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Unregulated Lending, Mortgage Regulations and Monetary Policy

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Abstract

Macroprudential policies are often aimed at the traditional banking sector while non-depository financial institutions or shadow banks have limited or no prudential regulations. This paper studies the macroeconomic impact of household-side macroprudential tightening in the presence of unregulated lenders. Our result shows that the presence of unregulated lenders dampens the impact of the policies on house prices and household debt. We also find that leakage to the unregulated sector increases when monetary policy is tightened.

Topics: Monetary policy transmission; Financial institutions; Financial system regulation and policies

JEL codes: E44, E50, E52, E58, G21, G23, G28

1 Introduction

In an attempt to enhance the resilience of the financial system, regulators have introduced macroprudential tools geared at the housing market (see Allen et al., 2017; Galati and Moessner, 2012; and Claessens, 2015). The effectiveness of these regulations depends on the extent of regulatory leakage (see Aiyar et al., 2014; Bengui and Bianchi, 2018). In this paper, we propose a dynamic stochastic general equilibrium (DSGE) model with interacting traditional and non-traditional lenders (shadow banks). Given the lack/looser set of regulatory standards, shadow banks are able to lend to consumers previously excluded from the lending market. To the extent, however, that it allows for regulatory arbitrage, there are concerns that shadow banks could alter the scope and consequences of macroprudential policy. Similar to Bengui and Bianchi (2018), our paper addresses the issue of imperfect regulation and the consequences.

To evaluate the effectiveness of macroprudential policies in the presence of asymmetric regulations, we extend Allen and Greenwald (2022) by including financial intermediaries – traditional (regulated) and shadow (unregulated) banks. Both intermediaries engage in similar intermediation activities on the aggregate level but differ in several ways. First, traditional banks are prudentially regulated while shadow banks are unregulated. Second, traditional banks have access to government-supported deposit insurance and central bank lending facilities. From the household perspective, they are less risky. In our model, this perceived riskiness leads to a positive spread between the rates households demand from shadow banks for their deposits, compared to traditional banks. Finally, we treat the traditional banking sector as monopolistically competitive and the shadow banks as a competitive fringe and therefore price takers. Our modeling framework is designed to reflect the Canadian mortgage system, which for the traditional banks is divided between a government-insured sector that offers high loan-to-value (LTV) mortgages (maximum of 95%) but imposes tight payment-to-income (PTI) requirements (maximum ratio of total debt payments to gross income of 44%), and an uninsured sector that uses much tighter LTV limits (maximum of 80%) but leaves PTI ratios essentially unlimited. Shadow banks only have access to an uninsured sector with LTV limits (maximum of 80%) and unlimited PTI. While we focus on Canada, it is worth noting that such segmentation is common and can be seen, for example, in the US, where the Federal Housing Administration (FHA), government-sponsored enterprises (GSEs) and private-label securitizers all employed different underwriting policies during the US housing boom.

Using the model, we analyze and answer questions on the possible limitations of macroprudential policies due to imperfect regulation enforcement. The theoretical literature has so far overlooked a key feature of many mortgage markets, a segmented market, while addressing this issue. Our main contribution is to write down a theoretical model that allows heterogeneity among financial intermediaries and at the same time capture a segmented mortgage market and use the framework to answer questions on policy leakages. We also contribute to the literature by showing how different tightening of policies in different mortgage market segments affects house prices, output and total mortgage origination. Our results show that leakage of macroprudential policy to shadow banks depends on the mortgage submarket the policy is intended for. For instance, tightening of the PTI limit on the insured mortgage does not lead to any policy leakage to the shadow sector; on the other hand, tightening the PTI limit in the uninsured mortgage sector leads to policy leakage to the shadow banking sector.

2 Related Literature

Our work is related to macroeconomic literature that incorporates heterogeneity among financial intermediaries in economic models. In a recent study, Meeks, Nelson and Alessandri (2017) developed a dynamic general equilibrium model in which traditional and shadow banking sectors trade securitized assets; they analyzed the consequences of securitized banking under financial shocks. Gertler et al. (2016) studied how both anticipated and unanticipated bank runs affect shadow banking. Verona et al. (2013) developed a model of shadow banking that shows that in response to an extended period of expansionary monetary policy, incorporating shadow banks increases the magnitude of boom-bust dynamics. Mazelis and Gebauer (2020) studied the effect of tightening commercial bank regulation on the shadow banking sector and found that coordinating the policy tightening with monetary easing can limit policy leakage. Mazelis (2016) studied the relevance of different types of credit on macroeconomic volatility using a model with traditional banks, shadow banks and investment funds and found that regulating shadow banks leads to milder recession when the interest rate is at the zero lower bound. Begenau and Landvoigt (2021) and Fève, Moura and Pierrard (2019) showed how the existence of shadow banks could affect the proposed capital requirements by the Basel framework. They showed that there is leakage of intermediation towards shadow banks and concluded that the existence of shadow banks can dampen the intended stabilizing effect of higher capital requirement for traditional banks. Chen, Ren and Zha (2018) studied how monetary policy in China influences shadow banking activities and found that contractionary monetary policy during 2009-2015 caused rapid rise in shadow banking loans.

Our paper also complements other policy models that analyze the effect of the interaction between macroprudential and monetary policy on the Canadian economy. Alpanda, Cateau and Meh (2018) built a medium-scale small open economy DSGE model to study the effectiveness of macroprudential and monetary policies in reducing household debt using financial shocks. The approach of this paper differs from those of the other papers in the sense that our model features a segmented mortgage market and unregulated lenders.

This paper is also related to studies that consider a broader set of credit frictions consisting of both LTV limits and PTI limits on borrowing. The LTV limits constrain the loans issued to borrowers to be a net of downpayment fraction of the expected future value of the housing, whereas the PTI limits constrain interest payments on the loans to be a specific fraction of the borrowers' labor income. Although the early literature on credit frictions is mostly focused on the effects of LTV limits, several recent papers have also considered PTI limits and their interaction with LTV limits. Among others, Corbae and Quintin (2015) introduced a PTI constraint in their model and used it to explain the housing boom in the US and its relationship to default risk and credit growth. Greenwald (2018) considered random combinations of LTV and PTI constraints in order to study how the structure of the US mortgage market influences macroeconomic dynamics. Kaplan et al. (2020) considered the effect of LTV and PTI credit frictions for the rise and collapse of US housing prices around the Great Recession. Grodecka (2020) showed that in the presence of a PTI constraint, a policy-induced reduction in the LTV ratio may increase rather than decrease housing prices.

This work is also related to papers that studied how monetary policy environment affects mortgage originations of unregulated lenders. Pescatori and Sole (2016) studied the effect of higher interest rate on the US financial system and found evidence that higher interest rate leads to a shift of intermediation to more weakly regulated sector. Den Haan and Sterk (2011) studied the changes in the time series properties of key financial and macro variables and found that following a monetary tightening, bank mortgages declined, while mortgages held by other institutions increased. Duca (2016) studied what drove the long-run and short-run movements in the relative importance of shadow bank funding and found that shadow bank share rose in the short run when deposit interest ceilings were binding on traditional banks.

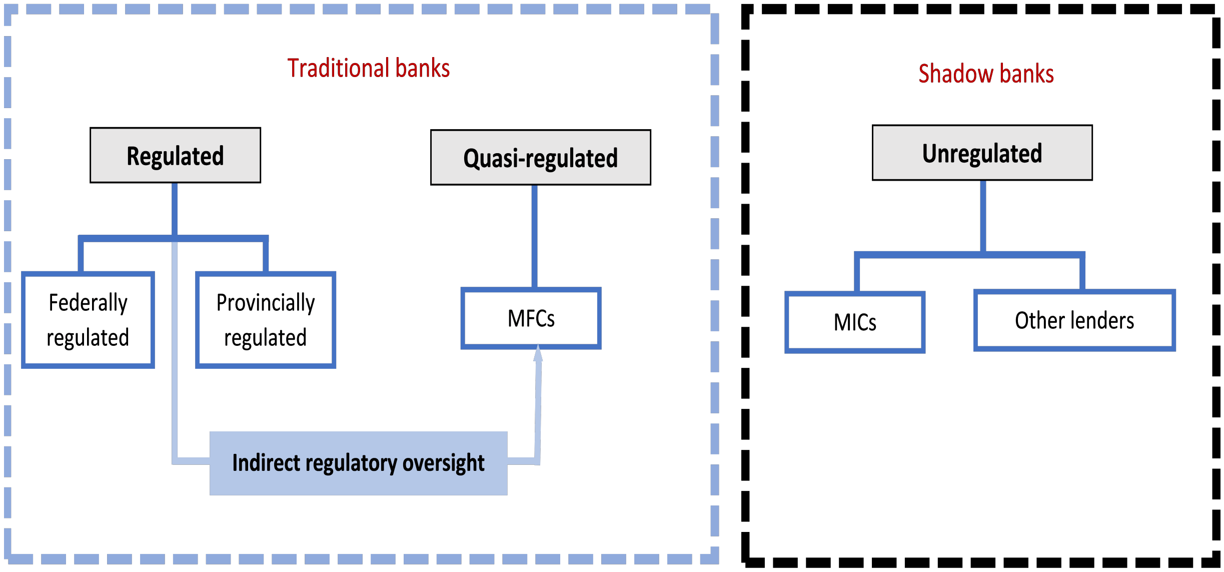
Our paper is closely related to Allen and Greenwald (2022), which, in addition to considering a combination of LTV and PTI limits, included a segmented mortgage market. We extend Allen and Greenwald (2022) by including heterogenous financial intermediaries. None of the papers above studied how tightening credit constraints and monetary policy affect policy leakage to the shadow banks in a segmented mortgage market, which is one of the main contributions of our paper.

3 Background: The Canadian Mortgage Market

3.1 Policy Framework Impact on Residential Mortgage Lending

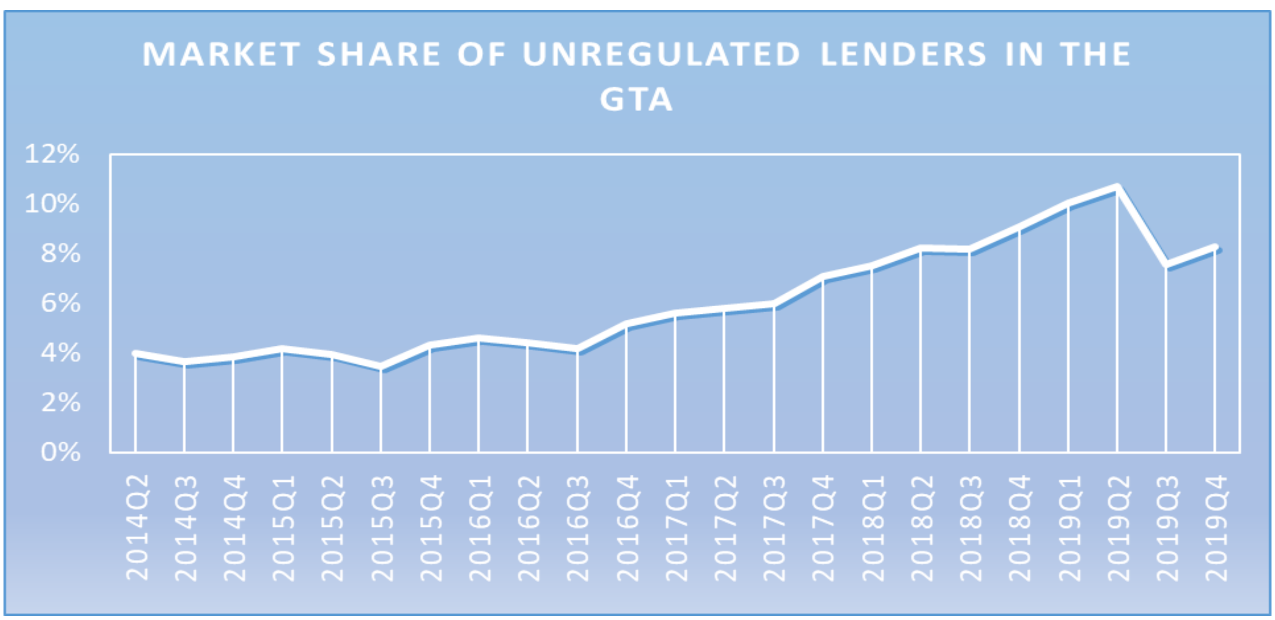
The underwriting standards in Canada are strongly affected by the regulatory and supervisory framework. The federal agency, Office of the Superintendent of Financial Institutions (OSFI), regulates domestic systemically important banks that originate about 74% of mortgages. In addition to federally regulated financial institutions, many of the other institutions that issue mortgages, such as credit unions and caisses populaires, are provincially regulated and are considered regulated lenders. Mortgage underwriting also takes place at institutions that are not directly subject to prudential regulation. Mortgage finance companies (MFCs) are indirectly subject to OSFI regulation because they underwrite insured mortgages that end up either being sold to OSFI-regulated lenders or securitized through National Housing Act mortgage-backed securities (NHA MBS). The shadow banks are made up of the mortgage investment corporations (MICs) and other lenders that issue uninsured and non-conforming mortgages that operate outside the purview of any prudential supervision. Our classification of shadow banks is based on the regulatory framework as shown in Figure 1.[1] The MICs industry experienced huge growth in the last decade; its mortgage lending has steadily increased, reaching $10 billion in 2017 (see Bédard-Pagé, 2019). The federal supervisory framework has supported the resilience of the Canadian mortgage market by implementing a number of measures to ensure that mortgage lenders adopt prudent lending practices; this has led to concerns that the measures could motivate a shift of credit intermediation towards the shadow banking sector.

[1] This figure was adapted from Canada Mortgage and Housing Corporation (2016) and Bédard-Pagé (2019).



Source: Adapted from Bédard-Pagé 2019

Figure 1 Scope of residential mortgage lending included in shadow banks



Source: Teranet Deeds data

Figure 2 Mortgage market share of lenders in the GTA

The market share of mortgages in the Greater Toronto Area (GTA) from all lender types and for unregulated lenders is shown in Figure 2. Our model captures how regulations and the monetary policy environment affect the shift of credit intermediation to unregulated lenders.

