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Exports and the Exchange Rate: A General Equilibrium Perspective

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Abstract

Understanding and predicting the evolution of exports after a change in the nominal exchange rate is of central importance in international economics. Most of the literature focuses on estimating this relationship by reduced form, with the aim of uncovering a single structural parameter, but theory suggests it could differ depending on the shock that drives the movement in the exchange rate. Building on this insight, we develop a small-open-economy SVAR model to derive structural shocks that affect the exchange rate. We then estimate this model using Canadian data and construct the response of exports relative to the response of the exchange rate, conditional on each shock. Our findings suggest that this relationship differs greatly from one shock to another, where domestic shocks generate a much weaker relationship than global shocks. We show that these differences can be reconciled with theoretical results from a small-open-economy New Keynesian model where Canadian exports are largely invoiced in US dollars. Finally, we highlight how our results help to inform recent discussions on the evolution of the exchange rate elasticity over time, the benefits of a flexible exchange rate, and the impact of terms of trade movements on exports.

Topics: Exchange rates; International topics; Trade integration; Exchange rate regimes; Balance of payments and components; Business fluctuations and cycles; Monetary policy transmission
JEL codes: F31, F32, F33, F41

1 Introduction

Nominal exchange rates move over time, and these movements are viewed as having important implications for exports in small open economies. According to standard theory, an exchange rate depreciation should lower the relative price of a given country's goods in the global market, and through expenditure switching, lead to higher exports. A large literature has focused on estimating the strength of this responsiveness, which is often called the "exchange rate elasticity of exports" (EREE). The size of the EREE has important implications for the effectiveness of monetary policy, the resolution of trade imbalances, and macroeconomic stability [Friedman (1953), Leigh et al. (2015)].

In the trade literature, the typical approach to estimating the EREE is through reduced-form estimation, where the aim is to uncover a structural parameter that reflects the *ceteris paribus* response of exports to exchange rate movements [Leigh et al. (2015), Bussière, Gaulier and Steingress (2020), Fitzgerald and Haller (2018), Auer, Burstein, Erhardt and Lein (2019)]. However, satisfying the *ceteris paribus* requirement in this approach faces significant econometric challenges related to the endogeneity of exchange rates. First, finding good proxies to use when identifying exogeneous movements in exchange rates often relies on unusual circumstances that might not be broadly applicable. Second, exchange rate movements usually generate contemporaneous responses in other important macro-variables that impact exports through additional and well-known economic channels. For example, exchange rate movements can pass through to consumer prices, and solicit a policy response that could independently affect exports. As a result, even if one succeeds in identifying an exogenous movement in exchange rates, the estimated effects of these changes on exports can be confounded with multiple other channels of transmission.

In this paper, we take a different approach and focus on estimating the *general equilibrium* relationship between movements in the exchange rate and movements in exports. Importantly, this general equilibrium relationship is, by nature, a conditional relationship that may differ depending on the underlying shock that generates volatility in the economy. For example, oil price shocks have different effects across the economy compared to monetary policy shocks in general, so estimates of the conditional exchange rate elasticity of exports (CEREE) in response to these two types of shocks might be different if other channels besides the exchange rate channel are influencing exports.

We argue that estimating conditional exchange rate elasticities of exports (CEREEs) in this manner provides several advantages. First, this approach does not rely on *ceteris paribus* criteria that are difficult to convincingly satisfy in the case of reduced-form EREE estimation. Second, this approach provides a strong indication of how important endogeneity concerns are for reduced-form estimates of the EREE. This is because any major differences between the estimated CEREEs will indicate that additional channels besides the exchange rate channel are influencing exports when exchange rate movements occur. And third, policy-makers are often interested not only in the *ceteris paribus* response of exports to exchange rate movements, but also in understanding or predicting how exports change *after* a movement in the exchange rate that is caused by a particular shock in the economy. Our approach provides CEREE estimates that indicate exactly this.

To estimate CEREEs, we propose a structural vector auto-regression (SVAR) model that identifies a broad set of macroeconomic shocks that influence the exchange rate for a small open economy. We include seven shocks: four domestic (demand shock, supply shock, monetary policy shock, and "exogenous" exchange rate shock) and three global (transitory US income shock, persistent US income shock, and oil price shock).[1] We estimate the model using Canadian data,[2] and construct the export response for a 1% exchange rate appreciation from each shock, which we refer to throughout the paper as CEREEs. We then investigate whether CEREE estimates differ from one another, and use theory as guidance to

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[1] Transitory and persistent US income shocks are seen as enveloping several types of shocks that originate in the US economy. For example, a US monetary policy shock will be captured as a transitory US income shock, and a US technology shock will be captured as a persistent US income shock.

[2] We view Canada as an ideal candidate for our exercise. It is a small open economy that experiences sizable exchange rate movements driven by several types of shocks, where the central bank has a clear monetary policy framework and does not engage in exchange rate interventions. Canada is also consistently a net oil exporter, and therefore oil price movements have clear implications for the terms of trade and the exchange rate. Since over 70% of Canadian exports go to the US, we can comfortably treat the US economy alone as the "foreign" block and use the bilateral Canada–US exchange rate and US GDP as our global variables. This avoids the need for a measure of the "effective exchange rate," the use of which introduces additional identification and measurement issues [Mayer and Steingress (2020)], and the need for a weighted measure of foreign activity. Finally, our empirical approach requires a long time series for inference, and data for Canada are available going back to the 1980s and up to very recent periods.

explain why these differences emerge.

Our findings reveal that different shocks generate significantly different estimates for the CEREE. The CEREE is weak, in a statistical sense, for all of the domestic shocks. In contrast, the CEREE is statistically significant for global shocks, and economically very different depending on the shock. For oil price shocks, the CEREE is negative and significant; but for persistent US income shocks, the CEREE is positive and significant.[3]

Our analysis distinguishes between the three drivers of export growth: the exchange rate channel, the export price channel, and the foreign demand channel. We find that the foreign demand channel is the only one of the three that consistently impacts Canadian exports in a significant way. The "exogenous" exchange rate shock in our model provides a close proxy for capturing the exchange rate channel, which is the channel that the reduced-form literature has typically tried to isolate. Our estimated CEREE from this shock is negative but statistically insignificant, suggesting that the exchange rate channel is not very strong. We propose that the weakness of the exchange rate channel likely stems from the invoicing currency of Canadian exports, which is predominantly in US dollars. Given this invoicing pattern, exchange rate movements have no direct short-run impact on Canadian export prices in the US, the main destination market. This, combined with nominal price rigidities, prevents pass-through of exchange rate movements to the foreign price of Canadian exports, and therefore prevents any relative price movement that would typically lead to expenditure switching and changes in real exports. This explanation is characteristic of the "dominant currency paradigm" (DCP) that has been recently highlighted in the literature [Gopinath et al. (2020)]. In the Appendix, we show that a version of the small-open-economy DCP model proposed by Gopinath et al. (2020) that is calibrated to match the composition and invoicing patterns in Canada's trade can largely reproduce the patterns that we find based on our SVAR estimates for the CEREEs.

Our results are relevant for understanding the relationship between oil price movements and exports. We find that whether or not oil price appreciations correspond with lower exports depends crucially on

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[3] In this paper, we define the exchange rate as the price of foreign currency in terms of domestic currency so that a rise in the exchange rate represents an appreciation in the domestic currency. The conventional wisdom is that the EREE is negative, so that an appreciation in the domestic currency coincides with a decline in exports.

the source of the oil price movement. If oil prices appreciate due to an oil price shock, which, in our formulation, is consistent with either an actual or speculated decrease in oil supply, then higher oil prices lead to an appreciation in the exchange rate and lower exports, as is often presumed. However, if higher oil prices are driven by a US income shock, then they correspond with an appreciation in the exchange rate, but an increase in Canadian exports. The crucial difference between these two cases is the evolution of US demand. When oil price shocks emerge, the impact on US income is negative, and hence this, combined with the exchange rate appreciation, leads to lower exports. However, US income shocks correspond to higher US income, which is a factor that increases Canadian exports, sufficiently so to counteract the drag from the higher exchange rate. These results are consistent with previous evidence from Charnavoki and Dolado (2014) who come to similar conclusions using different methods.

Some studies have found that the EREE has been weaker in recent decades due to, for example, the rise of global supply chains [Swarnali, Maximiliano and Michele (2017), Ollivaud, Rusticelli and Schwellnus (2015)]. One possible explanation for this finding is that the nature of the shocks that affect the economy have changed over time, and hence the weighted-average of the CEREEs might be different today than in the past. To investigate this possibility, we calculate the weighted-average of the CEREEs separately for the pre-2000 and post-2000 periods, based on the contribution of each shock to exchange rate variation over these two periods. We find little evidence that this weighted-average is significantly weaker (less negative) in more-recent decades. This finding is consistent with Leigh et al. (2015), who find that exports and exchange rates are as connected in recent years as they were in the past. For Canada, we find that both the reduced-form and the weighted-average relationship are weak over our sample period. That said, this relationship has been relatively strong (more negative) in recent years, as oil price shocks, which push the weighted-average more negative, have become more important. In contrast, persistent US income shocks, which push the weighted-average more positive, have become less so.

In terms of the existing literature, our study builds on a recent body of work that challenges the common practice of estimating exchange rate pass-through to prices as a single unconditional relationship, and instead proposes conditional estimates of this relationship [Shambaugh (2008), Cunningham, Friedrich, Hess and Kim (2017), Forbes, Hjortsoe and Nenova (2018), Garcia-Schmidt and Garcia-Cicco (2020), Ha, Marc Stocker and Yilmazkuday (2020)]. Our approach is very similar to some of the approaches taken in these articles, but ours is distinct in that we focus on the EREE rather than exchange rate pass-through to domestic prices.

Our findings also contribute to a large literature that studies how exports respond to exchange rate movements.[4] Generally, EREE estimates from this literature tend fall in the 0 to -0.75 range, depending on the set of countries, level of aggregation, and time period considered, although estimates for Canada tend to be on the higher end of this range (closer to 0).[5] We find that the weighted-average of the CEREE estimates, based on the contribution of each shock to exchange rate variation over our sample period, falls in the 0.2 to -0.2 range, as does a comparable reduced-form EREE estimate calculated over the same period. However, individual CEREE estimates go beyond this 0.2 to -0.2 range, or the conventional 0 to -0.75 range found the literature. This suggests that it would be incorrect to assume that the weighted-average will always fall within either of these ranges, since instances where one shock dominates could push this weighted-average outside of these ranges at any given point in time.

The rest of the paper proceeds as follows. In Section 2, we describe a simple analytical framework to guide our analysis. In Section 3, we discuss our identification assumptions, the data, and the estimation technique. In Section 4, we report our estimation results. In Section 5, we describe robustness exercises. In Section 6, we discuss the implications of our results for issues raised in the literature. Section 7 concludes. The Appendix follows.

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[4] Some recent contributions to this literature include Alessandria and Choi (2019), Alessandria, Pratap and Yue (2013), Amiti, Itskhoki and Konings (2014), Bussière, Gaulier and Steingress (2020), Freund and Pierola (2012), Kohn, Leibovici and Szkup (2020), Leigh et al. (2015) and Mayer and Steingress (2020).

[5] For example, Leigh et al. (2015) estimate the EREE from aggregate exports for a set of sixty countries, finding an average value of -0.23. Bussière, Gaulier and Steingress (2020) estimate the EREE at the product level for several dozen countries, and find average estimates in the -0.35 to -0.47 range. For Canada, their estimates are lower at -0.11 to -0.28.

2 A simple analytical framework

Figure 1 depicts year-over-year growth in the Canada-US exchange rate and Canadian goods exports. From the figure, it is clear that the degree of co-movement between the two variables varies over time — while in most of the 1990s and in the mid-2010s the two series display a negative relationship, there are periods in the late 1980s and 2000s where the two series were seemingly moving in tandem. In this section, we set out a basic framework that explains why sometimes the relationship can be negative, and other times less so.

Changes in Canadian exports can be driven through three basic channels following a shock to the economy — (1) the *exchange rate* channel, (2) the *domestic export price* channel, and (3) the *foreign demand* channel:

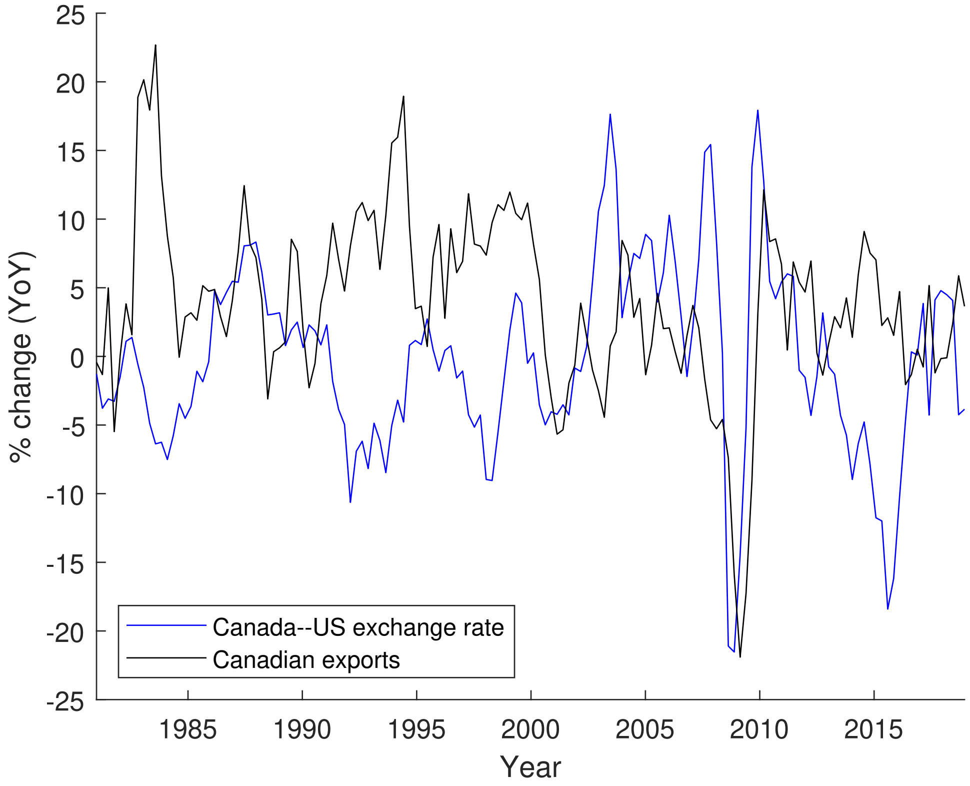
$$\Delta\,\mathrm{Exports} = f(\underset{-}{\Delta\,\mathrm{Exchange\ rate}}, \underset{-}{\Delta\,\mathrm{Domestic\ export\ price}}, \underset{+}{\Delta\,\mathrm{Foreign\ demand}}) \tag{1}$$

The nature of macroeconomic shocks and institutional arrangements, such as the predominant currency of invoicing for export goods, can affect the mix and magnitude of these channels, which in turn determines the degree of correlation between exchange rates and exports.

Consider the *exchange rate* channel. In a producer currency pricing regime, an appreciation of the Canadian dollar would lead to a rise in the foreign price of export goods, and through expenditure switching, real exports should decline. This channel implies a straightforward negative relationship between exchange rates and real exports. Now, consider the *domestic export price* channel. If a shock that generates an appreciation in the Canadian dollar also results in an increase (decrease) in the domestic export prices, then all else equal, the negative relationship between exchange rate and exports will be strengthened (lessened). Finally, any shock that affects *foreign demand* would have an independent impact on exports, where higher (lower) foreign demand is expected to lead to higher (lower) exports.

Standard theory informs our expectations regarding the mix of these channels in response to different shocks. For example, in a small open economy, foreign demand will be unaffected by domestic shocks,

Figure 1: Canada-US exchange rate and Canadian exports (% change)



Note: Depicts year-over-year growth in the Canada-US nominal exchange rate and Canadian goods exports and only the *exchange rate* and *domestic export price* channels will influence exports when these shocks arise.

As as specific example, consider a positive domestic demand shock in Canada. An increase in domestic demand would result in higher domestic prices and wages, which would put upward pressure on domestic input prices for exporters, and hence downward pressure on exports through the *export price* channel. At the same time, if the domestic monetary authority responds by endogenously increasing policy rates to keep inflation at bay, the exchange rate may end up appreciating, putting further downward pressure on exports through the *exchange rate* channel. Consider now a contractionary monetary policy shock in Canada. The increase in interest rates would reduce domestic prices and wages, and the domestic currency would appreciate. The former would put upward pressure on exports through the *export price* channel (inputs to production are cheaper), while the latter would put downward pressure on exports through the *exchange rate* channel.

Moreover, in both cases, imported input prices can have an important effect. An exchange rate appreciation would normally imply that imported inputs into production are cheaper, which if passed on to domestic export prices, would lead to an upward pressure on exports. Which of these effects dominates, and what the final relationship between exchange rates and exports will end up being, is therefore an empirical question.

Next, consider a shock to Canada's main trading partner — the US. If the US is hit by a positive demand shock, we expect the demand for Canadian exports to rise through the *foreign demand* channel. At the same time, an endogenous policy reaction in the US may result in a rise in US rates, which would put upward pressure on the US dollar, and hence a depreciation in the Canadian dollar. This would further boost Canadian exports through the *exchange rate* channel. If the US is hit by a positive supply shock, the result may be different. Higher income in the US may still imply higher demand for Canadian exports through the *foreign demand* channel. However, there is no clear reason to expect exchange rates to go in a certain direction. Moreover, if the positive supply shock is due to technological advancements that are readily transferable to Canadian production processes, this may result in a decline in *domestic export* *prices*, further bolstering Canadian exports.

The Canadian case is further complicated by the fact that Canada is an oil exporting country. An exogenous increase in global oil prices, due either to an actual or perceived future reduction in oil supply, would affect Canada through all the channels described above. A higher oil price would reduce foreign demand for Canadian non-oil exports, and increase domestic input prices in production — both of which would lead to lower exports. Moreover, higher oil prices give rise to an appreciation in the Canadian dollar, which further reduces exports through the *exchange rate* channel.

Institutional arrangements like the currency of invoicing can also affect the magnitude of these channels. Consider, for example, an extreme case where all of Canada's exports are invoiced in US dollars. In this case an exogenous appreciation of the Canadian dollar will not affect the foreign price of exports, hence the *exchange rate* channel will be muted, and the negative relationship between exports and exchange rates will be weakened. If imports are also invoiced entirely in US dollars, the appreciation can result in cheaper imported inputs, and a decline in domestic export costs. If exporters pass on these costs to lower domestic export prices, then the *export price* channel would suggest stronger exports, further weakening the negative relationship between exports and exchange rates.

Overall, the above discussion makes it clear that there is no reason to believe that the correlation between exports and the exchange rate should remain the same across different macroeconomic shocks. These conditional correlations are exactly our empirical objects of interest, which we call CEREEs.

