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CANVAS: A Canadian Behavioral Agent-Based Model

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Abstract

We develop a Canadian behavioral agent-based model (CANVAS) that utilizes Canadian micro- and macroeconomic data for forecasting and policy analysis. CANVAS represents a next-generation modelling effort, as it improves upon the previous generation of models in three dimensions: introducing household and firm heterogeneity, departing from rational expectations, and explicitly modelling the Canadian production network. This modelling capacity is achieved by harnessing large-scale Canadian micro- and macroeconomic datasets (of financial flows and national balance sheet accounts, input-output tables, government finance statistics, and the Labour Force Survey). By incorporating adaptive learning and heuristics, we equip the model to examine macroeconomic dynamics under significant uncertainty. We assess the out-of-sample forecasting performance of CANVAS against a benchmark vector auto-regressive (VAR) model and a DSGE model (Terms of Trade Economic Model, ToTEM). CANVAS advances several new frontiers of macroeconomic modelling for the Canadian economy. First, the detailed structure of the model allows for forecasting of the medium-run macroeconomic effects of the economy at the sector level. For instance, this structure allows us to assess the macroeconomic impact of the COVID-19 pandemic in Canada. Second, the realistic agent behaviour in CANVAS makes the model an ideal candidate for evaluating the effects of multiple macroeconomic policies. Third, the enriched modelling of the financial market structure allows policy-makers to conduct stress testing and assess the implication of macroprudential policies in Canada.

Topics: Central bank research, Econometric and statistical methods, Economic models, Firms dynamics, Inflation and prices

JEL codes: C, D22, D83, E, E17

Contents

1 Introduction

Economic models make a valuable contribution to the policymaking process at central banks. They help policymakers develop insights about economic developments, weigh the pros and cons of policy options, assess risks, and provide numeric projections for key macroeconomic variables like GDP and inflation. In this paper, we develop an alternative economic model, CANVAS, based on an agent-based modelling (ABM) approach that explains the evolution of an economy by simulating the micro-level behaviour of heterogeneous individual agents to provide a macro-level picture. This approach has the potential to make sense of the types of extreme macroeconomic movements the world has witnessed during the financial crisis and COVID-19 pandemic (Haldane and Turrell, 2018). To demonstrate this potential, we apply CANVAS to assess the macroeconomic effects of the COVID-19 pandemic in Canada.

Staff at the Bank of Canada have been using economic models to produce economic projections and policy analysis for decades (Poloz, 2017). Bank staff generate projections to achieve two goals: (1) to construct a logically consistent forecast of key macroeconomic variables that is as accurate as possible; and (2) to set a path of policy interest rates that will keep information as close as possible to the target without generating excessive fluctuations in the economy. Both objectives are determined simultaneously in that the projection embodied in a model is conditional on the policy setting, and the policy rate path is a function of the macro variables forecast as well as the structure of the model.

In addition to conducting economic projections, models have been used in Bank policy to foster a rigorous understanding of the evolving economic narrative and to serve as an effective communication vehicle with the public. Among the suite of models used over the years, the current economic modelling at the Bank features the usage of two complementary large-scale models: a DSGE model, ToTEM, and a semi-structural reduced-form model, LENS, for projection and policy analysis.

CANVAS complements the suite of models at the Bank with an ABM that fits micro and macroeconomic data of the Canadian economy and allows for forecasting of main variables as well as policy analysis through scenarios. This model extends the ABM framework of Poledna et al. (2022), who pioneered the development of a small open economy ABM for Austria. The model economy is structured into four mutually exclusive domestic institutional sectors: (1) non-financial corporations (firms), (2) financial corporations and the central bank (the financial sector), (3) the general government, and (4) individuals (the household sector). These four sectors make up the total domestic block of the Canadian economy and interact with (5) the rest of the world through trade linkages modelling imports and exports. Each sector is populated by heterogeneous agents who represent natural persons or legal entities (individuals, corporations, and government entities) and have separate balance sheets that depict assets, liabilities, and ownership structures. We use a scale of 1:100 between the model and the data so that each agent in the model represents 100 individuals or businesses in the Canadian economy. This level of heterogeneity offers a great advantage, since CANVAS is directly linked to Canadian microeconomic data and model parameters are either pinned down by data or calculated from accounting identities.

There are three main features of CANVAS that enhance the Austrian ABM in Poledna et al. (2022). First, in this paper, we model income heterogeneity in the household sector, and persons in the labor force are associated with additional characteristics such as their age, sex, and employed industry. Second, in the Austrian model, the policy rate is set exogenously to reflect Austria's membership in the Eurozone where the ECB sets the monetary policy of the entire zone. In CANVAS, we follow Blattner and Margaritov (2010) to model monetary policy following an augmented Taylor rule specification where both the inflation deviation from the 2 percent target and the growth rate enter the reaction function. Monetary policy of the foreign block of the model is characterized to follow the regime from the Fed prior to its framework change in 2020. This setup allows for analysis of transmission of foreign shocks. Third, in addition to continuing to model the adaptive learning behavior of agents, this paper also features a price-setting rule of firm agents that allows for a decomposition of inflation into three sources: (1) demand-pull inflation from increases in demand, (2) inflation expectations following adaptive learning, and (3) cost-push inflation from increased prices of intermediate production inputs. For an inflation targeting central bank, this capacity is of particular benefit.

Our discussion of the model proceeds as follows: Section 2 provides a literature review of agent-based models at central banks. Section 3 presents the model specification, derived from Poledna et al. (2022). Section 4 explains the calibration strategy and provides an overview of data used in the model. Section 5 evaluates the out-of-sample prediction performance of CANVAS against different macroeconomic modelling approaches.Section 6 assesses the macroeconomic effects of the COVID-19 pandemic to demonstrate a potential application of our agent-based model, and Section 7 concludes.

2 Economic Models at Central Banks

Economic projections and models at a central bank offer important benefits for monetary policy decision making. With each projection, the central bank lays out a plan for policy conditional on a view of how the economy works and a set of explicit assumptions about exogenous factors. It updates its plan as new information about the nature of the shocks arrives and as its understanding of the economy evolves. By providing a plausible causal narrative about economic developments and being transparent about credible plans to respond to shocks, central banks condition expectations in a manner that is consistent with achieving its objectives. In this way economic projections at central banks are more like optimal control problems than forecasting exercises. This distinction is of vital importance and reflects the fact that central banks' actions influence the outcomes. In fact, sometimes central banks cause behaviour to change in systematic ways.

Economic models play a central role in the policymaking process of central banks. They do this by imposing discipline on the analysis and helping eliminate fuzzy thinking. This added clarity helps sharpen both internal discussions and public communications, lending more credibility to the analysis. Models force policymakers to be clear about:

- how they think the economy and monetary policy work,
- their assumptions about factors determined outside of the model (i.e., geopolitical developments), and
- the role of idiosyncratic factors or information coming from outside of the model.

Models also settle debates that cannot be settled by theory alone. Economic theory often suggests that potentially offsetting influences are at work in the economy. Models help quantify the relative importance of each factor, thereby providing a net impact of the offsetting influences.

Ever since the seminal work by Smets and Wouters (2003, 2007), New Keynesian DSGE models that employ Bayesian estimation techniques have been shown to exhibit a similar forecast performance to comparable time series models (Del Negro and Schorfheide, 2013). These DSGE models have become the workhorse framework for central banks and other institutions to engage in economic forecasting and policy analysis on a sound theoretical basis and should be regarded as a minimum standard when it comes to studying business cycles in a general equilibrium framework (Christiano et al., 2018; Brunnermeier et al., 2013).

To complement the current generation of models and bridge some identified modelling gaps, some economists have been advocating ABMs as a new, promising direction for macroeconomic modelling.[1] Haldane and Turrell (2018), for example, suggest that ABMs can be complementary to existing approaches for answering macroeconomic questions where complexity, heterogeneity, networks, and heuristics play an important role. Farmer and Foley (2009), in particular, suggest that it might be possible to conduct economic forecasts with a macroeconomic ABM, although they consider this to be ambitious. For a state of the art survey of ABMs see Axtell and Farmer (2022).

2.1 Current Core Models of the Bank of Canada

Structural models like ToTEM, with its third generation introduced in 2018, are particularly useful for telling rich stories and for designing risk scenarios. For instance, Bank of Canada staff can point to the implications of shocks to labor supply and demand as opposed to analyzing the implications of unexplained movements in employment and wages. Adding more economic structure to models also renders the policy advice from staff projections less vulnerable to the Lucas critique. On the other hand, the added restrictions imposed by incorrect or incomplete theories can lead to a decline in forecasting ability and can make it more difficult for Bank staff to impose its views on the outlook. This trade-off has given rise to interest in semi-structural models, like LENS, which put a greater value on empirical fit over theoretical richness. LENS also allows for the addition of ad hoc elements that help with model fit. The magnitude and importance of these trade-offs in our current models are open questions to be investigated as part of this review.

ToTEM is a large-scale, open-economy, dynamic stochastic general equilibrium (DSGE) model that is built around the New Keynesian paradigm (Murchison and Rennison, 2006). Its greatest strength comes from its strong theoretical structure, which provides a disciplining framework that ensures its forecasts are internally consistent. The model's multi-sector optimization framework is particularly useful for assessing

[1] Some examples include Freeman (1998), Gintis (2007), Colander et al. (2008), LeBaron and Tesfatsion (2008), Farmer and Foley (2009), Trichet (2010), Stiglitz and Gallegati (2011), and Haldane and Turrell (2018).

the impact of open-economy shocks, especially terms-of-trade shocks, which frequently hit the Canadian economy.

Bank staff have used ToTEM to produce projections and analysis of the Canadian macroeconomy since 2005. In June 2011, an updated version of the model, ToTEM II, replaced ToTEM I as the Bank of Canada's quarterly projection model (Dorich et al., 2013). ToTEM II made improvements in various dimensions of the model structure and was estimated using maximum likelihood techniques. In 2018, a newly updated version of the model, ToTEM III, was introduced, aiming to better capture linkages between the real economy and financial variables in policy analysis and projection (Corrigan et al., 2021). ToTEM III is estimated over the 1995-2016 period using Bayesian methods. The model uses 50 observable variables, including new observables such as collateralized household debt, disposable income, and residential investment price deflator. Like LENS, ToTEM III also uses a new measure of core inflation equal to the average of the Bank's three preferred core measures. ToTEM III shows improved forecasting performance over ToTEM II as reflected in a much improved in-sample goodness of fit.

Together, ToTEM and LENS represent a powerful, modern approach to economic modelling at the Bank, with the former benefiting from theoretical consistency and the latter designed to complement ToTEM and guard against different types of forecast risks (Poloz, 2017). Because of the importance of both models in the policymaking process, it is essential to assess the forecasting performance to enhance the transparency and accountability of the resulting monetary policy.

It is common in the literature to assess empirical coherence by focusing on pseudo out-of-sample forecasting accuracy, typically summarized by RMSEs. Forecasting performance is assessed by comparisons with simple univariate statistical models, other macroeconomic models, or (when done in real-time) consensus forecasts (i.e., Diebold-Mariano tests (Diebold and Mariano, 1995)). Out-of-sample evaluation is preferred because in-sample predictive content does not necessarily translate into out-of-sample predictive ability, nor does it ensure the stability of the predictive relation over time (Giacomini and Rossi, 2010). This includes the risk of in-sample overfitting (tailoring fitted models to sample idiosyncrasies), particularly in models that are less constrained by theory.

Many simple and easy-to-use time series models have forecasting power that is as good as or better than the alternatives but have insufficient structure to make them useful for monetary policy purposes. Once you get beyond the "nowcast," central banks have a poor performance record. RMSEs are very close to the sample standard deviations of the series that are being forecast. Champagne et al. (2018) show that over the inflation targeting period, there is essentially no difference in accuracy between the Bank of Canada staff's 4 or 8 period forecasts for inflation and GDP growth and a simple autoregressive model.

Similarly Edge et al. (2010) and Edge and Gurkaynak (2010) examine the forecasting ability of various models for the US economy. The general conclusion is that the forecasting power for inflation and GDP growth is similarly poor across all methods beyond the Greenbook "nowcast." The well-documented decline in forecastability of inflation and output growth in the US, Canada, and many other economies over the inflation targeting period has left RMSE-based tests with little power to distinguish "good" models from "poor" ones (Edge and Gurkaynak, 2010).

Central banks need models that do much more than forecast well. For starters, they need models that clearly articulate an important role for monetary policy in inflation determination. This motivated us to explore the next generation of macroeconomic models that could offer some additional insights for central banks.

2.2 Developing Next-Generation Models

Future macroeconomic models require relaxing the requirement of rational expectations; introducing heterogeneous agents; underpinning the model with more appropriate micro-foundations; and incorporating financial frictions rather than assuming that financial intermediation is costless (Vines and Wills, 2018). There are three key areas we identify as relevant for developing the next generation of the Bank of Canada models. These include (1) heterogeneity, (2) departure from rational expectations, and (3) non-linearities.

2.2.1 Heterogeneity

First, we would like to consider enriching the modelling of household heterogeneity. We know from Canadian data that monetary policy affects aggregate leverage through multiple channels. First, during a period of loose policy, households are taking on more debt by purchasing bigger and more expensive homes; this is what we call the intensive margin. At the same time, as interest rates go down, there are also likely more first time home buyers entering the market, which indicates that monetary policy may also have some potential impact on the extensive margin. In ToTEM, for instance, housing is an asset that can be used as collateral against borrowing.

DSGE models used to study aggregate business cycles traditionally rely on the representative agent assumption. Currently, we only assume a fixed share of borrowers that are subject to credit constraint. This setting does not allow us to analyze the extensive margin effect of monetary policy. In addition, we also don't consider other richer sources for heterogeneity, such as different characteristics in the labor market. The COVID-19 pandemic has definitely intensified the need for a better understanding of the role of precautionary saving motives of the vulnerable labor force (workers over 55 years old, women, and low-income earners) during economic downturns.

In response to criticism, a parallel research program addressing some perceived shortcomings of DSGE models by exploring the effects of agent heterogeneity in a general equilibrium framework has led to the development of heterogeneous agent New Keynesian (HANK) models.[2]

HANK models offer useful insights to central banks, particularly related to their assessment of alternative monetary policy frameworks. For example, history-dependent policy frameworks, or "makeup" strategies, have recently been highlighted for their promise in weathering effective lower bound (ELB) episodes (Budianto et al., 2020; Amano et al., 2020). These frameworks ensure that past inflation shortfalls are followed by temporarily higher future inflation and through the expectations channel; this results in better inflation stabilization, specifically at the ELB. The secular decline in the neutral rates of interest in industrial economies suggest that these economies could encounter the ELB constraint more frequently in the future. This has prompted a review of alternative monetary policy strategies, highlighting the importance of history-dependent frameworks in a number of central banks—notably, the US Federal Reserve (Svensson, 2020), the European Central Bank, and the Bank of Canada (Dorich et al., 2021).

While most of the discussions on alternative policy frameworks have so far focused on the inflation and output trade-off facing the central bank, little research is available about inequality.[3] Recent Bank work in this area (Djeutem et al., 2022) contribute to this literature by introducing an additional trade-off involving inequality that originates from incomplete financial markets. We adopt the tractable HANK model of Acharya and Dogra (2020), where incomplete financial markets and idiosyncratic earnings risk lead to consumption inequality in response to aggregate shocks at business cycle frequencies. From a societal perspective, inequality arising from a market failure is costly.

HANK models have been used to show that household and firm heterogeneity affect macroeconomic aggregates, but they have rarely been used to forecast economic aggregates, where representative agent New Keynesian (RANK) models have remained the benchmark (Kaplan and Violante, 2018; Christiano et al., 2018; Del Negro and Schorfheide, 2013). It is well recognized as a significant effort, theoretically and computationally, to undertake a full-fledged HANK framework. The properties inherent to DSGE models due to their grounding in general equilibrium theory has led to criticism that HANK DSGE models are restricted to a mild form of heterogeneity (Fagiolo and Roventini, 2017). All of these simplifications in the current generation of central bank models may have led to underestimation of the importance of monetary policy.

2.2.2 Departure from rational expectations

The second motivation for exploring next-generation models at central banks is related to the consideration of modelling expectations.

Asset prices in current policy models rely on rational expectations (RE). Forward-looking economic agents also plan with perfect knowledge of the future. These statements sound far from reality, especially given recent experience in the context of COVID-19. There is a fairly small literature looking at how relaxing the conventional RE assumption affects households' consumption-savings decisions.

Recently, increasing empirical evidence points to the weakness of RE models in representing how market participants understand and act on the economic information they receive, particularly with respect to history-dependent frameworks. Kostyshyna et al. (2022) use laboratory experiments with university students in an artificial economy setting to evaluate the performance of alternative monetary policy rules. They show that in learning-to-forecast experiments with stronger trend-chasing expectations, history-dependent rules such as Average Inflation Targeting (AIT), Price-Level Targeting (PLT), and Nominal GDP Level Targeting (NGDPL) perform poorly relative to Flexible Inflation Targeting (FIT) following a large shock that leads to a binding ELB. In particular, under PLT and NGDPL targeting,

[2] A non-exhaustive list of prominent examples includes Kaplan et al. (2018); Kaplan and Violante (2014); McKay and Reis (2016); Khan and Thomas (2008); Chatterjee et al. (2007).

[3] A notable exception is Feivson et al. (2020), who suggest that stabilization benefits from makeup strategies could generate disproportionate improvements for historically disadvantaged households.

the economy may degenerate into deflationary spirals when the ELB binds for an extended period. Such evidence is attributed primarily to the significant share of identified extrapolative expectations in experimental participants.

Recent bank work (Wagner et al., 2022) builds upon Gabaix (2020), who adopts a parsimonious way to introduce bounded rationality. In this otherwise standard New Keynesian model, agents are myopic toward future information in the sense that they discount macroeconomic changes more than a rational agent would. This approach offers a tractable way for us to quantify the impact when economic agents operate under the assumption of bounded rationality. Myopic behavior effectively reduces the weights agents place on events further in the future when making decisions today.

Relying on dynamics generated by key shocks that reproduce key Canadian macroeconomic data since 1995, the authors find that an overall higher degree of myopia reverses the ranking of strongly history-dependent rules such as PLT and NGDPL for a class of loss functions. The relatively inferior performance of these frameworks can be attributed to a weakened expectation channel. Importantly, they find that when the degree of departure from rational expectation is sufficiently large, there is very little difference in adopting history-dependent policy frameworks over flexible inflation targeting.

The evidence from preliminary exploration suggests the importance of accounting for a realistic expectation formation process in central bank models. Nevertheless, so far Bank staff have not formally incorporated these realistic expectations into the Bank's policy models.

2.2.3 Non-linearities

In March 2020, Canada, like many other advanced economies, entered the ELB episode for the second time in recent history. The Bank now operates its projection and policy models with the occasional binding constraint on the overnight rate.

The ELB in these models have been solved using a standard solution method (e.g., the perfect foresight approach). Under perfect foresight it is conventionally found that the presence of the ELB implies a higher average interest rate and lower inflation (and output gap) than under the case of no ELB. The perfect foresight method, however, assumes no long-run ELB risk, which is at odds with theory. The missing and important role of ELB risk can be partially recovered using a stochastic extended-path approach, which captures the following medium-run precautionary effect: Firms realize the ELB could bind in the future, which possibly lowers their inflation expectations. This feeds into lower current inflation when away from the effective lower bound. This lower current inflation then leads to lower current interest rates. On average, these interest rates are still higher than in the linear model. However, even with a horizon up to four years, Bank staff cannot capture long-run risk of the ELB in its policy models.

This assumption abstracts from the risk of the ELB binding in the future and may potentially miss important long-run inflation risks associated with the ELB. The log-linearization of the DSGE model solution, in general, still ignores risk premium, precautionary savings, and other important asymmetry. Nevertheless, we have found that in a small New Keynesian model, even when extending future uncertainty up to four years, the unconditional moments are virtually the same as under perfect foresight. This motivates the exploration of global solution methods to account for the long-run risk.

There is also increasing demand for design of non-linear responses in monetary policy reaction function. The Bank of Canada currently operates under a flexible inflation-targeting regime in which the response to both inflation and the output gap is symmetric. During exceptional times when the interest rate is constrained at the ELB, policy rules that allow for some form of asymmetry can be beneficial in providing additional economic stimulus. A policy reaction function that incorporates some form of asymmetry would allow for the policy rate to respond to negative deviations in the output gap but not to positive deviations. On average, this policy rate will be lower than under a symmetric policy rule, thereby providing more stimulus to the economy.

Alternatively, the central bank can also react to positive and negative deviations in inflation in varying magnitudes. For example, one could propose setting the weight on positive inflation deviations to be less than the weight on negative inflation deviations. On average, this would yield a lower policy rate path as central banks allow for greater tolerance on inflation overshoot. Similarly, the central bank can also put greater weight on negative deviations in inflation in order to prevent inflation undershoot and bring inflation back to the target faster than otherwise.

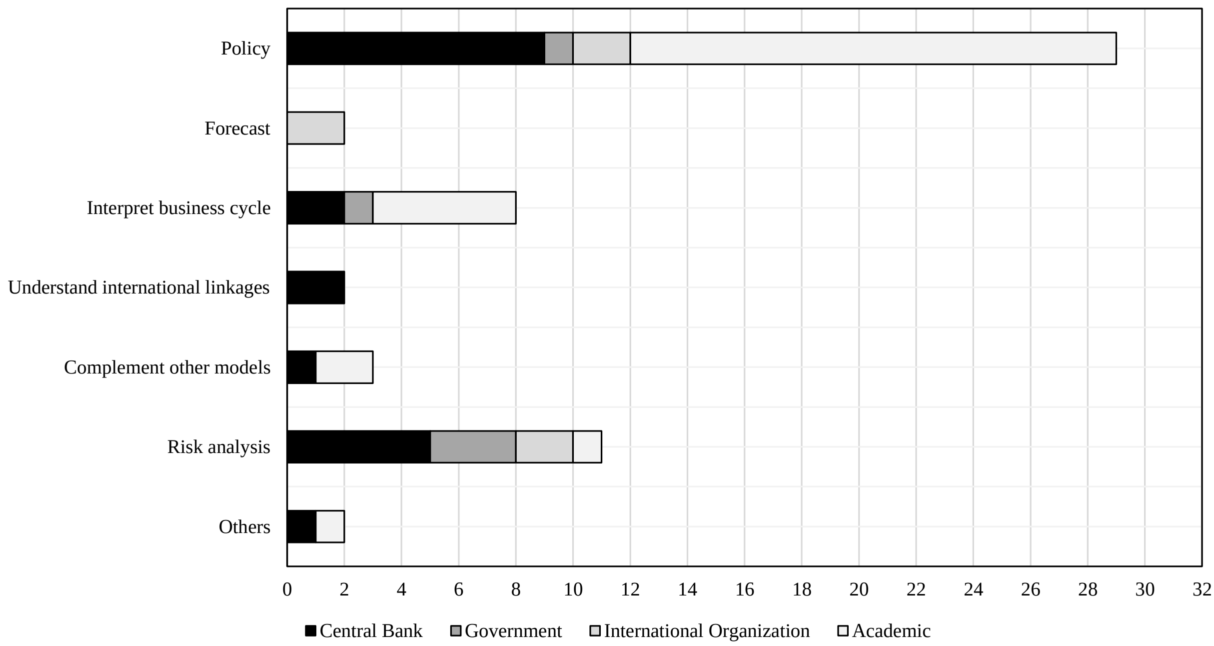
Finally, the prolonged low interest rate may induce more risk-taking behavior and have implications for financial stability.

2.3 Incorporation of Agent-Based Models at Central Banks

ABMs are a promising complement to the current crop of macroeconomic models for central banks that could fulfill the above requirements. Complementary to DSGE models, ABMs are suited to answering macroeconomic questions where heterogeneity, heuristics, and non-linearities play an important role (Haldane and Turrell, 2018).

However, to date, very few ABMs have been developed and deployed in practice within the central bank community. We have reviewed literature of 64 research papers published between 2008 and 2021 and provided an overview of the key themes of focus. As shown in Figure 1, the vast majority of ABM applications are for policy or risk analysis, followed by work aiming to interpret business cycles. Among these works, less than half are published research from central banks, indicating great potential of ABM as a tool for policy analysis, as well as economic forecasting. Our work intends to contribute to this stream of literature.