3.2 Qualifying LTV and PTI Ratios

The Canadian mortgage market is divided into two sectors. First, there is the "insured" sector, in which default insurance on mortgages is mandatory and guaranteed by the government. This sector allows borrowers to obtain loans with up to 95% LTV ratios but restricts them to a maximum PTI ratio of 44%.[2] The insurance is paid by the borrowers at the point of origination and is rolled into the mortgage. Second, there is an "uninsured" sector, in which lenders face default risk.

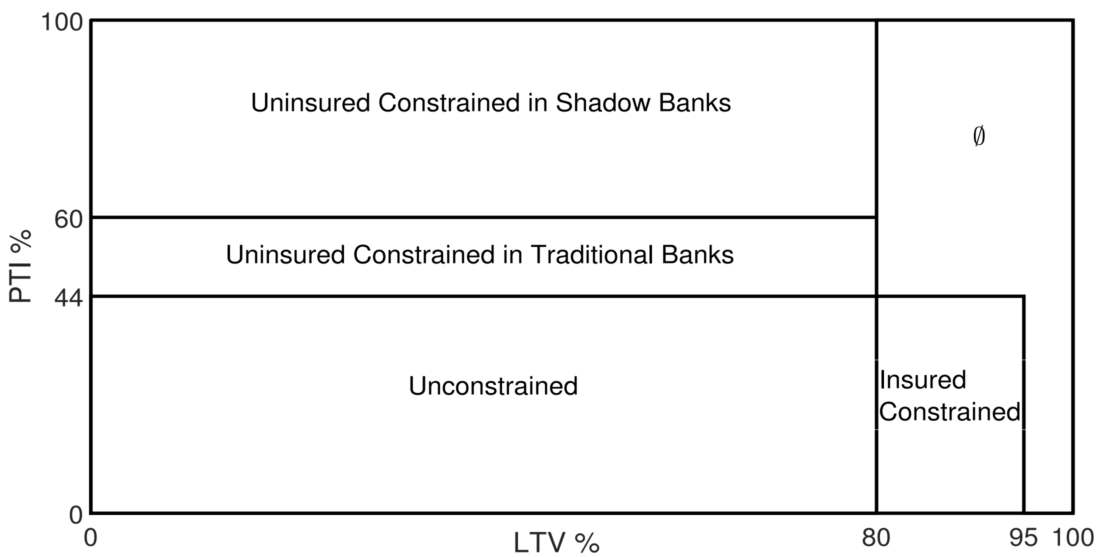
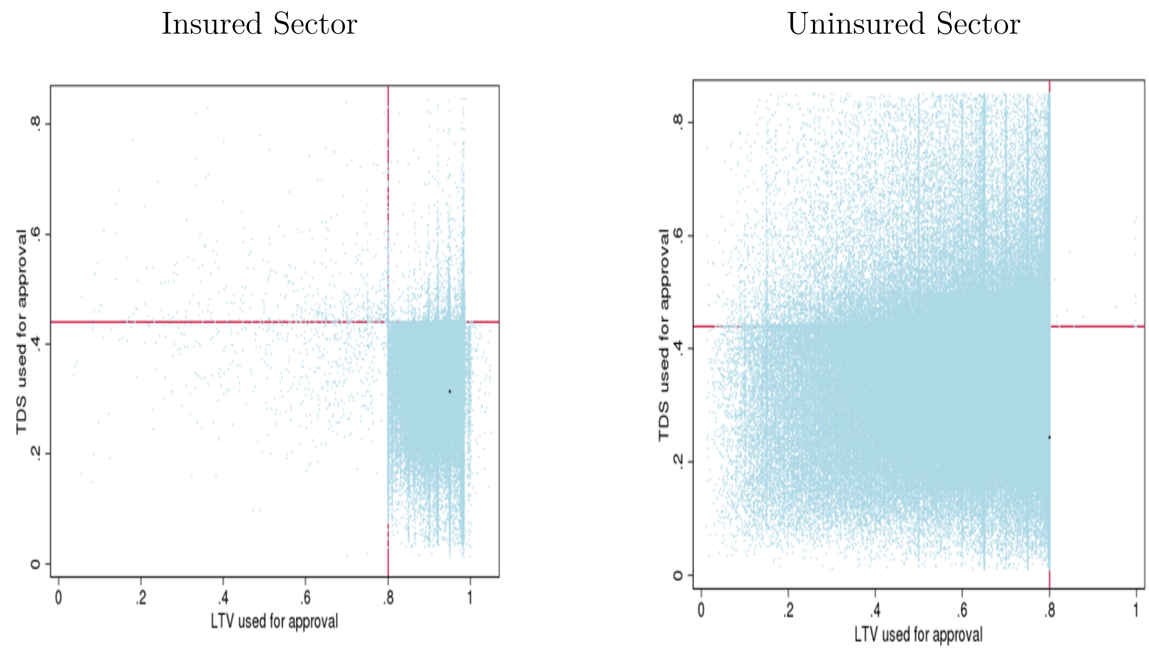


Figure 3 Borrowing constraints by sector

Mortgages in this sector are required to have an LTV ratio below 80% but do not have any formal cap on PTI limits. In Canada, non-banking financial intermediaries, commonly referred to as shadow banks, provide an alternative to banks for mortgage loans, but some of these shadow banks, such as MICs, are not prudentially regulated. The resulting system is captured by Figure 3. The shadow banks have only an LTV constraint and no PTI limits. To make sure that this definition fits with real-world behavior, Figure 4 displays the LTV and PTI (TDS) ratios for new purchase loans by sector in the traditional banks.

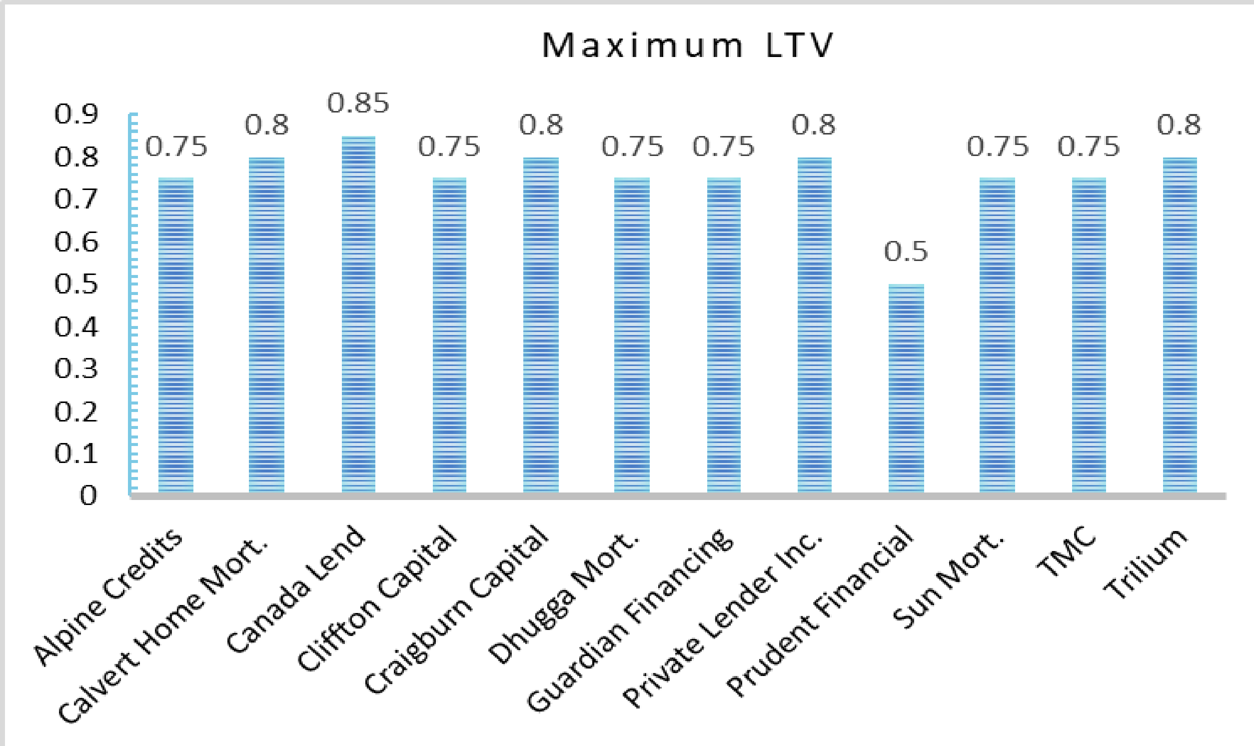
[2] PTI limit is also known as total debt service (TDS) limit.



Source: Allen and Greenwald 2022

Figure 4 Qualifying LTV and TDS ratios for traditional banks

Corresponding almost perfectly with our definitions in 3, these data confirm that nearly all high-LTV borrowers go to the insured sector; among these, extremely few surpass the typical 44% PTI limit. Similarly, nearly all high-PTI borrowers go to the uninsured sector; among these, virtually none take on more than 80% LTV, but PTI limit varies widely. Figure 5 displays the qualifying LTV for new mortgage loans for select shadow banks in Canada. The patterns in Figures 3 and 5 therefore validate our stylized definition of the three sectors, allowing us to proceed with construction of the model.



Source: Wowa.ca

Figure 5 Qualifying LTV ratios for some shadow banks in Canada

4 Model

This section constructs the modeling framework. We take the framework of Greenwald (2018) and Allen and Greenwald (2022), with the main innovations appearing in the treatment of the mortgage submarkets and inclusion of banking sectors. Income heterogeneity sorts borrowers into different mortgage markets and lender types.

Demographics. The economy is populated by two families of households, borrowers and savers, who are denoted by the subscripts b and s respectively. Households are infinitely lived and types are permanent, with fixed measures χ_b of borrowers and $\chi_s = 1 - \chi_b$ of savers. Both types of agent supply perfectly substitutable labor. The model features traditional and shadow banks that intermediate funds between savers and borrowers by lending in fixed-rate mortgages to borrowers. Traditional banks intermediate between savers and borrowers by raising term deposits, d_t, in the saver market and issuing insured and uninsured mortgages, m_{jt}^{TB}, in the borrower market. Shadow banks also issue uninsured mortgages; these are financed via their net worth, nw_t^{SB}, and deposits, s_t^{SB}. Shadow banks' ability to get external financing is constrained by a moral hazard problem that limits the willingness of creditors to provide funding; this leads to positive spread between the rates savers demand from shadow banks, for their deposits, compared to traditional banks.

Preferences. To capture the value of liquidity services produced by bank deposits, we follow Begenau and Landvoigt (2021) and assume that in addition to consumption and housing, savers derive utility from bank deposits. They maximize expected utility of the form

$$V_s = \mathbb{E}_t \sum_{k=0}^{\infty} \beta_s^k u(c_{s,t+k}, h_{s,t+k}, d_{t+k}, n_{s,t+k}), \tag{1}$$

where the inputs to the utility function are nondurable consumption c, housing services h, labor supply n and deposits at banks d. Borrowers maximize expected utility of the form

$$V_b = \mathbb{E}_t \sum_{k=0}^{\infty} \beta_b^k u(c_{b,t+k}, h_{b,t+k}, n_{b,t+k}) \tag{2}$$

and have preference over nondurable consumption c, housing services h and labor supply n.

Mortgages. The mortgage sector consists of two submarkets: one for government-insured mortgages, denoted I, and one for uninsured mortgages, denoted U. Borrowers obtaining new loans choose freely which submarket they prefer to enter. A mortgage in sector j is a nominal perpetuity with a fixed interest rate and geometrically decaying coupon. That is, a borrower in submarket j pays back a constant fraction ν_j of the outstanding principal in each period, so that the payment at time $t+h$ on \$1 of debt issued at time t is $\$(1-\nu_j)^h(r_t^* + \nu_j)$ for all h until the loan is renewed, where r_t^* is the fixed interest rate. A mortgage in submarket j is renewed each period with probability ρ, at which time the borrower prepays her existing balance and can take out a new loan. To induce changes in real mortgage rates similar to shifts in mortgage spreads or term premia, we introduce a proportional tax $\Delta_{q,t}$ on all future mortgage payments received by the saver on loans in submarket j originated at time t, subject to the process

$$\Delta_{q,t} = (1-\phi_q)\mu_q + \phi_q \Delta_{q,t-1} + \varepsilon_{q,t}. \tag{3}$$

This tax does not correspond to any real-world policy but is instead a parsimonious way to create a wedge between long rates and average future short rates, which is needed to match the large and volatile discrepancy between these two rates in the data. As a result, we rebate the proceeds from the tax back to the savers in lump-sum fashion each period. The size of a new loan, for borrower i, from a traditional bank is limited by both an LTV constraint and a PTI constraint, defined by

$$\frac{m_{i,t}^{*TB}}{p_t^h h_{i,t}^*} \leq \theta_{j,t}^{LTV_{TB}}, \qquad \frac{(r_t^* + \nu_j + \alpha) m_{i,t}^{*TB}}{w_t n_{i,t} e_{i,t}} + \omega \leq \theta_{j,t}^{PTI_{TB}}$$

respectively, $j \in (I, U)$. The loan from the shadow banks is only limited by the LTV constraint:

$$\frac{m_{i,t}^{*SB}}{p_t^h h_{i,t}^*} \leq \theta_{j,t}^{LTV_{SB}}.$$

The LTV constraint caps the ratio of the balance on the new loan $m_{i,t}^{*k}$, $k \in \{SB, TB\}$, against that borrower's housing collateral $p_t^h h_{i,t}^*$, where $h_{i,t}^*$ is the quantity of newly purchased housing. These constraints are applied at origination only. The PTI constraint caps the ratio of the borrower's debt and related payments to her income. The numerator on the left-hand side is the initial mortgage payment, where the offset term α is used in the calibration to adjust for property taxes and insurance payments, as well as to adjust for the difference in amortization between true fixed-rate mortgages and the model's geometrically decaying coupon loans. The denominator is equal to labor income, which is shifted for each borrower by an idiosyncratic income shock $e_{i,t}$, [3] drawn i.i.d. across borrowers and time from a distribution with c.d.f. Γ_e. Due to our assumption that markets are complete within the borrower family, this income shock has no impact on borrower consumption allocations but is instead used to create variation among borrowers, allowing for endogenous sorting into the two sub-markets and ensuring that endogenous fractions are limited by each constraint. Finally, ω is used in the calibration to adjust for debt obligations other than mortgages. For insured mortgages, the LTV and PTI limits are the same irrespective of the type of lending institution, and the mortgage rate charged by the two types of lending institution are the same. The mortgage rate affects the new-loan size a borrower can get based on the PTI limit in the traditional banks. For uninsured mortgages, the LTV limit is the same irrespective of lender type. These constraints imply the following maximum loan balances,

$$\bar{m}_{i,j,t}^{LTV_k} = \theta_{j,t}^{LTV_k} p_t^h h_{i,t}^*, \qquad \bar{m}_{i,j,t}^{PTI_{TB}} = \frac{(\theta_{j,t}^{PTI_{TB}} - \omega) w_t n_{i,t} e_{i,t}}{r_t^* + \nu_j + \alpha},$$

which define the maximum loan that borrower i can obtain in submarket j under each limit.

