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The Canadian Neutral Rate of Interest through the Lens of an Overlapping-Generations Model

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Abstract

The neutral rate of interest is an important concept and communication tool for central banks. We develop a small open economy model with overlapping generations to study the determinants of the neutral real rate of interest in a small open economy. The model captures domestic factors such as population aging, declining productivity, rising government debt and inequality. Foreign factors are captured by changes in the global neutral real rate. We use the model to evaluate secular dynamics of the neutral rate in Canada from 1980 to 2018. We find that changes in both foreign and domestic factors resulted in a protracted decline in the neutral rate.

Topics: Economic models; Interest rates; Monetary policy
JEL codes: E21, E22, E43, E50, E52, E58, F41

Résumé

Le taux d'intérêt neutre est un concept et un outil de communication important pour les banques centrales. Nous élaborons un modèle de petite économie ouverte à générations imbriquées pour étudier les déterminants du taux d'intérêt neutre dans une telle économie. Le modèle prend en compte de facteurs intérieurs comme le vieillissement de la population, la baisse de la productivité, l'accroissement de la dette publique et les inégalités. Il intègre également les variations du taux neutre réel mondial, qui englobe des facteurs étrangers. Nous utilisons le modèle pour évaluer la dynamique à long terme du taux d'intérêt neutre au Canada de 1980 à 2018. Nous constatons que les variations des facteurs tant étrangers qu'intérieurs ont entraîné une diminution prolongée de ce taux.

Sujets : Modèles économiques, Taux d'intérêt, Politique monétaire
Codes JEL : *E21, E22, E43, E50, E52, E58, F41*

1 Introduction

The neutral rate of interest is a prominent concept that informs monetary policy decision-making and communications to the public. It is often defined as the policy interest rate needed to maintain economic output at its potential level and inflation at target in the long run, after the effects of cyclical shocks have dissipated.[1] Bank of Canada staff adopted this definition in the pioneering analysis by Mendes 2014 and currently update their estimate of the neutral rate each year (see Faucher et al. [2022] for the 2022 assessment). The sign and size of the gap between the policy rate and the neutral rate can be used to assess the stance of monetary policy to the extent that it shows how monetary policy is offsetting temporary cyclical shocks. The output gap—which is driven by both temporary cyclical shocks and monetary policy—does not convey such information.

This long-run concept of the neutral rate depends on slow-moving domestic factors that are commonly thought to be outside the control of central banks. These factors include demographic trends, the rate of technological progress and secular shifts in the levels of public debt and inequality. In small open economies like Canada, similar factors around the world also affect the neutral rate through changes in the global neutral rate. We provide a tractable framework that characterizes the relationship between the neutral real rate of interest and its key factors.[2] We then study how the Canadian neutral rate changed between 1980 and 2018.

We build on a structural general equilibrium model of a small open economy with overlapping generations (OLG). Three generations of households borrow and save using safe assets. Households also own capital stock and rent it to firms that combine it with labour to produce consumption and investment goods. Each generation includes two types of households with ex-ante different incomes. At the end of life, old households leave bequests to other generations. The model also includes the government that funds public consumption and pension transfers by levying taxes and issuing public debt. We extend the OLG model in Dorich, Mavalwalla and Mendes (2014), which was previously used to assess the neutral rate in Canada.

As in Dorich, Mavalwalla and Mendes (2014), both domestic and foreign factors drive the Canadian neutral rate. In particular, the domestic neutral rate is specified as a function of the global neutral rate—which captures all the foreign factors—and an endogenous country-

[1] A related concept is the natural rate of interest: a real interest rate that would prevail in the short run if prices and wages were flexible (Woodford 2003). The natural rate fluctuates in response to temporary shocks and, depending on the shock, may or may not represent a benchmark for setting the policy rate.

[2] Throughout this paper, we use real interest rates. The neutral real rate can easily be reflated to obtain the nominal counterpart using the inflation target.

specific premium. The country-specific premium monotonically increases in the long-run level of external indebtedness relative to gross domestic product (GDP), which depends on the composition of demand and supply of domestic assets. Our model accounts for additional factors behind these demand and supply schedules, such as population longevity, income inequality and public borrowing. As we discuss below, these factors have been identified as prominent drivers of the neutral rates in other advanced economies.

Bringing the model to the data, we find a secular decline in the Canadian neutral rate from 1980–2018. Importantly, similar downward trends in the global and Canadian neutral rates do not simply result from the Canadian neutral rate being driven primarily by foreign factors. Our results imply that both foreign and domestic factors contributed significantly to the decline in the Canadian neutral rate. Hence, the comovement in the two neutral rates results from shared trends across the foreign and domestic factors. The most important domestic factors are those related to demographics: slower growth in trend labour input and longer longevity. Slower growth in trend labour productivity also significantly contributed to the reduction in the neutral rate. In contrast, higher government debt partially offsets the decline in the neutral rate, but slightly higher income inequality had only a negligible effect.

Our work pertains to the growing literature that examines the long-run evolution of interest rates.[3] The OLG structure in our model shares key features with the models that were used to explain a sustained decline in the neutral rate (Coeurdacier, Guibaud and Jin 2015) and a contemporaneous increase of the risk premium (Marx, Mojon and Velde 2021). More broadly, the qualitative effects of individual factors in our analysis are consistent with the following papers that emphasize the role of specific factors in driving the neutral rate. First, there is a long tradition of attributing changes in the neutral rate to demographic factors, see Carvalho, Ferrero and Nechio (2016) and Auclert et al. (2021) for recent contributions. Second, the role of fiscal policy—and government debt dynamics in particular—has been stressed by Rachel and Summers (2019). Finally, Mian, Straub and Sufi (2021) have argued that income inequality is another important factor behind the neutral rate.

Most of this literature focuses on the evolution of the global neutral rate or the neutral rate in a given large closed economy (e.g., the United States) independently from the rest of the world. Instead, we offer analysis from the perspective of a small open economy where the domestic neutral rate depends on the global neutral rate while possibly being different from it due to domestic factors. Our contributions to the literature are twofold. First, we develop a tractable framework for estimating the neutral rate in a small open economy. Second, our

[3] Cacciatore and Ozhan (forthcoming) provide a comprehensive thematic review of the literature on the neutral rate, including a discussion of alternative approaches for estimating the neutral rate.

results for Canada contribute to a discussion of the quantitative importance of various factors for the declining neutral rates in small open economies. Similarly to Carvalho, Ferrero and Nechio (2016) and Auclert et al. (2021), we find that demographic factors in Canada were among the most important reasons for the decline in the neutral rate. But we do not find a sizable effect of inequality in Canada on the neutral rate unlike the studies focusing on the United States (Mian, Straub and Sufi 2021).

The remainder of this paper is organized as follows. Section 2 introduces the OLG model. Section 3 characterizes the neutral rate and discusses channels through which various factors affect the neutral rate. Section 4 discusses the calibration of the model and shows the results of the quantitative analysis that identifies the secular decline in neutral rate over 1980–2018 and quantifies the contributions of different factors to this decline. Section 5 concludes.

2 The model

We consider a small open economy model populated by overlapping generations of households. The production sector of the economy consists of a continuum of firms that produce goods using capital and labour. The produced goods are homogeneous and can be used for consumption, investment and international trade in a perfectly competitive international market. There is no home bias in consumption, and the purchasing power parity holds, so the real exchange rate and the terms of trade always equal one. The government buys goods for public consumption and makes transfers to households. Government spending is financed using a mix of taxes and public debt. The government trades in risk-free bonds with households and with foreign investors in an international asset market. The model is deterministic, with no aggregate or idiosyncratic risk.[4] The remainder of this section describes the model in detail.

