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Regulatory Requirements of Banks and Arbitrage in the Post-Crisis Federal Funds Market

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Abstract

This paper explains the nature of interest rates in the U.S. federal funds market after the 2007-09 financial crisis. We build a model of the over-the-counter lending market that incorporates new aspects of the financial system: abundance of liquidity, different regulatory standards for banks, and arbitrage opportunities created by limited access to the facility granting interest on excess reserves. The model determines the equilibrium federal funds rate as a function of the policy rates and explains the "leaky floor" phenomenon in which we observe federal funds rates that are strictly below the interest rate paid on reserves. Using the model, we explain the impact of raising government yields and tightening the Liquidity Coverage Ratio (LCR) and the Supplementary Leverage Ratio (SLR) requirements on the federal funds rates.

Topics: Central bank research; Economic models; Financial institutions; Financial markets; Financial stability; Wholesale funding; Financial system regulation and policies
JEL codes: E42, E58, G2, G28

Résumé

Le présent document explique la nature des taux d'intérêt sur le marché des fonds fédéraux aux États-Unis après la crise financière de 2007-2009. Nous construisons un modèle du marché hors cote qui intègre de nouveaux aspects du système financier : abondance de liquidités, normes réglementaires variées applicables aux banques et possibilités d'arbitrage créées par l'accès limité au mécanisme octroyant des intérêts sur les réserves excédentaires. Le modèle établit le taux d'équilibre des fonds fédéraux en fonction des taux directeurs et explique le phénomène du « plancher poreux » qui se caractérise par des taux des fonds fédéraux strictement inférieurs au taux d'intérêt appliqué sur les réserves. À l'aide du modèle, nous montrons l'incidence qu'ont sur les taux des fonds fédéraux les hausses de rendement des titres d'État et le resserrement des exigences de ratio de liquidité à court terme et de ratio de levier supplémentaire.

Sujets : Recherches menées par les banques centrales; Modèles économiques; Institutions financières; Marchés financiers; Stabilité financière; Financement de gros; Réglementation et politiques relatives au système financier
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1. Introduction

The federal funds market has historically played a central role in the transmission of U.S. monetary policy to the financial system. Following the global financial crisis of 2007-2009, the structure of the market changed substantially. Interbank intermediation disappeared in its original form, and the liquidity facilities established by the Federal Reserve became the driving forces of liquidity flows. This led to the formation of the current federal funds market, which is mostly bipartite in structure and nourished by arbitrage activity rather than by liquidity needs.

We conduct analysis that seeks to explain the efficacy of the new system for the transmission of monetary policy under various financial conditions. In particular, we build a model that shows how pairwise rates for federal funds are negotiated in the context of two key policy rates: the interest on excess reserves (IOER) rate and the overnight reverse repurchase agreements (ONRRP) rate.

IOER was originally intended as a floor on interest rates in the federal funds market. Federal funds market participants that earn IOER have no incentive to lend in the federal funds market at a rate below the IOER rate. In reality, however, the market observed a "leaky" floor, where trades predominately occurred at rates below the IOER rate. This happened because an important segment of lenders in the federal funds market, Federal Home Loan Banks (FHLBs), were not eligible to earn IOER. This created an arbitrage market with IOER-earning banks passing liquidity from FHLBs to the IOER facility. Interestingly, the arbitrage activity by IOER-earning banks was not sufficient to drive rates in this market up to the level of IOER.

Why was it the case that competition among borrowers did not drive the federal funds rates up to the IOER rate? The first part of the story is that FHLBs undertake unsecured lending only to a selected group of counterparties that meet certain credit standards and for whom they have established monitoring relationships.[2] The second part of the story is that these acceptable borrowers do not want to borrow an unlimited amount. Excessive borrowing of federal funds could create imbalances between ratios of assets and liabilities for the participating bank and either lead to elevated leverage or require the bank to adjust balance sheets. Both of these outcomes are costly. Tightened regulatory standards on banks is one of the explanations for such costs. Following the global financial crisis, a series of regulatory requirements were imposed to ensure the overall financial stability of the U.S. banking system. Among other things, the new policies focused on high indebtedness of banks and their liquidity management. The updated leverage regulation required banks to hold more capital for each dollar on their balance sheets. Thus, the leverage requirement discourages banks from excessive borrowing in the interbank market. At the same time, the new liquidity coverage ratio obliged banks to hold a stock of high-quality liquid assets (HQLA)

[2] This explanation resonates with similar observations Chiu et al. (2020) make for the European interbank market in the context of relationship lending.

to compensate for the expected total net cash outflows over a 30-day period. This regulation encouraged federal funds borrowing. Since the leverage requirement eventually dominated liquidity needs, banks were limited in their desire to earn arbitrage profits, which often kept interbank interest rates below IOER.

The ONRRP facility was introduced in 2014 as a pilot program to put a "subfloor" under the leaky floor created by IOER. The ONRRP facility effectively increased the set of counterparties that could earn interest from the Fed on their reserves. Previous regulations prevented government-sponsored enterprises, money market funds and other financial institutions from earning IOER; however, a group of these institutions were eligible to make secured loans to the Fed in the form of overnight reverse repurchase agreements through the ONRRP facility.[3] Included in this group were the FHLBs, which, as mentioned above, were the main lenders in the federal funds market.

The ONRRP rate determined a "threat point" in the negotiation between lenders and borrowers in the arbitrage market. In principle, lenders in the federal funds market that have access to the ONRRP program should not be willing to lend at a rate below the ONRRP rate plus the amount needed to compensate them for any credit risk (since the federal funds market is an unsecured market). At times, the role of the ONRRP rate has diminished as alternative short-term secured rates have risen above the ONRRP rate and taken its place as the lenders' threat point. However, during much of the post-crisis era, the ONRRP rate was the best alternative rate that was available to lenders in the federal funds market who were not eligible for IOER.

We model arbitrage activity in the federal funds market by considering multiple subnetworks, which capture long-term relationships between borrowing banks and lending FHLBs. Within each subnetwork, our model assumes a structured bargaining procedure. In particular, heterogeneous borrowing banks make offers to heterogeneous lenders specifying the rate and maximum amount they are willing to borrow; the lenders accept or reject these offers and specify the exact loan amount. Garratt et al. (2015) argue that this structure reflects industry practices around the time of lift-off. Rates are thus determined by competition among borrowers and the threat point of the lenders, which is their best alternative rate. In subnetworks where the supply of funds from lenders exceeds the limited demand by eligible borrowers, rates in the arbitrage market will track the lower bound of the band determined by the ONRRP rate, or the best alternative secured rate, with some adjustments for credit risk. In subnetworks where the supply of funds is not sufficient to satisfy banks' needs for liquidity, the model predicts that rates will increase above the ONRRP rate (and possibly even above the IOER rate) and will be determined by the ability of banks to stay liquidity and leverage compliant.

In this setup, the equilibrium bilateral federal funds rate depends on the riskiness of the borrowing bank and on whether or not there is an excess supply of funds in each subnetwork. The supply of federal

[3] A complete list of eligible counterparties is available here: https://www.newyorkfed.org/markets/rrp$_{c}ounterparties$.

funds in any subnetwork is largely exogenous (it is determined by the available cash of large lenders, namely FHLBs). Excess supply may arise endogenously in any subnetwork because demand is limited by the needs of the borrowers to comply with their internal liquidity and leverage standards aligned with two post-crisis regulatory requirements: the Liquidity Coverage Ratio (LCR) and the Supplementary Leverage Ratio (SLR). The demand schedule for banks in each subnetwork, and thus their equilibrium federal funds exposures, is determined endogenously in the equilibrium of the game. It is a function of the interest rate policy of the central bank, initial liquidity and leverage conditions, and level of funding volatility specific to the current period. With the equilibrium rates and volumes, we specify how banks manage their liquidity shocks conditional on their balance sheets and federal funds exposures using balance sheet rebalancing.

We highlight three additional results that the model delivers in applications. First, it provides conditions for the external margin participation in the federal funds market. In particular, we show that leverage-constrained banks will prefer not to participate in the federal funds market at lower rates. Higher rates lead to higher profits and hence more participation. This indicates that the benchmark federal funds rate conveys information only about the subset of leverage-healthy banks; thus, it may not be an objective indicator of the overall financial market risk.

Second, we aggregate the equilibrium results across subnetworks to produce the formula for the federal funds benchmark rate and provide context to the historical events in the federal funds market using our model. We focus particularly on the 2016-2019 interest rate lift-off period. The model explains movements of the federal funds rate within the IOER-ONRRP band and above IOER.

Finally, we show how tightening of regulatory requirements impacts the federal funds rate. We find that qualitative predictions are different for stricter leverage and liquidity regulations. In both cases, the degree to which the federal funds rate is impacted by the policy changes depends on the business models of banks, some of which choose to comply with regulatory requirements with excess buffers, while others barely meet their targets by the time of regulatory reporting. Transitioning to a higher leverage requirement will put downward pressure on the federal funds rate only if banks have more difficulties complying with the new regulation. Otherwise, if banks do not change their business model and adjust to the new regulation accordingly, no impact on the federal funds rate takes place. In contrast, a stricter liquidity requirement is predicted to increase the federal funds rate even if banks maintain the same liquidity buffers with respect to the new regulatory requirement as they had with respect to the old one.

Our analysis of the post-crisis federal funds market focuses on the arbitrage market. This means that the qualitative predictions are made ignoring federal funds trades between the banks that have access to the IOER facility. We cannot determine empirically the share of market trades that are derived from arbitrage, but based on our knowledge of the players involved in the market, it appears to be quite large. Afonso and Stern (2016) report that the share of lending by FHLBs rose from 52% in 2008 to over 80% in 2015. Over the same period, the share of funds purchased by foreign borrowers increased from 50%

to almost 70%. Foreign borrowers are not subject to Federal Deposit Insurance Corporation (FDIC) assessment fees and face weaker reporting requirements, meaning arbitrage trades are more profitable for them. Both shifts suggest an increase in arbitrage trades. Likewise, Potter et al. (2016) mention that during the first half of 2016 less than 5% of fed funds transactions were interbank transactions. In short, evidence strongly suggests that the majority of federal funds trades are related to arbitrage.[4]

2. Related literature

A limited number of papers model the post-crisis interbank market. Our work is most closely related to Bech and Klee (2011), Garratt et al. (2015), Bech and Keister (2017) and Afonso et al. (2019), since these papers examine the federal funds market in the context of high excess reserves and explore the implications of different policies and regulations on the evolution of the federal funds rate.

Bech and Klee (2011) were the first to make an important distinction in the post-crisis federal funds market: government-sponsored enterprises (GSEs) do not earn interest on excess reserves. This divides the federal funds market into two segments, a bank-to-bank (b2b) segment where banks trade reserves to meet reserve requirements and an GSE-to-bank (g2b) segment where banks engage in IOER arbitrage. Bech and Klee (2011) model federal funds market activity as a combination of bargaining activities within these two segments. Thus, this paper represents an important departure from the classic Poole model (Poole (1968)) in which banks face payment shocks after the interbank market closes and demand reserves to meet liquidity requirements in a competitive market. Bech and Klee's main focus is on thinking about how future reductions in aggregate reserves and/or increases in IOER might change bargaining outcomes and therefore the effective federal funds rate. Bech and Klee's (2011) paper was written before the ONRRP facility was introduced. The authors recognize that current market outcomes seemed to reflect a lack of "unexploited" arbitrage opportunities, but they do not give a formal explanation for this.

Garratt et al. (2015) build off the Bech and Klee (2011) model but formally model the demand for reserves by banks in the b2b market. More specifically, Garratt et al. (2015) add balance sheet costs to the Poole model, which was already modified by Ennis and Keister (2008) to include IOER, and introduce the ONRRP as the threat point of GSE lenders and assume that borrowers in the g2b market have all of the bargaining power.[5] The main point of Garratt et al. (2015) is to argue that allowing banks to have segregated master accounts, meaning that they could borrow without credit risk, would reduce the bargaining power of existing borrowers and hence improve IOER arbitrage.

[4]Since March 2, 2016, the federal funds rate has been computed as the volume-weighted median of rates reported by trades from the FR 2420 borrower survey; hence, it may only represent arbitrage activity. See Statement Regarding the Calculation Methodology for the Effective Federal Funds Rate and Overnight Bank Funding Rate, 2015, Federal Reserve Bank of New York.

[5]They justify this based on stylized facts about the market.

Bech and Keister (2017) are similar to Garratt et al. (2015) in that both use the Poole model, and both model a single equilibrium rate in the b2b market. However, Bech and Keister (2017) do not address the leaky floor aspect of IOER or consider the ONRRP, and they do not consider a separate g2b segment of the federal funds market. They focus on the the impact of LCR regulation on the b2b market. LCR regulation enters as an additional constraint on end-of-day reserve holdings that impacts banks' demand for reserves in the interbank market. There are two types of interbank loans, overnight and term, with the latter being favored for meeting LCR. They show that the impact of LCR on market interest rates depends on which constraint is binding. It is only in the case where both constraints are binding that both overnight and terms rates rise above IOER. More generally, however, LCR can weaken the demand for overnight funds because liquidity obtained to meet LCR makes it more likely that a bank will over-satisfy its reserve requirement.

