A Tractable Income Process for Business Cycle Analysis

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Today: Three Questions

- Growing interest in heterogeneity/inequality in business cycle analyses (Kaplan et al. (2018), McKay and Reis (2021), Auclert et al. (2022)).
  - Aggregate and distributional effects of monetary policy
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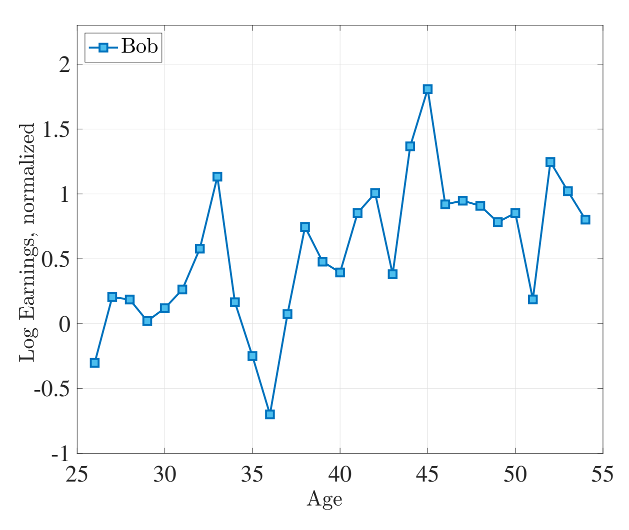
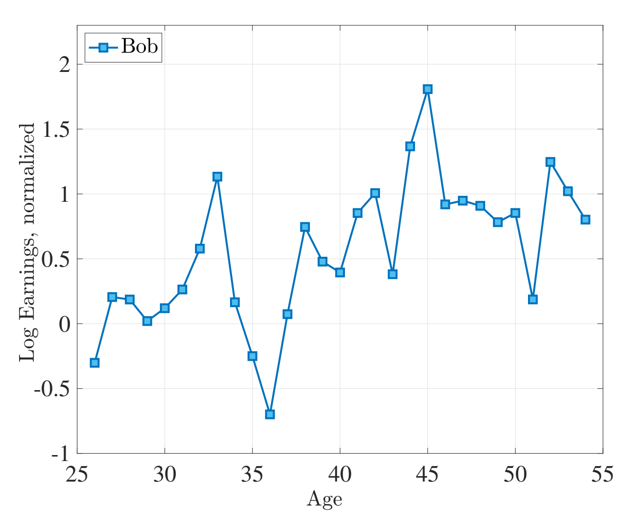
  3. How does idiosyncratic risk change over the business cycle?

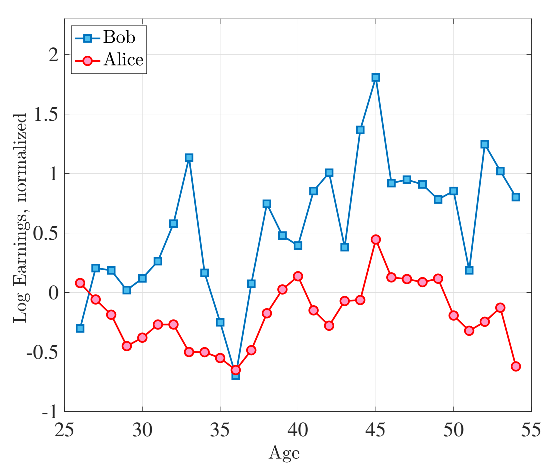
Example: Earnings History of Bob



Volatility of Annual Earnings Change ≈ 54% (US), 53% (Canada)!



Earnings Histories of Bob and Alice



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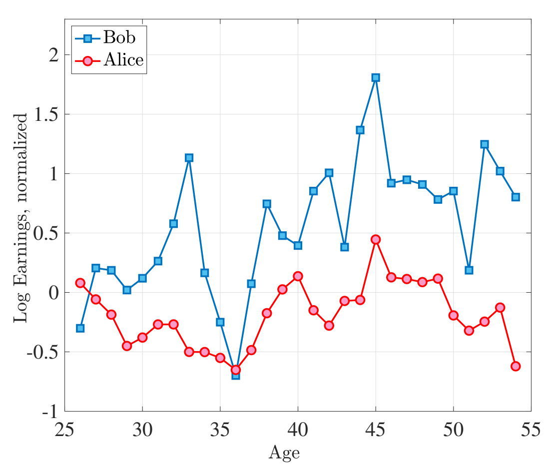

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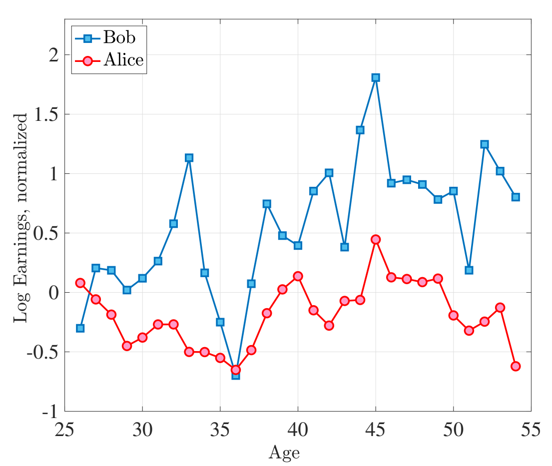