2.1 Households

Demographics. At every period of time, a new generation of households is born. Households of each generation live for two or three consecutive periods. In the first period, households are young (y), in the second period they are middle-aged (m), and in the third period they are old (o). The size of the generation born in period t is $N_t = g_{L,t} N_{t-1}$, where

[4] Effects of aggregate economic risk on the Canadian neutral rate are assessed using the risk-augmented neoclassical growth model introduced in Carter, Chen and Dorich (2019) by building on Farhi and Gourio (2018).

g_L is an exogenously given growth rate. The probability of middle-aged households born in period t surviving into old age is s_{t+1}. This overlapping generations setup allows us to model the effects of demographic factors—particularly declining fertility (lower g_L) and increasing longevity (higher s)—on the neutral rate.

Inequality. To estimate the effects of higher income inequality on the neutral rate, the model features additional heterogeneity among households. Specifically, each generation has two types of households, $i \in \{l, h\}$, that have ex-ante different life-cycle income profiles due to differences in individual labour productivity. Young households of both types have the same income, which is lower than that of middle-aged households. Young households also know with certainty that type-h households will receive a high income when reaching middle age, whereas type-l will receive a low income. Differences in income stem from differences in individual labour productivity that satisfy the following conditions $e_t^{y,l} = e_t^{y,h} < e_t^{m,l} < 1 < e_t^{m,h}$, where superscripts denote age and type. Moreover, the average productivity of middle-aged households is normalized to one: $\sum_i \mu_i e_t^{m,i} = 1$, where μ_i is the constant share of type-i households in every generation.

Income. Young and middle-aged households earn wage w_t by inelastically supplying one unit of effective labour $e_t^{j,i}$ for $i \in \{l, h\}$ and $j \in \{y, m\}$. Labour income is taxed at the rate τ_t. All households receive wage-indexed type- and age-specific government transfers, $\text{tr}_t^{j,i}$ for $i \in \{l, h\}$ and $j \in \{y, m\}$, that capture social insurance and pension. We define the total labour income after tax and transfer of the type-i household of age $j \in \{y, m\}$ as $I_t^{j,i} \equiv w_t\Big[(1-\tau_t)e_t^{j,i} + \text{tr}_t^{j,i}\Big]$.

Utility. Preferences of type-i households from generation t take the following form:

$$u(c_t^{y,i}) + \beta u(c_{t+1}^{m,i}) + s_{t+1}\beta^2 \left[u(c_{t+2}^{o,i}) + \kappa\nu(a_{t+3}^{o,i}, \Delta_{t+2})\right], \tag{2.1}$$

where c denotes individual consumption in three consecutive periods, superscripts denote the age and the productivity type. The last term in (2.1) reflects utility from bequests a_{t+3} left at the end of life, with κ being a scalar and Δ_{t+2} being a growth-scaling variable that makes the savings rate of the old households stationary. In practice, we use aggregate economic output Y_{t+2} as the growth-scaling variable, $\Delta_{t+2} = Y_{t+2}$. Such utility specification implies that old households value their bequest savings relative to the size of the economy. We assume that the functional form of the utility from consumption is $u(c) = \frac{(c)^{1-\sigma}}{1-\sigma}$ and the utility function from bequests is $\nu(a,\Delta) = \frac{(a)^{1-\varepsilon}(\Delta)^{\varepsilon-\sigma}}{1-\varepsilon}$, where σ is the inverse elasticity of intertemporal substitution of consumption and ε is bequest-specific elasticity.

Budget. Young households use their income, received bequests and funds borrowed ($b_{t+1}^{y,i} > 0$) against their future income to buy consumption goods. Middle-aged households use their income to repay debt, buy consumption goods, save for retirement ($b_{t+1}^{m,i} < 0$) and buy capital ($k_{t+1}^{m,i} > 0$). Old households use all of their available resources to buy consumption goods and leave bequests ($a_{t+1}^{o,i} > 0$).[5] Budget constraints of the households in period t read as follows:

$$c_t^{y,i} = I_t^{y,i} + b_{t+1}^{y,i} + (s_{t-2}N_{t-3}/N_t)R_t a_{t-1}^{o,i}, \tag{2.2}$$
$$c_t^{m,i} = I_t^{m,i} + b_{t+1}^{m,i} - R_t b_t^{y,i} - p_t^k k_{t+1}^{m,i}, \tag{2.3}$$
$$c_t^{o,i} = \Big(p_t^k(1-\delta) + r_t^k\Big)k_t^{m,i} - R_t b_t^{m,i} + \mathrm{tr}_t^{o,i} w_t - a_{t+1}^{o,i} + T_t^{s,i}, \tag{2.4}$$

where R_t is the gross real interest rate on debt, p_t^k is the relative price of investment goods, r_t^k and δ are the rental and depreciation rates of capital. The relative price of investment goods is exogenous and changes at the deterministic rate $g_{I,t} \equiv p_t^k/p_{t-1}^k$. Also, note that old households of type-i receive lump-sum transfers that proportionally redistribute assets of the deceased type-i households from the same generation: $T_t^{s,i} = (1-s_{t-1})/s_{t-1}\Big[\Big(p_t^k(1-\delta) + r_t^k\Big)k_t^{m,i} - R_t b_t^{m,i}\Big]$.

We also assume that young households are subject to credit constraints that restrict their gross debt to be no more than a fraction θ_t of the next-period income:

$$R_{t+1}b_{t+1}^{y,i} \leq \theta_t w_{t+1}\Big[(1-\tau_t)e_{t+1}^{m,i} + \mathrm{tr}_{t+1}^{m,i}\Big]. \tag{2.5}$$

In what follows, we analyze the case in which (2.5) is binding for $i \in \{l,h\}$.[6]

Decisions. Households of every generation maximize utility (2.1) by choosing purchases of consumption and capital, as well as borrowing and lending levels, subject to the constraints (2.2)–(2.5) at the relevant periods of time. The optimality conditions corresponding to the

[5] We assume the borrowing and lending positions of different households are outcomes held in equilibrium.
[6] As in Coeurdacier, Guibaud and Jin (2015), the necessary condition for (2.5) to be binding is a tight enough borrowing constraint and steep enough lifetime income profiles for both types of households.

decisions made by a middle-aged household born in period $t-1$ are as follows:

$$\left(c_t^{m,i}\right)^{-\sigma} = R_{t+1} s_t \beta \left(c_{t+1}^{o,i}\right)^{-\sigma}, \tag{2.6}$$

$$R_{t+1} = \left[r_{t+1}^k + p_{t+1}^k(1-\delta)\right]/p_t^k, \tag{2.7}$$

where (2.6) is the middle-age consumption-savings equation, and (2.7) is the no-arbitrage condition that equates total return on capital investment and the interest rate R_{t+1}.

The optimality condition that characterizes the same household's decisions in their old age is as follows:

$$a_{t+2}^{o,i} = \left[\kappa\eta\left(\frac{a_{t+2}^{o,i}}{\Delta_{t+1}}\right)\right]^{\frac{1}{\sigma}} c_{t+1}^{o,i}, \tag{2.8}$$

where $\eta(\cdot) = \nu_a(\cdot)/u_c(\cdot) = (\cdot)^{\sigma-\varepsilon}$ captures the marginal utility trade-off between bequest savings and consumption, similarly to Mian, Straub and Sufi (2021). If $\varepsilon = \sigma$, then $\eta(\cdot) = 1$ and bequests are proportional to consumption. In this case, savings rates are independent of income level. The distribution of income across households therefore does not affect the aggregate savings rate. When $\sigma > \varepsilon$, then $\eta(\cdot)$ is an increasing function, implying that bequests become relatively more valuable as they grow. As a result, households with higher incomes allocate a larger share of their income to savings. In turn, the aggregate savings rate is higher when income is more concentrated at the top of the income distribution.

2.2 Firms

A continuum of identical firms produce goods and sell them in a perfectly competitive market.[7] A representative firm uses two factors as inputs for production: labour supplied by the young and middle-aged households and capital rented out by old households and foreign investors. Labour, L_t, and capital, K_t, are turned into aggregate output, Y_t, using the following Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}\left[A_t L_t\right]^{1-\alpha}, \tag{2.9}$$

where $\alpha \in (0,1)$ is the output elasticity of capital, and A_t is the trend of labour-augmenting technological progress that grows at the exogenous growth rate $g_{A,t} \equiv A_t/A_{t-1}$. The pace of

[7] Extension to monopolistic competition in the goods market is discussed in Marx, Mojon and Velde (2021), who show that markups do not affect the neutral rate. A similar result holds in Farhi and Gourio (2018).

technological progress also affects the neutral rate.