Figure 1: Stated objectives of ABMs



Napoletano (2018) and Turrell (2016) provide comprehensive overviews of the generativist approach followed by agent-based models, contrasting them to the reductionist approach followed by mainstream central bank policy models relying on DSGE frameworks. In the latter, the explanation of a macroeconomic phenomenon can be reduced to some fundamental laws governing the behavior of a single component (household, firms) of the economic system. This clear mapping is an appealing feature to central bank policy analysis, where often agent-based models are perceived as "black boxes" where the causes and mechanism driving model results are somewhat blurred.

While agent-based models start to emerge as a complementary tool for macroeconomic analysis, several limitations remain to be addressed.

2.3.1 The "ad hocerism" critique

First, ABM models may suffer from too much freedom and therefore may lead to one being lost in the "wilderness of bounded rationality." It is helpful to recognize that while there exists great modelling flexibility, the ad hocerism in ABM and other models with optimizing agents only varies in degrees. For instance, in RANK models, maximization of objective of economic agents (such as households, firms, and central banks) are modelled through respective constrained optimization problems. It is equally important that ABMs are micro-founded on the behavioral rules of different agents by using empirical or experimental evidence of true agents' behavior.

2.3.2 The "Lucas critique"

ABMs don't assume rational expectations. Instead, agents operate with sticky behavioral routines and/or naive expectations. Given vast literature on both empirical and experimental evidence, ABMs may be a good proxy model environment to consider "what if" scenarios where agents' expectations are of little importance, such as (1) when agents face constraints in obtaining and processing relevant information about economic variables; or (2) when financial, income, or interest rate constraints bind.

To understand monetary policy and expectations formation, a recent literature on macroeconomic laboratory economics has recently developed. Learning-to-forecast experiments have been employed to understand how individuals coordinate their forecasts in macroeconomic economies largely driven by aggregate expectations (Adam, 2007). Learning-to-forecast experiments have demonstrated how the nature of inflation-targeting monetary policies can influence the nature of expectation formation and macroeconomic stability (Pfajfar and Zakelj, 2014; Kryvtsov and Peterson, 2013). Assenza et al. (2021) show that heterogeneous expectations tend to self-organize on different forecasting rules depending on monetary policy. Subjects are generally found to form non-rational expectations in that they use historical information, rather than relevant current shocks and the data-generating process (including the monetary policy rule), to formulate their forecasts. More aggressive reaction coefficients on inflation and output gap in the central bank's Taylor rule encourage more stable forecasting behavior and aggregate stability. Macroeconomic experiments have also been used to identify the heuristics individuals and groups use to forecast. Anufriev and Tuinstra (2013) show that the stability of a system depends on the composition of forecasting rules.

Along with the empirical and laboratory evidence, recent advances in ABMs have tried to address the Lucas critique to introduce agents with more sophisticated expectation rules taken from literature on adaptive learning in macroeconomics (Evans and Honkapohja, 2001). The work of Arifovic et al. (2010), Salle (2015), and Dosi et al. (2020) provide new examples of an emerging research stream on learning in agent-based macroeconomic applications.

2.3.3 The "data validation" critique

Empirical validation of ABMs remains a difficult task. While DSGE models match the historical evolution of variables, macroeconomic ABMs typically perform indirect validation tests such that the calibrated (or estimated) model is able to reproduce a large set of stylized facts at both the micro- and macroeconomic level (i.e., time series properties of output fluctuations and growth, as well as cross-sectional distributional characteristics of firms (Dosi et al., 2017; Axtell, 2018). Due to over-parameterization and the corresponding degrees of freedom, almost any simulation output can be generated with an ABM, and thus replication of stylized facts only represents a weak test for the validity of ABMs (Fagiolo and Roventini, 2017).

2.3.4 The "black box" critique

This remark of ABMs being like a "black box" largely reflects the differences between the generativist and reductionist approaches as traditionally used in macroeconomics. Because the system is evolving over time and there exist non-clearing markets with potential multiple equilibria, it is often more challenging to tell stories with many moving parts.

On the other hand, even with very complex ABMs, causal mechanisms can be derived through counterfactual analyses. This, in fact, is an appealing feature for central banks as it allows one to control the presence of some model dynamics through different settings and/or calibration. For example, Gualdi et al. (2015) examine the efficacy of fiscal and monetary policies with varying types of matching protocols in labor and goods markets. This may offer a promising model environment to study the interaction of multiple (monetary, fiscal, and macroprudential as well as public health or even climate policies) in an increasingly complex economic ecosystem.

Ultimately, CANVAS addresses some limitations of the current generation of Bank of Canada core macroeconomic models by departing from strict expectation assumptions, allowing for greater household and firm heterogeneity, as well as encompassing potential non-linearities in multiple dimensions. At the same time, we address some of the conventional critiques to ABMs by making a significant effort to calibrate the model with large amounts of micro data and enhance the model with estimation. Most importantly, we have taken the approach to micro-founded behavioral rules that are consistent with empirical and laboratory evidence through the learning literature. As a result, our ABM has the potential to analyze different policy scenarios at both the aggregate and granular level. Further, the counterfactual analysis feature of our ABM has a greater capability to determine causality and direct impacts of macroeconomic shocks.

3 The Model

In this section, we present the full specification of the agent-based model, derived from Poledna et al. (2022). Three elements of this model are unique to Canada. First, in this paper, we model income heterogeneity in the household sector, and persons in the labor force are associated with additional characteristics such as their age, sex, and employed industry. Second, the policy rate is determined by an augmented Taylor rule, where the central bank agent learns the optimal parameters. Third, firms make their price-quantity decision according to a simple heuristic rule and change their supply or price based on their perceived local market conditions. Minor differences with respect to the model developed in Poledna et al. (2022) include an inventory cycle caused by the accumulation and selling of inventories and the inclusion of exogenous price deflators for imports and exports.

3.1 Households

The household sector consists of all employed, unemployed, and inactive persons in Canada who are 15 years of age or older, with respective populations obtained from census data and labour force surveys. We model income heterogeneity in the household sector within the labor force. Persons in the labor force (employed and unemployed) are associated with additional characteristics, such as their age, sex, and employed industry. Employed persons supply labor to firms and receive sector-specific wages. Unemployed persons search for employment and receive unemployment benefits, which are a fraction of previously received wages. Inactive persons (e.g., pensioners) do not participate in the labor market and receive old age benefits.

Outside of the labor force, we also model heterogeneity in terms of firm ownership and transfers. We assume that each firm is owned by an "investor," who is also part of the household sector. Investors obtain dividend income from firm ownership. Each person in the model economy receives social transfers from the general government. All persons spend their income on personal consumption and residential investment.

3.1.1 Activity Status

The household sector consists of a total number of H $(h = 1, 2, \dots, H)$ persons. Every person in the household sector has an *activity status*, that is, a type of economic activity from which she receives an income. The activity status is categorized into H^{act} economically active and H^{inact} economically inactive persons.[4] Economically active persons are H^W persons in the labor force, and $I + 1$ investors (the number of investors equals the number of firms and banks and is constant). Within the first set of persons in the labor force, there are $H^{\mathrm{E}}(t)$ employed and $H^{\mathrm{U}}(t)$ unemployed persons in each industry. $H^{\mathrm{E}}(t)$ and $H^{\mathrm{U}}(t)$ are endogenous since we assume that agents' employment status may switch between employed/unemployed when they are dismissed from their current job or are hired for a new position. Each person in the labor force (employed and unemployed) is also associated with additional characteristics, such as an age group, sex, and industry of employment. Economically inactive persons include persons (over the age of 15) who are not part of the labor force (e.g., retirees).

Each person in the labor force either supplies labor to a firm when employed or remains unemployed. If unemployed, the person looks for a job on the labor market from firms with open vacancies in random order and applies for a job through a *search-and-matching* process. Because the worker has occupation-specific skills, job matching occurs when the person accepts a job from the first hiring firm in the same industry as their previous employer. Search-and-matching frictions may occur: a worker remains unemployed if there are no more open vacancies left in the same industry as her former employer. On the other hand, if there are no longer any unemployed in the industry of the searching firm, vacant positions will remain open. For simplicity, we do not consider hiring or firing costs for firms, and dismissed employees start searching for new jobs in the same period that they become unemployed.

3.1.2 Income

Every period t, each person receives income according to their activity status. Employed persons are remunerated with the wage $w_i(t)$ of firm i that provides their current employment. If the worker has

[4]In this manuscript subscripts are used for indices referring to an agent in the model, while superscripts generally indicate a behavioral relation for a variable. For example, a quantity X referring to a household is denoted by X_h, expectations for a quantity X are written as X^{e}, or demand for a quantity X is indicated by X^{d}. Additionally, superscripts in capital letters are used to further distinguish related variables, e.g., $\bar{P}^{\mathrm{HH}}(t)$ denotes the consumer price index while $\bar{P}^{\mathrm{CF}}(t)$ is the capital formation price index.

changed jobs in period t, the current wage rate $w_i(t)$ would differ from wage rate $w_h(t-1)$ from the previous employer:

$$w_h(t) = \begin{cases} w_i(t) & \text{if employed by firm } i \\ w_h(t-1) & \text{otherwise.} \end{cases} \tag{1}$$

Unemployed persons receive unemployment benefits, which are a fraction (θ^{UB}) of the labor income from the last period of employment. A subset of people are investors in the model, with each investor receiving income in the form of dividends in the event that the firm she owns makes profits after interest and tax payments. There is a fixed share of economically inactive population. Each person h within this group receives social benefits $sb^{\mathrm{inact}}(t)$ and does not look for a job:

$$sb^{\mathrm{inact}}(t) = sb^{\mathrm{inact}}(t-1)(1+\gamma^{\mathrm{e}}(t)) \, . \tag{2}$$

We model social benefits in our model by assuming that each person in the population receives a fixed-amount social transfer $sb^{\mathrm{other}}(t)$ (such as family and childcare support, sickness benefits, etc.) from the government:

$$sb^{\mathrm{other}}(t) = sb^{\mathrm{other}}(t-1)(1+\gamma^{\mathrm{e}}(t)) \, . \tag{3}$$

After considering all income sources, the net nominal disposable income $Y_h(t)$ of the h-th individual is defined as the sum of after-tax income and social transfers (including unemployment benefits):

$$Y_h(t) = \begin{cases} \left(w_h(t)\left(1-\tau^{\mathrm{SIW}}-\tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\right) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t) & \text{if employed} \\ \left(\theta^{\mathrm{UB}} w_h(t)\left(1-\tau^{\mathrm{SIW}}-\tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\right) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t) & \text{if unemployed} \\ \left(sb^{\mathrm{inact}}(t) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t) & \text{if not economically active} \\ \theta^{\mathrm{DIV}}(1-\tau^{\mathrm{INC}})(1-\tau^{\mathrm{FIRM}})\max(0,\Pi_i(t)) + sb^{\mathrm{other}}(t)\bar{P}^{\mathrm{HH}}(t) & \text{if investor in firm } i \\ \theta^{\mathrm{DIV}}(1-\tau^{\mathrm{INC}})(1-\tau^{\mathrm{FIRM}})\max(0,\Pi_k(t)) + sb^{\mathrm{other}}(t)\bar{P}^{\mathrm{HH}}(t) & \text{if a bank investor.} \end{cases} \tag{4}$$

3.1.3 Consumption

Each person purchases consumption goods with a certain budget constraint. Consumers' behavior is assumed to be bounded rational in that they follow a heuristic rule by consuming a fraction of their expected net disposable income ($Y_h^{\mathrm{e}}(t)$). Expected net disposable income is determined according to the household's activity status and the associated labor income stream, expected profits or social benefits and tax payments, the consumer price index of the last period, and expectations of the inflation rate $\pi^{\mathrm{e}}(t)$ formed using an AR(1) model[5] (see Equation (15)):

$$Y_h^{\mathrm{e}}(t) = \begin{cases} \left(w_h(t)\left(1-\tau^{\mathrm{SIW}}-\tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\right) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t-1)(1+\pi^{\mathrm{e}}(t)) & \text{if employed} \\ \left(\theta^{\mathrm{UB}} w_h(t)\left(1-\tau^{\mathrm{SIW}}-\tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\right) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t-1)(1+\pi^{\mathrm{e}}(t)) & \text{if unemployed} \\ \left(sb^{\mathrm{inact}}(t) + sb^{\mathrm{other}}(t)\right)\bar{P}^{\mathrm{HH}}(t-1)(1+\pi^{\mathrm{e}}(t)) & \text{if not economically active} \, , \\ \theta^{\mathrm{DIV}}(1-\tau^{\mathrm{INC}})(1-\tau^{\mathrm{FIRM}})\max(0,\Pi_i^{\mathrm{e}}(t)) + sb^{\mathrm{other}}(t)\bar{P}^{\mathrm{HH}}(t-1)(1+\pi^{\mathrm{e}}(t)) & \text{if investor in firm } i \\ \theta^{\mathrm{DIV}}(1-\tau^{\mathrm{INC}})(1-\tau^{\mathrm{FIRM}})\max(0,\Pi_k^{\mathrm{e}}(t)) + sb^{\mathrm{other}}(t)\bar{P}^{\mathrm{HH}}(t-1)(1+\pi^{\mathrm{e}}(t)) & \text{if a bank investor} \end{cases} \tag{5}$$

where $\Pi_i^{\mathrm{e}}(t)$ (see Equation (39)) and

$$\Pi_k^{\mathrm{e}}(t) = \Pi_k(t-1)(1+\gamma^{\mathrm{e}}(t))(1+\pi^{\mathrm{e}}(t)) \tag{6}$$

is the expected profit based on received profits from firm i and the banking sector in the last period, respectively; τ^{INC} denotes the income tax rate, τ^{SIW} is the social insurance contribution rates to be paid by the employee, θ^{DIV} is the dividend payout ratio, and τ^{FIRM} denotes the corporate tax rate.

The consumption budget of household h ($C_h^{\mathrm{d}}(t)$) is thus given by:

$$C_h^{\mathrm{d}}(t) = \frac{\psi Y_h^{\mathrm{e}}(t)}{1+\tau^{\mathrm{VAT}}} \, , \tag{7}$$

where $\psi \in (0,1)$ is the propensity to consume out of expected income and τ^{VAT} is a value added tax rate on consumption.

We motivate the formulation of the consumption function by the literature of bounded rationality and lab evidence that household consumption behavior follows simple heuristics, as in Delli Gatti et al. (2011).

[5] As a robustness check, we allowed households to form their expectation of the inflation rate based on CPI inflation instead of GDP deflator inflation, and the differences are negligible.

Consumers allocate their consumption budget to purchase different goods from firms. The consumption budget of the h-th household to purchase the g-th good is

$$C^{\mathrm{d}}_{hg}(t) = \frac{b^{\mathrm{HH}}_g \bar{P}_g(t-1)}{\bar{P}^{\mathrm{HH}}(t-1)} C^{\mathrm{d}}_h(t)\,, \tag{8}$$

where b^{HH}_g is the consumption coefficient for the g-th product of households.[6]

Once households determine their consumption budget, they visit firms to purchase goods and services according to the search-and-matching mechanism (see Section 3.2.3). Whether an individual firm can accommodate consumers' demand depends on its production and inventory stock. Thus realized consumption of household h follows:

$$C_h(t) \begin{cases} = \sum_g C^{\mathrm{d}}_{hg}(t) & \text{if consumers' demand is successfully met, and} \\ < \sum_g C^{\mathrm{d}}_{hg}(t) & \text{if all firms visited could not satisfy consumers' demand.} \end{cases} \tag{9}$$

3.1.4 Residential Investment

The housing market is also modelled by a simple heuristic that each person uses a portion of their income for residential investment. Similar to Equation (7), we assume residential investment occurs according to a fixed rate ψ^{H} on expected net disposable income:

$$I^{\mathrm{d}}_h(t) = \frac{\psi^{\mathrm{H}} Y^{\mathrm{e}}_h(t)}{1+\tau^{\mathrm{CF}}}\,, \tag{10}$$

where τ^{CF} can be considered as property tax rate on residential investment. Demand of individual h for product g net of taxes ($I^{\mathrm{d}}_{hg}(t)$) is determined by fixed weights b^{CFH}_g:

$$I^{\mathrm{d}}_{hg}(t) = \frac{b^{\mathrm{CFH}}_g \bar{P}_g(t-1)}{\sum_g b^{\mathrm{CFH}}_g \bar{P}_g(t-1)} I^{\mathrm{d}}_h(t)\,. \tag{11}$$

Realized sales of residential investment goods purchased by individuals are an outcome of the search-and-matching process on the housing market:

$$I_h(t) \begin{cases} = \sum_g I^{\mathrm{d}}_{hg}(t) & \text{if the individual successfully realized the investment plan, and} \\ < \sum_g I^{\mathrm{d}}_{hg}(t) & \text{if all firms visited could not satisfy its demand.} \end{cases} \tag{12}$$

The stock of residential structure owned by individual h then follows:

$$K_h(t) = K_h(t-1) + I_h(t)\,. \tag{13}$$

3.1.5 Savings

Savings in period t is the difference between current disposable income $Y_h(t)$ and realized consumption expenditure $C_h(t)$ that includes realized housing investment $I_h(t)$. Savings are used to accumulate financial wealth:[7]

$$\begin{aligned} D_h(t) = D_h(t-1) + &\underbrace{Y_h(t) - (1+\tau^{\mathrm{VAT}})C_h(t) - (1+\tau^{\mathrm{CF}})I_h(t)}_{\text{Savings}} \\ + &\underbrace{\bar{r}(t)\max(0, D_h(t-1))}_{\text{Interest received}} - \underbrace{r(t)\max(0, -D_h(t-1))}_{\text{Interest payments}} \end{aligned} \tag{14}$$

The stock of deposits therefore reflects interest payments on overdrafts of the household's deposit account ($D_h(t-1) < 0$) and interest received on deposits held with the bank ($D_h(t-1) > 0$).[8]

[6]Similar to how the Consumer Price Index (CPI) is measured in Canada, we assume all households buy the same set of goods, independent of the amount they spend on consumption. The consumption basket includes all goods and services purchased by households in Canada, including the groceries households buy, the electricity and water rates they pay, the haircuts they get, the hotels they stay in, etc. In CPI calculations, each item in the households' consumption basket receives a relative importance, or basket weight, that represents the proportion households' spend on each item. For example, a much larger share of Canadians' spending goes to gasoline than to milk; therefore, gasoline will receive a larger weight than milk in the CPI basket. We rely on Canadian input-output tables to calibrate these weights and will provide a detailed discussion in Section 4.

[7]Savings can also be negative in our model, in which case the person h would decumulate her financial wealth to finance her consumption needs.

[8]We assume that these interest payments or receipts do not enter the household's consumption decision and thus generates no wealth effects on consumption.

3.2 Firms

The firm sector is populated by heterogeneous agents that represent all firms from every industry in the Canadian economy. The firm population of each industry s is obtained from business demography data, and the size distribution is chosen to approximately correspond to the firm size distribution in Canada (for details, see Section 4.2.2).

There are s ($s = 1, 2, \ldots, S$) industries populated with I_s firms in each industry. Each firm i ($i = 1, 2, \ldots, I = \sum_s I_s$) produces a principal product g ($g = 1, 2, \ldots, G$) using labor, capital, and intermediate inputs from other firms.[9]

3.2.1 Firms' expectations for aggregate economy

Firms' expectations regarding economic growth and inflation are formed using simple but optimal AR(1) forecasting heuristics.[10] Agents learn the optimal AR(1) rules with parameters consistent with two observable statistics, the sample mean and the first-order sample autocorrelation (Hommes and Zhu (2014)). Equation (15) summarizes the firm's expectations of the real GDP growth rate ($\gamma^e(t)$) and inflation ($\pi^e(t)$), measured by log first difference of the GDP deflator:

$$\gamma^e(t) = \mathrm{e}^{\alpha^{\gamma}(t-1)\gamma(t-1)+\beta^{\gamma}(t-1)+\epsilon^{\gamma}(t-1)} - 1 \tag{15a}$$

$$\pi^e(t) = \mathrm{e}^{\alpha^{\pi}(t-1)\pi(t-1)+\beta^{\pi}(t-1)+\epsilon^{\pi}(t-1)} - 1\,, \tag{15b}$$

where parameters $\alpha^{\gamma}(t-1)$, $\alpha^{\pi}(t-1)$, $\beta^{\gamma}(t-1)$, and $\beta^{\pi}(t-1)$ are re-estimated every period with the time series of output growth $\gamma(t')$ and inflation $\pi(t')$ where $t' = -T', -T'+1, -T'+2, \ldots, 0, 1, 2, \ldots, t-1$. $\epsilon^{\gamma}(t-1)$, and $\epsilon^{\pi}(t-1)$ are random shocks with zero mean and variance re-estimated every period from past observations over the last $T'+t-1$ periods.

In a complex environment of fundamental uncertainty and imperfect information, obtaining accurate forecasts is nearly impossible. Firms in such a complex environment may choose simple forecasting methods that closely monitor the relevant variables, even if such methods fail to understand the complete model of the economy. This approach fits within the concept of procedural rationality (Gigerenzer, 2015). A forecasting method that meets these requirements is the AR(1) forecasting rule: this is a simple procedure for projecting past trends into the future while its forecasting capabilities are relatively high.

Under adaptive learning, the gaps between expected and realized values of state variables will close gradually. This ensures that the unconditional mean and autocorrelations of the unknown non-linear stochastic process—which describe the actual law of motion of the model economy—concur with the unconditional mean and autocorrelations of the AR(1) process in the long run. In fact, adaptive learning with the AR(1) rule leads to convergence to a behavioral learning equilibrium (BLE) in the complex ABM economy, one of the simplest types of misspecification equilibrium put forth in the adaptive learning literature (Hommes and Zhu, 2014).

3.2.2 Production

In each period t, firm i from industry s produces real output ($Y_i(t)$) of a principal product g with Leontief technology that combines intermediate goods, labor and capital, labor input ($N_i(t)$, in the number of persons employed), real intermediate goods/services and raw materials ($M_i(t)$), as well as real capital ($K_i(t-1)$):

$$Y_i(t) = \min\left(Q^{\mathrm{s}}_i(t), \beta_i M_i(t), \alpha_i(t) N_i(t), \kappa_i K_i(t-1)\right), \tag{16}$$

where $\alpha_i(t)$ is the labor-specific productivity of firm $i \in I_s$ (see Equation (20)) and β_i and κ_i are productivity coefficients for intermediate inputs and capital, respectively.[11]

[9] We assume a one-to-one correspondence between the sets of industries s and products g, meaning that the n-th sector produces only the n-th good, and $S = G$. Formally, the correspondence between goods g being produced in sector s would be represented by a unity matrix.

[10] This modelling choice is comparable to other adaptive mechanisms, such as VAR expectations as used in the US Federal Reserve's FRB/US macroeconomic model (Brayton et al., 1997), or expectations according to an exponential moving average (EMA) model, as in Assenza et al. (2015).

[11] The assumption of a Leontief production technology is consistent with the data and is in line with the literature (Assenza et al., 2015). Moreover, as our explicit aim was to derive the simplest possible ABM that has the features we desire, we relegate all further extensions of the model, such as assumptions on technological progress that change technology coefficients, to further research.

Demand for labor Each firm i uses labor input $N_i(t)$ for production where employment is measured by the number of persons employed. Demand for labor in time t, $N_i^{\mathrm{d}}(t)$, is determined according to the firm's desired scale of activity ($Q_i^{\mathrm{s}}(t)$) and its average labor productivity ($\bar{\alpha}_i$):

$$N_i^{\mathrm{d}}(t) = \max\left(1, \mathrm{round}\left(\frac{\min\left(Q_i^{\mathrm{s}}(t), \kappa_i K_i(t-1)\right)}{\bar{\alpha}_i}\right)\right). \tag{17}$$

We introduce some simple heuristics here to characterize firms' behavior. We consider four different scenarios: (1) if the additional labor demand of firm i is less than a half-time position, labor demand is left unchanged; (2) if the additional production needs of the firm i exceed a half-time occupation, a new employee is hired; (3) if the operating workforce at the beginning of period t ($N_i(t-1)$), i.e., the number of persons employed in $t-1$, is higher than the desired workforce, the firm fires $N_i(t-1)-N_i^{\mathrm{d}}(t)$ randomly chosen employees (accounting for production constraints due possibly to a shortage of capital); and (4) if the demand for labor to reach the desired scale of activity is greater than the operating workforce, the firm posts labor vacancies.