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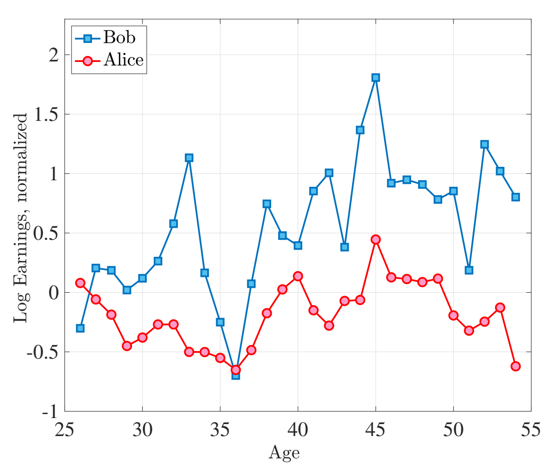

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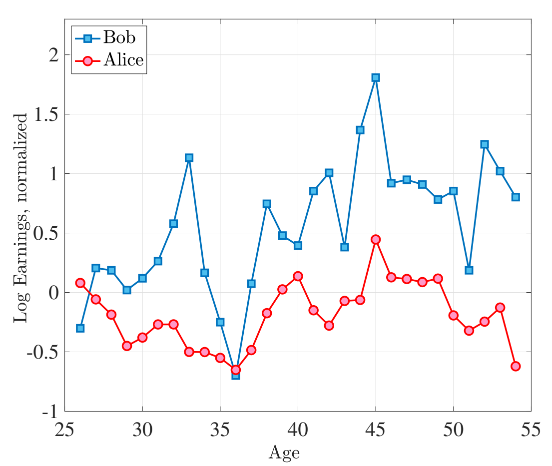

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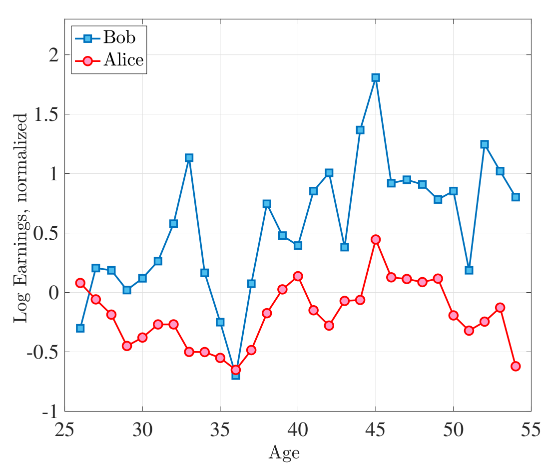

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  2. *Group-level* shocks: to employer, industry, occupation, local economy, etc.?
  3. Different sensitivities to (1) and (2)? *"Worker Betas"*
  4. *Purely idiosyncratic* shocks: Promotion, demotion, health, divorce, job loss/change, etc.

Canonical Approach: An Overview

- Let y_t^i denote *net log earnings* of individual i in year t :

$$\underbrace{y_t^i}_{\text{net log earnings}} = \underbrace{\tilde{y}_t}_{\text{log earnings}} - \underbrace{X_t^i\theta}_{X_t=\{\text{age, education}\}} \quad (1)$$

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$$\underbrace{y_t^i}_{\text{net log earnings}} = \underbrace{d_t}_{\text{year dummy}} + \underbrace{\nu_t^i}_{\text{residual earnings}} \tag{1}$$

Canonical Approach: An Overview

- Let y_t^i denote *net log earnings* of individual i in year t :

$$\underbrace{y_t^i}_{\text{net log earnings}} = \underbrace{d_t}_{\text{systematic risk}} + \underbrace{\nu_t^i}_{\text{idiosyncratic risk}} \tag{1}$$

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  1. d_t is typically taken as systematic risk

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     - ***No aggregate factor structure:*** $\beta^i \times d_t$ or other systematic factors (industry, employer, occupation-specific, regional, etc, risks)

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     - ***No aggregate factor structure:*** $\beta^i \times d_t$ or other systematic factors (industry, employer, occupation-specific, regional, etc, risks)
  2. But $\sigma(d_t) \ll \sigma(y_t^i) \rightarrow$ Most earnings fluctuations attributed to idiosyncratic shocks, ν_t^i.
     - All focus on modeling ex post risk, ν_t^i
     - $\therefore$ ***Income risk is (largely) unpredictable:*** recession arrives, impossible to tell who will suffer more (beyond a few demographics)

Canonical Approach: An Overview

- Idiosyncratic Risk:
  - Typically modeled as a (linear) ARMA(p,q) process.
  - Most often: Fixed effect + AR(1) + transitory shock

$$\nu_t^i = \alpha^i + z_t^i + \varepsilon_t^i$$
$$z_t^i = \rho z_{t-1}^i + \eta_t^i$$

with Gaussian innovations: $\varepsilon_t^i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\eta_t^i \sim \mathcal{N}(0, \sigma_\eta^2)$.

