) economic outlook, all else being equal, because a stock market index is in part a reflection of stronger (weaker) economic activity in the future. The same logic can be applied to the exchange rate and oil prices given the oil industry's large share in the Canadian economy. The term spread is widely regarded as a reliable predictor of growth and, to a lesser extent, of inflation.[4]

News variables are highly useful in updating the shape of the distribution. However, they may not be informative about the position of the distribution because the size, or the direction of the surprise, does not influence its level. For example, if the only information given was that recent inflation data missed the expectation by +/- 0.2%, it would not be clear whether inflation was closer to 2% or 4%. Furthermore, data releases contain useful information beyond the headline news (Feunou, Kyeong and Leiderman 2018). At times, markets react more to the details of the news than to the headline news. This is often the case with all four macroeconomic news variables we considered. For example, markets always pay close attention to core inflation numbers and any temporary factors that may have affected the headline inflation number.

For this reason, we augment the set of covariates with expectations of macroeconomic and financial variables. We obtain macroeconomic expectations from Bloomberg and assume that they are formed a week before the data release. This means that the impact of the change in macroeconomic expectations will occur a week before the data release, with additional impact occurring on the day of the data

[4] While the relationship between the term spread and inflation has not been as extensively documented, the term spread should co-move with inflation given its cyclical nature.

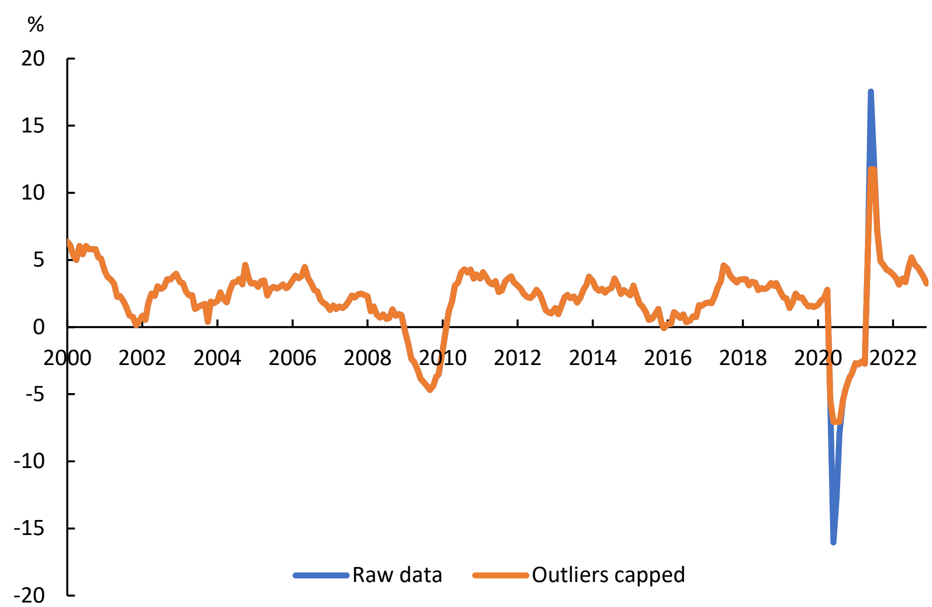
release should there be any surprise in both the headline and non-headline news. In terms of expectations of financial variables, we assume that they are best proxied by their current levels.

We include only three levels of financial variables (term spread, credit spread and CISS) and exclude the levels of the S&P/TSX, USD/CAD exchange rate and WTI given that they are non-stationary. The levels of both term and credit spreads are widely used in the literature and by market participants to forecast future growth. Following Boyarchenko et al. (2023), we also include the level of CISS, which complements credit spreads by capturing different facets of financial conditions. In summary, our covariate set includes seven expectations variables (four macroeconomic and three financial) and 10 news variables. A high number of covariates raises the possibility of over-parametrization, which typically leads to poor out-of-sample results. Although linking the estimated moments across different horizons helps mitigate this potential issue, it comes at the expense of goodness of fit.

3.2. Treatment of outliers

We make small adjustments to the underlying dataset to mitigate the effects of outliers in our relatively short sample. The adjustment mainly applies to the GFC and COVID-19 periods. For instance, real GDP growth swings from -16% in 2020 to +17% in 2021. We take two steps to mitigate the effect of outliers on the results. First, we limit the absolute values of inflation and GDP growth at their mean plus a standard deviation of 3.5. This adjustment has no impact on the inflation data because the entire sample is within this standard deviation. However, the adjustment excludes 2% of the GDP growth data, which occurred during the pandemic period (**Chart 7**).

Chart 7: Removing outliers in GDP growth data



Sources: Haver Analytics and Bank of Canada calculations Last observation: December 2022

Second, we take additional steps to ensure that rolling moments of realized inflation and GDP growth are not too sensitive to a one-off shock like the GFC or the pandemic. We do this because our use of a

20-year rolling window for skewness and kurtosis implies that high sensitivity to shocks can have a long-lasting impact on the parameter estimation. We regress the conventional measure of skewness on three alternative measures of skewness that collectively provide greater robustness. We then use the fitted values from this regression as our new measure of skewness (**Chart 8**).

The regression equation for skewness is given by:

$$SK_t = \beta_0 + \beta_1 sk_1 + \beta_2 sk_2 + \beta_3 sk_3 + u_t, \qquad (6)$$

where

- $sk_1 = \frac{mean-mode}{s\tan dard\ deviation}$

- $sk_2 = \frac{3(mean-median)}{s\tan dard\ deviation}$

- $sk_3 = \frac{Q(0.75)+Q(0.25)-2Q(0.50)}{Q(0.75)-Q(0.25)}$, where Q is the quantile function.

The measures of skewness shown above are Pearson's first and second skewness coefficients and Bowley's measure of skewness, also known as Yule's coefficient, respectively.

Similarly, to obtain a robust measure of kurtosis, we use the fitted values from the following regression as our new measure of kurtosis (**Chart 9**).

The regression equation for kurtosis is given by:

$$KR_t = \beta_0 + \beta_1 kr_1 + \beta_2 kr_2 + \beta_3 kr_3 + u_t, \qquad (7)$$

where

- $kr_1 = \frac{(E_7-E_5)+(E_3-E_1)}{E_6-E_2}$, where E_i is the i th octile

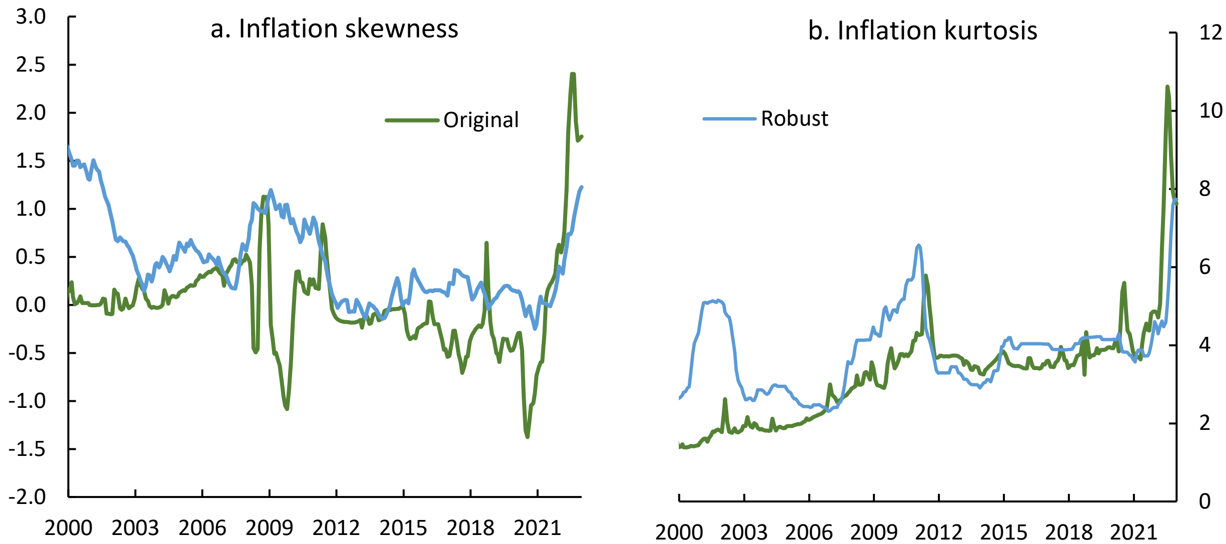
- $kr_2 = \frac{F^{-1}(0.975)+F^{-1}(0.025)}{F^{-1}(0.75)-F^{-1}(0.25)} - 2.91$, where F is the cumulative distribution function

- $kr_3 = \frac{U_{0.05}-L_{0.05}}{U_{0.5}-L_{0.5}} - 2.59$, where $U_\alpha(L_\alpha)$ is the average of the upper(lower) α quantiles, defined as:

  $$U_\alpha = \frac{1}{\alpha}\int_{1-\alpha}^{1} F^{-1}(y)dy,\ L_\alpha = \frac{1}{\alpha}\int_{0}^{\alpha} F^{-1}(y)dy.$$

The independent measures of kurtosis shown in equation 7 are proposed by Moors (1988), Hogg (1972) and Crow and Siddiqui (1967), respectively.