The demand for new labor can be represented as:

$$V_i(t) = N_i^{\mathrm{d}}(t) - N_i(t-1). \tag{18}$$

Recalling the process of individuals' seeking employment, where whether vacancies are filled or not depends on the search-and-matching mechanism in the labor market (see Section 3.1.1), we obtain labur input:

$$N_i(t) \begin{cases} = N_i^{\mathrm{d}}(t) & \text{if firm } i \text{ successfully fills all vacancies, and} \\ < N_i^{\mathrm{d}}(t) & \text{if there are unfilled vacancies.} \end{cases} \tag{19}$$

As employees are either employed full-time, part-time, or work overtime, the average productivity of labor $\alpha_i(t)$ of firm i reflects the distribution of hours worked by all employees:

$$\alpha_i(t) = \bar{\alpha}_i \min\left(1.5, \frac{\min\left(Q_i^{\mathrm{s}}(t), \beta_i M_i(t-1), \kappa_i K_i(t-1)\right)}{N_i(t)\bar{\alpha}_i}\right), \tag{20}$$

where the maximum work effort is 150 percent of a full position (which is the maximum working time legally allowed in Canada for a limited duration). To remunerate increased or decreased work effort as compared to a full-time position, the average wage $\bar{w}_i$ of firm i is set accordingly:

$$w_i(t) = \bar{w}_i \min\left(1.5, \frac{\min\left(Q_i^{\mathrm{s}}(t), \beta_i M_i(t-1), \kappa_i K_i(t-1)\right)}{N_i(t)\bar{\alpha}_i}\right), \tag{21}$$

where $w_i(t)$ is the real wage paid by firm i.[12]

Demand for capital goods and business investment Each period, the i-th firm chooses real investment ($I_i^{\mathrm{d}}(t)$), which adjusts the real capital stock $K_i(t)$. As in standard DSGE models, capital adjustment in CANVAS is not immediate and is time-consuming.

New capital goods acquired at the time t become part of the capital stock only in the next period $t+1$. This makes capital a durable and sticky input. The desired business investment in period t is

$$I_i^{\mathrm{d}}(t) = \frac{\delta_i}{\kappa_i} \min\left(Q_i^{\mathrm{s}}(t), \kappa_i K_i(t-1)\right), \tag{22}$$

where δ_i is a firm-specific capital depreciation rate. The capital stock, as an aggregate of all goods g, evolves according to depreciation rates and the investment law of motion:

$$K_i(t) = K_i(t-1) - \frac{\delta_i}{\kappa_i} Y_i(t) + I_i(t). \tag{23}$$

The firms' business investment decision process is modelled by a simple heuristic that accounts for both expectations of the aggregate economy and firm-specific conditions. Each period, firm i observes realized demand and makes a forecast of future demand according to the expected rate of economic growth. Conditional on demand for business investment, the firm's capital stock adjusts accordingly when accounting for depreciation. Under adaptive learning, should realized growth rates surpass growth

[12] This differs from the nominal wage rate that affects households' disposable income; see Section 3.1.2

expectations, investment in subsequent periods will adapt to the approximate trend equilibrium level, and vice versa. As a result, the resulting trend of business investment tends to approximate the trend equilibrium path of this model economy.

We assume a homogeneous capital stock for all firms and thus fixed weights b_g^{CF}, namely, each firm i—irrespective of the sector s firm i is part of—demands $b_g^{\mathrm{CF}} I_i^{\mathrm{d}}(t)$ as its real investment from firms producing good g:

$$I_{ig}^{\mathrm{d}}(t) = b_g^{\mathrm{CF}} I_i^{\mathrm{d}}(t)\,. \tag{24}$$

Realized business investment depends on the search-and-matching process on the capital goods market (see Section 3.2.3):

$$I_i(t) \begin{cases} = \sum_g I_{ig}^{\mathrm{d}}(t) & \text{if the firm successfully realized the investment plan, and} \\ < \sum_g I_{ig}^{\mathrm{d}}(t) & \text{if all firms visited could not satisfy its demand} \end{cases} \tag{25}$$

In the case where firm i cannot realize its investment plan, it will have to scale down future activity (see Equation (16)).

Demand for intermediate inputs Each firm needs intermediate inputs for production. We assume that firm i holds an inventory stock of input goods $M_i(t)$ (in real terms) for each type of good g. Each period, the firm follows a heuristic of maintaining its inventory stock in positive supply and choosing the desired amount of intermediate goods and raw materials ($\Delta M_i^{\mathrm{d}}(t)$) to avoid shortfalls of material input that would impede production:

$$\Delta M_i^{\mathrm{d}}(t) \;=\; \frac{\min\left(Q_i^{\mathrm{s}}(t), \kappa_i K_i(t-1)\right)}{\beta_i} \tag{26}$$

where the sector-specific technology, a_{sg}, is given by

$$\Delta M_{ig}^{\mathrm{d}}(t) \;= a_{sg} \Delta M_i^{\mathrm{d}}(t) \quad \forall i \in I_s\,. \tag{27}$$

The realized demand for intermediate goods depends on a search-and-matching process (see Section 3.2.3):

$$\Delta M_i(t) \begin{cases} = \sum_g \Delta M_{ig}^{\mathrm{d}}(t) & \text{if the firm successfully realized its plan, and} \\ < \sum_g \Delta M_{ig}^{\mathrm{d}}(t) & \text{if all firms visited could not satisfy its demand.} \end{cases} \tag{28}$$

If firm i does not succeed in acquiring the materials it intends to purchase, it will be limited in its production possibilities. The inventory stock of intermediate inputs evolves according to the material use in production and realized acquisitions of new intermediate goods:

$$M_i(t) = M_i(t-1) - \frac{Y_i(t)}{\beta_i} + \Delta M_i(t)\,. \tag{29}$$

For simplicity, we assume that the raw material stock does not depreciate since firms have a steady use.

3.2.3 Sales

Demand for products of firm i consist of three sources: (1) final consumption goods, (2) capital goods, as well as (3) material or intermediate input goods. Firms face uncertainty regarding the main determinants of their individual success on the market: future sales, market prices, the availability of inputs for the production process (labor, capital, intermediate inputs), wages, cash flow, and their access to external finance, among others, are unknown. Consequently, each firm has to form expectations about the future that may not correspond to actual realizations. We elaborate on the modelling of these aspects in the following section.

Firms face demand from other agents, including (1) an individual h, (2) the government j, and (3) another firm demanding capital or intermediate input goods. Consumption demand $Q_i^{\mathrm{d}}(t)$ is determined only after the firm has set its price and carried out production plan $Y_i(t)$. The process of firm i generating sales is modelled by a search-and-matching mechanism specifying the product supply relative to the demand of consumers:

$$Q_i^{\mathrm{d}}(t) \begin{cases} < Y_i(t) + S_i(t-1) & \text{if demand from consumers is smaller than supply from firm } i, \\ = Y_i(t) + S_i(t-1) & \text{if demand from consumers exactly matches supply from firm } i, \text{ and} \\ > Y_i(t) + S_i(t-1) & \text{if demand from consumers is larger than supply from firm } i, \end{cases} \tag{30}$$

where $S_i(t-1)$ is the inventory of finished goods.

In this search-and-matching process, every consumer searches for the best bargain for each of products g to satisfy its demand. The best bargain is characterized as a successful matching at a product's lowest market price. The consumption demand and supply in the model are formed within the firm's network: in each period, consumers visit a number of randomly chosen foreign or domestic firms that sell the good g (see Section 3.6.1 for details on foreign firms). The total probability of firm i being selected in this process, $pr_i^{\mathrm{cum}}(t)$, is the average of two probabilities: (1) the probability of firm i being selected due to its offering price $pr_i^{\mathrm{price}}(t)$; and (2) the probability of being chosen conditional on firm i's relative size to other firms $pr_i^{\mathrm{size}}(t)$:

$$\begin{aligned} pr_i^{\mathrm{price}}(t) &= \frac{e^{-2P_i(t)}}{\sum_{i \in I_{s=g}} e^{-2P_i(t)}} \\ pr_i^{\mathrm{size}}(t) &= \frac{Y_i(t)}{\sum_{i \in I_{s=g}} Y_i(t)} \\ pr_i^{\mathrm{cum}}(t) &= \frac{pr_i^{\mathrm{price}}(t) + pr_i^{\mathrm{size}}(t)}{2} \end{aligned}$$

where $Y_i(t)$ is the production of goods by firm i (see Equation (16)). Since the probability of a firm being selected is a function of the size and offering price, a firm charging a relatively lower price than its competitors is more likely to be picked by consumers and generate sales. Similarly, a larger firm tends to have a higher probability to be picked by consumers. After the consumer identifies the firm to purchase from, it satisfies all its demand with the first firm. When the consumer discovers that the most preferred firm is in short supply, she resorts to the remaining firms to ensure all demand can be met by a firm. If a consumer does not succeed in satisfying her demand for a specific product g, she saves involuntarily within this period.

Thus sales are the realized demand, which is the lesser of (1) firm i's total supply of goods, $Q_i^{\mathrm{o}}(t)$, and (2) consumer demand, $Q_i^{\mathrm{d}}(t)$:

$$Q_i(t) = \min(Q_i^{\mathrm{o}}(t), Q_i^{\mathrm{d}}(t)), \tag{31}$$

where $Q_i^{\mathrm{o}}(t) = Y_i(t) + S_i(t-1)$ denotes total supply at time t, which consists of production in the current period, $Y_i(t)$ and the inventory from last period, $S_i(t-1)$.

As a reflection of firms' expectation error concerning demand, "excess supply" is defined as the difference between production and sales:

$$\Delta S_i(t) = (1 - \delta_i^{\mathrm{S}})(Y_i(t) - Q_i(t)), \tag{32}$$

where δ_i^{S} is the depreciation rate of inventories. The stock of inventories evolves as excess supply accumulates over time, following

$$S_i(t) = S_i(t-1) + \Delta S_i(t). \tag{33}$$

At the next period, accumulated inventories are combined together with newly produced goods as goods supply.

3.2.4 Price and quantity setting

Firms set prices and determine supply based on the expectations for aggregate economic growth and inflation, as well as market expectations for the specific product they supply g.

Aggregate expectations under behavioral learning equilibrium Firms' expectations of the real GDP growth rate ($\gamma^e(t)$) and inflation ($\pi^e(t)$) are measured by log first difference of the GDP deflator. Consider an example of excess demand at time t where a firm realizes that the realized growth rates have exceeded its expected growth expectations last period. Recognizing its forecasting error from last period, it will start increasing production to meet the demand. Production is increased to a point where supply and demand will converge to equilibrium. In contrast, when firms discover they have excess supply where realized growth falls short of expectations, firms will gradually reduce production in order to avoid excessive inventories. Similar to production quantity adjustment, firms can also adjust their prices to facilitate the convergence to equilibrium.

Should a smaller or larger shock—such as an (endogenous) bankruptcy of a firm or an exogenous demand or supply shock (e.g., the COVID-19 pandemic or an export shock)—pull the economy off the trend, path dependencies might ensue that change the long-term BLE of this model economy. However, after the medium to long turn, adaptive learning will steer the model toward this new BLE, as can be observed in our applications to the economic effects of the lockdown following COVID-19 pandemic (see Section 6).

Firm-specific expectations with simple heuristics In addition to setting their expectations of the general economy (in terms of GDP growth and inflation), firm i also chooses to alter their previous period's quantity or price based on their perceived market conditions. We assume that due to asymmetric information and search costs, each firm has a certain degree of pricing power in their local market, so that the law of one price does not apply. We also assume that firms cannot change their quantity and price at the same time.[13]

Firms form their expectations of market conditions using two indicators that both rely on observed information from the previous period $t-1$: (1) the level of excess supply, which is the difference between the previous period's supply $Q_i^o(t-1)$ and realized demand $Q_i^d(t-1)$; and (2) the deviation of the firm's price $P_i(t-1)$ from the average price among competitors $\bar{P}_g(t-1)$ for product g. Each firm considers four possible price-quantity setting scenarios (depicted in Figure 2):

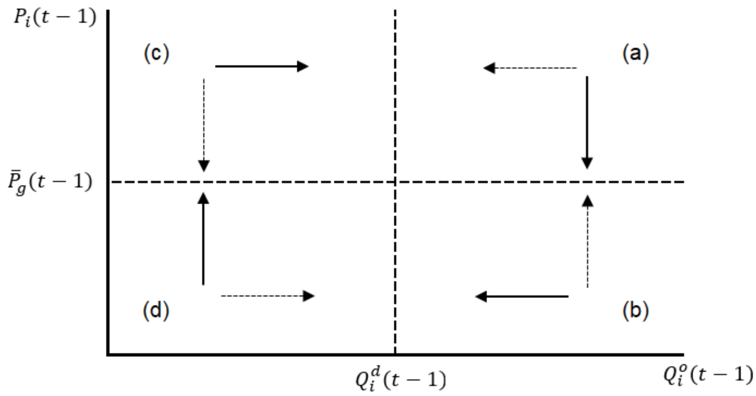
a) If there is excess supply and the charged price is higher than market average, reduce the price and maintain the production.

b) If there is excess supply and the charged price is lower than market average, maintain the price and reduce the production.

c) If there is excess demand and the charged price is higher than market average, maintain the price and increase the production.

d) If there is excess demand and the charged price is lower than market average, increase the price and maintain the production.

Figure 2: Firm quantity and price setting



Effectively, the growth rate of quantity only varies from zero in scenarios (b) and (c):

$$\gamma_i^d(t) = \begin{cases} \frac{Q_i^d(t-1)}{Q_i^o(t-1)} - 1 & \text{if } Q_i^o(t-1) \leq Q_i^d(t-1) \text{ and } P_i(t-1) \geq \bar{P}_g(t-1) \\ & \text{or if } Q_i^o(t-1) > Q_i^d(t-1) \text{ and } P_i(t-1) < \bar{P}_g(t-1) \\ 0 & \text{otherwise.} \end{cases} \tag{34}$$

Firm i's supply choice depends on its expectations of the aggregate economy and individual supply/demand versus pricing power. In our model, it is driven by three sources: (1) its product supply

[13]This modelling choice is motivated by empirical surveys of managers' pricing and quantity decisions; see, e.g., Kawasaki et al. (1982) and Bhaskar et al. (1993).

from last period $Q_i^{\rm o}(t-1)$; (2) the expected, economy-wide economic growth rate ($\gamma^e(t)$); and (3) the realized, firm-specific growth rate of quantity from the previous period, $\gamma_i^d(t)$:

$$Q_i^{\rm s}(t) = Q_i^{\rm o}(t-1)(1+\gamma^e(t))(1+\gamma_i^d(t))\,. \tag{35}$$

Similar to Equation (34), firms adjust their prices in the following manner:

$$\pi_i^d(t) = \begin{cases} \frac{Q_i^d(t-1)}{Q_i^{\rm o}(t-1)} - 1 & \text{if } Q_i^{\rm o}(t-1) \leq Q_i^d(t-1) \text{ and } P_i(t-1) < \bar{P}_g(t-1) \\ & \text{or if } Q_i^{\rm o}(t-1) > Q_i^d(t-1) \text{ and } P_i(t-1) \geq \bar{P}_g(t-1) \\ 0 & \text{otherwise.} \end{cases} \tag{36}$$

Inflation determination With some aggregation and simplification, firm i's nominal price $P_i(t)$ in our model can be decomposed to three components: (1) expectations of economy-wide inflation $\pi^e(t)$ (aggregate inflation expectation); (2) the cost structure of the firm $\pi_i^c(t)$ (cost-push inflation); and (3) the change from the previous period's price $\pi_i^d(t)$ (demand-pull inflation):

$$P_i(t) = P_i(t-1)\cdot \underbrace{(1+\pi_i^d(t))}_{\substack{\text{Demand-pull}\\ \text{inflation}}} \cdot \underbrace{(1+\pi_i^c(t))}_{\substack{\text{Cost-push}\\ \text{inflation}}} \cdot \underbrace{(1+\pi^e(t))}_{\substack{\text{Aggregate}\\ \text{inflation expectation}}} \tag{37}$$

We define the cost-push inflation $\pi_i^c(t)$ as:

$$\pi_i^c(t) = \underbrace{\frac{(1+\tau^{SIF})\bar{w}_i}{\bar{\alpha}_i}\left(\frac{\bar{P}^{HH}(t-1)}{\bar{P}_{g=s}(t-1)} - 1\right)}_{\text{Unit labor costs}} + \underbrace{\frac{1}{\beta_i}\left(\frac{\sum_g a_{sg}\bar{P}_g(t-1)}{\bar{P}_{g=s}(t-1)} - 1\right)}_{\text{Unit production material costs}} + \underbrace{\frac{\delta_i}{\kappa_i}\left(\frac{\bar{P}^{CF}(t-1)}{\bar{P}_{g=s}(t-1)} - 1\right)}_{\text{Unit capital costs}} \quad \forall i \in I_s \tag{38}$$

where, $\bar{\alpha}_i$ indicates the average labor productivity and $\bar{w}_i$ is the average real wage, defined as gross wages, which include both salary costs and employers' contributions to social insurance charged with a rate τ^{SIF}. $\frac{1}{\beta_i}\sum_g a_{sg}$ is real unit expenditures on intermediate production input by industry s on good g, weighted by the average product price index $\bar{P}_g(t)$ (see Equation (44)). δ_i/κ_i are unit capital costs, conditional on the average price of capital goods ($\bar{P}^{CF}(t)$) and capital depreciation relative to productivity growth (where δ_i is the firm-specific capital depreciation rate and κ_i is the productivity coefficient for capital).

3.2.5 Financial linkage

External finance The firm's expectation for its profit is determined with a heuristic:

$$\Pi_i^{\rm e}(t) = \Pi_i(t-1)(1+\gamma^{\rm e}(t))(1+\pi^{\rm e}(t))\,, \tag{39}$$

where firm i's expected profit $Pi_i^{\rm e}(t)$ is based on (1) the profit in the previous period,$Pi_i(t-1)$; (2) the expected, economy-wide economic growth rate ($\gamma^e(t)$); and (3) aggregate inflation expectations $\pi^e(t)$ ("n").

Thus, each firm i forms an expectation on its future cash flow $\Delta D_i^{\rm e}(t)$, that is, the expected change of deposits $D_i(t)$:

$$\Delta D_i^{\rm e}(t) = \underbrace{\Pi_i^{\rm e}(t)}_{\text{Exp. profit}} - \underbrace{\theta L_i(t-1)}_{\text{Debt instalment}} - \underbrace{\tau^{\rm FIRM}\max(0,\Pi_i^{\rm e}(t))}_{\text{Corporate taxes}} - \underbrace{\theta^{\rm DIV}(1-\tau^{\rm FIRM})\max(0,\Pi_i^{\rm e}(t))}_{\text{Dividend payout}}\,, \tag{40}$$

where θ is the rate of debt instalment on firm i's outstanding loans $L_i(t-1)$, $\tau^{\rm FIRM}$ is the corporate tax rate, and $\theta^{\rm DIV}$ is the dividend payout ratio. If the internal financing from profits are insufficient to finance the firm's expenditures, it seeks external financial resources to finance current or future expenditures. New credit (in form of bank loans) $\Delta L_i^{\rm d}(t)$ is determined as:

$$\Delta L_i^{\rm d}(t) = \max(0, -\Delta D_i^{\rm e}(t) - D_i(t-1))\,. \tag{41}$$

The availability of bank credit depends on the capitalization of the banking sector and the arrival of firms to ask for a loan (see Section 3.5.1 for details). If the firm cannot obtain a loan on the credit market, it might become credit constrained (see Equation (70)). If the firm does not obtain the desired loan, it may become insolvent (see Equation (49)).

Accounting Firm profits $\Pi_i(t)$ are an accounting measure that is defined as revenues from sales plus changes in inventories minus expenditures on labor, material, depreciation, interest payments, and taxes:

$$\begin{aligned}\Pi_i(t) = &\underbrace{P_i(t)Q_i(t)}_{\text{Sales}} + \underbrace{P_i(t)\Delta S_i(t)}_{\text{Inventory change}} - \underbrace{(1+\tau^{\text{SIF}})w_i(t)N_i(t)\bar{P}^{\text{HH}}(t)}_{\text{Labor costs}} \\ &- \underbrace{\frac{1}{\beta_i}\bar{P}_i(t)Y_i(t)}_{\text{Material costs}} - \underbrace{\frac{\delta_i}{\kappa_i}P_i^{\text{CF}}(t)Y_i(t)}_{\text{Depreciation}} - \underbrace{\tau_i^{\text{Y}}P_i(t)Y_i(t) - \tau_i^{\text{K}}P_i(t)Y_i(t)}_{\text{Net taxes/subsidies on products/production}} \\ &- \underbrace{r(t)(L_i(t-1) + \max(0, -D_i(t-1)))}_{\text{Interest payments}} + \underbrace{\bar{r}(t)\max(0, D_i(t-1))}_{\text{Interest received}},\end{aligned} \tag{42}$$

where $r(t)$ is the interest rate paid on outstanding loans (see Equation (72)). $\bar{P}_i(t)$ and $P_i^{\text{CF}}(t)$ are the actual prices paid by firm i for intermediate goods and investment in capital goods, respectively, which are both an outcome of the search-and-matching process. $\bar{P}^{\text{HH}}(t)$ is the consumer price index (CPI):

$$\bar{P}^{\text{HH}}(t) = \sum_g b_g^{\text{HH}}\bar{P}_g(t), \tag{43}$$

where b_g^{HH} is the household consumption coefficient for product g and $\bar{P}_g(t)$ is the producer price index for the principal good g:

$$\bar{P}_g(t) = \frac{\sum_{i\in I_{s=g}} P_i(t)Q_i(t) + P_{m=g}(t)Q_{m=g}(t)}{\sum_{i\in I_{s=g}} Q_i(t) + Q_{m=g}(t)}. \tag{44}$$

Firm net cash flow reflects the amount of liquidity moving in or out of its deposit account:

$$\begin{aligned}\Delta D_i(t) = &\underbrace{P_i(t)Q_i(t)}_{\text{Sales}} - \underbrace{(1+\tau^{\text{SIF}})w_i(t)N_i(t)\bar{P}^{\text{HH}}(t)}_{\text{Labor costs}} - \underbrace{\bar{P}_i(t)\Delta M_i(t)}_{\text{Material costs}} \\ &- \underbrace{\tau_i^{\text{Y}}P_i(t)Y_i(t) - \tau_i^{\text{K}}P_i(t)Y_i(t)}_{\text{Net taxes/subsidies on products and production}} - \underbrace{\tau^{\text{FIRM}}\max(0, \Pi_i(t))}_{\text{Corporate tax payments}} \\ &- \underbrace{\theta^{\text{DIV}}(1-\tau^{\text{FIRM}})\max(0, \Pi_i(t))}_{\text{Dividend payments}} - \underbrace{r(t)(L_i(t-1) + \max(0, -D_i(t-1)))}_{\text{Interest payments}} \\ &+ \underbrace{\bar{r}(t)\max(0, D_i(t-1))}_{\text{Interest received}} - \underbrace{P_i^{\text{CF}}(t)I_i(t)}_{\text{Investment costs}} + \underbrace{\Delta L_i(t)}_{\text{New credit}} - \underbrace{\theta L_i(t-1)}_{\text{Debt instalment}}.\end{aligned} \tag{45}$$

Furthermore, firm i pays interest on outstanding loans and overdrafts on firm i's deposit account (in case $D_i(t) < 0$) at the same rate $r(t)$, which includes the bank's markup rate. In the opposite case, when the firm holds (positive) deposits with the bank (i.e., $D_i(t) > 0$), the interest rate received is lower and corresponds to the policy rate set by the central bank (see Section 3.5).

Firm deposits are then previous deposits plus net cash flow:

$$D_i(t) = D_i(t-1) + \Delta D_i(t). \tag{46}$$

Similarly, overall debt is updated as follows:

$$L_i(t) = (1-\theta)L_i(t-1) + \Delta L_i(t). \tag{47}$$

Finally, firm equity $E_i(t)$ evolves as the balancing item on the firm's balance sheet, where all stocks are accounted for mark-to-market:

$$E_i(t) = D_i(t) + \sum_g a_{sg}\bar{P}_g(t)M_i(t) + P_i(t)S_i(t) + \bar{P}^{\text{CF}}(t)K_i(t) - L_i(t) \quad \forall i \in I_s. \tag{48}$$

$\bar{P}^{\text{CF}}(t)$ is the capital goods price index (CGPI), defined as

$$\bar{P}^{\text{CF}}(t) = \sum_g b_g^{\text{CF}}\bar{P}_g(t), \tag{49}$$

where b_g^{CF} is the capital formation coefficient for product g.