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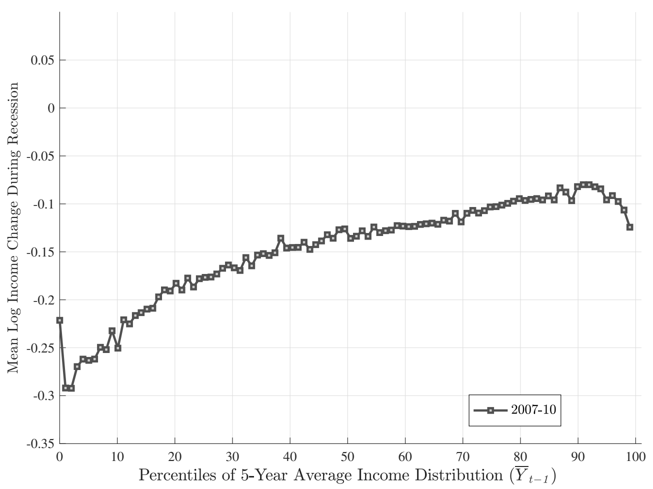
- Canonical model goes back to 1980s (Lillard and Weiss (1979), MaCurdy (1982)) and changed little until recently.

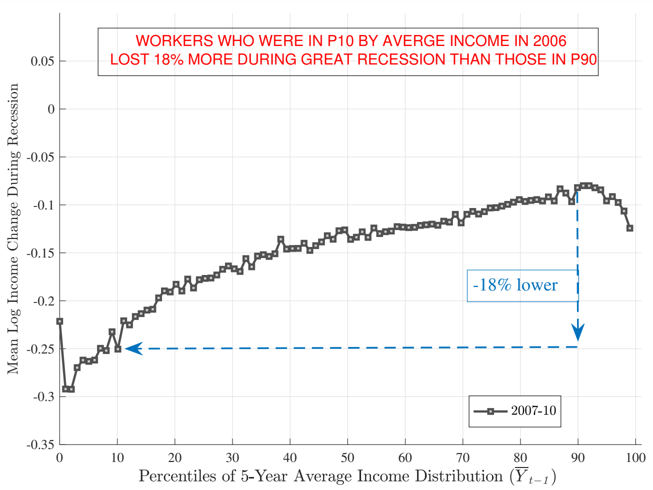
Q1: What is the Nature of Systematic Risk?

Factor Structure, Great Recession



Source: Guvenen, Ozkan, Song (JPE, 2014)

Business Cycle Risk *IS* Predictable



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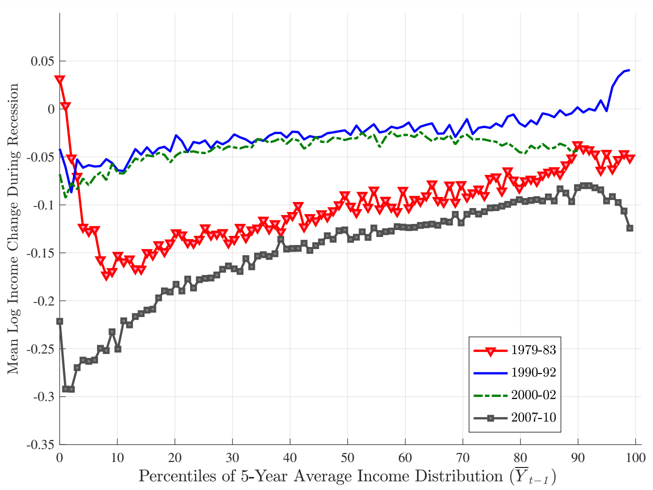
Four Recessions: Prime Age Males



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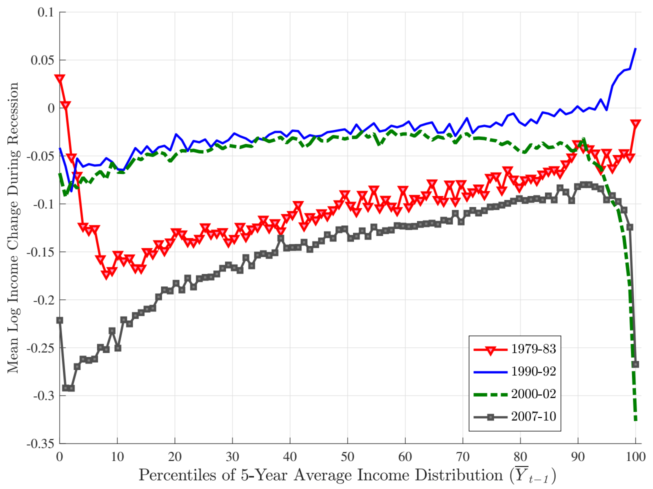
How About the Top 1%?