[3] The shock $e_{i,t}$ could stand for any shock that varies the ratio of house price to income from the lender's perspective.

Since both limits must be satisfied simultaneously, the maximum loan balance in submarket j from a traditional bank is defined by $\bar{m}_{i,j,t} = \min\left(m_{i,j,t}^{LTV_{TB}}, \bar{m}_{i,j,t}^{PTI_{TB}}\right)$.

At equilibrium, the optimal policy will be to choose the insured space, which has looser income-based constraints and tighter collateral-based constraints, if and only if $e_{i,t}$ exceeds an endogenous threshold e_t^*. We define $F_{j,t}$ to be the fraction of borrowers who choose sector j, so that $F_{U,t} = \Gamma_e(e_t^*)$ and $F_{I,t} = 1 - \Gamma_e(e_t^*)$.

Unregulated Lender Cost. We have assumed, for tractability purposes, that mortgage rates are the same for both shadow and traditional banks. Without additional heterogeneity, the model will not be able to generate a time-varying fraction of borrowers that take out a loan from the traditional banks. Given that borrowing restrictions are less stringent in the shadow bank sector, if rates are equalized across sectors, the demand for mortgages from traditional banks would be zero. We therefore impose a cost to borrowers from taking out a shadow bank mortgage. We assume that if agent i borrows from shadow banks, he pays a cost proportional to his income shock, $e_{i,t}$. This cost represents mortgage origination and broker fees and search cost. It captures the idea that some uninsured mortgages could not be renewed with regulated lenders, and these borrowers move to unregulated lenders at a cost. The optimal policy will be for all borrowers with realized income shock $e_{i,t} < \bar{e}_t^{SB}$ to choose a shadow bank loan, where $\bar{e}_t^{SB}$ is the threshold value of the shock for which borrowers are indifferent between getting a loan from the shadow or the traditional banks.

Mortgage Default. Borrowers can default on their mortgages, and the default rate for mortgages taken from traditional banks is lower than the default rate on loans from shadow banks. This captures the fact that households that borrow from shadow banks have a riskier profile in the data. The default rate is included to recognize the fact that loan repayment changes over time in a way that depends on the aggregate state of the macroeconomy. In each period, the proportion of loan repaid by borrowers is ϖ_t^{SB} and ϖ_t^{TB} for loans taken from shadow and traditional banks, respectively. These proportions are endogenous; following Agenor et al. (2012), we assume that the probabilities enter the model in reduced form and depend on an output gap, a measure of the inverse leverage gap, and an exogenous shock interpreted as a shock to the financial fragility of the debtors:

$$\varpi_t^{SB} = \varpi_0^{SB}\left(\frac{y_t}{\tilde{y}}\right)^{\phi_y}\left(\frac{p_t^h h_{i,t}^*}{m_t^{*SB}} / \frac{\tilde{p}^h \tilde{h}^*}{\tilde{m}^{*SB}}\right)^{\phi_h} exp(\varepsilon_t^{SB}),$$

$$\varpi_t^{TB} = \varpi_0^{TB}(\frac{y_t}{\tilde{y}})^{\phi_y}(\frac{p_t^h h_{i,t}^*}{m_t^{*TB}}/\frac{\tilde{p}^h\tilde{h}^*}{\tilde{m}^{*TB}})^{\phi_h} exp(\varepsilon_t^{TB}),$$

where ϖ_0^{SB} and ϖ_0^{TB} are steady-state values of repayment probabilities for loans originating from shadow and traditional banks, respectively. A tilde over a variable denotes its steady-state value, and ϕ_y and ϕ_h are fixed parameters greater than zero. The shocks ε_t^{SB} and ε_t^{TB} follow an AR(1) process with a common persistence parameter ρ_ε. The output gap captures the view that in a period of low levels of economic activity, the incentive to default increases. The inverse leverage gap relates the repayment probability to a borrower's net worth; it increases with the effective collateral provided by borrowers and decreases with the amount borrowed.

Monetary Policy. Monetary policy follows Greenwald (2018), and the Taylor rule is of the form

$$\log R_t = \log \bar{\pi}_t + \phi_r (\log R_{t-1} - \log \bar{\pi}_{t-1}) + (1-\phi_r)\Big[(\log R_{ss} - \log \pi_{ss}) + \psi_\pi(\log \pi_t - \log \bar{\pi}_t)\Big],$$

where $\bar{\pi}_t$ is a time varying inflation target defined by

$$\log \bar{\pi}_t = (1-\psi_{\bar{\pi}}) \log \pi_{ss} + \psi_{\bar{\pi}} \log \bar{\pi}_{t-1} + \epsilon_{\bar{\pi},t}.$$

Macroprudential Shocks. To allow experimentation with macroprudential policies, the maximum LTV and PTI ratios for each submarket are allowed to vary. In particular, we specify the following AR(1) processes to generate time-variation in debt limits:

$$\begin{aligned}
\log \theta_{j,t}^{LTV_{TB}} &= (1-\phi_\theta) \log \bar{\theta}_j^{LTV_{TB}} + \phi_\theta \theta_{j,t-1}^{LTV_{TB}} + \varepsilon_{j,t}^{LTV_{TB}}, \\
\log \theta_{U,t}^{LTV_{SB}} &= (1-\phi_\theta) \log \bar{\theta}_U^{LTV_{SB}} + \phi_\theta \theta_{U,t-1}^{LTV_{SB}} + \varepsilon_t^{LTV_{SB}}, \\
\log \theta_{j,t}^{PTI_{TB}} &= (1-\phi_\theta) \log \bar{\theta}_j^{PTI_{TB}} + \phi_\theta \theta_{j,t-1}^{PTI_{TB}} + \varepsilon_{j,t}^{PTI_{TB}}.
\end{aligned}$$

Housing. The final asset in the economy is housing, which is divisible in fixed total supply $\bar{H}$ and produces a service flow equal to its stock.

Taxation. Each household's labor income is subject to proportional taxation at rate τ_y. Tax revenues are returned to borrowers and savers in lump-sum transfers equal to the average amount paid by that type.

Financial Assets. We introduce a risk-free one-period bond that can be used as a policy instrument by the central bank. An investment of \$1 at time t yields a guaranteed nominal payoff of $\$R_t$ at time $t+1$. This bond is in zero net supply and cannot be shorted, which means that it will be held by savers only at equilibrium. In order to finance their assets, traditional banks issue one-period nominal deposits and bank bonds to savers. Similar to Gertler and Karadi (2011) and Polo (2021), we assume that the bank's bonds are not perfectly substitutable with government bonds due to a portfolio adjustment cost. Since large banks are known to be not fully deposit-funded, we assume that only non-negative holding of bonds are admissible. Savers invest in shadow bank securities at a higher interest rate than bank bonds because of perceived risk due to lack of regulation.

4.1 Representative Saver's Problem

The savers deposit funds in traditional banks and invest in government and bank bonds. The individual saver's problem aggregates to that of a representative saver. The representative saver chooses nondurable consumption $c_{s,t}$, labor supply $n_{s,t}$, holding of government bonds b_t^g, holding of banks' bonds b_t, investment in shadow bank's securities s_t^{SB}, bank deposit d_t, and capital k_t, which is rented to firms and depreciates at the rate δ_k to maximize a period utility that takes the separable form

$$u_s(c_{s,t}, h_{s,t}, n_{s,t}, d_t) = \Gamma_s^c \log\left(\frac{c_{s,t} - \varepsilon_c c_{s,t-1}}{\chi_s}\right) + \xi \log\left(\frac{h_s}{\chi_s}\right) + \xi_d \log\left(\frac{d_t}{\chi_s}\right) - \eta_s \frac{n_{s,t}^{1+\varphi}}{1+\varphi} \tag{4}$$

subject to the budget constraint

$$\begin{aligned} c_{s,t} \leq\ & \underbrace{(1-\tau_y) w_t n_{s,t}}_{\text{labor income}} - \underbrace{\delta p_t^h h_s}_{\text{maintenance}} - \underbrace{(b_t^g - \frac{R_{t-1} b_{t-1}^g}{\pi_t})}_{\text{net govt. bond}} \\ & - \underbrace{(b_t - \frac{R_{t-1}^b b_{t-1}}{\pi_t})}_{\text{net bank's bond}} + \underbrace{\Xi_t}_{\text{profits}} + \underbrace{T_{s,t}}_{\text{transfers}} - \underbrace{i_t}_{\text{investment}} \\ & + \underbrace{r_{k,t} k_{t-1}}_{\text{capital rent}} - \underbrace{(d_t - \frac{R_{t-1}^d d_{t-1}}{\pi_t})}_{\text{net deposit TB}} - \underbrace{(s_t^{SB} - \frac{R_{t-1}^s s_{t-1}^{SB}}{\pi_t})}_{\text{net invest. SB}} - \underbrace{f(b_t, m_t^{TB})}_{\text{port. adj. cost}}, \end{aligned}$$

where $r_{k,t}$ is the real rental rate of capital, s_t^{SB} is the investment in shadow banks, Ξ_t are profits from bank dividends and intermediate firm profits, and $T_{s,t}$ are rebate saver taxes. Habit in consumption is measured by ε_c, and Γ_s^c is a scaling factor that ensures that the marginal utilities of consumption are independent of habits in the stochastic steady state ($\Gamma_s^c = \frac{1-\varepsilon_c}{1-\beta_s\varepsilon_c}$).

Investment and capital are linked as follows:

$$k_t = i_t \left(1 - \frac{\phi}{2}(i_t/i_{t-1} - 1)^2\right) + (1 - \delta_k)k_{t-1}, \tag{5}$$

Following Gebauer and Mazelis (2020), we assume a non-negative spread between the interest rates, R_t^s, earned on shadow banks' investment, and on the rates, R_t^d, earned on deposits that savers can place with traditional banks. This spread is determined by the parameter τ^s, and the relationship between the rates is specified as follows:

$$1 + r_t^s = \frac{1 + r_t^d}{1 - \tau^s \epsilon_t^{\tau^s}}, \qquad\qquad 0 < \tau^s < 1,$$

where $R_t^s = 1 + r_t^s$ and $R_t^d = 1 + r_t^d$. The positive spread is as a result of higher probability of default for shadow banks; we follow Mazelis and Gebauer (2020) to assume an existence of a spread shock which captures exogenous fluctuations in the interest rate spread. The shock, $\epsilon_t^{\tau^s}$, follows an auto-regressive process.

Denoting the saver's stochastic discount factor as

$$\Lambda_{t,t+1}^s = \beta_s \frac{u_{s,t+1}^c}{u_{s,t}^c},$$

the first-order condition for government bond holdings is the standard Euler equation

$$1 = R_t \mathbb{E}_t \left[\Lambda_{t,t+1}^s \pi_{t+1}^{-1}\right].$$

The Euler equation for bank bonds is

$$\mathbb{E}_t \frac{\Lambda_{t,t+1}^s}{\pi_{t+1}} (R_t^b - R_t) = f'(b_t, m_t^{TB}),$$

with a positive value for $f'(b_t, m_{TB})$ in the deterministic steady state. The financial friction captures in reduced form that savers are not willing to hold any amount of banks' bonds at the risk-free rate because of rollover risk concerns, where the portfolio adjustment cost is of the form

$$f(b_t, m_t^{TB}) = \frac{\theta_b}{2} \left(\frac{b_t}{m_t^{TB}} - \nu_b\right)^2 m_t^{TB}.$$

The saver's problem also yields a Euler equation for deposits at a bank

$$\mathbb{E}_t \frac{\Lambda^s_{t,t+1}}{\pi_{t+1}}(R_t - R^d_t) = \frac{u^d_{s,t}}{u^c_{s,t}},$$

which sets the marginal cost of holding deposits at a traditional bank equal to its marginal benefit in equilibrium.

4.2 Traditional Banks

Traditional banks (TBs) are owned by savers. TBs engage in duration transformation (Begenau et al. (2021) and Polo (2021)). We capture this core feature by assuming that traditional banks finance their investments in fixed-rate mortgages issued to borrowers in the past and not yet prepaid, as well as new mortgages issued to borrowers in t, by borrowing in one-period deposits, d_t, from savers. The representative TB enters period t with total principal on outstanding mortgage m^{TB}_{t-1} and total payments to be collected from borrowers on outstanding mortgages x^{TB}_{t-1}. Note that we did not use the TB superscript on the total payment on outstanding mortgages in the insured sector, $x_{I,t}$, because we only have insured mortgages in the TBs. Let ρ be the fraction of mortgages prepaid in period t, and considering that a fraction $\nu_j,\ \ j \in \{I, U\}$ of outstanding principal in sector j is repaid in each period by borrowers, the total value of mortgages that the bank has to finance in period t in each of the sub-markets are

$$m^{TB}_{U,t} = \rho m^{*TB}_{U,t} + (1-\rho)(1-\varpi^{TB}_t \nu_U) m^{TB}_{U,t-1}\pi^{-1}_t \tag{6}$$

$$m_{I,t} = \rho m^*_{I,t} + (1-\rho)(1-\varpi^{TB}_t \nu_I) m_{I,t-1}\pi^{-1}_t \tag{7}$$

and the laws of motion for mortgage payments are

$$x^{TB}_{U,t} = \rho(r^*_t - \Delta_{q,t}) m^{*TB}_{U,t} + (1-\rho)(1-\varpi^{TB}_t \nu_U) x^{TB}_{U,t-1}\pi^{-1}_t \tag{8}$$

$$x_{I,t} = \rho(r^*_t - \Delta_{q,t}) m^*_{I,t} + (1-\rho)(1-\varpi^{TB}_t \nu_I) x_{I,t-1}\pi^{-1}_t. \tag{9}$$

The total mortgages and total payment made to the TBs are

$$m^{TB}_t = m_{I,t} + m^{TB}_{U,t} \tag{10}$$

$$x^{TB}_t = x_{I,t} + x^{TB}_{U,t}, \tag{11}$$

where r_t^* is the rate on new mortgages originating at time t. The balance sheet of the bank requires that in each period, it collects enough deposits and bonds to finance its book of mortgages.