Insolvency If a firm is cash flow insolvent (i.e., $D_i(t) < 0$) and balance sheet insolvent (i.e., $E_i(t) < 0$) at the same time, it goes bankrupt. For simplicity, we maintain the total firms' distribution by assuming that the bankrupted firm is replaced by a new firm that enters the market in the same period. We assume that the real capital stock of the bankrupt firm is left to the entrant firm at zero costs, but that the new firm has to take over a part of the bankrupt firm's liabilities. Therefore, a part of loans taken out by the bankrupt firm is written off so that the remaining liabilities of firm i amount to a fraction ζ^{b} of its real capital stock. After this partial debt cancellation, the remaining liabilities of the bankrupt firm are transferred to the balance sheet of the entrant firm. In the next period $(t+1)$ liabilities of firm i are initialized with

$$L_i(t+1) = \zeta^{\mathrm{b}} \bar{P}_i^{\mathrm{CF}}(t) K_i(t) \tag{50}$$

and firm deposits with

$$D_i(t+1) = 0\,. \tag{51}$$

Correspondingly, in the next period $(t+1)$ equity of the new firm i is initialized with

$$E_i(t+1) = E_i(t) + \big(L_i(t) - D_i(t) - \zeta^{\mathrm{b}} \bar{P}_i^{\mathrm{CF}}(t) K_i(t)\big)\,. \tag{52}$$

3.3 Monetary Policy

The central bank (CB) sets the policy rate $\bar{r}(t)$ based on implicit inflation and growth targets, provides liquidity to the banking system (advances to the bank), and takes deposits from the bank in the form of reserves deposited at the central bank. Furthermore, the central bank purchases external assets (government bonds) and thus acts as a creditor to the government.

3.3.1 Policy rule

The policy rate is determined by an augmented Taylor rule (Taylor, 1993), where the central bank agent learns the optimal parameters. Following Blattner and Margaritov (2010), we include forecasted quarter-over-quarter inflation and real GDP growth in the reaction function:

$$\bar{r}(t) = \rho(t)\bar{r}(t-1) + (1-\rho(t-1))(r^*(t-1) + \pi^* + \xi^{\pi}(t-1)(\pi^e(t) - \pi^*) + \xi^{\gamma}(t-1)\gamma^e(t))\,, \tag{53}$$

where $\rho(t-1)$ is the interest rate smoothing parameter that reflects the gradual adjustment to the policy rate, $r^*(t-1)$ is the real equilibrium interest rate, π^* is the inflation target, $\xi^{\pi}(t-1)$ is the policy parameter on inflation deviations from the target, and $\xi^{\gamma}(t-1)$ is the weight on the forecasted real GDP growth rate. $\rho(t-1)$, $r^*(t-1)$, $\xi^{\pi}(t-1)$, and $\xi^{\gamma}(t-1)$ are re-estimated every period on time series of the real GDP growth rate $\gamma(t')$, inflation $\pi(t')$, and $\bar{r}(t')$ where $t' = -T', -T'+1, -T'+2, \ldots, 0, 1, 2, \ldots, t-1$. As initial conditions for $t' = -T', -T'+1, -T'+2, \ldots, 0$, we use the log differences of real GDP, the GDP deflator (inflation), as well as the Bank of Canada's policy interest rate.

3.3.2 Central bank balance sheet

The central bank's profits $\Pi^{\mathrm{CB}}(t)$ are computed as the difference between revenues from interest payments on government debt, as well as revenues ($D_k(t) < 0$) or costs ($D_k(t) > 0$) due to the net position in advances/reserves vis-à-vis the banking system:

$$\Pi^{\mathrm{CB}}(t) = r^{\mathrm{G}} L^{\mathrm{G}}(t-1) - \bar{r}(t) D_k(t-1)\,. \tag{54}$$

The central bank's equity $E^{\mathrm{CB}}(t)$ evolves according to its profits or losses and its past equity and is given by

$$E^{\mathrm{CB}}(t) = E^{\mathrm{CB}}(t-1) + \Pi^{\mathrm{CB}}(t)\,. \tag{55}$$

The net creditor/debtor position of the national economy to the rest of the world ($L^{\mathrm{RoW}}(t)$)[14] evolves according to the following law of motion:

$$L^{\mathrm{RoW}}(t) = L^{\mathrm{RoW}}(t-1) + \underbrace{(1+\tau^{\mathrm{EXPORT}})\sum_l C_l(t)}_{\text{Exports}} - \underbrace{\sum_m P_m(t) Q_m(t)}_{\text{Imports}}\,. \tag{56}$$

[14] If $L^{\mathrm{RoW}}(t) < 0$, the national economy is a net creditor of the RoW; if $L^{\mathrm{RoW}}(t) > 0$, the national economy is a net debtor to the RoW.

Here, for example, a balance of trade surplus (deficit) enters with a negative (positive) sign, since $L^{\mathrm{RoW}}(t)$ is on the asset side of the CB's balance sheet. Thus a trade surplus (deficit), i.e., an inflow (outflow) of money into (out of) the national economy, would reduce (increase) national liabilities versus the RoW. Inherent stock-flow consistency relating to the accounting principles incorporated into our model implies that our financial system is closed via the accounting identity that connects the change in the amount of deposits in the banking system[15] to the government deficit (surplus)[16] and to the balance of trade:[17]

$$\begin{aligned} E^{\mathrm{CB}}(t) - L^{\mathrm{RoW}}(t) &= L^{\mathrm{G}}(t) - D_k(t) \\ &= L^{\mathrm{G}}(t) - \sum_{i=1}^{I} D_i(t) - \sum_{h=1}^{H} D_h(t) - E_k(t) + \sum_{i=1}^{I} L_i(t) \,. \end{aligned} \tag{57}$$

3.4 Fiscal Policy

The government sector is modelled after a large welfare state. Effectively, in our model, the government takes two functions: as a consumer on the retail market (government consumption), and as a redistributive entity that levies taxes and social contributions to provide social services and benefits to its citizens. We assume that government consumption is exogenous and attributed to individual government entities. Government expenditures, revenues, the deficit, and the public debt, however, are accounted for at the aggregate level (i.e., for the general government).

3.4.1 Government expenditure

Individual government entities j ($j = 1, 2, \ldots, J$) participate in the goods market as consumers. These entities represent the central government, state government, local governments, and social security funds. The growth rate of real final consumption expenditure of the general government ($C^{\mathrm{G}}(t)$) is assumed to follow an autoregressive process of lag order one (AR(1)):

$$\gamma^G(t) = \alpha^{\gamma,\mathrm{G}} \gamma^{\mathrm{G}}(t-1) + \beta^{\gamma,\mathrm{G}} + \epsilon^{\gamma,\mathrm{G}}(t-1) \,, \tag{58}$$

where $\epsilon^{\gamma,\mathrm{G}}(t-1)$ is a random shock with zero mean and variance that takes the extent to which the shocks are common to the other exogenous variables, as reflected by the covariance matrix C, into account, so that

$$C^{\mathrm{G}}(t) = C^{\mathrm{G}}(t-1) \mathrm{e}^{\gamma^G(t)} \,. \tag{59}$$

Similarly, the government's expected growth of prices is defined as

$$\pi^G(t) = \alpha^{\pi,\mathrm{G}} \pi^{\mathrm{G}}(t-1) + \beta^{\pi,\mathrm{G}} + \epsilon^{\pi,\mathrm{G}}(t-1) \,, \tag{60}$$

where $\epsilon^{\pi,\mathrm{G}}(t-1)$ is again a random shock with zero mean and variance that takes the extent to which the shocks are common to the other exogenous variables, as reflected by the covariance matrix C, into account. Thus, the government price index evolves in the following manner:

$$P^{\mathrm{G}}(t) = P^{\mathrm{G}}(t-1) \mathrm{e}^{\pi^G(t)} \,. \tag{61}$$

The total nominal government consumption demand is then uniformly distributed to the J government entities and attributed to goods g:

$$C_j^{\mathrm{d}}(t) = \frac{C^{\mathrm{G}}(t) \sum_g c_g^{\mathrm{G}} \bar{P}_g(t-1)(1 + \pi^{\mathrm{e}}(t))}{J} \,, \tag{62}$$

and the consumption budget of the j-th government entity to purchase the g-th good is given by

$$C_{jg}^{\mathrm{d}}(t) = \frac{c_g^{\mathrm{G}} \bar{P}_g(t-1)}{\sum_g c_g^{\mathrm{G}} \bar{P}_g(t-1)} C_j^{\mathrm{d}}(t) \,, \tag{63}$$

[15] These changes in the amount of deposits in the banking system directly correspond to changes in net central bank reserves $D_k(t)$, which in turn depend the private sector's surplus or deficit in relation to both the government and the RoW.

[16] Financial flows relating to a deficit (surplus) on the part of the government sector either accrue to (are paid by) the private sector (households and firms), or have to flow to (in from) the RoW, in the first case by increasing (decreasing) deposits, and in the second case by increasing (decreasing) D^{ROW}.

[17] A positive (negative) balance of trade will either increase (decrease) deposits held by the private sector, or reduce (increase) the amount of government debt by, e.g., reducing (increasing) the amount of government deficit.

where c_g^{G} is the fraction of goods of type g demanded by the government. Realized government consumption is then another outcome of the search-and-matching process on the consumption goods market:

$$C_j(t) \begin{cases} = \sum_g C_{jg}^{\mathrm{d}}(t) & \text{if the government successfully realized the consumption plan, and} \\ < \sum_g C_{jg}^{\mathrm{d}}(t) & \text{if all firms visited could not satisfy its demand.} \end{cases} \tag{64}$$

Other expenditures of the general government include interest payments, social benefits other than social transfers in kind, and subsidies. Interest payments by the general government are made with a fixed average interest rate r^{G} on loans taken out by the government $L^{\mathrm{G}}(t-1)$. Social transfers by the government consist of social benefits for inactive households ($\sum_{h \in H^{\mathrm{inact}}} sb^{\mathrm{inact}}(t)$) such as pension payments or social exclusion benefits, social benefits for any household h ($\sum_h sb^{\mathrm{other}}(t)$) such as relating to family, sickness, or housing, and unemployment benefits for unemployed households ($\sum_{h \in H^{\mathrm{U}}(t)} w_h(t)$). Subsidies are paid to firms with subsidy rates (uniform for each industry, but different across industries) on products and production and are incorporated in the net tax rates on products (τ_i^{Y}) and production (τ_i^{K}), respectively.[18]

3.4.2 Government revenues

Revenues of the general government are generated through taxes, social contributions, and other transfers from all sectors:

$$\begin{aligned} Y^{\mathrm{G}}(t) = & \underbrace{(\tau^{\mathrm{SIF}} + \tau^{\mathrm{SIW}})\bar{P}^{\mathrm{HH}}(t) \sum_{h \in H^{\mathrm{E}}(t)} w_h(t)}_{\text{Social security contributions}} + \underbrace{\tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\bar{P}^{\mathrm{HH}}(t) \sum_{h \in H^{\mathrm{E}}(t)} w_h(t)}_{\text{Labor income taxes}} \\ & + \underbrace{\tau^{\mathrm{VAT}} \sum_h C_h(t)}_{\text{Value added taxes}} + \underbrace{\tau^{\mathrm{INC}}(1-\tau^{\mathrm{FIRM}})\theta^{\mathrm{DIV}} \left(\sum_i \max(0,\Pi_i(t)) + \max(0,\Pi_k(t)) \right)}_{\text{Capital income taxes}} \\ & + \underbrace{\tau^{\mathrm{FIRM}} \left(\sum_i \max(0,\Pi_i(t)) + \max(0,\Pi_k(t)) \right)}_{\text{Corporate income taxes}} + \underbrace{\tau^{\mathrm{CF}} \sum_h I_h(t)}_{\text{Taxes on capital formation}} \\ & + \underbrace{\sum_i \tau_i^{\mathrm{Y}} P_i(t) Y_i(t)}_{\text{Net taxes/subsidies on products}} + \underbrace{\sum_i \tau_i^{\mathrm{K}} P_i(t) Y_i(t)}_{\text{Net taxes/subsidies on production}} + \underbrace{\tau^{\mathrm{EXPORT}} \sum_l C_l(t)}_{\text{Export taxes}} . \end{aligned} \tag{65}$$

3.4.3 Government deficit

The government deficit (or surplus) resulting from its redistributive activities is

$$\begin{aligned} & \Pi^{\mathrm{G}}(t) \\ & = \underbrace{\sum_{h \in H^{\mathrm{inact}}} \bar{P}^{\mathrm{HH}}(t) sb^{\mathrm{inact}}(t) + \sum_{h \in H^{\mathrm{U}}(t)} \bar{P}^{\mathrm{HH}}(t)\theta^{\mathrm{UB}} w_h(t) \left(1 - \tau^{\mathrm{SIW}} - \tau^{\mathrm{INC}}(1-\tau^{\mathrm{SIW}})\right) + \sum_h \bar{P}^{\mathrm{HH}}(t) sb^{\mathrm{other}}(t)}_{\text{Social benefits and transfers}} \\ & \quad + \underbrace{\sum_j C_j(t)}_{\text{Government consumption}} + \underbrace{r^{\mathrm{G}} L^{\mathrm{G}}(t-1)}_{\text{Interest payments}} - \underbrace{Y^{\mathrm{G}}(t)}_{\text{Government revenues}} . \end{aligned} \tag{66}$$

3.4.4 Government debt

The government debt as a stock variable is determined by the year-to-year deficits/surpluses of the government sector:

$$L^{\mathrm{G}}(t) = L^{\mathrm{G}}(t-1) + \Pi^{\mathrm{G}}(t) . \tag{67}$$

For reasons of model parsimony, we assume that the government sells its debt contracts to the central bank.

[18] The latter can therefore also have negative values if a sector receives more subsidies on products or production than it has to pay in taxes.

3.5 Financial System

For reasons of simplicity we assume that there is one representative financial intermediary for the Canadian economy.[19] The representative bank takes deposits from households and firms, extends loans to firms, and receives advances from (or deposits reserves at) the central bank. The interest rates for loans are set by a fixed markup on the policy rate. Capital of the banking sector grows or shrinks according to bank profits or losses and the write-off of bad debt. The bank is subject to macroprudential policies on both leverage and bank capital. Provision of loans is conditional on a minimum loan-to-value (LTV) ratio, and credit creation is limited by minimum capital requirements.

3.5.1 Provision of loans

The bank extends loans to firms according to a risk assessment of potential borrowers and is subject to a maximum LTV ratio and a minimum capital requirement imposed by the regulator. The bank has imperfect knowledge of the realized value of either its own equity capital or loans extended to the individual firm i. Similar to households and firms, the bank uses heuristics by forming expectations of equity capital ($E_k^{\mathrm{e}}(t)$) and loans as follows: ($\sum_{i=1}^{I}(L_i^{\mathrm{e}}(t)+\Delta L_i(t))$):

$$\frac{E_k^{\mathrm{e}}(t)}{\sum_{i=1}^{I}(L_i^{\mathrm{e}}(t)+\Delta L_i(t))}=\frac{E_k(t-1)}{\sum_{i=1}^{I}((1-\theta)L_i(t-1)+\Delta L_i(t))}\geq\zeta, \tag{68}$$

where $0<\zeta<1$ is a minimum capital requirement coefficient with $1/\zeta$ representing the maximum allowable leverage. $\Delta L_i(t)$ is the realized new loans to firm i in period t, which is equivalent to the new credit demanded by firms ($\Delta L_i^{\mathrm{d}}(t)$, see Equation (41)) if the bank capital requirements are met and the firm borrows within the maximum LTV (see Equation (69) and Equation (70)). When bank capital and leverage requirements are not met, no lending can take place.

Furthermore, the bank forms a risk assessment of a potential default on the part of firm i before granting a loan. This risk assessment is based on the borrower's leverage as measured by its LTV, i.e., the ratio of market value of loans over its capital stock. The bank will grant a loan to firm i only up to the point where the borrower's LTV ratio remains below a regulated maximum level at ζ^{LTV}. However, due to uncertainty, the bank has to form expectations of the value of firm i's capital stock ($K_i^{\mathrm{e}}(t)$):

$$\frac{L_i^{\mathrm{e}}(t)+\Delta L_i(t)}{K_i^{\mathrm{e}}(t)}=\frac{\overbrace{(1-\theta)L_i(t-1)}^{=L_i^{\mathrm{e}}(t)}+\Delta L_i(t)}{\underbrace{\bar{P}^{\mathrm{CF}}(t-1)\,(1+\pi^{\mathrm{e}}(t))\,K_i(t-1)}_{=K_i^{\mathrm{e}}(t)}}\leq\zeta^{\mathrm{LTV}}. \tag{69}$$

where $K_i^{\mathrm{e}}(t)$ denotes firm i's amount of available collateral and $L_i^{\mathrm{e}}(t)$ the amount of outstanding debt. The amount of new credit extended to firm i by the bank ($\Delta L_i(t)$) depends on the firm's credit demand, the bank's risk assessment regarding its own capital adequacy, and the leverage of firms requesting the loans:

$$\Delta L_i(t)\begin{cases}=\Delta L_i^{\mathrm{d}}(t) & \text{if the borrower's loan-to-value ratio (eq. (69))}\\ & \text{and capital requirements (eq. (68)) are satisfied}\\ <\Delta L_i^{\mathrm{d}}(t) & \text{otherwise.}\end{cases} \tag{70}$$

The order of arrival of firms at the bank is assumed to be random. A financially robust (low leverage) firm, which in principle deserves a large chunk of bank loans, may be denied credit if it arrives "too late" (i.e., after other less robust firms).

[19]This assumption of one representative bank is above all due to national accounting conventions. From national annual sector accounts, which determine the logic of financial flows between the aggregate sectors for our model (households, non-financial corporations, financial corporations, government, and the rest of the world), we obtain balance sheet positions (credit and debt) as well as interest payment flows between firms and the financial sector (banks) on an aggregate level. Since we do not have information on financial relations between individual firms (or industry sectors) and banks for this model, we have no empirically based method to determine credit and debt relations, acquisition and provision of credit, as well as interest payments, between individual firms (or industry sectors) and individual banks. Therefore, we account for credit relations and financial flows between individual firms and banks on an aggregate level for the banking sector, i.e., we assume a representative bank extending credit to individual firms according to the amount of firms' real capital stock, while we account for the value added generated by financial corporations in the real economy according to the logic of input-out tables as separate industries within the firm sector.

3.5.2 Banks' balance sheets

The bank's profits are computed as the difference between interest revenues from loans provisions (including overdrafts on deposit accounts incurred by firms and households ($D_{i,h}(t-1) < 0$)) and costs due to interest payments on deposits held with the bank by firms and households ($D_{i,h}(t-1) > 0$):

$$\Pi_k(t) = \underbrace{r(t)\left(\sum_{i=1}^{I} L_i(t-1) + \max(0, -D_i(t-1)\right) + r(t)\sum_{h=1}^{H} \max(0, -D_h(t-1)) + \bar{r}(t)\max(0, D_k(t-1))}_{\text{Interest received}}$$
$$\underbrace{-\,\bar{r}(t)\sum_{i=1}^{I} \max(0, D_i(t-1)) - \bar{r}(t)\sum_{h=1}^{H} \max(0, D_h(t-1)) - \bar{r}(t)\max(0, -D_k(t-1))}_{\text{Interest payments}} \tag{71}$$

Deposits are remunerated at the policy rate $\bar{r}(t)$, which we assume to be set exogenously by the central bank.[20] The effective interest rate $r(t)$ faced by firms for bank credit is determined by a fixed markup μ over the policy rate $\bar{r}(t)$:

$$r(t) = \bar{r}(t) + \mu\,. \tag{72}$$

Bank equity evolves according to bank profits or losses and is given by

$$E_k(t) = E_k(t-1) + \Pi_k(t) - \underbrace{\theta^{\text{DIV}}(1-\tau^{\text{FIRM}})\max(0, \Pi_k(t))}_{\text{Dividend payments}}$$
$$-\underbrace{\tau^{\text{FIRM}}\max(0, \Pi_k(t))}_{\text{Corporate taxes}} - \underbrace{\sum_{i \in I'}(L_i(t) - D_i(t) - \zeta^{\text{b}}\bar{P}_i^{\text{CF}}(t)K_i(t))}_{\text{Write-off of bad debt}}\,, \tag{73}$$

where I' is the set of insolvent borrowers, and we assume that outstanding overdraft of firm i's deposit account as well as a fraction $(1-\zeta^{\text{b}})\bar{P}_i^{\text{CF}}(t)K_i(t)$ of loans extended to firm i have to be written off from the bank's balance sheet. The bank's balance sheet includes (1) loans to firms (on the asset side); (2) deposits received (on the liability side), (3) equity capital; and (4) (net) central bank reserves held, $D_k(t)$, or advances obtained by the bank from the central bank:[21]

$$D_k(t) = \sum_{i=1}^{I} D_i(t) + \sum_{h=1}^{H} D_h(t) + E_k(t) - \sum_{i=1}^{I} L_i(t)\,. \tag{74}$$

3.6 Foreign Linkages

Foreign linkages of CANVAS are introduced in the form of trade linkage as in the Bank of Canada's main DSGE model, ToTEM (Corrigan et al., 2021). We simplify the modelling of the foreign economy by assuming one representative foreign firm in each sector supplies Canadian imported goods (raw materials, capital, and consumption goods). We also assume homogeneous consumers in the foreign economy who demand Canadian exports. Similar to ToTEM, we assume the foreign demand for Canadian exports and the supply of foreign goods to be exogenously given, since Canada is a small open economy. Canadian demand for imports is endogenous and is subject to supply constraint.

[20] For simplicity, we abstract from modelling the ELB in this version. We are currently working on an extension to consider the implications of extended monetary policy when incorporating the ELB.

[21] A positive net central bank reserve ($D_k(t) > 0$) implies the bank holds more central bank reserves than advances and is thus a net creditor to the central bank. In contrast, a negative net central bank reserve ($D_k(t) < 0$) implies that the bank is a net debtor to the central bank. The possibility of an inequality of advances and reserves, or, for that matter, an inequality of loans and deposits, is due to the fact that we do not explicitly distinguish between deposits and reserves for reasons of model parsimony. Rather, we use the central bank as a "clearing house" for flows of reserves and deposits between the national economy and the foreign economy (see Equation (57)).

3.6.1 Imports

We model, at the individual firm level, the Canadian economy as an open economy where a segment of the firm's sector participates in international trade. The growth rate of the supply of imports ($Y^{\mathrm{I}}(t)$) is assumed to follow an autoregressive process of lag order one (AR(1)):

$$\gamma^{\mathrm{I}}(t) = \alpha^{\gamma,\mathrm{I}}\gamma^{\mathrm{I}}(t-1) + \beta^{\gamma,\mathrm{I}} + \epsilon^{\gamma,\mathrm{I}}(t-1)\,, \tag{75}$$

where $\epsilon^{\gamma,\mathrm{I}}(t-1)$ is a random shock with zero mean. This means that the total supply of imports evolves in the following manner:

$$Y^{\mathrm{I}}(t) = Y^{\mathrm{I}}(t-1)\mathrm{e}^{\gamma^{\mathrm{I}}(t)}\,. \tag{76}$$

A representative foreign firm for each sector imports goods from the rest of the world and supplies them to domestic markets. Thus the m-th, ($m = 1, 2, \ldots, S$), foreign firm representing an industry s imports the principal product g:[22]

$$Y_m(t) = c^I_{g=s}Y^{\mathrm{I}}(t)\,, \tag{77}$$

where $c^I_{g=s}$ is the fraction of imported goods of type g as part of total imports.