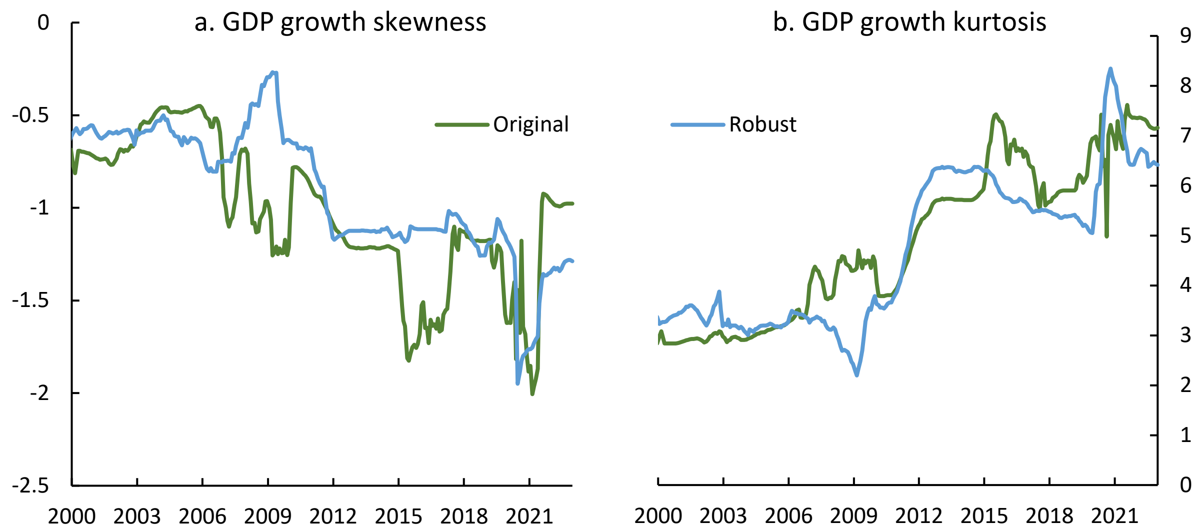
Chart 8: Original and robust measures of skewness and kurtosis for inflation



Sources: Haver Analytics and Bank of Canada calculations

Last observation: December 2022

Chart 9: Original and robust measures of skewness and kurtosis for GDP growth



Sources: Haver Analytics and Bank of Canada calculations

Last observation: December 2022

4. Empirical results

We now examine the conditional moments of y-o-y inflation and GDP growth and their goodness of fit. In this analysis, we focus mainly on the results for a 12-month forecast horizon because we observe that the conditional distribution widens considerably beyond this horizon.

4.1. Goodness of fit

In **Table 1** we report the gamma estimates of news variables for inflation and GDP growth.

The news of y-o-y inflation has significantly positive impacts on the first moment of inflation. Inflation news can influence future inflation by affecting the behaviour of economic players. When inflation news is higher than expected, consumers may buy goods and services sooner to avoid paying higher prices later, while businesses may raise their prices to offset higher costs.

The news of m-o-m GDP growth also has a significantly positive impact on inflation for similar reasons. Stronger-than-expected growth or expectations of higher growth could signal more demand for goods and services, which could put upward pressure on prices and increase the likelihood of inflation.

Retail sales have the largest significantly positive effect on the first moment of GDP growth and the second largest effect on the first moment of inflation. This is intuitive: retail sales are an indicator of consumer spending, a major component of GDP. Therefore, changes in retail sales can provide an early indication of future GDP growth. Retail sales can also be an important indicator of future inflation. For example, strong retail sales suggest high demand for goods and services, which can lead to price increases.

Oil price movements have a significant positive impact on the mean of both inflation and GDP growth. This is not surprising because the movements in oil prices contribute to changes in the consumer price index (CPI) directly and indirectly. Because energy accounts for a part of CPI, higher crude oil prices would lead to a direct increase in CPI. Higher crude oil prices also impact CPI indirectly since crude oil is an important input into a range of products (e.g., plastics) and its price affects transportation costs, another component of CPI. The positive impact on GDP growth can be explained by the large size of Canada’s oil industry.

The USD/CAD exchange rate has a significant positive impact on the inflation mean. When the exchange rate increases, indicating a depreciation of the Canadian dollar, it makes imports more expensive and creates inflationary pressures on domestic goods. Additionally, a positive relationship with GDP growth is intuitive because lower demand for imports and higher demand for exports result in an increase in net exports.

Table 1: Gamma estimates of news variables for inflation and GDP growth, 12 months

Gamma	Inflation				GDP growth			
News	Mean	Volatility	Skewness	Kurtosis	Mean	Volatility	Skewness	Kurtosis
CPI (year-over-year)	5.93e-02***	5.18e-03***	2.15e-03***	1.21e-02***	-5.47e-03	1.45e-02***	-4.07e-03***	6.38e-03***
	(8.1)	(5.7)	(3.9)	(7.6)	(0.8)	(9.8)	(-8.9)	(3.6)
GDP growth (month-over-month)	4.28e-03***	1.14e-03**	-8.49e-03***	6.34e-03***	5.02e-02***	1.58e-02***	-1.43e-02***	1.69e-02***
	(6.4)	(1.8)	(-7.5)	(6.6)	(3.3)	(7.2)	(-44.2)	(11.0)
Unemployment rate	3.00e-02	-3.82e-03***	-3.63e-03***	-1.73e-03***	5.69e-02***	-1.17e-02***	2.17e-03***	9.59e-04
	(-0.8)	(-5.4)	(-4.1)	(-2.6)	(7.9)	(-3.1)	(-3.8)	(0.7)
Retail sales (month-over-month)	4.17e-02***	1.39e-03**	4.24e-03***	5.50e-03***	8.08e-02***	1.09e-02***	-5.64e-04	6.75e-03***
	(6.1)	(1.7)	(4.4)	(4.4)	(8.5)	(11.3)	(-1.2)	(5.4)
S&P/TSX Composite Index	-6.02e-03***	3.59e-04	3.51e-03***	7.37e-04*	-1.19e-02***	1.23e-03***	1.65e-03***	-8.09e-04
	(-3.1)	(1.0)	(7.0)	(1.3)	(-2.5)	(4.8)	(9.8)	(-1.2)
USD/CAD exchange rate	7.20e-03***	1.07e-04	-4.17e-04***	1.59e-03***	3.48e-03*	5.68e-04***	-3.93e-03***	4.24e-03***
	(4.5)	(0.4)	(-5.1)	(5.2)	(1.6)	(3.9)	(-34.6)	(7.3)
West Texas Intermediate (3-month futures)	2.04e-02***	6.04e-04***	2.75e-03***	5.38e-04*	2.03e-02***	1.61e-03***	2.16e-04*	-1.19e-03**
	(9.4)	(2.7)	(5.0)	(1.6)	(6.5)	(2.5)	(1.4)	(-2.1)
Canadian term spread	-3.56e-03***	3.75e-04**	-5.60e-04	2.15e-05	-1.52e-02***	6.79e-04	-1.73e-04***	2.37e-05
	(-4.2)	(2.0)	(1.2)	(0.3)	(-5.3)	(-1.1)	(-4.8)	(0.0)
US credit spread	4.16e-04***	-1.06e-03***	1.61e-03	-5.53e-04***	1.31e-02***	-1.15e-03**	4.55e-04***	-1.66e-03***
	(4.2)	(-5.5)	(0.8)	(-3.5)	(4.8)	(-2.3)	(9.2)	(-7.1)
Composite Indicator of Systemic Stress	4.44e-03***	7.56e-04***	9.98e-04***	-8.47e-04***	-7.74e-03*	5.49e-04***	2.00e-03***	-4.24e-04
	(3.2)	(3.3)	(7.7)	(-3.8)	(-1.5)	(6.1)	(16.3)	(-0.9)

Note: Canadian term spread is the difference between the Government of Canada 10-year and 2-year bond yields; US credit spread is the Bloomberg US investment grade spread.

*p<.1; **p<.05; ***p<.01

Table 2 reports the gamma estimates of expectations variables for inflation and GDP growth.

The coefficient of y-o-y inflation expectations is positive and statistically significant on the first moment of inflation. This is because expectations provide useful information about the central tendency of the distribution, which cannot be captured by the news variable alone. Similarly, the coefficient of m-o-m GDP growth expectations is also positively and significantly associated with the first moment of inflation, albeit less directly. Expectations of higher growth may signal stronger demand for goods and services, which could raise prices and increase the likelihood of higher inflation.

The expectations for the term spread have a positive effect on the first three moments of inflation. Given that long-term interest rates reflect market expectations of future inflation, while short-term interest rates are influenced more by current economic conditions and monetary policy, a wider term spread suggests that market participants expect inflation to be higher in the future. Conversely, when the term spread is narrow or negative, it suggests that market participants expect inflation to be lower in the future. This is consistent with the literature: Ang, Bekaert and Wei (2007) find that a wide term spread is associated with higher inflation in the future, while a narrow term spread is associated with lower inflation. Stock and Watson (2003) determine that the term spread has strong predictive power for inflation up to two years ahead.

The expectations for the term spread is a significant indicator for all four moments of GDP growth. It has a positive impact on the first moment and a negative impact on volatility, indicating that a wider term spread (i.e., steeper yield curve) is associated with higher and more stable GDP growth. This is consistent with macroeconomic theory, which states that higher short-term rates lead to the postponement of investment and consumption, thus decreasing GDP growth. Given that the short-term interest rates are influenced by current economic conditions, while long-term interest rates reflect the future expectations of economic activity, a wide term spread indicates that investors expect the economy to grow in the future because they are willing to invest in long-term bonds with higher yields. In contrast, a narrow term spread indicates that investors are less confident in the future growth of the economy. Our findings corroborate the findings of Estrella and Mishkin (1998) and Rudebusch and Williams (2009). They study the relationship between the term spread and GDP and find that the term spread is a significant predictor of GDP, and that a flat or inverted yield curve is a significant indicator of a future recession.

