BANK OF CANADA
BANQUE DU CANADA

Staff Working Paper/Document de travail du personnel—2022-11

Last updated: March 4, 2022

The Central Bank Strikes Back! Credibility of Monetary Policy under Fiscal Influence

by Antoine Camous[1] and Dmitry Matveev[2]

[1]Department of Economics
University of Mannheim
camous@uni-mannheim.de

[2]Canadian Economic Analysis Department
Bank of Canada
matv@bankofcanada.ca

DOI: https://doi.org/10.34989/swp-2022-11 | **ISSN** 1701-9397

Acknowledgements

We thank Klaus Adam, Jean Barthelemy, Russell Cooper, Stefano Gnocchi, Hubert Kempf, Ramon Marimon, and seminar participants at various institutions and conferences for useful suggestions on this project: T2M Paris; CREST-X; University of Mannheim; CESifo Area Conference on Macro, Money, and International Finance; Banque de France; Swiss Macro Workshop; Boston College; Harvard CES; Federal Reserve Bank of Atlanta; Federal Reserve Bank of Minneapolis; Duke University; ESCP Business School – Paris; Society for Economic Dynamics 2021; and the Bank of Canada. Raphaela Hotten provided excellent research assistance. Comments from the editor and three anonymous referees are greatly appreciated. The views expressed in this paper are solely those of the authors, and no responsibility for them should be attributed to the Bank of Canada.

Abstract

How should independent central banks react if pressured by fiscal policymakers? We study an environment with strategic monetary-fiscal interactions where the central bank has a limited degree of commitment to follow policies over time and the fiscal authority has none. We contrast the implications of two monetary frameworks: one where the central bank follows a standard rule aiming exclusively at price stability against the other, where monetary policy additionally leans against fiscal influence. The latter rule improves economic outcomes by providing appropriate incentives to the fiscal authority. More importantly, the additional fiscal conditionality can enhance the credibility of the central bank to achieve price stability. We emphasize how the level and structure of government debt emerge as key factors affecting the credibility of monetary policy with fiscal conditionality.

Topics: Monetary policy; Fiscal policy; Credibility

JEL codes: E02, E52, E58, E61, E62

Irresponsible fiscal policies can jeopardise [monetary] credibility, as higher inflation becomes desirable to reduce the real value of government debt.

Draghi (2012)

1 Introduction

Central banks in many advanced economies enjoy a high degree of institutional and policy independence to shield monetary decisions from political influence. Under the predominant arrangements, politicians remain responsible for taxes, debt, and deficit, while central banks' mandates are tailored around different concepts of price stability. This policy regime, with a strict separation of assignments, also known as *monetary dominance*, is widely seen as having been instrumental in reducing the inflationary bias of monetary decisions.[1]

However, rising levels of public debt have triggered mounting political pressure and government interference with central banks.[2] This tension creates a risk of moving away from the conventional policy assignment to a regime of *fiscal dominance*, where monetary policy becomes predominantly subordinate to fiscal decisions. In this context, we examine whether a central bank should lean against fiscal pressure through the design of conditional monetary policy. Our answer is positive for two reasons: providing appropriate incentives to the fiscal authority not only improves economic outcomes but can also enhance the credibility of a central bank to support price stability.[3] We also discuss how debt level, indexation, and maturity influence the credibility of monetary policy with fiscal conditionality.

The Eurozone provides a prominent example of a monetary framework that includes provisions conditional on fiscal decisions. The Eurosystem collateral framework—a key mechanism determining the stance of monetary policy—effectively imposes conditions on fiscal sustainability. Indeed, eligibility and refinancing conditions for financial institutions are tied to a credit rating of public debt posted as collateral.[4] This design not only protects the central bank balance sheet but also provides appropriate incentives for governments to ensure fiscal sustainability. These considerations are also at the core of the Outright Monetary Transactions (OMT) program

[1]The terminology *monetary dominance*, as opposed to *fiscal dominance*, is commonly used in policy discussions to refer to the traditional policy assignment and statutory independence of central banks: see, for instance, Schnabel (2020): *"The euro has been built on the principle of monetary dominance (...) high government debt was seen as a major threat to central bank independence, and it was feared that fiscal dominance could induce a central bank to deviate from its monetary policy objectives, endangering price stability."*

[2]Binder (2021) studies political pressure using a sample of 118 central banks between 2010 and 2018. The pressure is widespread: 39% of central banks experienced at least one event of pressure. In an average year, there are reports of political pressure on over 10% of the central banks.

[3]Other institutions, such as fiscal councils, have been proposed to provide incentives to fiscal authorities. In practice, the experience with the introduction of fiscal rules and independent fiscal councils has been mixed (see, e.g., Beetsma and Debrun (2018)). Note, however, that fiscal rules may improve outcomes even when not respected, as argued by Piguillem and Riboni (2021).

[4]See Claeys and Goncalves Raposo (2018) for details on the collateral framework policies of the Eurosystem.

of the European Central Bank (ECB), which was introduced in 2012 to preserve monetary policy transmission during the European debt crisis. An important feature of the OMT is the conditionality on participating in the European Stability Mechanism (ESM) program to promote incentives for sound fiscal policies.

The conditionality of the OMT program has triggered debates among economists and policymakers. On the one hand, the associated provision of incentives to treasuries is considered an important element required to preserve "*a regime of "monetary dominance", in which governments retain the responsibility of balancing their budgets over the medium term and the ECB remains free to set the interest rate so as to ensure the maintenance of price stability*" (Coeure (2012)). In contrast, others argue that by connecting monetary policy to fiscal decisions through conditionality, the central bank increases its exposure to fiscal influence, which could undermine credibility: "*One can question how independent monetary policy is when it links its actions in this way to economic and fiscal policy processes. How credible this conditional path is remains to be seen*" (Weidmann (2012)).

We contribute to this discussion with a formal analysis of a non-cooperative game between a monetary and a fiscal authority. Our institutional set-up highlights the incentives of the treasury to exert influence over an independent central bank. Specifically, the central bank has some degree of commitment to follow policies over time, while the treasury acts in a discretionary way. At a commitment stage, the central bank discloses its operational framework and announces a monetary rule, which specifies policy decisions for different states of the economy. Then, at each point in time, the fiscal authority moves first and implements a policy decision. Given this fiscal choice, the monetary authority faces ex-post the following alternative: either to set policy according to the announced rule or to renounce the promise and revise its policy plan against a cost. The magnitude of the cost captures the degree of commitment of the monetary authority.[5]

Anticipating monetary responses, the fiscal authority might choose policy to induce the central bank to renounce its rule and reap the short-term benefits of policy discretion. The *credibility cut-off* of a monetary rule is then defined as the minimum degree of commitment that makes the central bank follow the preannounced policy, thereby effectively eliminating fiscal incentives to challenge the monetary rule.

We contrast the welfare implications and credibility of two monetary rules. Following a *standard rule*, the central bank commits to pursue price stability without responding to fiscal decisions, reflecting the conventional policy assignment. A *strategic rule* instead prescribes to set monetary policy explicitly conditional on fiscal decisions, so as to provide incentives for

[5]Monetary commitment reflects institutional features of an independent central bank (e.g., mandates and appointment procedures), which are meant to curb inflationary bias and insulate decisions from fiscal influence.

sound fiscal policy while minimizing the credibility cut-off required to support price stability. When designed appropriately, *strategic rules* dominate *standard rules* from a welfare perspective. However, what is the effect of added conditionality on the credibility cut-off? The answer is a priori unclear since a particular specification of conditionality affects both the equilibrium outcome and the off-equilibrium fiscal incentives to challenge the rule.

Our first key result is that comparison between the credibility of the standard and strategic rules depends on the relative intensity of the time-consistency problem of monetary policy. We show this using a static linear-quadratic environment in the tradition of Barro and Gordon (1983). Absent a commitment technology, both monetary and fiscal authorities are confronted with time-consistency problems that give rise to an *inflation bias* and, by analogy, a *fiscal bias*. A *standard* rule prescribes the central bank to unconditionally deliver an inflation target. A *strategic* rule is designed not only to achieve the same inflation target but also to eliminate the fiscal bias by providing proper incentives. If the fiscal bias is strong enough relative to the inflation bias, then the built-in conditionality of the strategic rule may undermine its credibility compared to the standard rule. As the inflation bias becomes stronger, while the credibility cut-off of each rule increases, the standard rule eventually becomes more prone to fiscal pressure. Hence, the strategic rule enhances monetary credibility to reach its inflation target precisely when the monetary authority is most exposed to the possibility of successful fiscal influence.[6]

As highlighted in the opening quote, public debt is a key factor affecting monetary-fiscal interactions. Our second set of results accounts explicitly for the role of public debt in shaping credibility of the strategic rule. We fit the policy game into a dynamic cash-credit economy, as in Lucas and Stokey (1983). The cash-credit economy brings together concerns for the conduct of monetary and fiscal policy under lack of full commitment. The monetary authority is tempted to generate unexpected inflation to inflate away outstanding debt, as, e.g., in Nicolini (1998), Diaz-Gimenez et al. (2008), and Martin (2009). The fiscal authority is tempted to set tax policy to manipulate interest rates and the price of newly issued debt, as, e.g., in Debortoli and Nunes (2013). With public debt, the monetary-fiscal game is dynamic: credibility is evaluated against the level of newly issued debt, both on and off equilibrium paths. As before, we contrast equilibrium outcomes under two classes of monetary rules: one where the central bank commits to a *standard* constant money growth rate and one where the central bank designs a *strategic* rule, with the objective to eliminate discretionary incentives of the fiscal authority to manipulate interest rates.

In an economy with short-term nominal debt, the standard constant money growth rate rule

[6] We also characterize an optimal monetary rule when the degree of commitment falls short of making the strategic rule credible. This rule deviates from optimal policy targets to restore credibility at minimum cost.

also eliminates these discretionary fiscal incentives: indeed, any fiscal attempt to influence the interest rate is offset by a revaluation of outstanding liabilities.[7] However, pursuing such an unconditional rule requires a higher degree of commitment than following a strategic rule, where monetary policy is set explicitly conditional on fiscal choices. Importantly, the credibility of the strategic monetary rule is tied to the level of public debt, since discretionary fiscal incentives and the relative gains to renounce the monetary rule are increasing in outstanding debt.

Further, we analyze the implications of debt characteristics on the credibility of the strategic rule. Generally, the ability to inflate away outstanding debt is weaker the more debt is indexed to inflation. Also, as discussed in Debortoli, Nunes and Yared (2017), long-term real debt mitigates fiscal incentives to manipulate interest rates. The sensitivity of the credibility cut-off of strategic rules reflects these respective incentives. Approaching the limit case where public debt is issued as long-term consol bonds indexed to inflation, credibility of the strategic rule goes to zero.

Additionally, we account for the effect of debt adjustments to fiscal shocks on the credibility of the strategic rule. As in Lucas and Stokey (1983), the hedging of government budget against a variation in public spending calls for a debt structure that makes outstanding public liabilities low during times of high spending, and vice versa. In turn, this leads to variation in monetary and fiscal discretionary incentives. As a result, times when outstanding debt is at its highest—for instance, in the aftermath of a war or a pandemic—require the largest degree of commitment to sustain the strategic rule, i.e., to enforce price stability and resist fiscal influence.