As in Equation (76), import price growth also follows an AR(1) process:

$$\pi^{\mathrm{I}}(t) = \alpha^{\pi,\mathrm{I}}\pi^{\mathrm{I}}(t-1) + \beta^{\pi,\mathrm{I}} + \epsilon^{\pi,\mathrm{I}}(t-1)\,, \tag{78}$$

where $\epsilon^{\pi,\mathrm{I}}(t-1)$ is again a random shock with zero mean and estimated variance. Import price is determined exogenously:

$$P_m(t) = P_m(t-1)\mathrm{e}^{\pi^m(t)}\,. \tag{79}$$

Sales of imports are then the realized demand as an outcome of the search-and-matching process on the goods markets (i.e., the minimum amount of import demand that can satisfy import supply, and vice versa):

$$Q_m(t) = \min(Y_m(t), Q^d_m(t))\,, \tag{80}$$

where $Q^{\mathrm{d}}_m(t)$ is subject to the search-and-matching mechanism specifying the demand by consumers from foreign firm m:

$$Q^{\mathrm{d}}_m(t) \begin{cases} < Y_m(t) & \text{if demand from consumers is smaller than supply from foreign firm } m, \\ = Y_m(t) & \text{if demand from consumers exactly matches supply from foreign firm } m, \text{ and} \\ > Y_m(t) & \text{if demand from consumers is larger than supply from foreign firm } m. \end{cases} \tag{81}$$

3.6.2 Exports

The l-th (l = 1, 2, . . . , L) foreign agent, be it a foreign firm, household, or government entity, participates in the domestic goods market as a consumer. Total sales to these foreign consumers on domestic markets represent exports to the rest of the world. Analogous to imports, the growth rate of the total demand for exports ($C^{\mathrm{E}}(t)$) is assumed to follow an autoregressive process of lag order one (AR(1)):

$$\gamma^{\mathrm{E}}(t) = \alpha^{\gamma,\mathrm{E}}\gamma^{\mathrm{E}}(t-1) + \beta^{\gamma,\mathrm{E}} + \epsilon^{\gamma,\mathrm{E}}(t-1)\,, \tag{82}$$

where $\epsilon^{\gamma,\mathrm{E}}(t-1)$ is a random shock with zero mean. Total demand for exports thus becomes:

$$C^{\mathrm{E}}(t) = C^{\mathrm{E}}(t-1)\mathrm{e}^{\gamma^{\mathrm{E}}(t)}\,. \tag{83}$$

The growth rate of export prices is analogous to Equation (82):

$$\pi^{\mathrm{E}}(t) = \alpha^{\pi,\mathrm{E}}\pi^{\mathrm{E}}(t-1) + \beta^{\pi,\mathrm{E}} + \epsilon^{\pi,\mathrm{E}}(t-1)\,, \tag{84}$$

where $\epsilon^{\pi,\mathrm{E}}(t-1)$ is again a random shock with zero mean. Thus, the export price index is determined exogenously:

$$P^{\mathrm{E}}(t) = P^{\mathrm{E}}(t-1)\mathrm{e}^{\pi^{\mathrm{E}}(t)}. \tag{85}$$

[22] As for domestic firms, we suppose there is a one-to-one correspondence between the sets of industries s and products g, meaning that the n-th sector produces only the n-th good, and $S = G$.

Total demand for exports is uniformly distributed to L foreign consumers and attributed to good g. The demand for exported goods by the l-th foreign consumer is

$$C_l^d(t) = \frac{P^{\mathrm{E}}(t) \cdot C^{\mathrm{E}}(t)}{L} \tag{86}$$

and the demand for exports by the l-th foreign consumer to purchase the g-th good is given by

$$C_{lg}^{\mathrm{d}}(t) = \frac{c_g^{\mathrm{E}} \bar{P}_g(t-1)}{\sum_g c_g^{\mathrm{E}} \bar{P}_g(t-1)} C_l^{\mathrm{d}}(t) \tag{87}$$

where $c_g^{\mathrm{E}}(t)$ is the fraction of exports of type g goods.

Final realized consumption by foreign consumers is an outcome of the search-and-matching process on the goods market:

$$C^l(t) \begin{cases} = \sum_g C_{lg}^d(t) & \text{if the foreign consumer successfully realized the consumption plan, and} \\ < \sum_g C_{lg}^d(t) & \text{if none of Canadian exporting firms visited could satisfy its demand.} \end{cases} \tag{88}$$

3.7 Macroeconomic Aggregates

GDP in the model can be calculated by aggregating the value of all final goods and services produced and purchased by agents in the model in a given period. The nominal GDP and its components of each period t can be defined by three different approaches: (1) production; (2) expenditure based, and (3) income based:

$$\begin{aligned}
GDP(t) &= \underbrace{\sum_i \tau_i^{\mathrm{Y}} P_i(t) Y_i(t) + \sum_h \tau^{\mathrm{VAT}} C_h(t) + \sum_h \tau^{\mathrm{CF}} I_h(t) + \sum_j \tau^{\mathrm{G}} C_j(t) + \sum_l \tau^{\mathrm{EXPORT}} C_l(t)}_{\text{Taxes on products}} \\
&\quad + \underbrace{\sum_i (1-\tau_i^{\mathrm{Y}}) P_i(t) Y_i(t)}_{\text{Total sales of goods and services}} \; - \underbrace{\sum_i \frac{1}{\beta_i} \bar{P}_i(t) Y_i(t)}_{\text{Intermediate inputs}} \quad \text{(Production approach)} \\
&= \underbrace{\sum_h (1+\tau^{\mathrm{VAT}}) C_h(t)}_{\text{Household consumption}} + \underbrace{\sum_j (1+\tau^{\mathrm{G}}) C_j(t)}_{\text{Government consumption}} + \underbrace{\sum_h (1+\tau^{\mathrm{CF}}) I_h(t) + \sum_i P_i^{\mathrm{CF}}(t) I_i(t)}_{\text{Gross fixed capital formation}} \\
&\quad + \underbrace{\sum_i P_i(t)(Y_i(t) - Q_i(t)) + \bar{P}_i(t) \left(\Delta M_i(t) - \frac{1}{\beta_i} Y_i(t) \right)}_{\text{Changes in inventories}} \\
&\quad + \underbrace{\sum_l (1+\tau^{\mathrm{EXPORT}}) C_l(t)}_{\text{Exports}} - \underbrace{\sum_m P_m(t) Q_m(t))}_{\text{Imports}} \quad \text{(Expenditure approach)} \\
&= \underbrace{\sum_i \tau_i^{\mathrm{Y}} P_i(t) Y_i(t) + \sum_h \tau^{\mathrm{VAT}} C_h(t) + \sum_h \tau^{\mathrm{CF}} I_h(t) + \sum_j \tau^{\mathrm{G}} C_j(t) + \sum_l \tau^{\mathrm{EXPORT}} C_l(t)}_{\text{Taxes on products}} \\
&\quad + \underbrace{\sum_i P_i(t) Y_i(t) - (1+\tau^{\mathrm{SIF}}) \bar{P}^{\mathrm{HH}}(t) N_i(t) w_i(t) - \frac{1}{\beta_i} \bar{P}_i(t) Y_i(t) - \tau_i^{\mathrm{Y}} P_i(t) Y_i(t) - \tau_i^{\mathrm{K}} P_i(t) Y_i(t)}_{\text{Gross operating surplus and mixed income}} \\
&\quad + \underbrace{\sum_i (1+\tau^{\mathrm{SIF}}) \bar{P}^{\mathrm{HH}}(t) N_i(t) w_i(t)}_{\text{Compensation of employees}} + \underbrace{\sum_i \tau_i^{\mathrm{K}} P_i(t) Y_i(t)}_{\text{Net taxes on production}} \quad \text{(Income approach)}
\end{aligned} \tag{89}$$

The GDP deflator is the economy-wide average producer price of all goods and services produced and sold, where all individual prices and sales are determined on the agent level by our search-and-matching mechanism. In our model, the GDP deflator is defined as nominal GDP divided by real GDP:

$$\mathit{GDP\ deflator}(t) = \frac{\mathit{GDP}(t)}{\mathit{real\ GDP}(t)}\,, \tag{90}$$

where

$$\begin{aligned}\mathit{real\ GDP}(t) = &\sum_i \tau_i^{\mathrm{Y}} Y_i(t) + \sum_h \tau^{\mathrm{VAT}} \frac{C_h(t)}{\bar{P}_h(t)} + \sum_h \tau^{\mathrm{CF}} \frac{I_h(t)}{\bar{P}_h^{\mathrm{CF}}(t)} + \sum_j \tau^{\mathrm{G}} \frac{C_j(t)}{\bar{P}_j(t)} \\ &+ \sum_l \tau^{\mathrm{EXPORT}} \frac{C_l(t)}{\bar{P}_l(t)} + \sum_i (1-\tau_i^{\mathrm{Y}}) Y_i(t) - \sum_i \frac{1}{\beta_i} Y_i(t)\,.\end{aligned} \tag{91}$$

4 Data & Calibration

In this section, we discuss the calibration of CANVAS. We start with a discussion of the calibration procedure from the data sources, similar to that from Poledna et al. (2022), followed by the specification of the initial conditions.

Table 1: Statistics Canada data tables

Name	Identifier	CANSIM	Frequency	Start	End
Labour force characteristics by industry, annual	14100023	282-0008	Annual	1/1/76	1/1/21
Canadian Business Counts, with employees	14100304		Semi-annual	7/1/14	7/1/20
Employment by industry, Annual, Canada	14100202	281-0024	Annual	1/1/01	1/1/20
Symmetric input-output tables, summary level	36100084	381-0038	Annual	1/1/13	1/1/19
Flows and stocks of fixed non-residential capital, by sector of industry and type of asset, Canada	36100097	031-0006	Annual	1/1/61	1/1/20
National Balance Sheet Accounts	36100580	378-0121	Quarterly	1/1/90	7/1/21
Current and capital accounts - Households, Canada, quarterly	36100112	380-0072	Quarterly	1/1/61	7/1/21
Current and capital accounts - Corporations, Canada, quarterly	36100116	380-0076	Quarterly	1/1/61	7/1/21
Current and capital accounts - General governments, Canada, quarterly	36100118	380-0079	Quarterly	1/1/61	7/1/21
Canadian Classification of Functions of Government (CCOFOG) by consolidated government component	10100005	385-0041	Annual	1/1/08	1/1/20
Canadian Classification of Functions of Government (CCOFOG) by consolidated government component, Canada	10100005	385-0041	Quarterly	1/1/08	1/1/20
Gross domestic product, expenditure-based, Canada, quarterly	36100104	380-0064	Quarterly	1/1/61	7/1/21

Note: The identifier under which the respective data tables are available from Statistics Canada (such as, e.g., 36100104) are shown in the second column. Links to each of the data sources on the Statistics Canada website can be accessed through the series names.

4.1 Calibration

The model is calibrated to the Canadian economy for 39 reference quarters from 2010:Q1 to 2019:Q3. Parameters of the model are calibrated so that a period t is one quarter. For each quarter, a wide range of parameters and initial conditions are calibrated to the data so that the model replicates the state of the economy in terms of aggregate GDP, GDP components, and industry sizes.[23]

In general, the calibration procedure involves either taking parameter values directly from the data or calculating them from national accounting identities. For exogenous processes such as imports and exports, parameters are estimated from national accounts. The parameters of the model are summarized in Table 3 where parameter values are shown, as an example, for 2019:Q4. Calibrated sector-specific parameters reported in Table 4 match 19 industries based on the North American Industry Classification System (NAICS) (see Table 2).

Data are obtained from Statistics Canada and the Bank of Canada and include *(1) census and business demography*; *(2) labour force survey (3) input-output tables*; *(4) government statistics and sector accounts*; and *(5) national accounts (GDP and main components) and money market interest rates*. Additionally,

[23]Unlike standard agent-based modelling literature, our model is not calibrated to reproduce so-called stylized facts (moments) of time series but rather to reproduce (and forecast) the time series themselves.

Table 2: North American Industry Classification System (NAICS) Canada

Sector	Code	Short Code
Agriculture, forestry, fishing and hunting	BS110	11
Mining, quarrying, and oil and gas extraction	BS210	21
Utilities	BS220	22
Construction	BS230	23
Manufacturing	BS3A0	31-33
Wholesale trade	BS410	41
Retail trade	BS4A0	44-45
Transportation and warehousing	BS4B0	48-49
Information and cultural industries	BS510	51
Finance, insurance, real estate, rental and leasing and holding companies	BS5B0	52-53
Professional, scientific and technical services	BS540	54
Administrative and support, waste management and remediation services	BS560	56
Educational services	BS610	61
Health care and social assistance	BS620	62
Arts, entertainment and recreation	BS710	71
Accommodation and food services	BS720	72
Other services (except public administration)	BS810	81
Non-profit institutions serving households	NP000	NP000
Government sector	GS600	91

a number of parameters are calibrated according to *(6) statutory guidelines, financial regulations, and banking practices.*[24]. All data sources are reported in Table 1.

4.1.1 Census and business demography data

Parameters that specify the number of agents are taken directly from census and business demography data and are scaled appropriately. Several consolidated tables, including input-output tables, demographic data, and cross-classification tables, are compiled for Canada with a breakdown of 19 NAICS activities/products. Parameters concerning the numbers of firms in the s-th industry (I_s) are calibrated to the respective numbers in business demography data. Since business demography data are available annually, we calibrate I_s to the annual values for each reference quarter of a calendar year.

4.1.2 Labour force survey

Parameters related to the number of persons in the labor force (employed and unemployed) are obtained from the labour force survey and are scaled appropriately. The calibration of the labor force is aggregated based on the combination of different demographic groups: sex (males and females), age group (young (15–24), prime age (24–54), senior/older (55+)) within each industry. We calibrate H_s^{F15}, H_s^{F25}, H_s^{F55}, H_s^{M15}, H_s^{M25}, and H_s^{M55} to the annual values for each reference quarter of a calendar year.

4.1.3 Input-output tables

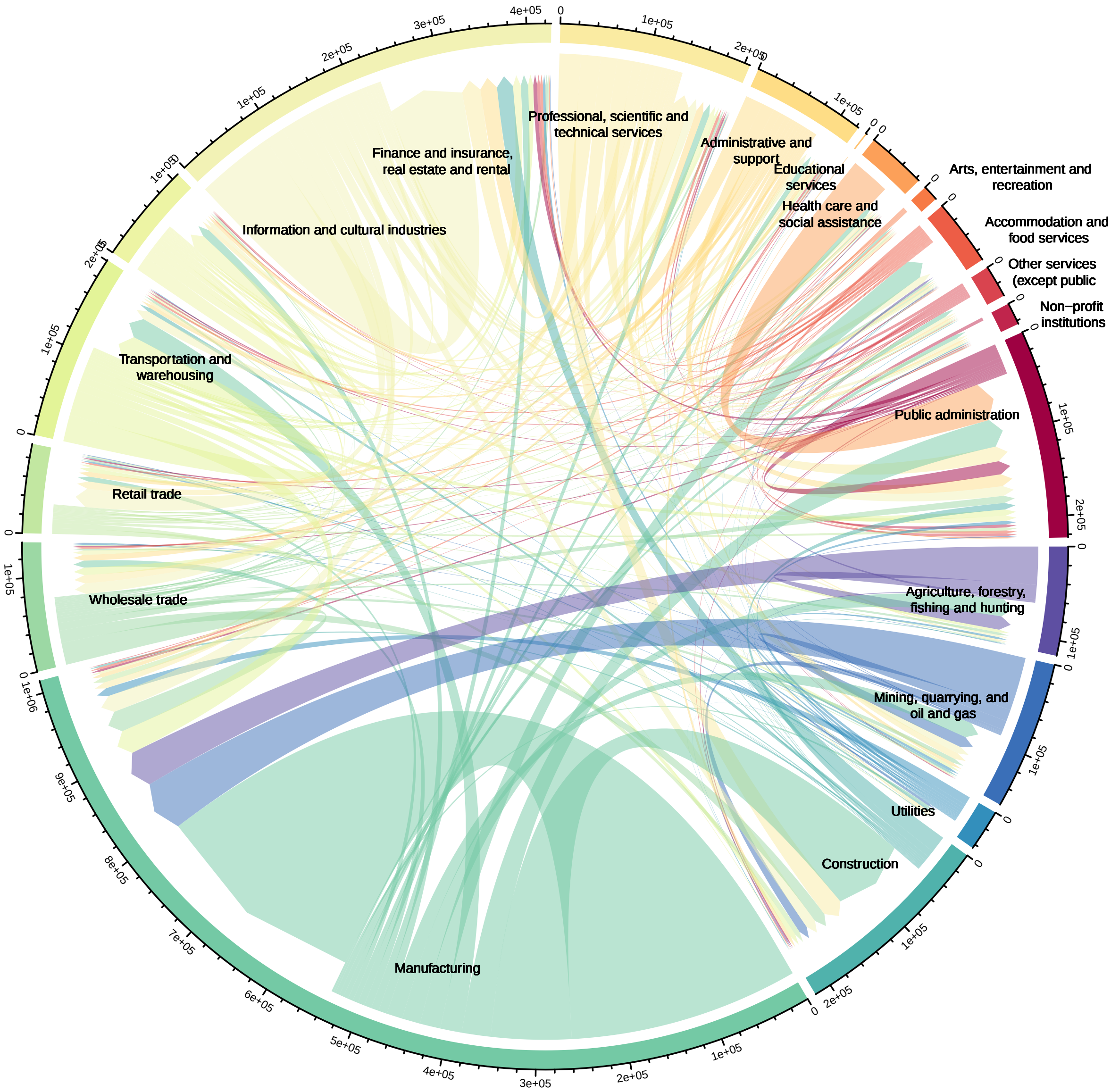
We derive parameters concerning productivity and technology coefficients, as well as capital formation and consumption coefficients from input-output tables. The parameters vary by NAICS classification and are calibrated to the annual values for each reference quarter of a calendar year. For parameters that are calibrated based on the data of input-output tables (such as the productivity coefficients for labor and capital ($\bar{\alpha}_i$, κ_i), the depreciation rate (δ_i), and the average wage rate ($\bar{w}_i$)), we use cross-classification tables and structural business statistics (business demography) that link information by industry and product to each sector (see the linkage in Figure 3).

4.1.4 Government statistics and sector accounts

Tax rates and marginal propensities to consume or invest are calculated from national accounting identities. These rates are set such that the financial flows observed in input-output tables, government

[24]Data obtained from Canada's national statistics agency can be found at `https://www.statcan.gc.ca/en/start` (last accessed March 25, 2022)

Figure 3: Canadian Production Network



Note: The transactions between industries is shown for the sectors: Agriculture, forestry, fishing and hunting [11]; Mining, quarrying, and oil and gas extraction [21]; Utilities [22]; Construction [23]; Manufacturing [31-33]; Wholesale trade [41]; Retail trade [44-45]; Transportation and warehousing [48-49]; Information and cultural industries [51]; Finance and insurance, real estate and rental and leasing [52-53]; Professional, scientific and technical services [54]; Administrative and support, waste management and remediation services [56]; Educational services [61]; Health care and social assistance [62]; Arts, entertainment and recreation [71]; Accommodation and food services [72]; Other services (except public administration) [81]; Non-profit institutions serving households [NP000]; Public administration [91].

statistics, and sector accounts are matched. Capital ratios and the inflation target of the monetary authority are set according to the literature. In the context of the model, we define an average tax rate as the aggregate tax flow paid by an institutional sector (firms, households, etc.) in a calendar year divided by the corresponding aggregate monetary flow that serves as the base for the tax and that is received by the same institutional sector (such as income, profit, output, fixed assets, etc.). This annual average tax rate obtained from macroeconomic aggregates is then applied to every individual agent in our model in the corresponding economic context. We thus calibrate tax rates and marginal propensities to consume or invest to the annual values for each reference quarter of a calendar year. Households' marginal propensity to consume (ψ) and invest (ψ^{H}) are calibrated such that consumption out of disposable income equals actual household consumption, and investment in dwellings is as obtained from input-output tables for

Canada.

4.1.5 Exogenously estimated from national accounts (GDP and main components)

For exogenous processes such as imports and exports, parameters are estimated from national accounts. The growth rates of real imports, real exports, and the final government expenditure of the general government, as well as the respective deflators, are assumed to follow AR(1) processes. The coefficients of the respective AR(1) models are estimated over the sample from 1997:Q4 to the respective reference quarter of the calibration. The sample 1997:Q1 to 1997:Q3 is used as a pre-sample period.

4.1.6 Statutory guidelines, financial regulation, and banking practices

A number of parameters are calibrated according to statutory guidelines, financial regulation (Basel III), and banking practices. Since the statutory guidelines and regulations did not change during the calibration period, these parameters are assumed to be constant for all reference quarters. The capital ratio (ζ) and the inflation target of the monetary authority (π^*) are set according to financial regulation (Basel III) and the statutes of the Bank of Canada (2 percent inflation target). We calibrate the unemployment benefit replacement rate (θ^{UB}) with the data of basic rate of the Employment Insurance (EI) program, at a level of 55%.

4.2 Initial Conditions for the Canadian Economy

Initial conditions, like model parameters, are set according to the procedure from Poledna et al. (2022) to represent the Canadian economy. All initial conditions in the model are collected in Table 6 where values are shown for 2019:Q4.

4.2.1 Households

The household sector consists of all employed, unemployed, and inactive persons in Canada who are 15 years of age or older. Persons in the labor force (employed and unemployed) are associated with an age group, sex, and industry as additional characteristics. The respective initial populations are obtained from census data and labour force surveys and are given in Tables 5 and 8 (the number of economically inactive persons is assumed to be constant and given in Table 3). Each household agent (person) is initially assigned to one of the respective cohorts such that the populations are matched. Additionally, each employed person is randomly assigned to a firm that is part of the industry associated with the employee, and investors are assigned to a firm from which they will earn dividends.

The initial wage of the h-th person ($w_h(0)$) is equal to the initial wage paid by firm i ($\bar{w}_i$) if i is the employer of person h; or it is equal to the initial unemployment benefits w^{UB} if the person is unemployed. Initial unemployment benefits are set by dividing the total flow of unemployment payments, as obtained from government expenditure by function, by the number of unemployed persons. Thus, $w_h(0)$ is determined as follows:

$$w_h(0) = \begin{cases} \bar{w}_i & \text{if employed by firm } i \\ \frac{w^{\mathrm{UB}}}{\theta^{\mathrm{UB}}} & \text{if unemployed} \end{cases} .$$

Initial personal assets (deposits) of the h-th person ($D_h(0)$) are obtained from national balance sheet accounts, which are disaggregated onto the individual level according to the share of each person's income of total income as a proxy for a person's wealth:

$$D_h(0) = D^{\mathrm{H}} \frac{Y_h(0)}{\sum_h Y_h(0)} ,$$

where D^{H} are the initial personal assets (deposits) of the household sector and $Y_h(0)$ is determined according to Equation (4). The initial capital stock (residential structures such as dwellings) of the h-th person ($K_h(0)$) is obtained from national balance sheet accounts (residential structures of the household sector) and is again disaggregated onto the individual level according to the share of each person's income of total income as a proxy for the person's wealth:

$$K_h(0) = K^{\mathrm{H}} \frac{Y_h(0)}{\sum_h Y_h(0)} ,$$

where K^{H} is the initial lump-sum capital (dwellings) of the household sector.