Our approach to modelling the post-crisis federal funds market is supported by theoretical findings of Kim et al. (2020), who find that interbank lending will constitute only a small fraction of overall federal funds activity in the post-crisis environment, characterized by excess reserves and high balance sheet costs. Differently from them, we also account for LCR regulation and focus on heterogenous borrowers that differ in terms of initial balance sheets and default risk and heterogeneous lenders that differ in terms of availability of funds.

Finally, Afonso et al. (2019) extend Garratt et al. (2015) and Bech and Klee (2011) by considering a segmented federal funds market with generalized Nash bargaining in which they introduce both ONRRP and balance sheet costs to the g2b market. They calibrate their model and use it to predict the evolution of federal funds interest rates and volumes under different scenarios for aggregate reserves.

We follow both Bech and Klee (2011) and Garratt et al. (2015) by thinking of the interbank market as being composed of two sub-markets, a competitive b2b market where banks trade excess reserves to meet reserve requirements and a non-competitive g2b arbitrage market. We focus our analysis on the g2b arbitrage market. Our emphasis on the g2b market is justified since, over the period we are studying, federal funds market interest rates were largely determined by trades in the arbitrage market.

Our paper is the only one to consider both LCR and leverage requirements. Bech and Keister (2017) formally model LCR but do so by examining the interplay between overnight and term lending in the b2b market. We do not consider the term interbank market. Instead, we capture the impact of LCR on the overnight market by making a distinction not present in Bech and Keister (2017). Specifically, we focus on the fact that the post-crisis interbank market is made up largely of arbitrage trades that involve loans from FHLBs. Borrowing from public sector entities, such as FHLBs, receives special treatment (lower runoff rates) under LCR, as the LCR rule considers these lenders to be "stickier"; see Potter (2018). This aspect of borrowing from GSEs was recognized but not emphasized by Afonso et al. (2019) (see their Appendix A), who argued that LCR does not play a role in the federal funds market. In contrast, we argue that LCR impacts overnight rates by increasing demand for funds in the overnight market. On balance, we find that leverage requirements often outweigh the impact of LCR. Our combined treatment of LCR and leverage requirements is crucial to explaining why borrowers in the g2b market have limited demand for funds and hence why arbitrage activity alone does not drive rates all the way up to IOER.

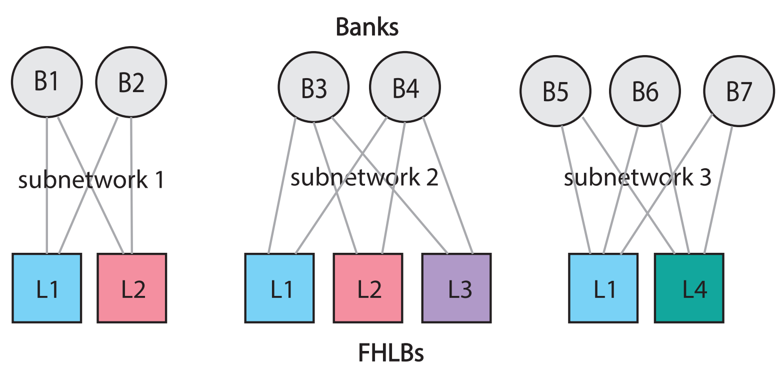
3. Model

3.1. Structure of the interbank arbitrage market

There is a set L of lenders and a set B of borrowers. Since our focus is on the arbitrage market, it is appropriate to think of the borrowers as banks and the lenders as FHLBs. FHLBs and banks deal with a subset of counterparties which can be smaller than the whole market. Selective interactions can be a result of long-term relationships, common exposures and geographical location.

We assume that the arbitrage market can be divided into subnetworks based on these selective interactions with at least two FHLBs and two banks in each subnetwork. Moreover, each FHLB in a subnetwork deals with all of the banks in the subnetwork, and each bank in a subnetwork deals with all of the FHLBs in the subnetwork. Thus the arbitrage market is modelled as a collection of disjointed complete bipartite subnetworks, as shown in Figure 1.

Figure 1: Federal funds market structure: stylized representation



The amount of funds FHLBs have available to lend in each subnetwork comes from their underlying business activities and is taken here as exogenous.[6]

3.2. Federal funds arbitrage

Banks earn interest on excess reserves (IOER) at net rate r^{ioer} for each dollar of excess reserves they hold in their master accounts. FHLBs do not earn IOER. However, they can lend funds in the federal

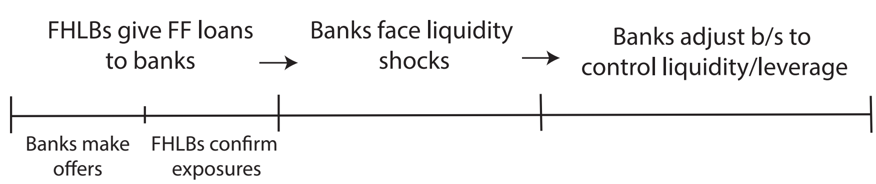
[6]Available funds are determined by the daily fluctuations of liquidity of FHLBs and to some extend the Federal Housing Finance Agency regulation that includes a limit on the amount of unsecured credit an individual FHLB may extend to a counterparty.

funds market or invest in an outside option. Let r^a denote the maximum expected (net) return that FHLBs can receive overnight from outside options, such as the overnight reverse repo facility (ONRRP), bilateral repo market, eurodollar market, or market of treasury bills.[7] When $r^{ioer} > r^a$, banks may earn arbitrage profits by serving as intermediaries between the FHLB lenders and the IOER facility. The size of the arbitrage profit that both types of institutions earn from this intermediation activity depends on the bargaining process between them and their balance sheets.

3.3. Timeline

The timing of our model of the federal funds market is shown in Figure 2. Banks start with some imbalances on liquidity and leverage positions, which may place them at, above or below their internal targets. FHLBs start with an amount of loanable funds. Given these positions, bargaining between lenders and borrowers takes place, which determines federal funds volumes and rates. Once the loans have been given, each bank faces a random liquidity shock, which may either stabilize or unbalance their liquidity and leverage positions. To comply with regulatory and internal requirements, the banks take rebalancing actions. Depending on the situation, banks may sell liquid or illiquid assets or borrow funds. Some rebalancing actions may be conflicting, so banks are driven by the cost-benefit considerations. We describe this process in detail in the remainder of the section in the reverse order.

Figure 2: Timeline of the model



3.4. Bargaining mechanism and strategies

Market participants bargain over rates and volumes in a sequential manner. Denote the set of lenders in a given subnetwork G^s as L^s and the set of borrowers as B^s. First, each borrower j sends a message to each connected lender i that contains the maximum amount, a_{ij}, it is willing to borrow and the corresponding net interest rate, r_{ij}, it is willing to pay. Once all offers are received, each lender i simultaneously responds to each borrower j with the amount, x_{ij}, it is willing to lend to j at the proposed rate. The counter-offer amounts need to be feasible, meaning that the lender does not offer more than each borrower asked for,

[7] While the maturity of treasury bills is longer than overnight, it is common for market participants to construct a portfolio of bills with different maturity to produce a contingent claim equivalent to the overnight loan.

$0 \le x_{ij} \le a_{ij}$, and the total volume proposed by the lender does not exceed the total amount of funds, w_i^s, that i has available:

$$\sum_{j \in B^s} x_{ij} \le w_i^s, \text{ for each } s \text{ if } i \in G^s. \tag{1}$$

If both borrower and lender are interested in trade, $a_{ij} \ge x_{ij} > 0$, loan (r_{ij}, x_{ij}) is issued by i to j at rate r_{ij}; otherwise, no loan is issued.[8].

Let (r_j, a_j) denote a vector of borrowing proposals made by bank j. Likewise, let x_i denote a vector of counter-offers given by lender i. Then, the strategies of all borrowers are defined as

$$(r, a) = \{(r_j, a_j) \text{ such that } a_{ij} \ge 0 \text{ and } r_{ij} \ge 0\}_{j \in B} \tag{2}$$

and the strategies of all lenders in each subgame (r, a) are defined as

$$x(r, a) = \{x_i \text{ such that } 0 \le x_{ij} \le a_{ij} \text{ for all } j \in B \text{ and } \sum_{j \in B^s} x_{ij} \le w_i^s \text{ for each } s \text{ if } i \in G^s\}_{i \in L}. \tag{3}$$

3.5. Payoff of lenders

Lenders are sensitive to the solvency status of borrowers. Borrower j's default probability is given by $1 - s_j \in [0, 1]$ such that credit score $s_j = 1$ corresponds to perfect solvency and $s_j = 0$ to certain failure. The default probabilities are public knowledge.[9] For simplicity, we proceed assuming full loss given default. This allows us to define the per-dollar profit that lender i could expect from a loan given to j as

$$\pi_{ij}(r_{ij}) = s_j r_{ij} - (1 - s_j). \tag{4}$$

The payoff of lender i is defined as the expected profit from utilizing available liquidity, w_i^s:

$$P_i^L((r, a), x(r, a)) = \sum_j \pi_{ij}(r_{ij}) x_{ij} + r^a (w_i^s - \sum_j x_{ij}). \tag{5}$$

If the bargaining outcome makes a lender indifferent between lending to the outside option and lending to the IOER arbitrage market, we assume that the lender strictly prefers the latter. One could justify this on the grounds that the lenders want to preserve relationships with borrowers.

[8] To simplify the notation, we will treat all lenders' offers of no trade as trades with zero volume and arbitrary interest rate

[9] In reality, it is possible that two different lenders will rate the same borrower differently. Different FHLBs utilize credit ratings of various nationally recognized statistical ratings organizations (NRSROs), and the sets of NRSROs used may differ across FHLBs. Moreover, FHLB credit departments may have different practices. For example, some may treat banks on negative watch as if they have already been downgraded. This means that there may be some discrepancy in terms of how an FHLB rates a potential borrower. However, we expect this discrepancy to be small, and hence we ignore it. NRSROs include Moodys Investor Services, Inc.; Standard and Poor's, Inc.; Fitch, Inc.; Dominion Bond Rating Service Limited (DBRS); and A.M. Best Company, Inc.

3.6. Payoff of borrowers

Borrowers value interbank loans not only as a source of arbitrage profit but also as a source of liquidity. By the end of the day, banks aim to maintain a buffer of liquid assets above their internally determined liquidity threshold. This liquidity threshold should at least exceed the LCR limit imposed by the regulator. A bank may also impose internal liquidity control that is stricter than the industry regulatory requirement, for instance, if liquidity intermediation is a crucial component of its business model. From day to day, there also can be some flexibility in the liquidity strategy of smaller banks, which are required to report only monthly calculations of the liquidity ratios. We assume that the internal liquidity practices of all banks are aligned with the regulatory requirements in how assets and liabilities are evaluated in terms of regulatory liquidity weights. However, the liquidity threshold, $\bar{\lambda}_i^{liq}$, may be different for each bank i.

Banks do not exploit federal funds arbitrage opportunities to an unlimited degree because this would lead to an expansion of their balance sheets and, as a result, increased leverage. High leverage not only increases riskiness but also sends a negative signal to the regulator and investors, which can result in less favourable borrowing terms for the bank and a loss of clients. Banks limit their borrowing to ensure that their end-of-day leverage ratio falls below their internal limit, $\bar{\lambda}_i^{lev}$. We assume that the internal leverage requirement is at least as strict as the supplemental leverage ratio imposed by the regulator.[10]

Banks face a challenge in controlling their leverage and liquidity ratios due to uncertainty about their end-of-day liquidity positions. Following interbank borrowing, bank j experiences a liquidity shock y_j applied to the deposits (as in Poole (1968)). Shock y_j is positive when the depositors withdraw liquidity, and negative when the depositors place liquidity at the bank. The shock may impact both the liquidity and leverage ratios of the bank. To counteract the shock and stay compliant with internal requirements, the bank may apply a combination of responses:

- raise liquidity in the endogenous amount x_j^d at net overnight rate r^d from the lender of last resort;
- decrease balance sheet size by repaying short-term debt prematurely with the endogenous cash amount x_j^c at marginal cost c, where c measures sacrificed net income margin above r^{ioer} that a bank could earn by keeping liquid assets (it also includes costs associated with the liquidation of safe assets)—thus, c is very small;[11]

[10] There may also be other reasons for banks to have limited borrowing capacity at the federal funds market. One of them is the presence of FDIC assessment fees and other supervisory oversight which typically bind for domestic firms operating in the federal funds market. The country-specific impact of such fees gives one a hint as to why foreign banks are so prevalent in the current interbank market. In addition, regulatory reporting is watched closely by analysts, and growing the size of the bank is not seen as neutral by equity analysts and rating agencies, even though the increased assets consist of reserves. Possibly this is because of the commingling of all the bank's assets, and the perception that overnight funding somehow increases the flightiness of a bank's funding overall. Finally, domestic banks in the U.S. are required to pay insurance-related costs, which are linked to the balance sheet size and thus add extra limits on the amount of borrowings in the interbank market.