Our last result shows that a central bank can improve the credibility of its promises by deviating along the equilibrium path from its optimal policy target: when the monetary authority commits to an inflation target higher than optimal, the relative benefits of renouncing the rule are lower, thus improving the credibility of the original promise. This result shows how the threat of fiscal dominance may influence the selection of an inflation target (see Schmitt-Grohé and Uribe (2010) for an overview of other factors).

Related literature. Our analysis assesses *"the risk (...) that pursuing multiple objectives simultaneously brings the central bank into the realm of politics. This can compromise its independence and risk losing sight of price stability"* (Orphanides (2013)). In contrast, we show that when the legal independence of the central bank is granted, as is the case particularly for the ECB, then confronting the risk of fiscal dominance with appropriate threats can enhance the credibility to deliver price stability.

Our analysis develops a non-cooperative game between the central bank and the treasury.

[7] If the central bank does not renounce the rule following a fiscal deviation, then the price level decreases following a tax cut, which increases the real value of nominal debt. This offsetting effect is absent when debt is indexed to inflation.

The institutional set-up features asymmetric commitment, in the sense that only the central bank has the ability to commit and respond to fiscal policy.[8] This approach is followed by Gnocchi (2013) and Gnocchi and Lambertini (2016): fiscal contingent monetary strategies clearly welfare-dominate restricted ones. Our focus is different and novel. Indeed, the introduction of strategic interactions with endogenous partial commitment allows us to evaluate central bank credibility across policy regimes and use it as an original criterion to guide policy design.[9]

Also, the game-theoretic implications of *strategic monetary rules* are similar to Bassetto (2005), Atkeson, Chari and Kehoe (2010), or Camous and Cooper (2019), where off-equilibrium policies influence equilibrium outcomes. These studies develop this idea in environments plagued by multiple equilibria with the objective to implement a unique and superior equilibrium outcome.[10] In contrast, we are interested in the effectiveness of this class of monetary interventions to eliminate the time inconsistency of optimal fiscal policy and the effect on the credibility of a central bank to deliver price stability.

The rest of the paper is organized as follows. Section 2 presents the institutional environment in a linear-quadratic framework, exposes the construction of monetary strategies, and discusses the main results. Section 3 then embeds the policy game in a dynamic cash credit economy and analyzes the influence of public debt on the design of strategic monetary rules and associated credibility. Section 4 discusses additional factors that influence the credibility of strategic monetary rules. Section 5 concludes. All proofs and additional details are in the Appendix.

2 Linear-Quadratic Framework

This section provides analysis of monetary-fiscal interactions with asymmetric commitment in a linear-quadratic framework following the tradition of Barro and Gordon (1983).[11] We pursue two objectives: first, we study the construction of *strategic* monetary rules designed to lean against fiscal influence; and second, we contrast the credibility of these rules with a *standard* monetary framework of strict inflation targeting.

[8] Recent studies of the monetary-fiscal games without commitment include analysis of the policy instruments assignment in Martin (2020) and a formal analysis of Wallace's game of chicken in Barthelemy et al. (2021). Bassetto and Sargent (2020) provide a review of the literature about monetary-fiscal interactions.

[9] Partial commitment in models with a single policymaker has been previously modeled either by introducing exogenous stochastic periods of reoptimization, as in Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2013), or by introducing commitment technology with an exogenous time limit, as in Clymo and Lanteri (2019).

[10] Bassetto (2005) highlights the shortcomings of standard Ramsey analyses for fiscal policy. Atkeson et al. (2010) study *sophisticated monetary policy* to implement a unique equilibrium outcome. Camous and Cooper (2019) study how monetary strategies can eliminate self-fulfilling debt crises.

[11] Variants of this environment to study monetary-fiscal interactions have been developed by Dixit and Lambertini (2003) and by Dixit (2000) in the context of a monetary union.

2.1 Economic Environment

We consider an economy with two policymakers. First, the central bank sets inflation π, wherein higher inflation is associated with more expansionary monetary policy. Second, the fiscal authority chooses a policy variable x, which can represent public consumption or investment, a tax cut or a subsidy; a larger x means a more expansionary fiscal policy.[12] The influence of policy decisions (x, π) on output y is captured by the following Phillips curve-type relation:

$$y(x^e, \pi^e, x, \pi) = x - x^e + \alpha(\pi - \pi^e), \tag{2.1}$$

where $\alpha > 0$ is the relative efficiency of monetary policy to stimulate the economy. As usual in this class of environments, expansionary policy choices (x, π) can stimulate output beyond its natural level (normalized to zero) only if unanticipated by private agents, whose expectations are captured by (x^e, π^e).

Economic outcomes are ranked according to the following loss function, shared by both monetary and fiscal authorities:

$$\mathcal{L}(x^e, \pi^e, x, \pi) = \frac{1}{2}\Big[(x - x^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma\Big(y(x^e, \pi^e, x, \pi) - y^*\Big)^2\Big], \tag{2.2}$$

where $\lambda > 0$ captures the cost of monetary deviation from an optimal target π^* relative to fiscal deviation from x^*, and $\gamma > 0$ stands for the relative cost of output deviation from its *first best* level $y^* > 0$.[13]

Private agents anticipate the conduct of public policy and act accordingly. In this environment, these actions are represented by the rational choice of expectations:

$$x^e = x \qquad\qquad \pi^e = \pi. \tag{2.3}$$

Each of these expectation terms are individually pinned down, but at the time of policy decisions, only a linear combination matters to policymakers. Using this linearity, we simplify notation by collecting the expectation of inflation and fiscal policy:

$$e = x^e + \alpha\pi^e, \tag{2.4}$$

and redefine output and the associated loss as functions of the summary of expectations e and policy decisions (x, π): $y(e, x, \pi)$ and $\mathcal{L}(e, x, \pi)$.

[12]There is no need to distinguish particular policy instruments as long as the choice is time inconsistent, as in a wide class of environments. However, interpreting x as a tax cut allows a direct parallel with the cash-credit economy analyzed in Section 3.

[13]The natural level of output is below the first best level, for instance, due to monopolistically competitive market for intermediate goods, or, as in the environment presented in Section 3, due to the need of financing public spending with a distortionary tax. This gives policymakers a motive to stimulate output.

2.2 Cooperative Optimal Policy

We first consider benchmark cases where the central bank and the fiscal authority cooperate over the choice of policy instruments with the same degree of commitment. This is equivalent to studying the joint choice of (x, π) by a consolidated government.

When the government operates under an infinite degree of commitment, it internalizes how policy choices (x^c, π^c) influence private agents' expectations $e^c = x^c + \alpha\pi^c$.[14] As a result, it recognizes that output cannot be stimulated beyond the natural level and chooses policy to minimize the loss function (2.2):

$$x^c = x^* \qquad \pi^c = \pi^* \qquad e^c = x^* + \alpha\pi^* \qquad y^c = 0. \tag{2.5}$$

This equilibrium outcome is not sensitive to parameters (λ, γ).

When the government lacks commitment, it makes policy decisions sequentially, i.e., after private agents form expectations. Under this regime of discretion, the government is tempted to stimulate output toward y^*. In equilibrium, though, private agents anticipate the conduct of public policy, and output is not stimulated:

$$x^d = x^* + \gamma y^* \qquad \pi^d = \pi^* + \frac{\alpha}{\lambda}\gamma y^* \qquad e^d = x^* + \alpha\pi^* + \Big(1 + \frac{\alpha^2}{\lambda}\Big)\gamma y^* \qquad y^d = 0. \tag{2.6}$$

In that case, policy choices are characterized by a *fiscal bias* and a *monetary bias* that make the loss higher than under commitment. Parameters α and λ drive discretionary incentives of the central bank, relative to those of the treasury. The monetary bias under discretion is increasing in α—the relative efficiency of monetary policy to stimulate output, and decreasing in λ—the relative cost of monetary deviation from the target π^*.

2.3 Non-cooperative Policy Game with Asymmetric Commitment

We now set up a non-cooperative game between monetary and fiscal authorities, where policy institutions have different degrees of commitment. This game lends itself to characterizing the credibility of various monetary rules under fiscal influence.

Timing and decisions. Initially, at a *commitment stage*, the central bank announces a *policy rule* $\pi^k(\mathcal{S})$. The rule prescribes the choice of π as a function of the state $\mathcal{S} = (e, x)$, which consists of private agents' expectations e and fiscal decisions x. Then, the following sequence of actions takes place:

i. Private agents form expectations $e = x^e + \alpha\pi^e$;

[14] Formally, under *commitment*, the government moves first and decides upon (x^c, π^c), then private agents form expectations, and finally the government implements its preannounced policy. See Appendix A.1 for details.

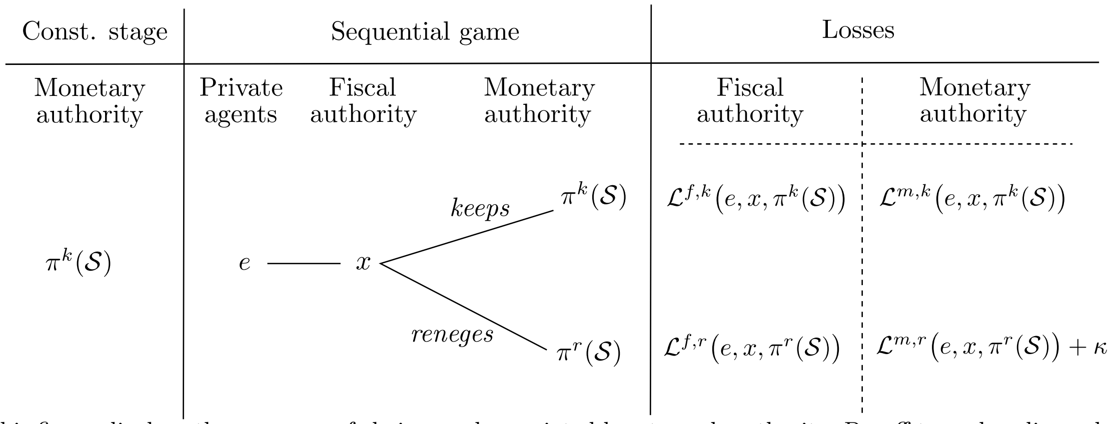
ii. The fiscal authority sets x;

iii. Given $\mathcal{S} = (e, x)$, the central bank either

- *keeps* its promise and follows its policy rule $\pi^k(\mathcal{S})$, or
- *reneges*, incurs a cost $\kappa \geq 0$, and implements an alternative policy $\pi^r(\mathcal{S})$.

Policy preferences. Both monetary and fiscal authorities rank economic outcomes using the loss function $\mathcal{L}(e, x, \pi)$.[15] To keep track of the sequential nature of the game, we index the loss function with the identity of the policymaker and the decision of the central bank to *keep* or *renege* on its promised policy rule. For instance, $\mathcal{L}^{f,k}(\cdot)$ is the loss as evaluated by the fiscal authority conditional on the central bank keeping its promise, and $\mathcal{L}^{m,r}(\cdot)$ is the loss as evaluated by the monetary authority conditional on reneging. Figure 1 graphically summarizes the interactions of monetary and fiscal authorities.

Figure 1: Structure of the Monetary-Fiscal Game



This figure displays the sequence of choices and associated loss to each authority. Payoff to each policymaker is indexed by the identity of the policymaker $\{f, m\}$ and the decision of the central bank $\{k, r\}$ to *keep* or *renege* on its rule.