Table 3: Model parameters

Parameter	Description	Value	Source
G/S	Number of products/industries	19	census data, labour force survey, business demography data
H_s^{F15}	Number of women 15 to 24 years in the labor force of the s^{th} industry	see Table 5	
H_s^{F25}	Number of women 25 to 54 years in the labor force of the s^{th} industry	see Table 5	
H_s^{F55}	Number of women over 54 years in the labor force of the s^{th} industry	see Table 5	
H_s^{M15}	Number of men 15 to 24 years in the labor force of the s^{th} industry	see Table 5	
H_s^{M25}	Number of men 25 to 54 years in the labor force of the s^{th} industry	see Table 5	
H_s^{M55}	Number of men over 54 years in the labor force of the s^{th} industry	see Table 5	
H^{inact}	Number of economically inactive persons	190855	
J	Number of government entities	3305	
L	Number of foreign consumers	6610	
I_s	Number of firms/investors in the s^{th} industry	see Table 4	
$\bar{\alpha}_i$	Average productivity of labor of the i^{th} firm		input-output tables, flows and stocks capital, national balance sheet; parameters are firm and sector specific
κ_i	Productivity of capital of the i^{th} firm		
β_i	Productivity of intermediate consumption of the i^{th} firm		
δ_i	Depreciation rate for capital of the i^{th} firm		
δ_i^{S}	Depreciation rate for inventories of the i^{th} firm		
$\bar{w}_i$	Average wage rate of firm i		
a_{sg}	Technology coefficient of the g^{th} product in the s^{th} industry		
τ_i^{Y}	Net tax rate on products of the i^{th} firm		
τ_i^{K}	Net tax rate on production of the i^{th} firm		
b_g^{CF}	Capital formation coefficient of the g^{th} product (firm investment)	see Table 4	
b_g^{CFH}	Household investment coefficient of the g^{th} product	see Table 4	
b_g^{HH}	Consumption coefficient of the g^{th} product of households	see Table 4	
c_g^{G}	Consumption of the g^{th} product of the government in mln. CAD	see Table 4	
c_g^{E}	Exports of the g^{th} product in mln. CAD	see Table 4	
c_g^{I}	Imports of the g^{th} product in mln. CAD	see Table 4	
τ^{INC}	Income tax rate	0.1454	government statistics, sector accounts
τ^{FIRM}	Corporate tax rate	0.1551	
τ^{VAT}	Value-added tax rate	0.0902	
τ^{SIF}	Social insurance rate (employers' contributions)	0	
τ^{SIW}	Social insurance rate (employees' contributions)	0.0908	
τ^{EXPORT}	Export tax rate	0.0001	
τ^{CF}	Tax rate on capital formation	0.1338	
τ^{G}	Tax rate on government consumption	0	
r^{G}	Interest rate on government bonds	0.0063	
μ	Risk premium on policy rate	0.0108	
ψ	Fraction of income devoted to consumption	1.0176	
ψ^{H}	Fraction of income devoted to investment in housing	0.1285	
θ^{DIV}	Dividend payout ratio	0.7228	
θ^{UB}	Unemployment benefit replacement rate	0.55	
θ	Rate of instalment on debt	0.05	Basel III, BoC statutes, banking practices, literature, etc.
ζ	Banks' capital requirement coefficient	0.03	
ζ^{LTV}	Loan-to-value (LTV) ratio	0.6	
ζ^{b}	Loan-to-capital ratio for new firms after bankruptcy	0.5	
π^*	Inflation target of the monetary authority	0.005	
$\alpha^{\gamma,\mathrm{G}}$	Autoregressive coefficient for government consumption	-0.1376	national accounts (exogenous estimated)
$\beta^{\gamma,\mathrm{G}}$	Scalar constant for government consumption	0.0058	
$\alpha^{\pi,\mathrm{G}}$	Autoregressive coefficient for government prices	0.2659	
$\beta^{\pi,\mathrm{G}}$	Scalar constant for government prices	0.0038	
$\alpha^{\gamma,\mathrm{E}}$	Autoregressive coefficient for exports	0.4016	
$\beta^{\gamma,\mathrm{E}}$	Scalar constant for exports	0.0044	
$\alpha^{\pi,\mathrm{E}}$	Autoregressive coefficient for export prices	0.0292	
$\beta^{\pi,\mathrm{E}}$	Scalar constant for export prices	0.006	
$\alpha^{\gamma,\mathrm{I}}$	Autoregressive coefficient for imports	0.2259	
$\beta^{\gamma,\mathrm{I}}$	Scalar constant for imports	0.0023	
$\alpha^{\pi,\mathrm{I}}$	Autoregressive coefficient for import prices	0.2935	
$\beta^{\pi,\mathrm{I}}$	Scalar constant for import prices	0.0011	
C	Covariance matrix of exogenous variables		

Note: Model parameters are calculated for 2019:Q4. Exogenous autoregressive parameters are estimated starting in 1997:Q1.

Table 4: Sectoral parameters

	I_s	N_s	α_s	β_s	κ_s	δ_s	δ_s^{S}	w_s	τ_s^{Y}	τ_s^{K}	b_g^{CF}	b_g^{CFH}	b_g^{HH}	c_g^{G}	c_g^{E}	c_g^{I}
BS110	496	3586	7.1069	1.6615	0.4337	0.0458	1	0.8043	-0.0026	0.0029	0.0003	0	0.0143	0	0.0341	0.0163
BS210	87	2665	19.8702	2.3689	0.0711	0.0288	1	3.3249	0.0042	0.0141	0.0148	0	0.0023	0	0.1607	0.041
BS220	14	1403	11.2352	3.425	0.0472	0.0223	1	2.812	-0.0165	0.0679	0.0024	0.0001	0.0172	0	0.0041	0.0009
BS230	1499	14694	6.1363	1.8601	2.2919	0.062	1	1.7475	0.0082	0.0371	0.5142	0.8141	0.0004	0	0.0005	0.022
BS3A0	517	17402	11.1372	1.4154	1.3095	0.0622	1	1.939	0.0021	0.0043	0.2612	0.0001	0.1967	0	0.5475	0.729
BS410	574	6270	7.2998	2.674	1.3017	0.0698	1	2.6558	0.006	0.0149	0.0425	0.0003	0.0393	0	0.0474	0.0036
BS4A0	1451	21977	2.0925	2.5929	0.7446	0.0518	1	0.9112	0.0048	0.0177	0	0	0.1194	0	0.0058	0.0119
BS4B0	722	10371	5.0758	1.879	0.2106	0.0313	1	1.4025	-0.0174	0.0142	0.0086	0	0.0359	0	0.0584	0.0231
BS510	186	4061	7.5499	2.0879	0.3898	0.0679	1	1.8432	-0.0037	-0.0044	0.0172	0	0.0361	0	0.0165	0.0162
BS5B0	1001	12028	13.8111	2.9934	1.219	0.0635	1	2.2545	0.0132	0.092	0.0147	0.155	0.3117	0	0.0266	0.0395
BS540	1491	15373	3.447	2.6895	1.7865	0.0975	1	1.3876	-0.0003	0.0041	0.0705	0.0279	0.0068	0	0.0398	0.0286
BS560	544	7681	3.3473	2.582	1.7806	0.065	1	1.3378	0.0062	0.0054	0.0012	0	0.0055	0	0.0126	0.0239
BS610	151	13589	0.1647	2.2701	0.6302	0.0361	1	0.0607	-0.0071	-0.0058	0.0001	0	0.0035	0	0.0001	0.0002
BS620	1220	24962	0.8741	3.6268	1.2125	0.0419	1	0.2956	-0.0101	0.0066	0.0004	0	0.0282	0	0	0.0002
BS710	192	3563	2.0804	1.9501	0.3817	0.0408	1	0.6572	0.005	0.0063	0.0003	0	0.0171	0	0.0047	0.0064
BS720	843	12098	2.2126	1.9607	0.7836	0.0409	1	0.7955	0.0162	0.0119	0.0001	0	0.0678	0	0.0217	0.0293
BS810	1132	8125	1.6944	2.6091	2.3794	0.0669	1	0.7462	0.0038	0.0031	0.0007	0	0.0256	0	0.0004	0.0015
NP000	100	1000	14.2901	2.2927	1.2124	0.0324	1	7.2995	0.021	0.0068	0.0006	0.0002	0.0397	0	0.002	0.0007
GS600	1000	10007	15.0565	2.7331	0.2284	0.0292	1	7.3461	0.0056	0.0108	0.0502	0.0024	0.0326	1	0.0171	0.0056

Note: Sectoral parameters are calculated for 2019:Q4. The contribution of industries is shown for the sectors: Agriculture, forestry, fishing and hunting [BS110]; Mining, quarrying, and oil and gas extraction [BS210]; Utilities [BS220]; Construction [BS230]; Manufacturing [BS3A0]; Wholesale trade [BS410]; Retail trade [BS4A0]; Transportation and warehousing [BS4B0]; Information and cultural industries [BS510]; Finance and insurance, real estate and rental and leasing [BS5B0]; Professional, scientific and technical services [BS540]; Administrative and support, waste management and remediation services [BS560]; Educational services [BS610]; Health care and social assistance [BS620]; Arts, entertainment and recreation [BS710]; Accommodation and food services [BS720]; Other services (except public administration) [BS810]; Non-profit institutions serving households [NP000]; Government sector [GS600].

Table 5: Number of persons in the labor force (employment and unemployment) in the sectors

	H_s^{F15}	H_s^{F25}	H_s^{F55}	H_s^{M15}	H_s^{M25}	H_s^{M55}
BS110	283	1200	404	322	1344	453
BS210	60	413	100	168	1723	418
BS220	96	440	147	107	488	164
BS230	325	1716	515	1714	9249	2854
BS3A0	402	3608	1155	1318	8507	3466
BS410	173	1451	455	329	2914	1200
BS4A0	3844	6031	2358	3378	5697	2141
BS4B0	245	1812	611	684	5313	2241
BS510	566	1148	346	530	1407	442
BS5B0	421	4597	1381	439	4159	1342
BS540	589	4835	1322	691	6542	2071
BS560	371	2413	1010	770	2878	1045
BS610	854	7169	1979	438	2910	1066
BS620	1782	14507	4239	404	3387	1242
BS710	496	1007	304	465	1234	388
BS720	3154	3338	938	2438	2747	621
BS810	485	3041	1135	431	2384	1024
NP000	84	344	116	97	387	131
GS600	320	3797	1026	342	3747	1112

Note: Number of employed and unemployed persons (women 15 to 24 years [F15], men 15 to 24 years [M15], women 25 to 54 years [F25], men 25 to 54 years [M25], women over 54 years [F55], men over 54 years [M55]) are shown for 2019:Q4. The contribution of industries is shown for the sectors: Agriculture, forestry, fishing and hunting [BS110]; Mining, quarrying, and oil and gas extraction [BS210]; Utilities [BS220]; Construction [BS230]; Manufacturing [BS3A0]; Wholesale trade [BS410]; Retail trade [BS4A0]; Transportation and warehousing [BS4B0]; Information and cultural industries [BS510]; Finance and insurance, real estate and rental and leasing [BS5B0]; Professional, scientific and technical services [BS540]; Administrative and support, waste management and remediation services [BS560]; Educational services [BS610]; Health care and social assistance [BS620]; Arts, entertainment and recreation [BS710]; Accommodation and food services [BS720]; Other services (except public administration) [BS810]; Non-profit institutions serving households [NP000]; Government sector [GS600].

4.2.2 Firms

The distribution of firm sizes in industrial countries is well known to be highly skewed, with large numbers of small firms coexisting with small numbers of large firms (Ijiri and Simon, 1977; Axtell, 2001). Initial employment of firm i ($N_i(0) \quad \forall i \in I_s$) is therefore drawn from a power law distribution with exponent -2 (where $\sum_{i\in I_s} N_i(0) = N_s$ and $N_i(0) > 0$), which approximately corresponds to firm size distribution in Canada. To determine initial production $Y_i(0)$ of the i-th firm, we use the initial employment by firm $N_i(0)$, and compute the corresponding amount of production by the productivity of labor per unit of output $\bar{\alpha}_i$:

$$Y_i(0) = Q_i^{\mathrm{d}}(0) = \bar{\alpha}_i N_i(0) \ .$$

The initial capital of firm i, $K_i(0)$, (i is part of industry s) is then obtained by dividing firm i's initial level of production $Y_i(0)$ by the productivity of capital κ_i and the desired rate of capacity utilization ω:

$$K_i(0) = \frac{Y_i(0)}{\kappa_i \omega} \ .$$

Thus, it is the share of the capital of the i-th firm in sector s as measured by production, accounting for the reserve capacity of its capital stock targeted by firm i. The initial stocks of raw materials, consumables, supplies, and spare parts (i.e., intermediate inputs) of the i-th firm ($M_i(0)$) are set such that firms hold sufficient intermediate inputs for expected production without stock-out:

$$M_i(0) = \frac{Y_i(0)}{\omega \beta_i}$$

where $Y_i(0)$ is the initial level of production by firm i, β_i denotes the productivity of intermediate inputs, and $1/\omega$ is the buffer stock of material inputs.

Since a breakdown of financial and current assets for the 19 NAICS sectors is not readily available, we calibrate initial debt $L_i(0)$ to i-th individual firms by disaggregating total firm debts according to the share of the firms' capital stock $K_i(0)$ in the total capital stock $\sum_i K_i(0)$:

$$L_i(0) = L^{\mathrm{I}} \frac{K_i(0)}{\sum_i K_i(0)} \ ,$$

where the total amount of firm debt L^{I} is obtained from national balance sheet accounts. The total initial liquidity (deposits) of all firms as an aggregate, D^{I}, is set according to national balance sheet accounts. This aggregate is broken down onto single firms by the share of firm i's operating surplus in the overall operating surplus, where we assume that firm liquidity (deposits) moves in line with its production as a liquid form of working capital used for current expenditures:

$$D_i(0) = D^{\mathrm{I}} \frac{\max(\bar{\pi}_i Y_i(0), 0)}{\sum_i \max(\bar{\pi}_i Y_i(0), 0)} \ ,$$

where $\bar{\pi}_i = 1 - \left(1 + \tau^{\mathrm{SIF}}\right) \frac{\bar{w}_i}{\bar{\alpha}_i} - \frac{\delta_i}{\kappa_i} - \frac{1}{\beta_i} - \tau_i^{\mathrm{K}} - \tau_i^{\mathrm{Y}}$ is the operating margin. Initial profit of the i-th firm is given by the initial operating surplus and the initial income from interest accrued from deposits less interest payments for bank loans:

$$\Pi_i(0) = \bar{\pi}_i Y_i(0) - r(0) L_i(0) + \bar{r}(0) D_i(0) \, .$$

The initial inventories of finished goods $S_i(t)$ of firm i is assumed to be zero due to a lack of reliable data sources. The initial price of the i-th firm $P_i(0)$ is set to one.

4.2.3 The central bank

Initial central bank's equity ($E^{\mathrm{CB}}(0)$) is the residual on the central bank's passive side, obtained by deducting initial bank reserves held ($D_k(0)$) and the initial net creditor/debtor position with the rest of the world ($D^{\mathrm{RoW}}(0)$) from the central bank's assets (initial government debt ($L^{\mathrm{G}}(0)$)). Thus, the initial central bank's equity ($E^{\mathrm{CB}}(0)$) is set according to Equation (57) where the initial balance of trade with the rest of the world ($D^{\mathrm{RoW}}(0)$) is assumed to be zero and the initial bank reserves held ($D_k(0)$) are set according to Equation (74).

4.2.4 The general government

Initial government debt ($L^{\mathrm{G}}(0)$) is set according to the Canadian government's consolidated gross debt.

4.2.5 The financial system

The initial bank's equity ($E_k(0)$) is obtained from national accounting data, and the initial bank's profits are given by the initial income from interest less interest payments:

$$\Pi_k(0) \ = \mu \sum_i L_i(0) + \bar{r}(0) E_k(0) \,,$$

where initial advances from the central bank ($D_k(0)$) are set according to Equation (74).

Table 6: Initial conditions

Initial condition	Description	Value
$P_i(0)$	Initial price of the i^{th} firm	initial conditions are firm (i) and household (h) specific
$Y_i(0)/Q_i^{\text{d}}(0)$	Initial production/demand of the i^{th} firm (in mln. CAD)	
$K_i(0)$	Initial capital of the i^{th} firm (in mln. CAD)	
$M_i(0)$	Initial stocks of raw materials, consumables, supplies of the i^{th} firm (in mln. CAD)	
$S_i(0)$	Initial stocks of finished goods of the i^{th} firm (in mln. CAD)	
$N_i(0)$	Initial number of employees of the i^{th} firm	
$D_i(0)$	Initial liquidity (deposits) of the i^{th} firm (in mln. CAD)	
$L_i(0)$	Initial debt of the i^{th} firm (in mln. CAD)	
$\Pi_i(0)$	Initial profits of the i^{th} firm (in mln. CAD)	
$D_h(0)$	Initial personal assets (deposits) of the h^{th} household (in mln. CAD)	
$K_h(0)$	Initial household capital (in mln. CAD)	
$w_h(0)$	Initial wage of the h^{th} household (in mln. CAD)	
$sb^{\text{inact}}(0)$	Initial pension/social benefits in mln. CAD	0.0795
$sb^{\text{other}}(0)$	Initial social benefits received by all households in mln. CAD	0.019
$L^{\text{G}}(0)$	Initial government debt (in mln. CAD)	2658545
$\Pi_k(0)$	Initial banks' profits (in mln. CAD)	bank-specific
$E_k(0)$	Initial banks' equity (in mln. CAD)	593541
$E^{\text{CB}}(0)$	Initial central banks' equity (in mln. CAD)	1879086
$L^{\text{RoW}}(0)$	Initial net creditor/debtor position of the national economy to RoW (in mln. CAD)	0

Note: Initial condition are shown for 2019:Q4.

Table 7: Initial conditions for the institutional sectors

Initial condition	Description	Value
D^{I}	Initial liquidity (deposits) of the firm sector (in mln. CAD)	560984
L^{I}	Initial debt of the firm sector (in mln. CAD)	1937189
ω	Desired capacity utilization rate	0.85
w^{UB}	Initial unemployment benefits (in mln. CAD)	0.2839
D^{H}	Initial personal assets (deposits) of the household sector (in mln. CAD)	1562123
K^{H}	Initial capital (dwellings) of the household sector (in mln. CAD)	3314388

Note: Initial condition are shown for 2019:Q4.

Table 8: Number of employed persons in the sectors

	N_s^{F15}	N_s^{F25}	N_s^{F55}	N_s^{M15}	N_s^{M25}	N_s^{M55}
BS110	232	1102	369	257	1219	408
BS210	34	362	82	134	1658	395
BS220	91	431	144	101	477	160
BS230	121	1324	374	1453	8748	2674
BS3A0	358	3409	1087	1138	8089	3321
BS410	142	1392	434	290	2839	1173
BS4A0	3523	5685	2259	3059	5400	2051
BS4B0	180	1687	566	601	5153	2184
BS510	508	1075	320	450	1315	393
BS5B0	383	4524	1355	391	4066	1309
BS540	565	4638	1267	634	6269	2000
BS560	317	2231	942	620	2603	968
BS610	780	6787	1870	381	2754	1017
BS620	1709	14367	4189	311	3208	1178
BS710	445	943	281	395	1154	345
BS720	2865	3084	856	2190	2539	564
BS810	434	2937	1096	395	2278	985
NP000	65	307	103	72	340	114
GS600	279	3728	985	306	3649	1060

Note: Number of employed persons (women 15 to 24 years [F15], men 15 to 24 years [M15], women 25 to 54 years [F25], men 25 to 54 years [M25], women over 54 years [F55], men over 54 years [M55]) are shown for 2019:Q4. The contribution of industries is shown for the sectors: Agriculture, forestry, fishing and hunting [BS110]; Mining, quarrying, and oil and gas extraction [BS210]; Utilities [BS220]; Construction [BS230]; Manufacturing [BS3A0]; Wholesale trade [BS410]; Retail trade [BS4A0]; Transportation and warehousing [BS4B0]; Information and cultural industries [BS510]; Finance and insurance, real estate and rental and leasing [BS5B0]; Professional, scientific and technical services [BS540]; Administrative and support, waste management and remediation services [BS560]; Educational services [BS610]; Health care and social assistance [BS620]; Arts, entertainment and recreation [BS710]; Accommodation and food services [BS720]; Other services (except public administration) [BS810]; Non-profit institutions serving households [NP000]; Government sector [GS600].

5 Forecast Performance

We conduct a series of forecasting exercises to evaluate the out-of-sample forecasting performance of CANVAS in comparison with standard macroeconomic modelling approaches. The out-of-sample forecasting exercise is constructed along the lines of Smets and Wouters (2007), who compare a Bayesian DSGE model to an unconstrained VAR as well as to Bayesian VAR (BVAR) models.[25]

We compare the out-of-sample forecast performance of CANVAS to that of an unconstrained (non-theoretical) VAR(1) model,[26] the Bank of Canada's main DSGE model (ToTEM), and an AR(1) model. We do this by means of the root mean squared forecast error (RMSE).[27] To test whether the models' forecasts are significantly different from that of the VAR(1) forecasts, we conduct Diebold-Mariano tests correcting for the overall length of the forecasting horizon (Harvey et al., 1997). In this test, the null hypothesis is that each model and the VAR(1) generate forecasts of equal accuracy.

We use the VAR model as the benchmark for the forecast performance comparison as the VAR is guaranteed to summarize the data in-sample. A drawback, however, is that even very small VAR models have a large number of parameters, whose estimates tend to be imprecise when estimated with short samples.

The benchmark VAR is estimated by including the log differences of real GDP, real household consumption, real capital fixed investment, real exports and real imports of Canada, as well as the log difference of the GDP deflator (a focused measure of inflation) and the Canadian Overnight Repo Rate Average. The VAR is initially estimated over the sample 1997:Q1 to 2010:Q1 and used to produce 12-quarters-ahead out-of-sample forecasts using 2010:Q1 as the last observation. The VAR is then re-estimated recursively for each quarter by extending the last observation from 2010:Q2 to 2019:Q3. For each observation between 2010:Q1 and 2019:Q3, a set of 12-quarters-ahead out-of-sample forecasts are produced. The average RMSEs of these forecasts by horizon are reported in Table 9.

We conduct a similar out-of-sample forecasting exercise in ToTEM, a large-scale open-economy DSGE model of the Canadian economy. Since ToTEM features an enriched firm- and individual-level heterogeneity, it serves as a relevant DSGE candidate to assess the relative performance of CANVAS. To help interpret the forecasting performance results, we provide a brief introduction of ToTEM's modelling structure.

On the firm side, the model features five distinct sectors producing final goods for consumption, residential investment, business investment, government spending, and non-commodity exports. The model also includes a separate commodity-producing sector, from which output is mostly exported (as is the case in the data). This elaborate sectoral structure helps the model capture the composition of Canadian GDP and dynamics of its components.

The firms responsible for producing final goods face nominal rigidities when setting their prices. More specifically, in a given final-good-producing sector, some of the firms re-optimize their prices in a forward-looking but staggered fashion, as in the literature following Calvo (1983), while the other firms set their prices using a rule of thumb (RoT) similar to that in Galı and Gertler (1999).

The model features three prominent household types differing in the financial markets they have access to and in their status as savers or borrowers in those markets. On the saver side, the model allows short- and long-term interest rates to influence aggregate household spending in distinct ways, following Andres et al. (2004) and Chen et al. (2012). Borrowers have been modelled in line with Alpanda and Zubairy (2017). They are assumed to face a collateral constraint under which new loans must be backed by some combination of new housing investment and home equity.

Turning to the labor market, ToTEM follows most of the DSGE literature in assuming that workers enjoy some degree of wage-setting power but are subject to nominal rigidities similar to those faced by price setters. Given the structure of the wage-setting process, coupled with the labor demand profile arising on the firm side, the model pins down both the aggregate wage and total number of hours worked in the economy.

Analogously to the VAR, ToTEM is initially estimated over the sample 1997:Q1 to 2010:Q1 to pro-

[25] Following Smets and Wouters (2007), and for reasons of data availability, we are restricted to using the latest vintage of data available from Statistics Canada at the time of model estimation. Since in this study, as in Smets and Wouters (2007), we are primarily interested in how well CANVAS fits the data of the Canadian economy, and not in benchmarking its forecasting performance with potentially inconsistent real-time data, conducting a real-time forecast evaluation along the lines of, e.g., Diebold et al. (2017) is left to future research.

[26] To determine the optimal lag length of the VAR model, the Bayesian information criterion (BIC) is minimized. For the entire period from 2010:Q1 to 2019:Q4, VAR models of lag order one minimize the BIC.

[27] The root mean squared error is defined as follows: $RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{T}(\hat{x}_t - x_t)^2}$, where $\hat{x_t}$ is the forecast value and x_t is the observed data point for period t.

duce 12-quarters-ahead out-of-sample forecasts and is then re-estimated recursively for each quarter by extending the last observation from 2010:Q2 to 2019:Q3.

We also compare the ABM's forecasting performance to more tightly parameterized univariate AR models. As in the VAR model, we use a similar methodology to determine the optimal lag length for the AR models. For the entire period from 2010:Q1 to 2019:Q3 and for all variables, AR models of lag order one minimize both the AIC and BIC.

Similar to the VAR, AR, and DSGE, CANVAS is calibrated 39 times to different reference quarters over the calibration period from 2010:Q1 to 2019:Q3. Once the model is calibrated to a reference quarter, it reproduces exactly the state of the economy in that quarter, in terms of aggregate GDP, GDP components, and industry sizes. Starting from each reference quarter, the model is run for 12 quarters, allowing it to collect model-based out-of-sample forecasts. The average of 500 Monte Carlo simulations are used as our point estimate for evaluating forecasting accuracy.