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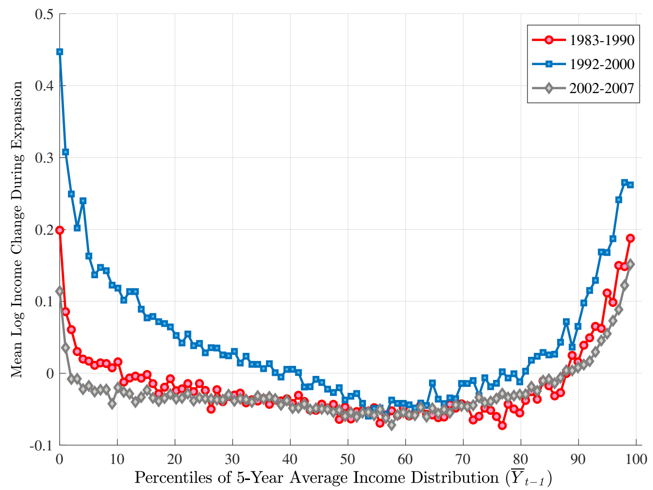
Three Expansions: Prime-Age Males



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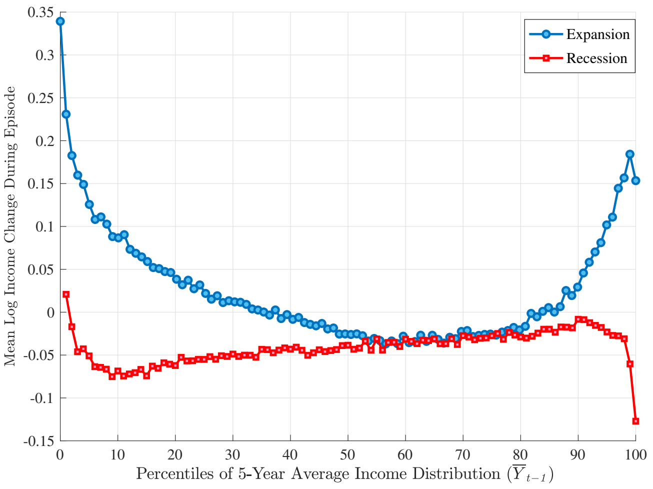
Putting Together: Expansions vs Recessions



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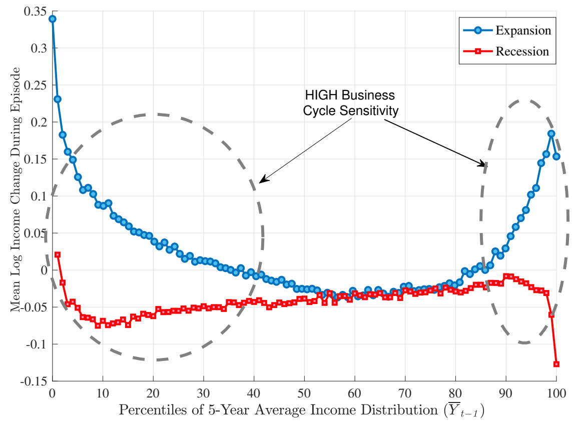

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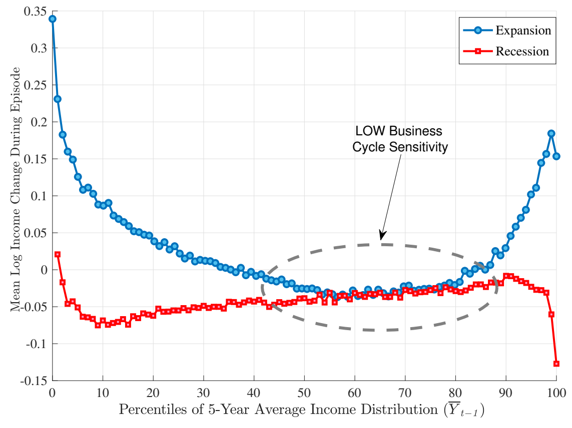

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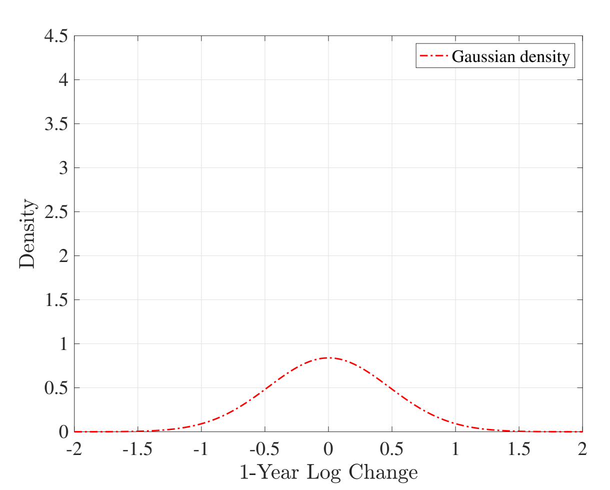
Q2: What is the Nature of Idiosyncratic Income Risk?

Is Earnings Growth Log Normal?