3 Identification strategy

We focus on the perspective of a small open economy that is subject to a standard set of domestic and foreign shocks. Our empirical model uses Bayesian VAR estimation with short-run, long-run, and sign restrictions to identify each shock.

We apply the model to Canada. This country is an ideal candidate because it has well-defined institutional and economic features that allow for a set of credible identifying restrictions, including those used to identify US shocks.[6] Since over 70% of Canada's exports are to the US, we treat the US economy alone as the "foreign" block in our model, which simplifies data construction issues.

3.1 Data

We estimate our model using quarterly data spanning from 1981–2018. The starting year 1981 is dictated by consistent data availability from Statistics Canada. Our benchmark specification is an eight-variable VAR that includes Canadian real gross domestic product (GDP), consumer price index (CPI), monetary policy interest rate, real goods exports and goods-export price index, as well as the nominal Canada-US exchange rate, US real GDP, and the price of oil in US dollars. We also estimate a version of our model with non-energy goods exports, rather than total goods exports. Including commodity exports in our measure could introduce a confounding impact of commodity prices directly on exports, and makes it difficult to isolate the exchange rate channel effect on exports [Kilian, Rebucci and Spatafora (2009)]. We describe results for this model in Section 6 and in the Appendix.

The Canadian data comes from Statistics Canada. For real GDP, we use an expenditure-based chained series (Table 36-10-0104). For CPI, we use a monthly series (Table 18-10-0004) (aggregated to quarterly frequency). For real exports, we use an expenditure-based chained series from national accounts for goods exports (Table 36-10-0104). For export prices, we use the implicit price index for goods exports (Table 36-10-0106) associated with the national accounts goods exports series. For the monetary policy interest rate, we use a monthly series (aggregated to quarterly frequency) for the average ceiling for the target overnight interest rate, also known as the bank lending rate, publicly available from the Bank of Canada. For the nominal exchange rate, we use a monthly series (aggregated to quarterly frequency) for the average nominal spot Canada–US exchange rate, also available from the Bank of Canada.

For US real GDP, we use the chained series from Table 3 of the US National Accounts provided by the Bureau of Economic Analysis. Finally, for oil prices, we use a series for the average market price of West Texas Intermediate (WTI) in US dollars, provided by the IMF's International Financial Statistics.

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[6] Canada has maintained a flexible exchange rate regime since 1970; see Powell (2005).

Except for the series of interest rates, all series are transformed into quarterly log differences. For interest rates, we use the de-trended level.

For measures of the exchange rate and foreign activity, we rely strictly on US variables for several reasons. First, nearly all Canadian goods exports are denominated in US dollars (even those destined for outside the US), hence our view is that the bilateral Canada–US rate is most relevant for export transactions. Second, beyond the dominant currency issue, over 70% of Canadian goods exports are destined for the US market, so introducing non-US prices into a trade-weighted measure would not yield a significantly different variable from the simple bilateral measure. Third, given that integrating a multitude of variables into an index introduces problems related to standardization and other issues [Mayer and Steingress (2020)], we view the use of US variables as, on balance, a preferable choice.

3.2 Sign and zero restrictions

We identify seven shocks in our small open economy VAR model: domestic supply, domestic demand, domestic monetary policy, "exogenous" exchange rate, transitory US income, persistent US income, and oil price. The choice of these shocks is consistent with recent empirical literature on exchange rate effects in small open economies, such as Forbes, Hjortsoe and Nenova (2018), who focus on exchange rate pass-through to domestic prices in the UK. We identify these shocks through a combination of sign restrictions and both short-run and long-run zero restrictions. Since we identify seven shocks with eight economic variables, we are left with a shock that is unidentified. We describe the underlying assumptions below and summarize the imposed restrictions in Table 1.

As a small open economy, the three domestic shocks and the "exogenous" exchange rate shock in Canada are assumed to have no effect on US GDP in the short or long run, and no influence on WTI oil prices in the long run. In other words, we impose that the long-run effect of all four shocks are zero on US GDP and global oil prices. Furthermore, the short-run impact of all four shocks on the US GDP is also restricted to be zero.

Domestic supply shocks are imposed to have a positive impact on domestic GDP and a negative impact on domestic CPI inflation in the short run, with no restrictions on the long-run path of these impacts.[7] This sign pattern is consistent with technological changes that are persistent in nature, increasing the domestic economy's productive capacity and lowering equilibrium prices in the long run.

Domestic demand shocks are assumed to have a positive impact on both domestic GDP and domestic CPI inflation in the short run, which is a standard characterization in macroeconomic models. We follow Blanchard and Quah (1989) and Gali (1999) and impose that the impact on GDP converges to zero in the long run as demand shocks should, by definition, have no impact on long-run productive capacity. We impose that monetary policy reacts by increasing policy rates in response to the demand-driven increases in domestic output and inflation in the short run. Since the shock is orthogonal to global shocks in the framework, we finally impose that the increase in policy rates is accompanied by an appreciation of the domestic exchange rate in the short run. This combination of sign and zero restrictions is consistent with Forbes, Hjortsoe and Nenova (2018).

We define monetary policy shocks as having a positive impact on the policy rate, and a negative impact on domestic GDP and CPI inflation over the short run, consistent with the standard transmission in macroeconomic models [Christiano, Eichenbaum and Evans (2005)] and with empirical evidence [Romer and Romer (2004)]. As the classic interest parity condition proposes, a higher overnight interest rate in Canada is assumed to attract foreign capital, generating an appreciation in the Canadian dollar and an increase in the Canada–US exchange rate.

"Exogenous" exchange rate shocks are assumed to correspond with an appreciation in the Canadian dollar and a rise in the Canada–US exchange rate. This appreciation is assumed to pass-through to domestic CPI, generating negative inflation. We impose that the inflation targeting central bank reacts to the decline in domestic inflation by raising the policy rate rate in the short run. Similar to the logic for demand shocks, we impose that neither the domestic monetary policy shock, nor the "exogenous" exchange rate shock, has any impact on Canadian GDP in the long run.

We now focus on the remaining three foreign shocks. The persistent US income shock and transitory

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[7] We apply our short-run sign restrictions for two quarters, following Shambaugh (2008) and Forbes, Hjortsoe and Nenova (2018).

US income shock have identical assumptions in the short run: both are assumed to have a positive effect on US GDP and oil prices. The only distinction between these shocks is that the transitory shock is assumed to have no impact on US GDP or oil prices in the long run. This distinction conceptually separates "short-run" shocks, such as US demand shocks or US monetary policy shocks, from "long-run" shocks, such as US technological change. The important aspect, from our perspective, is that these two US shocks are distinct from oil price shocks and, together, represent what the existing literature refers to a "flow/aggregate/global demand shock," where a shock to global income leads to greater demand for oil and higher oil prices [Kilian and Murphy (2012), Kilian and Murphy (2014), Charnavoki and Dolado (2014)].

This allows us to identify our third foreign shock, the oil price shock, which is assumed to correspond to an appreciation in oil prices that is fundamentally exogenous to US GDP. As a net-oil importer, higher oil prices are assumed to have a negative impact on US GDP. This shock can be viewed as representing either an "oil supply shock" or an "oil-specific demand shock" based on the classification in Kilian and Murphy (2012), both of which are fundamentally exogenous with respect to US GDP, but impact US GDP in a negative way.[8]

3.3 Estimation

We estimate a reduced-form VAR model with two lags using Gibbs sampling Bayesian estimation. We invoke the classic Minnesota-style prior for the beta coefficient matrix. This prior formulation assumes that the variables included in the VAR can be modelled sufficiently well as a unit-root AR(1) process, where coefficients on all other variables' lags are set to zero. For our prior on the residual covariance matrix, we follow Forbes, Hjortsoe and Nenova (2018) and set the hyperparameters for the constant term, the own lag, the other variable lag, and the second lags to 10,000, 0.2, 0.5, and 1, respectively, and derive a diagonal matrix for our prior covariance matrix based on these parameter settings. We

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[8] We follow Charnavoki and Dolado (2014) in assuming that the sign restriction for the oil price shock applies after four quarters, which allows the impulse response to vary differently in the initial period after the shock.

then draw a posterior beta coefficient matrix from a Normal posterior distribution, and draw a residual covariance matrix from an Inverse Wishart distribution.

We obtain structural shocks from the reduced-form residual covariance matrix by applying the algorithm developed in Binning (2013) for under-identified models, which builds on the contributions in Rubio-Ramírez, Waggoner and Zha (2010). In this two-step algorithm, we first find a random rotation matrix via QR decomposition that satisfies the short- and long-run zero restrictions subject to the estimated error co-variance matrix. The zero restrictions are imposed on the short-run impact matrix and the long-run cumulative impact matrix using the rank conditions developed in Rubio-Ramírez, Waggoner and Zha (2010). In the second-step, we use the rotation matrix to generate impulse responses, subject to the estimated error co-variance matrix. If the resulting impulse responses satisfy all the specified sign restrictions, they are saved; otherwise they are discarded, and a new random rotation matrix is chosen.

We repeat this procedure 11,500 times, discarding the first 10,000 and keeping the last 500 simulations, which yields 500 separate impulse responses that each satisfy the restrictions imposed in Table 1. In our estimation, we follow Forbes, Hjortsoe and Nenova (2018) and impose that sign restrictions hold for two periods including the period when the shock occurs and the subsequent period,[9] and that long-run restrictions are imposed as long-run convergence criteria. In the end, for each simulation, our derived impulse response functions can be used to map out impulse responses for each endogenous variable k to any of our seven specified exogenous shocks j. For each shock, we then construct the ratio of the impulse response for exports (E) to the impulse response for the exchange rate (ER) to arrive at an estimate for the CEREE:

$$CEREE_T^j = \frac{CIRF_T^{E,j}}{CIRF_T^{ER,j}}, \tag{2}$$

where $CEREE_T^j$ denotes the CEREE conditional on shock j, T periods after the shock; and $CIRF_T^{k,j}$ denotes the conditional impulse response of variable k to shock j, T periods after the shock. Note that

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[9] One exception here is the oil price shock, where we follow Charnavoki and Dolado (2014) in assuming that the sign restrictions holds for the fourth period after the shock.

we can derive 500 estimates from equation (2), one for each simulation that we keep.

This structural estimation is compared with a standard reduced-form estimating equation of the following form:

$$\Delta ln(E_t) = \alpha + \sum_{k=0}^{T} \beta_k \Delta ln(ER_{t-k}) + \lambda \Delta X_t + \epsilon_t, \tag{3}$$

where E_t denotes exports in period t, and ER_t denotes the nominal exchange rate. The matrix X_t denotes a set of control variables.

The reduced-form EREE is calculated by the sum of the estimated β_k coefficients for all lags included in the regression (T), where we include four lags in our estimation:

$$EREE_T^{RF} = \sum_{k=0}^{T} \beta_k = \sum_{k=0}^{4} \beta_k \tag{4}$$

4 Transmission of shocks to exports and the exchange rate

The methodology described in Section 3 permits us to derive a band of impulse responses for each of the eight variables in the system, for each of the seven structural shocks that we identify, as well as the median of these bands. We also construct a band of CEREE estimates for each shock according to equation (2), and from that band we can report the median for each shock.

Before examining these impulse responses and CEREEs, however, it is useful to first look at a reduced-form estimate for an unconditional EREE based on equation (4). We estimate two versions of this equation. In the first version, we do not include any control variables and assume that all movements in the exchange rate are exogenous from the perspective of Canadian exporters. In the second, we include Canadian export prices to control for costs or markup changes that might be associated with the exchange rate change, and US GDP to control for any changes in foreign demand that might correlate with the exchange rate change.

Our reduced-form estimates conform closely to standard estimates of the EREE for Canada. These reduced-form estimates are reported in Figures 2 and 3. Both are negative in the first period, but converge

Figure 2: EREE, reduced form

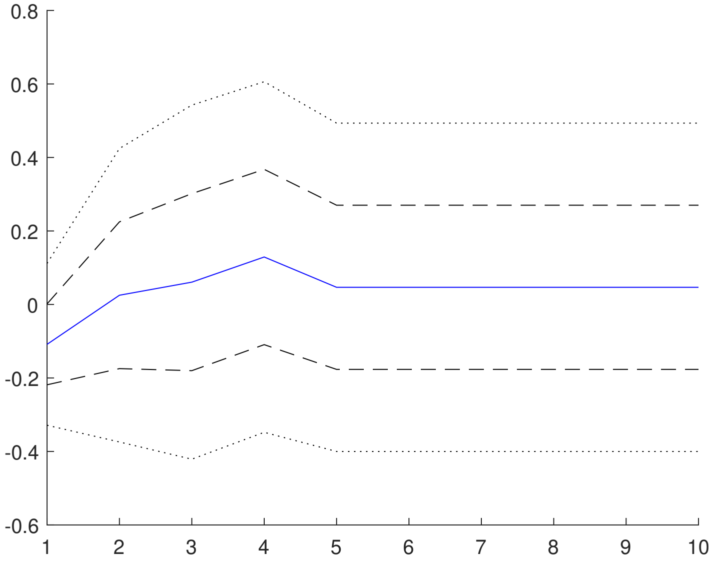
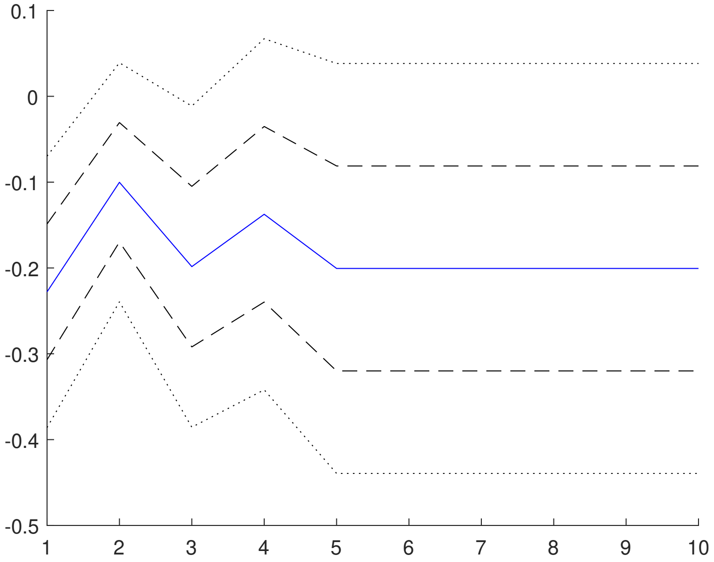


Figure 3: EREE, reduced form (w controls)



Note: Reduced-form estimates for the EREE based on equation (4) without controls (Figure 2) and with controls (Figure 3), where the dashed and dotted lines represent 65% and 95% confidence bands, respectively.

towards zero after several quarters. Only the estimate in Figure 3 is statistically significant at the 65% or 95% confidence level, and this too converges to lower significance in the longer run. These estimates are comparable to findings in the recent literature. For example, Leigh et al. (2015) estimate the EREE using aggregate exports for a set of sixty countries, and report an average long-run value of roughly -0.33. Bussière, Gaulier and Steingress (2020) find average values in the -0.35 to -0.47 range for a set of several dozen countries, and within the -0.11 to -0.28 range for Canada. The latter is particularly consistent with our estimates presented in Figure 3.

In the next sub-section, we discuss the CEREEs and other impulse responses from our set of structural shocks, starting with the domestic shocks.

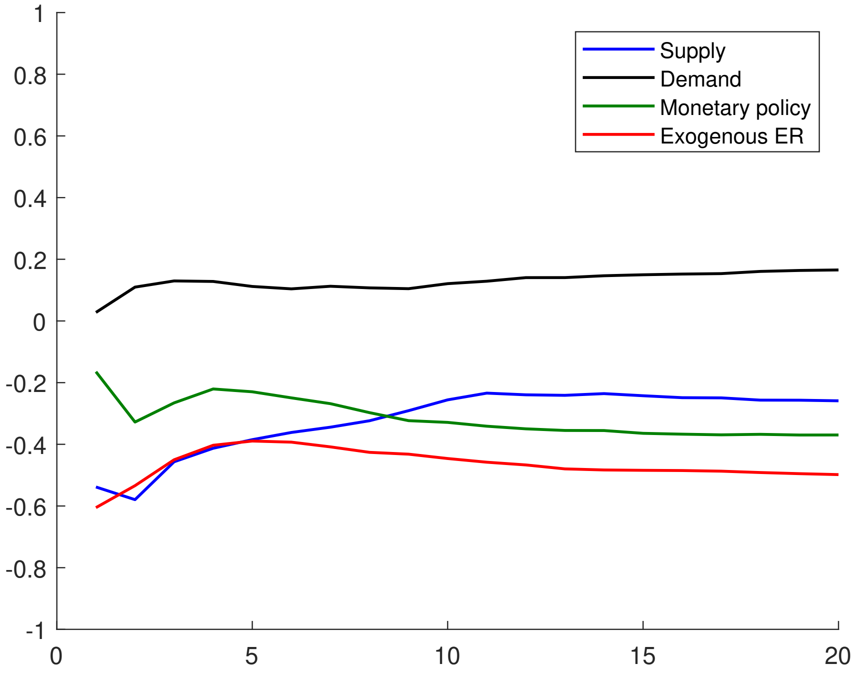
4.1 Domestic shocks

Figure 4 shows median estimates for the CEREEs conditional on orthogonal shocks to domestic supply, demand, monetary policy, and the exchange rate, calculated according to equation (2). We see that for three of the four shocks, the median CEREE estimate falls within the -0.1 to -0.7 range in the short run, and -0.2 to -0.5 range in the long run, in line with the findings in Leigh et al. (2015) and slightly stronger than the Canada-specific numbers in Bussière, Gaulier and Steingress (2020). However, the CEREE estimate for the demand shock is small and positive.

To understand the difference between these CEREEs, we refer to the impulse responses for each the variables included in the SVAR, for each domestic shock, presented in Figure 5. The dark solid line reports the median of the impulse responses, and the dotted lines indicate the 68% coverage bands.