Fiscal Influence and Monetary Commitment. The set-up with asymmetric and limited commitment provides scope for the fiscal authority to influence the central bank to renounce its promised rule. Indeed, at the time of setting policy, monetary and fiscal authorities do not share the same policy incentives. As Stackelberg leader during the game, the treasury wants to follow discretionary incentives and induce the central bank to do so as well. The treasury can strategically choose some policy x that makes the central bank renege on the rule and implement the sequential optimal policy given (e, x): $\pi^r(\mathcal{S}) = \text{argmin}_\pi \ \mathcal{L}(e, x, \pi)$.

The extent to which the central bank resists the temptation to renounce and reoptimize depends on its degree of commitment captured by the cost κ. If $\kappa = 0$, then no monetary rule can

[15] In Appendix A.5.1, we discuss the sensitivity of our results to relaxing the assumption of shared loss function and consider the case of *monetary conservatism* and *fiscal self-interest*.

be credibly implemented, and the equilibrium outcome coincides with (2.6) when a consolidated government chooses policy under discretion. If κ is arbitrarily large, then any monetary rule can be credibly implemented. Next, we characterize the credibility of a generic monetary rule allowing for intermediate values of $\kappa \in (0, \infty)$.[16]

Credibility of a Monetary Rule. The characterization of credibility under asymmetric commitment requires the study of fiscal and monetary decisions on and off equilibrium paths.

If the fiscal authority anticipates that the central bank keeps its promise and follows its rule $\pi^k(\mathcal{S})$, it chooses $\tilde{x}$ defined as:

$$\tilde{x} = \text{argmin}_x \ \mathcal{L}^{f,k}(e, x, \pi^k(\mathcal{S})). \tag{2.7}$$

Along this equilibrium path, private agents' expectations satisfy $\tilde{e} = \tilde{x} + \alpha\pi^k(\tilde{e}, \tilde{x})$.[17]

Other choices of x could dominate $\tilde{x}$ from the perspective of the fiscal authority if they were to induce the central bank to renege on its rule $\pi^k(\mathcal{S})$ and implement $\pi^r(\mathcal{S})$. We capture these fiscal incentives to challenge the monetary rule with a set of *fiscal influence* $\mathrm{T}(\tilde{e})$. Formally, it is the set of fiscal decisions x such that if these decisions lead the monetary authority to renege on its commitment, the fiscal authority is better off:

$$\mathrm{T}(\tilde{e}) = \{x \ \mid \mathcal{L}^{f,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)) \leq \mathcal{L}^{f,k}(\tilde{e}, \tilde{x}, \pi^k(\tilde{e}, \tilde{x}))\}. \tag{2.8}$$

Whether the monetary authority decides to follow its rule against a given attempt of fiscal influence $x \in \mathrm{T}(\tilde{e})$ depends on the degree of commitment $\kappa > 0$. Hence, a high enough degree of commitment would deter the temptation of deviating from the rule in response to every possible instance of fiscal influence. Whenever the central bank has no incentives to renounce the rule, we refer to this rule as credible.

Definition 1. *A monetary rule $\pi^k(\mathcal{S})$ is credible if for all fiscal decisions x in the set of fiscal influence* $\mathrm{T}(\tilde{e})$*:*

$$\mathcal{L}^{m,k}(\tilde{e}, x, \pi^k(\tilde{e}, x)) \leq \mathcal{L}^{m,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)) + \kappa,$$

where $\pi^r(\mathcal{S})$ =argmin$_\pi$ $\mathcal{L}(e, x, \pi)$*.*

We define a *credibility cut-off* as the minimum degree of commitment κ under which the monetary policy rule is credible against attempts of fiscal influence.

[16] A similar modeling approach for limited commitment is followed by Farhi et al. (2012) and Scheuer and Wolitzky (2016). In Appendix A.5.2, we discuss conditions under which κ can be interpreted as a reduced-form reputation component that supports monetary pledges.

[17] In what follows, we assume that expectations of private agents remain anchored on this equilibrium path to focus on the game between policy institutions. In Appendix A.6, we discuss implications of allowing for sunspot equilibria as well as a simple adjustment of monetary rules that can eliminates expectation-driven equilibria.

Definition 2. *Let $\bar{\kappa}$ be the credibility cut-off of a monetary rule $\pi^k(\mathcal{S})$, defined as:*

$$\bar{\kappa} = \max_{x \in \mathrm{T}(\tilde{e})} \mathcal{L}^{m,k}(\tilde{e}, x, \pi^k(\tilde{e}, x)) - \mathcal{L}^{m,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)). \tag{2.9}$$

Importantly, the specific off-equilibrium path of fiscal-monetary interaction that determines the credibility cut-off depends on the monetary rule.[18] The role of fiscal conditionality in shaping these off-equilibrium paths is central in our analysis of specific monetary rules.

2.4 Comparative Analysis of Monetary Rules

The policy game is simple, yet it has all the necessary ingredients to study whether a central bank can design a credible rule to lean against fiscal influence and reach its monetary target π^*. We approach this question by contrasting the equilibrium implications and credibility cut-offs of two alternative monetary rules:

1. a *standard* rule, which prescribes the central bank to target inflation π^* unconditionally;
2. a *strategic* rule, which prescribes the central bank to target inflation conditional on fiscal policy, with the objective to deliver π^* and induce the fiscal authority to implement x^*.

The standard rule reflects a conventional monetary-fiscal assignment – where the monetary authority operates without adjusting its policy to fiscal decisions, while the strategic rule is designed to provide appropriate incentives to the treasury and shield the monetary authority from fiscal influence.

2.4.1 Standard Monetary Rule

The standard rule of setting inflation on target unconditionally is defined formally as follows:

$$\forall \mathcal{S},\ \pi^k(\mathcal{S}) = \pi^*. \tag{2.10}$$

If credible, this policy rule results in the following equilibrium outcome:[19]

$$x_1 = x^* + \gamma y^*, \qquad \pi_1 = \pi^*, \qquad e_1 = x^* + \gamma y^* + \alpha \pi^*, \qquad y_1 = 0. \tag{2.11}$$

The central bank naturally implements its policy target π^*, while the fiscal decision is characterized by the *fiscal bias*. Private expectations reflect policy choices, and output is not stimulated beyond its natural level.

Recall that the central bank might be tempted to renounce the promised rule, and the fiscal authority has incentives to influence this decision. Sustaining equilibrium (2.11) requires a high enough degree of commitment κ to eliminate fiscal incentives to challenge the rule.

[18] The argument of the maximum in (2.9) is rule-specific due to the effect of $\pi^k(\mathcal{S})$ both on the magnitude of the monetary temptation to reoptimize for a given $x \in \mathrm{T}(\tilde{e})$ and on the set of fiscal influence $\mathrm{T}(\tilde{e})$ itself. As a result, it does not necessarily coincide with the best fiscal deviation conditional on the central bank reneging on its rule.

[19] See formal derivation in Appendix A.2.

Proposition 1. *The credibility cut-off for the standard monetary rule* (2.10) *is*

$$\bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1+\gamma)\Big(\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta}\Big)^2,$$

where $\eta \equiv \frac{\lambda\gamma}{\lambda+\gamma\alpha^2}$*. In addition:*

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \qquad \textit{and} \qquad \frac{d\bar{\kappa}_1}{d\alpha} > 0.$$

Proof. See Appendix A.2 ∎

The credibility cut-off $\bar{\kappa}_1$ is pinned down by the off-equilibrium path where the treasury undertakes the most contractionary attempt of fiscal influence $x_l = \min\{\mathrm{T}(e_1)\} < x_1$. Indeed, out of all the possible attempts of fiscal influence $x \in \mathrm{T}(e_1)$, this fiscal choice is associated with the strongest temptation of the central bank to renounce the rule and offset the contractionary effect of fiscal deviation on output. Additionally, the relative gains of renouncing the rule increase as parameters λ and α become smaller and larger, respectively. The lower is λ, the less costly is a marginal increase in inflation. The higher is α, the more sensitive is output to surprise inflation. Either way, the degree of commitment required to follow the rule against fiscal influence increases.

2.4.2 Strategic Monetary Rule

We now consider *strategic* monetary rules, which set monetary policy conditional on fiscal decisions. The construction of these rules rests on two components. First, the central bank picks as desired equilibrium outcome the one under cooperation and full commitment:

$$x_2 = x^*, \qquad \pi_2 = \pi^*, \qquad e_2 = x^* + \alpha\pi^*, \qquad y_2 = 0. \tag{2.12}$$

Second, these rules are strategic because the central bank includes off-equilibrium threats into $\pi^k(\mathcal{S})$ to discourage fiscal deviations from $x = x^*$. Formally,

$$\mathcal{L}^{f,k}(e_2, x, \pi^k(e_2, x)) \geq \mathcal{L}^{f,k}(e_2, x^*, \pi^*), \quad \forall x. \tag{2.13}$$

In what follows, we characterize strategic rules that minimize the associated credibility cut-off.

Proposition 2. *The lowest credibility cut-off for a strategic monetary rule that implements* (2.12) *is:*

$$\bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2}\frac{\gamma(\lambda+\alpha^2)}{\lambda+\gamma\alpha^2+\lambda\gamma}.$$

In addition:

$$\frac{d\bar{\kappa}_2}{d\lambda} < 0 \qquad \textit{and} \qquad \frac{d\bar{\kappa}_2}{d\alpha} > 0.$$

Proof. See Appendix A.3 ∎

The specific threats of a strategic rule associated to this result are such that the fiscal incentive constraint (2.13) is binding for all attempts of fiscal influence,[20]

$$\mathcal{L}^{f,k}(e_2, x, \pi^k(e_2, x)) = \mathcal{L}^{f,k}(e_2, x^*, \pi^*), \quad \forall x \in \mathrm{T}(e_2). \tag{2.14}$$

These threats are the least costly to the central bank, conditional on keeping the rule. Accordingly, the credibility cut-off of a strategic rule that features these specific threats is the lowest.

Note that the off-equilibrium path that determines the credibility cut-off $\bar{\kappa}_2$ is different from the one that determines the credibility cut-off $\bar{\kappa}_1$ of the standard rule. By design, following the strategic rule yields the same loss to the monetary authority in response to any attempt of fiscal influence. Hence, the strongest temptation of the central bank to renounce the rule is associated with an attempt of fiscal influence that encourages the most profitable monetary deviation.[21]

Parameters λ and α that control discretionary incentives of the central bank have the same qualitative effects on the credibility cut-off $\bar{\kappa}_2$ as under the standard rule. More importantly, the proposition below shows how these parameters influence the relative magnitude of the credibility cut-offs for the standard and the strategic rules.

Proposition 3. *The credibility cut-off $\bar{\kappa}_2$ associated with strategic monetary rules is lower than the credibility cut-off $\bar{\kappa}_1$ of the standard rule, for relatively low values of λ and high values of α. Formally,*

- *for every $\lambda > 0$, there is a threshold $\bar{\alpha} > 0$ such that $\bar{\kappa}_1 > \bar{\kappa}_2$ if and only if $\alpha > \bar{\alpha}$,*
- *for every $\alpha > 0$, there is threshold $\bar{\lambda} > 0$ such that $\bar{\kappa}_1 > \bar{\kappa}_2$ if and only if $\lambda > \bar{\lambda}$.*

Proof. See Appendix A.3 ∎

The credibility cut-off of the strategic rule is not necessarily lower than that of the standard rule. When discretionary incentives of the central bank are absent (e.g., $\alpha = 0$), the standard rule is time-consistent and does not require any commitment to be credible ($\bar{\kappa}_1 = 0$). Additionally, there is no scope for fiscal influence in this case, and the corresponding set of fiscal influence is degenerate. In contrast, the strategic rule exposes the central bank to fiscal influence: the off-equilibrium conditionality in response to fiscal influence requires a non-trivial degree of commitment to be credible ($\bar{\kappa}_2 > 0$).