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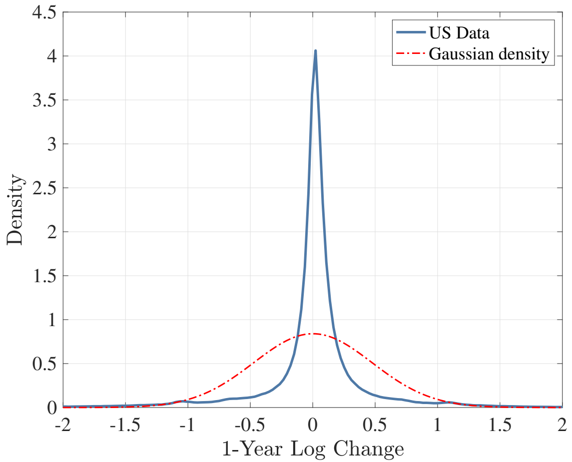

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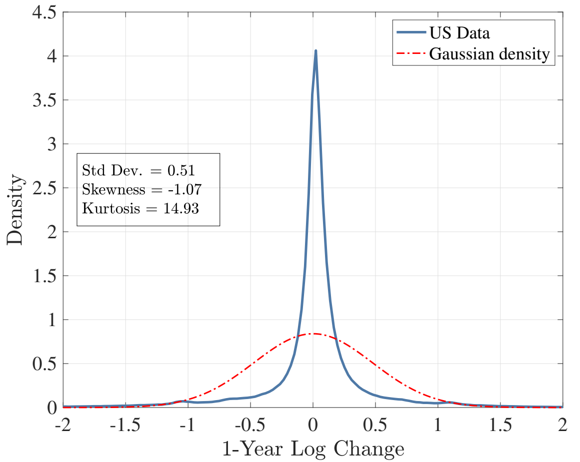

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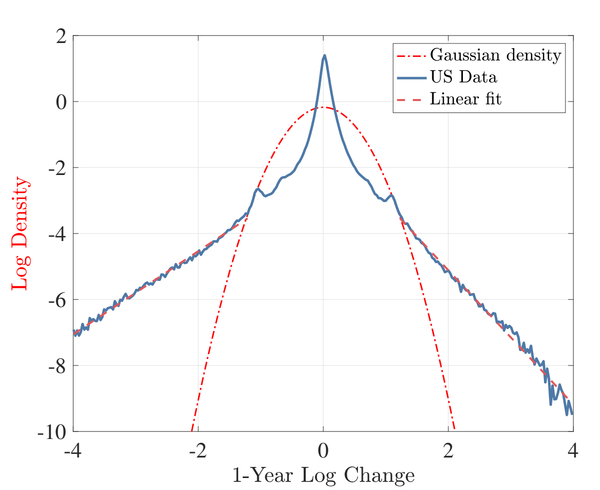

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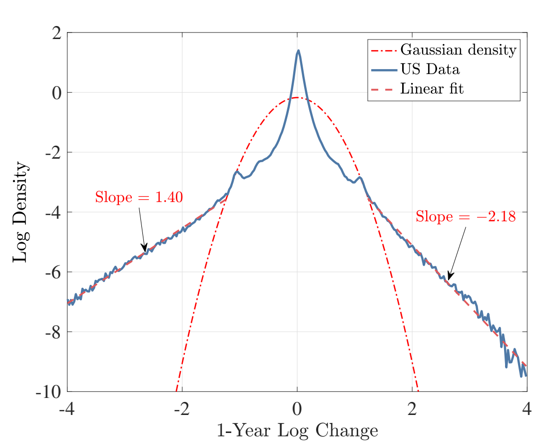

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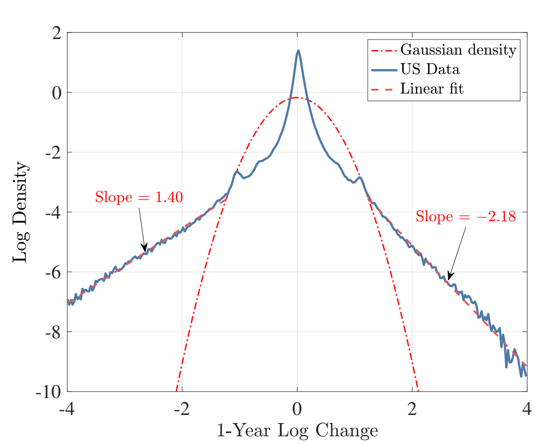

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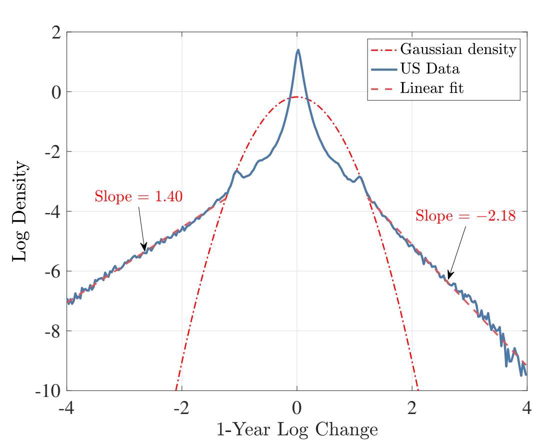