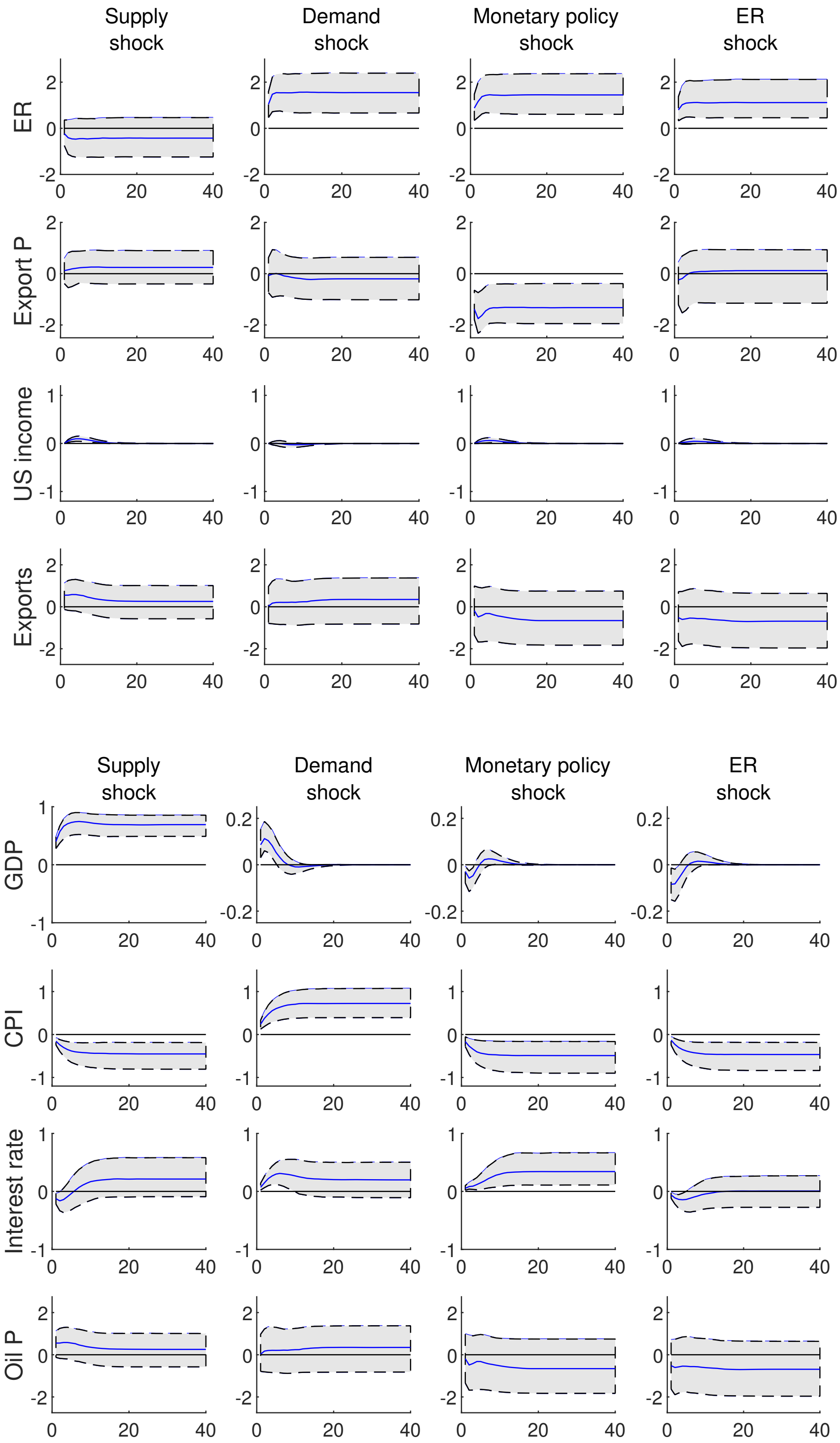
The domestic supply shock is defined to have a positive response for GDP and a negative response for CPI, with the possibility that the effect on GDP can persist in the long run. Of course, the effect on US income is restricted to be zero in the short and long run and the effect on oil prices is restricted to be zero in the long run. We remain agnostic about the responses of the remaining variables. We see that the monetary authority responds to the supply-driven decline in prices by lowering the policy rate initially, and eventually raising it. The lower interest rate results in a depreciation in the exchange

Figure 4: Main results: CEREE estimates from domestic shocks



Note: Median estimate for the CEREE, based on equation (2), for each of the four domestic shocks

Figure 5: Main results: Responses to domestic shocks



Note: Median estimates for the impulse response of each variable (rows) to each domestic shock (columns) based on the SVAR described in Section 3, where dashed lines represent 65% confidence bands.

rate (in our configuration, a decline in the IRF corresponds to a depreciation of the CAD), but one that is not significant judging by the range from the confidence bands. From our discussion in Section 2, we know that a domestic shock does not invoke a response through the foreign demand channel, and that the exchange rate channel implies that a depreciation of the CAD should lead to an increase in exports. Figure 5 shows that the identified supply shock leads to a mild increase in export prices, which may dampen the positive response on exports, especially in the long run. Overall, however, our results suggest that the exchange rate channel dominates since the estimated CEREE is negative. This finding is, however, surprising because the exchange rate response is not very significant in this case, and therefore other factors beyond the ones outlined in Section 2 could be at play.[10]

The domestic demand shock, by construction, results in a temporary rise in GDP and inflation, as well as an increase in the policy rate and the Canadian dollar. Through the exchange rate channel, the appreciation of the dollar is expected to lead to a decline in exports. However, domestic prices of exported goods decline slightly, suggesting a mitigation in the expected boost in exports. Interestingly, exports do not decline, as one would expect. Rather, exports modestly rise, leading to a slightly positive CEREE.

Like the domestic demand shock, the domestic monetary policy shock leads to an increase in the exchange rate, which should yield lower exports. Export prices decline in response to the shock. In terms of prices, the key difference in this case relative to the demand shock is that domestic prices (CPI) fall in response to the shock, and hence both domestic and imported input cost channels lead to lower export prices. Overall, higher exchange rate should lead to lower exports, and lower export prices should lead to higher exports, so we have counteracting effects on balance. The export response turns out to be negative, suggesting that the exchange rate channel could be dominant here. However, the bands around this export response are broad, as they are for the response from the domestic demand shock, so this is not a strong result.

The "exogenous" exchange rate shock corresponds to an increase in the exchange rate, which should lead to lower exports all else equal. By assumption, there is no change in foreign income. Although

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[10] One possibility here is positive supply shocks are associated with quality upgrading. This could correspond with higher export prices and generate higher exports due to factors unrelated to the exchange rate.

numerous channels could potentially affect export prices (see Section 2), we see no significant change in export prices. Hence, there is no operative channel that should meaningfully counteract the exchange rate channel. The export response is negative, with an implied median CEREE of between -0.3 and -0.4. But, from Figure 5 we see clearly that the bands around the export response are again large, suggesting that the statistical precision of here is not high.

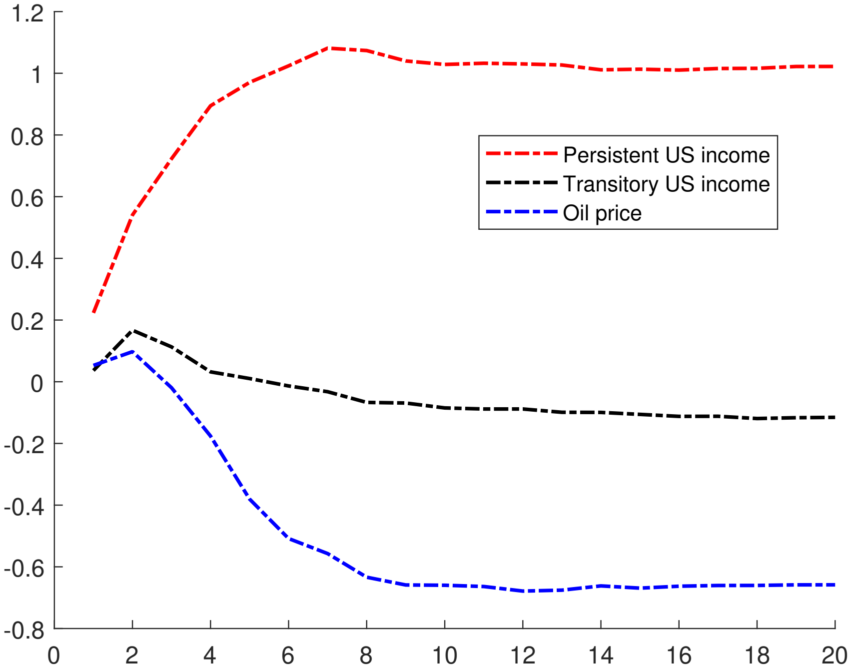
Overall, for the domestic demand shock and the "exogenous" exchange rate shock, export prices do not respond much to counteract the exchange rate channel, and yet the export response is either slightly positive (demand shock) or negative but statistically insignificant (exchange rate shock), suggesting that the exchange rate channel may not be very strong in these cases. One explanation for this finding could rely on the dominant currency paradigm [Gopinath et al. (2020)]. Since most Canadian exports are invoiced in the US currency, US import prices for Canadian-produced goods do not change much despite fluctuations in the Canadian currency. As a result, exports do not change much, and the CEREEs are muted.

But how important are these shocks in explaining the volatility of the Canadian dollar? The importance of each domestic shock in explaining exchange rate variation over our sample period is given in the factor error variance decomposition (FEVD) reported in Table 2. Comparing these shocks, demand shocks explain more variation than any other shock (roughly 20%); monetary policy shocks and "exogenous" exchange rate shocks are also somewhat important (roughly 14% and 12%, respectively). Explained exchange rate variation from supply shocks is comparatively small at 4%. Overall, a positive (or negligible) CEREE for demand shocks, combined with its importance in explaining exchange rate movements, and relatively modest CEREEs for monetary policy shocks and "exogenous" exchange rate shocks, can help explain why our reduced form EREE estimates in Figures 2 and 3 are so low.

4.2 Foreign shocks

We now turn to the median estimates of the CEREEs for the foreign shocks, presented in Figure 6. Remarkably, the CEREE from the persistent US income shock is positive: the sign of this estimate is opposite from the conventional EREE estimate found in the literature. In contrast, the CEREE from the oil price shock is negative and lower than -0.6 in the long run — stronger than the average reduced form EREE estimate for Canada found in the literature. To understand these differences, we refer to the impulse responses for all the variables included in the VAR estimation, presented in Figure 7, and the three factors outlined in Section 2.

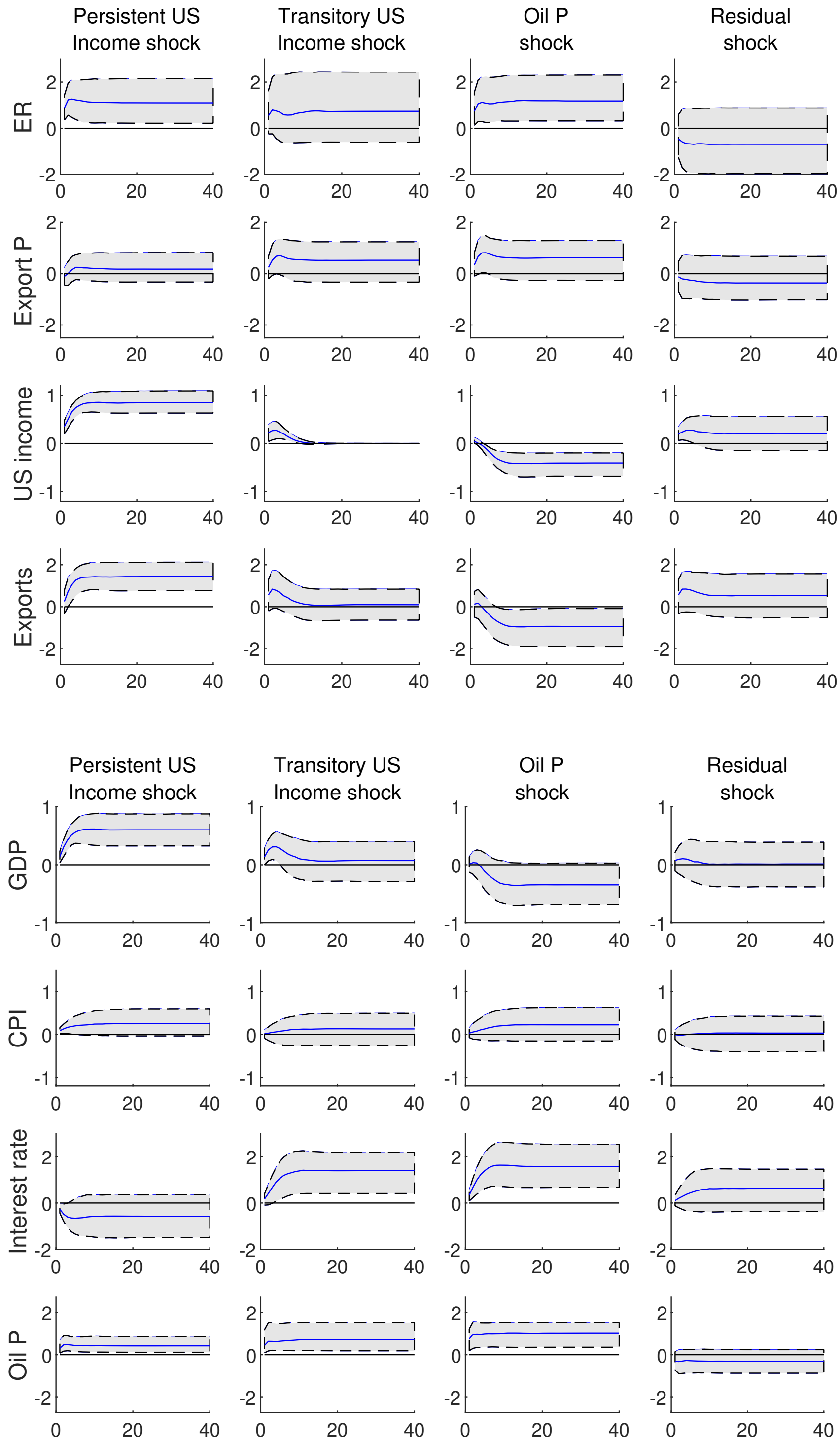
Figure 6: Main results: CEREE estimates from global shocks



Note: Median estimate for CEREE, based on equation (2), for each of the three global shocks.

The persistent US income shock leads to an increase in the exchange rate that, all else equal, is expected to generate a decline in exports. In contrast to the domestic shocks, in this case US income is affected by the shock and, consistent with assumptions, increases significantly. This should lead to more demand for exports and export growth, which counters the direct effect of the exchange rate. Export prices rise slightly after the shock, consistent with an increase in domestic input costs due to higher output growth and domestic price inflation, as well as higher global oil prices. Importantly, oil prices

Figure 7: Main results: Responses to global shocks



Note: Median estimates for the impulse response of each variable (rows) to each global shock (columns) based on the SVAR described in Section 3, where dashed lines represent 65% confidence bands.

rise after this shock and, since Canada exports oil, this direct effect leads to higher export prices in this case.[11] In the end, higher export price growth and exchange rate appreciation should lead to lower exports, supporting the direct effect of the exchange rate and domestic export price channels. However, neither of these factors is large enough to counteract the foreign demand channel, as Canadian exports rise after the persistent US shock despite their higher price.

The transitory US income shock yields similar responses as the persistent the US income shock, although mostly less pronounced. We observe a mild increase in the exchange rate, and a smaller and less persistent increase in US income. These two factors have opposite impacts on exports. Export prices increase after the shock. Finally, oil prices increase significantly, which should contribute to domestic input cost growth. Overall, exports rise after the shock, although only temporarily, suggesting that the US income factor has a larger influence on exports than the export price and exchange rate factors in this instance.

The oil price shock leads to an appreciation in the exchange rate that should put downward pressure on export growth. The shock leads to a decline in US income that, like the exchange rate factor, should lead to lower exports. In terms of export prices, both domestic inflation and oil prices increase after the shock, and therefore the domestic input cost channel is placing upward pressure on export prices. The import input channel should be placing downward pressure on export prices due to exchange rate pass-through. On balance, export prices rise after the shock, indicating that the impacts of inflation and oil prices dominate the imported input cost channel in this case. In terms of the three factors summarized in (1), all three are working to lower exports in this instance and, predictably, exports decline after the shock.

To get a sense of the importance of foreign shocks in explaining exchange rate movements, we again refer to Table 2. All three of the foreign shocks are relatively important, with none of them explaining less than 10% of the total exchange rate variation. The persistent US income shock is most important among these, explaining 16% of the variation. The oil price shock and transitory US income shock explain 13%

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[11] If we look at the non-energy export price response in Figure 8, we observe that these prices fall slightly after the persistent US income shock. We discuss this result in Section 5.

and 10% of the total exchange rate variation, respectively.

Overall, we stress several takeaways from these foreign shock results. Compared to the domestic shocks, foreign shocks generally have a significant impact on both the exchange rate and exports. Hence, to understand and quantify the relationship between exports and the exchange rate, it is crucial to focus on shocks that are foreign in nature. It is also clear that the foreign demand channel can easily overturn the effects of the exchange rate and domestic export price channels. The evolution of US income is therefore a key factor in driving Canadian exports when foreign shocks impact the economy, and hence the response of this variable offers the most guidance for understanding the export response during these episodes. Like for the domestic shocks, we find no firm evidence that the exchange rate channel is playing an influential role in determining the path of exports after foreign shocks.

5 Robustness

We conduct two robustness exercises. First, we replace total Canadian exports with Canadian non-energy exports. Since Canada is a net exporter of oil, and one of the shocks that we consider is an oil price shock, it makes sense to consider whether exports excluding oil behave differently from total exports after these shocks.

Results from this exercise are reported in Figure 8. For responses to domestic shocks, these results are similar to results from Figure 5 for total goods exports, hence we will not discuss these in detail.[12]

For responses to global shocks, the exchange rate response is positive for both US income shocks and the oil price shock, similar to what was found for estimates using total exports. This channel should lead to lower non-energy exports in all three cases, due to the exchange rate channel discussed in Section 2.

