Are Working Hours Complements in Production?

by Lin Shao,[1] Faisal Sohail[2] and Emircan Yurdagul[3]

[1] International Economic Analysis Department
Bank of Canada
lshao@bankofcanada.ca

[2] University of Melbourne
faisal.sohail@unimelb.edu.au

[3] Universidad Carlos III de Madrid and
Centre for Economic Policy Research
emircan.yurdagul@uc3m.es

Acknowledgements

Abstract

This paper uses Canadian matched employer-employee data to show that working hours are gross complements in production rather than perfect substitutes, as is typically assumed. We exploit within-establishment and individual-level variation in hours and wages to document novel evidence consistent with complementarities in hours worked. Next, we estimate an elasticity of substitution in working hours of 0.69 in the aggregate and between 0.52 and 1.04 at the industry level. We validate our estimates by showing that industries with higher elasticities exhibit greater flexibility in hours. Our findings have important implications for research on labor supply and the efficacy of policies that aim to influence it.

Topics: Labour markets; Economic models
JEL codes: E23, J22, J23, J31

Résumé

Dans cette étude, nous utilisons des données canadiennes appariées entre employeurs et employés qui permettent de montrer que les heures de travail sont des compléments bruts dans la production plutôt que de parfaits substituts, comme ce qui est souvent postulé. Nous étudions les variations du nombre d'heures travaillées et des salaires au sein des entreprises et d'un travailleur à l'autre et obtenons des résultats inédits qui mettent en évidence la complémentarité des heures de travail. Ensuite, nous estimons une élasticité de substitution du nombre d'heures travaillées que nous chiffrons à 0,69 au niveau agrégé et entre 0,52 et 1,04 au niveau sectoriel. Nous corroborons nos estimations en montrant que les secteurs où l'élasticité est plus élevée font preuve d'une plus grande flexibilité quant aux heures de travail. Nos résultats ont des conséquences importantes pour les recherches sur l'offre de travail et l'efficacité des politiques qui visent à influencer cette offre.

Sujets : Marchés du travail; Modèles économiques
Codes JEL : E23, J22, J23, J31

1   Introduction

An implicit assumption in most macroeconomic models of production is that the number of working hours of different workers are perfect substitutes. However, the production process requires coordination between workers who work on different tasks. With this in mind, the assumption of perfect substitutability between working hours becomes less natural. Indeed, a consequence of the need to coordinate tasks is the need to coordinate hours worked. For instance, is a worker who works 40 hours a week equally productive regardless of whether her coworkers work 25, 40 or 60 hours? Recent work suggests that the answer is no. Specifically, Bick et al. (2020) and Yurdagul (2017) document an aggregate, non-linear relationship between hours and wages, whereby workers earn higher wages when they work near the modal hours in the economy. Working hours being complements in production would imply precisely this relationship, as workers would be more productive and therefore earn higher wages when they work a similar number of hours to their coworkers.

Knowing whether and to what extent working hours are complements is essential for understanding the labor supply decisions of individuals and their response to policies. Indeed, the degree of substitutability between working hours will constrain the ability of an individual to respond to changes in income taxes and child-care provision or to idiosyncratic shocks. Yet, despite its relevance, there has been little work exploring whether the working hours of workers are complements in production. In this paper, we use matched employer-employee data from the Canadian Workplace and Employee Survey (WES) to document such evidence.

The WES, unlike many other employer-employee linked data, is ideally suited to study complementarity in working hours within the same workplace as it includes information on both working hours – measured by usual weekly hours rather than contracted hours – and average hourly wage. Our analysis of the WES is divided into two parts. First, we document a series of new facts that support the presence of complementarity in working hours. We begin by showing that individual working hours are significantly positively correlated with their average coworkers' hours. We document that wages *within* establishments exhibit the

same "hump-shaped" pattern across hours worked as observed in the aggregate data. We also find that the hours at which wages are maximized – that is, the peak of the hump shape – are strongly positively correlated with the median (or average) hours worked in the establishment. By tracking employees in the same establishment over time, we show that movements away from either the median or the wage-maximizing hours are associated with significant wage reductions. These results are robust to controlling for observable characteristics and consistent with a production process where the working hours of workers are complements.

In the second part of our analysis, we estimate the degree of working hours complementarities in production. To do this, we propose a simple CES production function that features imperfect substitutability between working hours and use the WES to estimate the elasticity of substitution between the working hours of different workers. The CES production function delivers an endogenous non-linear wage schedule. In particular, as observed in the data, wages are highest at an intermediate level of hours and decline if workers either increase or decrease their hours worked. The endogenous relationship between hours and wages underpins our estimation strategy, which follows an extensive literature by applying the optimal conditions of firms to the data in order to estimate the parameters of production functions (e.g., Katz and Murphy, 1992 and Acemoglu, 2002).[1]

In the aggregate, we estimate the elasticity of substitution between working hours to be 0.69, implying that workers' hours are gross complements in production. We also document significant heterogeneity in this elasticity across industries. Our estimates range from 0.52 to 1.04, with working hours being gross complements in all but one industry. We exploit the heterogeneity across industries to validate our estimation results. Specifically, we document an intuitive positive relationship between proxy measures of hours flexibility and our estimated elasticities.

These results challenge the canonical assumption of perfect substitutability between work-

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[1]Although our data is at the establishment level, we assume that all establishments in a firm share a production function. As such, we use the terms "firm" and "establishment" interchangeably.

ing hours and instead provide strong support for complementarities in working hours. Working hours being gross complements in production may act as effective constraints on labor supply. A growing literature has studied the implications of constraints on working hours. Altonji and Paxson (1988) and Chetty et al. (2011) show that the (in)ability to choose working hours has important implications for job mobility and estimates of labor supply elasticities, respectively. More recently, Labanca and Pozzoli (Labanca and Pozzoli) studied the impact of hours constraints on labor supply decisions using Danish administrative data. Our findings complement this literature by providing direct evidence that apparent constraints on hours may be driven by a feature of the production process – workers are more productive when they work a similar number of hours to each other.

Naturally, this paper relates and contributes to the literature studying the degree of complementarity and coordination between workers. The production function underpinning our estimation is a general version of a Leontief production function studied in Becker and Murphy (1992) and is most closely related to Yurdagul (2017), which studies the flexibility motive behind entrepreneurship. Rogerson (2011) introduces the notion of coordinated working hours by imposing (exogenous) constraints on labor supply on the worker side rather than on the firm side as in this paper. Cubas et al. (2019) use worker-level data to estimate an occupation-specific measure of work schedule coordination. Their measure focuses on a strict type of hours complementarity: namely, the extent to which occupations feature workers working *at the same time* rather than the same number of hours. In contrast, we focus on workers working the same number of hours, and our matched employer-employee data allows us to estimate this more general form of complementarity in the same workplace, the estimation of which is not feasible using worker-level data. Battisti et al. (2021) use a similar production function as ours to argue that the degree of complementarities in hours has important implications for estimating the Frisch elasticity.