As discretionary incentives of the central bank increase, the relative gains of *renouncing* either of the rules goes up. Importantly, the cost of following the standard rule against fiscal

[20]For fiscal decisions outside the set of fiscal influence $\mathrm{T}(e_2)$, strategic rules can specify any policy $\pi \geq \pi^r(S)$, because the fiscal authority would not contemplate these paths to challenge the monetary rule.

[21]Formally, the attempt of fiscal influence that characterizes $\bar{\kappa}_2$ is $x_b = \mathrm{argmin}_x \mathcal{L}^{m,r}(e_2, x, \pi^r(e_2, x)) > x_2$. It is also the optimal deviation from the perspective of the fiscal authority if the central bank were to reoptimize. As a result, x_b and $\pi^r(e_2, x_b)$ coincide with the optimal deviation under policy cooperation.

influence increases due to the expansion of the set of fiscal influence. In contrast, the cost of following the strategic rule against fiscal influence does not change by design. As a result, the credibility cut-off of the standard rule eventually (e.g., $\alpha > \bar{\alpha}$) exceeds that of the strategic rule ($\bar{\kappa}_1 > \bar{\kappa}_2$). Based on this, a key result is that the strategic rule requires a lower degree of commitment to be sustained precisely when the credibility problem of monetary policy is severe, and the central bank is more prone to fiscal influence.

2.5 Commitment-constrained Strategic Rule

From the perspective of policy design, it is straightforward to see that a strategic rule that credibly implements (x^*, π^*) is an optimal choice of policy rule for the central bank with a degree of commitment $\kappa \geq \bar{\kappa}_2$. This section extends the analysis of optimal policy design to the case where the degree of monetary commitment falls short of making the strategic rule credible, $\kappa < \bar{\kappa}_2$. To this end, we generalize the strategic rule to allow for an equilibrium outcome associated with policy choices $(\tilde{x}, \tilde{\pi})$ different from (x^*, π^*). The optimal configuration of this rule minimizes economic loss while ensuring that the rule is implemented given the constrained degree of monetary commitment κ.

Proposition 4. *Given $\kappa < \bar{\kappa}_2$, the optimal strategic monetary rule achieves a desired equilibrium outcome associated with policy choices $\tilde{x} > x^*$ and $\tilde{\pi} > \pi^*$. These equilibrium policy choices depend on the degree of commitment as follows:*

$$\frac{d\tilde{x}}{d\kappa} < 0 \qquad\qquad \frac{d\tilde{\pi}}{d\kappa} < 0.$$

In addition, the effects of parameters λ and α on the fiscal policy target $\tilde{x}$

$$\frac{d\tilde{x}}{d\lambda} > 0 \qquad\qquad \frac{d\tilde{x}}{d\alpha} < 0,$$

are the opposite to their effects on the monetary policy target $\tilde{\pi}$

$$\frac{d\tilde{\pi}}{d\lambda} < 0 \qquad\qquad \frac{d\tilde{\pi}}{d\alpha} > 0.$$

Proof. See Appendix A.4 ∎

Similar to (2.14), conditionality embedded in the commitment-constrained strategic monetary rule provides off-equilibrium threats that discourage deviations from $\tilde{x}$ by making the fiscal incentive constraint binding for all attempts of fiscal influence:

$$\mathcal{L}^{f,k}(\tilde{e}, x, \pi^k(\tilde{e}, x)) = \mathcal{L}^{f,k}(\tilde{e}, \tilde{x}, \tilde{\pi}), \quad \forall x \in \mathrm{T}(\tilde{e}). \tag{2.15}$$

The equilibrium choice of policy targets $(\tilde{x}, \tilde{\pi})$ different from (x^*, π^*) leads to an economic loss, however, it is needed for the commitment-constrained strategic rule to be credible. The lower the degree of commitment κ, the closer equilibrium policy choices become to their counterparts (x^d, π^d) under full discretion. In turn, the relative gains from renouncing the rule decline enough to preserve the incentives of the monetary authority to follow the rule. The specific choice of $(\tilde{x}, \tilde{\pi})$ depends on the relative difficulty of the central bank to follow its rule against strategic fiscal influence. Specifically, the difference between $\tilde{\pi}$ and π^* goes up while the difference between $\tilde{x}$ and x^* goes down when the credibility problem of monetary policy worsens, i.e., as α gets higher or λ gets lower.

Finally, note that the optimally designed monetary rule with fiscal conditionality eliminates inefficiency due to the asymmetry of commitment. Recall that for $\kappa \geq \bar{\kappa}_2$, equilibrium under the strategic rule coincides with equilibrium under cooperation with full commitment. When $\kappa < \bar{\kappa}_2$, the commitment-constrained strategic rule attains equilibrium as under cooperation with symmetric but limited degree of commitment κ (see the proof of Proposition 4). Hence, for any degree of commitment, strategic rules allow the central bank to share its commitment technology with the treasury, as if decisions were implemented jointly under cooperation.

Overall, Propositions 2 and 4 combined provide a full characterization of the optimal monetary rule under fiscal influence as a function of the degree of monetary commitment κ.

3 Dynamic Cash-Credit Economy

The institutional environment developed in Section 2 is now introduced in a dynamic cash-credit economy as in Lucas and Stokey (1983). The objective is to study how public debt influences the credibility of *standard* and *strategic* monetary rules. Our main results below are presented using a baseline model with one-period nominal debt. Additional factors affecting credibility of the strategic rule, including the role of government debt structure, are studied in Section 4.

3.1 Economic Environment

Time is discrete, and each period is indexed with $t \geq 0$. The economy is populated by a representative agent and a government. The resource constraint of the economy is:

$$c_t + d_t + g = 1 - l_t, \tag{3.1}$$

where c_t is private consumption of a *credit good*, d_t is private consumption of a *cash good*, $g > 0$ is exogenous and constant public consumption, l_t is leisure, while production is linear in labor $y_t = 1 - l_t$.

Private agents. A representative household consumes both cash and credit goods and supplies labor to competitive firms that produce both type of goods. She earns a real wage equal to the (unitary) marginal product of labor and is taxed at a linear rate τ_t. At a price q_t, she can buy or sell nominal one-period risk-free government bonds B_t^h. The nominal flow budget constraint in period t reads:

$$P_t c_t + P_t d_t + q_t B_t^h + M_t^h = P_t(1-\tau_t)(1-l_t) + B_{t-1}^h + M_{t-1}^h, \tag{3.2}$$

where P_t is the price level, and M_t^h is the stock of money, carried over from period t into next period. The purchase of cash good d_t is subject to a cash-in-advance constraint:[22]

$$P_t d_t \leq M_{t-1}^h. \tag{3.3}$$

The household enjoys utility from private consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, d_t, l_t) = \sum_{t=0}^{\infty} \beta^t (\alpha \log c_t + (1-\alpha)\log d_t + \gamma l_t), \tag{3.4}$$

where $\beta \in (0,1)$ is the time discount factor. This specification of preferences implies a unitary elasticity of intertemporal substitution. As a result, the incentive to adjust next-period consumption of cash goods in response to an expected marginal change in price level is as strong as the ex-post need to adjust the consumption of cash goods to satisfy the cash-in-advance constraint (3.3).[23] As explained in Diaz-Gimenez et al. (2008), this alignment eliminates policy motives to renege on the price level other than for the purpose of reducing public debt burden.

Government. The government consists of a fiscal and a monetary authority. The treasury controls the tax rate τ_t on labor income and the supply of government bonds B_t. The central bank controls the growth rate σ_t of money supply $M_t = M_{t-1}(1+\sigma_t)$. The cash-in-advance constraint imposes a bound on the nominal interest rate $q_t^{-1} \geq 1$, which induces the following restriction on the money growth rate: $(1+\sigma_t) \geq \beta$.[24] Every period, policies satisfy the budget constraint of the government, which in nominal terms reads:

$$q_t B_t + M_t + P_t \tau_t (1-l_t) = P_t g + B_{t-1} + M_{t-1}. \tag{3.5}$$

Initial outstanding debt $B_{-1} = B_{-1}^h$, and stock of money $M_{-1} = M_{-1}^h$, are exogenous and non-negative.

[22] The stock of money at the beginning of a period constrains the purchase of cash goods, as in Svensson (1985), Nicolini (1998), Diaz-Gimenez et al. (2008), or Martin (2009). Also, exogenous debt limits are in place to prevent Ponzi schemes, but they do not bind in equilibrium.

[23] Given that consumption of the cash goods reflects demand for real money balances, Nicolini (1998) additionally describes this case as the one with equal elasticities of a 'short-run' and a 'long-run' demand for money balances.

[24] When $q_t = 1$, we assume that households keep the minimum amount of money required to purchase goods, hence the cash-in-advance constraint always holds with equality.

Competitive equilibrium. Our analysis considers standard competitive equilibria that arise in this economy (the definition is provided in Appendix B.1). These competitive equilibria differ depending on government policies, which creates scope for policy analysis. The remainder of this section presents results that underpin our analysis of strategic monetary and fiscal decisions. First, note that the price level in equilibrium is determined as the outcome of interactions between monetary and fiscal policy.

Lemma 1. *In a competitive equilibrium, the price level P_t satisfies:*

$$P_t = \frac{\gamma}{\beta(1-\alpha)} \frac{M_t}{(1-\tau_t)}. \tag{3.6}$$

Proof. See Appendix B.1. ∎

A tax cut could be associated with a decline of the price level if money supply stays unchanged, or an increase in the price level if money supply increases.

Second, a convenient way to characterize competitive equilibria is to combine equilibrium conditions to derive an implementability constraint in terms of policy instruments.[25]

Lemma 2. *A sequence of tax rates and money growth rates, $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, supports a competitive equilibrium when government debt is nominal if and only if the following constraint is satisfied for all $t \geq 0$ given z_{-1}:*

$$\beta \left[\frac{(1-\alpha)\beta}{1+\sigma_{t+1}}\right] z_t - \alpha(1-\tau_t) - (1-\alpha)\beta \frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right] z_{t-1}, \tag{3.7}$$

where the bond-to-money ratio $z_t \equiv B_t/M_t$ is the state variable that captures the real level of government debt, $\Phi \equiv (\beta(1-\alpha) + \alpha - \gamma g)$ is a constant, and Ponzi schemes are ruled out by exogenous debt limits.

Proof. See Appendix B.1 ∎

Additionally, welfare associated with the equilibrium outcome can be evaluated via an indirect utility function.

Lemma 3. *In the competitive equilibrium induced by a policy sequence $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, the flow utility of the representative household is given by the following indirect utility function:*

$$U(\tau_t, \sigma_t) = \alpha \big[\log(1-\tau_t) - (1-\tau_t) \big] + (1-\alpha) \left[\log\left(\beta \frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta \frac{(1-\tau_t)}{(1+\sigma_t)} \right]. \tag{3.8}$$

Proof. See Appendix B.1 ∎

[25] The optimal policy literature often uses the *primal approach*, whereby one substitutes away prices and policy instruments using equilibrium conditions to obtain an implementability constraint in terms of allocation. Instead, we substitute away prices and allocation to highlight interactions of policymakers with different policy instruments.