Non-energy export prices respond negatively to persistent and transitory US income shocks, in contrast to the muted or positive response of total export prices to these shocks. These negative responses

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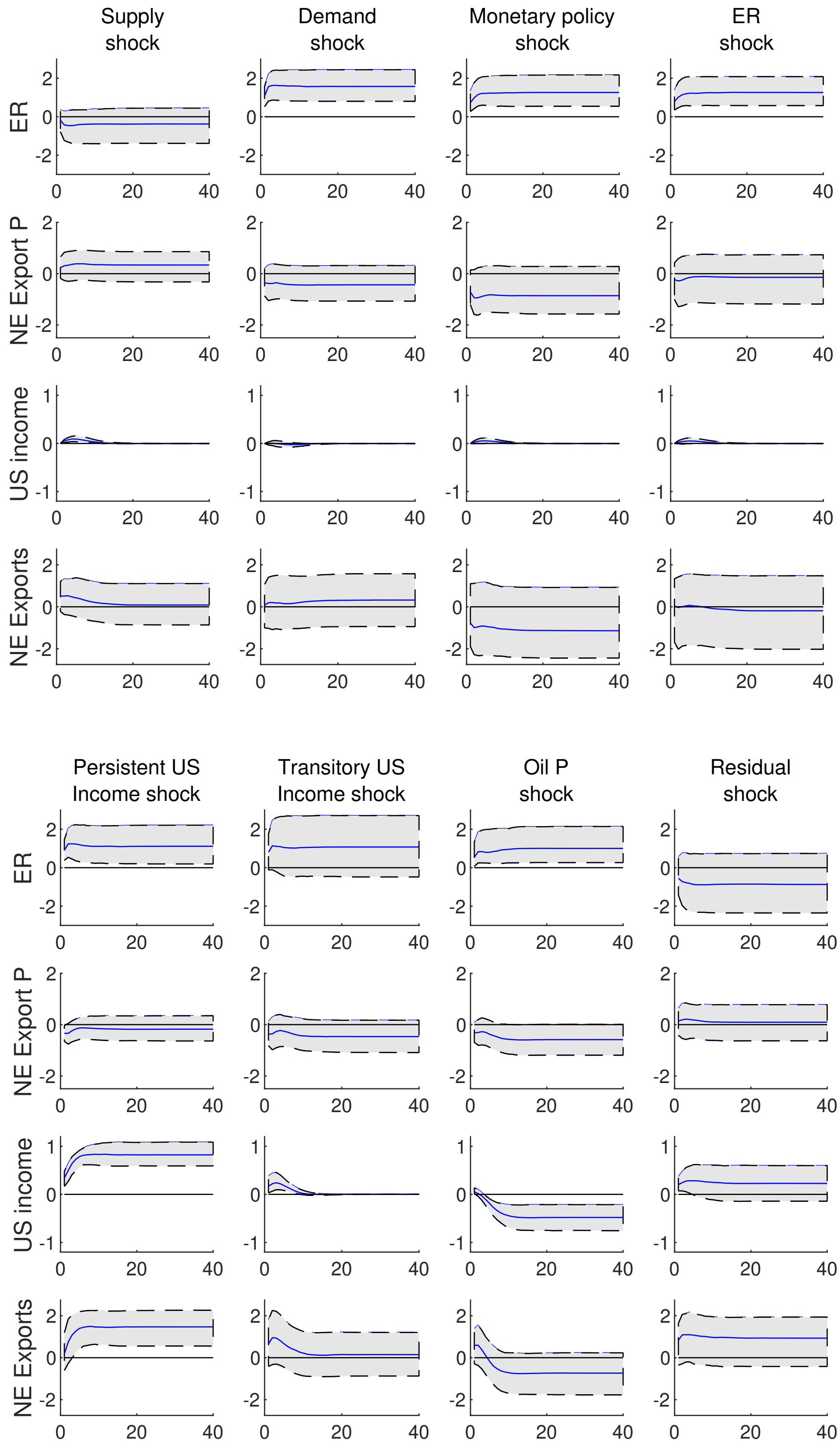
[12] The similarity of our results for these two cases is to be expected, since the domestic shocks have very limited effects on oil prices (by assumption), and hence non-energy export prices and quantities should not respond in a manner that is significantly different from total export prices and quantities.

Figure 8: Robustness: Non-energy exports responses to shocks



Note: Median estimates for the impulse response of each variable (rows) to each shock (columns) based on the SVAR described in Section 3, where dashed lines represent 65% confidence bands.

should place upward pressure on non-energy exports due to the export price channel, and therefore limit the negative impact of the exchange rate appreciation on non-energy exports. The US income response is, by assumption, positive for both the persistent and transitory US income shocks, and of a similar magnitude as found for total exports. Higher US income should lead to higher non-energy exports due to the foreign demand channel.

Looking at the response of non-energy exports to these two shocks, we find it is positive and significant, and of a similar magnitude as was found for total exports in Figure 5. This result is somewhat surprising since non-energy export prices respond differently than total export prices after these shocks, and the other channels that affect exports respond similarly across these two estimations. This suggests that the export price channel is not very important in these cases, and hence these results provide further support to the conclusions drawn from Section 4, that the foreign demand channel is the dominant driver of Canadian exports when foreign shocks affect the economy.

In response to the oil price shock, non-energy export prices decline substantially in Figure 8, in clear contrast to the positive response of total export prices to oil prices shocks reported in Figure 5. These differences reveal that oil prices are indeed directly driving higher total export prices in periods when oil price shocks occur, and therefore separately looking at responses for non-energy export prices and quantities is a useful exercise. Lower non-energy export prices should encourage higher exports through the export price channel, and thereby dampen the negative effect of the exchange rate appreciation on non-energy exports. The response of non-energy exports is negative, and only slightly less negative than the response of total exports to oil price shocks that we discussed in Section 4.

For the total exports cases, all three channels defined in Section 2 — the exchange rate channel, the export price channel, and the foreign demand channel — were placing downward pressure on exports. In the non-energy export cases, the exchange rate channel and the foreign demand channel are placing downward pressure, but the export price channel is placing upward pressure on exports, and yet the realized export response to the oil price shock is quite similar for the total export and non-energy export cases. This result suggests that the export price channel does not materially alter the path of exports when oil price shocks affect the Canadian economy, a message that is consistent with our main findings described in Section 4.

As a second robustness exercise, we estimate our model replacing total Canadian exports with total Canadian imports. This set of results is particularly useful for evaluating the performance of the model outlined in the Appendix, since in several cases the model predicts different responses for exports and imports. We discuss these results in greater detail in the Appendix.

6 Discussion

In this section, we review how our results relate to several prominent issues discussed in the literature. Specifically, we evaluate whether our findings are consistent with the weak reduced-form EREE found in previous studies, whether our results account for changes in the EREE found over time, and what our results imply for the relationship between oil prices and exports and for the benefits of a flexible exchange rate regime.

6.1 The weak reduced-form EREE

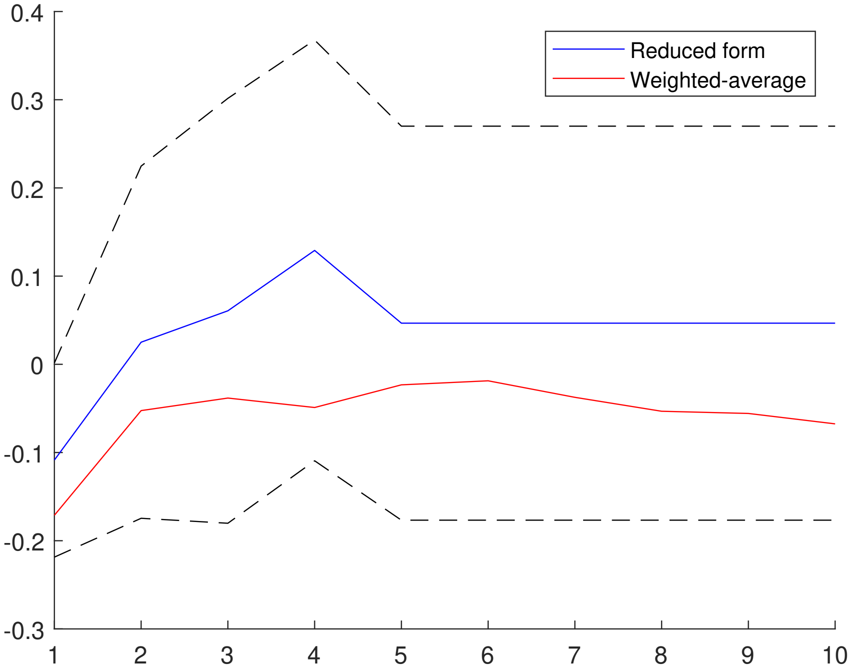
As reported in Figures 1, 2 and 3, and discussed in Section 2, the reduced-form EREE for Canada is very low and, in cases where confounding variables are not controlled for, statistically insignificant.

Using the FEVD shares reported in Table 2 for each shock j, denoted by $FEVD^j$, and the shock-specific impulse responses for each period k after the shock, reported in Figures 4 and 6 and denoted by $CEREE_k^j$, we construct a weighted-average impulse response function for the CEREEs based on the following formula:

$$CEREE_T^W = \sum_{k=0}^{T} \sum_{j} \left(FEVD^j\right)\left(CEREE_k^j\right) \tag{5}$$

We construct the impulse response from period 0 to 10 based on (5), and report the result in Figure 9. The figure includes both this constructed weighted-average and the reduced-form EREE reproduced from

Figure 9: Reduced-form EREE vs. weighted-average of the CEREEs



Note: The reduced-form estimate for the EREE, reproduced from Figure 2 where the dashed lines represent 65% confidence bands, and the weighted-average of the CEREEs based on equation (5).

Figure 2. The weighted-average of the CEREEs is lower than -0.1 in the initial period, but then converges to roughly zero in the long run. The reduced-form EREE is initially about -0.1, and soon converges to above zero before drifting back to about zero over the long run. The patterns for these two impulses are remarkable similar. This is not surprising, since the reduced-form estimate without controls is expected be close to a weighted-average of the structural estimates, but nevertheless we consider the similarity here encouraging from the perspective of model validity.

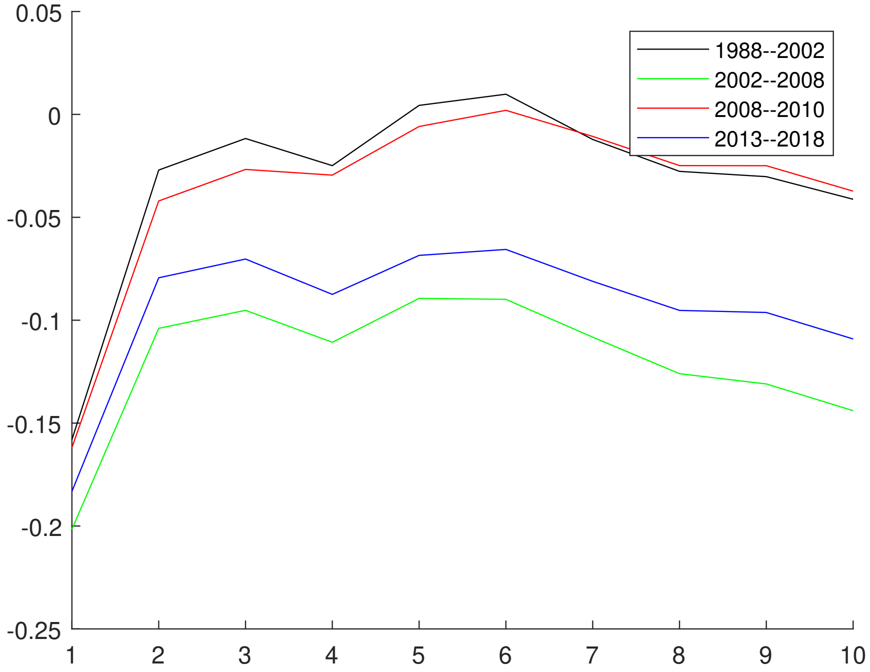
6.2 Changes in the EREE over time

A key finding in our analysis is that, despite the fact that the reduced-form EREE (and the weighted-average of the CEREEs) over our sample period is essentially zero, individual CEREEs can be significantly above or below zero depending on the shock.

This insight is well-reflected in the plotted relationship between the two variables in Figure 1 . In the figure, there are some periods where exports and the exchange rate are negatively correlated, like from 1990–1995 and from 2013–2016, and others where they seem to be positively correlated, like from 2008–2010.

To shed more light on this, we construct period-specific estimates of (5) for several key sub-periods of interest: the Canada–US FTA period of 1988–2002, the commodity price boom of 2002–2008, the global financial crisis period of 2008–2010, and the post-recovery period of 2013–2018. These estimates are reported in Figure 10. The weighted-averages are relatively similar, but different in ways that are intuitive given the patterns apparent in Figure 1. During the global financial crisis period of 2008–2010, the weighted-average is relatively high and close to zero in the long run. This reflects the importance of persistent US income shocks during this period, which generate a positive CEREE as reported in Figure 6. In contrast, during the commodity boom of 2002–2008 and the commodity crash of that occurred in the 2013–2018 period, oil price shocks are more important in driving movements in the exchange rate and, since these shocks generate a negative CEREE, the weighted-average is comparatively low. Finally, during the Canada–US FTA boom of 1988–2002, results suggest that the weighted-average was

Figure 10: Weighted-average of the CEREEs, notable periods



Note: The weighted-average of the CEREEs is based on equation (5), where this average is constructed over different notable periods.

relatively high, suggesting that the relatively low exchange rate during this period was not corresponding to significantly higher Canadian exports.

6.2.1 Secular decline in the EREE

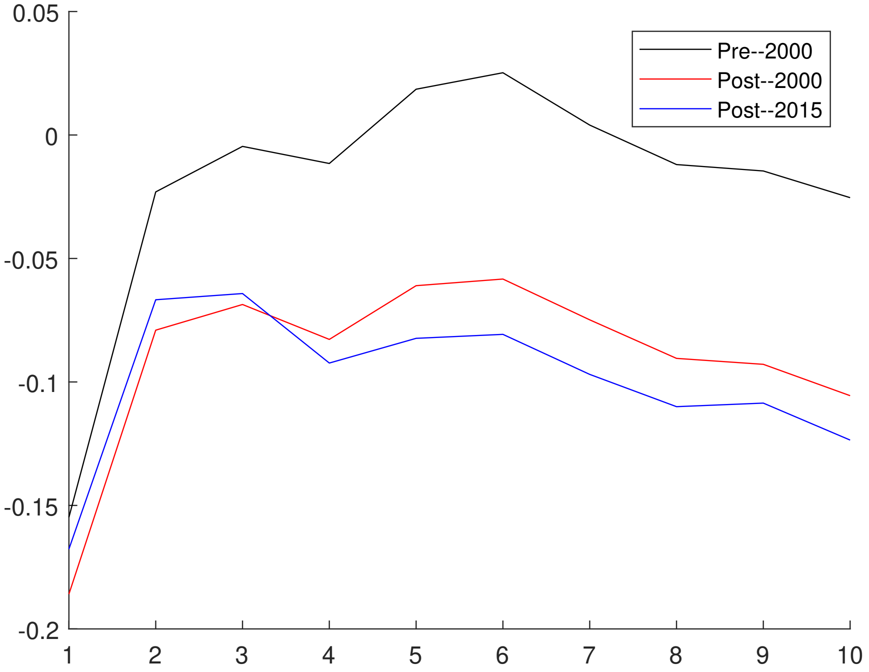
Another topic that our results relate to is the secular decline in the EREE over time. This theory became widely discussed during the 2013–2015 period, when oil prices experienced a large decline. At that time, policy-makers in net commodity exporting countries anticipated a boost to exports due to the exchange rate depreciation that coincided with the commodity price decline. However, global export growth was very tepid throughout this period, inspiring some, including Leigh et al. (2015), to investigate whether the EREE was weaker in the post-2000 period than it had been prior to this.

There are several reasons why exports might be less responsive to exchange rate movements in recent years. Many studies have found that the pass-through of exchange rates to prices has declined over time, and a variety of factors have been seen as contributing to this. If prices are less responsive to exchange rates, it stands to reason that exports might be as well. Related to this, the rise of global value chains implies that exchange rate movements have balancing effects on exports and imports, leaving a moderate net effect on the international price (inclusive of exchange rate effects) for any firm participating in these chains [Amiti, Itskhoki and Konings (2014), Swarnali, Maximiliano and Michele (2017) and Ollivaud, Rusticelli and Schwellnus (2015)]. In the Canadian context, export growth under-performed compared to expectations throughout the 2014–2017 period despite significant depreciation in the Canadian dollar [Brouillette et al. (2018)].

Our framework provides another possible explanation for weakness in the EREE in more recent years: the shocks that have driven exchange rate movements in recent years might be those that give rise to less-negative CEREEs than the shocks that drove exchange rate movements in the past. For example, if persistent US income shocks are more important in recent years, opposite responses from the conventional sign found in the literature are to be expected (see Figure 7).

To investigate this possibility, we again construct period-specific estimates of (5), but this time for

Figure 11: Weighted-average of the CEREEs, over time



Note: The weighted-average of the CEREEs based on equation (5), where this average is constructed over different time periods.

the pre-2000, post-2000, and post-2015 periods, and construct corresponding impulse responses for the weighted-average of the CEREEs, reported in Figure 11. From the figure, it is apparent that the long-run weighted-average is becoming more negative over time, not more positive. This is intuitive since, as revealed in Table 2, oil price shocks have become more important, and persistent US income shocks less important, over time, and the former does relatively more to push the overall weighted-average into negative territory.

These results suggest that the composition of shocks that have driven exchange rate movements have not contributed to a higher (less negative) EREE over time for the Canada, and in fact provide evidence of a stronger negative EREE in recent years than in the past.

6.2.2 The international elasticity puzzle

Several studies have found disparities between estimates of the export response to exchange rate movements and the export response to other international shocks, such as changes in tariffs and export prices [Fontagné, Martin and Orefice (2018), Fitzgerald and Haller (2018), and Ruhl (2008)]. These studies typically find that the elasticity of exports in response to changes in tariffs is lower than -2, but that the elasticity of exports in response to exchange rate movements is in the 0 to -0.75 range, whether measured at the aggregate level or firm level. Differences across these types of shocks should not exist according to standard theory, and so these disparities are deemed a puzzle, coined the "international elasticity puzzle."

In our analysis, we do not attempt to isolate the *ceteris paribus* elasticity of exports to exchange rate (or any other international price) movements. Instead, a fundamental message in our analysis is that estimates for the relationship between exchange rates and exports are generally affected by third variables. Hence any attempts to estimate a deep parameter to reflect the *ceteris paribus* effect of exchange rate movements on exports will typically face issues of endogeneity bias. Other studies try to overcome this endogeneity bias by using natural experiments such as exogenous exchange rate shocks [Auer, Burstein, Erhardt and Lein (2019)] or by controlling for endogenous variables by either including observable proxies or including fixed-effects in regression analysis [Bussière, Gaulier and Steingress (2020)].

Our approach has two distinguishing features. First, we accept this endogeneity, and provide an accounting of the driving forces of the exchange rate, exports, and their covariance. Through this approach, our results are able to speak to the issue of whether the *ceteris paribus* elasticity of exports to exchange rate movements is the governing factor in determining the relationship between exchange rates and exports. Second, we specify an "exogenous" exchange rate shock that is unrelated to domestic GDP in the long run and, by definition, orthogonal to all other shocks that we identify, and hence free from many of the endogeneity biases that are typically described in the literature.

Together, these two features of our approach provide a common message. We find little evidence that the *ceteris paribus* effect of exchange rate movements on exports determine the path of exports in the midst of exchange rate changes. In general, this effect appears to be either very small or insignificant, and other factors such as foreign demand are more important. In the case of the "exogenous" exchange rate shock, our estimate of the CEREE is negative, but statistically insignificant. In terms of magnitude, our point estimate is roughly -0.3 to -0.4, which falls squarely in the range found in other existing studies [Bussière, Gaulier and Steingress (2020)] and is much smaller than long-run estimates for the price elasticity of import demand found in the literature more broadly, which are on average around -4. As such, we conclude that the international elasticity puzzle lives on through our results.