[11] This repayment x_j^c includes neither after-shock repayment $y_j > 0$ nor increase in cash resources $y_j < 0$. Shock is treated

- repay long-term debt prematurely with the cash raised from selling illiquid assets in the endogenous amount x_j^{α} at marginal cost α (the cost α includes both sacrificed net income margin from keeping illiquid assets and the liquidation cost for these assets).[12][13]

Given the profile of strategies (r, a) and $x(r, a)$, the payoff to borrower j is defined as the expected profit from arbitrage minus the expected compliance costs:

$$P_j^B((r,a),x) = \sum_i (r^{ioer} - r_{ij})x_{ij} - (c + r^{ioer})E[x_j^c] - (r^d - r^{ioer})E[x_j^d] - \alpha E[x_j^{\alpha}]. \tag{6}$$

3.7. Leverage and liquidity requirements of banks

Liquidity ratio. The liquidity ratio defines the ability of a bank to repay monthly withdrawals with available cash and liquid assets. For any set of pre-shock actions $((r, a), x)$ and post-shock actions $(x_j^d, x_j^c, x_j^{\alpha})$ of bank j, the liquidity ratio of bank j at the end of the day is defined as follows:

$$\lambda_j^{liq,eod} = \frac{hqla_j + \sum_i x_{ij} + x_j^d - x_j^c - y_j}{ncof_j + \rho_x(\sum_i x_{ij} + x_j^d - x_j^c) - \rho_y y_j}, \tag{7}$$

where $hqla_j$ denotes the exogenously given value of bank j's high-quality liquid assets and $ncof_j$ denotes the exogenously given expected 30-day net cash outflow of bank j before the arbitrage game takes place. Consistent with reality, we assume that more liquid funding has a higher run-off rate, $\rho_x > \rho_y$, and overnight loans are considered perfectly liquid.

Bank j is required to stay above the required liquidity ratio: $\lambda_j^{liq,eod} \geq \bar{\lambda}_j^{liq} \geq 1$. We define q_j^{liq} as the amount that needs to be raised at the beginning of the day to satisfy the liquidity ratio:

$$q_j^{liq} = \frac{\bar{\lambda}_j^{liq} ncof_j - hqla_j}{1 - \bar{\lambda}_j^{liq}\rho_x}.$$

The initial liquidity gap, q_j^{liq}, can be both positive and negative: a positive value means bank j starts the day with LCR below $\bar{\lambda}_j^{liq}$, and a negative value means bank j starts the day with LCR above $\bar{\lambda}_j^{liq}$. After the shock, the values of x_j^d and x_j^c must be chosen to satisfy the liquidity requirement, which can be

separately on the balance sheet. Banks do not control depositors and always apply the shock to their balance sheet.

[12]Illiquid assets include low-rated corporate debt or high-rated corporate debt and common equity shares beyond certain limits.

[13]We focus on limited management actions rather than compare all possible balance sheet adjustments. For instance, we do not consider a possibility when a bank sells illiquid assets simply to increase its amount of liquid assets (without repaying funding). Instead, we consider borrowing from the lender of last resort with simultaneous adjustment of illiquid exposures as an alternative, which allows for matching maturities of assets and liabilities for each action taken. In this way, we stay consistent with the previous banking literature relying on Poole's (1968) model and keep the derivations simple.

rewritten in the linear form

$$q_j^{liq} - \sum_i x_{ij} - x_j^d + x_j^c + \rho y_j \leq 0, \tag{8}$$

where we adopt the notation

$$\rho = \frac{1 - \bar{\lambda}_j^{liq} \rho_y}{1 - \bar{\lambda}_j^{liq} \rho_x} > 1$$

for the relative LCR weight.

Leverage ratio. For any set of pre-shock actions $((r, a), x)$ and post-shock actions $(x_j^d, x_j^c, x_j^\alpha)$ of bank j, the leverage ratio at the end of the day is defined as

$$\lambda_j^{lev,eod} = \frac{e_j}{TA_j + \sum_i x_{ij} + x_j^d - x_j^c - x_j^\alpha - y_j}, \tag{9}$$

where e_j is equity and TA_j stands for non-weighted total assets of banks j. Bank j is required to keep $\lambda_j^{lev,eod} \geq \bar{\lambda}_j^{lev}$. We define q_j^{lev} as the amount that needs to be reduced from the balance sheet before the deposit shock and banks' actions take place in order for the leverage ratio to bind:

$$q_j^{lev} = TA_j - \frac{e_j}{\bar{\lambda}_j^{lev}}.$$

Variable q_j^{lev} is the initial leverage gap: positive q_j^{lev} means bank j starts the day with a leverage ratio below $\bar{\lambda}_j^{lev}$, and negative q_j^{lev} means bank j starts the day with a leverage ratio above $\bar{\lambda}_j^{lev}$.

After the shock, the strategically optimal values of x_j^d, x_j^c and x_j^α must be chosen to satisfy the leverage requirement, which can be written in the linear form

$$q_j^{lev} + \sum_i x_{ij} + x_j^d - x_j^c - x_j^\alpha - y_j \leq 0. \tag{10}$$

Next, we show that given federal funds exposure $\sum_i x_{ij}$ and the level of the shock y_j, the rebalancing actions x_j^d, x_j^c and x_j^α are uniquely determined for any bank j.

3.8. Rebalancing actions of banks

Following the realization of the liquidity shock, the bank may find itself in a position where one or both of its internal constraints are violated. The optimal rebalancing strategy of the bank depends on the initial leverage and liquidity gaps, federal funds borrowing, and the liquidity shock itself. It is convenient to think about the rebalancing strategy by splitting parameter space into six regions (Figure 3), with axes being federal funds exposure $\sum_i x_{ij}$ and the liquidity shock y_j. The regions are separated by two upward-sloping lines: the liquidity threshold

$$y_j = \frac{1}{\rho} \left(\sum_i x_{ij} - q_j^{liq} \right)$$

and the leverage threshold

$$y_j = q_j^{lev} + \sum_i x_{ij},$$

and the horizontal threshold on the interbank borrowing $\sum_i x_{ij}$, for which both constraints bind at the same time:

$$y_j = \frac{1}{\rho - 1}\left(-q_j^{liq} - q_j^{lev}\right).$$

The next proposition summarizes how rebalancing strategies depend on federal funds exposure and the initial balance sheet. All proofs are provided in the Appendix.

Proposition 1. *Given federal funds borrowing $\sum_i x_{ij}$ and liquidity shock y_j, the optimal choice of rebalancing actions x_j^d, x_j^c and x_j^α for bank j is unique in each of the parameter regions (I)-(VI) of the space $(\sum_i x_{ij}, y_j)$, as shown in Figure 3. In the regions (I)-(III), bank j does not borrow from the lending facility: $x_j^d = 0$. In the regions (IV)-(V), bank j borrows from the lender of last resort in the amount*

$$x_j^d = q_j^{liq} - \sum_i x_{ij} + \rho y_j.$$

In the regions (III) and (IV), bank j additionally sells non-liquid assets in the amount

$$x_j^\alpha = q_j^{liq} + q_j^{lev} + (\rho - 1) y_j.$$

In the regions (II) and (III), bank j sheds liquid assets to pay liabilities and decreases balance sheet size by the respective amounts

$$x_j^c = q_j^{lev} + \sum_i x_{ij} - y_j$$

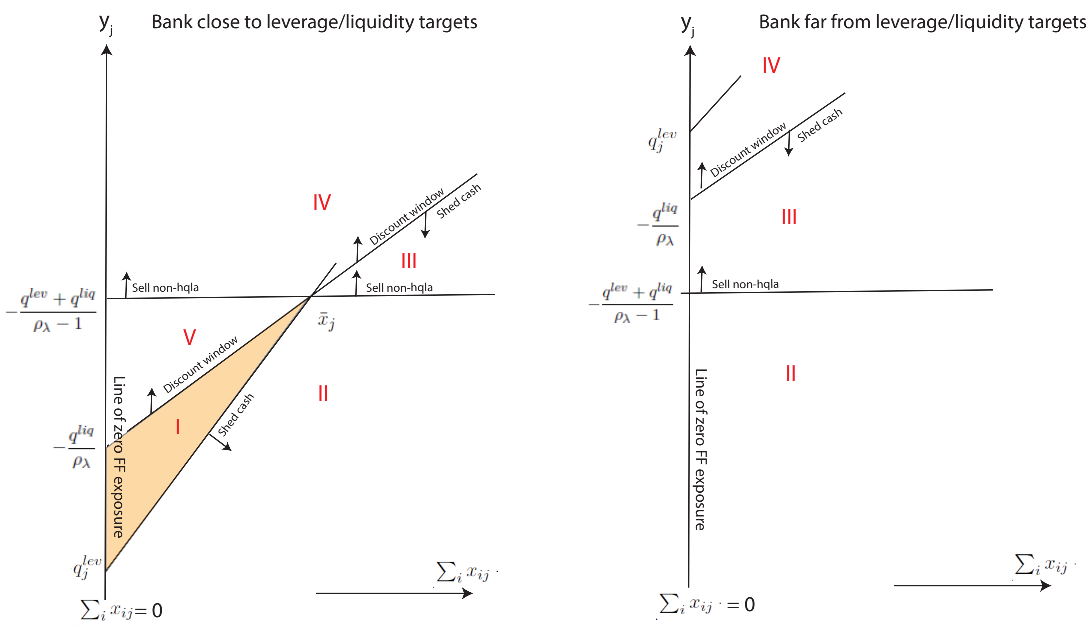
and

$$x_j^c = -q_j^{liq} + \sum_i x_{ij} - \rho y_j.$$

The set of optimal rebalancing actions of the bank can be summarized as follows. If both regulatory constraints are met, then the bank does nothing. In the case where the liquidity constraint is met with a surplus and the leverage constraint is not met, the bank liquidates HQLA to decrease the size of its balance sheet. Liquidating high-quality liquid assets reduces the liquidity ratio and increases the leverage ratio. If there is enough of a buffer of liquid assets beyond the limit to meet the leverage constraint, then the bank does not need to do anything else. However, if all liquid assets beyond the liquidity requirement have been sold and the leverage ratio is still not met, then the bank sells non-liquid assets until the leverage condition is met. In the case where the leverage constraint is met and liquidity is below the required limit, the bank starts borrowing from the central bank. If there is enough room in the leverage constraint, then this solves the problem. Otherwise, the bank has to adjust other strategies, such as selling non-liquid assets. If neither constraint is met initially, then the bank sequentially satisfies its liquidity and leverage ratios.

Proposition 1 and the accompanying Figure 3 reveal that the behavior of an imbalanced bank differs

Figure 3: Regions for behavioral responses of bank j to liquidity shock y_j given federal funds loans $\sum_i x_{ij}$.



from that of a bank that is close to its target ratios. Excessive arbitrage in the federal funds market may further exacerbate the positions of the bank for various deposit shocks. For instance, if the bank borrows in the federal fund market above critical value

$$\bar{x}_j = -\frac{1}{\rho - 1} q_j^{liq} - \frac{\rho}{\rho - 1} q_j^{lev}, \tag{11}$$

then regardless of whether or not there is a deposit shock, the bank will have to take counterbalancing actions. Conversely, if the bank borrows below $\bar{x}_j$, there is an interval of shock values for which the bank does not need to take any counterbalancing actions to stay within the targeted ratios. Therefore, the necessity of satisfying liquidity and leverage constraints limits the extent to which banks seek to take advantage of arbitrage opportunities.

Given bank j's initial leverage and liquidity positions, its interbank borrowing vector x_j and optimal rebalancing actions x_j^d, x_j^c and x_j^α, we can compute its expected borrowing from the discount window, $E[x_j^d]$, expected liquidations of non-liquid assets, $E[x_j^\alpha]$, and expected cash depletion amount, $E[x_j^c]$, as described in Proposition 1.