Firms maximize profits—revenue from selling output minus wage and capital rental bills—by choosing the amount of capital and labour inputs. Capital and labour markets are perfectly competitive, so both inputs are acquired at the price of the corresponding marginal revenue product (and firm profits are zero):

$$w_t = (1-\alpha)A_t^{1-\alpha}k_t^{\alpha}, \tag{2.10}$$

$$r_t^k = \alpha A_t^{1-\alpha}k_t^{\alpha-1}, \tag{2.11}$$

where $k_t \equiv K_t/L_t$ is the capital-to-labour ratio.

2.3 Government

The government supplies assets by issuing public debt in an amount consistent with the fiscal policy. Specifically, the government borrows B_t^g and levies proportional income tax for labour τ_t to fund public consumption G_t and transfers tr_t. The government budget constraint reads as:

$$\begin{aligned} G_t + \Big[N_t \sum_i \mu_i \mathrm{tr}_t^{y,i} + N_{t-1} \sum_i \mu_i \mathrm{tr}_t^{m,i} + s_{t-1} N_{t-2} \sum_i \mu_i \mathrm{tr}_t^{o,i}\Big] + R_t B_t^G \\ = \tau_t w_t \Big[N_t \sum_i \mu_i e_t^{y,i} + N_{t-1} \sum_i \mu_i e_t^{m,i}\Big] + B_{t+1}^G. \end{aligned} \tag{2.12}$$

As in Auclert et al. (2021), we set taxes and transfers as given, whereas we adjust public consumption to ensure that the ratio of debt to output $\varphi_t \equiv B_{t+1}^G/Y_t$ follows a given path.

2.4 Trade balance and net foreign assets

The domestic economy trades goods and assets with the global economy. Trade balance TB_t measures the difference between exports and imports of goods. The trade balance must equal the difference between the amount of goods produced domestically and the amount of goods absorbed domestically for consumption and investment purposes:

$$TB_t = Y_t - C_t - G_t - p_t^k(K_{t+1} - (1-\delta)K_t), \tag{2.13}$$

where $C_t \equiv N_t \sum_i \mu_i c_t^{y,i} + N_{t-1} \sum_i \mu_i c_t^{m,i} + s_{t-1} N_{t-2} \sum_i \mu_i c_t^{o,i}$ is total private consumption.

On the asset side, domestic households operate with risk-free debt and capital. All households trade in risk-free bonds with each other and with the government and foreign investors in an international asset market. Middle-aged households combine new investment goods with the outstanding stock of capital acquired from old households to rent out both domestically and in the rest of the world. Since middle-aged households are the only ones that hold both bonds and capital at the end of every period, it is convenient to denote their total assets by $a_{t+1}^{m,i} \equiv p_t^k k_{t+1}^{m,i} - b_{t+1}^{m,i}$. One can then show that the trade balance is linked to the net foreign asset position, X_t, through the balance of payment:

$$TB_t = X_{t+1} - R_t X_t, \tag{2.14}$$

where the net foreign asset position at the end of period t captures the excess of savings over debt and capital used for production domestically:

$$X_{t+1} = \underbrace{\left[N_{t-1} \sum_i \mu_i a_{t+1}^{m,i} + s_{t-1} N_{t-2} \sum_i \mu_i a_{t+1}^{o,i} \right]}_{\text{Asset demand by middle-aged and old households}} - \underbrace{\left[p_t^k K_{t+1} + N_t \sum_i \mu_i b_{t+1}^{y,i} + B_{t+1}^G \right]}_{\text{Asset supply: capital + debt of young households and government}} \tag{2.15}$$

Note that in our environment without risk, households are indifferent to a choice between holding bonds and holding capital. We therefore do not determine the bilateral bond holdings across households, the government and foreign investors. Furthermore, we also do not determine how the portfolios of middle-aged households and the net foreign asset position are split between bonds and capital.

Finally, we assume that international asset flows do not adjust with infinite elasticity to equalize domestic interest rate R_t with the exogenously given global interest rate R_t^W. Instead, the model features a downward-sloping supply schedule for net foreign assets, reflecting various market imperfections and barriers to global asset allocation (such as those discussed in Pellegrino, Spolaore and Wacziarg [2022]). This feature is introduced through the following parametric form for the domestic interest rate as the function of the global interest rate and a country-specific premium increasing in the level of external indebtedness relative to domestic output:

$$R_{t+1} = R_{t+1}^W \psi_1 \exp\{-\psi_2 X_{t+1}/Y_t\}, \tag{2.16}$$

where ψ_1 is the unconditional component of the premium and $\psi_2 > 0$ is the elasticity of the premium to the net foreign asset position. It is common to assume that the domestic interest rate is increasing in the country's net foreign debt in order to close small open economy models following Schmitt-Grohe and Uribe (2003).[8] Additionally, note that net demand for foreign assets is driven by the individual components of domestic asset supply and demand represented by the right-hand-side terms in (2.15).

2.5 Equilibrium

An equilibrium is a sequence of quantities and prices in the economy described above such that, when households and firms take prices as given and solve their decision problems, markets are cleared.

Definition 1. Given exogenous growth rates of population, labour-augmenting aggregate productivity and relative price of investment $\{g_{L,t}, g_{A,t}, g_{I,t}\}_t$, global interest rates $\{R_t^W\}_t$, individual labour productivity levels $\{e_t^{y,i}, e_t^{m,i}\}_{i,t}$, life-expectancy rates $\{s_t\}_t$, tax-transfer system $\{\tau_t, tr_t^{y,i}, tr_t^{m,i}, tr_t^{o,i}\}_{i,t}$ and debt-to-GDP ratio path $\{\varphi_t\}_t$, the competitive equilibrium consists of prices $\{w_t, r_t^k, R_t\}_t$, private sector allocation $\{c_t^{y,i}, c_t^{m,i}, c_t^{o,i}, b_t^{y,i}, a_t^{m,i}, a_t^{o,i}, K_t, L_t, Y_t\}_{i,t}$, public spending and borrowing $\{G_t, B_{t+1}^G\}_t$, and international balances $\{TB_t, X_t\}_t$ such that:

1. given prices, private sector allocation solves decision problems of households and firms (i.e., optimality conditions (2.2)–(2.11) are satisfied);

2. given prices and private sector allocation, public spending and borrowing satisfy budget constraints (2.12) and imply a given debt-to-GDP ratio path φ_t;

3. market clearing conditions for labour ($L_t = e_t^y N_t + N_{t-1}$), goods (2.13) and assets (2.15)–(2.16) hold.[9]

3 Neutral rate characterization

The neutral rate R^* is defined as the real interest rate R_t that clears the asset market in the long run, when the economy converges to a steady (balanced) growth path. Before defining the neutral rate formally and discussing its main drivers, we find it insightful to analyze the

[8] As in the literature following Schmitt-Grohe and Uribe (2003), we do not derive this relation from microfoundations. Unlike the common approach in this literature, we do not impose equality between domestic and global rates at the steady state. This generalization allows us to capture persistent international asset market imperfections in a parsimonious way.

[9] Balance of payments (2.14) holds in equilibrium by Walras's law.

components of demand for net foreign assets and relate them to the income and savings rates of various household types that populate the economy.

3.1 The components of net foreign asset demand

Recall that net foreign asset demand comprises domestic asset demand and supply of households, firms and the government. In what follows, we discuss these components individually.