3.2 Cooperative Optimal Policy

As in the linear-quadratic economy, we first consider an arrangement where monetary and fiscal authorities cooperate over the choice of policy instruments. Consider a benevolent government with an infinite degree of commitment that pursues a dynamic policy plan $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ defined at $t = 0$ to maximize discounted sum of (3.8) subject to the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left\{ \Phi - \alpha(1-\tau_t) - (1-\alpha)\beta \frac{(1-\tau_t)}{(1+\sigma_t)} \right\} = \left[\frac{(1-\alpha)\beta}{(1+\sigma_0)} \right] z_{-1} \tag{3.9}$$

that is derived by forward substitution of (3.7). The following proposition characterizes properties of this optimal *Ramsey* plan and highlights the nature of its time inconsistency.

Proposition 5. *Given positive initial government debt $z_{-1} > 0$, the Ramsey policy plan $\{\tau_t^*, \sigma_t^*\}_{t=0}^{\infty}$ consists of two segments:*

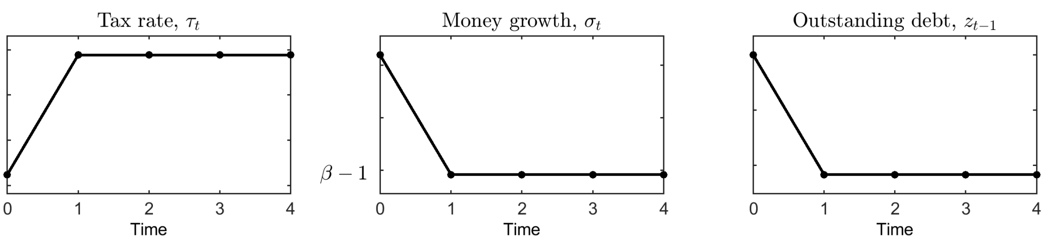
- *"tail policy": for all $t \geq 1$, the labor tax rate τ_t^* and the money growth rate σ_t^* are constant over time, with the latter set according to the Friedman rule ($\sigma_t^* = \beta - 1$); the associated outstanding debt z_{t-1}^* is also constant; and*

- *"initial policy": at $t = 0$, policy choices differ from the tail levels. In particular, $\tau_0^* < \tau_1^*$ and $\sigma_0^* > \sigma_1^*$; the resulting debt dynamics satisfies $z_{-1} > z_0^*$.*

Proof. See Appendix B.2. ∎

This characterization of optimal policy, illustrated in Figure 2, is typical for this class of economies (see, e.g., Chari, Christiano and Kehoe (1991) and Ljungqvist and Sargent (2018)). The constant path of policies starting from $t = 1$ reflects a motive to smooth tax distortions over time. Following the Friedman rule eliminates the inefficient opportunity cost of holding money induced by the cash in advance constraint. This tail policy requires commitment to be implemented, because its choice in every period internalizes the effects on equilibrium outcomes in the preceding periods.

Figure 2: Ramsey Equilibrium: Joint Government with Commitment



This figure illustrates optimal policy under cooperation and commitment characterized in Proposition 5.

The *initial* policy choices at $t = 0$ are different because there is no need for the government to internalize the effect of these choices on past outcomes. These differences are indicative of the time inconsistency of the Ramsey plan, motivated by the incentive to reduce the burden of initial government debt $B_{-1} > 0$.[26] One mechanism driving this incentive is the ability to inflate debt away with an increase in the price level. Additionally, a temporarily low tax rate increases the price of newly issued bonds through a lower marginal utility of credit-good consumption at $t = 0$.[27]

Our objective is to study institutional environments where independent policymakers do not give in to the sequential incentives to deviate from the optimal Ramsey plan. Accordingly, the analysis of non-cooperative policy below treats the Ramsey plan as the normative benchmark to evaluate the performance of different policy regimes. Let $W^{rp}(z)$ be the welfare of the representative household under the Ramsey plan at $t = 0$ when $z_{-1} = z$. Given the two-step path of the Ramsey plan, it can be characterized as a solution of the following Bellman equation:

$$W^{rp}(z_{-1}) = \max_{\tau_0, \sigma_0, z_0} U(\tau_0, \sigma_0) + \beta W^{ta}(z_0), \tag{3.10}$$

where maximization over τ_0, σ_0 and z_0 is subject to the implementability constraint (3.7) at $t = 0$ with the tail growth rate of money $\sigma_1 = \beta - 1$. Furthermore, $W^{ta}(z)$ is the continuation welfare under tail policy at $z_0 = z$:

$$W^{ta}(z_0) = \frac{1}{1-\beta} U(\tau^*(z_0), \sigma^*), \tag{3.11}$$

where $\tau^*(z)$ is the tax rate required to sustain debt at z permanently when monetary policy is set according to the Friedman rule $\sigma^* = \beta - 1$.

3.3 Non-cooperative Policy Game with Asymmetric Commitment

The institutional set-up with asymmetric commitment from Section 2.3 is now applied to the dynamic cash-credit good economy to contrast the credibility of different monetary rules in an environment with public debt.

Timing and decisions. At a commitment stage before $t = 0$, the central bank announces a monetary rule $\varrho^{Mk} = \{\varrho_t^{Mk}\}_{t=0}^{\infty}$ for setting money growth rates σ_t at all times in the future.[28] After the commitment stage, the game unfolds dynamically. In every period $t \geq 0$, the fiscal authority moves first and chooses the tax rate τ_t. The central bank moves second and chooses

[26] In equilibrium with policy cooperation but without commitment, systematic discretionary incentives gradually drive government debt to zero where discretionary incentives are no longer present; see Appendix B.3 for details.

[27] The right-hand side of (3.9) can be rewritten as $u_{c,t} B_{t-1}/P_t$ to highlight both mechanisms.

[28] Superscript M in ϱ^{Mk} stands for *monetary* authority and, as made clear below, k indicates that the central bank *keeps* its promise when it decides to follow this rule over time.

the money growth rate σ_t. The central bank can *keep* the promise and follow the preannounced rule ϱ^{Mk} or *renege* on the promise and deviate from the rule. Given policy choices, the household chooses consumption, leisure, and savings. The resulting dynamic transition of government debt is represented by a function ϱ^s such that $z_t = \varrho^s(z_{t-1}, \tau_t, \sigma_t)$.[29]

Policy objectives and commitment technology. Policymakers are benevolent but differ in their degree of commitment. Every period, the fiscal authority sets the tax rate so as to maximize the discounted utility of the household. The central bank then acts according to its rule ϱ^{Mk} unless it finds it profitable to renege its promise, namely if it improves the utility of the representative household starting from this period net of an institutional loss $\kappa \geq 0$. This cost—born only by the central bank—reflects the degree of commitment of the monetary authority.[30]

Policy rules and strategies. We study monetary rules that set the money growth rate σ_t as a function ϱ_t^{Mk} of outstanding debt z_{t-1} and the tax rate τ_t. The sequential actions of all players involved in the game are described by Markovian strategies. First, the fiscal authority sets the tax rate as a function of outstanding liabilities, $\tau_t = \varrho_t^F(z_{t-1})$. Second, the decision of the central bank of whether or not to follow the preannounced rule is described by an indicator function $\mathbb{I}_t^r(z_{t-1}, \tau_t)$, which is equal to one when the central bank reneges on the promise and zero otherwise. When reneging, the choice of the money growth rate by the central bank is described by a policy strategy $\varrho_t^{Mr}(z_{t-1}, \tau_t)$. To simplify the notation, from now on we omit the indices that reflect time-dependence of the rules, strategies, and associated value functions. Formally, an equilibrium of this game is as follows.

Definition 3. *Given a monetary rule ϱ^{Mk}, a Markov-perfect equilibrium of the policy game consists of a transition function ϱ^s, policy strategies $(\varrho^{Mr}, \mathbb{I}^r)$ and ϱ^F, and value functions V^M, V^F such that:*

(i) Given $(\varrho^s, \varrho^{Mr}, \mathbb{I}^r)$, the fiscal policy ϱ^F and value function V^F solve the following Bellman equation:

$$V^F(z_{t-1}) = \max_{\tau_t} \; U(\tau_t, \sigma_t) + \beta V^F(\varrho^s(z_{t-1}, \tau_t, \sigma_t)),$$

where

$$\sigma_t = [1 - \mathbb{I}^r(z_{t-1}, \tau_t)]\varrho^{Mk}(z_{t-1}, \tau_t) + \mathbb{I}^r(z_{t-1}, \tau_t)\varrho^{Mr}(z_{t-1}, \tau_t);$$

[29] With our focus on the interactions across policy authorities, we treat the household as non-strategic. The transition function is implicitly determined by the conditions associated to the competitive equilibrium Definition 6 and characterized in Lemma 2.

[30] The extreme cases where $\kappa = +\infty$ and $\kappa = 0$ correspond respectively to full monetary commitment and no monetary commitment.

(ii) Given (ϱ^s, ϱ^F), the <u>*monetary policies*</u> *$(\varrho^{Mr}, \mathbb{I}^r)$ and value function V^M solve the following Bellman equation:*

$$V^M(z_{t-1}, \tau_t) = \max_{\mathbb{I}_t^r \in \{0,1\}} \; [1 - \mathbb{I}_t^r] V^{Mk}(z_{t-1}, \tau_t) + \mathbb{I}_t^r [V^{Mr}(z_{t-1}, \tau_t) - \kappa],$$

with the value functions V^{Mk} and V^{Mr} corresponding respectively to the central bank keeping or reneging on its rule ϱ^{Mk}:

$$V^{Mk}(z_{t-1}, \tau_t) = U(\tau_t, \varrho^{Mk}(z_{t-1}, \tau_t)) + \beta V^M\Big(\underbrace{\varrho^s(z_{t-1}, \tau_t, \varrho^{Mk}(z_{t-1}, \tau_t))}_{z_t}, \underbrace{\varrho^F\left(\varrho^s(z_{t-1}, \tau_t, \varrho^{Mk}(z_{t-1}, \tau_t))\right)}_{\tau_{t+1}}\Big),$$

$$V^{Mr}(z_{t-1}, \tau_t) = \max_{\sigma_t} \; U(\tau_t, \sigma_t) + \beta V^M\left(\varrho^s(z_{t-1}, \tau_t, \sigma_t), \varrho^F\left(\varrho^s(z_{t-1}, \tau_t, \sigma_t)\right)\right);$$

(iii) Given $(\varrho^F, \varrho^{Mr}, \mathbb{I}^r)$, the <u>*transition function*</u> *ϱ^s induces a competitive equilibrium for any $(z_{t-1}, \tau_t, \sigma_t)$:*

$$0 = f(z_t, z_{t-1}, \sigma_{t+1}, \sigma_t, \varrho^F(z_t), \tau_t),$$

where $f(\cdot)$ is a generic representation of implementability constraint (3.7) *and*

$$\sigma_{t+1} = \mathbb{I}^r(z_t, \varrho^F(z_t))\varrho^{Mr}(z_t, \varrho^F(z_t)) + [1 - \mathbb{I}^r(z_t, \varrho^F(z_t))]\varrho^{Mk}(z_t, \varrho^F(z_t)).$$

This equilibrium definition highlights the sequential nature of policy moves within periods, where a leading fiscal authority can challenge the preannounced monetary rule ϱ^{Mk}.[31] Finally, the transition function reflects the intertemporal strategic interactions of policymakers bound by outstanding debt z_t.