For computational purposes, assume that $y_j \sim U[-v, v]$ and $q_j^{lev}, -q_j^{lev} - q_j^{liq} \in [-v, v]$, meaning that the shock can unbalance the banks more than they are initially unbalanced. Then borrower j's payoff under the strategy profile $((r, a), x)$ is defined as

$$
\begin{aligned}
P_j^B = & \sum_i (r^{ioer} - r_{ij}) x_{ij} \\
& - \frac{c}{4v} \left(v + q_j^{lev} + \sum_i x_{ij} \right)^2 \\
& - \frac{r^d - r^{ioer}}{4v} \frac{1}{\rho} \left(\rho v + q_j^{liq} - \sum_i x_{ij} \right)^2 \\
& - \frac{\alpha}{4v} \frac{1}{\rho - 1} \left((\rho - 1) v + q_j^{lev} + q_j^{liq} \right)^2
\end{aligned} \tag{12}
$$

when bank j borrows below threshold $\bar{x}_j$, and

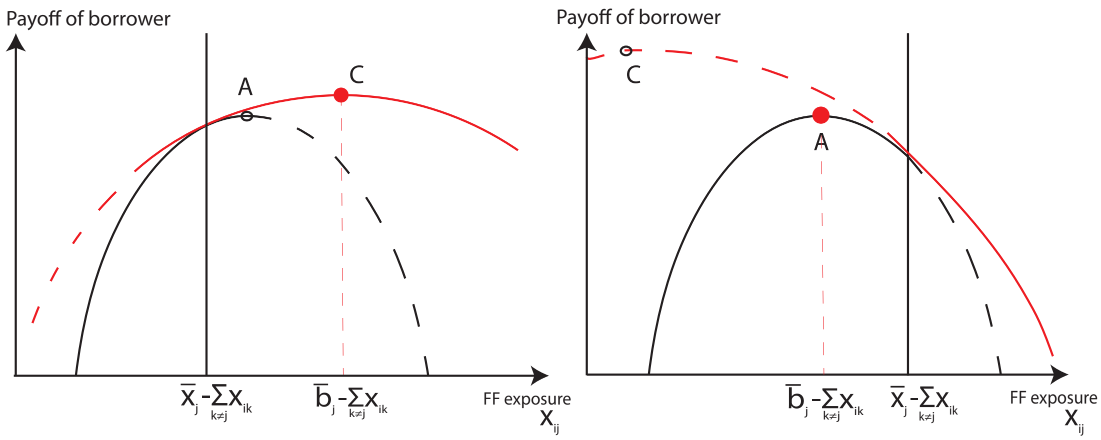
$$
\begin{aligned}
P_j^B = & \sum_i (r^{ioer} - r_{ij}) x_{ij} \\
& - \frac{c}{4v} \left(v + q_j^{lev} + \sum_i x_{ij} \right)^2 + \frac{c}{4v} \left(\frac{\rho - 1}{\rho} \sum_i x_{ij} + \frac{1}{\rho} q_j^{liq} + q_j^{lev} \right)^2 \\
& - \frac{r^d - r^{ioer}}{4v} \frac{1}{\rho} \left(\rho v + q_j^{liq} - \sum_i x_{ij} \right)^2 \\
& - \frac{\alpha}{4v} \frac{1}{\rho - 1} \left((\rho - 1) v + q_j^{lev} + q_j^{liq} \right)^2
\end{aligned} \tag{13}
$$

when bank i borrows above threshold $\bar{x}_j$. While rebalancing actions may be different in the case where a bank does too much arbitrage versus the case where it does too little (see Figure 3), a bank's ex ante expected payoff will be almost the same in both cases whenever the cost of safe assets liquidation c is very small.

For a given vector of rates r_j, and holding fixed the borrowing amounts x_{-ij}, we can define the marginal payoff borrower j earns from borrowing an extra dollar from lender i. For a broad range of policy rates and costs, payoff functions (12) and (13) are parabolas in x_{ij} that open downwards, and parabola (12) is not above parabola (13). The implication is that whenever borrower j decides to borrow a total amount in the federal funds market that is below the threshold $\bar{x}_j$, it is in its best interest to borrow the amount $b_j^{lev}(r_{ij})$ (point A in the left part of Figure 4). Alternatively, whenever borrower j decides to borrow an amount in the federal funds market that is above the threshold $\bar{x}_j$, it is in its best interest to borrow the amount $b_j^{liq}(r_{ij})$ (point C in the left part of Figure 4). Thresholds $b_j^{liq}(r_{ij})$ and $b_j^{lev}(r_{ij})$ are as defined below:

$$
b_j^{liq}(r_{ij}) = q_j^{liq} + \frac{2(r^{ioer} - r_{ij}) - c + r^d - r^{ioer}}{c + r^d - r^{ioer}} \rho v \tag{14}
$$

Figure 4: Payoff of banks as a function of their federal funds exposure



The payoff of a borrowing bank j as a function of its federal funds exposure x_{ij} is depicted with a solid line (composed of two parabolas). The marginal payoff has a structural break when the bank exceeds federal funds exposure $\sum_j x_{ij} = \bar{x}_j$. The left figure shows the case of bank that is more liquidity-constrained, and the right figure shows the case of a bank that is more leverage-constrained when the market opens.

$$b_j^{lev}(r_{ij}) = q_j^{liq} + \frac{2(r^{ioer} - r_{ij}) - c + r^d - r^{ioer}}{\rho c + r^d - r^{ioer}}\rho v - \frac{c\rho(q_j^{liq} + q_j^{lev})}{\rho c + r^d - r^{ioer}} \tag{15}$$

By combining these two results, we get the maximum desired exposure of the bank conditional on the offered rate.

Proposition 2. *Bank j earns positive marginal profit from an interbank loan taken from* any *lender i at rate r_{ij} if and only if bank j's total interbank borrowing $\sum_k x_{kj}$ is below*

$$b_j(r_{ij}) = max(b_j^{liq}(r_{ij}), b_j^{lev}(r_{ij})). \tag{16}$$

The cut-off on federal funds borrowing of bank j in (16) is decreasing in the federal funds rate r_{ij} for all i, meaning that bank j is willing to borrow less from every FHLB i at higher rates. The intuition is simply that if arbitrage profits are reduced, the bank is less willing to take additional liquidity risk because of higher r_{ij}.

We can use (16) to define an upper limit on the total borrowing of any borrower j. First, we observe that for any two lenders i and k trading at the same rates with j, $r_{ij} = r_{kj}$, the cut off is the same, $b_j(r_{ij}) = b_j(r_{kj})$. Second, we define the lowest possible rate that borrower j can offer and expect to be accepted by lenders. To do this, we define the credit risk premium over the rate r^a that makes lenders indifferent between lending and not lending to borrower j (i.e., $\pi_{ij} = r^a$):

$$\frac{(1 + r^a)(1 - s_j)}{s_j}.$$

Thus the rate

$$\underline{r}_j = r^a + \frac{(1+r^a)(1-s_j)}{s_j} \tag{17}$$

determines the lower bound on the equilibrium rates of borrower j. We assume that all borrowers have a sufficiently low probability of default so that $\underline{r_j} \leq r^{ioer}$. We can now define the maximum amount that borrower j can ever expect to borrow as

$$\bar{b}_j = b_j(\underline{r}_j).$$

This value will be useful for characterizing equilibrium outcomes in the arbitrage market. We will refer to $\bar{b}_j$ as j's maximum desired exposure to the federal funds market.

In a special case when the liquidation of safe assets is cost-free, $c = 0$, the bank benefits most when its total exposure to the federal funds market at a given rate r_{ij} is at level

$$b_j(r_{ij}) = q_j^{liq} + \rho v \left(1 - 2\frac{r_{ij} - r^{ioer}}{r^d - r^{ioer}}\right).$$

The desired exposure, $b_j(r_{ij})$, is equal to the sum of the initial liquidity gap, q_j^{liq}, and an additional buffer which insures against market volatility ranging from $[-v, v]$, weighted by the costs of the rebalancing actions. In this case, the leverage position of the bank does not impact federal funds borrowing because a bank that has abundant safe assets can reduce its HQLA at the end of the day at zero cost whenever the leverage is binding.

3.9. Equilibrium

We first define the Nash equilibrium of the lender subgame and then provide the definition of the subgame perfect equilibrium for the full game.

Definition 1. *Strategies $x^*(r,a)$ constitute a pure strategy Nash equilibrium of the lender subgame specified by (r, a) if*

$$P_i^L((x_i^*, x_{-i}^*), r, a) \geq P_i^L((x_i, x_{-i}^*), r, a)$$

for all $x_i \neq x_i^*$ and $i \in L$.

The equilibrium of the full game is defined in the standard way.

Definition 2. *A subgame perfect equilibrium of the arbitrage game is a strategy profile $(x^*, (r^*, a^*))$ such that, for each j, strategy (r_j, a_j) maximizes the payoff*

$$P_j^B(x^*(r^*, a^*), r^*, a^*) \geq P_j^B(x^*((r_j, r_{-j}^*), (a_j, a_{-j}^*)), (r_j, r_{-j}^*), (a_j, a_{-j}^*))$$

for all $(r_j, a_j) \neq (r_j^*, a_j^*)$, $j \in B$, and $x^*(r, a)$ being a pure strategy Nash equilibrium strategy profile of the relevant subgame specified by (r, a).

4. Equilibrium results and observations

The theoretical results of the paper are summarized in Theorem 1 and are valid for each subnetwork. Corrollary 1 is an aggregated version of Theorem 1.

Theorem 1. *In each subnetwork G^s, the set of subgame perfect equilibria is non-empty, and any equilibrium satisfies the following conditions:*

i) all borrowers with $\bar{b}_j \leq 0$ will not participate in the interbank market;

ii) all borrowers with $\bar{b}_j > 0$, defined as set $J^s \in B^s$, will participate in the interbank market with the following terms.

a) The rate of any trading pair (i, j) is equal to the minimum risk-adjusted rate $\underline{r}_j$ plus the accumulated market spread adjusted for bank-specific solvency risk:

$$r_{ij} = \underline{r_j} + \Delta r^* \frac{\frac{1}{s_j}}{\sum_{k \in J^s} \frac{1}{s_k}},$$

where the accumulated market spread is

$$\Delta r^* = \left(r^d - r^{ioer} + c\frac{\rho + 1}{2} \right) \frac{max(\sum_{j \in J^s} \bar{b}_j - \sum_i w_i^s, 0)}{4\rho v}.$$

b) The loan volume borrowed by bank j is equal to

$$\sum_i x_{ij} = max \left(\bar{b}_j^{lev} - 2\frac{\rho v \Delta r^*}{c\rho + r^d - r^{ioer}} \frac{1/s_j}{\sum_{k \in J^s} 1/s_k}, \bar{b}_j^{liq} - 2\frac{\rho v \Delta r^*}{c + r^d - r^{ioer}} \frac{1/s_j}{\sum_{k \in J^s} 1/s_k} \right).$$

c) The total volume borrowed in the subnetwork is $min(\sum_{j \in J^s} \bar{b}_j, \sum_i w_i^s)$.

d) Each lender i and borrower j, credit unused by lender i with excluded j, is positive:

$$\sum_{k \neq j} (a_{ik} - x_{ik}) > 0. \tag{18}$$

As is clear from the statement of the theorem, the federal funds rate of each trading bank j is adjusted upward from the minimum proposed rate $\underline{r}_j$ whenever borrowers need more liquidity than the lenders are willing to provide. Such needs may appear in the short run if there is an urgent liquidity breach in the bank's balance sheet or an increase in daily volatility of liquidity shocks against which the bank is trying to insure. In the long run, this outcome will appear if there is a general tendency of the bank to operate close to its liquidity target or if the IOER and ONRRP interest rates are selected to provide sufficient incentives for banks to arbitrage.

To highlight additional intuition of the theorem, from now on we focus on the case where federal funds lending is strictly preferred by the FHLBs to investing in the alternative option. This will allow us to not carry the max operator and to focus on the interior solution of the problem. Because the desire of banks to engage in arbitrage and get funding ($\bar{b}$) is determined by the policy interest rates, this assumption is equivalent to the one where the central bank chooses policy rates for FHLBs to earn sufficient profits and get involved.

While the assumption of no excess liquidity supply seems counterintuitive during an era of ample reserves, two arguments may support this assumption. First, banks, by definition, operate near their internal liquidity and leverage targets on a daily basis. Since holding safe assets and capital in excess amounts is costly for the bank, it is natural to assume that banks approach their targets from below and use federal funds to close the breach when needed.[14] Second, funds allocated by FHLBs for lending at the beginning of each day cannot be too far from what will be given out because they plan for efficient uses of resources ahead of time.

Define the set of banks participating in the market as $J = \cup_s J^s$, and the cardinality of it as $|J|$. Allow for multiple subnetworks of different size to coexist in the market. Then the generalization of Theorem 1 follows.