The expectations for the credit spread (which captures market participants' assessment of default risk) have a significant negative impact on the first moments of both GDP growth and inflation. A wider credit spread suggests an increase in default risk, which would hamper economic growth. This transmission can be explained by the "financial accelerator" theory developed by Bernanke, Gertler and Gilchrist (1996). The theory suggests that an increase in the external finance premium makes borrowing more costly, thus reducing borrowers' spending and production, which in turn leads to a decrease in GDP growth. The resulting economic slack leads to the fall of inflation over time. Although this premium is not directly observable, credit spreads are considered a good proxy to approximate it (Mueller 2009). Therefore, a wider credit spread would predict downturns in economic activity. The negative relationship is consistent with the findings of Gilchrist and Zakrajšek (2012), Bedock and Stevanović (2017) and Lopez-Salido and Loria (2020).

Both the news and the expectations of CISS have a negative (but not significant) impact on the mean of GDP growth and a positive and significant impact on its volatility. This is intuitive since higher systemic risk increases the risk of lower and less stable economic growth. The negative relationship is consistent with the findings of Figueres and Jarociński (2020).

Table 2: Gamma estimates for expectations for inflation and GDP growth, 12 months

Gamma	Inflation				GDP growth			
Expectations	Mean	Volatility	Skewness	Kurtosis	Mean	Volatility	Skewness	Kurtosis
CPI (year-over-year)	6.89e-03***	1.32e-01***	2.78e-03***	-9.14e-03**	7.50e-02***	8.41e-02***	2.25e-02***	-2.25e-03***
	(31.8)	(28.6)	(39.4)	(-2.1)	(-4.2)	(5.5)	(4.9)	(-12.6)
GDP growth (month-over-month)	1.85e-03***	-4.75e-03*	-2.97e-03***	-1.70e-02***	5.07e-02*	-7.85e-03***	-1.05e-02***	1.83e-03***
	(3.1)	(-1.3)	(-49.9)	(-7.3)	(1.5)	(-13.1)	(-10.5)	(16.5)
Unemployment rate	1.21e-02***	4.69e-02***	-1.32e-03***	-2.40e-02***	1.69e-02***	8.93e-02***	-4.16e-02***	1.29e-03***
	(31.1)	(31.2)	(-54.3)	(-48.1)	(23.3)	(32.1)	(-62.9)	(16.9)
Retail sales (month-over-month)	1.45e-04***	2.35e-03***	-7.26e-04***	-2.38e-03***	3.02e-02***	6.10e-03***	1.79e-03	-2.46e-04***
	(8.8)	(6.5)	(10.6)	(-4.8)	(16.7)	(-11.5)	(-0.6)	(-5.3)
Canadian term spread	-3.54e-03***	6.12e-02***	1.07e-03***	-1.43e-02	1.07e+00***	3.06e-02***	1.16e-01***	-1.50e-03***
	(11.6)	(13.4)	(13.7)	(-0.6)	(7.1)	(-17.6)	(60.0)	(-11.8)
US credit spread	1.26e-03***	4.96e-02***	2.28e-03***	-7.20e-03***	-1.44e+00***	4.32e-03**	2.01e-01***	-3.44e-03***
	(-18.1)	(5.6)	(45.5)	(-4.6)	(-9.1)	(2.0)	(48.9)	(-16.3)
Composite Indicator of Systemic Stress	-4.67e-03**	-2.39e-03	-1.70e-03***	1.17e-01***	4.96e-02	8.80e-02***	-1.28e-01***	3.74e-03***
	(-1.8)	(-0.3)	(-15.6)	(39.1)	(-0.8)	(8.0)	(-41.9)	(17.5)

Note: Canadian term spread is the difference between the Government of Canada 10-year and 2-year bond yields; US credit spread is the Bloomberg US investment grade spread.

*p<.1; **p<.05; ***p<.01

Table 3 shows the estimates of beta for the four moments of inflation and GDP growth. Recall that the moments are modelled as moving averages of news (short-run) and expectations (long-run), where betas show the persistence of new information or expectations in shaping the distribution. Beta estimates are close to 1 with high t-statistics for most cases, which means that recent news or expectations about the economy captured in the set of covariates is highly persistent in shaping the distribution of inflation and GDP growth. The main exception is the inflation expectation dynamic where the beta estimates are zero, suggesting that the most recent expectations formed by market participants tend to be the only useful information in forming the long-run distribution of inflation moments.

Table 3: Beta estimates at the 12-month forecast horizon

Beta	News		Expectations	
	Inflation	GDP growth	Inflation	GDP growth
Mean	0.99 (4.7e+0.3)	0.99 (3.3e+0.3)	0.00 (0.0e+0)	0.99 (2.7e+0.3)
Variance	0.99 (4.7e+0.3)	0.99 (3.5e+0.3)	0.00 (0.0e+0)	0.99 (2.8e+0.3)
Skewness	0.99 (4.6e+0.3)	0.99 (2.0e+0.3)	0.00 (0.0e+0)	0.99 (3.1e+0.3)
Kurtosis	0.99 (4.8e+0.3)	0.99 (3.7e+0.3)	0.00 (0.0e+0)	0.99 (3.0e+0.3)

To further assess the model's performance, we examine its statistical fit for other forecasting horizons. Recall that the conditional moments are linked across different horizons, allowing us to compute conditional moments for different horizons using the same set of parameters.

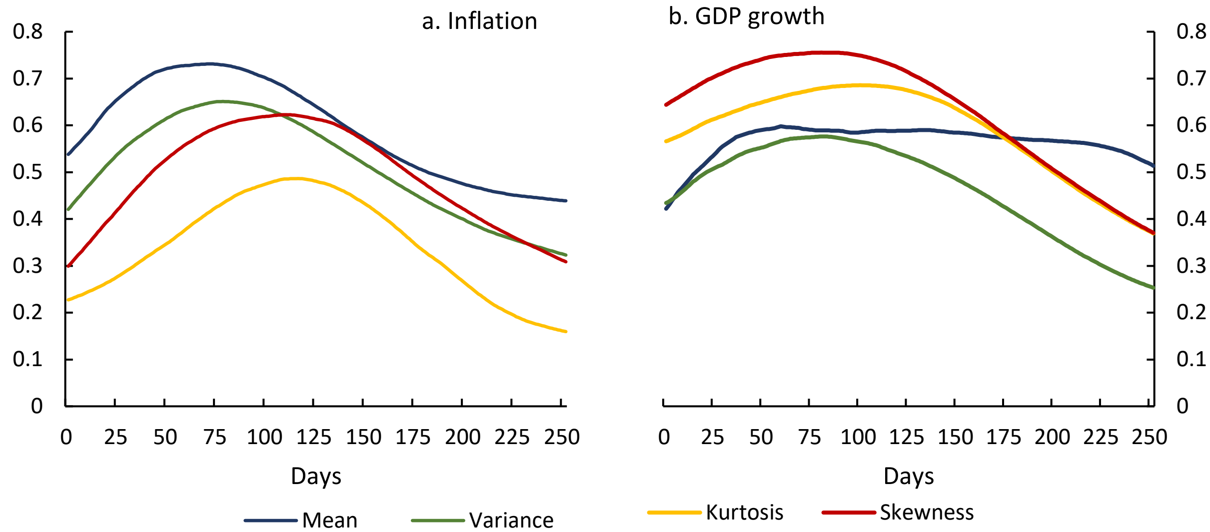
We conduct a univariate regression by regressing the realized moments on their corresponding conditional moments for both inflation and GDP growth. The resulting summary statistics are presented in the **Table 4**. As expected, the coefficients of the conditional moments are close to 1, meaning that the fitted moments move in near lockstep with the realized moments. They are all statistically significant at the 5% level.

Table 4: Summary statistics for conditional moments of inflation and GDP growth

	News				Expectations			
	Coefficients							
	Mean	**Variance**	**Skewness**	**Kurtosis**	**Mean**	**Variance**	**Skewness**	**Kurtosis**
3 months	0.84	0.98	0.86	0.85	0.81	0.80	0.86	0.79
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01	(0.01)	(0.01)	(0.01)
6 months	1.16	1.26	1.16	1.39	1.11	1.38	1.19	1.29
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)
9 months	1.22	1.25	1.09	1.26	1.19	1.76	1.22	1.51
	(0.01)	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)
12 months	1.20	1.14	0.91	0.89	1.14	1.68	1.07	1.40
	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.04)	(0.02)	(0.03)
	Intercepts							
3 months	1.20	0.01	0.04	0.57	0.47	0.33	-0.13	1.05
	(0.01)	(0.01)	(0.00)	(0.06)	(0.03)	(0.02)	(0.01)	(0.04)
6 months	1.20	-0.17	-0.06	-1.43	-0.18	-0.41	0.18	-1.31
	(0.01)	(0.01)	(0.01)	(0.08)	(0.03)	(0.03)	(0.01)	(0.06)
9 months	1.20	-0.15	-0.03	-0.93	-0.42	-0.88	0.21	-2.41
	(0.01)	(0.02)	(0.01)	(0.1)	(0.04)	(0.04)	(0.02)	(0.1)
12 months	1.20	-0.06	0.036	0.46	-0.35	-0.75	0.07	-1.87
	(0.01)	(0.02)	(0.01)	(0.11)	(0.04)	(0.06)	(0.02)	(0.12)
	R-squared values							
3 months	0.73	0.64	0.57	0.39	0.6	0.57	0.75	0.66
6 months	0.64	0.58	0.61	0.48	0.59	0.53	0.71	0.67
9 months	0.49	0.42	0.45	0.31	0.57	0.39	0.54	0.53
12 months	0.44	0.32	0.31	0.16	0.51	0.25	0.37	0.37

The overall goodness of fit as indicated by the R-squared values is above 0.5 for most cases, and it generally deteriorates as the forecast horizon increases. We repeat this regression analysis for each horizon length up to a year and show their R-squared values and coefficient estimates in **Chart 10** and **Chart 11**, respectively. We find that the goodness of fit peaks at around the three-month mark across all moments for both inflation and GDP growth. Similarly, the coefficient estimates peak at around the four-month mark, suggesting that the conditional moments as estimators become unbiased at that horizon. This reflects the trade-off between the goodness of fit and the unbiasedness of the coefficient estimator as horizons change.