A TB, in each period, uses $m_{U,t}^{*TB}, m_{I,t}^*$ and d_t to maximize the expected discounted value of net dividend paid to savers:

$$\underbrace{\max}_{m_t^{*TB}, s_t^{TB}, d_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t+1}^s div_{t+1}$$

$$div_{t+1} = \pi_{t+1}^{-1} \left(x_{U,t}^{TB} + x_{I,t} - \varpi_{t+1}^{TB} \nu_U m_{U,t} - \varpi_{t+1}^{TB} \nu_I m_{I,t} - (r_t^d + \kappa) d_t - r_t^b b_t \right) - f(div_{t+1})$$

$$f(div_t) = \frac{\theta^{div}}{2} \left(div_t - \bar{div} \right)^2,$$

subject to the laws of motion (6), (7), (8), (9) and bank balance sheet ($m_t^{TB} = d_t + b_t$). The term x_{t-1}^{TB} is the interest income earned on book of mortgages issued by TBs. The TBs also pay interest to savers on deposits at the rate $R_t^d = (1 + r_t^d)$. They incur a marginal cost κ in offering one dollar of deposit. Following Jermann and Quadrini (2012), Begenau (2020) and Polo (2021), we include $f(div_t)$ as a cost to paying dividends, where $\bar{div}$ is the target level of dividend corresponding to the steady-state level of dividends.

4.3 Shadow Banks

Shadow banks (SBs) consist of differentiated business entities that engage in financial intermediation. We assume shadow banks operate under perfect competition. They issue security s_t^{SB} in order to fund mortgages. To capture the dependence of shadow banks on market funding, we draw on the incentive constraint in Gertler and Karadi (2011) and assume that a lack of regulation means that shadow banks are constrained by a standard moral hazard problem which limits the willingness of savers to provide funding. We assume that the lack of regulation is similar to the risk that shadow banks can divert a share of funds, defaulting on the remaining liabilities whenever the benefit from doing so exceeds the returns from behaving honestly. They have an incentive to exit the market and leave investors with losses on their investments. Savers are aware of this risk and limit their funding to the amount that motivates the shadow banks to continue operations rather than defaulting. Like traditional banks, the representative shadow bank enters period t with total principal on outstanding mortgage m_t^{SB} and total payments to be collected from borrowers on outstanding mortgages x_{t-1}^{SB}. The law of motion for principal and mortgage payments is respectively

$$m_{U,t}^{SB} = \rho m_{U,t}^{*SB} + (1-\rho)(1-\varpi_t^S \nu_U) m_{U,t-1}^{SB} \pi_t^{-1} \tag{12}$$

$$x_{U,t}^{SB} = (r_t^* - \Delta_q)\rho_t m_{U,t}^{*SB} + (1-\rho)(1-\varpi_t^{SB}\nu_U) x_{U,t-1}^{SB} \pi_t^{-1}. \tag{13}$$

We assume that shadow banks do not issue insured mortgages, since insured mortgages are regulated through securitization. The shadow bank's balance sheet is given by

$$m_{U,t}^{SB} = m_t^{SB} = s_t^{SB} + nw_t^{SB}, \tag{14}$$

where nw_t^{SB} is the shadow bank's net worth.
The net worth of shadow banks is specified as follows:

$$nw_t^{SB} = (1 + r_{t-1}^*)\varpi_t^{SB} m_{U,t-1}^{SB} - (1 + r_{t-1}^s) s_{t-1}^{SB}, \tag{15}$$

where $(1+r_{t-1}^*)m_{t-1}^{SB}$ is the interest earned on issued mortgages and $(1+r_{t-1}^s)s_{t-1}^{SB}$ is the interest paid to the holders of their securities. The difference between the real earnings on mortgage loans and real interest payment to creditors determine the evolution of bank capital:

$$nw_{t+1}^{SB} = (1 + r_t^*)\varpi_{t+1}^{SB} m_t^{SB} - (1 + r_t^s) s_t^{SB} = (r_t^* - r_t^s)\varpi_{t+1}^{SB} m_t^S + (1 + r_t^s) nw_t^S. \tag{16}$$

Shadow banks will exist as long as bank real return on lending $(r_t^* - r_t^s)$ is positive; if not, it exits the shadow banking sector. Each shadow bank has a survival probability γ^S with which it operates in the next period. Thus the shadow bank's objective is to maximize expected terminal wealth W_t:

$$W_t = \max \mathbb{E}_t \sum_{k=0}^{\infty} (1-\gamma^S)\gamma^{S^k} \beta^{S^{k+1}} nw_{t+1+k}^{SB}. \tag{17}$$

We introduce a moral hazard problem by allowing for the possibility that shadow banks divert a fraction of available funds, θ^S, for private benefit. Diverting funds and exiting the market is equivalent to declaring bankruptcy; the shadow banks will do so only if the return of declaring bankruptcy is larger than the discounted future return of behaving honestly and continuing:

$$W_t \geq \theta^S m_{U,t}^{SB}. \tag{18}$$

The shadow bank will lose W_t if it diverts a fraction of the asset but will gain $\theta^S m_{U,t}^{SB}$ if it does so. Following Gertler and Karadi (2011), we can write (18) as

$$W_t = v_t m_{U,t}^{SB} + \eta_t^S n_{w,t}^{SB}, \tag{19}$$

where v_t is the expected discounted marginal gain to expand $m_{U,t}^{SB}$ by a unit, holding net worth constant, and η_t^S is the expected discounted value of having another unit of $n_{w,t}^{SB}$.

4.4 Borrower's Problem

Members of each borrowing household are ex ante identical but heterogeneous following a random shock to their labor income. Similar to Greenwald (2018), the income shock serves to induce heterogeneity among borrowers such that an endogenous fraction is limited by each constraint in equilibrium. The idiosyncratic income shock determines a borrowers lender type and which mortgage submarket to sort into. To simplify the model, we follow Greenwald (2018) and assume that there exists some implicit financial arrangement to insure against realizations to the idiosyncratic income shock. Due to this assumption of perfect insurance within the borrower family, the problem of the borrowers aggregates to that of a representative borrower. When the realized income shock is low, the borrower obtains loans from the shadow banks; if the shock reaches a certain threshold, the borrower is better off obtaining a loan from the traditional bank. This is because the borrowing constraints are less stringent in the shadow banks, as shadow bank borrowers are only constrained by the LTV limits and have no PTI constraint. However, if a borrower takes out a shadow bank loan, he pays a cost proportional to the realized shock $e_{i,t}$. In each period, the proportion of loans repaid by borrowers is ϖ_t^{SB} and ϖ_t^{TB} for loans taken from shadow and traditional banks, respectively. These proportions are endogenous. Borrowers with income shock $e_{i,t} < \bar{e}_{U,t}^S$ borrow from the shadow bank, where $\bar{e}_{U,t}^S$ is the threshold value of the shock such that borrowers are indifferent between getting a loan from either a shadow bank or a traditional bank. A representative borrower chooses non-durable consumption $c_{b,t}$, labor supply $n_{b,t}$, the size of newly purchased houses $h_{b,t}^*$ and the face value of newly issued mortgages in each sector $m_{j,t}^{*k}$ to maximize

$$u_j(c_{b,t}, h_{b,t}, n_{b,t}) = \Gamma_b^c \log\left(\frac{c_{b,t} - \varepsilon_c c_{b,t-1}}{\chi_b}\right) + \xi \log\left(\frac{h_{b,t}}{\chi_b}\right) - \eta_b \frac{n_{b,t}^{1+\varphi}}{1+\varphi}, \tag{20}$$

subject to the budget constraint

$$
\begin{aligned}
c_{b,t} \leq \; & \underbrace{(1-\tau_y)w_t n_{b,t}}_{\text{labor income}} - \underbrace{\rho p_t^h(h_{b,t}^* - h_{b,t-1})}_{\text{housing purchase}} + \underbrace{\sum_{I,U}\{\rho \sum_{k=SB,TB}(m_{j,t}^{*k} - (1-\varpi_t^k \nu_j)\pi_t^{-1} m_{j,t-1}^k)}_{\text{net new debt issuance}} \\
- \; & \underbrace{\pi_t^{-1}\sum_{k=SB,TB}\varpi_t^k x_{jb,t-1}^k}_{\text{interest payment}} - \underbrace{\pi_t^{-1}\sum_{k=SB,TB}\varpi_t^k \nu_j m_{j,t-1}^k\}}_{\text{principal payment}} - \underbrace{\delta p_t^h h_{b,t-1}}_{\text{maintenance}} + \underbrace{T_{b,t}}_{\text{transfers}} \\
+ \; & \underbrace{\rho \Psi_t^{SB}}_{\text{unregulated lender cost}}
\end{aligned}
$$

and the aggregate borrowing constraints

$$
\begin{aligned}
m_{U,t}^* &\leq \bar{m}_{U,t}^{LTV}\Gamma_e(\bar{e}_{U,t}^{SB}) + \int_{\bar{e}_{U,t}^{SB}}^{\bar{e}_{U,t}^{TB}} \bar{m}_{U,t}^{PTI_{TB}} d\Gamma_e(e_i) + \bar{m}_{U,t}^{LTV_{TB}}(\Gamma_e(e_t^*) - \Gamma_e(\bar{e}_{U,t}^{TB})) \\
m_{I,t}^* &\leq \int_{e_t^*}^{\bar{e}_{I,t}} \bar{m}_{I,t}^{PTI} d\Gamma_e(e_i) + \bar{m}_{I,t}^{LTV}(1 - \Gamma_e(\bar{e}_{I,t}))
\end{aligned}
$$

and Ψ_t^{SB} is the aggregate value of the unregulated lender cost given by

$$
\Psi_t^{SB} = \Psi_1 \int_0^{\bar{e}_t^{SB}} (e - \bar{e}_{U,t}^{SB}) d\Gamma_e - \Psi_2.
$$

Since the interest rates are the same, the borrower will sort into the submarket where he gets the maximum loan. Therefore the threshold values of the shock are

$$
\begin{aligned}
\bar{e}_{U,t}^{SB} &= \frac{\bar{m}_{U,t}^{LTV_{SB}} - \Psi_2}{\bar{m}_{U,t}^{PTI_{TB}}} \\
\bar{e}_{U,t}^{TB} &= \frac{\bar{m}_{U,t}^{LTV_{TB}}}{\bar{m}_{U,t}^{PTI_{TB}}} \\
\bar{e}_{I,t} &= \frac{\bar{m}_{I,t}^{LTV_{TB}}}{\bar{m}_{I,t}^{PTI_{TB}}}
\end{aligned}
$$

and

$$
e_t^* = \frac{\mu_{U,t}\bar{m}_{U,t}^{LTV_{TB}}}{\mu_{I,t}\bar{m}_{I,t}^{PTI_{TB}}},
$$

where $\mu_{I,t}$ and $\mu_{U,t}$ are multipliers on the debt limit.

The principal balances and promised interest payments for each submarket and lender type and total borrower housing follow the laws of motion:

$$
\begin{aligned}
m^{k}_{U,t} &= \rho m^{*k}_{U,t} + (1-\rho)(1-\varpi^{k}_{t}\nu_{U})\pi^{-1}_{t} m^{k}_{U,t-1} \\
m_{I,t} &= \rho m^{*}_{I,t} + (1-\rho)(1-\varpi^{TB}_{t}\nu_{I})\pi^{-1}_{t} m_{I,t-1} \\
x^{k_b}_{U,t} &= \rho r^{*}_{t} m^{*k}_{U,t} + (1-\rho)(1-\varpi^{k}_{t}\nu_{I})\pi^{-1}_{t} x^{k_b}_{U,t-1} \\
x_{I,t} &= \rho r^{*}_{t} m^{*}_{I,t} + (1-\rho)(1-\varpi^{TB}_{t}\nu_{I})\pi^{-1}_{t} x_{I,t-1} \\
h_{b,t} &= \rho h^{*}_{b,t} + (1-\rho) h_{b,t-1}
\end{aligned}
$$

and the total balance and promised payment are

$$
\begin{aligned}
m^{*}_{U,t} &= \sum_{k=SB,TB} m^{*k}_{U,t} \\
m^{*}_{t} &= m^{*}_{U,t} + m^{*}_{I,t}.
\end{aligned}
$$

4.5 Productive Technology

The productive technology follows the standard New Keynesian assumptions of a competitive final goods producer and a continuum of monopolistic competitive intermediate goods producers. Both types of firm are owned by the saver. The final goods producer solves the static problem

$$
\underbrace{\max P_t}_{y_t(i)} \left(\int y_t(i)^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}} - \int P_t(i) y_t(i) di,
$$

where $y_t(i)$ is the intermediate good produced by firm i, $P_t(i)$ is the price of that good, P_t is the price of the final good and

$$
\left(\int y_t(i)^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}} \tag{21}
$$

is the production function operated by the final goods producer. Profit maximization leads to the following demand function for each intermediate good i:

$$
y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\lambda} Y_t. \tag{22}
$$

Intermediate firms rent capital from savers and hire labour supplied by the two types of households to produce the intermediate goods. They solve

$$
\max \frac{y_t}{p^m_t} - w_t n_t - r_{k,t} k_{t-1}, \tag{23}
$$

where p_t^m is the price markup of final over intermediate goods. The intermediate good producer operates the linear production function

$$y_t = n_t^{(1-\alpha_p)} k_{t-1}^{\alpha_p} \tag{24}$$

to meet the demand of the final good producer, where output is produced with labor and capital; as in the previous sections χ_b measures the share of borrowers, $0 < \alpha_p < 1$ is the share of capital in intermediate good production. Intermediate goods producers are subject to pricing frictions of the Calvo-Yun form, meaning that fraction ζ of firms cannot adjust their price in a given period, while the other $1 - \zeta$ fraction are free to do so.