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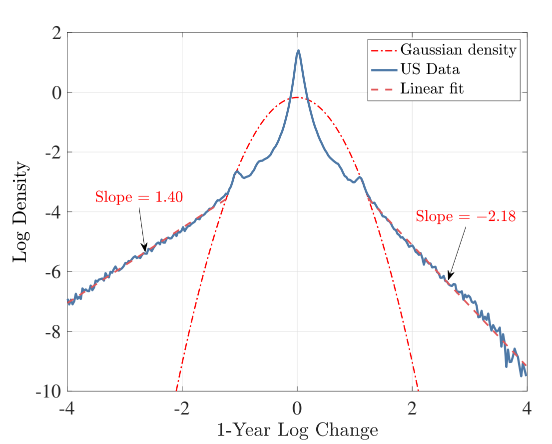
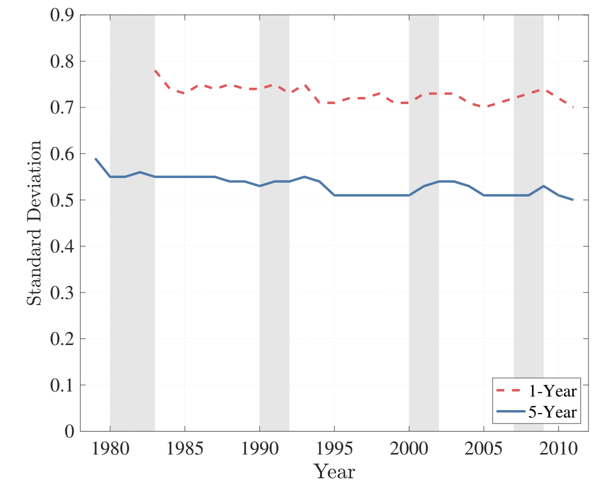


Important deviations from Normality:

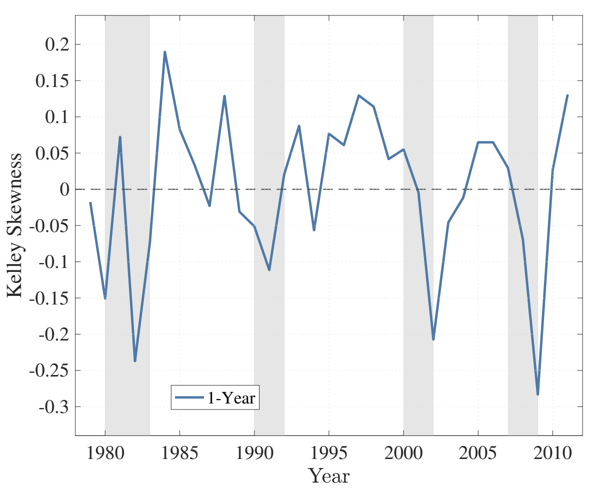
- Sharp peak in center, long tails → very high kurtosis (~15)
- Tails follow a straight line → Double-Pareto distribution
- Left tail thicker than right tail → negative skewness

Q3: How Does Idiosyncratic Risk Change Over the Business Cycle?

Variance: Flat and Acyclical



Skewness: Volatile and Procyclical



Our income process

- Target six key features of income dynamics
  1. Flat and acyclical variance
  2. Volatile and procyclical skewness
  3. High kurtosis
  4. Slopes of tails of log density
  5. Factor structure
  6. Moderate rise in cross-sectional inequality over the life cycle
- Moment data taken from Guvenen et al. (2014) and Guvenen et al. (2021)
  - Based on 36 year earnings histories for male workers from US SSA data

Three key features

Estimate income process with three departures from workhorse linear-Gaussian model:

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- Normal mixture process for persistent income innovations
  - Procyclical skewness
  - High kurtosis
- Factor Structure
  - Income position affects aggregate shock exposure

Income Process: 1. Scarring Effects

$$\underbrace{y_{i,t}}_{\text{log earnings}} = \gamma_i + z_{i,t} + (1-\psi)\zeta_{i,t} + [1 + f(\gamma_i + z_{i,t})]\, w_t$$

$$z_{i,t} = \rho z_{i,t-1} + \tilde{\eta}_{i,t} + \underbrace{\psi\zeta_{i,t}}_{\text{scarring effect}}$$

Income Process: 1. Scarring Effects

Non-employment shock: transitory wage loss + long lasting "scar"

$$\underbrace{y_{i,t}}_{\text{log earnings}} = \gamma_i + z_{i,t} + \underbrace{(1-\psi)\zeta_{i,t}}_{\text{transient shock}} + [1 + f(\gamma_i + z_{i,t})]\, w_t$$

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- Scarring Effect:

$$\zeta_{i,t} = \begin{cases} 0 & \text{with prob. } p^{\zeta} \\ \log(1-\ell_{i,t}) & \text{with prob. } 1-p^{\zeta} \end{cases}$$