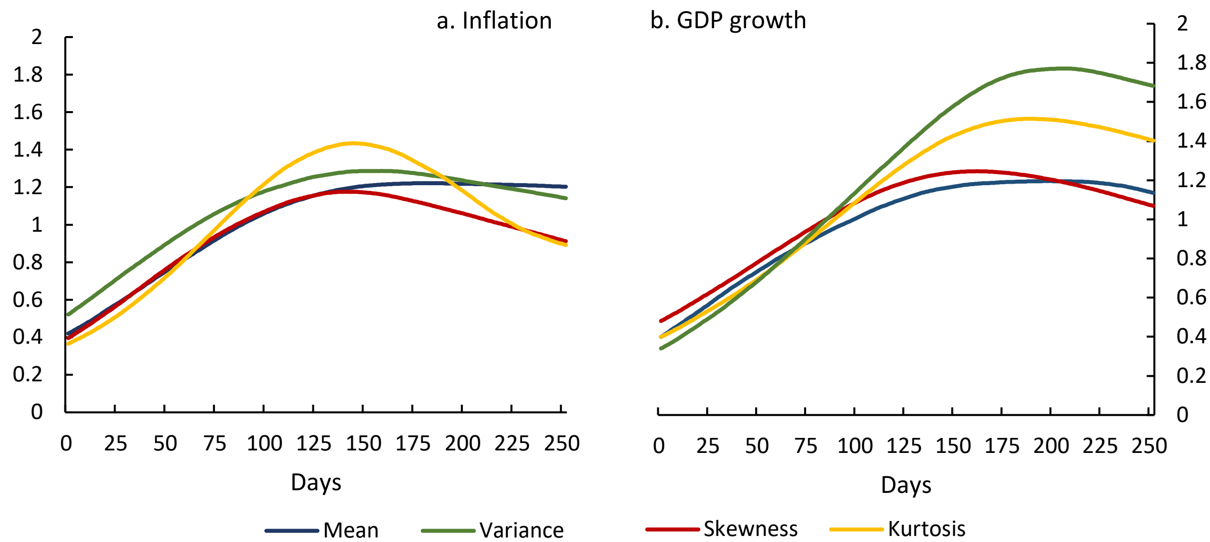
Chart 10: R-squared measures by forecasting horizons for inflation and GDP growth



Source: Bank of Canada calculations

Last observation: December 31, 2022

Chart 11: Coefficients by forecasting horizons for inflation and GDP growth



Source: Bank of Canada calculations

Last observation: December 31, 2022

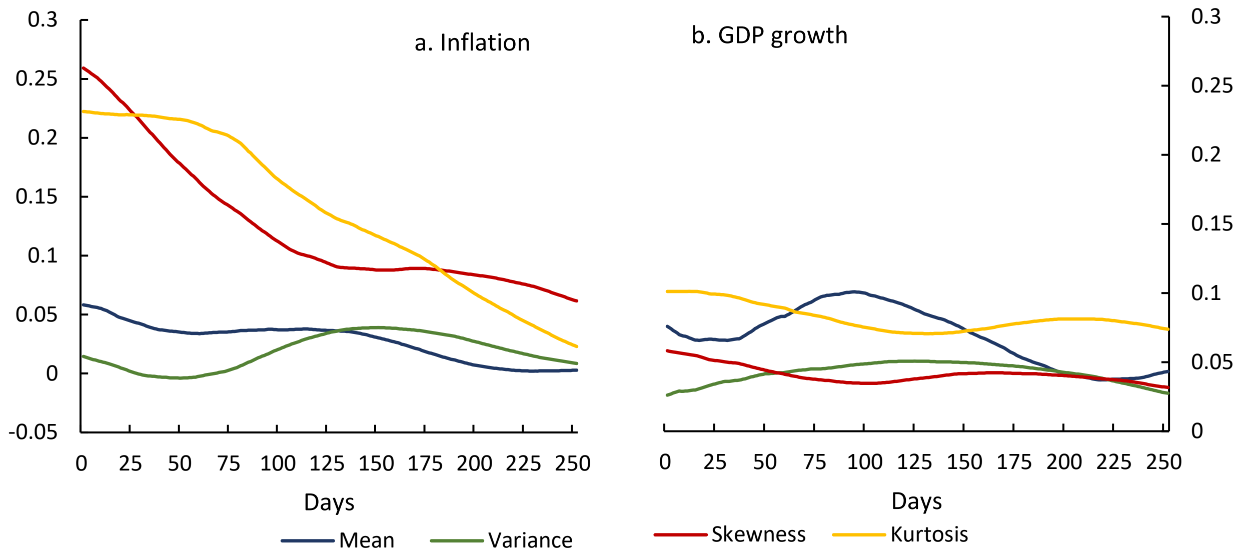
4.2. Potential role of asymmetry in news

So far, we have implicitly assumed that positive and negative news have the same absolute impact on the conditional moments. However, many empirical studies show that responses to positive and negative news are asymmetric because negative news tends to have a greater impact on agents than positive news does.[5] Therefore, we investigate the potential role of asymmetry in our work by separating each of the news variables into positive and negative news variables. Positive news variables

[5] A partial list of contributions includes Soroka (2006) and Gambetti, Maffei-Faccioli and Zoi (2023).

are derived exclusively from the positive values of the original news variables, indicating instances where the realized data exceeded expectations. Similarly, negative news variables are derived exclusively from the negative values of the original news variables.

Chart 12: Difference between R-squared values for level and asymmetry models



Source: Bank of Canada calculations

Last observation: December 31, 2022

Since this doubles the number of news variables, the number of covariates is higher, and the goodness of fit, as indicated by the R-squared values, improves across all horizons and moments but to varying degrees (**Chart 12**). First, the improvement is more pronounced for inflation than for GDP growth. One possible explanation is that macrofinancial variables are more tightly linked to inflation (or its expectations) than GDP data, given inflation's stronger implication for monetary policy. Therefore, financial variables have a greater scope for an asymmetric impact on the conditional distribution of inflation than that of GDP growth. Second, the improvement is more pronounced for conditional skewness and kurtosis. This is likely because a negative (positive) asymmetric reaction to news is generally associated with increased risks to the downside (upside) but not associated with a material shift in the conditional mean or variance. Third, the improvement is generally greater at a shorter horizon, suggesting that the role of asymmetry largely disappears at longer horizons.

The Gamma estimates with asymmetry are shown in **Table 5** and **Table 6**. We can observe a difference in the direction of the gamma estimates for some of the positive and negative news (**Table 7**).

Table 5: Gamma estimates with asymmetry—positive news, 12 months

Gamma	Inflation				GDP growth			
Positive news	Mean	Volatility	Skewness	Kurtosis	Mean	Volatility	Skewness	Kurtosis
CPI (year-over-year)	2.47e-02*** (3.9)	6.47e-03*** (3.5)	-9.63e-03*** (-4.1)	3.36e-03** (2.1)	1.13e-01*** (4.8)	7.31e-03*** (3.1)	-6.01e-03*** (-7.4)	7.28e-03** (2.2)
GDP growth (month-over-month)	1.80e-02*** (3.7)	-5.63e-03*** (-3.6)	-2.20e-03* (-1.3)	7.47e-03*** (5.0)	-4.29e-02*** (-4.1)	-3.15e-03*** (-2.4)	-4.01e-03*** (-5.9)	6.21e-03*** (3.2)
Unemployment rate	-6.52e-02*** (-5.6)	1.79e-03 (0.5)	-3.92e-02*** (-7.9)	-1.41e-02*** (-4.2)	-4.74e-02 (-1.2)	1.32e-04 (0.0)	-1.51e-02*** (-7.7)	7.38e-03 (1.0)
Retail sales (month-over-month)	1.64e-02** (2.0)	5.69e-03** (2.2)	-2.19e-03 (-0.8)	6.65e-03*** (3.1)	4.28e-02* (1.3)	1.40e-02*** (4.3)	-9.37e-04 (-0.8)	4.14e-03 (0.8)
S&P/TSX Composite Index	4.16e-03*** (2.7)	9.25e-04** (2.3)	3.59e-03*** (6.2)	-1.43e-03*** (-3.4)	-1.42e-02*** (-2.6)	1.80e-04 (0.3)	2.65e-03*** (13.0)	-1.40e-03* (-1.6)
USD/CAD exchange rate	4.37e-03*** (3.9)	1.36e-03*** (4.1)	1.54e-03*** (4.2)	3.12e-03*** (11.9)	9.01e-03** (2.1)	1.91e-03*** (3.5)	-1.69e-03*** (-9.4)	2.54e-03*** (3.6)
West Texas Intermediate (3-month futures)	1.48e-02*** (9.4)	-3.57e-07 (-0.0)	1.54e-03*** (4.2)	-9.09e-04*** (-2.9)	1.87e-02*** (4.2)	7.76e-04* (1.6)	-3.57e-04** (-2.2)	-6.71e-04 (-1.0)
Canadian term spread	3.36e-04 (0.4)	7.33e-04*** (3.0)	-4.14e-04* (-1.5)	8.14e-04*** (4.0)	-8.34e-03*** (-3.1)	5.35e-04* (1.5)	-1.50e-04* (-1.3)	1.04e-03*** (2.4)
US credit spread	1.62e-04 (0.3)	-2.14e-04* (-1.3)	8.12e-04*** (3.6)	-2.43e-04* (-1.6)	5.54e-03*** (2.6)	-1.16e-04 (-0.4)	9.02e-04*** (10.3)	-1.15e-03*** (-3.3)
Composite Indicator of Systemic Stress	1.61e-03** (1.9)	6.55e-04*** (3.2)	3.67e-04* (1.5)	-9.04e-04*** (-4.3)	-7.96e-03*** (-2.9)	1.13e-03*** (3.2)	2.10e-04** (1.8)	-5.79e-04 (-1.2)

Note: Canadian term spread is the difference between the Government of Canada 10-year and 2-year bond yields; US credit spread is the Bloomberg US investment grade spread.