Our findings are also relevant to the literature on labor market sorting. As noted in Bonhomme (2021), complementarities are crucial in driving sorting patterns. For example,

Calvo et al. (2021) document complementarity between partners' hours in home production and show that this shapes sorting in marriage and labor markets. Shao et al. (2021) show that complementarity in workers' hours impacts matching between heterogeneous firms and workers. They find that complementarity in hours and the implied sorting amplify existing heterogeneity across firms, impacting income inequality and welfare.

An important result of our analysis is that the degree of complementarity in the production process varies across industries. Specifically, we find that the manufacturing and construction sectors feature a higher degree of hours complementarity than service sectors, especially low-skill service sectors such as retail. While this finding may seem intuitive, this is the first paper providing empirical evidence to support it. This cross-industry difference in complementarity is also related to the literature on structural change and the evolution of gender inequality, which argues that women have a comparative advantage in the service sector. Hence, a structural shift from manufacturing to services benefits women more than men (Ngai and Petrongolo, 2017). Our results suggest that, as women value flexibility more than men (Goldin, 2014), this comparative advantage could stem from the relative flexibility in hours afforded in service sectors. This result complements existing literature which emphasizes the comparative advantage based on sector-specific skill requirements (Rendall, 2018).

Finally, the findings in this paper provide insights that may be leveraged by policymakers. First, heterogeneity in the degree of complementarity in working hours implies that changes in income tax or other labor supply policies will have differential impacts depending on the industry. Second, taking into account the inherent complementarities in working hours may be particularly important when considering policies related to labor supply over the life cycle or by gender. For instance, Cubas et al. (2019) argue that the requirement to work the same hours contributes to the majority of the gender wage gap, while Ameriks et al. (2020) find that older workers would work longer if they could choose their hours. Mas and Pallais (2017, 2020) argue that alternative work arrangements, particularly those that provide more

flexibility, play an important role in workers' labor supply decisions. Complementarities in working hours introduce a trade-off for employers if they consider providing more flexible contracts. Indeed, if coworker hours are complements, employees may be less productive, on average, when they work different hours. Policies that intend to address gender inequity or labor force participation through increased flexibility should internalize this trade-off.

The rest of this paper is organized as follows. In Section 2, we describe our data and present evidence for the presence of complementarities between the hours of coworkers. In Section 3 we propose a production function and use it to estimate the degree of complementarity between hours within firms. In Section 4, we validate our estimation by relating proxy measures of work schedule coordination to our estimates across industries. We conclude in Section 5.

2 Evidence of Complementarities in Working Hours

Our empirical analysis uses matched employer-employee data from the Canadian Workplace and Employee Survey (WES). In this section, we use the WES to present novel evidence supporting the presence of complementarities in working hours.

The WES is an annual survey of Canadian establishments and their workers with a longitudinal design. The survey tracks surviving employers for six years, from 1999 to 2006. In even-numbered years, a sample of employees from each employer is interviewed and followed for two years. A maximum of 24 employees are sampled, and in workplaces with fewer than four employees, all employees are interviewed. We restrict attention to individuals aged between 25 and 64 and exclude those who usually work less than 10 hours a week or earn less than half the federal minimum wage. Our final sample includes just over 120,000 employer-employee-year observations.

Importantly, and in contrast to many other matched employer-employee datasets, we observe both the usual weekly hours worked and average hourly wage of employees, which

makes the WES particularly well-suited to addressing our primary research question. Our measure of hourly wage includes overtime pay, commissions, and tips. However, our results are robust to an alternative measure of wages that excludes such extra pay. We also observe employee occupation, education and a number of other demographic characteristics such as age, marital status, immigration status and parenthood.[2]

2.1 Correlations Between Own and Coworker Hours

Complementarities between working hours would imply a positive correlation between the hours of individual workers and those of their coworkers. This is due to the sorting of workers with similar desires into the same establishments and the wage schedules of individual workers pushing workers towards their peers' hours. Indeed, the crude correlation between a worker's hours and the average hours of her coworkers in our data is 0.33.

While suggestive, the unconditional positive correlation need not be due to complementarities but instead may be driven by other factors such as sorting of similar workers into production units or may be explained by characteristics of establishments. To control for such factors, we conduct a more formal evaluation of the correlation between the hours of a worker and those of her coworkers by estimating the following regression,

$$\log(h_{ist}) \quad = \quad \alpha + \gamma \log\left(\overline{h}_{s_{-i}}\right) + \delta X_i + \eta Y_s + \mathbf{B}_t + \epsilon_{ist}, \tag{1}$$

where h_{ist} are hours worked by worker i employed by establishment s in year t and $\overline{h}_{s_{-i}}$ is the average hours worked among i's coworkers in establishment s. X_i is a vector of individual-level control variables which include a quadratic in age, dummy variables for educational status (college degree or not) as well as indicators for marital and immigration status. Y_s is a vector of establishment-level controls that includes establishment age, size and industry and the average establishment wage. $\mathbf{B}_t$ captures year fixed effects.

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[2]Summary statistics are reported in Table A.1 in the Appendix.

Table 1 reports the coefficient γ – the elasticity of worker i's hours with respect to those of her coworkers. Under all specifications, we document a significant and positive correlation between own and coworker hours with an estimated elasticity between 0.28 and 0.35. In other words, having coworkers that work longer (shorter) hours is associated with one's own hours being longer (shorter) – a key implication of a production process where working hours are complementary. Having said this, we do not interpret this finding to be conclusive in establishing the presence of working hours complementarities. For example, we cannot rule out workers sorting on unobserved individual or establishment characteristics. Instead, we take this evidence to be suggestive, which, combined with the evidence on earnings that we present below, strengthens the case for the hours of workers being complements in production.

2.2   Wage-Hour Profiles within Establishments

If working hours are complements in production, we expect workers to be less productive if they work longer or shorter hours than their coworkers. This lower productivity should be reflected in workers' earnings with relatively lower wages for workers working shorter or longer hours. Such a relationship between wages and hours has been previously documented in the aggregate by Yurdagul (2017) and Bick et al. (2020) using US labor force surveys.[3] These papers' findings point to wage penalties for working either short or long hours among all workers in the economy. To test whether such penalties are present *within* establishments, we use the WES to estimate the following specification,

$$\log(w_{ist}) = \alpha + \left(\sum_{h \in H} \gamma_h \mathbb{I}_{i,h}\right) + \delta X_i + \mathbf{A}_s + \mathbf{B}_t + \epsilon_{ist}, \tag{2}$$

where w_{ist} is the hourly wage of worker i in establishment s at time t. The indicator variable $\mathbb{I}_{i,h}$ is equal to one if an individual works h hours. We partition weekly hours into a set H

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[3]Bick et al. (2020) and Yurdagul (2017) use data from the Current Population Survey (CPS) and Survey of Income and Program Participation, respectively. Figure A.1 in the Appendix reports the wage-hours profile estimated from the US CPS and Labor Force Survey (LFS) in Canada.

by grouping hours in 5-hour bins. We choose the category $40 - 44$ as the reference category of hours worked as most workers work these hours. Then, the coefficients γ_h capture the relative wage penalty/premium from working either more or less than the reference hours bin. X_i is a vector of individual-level control variables which include a quadratic function of age, dummy variables for educational status (college degree or not) as well as indicators for marital and immigration status. $\mathbf{A}_s$ and $\mathbf{B}_t$ are establishment and year fixed effects, respectively.

