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Job Ladder and Business Cycles

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Abstract

I build a Heterogeneous Agents New Keynesian model with rich labor market dynamics. Workers search both off- and on-the-job, giving rise to a job ladder, where employed workers slowly move toward more productive and better paying jobs through job-to-job transitions, while negative shocks occasionally throw them back into unemployment. The state of the economy includes the distribution of workers over wealth, labor earnings and match productivities. In the wake of an adverse financial shock calibrated to mimic the US Great Recession unemployment dynamics, firms reduce hiring, causing the job ladder to all but "stop working." This leaves wages stagnant for several years, triggering a sharp contraction and slow recovery in consumption and output. On the supply side, the slow pace in worker turnover leaves workers stuck at the bottom of the ladder, effectively cutting labor productivity growth in the aggregate. The interaction between weak demand and low productivity leads to inflation dynamics that resemble the missing disinflation of that period.

Topics: Business fluctuations and cycles; Labour markets; Productivity; Inflation and prices
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1 Introduction

Search and matching frictions plague labor markets, leading to misallocation of resources and deviations from the "law of one price." Unemployed workers are temporarily left out of production against their will, while employed workers are not necessarily allocated to the most productive firms nor paid their marginal productivity. Instead, workers must slowly move toward more productive and better-paying job opportunities through job-to-job transitions, in a turnover process usually described as the job ladder. As the pace of worker turnover picks up during expansions and cools down during recessions, the job ladder plays a fundamental role in transmitting aggregate shocks to productivity and wages—a process Moscarini and Postel-Vinay (2017a) calls the *cyclical job ladder*.

To study how these forces play out in equilibrium and over the cycle, I develop a Heterogeneous Agents New Keynesian (HANK) model with search and matching frictions. Workers are risk averse, *ex-ante* identical and search both off- and on-the-job for vacancies posted by firms. The productivity of firms (matches) are heterogeneous, but fixed within the match. Firms are allowed to counter outside offers received by the worker, like in the sequential auction framework in Postel-Vinay and Robin (2002). Match heterogeneity and between-firm competition for employed workers give rise to a distribution over jobs and wages—workers ascend it via job-to-job transitions, but occasionally suffer negative shocks that throw them back to unemployment. The income risk stemming from climbing and falling off the ladder is uninsurable, and workers' only available instrument for self-insurance is a risk-free real government bond.

The remaining blocks of the model closely follow the New Keynesian tradition. The output of the worker–firm match, which I denote by "labor services," is an input to the production of monopolistically competitive retailers. Retailers produce specialized goods by combining labor services and intermediate material goods and are subject to nominal rigidities. A final good producer combines the varieties produced by retailers to produce the final good, which can be used for final consumption or as an input into production (materials). A government runs an unemployment insurance program, and monetary policy follows a Taylor rule.

I calibrate the model to the US economy. I use aggregate data on labor market flows to discipline the magnitude of search frictions, and choose the firm productivity distribution to match moments of the residual wage inequality. I constrain the supply of government debt to target a large aggregate marginal propensity to consume (MPC), as documented in Johnson, Parker, and Souleles (2006) and Parker, Souleles, Johnson, and McClelland (2013). Wages are endogenous and depend on the interaction of search frictions, the distribution of firm productivity and the assumptions on firm competition.[1] For the stationary equilibrium,

[1]This distinguishes the current setup from the majority of the heterogeneous agent incomplete markets literature, which generally assumes an exogenous earnings process calibrated to replicate main moments of the data.

I show that the model-implied wage process replicates two salient features of the data: (i) it generates a negatively skewed and fat-tailed distribution of annual earnings growth; and (ii) it predicts large and persistent wage losses following job displacement. The ladder lies at the heart of both findings. Employed workers experience wage growth only through the arrival of outside offers, which are infrequent and therefore contribute to the excess kurtosis. Separations into unemployment trigger large earnings losses, explaining the negative skewness in earnings growth. Following displacement, workers are thrown to the bottom of the ladder, which they must climb all over again through on-the-job search—a process that can keep wages depressed long after workers have regained employment, explaining the persistent wage losses.

My main quantitative exercise focuses on the period of the US Great Recession, during which worker reallocation up the ladder all but collapsed—as Moscarini and Postel-Vinay (2016a) describes, the job ladder seems to have failed during this period. In the model, the recession is engineered by an increase in the discount rate of firms—a shortcut for worsening financial conditions—which mechanically results in less firm entry, higher unemployment and fewer job-to-job transitions. The result of the shock is a sharp and very persistent reduction in consumption and labor productivity, while inflation falls only temporarily.[2] Inflation is linked to the failure of the ladder, whose effect changes from initially disinflationary to inflationary later in transition. Consumption falls sharply in the initial periods following the shock, which (due to price rigidity) leads to a large contraction of marginal costs and hence of inflation. As time passes, however, the decline in job-to-job transitions slows down worker reallocation up the ladder, causing labor productivity to fall.[3] This effect is persistent—in particular, even more persistent than the effect on employment—and puts upward pressure on marginal costs at longer horizons, a countervailing force that acts to keep inflation from falling too much at the outset.

The remainder of the paper explores the demand- and supply-side channels operating through the job ladder. On the supply-side, I study a counterfactual equilibrium where unemployment and labor earnings move as in the baseline, but where the productivity effects of the ladder are turned off. Compared to the full model, the recovery in the counterfactual equilibrium is much faster and inflationary pressures arise much sooner. The rationale for this result is simple. The reduction in labor mobility during a recession not only increases unemployment, but also leaves employed workers stuck at low-productivity jobs. The distribution of employment across jobs is a slow-moving state that impairs production even after the direct effects of the shock have died out, which delays the return of the economy to steady state.

[2] The joint behavior of unemployment and inflation during the Great Recession, in particular the small disinflation in face of large and persistent unemployment, has came to be known as the *missing disinflation puzzle*. I discuss why it is viewed as missing in Section 6.

[3] The productivity consequences of job-to-job transitions were first raised by Barlevy (2002), who named it the *sullying effect* of recessions. This contrasts with the so-called "cleansing effect" of recessions, according to which recessions may increase labor productivity through the destruction of the least-productive jobs.

Next, I look at the effects of the collapse of the ladder on the demand side. A decomposition of aggregate consumption response reveals that the reduction in the contact rates for both employed and unemployed workers is the main driving force behind the sharp contraction and slow recovery.[4] This is because the reduction in the pace of transitions leads to a sizable cut to workers' expected future income—as meetings with potential employers become scarce, wages remain stagnant, forcing workers to revise down their consumption plans. The importance of income shocks in driving the consumption response echoes the findings in Kaplan, Moll, and Violante (2018) and Auclert, Rognlie, and Straub (2018). In these cases, however, the finding lies on the strong sensitivity of *current* consumption in response to *transitory* income shocks—a moment captured by the MPC. The collapse of the ladder is felt by workers mostly as a shock to their *future* income instead. Also, shocks are much more persistent in nature—an unrealized wage increase affects workers' income at the time of the event as well as its entire future path. The distinction is important as consumption sensitivity to current versus future income changes varies with workers' liquid wealth holdings—liquidity-constrained hand-to-mouth workers feature high MPC out of transitory income changes, but are not sensitive to what happens to their future income distribution. In line with this reasoning, I find that the workers whose consumption adjusts the most on impact are the *unconstrained* who happen to be at the lower rungs of the ladder.

Outline The rest of the paper proceeds as follows. Section 2 reviews some of the empirical literature documenting the importance of job-to-job flows for productivity and earnings growth, and relates this paper to the (scarce) literature that includes the job ladder in a business cycle model. Section 3 outlines the model and defines the equilibrium. Sections 4 and 5 explain the adopted calibration strategy and evaluate the model's implications for earnings and consumption dynamics in the stationary equilibrium. Section 6 discusses the results for the Great Recession exercise, while Section 7 unpacks the demand and supply effects of the ladder. Section 8 concludes.

2 Literature Review

The defining characteristic of the job ladder is that "workers agree on a common ranking of available jobs which they aspire to climb through job search, while being occasionally thrown back into unemployment."[5] When given the opportunity, workers tend to move toward "better jobs." Therefore, a robust implication of the ladder is that higher-ranked firms should be more successful in attracting and retaining workers. Bagger and Lentz (2019) uses this insight to rank firms in Danish matched employer-employee data by the fraction of their hires filled by workers coming from other jobs, as opposed to unemployment. They show that a firm's position in this "poaching rank" is stable over time and positively correlated

[4] The decomposition exercise follows Kaplan, Moll, and Violante (2018), which presents the exercise as a way to learn about the transmission mechanism of monetary shocks to consumption.

[5] Citation from Moscarini and Postel-Vinay (2017b).

with the firm's value-added per worker, suggesting that firms high up in the ladder are also more productive. Looking over the cycle, Crane, Hyatt, and Murray (2020) ranks firms by productivity using matched employer-employee US data and finds that the firm productivity distribution shifts down in recessions. Also looking at the US, Haltiwanger, Hyatt, Kahn, and McEntarfer (2018) documents the presence of a robust wage ladder with highly procyclical net flows from low- to high-wage firms. In the context of the Great Recession, Haltiwanger, Hyatt, Kahn, and McEntarfer (2018) and Moscarini and Postel-Vinay (2016b) show that the job ladder all but stops working.

As for the impact of job-to-job transitions on earnings, there is extensive empirical evidence documenting that workers experience wage increases when they undergo a job-to-job transition.[6] Just as important, employed workers who do not switch jobs may still benefit from outside offers, as those can be used to increase their wages at their current jobs. As evidence of the latter mechanism, Moscarini and Postel-Vinay (2017a) finds, using longitudinal microdata from the Survey of Income and Program Participation (SIPP), that earnings growth covaries with "predicted" job-to-job transitions even among workers who do not actually experience one. The "predicted" rate means to capture how likely it is for a worker to undergo a job-to-job transition based on effective transitions experienced by observationally similar workers. The authors interpret the positive correlation as evidence of workers gaining surplus via outside offers, as they would in a sequential auction model like that in Postel-Vinay and Robin (2002).

Next, I discuss how this paper relates to the literature. By featuring risk-averse workers making consumption and savings decisions in an environment with search frictions and on-the-job search, this paper relates to Lise (2012). His partial equilibrium analysis is the building block of the demand side of my model, as the regular income fluctuation problem in the traditional heterogenous agent incomplete markets model. This paper also relates to the extensive labor literature studying cyclical movements in labor market flows. The literature tends to feature workers with linear utility and does not address the impact of the job ladder on aggregate variables outside the labor market (see Menzio and Shi, 2011; Robin, 2011; Lise and Robin, 2017; Moscarini and Postel-Vinay, 2018). In the few cases where labor market frictions are incorporated into business cycle frameworks with consumption decisions and nominal rigidities, models tend to abstract from job-to-job flows (see Christiano, Eichenbaum, and Trabandt, 2016).

An exception is the work in Moscarini and Postel-Vinay (2019), which heavily motivates this paper. It is the first to introduce a job ladder into a DSGE New-Keynesian model and study the aggregate responses to productivity, preference and monetary shocks. Backed by their previous empirical work uncovering a positive relation between job-to-job transitions

[6] See for example Topel and Ward (1992), Hyatt and McEntarfer (2012), Moscarini and Postel-Vinay (2017a) and Hahn, Hyatt, and Janicki (2021). Gertler, Huckfeldt, and Trigari (2018) estimates the average wage changes of job changers is about plus 4.5%. The average hides lots of heterogeneity, with conditional wage changes equal to plus 30% for workers realizing wage gains and minus 23% for workers realizing wage losses.

and wage inflation,[7] the authors use the model as a laboratory to test the predictive power of labor market flows on future inflation. While I share their motivation to study the role of the job ladder over business cycles, this paper differs from theirs in two respects. First, I examine the economy's response to an adverse financial shock and show that the job ladder helps account for the aggregate behavior during and after the Great Recession, an exercise they do not consider. Second, on the modeling side, I assume that labor earnings risk is uninsurable. I show that this assumption affects the transmission of aggregate shocks to consumption, with workers reducing their consumption expenditures when the job ladder breaks down. This work also relates to Faccini and Melosi (2020). These authors empirically evaluate a simpler version of the Moscarini and Postel-Vinay (2019) model for the US during the post-Great Recession period, but focus mainly on the missing inflation following the recession instead of the missing disinflation during the recession, which is the main focus of this paper.

This paper also contributes to the burgeoning literature on Heterogenous Agent New Keynesian (HANK) models by adding realistic labor market flows to this framework.[8] Den Haan, Rendahl, and Riegler (2017), Gornemann, Kuester, and Nakajima (2016) and Kekre (2019) also study HANK models with labor market frictions, but none considers that the employed face search frictions through on-the-job search. In an analytically tractable HANK model with unemployment, Ravn and Sterk (2018) highlights that the precautionary savings response to *countercyclical* unemployment risk amplifies the consumption response to shocks compared to a complete market economy. This result contrasts with the dampening in consumption I find in response to the financial shock. There are two main differences between the model I develop here and their analysis. First, the cyclicality of the earnings risk in this paper is much more complex and takes into account wage fluctuations while employed, as well as unemployment risk.[9] Moreover, the model features a full distribution of marginal propensities to consume (MPCs), introducing a *redistribution channel* (Auclert, 2019) to any aggregate shock that unevenly affects workers.

3 Model

In this section, I lay out the Heterogeneous Agent New Keynesian (HANK) model I use to study the aggregate implications of labor market flows.

Goods, Technology, Agents Time is continuous. There are three vertically integrated sectors in the economy, each producing a different type of good that can be used either as an

[7]See Moscarini and Postel-Vinay (2017a).

[8]The recent literature that incorporates micro-heterogeneity into New Keynesian models of the macroeconomy include, among others, Guerrieri and Lorenzoni (2017), Bayer, Luetticke, Pham-Dao, and Tjaden (2019), McKay and Reis (2016), Auclert (2019), McKay, Nakamura, and Steinsson (2016), Ravn and Sterk (2018), and Kaplan, Moll, and Violante (2018).