3.3.1 Credibility of the Central Bank

An equilibrium path of this game is the outcome of non-cooperative actions by policymakers. Both authorities are interested in minimizing tax distortions and intertemporal losses, but only the central bank is endowed with a commitment technology. Intuitively, a monetary rule ϱ^{Mk} announced at the commitment stage is credible if the central bank keeps the promise along the equilibrium path.

Definition 4. *Let z_{-1} be initial government liability and $\{\tilde{\sigma}_t, \tilde{\tau}_t, \tilde{z}_t\}_{t=0}^{\infty}$ an equilibrium outcome of the policy game given a monetary rule ϱ^{Mk}. The rule ϱ^{Mk} is credible given z_{-1} if $\tilde{\sigma}_t = \varrho^{Mk}(\tilde{z}_{t-1}, \tilde{\tau}_t)$ $\forall t \geq 0$.*

To characterize the degree of commitment required to enforce a monetary rule, we specify the sequential incentives of fiscal and monetary authorities on and off the equilibrium path.

[31] Importantly, the equilibrium definition stipulates that the continuation game after an event of central bank *reneging* does not carry any reputational stigma: in the next periods, the history of past decisions does not change fiscal and monetary decisions, except through its impact on the state of the economy.

Start with the fiscal authority and let $V^{Fk}(z_{t-1},\tau_t)$ be the value to the fiscal authority of choosing a tax rate τ_t, conditional on the monetary policy *keeping* its promise to follow ϱ^{Mk}:

$$V^{Fk}(z_{t-1},\tau_t) = U\big(\tau_t, \varrho^{Mk}(z_{t-1},\tau_t)\big) + \beta V^F\Big(\varrho^s\big(z_{t-1},\tau_t,\varrho^{Mk}(z_{t-1},\tau_t)\big)\Big). \tag{3.12}$$

Similarly, let $V^{Fr}(z_{t-1},\tau_t)$ be the value function to the fiscal authority when setting a tax rate τ_t conditional on monetary policy *renouncing* the preannounced rule:

$$V^{Fr}(z_{t-1},\tau_t) = U\big(\tau_t, \varrho^{Mr}(z_{t-1},\tau_t)\big) + \beta V^F\Big(\varrho^s\big(z_{t-1},\tau_t,\varrho^{Mr}(z_{t-1},\tau_t)\big)\Big). \tag{3.13}$$

Consider an equilibrium path $\{\tilde{\sigma}_t,\tilde{\tau}_t,\tilde{z}_t\}_{t=0}^{\infty}$ under a credible monetary rule. Along this path, the choices of the fiscal authority satisfy:

$$\tilde{\tau}_t = \operatorname{argmax}_{\tau_t} V^{Fk}(\tilde{z}_{t-1},\tau_t) \qquad \text{and} \qquad V^F(\tilde{z}_{t-1}) = V^{Fk}(\tilde{z}_{t-1},\tilde{\tau}_t). \tag{3.14}$$

These expressions also reflect the monetary decision to follow its rule along the equilibrium path.

At $\tilde{z}_{t-1}$, the fiscal authority would deviate from $\tilde{\tau}_t$ and set a tax rate $\tau_t \neq \tilde{\tau}_t$ if it were to lead to a welfare improvement, conditional on the central bank renouncing its promise. Define accordingly $\mathrm{T}(\tilde{z}_{t-1})$, the set of *fiscal influence at* $\tilde{z}_{t-1}$:

$$\mathrm{T}(\tilde{z}_{t-1}) = \{\tau_t \mid V^{Fr}(\tilde{z}_{t-1},\tau_t) \geq V^F(\tilde{z}_{t-1})\}. \tag{3.15}$$

Consider now the central bank: its incentives to *renounce* its promise at $(\tilde{z}_{t-1},\tau_t)$ for all $\tau_t \in \mathrm{T}(\tilde{z}_{t-1})$ are evaluated against the value to *keep* its promise and follow the rule. The central bank follows the rule at $\tilde{z}_{t-1}$ when

$$\kappa \geq \Delta(\tilde{z}_{t-1}) = \max_{\tau_t \in \mathrm{T}(\tilde{z}_{t-1})} \{V^{Mr}(\tilde{z}_{t-1},\tau_t) - V^{Mk}(\tilde{z}_{t-1},\tau_t)\}. \tag{3.16}$$

In words, the rule is implemented at $\tilde{z}_{t-1}$ if the degree of commitment is high enough to eliminate all fiscal incentives to challenge the rule and monetary incentives to renounce the rule.

We define the credibility cut-off $\bar{\kappa}(z_{-1})$ as the minimum degree of commitment that supports the credible implementation of the monetary rule.

Definition 5. *Given initial liabilities z_{-1}, the credibility cut-off of the monetary rule ϱ^{Mk} is defined as:*

$$\bar{\kappa}(z_{-1}) = \min\{\kappa \mid \kappa \geq \Delta(\tilde{z}_{t-1}) \ \forall t \geq 0\}. \tag{3.17}$$

Note that the evaluation of credibility involves off equilibrium paths, hence $\Delta = \max_{\{\tilde{z}_{t-1}\}_{t=1}^{\infty}} \Delta(\tilde{z}_{t-1})$ might be a function of the degree of commitment κ.[32]

[32] If it is the case, provided the dependence is monotone, the credibility cut-off $\bar{\kappa}$ is a fixed point of $\Delta(\kappa)$. Otherwise, the credibility cut-off is simply equal to Δ.

3.4 Standard and Strategic Monetary Rules

As in Section 2, we contrast the equilibrium implications of two classes of monetary rules.

3.4.1 Standard Monetary Rule

A standard rule prescribes the central bank to set the growth rate of money unconditionally according to a prespecified path $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$:

$$\varrho_t^{Mk}(z_{t-1}, \tau_t) = \tilde{\sigma}_t \geq \beta - 1, \ \forall t \geq 0. \tag{3.18}$$

Consider first a standard rule that prescribes a constant path of money growth rates set according to the Friedman rule. We establish a connection between this rule and fiscal decisions to pursue tax smoothing.

Lemma 4. *Let κ be arbitrarily large. If the central bank commits to a standard rule with constant money growth rate $\tilde{\sigma}_t = \beta - 1$, then the policy game leads to an equilibrium outcome with the constant tax rate $\tilde{\tau}_t = \tau^*(\tilde{z}_{t-1})$ and debt $\tilde{z}_{t-1}$ sustained at the initial outstanding level for all t.*

Proof. See Appendix B.4 ∎

The credible commitment of the central bank to keep the money growth rate constant induces tax smoothing on the part of fiscal authority.[33] While the fiscal authority lacks commitment and makes decisions sequentially, it does not lower the tax rate to influence the price of newly issued bonds. Indeed, a tax cut would lead to a decline in the price level, as follows from Lemma 1. In turn, the real value of outstanding nominal debt would then increase and offset the beneficial impact of a temporarily low tax rate on the price of newly issued debt. Hence, the possible revaluation of outstanding nominal debt under the standard rule eliminates fiscal incentives to deviate from tax smoothing.

Consider now a standard rule that prescribes the path of money growth rates as in the Ramsey plan, with an initial rate $\tilde{\sigma}_0 = \sigma_0^* > \beta - 1$ followed by the constant tail rate $\tilde{\sigma}_t = \beta - 1$ for $t > 0$. As follows from Lemma 4, a credible monetary commitment to the constant tail rate induces a constant path of taxes, as in the Ramsey tail plan. Paired with the optimal initial money growth rate, such a standard rule induces an overall dynamic discretionary fiscal response consistent with the fiscal part of the Ramsey plan under commitment: $\tilde{\tau}_0 = \tau_0^*$ and $\tilde{\tau}_t = \tau^*(\tilde{z}_{t-1})$ with $\tilde{z}_{t-1} = z_0^*$ for $t > 0$.

[33]The result in Lemma 4 holds more generally: any credible constant money growth rate induces a path of constant tax rates.

Proposition 6. *Let κ be arbitrarily large. Given z_{-1}, if the central bank commits to the standard rule that targets the path of money growth rates prescribed by the Ramsey plan, then the equilibrium outcome of the policy game coincides with the Ramsey plan.*

Proof. See Appendix B.4 ∎

This proposition shows that a standard monetary rule can support the optimal economic outcome even when the fiscal authority lacks commitment. Hence, a credible standard rule makes the central bank effectively share its commitment with the fiscal authority. Importantly, these results hinge on debt being nominal and sensitive to variations in the price level, and not indexed to inflation.

Naturally, following the standard rule requires a high enough degree of monetary commitment to resist fiscal influence. Indeed, when $z_{-1} > 0$, the fiscal authority without commitment has sequential incentives to deviate from the policy prescribed by the Ramsey plan if it were to induce the central bank to renege its rule. Next, we consider strategic rules designed to also induce the Ramsey plan but with the additional objective to minimize the required degree of commitment.

3.4.2 Strategic Monetary Rule

Differently from standard rules, strategic rules allow to set monetary policy conditional on fiscal decisions. As in Section 2.4.2, these rules target optimal policy plans, and appropriate off-equilibrium threats are designed to discourage discretionary attempts of fiscal influence.

As with standard rules, we proceed in two steps. First, we consider strategic rules designed to implement a stationary equilibrium outcome that mimics the tail segment of a Ramsey plan. Starting at any z_{t-1}, as long as fiscal policy is set to smooth taxes, the monetary rule prescribes to set the money growth rate according to the Friedman rule:

(p.1) if $\tau_t = \tau^*(z_{t-1})$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \beta - 1, \quad \forall t \geq 0.$

If fiscal policy is set according to $\tau^*(z_{t-1})$ and the central bank keeps its promise, the level of debt remains unchanged. Tax rates are then constant over time because, when faced with the same state next period, the fiscal authority would set the same tax rate. Thus, the associated values to both policy authorities, $V^{Fk}(z_{t-1}, \tau^*(z_{t-1}))$ and $V^{Mk}(z_{t-1}, \tau^*(z_{t-1}))$, coincide with the welfare under the tail segment of a Ramsey plan, $W^{ta}(z_{t-1})$.

Additionally, the central bank designs threats to discourage fiscal deviations from tax smoothing. As highlighted in Section 2, the credibility of a rule is related to the strength of these threats. To minimize the associated credibility cut-off, the central bank calibrates its off-equilibrium reaction to make the fiscal incentive constraint binding for all deviations from $\tau^*(z)$:

(p.2) if $\tau_t \neq \tau^*(z_{t-1})$ and $\tau_t \in \mathrm{T}(z_{t-1})$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(z_{t-1}, \tau_t) = W^{ta}(z_{t-1}), \tag{3.19}$$

where the set of fiscal influence $\mathrm{T}(z_{t-1})$ is formally defined as:

$$\mathrm{T}(z_{t-1}) = \{\tau_t \; | V^{Fr}(z_{t-1}, \tau_t) \geq V^{Fk}(z_{t-1}, \tau^*(z_{t-1})) = W^{ta}(z_{t-1})\}. \tag{3.20}$$

If such a strategic rule is credible, it leads to a stationary equilibrium outcome with a constant growth rate of money $\tilde{\sigma_t} = \beta - 1$, a constant tax rate $\tilde{\tau}_t = \tau^*(\tilde{z}_{t-1})$, and debt $\tilde{z}_{t-1}$ sustained at the initial level for all t.