Panels (a) and (b) of Figure 1 report the results from estimating equation (2). Panel (a) plots the wage-hours profile – that is, the coefficients γ_h – when we exclude establishment fixed effects but include controls for establishment size, age and industry. Consistent with existing evidence from worker-level data, the estimated wage-hours profile features a kinked or hump-shaped pattern, with relatively long and short hours exhibiting lower wages than those obtained in intermediate hours.

Panel (b) plots the coefficients γ_h when establishment fixed effects are included. This captures the wage-hours profile *within* establishments – a measure that, to our knowledge, is novel. Importantly, we find that short and long hours penalties exist *within* establishments. Indeed, workers earn around 5% lower wages when working either 25 or 60 hours per week relative to those working around 40 hours. Compared to Panel (a), which excludes establishment fixed effects, the penalties from working short or long hours are relatively smaller but still statistically significant.

While a hump-shaped wage-hours profile is consistent with complementarities in working hours, there may be other, unrelated drivers of the observed short and long hours penalties within establishments. For instance, short hours penalties may reflect a (time) startup cost of working, while long hours penalties may be due to diminishing returns. A more specific litmus test for the presence of complementarities is to ask whether the hours at which wages are maximized relate to the usual hours worked within an establishment. Complementarities in working hours would imply a positive relationship between the wage-maximizing hours

and usual hours worked. We test this prediction and find a significant positive correlation between the wage-maximizing hours and two measures of usual establishment hours: the median and average hours worked. Indeed, the correlation between median (average) hours and the hours at which wages are highest is 0.84 (0.79) in the WES.[4]

Another important and related implication of complementarities in working hours is that deviating from the hours that most workers work impacts one's productivity. In particular, the more a worker deviates from her coworkers, the less productive she will be. To study whether this is observed in the WES, we test whether deviations from the median or wage-maximizing hours result in wage penalties and whether these penalties increase as the magnitude of deviation increases. We do this by estimating the following,

$$\log(w_{ist}) = \alpha + \left(\sum_{h \in H} \gamma_{\Delta h} \mathbb{I}_{i,\Delta h}\right) + \delta X_i + \mathbf{A}_{st} + \epsilon_{ist}, \tag{3}$$

where $\mathbb{I}_{\Delta h}$ indicates *deviations* of a worker's own hours from a reference level of hours which is either the median or wage-maximizing hours in an establishment. $\mathbf{A}_{st}$ is an interaction of establishment and year fixed effects, and all other regressors are as in equation (2).

Panels (c) and (d) of Figure 1 report the results from estimating this alternative regression. Panel (c) shows the relationship between relative wages and deviations from the median hours worked in an establishment. We find that wages are the highest when workers work around 10 hours longer than the establishment median hours, with larger deviations from this level resulting in larger wage reductions.

Panel (d) shows the relationship between relative wages and deviations from the wage-maximizing hours in an establishment. By construction, wages are the highest when workers remain at those hours that deliver the maximal hourly wage. Consistent with complementarities in hours, wage penalties monotonically increase as workers' hours deviate from the wage-maximizing hours. Furthermore, the wage decrease is roughly symmetric regardless of

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[4]Figure A.2 in the Appendix shows a strong positive relationship between establishment-level wage-maximizing hours and average or median hours worked. A simple linear regression between average (median) hours and wage-maximizing hours gives a statically significant slope coefficient of 0.88 (0.94).

whether a worker works longer or shorter hours relative to the wage-maximizing hours.

2.3 Changes in Wages When Hours Change

A concern with the above analysis of hours and wages is that the relationships we document may be driven by unobserved individual characteristics. While we cannot fully address this concern, we can control for the fixed unobservable traits of workers by exploiting the short panel nature of the WES. Related to the intuition above, if coworkers' hours are complements in production, the same worker should earn lower wages if they deviate further from the hours of their coworkers between two periods. By tracking workers over time, we control for the time-invariant unobserved characteristics of workers.

We test the relationship between changes in wages and hours worked at the individual level in the WES. More formally, for each worker i working in establishment s in year t, we first compute the absolute log difference between their own hours, $h_{i,t}$, and the median or wage-maximizing hours, $\overline{h}_{s,t}$: $|\log h_{i,t} - \log \overline{h}_{s,t}|$. We then compute the changes in this measure between period t and $t+1$: $\Delta h_{ist} = |\log h_{i,t+1} - \log \overline{h}_{s,t+1}| - |\log h_{i,t} - \log \overline{h}_{s,t}|$. A positive (negative) value for the difference in differences, Δh_{ist}, indicates that worker i moved further (closer) from the reference hours between period t and $t+1$. Similarly, we can compute the corresponding changes in wages between t and $t+1$, $\Delta w_{ist} = (\log w_{i,t+1} - \log w_{i,t})$.

The presence of complementarities would imply a negative correlation between Δh_{ist} and changes in wages Δw_{ist}. That is, workers who move further away from their establishments' median or wage-maximizing hours suffer wage losses, while those who move closer experience wage gains.

We examine this relationship estimating the following,

$$\Delta w_{ist} = \alpha + \left(\sum_{\Delta h_{ist} \in \mathcal{H}} \gamma_{\Delta h_{ist}} \mathbb{I}_{\Delta h_{ist}} \right) + \delta X_i + \mathbf{A}_{st} + \epsilon_{ist}, \tag{4}$$

where the indicator variable $\mathbb{I}_{\Delta h_{ist}}$ is equal to one if the relative change in difference hours

over time is Δh_{ist}. We partition this "difference in difference" measure into a set $\mathcal{H}$ by grouping hours changes into 10% bins. We choose the 0 to +10% bin as the reference group since most workers fall into this category of hours changes. All other regressors are as in equation (3).

Figure 2 reports the coefficient $\gamma_{\Delta h_{ist}}$ when using the median (Panel (a)) or wage-maximizing hours (Panel (b)) as the reference hours $\overline{h}_{s,t}$. For both reference hours, we find a clear negative correlation between changes in wages and changes in hours worked over time – in line with the presence of complementarities in hours worked. Indeed, moving 10% further from the median and wage-maximizing hours results in a 2% and 5% penalty in wages, respectively.

The evidence presented in this section strongly supports the presence of complementarities in working hours in production. In the next section, we move beyond this reduced-form evidence and estimate the extent of these complementarities using the same within-firm variation in hours and wages in a more structural framework.

3 Estimating Complementarities in Working Hours

In this section, we first propose a model, which we then use to estimate the parameter that governs complementarities in working hours.