The intermediate goods firms try to minimize the costs of production and maximize profits.

Firms' Cost Minimization:

The intermediate goods firms are not freely able to adjust prices so as to maximize profit each period but will always act to minimize cost. They minimize total cost subject to the constraint of producing enough to meet demand:

$$\underbrace{\max}_{n_t(i),k_t(i)} \quad w_t n_t(i) + r_{k,t}(i) k_{t-1}(i) \tag{25}$$

subject to

$$n_t(i)^{(1-\alpha_p)} k_{t-1}(i)^{\alpha_p} \geq \left(\frac{P_t(i)}{P_t} \right)^{-\lambda} Y_t. \tag{26}$$

The first-order condition implies that

$$\varphi_t = \frac{w_t}{(1-\alpha_p)\left(\frac{k_{t-1}}{n_t}\right)^{\alpha_p}}.$$

The i reference is dropped because wages are the same across firms and the production functions have constant returns to scale, and because capital and labor can flow freely across firms, firms choose the same capital-labor ratios. The real marginal cost $mc_t = \varphi_t$.

Firms' Profit Maximization:

Real flow of profit for intermediate producer i is

$$\Pi_t = \frac{P_t(i)}{P_t} y_t(i) - w_t n_t(i) - r_{k,t}(i) k_{t-1}(i). \tag{27}$$

Following Calvo (1983), in each period, a fraction ζ of firms is not able to change its price and has to stick to the price chosen in the previous period. The dynamic problem of an updating firm can be written

$$\underbrace{\max}_{P_t(i),n_{j,t}(i),k_{t-1}(i)} \quad \mathbb{E}_0\left(\sum_{t=0}^{\infty}\zeta\Lambda_{s,t}\left[\left(\frac{P_t(i)}{P_t}\right)^{1-\lambda}y_t-\frac{w_t}{P_t}n_t(i)-\frac{r_{k,t}}{P_t}(i)k_{t-1}(i)\right]\right). \tag{28}$$

Substituting the equations from the first-order conditions and replacing $y_t(i)$ with (22), we get

$$\underbrace{\max}_{P_t(i)}\mathbb{E}_t\left(\sum_{h=0}^{\infty}(\zeta\beta)^h\frac{\lambda_{s,t+h}}{\lambda_{s,t}}\left[\left(\frac{P_t(i)}{P_{t+h}}\right)^{1-\lambda}y_{t+h}-mc_{t+h}\left(\frac{P_t(i)}{P_{t+h}}\right)^{-\lambda}y_{t+h}\right]\right). \tag{29}$$

Note that $P_t(i)$ is not moved forward to $P_{t+h}(i)$, since firms choose their price in the current period under the constraint that they might not be able to change this price in future periods. Rewriting the maximization problem, we get

$$\underbrace{\max}_{P_t(i)}\mathbb{E}_t\left(\sum_{h=0}^{\infty}(\zeta\beta)^h\frac{\lambda_{s,t+h}}{\lambda_{s,t}}\left[(P_t(i))^{1-\lambda}(P_{t+h})^{\lambda-1}y_{t+h}-mc_{t+h}(P_t(i))^{-\lambda}(P_{t+h})^{\lambda}Y_{t+h}\right]\right). \tag{30}$$

The production sector optimality conditions are shown in Appendix A6.

4.6 Equilibrium

A competitive equilibrium in this model is defined as a sequence of endogenous states ($m^k_{j,t-1}$, $x^k_{j,t-1}$), allocations ($c_{j,t}$,$n_{j,t}$), mortgage and housing market quantities ($h^*_{b,t}$,$m^{*k}_{j,t}$) and prices ($\pi_t, w_t, p^h_t, R^d_t, R^s_t, R_t, r^*_t$) that satisfy borrower, saver, firm optimality and market clearing for resources, bonds, housing, labor and profits.

5 Calibration

The model is calibrated to Canadian data at quarterly frequency. Some of the parameters are set to standard values picked from the related literature, while some are calibrated to match particular Canadian housing data moments. Most parameter values are listed in Table 1. The fraction of borrower household is set to 0.36 in order to match the fraction of households with less than three months' worth of expenses in liquid assets (2019 Canadian financial capability survey). The housing preference parameter ξ is set to 0.24 to target housing expenditure share of 20%. The saver's discount factor β_s is set to 0.994 to match the average of the trend for the real risk-free short-term rate from 1995 to 2015 of 2.4%, borrower's discount factor is set to 0.981 to match the steady-state fraction of uninsured mortgages of 65%, an intermediate value between 63% in the Canada Mortgage and Housing Corporation 2020 residential mortgage industry report and average share of 67.8% from the Bank of Canada quarterly household credit monitoring data. The inflation parameter, π_{ss}, is set to 1.005 to match a 2% annual inflation target, and the coefficient on inflation in the Taylor rule ψ_π is set at 1.5, which is a standard value used in the literature. The inflation target persistence parameter ψ_π is set at 0.994, and the Taylor rule smoothing parameter is set at 0.89.

For the debt limit parameters, these are set to institutional limits for traditional banks. We therefore set $\theta_U^{LTV_{TB}}$ to 0.80, the institutional required LTV limit for uninsured mortgages in the traditional banks, and for insured mortgages, the LTV limit is set to 0.95, which is also the institutional limit. The PTI limit for insured borrowers is 0.44. There is no PTI limit for uninsured mortgages; banks qualify these borrowers at a limit subject to their discretion. Microdata shows that banks have typically imposed a PTI limit of 44%, with about 10-20% of uninsured borrowers with no caps at all, as shown in Figure 13 in Appendix D; hence, we set the PTI limit to 0.6 for the uninsured space overall. We also set the LTV for uninsured mortgages in the shadow banks $\theta_U^{LTV_{SB}}$ to 0.80. Turning to other mortgage parameters, ν_U and ν_I are set to 0.1% and 0.083%, respectively, to match the average share of principal paid on uninsured and insured mortgages for 25- and 30-year amortization, respectively. The PTI offset parameter, α, is set such that $r^* + \alpha$ is equal to the interest and principal payment for fixed-rate mortgage, plus 1.75% annually for taxes and insurance.

Table 1: Parameter Settings

Parameter	Name	Value	Source
		Demographics and Preferences	
Fraction of borrowers	χ_b	0.36	2019 Canadian financial capability survey
Income dispersion	σ_e	0.22	set to match 9% share of SB borrowers
Borrowers discount factor	β_b	0.981	Set to match 65% uninsured borrowers share
Savers discount factor	β_s	0.994	avg. short-term rate
Housing preference	ξ	0.24	housing expenditure share of 20%.
Borr. labor disutility	η_b	3.375	$\frac{n_{b,ss}}{\chi_b} = \frac{1}{3}$
Saver labor disutility	η_s	7.512	$\frac{n_{s,ss}}{\chi_s} = \frac{1}{3}$
Inv. Frisch elasticity	φ	1.0	standard
Habit (pers.) cons.	ε_c	0.8	standard
TB benefit parameter	Ψ_2	0.005	set to match 0.5% mortgage broker fee
		Housing and Mortgages	
Mortgage amortization (U)	ν_u	0.0083	30 years amortization
Mortgage amortization (I)	ν_I	0.01	25 years amortization
Income tax rate	τ_y	0.232	Bradbury et al.
Max LTV ratio (I)	$\bar{\theta}_I^{LTV}$	0.95	Canadian policy
Max LTV ratio (U)	$\bar{\theta}_u^{LTV}$	0.80	Canadian policy
Max PTI ratio (I)	$\bar{\theta}_I^{PTI}$	0.44	Canadian policy
Max PTI ratio (U) TB	$\bar{\theta}_U^{PTI_{TB}}$	0.60	see text
Prepayment rate	ρ	0.02	cash-out refinancing rate of 2%
PTI offset (taxes, etc.)	α	0.313%	$r_{ss} + \alpha = 9.97\%$ (annualized)
PTI offset (other debt)	ω	0.05	see text
Term premium (mean)	μ_q	0.02%	interest rate spread
Term premium (pers.)	ϕ_q	0.852*	autocorr. of (mort. rate - 1Y rate)
Log housing stock	$\log \bar{H}$	6.464	$p_{ss}^h = 1$
Housing depreciation	δ_h	0.004	Statistics Canada
		Productive Technology and Monetary Policy	
Variety elasticity	λ	6.0	standard
Price stickiness	ζ	0.75	standard
Capital depreciation	δ_k	0.025	standard
share of capital	α_p	0.33	standard
Steady state inflation	π_{ss}	1.005	avg. infl. expectations
Taylor rule (inflation)	ψ_π	1.5	Allen & Greenwald
Taylor rule (smoothing)	ϕ_r	0.89	Allen & Greenwald
Infl. target (pers.)	$\phi_{\bar{\pi}}$	0.994	Allen & Greenwald
Macropru. policy (pers.)	ϕ_θ	0.975	standard

The depreciation rate of housing, δ, is set equal to 0.004, an approximate quarterly value for yearly depreciation rate of 1.5% from the prices analytical series on shelter in the Canadian CPI by Statistics Canada. The depreciation rate of capital δ_k and the share of capital in production, α_p, are set to 0.025 and 0.33, respectively, which are standard values used in literature. The variety elasticity and price stickiness parameters are set to 0.6 and 0.75, respectively, which are standard values in the literature. The steady-state value of the repayment probabilities are 0.97 and 0.95, respectively, for traditional and shadow banks; equivalently, the default probabilities are 3% and 5% respectively. We calibrate τ^s such that the resulting annualized spread between shadow bank and traditional bank deposit rates is approximately 1% in steady state. The habit parameter for nondurable consumption is set to 0.8 (Dorich et al., 2021). The housing stock $\bar{H}$ and saver housing demand $\bar{H}_s$ are calibrated to keep the price of housing to unity at steady state. Following Greenwald (2018), we choose the log-normal specification $\log e_{i,t} \sim \mathcal{N}(\sigma_e/2, \sigma_e^2)$, which implies that

$$\int^{\bar{e}_t} e_i d\Gamma(e_i) = \Phi\left(\frac{\log \bar{e}_t - \sigma_e^2/2}{\sigma_e}\right),$$

where Φ is the standard normal c.d.f. to capture dispersion in which constraint is binding and σ_e is set to 0.22 to match an approximate 9% average share of shadow banks' mortgage volumes relative to total mortgage volumes in the GTA in 2019Q1 to 2019Q4.

6 Results

This section illustrates how borrower-based macroprudential policies are affected by lenders outside the purview of regulation (shadow banks) and rising interest rate. These quantitative results are computed as nonlinear perfect foresight paths using the deterministic simulation algorithm implemented in Dynare using the 'simul' command.

6.1 Effect of Contractionary Monetary Policy

We consider a persistent inflation-target shock which corresponds to persistent changes in monetary policy as in Smets and Wouters (2003), Garriga et al. (2017) and Greenwald (2018). As shown in Greenwald (2018), inflation target shocks move nominal rates while having very little influence on real rates, making them convenient for analyzing the effect of changing nominal rates in isolation. First, we show in Figure 6 the impulse responses to 0.25 bps shock to the inflation target for the models with and without shadow banks, shown in terms of percentage deviations from steady state for all variables except the fractions that are presented in absolute deviations from steady state. The result shows that in the presence of shadow banks, the impact of the shock leads to a smaller decline in house prices and total mortgage origination. This is because of the increase in new mortgage origination in the shadow banks. This occurs because a rise in the interest rate affects the PTI limit much more than the LTV limit. A tighter PTI limit in the uninsured segment leads some borrowers to the shadow bank sector. In contrast, debt limits in the LTV economy are only indirectly affected by interest rates through house prices. Lower house prices decrease the collateral value and tighten the LTV limit. From the impulse responses, the price effect is more in the model without shadow banks. Their effect on consumption and output is similar in both models, with output declining more on impact in the model without shadow banks, and the loan-to-income (LTI) ratio, defined as the ratio of total new mortgages to the borrower's income, also declines.

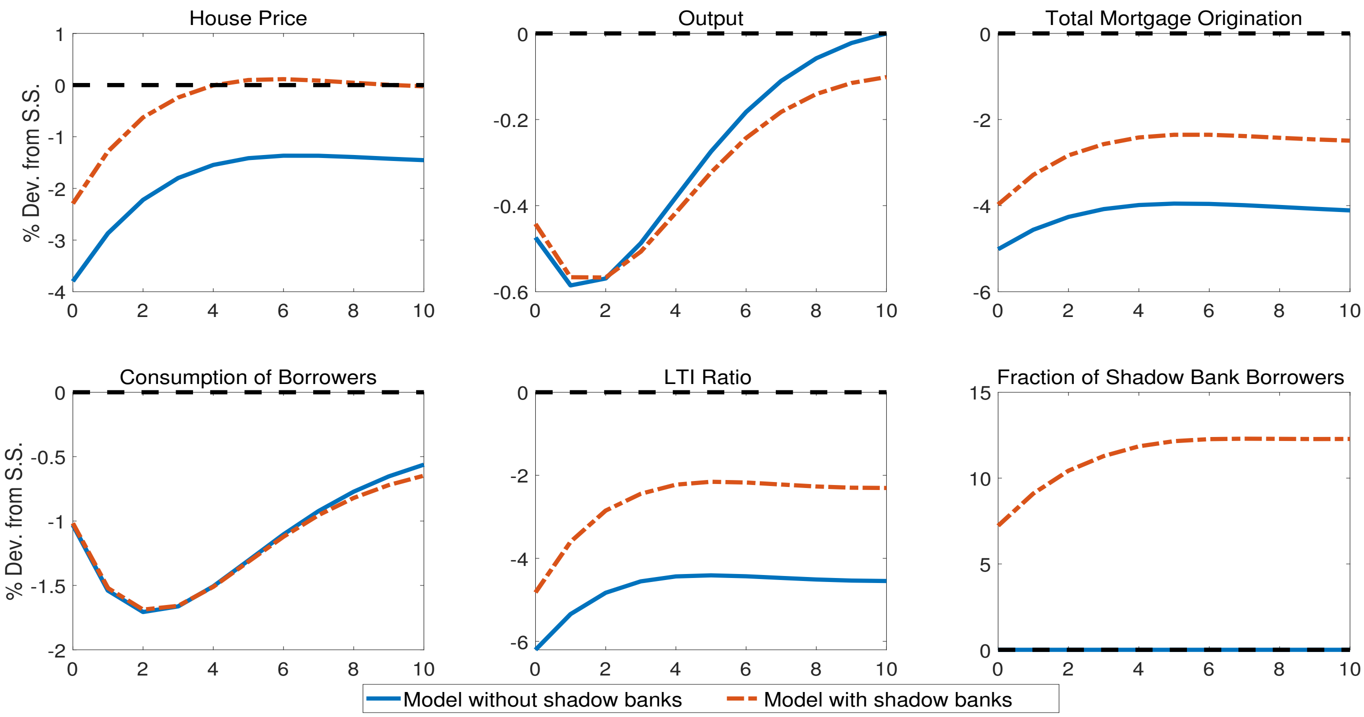


Figure 6 Responses to 25bps monetary tightening. Responses are in percentage deviation from steady state, except for fractions, which are measured in percentage points. LTI is the ratio of new loans to income of borrowers.