$\ell_{i,t}$ ~exponentional (λ), conditional on $\ell \in [0,1]$

Income Process: 2. Time Varying Shocks

$$\underbrace{y_{i,t}}_{\text{log earnings}} = \gamma_i + \underbrace{z_{i,t}}_{\text{persistent shock}} + (1-\psi)\zeta_{i,t} + [1 + f(\gamma_i + z_{i,t})]\, w_t + \kappa_i (t - h_i)$$

$$z_{i,t} = \rho z_{i,t-1} + \underbrace{\eta_{i,t}}_{\text{persistent innovation}} + \psi \zeta_{i,t}$$

- Persistent shocks drawn from a Normal mixture distribution:

$$\eta_{i,t} \sim \begin{cases} \mathcal{N}(\mu^{\eta}_{1,t}, \sigma^{\eta}_{1}) & \text{with prob. } p^{\eta}_{1}, \\ \mathcal{N}(\mu^{\eta}_{2,t}, \sigma^{\eta}_{2}) & \text{with prob. } p^{\eta}_{2}, \\ \mathcal{N}(\mu^{\eta}_{3,t}, \sigma^{\eta}_{3}) & \text{with prob. } 1 - p^{\eta}_{1} - p^{\eta}_{2}, \end{cases}$$

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$$z_{i,t} = \rho z_{i,t-1} + \underbrace{\eta_{i,t}}_{\text{persistent innovation}} + \psi\zeta_{i,t}$$

- Introduce business cycle variation: Distribution means $(\mu_1^\eta, \mu_2^\eta, \mu_3^\eta)$ fluctuate:

$$\begin{aligned} \mu_{1,t}^\eta &= \bar{\mu}_t^\eta, \\ \mu_{2,t}^\eta &= \bar{\mu}_t^\eta + \mu_2^\eta - x_t, \\ \mu_{3,t}^\eta &= \bar{\mu}_t^\eta + \mu_3^\eta. \end{aligned}$$

where $x_t = \beta \Delta w_t$ (can be GDP, unemployment rate, average wage, etc.)

Income Process: 3. Factor Structure

$$\underbrace{y_{i,t}}_{\text{log earnings}} = \gamma_i + z_{i,t} + (1-\psi)\zeta_{i,t} + \left[\underbrace{1 + f(\gamma_i + z_{i,t})}_{\text{Factor Structure}}\right] w_t + \kappa_i (t - h_i)$$

$$z_{i,t} = \rho z_{i,t-1} + \psi \zeta_{i,t}$$

- Factor Structure:
  - Aggregate shock (w_t) exposure depends on income position.
  - Assume piecewise linear function:

$$f(q) = \begin{cases} \alpha_1 q & \text{if } q < \bar{q} \\ \alpha_2 (q - \bar{q}) + \alpha_1 \bar{q} & \text{if } q \geq \bar{q}, \end{cases}$$

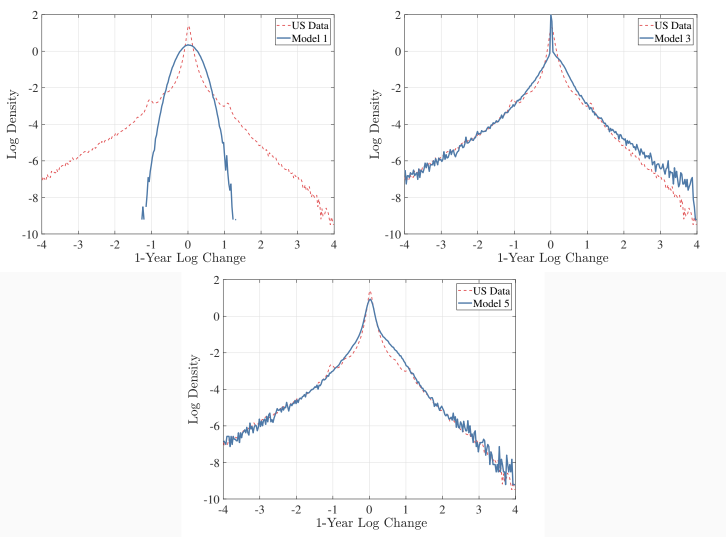
where $q \equiv \gamma_i + z_{i,t}$

Estimated Models - Specification Summary

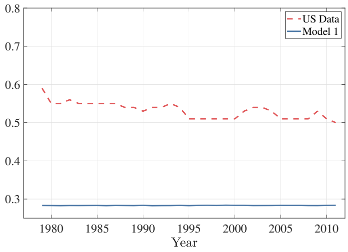
	Key Components of Stochastic Process					
Model	ζ	η	ψ	ρ	σ^{κ}	Factor Str.
(1)	Gaussian	Gaussian	$= 0$	$= 1$	$= 0$	
(2)	Non-emp.	Gaussian	> 0	$= 1$	$= 0$	
(3)	Non-emp.	Mixture	> 0	$= 1$	$= 0$	
(4)	Non-emp.	Mixture	> 0	$= 1$	$= 0$	✓
(5)	Non-emp.	Mixture	> 0	≤ 1	> 0	
(6)	Non-emp.	Mixture	> 0	≤ 1	> 0	✓