[9]Ravn and Sterk (2018) also feature *aggregate* wage fluctuations that impact the cyclicality of earnings risk in their model. My point here refers to the piece-rate wage changes induced by the job ladder, which introduces a complex mapping between labor market flows and workers' labor income processes that varies over the cycle.

input by other sectors or consumed.[10]

At the bottom of this supply chain, *labor intermediaries* hire workers in a frictional labor market. Technology is linear in labor, with a unit of labor mapping to z units of labor services (thought as an intermediate input), which is then sold in a competitive market at price φ_t. Productivity z is specific to the worker–firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0, 1]$.

A measure one of *retailers* indexed by $j \in [0, 1]$ lies above the intermediate sector. Each retailer produces a *specialized input* $\tilde{Y}_j$ with a constant returns-to-scale technology in two inputs: labor services and materials.[11] The specialized inputs are then aggregated by a competitive representative firm to produce the final good $\tilde{Y}_t$.

The economy is populated by a continuum of *ex-ante* identical risk-averse workers indexed by $i \in [0, 1]$. Labor market risk makes workers heterogeneous in their employment status, labor income and wealth. A government issues debt and taxes labor income in order to finance government expenditures and an unemployment insurance program. I start by describing the worker's problem.

Workers Workers receive utility flow u from consuming c_{it} and do not value leisure. Preferences are time-separable, and the future is discounted at rate ρ

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt, \tag{1}$$

where the expectation reflects individual-level uncertainty in labor income.

An unemployed worker receives unemployment insurance (UI) benefits in the amount of $b \times \varphi_t$. An employed worker in a match of productivity z receives as a wage $y \times \varphi_t$, where the piece-rate $y \leq z$ depends on the worker's history in the labor market. Workers receive lump-sum dividends in the amount of d_{it}, save through a riskless government bond at flow real rate r_t and are subject to a no-borrowing constraint. Wealth a_{it} evolves according to

$$\begin{aligned} \dot{a}_{it} &= (1-\tau)\varphi_t \Big(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \Big) + r_t a_{it} + d_{it} - c_{it} - \tau^0_t, \\ a_{it} &\geq 0, \end{aligned} \tag{2}$$

where $\mathbb{1}^u_{it}$ is an indicator for unemployment status, τ^0_t is a government lump-sum transfer and τ is a proportional tax. The distribution of dividends across workers is a crucial determinant of the aggregate consumption response in HANK models (e.g., Bilbiie, 2018; Broer, Hansen, and Krusell, 2018; Werning, 2015). I follow Kaplan, Moll, and Violante (2018) and distribute

[10] See Christiano, Eichenbaum, and Trabandt (2016) and Moscarini and Postel-Vinay (2019) for a similar supply-side structure.

[11] Materials are converted one-for-one from the final good. I discuss the importance of materials in the retailer's problem. See Christiano, Trabandt, and Walentin (2010) for a standard New Keynesian model with materials.

profits in proportion to individuals' labor income

$$d_{it} = \frac{\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it}}{\int \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \right) di} D_t, \tag{3}$$

where D_t denotes aggregate profits.[12]

Workers maximize their lifetime utility given in (1) subject to the wealth accumulation process in (2), the labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{t\geq 0}$, dividends payouts $\{d_{it}\}_{t\geq 0}$ and paths of $\{r_t, \varphi_t, \tau^0_t\}_{t\geq 0}$, which they take as given.[13] At steady state, the recursive solution to this problem consists of value functions and consumption decision rules for the unemployed and the employed worker $\{c^u(a), c^e(a, y)\}$.[14] The worker's consumption policy function together with labor market transition rates and wage contracts induce a stationary distribution over wealth, labor income and match productivities $\Psi(a, y, z)$. With a slight abuse of notation, I denote marginal distributions by Ψ as well. Outside steady state, distributions and policies are time varying and described by a Kolmogorov forward and a Hamilton–Jacobi–Bellman equation. I indicate that dependence when necessary by adding a t subscript to equilibrium variables.

Search Frictions in the Labor Market The labor market features search frictions. Labor intermediaries post vacancies v_t to match with workers. Employed and unemployed workers search for open job vacancies. The searching effort of unemployed workers is normalized to one, while employed workers search with lower intensity s_e. Combined, they produce a search effort of

$$\mathcal{S}_t = u_t + s_e(1 - u_t). \tag{4}$$

Effective job market tightness is therefore

$$\theta_t = \frac{v_t}{u_t + s_e(1 - u_t)}. \tag{5}$$

[12] Aggregate profits include profits earned both by monopolistically competitive firms and labor intermediaries. Rewriting the worker's budget constraint under this profit distribution rule, we get

$$\dot{a}_{it} = \left((1 - \tau)\varphi_t + \frac{D_t}{\int \left(\mathbb{1}^u_{it} + (1 - \mathbb{1}^u_{it}) y_{it} \right) di} \right) \left(\mathbb{1}^u_{it} b + (1 - \mathbb{1}^u_{it}) y_{it} \right) + r_t a_{it} - c_{it} - \tau^0_t.$$

Hence, distributing profits in proportion to labor earnings neutralizes the redistribution effects by making all workers equally exposed to its fluctuations. Overall, dividends D_t and price of labor services φ_t enter in the same way in the budget constraint by multiplying the idiosyncratic worker labor market state $(\mathbb{1}^u_{it}, y_{it})$.

[13] Appendix A.2 derives the Hamilton–Jacobi–Bellman equation associated with the household problem and discusses the impact of the job ladder on consumption and savings decisions.

[14] Note that policy functions depend on wealth and the piece rate wage only. The attentive reader may notice the lack of match productivity z in the worker's state space, that, even if not a direct payoff relevant variable, still contains information about future labor income distribution. As I discuss below, the worker does not observe the productivity of its current match, making income and wealth the only state variables in the worker problem.

The flow of meetings at time t is given by a constant returns-to-scale matching function $\mathcal{M}(v_t, \mathcal{S}_t)$. Define $\lambda_t := \frac{\mathcal{M}(v_t,\mathcal{S}_t)}{\mathcal{S}_t}$ as the rate at which an unemployed worker meets a vacancy, while employed workers contact outside firms at a rate $\lambda_{et} = s_e \lambda_t$. A vacancy contacts a worker with intensity $q_t := \lambda_t / \theta_t$. Once a worker and firm meet, the firm makes a wage offer (details below) that may or may not be accepted by the worker. Finally, all matches are subject to a destruction shock at an exogenous flow probability δ.

Wage Contract Firms are restricted to offering workers piece-rate wage contracts that can be renegotiated only if the worker receives a better outside offer.[15] When the firm contacts a worker, it observes the worker's employment status and incumbent match productivity in case the worker is already employed. In contrast, the worker is uninformed about their match productivity, but learn about it from labor market transitions and wage offers—I discuss this assumption in the next section. In what follows, I describe wage offers to employed and unemployed workers.

Employed Worker Consider a worker employed at a match of productivity z who contacts an outside firm with which the match productivity draw is z'. The two firms Bertrand compete for the worker's services over piece-rate wage contracts, with the more productive firm winning the bidding for the worker.

First, consider the case where $z' > z$; that is, when the poacher is more productive than the incumbent firm. The incumbent's maximum wage offer is to promise the worker the whole output flow of the match—i.e., offer a piece-rate $y = z$. The poaching firm z' attracts the worker by outbidding incumbent's piece-rate wage offer by ϵ, which results in the worker moving to firm z' at a piece-rate wage of $z + \epsilon$. In the solution of the model, I take ϵ to be an arbitrarily small number.[16]

Now, suppose instead that $z' < z$. The competition between the two firms has the worker staying with the incumbent, but the wage contract can still be renegotiated if the poaching firm's maximum wage offer is above the worker's current piece-rate (i.e., if $z' > y$). In this case, the worker's piece-rate wage from the incumbent firm increases to $z' + \epsilon$.

Unemployed Worker Upon meeting an unemployed worker, I assume that the firm makes a piece-rate offer of $\underline{z}$; that is, the firm offers the unemployed the full production of the least productive firm. In the calibration, I choose the unemployment insurance replacement rate b

[15] Note that piece-rates are usually defined in terms of a share of the match output flow, so if the match produces X, a piece-rate p would entail a wage of pX with $p \leq 1$. I define the piece-rate in terms of the price of labor services instead, so the wage of a worker in match z with piece-rate of y is $y\varphi_t$, with the restriction $y \leq z$. See Bagger, Fontaine, Postel-Vinay, and Robin (2014) for an implementation of a piece-rate version of the sequential auction framework in a standard labor market model that abstracts from incomplete markets and consumption and savings decisions.

[16] Note that this assumption departs from Postel-Vinay and Robin (2002) as the more productive firm attracts the worker by matching the *wage offer* of the less productive firm, as opposed to matching the worker's value of staying at the incumbent firm under the maximum wage offer. The same assumption is made in Graber and Lise (2015) and is intended to keep the problem tractable in the presence of a non-degenerate wealth distribution on the worker side.

to be equal to $\underline{z}$, so firms effectively offer the unemployment insurance rate to unemployed workers.

In the description above, I have treated the worker's acceptance decision as given. In particular, I implicitly assume that (i) the unemployed worker accepts the initial wage offer coming from any firm, and (ii) the employed worker always moves/stays in the firm offering the highest wage. While (ii) is a natural assumption in the current setup where more productive matches also offer higher wages, it is not clear that (i) would hold without any additional assumptions. In what follows, I discuss the unemployed worker's reservation strategy in the presence of such wage contracts.

Worker's Reservation Strategy While firms offer the same initial wage contract to workers coming out of unemployment, the unemployed workers' value of meeting a vacancy increases with the productivity of the match. This is because being hired by a firm with greater productivity implies a better (in the first-order sense) distribution of future wages.[17]

Because the unemployed search intensity is greater than that of the employed ($\lambda > \lambda_e$), there is an *option value* associated with waiting to meet more productive firms. The value of remaining in unemployment and waiting for better matches versus accepting an offer at a match of productivity z will depend on the worker's assets, leading to a reservation productivity policy that depends on wealth.

The extent to which search decisions depend on the worker's wealth is certainly an important question.[18] My main interest here, however, is not to analyze how incomplete markets impact search decisions, but instead to study how a "realistic" model of the labor market transmits aggregate shocks to consumption. Therefore, I simplify the worker's reservation decisions by assuming that the worker never gets to observe the productivity z of its own match.[19] This transforms the reservation decision of the unemployed into a trivial one: by making all offers coming out of unemployment identical—meaning that all firms offer the same wage, so they all look the same to the unemployed worker—they are either all accepted or all rejected. Since being employed entails a higher present value of earnings than being unemployed, all offers will be all accepted by the worker.

Making the productivity a hidden state adds a learning/filtering dimension to the worker's problem, who still gets to observe their wage history in the labor market. I describe this problem in Appendix A.1. Next, I turn to the supply side of the economy.

[17]To see this, consider the future path of wages for a recently hired worker at matches of productivity z_1, z_2, with $z_1 > z_2$, in the circumstance where the worker meets an outside firm of productivity $z_3 \in [z_1, z_2]$. If employed at firm z_1, the worker switches jobs and the piece-rate wage changes to z_1. If employed at firm z_2, however, the worker stays in the firm and the wage increases to $z_3 > z_1$.

[18]For examples of papers that study this, see Lentz and Tranæs (2005) and Eeckhout and Sepahsalari (2021).

[19]A simpler way to eliminate the option value would be to assume that the search intensity is the same for the employed and unemployed, $s_e = 1$. This, however, would preclude the model from matching the small flow of employer to employer transitions relative to unemployment to employment flows. But, as I show in the experiments, it is the slow reallocation along the ladder that generates long-lasting impacts of misallocation—one of the main points of the paper.

Final Good Producer A competitive representative final good producer aggregates a continuum of specialized inputs, $\tilde{Y}_{j,t}$, using the technology

$$\tilde{Y}_t = \left(\int_0^1 \tilde{Y}_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $\epsilon > 0$ is the elasticity of substitution across goods. The firm's first-order condition for the jth input is

$$\tilde{Y}_{j,t}(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} \tilde{Y}_t, \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{7}$$

Retailers The jth input good in (6) is produced by a *retailer*, who is a monopolist in the product market. Following Basu (1995) and Nakamura and Steinsson (2010), each retailer produces their specialized good by combining materials $M_{j,t}$ and labor services $N^e_{j,t}$ according to the production function

$$\tilde{Y}_{j,t} = M_{j,t}^{\gamma} (Z_t N^e_{j,t})^{1-\gamma}, \tag{8}$$

where Z_t is an aggregate productivity component. Materials are converted one-for-one from the final good $\tilde{Y}_t$ in (6), so each retailer effectively uses the output of all other retailers as input to production. Retailers buy labor services at the competitive price φ_t and materials for the real price of one.

Cost minimization implies a common marginal cost across all retailers, given by

$$m_t = \left(\frac{1}{\gamma} \right)^{\gamma} \left(\frac{\varphi_t / Z_t}{1-\gamma} \right)^{1-\gamma}. \tag{9}$$

Cost minimization also implies that the relative price of labor services and materials must be equal to the ratio of their marginal productivities

$$\frac{\varphi_t / Z_t}{1} \frac{\gamma}{1-\gamma} = \frac{M_{jt}}{Z_t N_{jt}}. \tag{10}$$

Each retailer must also choose a price $P_{j,t}$ to maximize profits subject to demand curve (7) and price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the firm's rate of price change $\dot{P}_{j,t}/P_{j,t}$ and expressed as a fraction of gross output $\tilde{Y}_t$ as

$$\Theta_t \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right) = \frac{\theta}{2} \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right)^2 \tilde{Y}_t, \tag{11}$$

where $\theta > 0$.[20] Therefore, each retailer chooses $\{P_{j,t}\}_{t\geq 0}$ to maximize

$$\int_0^\infty e^{-\int_0^t r_s ds} \left\{ \tilde{\Pi}_t(P_{j,t}) - \Theta_t \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right) \right\} dt,$$

where retailers discount profits at the real rate $\{r_t\}_{t\geq 0}$ and

$$\tilde{\Pi}_t(P_{j,t}) = \left(\frac{P_{j,t}}{P_t} - m_t \right) \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t$$

are flow profits before price adjustment costs.