In the Appendix, we calibrate a model based on Gopinath et al. (2020), where the partial elasticity of import demand is set to -2, but the correlation between exports and the exchange rate after an "exogeneous" exchange rate shock is higher than -1 (closer to 0) in general equilibrium. In that model, dominant currency pricing is a key factor that leads to a weak relationship here, but other factors could be at play in explaining the puzzle more broadly.

6.3 The relationship between oil prices and exports

A perennial question for small-open commodity-exporting economies pertains to how oil price increases impact exports. The typical concern is that higher oil prices lead to exchange rate appreciation that raises the foreign price of exports and, in turn, leads to lower exports.[13]

Our findings suggest that whether or not an oil price appreciation corresponds with lower exports depends crucially on the source of the oil price movement. If oil price appreciation stems from an oil price shock (e.g., supply-driven shock), then higher oil prices lead to a higher exchange rate and lower exports. On the other hand, if oil price appreciation is driven by a US income shock (transitory or persistent), then higher oil prices correspond with a higher exchange rate but higher exports (see Figure 6). The crucial difference between these two cases is that oil price shocks lead to lower US income growth, whereas US income shocks correspond, by definition, to higher US income growth. In both cases, US demand growth appears to drive Canadian exports growth, and since US demand is moving in different directions in the two cases, exports also grow in different directions. Table 2 reports the factor error variance for oil prices based on our model, where roughly 44% of oil price variation originates from oil price shocks, compared to roughly 30% for US income shocks (persistent and transitory). These results suggest that concerns over the negative impact of oil price booms on the Canadian export sector are often valid, however the source of the export weakness is not the exchange rate channel, but rather weak foreign demand.

Although the existing literature includes both supportive and un-supportive findings regarding the Dutch Disease,[14] findings from Charnavoki and Dolado (2014) warrant special attention as they focus specifically on Canada. Similar to our study, these authors take inspiration from Kilian and Murphy (2012) and Kilian and Murphy (2014) in using sign restrictions to separate global shocks into a demand-driven shock, a supply-driven shock, and a commodity supply-driven shock. Crucially, they find, as we do, that distinguishing between these shocks is highly relevant for understanding how exports perform in the midst of oil price movements. Their results are largely consistent with ours. They find that when oil prices increase due to a demand-driven global income shock, the exchange rate appreciates but Canadian exports increase rather than decrease, which is similar to what we find after a persistent US income shock.

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[13] For examples of media articles that discuss this narrative in the Canadian context, see Vallée and Bimenyimana (2011) and Cross (2013).

[14] See Stijns (2003) for a supportive finding, and Spatafora and Warner (1995) for an un-supportive finding.

These authors also find, as we do, that when oil prices increase due to an oil price shock, Canadian exports decline in the face of exchange rate appreciation. This consistency is particularly noteworthy given that Charnavoki and Dolado (2014) use a different empirical approach from ours.

6.4 Exchange rate flexibility and exports

According to the conventional wisdom, as described in Friedman (1953), a flexible exchange rate promotes economic stability in small open economies by (i) aiding the adjustment to external shocks and (ii) facilitating domestic monetary policy independence and transmission.[15]

The first benefit relies, in part, on the exchange rate channel described in Section 2, and we find that this channel as has limited influence in the Canadian context. For example, in events where a negative US income shock leads to lower global oil prices and a depreciation in the Canadian dollar, we find that the exchange rate channel is not sufficiently strong to raise export growth to support economic output. For this reason, (i) appears to be not as potent for Canada as the conventional wisdom assumes. In terms of (ii), we do not provide a fulsome examination of Canadian monetary policy independence across exchange rate regimes, but we do identify a monetary policy shock and study the responses to it. One channel of monetary policy transmission, according to standard models, is through the exchange rate channel, where a monetary policy shock leads to an increase in the exchange rate, which generates a decrease in exports and contributes to lowering domestic GDP. As reported in Figure 4, exports do contract as the exchange rate appreciates after a monetary policy shock, however this response is not terribly significant. The shock also leads to a depreciation in export prices, and this should also affect exports in the opposite way from the exchange rate appreciation. The fact that exports decline despite this confounding factor suggests that the exchange rate channel is having some effect here, however the net effect is not statistically significant.[16] These findings relate to recent evidence from Corsetti,

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[15] Some of these arguments, as well as others, are provided in Schembri (2019).

[16] Note that in Figure 5, the monetary policy shock gives rise to lower prices and lower GDP, hence our findings are consistent with the overall independence and effectiveness of Canadian monetary policy. Moreover, the weakness of the exchange rate channel that we find in some cases does not necessarily imply that there are not benefits to exchange rate flexibility. As formalized in Egorov and Mukhin (2020), a degree of exchange rate flexibility is optimal for welfare in small

Kuester, Müller and Schmidt (2021) who examine the impact of Euro-zone-originated shocks on external economies. They find that exchange rate flexibility has limited bearing on adjustment to these shocks, and further articulate a model where flexible exchange rates have little short- to medium-run impact on exports due to dominant currency invoicing.[17] Our results suggest that exchange rate flexibility has a bearing on adjustment to some shocks, like monetary policy shocks, but overall we find little evidence that the exchange rate channel is sufficiently important to deliver responses that are on the order of what would be expected in standard models.

Our study finds suggestive evidence that dominant currency invoicing could also be playing a significant role for Canadian exports. Among the shocks that we identify, two of them — the domestic demand shock and the "exogenous" exchange rate shock — lead to a significant increase in the exchange rate with no significant change in other potential factors that influence exports, export prices, and foreign demand. As such, these two shocks provide strong identification for the *ceteris paribus* impact of exchange rate movements on Canadian exports. What we find is that there is no statistically significant change in exports after either of these shocks, which implies that either the exchange rate channel is weak due to a low partial elasticity, or that dominant currency pricing prevents the exchange rate channel from fully operating. We suspect that the latter is driving the results for Canada, as empirical studies provide evidence that that the majority of Canadian exports are invoiced in US dollars [Cao, Dong and Tomlin (2015), Gopinath (2015)]. To provide a more rigorous treatment of this view, in the Appendix we calibrate a version of the model from Gopinath et al. (2020) to closely match the evidence on US-dollar invoicing for Canadian trade, and simulate several shocks aimed to mimic the shocks that we identify from our empirical model. From that exercise, we find that the calibrated model generates a weak general equilibrium relationship between exports and the exchange rate after domestic monetary policy shocks and exogenous exchange rate shocks, similar to what we find in our empirical results.

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open economies even in a dominant currency environment where the exchange rate channel is partially shut down.

[17] Gopinath et al. (2020) also find evidence that exports are less responsive to bilateral exchange rate movements when the share of dominant currency invoicing is high, and propose a small-open-economy model with dominant currency invoicing that yields similar results to Corsetti, Kuester, Müller and Schmidt (2021).

7 Conclusion

This paper examines the relationship between exports and exchange rate movements in a small open economy. To understand this relationship, we propose an SVAR model that separately identifies the CEREE that emerges after each of a set of domestic and global shocks, and estimate the model using Canadian data.

We find that different shocks generate significantly different CEREE estimates. The CEREE is weak, in a statistical sense, for all of the domestic shocks. In contrast, the CEREE is statistically significant for global shocks, and economically very different depending on the shock. For oil price shocks, the CEREE is negative and significant; but for persistent US income shocks, the CEREE is positive and significant.

In terms of the channels that generate these results, our analysis distinguishes among the three drivers of export growth: the exchange rate channel, the export price channel, and the foreign demand channel. We find that the foreign demand channel is the only one of the three that meaningfully impacts exports after shocks to the Canadian economy. We believe that the weakness of the exchange rate channel can likely be accounted for by the invoicing currency of Canadian exports, which is predominantly in US dollars. Given this invoicing currency, exchange rate movements have no mechanical impact on export prices from the perspective of foreign buyers, and hence this, combined with nominal price rigidities, prevents pass-through of exchange rate movements to the foreign price of Canadian exports, reflective of the "dominant currency paradigm" [Gopinath et al. (2020)].

Our results suggest that neither the benefits nor the costs of exchange rate flexibility are what they are presumed to be under conventional wisdom. In terms of the benefits, we find little evidence that the exchange rate channel is significant in transmitting monetary policy or in absorbing external shocks. In terms of the costs, we find little evidence that exchange rate appreciation due to oil price growth has a direct negative impact on exports, and hence concerns about this phenomenon in Canada are overstated.

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Table 1: Identification restrictions

	Domestic supply shock	Domestic demand shock	Domestic monetary policy shock	Exogenous exchange rate shock	Persistent US income shock	Transitory US income shock	Oil price shock	Residual shock
		Short-run restrictions						
Domestic GDP	+	+	-					
Domestic CPI	-	+	-	-				
Domestic interest rate		+	+	-				
Nominal ER		+	+	+			+	
Exports								
US GDP	0	0	0	0	+	+	-	
Oil price					+	+	+	
Export prices								
		Long-run restrictions						
Domestic GDP		0	0	0				
Domestic CPI								
Domestic interest rate								
Nominal ER								
Exports								
US GDP	0	0	0	0		0		
Oil price	0	0	0	0				
Export prices								

Note: A ‘+’ (‘-’) sign indicates that the impulse response of the varible is restricted to be positive (negative) in the quarter the shock occurs and in the following quarter. One exception here is the oil price shock, where we make the sign restrictions hold for the fourth period after the shock. A ‘0’ indicates that the response of the variable is restricted to be zero (either in the quarter the shock occurs or in the long run). A blank indicates that no restrictions are imposed on the variable.

Table 2: Factor error variance decompositions

Time	Supply shock	Demand shock	MP shock	Exog ER shock	P US inc shock	T US inc shock	Oil P shock	Resid shock
Exchange rate								
1 period	3.3%	19.6%	13.4%	12.1%	16.1%	10.4%	13.4%	11.8%
5 periods	3.8%	19.3%	13.6%	11.8%	15.9%	10.3%	13.6%	11.8%
20 periods	3.9%	19.2%	13.5%	11.7%	15.9%	10.4%	13.6%	11.8%
Exports								
1 period	10.7%	13.2%	19.0%	23.6%	5.3%	12.7%	5.3%	10.3%
5 periods	9.5%	11.5%	17.2%	20.7%	9.8%	13.2%	7.2%	10.9%
20 periods	9.4%	11.3%	16.9%	20.3%	9.8%	13.5%	7.8%	11.0%
Oil Price								
1 period	1.2%	1.9%	4.9%	2.7%	10.7%	19.5%	44.6%	14.5%
5 periods	1.5%	2.1%	5.7%	3.1%	11.0%	19.4%	43.0%	14.1%
20 periods	1.5%	2.2%	5.7%	3.2%	11.0%	19.4%	42.9%	14.1%

Note: FEVDs for the exchange rate, exports and oil price are constructed using mean SVAR estimates based on the approach described in Section 3, with constructs for 1, 5, and 20 periods after the shocks. Columns report the share of total error variance attributed to each shock.

A A small-open-economy model under different pricing regimes

In this section, we provide a brief description of a small-open-economy model that can reproduce several of the shock-dependant CEREE results that we find in our empirical analysis. Specifically, we show that a version of this model that is calibrated to match evidence on Canada's cross-country trade shares and the currency of invoicing for trade predicts weak CEREEs when shocks originate in the Canadian economy, but strong CEREEs when shocks originate in the global economy, which is consistent with our empirical findings.

The model setup closely follows a version of the model postulated in Gopinath et al. (2020). There are three countries indexed by $j \in \{H, U, R\}$, where H refers to the home country, U refers to the dominant currency country, and R refers to a "rest of the world" region. We assume that both U and R trade with H but, for simplicity, we assume they do not directly trade with one another. For the remainder of the exposition, we concentrate on the home country, H, and to preserve simplicity of notation, omit indexing home-country-specific variables with H wherever possible.

The model specifies monopolistically competitive markets, nominal rigidities in prices and wages, and intermediate goods as inputs to production. Aggregation for consumption goods and intermediate inputs follow Kimball (1995), which allows "strategic complementarities" in pricing decisions, where price setters' optimal markup decisions depend not only on demand conditions, but also on the prices of competitors in their destination markets. Export prices can be specified as based on domestic currency, destination currency, or the dominant currency.

B Households

The home country is populated by a continuum of symmetric households $h \in [0, 1]$, which consume a variety of goods $C_{i,t}(h)$ and supply an individual variety of labour $L_t(h)$ to firms. Households own all domestic firms and earn profits. They invest in state contingent domestic bonds, B_t as well as US bonds

$B_{U,t}$. Households maximize the discounted sum of utilities:

$$\max_{C_t, N_t, B_{U,t+1}, B_{t+1}(s')} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta \xi_t)^t \, U(C_t, N_t)$$

where the per-period utility function is given by:

$$U(C_t, N_t) = \frac{1}{1-\sigma_c} (C_t)^{1-\sigma_c} - \frac{\kappa}{1-\varphi} (N_t)^{1+\varphi}, \tag{6}$$

Here, $\mathbb{E}_t$ is the expectations operator at time t, σ_c is the household's coefficient of relative risk aversion, φ is the inverse of the Frisch elasticity of labour supply, κ is a scale parameter for the disutility of labour, and ξ_t denotes a preference shock.

Aggregate consumption C_t is implicitly defined by a Kimball (1995) demand aggregator:

$$\sum_i \frac{1}{|\Omega_i|} \int_{\omega \in \Omega_i} \gamma_i \Upsilon \left(\frac{|\Omega_i| C_{i,t}(\omega)}{\gamma_i C_t} \right) d\omega \quad = \quad 1 \tag{7}$$

where $C_{i,t}(\omega)$ denotes household consumption of goods of variety ω originating from country i, γ_i denotes a preference weight for goods from i (where $\sum_i \gamma_i = 1$), and $|\Omega_i|$ represents the measure of varieties produced in i. The function $\Upsilon(\cdot)$ has the properties that $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$. This demand function yields "strategic complementarities" in pricing decisions, where producers optimally adjust their markups in setting prices depending on demand conditions and the prices of competitors in destination markets, a phenomenon typically referred to a pricing-to-market.

Households face a budget constraint, expressed in home currency:

$$\begin{aligned} P_t C_t + \epsilon_{U,t} (1 + i_{U,t-1}) B_{U,t} + B_t \quad &= \quad W_t(h) N_t(h) + \Pi_t + \epsilon_{U,t} B_{U,t+1} \\ &+ \sum_{s' \in S} Q_t(s') B_{t+1}(s') + \epsilon_{U,t} \zeta_t \end{aligned}$$

where P_t is the price of domestic aggregate consumption, $\epsilon_{U,t}$ is the nominal exchange rate between the home country and the US, and W_t and N_t are the wage and labour supplied domestically. Π_t denotes domestic profits that are transferred to households. $B_{U,t+1}$ denotes home country purchases of internationally-traded US bonds that pay interest rate $i_{U,t}$ in US currency in the next period. Households also have access to the full set of domestic state contingent bonds, where $Q_{t+1}(s')$ denotes the price and $B_{t+1}(s')$ the purchased quantity of bonds that pay one unit of home currency in period $t+1$ in state s. The variable ζ_t represents an exogenous shock to the household's budget constraint, and is denominated in foreign currency to mimic the effect of a global commodity price shock.

We can think of the household's problem in two-stages. In the first stage, households maximize discounted sum of utilities subject to the aggregate budget constraint to determine aggregate consumption, labour, and investments in bonds. The household's first order conditions with respect to B_{t+1} and $B_{U,t+1}$ yields the following Euler equations:

$$C_t^{-\sigma_c} = \beta\left(1+i_t\right)C_{t+1}^{-\sigma_c}\frac{P_t}{P_{t+1}} \tag{8}$$

$$C_t^{-\sigma_c} = \beta\left(1+i_{U,t}\right)C_{t+1}^{-\sigma_c}\frac{P_t}{P_{t+1}}\frac{\epsilon_{U,t+1}}{\epsilon_{U,t}} \tag{9}$$

where $(1+i_t) \equiv \left(\sum_{s'\in S} Q_t(s')\right)^{-1}$.

In the second stage, households choose individual variety of consumption $C_{i,t}(\omega)$ conditional of total consumption C_t from the first-stage problem. Taking the first order conditions of the household problem w.r.t. $C_{i,t}$ and C_t separately and simplifying, we get:

$$C_{i,t}\left(\omega\right) = \gamma_i C_t \Upsilon'^{-1}\left(\frac{P_{i,t}}{P_t}D_t\right)$$

where $D_t = \sum_i \int_{\omega\in\Omega_i} \Upsilon'\left(\cdot\right)\frac{C_{i,t}(\omega)}{C_t}d\omega$.

We parameterize the function $\Upsilon(x)$ for any variable x by adopting the Klenow and Willis (2016) flexible form as follows:

$$\Upsilon\left(x\right) \;=\; \left(\sigma-1\right) exp\left(\frac{1}{\varepsilon}\right)\epsilon^{\left(\frac{\sigma}{\epsilon}-1\right)}\left(\Gamma\left(\frac{\sigma}{\epsilon},\frac{1}{\epsilon}\right)-\Gamma\left(\frac{\sigma}{\epsilon},\frac{x^{\frac{\epsilon}{\sigma}}}{\epsilon}\right)\right)$$

where $\Gamma\left(u,z\right)$ is the incomplete gamma function:

$$\Gamma\left(u,z\right)\equiv\int_{z}^{\infty}s^{u-1}e^{-s}ds=\int_{\frac{x^{\epsilon/\sigma}}{\epsilon}}^{\infty}s^{\frac{\sigma}{\epsilon}-1}e^{-s}ds$$

where σ and ϵ are two parameters that determine the elasticity of demand and the elasticity of the mark-up.