Domestic asset demand. Middle-aged and old households are sources of domestic asset demand. Total asset demand by the middle-aged household of type i is proportional to its total labour income after tax and transfer:

$$a_{t+1}^{m,i} = \underbrace{[1-\theta_{t-1}]\left[1+\left(R_{t+1}^{1-\sigma}s_t^{1+\sigma}\beta\right)^{-\frac{1}{\sigma}}\left(\frac{1+\phi_{t+1}^i}{1+\xi_{t+1}^i}\right)\right]^{-1}}_{\equiv\gamma_t^i} I_t^{m,i}, \tag{3.1}$$

where γ_t^i is the savings rate, ϕ_{t+1}^i is the ratio of expected public pension income to private income of old and ξ_{t+1}^i captures the bequest saving motive.[10] Intuitively, a greater probability of surviving into the old age s_t and higher preferences for bequests captured by higher ξ_{t+1}^i increase the savings rate of middle-aged households γ_t^i. In contrast, a more generous public pension system measured by a higher ratio of expected public pension income to the private income of old ϕ_{t+1}^i reduces the private savings rate of middle-aged households γ_t^i.

Asset demand by the old household of type i is proportional to its past total net income at middle age:[11]

$$a_{t+1}^{o,i} = \left[\left(\frac{1+\phi_t^i}{1+\xi_t^i}\right)\frac{\xi_t^i}{s_{t-1}}\right] R_t\gamma_{t-1}^i I_{t-1}^{m,i}. \tag{3.2}$$

Bequests of the old household increase in the realized return on the savings from middle age $R_t\gamma_{t-1}^i I_{t-1}^{m,i}$, the generosity of the public pension system measured by the ratio of expected public pension income to the private income of old households ϕ_{t+1}^i and in the bequest motive ξ_{t+1}^i.

[10] See Appendix A.1 for a proof that also defines the composite variables ϕ_{t+1}^i, and ξ_{t+1}^i.
[11] See Appendix A.1 for a proof.

Domestic asset supply. Young and old households, firms and the government are sources of domestic asset supply. First, consider young households who borrow against their future income. The supply of assets by the young household of type i is proportional to its future total net income at middle age:[12]

$$b_{t+1}^{y,i} = \left(\frac{\theta_t}{R_{t+1}} \right) I_{t+1}^{m,i}. \tag{3.3}$$

Old households supply assets by selling the depreciated stock of capital. This capital stock combined with the supply of new investment goods by firms constitutes total supply of capital. Finally, the government supplies assets by issuing debt to balance its budget.

3.2 The neutral rate and its drivers

We now use the net foreign asset demand components from the previous section to define the neutral rate and discuss its drivers. To do so, we rewrite (2.15) using (2.16) and the decomposition of net foreign asset demand to express the asset market clearing condition. This condition in the long run—equation (3.4)—implicitly defines the neutral rate of interest.

Proposition 1. The neutral rate R^* is the value of the interest rate $R = R^*$ that solves

$$\begin{aligned} \frac{1}{\psi_2} \log \left(\frac{\psi_1 R^W}{R} \right) = & \left[\frac{1-\alpha}{1+e^y g_L} \right] \Bigg(\overbrace{\left[\sum_i \mu_i \gamma^i \tilde{I}^{m,i} \right]}^{\text{Assets of middle aged}} + \overbrace{\left[\sum_i \mu_i \gamma^i \tilde{I}^{m,i} \frac{\xi^i (1+\phi^i)}{(1+\xi^i)} \right] \left[\frac{R}{g_L g_I^{\frac{\alpha}{\alpha-1}} g_A} \right]}^{\text{Assets of old}} \Bigg) \\ & - \left[\frac{1}{1+e^y g_L} \right] \left[\frac{g_L g_I^{\frac{\alpha}{\alpha-1}} g_A}{R - g_I(1-\delta)} \right] \Bigg(\underbrace{\alpha(1+e^y g_L)}_{\text{Capital}} + \underbrace{(1-\alpha) \left(1 - g_I \frac{1-\delta}{R} \right) \sum_i \mu_i \theta \tilde{I}^{m,i}}_{\text{Debt of young}} \Bigg) - \frac{B^G}{Y}, \end{aligned} \tag{3.4}$$

where $\tilde{I}^{m,i} \equiv (1-\tau) e^{m,i} + \text{tr}^{m,i}$, and the balanced-growth-path values of the variables are denoted by omitting the time subscript.[13]

We proceed with a discussion of factors that affect the neutral rate.

Equation (3.4) can be illustrated graphically by Figure 1. Net foreign asset demand on the right-hand side of (3.4) is represented by the upward-sloping curve. Net foreign asset

[12] This follows directly from the binding borrowing constraint.

[13] See Appendix A.2 for a proof. Note that ξ^i and ϕ^i depend on R. Hence, a system of equations needs to be solved.

supply on the left-hand side of (3.4) is represented by the downward-sloping curve (panel A) that turns vertical (see panel B) in the limit case where the country premium is inelastic to the net foreign asset position $\psi_2 = 0$. In this limit case, domestic economic factors that shift net foreign asset demand do not affect the neutral rate because international asset flows are infinitely elastic at $R^* = \psi_1 R^W$. Hence, foreign factors summarized by the global neutral rate are the sole drivers of the domestic neutral rate.

Next, consider our baseline case with a finite elasticity of international asset flows ($\psi_2 > 0$). Domestic economic factors that shift net foreign asset demand become relevant for determining the neutral rate of interest. The extent to which R^* is sensitive to a change in a given domestic factor depends on (i) the magnitude of this change, (ii) the sensitivity of the demand curve to this change, and (iii) the steepness of the supply curve. The latter crucially depends on the elasticity of the country premium to the net foreign asset position ψ_2. In particular, the smaller ψ_2 is, the steeper the supply curve is and, therefore, the smaller the effect of changing demand on R^* is. The remainder of this section highlights domestic economic factors that we later examine as potential factors behind the historical evolution of the neutral rate in Canada.

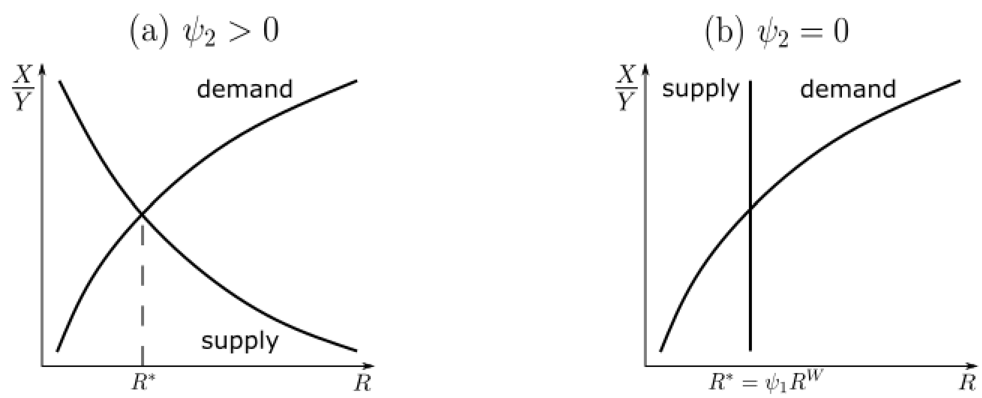


Figure 1: The neutral rate of interest (R^*) is the price that clears the asset market. Source: Proposition 1

A faster rate of technological progress g_A and an accelerated rate of growth of the demographic trend g_L (driven, for instance, by higher fertility or immigration at young age) reduce net foreign asset demand and move the associated curve in Figure 1 down through several channels. First, the relative size of young households' debt rises due to the composition effect (higher g_L increases the relative weight of the young in the population) and due to an increase in wages (induced by higher g_A). Higher wages, in turn, relax the borrowing constraint of young households. Second, faster growth of labour and labour-augmenting technological progress boost domestic supply of capital. Finally, because the same factors increase output, they reduce the ratio of bequests to output. The neutral rate would rise in response to such domestic economic changes.

Another domestic factor is longevity. A higher probability of middle-aged households surviving into old age, s, increases demand for net foreign assets and moves the associated curve up by boosting savings of the middle-aged households. Indeed, because households expect to live longer after being retired, the incentive to smooth consumption makes them increase savings in middle age when they are productive. As a result, the neutral rate would decrease in response to an increase in longevity.