In a symmetric equilibrium, all firms choose the same price $P_{j,t} = P_t$ and produce the same amount of goods $\tilde{Y}_{j,t} = \tilde{Y}_t$. Moreover, as shown in Kaplan, Moll, and Violante (2018), the quadratic price adjustment costs in a continuous-time setting yields a simple equation characterizing the evolution of aggregate inflation $\pi_t := \dot{P}_t / P_t$

$$\left(r_t - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon}. \tag{12}$$

Equation (12) is the New Keynesian Phillips curve, which can also be represented in present-value form as

$$\pi_t = \frac{\varepsilon}{\theta} \int_t^\infty e^{-\int_t^s r_\tau d\tau} \frac{\tilde{Y}_s}{\tilde{Y}_t} (m_s - m^*) ds. \tag{13}$$

The presence of materials adds a flexible factor input into production, which allows output to change immediately (at time 0) upon aggregate shocks. To see this, substitute (10) into (8) and (6) evaluated at the symmetric equilibrium. This gives an aggregate *restriction* between aggregate production $\tilde{Y}_t$, marginal costs m_t and labor services N_t^e

$$\tilde{Y}_t = (m_t \gamma)^{\frac{\gamma}{1-\gamma}} Z_t N_t^e. \tag{14}$$

So, production changes in equilibrium if (i) productivity Z changes, (ii) marginal costs m changes or (iii) labor services inputs N^e change. Market clearing in the market for labor services—see equilibrium definition in Section 3.1—imposes that all the supply of labor service must be employed by retailers. This is a stock (state variable), however, so it cannot adjust at the impact of an aggregate shock—retailers, while individually allowed to reduce their usage of labor services, cannot do so in the aggregate immediately following the shock. Labor service competitive price φ_0 must therefore adjust to make retailers willing to hire the labor service stock in the economy. As φ_0 changes to clear the labor market, retailers adjust their materials-labor ratio according to (10), which leads production to adjust.

[20] I follow Hagedorn, Manovskii, and Mitman (2019) in assuming that price adjustment costs are "virtual," meaning that they affect optimal choices but do not cause real resources to be expended. That is why pricing costs do not appear in the goods market clearing condition in the definition of equilibrium.

Labor Intermediaries A firm in the intermediate sector can post a vacancy at a flow cost of κ^f, expressed in units of the consumption good. Upon meeting a worker, the firm must pay an additional fixed screening/training cost to learn the match productivity and start producing.[21] This cost is allowed to depend on the employment status of the worker, but, different from the vacancy cost, it does not expend any real resources and does not show up in any budget constraint.[22]

Firms discount their profit flow at the rate $r_t + \chi_t$, where χ_t is a (exogenous) spread between the return of vacancy posting investments and the risk-free rate.[23] Let $\mathcal{J}_t(z, y)$ denote the expected present discounted value of dividends for a firm with match productivity z currently offering the worker a piece-rate contract y. The firm's value function is defined recursively in Appendix A.3.

The measure of vacancies v_t is pinned down in equilibrium by the following free-entry condition

$$\frac{\kappa^f}{q_t} = \int \Bigg\{ \frac{u_t}{u_t + s_e(1-u_t)} \left[\mathcal{J}_t(z, \underline{z}) - \tilde{\kappa}^{su} \right] + \\ + \frac{s_e(1-u_t)}{u_t + s_e(1-u_t)} \left[\int_{\underline{z}}^{z} \mathcal{J}_t(z, z') \frac{d\Psi_t(z')}{1-u_t} - \tilde{\kappa}^{se} \right] \Bigg\} d\Gamma(z), \tag{15}$$

which equates the expected flow cost of hiring a worker, $\frac{\kappa^f}{q_t}$, to the expected value of a match. The latter accounts for the probability of meeting an unemployed worker, an event that the firm values by $\mathcal{J}(z, \underline{z})$, and the probability of meeting an employed worker matched with a firm of productivity z', which has a value of $\mathcal{J}(z, z')$ if $z > z'$ and otherwise is zero.

The distribution of workers in the labor market—the measure u_t of unemployed workers and the distribution $\Psi_t(z)$ of employed workers—affects firms' incentives by changing their expectations of the type of worker they will encounter. At the same time, the distribution of employment depends on the measure of vacancies posted through the market tightness.

Monetary Authority The monetary authority sets the nominal interest rate on nominal government bonds i_t according to a Taylor rule

$$i_t = \bar{r} + \phi_\pi \pi_t + \epsilon_t, \tag{16}$$

where $\phi > 1$ and ϵ_t is a monetary policy shock. Given inflation and the nominal interest rate, the real return on the government bonds r_t is determined by the Fisher equation $r^b = i_t - \pi_t$.

[21] As suggested in Pissarides (2009), and exploited in Christiano, Eichenbaum, and Trabandt (2016) in an estimated model without OJS, screening costs raise amplification of unemployment fluctuations to aggregate shocks by insulating hiring costs from vacancy congestion coming from the matching function.

[22] These costs can be thought of as utility costs associated with the training/screening of workers. See Moscarini and Postel-Vinay (2019) for a similar assumption.

[23] At steady state, I set χ to zero. Outside steady state, I interpret shocks to χ_t as a reduced form of financial shock.

Government The government issues *real* bonds of infinitesimal maturity B_t^g, with positive values denoting debt. This is the only savings instrument available to workers. The government taxes workers' labor income at rate τ and uses this revenue to finance unemployment insurance, government expenditures G_t and interest payments on its debt. The government fiscal policy must satisfy the sequence of budget constraints

$$\dot{B}_t^g = r_t B_t^g + G_t + u_t(1-\tau)\varphi_t b - \tau\varphi_t \int y d\Psi_t(y) - \tau_t^0, \text{ for all } t. \tag{17}$$

At steady state, lump-sum transfers τ^0 are set to zero. Outside steady state, I let lump-sum transfers be the fiscal instrument that adjusts in order to keep government debt B_t^g constant at the steady-state level.[24]

3.1 Equilibrium

The rich worker heterogeneity over jobs, earnings and wealth shows up in the equilibrium definition below only through a small number of functions that integrate workers' decisions and states over its distribution. For example, while the consumption of workers varies across earnings and wealth, equilibrium conditions only depend on an *aggregate consumption function*

$$\mathcal{C}_t := \int c_t(a,y) d\Psi_t(a,y),$$

where the time index t subsumes the dependency of policies and distributions on the whole sequence of equilibrium prices and quantities entering the worker's problem.[25] In a similar way, the *aggregate labor services supply*

$$\mathcal{N}_t^e := \int z d\Psi_t(z)$$

is all that enters the market clearing of labor services. Notwithstanding all the complexity involved in *evaluating* those functions,[26] they still constitute a mapping from *aggregate* sequences of equilibrium prices and quantities (like real rate) into other *aggregate* sequences (like consumption), which in turn must satisfy certain equilibrium conditions.[27] This observation is the basis of the numerical algorithm used to solve the model—see Appendix C for details. I now turn to the equilibrium definition.

[24] See Kaplan and Violante (2018) for a discussion on the importance of fiscal adjustment in HANK models.

[25] Specifically, the worker cares about the evolution of $\{r_t, \varphi_t, d_t, \tau_t, \tau_t^0, \lambda_t, \lambda_{et}\}_{t\geq 0}$.

[26] Aggregate consumption at time t, for instance, is the summation of consumption decisions $c_t(a,y)$, itself a function of the whole sequence of prices, labor market transitions and fiscal policy, across wealth and earnings distribution, the evolution of which depends on the consumption decisions and labor market transitions up to time t.

[27] Even though continuous time perfect-foresight transition equilibrium objects consists of *real valued functions* $X : [0,\infty) \to \mathbb{R}$ and not really *sequences* $Y : \mathbb{N} \to \mathbb{R}$, I use sequences when describing those in the text since this agrees with the more commonly used discrete time convention.

Definition 1 (Equilibium) *Given an initial government debt B^g, an initial distribution Ψ_0 over wealth, labor income and match productivity, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, a general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N_t^e, M_t, u_t, v_t, D_t\}_{t\geq 0}$, labor market transition rates $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$, government policies $\{G_t, B_t^g, \tau_t, \tau_t^0, i_t\}_{t\geq 0}$, labor income process $\{\mathbb{1}_{it}^u, y_{it}\}_{i\in[0,1],t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t, \mathcal{N}_t^e\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that workers optimize, firms optimize, monetary and fiscal policy follow their rules, the labor income process is the result of labor market transitions and wage-setting, worker aggregate functions and distributions are consistent with labor market transition rates and worker's decision rules,*

- *the free-entry condition (15) holds,*
- *and all markets clear: the asset market $\mathcal{A}_t = B_t^g$, the labor services market $N_t^e = \mathcal{N}_t^e$ and the good market $\mathcal{C}_t + G_t + \kappa^f v_t = Y_t = \tilde{Y}_t - M_t$.*

4 Calibration

I calibrate the model at a monthly frequency. The calibration strategy is divided into four main steps. First, I calibrate the labor market transition rates to match estimated flows and choose the firm productivity distribution to match the dispersion in the residual wage distribution. Second, I choose the vacancy costs and the relative importance of screening versus flow costs. Third, I use the overall amount of liquidity, which in the economy takes the form of government bonds, to directly target average MPC in the data. Finally, I calibrate the parameters of the production and monetary side to standard values used in the New Keynesian literature. The full list of parameter values and targeted moments is displayed in Table 1.

Labor Market (Transitions and Productivity) I assume a standard Cobb–Douglas matching function $\mathcal{M}(v, \mathcal{S}) = v^\alpha \mathcal{S}^{1-\alpha}$, with $\alpha = 0.5$, as in Moscarini and Postel-Vinay (2018). I target a job finding rate λ of 0.45, which implies a monthly job finding probability of $1 - exp(\lambda) = 0.36$. I set δ to match the monthly probability of transitioning from employment to unemployment. These two flows imply a steady-state rate of $\frac{\delta}{\delta+\lambda} = 5\%$. The relative search efficiency of employed worker s_e is set so the steady-state monthly job-to-job transition rate equals 2.4%.

The productivity distribution Γ is assumed to be an affine transformation of Beta distribution; that is, a match productivity $z = c_0 + c_1 X$, where $X \sim \text{Beta}(\beta^1, \beta^2)$. This reduces the distribution of firm productivity to four parameters $(c_0, c_1, \beta^1, \beta^2)$, which I calibrate as follows. I fix $c_0 = 0.3$, since it is just a normalization. I use the evidence on the frictional wage dispersion to pin down the remaining parameters (c_1, β^1, β^2). In the data, this is understood as the dispersion in wages after eliminating all variation due to observable and unobservable individual characteristics (e.g., experience, education, marital status, gender and ability).[28] The dispersion left in the residual is the correct empirical counterpart for the *overall* wage

[28] That is why the literature also refers to this measure as *residual* wage dispersion.

Table 1: List of parameter values and targeted moments

Variable		Value	Target
Labor market			
$\mathcal{M}$	matching function	$v^{0.5}\mathcal{S}^{0.5}$	—
δ	destruction rate	0.024	—
λ	job finding prob.	0.412	unemployment of 5%
s_e	employed search intensity	0.127	ee transition of 0.024
b	replacement rate	$\underline{z}$	UI replacement rate of 50%
$z = c_0 + c_1 X$ $X \sim \text{Beta}(\beta^1, \beta^2)$	productivity grid	$(0.30, 2.61)$ $(1.0, 10.0)$	*residual wage dispersion* $p^{50}/p^{10}, p^{90}/p^{10} = (0.64, 1.10)^{\text{A}}$
$\kappa^f, \kappa^{su}, \kappa^{se}$	vacancy costs	$0.34, 3.4, 1.0$	see text
Preferences and Liquidity			
ρ	discount rate	$0.08/12.0$	$r^{ann} = 0.02$
$u(\bullet)$	utility function	$\log(\bullet)$	—
B^g/Y^{ann}		≈ 0.30	*target quarterly MPC of 0.25*[B]
Retailers, Final Good and Government			
γ	material share	0.50	share of materials in gross output
ϵ	elasticity of substitution	10.0	—
ϵ/θ	slope of Phillips curve	0.0067	price rigidity of 12 months
τ	tax rate	0.25	$G/Y \approx 0.20$
ϕ_π	Taylor rule coefficient	1.50	—

[A] Lemieux (2006) and Autor, Katz, and Kearney (2008).
[B] Johnson, Parker, and Souleles (2006) and Parker, Souleles, Johnson, and McClelland (2013) report quarterly MPC estimates around $[0.15, 0.30]$.

dispersion in the model, where workers are *ex-ante* identical and wage differentials arise from different labor market histories. At the chosen calibration ($c_1 = 2.61, \beta^1 = 1, \beta^2 = 10.0$), log differences between the 50/10 and 90/10 percentiles of the wage distribution are 0.64 and 1.10 respectively, in line with the estimates reported in Lemieux (2006) (Figure 1A) and Autor, Katz, and Kearney (2008) (Figure 8). Finally, I set $b = \underline{z}$ so the unemployed agent earns as much as a recently employed agent. This delivers a UI replacement rate of approximately 50%, which is within the range of values used in the literature.

Labor Market (Vacancy Costs) The canonical search and matching model fails to match the cyclical volatility in the job finding rate—a point initially noted by Shimer (2005). The same difficulty is also present in a model with on-the-job search—see Moscarini and Postel-Vinay (2018) for a detailed comparison of the canonical model versus a model with on-the-job search. Since one of my objectives is to study the impact of labor market fluctuations on consumption, it is crucial that fluctuations in unemployment and earnings risks approximate those in the data.