To get the variety-specific conditional demand function, start by noting that:

$$\begin{aligned}\Upsilon'(x) &= -\left(\sigma-1\right)e^{\frac{1}{\epsilon}}\epsilon^{\left(\frac{\sigma}{\epsilon}-1\right)}\Gamma'\left(x\right)\\ &= \frac{\sigma-1}{\sigma}e^{\frac{1-x^{\frac{\epsilon}{\sigma}}}{\epsilon}}\end{aligned}$$

Taking the inverse of $\Upsilon'(x)$, and replacing x with $\frac{P_{i,t}}{P_t}D_t$ for the current scenario, we get:

$$C_{i,t}\left(\omega\right) \;=\; \gamma_i\left(1+\epsilon\ln\left(\frac{\sigma-1}{\sigma}\right)-\epsilon\ln\left[\frac{P_{i,t}\left(\omega\right)}{P_t}D_t\right]\right)^{\sigma/\epsilon}C_t \qquad (10)$$

The elasticity of demand for the Kimball aggregator depends on the relative price $\frac{P_{i,t}(\omega)}{P_t}$, rather than being a constant:

$$\begin{aligned}\sigma_{iH,t} &= -\frac{dY_{iH,t}/dP_{iH,t}(\omega)}{Y_{iH,t}/P_{iH,t}}\\ &= \frac{\sigma}{\left[1+\epsilon\ln\left(\frac{\sigma-1}{\sigma}\right)-\epsilon\ln\left(\frac{P_{i,t}(\omega)}{P_t}D_t\right)\right]}\end{aligned}$$

If we define the markup as $\mu_{iH,t} = \frac{\sigma_{iH,t}}{\sigma_{iH,t}-1}$, we can derive the elasticity of the markup as follows:

$$\begin{aligned}
\Gamma_{iH,t} &= -\frac{d\mu_{iH,t}/dp_{iH,t}}{\mu_{iH,t}/p_{iH,t}} \\
&= -\frac{p_{iH,t}\left(\sigma_{iH,t}-1\right)}{\sigma_{iH,t}}\left[\frac{\left(d\sigma_{iH,t}/dp_{iH,t}\right)}{\sigma_{iH,t}-1} - \frac{\sigma_{iH,t}\left(d\sigma_{iH,t}/dp_{iH,t}\right)}{\left(\sigma_{iH,t}-1\right)^2}\right] \\
&= \frac{\epsilon}{\sigma - 1 - \epsilon\ln\left(\frac{\sigma-1}{\sigma}\right) - \epsilon\ln\left(\frac{P_{i,t}(\omega)}{P_t}D_t\right)}
\end{aligned}$$

Note that in the symmetric steady-state, $\left(\frac{P_{i,t}(\omega)}{P_t}D_t\right) = \frac{\sigma-1}{\sigma}$, which implies that the elasticity of demand $\sigma_{iH,t} = \sigma$ and the elasticity of the markup $\Gamma_{iH,t} = \frac{\epsilon}{\sigma-1}$ are both constant.

C Firms

There is a continuum of firms in each country that produce varieties ω that are available to both domestic and foreign buyers. Output from production is used for household consumtion $C_t(\omega)$ and as intermediate inputs into production $X_t(\omega)$. Producers use a standard Cobb-Douglas technology:

$$Y_t = A_t L_t^{1-\alpha} X_t^{\alpha} \tag{11}$$

where α denotes the share of intermediate inputs in production and A_t is aggregate productivity. Intermediate input bundle X_t is a combination of varieties that are assembled using the same Kimball (1995) aggregator as for consumption:

$$\sum_i \frac{1}{|\Omega_i|}\int_{\omega\epsilon|\Omega_i|} \gamma_i \Upsilon\left(\frac{|\Omega_i| X_{i,t}(\omega)}{\gamma_i X_t}\right) d\omega \tag{12}$$

where $X_{i,t}$ denotes purchases by firms of variety ω that originate from country i. Labour input L_t consists of a CES bundle of differentiated household labour inputs $L_t(h)$:

$$L_t = \left[\int_0^1 L_t(h)^{\frac{\vartheta-1}{\vartheta}} dj\right]^{\frac{\vartheta}{\vartheta-1}} \tag{13}$$

where $\vartheta > 1$ is the elasticity of substitution across different types of labour.

The firms' problem can also be thought of in two steps. In the first, the firm minimizes costs:

$$W_t L_t + P_t X_t$$

subject to the Cobb-Douglas production function given above. The first order conditions, after some algebra, gives us the conditional input demand functions:

$$\begin{aligned} X_t &= \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{W_t}{P_t}\right)^{1-\alpha} \frac{Y_t}{A_t} \\ L_t &= \left(\frac{\alpha}{-\alpha}\right)^{1-\alpha} \left(\frac{W_t}{P_t}\right)^{-\alpha} \frac{Y_t}{A_t} \end{aligned}$$

which, when plugged into the cost expression, give us the following cost function:

$$\begin{aligned} \mathcal{C}_t &= W_t L_t + P_t X_t \\ &= \frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} \frac{P_t^{\alpha} W_t^{1-\alpha}}{A_t} Y_t \end{aligned}$$

So that the marginal cost is given by:

$$_t = \frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} \frac{P_t^{\alpha} W_t^{1-\alpha}}{A_t} \tag{14}$$

In the second step, firms choose the variety of intermediate inputs $X_{i,t}\left(\omega\right)$, conditional on aggregate input demand X_t, and the Kimball (1995) aggregator. Similar to the household problem, this yields the following conditional intermediate input demand function:

$$X_{i,t}(\omega) \quad = \quad \gamma_i \left(1 + \epsilon \ln\left(\frac{\sigma - 1}{\sigma}\right) - \epsilon \ln\left[\frac{P_{i,t}(\omega)}{P_t} D_t\right]\right)^{\sigma/\epsilon} X_t \tag{15}$$

D Wage determination

Households face Calvo friction in setting wages — they adjust wages with probability $1 - \delta_w$, and a demand for the specific variety h of labour supply given by: $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\vartheta} N_t$, where N_t is aggregate labour supplied. The wage setting problem is then to choose a wage rate, $W_t(h)$ today to maximize the discounted sum of real earnings flow, net of utility costs of labour supplied:

$$\max_{W_t(h)} E_t \sum_{s=t}^{\infty} (\beta\delta_w)^{s-t} \left[U_s^c \frac{W_t(h)}{P_s} N_{s|t}(h) - \frac{\kappa}{1+\varphi} N_{s|t}(h)^{1+\varphi}\right]$$

Here, the first term within brackets captures the value of real income from employment at time s evaluated at marginal utility $U_s^c = C_s^{-\sigma}$ at time s, and the second term captures the utility cost of labour provided at time s. Households maximize this discounted sum subject to the demand for labour of type h at time s, provided the wage is set in time t:

$$N_{s|t}(h) = \left(\frac{W_t(h)}{W_s}\right)^{-\vartheta} N_{s|t}$$

The first-order condition with respect to $W_t(h)$ yields:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\delta_w)^{s-t} \left[\frac{C_s^{-\sigma}}{P_s}(1 - \vartheta) N_{s|t}(h) + \kappa\vartheta \frac{N_{s|t}(h)^{1+\varphi}}{W_t(h)}\right] \quad = \quad 0$$

After some algebra, we get:

$$\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\frac{W_t(h)}{P_s}C_s^{-\sigma}N_{s|t}(h) \quad = \quad \kappa\left(\frac{\vartheta}{\vartheta-1}\right)\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}N_{s|t}(h)N_{s|t}(h)^{\varphi}$$

Expressing log form variables in lower cases, we can re-write the above as:

$$\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\exp\left[-\sigma c_s+n_{s|t}(h)+w_t(h)-p_s\right]=$$
$$\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\exp\left[n_{s|t}(h)+\log\kappa+\log\left(\frac{\vartheta}{\vartheta-1}\right)+\varphi n_{s|t}(h)\right]$$

Taking a Taylor series expansion around the steady-state on both sides, we get:

$$\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\left[1-\sigma\left(c_s-c\right)+\left(n_{s|t}(h)-n\right)+\left(w_t(h)-w_t\right)-\left(p_s-p_t\right)\right]=$$
$$\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\left[1+\left(n_{s|t}(h)-n\right)+\varphi\left(n_{s|t}(h)-n\right)\right]$$

Isolating $w_t(h)$ on the LHS and simplifying, we get:

$$\begin{aligned} w_t(h)\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t} &\equiv w_t(h)\left(\frac{1}{1-\beta\delta_w}\right) \\ &= \mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}\left(\beta\delta_w\right)^{s-t}\left[\varphi n_{s|t}+\sigma c_s+p_s-\left(\varphi n+\sigma c-w_t+p_t\right)\right] \end{aligned}$$

Note that for the flexible wage and price case, wages and labour supply for each type h is the same.

Furthermore, in the zero price and wage inflation steady-state, we have:

$$\frac{W_t}{P_t} = \kappa \frac{\vartheta}{\vartheta - 1} N^{\varphi} C^{\sigma}$$

Log-linearizing, we get:

$$\begin{aligned} w_t - p_t &= \log \kappa + \log \left(\frac{\vartheta}{\vartheta - 1} \right) + \varphi n + \sigma c \\ \varphi n + \sigma c - w_t + p_t &= - \log \kappa - \log \left(\frac{\vartheta}{\vartheta - 1} \right) \end{aligned}$$

Plugging in the above expression, we get:

$$w_t(h) = (1 - \beta \delta_w) \mathop{\mathbb{E}}_t \sum_{s=t}^{\infty} (\beta \delta_w)^{s-t} \left[\varphi n_{s|t} + \sigma c_s + p_s + \log \kappa + \log \left(\frac{\vartheta}{\vartheta - 1} \right) \right] \tag{16}$$

Further simplifying, we can express current wages $w_t(h)$ as a function of future expected wages, as follows:

$$\begin{aligned} w_t(h) &= (1 - \beta \delta_w) \left[\varphi n_t + \sigma c_t + p_t + \log \kappa + \log \left(\frac{\vartheta}{\vartheta - 1} \right) \right] \\ &\quad + \beta \delta_w (1 - \beta \delta_w) \mathop{\mathbb{E}}_t \sum_{s=t}^{\infty} (\beta \delta_w)^{s+1-t} \left[\varphi n_{s+t|t} + \sigma c_{s+t|t} + p_{s+t|t} + \log \kappa + \log \left(\frac{\vartheta}{\vartheta - 1} \right) \right] \\ w_t(h) &= (1 - \beta \delta_w) \left[\varphi n_t + \sigma c_t + p_t + \log \kappa + \log \left(\frac{\vartheta}{\vartheta - 1} \right) \right] + \beta \delta_w \mathop{\mathbb{E}}_t w_{t+1}(h) \end{aligned}$$

Calvo pricing implies that current aggregate wage is a CES function of aggregate wage in the previous period and the newly set wage today, where the weights represent the probabilities of resetting wages:

$$\begin{aligned} W_t &= \left[\delta_w W_{t-1}^{1-\vartheta} + (1-\delta_w) W_t(h)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \\ \implies w_t - w_{t-1} &= (1-\delta_w)\left[w_t(h) - w_{t-1}\right] \end{aligned}$$

Plugging in the expression for $w_t(h)$ in the equation above gives:

$$w_t - w_{t-1} = (1-\delta_w)\left[(1-\beta\delta_w)\left[\varphi n_t + \sigma c_t + p_t + \log\kappa + \log\left(\frac{\vartheta}{\vartheta-1}\right)\right] + \beta\delta_w w_{t+1}(h) - w_{t-1}\right]$$

Simplifying, we get:

$$\delta_w(w_t - w_{t-1}) = \beta\delta_w\left[w_{t+1} - w_t\right] + (1-\delta_w)(1-\beta\delta_w)\left[\varphi n_t + \sigma c_t + p_t + \log\kappa + \log\left(\frac{\vartheta}{\vartheta-1}\right) - w_t\right]$$

which gives the following equation for wage inflation $w_t - w_{t-1}$:

$$w_t - w_{t-1} = \beta\,\mathop{\mathbb{E}}_t\left[w_{t+1} - w_t\right] + \frac{(1-\delta_w)(1-\beta\delta_w)}{\delta_w}\left[\varphi n_t + \sigma c_t + p_t - w_t + \log\kappa + \log\left(\frac{\vartheta}{\vartheta-1}\right)\right]$$

E Price setting

Destination markets are segmented so that firms can set different prices across destinations. Price setting is subject to a Calvo-like friction, where firms may adjust their prices in any given period with probabiliy $1-\delta_p$, and otherwise must maintain the previous period's price. Consider the pricing problem of a domestic firm selling variety ω in country i in currency j. Let the reset price in time t be $\bar{P}_{i,t}^{j}(\omega)$.

The firm's problem is then to maximize discounted sum of profits, expressed in the domestic currency:

$$\max_{\bar{P}^j_{Hi,t}} \mathbb{E}_t \sum_{s=t}^{\infty} \delta_p^{s-t} \beta^{s-t} \frac{U_{c,s}}{U_{c,t}} \left[\epsilon_{j,s} \bar{P}^j_{Hi,t} Y^j_{Hi,s|t} - \mathcal{C}_{s|t}\left(Y^j_{Hi,s|t} \right) \right]$$

where the demand for home goods in country i is given by:

$$Y^j_{Hi,s|t} = \gamma_H \left(1 + \epsilon \ln \left(\frac{\sigma - 1}{\sigma} \right) - \epsilon \ln \left(\frac{\epsilon_{j,s} \bar{P}^j_{Hi,t}}{\epsilon_{i,s} P_{i,s}} D \right) \right)^{\sigma/\epsilon} Y_{i,s}$$

Here, the nominal exchange rate between home currency and country i, $\epsilon_{i,s}$ is expressed as home currency per unit of foreign currency i, and $P_{i,s}$ is the aggregate price in country i. Note also that $Y^j_{Hi,s|t}$ is the demand for home-produced goods in country i at time s, whose price has been set at time t in currency j, and $Y_{i,s}$ is the aggregate demand for goods in country i at time s. The first-order condition from the firms' price setting problem gives:

$$\begin{aligned} 0 &= \mathbb{E}_t \sum_{s=t}^{\infty} (\beta\delta_p)^{s-t} \frac{U_{c,s}}{U_{c,t}} \left[\epsilon_{j,s} Y^j_{Hi,s|t} + \epsilon_{j,s} \bar{P}^j_{Hi,t} \frac{dY^j_{Hi,s|t}}{d\bar{P}^j_{Hi,t}} - \mathcal{MC}_{s|t} \frac{dY^j_{Hi,s|t}}{d\bar{P}^j_{Hi,t}} \right] \\ &= \mathbb{E}_t \sum_{s=t}^{\infty} (\beta\delta_p)^{s-t} \frac{U_{c,s}}{U_{c,t}} Y^j_{Hi,s|t} \left(1 - \sigma^j_{Hi,s} \right) \left[\epsilon_{j,s} \bar{P}^j_{Hi,t} - \frac{\sigma^j_{Hi,s}}{\sigma^j_{Hi,s} - 1} \mathcal{MC}_{s|t} \right] \end{aligned}$$

Re-arranging the above, we get:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\delta_p)^{s-t} \frac{U_{c,s}}{U_{c,t}} Y^j_{Hi,s|t} \left(1 - \sigma^j_{Hi,s} \right) \epsilon_{Hj,s} \bar{P}^j_{Hi,t} =$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\delta_p)^{s-t} \frac{U_{c,s}}{U_{c,t}} Y^j_{Hi,s|t} \left(1 - \sigma^j_{Hi,s} \right) \frac{\sigma^j_{Hi,s}}{\sigma^j_{Hi,s} - 1} \mathcal{MC}_{s|t}$$

Now, rewrite the above by expressing logged terms in lower case as follows:

$$\begin{aligned}\underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\exp\left[u_{c,s}-u_{c,t}+y^j_{Hi,s|t}+\log\left(1-\sigma^j_{Hi,s}\right)+\varepsilon_{j,s}+\bar{p}^j_{Hi,t}\right] &= \\ \underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\exp\left[u_{c,s}-u_{c,t}+y^j_{Hi,s|t}+\log\left(1-\sigma^j_{Hi,s}\right)+\log\mu^j_{Hi,s}+mc_{s|t}\right]&\end{aligned}$$

where, $\mu^j_{Hi,s}=\frac{\sigma^j_{Hi,s}}{\sigma^j_{Hi,s}-1}$ is the time-varying markup. Taking a Taylor series expansion around the zero inflation steady-state of the above, we get:

$$\begin{aligned}&\underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\Bigg[1+\left(u_{c,s}-u_c\right)-\left(u_{c,t}-u_c\right)+\left(y^j_{Hi,s|t}-y\right)+\frac{\partial\log\left(1-\sigma^j_{Hi}\right)}{\partial\bar{p}^j_{Hi}}\left(\bar{p}^j_{Hi,s}-p_{H,t}\right)\\ &\qquad\qquad+\left(\varepsilon_{j,s}-\varepsilon_j\right)+\left(\bar{p}^j_{Hi,t}-p_{H,t}\right)\Bigg]=\\ &\underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\Bigg[1+\left(u_{c,s}-u_c\right)-\left(u_{c,t}-u_c\right)+\left(y^j_{Hi,s|t}-y\right)+\frac{\partial\log\left(1-\sigma^j_{Hi}\right)}{\partial\bar{p}^j_{Hi}}\left(\bar{p}^j_{Hi,t}-p_{H,t}\right)\\ &\qquad+\frac{\partial\log\mu^j_{Hi}}{\partial\bar{p}^j_{Hi}}\left(\bar{p}^j_{Hi,t}-p_{H,t}\right)+\frac{\partial\log\mu^j_{Hi}}{\partial p_i}\left(p_{i,s}-p_{i,t}\right)+\frac{\partial\log\mu^j_{Hi}}{\partial\varepsilon_j}\left(\varepsilon_{j,s}-\varepsilon_j\right)\\ &\qquad\qquad+\frac{\partial\log\mu^j_{Hi}}{\partial\varepsilon_i}\left(\varepsilon_{i,s}-\varepsilon_i\right)+\left(mc_{s|t}-mc_t\right)\Bigg]\end{aligned}$$

Note that $\frac{\partial\log\mu^j_{Hi}}{\partial\bar{p}^j_{Hi}}=\frac{\partial\log\mu^j_{Hi}}{\partial\varepsilon_j}=-\frac{\partial\log\mu^j_{Hi}}{\partial p_i}=-\frac{\partial\log\mu^j_{Hi}}{\partial\varepsilon_i}=-\Gamma$. Moreover, in the steady-state, $\varepsilon_j=\log\epsilon_j=0\forall j$. Simplifying the above, we get:

$$\underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\left[\varepsilon_{j,s}+\left(\bar{p}^j_{Hi,t}-p_{H,t}\right)\right]=\underset{t}{\mathbb{E}}\sum_{s=t}^{\infty}\left(\beta\delta_p\right)^{s-t}\left[-\Gamma\bar{p}^j_{Hi,t}+\Gamma p^j_{i,s}-\Gamma\varepsilon_{j,t}+\Gamma\varepsilon_{i,s}+\left(mc_{s|t}-mc_t\right)\right]$$

Further simplifying, we get:

$$\begin{aligned}
(1+\Gamma)\,\bar{p}^{j}_{Hi,t}\,\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}(\beta\delta_p)^{s-t} &= \mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}(\beta\delta_p)^{s-t}\left[\Gamma\left(p_{i,s}-\varepsilon_{j,t}+\varepsilon_{i,s}\right)+\left(mc_{s|t}-mc_t\right)-\varepsilon_{Hj,s}+p_{H,t}\right]\\
\left(\frac{1+\Gamma}{1-\beta\delta_p}\right)\bar{p}^{j}_{Hi,t} &= \mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}(\beta\delta_p)^{s-t}\left[\left(mc_{s|t}-\varepsilon_{j,s}\right)+\Gamma\left(p_{i,s}+\varepsilon_{i,s}-\varepsilon_{j,s}\right)+\left(p_{H,t}-mc_t\right)\right]\\
\bar{p}^{j}_{Hi,t} &= \left(\frac{1-\beta\delta_p}{1+\Gamma}\right)\mathop{\mathbb{E}}_{t}\sum_{s=t}^{\infty}(\beta\delta_p)^{s-t}\left[\left(mc_{s|t}-\varepsilon_{j,s}\right)+\Gamma\left(p_{i,s}+\varepsilon_{i,s}-\varepsilon_{j,s}\right)+\log\mu\right]
\end{aligned}$$

where the last equation uses the fact that in the no-inflation steady-state, real marginal cost $mc_t - p_{H,t} = -\log\mu$ is a constant. Finally, we have the reset price as:

$$\bar{p}^{j}_{Hi,t} = \beta\delta_p\,\mathop{\mathbb{E}}_{t}\bar{p}^{j}_{Hi,t+1}+\left(\frac{1-\beta\delta_p}{1+\Gamma}\right)\left[\left(mc_t-\varepsilon_{j,t}\right)+\Gamma\left(p_{i,t}+\varepsilon_{i,t}-\varepsilon_{j,t}\right)+\log\mu\right] \tag{17}$$

where, $mc_t - \varepsilon_{j,t}$ is the log nominal marginal cost of home firms expressed in currency j, $p^{j}_{i,t} = p_{i,t} - \varepsilon_{ij,t}$ is the log of the aggregate price level in country i expressed in currency j, μ is the log of the steady-state desired gross markup, and Γ is the steady-state elasticity of that markup. In general, the marginal cost of the country of origin and the aggregate price level of the destination country must both be translated to the currency of invoice using the appropriate nominal exchange rates.