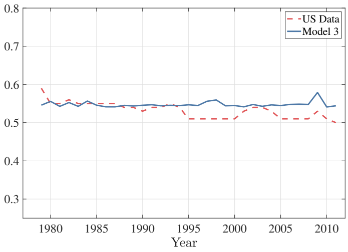
Results

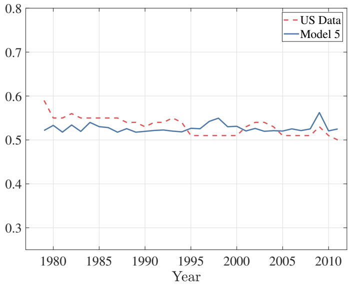
Histogram of 1-Year Log Earnings Growth



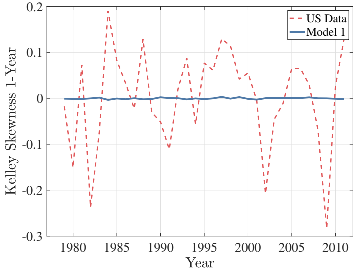
Time Series of 1-Year Standard Deviation

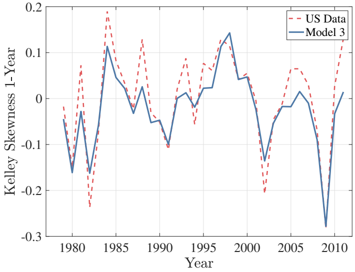


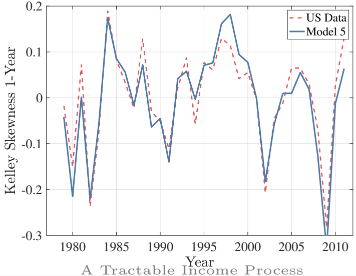




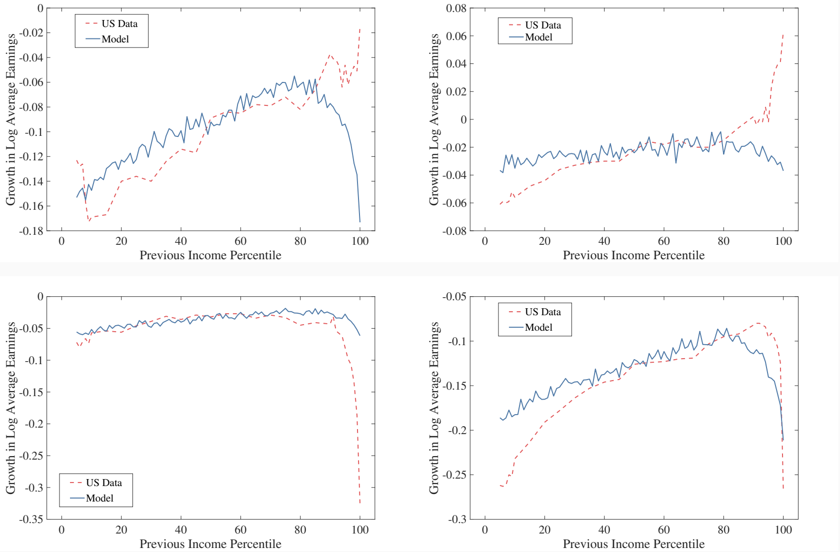
Time Series of 1-Year Kelley Skewness



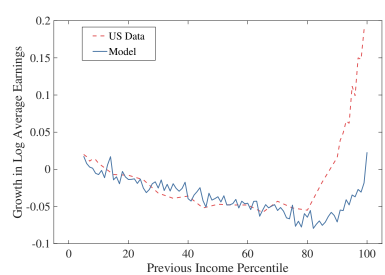


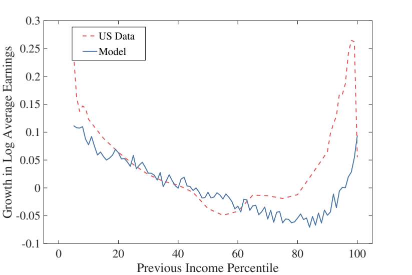


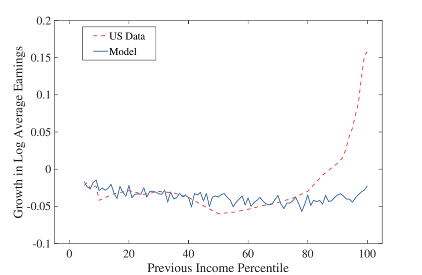
Factor Structure: Recessions



Factor Structure: Expansions







Applying the Process

Incorporating the Income Process into a Business Cycle Model

- Despite generating non-linear and non-Gaussian properties, the process features only 1 state variable.
  - Same as canonical linear-Gaussian model.
- So, solving a dynamic programming model with it is fairly straightforward:
  - Use quadrature for time-varying mixture of normals
  - Solution with standard methods:
    - Endogenous grid method
    - Value function iteration

Conclusions

- We introduce a new income process with 3 key features:
  - nonemployment shocks with scarring effects
  - normal mixture persistent shocks
  - factor structure with “betas” that depend on income levels
- The process matches many key features of income risk and how it varies with the business cycle.
- Features one state variable and fairly easy to incorporate into a business cycle model.

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