Corollary 1. *The average federal funds rate across participating banks is*

$$r^{ff} = \frac{\sum_{j\in J} \underline{r}_j}{|J|} + \frac{\Delta r^*}{|J|},$$

where the accumulated market spread is

$$\Delta r^* = \frac{1}{2}\sum_{j\in J}(r^{ioer} - \underline{r}_j - c) + \frac{1}{4}\frac{\sum_{j\in J}(\rho v + q_j^{liq}) - \sum_s \sum_i w_i^s}{\rho v}\left(r^d - r^{ioer} + c\frac{\rho+1}{2}\right) \tag{19}$$

$$+\frac{c(1-\rho)}{4}\max\left(-r^{ioer} + \underline{r}_j + \frac{\sum_{j\in J}((1-\rho)v - q_j^{liq} - q_j^{lev})}{(1-\rho)v}, r^{ioer} - \underline{r}_j\right).$$

4.1. Observations

Leaky floor is normal

We first look at what the theory predicts when overall banks have abundant liquidity and when leverage requirements are too far from binding. These assumptions replicate the times when IOER and ONRRP facilities were first introduced in the United States. At that time, the ONRRP facility paid a higher rate than the government bond market, collateral was available in large amounts, and the ONRRP served as an

[14] The cost of safe assets can be explained by loss of income on more profitable assets. The cost of capital arises from banks desiring more risk taking than is possible with high capital ratios. See for instance Myers and Majluf (1984), who develop a pecking order theory on incentives of banks for raising less capital, and Admati and Hellwig (2014) for more general intuition about why banks prefer debt to capital.

outside option in the federal funds market. That is exactly when the IOER floor was leaking: the federal funds rate stayed well below the IOER rate and close to the ONRRP rate.

To see how the model predicts the same outcome, consider a special case of cost-free liquidation of safe assets, $c = 0$.

$$\Delta r^* = \frac{1}{2}\sum_{j\in J}(r^{ioer} - \underline{r}_j) + \frac{1}{4}\frac{\sum_{j\in J}(q_j^{liq} + \rho v) - \sum_s \sum_i w_i^s}{\rho v}\left(r^d - r^{ioer}\right). \tag{20}$$

In this setup, if banks perfectly meet their targets at the end of the day, the interest rate will be positioned around the middle point of IOER and ONRRP. Alternatively, if banks have plenty of liquidity to withstand any daily shocks and reach internal targets, $\sum_{j\in J}(q_j^{liq} + \rho v) < \sum_s \sum_i w_i^s$, the interest rate would fall below the middle of the IOER-ONRRP range. Thus, during times of excess liquidity, competition between banks does not drive the federal funds rates all the way up to the IOER threshold.

Adding both the cost of liquidation of HQLA, $c > 0$, and tighter leverage requirements as in Corollary 1, push the interest rate even closer to the ONRRP rate.

Arbitrage is limited de facto

Our theory predicts that arbitrage opportunities will not be fully exploited by the regulated banks. Balance sheet management of banks driven by leverage requirements and internal regulation prevents banks from greatly expanding their federal funds market participation.

The federal funds rate is still informative

Pre-crisis, when the federal funds market worked as an interbank market with scarce reserves, changes in the federal funds rate reflected changes in the demand for liquidity. A sudden increase in rate signaled tighter liquidity conditions, and in this way informed the Federal Reserve about the state of the economy. In the new system, reserves are not scarce. However, our theory suggests that the federal funds rate still conveys information, since an increase in the liquidity needs of banks still leads to a higher federal funds rate within the ONRRP-IOER band.

Impact of liquidity regulation

Liquidity regulation may have two separate effects on the federal funds rates depending on the business model of a bank. First, banks that comply with higher regulatory requirements but do not adjust their business model appropriately may experience higher liquidity distance to the requirement q_j^{liq} at the beginning of the day. This increases the marginal utility that bank j receives from borrowing federal funds, because these funds build up the HQLA buffer. In this case an increase in the regulatory requirement stimulates the demand of bank j and pushes the federal fund rates higher:

$$\frac{\partial r_{ij}}{\partial \bar{q}_j^{liq}} > 0. \tag{21}$$

It also increases the rates of other banks in the subnetwork due to competition effects.

Second, if bank j adjusts its business model according to the new liquidity ratio, such that distance to the liquidity threshold $\bar{q}_j^{liq}$ stays the same, a tighter liquidity requirement $\bar{\lambda}_j^{liq}$ may still impact the federal funds market. This happens because the higher liquidity ratio exacerbates the maturity mismatch problem of the bank and leads to higher volatility in daily liquidity regulation. This can be illustrated with two examples. In both examples bank j complies with LCR regulation precisely. In the first example, bank j aims to meet $\bar{\lambda}_j^{liq} = 100\%$ ratio, while the unadjusted position is at 90%. In the second example, bank j complies with $\bar{\lambda}_j^{liq} = 120\%$ regulation, while the unadjusted position is 110%. Assuming all other parameters are equal, in the two examples $\bar{q}_j^{liq}$ and $\bar{q}_j^{liq}$ are the same. However the impact of liquidity shock y_j on the liquidity balance is higher when liquidity ratio $\bar{\lambda}_j^{liq}$ is higher because the marginal impact ρ of the liquidity shock on the balance sheet is higher whenever $\bar{\lambda}_j^{liq}$ is higher (it is clear from the end-of-day liquidity balance (8)). If the shock is negative, higher ρ may be better for the bank because it will reduce the liquidity gap marginally more. When the shock is positive, in contrast, it distances the bank from complying with the regulation. But because staying below the target is more costly than staying above the target, and the bank is risk-averse, higher ρ, ceteris paribus, implies higher liquidity demand by the bank. This explains why tighter liquidity regulation has an impact even on the banks that permanently increase their liquidity buffer by the same amount as the limit increase.

Impact of leverage regulation

A higher leverage requirement is represented in our model by higher $\bar{\lambda}_j^{lev}$. If the bank adjusts its balance sheet size accordingly, and this change is permanent, the leverage gap $\bar{q}_j^{lev}$ does not change, and the federal funds rate is not impacted. However, if the bank only targets its leverage ratio at the end of the day while starting the day well below the leverage requirement, a higher limit will discourage federal funds participation:

$$\frac{\partial r_{ij}}{\partial \bar{\lambda}_j^{lev}} < 0. \tag{22}$$

Externalities and heterogeneity among banks engaged in arbitrage

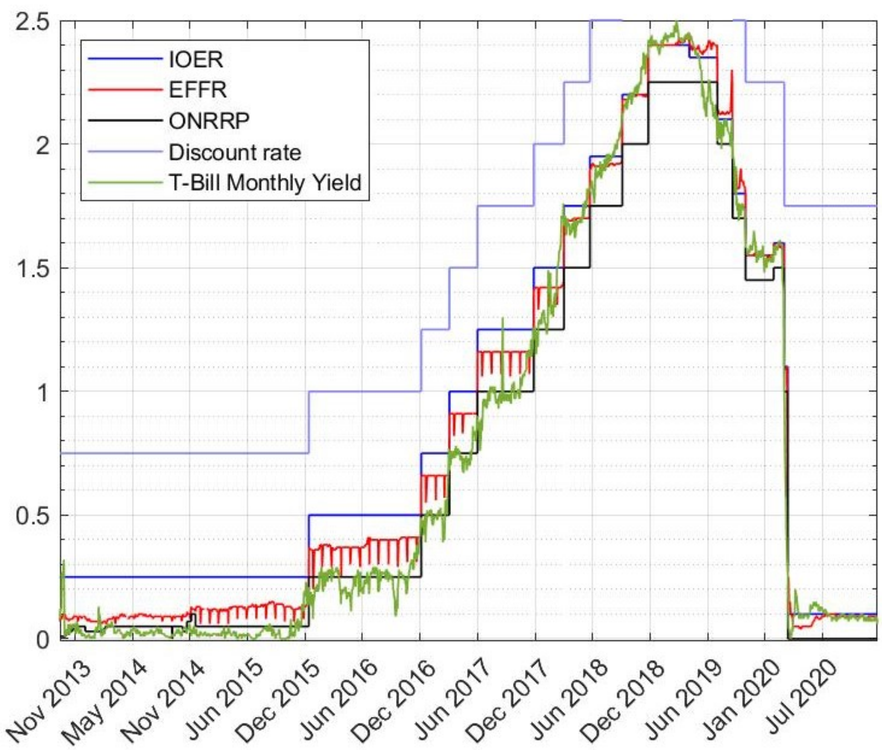
According to our predictions, interest rates are higher for the banks that are considered riskier by the lenders. Moreover, less solvent borrowers, which pay a higher spread on federal funds loans, get less funding from each lender. Finally, liquidity difficulties of some banks impose negative externalities on other banks by raising aggregate borrowing and pushing the interest rates higher. Banks that are considered by the lenders to be less solvent (have lower s_j) will be impacted by such externalities more than others.

5. Historical context

The current monetary policy regime started in 2008, when in order to mitigate systemic risk and the liquidity crunch, the Federal Reserve initiated several facilities flooding the financial system with liquidity and expanding the Fed's balance sheet.[15] At the peak, excess reserves in the banking system exceeded $2.7 trillion. In the absence of a market intervention, economic theory predicts that excess supply of reserves in the federal funds market will cause rates to fall to zero. In an attempt to prevent this, the Fed accelerated its policy on planned interest on reserves.[16] In October of 2008, U.S. banks began to receive interest on excess reserves kept at the central bank.

During the early years of the IOER facility, the federal funds rate and other risk-free rates (e.g. 3-month Treasury Bill) traded below IOER at rates that were often very close to zero. In order to raise the rates above zero, the Federal Reserve System introduced the ONRRP facility. Until 2014, federal funds market rates traded consistently in the range between the ONRRP and IOER rates, but generally much closer to the former (see Figure 5). The positioning of the federal funds rate within the ONRRP-IOER band first started to increase around the end of 2014. During this period, the rate on ONRRP remained constant.

Figure 5: U.S. short-term interest rates



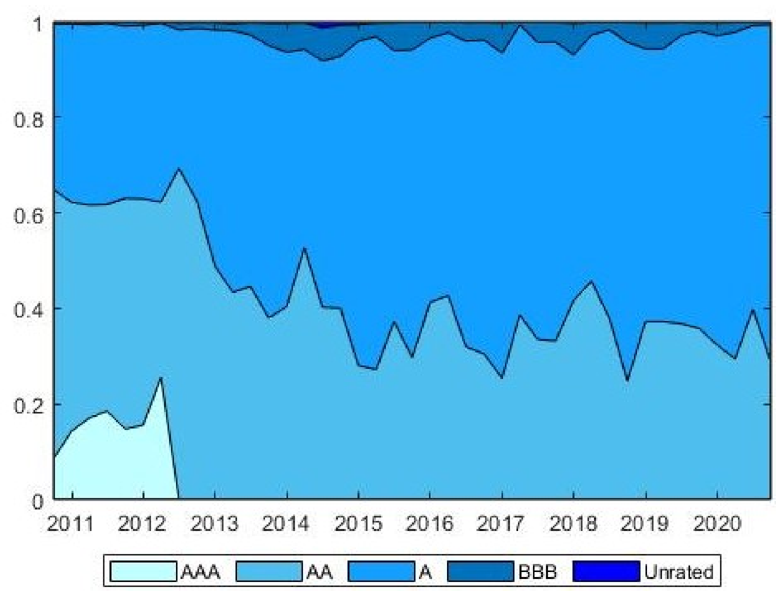
Sources: Federal Reserve Bank of New York and the Board of Governors of the Federal Reserve System

[15]These measures include the Term Auction Facility, Primary Dealer Credit Facility, Term Securities Lending Facility, and Term Asset-Backed Securities Loan Facility, as well as purchases of long-term Treasury securities and agency mortgage-backed securities.

[16]This policy was originally scheduled to begin on October 1, 2011.

Our analysis sheds light on the initial widening of the interest rate spread between the ONRRP rate and the federal funds rate and its further development. Our model suggests that one contributor to this interest rate rise was an increase in the credit risk of borrowers. According to Corollary 1 and equation (17), a 1% decrease in the credit rating of bank j increases the federal funds rate of borrower j by $0.01 \times \frac{1+r^a}{s_j} > 1\%$. This result holds independent of whether we assume excess liquidity supply or not and whether any regulatory constraints are critical for the banks. Data supports this hypothesis. As illustrated in Figure 6, the credit ratings of borrowers in the federal funds market deteriorated over the last several years.[17] The fraction of AAA-rated borrowers decreased to zero after 2012, and the proportions of BBB- and A-rated borrowers have been increasing steadily up to the last observations.[18]

Figure 6: Counterparty risk of FHLBs lending in the federal funds market



Source: Financial reports of FHLBs

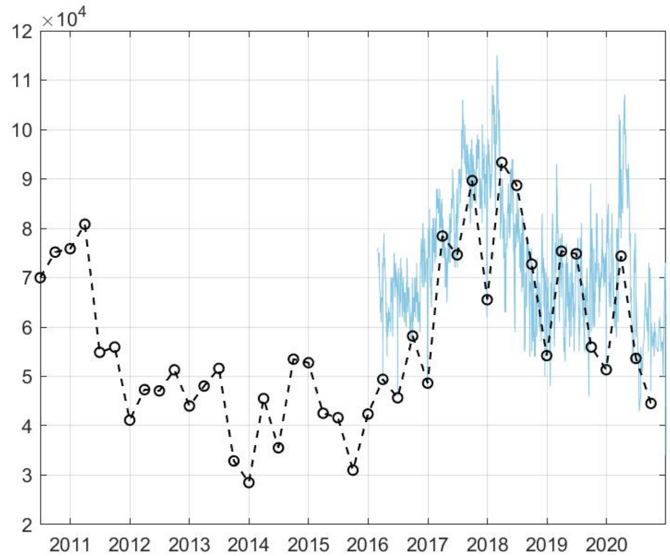
Another explanation has to do with liquidity demand. As shown in Figure 7, federal funds market volume increased dramatically from 2015 to 2018. The run-up to the peak is explained in our model by the LCR reform that was initiated in 2015.[19] As captured in section 4.1, LCR regulation increases

[17]We use the counterparty risk data for the FHLBs because these government-sponsored enterprises constitute the major lenders in the post-crisis federal funds market (see Figure 7).