Lemma 5. *For a given initial outstanding debt level z, the credibility cut-off of a strategic rule that satisfies (p.1) and (p.2) to implement a stationary equilibrium is characterized as follows:*

$$\bar{\kappa}^{ta}(z) = \underbrace{\max_{\tau_t, \sigma_t, z_t} U(\tau_t, \sigma_t) + \beta W^{ta}(z_t)}_{=W^{rp}(z)} - W^{ta}(z), \tag{3.21}$$

where maximization over τ_t, σ_t, and z_t is subject to the implementability constraint (3.7) *with $z_{t-1} = z$ and $\sigma_{t+1} = \beta - 1$. In addition,* $\frac{d\bar{\kappa}^{ta}(z)}{dz} > 0$.

Proof. See Appendix B.5. ∎

This characterization relies on welfare properties of the off-equilibrium paths that feature attempts of fiscal influence. The central bank's value $V^{Mk}(z, \tau_t)$ of following the rule in response to fiscal influence $\tau_t \in \mathrm{T}(z)$ equals the welfare associated with the stationary equilibrium outcome, $W^{ta}(z)$. Hence, the attempt of fiscal influence that seeks to induce the most profitable monetary deviation from the rule is the one that pins down the credibility cut-off.[34] This deviation coincides with the one that would occur under policy cooperation because the incentives of the two authorities are aligned conditional on monetary deviation from the rule. The central bank's value $V^{Mr}(z, \tau_t)$ along this specific off-equilibrium path is hence equal to the welfare associated with a "reset" Ramsey plan, i.e., $W^{rp}(z)$.[35]

Consider next a similar strategic rule, adjusted to induce the Ramsey plan as an equilibrium outcome. To this end, the promise to follow the Friedman rule conditional on tax smoothing (p.1) and the associated threats (p.2) are imposed only starting from period $t = 1$. In period $t = 0$,

[34] As highlighted in Section 2.4.2, this characterization is inherent in the strategic rule but does not hold in general for other rules.

[35] An important factor behind this result is that the continuation debt issued under the Ramsey plan is lower than the initial outstanding debt (see Proposition 5). It implies that the associated continuation welfare of the reset Ramsey plan coincides with the continuation values of the most profitable deviation in the policy game.

given initial outstanding debt z_{-1}, the rule prescribes to set the money growth rate at the initial value prescribed by the Ramsey plan when the fiscal policy is set accordingly:

(p.1') if $\tau_0 = \tau_0^*$, then $\varrho_0^{Mk}(z_{-1}, \tau_0) = \sigma_0^*$.

Under this rule, the fiscal authority finds it optimal to set the initial tax rate at τ_0^* if the degree of monetary commitment is high enough to support the Ramsey tail plan as the continuation path. In this case, there is no scope for fiscal influence at $t = 0$.[36]

Proposition 7. *Given z_{-1}, the credibility cut-off of a strategic rule that implements the Ramsey plan as equilibrium outcome of the game is:*

$$\bar{\kappa}_2(z_{-1}) = \bar{\kappa}^{ta}(z_0^*), \tag{3.22}$$

where $z_0^* < z_{-1}$ is the tail level of debt of the Ramsey plan, and $\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} > 0$.

Proof. See Appendix B.5. ∎

The credibility cut-off of the strategic rule that implements the Ramsey plan is tied to the credibility of supporting the associated tail allocation from $t = 1$ onward. The higher the initial outstanding level of debt, the larger the tail level of debt, which, in turn, increases the relative gains from renouncing the rule and makes the credibility cut-off higher. Without debt, there are no discretionary incentives to manipulate the interest rate or generate excessive inflation, hence $\bar{\kappa}_2(0) = 0$.

Finally, note that the credibility cut-off of the strategic rule is unambiguously lower than under the standard rule since both rules implement the same equilibrium outcome. This relative ranking contrasts with the general result from Section 2 and is due to the endogenous revaluation of nominal debt under monetary-fiscal interactions. This knife-edge result makes it possible to isolate the credibility-enhancing role of monetary conditionality from its contribution to improve equilibrium outcomes. Next, we study additional factors affecting credibility of strategic rules.

4 Credibility of Strategic Rule: Additional Factors

4.1 Debt characteristics

The analysis above studies strategic monetary-fiscal interactions driven by discretionary incentives to reduce the burden of government debt issued as one-period nominal bonds. Here we additionally discuss how the characteristics of government debt affect these incentives and the credibility

[36]Hence, one can specify any fiscal conditionality for $t = 0$, but it wouldn't affect the credibility cut-off since the corresponding set of fiscal influence is degenerate.

of strategic monetary rules. Indeed, the ability to inflate away outstanding debt is weaker the more debt is indexed to inflation, while the ability to manipulate the price of newly issued debt is decreasing as debt maturity increases, as studied in Debortoli et al. (2017). In the special case of fully indexed debt and long-term consol bonds, we get the following result.

Proposition 8. *The Ramsey policy plan is time consistent if government issues debt in the form of real consol bonds.*

Proof. See Appendix C.1. ∎

Note that this result resembles the limit case of the static model in Section 1 with parameters $\alpha = \gamma = 0$, where optimal policy is time consistent. This result also points to a more general dependence of the credibility cut-off of the strategic rule on debt indexation and debt maturity. Approaching the limit case with real consol bonds, credibility of the strategic rule goes to zero. Cases with partial debt indexation and shorter debt maturity have credibility cut-offs in between the real consol bonds and nominal one-period bonds. The qualitative mapping between these cases and the static model extends with $\alpha > 0$ corresponding to indexation and $\gamma > 0$ corresponding to maturity.

4.2 Fiscal Hedging

The seminal analysis in Lucas and Stokey (1983) highlights the benefits from smoothing taxes, not only across time, but also across states of the economy. Government budget can be hedged against the need to adjust tax rates in response to economic shocks by appropriately structuring public debt. To study the effects of fiscal hedging on the credibility of the strategic monetary rule, we follow Lucas and Stokey (1983) by introducing variation in public consumption and complete markets for state-contingent government debt. We summarize the key results below and provide a detailed analysis in Appendix C.2.

Let public consumption g vary exogenously according to a discrete stochastic process with a finite number N of possible states, each denoted $\hat{g}_n$. Each period t, the representative household trades with the government a complete set of N one-period nominal state-contingent bonds that pay one unit of account in the next period only if $g_{t+1} = \hat{g}_n$. We define $g^t \equiv \{g_0, g_1, \dots, g_t\}$ to be a history of public consumption realizations up until period t. Conditional on a given history, let $G_t \equiv (1-\beta)\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s g_{t+s}$ be the expected weighted average of public consumption.

In this environment, some key properties of the Ramsey plan remain unchanged. As in the case with constant g, it consists of initial and tail policy segments, where the latter prescribes to set a constant tax rate τ_t^* and the money growth rate that satisfies the Friedman rule $\sigma_t^* = \beta - 1$

for all $t \geq 1$.[37] Additionally, the Ramsey path of policy instruments (but not debt) and implied welfare are the same as in a doppelgänger economy with constant g equal to G_0.

The Ramsey tail is supported by a debt structure that hedges government budget against changes in spending and tax revenue.[38] Debt issuance is structured to make outstanding liabilities of the government high during times of low public consumption, and vice versa. In particular, let $z^*_{\max}$ denote the highest outstanding level of debt at any history of the Ramsey tail plan. The histories underlying $z^*_{\max}$ are characterized by having the lowest expected weighted average of public consumption denoted as $G_{\min}$.

Our focus is on highlighting the role of variation in public consumption for the credibility of a strategic monetary rule that implements a Ramsey policy plan. In such an environment, the promise to follow the Friedman rule conditional on tax smoothing and the associated off-equilibrium threats have to be imposed for all histories g^t. As before, the credibility of the strategic rule is tied to the credibility of supporting the Ramsey tail plan. Using policy-equivalence with an economy under constant public consumption, we characterize the continuation welfare associated with the Ramsey tail as $W^{ta}(\bar{z}^*_0, G_0)$, where $\bar{z}^*_0$ is the tail level of debt that would prevail if g_t were to remain permanently at G_0.

The relative gains from renouncing the rule vary with the level of debt. The most profitable deviation is associated with the histories when outstanding debt is at its highest, $z_{\max}$, and the expected flow of public consumption is at its lowest, $G_{\min}$. This is the state that requires the highest degree of commitment for the strategic rule to be credible. The corresponding deviation is characterized by a "reset" Ramsey plan that implies the same welfare $W^{rp}(z^*_{\max}, G_{\min})$ as the Ramsey plan in the economy with initial debt equal to $z^*_{\max}$ and constant public consumption equal to $G_{\min}$.

Proposition 9. *Given z_{-1} and the process for g_t, the credibility cut-off of a strategic rule that implements the Ramsey plan as equilibrium outcome of the game is:*

$$\bar{\kappa}_2 = W^{rp}(z^*_{\max}, G_{\min}) - W^{ta}(\bar{z}^*_0, G_0). \tag{4.1}$$

Proof. See Appendix C.2. ∎

The following examples illustrate practical aspects of fiscal policy in a way similar to Lucas and Stokey (1983). First, consider infrequent episodes of large public consumption that can be interpreted as wars (or pandemics). In the case of a perfectly foreseen war with $g_T > 0$ and $g_t = 0$ for $t \neq T$, the weighted average of public consumption G_t rises until reaching its

[37]Constant policy instruments imply constant levels of private consumption of both cash and credit goods.
[38]Tax revenue variation is due to changes in tax base driven by labor responding to changes in public consumption.

maximum in period T and then, one period after, it permanently falls to the minimum level. Hence, the aftermath of war at $T+1$ features the largest outstanding level of debt $z^*_{\max}$. In turn, this brings the relative gains from renouncing the rule to the highest level and sets the cut-off for the degree of commitment required to ensure the monetary rule is credible. This conclusion generalizes to the cases with perfectly foreseen cyclical wars and wars with uncertain occurrence or duration. In these cases, too, the maximum outstanding level of debt is reached during periods that immediately follow the war episodes, and the corresponding histories determine the credibility cut-off of the strategic monetary rule.

Second, consider small and frequent fluctuations in public consumption that can be thought of as corresponding to business cycles. A simple case is to let g_t follow an independently and identically distributed process. Realizations of the smallest state among $\hat{g}_n$ are the ones that correspond to $G_{\min}$ and $z^*_{\max}$. This result holds more generally if g_t follows a Markov process with persistence, such that the expected value of next-period public consumption is increasing in the current realization.[39] Hence, if public consumption is countercyclical, sustaining the rule requires the highest degree of commitment at business cycles peaks with large outstanding debt.

4.3 Money Growth Rate

Consider the baseline economy with nominal one-period debt and no variation in public consumption, where the degree of commitment κ is lower than the credibility cut-off required to sustain the Ramsey plan. Can the central bank adjust the strategic rule to enhance its credibility? Given that the credibility of the strategic rule is tied to the credibility of supporting a stationary tail policy plan, in this section we focus on the effect of adjusting the targeted equilibrium tail money growth rate (with additional details on this adjustment in Appendix C.3).