I achieve this by resorting to high fixed screening costs. Specifically, I need three restrictions to pin down the values of vacancy posting κ^f and screening costs $(\tilde{\kappa}^{se}, \tilde{\kappa}^{su})$. The targeted job finding rate $\lambda = 0.45$ imposes the first restriction—through the matching function, this implies a steady-state level of market tightness θ that must be consistent with the free-entry condition. I impose two additional restrictions by (i) making the firm indifferent between hiring an employed worker and hiring an unemployed worker at steady state,[29] and (ii) making screening costs 90% of the total hiring cost. The fixed cost's share of total cost is in line with Christiano, Eichenbaum, and Trabandt (2016), which estimates this to be 94%.

To understand the rationale behind (i), suppose I did not make the screening costs dependent on employment status. As the value of meeting an unemployed worker is greater than that of meeting an employed worker, firms would be more willing to post vacancies whenever unemployment is high because these are periods when firms face a higher probability of meeting an unemployed worker. This force, which is quite powerful in the model, accelerates transitions back to steady state and reduces the unemployment response to shocks. Hence, having the screening cost depend on the employment status of workers and satisfying restriction (i) mitigates this effect.

Preferences and Liquidity Workers have log utility over consumption with discount rate set to 8% annually. I assume that steady-state inflation is equal to zero and that the steady-state real interest rate equals 2%. The high value of discount rate relative to the real rate yields a low-liquidity economy, with government debt B^g amounting to only 28% of annual GDP.

As discussed in Kaplan, Moll, and Violante (2018), one-asset HANK models feature a tension between matching the high observed aggregate wealth-to-output ratio and generating average MPCs within the range of available estimates. Matching one of these two targets necessarily comes at the expense of getting the other one correct. If we calibrate the model to reflect the amount of wealth-to-output we observe, average MPC turns out to be only a fraction of the data; if we directly target the value of estimated MPCs instead, we end up with wealth-to-output ratios that are too small. Given the importance of (intertemporal) MPCs for the aggregate demand response to shocks, I opt for the second strategy and set the household's discount factor to directly target empirical evidence on the side of MPCs. Specifically, I target an average quarterly MPC out of a $500 unexpected transfer of 0.25, a value that lies within the range of values reported in the literature (Johnson, Parker, and Souleles, 2006; Parker, Souleles, Johnson, and McClelland, 2013).

Production The elasticity of substitution for the inputs produced by retailers ϵ is set to 10. The input share of materials γ is set to 0.5, which lies in the interval of values considered

[29] In terms of the values defined before, this restriction is written as

$$\int \left[\mathcal{J}(z,\underline{z}) - \kappa^u\right] d\Gamma(z) = \int \left\{ \left[\int_{\underline{z}}^{z} \mathcal{J}(z,z') \frac{d\Psi(z')}{1-u} - \tilde{\kappa}^{se} \right] \right\} d\Gamma(z).$$

in Nakamura and Steinsson (2010). I set the price adjustment cost θ coefficient to 1500, so the slope of the Phillips curve is given by 0.0067. The Phillips curve under Rotemberg or Calvo price rigidities has the same log-linear representation, so we can map the slope of the Rotemberg Phillips curve to the implied Calvo parameter determining the time between price changes. In that case, the slope of 0.0067 implies prices change once every 12 months, which is close to the Bayesian estimates in Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2014).[30]

Fiscal and Monetary Policy I set the labor income tax to 25%. Government expenditures are determined residually from the government budget constraint and amounts to around 20% of GDP. The Taylor rule coefficient is set to 1.5.

5 Earnings, Consumption and Job Ladder

Labor earnings in the model are a function of labor market transition rates, the distribution of firm productivity and firm competition for employed workers. The previous section discussed how to calibrate these ingredients using information on labor market flows and measures of frictional wage dispersion, but did not address its implication for earnings dynamics. I explore this point in this section.

First, I look at the high-order moments of labor earnings growth distribution. I show that the job ladder structure, although parsimonious, generates the negative skewness and excess kurtosis in the distribution of annual earnings changes that are documented in Guvenen, Karahan, Ozkan, and Song (2016) using the US Social Security Administration data. Second, I discuss how the model performs with respect to the empirical evidence on wage and consumption dynamics following a job displacement.

5.1 Earnings Dynamics

Let the economy rest at its stationary equilibrium. I follow the literature and look at the distribution of annual earnings changes. For worker i with piece-rate wage $\{y_{is}\}_{s\in[t,t+12]}$ during year t, yearly labor earnings are given by

$$y_{it}^{A} \equiv \varphi \int_{t}^{t+12} y_{is} ds.$$

[30] This mapping is given by

$$\frac{\epsilon}{\theta} = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha},$$

where β is the household discount factor, and $1-\alpha$ denotes the probability with which the firm gets to reset prices in the month. Setting $\beta = exp(-12.0r)$, $\epsilon/\theta = 0.0067$, which leads to $\alpha = 0.92$, meaning an expected price rigidity of $(1-\alpha)^{-1} \approx 12$ months.

Table 2: Moments of earnings change distribution

Moment	Data	Model
$Var[\Delta\tilde{y}^A]$	0.260	0.140
$Skew[\Delta\tilde{y}^A]$	-1.07	-0.721
$Kurt[\Delta\tilde{y}^A]$	14.93	5.907
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.05$	0.310	0.337
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.10$	0.490	0.434
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.20$	0.670	0.578
Fraction $\lvert\Delta\tilde{y}^A\rvert < 0.50$	0.830	0.838

Notes: **Data:** Moments of the one-year log labor earnings changes distribution computed from the Master Earnings File of the Social Security Administration data (values are taken from Guvenen, Karahan, Ozkan, and Song (2016)). **Model:** Moments of the one-year log labor earnings changes distribution computed by simulating a panel of 100,000 workers in the stationary equilibrium of the model. The measure of labor earnings in the model and data excludes unemployment insurance payments collected by workers.

Let $\tilde{y}_{it}^A \equiv \log y_{it}^A$ denote annual log earnings of individual i, so changes in annual log earnings are given by $\Delta\tilde{y}_{it}^A \equiv \tilde{y}_{i,t+1}^A - \tilde{y}_{it}^A$.[31] To compute this distribution, I simulate a panel of workers and record their earnings as well as their movements in the labor market.

Table 2 reports some moments of the simulated data along with estimates from Guvenen, Karahan, Ozkan, and Song (2016) from administrative Social Security data. The model successfully generates two key facts of the data: a strong negative skewness and kurtosis (in the case of kurtosis, the model value falls short of the estimates in the data, but is still above what would be expected from a normal distribution). The fraction of small earnings changes in the model (less than 5%, 10% and 20%) are also close to the data. These facts on higher-order moments—which are also highlighted in Guvenen, Karahan, Ozkan, and Song (2016)—are a natural consequence of the job ladder structure. The wage received by workers is fixed within the contract and grows only with the arrival of outside offers (which are infrequent but sizable, contributing to excess kurtosis), while occasional unemployment shocks lead to large earnings losses (contributing to the negative skewness).[32] Notwithstanding the model's ability to generate those facts, the magnitude of the shocks is much smaller than what we observe in the data—the variance of log earnings changes $Var[\Delta\tilde{y}^A]$ is only half of that in the data, with kurtosis also falling short. This is somewhat expected as the data compounds the influence of factors beyond the job ladder, such as idiosyncratic productivity (human capital). In the model, wage dispersion and earnings growth are solely due to search frictions, so we

[31] Notice that the measure of labor earnings ignores unemployment insurance payments. This ensures that the moments computed in the model are comparable to data from Guvenen, Karahan, Ozkan, and Song (2016), whose measure of earnings include only wages, salaries and bonuses.

[32] The ability of a job ladder model to reproduce the high-order moments documented by Guvenen, Karahan, Ozkan, and Song (2016) is also highlighted by Hubmer (2018).

should not expect it to capture all the risk contained in the data.

5.2 Earnings and Consumption upon Job Loss

I now turn to the model's predictions for the dynamics of earnings and consumption for workers who lose their job. These (nontargeted) moments are informative about the magnitude of the downside earnings risk faced by workers and their ability to smooth consumption in face of such events, and therefore help validate the calibration. I measure the effect of job-displacement following the methodology in Saporta-Eksten (2014). The author uses the 1999–2009 biennial waves of Panel Study of Income Dynamics (PSID) to document the dynamics of wages and consumption around a job displacement.[33] The main regression specification is

$$\log w_{it}^{A} = \alpha_0 + \sum_{k\geq -2}^{10} \delta_k D_{it}^{k} + u_{it}, \tag{18}$$

where the outcome variable w_{it}^{A} is the annual wage rate of individual i in year t (total labor earnings divided by total hours), and D_{it}^{k} is a set of dummy variables used to indicate a worker in their kth year before, during or after a job loss.[34] The coefficients δ_k capture the wage losses of workers who were displaced k years ago (or will be displaced in $-k$ years) relative to workers who have not experienced displacement at that time. These losses are estimated for the two years preceding the displacement ($k = -2$), the year of job loss ($k = 0$) and ten years following the displacement ($k = 2, 4, ..., 10$).

I run regression (18) on data simulated from the model.[35] Figure 1, Panel A reports the estimation results for wage dynamics on data simulated by the model together with numbers reported in Saporta-Eksten (2014). Panel B shows the coefficients from the regression with log consumption as the dependent variable. Overall, the model implied-behavior of wages and consumption following a job loss is comparable to the data.

5.3 Earnings Risk and Labor Market Flows

A natural follow-up question is how do earnings changes vary depending on the type of transitions experienced by workers in the labor market. Figure 2, Panel (B), shows exactly

[33] While there is an extensive literature exploring the effects of job displacement on wages and earnings (see Jacobson, LaLonde, and Sullivan, 1993; Stevens, 1997), there are fewer studies that also explore the consumption response. This has to due with the difficulty of coming up with panel data with information on both consumption and employment. Starting in 1999, the PSID began to collect data on a wide variety of consumption categories, offering an unique view of the consumption dynamics around a job loss.

[34] A job loss is defined as in Saporta-Eksten (2014): a job loser is an individual who reports being unemployed at the time of the interview due to involuntary unemployment of firm closure (all transitions to unemployment are involuntary in the model, the last condition does not apply in the model context). The original specification also includes time fixed effects and a set of variables capturing worker's observable characteristics. Since my simulated data come from the stationary equilibrium and agents are all *ex-ante* identical in the model, I omit those terms in the regressions for model generated data.

[35] Details on the simulation and data adjustments are discussed in Appendix C.1.

that. It plots the distribution of earnings changes for three different groups of workers: "stayers" corresponds to workers employed *at the same* firm throughout years t and $t+1$; "ee" corresponds to workers employed throughout years t and $t+1$ who experience at least one job-to-job transition; and "ue,eu" includes any worker who has experienced least one unemployment spell during either year t or $t+1$. In terms of the likelihood of those events, note that 46% undergo at least one "eu" or "ue" type of transition during the two-year period. Among those who remain employed throughout the period (54% of workers), 53% experience a job-to-job transition. The left tail of the earnings changes distribution comes from workers transitioning through unemployment and is the result of both (i) lack of earnings during the unemployment spell and (ii) low re-entry wages. Workers who do not suffer an unemployment spell experience positive earnings growth, but the gains are higher for workers who experience a job-to-job transition.

6 Results

In this section, I study the impact of an adverse financial shock aimed at capturing labor market movements during the Great Recession (GR). This is the perfect-foresight solution to an unanticipated shock to firms' discount rate ("MIT shocks").[36] First, I will describe the behavior of some key macro variables during and following the Great Recession.

6.1 Great Recession in the Data

Unemployment and inflation Figure 3 shows the behavior of some aggregate variables during and after the GR. From the last quarter of 2007 until the second quarter of 2009, the US experienced a severe economic downturn: the unemployment rate more than doubled, reaching 10%, job-to-job transitions fell by 0.6 percentage points and consumption dropped by almost 4%. Recovery has been very slow. Unemployment took six years to go back to its steady-state level, while job-to-job transitions have failed to do so to this date. Figure 3, Panel (C), which plots log-deviations of consumption from a linear trend estimated from 1984, shows that consumption growth during the recovery has not been high enough to close the negative gap opened during the GR. Despite the depth of the downturn, inflation only fell modestly—with the exception of the last quarter of 2008, when prices fell by 6%—fluctuating between 1–3% during most of the recovery. The limited amount of disinflation in face of the large contraction in economic activity is seen as puzzling.[37] In particular, inflation behavior is surprising if viewed through the lens of the Phillips curve, here thought of both as an empirical and theoretical relation connecting real variables (like unemployment, marginal cost or other measure of slackness) to inflation. Coibion and Gorodnichenko (2013) makes

[36] See Appendix C for a description of the numerical algorithm.

[37] Hall (2011), for instance, argues that popular DSGE models based on the simple New Keynesian Phillips curve "cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack."

this point by showing that a Phillips curve relating inflation and unemployment estimated from 1960 to 2007 consistently underpredicts inflation by 2–3% in the years following the GR. This fact is usually referred to as *missing disinflation*.

Labor productivity Labor productivity, Figure 3, Panel (E), starts to decrease sometime before the Great Recession, and features a short-lived spike in 2009–2010, only to slow down again around 2012. The slowdown in labor productivity, also highlighted in Christiano, Eichenbaum, and Trabandt (2014), Reifschneider, Wascher, and Wilcox (2015) and Fernald, Hall, Stock, and Watson (2017), is often cited as contributing to the slow recovery following the recession. The causes behind it are a matter of debate. One view considers that productivity behavior could be a direct result of the crisis, which led firms to reduce their productivity-enhancing investments.[38] A second view, articulated in Fernald, Hall, Stock, and Watson (2017), considers the fall to be unrelated to the factors leading to the GR and simply the result of poor luck (i.e., of exogenous negative shocks to TFP). As I discuss next, the job ladder provides an alternative (complementary) explanation that ties the fall in labor productivity to the slowdown in labor reallocation.