Our results are consistent with findings of Pescatori and Sole (2016), which show that monetary policy decreases aggregate lending activities even though the size of the nonbank sector increases, indicating a relative dampening of the transmission channel as nonbanks step in as lenders whenever traditional banks reduce credit provisions. Den Haan and Sterk (2011), using US flow-of-funds data, similarly show that nonbank asset holdings increase in response to monetary tightening, even though overall credit declines or stays relatively flat. Our result confirms the presence of credit leakage towards shadow banks in response to monetary policy tightening. This rise in shadow banks' share has a strong correlation with policy rate. In Figure 7, we show the plot of shadow banks' share and policy rate from 2015Q1 to 2019 Q1.

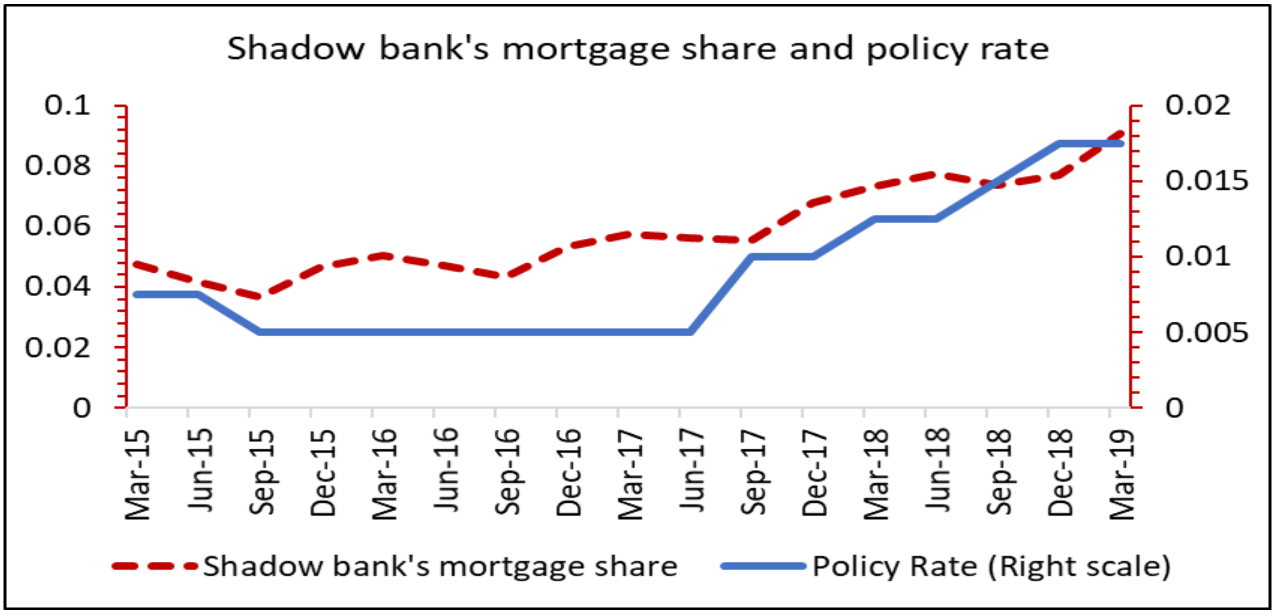


Figure 7 This figure shows the share of shadow bank mortgages and policy rate from 2015 to 2019. The data are quarterly. Shadow bank shares are from Teranet deeds data, and policy rates are from the Bank of Canada.

6.2 Effect of macroprudential tightening

We next consider the impact of 2% tightening of PTI limits in the insured and uninsured mortgage sectors. With the exception of the fractions of insured, uninsured and shadow bank borrowers, these results are presented in percentage deviations from steady state. The impact of LTV tightening is shown in Appendix B.

6.2.1 PTI Tightening in the Uninsured Sector

Tightening of PTI limit in the uninsured mortgage market segment drives more borrowers to hit their PTI limits, and we have more PTI-constrained borrowers in the uninsured space. This depresses house prices by reducing the marginal collateral value of housing $\mathcal{C}_t$ – the benefit the borrower would receive from an additional dollar of housing through its ability to relax her debt limit. The impact of the shock on house prices, output new loans and other variables is much smaller in the model with shadow banks because of policy leakage. Figure 8 compares the impulse responses to the values from the model without shadow banks.

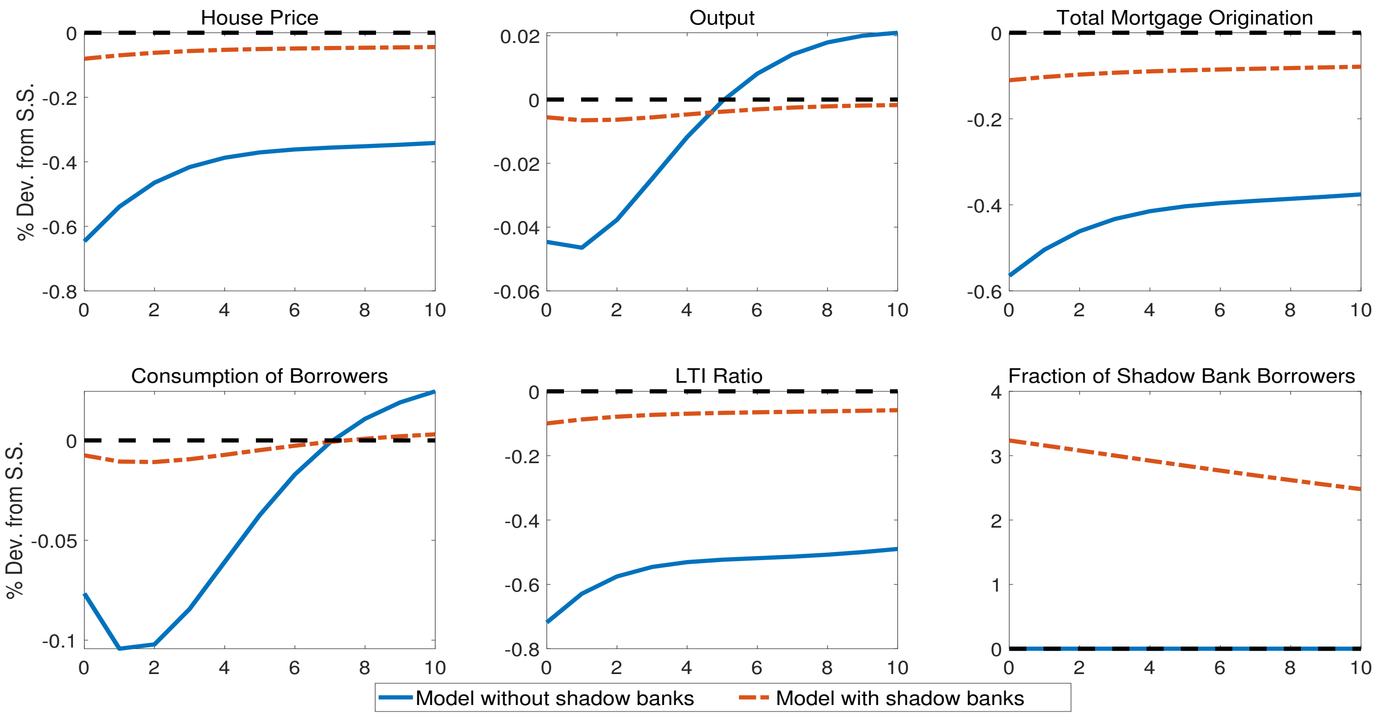


Figure 8 Responses to 2pp PTI limit tightening in the uninsured sector. Responses are in percentage deviation from steady state, except for fractions, which are measured in percentage points.

6.2.2 PTI Tightening in the Insured Sector

The impulse response functions in Figure 9 indicate that the effect of tightening PTI limits in the submarket in which PTI limits are already relatively tight can be substantially weakened as borrowers exercise their option to switch submarkets. Tightening PTI limit in the insured sector reduces house prices as demand for collateral weakens due to the fact that more borrowers in the insured sector are now constrained by the PTI limit. A fraction of these borrowers switch to the uninsured space to escape the tighter limit in the insured space. These borrowers become LTV constrained in the uninsured space; this increases the fraction of LTV-constrained borrowers in the uninsured space increasing the demand for collateral. Due to general equilibrium effects, reduction in house prices makes the LTV constraint slightly tighter, leading to a reduction in new loans and fraction of shadow bank borrowers who are constrained only by an LTV limit.

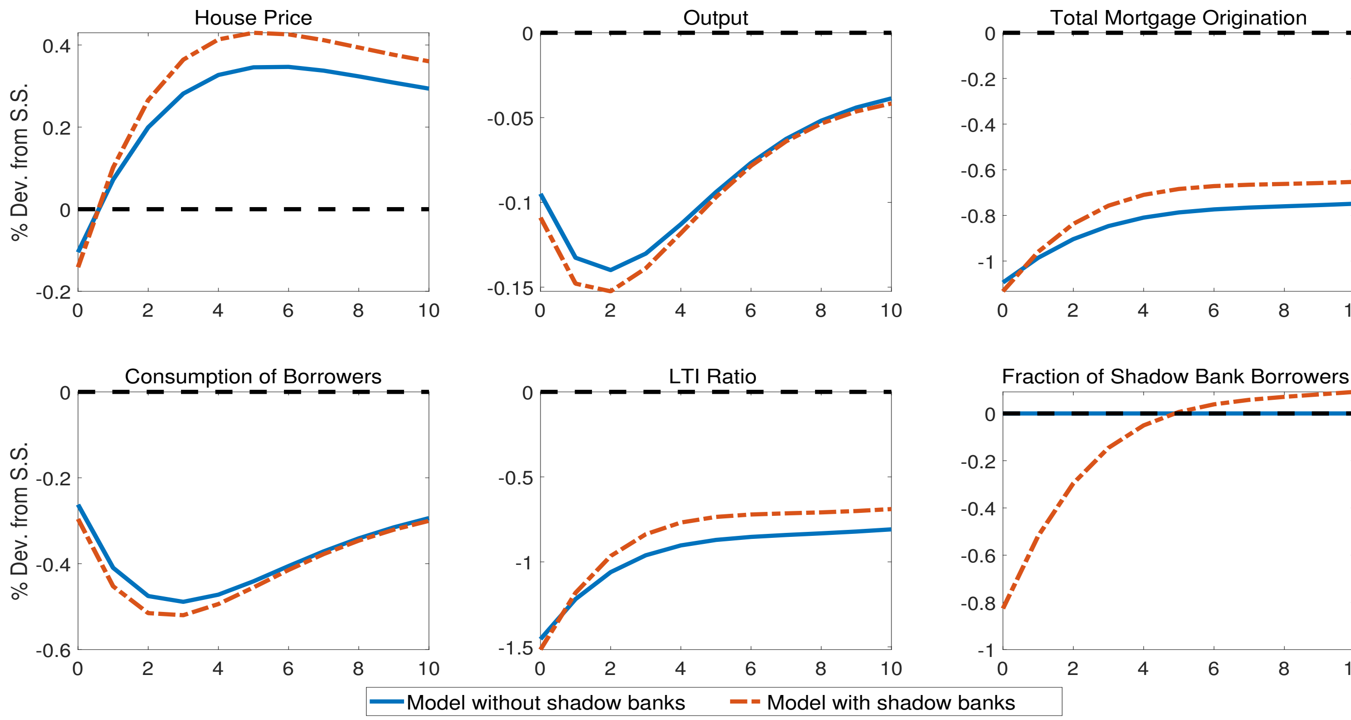


Figure 9 Responses to PTI limit tightening in the insured sector. Responses are in percentage deviation from steady state, except for fractions, which are measured in percentage points.

6.3 Sensitivity Analysis

In this subsection, we consider the sensitivity of the quantitative results to changes in the shadow banks' LTV limit. Figure 11 shows the impulse responses to inflation target and uninsured PTI tightening shocks for the models with and without shadow banking. The shadow banks' LTV limits (θ_{LTV}^{SB}) are 0.80 and 0.75 respectively. It is shown that there is more decline in house prices, output and new mortgage originations and a smaller shift of borrowers to the shadow banks when the θ_{LTV}^{SB} is 0.75 relative to 0.80. This shows that even though the shadow banks are not PTI constrained, a tighter LTV regulation reduces the level of policy leakage to the shadow banks.