*p<.1; **p<.05; ***p<.01

Table 6: Gamma estimates with asymmetry—negative news, 12 months

Gamma	Inflation				GDP growth			
Negative news	Mean	Volatility	Skewness	Kurtosis	Mean	Volatility	Skewness	Kurtosis
CPI (year-over-year)	4.52e-02***	7.51e-03***	2.00e-02***	2.17e-02***	-9.43e-02***	2.83e-02***	-1.90e-03**	4.77e-04
	(6.6)	(5.1)	(8.2)	(11.7)	(-4.1)	(9.5)	(-1.9)	(0.1)
GDP growth (month-over-month)	1.62e-02***	-3.40e-03**	-4.10e-03**	7.70e-03***	8.57e-02***	1.67e-02***	-1.49e-02***	1.71e-02***
	(2.9)	(-2.1)	(-2.0)	(4.2)	(3.6)	(5.4)	(-15.4)	(4.8)
Unemployment rate	2.01e-02***	-2.48e-03**	-1.30e-03	-3.79e-03***	5.85e-02***	-7.41e-03***	1.75e-03***	-9.27e-04
	(5.3)	(-2.2)	(-1.0)	(-4.9)	(4.9)	(-4.2)	(2.8)	(-0.5)
Retail sales (month-over-month)	1.27e-02***	2.99e-03**	4.08e-04	3.72e-03***	9.66e-02***	9.01e-03***	-2.39e-03***	4.36e-03**
	(2.5)	(2.1)	(0.2)	(2.6)	(7.2)	(5.9)	(-3.3)	(2.3)
S&P/TSX Composite Index	-3.96e-03***	1.83e-03***	2.76e-03***	1.32e-03***	-1.32e-02***	2.22e-03***	1.31e-03***	-4.48e-04
	(-3.1)	(4.4)	(5.5)	(3.5)	(-2.4)	(3.3)	(6.1)	(-0.6)
USD/CAD exchange rate	8.92e-03***	1.11e-03***	-1.48e-03***	3.13e-04	3.23e-03	2.81e-04	-3.49e-03***	4.72e-03***
	(7.1)	(3.4)	(-4.0)	(1.2)	(0.8)	(0.6)	(-18.7)	(6.2)
West Texas Intermediate (3-month futures)	1.77e-02***	6.71e-04***	3.05e-03***	-6.72e-05	2.35e-02***	8.95e-04**	1.14e-03***	-1.61e-03***
	(11.7)	(2.4)	(8.5)	(-0.3)	(5.5)	(1.7)	(7.2)	(-2.6)
Canadian term spread	-5.12e-03***	2.50e-05	-7.29e-04**	5.86e-04**	-1.70e-02***	-1.46e-03***	1.15e-03***	-1.28e-03**
	(-5.8)	(0.1)	(-2.3)	(2.2)	(-4.5)	(-3.8)	(6.9)	(-2.2)
US credit spread	2.50e-03***	-8.20e-04***	1.03e-03***	-1.49e-03***	1.02e-02***	-8.84e-04***	1.92e-03***	-1.50e-03***
	(4.1)	(-4.4)	(4.1)	(-8.4)	(3.9)	(-2.5)	(16.4)	(-4.3)
Composite Indicator of Systemic Stress	1.70e-03**	7.49e-04***	8.20e-04***	-9.70e-04***	-2.92e-03	2.15e-03***	-2.58e-04**	-3.14e-04
	(2.1)	(3.4)	(3.3)	(-5.0)	(-1.0)	(5.3)	(-1.9)	(-0.6)

Note: Canadian term spread is the difference between the Government of Canada 10-year and 2-year bond yields; US credit spread is the Bloomberg US investment grade spread.

*p<.1; **p<.05; ***p<.01

Table 7: Difference in gamma estimates between positive and negative news, 12 months

	Inflation				GDP growth			
Difference	Mean	Volatility	Skewness	Kurtosis	Mean	Volatility	Skewness	Kurtosis
CPI (year-over-year)	-6.05e-03	-8.85e-04	-2.05e-02***	-1.67e-02***	1.38e-01***	-2.63e-02***	2.64e-03***	6.97e-03
	(-0.5)	(-0.4)	(-6.2)	(-6.9)	(4.4)	(-8.8)	(2.5)	(1.2)
GDP growth (month-over-month)	-4.08e-02***	-2.88e-03*	8.26e-04	-1.22e-03	-1.74e-02	4.80e-03**	5.43e-03***	-1.29e-02***
	(-3.8)	(-1.3)	(0.2)	(-0.4)	(-0.8)	(1.8)	(5.3)	(-2.9)
Unemployment rate	-9.78e-02***	4.39e-03	-3.88e-02***	-9.34e-03***	-9.78e-02***	1.91e-03	-2.13e-02***	8.43e-03
	(-5.6)	(1.2)	(-8.1)	(-2.7)	(-2.6)	(0.4)	(-13.0)	(1.1)
Retail sales (month-over-month)	-1.93e-02*	2.75e-03	-7.56e-03**	3.81e-03*	1.88e-02	2.37e-02***	-4.79e-03***	-8.75e-04
	(-1.6)	(1.0)	(-2.2)	(1.5)	(0.6)	(9.2)	(-4.5)	(-0.2)
S&P/TSX Composite Index	2.70e-03**	-8.35e-04**	6.07e-04	-2.87e-03***	5.52e-03*	4.46e-04	1.15e-03***	-8.30e-04
	(1.7)	(-2.2)	(1.2)	(-8.1)	(1.3)	(1.0)	(8.5)	(-1.1)
USD/CAD exchange rate	-2.55e-03***	1.78e-04	2.04e-03***	2.82e-03***	6.48e-03***	2.18e-04	1.22e-03***	-2.37e-03***
	(-2.6)	(0.7)	(8.6)	(12.2)	(2.7)	(0.8)	(13.8)	(-4.0)
West Texas Intermediate (3-month futures)	-6.46e-04	-6.98e-04***	-1.15e-04	-7.81e-04***	-1.30e-02***	-1.28e-03***	-7.09e-04***	9.62e-04**
	(-0.7)	(-3.0)	(-0.4)	(-3.5)	(-5.6)	(-5.4)	(-9.6)	(2.3)
Canadian term spread	7.48e-03***	8.39e-04***	3.90e-04*	9.75e-05	7.81e-04	2.65e-04	-3.23e-04***	2.61e-03***
	(5.8)	(3.0)	(1.4)	(0.4)	(0.3)	(0.9)	(-3.5)	(4.9)
US credit spread	-4.60e-04	5.88e-04***	3.88e-04	1.25e-03***	-5.95e-03**	2.29e-04	-4.65e-04***	2.98e-04
	(-0.4)	(2.4)	(1.1)	(5.1)	(-2.1)	(0.8)	(-4.6)	(0.6)
Composite Indicator of Systemic Stress	-2.30e-05	-1.19e-04	-1.16e-03***	1.89e-04**	6.97e-04	2.77e-04***	-7.28e-04***	-2.19e-04
	(-0.0)	(-0.9)	(-9.6)	(2.0)	(0.5)	(2.4)	(-16.6)	(-0.6)

Note: Canadian term spread is the difference between the Government of Canada 10-year and 2-year bond yields; US credit spread is the Bloomberg US investment grade spread.

*p<.1; **p<.05; ***p<.01

A positive surprise in the unemployment rate news (i.e., the unemployment rate is higher than expected) has a negative impact on the first moment of inflation. This is consistent with higher unemployment potentially leading to lower wages and lower production costs, ultimately leading to lower prices. Higher unemployment can lead to less consumer spending, which would reduce GDP growth. Conversely, a lower-than-expected unemployment rate suggests more upward pressure on wages, which can lead to higher inflation. With higher employment, consumer spending could increase, which would in turn result in higher economic growth. This difference in the impact of positive and negative news is statistically significant, indicating that the unemployment rate has an asymmetric effect on the first moments.

Estimates of beta for the distribution of inflation and GDP growth using the asymmetry model are shown in **Table 8**. The estimates for news are consistent with the estimates we obtained without accounting for the asymmetry of the news. That is, all the beta estimates of news are close to 1, with high statistical significance, indicating that the content of news information has a long-lasting impact on future forecasts of inflation and GDP growth. The beta estimates for inflation expectations change to be close to 1 with the addition of asymmetry.