6.2 Great Recession in the Model

While the model does not portray financial frictions explicitly, I consider a shock that transmits through the economy in a manner similar to that of a financial shock.[39] Specifically, the shock I have in mind is one that raises the spread χ_t in the discount rate of labor intermediaries. This bumps the required rate of return for their vacancy-posting investment decisions, directly reducing firms' incentives to enter the labor market. In a similar exercise, likewise trying to understand the GR, Christiano, Eichenbaum, and Trabandt (2014) models a financial shock as a "wedge" to the household intertemporal Euler equation for capital investment, which drives a spread between the rate of return of capital and the risk-free rate.[40] In my model, investment occurs through vacancy creation: firms must expend resources to post vacancies, which can lead to the creation of a worker–firm match providing a long-lived profit stream to the firm. The financial shock then raises the required rate of return for this investment, as would the investment wedge in a model with capital.[41]

[38] An example of such is Anzoategui, Comin, Gertler, and Martinez (2019), which develops a model of R&D and technology adoption. In this environment, the fall in TFP becomes an endogenous outcome of a financial shock.

[39] Although the fundamental cause of the GR is still a matter of debate, it is clear that a shock to the financial sector played a crucial role.

[40] More generally, this shock relates to the investment wedges from business cycle accounting literature explored in Chari, Kehoe, and McGrattan (2007), which shows that popular theories of financial frictions, such as Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), manifest themselves as wedges to the investment Euler equation.

[41] Versions of the search and matching model in which a firm's discount factor fluctuates in response to aggregate shocks have been recently explored in Hall (2017), Kehoe, Midrigan, and Pastorino (2017) and Borovicka and Borovickova (2018). Time-varying discount rates considerably increase the model's unemployment volatility compared with the risk-neutral textbook search and matching model. In these examples, however, the firm's

Figure 4 shows the impulse response to an increase in the spread of labor intermediaries. The shock is calibrated to target unemployment dynamics during the Great Recession.[42] The financial shock has a direct effect on the labor market. A higher discount rate reduces the present value of a match to the firm, which causes vacancies to decrease to satisfy free-entry condition (15). As firms reduce their vacancies, unemployment surges (Panel A) while job-to-job transitions come to a halt (Panel B).

Overall unemployment increases by 5 percentage points, consumption falls 10% at the trough and labor productivity—measured as output divided by the measure of employed workers—falls by 6%. The overall behavior predicted by the model is similar to that during the Great Recession. Figure 4 also shows the behavior of marginal costs and inflation. The model predicts a sharp initial drop of marginal costs. Inflation, however, falls only momentarily and quickly reverts above the steady state.

What explains these results? The reduction in job-to-job flows leaves workers stuck at the low rungs of the job ladder, leading to *low wage and productivity* growth. This misallocation in the employment distribution explains the aggregate labor productivity movements in Panel (B), which fall even though total factor productivity Z_t has not changed.[43] The effects of misallocation are persistent and prevail even after the unemployment rate returns to its steady-state value. Similar to an adverse technological shock, the misallocation exerts upward pressures on marginal costs, which explains the inflationary pressures during the recovery.

At the moment of the shock, however, the supply of labor services has not yet changed.[44] So the response over initial periods is mainly driven by a fall in aggregate demand that responds to the lower future incomes and higher real interest rates. Since the supply of labor

discount rate varies endogenously in response to technological shock or a credit tightening shock. Here, I consider exogenous variations in the wedge χ and interpret those as standing for a financial shock.

[42]I consider paths for χ_t of the form

$$\begin{cases} \chi_0 & \text{if } t < \bar{T} \\ \chi_0 \exp(-\chi_1 t) & \text{if } t > \bar{T} \end{cases} \tag{19}$$

I explore different combinations of $\bar{T}, \chi_0, \chi_1$ and choose the one that more closely matches the unemployment dynamics during the GR. Getting the persistence of unemployment is particularly hard, since the misallocation induced by the shock is itself a force that pushes unemployment back to the steady state. See the calibration section for an explanation of this point.

[43]The labor productivity measure captures changes both in materials input usage and to the average match productivity of employed workers. Using the production function of retailers, we can show that model-implied labor productivity is given by

$$\frac{Y_t}{1-u_t} = (1-\gamma mc_t)(m_c\gamma)^{\frac{\gamma}{1-\gamma}} Z_t \frac{\mathcal{N}_t^e}{1-u_t}.$$

So, labor productivity can fall due to (i) a fall in TFP component Z_t; (ii) a decline in marginal costs, which induces a decline in materials; or (iii) a decline in the average match productivity of employed workers $\mathcal{N}_t^e/1-u_t$. Since $\mathcal{N}^e$ is a state variable in the model, the initial drop in labor productivity comes entirely through a reduction in materials. Along the recovery, marginal costs rise *above* the steady state, so the labor productivity fall is entirely due to the lower average match productivity of employed workers.

[44]Remember that the supply of labor services is given by $\int z \, \mathrm{d}\Psi_t(z)$. At $t=0$, the distribution Ψ_0 is a state variable, so labor services are equal to their steady-state value.

services takes time to adjust, most of the initial reaction occurs via the usage of material inputs, driving down the price of labor services and of marginal costs. This does not result in a major disinflation because inflation depends on the whole discounted sum of future marginal costs—recall equation (13). Higher future marginal costs during the recovery therefore prevent inflation from falling too much at the outset. Several other papers offer related explanations for the missing disinflation.[45] Similar to those, I relate the missing disinflation to a fall in productivity. But in my case, the fall in labor productivity comes endogenously from the slowdown in employment reallocation in the labor market.

7 Unpacking the Mechanism

In this section, I compare the response from the full model to counterfactuals aimed to expose the impact of the job ladder on the supply and demand sides of the economy. Subsection 7.1 investigates the effects of the job ladder on supply through its impact on aggregate labor productivity, while Subsection 7.2 explores the impacts on the demand side through consumption.

7.1 Effects of the Job Ladder on Labor Productivity

What are the supply-side effects of worker reallocation in the job ladder? To answer this question, I consider a different notion of equilibrium, which I denote as *exogenous-Λ equilibrium*. The definition is analogous to the original equilibrium, except for the following modifications: I drop the free-entry condition (15) and treat both the supply of labor services $\mathcal{N}_t^e$ and workers' income process $\{\mathbb{1}_{it}^u, y_{it}\}$ as exogenous.[46] The full definition is laid out in the appendix.

To isolate the productivity effects coming from the job ladder, I compare the benchmark response with the exogenous-Λ equilibrium in which workers face the same equilibrium labor income processes $\{\mathbb{1}_{it}^u, y_{it}\}_{t\geq 0}$, but where the supply of labor services $\mathcal{N}_t^e$ varies only with the measure of employed workers, according to $(1-u_t)\mathcal{N}^{e,SS}$. This counterfactual neutralizes the impact of the job ladder on the supply of labor services by treating all employed workers as equally productive, while keeping the job ladder implications for labor earnings unchanged.[47]

[45]See Christiano, Eichenbaum, and Trabandt (2014) and Anzoategui, Comin, Gertler, and Martinez (2019) for explanations that rely on the slowdown on productivity growth, and Del Negro, Giannoni, and Schorfheide (2015) for an explanation that does not rely on supply-side considerations, but on monetary policy instead.

[46]Importantly, I do not impose that the exogenous paths/processes $\{\mathcal{N}_t^e, \mathbb{1}_{it}^u, y_{it}\}$ must be the outcome of a feasible path of transition rates Λ_t.

[47]An alternative way to answer this would be write down a model without on-the-job search and compute the economy's response to the same underlying shocks. There are a couple of difficulties with this strategy though. First, it is not clear how to incorporate the benchmark earnings process into a model without a job ladder. Second, the model without on-the-job search would feature a different response for variables in and out of the labor market, making the comparison of variables like inflation and consumption less transparent.

Figure 5 plots the evolution of labor market stocks for the benchmark and the counterfactual equilibrium. Panel (A) shows that unemployment fluctuations are the same in the two economies, as expected. Panel (B) plots the overall supply of labor services $\mathcal{N}_t^e$. In the counterfactual equilibrium (purple line), labor services mirror the movements in unemployment. In the benchmark (blue line), the stock of labor services suffer a larger and more persistent decline than unemployment, reflecting the misallocation that occurs among employed workers.

Figure 6 shows that this difference matters tremendously for the response of other aggregates. Consumption in the counterfactual economy is much less persistent and quickly recovers toward the steady state. The counterfactual also predicts inflation throughout the whole transition, with measured labor productivity rising above the steady state instead of falling.

7.2 Effects of the Job Ladder on Consumption

I now investigate the determinants of consumption by looking at the decomposition of aggregate consumption response, similar to what is done in Kaplan, Moll, and Violante (2018). I start by writing aggregate consumption, $\mathcal{C}_t$, explicitly as a function of the sequence of equilibrium prices, quantities and rates entering the worker's problem

$$\mathcal{C}_t(\{r_s, \varphi_s, d_s, \tau_s, \tau_s^0, \lambda_s, \lambda_{es}\}_{s\geq 0}) := \int c_t(a,y)d\Psi_t(a,y). \tag{20}$$

Totally differentiating (20) with respect to the sequences entering the worker's problem, we get

$$d\mathcal{C}_t = \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial r_\tau} dr_\tau d\tau + \sum_{i\in(\varphi,d,\tau_0)} \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial i_\tau} di_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_\tau} d\lambda_\tau d\tau + \int_{\tau=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \lambda_{e\tau}} d\lambda_{e\tau} d\tau, \tag{21}$$

which decomposes the total change in aggregate consumption $d\mathcal{C}_t$ into the partial response of consumption $\partial\mathcal{C}_t$ to the changes in (i) the real rate (r); (ii) the competitive price of labor services, dividends and government transfers (φ, d, τ_0), which I jointly refer as moving workers' *current* disposable income; and (iii) labor market transition rates—in other words, the job-finding rate (λ) and on-the-job contact rate (λ_e).[48]

Figure 7 displays the decomposition together with the equilibrium consumption response (blue line). The bulk of the consumption response is accounted for by changes in the contact rate on-the-job, especially at longer horizons. This is because wage growth for workers hinges on their capacity to meet with potential employers who bid up their wages through competition. The prolonged recession caused by the collapse of the ladder reduces the likelihood of

[48]In practice, each term of the decomposition is calculated as the *partial equilibrium* consumption response where some variables entering the aggregate consumption function $\mathcal{C}_t$ are allowed to vary as in the equilibrium while others are held fixed at their steady-state value.

such encounters, pushing down the entire path of expected future wages. Changes in the price of labor services, dividends and government transfers act as the second-most-important channel, while movements in the real rate explain only a small fraction of overall consumption response. These findings are reminiscent of what others have documented in the HANK literature, which have typically looked at the decomposition of consumption to monetary shocks into direct and indirect effects (see Kaplan, Moll, and Violante, 2018; Auclert, Rognlie, and Straub, 2020).

While the decomposition of aggregate consumption highlights the importance of the collapse of the ladder on consumption, it doesn't give us a sense of how the shock is distributed across workers. Figure 8 focuses on a particular dimension of the heterogeneous impacts by plotting the distribution of individual consumption changes at time zero (i.e., the distribution of log-consumption reaction that takes place immediately upon impact of the shock). The figure highlights that there is a significant amount of heterogeneity in responses, with reduction in consumption ranging from 2% to 10%.

Which worker characteristics help explain this heterogeneity in consumption response? Given the prominent role of contact rates in the overall consumption response, we can expect that workers standing at the bottom of the ladder—those who gain the most by meeting with potential employers—should be the ones most impacted by the shock. The extent to which these income shocks are transmitted to consumption should also depend on workers' holdings of liquid assets. In order to see which dimensions of heterogeneity are behind the initial consumption reaction, Figure 8 repeats the distribution of initial consumption adjustments but highlights workers based on their position in the ladder and on their level of savings. Specifically, I split workers into three disjoint groups: (i) liquidity-constrained unemployed and employed workers at the bottom of the ladder, (ii) unconstrained unemployed and employed workers also at the bottom of the ladder and (iii) all other employed workers.[49]

As the figure makes clear, low-wage and high-savings individuals are the ones cutting consumption the most upon the shock's impact. In contrast, workers earning low wages—hence subject to the same earnings losses as the previous group—but with low levels of wealth stand at the far right-end of the distribution, cutting consumption the least. Interestingly, the small sensitivity of initial consumption of the low-wages and low-savings groups occurs despite the fact that their MPC is much bigger than that of the former group of workers ($0.82 > 0.10$). The fact that low-MPC workers (conditional on their position in the ladder) reduce their consumption by more in response to the shock contrasts with the kind of analysis we see in HANK papers, where all the action in consumption comes from high-MPC individuals (Auclert, 2019; Kaplan and Violante, 2018). The difference has to do with the type of income shocks induced by the collapse of the job ladder versus the one brought by transitory monetary policy shocks—the focus of these papers. To better understand this point, notice that the consumption behavior of low-wage workers is driven by their expectation of

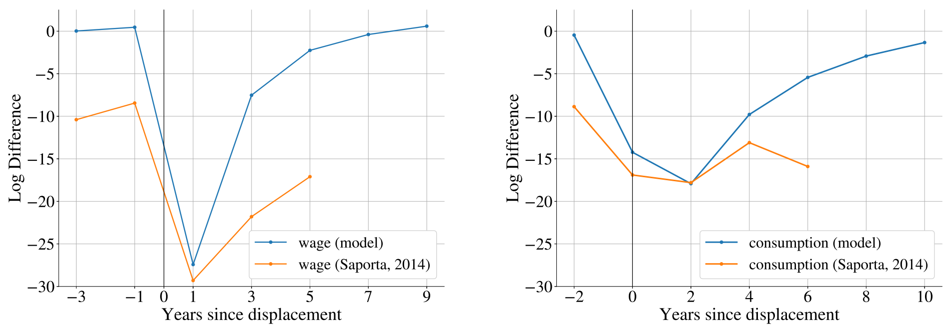
[49]The thresholds for savings and earnings are discussed in the legend of Figure 8.

earnings growth. Their low position in the ladder implies that the expectation of *future* wage increases brought by on-the-job search dominates the potential expected losses from a job loss. Given their expectation that income will be higher in the future than it is today, low-wage workers attempt to raise their consumption relative to their current levels of income to experience a smoother consumption path. Their ability to do so hinges on their level of savings—the closer they are to being constrained, the less able they are to anticipate their consumption. This explains why constrained households, despite their high-MPCs, don't adjust their initial consumption by much.