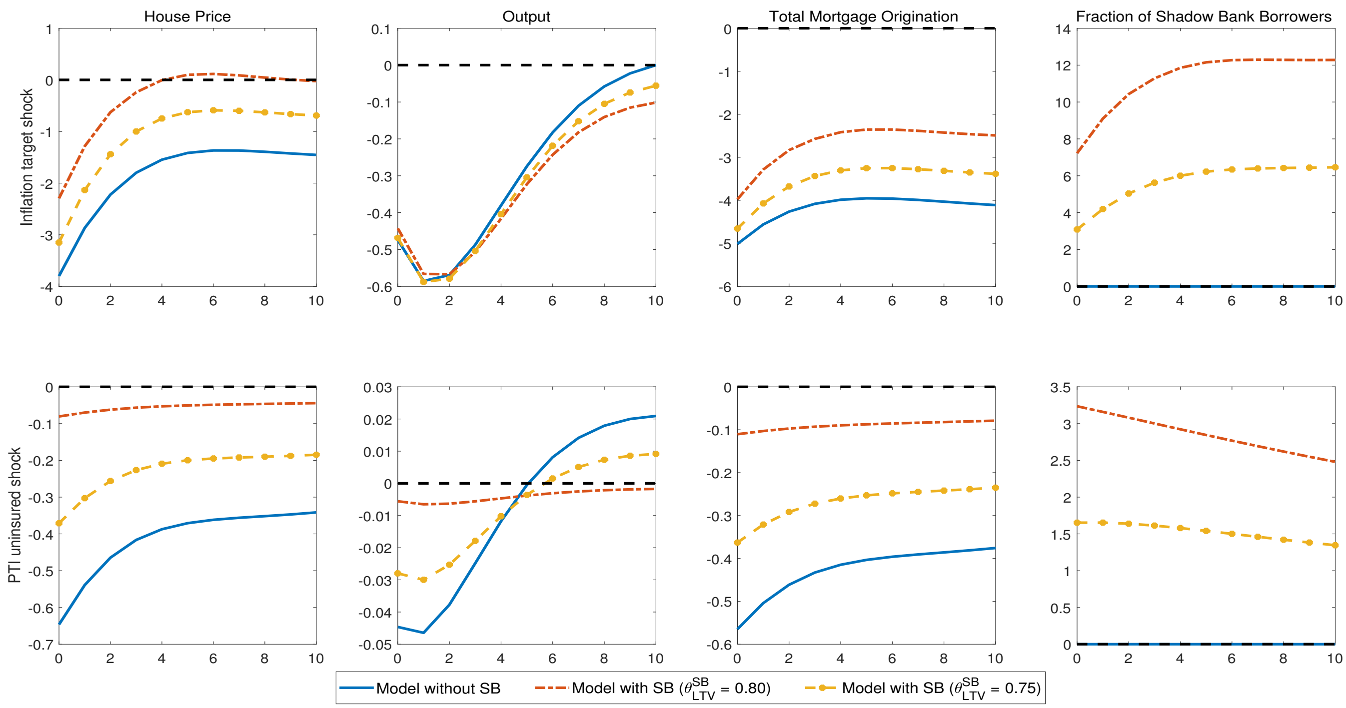


Figure 10 Responses to tighter LTV limit in the shadow banks. Responses are in percentage deviation from steady state, except for fractions, which are measured in percentage points.

7 Conclusion

We contribute to the literature on leakages from financial regulations by examining the impact of macroprudential policies in an economy with mortgage market segmentation and two credit constraints. We show that shadow banking affects the effectiveness of regulatory policies due to potential leakage across sub-sectors of the mortgage market. We also show that shadow banking affects monetary policy transmission through sub-sectors of the mortgage market. Specifically, we found that the impact of contractionary monetary policy is weakened in the presence of shadow banking, and policy leakage across the sub-sectors of the mortgage market depends on the sector that is targeted by the regulation. We document leakages to the shadow banking sector when the PTI limit in the uninsured sector of the traditional banks is tightened and when the policy rate increases. The model presented in this paper is important as it could be used as a tool to think about macroprudential policies in countries with a segmented mortgage market and both LTV and PTI constraints.

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Appendix

A: List of Equilibrium Conditions

A1: Savers:

Euler equation for government bonds

$$1 = R_t \mathbb{E}_t \left[\Lambda^s_{t,t+1} \pi^{-1}_{t+1} \right]$$

Euler equation for deposits

$$\mathbb{E}_t \frac{\Lambda^s_{t,t+1}}{\pi_{t+1}} (R_t - R^d_t) = \frac{u^d_{s,t}}{u^c_{s,t}},$$

Euler equation for bank bonds

$$\mathbb{E}_t \frac{\Lambda^s_{t,t+1}}{\pi_{t+1}} (R^b_t - R_t) = f'(b_t, m_t)$$

Intratemporal condition

$$-\frac{u^n_{s,t}}{u^c_{s,t}} = (1 - \tau_y) w_t$$

Capital accumulation equations

$$k_t = i_t \left(1 - \frac{\phi}{2} (i_t / i_{t-1} - 1)^2 \right) + (1 - \delta_k) k_{t-1},$$

$$\varrho_t = \mathbb{E}_t \Lambda^s_{t,t+1} \left((1 - \delta_k) \varrho_{t+1} + r_{k,t+1} \right)$$

$$\varrho_t \left(1 - \frac{\phi}{2} (\frac{i_t}{i_{t-1}} - 1)^2 - \phi (\frac{i_t}{i_{t-1}} - 1)(\frac{i_t}{i_{t-1}}) \right) = 1 - \mathbb{E}_t \Lambda^s_{t,t+1} \varrho_{t+1} \phi (\frac{i_{t+1}}{i_t} - 1)(\frac{i_t}{i_{t-1}})^2$$

Budget constraint

$$
\begin{aligned}
c_{s,t} \;\leq\; & \underbrace{(1-\tau_y)w_t n_{s,t}}_{\text{labor income}} - \underbrace{\delta p_t^h h_s}_{\text{maintenance}} - \underbrace{(b_t^g - \frac{R_{t-1} b_{t-1}^g}{\pi_t})}_{\text{net govt. bond}} \\
& - \underbrace{(b_t - \frac{R_{t-1}^b b_{t-1}}{\pi_t})}_{\text{net bank's bond}} + \underbrace{\Xi_t}_{\text{profits}} + \underbrace{T_{s,t}}_{\text{transfers}} - \underbrace{i_t}_{\text{investment}} \\
& + \underbrace{r_{k,t} k_{t-1}}_{\text{capital rent}} - \underbrace{(d_t - \frac{R_{t-1}^d d_{t-1}}{\pi_t})}_{\text{net deposit TB}} - \underbrace{(s_t^{SB} - \frac{R_{t-1}^s s_{t-1}^{SB}}{\pi_t})}_{\text{net invest. SB}} - \underbrace{f(b_t, m_t)}_{\text{port. adj. cost}}
\end{aligned}
$$

Positive spread between deposit and SB security rates

$$1 + r_t^s = \frac{1 + r_t^d}{1 - \tau^s \epsilon_t^{\tau^s}}$$

A2: Traditional Banks:

Total value of mortgages to finance in period t in each of the sub-markets

$$m_{U,t}^{TB} = \rho m_{U,t}^{*TB} + (1-\rho)(1 - \varpi_t^{TB} \nu_U) m_{U,t-1}^{TB} \pi_t^{-1}$$

$$m_{I,t} = \rho m_{I,t}^{*} + (1-\rho)(1 - \varpi_t^{TB} \nu_I) m_{I,t-1} \pi_t^{-1}$$

Laws of motion for mortgage payments

$$x_{U,t}^{TB} = \rho (r_t^* - \Delta_{q,t}) m_{U,t}^{*TB} + (1-\rho)(1 - \varpi_t^{TB} \nu_U) x_{U,t-1}^{TB} \pi_t^{-1}$$

$$x_{I,t} = \rho (r_t^* - \Delta_{q,t}) m_{I,t}^{*} + (1-\rho)(1 - \varpi_t^{TB} \nu_I) x_{I,t-1} \pi_t^{-1}$$

The total mortgages and payments made to the TBs

$$
\begin{aligned}
m_t^{TB} &= m_{I,t} + m_{U,t}^{TB} \\
x_t^{TB} &= x_{I,t} + x_{U,t}^{TB}
\end{aligned}
$$

Balance-sheet constraint

$$m_t^{TB} = d_t + b_t$$

Dividends

$$div_{t+1} = \pi_{t+1}^{-1}\left(x_{U,t}^{TB} + x_{I,t} - \varpi_{t+1}^{TB}\nu_U m_{U,t} - \varpi_{t+1}^{TB}\nu_I m_{I,t} - (r_t^d+\kappa)d_t - r_t^b b_t\right) - f(div_{t+1})$$

Dividend adjustment cost

$$f(div_t) = \frac{\theta^{div}}{2}\left(div_t - \bar{div}\right)^2$$

No-arbitrage condition

$$\frac{\mathbb{E}_t\Lambda_{t,t+1}^s\Omega_{t+1}^{div}}{\pi_{t+1}} r_t^b = \mathbb{E}_t\left[\Lambda_{t,t+1}^s\left((\Omega_{U,t}^x+\Omega_{I,t}^x)(r_t^*-\Delta_q)+\Omega_{U,t}^m+\Omega_{I,t}^m\right)\right]$$

where

$$\Omega_{U,t}^m = -\frac{\mathbb{E}_t\Lambda_{t,t+1}^s}{\pi_{t+1}}\left[\Omega_{U,t+1}^x(r_t^*-\Delta_q)(1-\rho)(1-\varpi_{t+1}^{TB}\nu_U)\right] - \frac{\varpi_{t+1}^{TB}\nu_U\Omega_t^{div}}{\pi_{t+1}}$$
$$\Omega_{I,t}^m = -\frac{\mathbb{E}_t\Lambda_{t,t+1}^s}{\pi_{t+1}}\left[\Omega_{I,t+1}^x(r_t^*-\Delta_q)(1-\rho)(1-\varpi_{t+1}^{TB}\nu_I)\right] - \frac{\varpi_{t+1}^{TB}\nu_I\Omega_t^{div}}{\pi_{t+1}}$$
$$\Omega_{U,t}^x = \frac{\mathbb{E}_t\Lambda_{t,t+1}^s}{\pi_{t+1}}\left[\Omega_{U,t+1}^x(1-\rho)(1-\varpi_{t+1}^{TB}\nu_U)\right] + \frac{\Omega_t^{div}}{\pi_{t+1}}$$
$$\Omega_{I,t}^x = \frac{\mathbb{E}_t\Lambda_{t,t+1}^s}{\pi_{t+1}}\left[\Omega_{I,t+1}^x(1-\rho)(1-\varpi_{t+1}^{TB}\nu_I)\right] + \frac{\Omega_t^{div}}{\pi_{t+1}}$$

Marginal value of profits

$$\Omega_t^{div} = \frac{1}{1+\theta^{div}(div_{t+1}-\bar{div})}$$

A3: Shadow Banks:

$$m_{U,t}^{SB} = \rho m_{U,t}^{*SB} + (1-\rho)(1-\varpi_t^S\nu_U)m_{U,t-1}^{SB}\pi_t^{-1}$$

$$x_{U,t}^{SB} = (r_t^*-\Delta_q)\rho_t m_{U,t}^{*SB} + (1-\rho)(1-\varpi_t^{SB}\nu_U)x_{U,t-1}^{SB}\pi_t^{-1}$$

Balance sheet

$$m_{U,t}^{SB} = m_t^{SB} = s_t^{SB} + nw_t^{SB}$$

Net worth

$$nw_t^{SB} = (1+r_{t-1}^*)\varpi_t^{SB}m_{U,t-1}^{SB} - (1+r_{t-1}^s)s_{t-1}^{SB}$$

Moral hazard condition

$$W_t = v_t m_{U,t}^{SB} + \eta_t^S n_{w,t}^{SB}$$

A4: Borrowers:

For notation, let, e.g., $u^c_{b,t}$ denote the derivative of the utility function of borrowers with respect to c, and let

$$\Lambda_{b,t+1} = \beta_b \frac{u^c_{b,t+1}}{u^c_{b,t}}$$

denote the stochastic discount factor of the borrower, we have the following equilibrium conditions for borrowers: Euler equation for new housing

$$p^h_t = \frac{\frac{u^h_{b,t}}{u^c_{b,t}} + \mathbb{E}_t \Lambda_{b,t+1} p^h_{t+1}[1 - \delta - (1-\rho)\mathcal{C}_{t+1}]}{1 - \mathcal{C}_t}$$

Marginal collateral value of housing

$$\mathcal{C}_t = \mu_{U,t}(\theta^{LTV_{SB}}_{U,t} F^{LTV_{SB}}_{U,t} + \theta^{LTV_{TB}}_{U,t} F^{LTV_{TB}}_{U,t}) + \mu_{I,t} F^{LTV_{TB}}_{I,t} \theta^{LTV_{TB}}_{I,t}$$

Intratemporal condition

$$\begin{aligned} -\frac{u^n_{b,t}}{u^c_{b,t}} &= (1-\tau_y)w_t + \mu_{U,t}\rho \left[\left(\frac{\left(\theta^{PTI_{TB}}_{U,t} - \omega\right) w_t}{r^*_t + \nu_U + \alpha} \right) \left[\Psi(\bar{e}^{TB}_{U,t}) - \Psi(\bar{e}^{SB}_{U,t}) \right] \right] \\ &+ \mu_{I,t}\rho \left((\frac{\left(\theta^{PTI}_{I,t} - \omega\right) w_t}{r^*_t + \nu_I + \alpha} \right) (\Psi(\bar{e}_{I,t}) - \Psi(\bar{e}^*_t)) \end{aligned}$$

Budget constraint

$$\begin{aligned} c_{b,t} \leq\ & \underbrace{(1-\tau_y)w_t n_{b,t}}_{\text{labor income}} - \underbrace{\rho p^h_t (h^*_{b,t} - h_{b,t-1})}_{\text{housing purchase}} + \underbrace{\sum_{I,U}\{\rho \sum_{k=SB,TB} (m^{*k}_{j,t} - (1-\varpi^k_t \nu_j)\pi^{-1}_t m^k_{j,t-1})}_{\text{net new debt issuance}} \\ & - \underbrace{\pi^{-1}_t \sum_{k=SB,TB} \varpi^k_t x^k_{jb,t-1}}_{\text{interest payment}} - \underbrace{\pi^{-1}_t \sum_{k=SB,TB} \varpi^k_t \nu_j m^k_{j,t-1}\}}_{\text{principal payment}} - \underbrace{\delta p^h_t h_{b,t-1}}_{\text{maintenance}} + \underbrace{T_{b,t}}_{\text{transfers}} \\ & + \underbrace{\rho \Psi^{SB}_t}_{\text{unregulated lender cost}} \end{aligned}$$