Fiscal policy affects the neutral rate through several channels. In our quantitative analysis, we focus on the effect of changes in the level of government debt relative to GDP B^G/Y while adjusting government spending to ensure that government budget constraint is satisfied. A larger supply of government debt would reduce demand for net foreign assets and move the associated curve down, therefore leading to a higher neutral rate. Another effect of fiscal policy on the neutral rate could arise from increased taxation τ or higher transfers to the old households tr^o, which would reduce demand for net foreign assets because of lower after-tax income or reduced need for private retirement savings respectively. This lower demand for net foreign assets would, in turn, lead to an increase in the neutral rate.

Finally, income inequality has implications for demand for net foreign assets and the neutral rate when a bequest motive exists. Following Mian, Straub and Sufi (2021), we consider the parametric case with $\sigma > \varepsilon$, which implies that households with higher income save a relatively larger fraction of their income. In this case, greater income inequality due to a mean-preserving rise in the relative productivity ratio $e^{m,h}/e^{m,l}$ increases the demand for net foreign assets and moves the corresponding curve up. Higher income inequality would therefore lead to a lower neutral rate.

4 The secular decline of neutral rate

We use the model to analyze the decline in the Canadian neutral real rate from 1980 to 2018 and to identify the role of various domestic and global factors in this decline.[14] In this section,

[14] The reason for ending in 2018 is that most proxies for the global neutral rate are not available after 2018. The New York Federal Reserve Bank suspended the posting of regular updates of the Laubach-Williams model estimates (Laubach and Williams 2003) due to "extraordinary volatility of GDP during the pandemic" (Federal Reserve Bank of New York 2020). We use other measures from Bauer and Rudebusch (2020) whose estimates end in 2018. See Faucher et al. (2022) for more recent estimates of the neutral rate for Canada.

we describe how we bring the model to the data. First, in Subsection 4.1 we calibrate the parameters of the model that are constant over the period studied. Second, in Subsection 4.2 we quantify the evolution of foreign and domestic factors between 1980 and 2018.

4.1 Model calibration

We calibrate the model parameters to match key characteristics of the Canadian economy. We match key data moments over the calibration period from 1990 to 2009. Some parameters are calibrated following previous literature. Table 1 in Appendix B reviews the parameter calibration and the rationale.

We set the annual discount factor β to the frequent benchmark in the literature of 0.98. The inverse of the intertemporal elasticity of substitution in consumption σ is set to 1.14 as in the Bank of Canada's Terms-of-Trade Economic Model (ToTEM) (Dorich et al. 2013). The benchmark risk-free rate in our model for the calibrated period is also set to match the steady-state annual net real rate in ToTEM of 3% (Dorich et al. 2013). The world risk-free rate is then set to 2.5% to match the spread between Canadian and US yields on 10-year government bonds of 0.5% over the calibration period. Note that for the exercise in Section 4, we use various measures of the global neutral rate. We then recalibrate the benchmark domestic real risk-free rate R^* to match the average of R^{W*} and the spread between Canadian and US government bonds over the calibration period.

We use the following regression to estimate the parameters in the reduced-form relationship between domestic and global risk-free rates that determine the country-specific risk premium:

$$\log\left(\frac{R_t^{CAN,10y}}{R_t^{US,10y}}\right) = \log(\psi_1) + \psi_2\left(-\frac{NFA_t}{GDP_t}\right) + \epsilon_t, \tag{4.1}$$

where $R_t^{CAN,10y}$ and $R_t^{US,10y}$ are yields on 10-year government bonds for Canada and the United States, respectively, and NFA_t/GDP_t is the ratio of Canadian net foreign assets to GDP. The confidence interval for the estimates of the elasticity of country premium to net foreign asset position, ψ_2, is relatively large. Thus, to show the sensitivity of results to ψ_2, we estimate the neutral rate for high elasticity and low elasticity separately. The high elasticity case uses estimate $\psi_2 = 6.2$ from regression (4.1) with $-NFA_t/GDP_t$ on the right-hand side. The low elasticity case uses estimate $\psi_2 = 1.2$ from regression (4.1) with External debt$_t/GDP_t$ on the right-hand side. These estimates converted to annual frequency imply that an increase of 1 percentage point (pp) in the ratio of the net foreign assets to annual GDP reduces the interest rate spread at the annual frequency by 1.6 basis points (bps) and 0.3 bps in the high and the low elasticity case, respectively.

Household heterogeneity and preference parameters are also set to match key moments or follow benchmarks in the literature. The average productivity of middle-aged households is normalized to one and the relative productivity of low-income and high-income middle-aged households $[e^{m,l}, e^{m,h}] = [0.78, 1.88]$ are set to match the 37.6% after-tax income share of top 20% of highest income households. To match this moment, the share of high-income households in the population μ^h is set to 20%. The relative productivity of young households e_y is set to 0.3 and their borrowing constraint θ to 0.035 as in Marx, Mojon and Velde (2021). The bequest weight in the utility $\kappa = 1.14$ and the curvature of the bequest utility function $\varepsilon = 0.67$ are set to clear the bonds market and match the wealth share if the top 20% richest households among the population older than 65, respectively.

Fiscal policy parameters are calibrated as follows. The labour tax rate $\tau = 0.31$ is set to match the average tax wedge on labour income of 31%, while transfers to young and middle-aged households are normalized to zero $tr^{m,i} = tr^{y,i} = 0 \; \forall i \in l, h$. Transfers to old households $tr^{o,i} = 0.19 \; \forall i \in l, h$ matches the ratio of public pension expenses to GDP of 4.2%.

Finally, the capital share $\alpha = 0.124$ is set to match the household net worth to GDP ratio of 285%.

4.2 Evolution of global and domestic factors

All global factors in our small open economy model are captured by the global neutral rate R^{W*}. We use the estimates of the US neutral rate as a proxy for the global neutral rate. The Bank of Canada estimates for the US neutral rate are available only for 2014 and then from 2017 onward (see Boutilier et al. [2022] for the latest estimates and overview of methodology). We therefore use a number of alternative estimates from the literature. First, we use the two-sided estimates of the Laubach-Williams model (Laubach and Williams 2003)—henceforth LW—produced by the Federal Reserve Bank of New York (Federal Reserve Bank of New York 2020). Second, we use other estimates of the neutral rate produced by Bauer and Rudebusch (2020). These authors also use estimates of the neutral rate from other published studies: Del Negro et al. (2017), henceforth DGGT; Johannsen and Mertens (2018), henceforth JM; Kiley (2015); and add their own estimates: the first uses univariate unobserved components model—henceforth "UC," the second adds to the UC model moving averages of real GDP and labour force growth—henceforth "Proxies," and the third is a state-space model with observed nominal rate and inflation and latent real rate and r^*—henceforth "SSM." Finally,

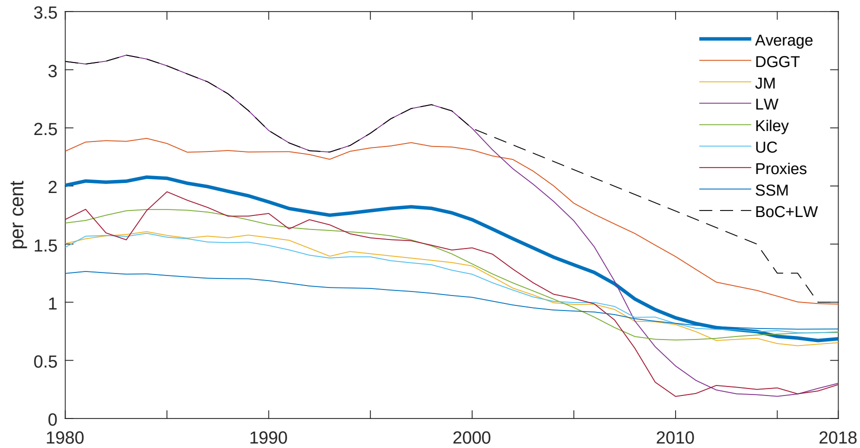


Chart 1: The path of estimates of the real global neutral rate. Sources of proxies for R^{W*}: Bauer and Rudebusch (2020), Federal Reserve Bank of New York (2020) and Bank of Canada estimates

we combine the Bank of Canada estimates for 2014–18 with the LW estimates for 1980–2002 with interpolation in between—henceforth "BoC+LW" estimate. Chart 1 shows the various proxies for the global neutral rate.