8 Conclusion

The economy's response to business cycles is shaped by what happens to the job ladder: the distribution of employment across jobs drives aggregate labor productivity, while the slow-down in the contact rate slows down earnings growth for both unemployed and employed workers. As Moscarini and Postel-Vinay (2017b) puts it, the *cyclical job ladder* shapes business cycles. In this paper, I've developed a Heterogeneous Agents New Keynesian (HANK) model with rich search frictions in the labor market giving rise to a job ladder. An adverse financial shock calibrated to mimic the dynamics around the Great Recession generates both the missing disinflation and slow recovery.

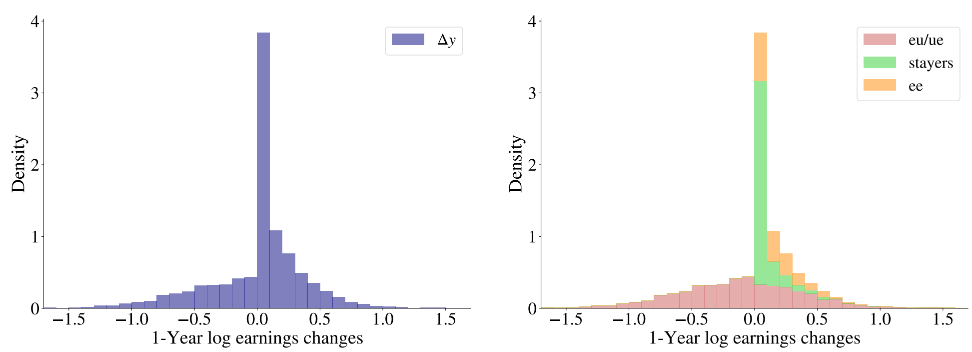
Figure 1: Effects of job displacement on wages and consumption



(A) Wages (B) Consumption

Notes: Coefficients $\{\delta_k\}$ from the distributed lag regression (18) for log wages (**Panel A**) and consumption (**Panel B**). The blue line corresponds to estimates using model simulated data. The orange line corresponds to estimates from Saporta-Eksten (2014) using PSID for the years 1999–2009. The PSID is conducted in biannual waves with earnings data collected for the year previous to the interview, while consumption is reported for the interview year (see text for more details on the timing). The sample includes all non-SEO male heads of households, 24–65 years, hourly wages above 0.5 the state minimum wage, with a minimum of 80 annual hours of employment. The sample in the model-simulated data follows as close as possible the sample selection and the timing restrictions.

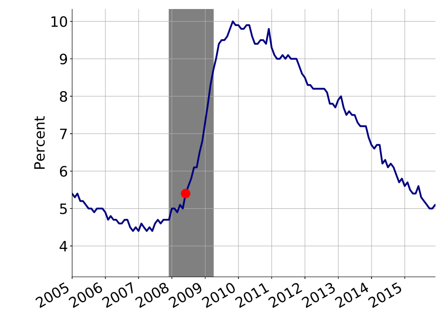
Figure 2: Model implied histogram of earnings changes



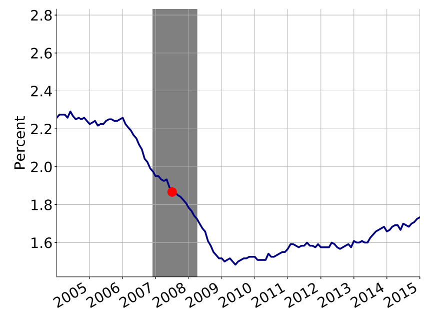
(A) Unconditional (B) Conditional

Notes: **Panel (A) Unconditional:** Model-implied histogram of one-year log earnings changes. **Panel (B) Conditional:** Same histogram, but with colors to indicate different groups of workers depending on their labor market experience in years $t, t+1$. The legends are defined as follows: "stayers" corresponds to workers employed *at the same* firm throughout years t and $t+1$; "ee" corresponds to workers employed throughout years t and $t+1$ who experience at least one job-to-job transition; "ue,eu" includes any worker that has experienced least one unemployment spell during either year t or $t+1$.

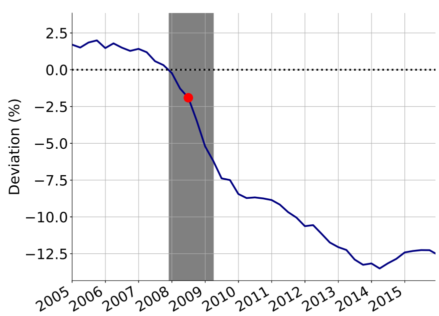
Figure 3: Great Recession aggregate series



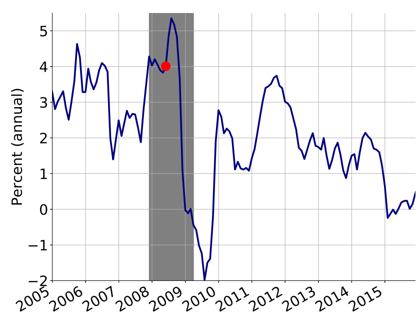
(A) Unemployment



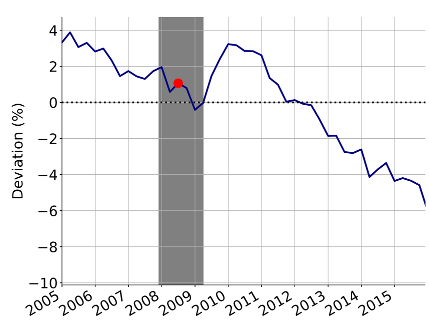
(B) Job-to-job



(C) Consumption



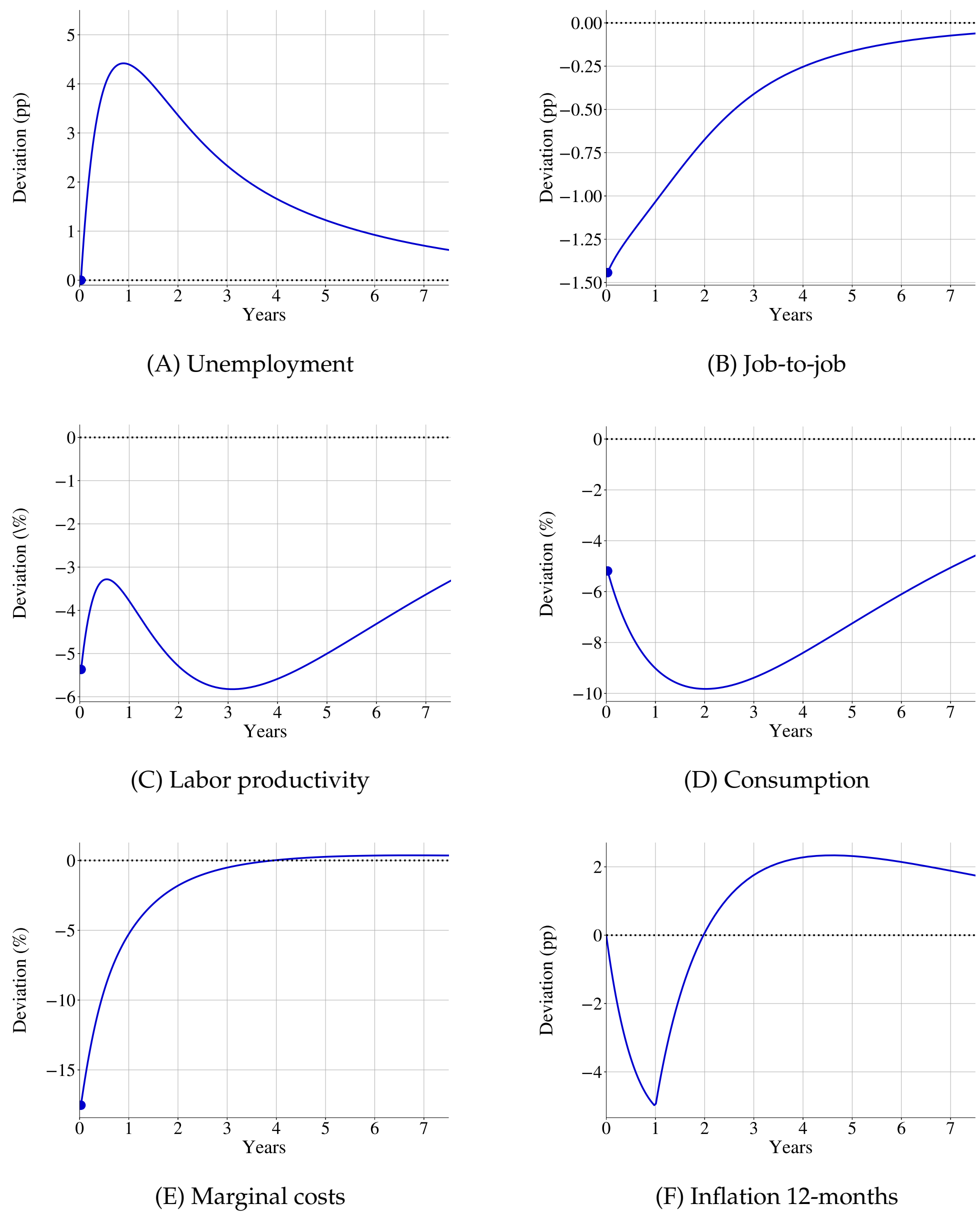
(D) Inflation 12-months



(E) Labor productivity

Notes: This figure plots the behavior of US aggregates for the GR period. Consumption and labor productivity are log-linearly detrended. Inflation corresponds to year-over-year changes in price level. All other variables are in levels. The red dot marks the second quarter of 2008, which I use as the time-0 steady state when comparing model IRFs to the data. See Appendix B for data sources.

Figure 4: Aggregate response to the firm spread shock



(A) Unemployment

(B) Job-to-job

(C) Labor productivity

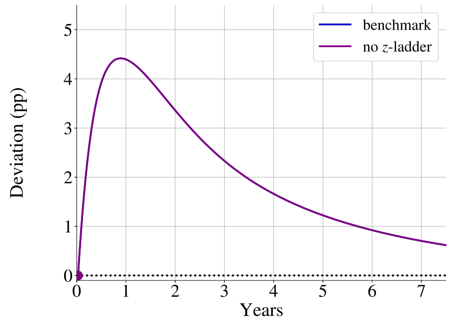
(D) Consumption

(E) Marginal costs

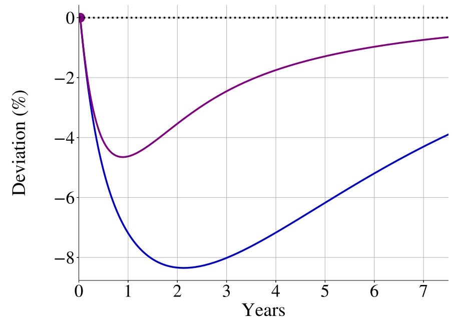
(F) Inflation 12-months

Notes: Model response to the financial shock defined in the text. Inflation 12-months denotes the year-over-year change in the price level. Unemployment and job-to-job transitions are in percentage point deviations. All other variables are plotted in log deviations from steady-state.

Figure 5: Aggregate responses to a spread shock



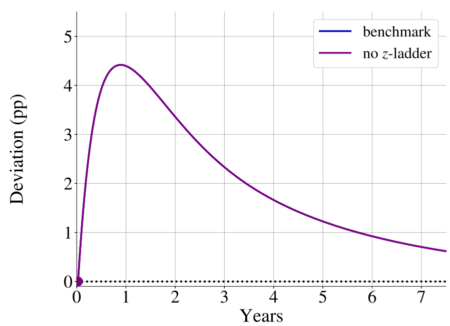
(A) Unemployment



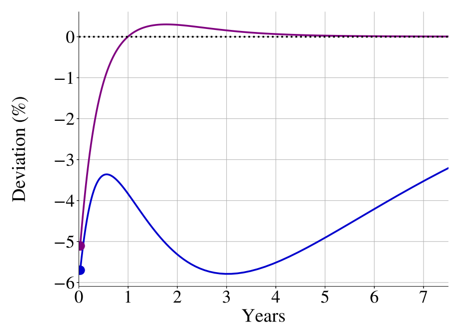
(B) Labor services

Notes: The blue line corresponds to the *benchmark* model response to the adverse financial shock. The purple line corresponds to the *exogenous-Λ equilibrium* response to the same shock. In this counterfactual, the supply of labor services varies only with the measure of employment, ignoring the distribution of workers across the z-ladder.