The aggregate price level is given from the Calvo-pricing log-linearized formula as follows:

$$p^{j}_{Hi,t} - p^{j}_{Hi,t-1} = (1-\delta_p)\left(\bar{p}^{j}_{Hi,t} - p^{j}_{Hi,t-1}\right) \tag{18}$$

Firms set their price in home (producer) currency, destination (local) currency, or US (dominant) currency. We denote θ^{k}_{ji} as the share of exports from country j to country k denoted in currency k, where $k \in \{j, i, \$\}$ and $\sum_k \theta^{k}_{ji} = 1$. We assume that all goods that both originate and are destined for the same country, i.e., domestic goods, are priced in producer currency, so that $\theta^{j}_{ji} = 1$ for all j. In our model analysis, we will focus on the dominant currency pricing environment, where $\theta^{\$}_{ji} = 1$ for all $i \neq j$. We compare this to the standard producer currency pricing environment, where $\theta^{j}_{ji} = 1$ for all i, j.

E.1 Bilateral trade and prices

Combining consumer demand from equation (10), input demand from equation (19), and aggregating together all varieties produced by a given country i, we arrive at an expression for the total demand in country j for goods produced by country i:

$$Y_{ij,t} = \sum_{\omega} \left[C_{ij,t}\left(\omega\right) + X_{ij,t}\left(\omega\right)\right] = \gamma_{ij} \left(1 + \epsilon \ln \left(\frac{\sigma - 1}{\sigma}\right) - \epsilon \ln \left[\frac{P_{ij,t}}{P_{j,t}} D_{j,t}\right]\right)^{\sigma/\epsilon} Y_{j,t} \tag{19}$$

Since the currency of invoicing affects the relative price in this expression, demand in i for goods produced in j will also depend on the currency in which these exports are invoiced. We take aggregate demand in U as given, and assume that $\gamma_{ij} = 1$ for all i, j and that the market clears so that bilateral demand is equated to bilateral exports. This gives rise to the currency-specific export and import equations. When trade is invoiced in producer currency, we have the following equations for country H exports and imports:

$$Y_{HU,t}^{H} = \left(1 - \varepsilon\left(p_{HU,t}^{H} - \log \epsilon_t^{U} - p_{U,t}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{U,t} \tag{20}$$

$$Y_{HR,t}^{H} = \left(1 - \varepsilon\left(p_{HR,t}^{H} - \log \epsilon_t^{R} - p_{R,t}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{R,t} \tag{21}$$

$$Y_{UH,t}^{U} = \left(1 - \varepsilon\left(p_{UH,t}^{U} + \log \epsilon_t^{U}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{H,t} \tag{22}$$

$$Y_{RH,t}^{R} = \left(1 - \varepsilon\left(p_{RH,t}^{R} + \log \epsilon_t^{R}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{H,t} \tag{23}$$

where $Y_{U,t} = \overline{Y_u^d} u_t^D$ and u_t^D is a shock to demand in U, and $Y_{R,t} = \overline{Y_u^d}$. When trade is invoiced in country

U currency, we have the following equations for country H exports and imports:

$$Y_{HU,t}^{U} = \left(1 - \varepsilon\left(p_{HU,t}^{U} - p_{U,t}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{U,t} \tag{24}$$

$$Y_{HR,t}^{U} = \left(1 - \varepsilon\left(p_{HR,t}^{U} - \log \epsilon_t^R + \log \epsilon_t^U - p_{R,t}\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{R,t} \tag{25}$$

$$Y_{UH,t}^{U} = \left(1 - \varepsilon\left(p_{UH,t}^{U} + \log \epsilon_t^U\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{H,t} \tag{26}$$

$$Y_{RH,t}^{U} = \left(1 - \varepsilon\left(p_{RH,t}^{U} + \log \epsilon_t^U\right)\right)^{\frac{\sigma}{\varepsilon}} Y_{H,t} \tag{27}$$

For exports from H, the difference between the equations (20) and (21) invoiced in producer currency (PCP) and equations (24) and (25) invoiced in U currency (DCP) largely boils down to the direct role of the nominal exchange rate. In the PCP case, a decrease in ϵ_t^U (ϵ_t^R) has a direct impact by raising the relative price of exports to U (R) and, therefore, lowers exports from H to U (R). In contrast, ϵ_t^U does not enter directly into the DCP equation (24), and for equation (25), exchange rates do enter but their impact is limited to the extent that there is positive co-movement between ϵ_t^U and ϵ_t^R. As a result, when exports are invoiced in DCP, movements in exchange rates will not have a large direct impact on exports.

For imports to H, we see slightly different insights. In cases where imports are invoiced in producer currency as in equations (22) and (23), a decrease in ϵ_t^U (ϵ_t^R) has a direct impact of lowering the relative price of imports from U (R) and, therefore, raises imports from U (R). In cases where imports are invoiced in currency U (DCP), imports from U behave very similarly as in the PCP case (see (26)) and imports from H respond directly to ϵ_t^U rather than to ϵ_t^R, hence a decrease in ϵ_t^U directly lowers the relative price of imports from both countries, and therefore raises imports from both sources.

To arrive at aggregate export and imports, we take the weighted average of these invoice-specific values:

$$Y_{H,t}^{X} = \sum_{j=U,R} \gamma_j Y_{Hj,t} = \sum_{j=U,R} \gamma_j \left[\theta_{Hj}^{U} \left(Y_{Hj,t}^{U}\right) + \left(1 - \theta_{Hj}^{U}\right) Y_{Hj,t}^{H}\right] \tag{28}$$

$$Y_{H,t}^{I} = \sum_{i=U,R} \gamma_i Y_{iH,t} = \sum_{i=U,R} \gamma_i \left[\theta_{iH}^{U} \left(Y_{iH,t}^{U}\right) + \left(1 - \theta_{iH}^{U}\right) Y_{iH,t}^{i}\right] \tag{29}$$

where θ_{ij}^{U} denotes the share of exports from i to j denoted in currency U.

We can also specify the optimal reset prices for traded goods depending on the currency of invoicing, based on the general reset price equation (17). When goods are invoiced in producer currency, we have the following equation for bilateral export and import prices:

$$\bar{p}_{HU,t}^{H} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}_{HU,t+1}^{H} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{H,t} + \Gamma\left(p_{U,t} + \varepsilon_t^{U}\right) + \mu\right) \tag{30}$$

$$\bar{p}_{HR,t}^{H} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}_{HR,t+1}^{H} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{H,t} + \Gamma\left(p_{R,t} + \varepsilon_t^{R}\right) + \mu\right) \tag{31}$$

$$\bar{p}_{UH,t}^{U} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}_{UH,t+1}^{U} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{U,t} + \Gamma\left(-\varepsilon_t^{U}\right) + \mu\right) \tag{32}$$

$$\bar{p}_{RH,t}^{R} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}_{RH,t+1}^{R} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{R,t} + \Gamma\left(-\varepsilon_t^{R}\right) + \mu\right) \tag{33}$$

where $\bar{p}_{ij,t}^{k}$ denotes the reset price of goods exported from country i to country j when invoiced in k currency. By contrast, when goods are invoiced in country U currency (DCP), we have the following equation for bilateral export and import prices:

$$\bar{p}^{U}_{HU,t} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}^{U}_{HU,t+1} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{H,t} - \varepsilon^{U}_{t} + \Gamma\left(p_{U,t}\right) + \mu\right) \tag{34}$$

$$\bar{p}^{U}_{HR,t} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}^{R}_{HU,t+1} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{H,t} - \varepsilon^{R}_{t} + \Gamma\left(p_{R,t}\right) + \mu\right) \tag{35}$$

$$\bar{p}^{U}_{UH,t} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}^{U}_{UH,t+1} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{U,t} + \Gamma\left(-\varepsilon^{U}_{t}\right) + \mu\right) \tag{36}$$

$$\bar{p}^{U}_{RH,t} = \beta\delta_p \mathop{\mathbb{E}}_{t} \bar{p}^{U}_{RH,t+1} + \frac{1-\beta\delta_p}{1+\Gamma}\left(mc_{R,t} - \varepsilon^{U}_{t} + \varepsilon^{R}_{t} + \Gamma\left(-\varepsilon^{U}_{t}\right) + \mu\right) \tag{37}$$

When exporters in H price in producer currency as in equation (30) and (31), a decrease in ε^{U}_{t} (ε^{R}_{t}) raises the relative price of these goods from the perspective of buyers in U (R) and lowers export demand; producers in H respond to this lower demand by lowering ts heir markup Γ, which leads to a lower reset price. By contrast, when exporters in H price in U currency as in equations (34) and (35), a decrease in ε^{U}_{t} (ε^{U}_{t}) has no impact on the relative price from perspective of buyers in U (R), but does raise the price of marginal costs (paid in H currency), and therefore leads to a higher reset price through that channel. For exporters in country U (R) that price in producer currency as in equations (32) and (33), a decrease in ε^{U}_{t} (ε^{R}_{t}) lowers the relative price of these goods from the perspective of buyers in H and raises import demand in that country; producers in U (R) respond to this higher demand by raising their markup Γ, which leads to a higher reset price. When exporters in country U (R) invoice in currency U as in equations (36), the outcome is equivalent to the PCP case for exports from this country. When exporters from R invoice in currency U as in (37), a decrease in ε^{U}_{t} lowers the relative price of these goods from the perspective of buyers in H and raises import demand in that country, leading to a higher markup, while the decrease in ε^{U}_{t} also raises the price of marginal costs (paid in H currency) unless movements in ε_t^U are perfectly correlated with movements in ε_t^R, in which case this latter effect cancels out.

As prices are subject to nominal rigidities, actual export prices will evolve sluggishly according to the Calvo-pricing formula described in equation (18). As such, the forces described in equations (30) to (37) will only have a modest impact on aggregate prices in the short-to-medium run. We will express all prices in a common currency H, consistent with the prices used in our empirical estimates (reported in domestic currency). Accordingly, the aggregate export price and aggregate import price in currency H are defined as follows:

$$p_{j,t}^X = \sum_{j=U,R} \gamma_j p_{Hj,t} = \sum_{j=U,R} \gamma_j \left[\theta_{Hj}^U \left(p_{Hj,t}^U + \varepsilon_t^U\right) + \left(1 - \theta_{Hj}^U\right) p_{Hj,t}^H\right], \tag{38}$$

$$p_{j,t}^I = \sum_{i=U,R} \gamma_j p_{iH,t} = \sum_{i=U,R} \gamma_i \left[\theta_{iH}^U \left(p_{iH,t}^U + \varepsilon_t^U\right) + \left(1 - \theta_{iH}^U\right) \left(p_{iH,t}^i + \varepsilon_t^i\right)\right] \tag{39}$$

The higher θ_{ij}^U is, the larger the mechanical effect of movements in the bilateral HU exchange rate on aggregate trade prices, where a decline in ε_t^U (an appreciation) lowers the price of export/import goods once converted to currency H.

E.2 Interest rates and the exchange rate

The home interest rate follows a simple inflation targeting rule:

$$i_t - \bar{i} \;=\; \rho^m \left(i_{t-1} - \bar{i}\right) + \left(1 - \rho^m\right) \left(\phi^m \pi_t + \phi^y \left(y_t - y_t^{flex}\right)\right) + u_t^M \tag{40}$$

where u_t^M denotes a shock to the home interest rate. The dollar interest rate faced by households is assumed to be an increasing function of the deviation of aggregate debt from its steady state:

$$i_t^U = i^* + \phi \left(e^{B_{t+1}^U - \bar{B}} - 1 \right) \tag{41}$$

A standard uncovered interest parity condition (UIP) relates changes in the bilateral HU exchange rate to differences between the domestic and foreign interest rate:

$$i_t - i_t^U = \epsilon_{t+1}^U - \epsilon_t^U + \pi_{t+1} + u_t^{UIP} \tag{42}$$

where u_t^{UIP} represents as shock to the UIP condition. Finally, we assume for simplicity that $\varepsilon_t^U = \varepsilon_t^R$ so that movements in the bilateral HU exchange rate are equivalent to movements in the bilateral HR exchange rate.

E.3 Shocks

In the model above, we describe four separate shocks: a monetary policy shock in H (u_t^M), a shock to the bilateral UIP condition that is intended to mimic an exogenous nominal exchange rate movement (u_t^{UIP}), a shock to total demand in country U (u_t^D), and an endowment shock in country H that is intended to represent an exogenous change in its terms of trade (u_t^C). The processes for u_t^M, u_t^{UIP}, and u_t^D are similar and can be generalized as follows:

$$u_t^k = \rho^k u_{t-1}^k + \varepsilon_t^k \tag{43}$$

for $k = M, UIP, D$, where ε_t^k represents an i.i.d. innovation. The process for u_t^C is slightly different and is defined as follows:

$$\ln \zeta_t - \ln \overline{\zeta} = \rho^C \left(\ln \zeta_{t-1} - \ln \overline{\zeta} \right) + \varepsilon_t^C \tag{44}$$

where ε_t^C denotes an i.i.d. innovation.

E.4 Model implications

To set up our quantitative experiments, it is worthwhile to discuss the implications for export and import price and quantity dynamics after shocks. These implications are also discussed in Gopinath et al. (2020). To illustrate this in a simple way, we take the extreme case where nominal prices are entirely rigid, which is a close approximation to the degree of rigidity applied in the actual model in the short run. In that case, the price channel is absent, and changes in country H export prices (based on equation (38)) can be expressed as the following:

$$\Delta p_{j,t}^X = \sum_{j=U,R} \gamma^j \Delta p_{Hj,t} = \sum_{j=U,R} \gamma^j \theta_{Hj}^U \Delta \varepsilon_t^U \tag{45}$$

That is, changes in export prices (expressed in country H currency) only depend on movements in the bilateral exchange rate with country U and the share of exports that are invoiced in country U currency. If exports are invoiced entirely in domestic currency (PCP), then we will see no co-movement with the exchange rate, and if exports are invoiced entirely in country U currency (DCP), then we will see perfect co-movement with the country U exchange rate.

For short-run changes in real exports, log-linear approximation of equation (28) around the steady state yields the following:

$$\Delta y_{H,t}^X = \sum_{j=U,R} \gamma_j \Delta y_{Hj,t} = \sum_{j=U,R} \gamma^j \left[-\sigma_{Hj} \left(\Delta p_{Hj,t} - \Delta \varepsilon_t^j - \Delta p_{j,t} \right) + \Delta y_{j,t} \right], \tag{46}$$

where σ_{Hj} denotes the elasticity of demand. This expression highlights the two main channels that drive real exports, the exchange rate channel (first half of the expression) and the foreign demand channel (second half of the expression). As mentioned above, when the share of exports invoiced in currency H is high (PCP), export prices (expressed in currency H) do not change in the short run, and hence

$\Delta p_{ij,t} - \Delta\varepsilon_t^j = -\Delta\varepsilon_t^j \neq 0$, and real exports will respond to exchange rate movements through the exchange rate channel. On the other hand, if exports are highly invoiced in country U currency (DCP), export prices will co-move with the exchange rate so that $\Delta p_{ij,t} - \Delta\varepsilon_t^j = 0$ and, therefore, exports will not respond to movements through the exchange rate channel. Regardless of the pricing regime, the exports do respond to the foreign demand channel.

For import prices, we see a similar expression for short-run changes as described in equation (45) (based on equation (39)):

$$\Delta p_{j,t}^I = \sum_{i=U,R} \gamma^i \Delta p_{iH,t} = \gamma^i \left[\Delta\theta_{iH}^U \Delta\varepsilon_t^U + \left(1 - \theta_{iH}^U\right)\Delta\varepsilon_t^i\right] \tag{47}$$

Changes in import prices (expressed in country H currency) depend on movements in both bilateral exchange rates and the share of imports that are invoiced in country U currency. If imports are invoiced entirely in producer currency (PCP), then we see a mixed co-movement with the bilateral exchange rate with country U and country R, and if imports are invoiced entirely in the country U currency (DCP), then we see perfect co-movement with the country U exchange rate.

For short-run changes in real imports, log-linear approximation of equation (29) around the steady state yields the following:

$$\Delta y_{H,t}^I = \sum_{i=U,R} \gamma_i \Delta y_{iH,t} = \sum_{i=U,R} \gamma^i \left[-\sigma_{iH}\left(\Delta p_{iH,t} - \Delta p_{H,t}\right) + \Delta y_{H,t}\right], \tag{48}$$

As mentioned above, whether imports are invoiced in producer currency (PCP) or country U currency (DCP), import prices do change in the short run in response to the exchange rate, and hence $\Delta p_{iH,t} \neq 0$ and real imports will respond to exchange rate movements through the exchange rate channel. Again, regardless of the pricing regime, the imports do respond to the foreign demand channel.

F Quantitative experiments

In this section, we subject the model economy to the four shocks described in the previous section. Our aim here is to compare impulse responses in this model economy to the impulse responses we estimated using the SVAR in our empirical analysis.

Before we report these results, we should provide a brief description of the calibrated parameters used in this analysis. They are listed in Tables 3 and 4. In general, the parameters applied here are identical to those used in the quantitative analysis conducted in Gopinath et al. (2020). However, there are a few exceptions. Gopinath et al. (2020) separately study the model economy under PCP, LCP, and DCP regimes, but the reality for Canada is closer to a blend of these extremes. For our purposes, we assume that 80% of trade (exports and imports) in the model is invoiced in country U currency (DCP), and 20% is invoiced in producer currency (PCP). These figures are roughly consistent with estimates for the share of Canada's exports and imports that is invoiced in US dollars versus producer currency.[18] We assume that 59% of country H goods are produced at home, 24% are produced in U, and 17% are produced in R, which is consistent with estimates for the share of Canadian spending on domestic-produced goods, US-produced goods, and goods from the rest of the world. Aside from these, all other parameters in Table 3 are identical to those applied in Gopinath et al. (2020).