Table 8: Beta estimates at a 12-month forecast horizon with asymmetry

Beta	News		Expectations	
	Inflation	GDP growth	Inflation	GDP growth
Mean	0.99 (3.6e+0.3)	0.99 (1.1e+0.3)	0.92 (4.4e+02)	0.99 (1.6e+0.3)
Variance	0.99 (1.0e+0.3)	0.99 (1.6e+0.3)	0.99 (1.0e+03)	0.99 (3.0e+0.3)
Skewness	0.99 (2.2e+0.3)	0.99 (3.6e+0.3)	0.99 (1.9e+03)	0.99 (2.7e+0.3)
Kurtosis	0.99 (3.9e+0.3)	0.99 (9.7e+0.3)	0.85 (6.8e+02)	0.99 (1.1e+0.3)

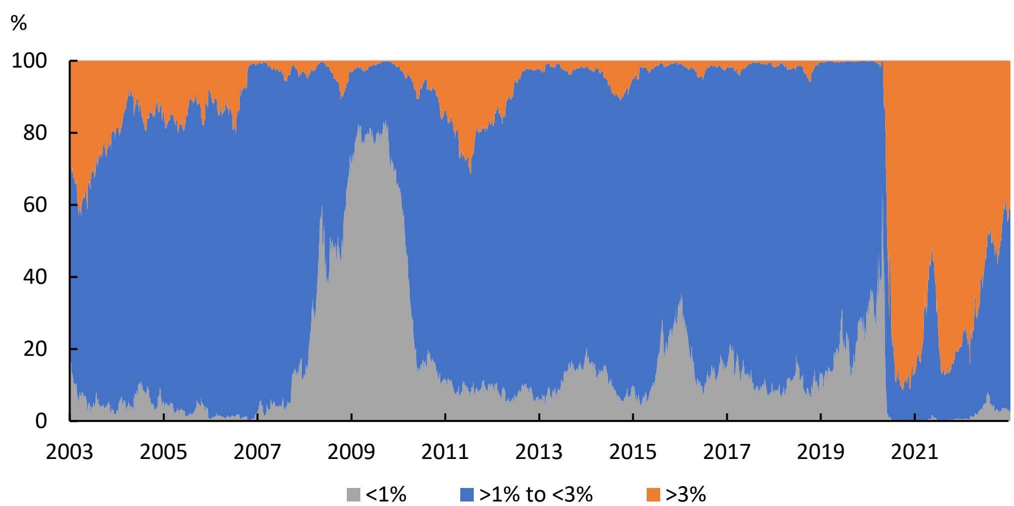
Therefore, while there appear to be some benefits to separating the impact of positive and negative news, they are not significant, especially at the longer horizon. Given the large cost (i.e., over-parameterization) associated with adding asymmetry, we revert to the original specification for the rest of the paper.

4.3. Inflation- and GDP-at-risk across time

Having estimated the conditional moments for inflation and GDP growth, we can generate their conditional density functions using the SGED function. The conditional tail risk, often referred to as inflation- or GDP-at-risk, is simply the probability mass below or above a specific threshold in its conditional density. For example, setting the threshold at 0% for GDP-at-risk provides the probability of a recession (or no recession). Since our conditional moments are at a daily frequency, our conditional tail risk measures are as well. This is a major upgrade to existing measures in the literature, which are typically at a quarterly frequency. A daily frequency measure of tail risk can be extremely useful to policy-makers, especially during uncertain or rapidly changing macroeconomic conditions.

The charts below show the in-sample estimate of conditional tail risks for inflation and GDP growth. They capture key events in the sample period, providing us with a good degree of confidence in our results. For example, **Chart 13** shows the estimated probability of inflation falling within the target range of 1% to 3% over the next 12 months. The probability sits well above 50% for most of the sample period, which is consistent with Canada's strong track record with the 2% inflation target before the pandemic. The probability drops significantly around the time of the GFC due to the heightened risk of deflation, but its decline following the start of the pandemic is due to the increased risk of high inflation. Toward the end of the sample period, the probability is on the rise again as high inflation risk starts to moderate. Our model forecasts an equal chance of inflation falling inside the target range in early 2024, which aligns with the July 2023 *Monetary Policy Report*'s projection of inflation staying around 3% for the next year and returning to 2% by the middle of 2025.

Chart 13: Probability of inflation outcomes in 12 months



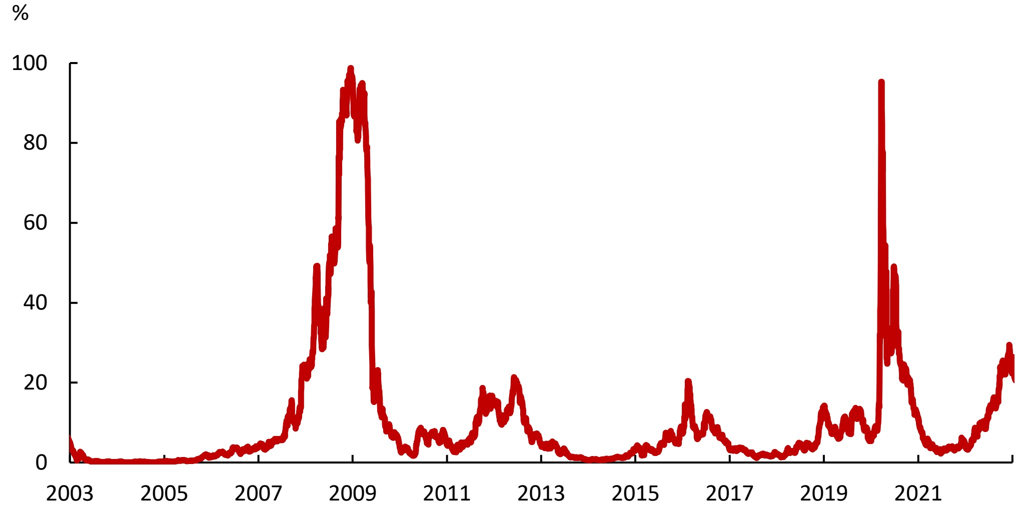
Sources: Bank of Canada calculations

Last observation: December 31, 2022

Chart 14 shows the probability of a recession in y-o-y terms over the next 12 months. As expected, recession probabilities increased significantly during the GFC and the pandemic. Conversely, the probabilities increased only marginally during the euro debt crisis around 2012 and the oil price shock in 2015. This is consistent with the C.D. Howe Institute Business Cycle Council findings that Canada did not enter a recession during those events due to their short duration and limited impact.[6] The recession probability is on the rise toward the end of the sample period as higher interest rates work their way through the economy to cool inflation by dampening excess demand.

[6] C.D. Howe Institute, "Evidence Mounts that 2015 Downturn was no Recession," Report of the C.D. Howe Institute Business Cycle Council (December 21, 2016).

Chart 14: Probability of a recession in 12 months



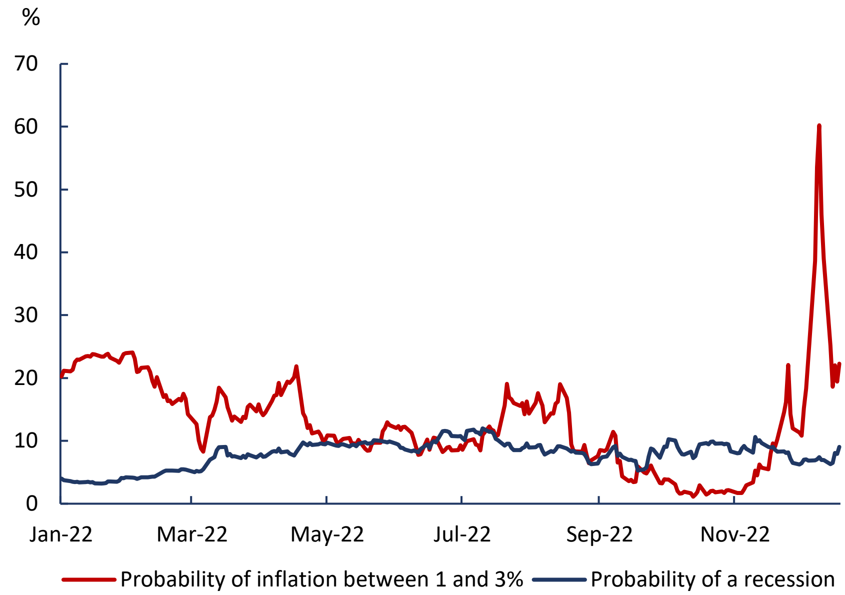
Source: Bank of Canada calculations

Last observation: December 31, 2022

Our analysis extends to examining the evolution of tail risks for specific dates, such as the end of a given year. As mentioned earlier, the flexible specification of our model allows us to link moments across different forecasting horizons and share a common set of parameters. **Chart 15** shows the change in the estimated probabilities of inflation and a recession at the end of 2022 over the course of the year. Initially, the model estimated a probability of around 20% for inflation falling back to the target range by the end of 2022. However, as the year progressed, this probability gradually declined to 0, reflecting persistent inflation above 3%. These findings provide valuable insights that differ from fixed-horizon forecasts, which indicated a steady increase in the probability throughout the year. Similarly, the y-o-y recession probability for the end of 2022 remained consistently low, hovering around 10%. This is because the Canadian economy continued to grow at a respectable pace. Again, this observation contrasts with fixed-horizon forecasts, which displayed a rising trend toward the end of the year.