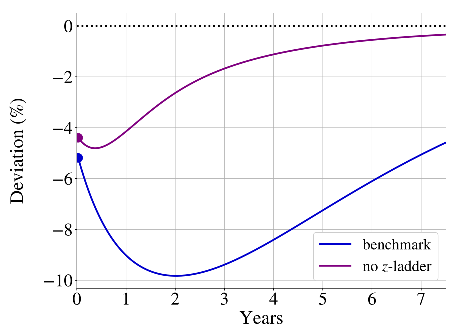
Figure 6: Aggregate responses to a spread shock



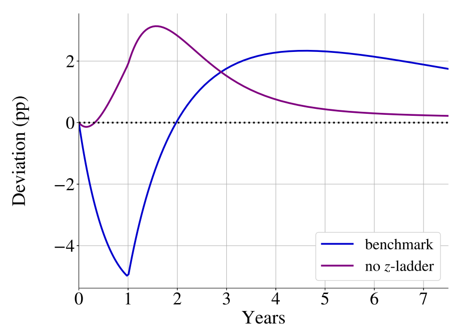
(A) Unemployment



(B) Labor productivity



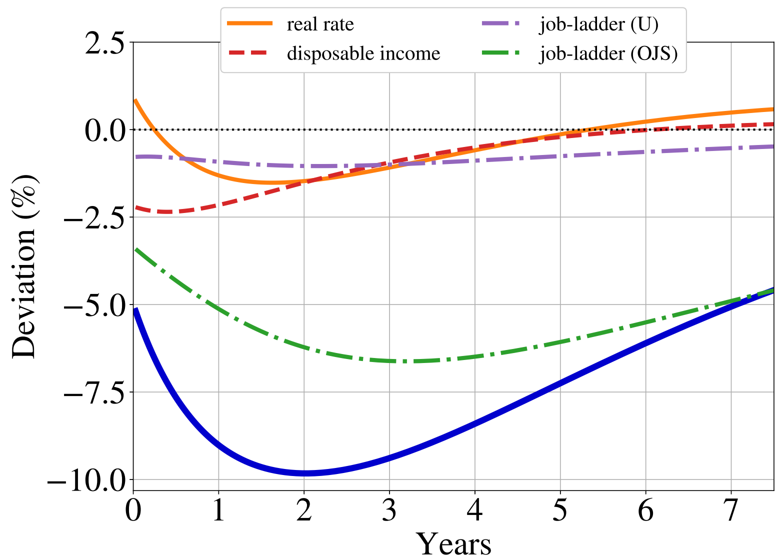
(C) Consumption



(D) Inflation 12-months

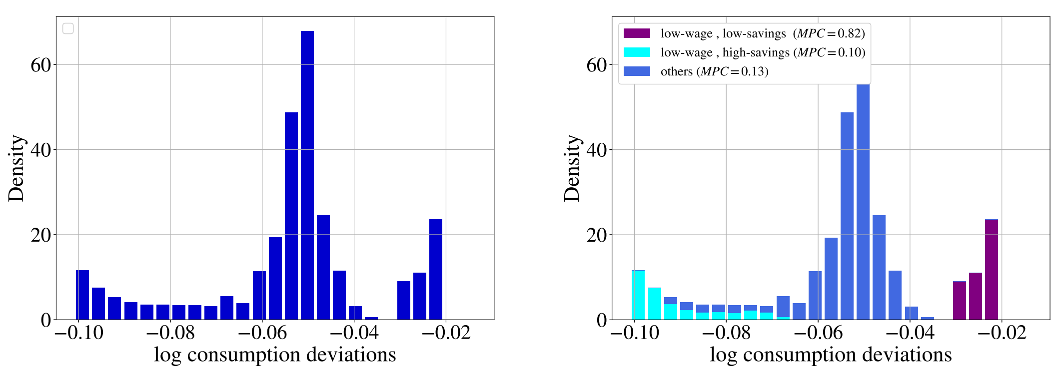
Notes: The blue line corresponds to the *benchmark* model response to the adverse financial shock. The purple line corresponds to the *exogenous-Λ equilibrium* response to the same shock. In this counterfactual, the supply of labor services varies only with the measure of employment, ignoring the distribution of workers across the z-ladder.

Figure 7: Consumption response decomposition



Notes: The blue line corresponds to the equilibrium consumption response to the financial shock. Other lines correspond to counterfactual consumption responses that allow for some equilibrium variables entering (20) to adjust as in equilibrium, while the remaining variables are kept at their steady-state values. Real rate refers to the case where only the real rate $\{r_t\}_{t\geq 0}$ adjusts. Disposable income refers to the case where price of labor services, lump-sum profits, taxes and transfers $\{\varphi_t, d_t, \tau_t, \tau_t^0\}_{t\geq 0}$ adjust. Job ladder (U) refers to the case where the job-finding rate $\{\lambda_t\}_{t\geq 0}$ adjusts. Job Ladder (OJS) refers to the case where the on-the-job contact rate $\{\lambda_{et}\}_{t\geq 0}$ adjusts.

Figure 8: Histogram for time-0 log-deviations of consumption



Notes: This figure explores the time-0 consumption reaction to the financial shock. The left panel features the histogram of time-0 consumption log-deviation from steady state for the cross-section of workers. The right panel splits this distribution in three different groups defined by their joint wage earnings and wealth holdings: *low-wage, low-savings* corresponds to workers with zero wealth earning wages below the 40*th* percentile; *low-wage, high-savings* corresponds to workers with savings above the 40*th* percentile and wages below the 40*th* percentile; *others* corresponds to all oher workers. In terms of their MPCs, the quarterly marginal propensity to consume out a $500 lump-sum transfer for each group is 0.82, 0.10 and 0.13 respectively.

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A Model derivation

A.1 Filtering problem

I start by writing down the process for productivity and piece-rate wage $\{z_t, y_t\}$ for the worker. For the derivations in this section, I consider the economy to be at the steady state, so transition rates are not indexed by t. Productivity z_t is specific to the worker-firm match and is drawn at origination from an exogenous distribution function $\Gamma : [\underline{z}, \bar{z}] \to [0, 1]$. The type of contract offered by firms and labor market transitions determines the evolution of piece-rate y_t. Let $(0, 0)$ stand in for the status of unemployed agent and $X = \{0\} \cup [\underline{z}, \bar{z}]$, so the state space for the Markov process $\{z_t, y_t\}$ is X^2. In what follows, I describe this process in recursive notation, letting $\cdot^*$ denote the new state.

The rate at which workers leave state (z, y) to a new state (z^*, y^*) depends only on the employment status and the type of transition: workers leave unemployment state $(0, 0)$ with intensity λ; employed workers contact other firms with intensity λ_e and suffer exogenous destruction shocks with intensity δ. Upon any of those events, the distribution of the worker's new state (z^*, y^*) is given by a *stochastic kernel funtion* $T_i : X^2 \times X^2 \to \mathbb{R}$, where $i \in \{ue, ee, eu\}$ indexes the different type of transitions.

The kernel when finding a job from unemployment T_{ue} or receiving a match destruction shock T_{eu} does not depend on the current state (z, y). I write them as

$$T_{ue}(z^*, y^*) = \gamma(z^*)\delta(y^* - \underline{z}) \tag{A.1}$$

$$T_{eu}(z^*, y^*) = \delta(z^* - 0)\delta(y^* - 0), \tag{A.2}$$

where $\delta(\cdot)$ is a Dirac delta function.[50] The match productivity of a worker moving out of unemployment is drawn from exogenous distribution Γ and its piece-rate wage is $\underline{z}$ no matter which firm they go to. An employed worker that receives a destruction shock moves to unemployment state $(0, 0) \in S$.

The stochastic kernel for an employed worker T_{ee} is more complicated as it depends on the worker's current state (z, y). Remember from the discussion in the main text that an employed worker with state (z, y) who receives an offer from outside firm will: (i) with probability $\Gamma(y)$ discard the offer since it is smaller than its current wage; (ii) with probability $\gamma(y')$ receive an wage offer of $y' \in (y, z)$ that is matched by its current firm, who offers $y' + \epsilon$

[50]The Dirac delta can be loosely thought of as a object with the following properties

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} g(x)\delta(x)\, dx = g(0)$$

I use the Dirac delta in the derivation whenever the worker transition is deterministic—for instance, when the worker loses their job, they transition to state $(0, 0)$ with certainty.

to the worker; and (iii) with probability $1 - \Gamma(z)$ meet a firm $z^* > z$ that poaches the worker by offering $z + \epsilon > z$, the maximum wage offer of the incumbent. So, taking $\epsilon \downarrow 0$, I write

$$T_{ee}(z^*, y^*|z, y) = \begin{cases} \gamma(z^*) \times \delta(y^* - z) & \text{for } z^* > z, \\ \gamma(y^*) \times \delta(z^* - z) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) \times \delta(z^* - z) & \text{otherwise} \end{cases}$$

Integrating out firm productivity z^* from stochastic kernel T_i, we recover the conditional density f_i for piece-rate wage y^*

$$f_{ue}(y^*) = \delta(y^* - \underline{z}), \quad f_{eu}(y^*) = \delta(y^* - 0)$$

$$f_{ee}(y^*|z, y) = \begin{cases} \bar{\Gamma}(z) \times \delta(y^* - z) & \text{for } y^* = z \\ \gamma(y^*) & \text{for } y^* \in (y, z) \\ \Gamma(y) \times \delta(y^* - y) & \text{for } y^* = y, \end{cases}$$

which ultimately is the distribution workers care about when deciding how much to consume. Importantly, f_i is a function of current match productivity z, which the worker is uninformed about.

Therefore, when making its consumption/savings decisions, the worker must hold beliefs about z to evaluate the probability distribution for the future piece-rate wages f_i. Let Φ be the worker's belief distribution regarding the firm's productivity, with ϕ denoting the (generalized) density function.[51] This distribution is a function of the whole history of job transition and wage offers experienced by the worker. Fortunately, Bayes's rule gives us a way to update this distribution in response to a new signal, so we can treat this problem recursively.[52] Using the same notation as before, let ϕ denote the pre-transition belief density and ϕ^* the updated belief following a labor market event (transition or a wage gain inside the firm). Again, transitions in and out of unemployment involve simple updates that are independent of the previous belief

$$\phi^*_{ue}(z^*) = \gamma(z^*) \tag{A.3}$$

$$\phi^*_{eu}(z^*) = \delta(z^* - 0), \tag{A.4}$$

where δ is again the Dirac delta function. An unemployed worker who meets a firm holds as belief the exogenous firm productivity distribution.

For an employed worker who meets an outside firm, the updated ϕ^* density function

[51] I use the generalized classification because some densities will be degenerate.
[52] The derivation in this section draws upon Hansen (2007).

given new wage offer y^* and transition status is determined by Bayes's rule according to

$$\phi^*_{ee}(z^*) = \begin{cases} \dfrac{\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z<z^*} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker switch jobs} \\ \\ \dfrac{\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)}{\int \left[\int_{z^*=z} T_{ee}(z^*, y^*|z, y) d\Phi(z)\right] dz^*} & \text{if worker does not switch jobs .} \end{cases} \tag{A.5}$$

Note that an employed worker gets to observe two signals: whether the highest wage offer comes from the incumbent or the poacher — in the former, the worker realizes that $z^* > z$, while if they stay in the same match $z^* = z$ — and the new piece-rate offer y^*. The filtering problem can thus be thought as a substituting the original Markov process $\{z_t, y_t\}$ by a new one where the hidden match productivity z is replaced by a distribution Φ over possible values, the evolution of which is determined by equations (A.3) to (A.5).

In this case, the conditional density for piece-rate wages y^* becomes a *compound lottery*

$$\bar{f}_i(y^*|y, \phi) = \int f_i(y^*|z, y) d\Phi(z). \tag{A.6}$$

The following proposition shows that distribution $\Phi(z)$ is fully characterized by the current piece-rate wage.

Proposition 1 *The belief ϕ for an unemployed is degenerate at $z = 0$. Piece-rate densities in case of a job destruction and job finding from unemployment are independent from z and agree with the full information case, i.e., $\bar{f}_{ue} = f_{ue}$ and $\bar{f}_{eu} = f_{eu}$. The belief ϕ for an employed worker is a function of piece-rate wage y only*

$$\phi(z; y) = \frac{\gamma(z)}{\bar{\Gamma}(y)} \text{ for } z > y, \tag{A.7}$$

with the condition piece-rate density in case of job-to-job transition given by

$$\bar{f}_{ee}(y^*|y) = \int f_{ee}(y^*|z, y) \times \frac{\gamma(z)}{\bar{\Gamma}(y)} dz. \tag{A.8}$$

Proof. Conditional density in the case of transitions in and out of unemployment f_{ue}, f_{eu} follow directly from the discussion in the text. For the employed worker, the proof simply applies Bayes rule for each possible transition.

Coming from unemployment When the worker is hired from unemployment, they receive wage $y = \underline{z}$ and hold belief $\phi = \gamma$ equal to exogenous distribution of match productivity. Note that this satisfies (A.7).

Employed worker with job transition Consider an employed worker with belief distribution Φ and piece-rate y who contacts an outside firm. Suppose that as an outcome of this contact, the worker receives an offer $y_1(+\epsilon)$ from the outside firm, while the incumbent offer is y_1.

The worker accepts the offer from the poacher and their belief over productivity z^* of the new match is given by (A.5)

$$\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y)d\Phi(z)}{\int \left[\int_{\{z<z^*\}} T_{ee}(z^*, y_1|z, y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z)d\Phi(z)}{\int \int_{\{z<z^*\}} \gamma(z^*)\delta(y_1 - z)d\Phi(z)dz^*} \\
&= \frac{\phi(y_1)\gamma(z^*)}{\phi(y_1) \int \mathbb{1}\{z^* > y_1\}\gamma(z^*)dz^*} \\
&= \frac{\gamma(z^*)}{\bar{\Gamma}(y_1)} \text{ for } z^* > y_1,
\end{aligned}$$

where the second line substitutes T_{ee}, the third integrates with respect to z^*.

Employed worker with wage increase in the firm Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along with the following outcome: the incumbent firm offers a wage increase $y_2(+\epsilon)$ above the poacher's offer of y_2. The worker stays in the incumbent under a higher wage and their belief evolves as

$$\begin{aligned}
\phi^*(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y_2|z, y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \mathbb{1}\{z > y_2\}\gamma(y_2)\delta(z^* - z)d\Phi(z)\right] dz^*} \\
&= \frac{\gamma(y_2)\phi(z^*)\mathbb{1}\{z^* > y_2\}}{\gamma(y_2) \int \mathbb{1}\{z^* > y_2\}\phi(z^*)dz^*} \\
&= \frac{\phi(z^*)}{\bar{\Phi}(y_2)} \text{ for } z^* > y_2.
\end{aligned}$$

Employed worker with discarded wage offer Consider an employed worker with belief distribution Φ and piece-rate wage y. Suppose a poaching firm comes along and offers a wage smaller than the current piece-rate y, which does induce a counteroffer from the incumbent, i.e., $y^* = y$. Applying Bayes rule one more time,

$$\begin{aligned}
\phi(z^*) &= \frac{\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} T_{ee}(z^*, y|z, y)d\Phi(z)\right] dz^*} \\
&= \frac{\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)}{\int \left[\int_{\{z=z^*\}} \Gamma(y)\delta(y^* - y)\delta(z^* - z)d\Phi(z)\right] dz^*} \\
&= \frac{\phi(z^*)}{\int \phi(z^*)dz^*} = \phi(z^*),
\end{aligned}$$

which is the expected result as the signal does not reveal any new information regarding the productivity of the current match.