3.1 Production Function

The model we use to estimate the degree of complementarity in working hours is intentionally simple and limited to the description of the production function and a labor market structure. Firms' production is represented by $Y = zL^\eta$, where L denotes the effective labor input and z is broadly defined as productivity, which can capture non-labor variables at the firm level such as technology, capital or intermediate inputs.

In contrast to standard neoclassical models of production, where the effective labor input

of a firm is the sum of total hours worked, we follow Yurdagul (2017) and allow for complementarities between the hours of workers. In particular, we assume that the labor input of workers is aggregated in a non-linear manner so that the aggregate labor input depends on the distribution of hours worked in a firm. The labor input L is given by

$$L = \left(\int_{i \in N} x_i l_i^\rho di \right)^{\frac{1}{\rho}} \left(\int_{i \in N} x_i di \right)^{1 - \frac{1}{\rho}},$$

where N is the set of workers, and $\{l_i\}_{i \in N}$ is their hours worked. Without loss of generality, we assume the working hours range from 0 and 1. The contribution of each worker to the labor input is scaled up by their efficiency units $x_i > 0$. To abstract from indices of workers, one can rewrite the aggregation in terms of the measure of workers employed at each level of hours worked,

$$L = \left(\int_{x \in B_x} \int_0^1 x \mu(l, x) l^\rho \, dl \, dx \right)^{\frac{1}{\rho}} \left(\int_{x \in B_x} \int_0^1 x \mu(l, x) \, dl \, dx \right)^{1 - \frac{1}{\rho}}, \tag{5}$$

where $\mu(l, x)$ is the measure of workers with efficiency x working l hours. The parameter ρ determines the elasticity of substitution $(\frac{1}{1-\rho})$ between hours of different workers and is our key parameter of interest to be estimated.

In order to estimate ρ, we must first provide additional structure on the economic environment. To this end, we assume that labor markets are segmented by firm type, z, and all firms within a sub-market compete for workers in perfectly competitive labor markets.[5] Under this market structure, firms take the wage schedule $w_z(l, x)$ in their market as given and choose the measure, $\mu(l, x)$, of workers with a given hour-efficiency combination to hire

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[5]The purpose of the partitioning is simply to accommodate heterogeneous equilibrium wage schedules between firms since perfect competition implies uniform wages within each sub-market. We could also allow for a more general segmentation whereby firms and workers are randomly allocated into an arbitrary number of sub-markets, with perfect competition within each sub-market. Even though we abstract from the possibility of imperfect competition in labor markets (e.g., Berger et al., 2022), our estimation only requires wages to be proportional to workers' marginal product and can accommodate constant wage markdowns which might result from firms having labor market power.

in order to maximize their static profits:

$$\pi = \max_{\mu(l,x)} zL^{\eta} - \int\limits_{x \in B_x} \int\limits_{0}^{1} w_z(l,x)\mu(l,x)ldldx,$$

where L is given by (5).

The first-order condition of the firm's maximization problem returns an expression for the equilibrium wage schedule,

$$w_z(l,x) = \eta z x L^{\eta-1} E(l^\rho)^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{l^{\rho-1}}{E(l^\rho)} + \left(1 - \frac{1}{\rho}\right)l^{-1} \right], \tag{6}$$

where the right-hand side is the marginal productivity of a worker with efficiency units of x working l hours. $E(l^\rho) \equiv \left(\int\limits_{x \in B_x} \int\limits_{0}^{1} x\mu(l,x)l^\rho dldx \right) \div \left(\int\limits_{x \in B_x} \int\limits_{0}^{1} x\mu(l,x)dldx \right)$ is a weighted average of l^ρ, and L is the aggregate labor input.[6]

Equation (6) shows that a worker's wage depends not only on her own hours but those of her coworkers. Indeed, for $\rho \in (-\infty, 1)$ the maximum hourly wage, for each efficiency group, is achieved at the same level of hours $l^* = E(l^\rho)^{\frac{1}{\rho}}$ and wages decrease as working hours move away from l^*.[7] This wage schedule would also generate a channel through which coworkers' hours are positively correlated – as observed in the data. If $\rho = 1$, so that workers' hours are perfect substitutes in production, the hourly wage for the same x-type workers will be the same regardless of their hours. On the other hand, as $\rho \to -\infty$, so that the production function approaches Leontief, the marginal product for all workers will be zero if they do not work l^* hours. As illustrated by these two extremes, different values of ρ represent different relationships between wages and hours worked, which we use to estimate this parameter.

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[6]Our estimation relies on variation within firms rather than between firms. Hence, for notational convenience, we omit indexing the variables L, $E(l^\rho)$, $\mu(l,x)$, Y, π by firms' type although they are all specific to a firm.

[7]Notice, l^* depends on ρ and would in general be different from the median or average hours worked in a firm – as in the data.

3.2 Estimation Strategy

Estimating ρ directly from equation (6) requires information on firm-level productivity z, the returns to scale parameter η as well as a measure of worker efficiency x. This poses a challenge since there are no natural counterparts to these measures in the WES. However, we can eliminate z and η by normalizing worker earnings $(w_z(l, x)l)$ relative to average earnings of her establishment $\overline{W} \equiv \eta z L^{\eta-1} E(l^\rho)^{\frac{1}{\rho}} \mathbb{E}(x)$ where $\mathbb{E}(x)$ is the mean worker efficiency across all workers in the firm. Then, relative worker earnings for a worker i is independent of measures that are fixed within an establishment and given by

$$\widetilde{W}_i \equiv \frac{w_z(l_i, x_i)l_i}{\overline{W}} = \frac{x_i}{\mathbb{E}(x)}\left[\frac{1}{\rho}\frac{l_i^\rho}{E(l^\rho)} + 1 - \frac{1}{\rho}\right]. \tag{7}$$

While simpler, estimating ρ from (7) requires information on worker type x, which is unobserved. Instead, we construct a proxy for worker efficiency, X, using a linear function of education, gender and age:

$$X_i = \theta_0 + \theta_1 \text{Education}_i + \theta_2 \text{Gender}_i + \theta_3 \text{Age}_i,$$

where Education_i, Gender_i and Age_i are dummy variables which indicate whether worker i has a college degree, is male or is over 45 years of age, respectively. In turn, we compute the efficiency of worker i relative to the firm average using $\widetilde{X}_i \equiv \frac{X_i}{\mathbb{E}(X)}$ so that the expression for relative earnings becomes

$$\widetilde{W}_i = \widetilde{X}_i \times \left[\frac{1}{\rho}\frac{l_i^\rho}{E(l^\rho)} + 1 - \frac{1}{\rho}\right].$$

Finally, we can substitute $E(l^\rho)$ with an observable measure that does not depend on ρ by recognizing that wages for each efficiency group in a firm are maximized when hours worked are $l^* = E(l^\rho)^{\frac{1}{\rho}}$. This allows us to replace $E(l^\rho)^{\frac{1}{\rho}}$ with the observed hours $\widetilde{l}^*$ that return the highest hourly wage in each establishment. Then, for each worker, we compute

the hours relative to this establishment-specific reference hour as $\widetilde{h}_i = \frac{l_i}{\widetilde{l}^*}$, which delivers our estimating equation:

$$\widetilde{W}_i \;=\; \widetilde{X}_i \times \left[\frac{1}{\rho}\widetilde{h}_i^{\,\rho} + 1 - \frac{1}{\rho}\right], \tag{8}$$

where $\widetilde{W}_i$ are the earnings of worker i relative to the average firm earnings, $\widetilde{X}_i$ is a proxy for relative worker efficiency and $\widetilde{h}_i$ are the hours worked of worker i relative to the wage-maximizing hours. We use this equation to estimate the substitution parameter ρ.