[18]Other explanations for this rate increase are also possible. For example, it is realistic that lenders increased their set of counterparties over this period, which, by increasing competition, would result in lenders getting more favorable terms. However, support for this hypothesis is difficult to obtain in the absence of detailed data. In addition, the decrease in credit ratings of FHLB borrowers can be in line with the overall decrease in credit ratings in the U.S. banking industry witnessed by Alp (2013).

[19]For large depository banks, the initial requirement was that covered institutions maintain sufficient HQLA to satisfy 80%

Figure 7: Brokered federal funds volume



Description: Blue - lending of FHLBs to Federal Funds market; black - Brokered Federal Funds Volume. Sources: Reports of FHLBs and Federal Reserve Bank of New York

banks' demand for liquid assets. However, equally important to our story is the fact that LCR regulation discriminates between the ways in which assets are funded. In his speech at the EMEAP Governors' Meeting in August of 2018, the Head of the Markets Group at the FRBNY, Simon Potter, pointed out that, in the LCR regime, borrowing from public-sector entities, in particular FHLBs, is assigned lower run-off rates. As a result, he stated, banks interested in improving their LCRs "might seek out, and pay up for, loans from the FHLBs and other LCR-favored lenders."[20]

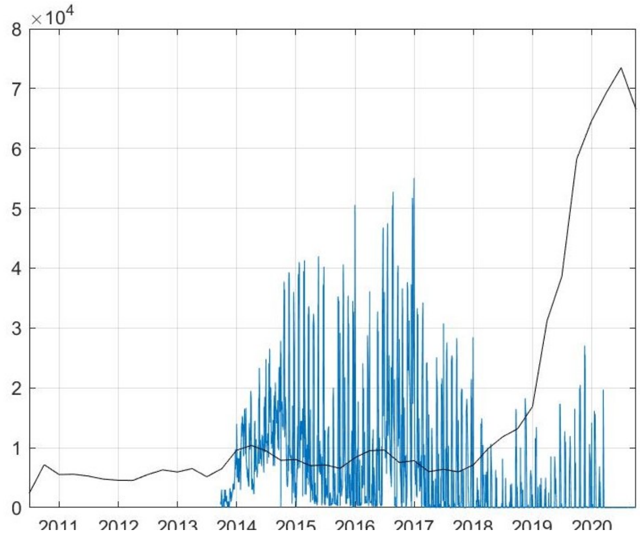
Leverage requirement regulation became effective in the United States in September 2014. However, key to understanding fed funds volumes is the fact that in January of 2018, regulators increased the minimum leverage requirements from 3% to 5% for eight U.S. global systemically important banks (G-SIBs). We argue in Section 4.1 that this works to counter the increase in demand for federal funds that arises from LCR and explains why federal funds market activity started to decline at the beginning of 2018.

In 2018, the federal funds rate moved significantly closer to the top of the ONRRP-IOER band. This is consistent with the change in demand for liquidity caused by the banking regulations. Despite that, our model predicts that the major increase in the rate is due to a change in the threat point of FHLB

of their projected net cash outflows over a 30-day period. This requirement rose to 90% in 2016 and 100% in 2017. It is also well known that banks utilize their own internal models to estimate short-term liquidity needs that can be even more demanding than LCR.

[20]See Potter (2018).

Figure 8: Holdings of alternative safe assets



Description: Black - U.S. obligations held by Federal Home Loan Banks; blue - ONRRP of Government Sponsored Enterprises. Sources: Financial reports of FHLBs and Federal Reserve Bank of New York

lenders. Specifically, around this time rates on treasury bills increased above the ONRRP rate (see Figure 5). Because FHLBs can invest overnight in the market of government debt by rolling over T-bill purchases, they likely benchmarked federal fund rates to the T-bill yields. In terms of our model, this is reflected in Theorem 1. This story is also confirmed empirically. Figure 8 shows that the timing of the increase in the federal funds rate to the top of the ONRRP-IOER band coincides with an increase in holdings of U.S. obligations by FHLBs.[21]

6. Concluding Remarks

A key aspect of our analysis is that we formally model the role leverage and liquidity regulations have on IOER-arbitrage activity. Due to leverage regulatory costs, banks limit the amount of arbitrage activity they are willing to undertake. Liquidity regulation, by contrast, increases federal funds rates and volumes. For much of the post-crisis era, this trade-off could explain the rates below IOER. We interpret additional dynamics within the IOER-ONRRP interval with changes in alternative rates and the market composition of credit risk.

Our theoretical results go beyond existing market conditions and can be applied to understand how the federal funds rate might evolve as the Federal Reserve reduces the quantity of excess reserves in the

[21] A similar story can be told by looking at movements in the Secured Overnight Financing Rate. See Figure 2 of "Observations on Implementing Monetary Policy in an Ample-Reserves Regime" by Lorie Logan, 2019, Federal Reserve Bank of New York.

system. Reduction in excess reserves can lead to higher equilibrium spreads in the arbitrage market. Rates should be higher especially for the banks that experience more daily volatility. Fewer reserves also means that banks need to find alternative high-quality liquid assets to satisfy regulatory and internal liquidity constraints. If federal funds market will be used for restoring liquidity reserves, the market rates will increase. As shown in the paper, it is even possible for the rate in the arbitrage market to exceed IOER. In addition, a reduction in reserves may impact the federal funds rate especially strongly if contractionary monetary policy is performed at the same time. Thus, for precision in the control over the federal funds rate, the Fed needs to coordinate changes in banking regulatory requirements, monetary policy, and the unwinding of the central bank's balance sheet.

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Appendices

Appendix A: Proof of Proposition 1

a) Case of $x_j^d > 0$: need to raise liquidity

If the liquidity ratio is below the required level, $\lambda_j^{liq} < \bar{\lambda}_j^{liq}$, the bank is forced to borrow $x^d > 0$. According to (8), such borrowing takes place only when the unexpected deposit withdrawals exceed the threshold:

$$y_j > \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right). \tag{23}$$

In this case, the amount of liquidity to be raised is

$$x_j^d = q_j^{liq} - \sum_i x_{ij} + \rho y_j. \tag{24}$$

Borrowing additional liquidity $x_j^d > 0$ impairs the bank's leverage ratio. If the illiquid bank also falls below the leverage requirement, $\lambda^{lev} < \bar{\lambda}_j^{lev}$, it faces a challenge decreasing its balance sheet by shedding cash, because cash is needed to stay liquid (the option of shedding cash and borrowing from the discount window is not considered due to cost inefficiency). Thus, such a bank is forced to sell non-HQLA assets to decrease the size of its balance sheet until (10) and (24) holds. The amount of non-HQLA sales necessary for the bank to be compliant is

$$x_j^\alpha = q_j^{liq} + q_j^{lev} + (\rho - 1)\, y_j. \tag{25}$$

Such liquidation takes place only when the deposit withdrawals exceed the threshold

$$y_j > \frac{1}{\rho - 1}\left(-q_j^{liq} - q_j^{lev}\right). \tag{26}$$

We can summarize the optimal response of the illiquid bank as follows:

– Borrowing from the lender of last resort and shedding long-term exposures take place ($x_j^d > 0$ and $x_j^\alpha > 0$) when liquidity shock y_j is restricted such that

$$\begin{cases} y_j > \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right) \\ y_j > \frac{1}{\rho-1}\left(-q_j^{liq} - q_j^{lev}\right). \end{cases} \tag{27}$$

– Borrowing from the lender of last resort but not shedding long-term exposures takes place ($x_j^d > 0$ and $x_j^\alpha = 0$) when liquidity shock y_j and parameters are restricted such that

$$\begin{cases} y_j > \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right) \\ y_j \le \frac{1}{\rho-1}\left(-q_j^{liq} - q_j^{lev}\right). \end{cases} \tag{28}$$

In this case, the amount to be borrowed is the same as in (24).

b) Case of $x_j^d = 0$: no need to raise liquidity

We now focus on the cases when the bank does not need to raise liquidity following the shock:

$$y_j \le \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right).$$

It means that the liquidity ratio following the deposit shock is above the required limit, so the bank has some extra cash it can spend to release the leverage ratio. The maximum amount it can use is determined by the threshold that keeps the liquidity ratio at the regulatory limit:

$$x_j^c \le -q_j^{liq} + \sum_i x_{ij} - \rho y_j.$$

The exact amount that will be repaid with the current liquidity depends on the leverage gap:
i) When the withdrawals are sufficiently large for the leverage gap (10) not to be critical,

$$y_j > q_j^{lev} + \sum_i x_{ij}, \tag{29}$$

the bank will keep all extra cash on the books, $x_j^c = 0$, because $\lambda_j^{lev} > \bar{\lambda}_j$ even when the actions are not taken.

ii) Consider the case when inequality (29) does not hold and the bank has enough extra liquidity to reach the required leverage ratio. Then the bank will shed amount x_j^c sufficient for the leverage ratio to bind:

$$x_j^c = q_j^{lev} + \sum_i x_{ij} - y_j.$$

iii) When inequality (29) does not hold and the amount of excess liquidity is insufficient, the bank may need to sell non-liquid assets. The condition for selling non-HQLA is

$$y_j > \frac{1}{\rho - 1} \left(-q_j^{lev} - q_j^{liq} \right). \tag{30}$$

The bank will first liquidate

$$x_j^c = -q_j^{liq} + \sum_i x_{ij} - \rho y_j.$$

The exact amount that will be sold should be sufficient for (10) to hold:

$$x_j^\alpha = q_j^{lev} + q_j^{liq} + (\rho - 1)\, y_j.$$

We can summarize the optimal response of the liquid bank as follows:

– No shedding of exposures take places ($x_j^c = 0$ and $x_j^\alpha = 0$) when

$$\begin{cases} y_j \leq \frac{1}{\rho} \left(\sum_i x_{ij} - q_j^{liq} \right) \\ y_j > q_j^{lev} + \sum_i x_{ij}. \end{cases} \tag{31}$$

– The bank sheds only liquid exposures ($x_j^c > 0$ and $x_j^\alpha = 0$) when

$$\begin{cases} y_j \leq \frac{1}{\rho} \left(\sum_i x_{ij} - q_j^{liq} \right) \\ y_j \leq q_j^{lev} + \sum_i x_{ij}. \\ y_j \leq \frac{1}{\rho - 1} \left(-q_j^{lev} - q_j^{liq} \right). \end{cases} \tag{32}$$

In this case, the cash to be repaid is

$$x_j^c = q_j^{lev} + \sum_i x_{ij} - y_j.$$

– The bank sheds both liquid and illiquid exposures ($x_j^c > 0$ and $x_j^\alpha > 0$) when

$$\begin{cases} y_j \leq \frac{1}{\rho} \left(\sum_i x_{ij} - q_j^{liq} \right) \\ y_j \leq q_j^{lev} + \sum_i x_{ij}. \\ y_j > \frac{1}{\rho - 1} \left(-q_j^{liq} - q_j^{lev} \right). \end{cases} \tag{33}$$

In this case, the amount to be repaid is

$$x_j^c = -q_j^{liq} + \sum_i x_{ij} - \rho y_j \tag{34}$$

and the amount to be sold is

$$x_j^\alpha = q_j^{lev} + q_j^{liq} + (\rho - 1) y_j.$$

Appendix B: Conditions for threshold on federal funds $\sum_i x_{ij}$ that determines the rebalancing strategy of bank j

Define the federal fund exposure threshold:

$$\bar{x}_j = -\frac{1}{\rho - 1}\left(q_j^{liq} + \rho q_j^{lev}\right).$$

According to this definition, both inequalities

$$q_j^{lev} + \sum_i x_{ij} < \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right) < \frac{1}{\rho - 1}\left(-q_j^{liq} - q_j^{lev}\right)$$

hold whenever $\sum_i x_{ij} < \bar{x}_j$, meaning the bank borrows "a little" in the interbank market.
Analogously, the reverse relationships,

$$q_j^{lev} + \sum_i x_{ij} > \frac{1}{\rho}\left(\sum_i x_{ij} - q_j^{liq}\right) > \frac{1}{\rho - 1}\left(-q_j^{liq} - q_j^{lev}\right),$$

hold whenever $\sum_i x_{ij} > \bar{x}_j$, meaning the bank borrows "a lot" in the interbank market.
The boundaries of the intervals above are identical to the critical values of liquidity shock y_j, which determine the rebalancing actions, so the statement of the theorem follows (see Figure 9).