Alternative estimates of R^{W*} imply different sizes and timings for the decline of the neutral rate in the United States. Most models imply the steepest decline in R^{W*} in the period around the great financial crisis. The decline is the largest for the LW estimate with a cumulative decline of 2.8 pp over 1980–2018, while the decline is the least pronounced and very gradual for SSM (-0.5 pp).

The key domestic factors in the model are the growth rate of technological progress $g_{A,t}$, the growth rate of the demographic trend $g_{L,t}$, the probability of middle-aged households surviving into the old age s_t, government debt B_t^G and the relative productivity of low-income to high-income middle-aged households $e_t^{m,l}/e_t^{m,h}$. We parameterize these domestic factors using a time series for trend labour productivity (TLP) growth, trend labour input (TLI) growth, life expectancy at the age of 65 that translates into the probability of middle-aged households surviving to old age (longevity), gross government debt to GDP and average after-tax income share of bottom 80% of the lowest-income households. Chart 2 shows these time series.

The demographic factors, TLI growth and life expectancy show a clear secular trend. The TLI growth declines, and life expectancy increases. While TLP growth declines over the entire period from 1980 to 2018, it temporarily accelerates in the 1990s. Similarly, government gross debt increases overall, despite its pronounced temporary fall in the late 1990s and early

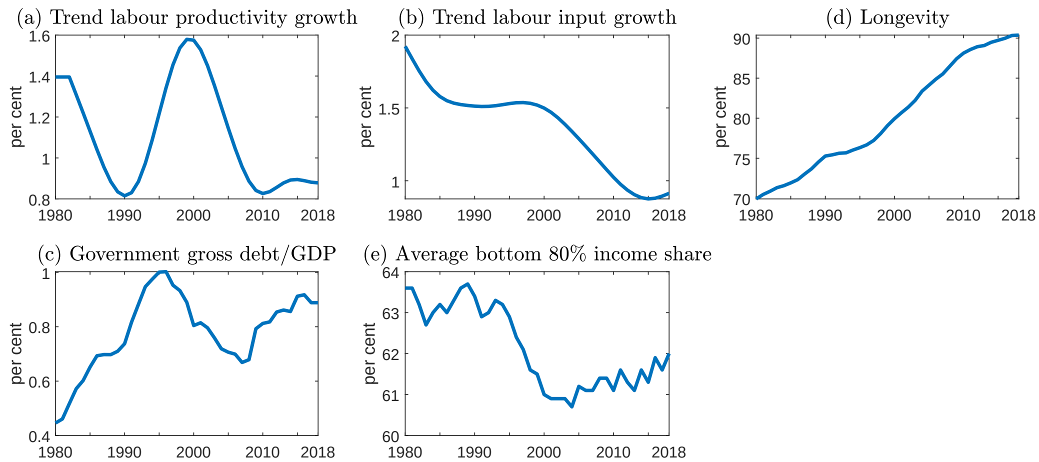


Chart 2: Evolution of domestic factors that affect the neutral rate. Note: Panel (d) plots the probability of survival of middle-aged households in the model calibrated to the data for life expectancy. Sources: Statistics Canada, International Monetary Fund, Bank of Canada and author calculations

2000s, after reaching the sample peak in 1996. Finally, our inequality measure does not seem to have a clear trend, with the exception of an increase in inequality in the 1990s.

4.3 Results

Chart 3 shows the estimates of the Canadian real neutral rate for different proxies of the global neutral rate and for high and low elasticity of the country premium to the net foreign asset position. The heterogeneity in R^{W*} implies differences in the level and the dynamics of the domestic neutral rate R^*. The average level of R^* for the calibration period 1990–2009 is recalibrated for every measure of R^{W*} such that the average spread between the two rates matches the rate spread between Canadian and US 10-year government bonds. The effect of domestic factors on the neutral rate estimates is sensitive to the elasticity of the country-specific premium on the net foreign asset position, ψ_2. Since this estimate has large confidence intervals, we estimate the neutral rate separately for high elasticity and low elasticity. All estimates suggest a decline in the Canadian neutral rate. In the high-elasticity case, domestic factors are the main reason for the evolution of the R^* estimates, as can be seen by strong comovement among estimates for alternative R^{W*} proxies. In the low-elasticity case, global factors are the main drivers of the evolution of R^*.

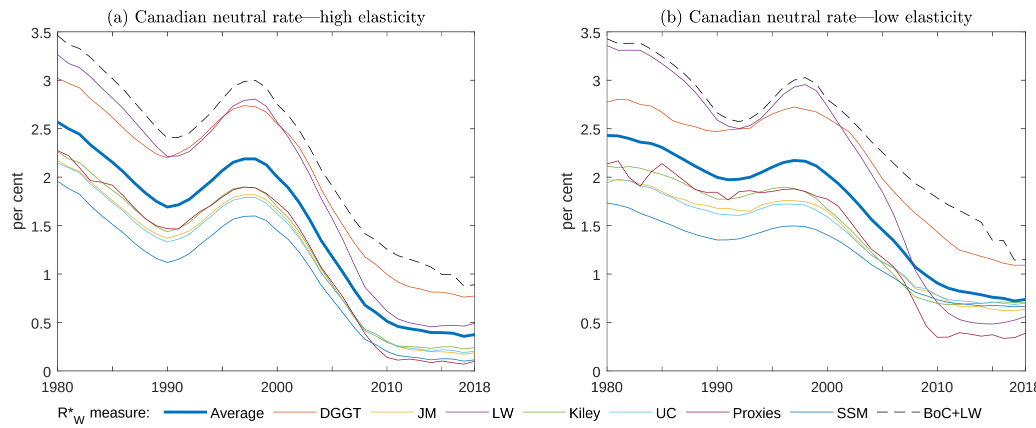


Chart 3: Estimates of the Canadian real net neutral rate for various R^{W*} measures and country-premium elasticites. Source: model output

Cumulative effects. In Chart 4 we report the overall decline in the Canadian neutral rate over 1980–2018 on average across various proxies for R^{W*} but separately for high- and low-elasticity cases. The bars in the figure report the average change, and the black line shows the interval between the maximum and minimum change across alternative proxies for R^{W*}. Our results imply the overall R^* decline by an amount between 1.8 pp and 2.8 pp in the high-elasticity case and between 1.1 pp and 2.8 pp in the low-elasticity case.

Figure 4 also shows the contributions of domestic and global factors. Simple observation of similar trends in the US neutral rate (a proxy for R^{W*}) and Canadian neutral rate in Figure 3 might be interpreted as the Canadian neutral rate closely following the US rate. However, our decomposition of changes in the R^* assigns significant weights to domestic factors in both high- and low-elasticity cases. Thus the comovement in the two neutral rates is likely caused by shared trends in the US and Canadian domestic factors rather than by insignificance of domestic factors driving the Canadian neutral rate. In the high-elasticity case, domestic factors are the main drivers in the R^* decline, while in the low-elasticity case domestic factors play a lower but still important role. The ranking and relative contributions among the domestic factors are the same in both cases.

The reduction in the global neutral rate contributed on average to a decline in R^* by 47 bps and 100 bps in the high- and low-elasticity case, respectively. The contribution of global factors to change in R^* varies significantly depending on the used proxy for the global neutral rate. The black lines in Figure 4 show the dispersion between the maximum effect for the LW measure that implies the largest decline in R^{W*} over 1980–2018 and minimum for SSM measure with the smallest change in R^{W*}.

The biggest domestic contributors to the neutral rate change were the two demographic factors. Firstly, higher domestic savings due to longer longevity contributed -83 bps and -34 bps to the R^* change in the high- and the low-elasticity case, respectively. Secondly, the reduction in TLI growth that implies lower investment and borrowing of young households contributed by -75 bps and -34 bps to the R^* change in the high- and low-elasticity case, respectively.