3.3   Results

We estimate equation (8) using a non-linear least squares regression and report our estimation results in Table 2. Panel A reports the estimates of ρ for the aggregate sample for three alternative proxy measures of worker skill x_i. When considering only education as a proxy for skill, we estimate ρ to be around -0.44, implying an elasticity of substitution of around 0.69. Including gender and age leads to a very similar elasticity. Importantly, regardless of the proxy for x_i, we find that the elasticity of substitution between working hours is below 1 – that is, working hours are gross complements in production.

We also separately estimate the working hours elasticity of substitution by industry by restricting our sample to a particular industry and then estimating the substitution parameter, ρ, using education, age and gender to proxy for worker skill, x_i. Panel B of Table 2 shows the results for each of the 14 industry groups in the WES. Except for one industry, our estimates imply working hours to be gross complements in production. Indeed, even in "communications and other utilities," where the elasticity of substitution is around 1.04, the 95% confidence interval does not preclude working hours being gross complements.[8]

Our results show significant heterogeneity in the extent of complementarity in working hours. Secondary and primary product manufacturing features the highest degree of comple-

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[8]Appendix Table A.2 outlines the NAICS industries that comprise each WES industry category.

mentarities with an estimated elasticity of substitution between working hours of 0.52 and 0.59, respectively.

Low-skill services sectors such as retail trade and consumer services and information and cultural industries exhibit the lowest (statistically significant) degree of complementarities with an elasticity of substitution of around 0.85. In contrast, education and health services, which require higher skills, feature higher complementarities with an elasticity of 0.62.

The manufacturing sector generally features stronger complementarities in working hours than the services.[9] This suggests that the manufacturing sector allows less flexibility in terms of allowing workers to choose a "non-standard" number of working hours than the service sector. Such flexibility may give women, who tend to value flexibility more than men (Goldin, 2014), a comparative advantage in service industries. Indeed, our findings complement existing works such as Ngai and Petrongolo (2017) and Rendall (2018) that highlight women's comparative advantage in the service sector. These estimates suggest that this comparative advantage could stem from a relative flexibility in service sector production. We further study how our estimates of complementarities vary with flexibility and female employment in the next section, which aims to validate our estimates.

4   Validation

This section explores whether our estimates of complementarities are systematically related to measures of hours coordination across industries. In particular, we use three measures to proxy for worker coordination at the industry level and test whether industries with a lower degree of complementarities in working hours also feature lower levels of coordination in work schedules. The three measures are i) the share of workers that work flexible hours, ii) the standard deviation of hours worked, and iii) the share of female workers in an industry. Figure 3 plots the industry average of these measures and the industry elasticity of substitution.

Panel (a) shows that industries with a larger share of workers with flexible working hours

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[9]The estimated elasticities for all manufacturing and service industries are 0.61 and 0.69, respectively.

also feature higher elasticities of substitution and, therefore, a lower degree of complementarity in working hours.[10] Indeed, the correlation between workers' shares and elasticity is 0.59. Industries with lower shares of flexible-hour workers, such as labor-intensive or primary product manufacturing, feature a higher degree of complementarity (a lower elasticity of substitution) than industries with higher shares of flexible-hour workers, such as information and cultural industries. This is intuitive since greater flexibility in choosing one's working hours may indicate less need for workers to coordinate their work schedule and hence a lower degree of complementarity in working hours.

Next, we examine how the industry's standard deviation of working hours relates to our estimates of working-hour complementarities. Consistent with Labanca and Pozzoli (Labanca and Pozzoli), we view a more significant variation in hours as indicating a production process that requires lower coordination of hours. We expect to see a positive relationship between the degree of substitution in working hours and dispersion across industries. Panel (b) plots these two measures and shows that, as expected, there is a positive correlation (0.40) between our estimated elasticity of substitution and the standard deviation of working hours.

Finally, we consider the share of female workers as a proxy for the degree of flexibility in hours in production. Since female workers tend to work fewer hours and choose more flexible occupations, a higher share of female workers in an industry may proxy for the degree of flexibility in working hours (Goldin, 2014).[11] Panel (c) shows that there exists a strong positive correlation (0.51) between the degree of working hours substitutability and the share of female workers across industries.

Importantly, the measures we considered in this section are not directly related to variation in earnings across workers, which underpinned our estimation strategy. Taken together, the findings in Figure 3 support our estimated measures of complementarities in hours.

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[10]The share of flexible workers in an industry is the share of workers giving an affirmative response to the question *Do you work flexible hours?* An affirmative answer implies that workers can vary their daily start and stop times as long as they work a full workweek. That is, on a daily basis, a "flexible" worker may not work the same number of hours as their coworkers nor work at the same time as them.

[11]The correlation between the share of flexible-hour workers and female workers is 0.35.

5 Conclusion

This paper uses matched employer-employee data from Canada to study complementarities between coworkers' hours in production. We provide novel reduced-form evidence that is consistent with the presence of such complementarities. We then use a simple model to estimate the degree of complementarities in working hours. The economy-wide elasticity of substitution is estimated to be 0.69, implying that working hours are gross complements in production. Although our industry-specific estimates exhibit significant variation, we find that hours are gross complements for almost all sectors in our data.

The complementarities that we estimate should be positively related to the degree of coordination in working hours. As a validity test of our results, we show that our estimated elasticities of substitution are positively correlated to industry-specific flexibility in hours, the share of female workers, and the level of dispersion in working hours.

Our results challenge a canonical assumption about the nature of production, namely, that the working hours of workers are perfect substitutes. We provide strong evidence supporting coworker hours being gross complements. Such complementarity has important implications for research on labor supply and the efficacy of policies that aim to influence it.

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Figures and Tables

Table 1: Correlation between Own and Coworker Hours

	(1)	(2)	(3)
Average Coworker Hours	0.345	0.285	0.277
	(0.014)	(0.014)	(0.014)
Individual Controls	Y	Y	Y
Establishment Controls	N	Y	Y
Average Wage	N	N	Y
N	120420	118336	118336
R^2	0.172	0.190	0.192

Notes: The table reports the coefficient γ from estimating equation (1). The regressions include a set of controls for worker and establishment characteristics, as indicated in the table. Robust standard errors are reported in the parentheses. *** indicates statistical significance at the 1% level.