Conclusion Whenever the worker moves from jobs or when they find a job from unemployment, their belief ϕ satisfies (A.7). When they receive a wage increase by the incumbent, the updated density ϕ^* is a function of the previous belief ϕ, which does not satisfy (A.7) for a generic ϕ. Note however that if we assume that ϕ is of form (A.7), then we get

$$\phi^*(z^*) = \frac{\phi(z^*)}{\bar{\Phi}(y_2)} = \frac{\gamma(z^*)/\bar{\Gamma}(y)}{\bar{\Gamma}(y_2)/\bar{\Gamma}(y)} = \frac{\gamma(z^*)}{\bar{\Gamma}(y_2)} = \phi(z; y_2).$$

Since the worker arrives at a firm either by job-to-job transition or from unemployment, ϕ must be of form (A.7). This proves the result. ■

I end this section by discussing the wage contracts. First, note that the specification adopted satisfies the worker's and the firm's *individual rationality* as both parts always prefer following the contract to dissolving the match. Second, match origination and job-to-job transitions are efficient with workers always moving toward more productive matches. However, contracts are not optimally designed.[53]

First, since workers are risk averse, firms would be willing to offer some insurance against aggregate fluctuations in the price of labor services. Moreover, the current contract has firms overpaying workers upon renegotiation. Since expected future earnings are increasing in the match productivity, more productive firms could in principle poach workers from less productive firms by offering less than the incumbent's maximum wage offer. By how much workers value smoother income paths and the timing of payments depends on their wealth, which makes the optimal contract a function of workers' assets. Not only do I regard wage contracts conditional on workers' asset holdings a poor representation of reality, implementing a wealth dependent wage would greatly complicate the determination of wages.

A.2 Worker Problem – Recursive Formulation

I present the household's Hamilton–Jacobi–Bellman (HJB) equation in this section. I focus on the stationary versions of these equations. Let $V^u(a), V(a, y)$ denote the optimal value of the unemployed and employed worker's original problem—see the description in the main text—starting from an initial level of assets a and, in the case of the employed worker, from earnings y. Furthermore, let $s^u(a, c), s(a, y, c)$ denote the savings of the employed and unemployed worker with assets a and labor earnings y (b for the unemployed) when they

[53] See Lentz (2014) for the derivation of the optimal contracts in an environment with risk-averse workers and where firms are allowed to make counter offers.

consume a flow c

$$s^u(a,c) := (1-\tau)\varphi b + ra + d(b) - c, \qquad s(a,y,c) := (1-\tau)\varphi y + ra + d(y) - c,$$

where I have already incorporated the fact that dividends are distributed in proportion to labor earnings. The HJB is thus given by

$$\Big(\rho + \lambda_u\Big) V^u(a) = \max_c \Big\{ u(c) + \partial_b V^u(a)\, s^u(a;c) \Big\} + \lambda_u V(a, \underline{z}) \tag{A.9}$$

$$\begin{aligned} \rho V(a,y) =& \max_c \Big\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \Big\} + \delta \big[V^u(b) - V(a,y) \big] \\ &+ \lambda_e \int_y \Big[\int_y^z \big[V(a,y^*) - V(a,y) \big] \, \mathrm{d}\Gamma(y^*) + \bar{\Gamma}(z) \big[V(a,z) - V(a,y) \big] \Big] \phi(z;y)\, \mathrm{d}z, \end{aligned} \tag{A.10}$$

where $\phi(z;y)$ is the household belief regarding the current match productivity. Remembering the definition of $\bar{f}$ in (A.8), we can rewrite the HJB of the employed as

$$\begin{aligned} \rho V(a,y) =& \max_c \Big\{ u(c) + \partial_a V(a,y)\, s(a,y;c) \Big\} + \delta \big[V^u(a) - V(a,y) \big] \\ &+ \lambda_e \int_y \big[V(a,y^*) - V(a,y) \big] \bar{f}(y^*|y) dy^*. \end{aligned} \tag{A.11}$$

A.3 Intermediate Firm Problem – Recursive Formulation

Consider a firm with productivity z under a piece-rate wage contract of y. The value of a match to the firm $\mathcal{J}_t(z,y)$ satisfies the following HJB

$$\begin{aligned} \Big(r_t + \chi_t\Big) \mathcal{J}_t(z,y) = \varphi_t(z-y) + \lambda_{et} \int_y^z \Big[\mathcal{J}_t(z,z') - \mathcal{J}_t(z,y) \Big] d\Gamma(z') \\ + (\lambda_{et} \bar{\Gamma}(z) + \delta)[0 - \mathcal{J}_t(z,y)] + \partial_t \mathcal{J}_t. \end{aligned} \tag{A.12}$$

The firm discount profit flows at rate $(r_t + \chi_t)$. With intensity λ_{et}, the worker meets an outside firm. If productivity z' of the poaching firm is between the current piece-rate y and the productivity of the match z, the firm makes a counteroffer z' to the worker who stays in the incumbent firm. This event changes the firm value to $\mathcal{J}(z,z')$. If the productivity z' of the poaching firm is above the productivity of the match, i.e., $z' > z$, the firms looses its worker and the match is dissolved, which leaves the firm with a value of 0. The same happens if the match is hit by a destruction shock δ. Finally, the value of the match changes with calendar time t by $\partial_t \mathcal{J}_t$.

We can interpret this HJB equation by treating the function $\mathcal{J}$ as the value of an asset with flow dividends $\{\varphi_t(z - y_t)\}$, where y is indexed by time to reflect that it evolves in the history of the match.[54] The return in this asset comes from two sources. The first is the flow dividends $\varphi_t(z - y_t)$. The second comes from capital and losses, which in the current context incorporate all right-hand side terms after dividends in (A.12). The asset loses value whenever the firm has to renegotiate the contract y, or the match is destroyed. The asset also appreciates/depreciates depending on the evolution of aggregate variables, which is captured by $\partial_t \mathcal{J}_t$. The HJB equation then states that return on this asset—right-hand side of (A.12)—must be equal to the required rate of return, $(r_t + \chi_t)\mathcal{J}$.

In order to derive some properties of the value of the firm, let me consider a stationary environment where $\lambda_{et}, r_t, \chi_t, \varphi_t$ do not vary with time. In this case, (A.12) simplifies to

$$\Big((r+\chi) + \delta + \lambda_e \bar{\Gamma}(y)\Big)\mathcal{J}(z,y) = \varphi(z-y) + \lambda_e \int_y^z \mathcal{J}(z,z')d\Gamma(z'). \tag{A.13}$$

The derivative of $\mathcal{J}$ with respect to piece-rate y is

$$\begin{aligned}(r+\delta+\lambda_e\bar{\Gamma}(y))\mathcal{J}_y(z,y) - \lambda_e\gamma(y)\mathcal{J}(z,y) &= -\varphi - \lambda_e\gamma(y)\mathcal{J}(z,y)\\ \mathcal{J}_y(z,y) &= -\frac{\varphi}{(r+\delta+\lambda_e\bar{\Gamma}(y))\mathcal{J}_y(z,y)}.\end{aligned}$$

Doing to same for productivity z, we get

$$\mathcal{J}_z(z,y) = \frac{\varphi}{\Big(r+\delta+\lambda_e\bar{\Gamma}(y)\Big)}.$$

Hence, I conclude $\mathcal{J}_z > 0, \mathcal{J}_y < 0$, that is, the value of the match is decreasing in the piece-rate wage and increasing in match productivity.

A.4 Equilibrium

Definition 2 ((Equilibrium)) *Given an initial government debt B^g, an initial distribution Ψ_0 over wealth, labor income and match productivity, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, an general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N_t^e, M_t, u_t, v_t, D_t\}_{t\geq 0}$, labor market transition rates $\{\lambda_t, \lambda_{et}\}_{t\geq 0}$, government policies $\{G_t, B_t^g, \tau_t, \tau_t^0, i_t\}_{t\geq 0}$, labor income process $\{\mathbb{1}_{it}^u, y_{it}\}_{i\in[0,1],t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t, \mathcal{N}_t^e\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that workers optimize, firms optimize, monetary and fiscal policy follow their rules, the labor income process is the result of labor market transitions and wage-setting, worker aggregate functions and distributions are consistent with labor market transition rates and worker's decision rules,*

- *the free-entry condition (15) holds,*
- *and all markets clear:*
    - *asset market*

$$\mathcal{A}_t = B_t^g$$

    - *labor services market*

$$N_t^e = \mathcal{N}_t^e$$

[54] See Acemoglu (2007, Chapter 7) for a similar argument.

- *goods market*

$$\mathcal{C}_t + G_t + \kappa^f v_t = Y_t = \tilde{Y}_t - M_t$$

Definition 3 ((Exogenous-Λ)) *Given an initial government debt B^g, an initial distribution Ψ_0 over assets and labor income, a sequence for exogenous shocks $\{Z_t, \epsilon_t, \chi_t\}_{t\geq 0}$, exogenous labor income process $\{\mathbb{1}^u_{it}, y_{it}\}_{i\in[0,1],t\geq 0}$ and an exogenous path of labor services supply $\{\mathcal{N}^e_t\}_{t\geq 0}$, a general equilibrium is a path for prices $\{\varphi_t, \pi_t, r_t\}_{t\geq 0}$, aggregates $\{\tilde{Y}_t, Y_t, N^e_t, M_t, u_t, D_t\}_{t\geq 0}$, government policies $\{G_t, B^g_t, T_t, \tau_t, \tau^0_t, i_t\}_{t\geq 0}$, worker aggregates $\{\mathcal{C}_t, \mathcal{A}_t\}_{t\geq 0}$, and joint distributions $\{\Psi_t\}_{t\geq 0}$, such that households optimize, firms optimize, monetary and fiscal policy follow their rules, the worker aggregates and distribution are consistent with the worker's decision rules and exogenous process for income, and all markets clear*

- *Asset market clearing*

$$\mathcal{A}_t = B^g_t$$

- *Labor services market clearing*

$$N^e_t = \mathcal{N}^e_t$$

- *Goods market clearing*

$$\mathcal{C}_t + G_t = Y_t$$

A.5 Complete Market Family

The complete market version of the model follows Merz (1995) in adopting a representative family construct, which allows for perfect consumption insurance. The family is composed of a continuum of workers who are either employed or unemployed. At time t, a measure u_t of its workers is unemployed and receives unemployment insurance in the amount of $b\varphi_t$ from the government. The distribution of employed workers inside a family is given by $\Psi_t(z,y)/(1-u_t)$, where again z denotes the productivity of the match and y the piece-rate contract earned by employed workers. The family pools all income earned by workers in the form of unemployment insurance and wages. Additionally, the firm receives profits D_t from its ownership of firms. The family then decides on consumption C_t for its members and saves through government bonds at rate of return r_t. The problem of the family is then

$$\max_{\{C_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(C_t) dt$$

$$\text{S.t. } \dot{A}_t = r_t A_t + (1-\tau_t)\varphi_t \left(\int y d\Psi_t(y) + b u_t \right) + D_t - C_t + \tau^0_t,$$

where τ^0_t are lump-sum transfers from the government.

B Data

Consumption is given by real personal consumption expenditures (GDPC) and inflation is the PCE deflator (PCECTPI). Both are produced by the Bureau of Economic Analysis (BEA) at quarterly frequency. For labor productivity, I use Nonfarm Business Sector Real Output

Per Hour of All Persons from the Bureau of Labor Statistics (BLS). Job-to-job transitions data comes from Fallick and Fleischman (2004).

C Numerical Implementation

C.1 Job Loss Simulation

Saporta-Eksten (2014) uses the 1999–2009 biennial waves of Panel Study of Income Dynamics (PSID) to document the dynamics of wages and consumption around a job displacement. To make the results comparable to the empirical estimates, I apply the same treatment to model simulated data as the author does to actual PSID data. First, while the model is simulated continuously in time, I use the (annually aggregated) measures for every other year in the regression to replicate the way data is collected in the PSID. A job loser is an individual who reports being unemployed at the time of the interview (assumed to take place between March and May). The timing of variables is also important. In the PSID, earnings variables used to compute wage rate refer to the year *prior to each survey*. Information on consumption expenditures is reported for "typical week or month," usually understood as reflecting consumption in the first quarter of the survey year. So for the sample year t, earnings and hours data refer to $t-1$, while consumption refers to the (annualized) first quarter of year t.

C.2 Transition Dynamics

The numerical solution adapts the Auclert, Bardóczy, Rognlie, and Straub (2019) method for solving nonlinear perfect-foresight transitions to a continuous time setting. The perfect-foresight equilibrium defined in Section 3.1 can be framed in the form of a *functional equation*. Let X be a the space of real-valued functions $x : [0, \infty) \mapsto \mathbb{R}$. Equilibrium restrictions form an operator $\mathcal{H} : X^n \to X^n$ for $n \in \mathbb{N}$ and an equilibrium is a set of real-values functions $y^* \in X^n$ such that $\mathcal{H}(y^*) = \mathbf{0}$. For instance, the real rate path $\{r_t\}_{t\geq 0} \in X$ is one of the n dimensions of the equilibrium vector y^*, while the asset market $\{\mathcal{A}_t - B_t^g\}_{t\geq 0}$ is one of the n dimensions (say $i \leq n$) of the image of y^* under $\mathcal{H}$, that is $\mathcal{H}(y^*)$. In equilibrium, the restriction that asset markets must clear is equivalent to the statement that $\mathcal{H}_i(y^*)(t) = 0$ for all $t \geq 0$.

Solving the model involves discretizing and truncating the time dimension, in which case the X turns into $\mathbb{R}^K$ for some finite K and $\mathcal{H}$ becomes a nonlinear system of equations $H : \mathbb{R}^{nK} \to \mathbb{R}^{nK}$. In this case, solving for the equilibrium is equivalent to solving a root-finding problem of a conventional (although potentially big) nonlinear system of equations. Spreading time points effectively and reducing dimension n by substituting equilibrium conditions makes solving this problem possible.