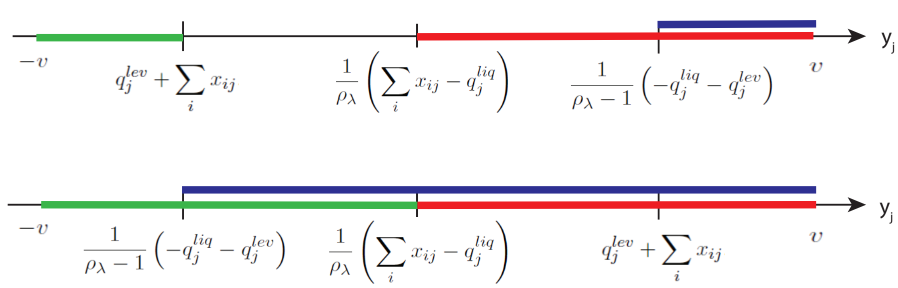

Figure 9: Values of deposit shock y_j and bank's actions. Top figure: if bank j borrows below threshold $\bar{x}_j$; bottom figure: if j borrows above threshold $\bar{x}_j$. Red – interval of borrowing from the central bank $x_j^d > 0$; blue – interval of non-HQLA sales $x_j^\alpha > 0$; green – interval of cash depletion $x_j^c > 0$.

Appendix C: Proof of Theorem 1

Part 1: We will first prove the statement of the theorem for the special case when FHLBs supply more liquidity than the banks are willing to borrow at the minimum rate offered to them:

$$\sum_i w_i^s \geq \sum_{j \in B^s} max(\bar{b}_j, 0). \tag{35}$$

Then the statements a) and b) of the theorem reduce to

a) the rate of any trading lender i and borrower j is equal to $\underline{r}_j$ $r_{ij} = \frac{r^a + (1-s_j)}{s_j}$;

b) the total amount borrowed by any participating borrower j is $\bar{b}_j$.

The proof of this special case proceeds as follows.

Proof. i) By definition, condition $\bar{b}_j \leq 0$ is equivalent to the condition on the marginal payoff that j would receive from each loan $x_{ij} > 0$:

$$\frac{\partial P_j^B}{\partial x_{ij}} \leq \left. \frac{\partial P_j^B}{\partial x_{ij}} \right|_{r_j = \underline{r}_j} \leq 0.$$

Thus, bank j with cut-off $\bar{b}_j \leq 0$ does not have incentives to borrow in the federal funds market at any rate $r_{ij} \geq \underline{r}_j$. If such a borrower ends up with a loan $x_{ij} > 0$, it will have incentives to deviate and reverse the offer $a_{ij} \geq x_{ij} > 0$ to $a_{ij} = 0$.

ii) In this part of the proof, we ignore all non-trading borrowers and focus only on those borrowers J^s that can benefit from borrowing in the interbank market under some conditions.

First, we notice that the set of equilibria of type a)-b) is non-empty. This is true because based on condition (35), it is possible to specify borrowing amounts a_{ij} for each pair (i, j) such that $\sum_k a_{kj} = \bar{b}_j$ for all $j \in J^s$ and $\sum_{m \in J^s} a_{im} \leq w_i^s$ for all i. We then specify offers for each borrower j as equal to $(\underline{r}_j, a_{ij})$. Lenders accept these offers because they are all indifferent between lending in the interbank market and lending at the alternative rate r^a. No borrower wishes to deviate to an alternative offer because minimum rates and maximum total volume maximize payoff of the borrower (13) over $r_{ij} \geq \underline{r}_j$ and $\sum_i x_{ij} \geq 0$.

To prove that statements a)-b) of the theorem hold for every equilibrium outcome, we first prove that given that condition (35) holds, under any equilibrium $(x, (r, a))$, there is at least one lender i' with excess liquidity supply, meaning

$$\sum_{j \in J^s} x_{i'j} < w_{i'}^s.$$

Suppose this is not the case and instead that $\sum_{j \in J^s} x_{ij} = w_i^s$ for all lenders i. Then the total volume traded in the market is

$$\sum_i \sum_{j \in J^s} x_{ij} = \sum_i w_i^s.$$

The excess supply condition of the theorem states that

$$\sum_i w_i^s > \sum_{j \in J^s} \bar{b}_j,$$

from which we conclude that

$$\sum_i \sum_{j \in J^s} x_{ij} > \sum_{j \in J^s} \bar{b}_j.$$

This is possible only when there is at least one actively trading borrower j' for which

$$\sum_i x_{ij'} > \bar{b}_{j'} = b_{j'}(\underline{r}_{j'}).$$

But this implies that under any potential equilibrium rate $r_{kj'} \geq \underline{r}_{j'}$, offered by borrower j', the marginal payoff of borrower j' is negative:

$$\frac{\partial P^B_{j'}}{\partial x_{kj'}} < 0.$$

As such, j' has an incentive to reduce its exposure to a lender or lenders trading with j' by at least some small amount, a contradiction. We therefore conclude that in the proposed setup, in any equilibrium there is at least one lender i' with some unused liquidity. From this result it follows immediately that the rates of all trades should be at the minimum to prevent a deviation by some borrowers. This proves result a) of the special case of the theorem.

To prove result b), notice that given a), each borrower faces the same marginal payoff from trades with any two existing or potential lenders i and k independent of how the volumes are split between these borrowers:

$$\frac{\partial P^B_j}{\partial x_{ij}} = \frac{\partial P^B_j}{\partial x_{kj}}.$$

That means that to prove result b), we need to prove that each participating borrower gets zero marginal payoff from all trades. Indeed, if the marginal payoff of some borrower j is not zero, this borrower has incentives to either reduce the offer $a_{i'j}$ to some lender i' or increase the proposal $a_{i''j}$ to a lender i'' who is abundant in liquidity. Therefore, in the equilibrium, it is always beneficial for the borrower to keep the trading volume at $\bar{b}_j$. This concludes the proof of result b) and the theorem overall for condition

$$\sum_i w^s_i > \sum_{j \in J^s} max(\bar{b}_j, 0).$$

□

Part 2: We now focus on the case of

$$\sum_i w^s_i < \sum_{j \in J^s} max(\bar{b}_j, 0). \tag{36}$$

Proof. Result i) of the general statement of the theorem can be proved in the same way as the special case in Part 1. From now on, we focus only on borrowers J^s that have incentives to participate in the federal funds market, $\bar{b}_j > 0$. For simplicity, assume all non-participating borrowers are excluded from the market.

To prove statement ii), we first take the existence result for granted and show that if the equilibrium set is non-empty, the traded rates r and volumes x are as specified in the theorem. We then show that there exist strategies a that make $((r, a), x)$ the equilibrium—strategies stable to deviations. This will prove that the equilibrium set is non-empty. Next, we define a necessary condition on all possible a that would make $((r,a),x)$ stable to deviations.

We advance the proof by proving multiple claims.

Claim 1: *If the equilibrium exists, each lender i trading with any two borrowers, $x_{ij} > 0$ and $x_{ik} > 0$, gets the same marginal expected payoffs from a loan with each counterparty: $\pi_{ij} = \pi_{ik}$.*

Proof. Suppose this is not the case for the equilibrium payoffs of lender i. Without loss of generality, assume borrower j delivers the maximum marginal payoff to i, and borrower k delivers the second largest marginal payoff to i: $\pi_{ik} < \pi_{ij}$. Then j has an incentive to deviate to a lower rate such that the updated marginal profit of lender i is $\pi_{ij}^{'} = \pi_{ik} + \varepsilon < \pi_{ij}$ for very small ε. This rate will still be accepted by the lender in the same trading amount because it generates higher marginal payoff than other offers and the alternative offer. Also, it will strictly benefit bank j because the bank's payoff is strictly decreasing in r_{ij}. By presenting a deviation that is profitable for j, we reached a contradiction to the statement that no deviations exist, which proves the claim. □

Claim 2: *If the equilibrium exists, any two lenders i and i' get the same marginal expected payoff from each loan they give: $\pi_{ij} = \pi_{i'j'}$ for any $j, j' \in J^s$.*

Proof. If this is not the case and one lender i receives a higher marginal payoff from $x_{ij} > 0$ than another lender i' from $x_{i'j'} > 0$, then borrower j has an incentive to offer lender i' volume $a_{i'j} = x_{i'j} + \varepsilon_a$ and a rate $\frac{r_{i'j}x_{i'j} + (r_{i'j'} + \varepsilon)\varepsilon_a}{x_{i'j} + \varepsilon_a}$, for some small $\varepsilon > 0$ and $\varepsilon_a > 0$, that will produce a higher profit for i' given that i' will redirect additional cash ε_a from j' to j. Borrower j will also redirect exposure ε_a from i to i' by offering $a_{ij} = x_{ij} - \varepsilon_a$ and getting the same amount of cash for lower overall cost, which will generate a payoff strictly above what j earned before the deviation. By presenting a deviation that is profitable for j, we reached a contradiction to the statement that no deviations exist, which proves the claim.

□

Claim 3: *If the equilibrium exists, at least one loan $x_{ij} > 0$ is traded at the rate above the minimum accepted rate: $r_{ij} > \underline{r_j}$.*

Proof. We need to prove that the outcome with all rates being at the minimum level is not stable. For the borrowers that borrow at the minimum accepted rates, the total federal funds volumes should deliver a marginal payoff which is non-negative, otherwise a borrower would deviate and borrow less. When the minimum rate is assumed, the condition of marginal payoff being non-negative is identical to the condition

$$\sum_i x_{ij} \leq \bar{b}_j. \tag{37}$$

In addition, we combine the no excess liquidity supply condition (36) with the total volume restriction (1) to obtain

$$\sum_i \sum_j x_{ij} \leq \sum_i w_i^s < \sum_j \bar{b}_j. \tag{38}$$

From (37) and (38) it follows that at least one borrower j borrows strictly below the optimal level:

$$\sum_i x_{ij} < \bar{b}_j. \tag{39}$$

This means that borrower j receives a positive marginal payoff on the loan it receives from one of its counterparties i: $\frac{\partial P_j^B}{\partial x_{ij}} > 0$. According to our initial assumption on rates being equal to $\underline{r}_j$, j gets the same marginal expected payoff from interactions with all lenders. Then the marginal payoff of borrower j is also strictly positive for another lender k: $\frac{\partial P_j^B}{\partial x_{kj}} > 0$. It is possible to choose a lender k that may or may not lend to j but importantly lends to someone apart from j. An increase in trading volume of x_{kj} would make borrower j strictly better off. Moreover, due to the continuity of the payoff function, for any small Δx_{kj} it is possible to find $\varepsilon > 0$ such that borrower k would prefer loan $(r_{kj} + \varepsilon, x_{kj} + \Delta x_{kj})$ to loan (r_{kj}, x_{kj}). Lender k would also be willing to redirect cash Δx_{ij} from another borrower to j for an extra return ε offered to i. This proves that there is a profitable deviation from the strategies with minimum rates. This contradicts our assumption that there exists an outcome with all rates being equal to the minimum accepted rates, which is an equilibrium. The statement of Claim 3 follows directly. □

From Claims 2 and 3 it follows that any lender i gets the same positive marginal payoff from each loan it gives. We introduce variable Δr^* such that

$$r_{ij} s_j + (1 - s_j) = r^a + \Delta r^* \sum_j \frac{1}{s_j}.$$

We will further use the rearranged version of this equation:

$$\sum_j r_{ij} = \sum_j \frac{r^a - (1 - s_j)}{s_j} + \Delta r^*,$$

and call Δr^* the accumulated market spread.

Claim 4: *If the equilibrium exists, and borrower j borrows positive amount $x_{ij} > 0$, the first-order condition is met:* $\frac{\partial P_j^B}{\partial x_{ij}} = 0$.

Proof. We have already proved that all lenders lend to j at the same rate r_j. If the first-order condition is not met for loan volume between i and j, then it is not met for every other FHLB in the subnetwork, and $\sum_i x_{ij} < b_j(r_j)$. Then borrower j has incentives to offer a slight rate increase to some FHLB i' trading with another bank j', together with a higher volume offer, $(r_{i'j} + \varepsilon, x_{i'j} + \varepsilon_a)$, with ε being much smaller than ε_a.[22] This offer will be accepted by the lender because, before the adjustments, lender i' got the same utility from loans with j and j' (see Claim 3). This will make both j and i' better off.

Consequently, for deviations of this sort not to exist in equilibrium, we require

$$\sum_i x_{ij} = b_j(r_j).$$

[22]We implicitly assumed that there are at least two trading borrowers.