Slower domestic TLP growth also had, on average, a significant negative impact on R^* of -37 bps and -15 bps in the high- and the low-elasticity case, respectively. The slowdown in the TLP growth over the period implies lower investment and borrowing of young that translates into lower R^*. The increase in government debt partially offsets the R^* decline. Over 1980–2018, the ratio of gross government debt to GDP increased by 44 pps, and this increased demand for savings tended to increase the R^* by 24 bps and 10 bps in the high- and the low-elasticity case, respectively.

The inequality factor has only a very small contribution to the decline in R^*. This is because, as measured by the share of after-tax income going to the bottom 80% of the income distribution, the increase in inequality was only very limited over 1980–2018 (Figure 2).

(a) High elasticity (b) Low elasticity

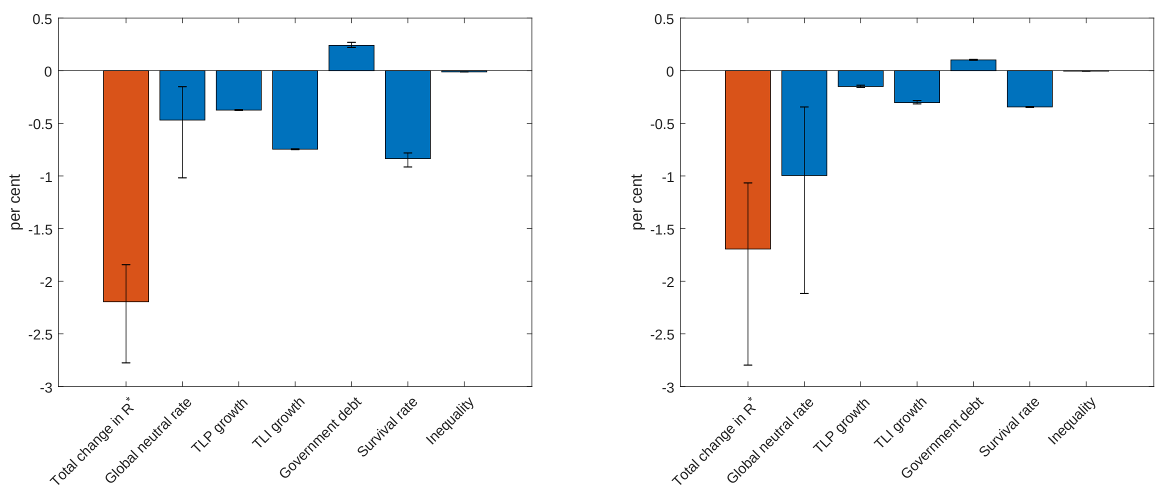


Chart 4: Contributions of factors to the change in neutral rate over 1980–2018. Panel (a) and panel (b) show estimates for high and low elasticity of the country-specific risk premium to net foreign asset position, respectively. Total change in R^* is represented by orange bars and contributions of factors by blue bars. Bars represent mean change across various proxies for global neutral rate R^{W*} and the black lines show the interval between the maximum and minimum change/contribution across alternative R^{W*}. Source: model output

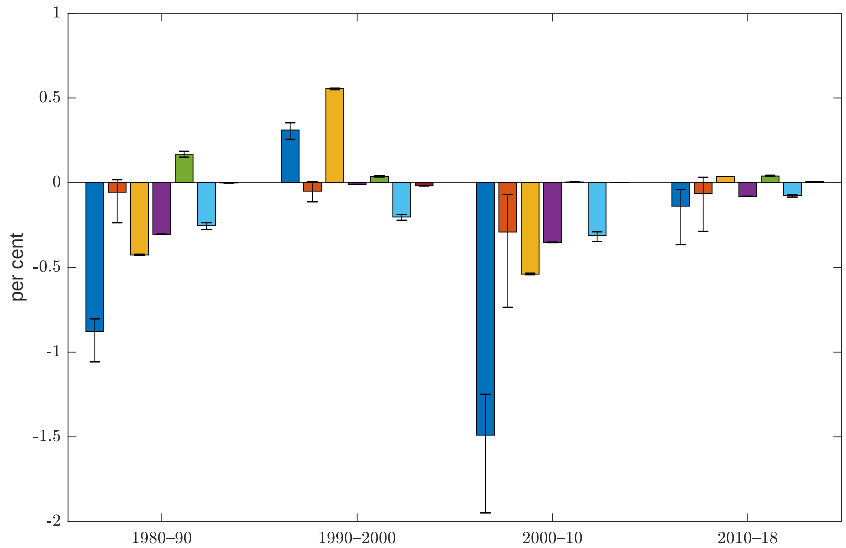
Evolution of R^* by decades. In Chart 5 we show the change in R^* and decomposition of this change by factors by decades. Our analysis suggests that R^* declined over 1980–90 by 88 bps and 43 bps, on average, in the high- and low-elasticity case, respectively. This was due to contributions of all factors but the increase in government debt, which offset some of the R^* decline. Over the following period, from 1990 to 2000, R^* increased by 31 bps in the high-elasticity case (3 bps in the low-elasticity case) due to an increase in TLP growth. The largest decrease in R^* by 149 bps in the high-elasticity case (112 bps in the low-elasticity case) is observed over 2000–10 likely as a result of the 2008–09 global financial crisis. The large dispersion in the total effect is given by differences in proxies for the global neutral rate, some of which imply that the decline in R^{W*} was smaller or partially delayed to the following decade. Finally, our analysis suggests a smaller average R^* decline of 14 bps in the high-elasticity case (17 bps in the low-elasticity case) in the last period 2010–18. This change is driven by lower R^{W*}, slower TLI and longer longevity but is partially offset by higher government debt and marginally faster TLP growth.

5 Conclusion

We have presented an overview of the updated small open economy model with overlapping generations, which we propose to use as one of the tools for assessing the neutral real rate of interest in Canada. The model allows us to capture most of the key factors that are prominent in the recent literature that analyzes the decline in neutral rates over the past few decades. Namely, the model can capture a decline in the rate of technological growth as well as demographic factors, such as lower growth of the labour force and longer longevity. We also model explicitly the impact of government debt and income inequality on the neutral rate. The small open economy nature of the model allows for distinguishing the role of domestic and foreign factors.

After calibrating the model to the Canadian data, we use it to evaluate the secular decline of the neutral rate from 1980 to 2018. We find a decline in the neutral rate by between 1.8 pps and 2.8 pps over this period. This suggests that both foreign and domestic factors contributed significantly to the cumulative decline in the neutral rate. The key domestic factors were demographic—slower growth of the labour force and longer longevity. A slower rate of domestic technological progress also significantly contributed to lowering neutral rate, while higher government debt partially offsets the decline in the neutral rate. Finally, our analysis suggests a limited role of inequality in the neutral rate decline in Canada.