Table 9: Out-of-sample forecast performance

	GDP	Inflation	Consumption	Investment	Exports	Imports
VAR(1)	*RMSE-statistic for different forecast horizons*					
1q	0.48	0.73	0.33	1.54	2.17	1.8
2q	0.76	0.68	0.54	2.7	2.98	2.68
4q	1.24	0.65	1.01	5.19	3.53	4.55
8q	1.9	0.69	1.66	9.95	4.57	9.22
12q	2.24	0.71	1.98	15.14	4.65	13.83
AR(1)	*Percentage gains (+) or losses (-) relative to VAR(1) model*					
1q	-0.4 (0.94)	10.4 (0.06*)	13.1 (0.15)	5.8 (0.52)	-2.2 (0.67)	14.9 (0.14)
2q	1.7 (0.77)	2.6 (0.35)	4.9 (0.46)	11.4 (0.19)	5.7 (0.39)	23.2 (0.03**)
4q	11.1 (0.00***)	-0.5 (0.85)	6.4 (0.33)	17.3 (0.11)	2.5 (0.45)	44.3 (0.01***)
8q	10.6 (0.00***)	3.3 (0.34)	7.7 (0.55)	21.2 (0.04**)	-5.7 (0.50)	60.4 (0.01***)
12q	18.6 (0.07*)	2.8 (0.26)	8.7 (0.72)	26 (0.00***)	-16.4 (0.38)	68.6 (0.06*)
ToTEM	*Percentage gains (+) or losses (-) relative to VAR(1) model*					
1q	-27.2 (0.09*)	14.4 (0.07*)	-49.2 (0.00***)	-18.8 (0.03**)	14.9 (0.05**)	24.2 (0.00***)
2q	-56 (0.01***)	7.3 (0.09*)	-77.5 (0.00***)	-28.7 (0.02**)	20.4 (0.16)	27.6 (0.01***)
4q	-73.4 (0.00***)	1.9 (0.71)	-76.7 (0.02**)	-16.8 (0.15)	6.8 (0.67)	30.1 (0.02**)
8q	-58.5 (0.03**)	8 (0.14)	-56.6 (0.18)	15.9 (0.50)	8.7 (0.78)	48 (0.00***)
12q	-33.8 (0.29)	6.4 (0.23)	-39.2 (0.07*)	41.5 (0.01***)	24.7 (0.19)	64.9 (0.03**)
CANVAS	*Percentage gains (+) or losses (-) relative to VAR(1) model*					
1q	0.6 (0.93)	10.1 (0.07*)	-51.5 (0.01***)	5.4 (0.49)	-0.7 (0.89)	13.5 (0.20)
2q	4 (0.46)	-0.6 (0.84)	-67.8 (0.02**)	13.3 (0.02**)	0.8 (0.90)	21.6 (0.07*)
4q	17.2 (0.02**)	-5.3 (0.27)	-25.3 (0.43)	23.6 (0.08*)	-6.1 (0.36)	42.3 (0.02**)
8q	20.6 (0.04**)	-6.4 (0.19)	7.7 (0.85)	33.5 (0.09*)	-15.5 (0.31)	65.9 (0.01***)
12q	33.4 (0.00***)	-2.4 (0.58)	31.8 (0.67)	43.3 (0.00***)	-38.5 (0.17)	79.6 (0.05**)

Note: The forecast period is 2010:Q2 to 2019:Q4. All models are re-estimated each quarter and the results are obtained as an average of 500 Monte Carlo simulations. In parentheses, we show p-values of (modified) Diebold-Mariano tests (Harvey et al., 1997), where we test whether forecasts are significantly different from the VAR(1) (the null hypothesis of the test is that CANVAS, AR(1), and ToTEM are less accurate than the VAR(1)). *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent level, respectively.

Table 9 shows the percentage difference between the RMSE of the benchmark VAR(1) and each of the three models (i.e., AR(1), ToTEM, and CANVAS). A positive sign means that the model has a lower RMSE than that of VAR(1), suggesting superior forecasting ability in the model. In parenthesis, the p-value of the Diebold-Mariano test is used to test if the difference in forecasting performance is statistically significant.

ToTEM has better but not statistically different forecast performance compared to all other models in forecasting inflation and exports for all horizons. The only exceptions are the one-quarter-ahead forecast for exports and the two-quarter-ahead forecast for inflation where its forecasting gains compared to all other models are significantly higher. The strength of ToTEM in forecasting inflation likely benefits from its enriched multi-stage production network with both nominal and real rigidities. In contrast, ToTEM fails to improve on VAR(1) and AR(1) at forecasting GDP and consumption, and only improves on the VAR(1) forecasts for investment at the longer horizons.

Both ToTEM and CANVAS forecast imports relatively well, improving upon the VAR(1) model. Moreover, both models generally perform better than VAR(1) and AR(1) at longer horizons, with a few exceptions. It is worth highlighting that CANVAS yielded the lowest RMSEs in predicting GDP and investment among all the models, and improves upon ToTEM's consumption forecasts, especially at the 8q- and 12q-ahead horizons.

Both CANVAS and ToTEM generally exhibit difficulties in explaining household consumption in Canada in the short run (i.e., over one year). Beyond one year, CANVAS shows great strength and outperforms ToTEM in forecasting consumption. This competitive forecasting performance relative to ToTEM is impressive, suggesting that incomplete markets, non-linear behavior of households in consumer goods, etc., are somewhat better encompassed in the behavioral model.

6 Application

To gain some insights into the economic effects of the recent COVID-19 pandemic in Canada, we apply CANVAS to model the evolution of the Canadian economy since 2020:Q1. Apart from the direct personal and social costs, the pandemic had far-reaching economic consequences, from supply-side manufacturing issues to decreased consumer activity due to lockdown measures, as well as higher unemployment and rising debt levels. To limit the impact of the pandemic, the Canadian economy has been supported by a range of extraordinary measures. Monetary policy has deployed a variety of targeted programs to ensure the proper functioning of credit and financial markets. In March 2020, the policy rate was lowered to its ELB. Fiscal policy has provided liquidity to households and businesses to prevent inefficient defaults and bankruptcies. In the April 2020 Monetary Policy Report, a summary of actions taken by the monetary and fiscal authorities was provided as a response to COVID-19.[28]

Our analysis demonstrates how CANVAS can provide insights into likely trajectories of an open economy when confronted with a variety of shocks. The level of detail of our model allows for the measurement of economic reactions to the COVID-related lockdown measures within particular industries and specific populations, as well as the tracking of the propagation of these measures through the economy. We focus on three aspects: (1) stages of the pandemic; (2) sector heterogeneity; and (3) labor market heterogeneity.

6.1 Scenario Analysis of the COVID-19 Pandemic in Canada

To isolate the economic effects at different stages of the pandemic, we investigate three scenarios: the "Baseline," the "Lockdown," and the "Lockdown and supply chain crisis." The "Baseline" scenario shows the macroeconomic dynamics without the pandemic. Conditional on the state of the economy in 2019:Q4, we use the model to conduct eight-quarters-ahead out-of-sample projections until Q4:2021. Represented by the green line in Figures 4 and 5, the mean model projections under this scenario are shown. In the baseline scenario (i.e., in the absence of the pandemic), Canadian GDP would have been on a steady growth path, with annual inflation remaining close to 2% (Figure 5).

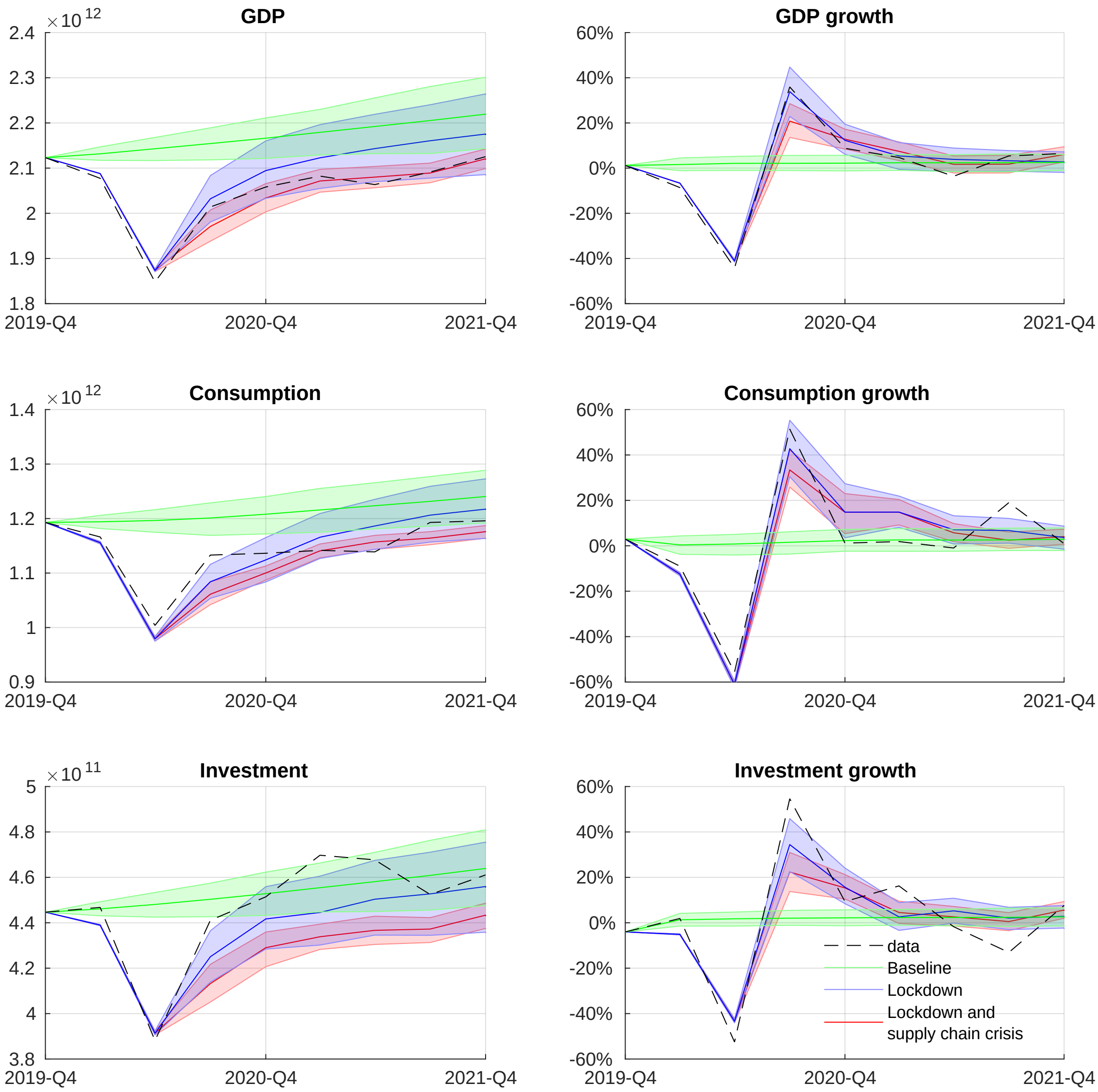
Following the initial shutdown that lasted until mid-May 2020, the Canadian economy suffered from a sharp contraction (see solid line showing "data" in Figure 4, upper right panel). This sharp contraction is reflected in the "Lockdown" scenario (the blue line in Figures 4 and 5). This includes the impact of lockdown measures in Canada and the rest of the world, which is modelled through a supply shock that exogenously reduces firms' supply in 2020:Q1 and 2020:Q2. In particular, this is implemented through a production contraction in domestic firms and a trade adjustment of foreign firms. To capture heterogeneous changes in production in different industries, industry-level GDP data from 2020:Q1 and 2020:Q2 is fed into the model. Realized import and export data, from the same quarters, account for the impact of foreign lockdowns. All other components from the baseline scenario remain the same. The result of this specification is that the economy experiences a large, initial negative shock but rapidly rebounds to a new growth path similar to the one under the baseline scenario (Figure 4). With regards to inflation (Figure 5), there is increased volatility in both measures. However, over the long term, annualized inflation rates remain close to the baseline forecasts.

The third and final scenario is labelled "Lockdown and supply chain crisis," represented by the red line in Figures 4 and 5. This scenario accounts for the development of global commodity prices since 2021 by adding observed imports and exports up to 2021Q4. All other components are identical to the lockdown scenario. This added export and import data affect the model in two ways: exports account for the impact of foreign (and thus aggregate) demand on the Canadian economy, while prices of imports reflect firms' production costs (since imports largely include intermediate outputs used in domestic goods production). Domestic prices of all sectors are determined endogenously, and export and import price deflators are exogenous in this application. Under this scenario, the Canadian economic recovery is slower to materialize and the long-run GDP is lower than in the first two scenarios. Inflation is similar to the outcome under the lockdown scenario, if not slightly more volatile at longer horizons.

In Canada, the inflation targeting framework is established around total CPI inflation. CPI is a measure of goods purchased by consumers. Since we don't model CPI directly in the model, we report two alternative measures of inflation rates from model simulations: (1) consumption deflator inflation and (2) GDP deflator inflation, as seen in Figure 5. As in Amiti et al. (2019), we use producer price indexes instead of consumer price indexes because a clear mapping between international products and domestic

[28] See Table 1 "Summary of Key Policy Measures," `https://www.bankofcanada.ca/wp-content/uploads/2020/04/mpr-2020-04-15.pdf`.

Figure 4: GDP and components

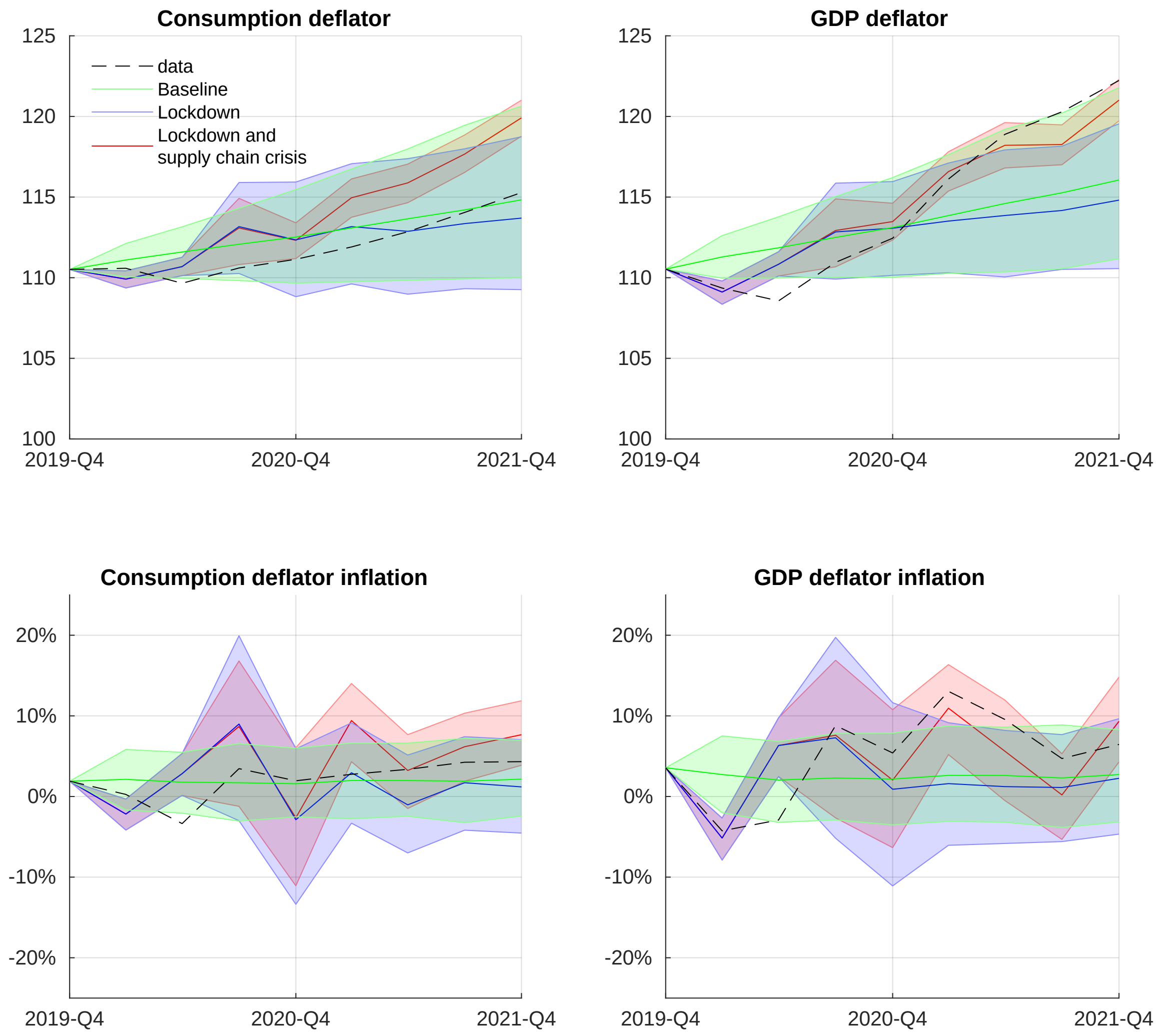


Note: Impact of economic scenarios on quarterly macroeconomic variables with respect to the baseline scenario. One standard deviation is plotted around the mean trajectory. Model results are obtained as an average of 500 Monte Carlo simulations. The left panel shows quarterly levels, whereas the right panel shows quarterly growth at annualized rates.

industry categories is only available for producer price categories. The producer price indexes therefore more effectively measure the price received by domestic firms for their goods or services, consisting of both intermediate and final goods.

Comparing between scenarios, Figure 5 shows that the lockdown scenario is able to explain the initial increase of GDP deflator inflation relative to the baseline scenario in 2020 (lower right panel). However, after an initial spike, inflation returns relatively close to the target under this scenario. In contrast, the model most closely tracks inflation and GDP components in the "Lockdown and supply chain crisis" scenario, when international commodity prices are considered (Figures 4 and 5). In particular, GDP deflator inflation experiences fluctuations into 2021, rising to a high of around 10 percent in 2021Q2. This suggests that post-pandemic inflation can be traced back to two main causes. The first cause is domestic and is related to the rapid increase in demand following the lifting of lockdown restrictions. Firms react to the demand shock by increasing prices. The second cause is the international increase in commodity prices observed in 2021, which raises firms' intermediate costs and is then translated into

Figure 5: Inflation



Note: Impact of economic scenarios on quarterly inflation measures with respect to the baseline scenario. One standard deviation is plotted around the mean trajectory. Model results are obtained as an average of 500 Monte Carlo simulations. The top panel shows quarterly GDP deflator and consumption deflator levels, whereas the bottom panel shows quarterly GDP deflator inflation and consumption deflator inflation, both at annualized rates.

their prices.

6.2 Uneven Impact of the Lockdown across Industries

CANVAS allows further complementary macroeconomic analysis with its ability to track dynamics at the firm and industry level. The sectoral decomposition of GDP dynamics in Figure 6 shows that hard-to-distance sectors (including construction, wholesale and retail trade, transportation, accommodation and food services, as well as arts, entertainment, recreation, and other activities) experience a much steeper decrease in output due to the shutdown.

The accommodation and food services sector, for instance, experiences a decline of 50 percent at a quarterly rate during the initial lockdown in 2020:Q1 and Q2, and a rapid recovery in 2021. The decline in output is only partially compensated for by the subsequent expansion in the three-year simulation period, so that sectoral output, especially for transportation, accommodation and food services, as well as arts, entertainment, recreation, and other activities, remains below the pre-pandemic output until the end of 2021. Apart from these sectors, most industries have returned close to the baseline growth path. This reflects solid fundamentals, notably that the recovery in the labor market, fiscal support, and favorable terms of trade are supporting the level of disposable income and household consumption over the near-term simulations.

Investigating more closely the hardest-hit sectors during the pandemic, we observe that some played a larger role than others in Canada's economic performance. For example, while the manufacturing sector is relatively small, making up 10% of the economy, it is important because it drives activity in other sectors. The sector itself is diverse, including food, machinery, transportation equipment, chemical products, and several others. During the containment, the manufacturing sector experienced a very sharp decline (of about 17 percent), and its sectoral growth remained below the level of other service sectors through 2020:Q3. In contrast, the high-contact sector of accommodation, food, and recreation experienced a very large initial contraction. However, since its share of Canadian GDP is only 3 percent, its overall impact on GDP is insignificant.

Figure 7 decomposes the role of individual industries in creating inflation in each of the scenarios. Manufacturing is shown to be the main industry contributing to inflation in Q3:2020. In contrast, service sectors play a more prominent role in Q1:2021. Commodity sectors caused, to a large extent, deflation in Q1 and Q2:2020 and were one of the main contributors to inflation from 2020:Q4 onward.

The energy commodity sector, including mining, quarrying, and oil and gas extraction, is a key sector of the Canadian economy. As the world's fourth largest oil producer in 2018, investment in the oil and gas sector accounts for about 30 percent of total business investment. Canada is also a net oil exporter, with about 75 percent of commodity production being exported. As a result, the sharp energy price increase had a significant impact on the Canadian economy through several channels and had important implications for household expenditures and balance sheets. In addition to the impact through terms of trade, crude oil also directly accounts for the largest weight in the Bank of Canada Commodity Price Index (BCPI) given its large share (about 45 percent) of the value of Canada's commodity production. With calibration and estimation, CANVAS is uniquely positioned to help us better understand the implication of the oil price movement and its impact on Canada.

6.3 Impact of the COVID-19 Pandemic on the Labor Market

Our ABM can also shed some light on labor market developments. In April 2020, the Canadian government rolled out the Canada Emergency Wage Subsidy (CEWS) to qualifying employers (of all sizes and across all sectors) whose revenues were impacted by COVID-19. CEWS was intended to provide an incentive for employers to pay their employees who were sent home due to lack of work or for health and safety reasons. This, in turn, prevented further job losses and enabled employers to rehire employees who had been laid off. Even though a significant proportion of Canadian companies have received fiscal support to avoid laying off their workers, the effects of the pandemic on labor markets are still tremendous.

The simulated unemployment rate in our model with CEWS rises to about 13 percent in 2020 (see Figure 8). It is particularly compelling to note that the labor market takes a longer time to recover than GDP: unemployment does not return to levels seen before the COVID-19 crisis until the end of the simulation period (winter 2021). As in other countries around the globe, massive amounts of additional government funding were required to support companies and households and keep the Canadian economy afloat.

Our findings can be related to the Bank of Canada's significant ongoing efforts to understand the labor market implications of the pandemic. Ens et al. (2021) provides a new, detailed approach to identify important areas of weakness (or strength) in the labor market. They construct the expanded labor market indicator (ELMI) by featuring changes in methodology and an expanded scope of variables to capture additional areas of slack. For instance, the authors examine the amount of disagreement between measures to more systematically track and quantify unevenness in the labor market. By addressing the drawbacks of traditional measures, they propose a framework for assessing the labor market recovery along three different dimensions: (1) overall labor market conditions; (2) labor market inclusiveness, and (3) job characteristics.