□

Claim 5: *If the equilibrium exists, liquidity needs of lenders determine the total volume traded by borrowers:* $\sum_j x_{ij} = w_i^s$.

Proof. Suppose this is not the case and there exists lender i that invests below the maximum possible amount w_i^s. If $\sum_k a_{ik} > \sum_k x_{ik}$, there is a deviation by this lender to a higher lending volume, so we reach a contradiction.

Suppose now that $\sum_{k \in B^s} a_{ik} = \sum_{k \in B^s} x_{ik}$. Then there is still a deviation by one of the borrowers, j, who is taking money from i, $x_{ij} > 0$. In particular, j decreases its offered rate from $r_{ij} > \underline{r}_j$ to $\underline{r}_j$. Because no other borrowers change their offers, lender i will accept an even less attractive rate. So the deviation will benefit borrower j—a contradiction. This completes the proof of Claim 5.

We combine the results of Claims 3 to 5 to find the equilibrium rates and volumes, assuming equilibrium exists. Let us call the set of banks for which $b_j(r_{ij}) = b_j^{liq}(r_{ij})$ liquidity-constrained banks B^{liq}, and the set of banks for which $b_j(r_{ij}) = b_j^{lev}(r_{ij})$ leverage-constrained banks B^{lev}. Suppose FHLBs lend total amount $\bar{w}^{liq}$ to the first group and $\bar{w}^{lev}$ to the second group of banks. According to Claim 4:

$$\bar{w}^{liq} = \sum_{j \in B^{liq}} b^{liq},$$

$$\bar{w}^{lev} = \sum_{j \in B^{lev}} b^{lev}.$$

Using representation of the interest rate as the total of the low bound and the spread adjusted for credit risk, we rewrite the two conditions above:

$$\Delta r = \frac{\sum_{j \in B^{liq}} \bar{b}_j^{liq} - \bar{w}^{liq}}{2\rho v}(c + r^d - r^{ioer}) \tag{40}$$

$$\Delta r = \frac{\sum_{j \in B^{lev}} \bar{b}_j^{lev} - \bar{w}^{lev}}{2\rho v}(c\rho + r^d - r^{ioer}) \tag{41}$$

According to Claim 3, $\bar{w}^{liq} + \bar{w}^{lev} = \sum_i w_i^s$, so the two equations above can be transformed as:

$$\Delta r^* = \frac{1}{2\rho v} \frac{(c\rho + r^d - r^{ioer})(c + r^d - r^{ioer})}{c(1+\rho) + 2(r^d - r^{ioer})} \left(\sum_{j \in B^{lev}} \bar{b}_j^{lev} + \sum_{j \in B^{liq}} \bar{b}_j^{liq} - \sum_i w_i^s \right).$$

Because c is very small, the first multiplier of the spread can be linearized around zero, which leads to the formula for Δr^* in the statement of the theorem.

□

We have proved that if an equilibrium exists, the traded rates and volumes (r, x) are as described in conditions a)-c). We next prove that there exist strategies $((r, a), x)$ that form an equilibrium, so the equilibrium set is non-empty.

Claim 6: *The set of equilibrium strategies is non-empty.*

An infinite number of bank offers lead to the same loan terms and payoffs as described in conditions a)-c) of the theorem statement. To see why this equilibrium set is non-empty, consider all strategies $((r,a),x)$ that satisfy the additional condition

$$\sum_{k\neq j} a_{ik} \geq \sum_{j} x_{ij}, \text{ for each bank j and lender i.} \tag{42}$$

This condition means that lender i can completely redistribute loan amount x_{ij} to other borrowers by utilizing unused buffers $\sum_{k\neq j} a_{ik} - \sum_{k\neq j} x_{ik}$ from all banks other than j. It implies that if there is a deviation by j to a lower rate offer $\hat{r}_{ij}$, while keeping a_{ij} the same, it is possible for lender i to secure expected profit received before the deviation by redistributing funds to other borrowers. As a reminder, lender i has an interest in doing so because in the original equilibrium, r_{ij} was chosen for i to receive the same marginal payoff loan with j as from any other loan it makes (see Claim 2), so a decrease in the interest rate below r_{ij} would imply that i strictly prefers to give loans to other banks. In conclusion, such a deviation of j will only lead to a loss of funds for bank j and thus not improve its payoff. Other deviations of j, including ones in which j varies both r_{ij} and a_{ij}, are not improving either, as this result will follow from Claim 7.

Suppose the initial equilibrium is made up of strategies $((r,a),x)$. Consider deviations by borrower j to strategies $(\hat{r}_j, \hat{a}_j)$, while strategies of other banks are fixed. Because bargaining is modelled as a sequential game, bank j takes into account the best responses of lenders $\hat{x}$ when deciding whether to deviate to a different strategy.

Claim 7: *If there is a deviation and best response $((\hat{r},\hat{a}),\hat{x})$ that strictly benefit j, deviation and best response $((\hat{\hat{r}},\hat{a}),\hat{x}))$ also strictly benefit j:*

$$P_j^B((\hat{\hat{r}},\hat{a}),\hat{x}) \geq P_j^B((\hat{r},\hat{a}),\hat{x}) > P_j^B((r,a),x), \tag{43}$$

where we defined

$$\hat{\hat{r}}_{ij} = \begin{cases} r_{ij} + \varepsilon, & \text{if } \hat{r}_{ij} > r_{ij} \\ r_{ij}, & \text{if } \hat{r}_{ij} = r_{ij} \\ \underline{r}_{ij}, & \text{if } \hat{r}_{ij} < r_{ij}. \end{cases}$$

for small enough $\varepsilon > 0$.

Proof. Consider lender i for which $\hat{r}_{ij} > r_{ij}$. We know that according to $((r,a),x)$, each lender gets the same expected profit from each loan, so any updated offer from j with a rate above r_{ij} would be accepted by i in the maximum amount possible, namely $\min(a_{ij}, w_i^s)$, even if the lender will need to reduce other borrowing amounts. So if $\hat{r}_{ij}$ is accepted in amount $\hat{x}_{ij}$, offer $r_{ij}+\varepsilon$ would be also accepted in the same amount.

Now consider lender i for which $\hat{r}_{ij} < r_{ij}$. If $\hat{r}_{ij}$ is accepted in positive amount $x_{ij} > 0$, it means that i was not able to redirect funds $\hat{x}_{ij}$ to any other borrowers that offer a higher payoff. Likewise, an offer with rate $\hat{\hat{r}}_{ij}$ will lead to the same outcome in terms of volumes $\hat{x}$, because i will again be strictly worse off by lending to j (simultaneous deviations of other borrowers are not considered).

So far we have shown that the best responses of lenders are identical in both cases and equal to $\hat{x}$. In addition, because P_j^B is a decreasing function of rates, and $\hat{\hat{r}} \leq \hat{r}$, the second type of deviation is more profitable for j:

$$P_j^B((\hat{\hat{r}}, \hat{a}), \hat{x}) \geq P_j^B((\hat{r}, \hat{a}), \hat{x}. \tag{44}$$

So if $P_j^B((\hat{r}, \hat{a}), \hat{x}) > P_j^B((r, a), x)$, then $P_j^B((\hat{\hat{r}}, \hat{a}), \hat{x}) > P_j^B((r, a), x)$, which proves the statement of Claim 7. $\square$

We now focus on all possible deviations of type $(\hat{\hat{r}}, \hat{a}), \hat{x})$ conducted by arbitrary chosen borrower j and finish the proof of Claim 6. We aim to show that such deviations do not benefit j, so no other deviations exist either.

Denote the set of lenders for which rate r_{ij} increases to $\hat{\hat{r}}_{ij} = r_j + \varepsilon$ as L^+, and the set of lenders for which the federal funds rate decreases to $\hat{\hat{r}}_{ij} = \underline{r}_j$ as L^-. Condition (42) guarantees that if set L^- is non-empty, no updated offers will be accepted by L^- lenders. Consequently, among all deviations of type $((\hat{\hat{r}}, \hat{a}), \hat{x})$, the deviation that brings P_j^B the maximum payoff to j is the one where set L^- is empty, and bank j lends only at rates at or strictly above r_j. Assuming bank j can borrow the optimal amount $\hat{x}_j$ it needs to maximize its utility P_j^B, given rates r, then a decrease from r to $\hat{\hat{r}}$ does not benefit bank j due to envelope conditions. Moreover, the optimal funds volume demanded by j would need to be less than the original volume, and thus would be feasible to achieve. This proves that any bank j does not benefit from a deviation of type $(\hat{\hat{r}}, \hat{a}), \hat{x})$ when set L^- is empty.

We arrive at a contradiction. Specifically, we assumed that (42) holds and beneficial deviation $((\hat{r}, \hat{a}), \hat{x})$ exists, and we proved that any strategy of j along $(\hat{r}, \hat{a})$ is dominated by strategy $((\hat{\hat{r}}, \hat{a}), \hat{x})$, so

$$P_j^B((\hat{\hat{r}}, \hat{a}), \hat{x}) \geq P_j^B((\hat{r}, \hat{a}), \hat{x}) > P_j^B((r, a), x).$$

We also showed that set L^- is empty, so by design, bank j borrows at a higher rate fewer funds, which leads to

$$P_j^B((\hat{\hat{r}}, \hat{a}), \hat{x}) < P_j^B((r, a), x).$$

So we reach a contradiction to the previous inequality and our initial assumption that there is a deviation from which j benefits strictly. This proves that if (42) holds, no deviation exists, and any strategies $((r, a), x)$ that satisfy (42) and conditions a)-c) constitute an equilibrium. This proves that the equilibrium set is non-empty. $\square$

Claim 8: *A necessary condition for $((r, s), x)$ to be an equilibrium is that for each i and j, credit unused by lender i with excluded j is positive:*

$$\sum_{k \neq j} (a_{ik} - x_{ik}) > 0. \tag{45}$$

Proof. If this is not true, then $a_{ik} = x_{ik}$ for all $k \neq i$ and some fixed pair (i, j). Then there is a deviation by borrower j from offers (r_{ij}, a_{ij}) to new offers $(\tilde{r}_{ij}, \tilde{a}_{ij}) = (\underline{r}_j, x_{ij})$, while keeping offers to other lenders fixed. Clearly, the new offers will be accepted by i at amount $\tilde{x}_{ij} = x_{ij}$ because the offered rate delivers equal marginal payoff as the alternative rate r^a (earlier we assumed that lenders split ties in favor of banks to maintain the relationships), and condition (45) guarantees that lenders do not have incentives to accept anything below $\tilde{a}_{ij}$ by redirecting the funds to other banks. This deviation will make j strictly better off because it will increase j's cost of borrowing while keeping the same volume borrowed. The existence of such a deviation completes the contradiction example, and in this way proves (45) is necessary for the equilibrium to be stable. □

The results of all of the above claims can be combined to prove that the proposed outcome is feasible and stable to deviations. The proposed strategies form an equilibrium whenever profitable deviations do not exist. First, consider a deviation by a lender in the counter-offer subgame. As we showed in Claim 2, lenders are indifferent between the loans that are offered to them and thus do not have incentives to deviate. Borrowers also do not have incentives to offer different volumes, while keeping rates the same, because the current volumes maximize their concave payoff function as proved in Claim 4. Therefore, we only need to check for deviations of each borrower involving change of rates (and potentially offered volumes) to one or multiple lenders. Ceteris paribus, the new volume and rate offers by j may incentivize a lender to redirect their funds from other borrowers to j only when j offers a higher rate than it is currently offered. Moreover, by slightly increasing the rate, the borrower can attract any amount of funding available from FHLBs. The optimal amount of funds that j would like to attract at given rates is defined by marginal payoff conditions. If j increases rate r_{ij} to $r_{ij} + \varepsilon$, the first-order conditions would require less lending from i, so the whole purpose of increasing some rates for attracting new funds, while keeping other funds constant, is meaningless. A decrease in rates for the purpose of attracting new funds, while keeping other rates the same, will also not work because lenders will not deviate from their existing positions. Finally, the only possibility left to consider is for borrower j to decrease the rate of some existing contracts, while increasing other rates. This deviation may involve changes in offered volumes. In Claims 6 and 7 we showed that it is possible to choose offers a, such that deviations of this kind are not beneficial for a borrower. This establishes that the proposed strategies are stable to deviations.

Appendix D: Proof of Corollary 1.

Proof. We obtain the predicted formulas using the fact that c is very small. Along these lines, we substitute $\bar{b}_j^{lev}$ and $\bar{b}_j^{liq}$ in the equilibrium conditions of Theorem 1, linearizing terms

$$\frac{r^d - r^{ioer} + c\frac{\rho+1}{2}}{r^d - r^{ioer} + \rho c} = 1 + c\frac{1-\rho}{2}$$

$$\frac{r^d - r^{ioer} + c\frac{\rho+1}{2}}{r^d - r^{ioer} + c} = 1 - c\frac{1-\rho}{2}$$

and dropping terms with c of order greater than 1. □