Euler equation for new borrowing

$$1 = \Omega^{mT}_{Ub,t} + \Omega^{xT}_{Ub,t} r^*_t + \mu_{U,t}$$

$$1 = \Omega^m_{Ib,t} + \Omega^x_{Ib,t} r^*_t + \mu_{I,t}$$

Where

$$\begin{aligned}
\Omega^{mT}_{Ub,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(\varpi^{TB}_{t+1}\nu_U + (1-\varpi^{TB}_{t+1}\nu_U)\rho + (1-\varpi^{TB}_{t+1}\nu_U)(1-\rho)\Omega^{mT}_{Ub,t+1}\right)\right] \\
\Omega^{mS}_{Ub,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(\varpi^{SB}_{t+1}\nu_U + (1-\varpi^{SB}_{t+1}\nu_U)\rho + (1-\varpi^{SB}_{t+1}\nu_U)(1-\rho)\Omega^{mS}_{Ub,t+1}\right)\right] \\
\Omega^{m}_{Ib,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(\varpi^{TB}_{t+1}\nu_I + (1-\varpi^{TB}_{t+1}\nu_I)\rho + (1-\varpi^{TB}_{t+1}\nu_I)(1-\rho)\Omega^{m}_{Ib,t+1}\right)\right] \\
\Omega^{xT}_{Ub,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(1 + (1-\varpi^{T}_{t+1}\nu_U)(1-\rho)\Omega^{xT}_{Ub,t+1}\right)\right] \\
\Omega^{xS}_{Ub,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(1 + (1-\varpi^{S}_{t+1}\nu_U)(1-\rho)\Omega^{xS}_{Ub,t+1}\right)\right] \\
\Omega^{x}_{Ib,t} &= \mathbb{E}_t\left[\Lambda_{b,t+1}\pi^{-1}_{t+1}\left(1 + (1-\varpi^{TB}_{t+1}\nu_I)(1-\rho)\Omega^{x}_{Ib,t+1}\right)\right]
\end{aligned}$$

A5: Producers:

$$\begin{aligned}
\mathcal{N}_t &= u^c_{s,t} y_t mc_t + \zeta\beta\mathbb{E}_t\left(\mathcal{N}_{t+1}\left(\frac{\pi_{t+1}}{\pi_{ss}}\right)^\lambda\right) \\
\mathcal{D}_t &= u^c_{s,t} y_t + \zeta\beta\mathbb{E}_t\left(\mathcal{D}_{t+1}\left(\frac{\pi_{t+1}}{\pi_{ss}}\right)^\lambda\right) \\
\tilde{p}_t &= \frac{\lambda}{\lambda-1}\frac{\mathcal{N}_t}{\mathcal{D}_t} \\
\Delta_t &= (1-\zeta)\tilde{p}_t^{-\lambda} + \zeta\left(\frac{\pi_t}{\pi_{ss}}\right)^\lambda \Delta_{t-1} \\
\pi_t &= \pi_{ss}\left[\frac{1-(1-\zeta)\tilde{p}_t^{1-\lambda}}{\zeta}\right]^{\frac{1}{\lambda-1}} \\
y_t &= \frac{n_t^{(1-\alpha_p)} k_{t-1}^{\alpha_p}}{\Delta_t} \\
w_t &= mc_t(1-\alpha_p)\left(\frac{k_{t-1}}{n_t}\right)^{\alpha_p} \\
r_{k,t} &= mc_t\alpha_p\left(\frac{k_{t-1}}{n_t}\right)^{\alpha_p-1} \\
\Pi_t &= y_t - w_t n_t - r_{k,t}k_{t-1} + div_t + \kappa d_{t-1}\pi_t^{-1}
\end{aligned}$$

Where $\mathcal{N}_t$ and $\mathcal{D}_t$ are auxiliary variables, $\tilde{p}_t$ is the ratio of the optimal price for resetting firms relative to the average price, and Δ_t is price dispersion.

A6: Market clearing conditions:

Resources: $y_t = c_{b,t} + c_{s,t} + i_t + \delta p_h \bar{H}$

Government bonds: $b^g_{s,t} = 0$

Housing: $\bar{H} = h_s + h_{b,t}$

Labor: $n_{b,t} + n_{s,t} = n_t$

B: LTV tightening

B1: LTV tightening in the insured sector

The threshold borrower in the insured space is PTI constrained, and the LTV limit in the insured sector is loose. Tightening the LTV limit in the insured sector of the traditional banks does not actually lead to borrowers moving to the uninsured space, but it rather increases the fraction of borrowers constrained by the LTV limit in the insured sector, as this policy does not directly affect the boundary between the insured and uninsured sectors. We find that this policy is ineffective because it induces an increase in collateral and housing demand, pushing the house prices up. A higher house price relaxes the LTV limit, increasing mortgage originations in the sectors where borrowers are constrained by the LTV limits.

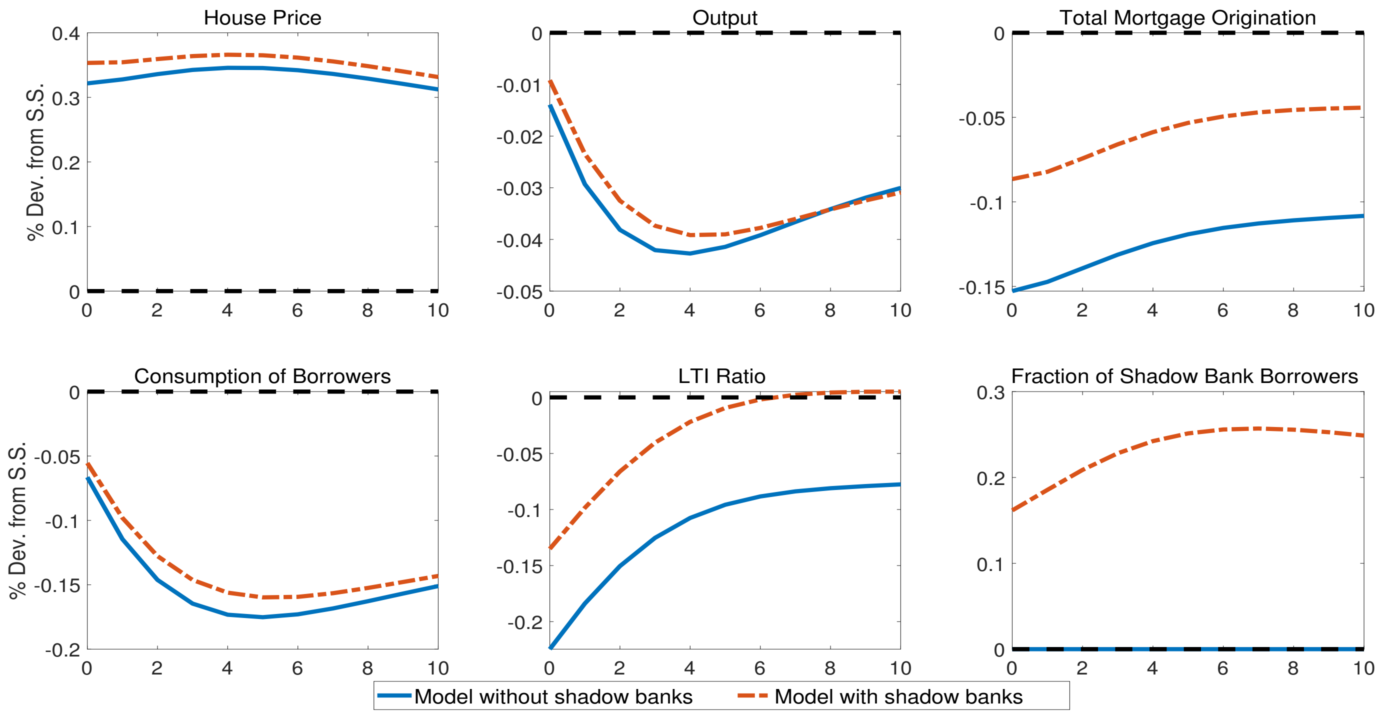


Figure 11 Responses to LTV limit tightening in the insured sector of the traditional banks

B2: LTV tightening in the uninsured sector

Tightening the LTV limit in the uninsured sector of the traditional banks reduces the income threshold at which borrowers switch from the uninsured to insured space, pushing borrowers out of the uninsured sector. These borrowers, upon entering the insured space, become PTI constrained, which puts downward pressure on housing demand and subsequently results in reduction in house price. A lower house price tightens the LTV limit, decreasing mortgage originations in the shadow banks.

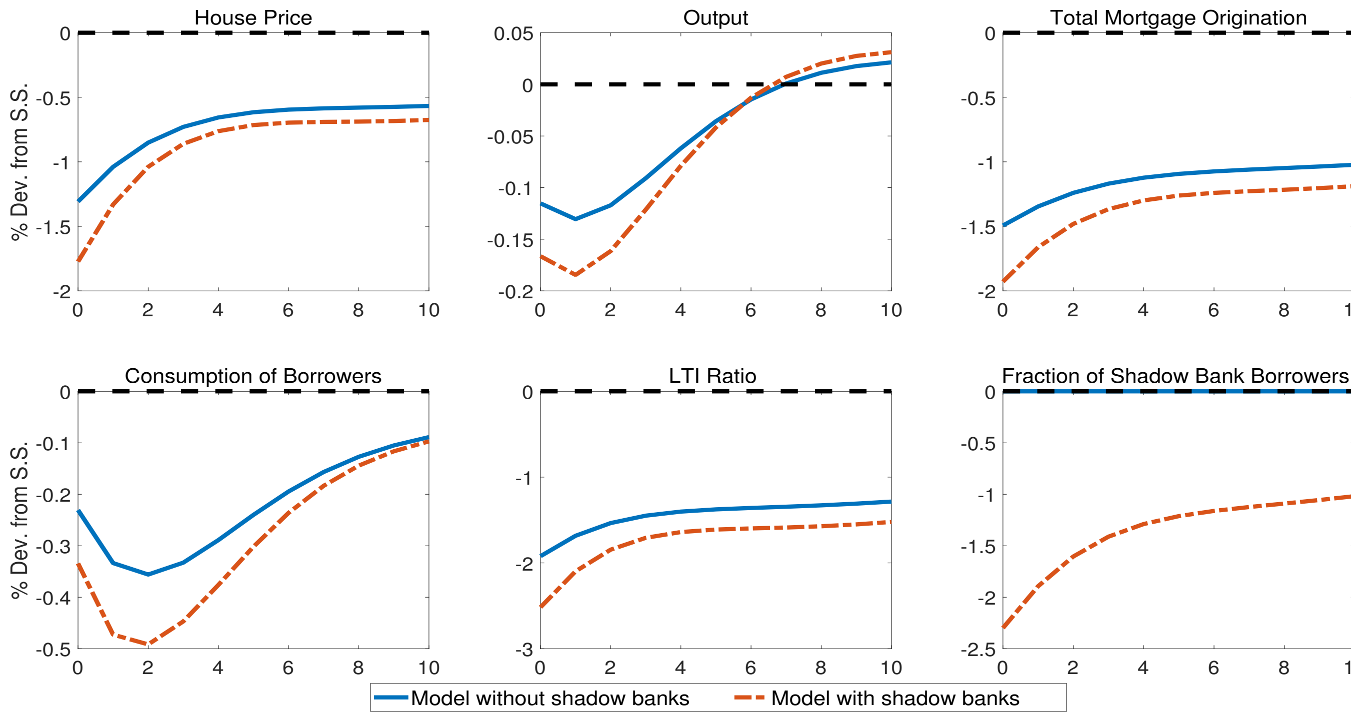


Figure 12 Responses to LTV limit tightening in the uninsured sector of the traditional banks

B3: Summary of results

Table 2: Impact of 25 bps and 2 pp contractionary monetary and macropru shocks respectively (in % dev. from S.S)

Shock	Model	House Prices	Output	Mortgage Origination
Monetary policy	SB	-2.294	-0.570	-3.974
	No SB	-3.800	-0.585	-5.013
PTI uninsured	SB	-0.081	-0.007	-0.110
	No SB	-0.647	-0.046	-0.565
PTI insured	SB	-0.141	-0.152	-1.133
	No SB	-0.105	-0.140	-1.094
LTV uninsured	SB	-1.773	-0.185	-1.929
	No SB	-1.308	-0.115	-1.494
LTV insured	SB	0.353	-0.039	-0.087
	No SB	0.322	-0.043	-0.153

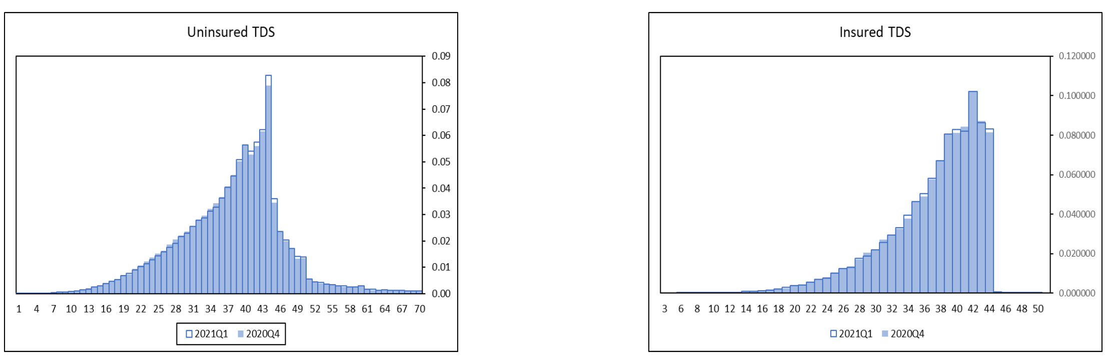
C: Data used for calibration

1. Fraction of borrowers households: https://www.canada.ca/en/financial-consumer-agency/programs/research/canadian-financial-capability-survey-2019.html
2. Fraction of uninsured mortgages: https://publications.gc.ca/collections/collection$_2$020/schl-cmhc/NH70-2-2020-eng.pdf and Bank of Canada Quarterly Household Credit Monitoring 2021 Q2
3. LTV and PTI limits: Bank of Canada summary of purchases and refinances by Federally Regulated Financial Institutions (FRFIs), 2021Q1
4. Income tax rate: https://www.oecd.org/canada/taxing-wages-canada.pdf
5. Housing depreciation rate: https://www150.statcan.gc.ca/n1/pub/62f0014m/62f0014m2017001-eng.htm

D: Additional Figures

Uninsured TDS

Insured TDS



Source: Summary of purchases and refinances by FRFIs

Figure 13 Qualifying TDS ratios for uninsured and insured mortgages for FRFIs