Recent Bank work with ToTEM by Chu et al. (2020) has also shown that the Canadian workforce can be classified by occupational risk to COVID-19 as "COVID-sensitive" or "COVID-neutral." The latter is defined as those who can work from home and thus have a relatively lower exposure to COVID transmission. In contrast, individuals who cannot work from home tend to have a higher risk of being exposed to COVID-19 (e.g., more face-to-face discussions, dealing with external customers, assisting and caring for others). As of August 2020, the share of "COVID-sensitive" industries was calibrated to be 43.7 percent using the VSE Risk Index, which is built upon national and provincial labor force microdata according to occupation. It is clear that in Canada, service industries (e.g., health care, accommodation and food services, and transportation) tend to be more "COVID-sensitive," while goods industries (e.g.,

Figure 6: Decomposition of GDP growth by industry (%)

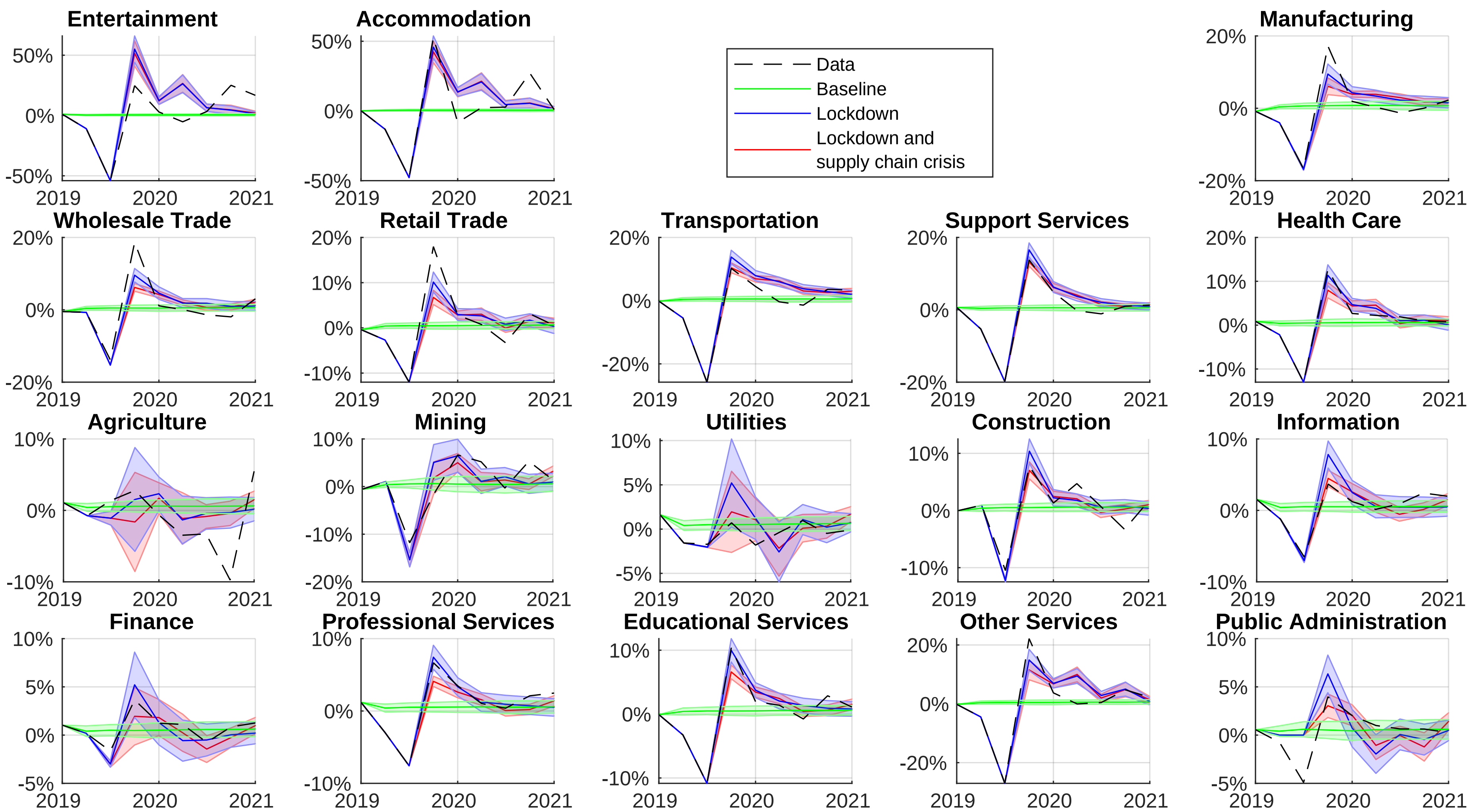
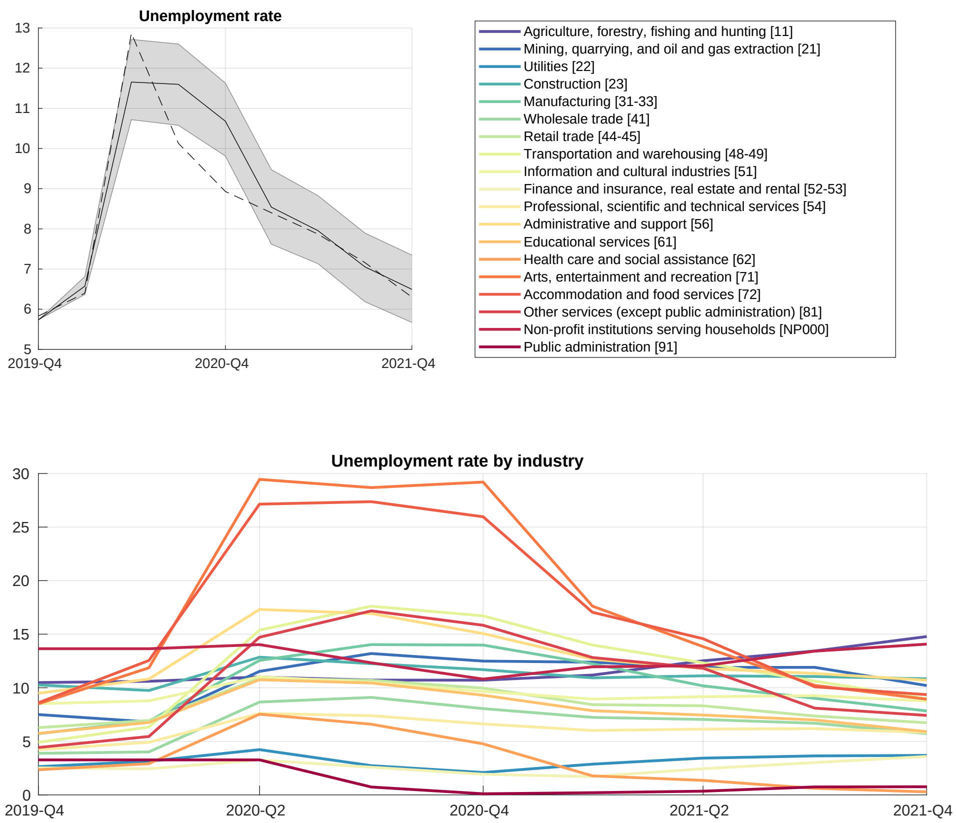


Figure 7: Decomposition of GDP deflator inflation by industry (%)

Note: Decomposition of GDP deflator inflation by industry is shown for the "Lockdown and supply chain crisis" scenario.

Figure 8: Sectoral unemployment rate (%)

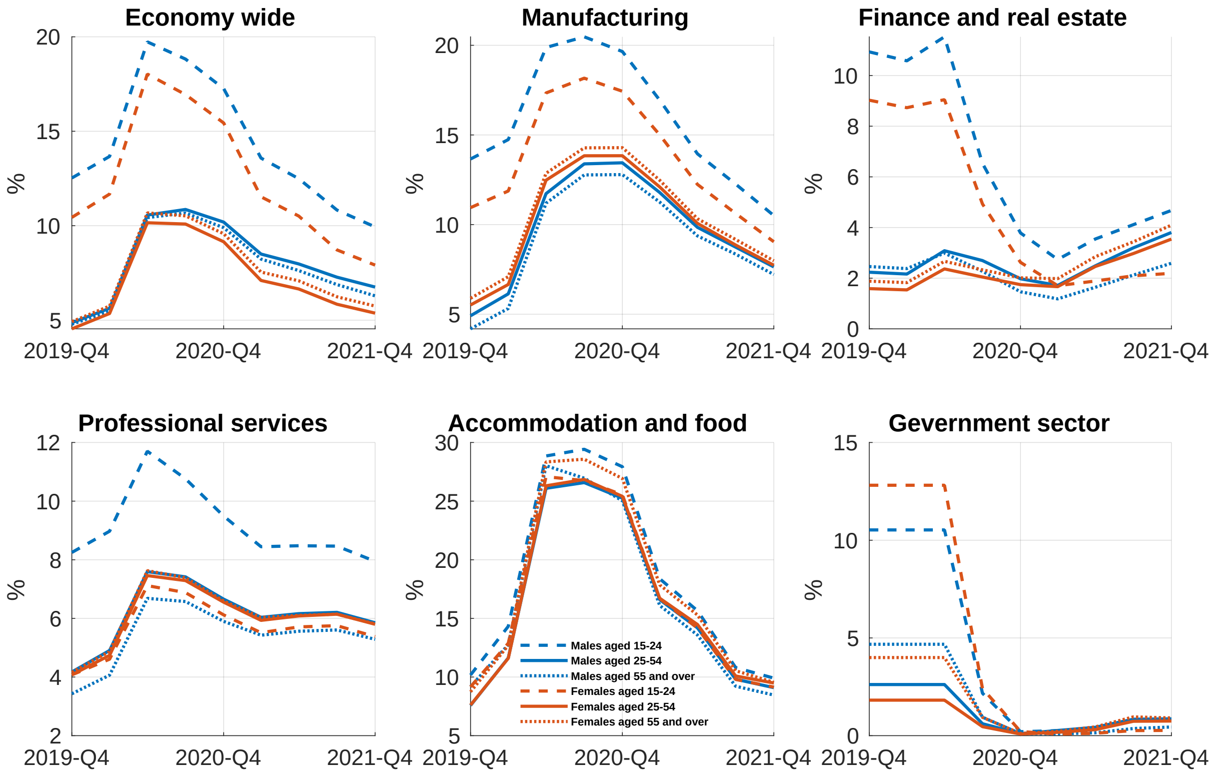


Note: Simulated dynamics of the unemployment rate at the aggregate level and by industry are shown for the "Lockdown and supply chain crisis" scenario.

forestry) tend to be more "COVID-neutral."

Although the current version of CANVAS does not differentiate the labor supply elasticity of "COVID sensitive" households by modelling a relatively greater decline in their working hours in response to negative shocks, current characteristics (sectoral working hours, sex, and ages) allows for some initial exploration of the model's capacity to address labor market developments.

Figure 9: Sectoral unemployment rate by sex and age (%)



Note: Simulated dynamics of the unemployment rate by sex and age in the "Lockdown and supply chain crisis" scenario, at the aggregate level and for select industries. All unemployment rates are annualized.

Our findings suggest the importance of considering household characteristics (such as sex, age, and balance sheets) in understanding their employment choices and spending patterns. The bottom panel of Figure 9 shows that younger people (age 14–25) and women in service sectors tend to bear greater and more prolonged impacts from the pandemic.

In this context, MacGee et al. (2020) have uncovered some Canadian evidence by constructing wealth, debt, and income for a set of households using the Survey of Financial Security. They show different effects on households across the income distribution. Low-income households have the highest risk of unemployment, but the Canada Emergency Response Benefit (CERB) provided a relatively high replacement of previous income. Middle-income households that lost jobs saw the fastest rise in debt, as the CERB only partially replaced income lost due to unemployment. High-income households had a lower probability of unemployment in this crisis and on average accumulated unplanned savings. The implication of unemployment by income level of households is not explored in our current version of analysis. We leave this for future research.

7 Conclusion

Since the inception of the first DSGE model, ToTEM, in 2005 as the Bank of Canada's main projection and policy analysis model, Bank staff have made considerable effort to improve macroeconomic models' ability to explain Canadian macroeconomic data as well as to expand its capacity to address a growing variety of pertinent policy questions.

The model-based projections and policy analysis tradition at the Bank has culminated in the development of the updated version of the DSGE model, ToTEM II, which replaced ToTEM in June 2011. The richer interest rate structure in ToTEM II has allowed Bank staff to include short-term and long-term risk spreads in the quarterly projection analysis and to conduct policy analysis related to the implementation of the extended monetary policy tool kit.

The latest Bank DSGE model, ToTEM III, incorporates new features that have been used to address many relevant policy questions. For example, the presence of borrowers in the model has allowed staff to assess how higher household indebtedness affects the sensitivity of consumption to interest rates.

ToTEM III's structure already incorporates some level of household heterogeneity through the introduction of borrowers and savers in a more elaborate modelling of the housing market. These new features allow Bank staff to explore a broader range of policy questions. Moreover, the new structure of the model and the enhanced estimation of the model's parameters have contributed to a significant improvement of the forecasting performance of the model. Nevertheless, introducing both household and firm heterogeneity in a large-scale DSGE model has proven to be extremely challenging for both theoretical development and computational execution.

These limitations have motivated Bank staff to devote effort into developing its first behavioral macroeconomic agent-based model—CANVAS. The goal of our development concentrates on incorporating policy-relevant advances of economic modelling as well as improving the model's forecasting ability to complement our current suite of in-house core macro models.

In this paper, we describe CANVAS's most important features, which include very detailed modelling of the Canadian economic structures using microdata and improvements in the modelling of agents' behavior through adaptive learning. More specifically, CANVAS's structure effectively encompasses both detailed national accounting of all individual households and firms' balance sheets. Leveraging a more granular level of heterogeneity offers elaborate modelling of demographic data such as sex, age, occupation, and household balance sheets. These new features allow Bank staff to explore a broader range of policy questions, including understanding the unevenness in both production and pricing behavior of firms during the COVID-19 pandemic and recovery. In addition to providing the capacity to understand dynamics from detailed production networks, CANVAS also helps shed some light on labor markets and the evolution of various inflation rates in Canada. Finally, the detailed structure of the model has also contributed to a significant improvement in the forecasting performance of the model, making it among the first class of ABM models that can compete with benchmark VAR and DSGE models in out-of-sample forecasting of macro variables.

Among many potential developments with CANVAS, heterogeneous learning such as modelling extrapolative expectations in the housing market, a more elaborate labor market, and further enriched financial frictions to analyze the efficacy of extended monetary policies such as quantitative easing and forward guidance, constitute our medium-term modelling effort.

8 References

Acharya, S. and Dogra, K. (2020). Understanding HANK: Insights from a PRANK. *Econometrica*, 88(3):113-1158.

Adam, K. (2007). Optimal monetary policy with imperfect common knowledge. *Journal of Monetary Economics*, 54(2):267-301.

Alpanda, S. and Zubairy, S. (2017). Addressing household indebtedness: Monetary, fiscal or macroprudential policy? *European Economic Review*, 92(C):47-73.

Amano, R., Gnocchi, S., Leduc, S. and Wagner, J. (2020). Average is good enough: Average-inflation targeting and the ELB. *Federal Reserve Bank of San Francisco Working Paper 2020-21.*

Amiti, S., Itskhoki, O. and Konings, J. (2019). International shocks, variable markups, and domestic prices. *The Review of Economic Studies*, 86(6):2356–2402.

Andrés, J., López-Salido, D. and Nelson, E. (2004). Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking*, 36(4):665-690.

Anufriev, M. and Tuinstra, J. (2013). The impact of short-selling constraints on financial market stability in a heterogeneous agents model. *Journal of Economic Dynamics and Control*, 37(8):1523-1543.

Arifovic, J., Dawid, H., Deissenberg, C. and Kostyshyna, O. (2010). Learning benevolent leadership in a heterogenous agents economy. *Journal of Economic Dynamics and Control*, 34(9):1768-1790.

Assenza, T., Delli Gatti, D., and Grazzini, J. (2015). Emergent dynamics of a macroeconomic agent based model with capital and credit. *Journal of Economic Dynamics and Control*, 50:5-28.

Assenza, T., Heemeijer, P., Hommes, C. and Massaro, D. (2021). Managing self-organization of expectations through monetary policy: A macro experiment. *Journal of Monetary Economics*, 117:170-186.

Axtell, R.L., 2001. Zipf distribution of US firm sizes. *Science*, 293(5536):1818-1820.

Axtell, R.L. (2018). Endogenous firm dynamics and labor flows via heterogeneous agents. *In: Hommes, C., LeBaron, B. (Eds.), Handbook of Computational Economics. Vol. 4 of Handbook of Computational Economics*, Elsevier, pp. 157–213.

Axtell, R.L. and Farmer, J.D. (2022). Agent-based modeling in economics and finance: Past, present, and future. *Journal of Economic Literature, forthcoming.*

Bhaskar, V., Machin, S. and Reid, G.C. (1993). Price and quantity adjustment over the business cycle: Evidence from survey data. *Oxford Economic Papers*, 45(2): 257–268.

Blanchard, O. (2016). Do DSGE models have a future? *PIIE Policy Brief 16-11.*

Blattner, T.S. and Margaritov, E. (2010). Towards a robust monetary policy rule for the euro area. *ECB Working Paper No. 1210.*

Brayton, F., Mauskopf, E., Reifschneider, D., Tinsley, P., Williams, J., Doyle, B. and Sumner, S. (1997). The role of expectations in the FRB/US macroeconomic model. *Federal Reserve Bulletin April 1997.*

Brunnermeier, M. K., Eisenbach, T. M. and Sannikov, Y. (2013). Macroeconomics with financial frictions: A survey. *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society*, New York: Cambridge University Press.

Budianto, F., Nakata, T. and Schmidt, S. (2020). Average inflation targeting and the interest rate lower bound. *BIS Working Paper No. 852.*

Calvo, G.A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Champagne, J., Poulin-Bellisle, G. and Sekkel, R. (2018). Evaluating the Bank of Canada staff economic projections using a new database of real-time data and forecasts. *Bank of Canada Staff Working Paper 2018-52.*

Chatterjee, S., Corbae, D., Nakajima, M. and Ríos-Rull, J.V. (2007). A quantitative theory of unsecured consumer credit with risk of default. *Econometrica*, 75(6): 1525-1589.

Chen, H., Cúrdia, V. and Ferrero, A. (2012). The macroeconomic effects of large-scale asset purchase programmes. *The Economic Journal*, 122(564): F289-F315.

Christiano, L. J., Eichenbaum, M. S. and Trabandt, M. (2018). On DSGE models. *Journal of Economic Perspectives*, 32(3):113–40.

Chu, V., Pavilanis, C. and Zhang, Y. (2020). A tale of two savings: Involuntary or precautionary savings during the pandemic. *Bank of Canada Internal Note.*

Colander, D., Howitt, P., Kirman, A., Leijonhufvud, A. and Mehrling, P. G. (2008). Beyond DSGE models: Toward an empirically based macroeconomics. *American Economic Review, Papers and Proceedings*, 98(2):236–240.

Corrigan, P., Desgagnés, H., Dorich, J., Lepetyuk, V., Miyamoto, W. and Zhang, Y. (2021). ToTEM III: The Bank of Canada's main DSGE model for projection and policy analysis. *Bank of Canada Technical Report No. 119.*

Del Negro, M. and Schorfheide, F. (2013). DSGE model-based forecasting. *Handbook of Economic Forecasting*, 2:57–140

Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P., and Gallegati, M. (2011). Macroeconomics from the bottom-up. Springer Milan, pp. 54-57.

Diebold, F. X. and Mariano, R. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, 13(3):253–263.

Diebold, F. X., Schorfheide, F. and Shin, M. (2017). Real-time forecast evaluation of DSGE models with stochastic volatility. *Journal of Econometrics*, 201(2):322–332.

Djeutem, E., He, M., Reza, A. and Zhang, Y. (2022). Household heterogeneity and the performance of monetary policy frameworks. *Bank of Canada Staff Working Paper 2022-12.*

Dorich, J., Johnston, M., Mendes, R., Murchison, S. and Zhang, Y. (2013). ToTEM II: An updated version of the Bank of Canada's quarterly projection model. *Bank of Canada Technical Report No. 100.*

Dorich, J., Mendes, R. and Zhang, Y. (2021). The Bank of Canada's "horse race" of alternative monetary policy frameworks: Some interim results from model simulations. *Bank of Canada Discussion Paper 2021-13.*

Dosi, G., Napoletano, M., Roventini, A. and Treibich, T. (2017). Micro and macro policies in the keynes+schumpeter evolutionary models. *Evolutionary Economics*, 27:63-90.

Dosi, G., Napoletano, M., Roventini, A., Stiglitz, J. E. and Treibich, T. (2020). Rational heuristics? Expectations and behaviors in evolving economies with heterogeneous interacting agents. *Economic Inquiry*, 58(30):1487-1516.

Edge, R., Kiley, M.T. and Laforte, J.-P. (2010). A comparison of forecast performance between federal reserve staff forecasts, simple reduced-form models, and a DSGE model. *Journal of Applied Econometrics*, 25(4):720-754.

Edge, R. and Gurkaynak, R.S. (2010). How useful are estimated DSGE model forecasts for central bankers? *Brookings Papers on Economic Activity*, 41(2):209-259.

Ens, E., Savoie-Chabot, L. and See, K.G. (2021). Assessing labour market slack for monetary policy. *Bank of Canada Staff Discussion Paper 2021-15.*

Evans, G. W. and Honkapohja, S. (2001). Learning and expectations in macroeconomics. *Princeton University Press.*

Fagiolo, G. and Roventini, A. (2017). Macroeconomic policy in dsge and agent-based models redux: New developments and challenges ahead. *Journal of Artificial Societies and Social Simulation*, 20(1).

Farmer, J. D. and Foley, D. (2009). The economy needs agent-based modelling. *Nature*, 460:685–686.

Feiveson, L., Goernemann, N., Hotchkiss, J., Mertens, K. and Sim, J. (2020). Distributional considerations for monetary policy strategy. *Finance and Economics Discussion Series 2020-073.*

Freeman, R.B. (1998). War of the models: Which labour market institutions for the 21st century? *Labour Economics*, 5:1-24.

Gabaix, X. (2020). A behavioral New Keynesian model. *American Economic Review*, 110(8):2271-2327.

Galı, J. and Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, 44(2):195-222.

Giacomini, R. and Rossi, B. (2010). Forecast comparisons in unstable environments. *Journal of Applied Econometrics*, 25(4):595-620.

Gigerenzer, G. (2015). Risk savvy: How to make good decisions. *Allen Lane.*

Gintis, H. (2007). The dynamics of general equilibrium. *The Economic Journal*, 117:1280-1309.

Gualdi S., Tarzia M., Zamponi F. and Bouchaud J.P. (2015). Tipping points in macroeconomic agent-based models. *Journal of Economic Dynamics and Control*, 50:29-61.

Haldane, A.G. and Turrell, A.E. (2018). An interdisciplinary model for macroeconomics. *Oxford Review of Economic Policy*, 34(1-2):219–251.

Harvey, D., Leybourne, S. and Newbold, P. (1997). Testing the equality of prediction mean squared errors. *International Journal of Forecasting*, 13(2):281–291

Hommes, C. and Zhu, M. (2014). Behavioral learning equilibria. *Journal of Economic Theory*, 150:778-814.

Ijiri, Y. and Simon, H. A., 1977. Skew distributions and the sizes of business firms. North-Holland.

Kaplan, G. and Violante, G. L. (2018). Microeconomic heterogeneity and macroeconomic shocks. *Journal of Economic Perspectives*, 32(3):167-94.

Kaplan, G. and Violante, G.L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199-1239.

Kaplan, G., Moll, B. and Violante, G.L. (2018). Monetary policy according to HANK. *American Economic Review*, 108(3):697-743.

Kawasaki, S., McMillan, J., and Zimmermann, K.F. (1982). Disequilibrium dynamics: An empirical study. *The American Economic Review*, 72(5): 992–1004.

Khan, A. and Thomas, J. (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica*, 76(2):395-436.

Kostyshyna, O., Petersen, L., Yang, J. (2022). A horse race of monetary policy regimes: An experimental investigation. *National Bureau of Economic Research Working Paper No. w30530.*

Kryvtsov, O. and Peterson, L. (2013). Expectations and monetary policy: Experimental evidence. *Bank of Canada Staff Working Paper 2013-44.*

LeBaron, B. and Tesfatsion, L. (2008). Modeling macroeconomies as open-ended dynamic systems of interacting agents. *American Economic Review Papers and Proceedings*, 98(2):246–250.

MacGee, J., Pugh, T.M. and See, K.G. (2020). The heterogeneous effects of COVID-19 on Canadian household consumption, debt and savings. *Bank of Canada Staff Working Paper 2020-51.*

McKay, A. and Reis, R. (2016). The role of automatic stabilizers in the U.S. business cycle. *Econometrica*, 84(1):141-194.

Murchison, S. and Rennison, A. (2006). ToTEM: The Bank of Canada's new quarterly projection model. *Bank of Canada Technical Report No. 97.*

Napoletano, M. (2018). A short walk on the wild side: Agent-based models and their implications for macroeconomic analysis. *Revue de l'OFCE*, 167(3):257-281.

Pfajfar, D. and Zakelj, B. (2014). Experimental evidence on inflation expectation formation. *Journal of Economic Dynamics and Control*, 44:147-168.

Poledna, S., Miess, M.G., Hommes, C.H., and Rabitsch, K. (2022). Economic forecasting with an agent-based model. *European Economic Review*.

Poloz, S. (2017). Models and the art and science of making monetary policy. *Speech, University of Alberta School of Business*.

Salle, I. (2015). Modeling expectations in agent-based models—An application to central bank's communication and monetary policy. *Economic Modelling*, 46:130-141.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Stiglitz, J.E. and Gallegati, M. (2011). Heterogeneous interacting agent models for understanding monetary economies. *Eastern Economic Journal*, 37:6-12.

Svensson, L.E.O. (2020). Monetary policy strategies for the Federal Reserve. *International Journal of Central Banking*, 16(1):133-193.

Taylor, J. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195-214.

Trichet, J.C. (2010). Reflections on the nature of monetary policy non-standard measures and finance theory. *Speech for the Opening address at the ECB Central Banking Conference, Frankfurt, 18 November 2010*.

Turrell, A. (2016). Agent-based models: Understanding the economy from the bottom up. *Bank of England Quarterly Bulletin*, 56(4):173-188.

Vines, D. and Wills, S. (2018). The rebuilding macroeconomic theory project: an analytical assessment. *Oxford Review of Economic Policy*, 34(1-2):1–42.

Wagner, J., Schlanger, T. and Zhang, Y. (2022). A horse race of alternative monetary policy regimes under bounded rationality. *Bank of Canada Staff Discussion Paper 2022-4*.