Table 2: Estimation Results

Panel A: Aggregate

	Substitution Parameter, ρ		Elasticity of Substitution $\frac{1}{1-\rho}$	
	Estimate	95% CI	Estimate	95% CI
Proxy for x_i using only Education	-0.443	[-0.476,-0.410]	0.693	[0.677,0.709]
Proxy for x_i using only Education and Gender	-0.459	[-0.491,-0.426]	0.686	[0.671,0.701]
Proxy for x_i using only Education, Gender and Age	-0.459	[-0.491,-0.426]	0.686	[0.671,0.701]

Panel B: Industry

	Substitution Parameter, ρ		Elasticity of Substitution $\frac{1}{1-\rho}$	
	Estimate	95% CI	Estimate	95% CI
Secondary product manufacturing	-0.943	[-1.269,-0.616]	0.515	[0.441,0.619]
Primary product manufacturing	-0.707	[-0.932,-0.482]	0.586	[0.518,0.675]
Construction	-0.683	[-0.799,-0.566]	0.594	[0.556,0.638]
Transportation, warehousing, wholesale	-0.646	[-0.749,-0.543]	0.608	[0.572,0.648]
Labor-intensive tertiary manufacturing	-0.621	[-0.823,-0.418]	0.617	[0.548,0.705]
Education and health services	-0.604	[-0.675,-0.533]	0.624	[0.597,0.653]
Real estate, rental and leasing operations	-0.483	[-0.660,-0.307]	0.674	[0.603,0.765]
Forestry, mining, oil, and gas extraction	-0.478	[-0.658,-0.297]	0.677	[0.603,0.771]
Capital intensive tertiary manufacturing	-0.403	[-0.683,-0.123]	0.713	[0.594,0.890]
Business services	-0.388	[-0.546,-0.229]	0.721	[0.647,0.813]
Finance and insurance	-0.248	[-0.379,-0.118]	0.801	[0.725,0.895]
Retail trade and consumer services	-0.182	[-0.290,-0.074]	0.846	[0.775,0.931]
Information and cultural industries	-0.178	[-0.345,-0.010]	0.849	[0.743,0.990]
Communication and other utilities	+0.040	[-0.138,+0.219]	1.042	[0.878,1.281]

Notes: The table reports estimates of the substitution parameter, ρ and the corresponding elasticity of substitution $\frac{1}{1-\rho}$ along with 95% confidence intervals, as estimated from equation (8) using non-linear least squares. Panel A reports these estimates for the aggregate sample. Panel B reports the same measures by establishment industry. To limit the influence of outliers, we trim the top and bottom 1% of the ratios $\tilde{W}_i$ and $\tilde{h}_i$ from equation (8).

(a) γ_h, excluding Establishment FE

(b) γ_h, including Establishment FE

(c) $\gamma_{\Delta h}$, deviations from Median Coworker Hours

(d) $\gamma_{\Delta h}$, deviations from Wage-Maximizing Hours
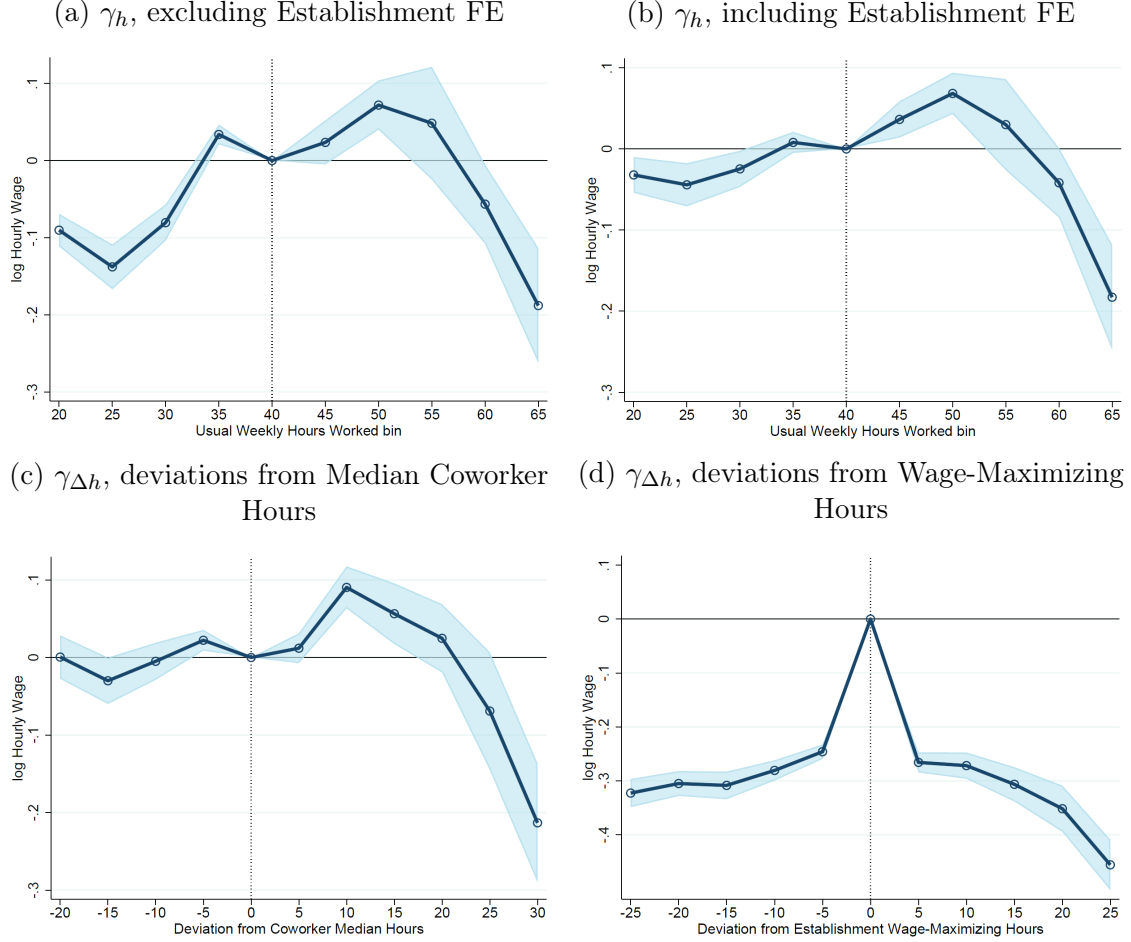
Figure 1: Relationship between wages and hours using within-establishment variation