While fixed-date forecasts can offer valuable information for policy-makers, their results at shorter horizons become more sensitive to financial market movements and thus must be interpreted with caution. For example, the probability of inflation falling within the target range temporarily spikes up to 60% in the final days of 2022 when risk assets were sold off on expectations of tighter monetary policy. Although it is statistically implausible for y-o-y inflation to drop from near 7% to below 3% and then back in a matter of weeks, it is natural that shorter-horizon models become more sensitive to financial variables, which are forward-looking. Therefore, it is important to recognize some inherent volatility leading up to the fixed date and exercise caution in attributing any significance to such fluctuations.

Chart 15: Evolution of estimated probabilities of inflation and a recession at the end of 2022



Source: Bank of Canada calculations

Last observation: December 31, 2022

4.4. Joint distribution and copula

Up to this point, we have focused our analysis on the marginal distributions of inflation and GDP growth. We take a step further and estimate their joint distribution to evaluate the risk of joint events, such as stagflation (i.e., slow growth and high inflation). If the two distributions are fully independent of each other, then the probability of a joint event is simply the product of the two marginal probabilities. However, it is clear from the previous figures that there is a strong dependence between inflation and GDP growth given that their conditional tail risks increase significantly around the crises. We therefore need to combine the two marginal densities with a copula. This offers a way to separate margins from the dependence structure and build more flexible multivariate distributions.

The joint distribution of inflation (π) and GDP growth (g) is:

$$f_t^{(\tau)}(\pi, g) = f_{\pi t}^{(\tau)}(\pi) f_{gt}^{(\tau)}(g) c_t^{(\tau)}(u_\pi, u_g),$$

where $f_{\pi t}^{(\tau)}(\pi)$ and $f_{gt}^{(\tau)}(g)$ are the marginal inflation and GDP growth densities, and

$u_\pi = F_{\pi t}^{(\tau)}(\pi)$ and u_g=$F_{gt}^{(\tau)}(g)$, where $F_{\pi t}^{(\tau)}(\pi)$ and $F_{gt}^{(\tau)}(g)$ are the corresponding cumulative distribution functions.

Having estimated the marginal densities and cumulative distribution functions in the previous section, the remaining task is to estimate the copula $c_t^{(\tau)}(u_\pi, u_g)$. To do this, we assume that the copula has two parameters that measure the strength of the dependence between the two variables in the joint lower or joint upper tails of their support (i.e., quantile dependence). This dependence is defined as:

$$\lambda^q = \begin{cases} Pr(U_\pi \leq q | U_g \leq q), & 0 < q \leq 1/2 \\ Pr(U_\pi > q | U_g > q), & 1/2 < q < 1 \end{cases}$$

An empirical estimate of this dependence can be assessed easily from the inflation and GDP growth data:

$$\hat{\lambda}^q = \begin{cases} \frac{1}{Tq}\sum_{t=1}^{T} 1_{\{U_{\pi t} \leq q,\ U_{gt} \leq q\}}, & 0 < q \leq 1/2 \\ \frac{1}{Tq}\sum_{t=1}^{T} 1_{\{U_{\pi t} > q,\ U_{gt} > q\}}, & 1/2 < q < 1 \end{cases}$$

Quantile dependence provides a richer description of the dependence structure of the two variables. By estimating the strength of the dependence between the two variables as we move from the centre ($q = 1/2$) to the tails, and by comparing the left tail ($q \leq 1/2$) with the right tail ($q > 1/2$), we are provided with more detailed information about the dependence structure than can be provided by a scalar measure such as linear correlation or rank correlation. Information on the importance of asymmetric dependence is useful because many copula models, such as the Normal and the Student's t, impose symmetric dependence.

Tail dependence is a measure of the dependence between extreme events, and population tail dependence can be obtained as the limit of population quantile dependence approaches $q \to 0$ or $q \to 1$:

$$\begin{cases} \lambda^L = \lim_{q \to 0} \lambda^q \\ \lambda^U = \lim_{q \to 1} \lambda^q \end{cases}$$

To account for time variations in λ^L and λ^U, we follow the same model structure used to account for variations in the four moments of the marginal densities, that is:

$$\hat{\lambda}_t^{L(\tau_0)} = \lambda_t^{L*} + \tilde{\lambda}_t^{L(\tau_0)};\ \tilde{\lambda}_t^{L(\tau_0)} = \beta_{L,n}\tilde{\lambda}_{t-1}^{L(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^L;\ \lambda_t^{L*} = \ \omega_L + \beta_{L,e}\lambda_{t-1}^{L*} + \mathrm{E}_t{}'\gamma_e^L$$

$$\hat{\lambda}_t^{L(\tau)} = \lambda_t^{L*} + \beta_{L,n}^{\tau-\tau_0}(\hat{\lambda}_t^{L(\tau_0)} - \lambda_t^{L*}), \text{ where } \hat{\lambda}_t^{L(\tau)} = -ln\left(\frac{1}{\lambda_t^{L(\tau)}} - 1\right)$$

$$\hat{\lambda}_t^{U(\tau_0)} = \lambda_t^{U*} + \tilde{\lambda}_t^{U(\tau_0)};\ \tilde{\lambda}_t^{U(\tau_0)} = \beta_{U,n}\tilde{\lambda}_{t-1}^{U(\tau_0)} + \ News_t{}'\gamma_{n,\tau_0}^U;\ \lambda_t^{U*} = \ \omega_U + \beta_{U,e}\lambda_{t-1}^{U*} + \mathrm{E}_t{}'\gamma_e^U$$

$$\hat{\lambda}_t^{U(\tau)} = \lambda_t^{U*} + \beta_{U,n}^{\tau-\tau_0}(\hat{\lambda}_t^{U(\tau_0)} - \lambda_t^{U*}), \text{ where } \hat{\lambda}_t^{U(\tau)} = -ln\left(\frac{1}{\lambda_t^{U(\tau)}} - 1\right)$$

Like marginal moments, we estimate the tail dependence by fitting the wedge between the model quantities $\lambda_t^{L(\tau)}$, $\lambda_t^{U(\tau)}$ and realized quantities $R\lambda_{t+\tau}^L$ and $R\lambda_{t+\tau}^U$:

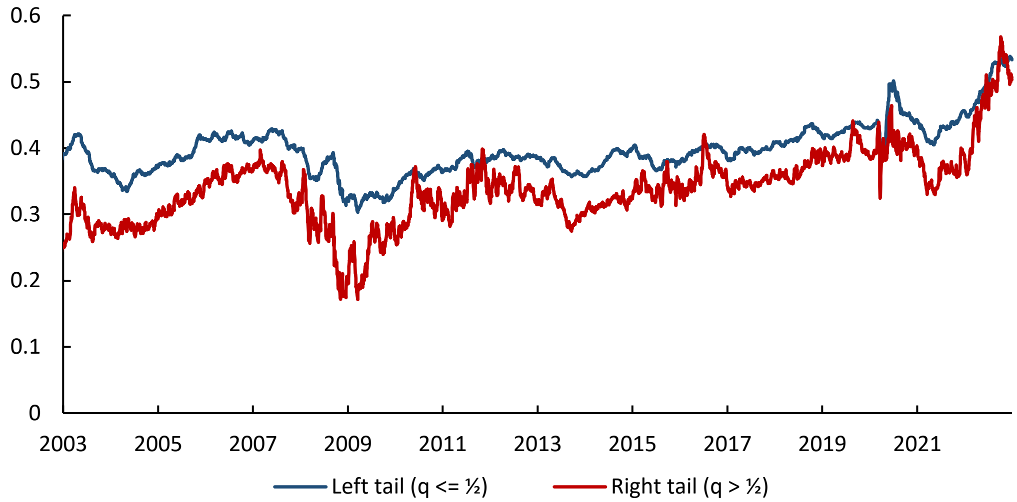
$$\begin{cases} R\lambda_t^L = \dfrac{1}{Tq}\displaystyle\sum_{s=t-T_L+1}^{t} 1_{\{U_{\pi s}\leq q,\ U_{gs}\leq q\}}, & 0 < q \leq 1/2 \\ R\lambda_t^U = \dfrac{1}{Tq}\displaystyle\sum_{s=t-T_L+1}^{t} 1_{\{U_{\pi s}> q,\ U_{gs}> q\}}, & 1/2 < q < 1 \end{cases}$$

$$\begin{aligned}\{\hat{\beta}_{L,n}; \hat{\gamma}_{n,\tau_0}^L; \widehat{\omega}_L; \hat{\beta}_{L,e}; \hat{\gamma}_e^L\} \\ = arg \min_{\beta_{L,n};\gamma_{n,\tau_0}^L;\omega_L;\beta_{L,e};\gamma_e^L} \left[\sum_{j=0}^{J-1}\left\{\sum_{t=1}^{T-\tau_j}\left(R\lambda_{t+\tau_j}^L - \lambda_t^{L(\tau_j)}\right)^2\right\}\right] \\ + \sum_{t=1}^{T}\left(\mathrm{E}_t{}'\gamma_e^L - \left((1-\beta_{L,e})\lambda_{t-1}^{L*} - \omega_L\right)\right)^2\end{aligned}$$

$$\begin{aligned}\{\hat{\beta}_{U,n}; \hat{\gamma}_{n,\tau_0}^U; \widehat{\omega}_U; \hat{\beta}_{U,e}; \hat{\gamma}_e^U\} \\ = arg \min_{\beta_{U,n};\gamma_{n,\tau_0}^U;\omega_U;\beta_{U,e};\gamma_e^U} \left[\sum_{j=0}^{J-1}\left\{\sum_{t=1}^{T-\tau_j}\left(R\lambda_{t+\tau_j}^U - \lambda_t^{U(\tau_j)}\right)^2\right\}\right] \\ + \sum_{t=1}^{T}\left(\mathrm{E}_t{}'\gamma_e^U - \left((1-\beta_{U,e})\lambda_{t-1}^{U*} - \omega_U\right)\right)^2\end{aligned}$$

Chart 16 shows the rolling estimates of upper and lower tail dependence. Both estimates are statistically different from zero and have increased in value over time. This validates our conjecture that there is a strong dependence structure between inflation and GDP growth, and thus it must be accounted for when estimating their joint distribution. It is also worth noting that the lower tail dependence (i.e., low inflation and low growth risk) is greater than the upper tail dependence (i.e., high inflation and high growth risk) throughout the sample period. This is consistent with the previously reported marginal probability results, which showed that downside risks to inflation typically increased when downside risks to growth also increased.