(a) High elasticity



(b) Low elasticity

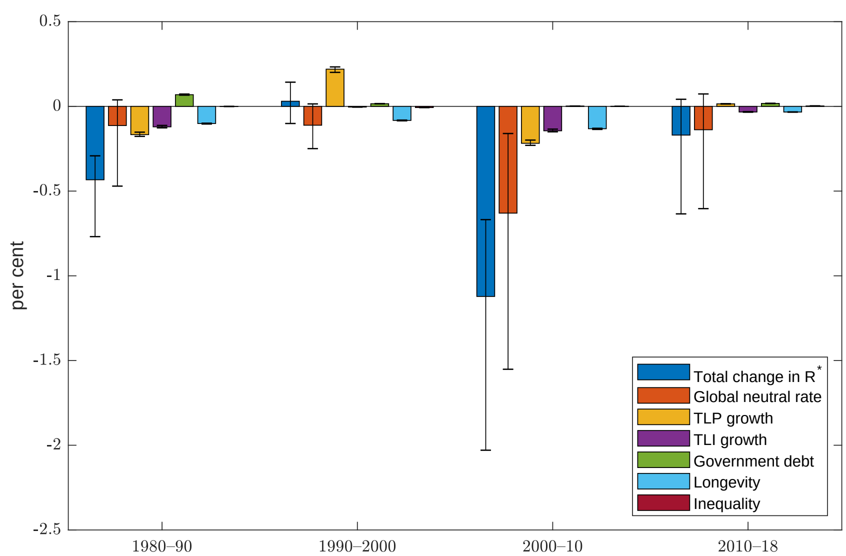


Chart 5: Contributions of factors to the change in neutral rate over by decades. Panel (a) and panel (b) show estimates for high and low elasticity of the country-specific risk premium to net foreign asset position, respectively. Bars represent mean change across alternative R^{W*} and the black lines show the interval between the maximum and minimum change/contribution across alternative R^{W*}. Source: model output

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A Model Appendix

A.1 Proofs—Components of demand for net foreign assets

Asset demand by middle-aged households. The budget constraint at middle age, (2.3), can be rewritten, using binding borrowing constraint (2.5), as follows:

$$c_t^{m,i} = [1 - \theta_{t-1}] I_t^{m,i} - a_{t+1}^{m,i}. \tag{A.1}$$

The budget constraint at old age, (2.4), can be rewritten, using optimality conditions (2.7) and (2.8), as follows:

$$c_{t+1}^{o,i} = \frac{R_{t+1} a_{t+1}^{m,i}}{s_t} \frac{(1 + \phi_{t+1}^i)}{(1 + \xi_{t+1}^i)}, \tag{A.2}$$

where the composite variable ϕ_{t+1}^i is the expected public pension income of old to private income of old and ξ_{t+1}^i captures the saving due to the bequest motive. These variables are defined as follows:

$$\phi_{t+1}^i \equiv \frac{\mathrm{tr}_{t+1}^{o,i} w_{t+1}}{R_{t+1} a_{t+1}^{m,i} / s_t}, \tag{A.3}$$

$$\xi_{t+1}^i \equiv \left[\kappa\eta \left(\frac{a_{t+2}^{o,i}}{\Delta_{t+1}} \right) \right]^{\frac{1}{\sigma}}. \tag{A.4}$$

One then derives (3.1) using (2.6), (A.1), and (A.2).

Asset demand by old households. The budget constraint at old age, (2.4), can be rewritten, using optimality conditions (2.7) and (2.8), as follows:

$$a_{t+1}^{o,i} = \left[\left(\frac{1 + \phi_t^i}{1 + \xi_t^i} \right) \frac{\xi_t^i}{s_{t-1}} \right] R_t a_t^{m,i}. \tag{A.5}$$

One then derives (3.2) using (3.1) and (A.5).

A.2 Proof of proposition 1

We start the proof by rewriting the middle-aged income $I_t^{m,i}$ of the type-i household as a function of R_t using equilibrium conditions. To do this, we first express it as a function of k_t

by using (2.10):

$$I_t^{m,i} = (1-\alpha)A_t^{1-\alpha}k_t^{\alpha}\Big[(1-\tau)e_t^{m,i} + \text{tr}_t^{m,i}\Big]. \tag{A.6}$$

Next, using no-arbitrage condition (2.7), we derive a relation between k_t and R_t out of (2.11):

$$k_t = \left[\Big(R_t - g_{I,t}(1-\delta)\Big)\frac{p_{t-1}^k}{\alpha A_t^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}. \tag{A.7}$$

We can then proceed to characterizing the equilibrium condition on the market for net foreign assets by expressing individual components of the net demand for foreign assets in (2.15) as functions of R_t using (A.6), (A.7) and (3.1)–(3.3):

$$\begin{aligned}
\frac{N_{t-1}\sum_i \mu_i a_{t+1}^{m,i}}{Y_t} &= \left[\frac{1-\alpha}{1+e_t^y g_{L,t}}\right]\left[\sum_i \mu_i \gamma_t^i \tilde{I}_t^{m,i}\right] \\
\frac{s_{t-1}N_{t-2}\sum_i \mu_i a_{t+1}^{o,i}}{Y_t} &= \left[\frac{1-\alpha}{1+e_t^y g_{L,t}}\right]\left[\sum_i \mu_i \gamma_{t-1}^i \tilde{I}_{t-1}^{m,i}\frac{\xi_t^i(1+\phi_t^i)}{(1+\xi_t^i)}\right]\left[\frac{k_{t-1}}{k_t}\right]^{\alpha}\left[\frac{R_t}{g_{L,t-1}g_{A,t}^{1-\alpha}}\right] \\
\frac{p_t^k K_{t+1}}{Y_t} &= \left[\frac{\alpha}{1+e_t^y g_{L,t}}\right]\left[\frac{k_{t+1}}{k_t}\right]\left[\frac{g_{L,t}g_{I,t}}{R_t - g_{I,t}(1-\delta)}\right]\left[(1+e_{t+1}^y g_{L,t+1})\right] \\
\frac{N_t\sum_i \mu_i b_{t+1}^{y,i}}{Y_t} &= \left[\frac{1-\alpha}{1+e_t^y g_{L,t}}\right]\left[\frac{k_{t+1}}{k_t}\right]\left[\frac{R_{t+1} - g_{I,t+1}(1-\delta)}{R_t - g_{I,t}(1-\delta)}\right]\left[\frac{g_{L,t}g_{I,t}}{R_{t+1}}\right]\left[\sum_i \mu_i \theta_t \tilde{I}_{t+1}^{m,i}\right]
\end{aligned}$$

where $\tilde{I}_t^{m,i} \equiv (1-\tau)e_t^{m,i} + \text{tr}_t^{m,i}$ and the capital-to-labour ratio k_t evolves following:

$$\frac{k_{t+1}}{k_t} = \frac{A_{t+1}}{A_t}\left[\frac{p_t^k}{p_{t-1}^k}\frac{R_{t+1} - g_{I,t+1}(1-\delta)}{R_t - g_{I,t}(1-\delta)}\right]^{\frac{1}{\alpha-1}}.$$

The remaining component is public debt, which we assume to follow an exogenous path supported by the fiscal policy that ensures the government budget constraint is always satisfied.

Substituting these expressions back into (2.15) (after normalizing by output) and equating to the net supply of foreign assets (2.16), leads to a difference equation in R_t. Along the balanced growth path, this equation then reduces to the equation (3.4). Also, note that (3.4) contains four other endogenous variables: ξ^i and φ^i for $i \in \{l, h\}$. Hence, in order to solve for the neutral rate, one should solve for a fixed point of the system of equations that includes (3.4) and four additional equations. These additional equations are constructed out of (A.3) and (A.4) by substituting away other endogenous variables using (2.10), (3.1), (A.5)–(A.7).

B Tables

Table 1. Calibrated Parameters

	Symbol	Value	Rationale / Data moments (90s-00s)
Discount factor[1]	β	0.98	Literature
Inverse EIS	σ	1.14	Dorich et al. (2013)
Capital depreciation rate[1]	δ	0.1	Literature
Domestic neutral rate[2]	R^*	3.00%	Dorich et al. (2013)
Foreign neutral rate[2]	R^{W*}	2.50%	10-year government bonds spread of 0.5%
Country premium	$[\psi_1, \psi_2]$	[1.00, 6.23]	Regression analysis
Middle-aged productivity	$\left[e^{m,l}, e^{m,h}\right]$	[0.78, 1.88]	After-tax income share of top 20% of 37.6%
Labour tax rate	τ	31%	Tax wedge on labour income of 31%
Bequest weight	κ	1.34	Bond market clearing
Bequest curvature	ϵ	0.66	Wealth share of top 20% (among 65y+) of 46%
Transfer to old	tr^o	0.19	Public pension expense / GDP of 4.2%
Productivity of young	e^y	0.3	Marx, Mojon and Velde (2021)
Borrowing constraint	θ	0.035	Marx, Mojon and Velde (2021)
Capital share	α	0.124	Household net worth / GDP of 285%

[1] Annualized rate. [2] Annualized net real rate.