Notes: Panels (a) and (b) display the coefficient γ_h from estimating variants of equation (2). In Panel (a), we do not include establishment fixed effects $\mathbf{A}_s$, but instead include a set of establishment characteristics including establishment size and age dummies as well as industry fixed effects. Panel (b) reports the coefficient γ_h when establishment fixed effects are included. Panels (c) and (d) report the coefficient $\gamma_{\Delta h}$ from estimating equation (3) when the reference hours are the establishment median and establishment wage-maximizing hours, respectively. The shaded area represents the 95% confidence interval of the coefficient using robust standard errors.
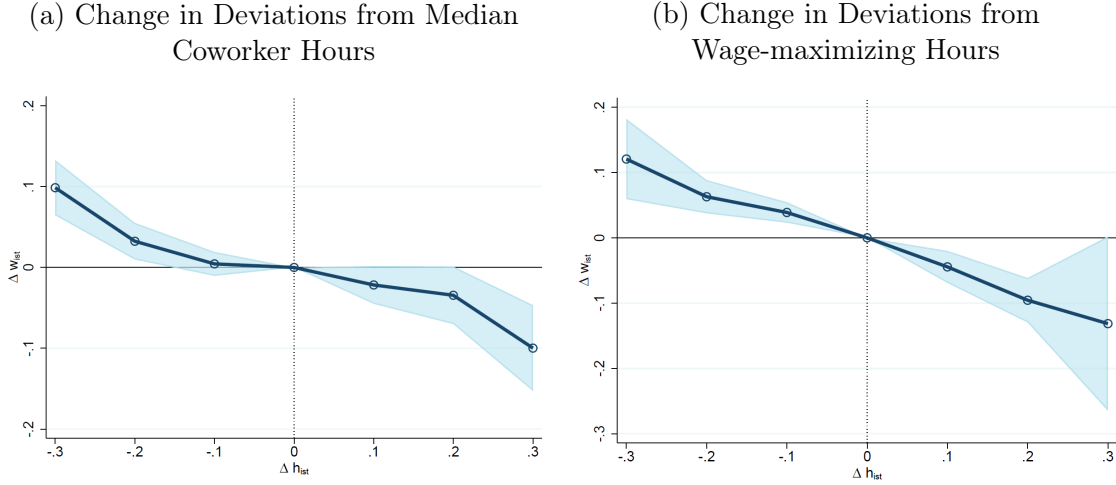
(a) Change in Deviations from Median Coworker Hours

(b) Change in Deviations from Wage-maximizing Hours

Figure 2: Dynamic Changes in Hours and Wages

Notes: The figure reports the coefficient $\gamma_{\Delta h_{ist}}$ from estimating equation (4). Panels (a) and (b) report this coefficient when the reference hours are the establishment median and establishment wage-maximizing hours, respectively. The shaded area represents the 95% confidence interval of the coefficient using robust standard errors.

(a) Flexible Hours

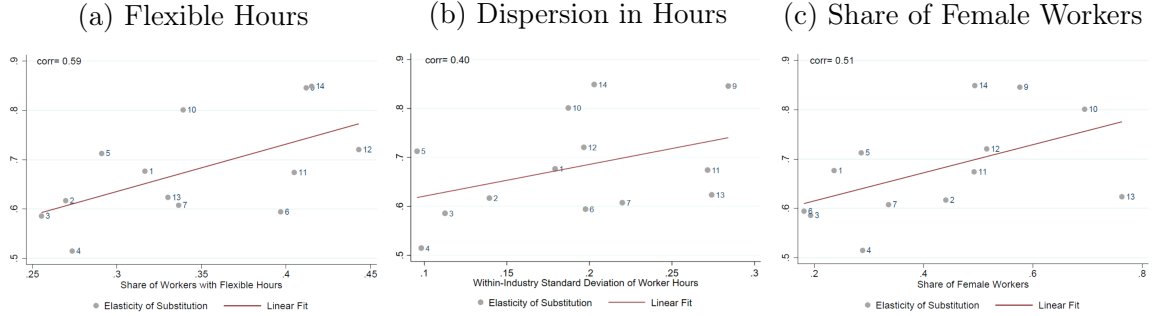(b) Dispersion in Hours

(c) Share of Female Workers

Figure 3: Elasticity of Substitution and Coordination Measures Across Industries

Notes: Each panel of the figure plots the industry-specific estimate of the elasticity of substitution on the vertical axis and a different measure of coordination on the horizontal axis. Panel (a) plots the share of workers with flexible hours, Panel (b) plots the within-industry standard deviation of hours worked, and Panel (c) plots the share of female workers. The correlation between elasticity of substitution and coordination measure is reported in the panel. The sample excludes "Communication and other utilities" since our estimate does not establish at the 5% significance level whether working hours are either gross complements or substitutes. The correlation between elasticity of substitution and share of flexible hours, the share of female workers, and the standard deviation of working is 0.32, 0.26, and 0.19, respectively, when all 14 sectors are included. The descriptions of the WES industry codes can be found in Table A.2.

Online Appendix for:

Are Working Hours Complements in Production?

Lin Shao   Faisal Sohail   Emircan Yurdagul

A   Additional Figures and Tables

Table A.1: Summary Statistics, WES

	Obs	Mean	Std. Dev.
Age	129,037	42.78	9.89
Usual Weekly Hours Worked	126,613	37.57	8.17
Hourly Wage (CAD)	126,613	20.96	12.94
Hourly Wage (no extra earnings, CAD)	105,005	20.08	11.44
High School Graduate	129,037	0.85	0.36
College Graduate	129,037	0.46	0.50
Establishment Age	126,911	21.77	22.13
Total # Employees in Firm	129,037	402.17	1088.71
Gross operating revenue (mil. CAD)	129,037	37.08	139.05
Average Usual Weekly Hours in Establishment	128,828	37.11	6.50
Average Hourly Wage in Establishment	128,828	20.71	9.75

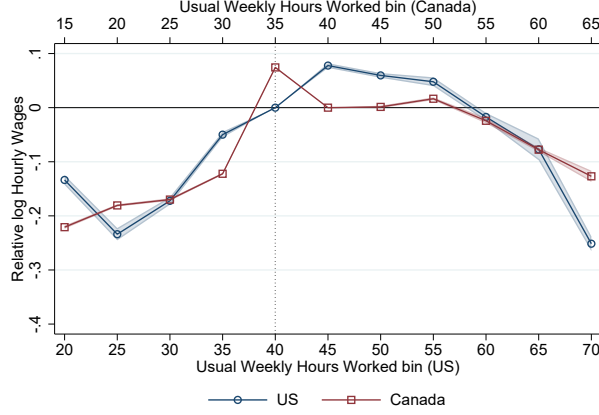Notes: The table reports a number of summary statistics from the WES sample.