Chart 16: Rolling estimates of tail dependence



Source: Bank of Canada calculations

Last observation: December 31, 2022

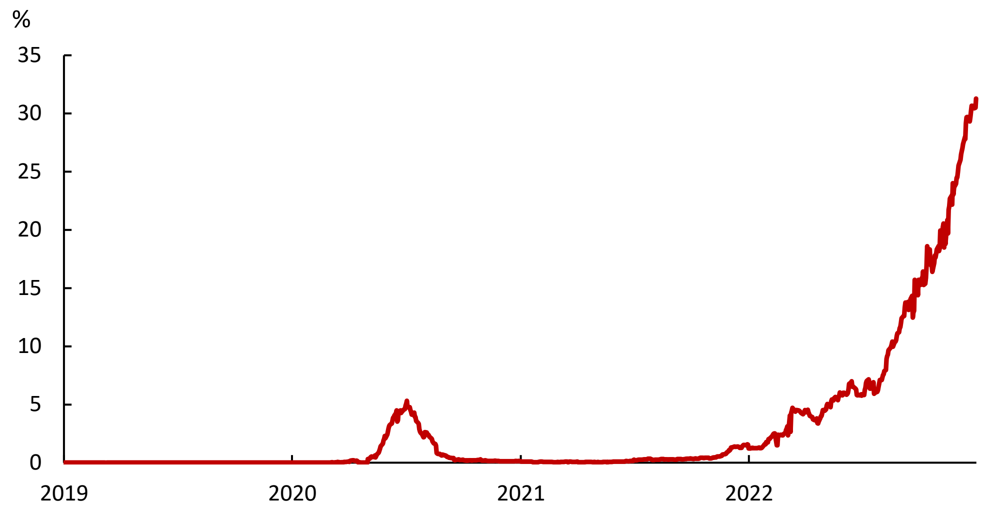
Next, we use the Joe-Clayton copula function (also known as the BB7 copula) to move from tail dependence measures $\lambda_t^{L(\tau)}$ and $\lambda_t^{U(\tau)}$ to copula density $c_t^{(\tau)}(u_\pi, u_g)$:

$$c_t^{(\tau)}(u_\pi, u_g) = \left[c_1(u_\pi)c_1(u_g)\right]^{-1-\breve{\lambda}_t^{L(\tau)}} c_2(u_\pi)c_2(u_g) h_1^{-\frac{2\left(1+\breve{\lambda}_t^{L(\tau)}\right)}{\breve{\lambda}_t^{L(\tau)}}} \left(1 - h_1^{-1/\breve{\lambda}_t^{L(\tau)}}\right)^{1/\breve{\lambda}_t^{U(\tau)}-2}$$
$$\times \left[\left(1+\breve{\lambda}_t^{L(\tau)}\right)\breve{\lambda}_t^{U(\tau)} h_1^{1/\breve{\lambda}_t^{L(\tau)}} - \breve{\lambda}_t^{L(\tau)}\breve{\lambda}_t^{U(\tau)} - 1\right],$$

where $c_1(u) = 1 - (1-u)^{\breve{\lambda}_t^{U(\tau)}}$, $c_2(u)$=$(1-u)^{\breve{\lambda}_t^{U(\tau)}-1}$, $h_1 = c_1(u_\pi)^{-\breve{\lambda}_t^{L(\tau)}} + c_1(u_g)^{-\breve{\lambda}_t^{L(\tau)}} - 1$, $\breve{\lambda}_t^{L(\tau)} = -\ln(2)/ln\left(\lambda_t^{L(\tau)}\right)$ and $\breve{\lambda}_t^{U(\tau)} = \ln(2)/ln\left(2-\lambda_t^{U(\tau)}\right)$.

Finally, to estimate the joint distributions of inflation and GDP growth, we multiply the two marginal and copula densities. This allows us to assess the risk of joint events, such as stagflation risk, which we define as negative y-o-y GDP growth and above 3% y-o-y inflation. **Chart 17** shows that the probability of stagflation risk was negligible until 2022. But the probability of stagflation risk rose significantly in 2022 as inflation surged to a multi-decade high of 8.1% in June 2022 and the recession risk increased due to the rapid tightening of monetary policy.

Chart 17: Probability of stagflation



Source: Bank of Canada calculations

Last observation: December 31, 2022

5. Conclusion

In this paper, we present an econometric approach to model the full distribution of future inflation and real GDP growth for the Canadian economy at a daily frequency. The estimated tail risk probabilities derived from the conditional distributions accurately captured key risk events during the sample period from 2002 to 2022. Our methodology offers high-frequency forecasts with flexible forecasting horizons, making it highly useful for policy-makers and market participants alike. In an environment of elevated uncertainty surrounding the inflation and growth outlook, our approach may be a valuable addition to a monitoring tool kit.

Although we have achieved promising results in this paper, our journey is far from finished. We plan to test the performance of our model on out-of-sample data, which were excluded from this paper given the relatively short sample period. Then, we will make any necessary adjustments to further enhance its accuracy. Furthermore, an avenue for future research is to expand our approach to other economies beyond Canada.

References

Adams, P., T. Adrian, N. Boyarchenko, D. Giannone, N. Liang and E. Qian. 2020. "What Do Financial Conditions Tell Us about Risks to GDP Growth?" *Liberty Street Economics.* Federal Reserve Bank of New York, May 21.

Adrian, T., N. Boyarchenko and D. Giannone. 2019. "Vulnerable Growth." *American Economic Review* 109 (4): 1263–1289.

Ang, A., G. Bekaert and M. Wei. 2007. "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?" *Journal of Monetary Economics* 54 (4): 1163–1212.

Bedock, N. and D. Stevanović. 2017. "An Empirical Study of Credit Shock Transmission in a Small Open Economy." *Canadian Journal of Economics/Revue canadienne d'économique* 50 (2): 541–570.

Bernanke, B., M. Gertler and S. Gilchrist. 1996. "The Financial Accelerator and the Flight to Quality." *Review of Economics and Statistics* 78 (1): 1–15.

Boyarchenko, N., R. Crump, L. Elias and I. L. Gaffney. 2023 "What Is "Outlook-at-Risk?" *Liberty Street Economics.* Federal Reserve Bank of New York, February 15.

Crow, E. L. and M. Siddiqui. 1967. "Robust Estimation of Location." *Journal of the American Statistical Association* 62 (318): 353–389.

Duprey, T. 2020. "Canadian Financial Stress and Macroeconomic Condition." *Canadian Public Policy* 46 (S3): S236–S260.

Estrella, A. and F. S. Mishkin. 1998. "Predicting U.S. Recessions: Financial Variables as Leading Indicators." *Review of Economics and Statistics* 80 (1): 45–61.

Feunou, B., J. Kyeong and R. Leiderman. 2018. "Markets Look Beyond the Headline." Bank of Canada Staff Analytical Note No. 2018-37.

Figueres, J. M. and M. Jarociński. 2020. "Vulnerable Growth in the Euro Area: Measuring the Financial Conditions." *Economics Letters* 191: 109126.

Gambetti, L., N. Maffei-Faccioli and S. Zoi. 2023. "Bad News, Good News: Coverage and Response Asymmetries." Federal Reserve Board Finance and Economics Discussion Series No. 2023-001.

Gilchrist, S. and E. Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review* 102 (4): 1692–1720.

Hogg, R.V. 1972. "More Light on the Kurtosis and Related Statistics." *Journal of the American Statistical Association* 67 (338): 422–424.

Holló, D., M. Kremer and M. Lo Duca. 2012. "CISS – A Composite Indicator of Systemic Stress in the Financial System." European Central Bank Working Paper Series No 1426.

Lopez-Salido, D. and F. Loria. 2020. "Inflation at Risk." FEDS Working Paper No. 2020-013.

Moors, J. J. A. 1988. “A Quantile Alternative to Kurtosis.” *Journal of the Royal Statistical Society. Series D (The Statistician)* 37 (1): 25–32.

Mueller, P. 2009. “Credit Spreads and Real Activity.” EFA 2008 Athens Meetings Paper.

Nelson, D. B. 1991. “Conditional Heteroskedasticity in Asset Returns: A New Approach.” *Econometrica* 59 (2): 347–370.

Rudebusch, G. D. and J. C. Williams. 2009. “Forecasting Recessions: The Puzzle of the Enduring Power of the Yield Curve.” *Journal of Business and Economic Statistics* 27 (4): 492–503

Soroka, S. N. 2006. “Good News and Bad News: Asymmetric Responses to Economic Information. *Journal of Politics* 68 (2): 372–385.

Stock, J. H. and M. W. Watson. 2003. “Has the Business Cycle Changed and Why?” In *NBER Macroeconomics Annual: 2002*, edited by M. Gertler and K. Rogoff, 159-230. Cambridge, MA: MIT Press.