Table 4 reports the parameters we use for the shock processes. We specify four shocks: a country H monetary policy shock, a shock to the country H – country U UIP condition, a shock to country U income, and a country H endowment shock. These four shocks are intended to represent a monetary policy shock in Canada, an exogenous Canada–US exchange rate shock, a US income shock, and an oil price shock, respectively. To mimic in our empirical identification strategy, we assume here that the shock to country U income is negatively correlated with the country H endowment shock. The UIP shock, which is not modelled in the original Gopinath et al. (2020) analysis, is assumed to have same persistence as the commodity price shock and country U income shock. Aside from these cases, all other shock parameters are identical to those applied in Gopinath et al. (2020).

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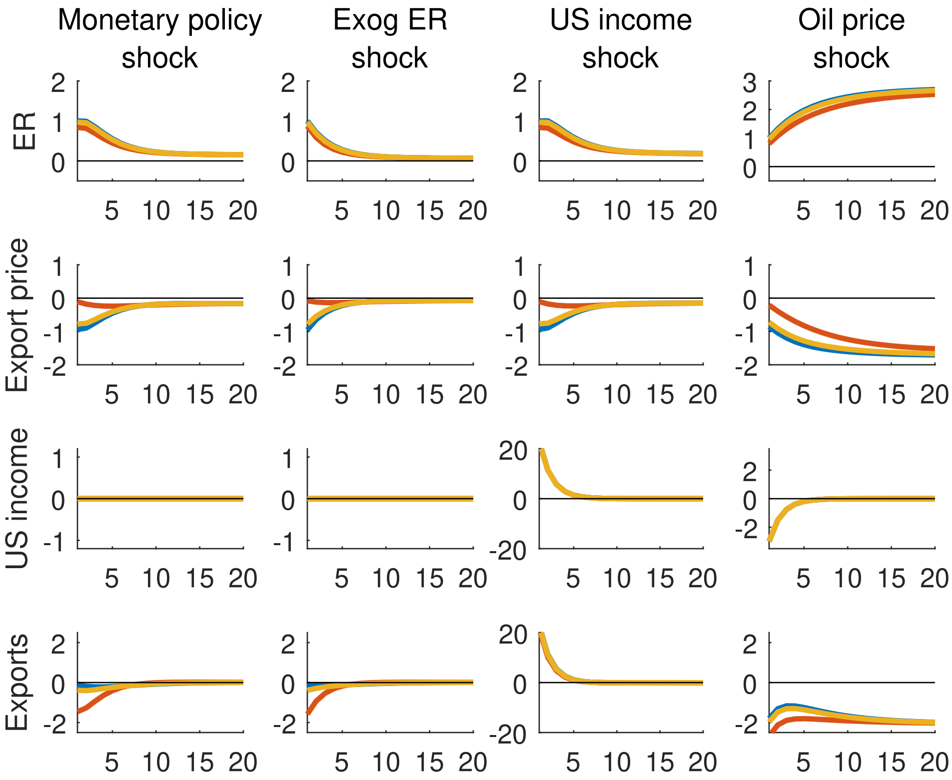
[18] See Cao, Dong and Tomlin (2015), Gopinath (2015) and Devereux, Dong and Tomlin (2017).

We simulate each of these four shocks, and examine impulse responses around the steady state for several key variables of interest. These variables are the bilateral HU exchange rate determined by equation (42), aggregate export and import prices for country H according to equations (38) and (39), aggregate income in country U, which is defined exogenously as $Y^d_{U,t} = \overline{Y^d_u} u^D_t$, and real exports and imports for country H, according to equations (28) and (29).

Impulse responses for the exchange rate, export prices, income in U, and real exports are provided in Figure 12. We label the shocks as "Monetary policy shock" for the country H monetary shock, "Exogenous ER shock" for the UIP shock, "US income shock" for the country U income shock, and "Oil price shock" for the country H endowment shock, to match the empirical equivalents from Figures 5 and 7 that we look to for comparison. For the sake of illustration, we include results under strict PCP (orange lines) and strict DCP (blue lines), but the results that we think should closely align with the empirical results for Canada are represented by the yellow lines, which depict a blend of 80% DCP and 20% PCP. In calibrating the size of these shocks, we aimed to match a 1% appreciation in country H currency in the immediate period after the shock, which is consistent with how we normalized our shocks relative to appreciation in the Canadian dollar in the empirical section of the paper. As such, impulse responses for the exchange rate in the first period are roughly consistent with empirical results by construction.

The main results we focus on are for real exports, reported in the bottom row of Figure 12. Our empirical results for responses from domestic shocks, reported in Figure 5, suggest a mild negative response to the monetary policy shock and the exogenous ER shock. These results are consistent with the responses from the model in Figure 12, where the yellow lines decline very modestly after these shocks. Importantly, the response of the orange line (PCP) is much more significant, reaching nearly -2% in the first period after both shocks. This larger response is on account of the exchange rate channel under PCP, where the exchange rate appreciation raises the price of exports from the perspective of foreign buyers, and therefore leads to lower demand. When exports are largely invoiced according to DCP, as with the yellow and blue lines, the exchange rate channel is almost absent, and since the foreign demand channel is, by assumption, unaffected by these two shocks, export demand does not change, and therefore exports

Figure 12: Model impulse responses to key shocks



Note: Model results under PCP (orange line), DCP (blue line) and a blend of 20% PCP and 80% DCP (yellow line), where the latter case closely approximates the distribution for Canadian exports.

are essentially unchanged when these domestic shocks affect the economy.

Moving to the export responses from global shocks, our empirical results reported in Figure 7 suggest a positive response for real exports after a US income shock (persistent or transitory) and a negative response after an oil price shock. Consistent with this, the real export response after the US income shock is positive and the response after an oil price shock is negative according to the model results in Figure 12, and both responses are much larger than the responses after the domestic shocks in the first two columns. In both cases, the exchange rate channel remains almost absent for the results represented by the yellow lines, but the foreign demand channel is not fixed, and this channel drives the export response regardless of the currency of invoicing. In the model results, exports appear to correspond almost 1–to–1 with foreign demand (in % terms), and this is very close to what we find in our empirical results in Figure 7.

Both qualitatively and quantitatively, the model predictions line up with our empirical results. An important ingredient for generating these predictions, at least for the case of the domestic shocks, is the currency of invoicing, which is largely US dollars for Canada.

We are also interested in results for export prices, real imports, and import prices. Across these three variables and the four shocks that we examine, we find that our empirical results are consistent with the model results in 10 out of 12 cases. For export prices, our empirical results are largely consistent with the model results for all four shocks. Here, we reference our empirical results in Figure 8, where non-energy exports and export prices are used.[19] Our empirical results indicate a decline in non-energy export prices after a monetary policy shock, an exogenous ER shock, a US income shock (persistent or transitory), and an oil price shock, although the size of the decline is fairly modest in the second case. In all four cases, the model results reported in Figure 12 also suggest a decline in export prices.

For real imports, empirical results are reported in Figure 13. Here, we find evidence of a modest increase in imports after a monetary policy shock, and a very modest increase in imports after an exogenous ER shock. By comparison, the model results in Figure 14 suggest that imports increase slightly after a

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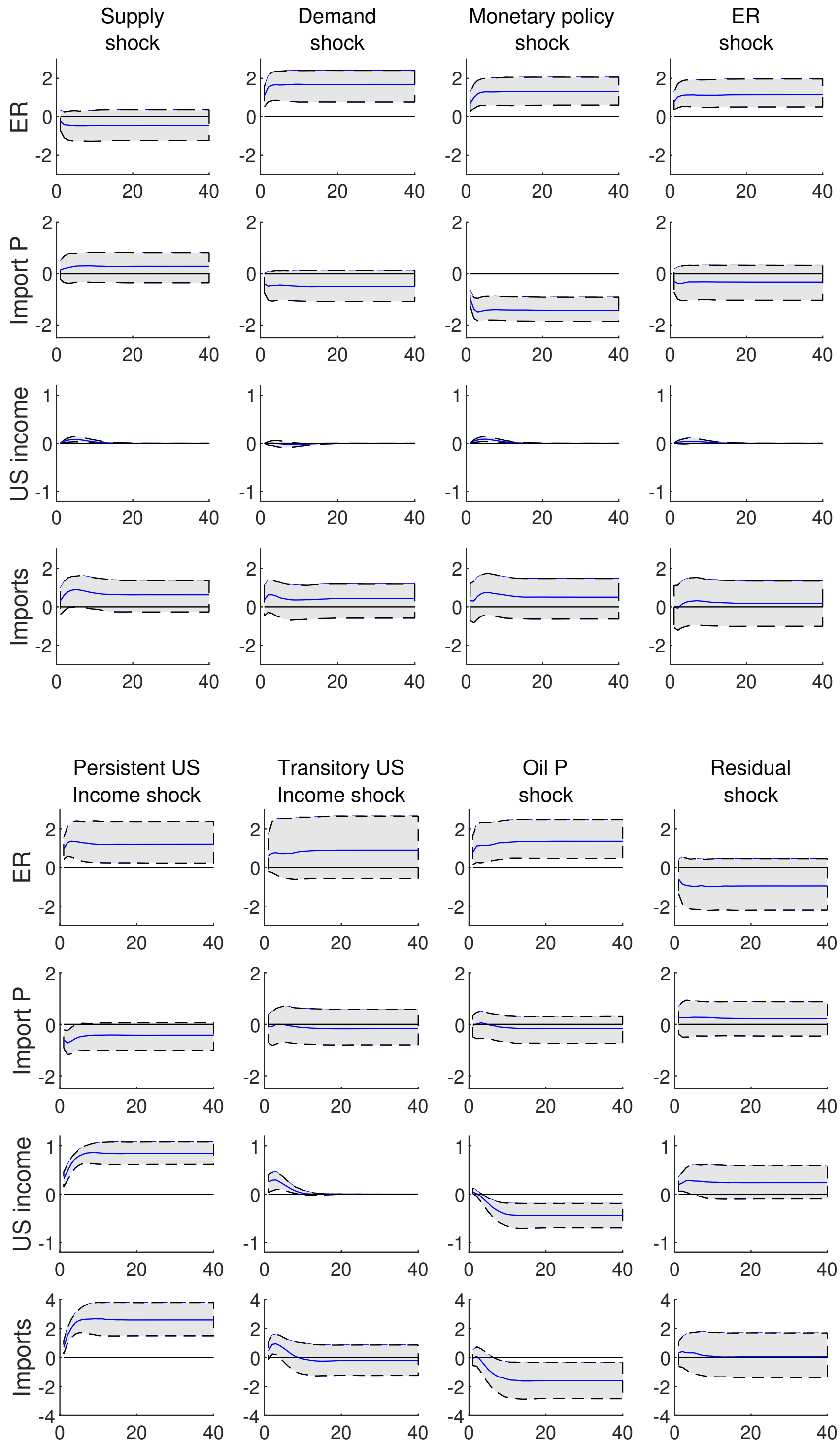
[19] The export price from the model does not include the price of the endowment good, and therefore focusing on empirical export prices that exclude energy prices is a more appropriate comparison.

monetary policy shock, and noticeably more than this after the exogenous ER shock, although still not as much as 2%, as one might naively expect based on our parameterization for the elasticity of demand. In both cases, the response for real imports is the opposite sign from the responses for real exports reported in Figure 12 (yellow lines), while the magnitude is slightly larger for imports than exports, although still quite small and below 1%. Our empirical results are consistent with the model results in terms of the sign of the response, but reveal a similarly weak response for imports as for exports in terms of magnitude. Our empirical results also indicate a significant positive response in imports after a US income shock (persistent or transitory), and this result is consistent with the model results in Figure 14. Finally, for import prices, our empirical results suggest that prices declines after a monetary policy shock, an exogenous ER shock, and a US income shock (persistent), and the model results in Figure 14 are consistent with these results.

The two cases where our empirical results are not consistent with the model predictions both pertain to oil price shocks. Our empirical results in Figure 14 reveal that real imports decline and import prices change very little after an oil price shock. By contrast, the model results suggest that real imports increase substantially and import prices decline after an oil price shock. For the case of real imports, the increase in the model stems from a wealth effect, where the endowment shock gives rise to more demand for imports in the home country. We believe that this depiction of the oil price shock is possibly too stylized to properly capture import dynamics as these appear in the data — in reality, the negative effect of the oil price shock on global demand could lead to lower global supply of imports, but this connection is not modelled in our framework. For the case of import prices, we are surprised to observe so little depreciation in prices when the exchange rate appreciates after an oil price shock in our empirical results. One possibility here is that, since Canadian imports do include some commodities, oil prices are correlated with the import price of these commodities and this is driving the increase in import prices when oil price shocks emerge. This would also help explain why real imports decline in these cases.

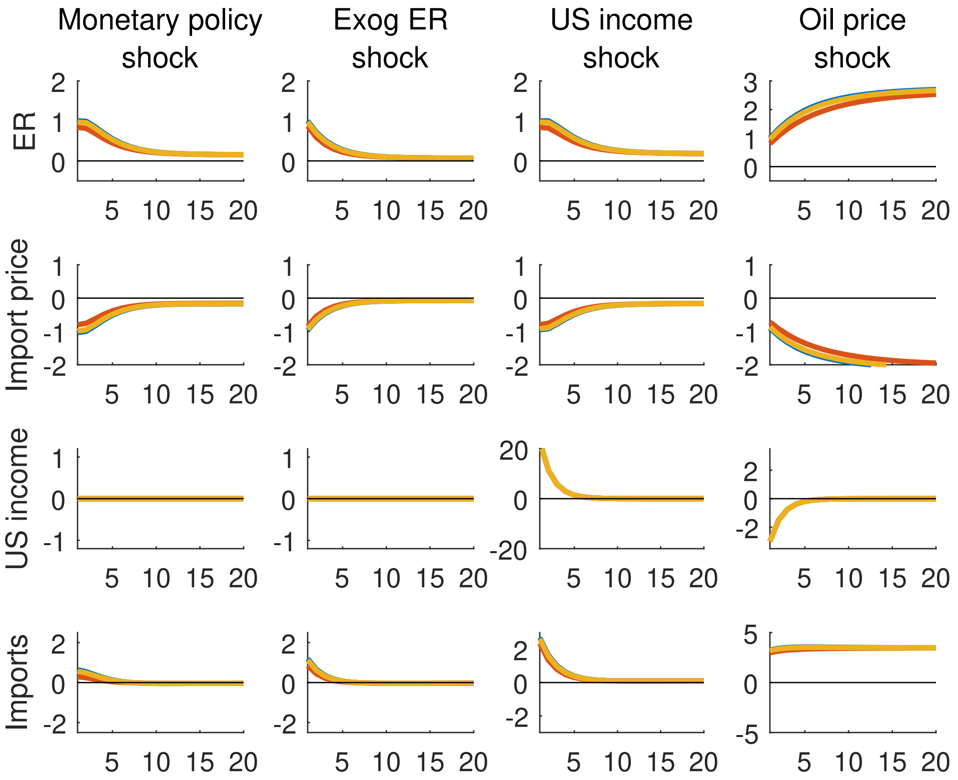
Overall, we judge that the blended version of the model, which is close to a DCP regime, lines up well with our empirical results. The model results indicate a weak relationship between real exports and

Figure 13: Robustness: Imports responses to shocks



Note: Median estimates for the impulse response of each variable (rows) to each domestic shock (columns) based on the SVAR described in Section 3, where dashed lines represent 65% confidence bands.

Figure 14: Model impulse responses to key shocks: imports



Notes: Model results under PCP (orange line), DCP (blue line), and a blend of 20% PCP and 80% DCP (yellow line), where the latter case closely approximates the distribution for Canadian imports.

exchange rate movements when the movements arise due to domestic shocks (a monetary policy shock or an exogenous ER shock), but a strong relationship here when the movements arise due to global shocks (a US income shock or an oil price shock). These results are consistent with our empirical findings reported in Figures 5 and 7. For the remainder of the impulse responses that we focused on in this section, we find that the model results are consistent with our empirical results in 10 out of 12 cases. For the two cases where the model results do not match well with our empirical results, this inconsistency is likely due to the simplified manner in which we have modelled oil supply shocks (an endowment shock) and the structure of the foreign economy (exogenous).

Still, the DCP model does fall short of fully accounting for export and import price dynamics in instances of exchange rate volatility. According to the model, export and import prices should respond fully to account for exchange rate movements when trade is invoiced in US dollars, but our empirical results suggest that these prices only respond partially to exchange rate movements, particularly in the case of "exogenous" exchange rate movements. Moreover, studies for Canada that use microdata also find that import and export prices only respond partially to exchange rate movements when trade is invoiced in US dollars [Cao, Dong and Tomlin (2015), Devereux, Dong and Tomlin (2017)], suggesting that some form of pricing-to-market that behaves differently from the form of strategic complementarity embedded in the DCP model used here could be at play.[20] We leave efforts to account for these inconsistencies to future work.

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[20] We experimented with versions of the DCP model that apply different parameters for nominal rigidities and strategic complementarities, but were unable to find a calibration that could successfully replicate both a partial import price and partial export price response to exchange rate movements for the exogenous exchange rate shock.

Table 3: Calibrated parameters

	Symbol	Value
Preferences		
Discount factor	β	0.99
Risk aversion	σ_c	2
Frish elasticity	φ^{-1}	0.5
Disutility of labor	κ	1
Labor demand elasticity	ϑ	4
Steady-state NFA	$\overline{\beta}$	0
Production		
Input share	α	2/3
(Log) productivity	a	1
Demand		
Elasticity	σ	2
Super elasticity	ϵ	1
Home bias	γ	0.7
Steady-state US/RoW demand	Y_U^D/Y_R^D	-2
Rigidities		
Wage	δ_w	0.85
Price	δ_p	0.75
Invoicing currency and trade shares		
DCP share exports	θ_{HU}^U	0.8
DCP share imports	θ_{UH}^U	0.8
Home bias	γ_H	0.59
U bias	γ_U	0.24
RoW bias	γ_H	0.17

Table 4: Calibrated shock parameters

	Symbol	Value
Monetary rule		
Inertia	ρ_m	0.5
Inflation sensitivity	ϕ_M	1.5
Output gap sensitivity	ϕ_Y	0.5/4
Shock persistence	ρ_ϵ	0.5
Steady-state interest rate	i*	$(1/\beta) - 1$
Commodity income		
Steady-state income	$\bar{\xi}$	-9
Shock persistence	ρ_ξ	0.74
Foreign income		
Steady-state income	Y_R^D	-2
Shock persistence	ρ_{UPS}	0.74
Correl with commodity inc shock		-0.001
UIP		
Shock persistence	ρ_{UIP}	0.74