Figure A.1: Aggregate Relationship between Hours and Wages, Data from US and Canada Worker Surveys

Notes: The figure plots the aggregate relationship between weekly hours worked and wages. In particular, it plots the coefficient γ_h as estimated from the following regression,

$$\log(w_i) = \alpha + \left(\sum_{h \in H} \gamma_h \mathbb{I}_{i,h} \right) + \delta X_i + \epsilon_i,$$

where $\log(w_i)$ is the log hourly wages of individual i. X_i is a vector of individual-level controls which includes demographic controls including gender, race and education dummies, a quadratic in years of experience as well as state, year, and industry fixed effects. The indicator variable $\mathbb{I}_{i,h}$ is equal to one if an individual works h hours. Weekly hours h are partitioned into a set $H = \{10 - 14, 15 - 19, \ldots, 65 - 69, 70 - 99\}$. As most workers work 40 hours, the category $40 - 44$ hours is the omitted (reference) category. Data from the US is from 1991 to 2018 Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). Data from Canada is from the 1997 to 2018 Canadian Labour Force Surveys (LFS). Additional details on the sample construction can be found in Shao et al. (2021). The bottom horizontal axis reports results from the CPS. The top horizontal axis is shifted by 5 hours and reports results from the LFS. The shaded regions indicate the 95% confidence interval.
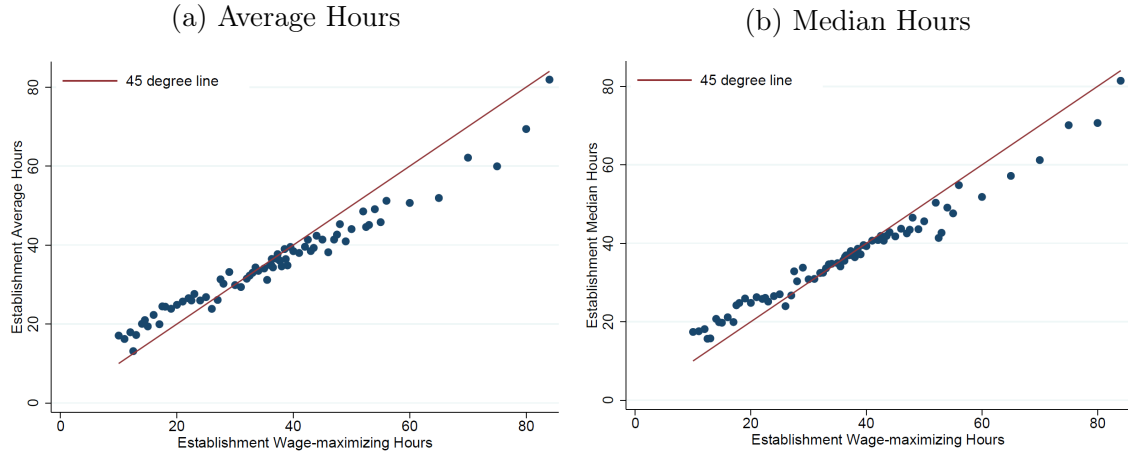
2

(a) Average Hours   (b) Median Hours

Figure A.2: Wage-Maximizing Hours and Average/Median Coworker Hours

Notes: The figure illustrates the relationship between establishment-level wage-maximizing hours and average/median hours. Each dot represents a group of establishments with the same wage-maximizing hours. Due to data confidentiality restrictions, the panels only display binned groups with at least 10 observations. The horizontal axis reports the wage-maximizing hours while the vertical axis reports the mean value of establishment-level average hours (Panel (a)) and median hours (Panel (b)) in each group.

Table A.2: WES and NAICS Industry Codes

WES Industry Code	WES Industry Description	NAICS Industry Description and Code
1	Forestry, mining, oil, and gas extraction	Forestry and Logging (113), Support Activities for Forestry (1153), Oil and Gas Extraction (211), Mining (except Oil and Gas) (212), Support Activities for Mining and Oil and Gas Extraction (213)
2	Labour intensive tertiary manufacturing	Food Manufacturing (311), Beverage and Tobacco Product Manufacturing (312), Textile Mills (313), Textile Product Mills (314), Clothing Manufacturing (315), Leather and Allied Product Manufacturing (316), Furniture and Related Product Manufacturing (337), Miscellaneous Manufacturing (339)
3	Primary product manufacturing	Wood Product Manufacturing (321), Paper Manufacturing (322), Petroleum and Coal Products Manufacturing (324), Non-Metallic Mineral Product Manufacturing (327), Primary Metal Manufacturing (331)
4	Secondary product manufacturing	Chemical Manufacturing (325), Plastics and Rubber Products Manufacturing (326), Fabricated Metal Product Manufacturing (332)
5	Capital intensive tertiary manufacturing	Printing and Related Support Activities (323), Machinery Manufacturing (333), Computer and Electronic Product Manufacturing (334), Electrical Equipment, Appliance and Component Manufacturing (335), Transportation Equipment Manufacturing (336)
6	Construction	Prime Contracting (231), Trade Contracting (232), Construction of Buildings (236), Heavy and Civil Engineering Construction (237), Specialty Trade Contractors (238)
7	Transportation, warehousing, wholesale	Farm Product Wholesaler-Distributors (411), Petroleum Product Wholesaler-Distributors (412), Food, Beverage and Tobacco Wholesaler-Distributors (413), Personal and Household Goods Wholesaler-Distributors (414), Motor Vehicle and Parts Wholesaler-Distributors (415), Building Material and Supplies Wholesaler-Distributors (416), Machinery, Equipment and Supplies Wholesaler-Distributors (417), Miscellaneous Wholesaler-Distributors (418), Wholesale Agents and Brokers (419), Air Transportation (481), Rail Transportation (482), Water Transportation (483), Truck Transportation (484), Transit and Ground Passenger Transportation (485), Pipeline Transportation (486), Scenic and Sightseeing Transportation (487), Support Activities for Transportation (488), Warehousing and Storage (493)
8	Communication and other utilities	Utilities (221), Postal Service (491), Couriers and Messengers (492), Waste Management and Remediation Services (562)
9	Retail trade and consumer services	Motor Vehicle and Parts Dealers (441), Furniture and Home Furnishings Stores (442), Electronics and Appliance Stores (443), Building Material and Garden Equipment and Supplies Dealers (444), Food and Beverage Stores (445), Health and Personal Care Stores (446), Gasoline Stations (447), Clothing and Clothing Accessories Stores (448), Sporting Goods, Hobby, Book, and Music Stores (451), General Merchandise Stores (452), Miscellaneous Store Retailers (453), Nonstore Retailers (454), Amusement, Gambling, and Recreation Industries (713), Accommodation (721), Food Services and Drinking Places (722), Repair and Maintenance (811), Personal and Laundry Services (812)
10	Finance and insurance	Monetary Authorities - Central Bank (521), Credit Intermediation and Related Activities (522), Securities, Commodity Contracts, and Other Financial Investments and Related Activities (523), Insurance Carriers and Related Activities (524), Funds and Other Financial Vehicles (526)
11	Real estate, rental and leasing operations	Real Estate (531), Rental and Leasing Services (532), Lessors of Nonfinancial Intangible Assets (except Copyrighted Works) (533)
12	Business services	Professional, Scientific, and Technical Services (541), Management of Companies and Enterprises (551), Administrative and Support Services (561)
13	Education and health services	Educational Services (611), Ambulatory Health Care Services (621), Hospitals (622), Nursing and Residential Care Facilities (623), Social Assistance (624), Grantmaking and Giving Services (8132), Social Advocacy Organizations (8133), Civic and Social Organizations (8134), Business, Professional, Labor, Political, and Similar Organizations (8139)
14	Information and cultural industries	Publishing Industries (except Internet) (511), Motion Picture and Sound Recording Industries (512), Broadcasting and Telecommunications (513), Information Services and Data Processing Services (514), Performing Arts, Spectator Sports, and Related Industries (711), Museums, Historical Sites, and Similar Institutions (712)

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