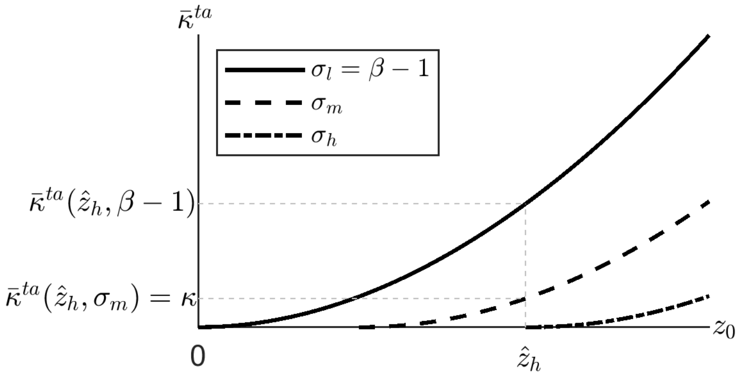
A specific tail level of debt, $\hat{z}_h$, can be sustained permanently by different equilibrium pairs of the monetary and fiscal instruments. On the one hand, there is the Ramsey pair with the money growth rate set according to the Friedman rule, $\beta - 1$, and the associated tax rate. This combination maximizes the economic value but, as discussed above, may lack credibility due to a low degree of commitment, $\kappa < \bar{\kappa}^{ta}(\hat{z}_h, \beta - 1)$. On the other hand, there is a 'break-even' pair, with $\sigma_h > \beta - 1$, that doesn't require any commitment because its associated relative gains from reneging are nil. While minimizing the credibility cut-off, the 'break-even' policy mix sacrifices economic value. When choosing policy targets for the adjusted strategic rule, it is then optimal to pick the intermediate case, with $\sigma_m \in (\beta - 1, \sigma_h)$, that balances economic loss

[39]Markov processes with monotone cumulative transition distributions exhibit this property. Conclusions are more nuanced when g_t follows a Markov process with mean-reversion such that low current values of public consumption lead to high expected values, and vice versa.

against credibility considerations. A graphic illustration is provided in Figure 3.

This analysis of the trade-off between equilibrium losses and credible provision of incentives echoes our characterization of the commitment-constrained strategic rule in Section 2. While the strategic rule with the adjusted growth rate of money may not minimize the economic loss given κ, its appeal is in mapping well into a policy discussion about the selection of an inflation target: a higher than optimal level of inflation enhances the credibility of monetary pledge against fiscal influence.[40]

Figure 3: Credibility of the Strategic Rule: Effect of the Tail Money Growth Rate



This figure displays tail credibility cut-offs $\bar{\kappa}^{ta}(z_0, \sigma)$ for strategic rules under three different tail equilibrium money growth rates $\sigma_l = (\beta - 1) < \sigma_m < \sigma_h$. The graph is based on numerical simulations using the following illustrative calibration. The economic parameters are set to target moments of the first best allocation, which is the solution to the maximization of (3.4) subject to the sequence of resource constraints (3.1). The implied moments are $g/(c+d) = 0.25$, the fraction of time devoted to leisure $l = 0.75$ and an equal consumption of cash and credit good. The resulting values are $\gamma = 5$, $\alpha = 0.5$, and $g = 0.05$; also we set $\beta = 0.96$. The values of the tail money growth rates σ_m and σ_h are set equal to 0 and $\beta^{-1} - 1$ respectively.

5 Conclusions

Should a central bank lean against fiscal influence? Does it compromise its capacity to ensure price stability? We show that a *strategic* monetary rule that includes explicit fiscal conditionality has several benefits. First, the central bank can rely on its commitment technology to stir the economy toward better economic outcomes. Second, this rule does not necessarily require a higher degree of commitment to be implemented than the standard rule without conditionality. In particular, the strategic rule is relatively more credible when the threat of fiscal influence is the highest. This is particularly reflected in the dependence of the credibility of the strategic rule on the level and structure of public debt.

Our economy with a representative agent abstracts from the distributive consequences of monetary-fiscal interactions. Introducing heterogeneous agents is an important direction for future research that would also make it possible to account for political economy considerations.

[40] More generally, one could show that it is possible to design a strategic rule that supports a benchmark equilibrium under cooperation with symmetric but limited degree of commitment κ, as in Section 2. However, finding this benchmark is more challenging in the current dynamic environment with the endogenous state variable.

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Appendix - For Online Publication Only

A Linear-Quadratic Framework

A.1 Policy Choices under Cooperation - Section 2.2

Under cooperation, a single benevolent authority decides on policy instruments. Under commitment, the government sets (x, π) before private agents form expectations $e = x^e + \alpha\pi^e$. The objective is to minimize the welfare loss (2.2), subject to the Phillips curve (2.1) and private agents' expectations (2.3). Substituting the constraints into the loss function yields:

$$\min_{x,\pi} \frac{1}{2}\Big[(x - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(0 - y^*)^2\Big]. \tag{A.1}$$

The first-order conditions naturally lead to $x^c = x^*$ and $\pi^c = \pi^*$.

When the government does not have a commitment technology, it takes policy decisions after private agents formed expectations $e = x^e + \alpha\pi^e$. The objective is to minimize the welfare loss (2.2), subject to the Phillips curve (2.1), given e:

$$\min_{x,\pi} \frac{1}{2}\Big[(x - x^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma\big(x - x^e + \alpha(\pi - \pi^e) - y^*\big)^2\Big]. \tag{A.2}$$

The first-order conditions give policy reactions to private agents' expectations:

$$x - x^* + \gamma\big(x - x^e + \alpha(\pi - \pi^e) - y^*\big) = 0 \tag{A.3}$$

$$\lambda(\pi - \pi^*) + \gamma\alpha\big(x - x^e + \alpha(\pi - \pi^e) - y^*\big) = 0 \tag{A.4}$$

In equilibrium, private agents hold rational expectations (2.3), which yields $x^d = x^* + \gamma y^*$ and $\pi^d = \pi^* + \frac{\gamma\alpha}{\lambda} y^*$.

A.2 Standard Monetary Rule - Section 2.4.1

Equilibrium outcome. Consider the case where the central bank degree of commitment is arbitrarily large: $\pi_1 = \pi^e = \pi^*$. The fiscal authority solves (2.7):

$$\tilde{x} = \text{argmin}_x\ \mathcal{L}(e, x, \pi^*), \tag{A.5}$$

subject to (2.1). The first-order condition yields:

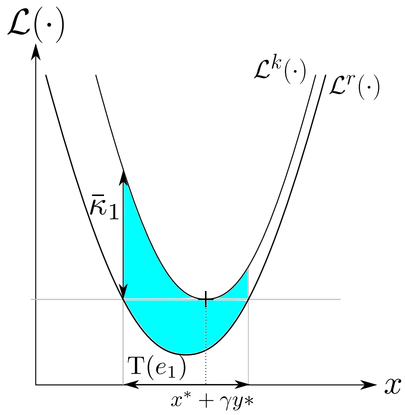
$$x - x^* + \gamma(x - x^e - y^*) = 0. \tag{A.6}$$

In equilibrium, $x^e = x$, which gives $x_1 = x^* + \gamma y^*$ and $e_1 = e^* + \gamma y^*$. The loss in equilibrium is then:

$$\mathcal{L}(e_1, x^* + \gamma y^*, \pi^*) = \frac{\gamma(1+\gamma)}{2}(y^*)^2. \tag{A.7}$$

Proposition 1. Given $e_1 = e^* + \gamma y^*$, we characterize $\pi^r(e_1, x)$, the set of fiscal influence $\mathrm{T}(e_1)$, and then derive the credibility cut-off $\bar{\kappa}_1$ to sustain the standard rule π^* against all attempts of fiscal influence $x \in \mathrm{T}(e_1)$. The proof is supported by Figure 4.

Figure 4: Credibility Cut-off under Standard Rule



This figure illustrates the characterization of credibility cut-off $\bar{\kappa}_1$, when the central bank follows a standard inflation target rule. Given the sequential nature of the game, credibility is evaluated at fiscal choices within the set of influence $\mathrm{T}(e_1)$, represented by the blue area. The credibility cut-off $\bar{\kappa}_1$ is then given by the maximum distance within that set between $\mathcal{L}^k(\cdot)$ and $\mathcal{L}^r(\cdot)$, corresponding respectively to the cases where the central bank *keeps* its promise (2.10) or *reneges* and implements $\pi^r(\mathcal{S})$.

1. Monetary reoptimization. Given (e, x) the central bank that reneges the rule implements $\pi^r(e, x) = \mathrm{argmin}_\pi \ \mathcal{L}^{m,r}(e, x, \pi)$. Simple computations lead to:

$$\pi^r(\mathcal{S}) = \frac{\lambda \pi^* + \gamma\alpha(y^* + e - x)}{\lambda + \gamma\alpha^2}. \tag{A.8}$$

2. Set of fiscal influence. Using (2.8) evaluated at (2.11):

$$\mathrm{T}(e_1) = \{x \quad | \quad \mathcal{L}^{f,r}(e_1, x, \pi^r(\mathcal{S})) \leq \mathcal{L}^{f,k}(e_1, x_1, \pi^*)\}, \tag{A.9}$$

where $\pi^r(\mathcal{S})$ is given by (A.8) evaluated at $e_1 = e^* + \gamma y^*$:

$$\pi^r(e_1, x) = \pi^* + \frac{\gamma\alpha}{\lambda + \gamma\alpha^2}((1+\gamma)y^* + x^* - x). \tag{A.10}$$

Since the loss function is quadratic, get $\mathrm{T}(e_1) = [x_l, x_h]$, where x_i are the solutions to:

$$\mathcal{L}(e_1, x, \pi^r(e_1, x)) = \frac{\gamma(1+\gamma)}{2}(y^*)^2. \tag{A.11}$$

Rewrite this expression as:

$$(x - x^*)^2 + \frac{\gamma\lambda}{\lambda + \gamma\alpha^2}\left(x - x^* - (1+\gamma)y^*\right)^2 = \gamma(1+\gamma)(y^*)^2, \tag{A.12}$$

and get

$$x_i - x^* = y^* \frac{\eta(1+\gamma) \pm \sqrt{(1+\gamma)(\gamma - \eta)}}{1 + \eta}, \tag{A.13}$$

where $\eta = \frac{\gamma\lambda}{\lambda + \gamma\alpha^2}$.

3. Credibility cut-off. Given $e_1 = e^* + \gamma y^*$ and applying Definition 2, the standard rule is credible if and only if:

$$\kappa \geq \max_{x \in \mathrm{T}(e_1)} \bar{\kappa}(x) = \mathcal{L}^{m,k}(e_1, x, \pi^*) - \mathcal{L}^{m,r}(e_1, x, \pi^r(e_1, x)). \tag{A.14}$$

Note that $\bar{\kappa}(x)$ is a second-order positive polynomial, so its highest value is reached for either x_l or x_h. As $\bar{\kappa}(x)$ is minimum for $x = x^* + (1+\gamma)y^*$ and that $[x_l, x_h]$ is centered on $x = x^* + (1+\gamma)y^* \frac{\gamma\lambda}{\lambda+\gamma\alpha^2+\gamma\lambda}$, with $\frac{\gamma\lambda}{\lambda+\gamma\alpha^2+\gamma\lambda} < 1$, we get $\bar{\kappa}(x_l) > \bar{\kappa}(x_h)$. Accordingly,

$$\bar{\kappa}_1 = \bar{\kappa}(x_l) = \mathcal{L}^k(e_1, x_l, \pi^*) - \mathcal{L}^r(e_1, x_l, \pi^d(e_1, x_l)). \tag{A.15}$$

Computations lead to:

$$\bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1+\gamma)\Big(\frac{\sqrt{1+\gamma} + \sqrt{\gamma - \eta}}{1+\eta}\Big)^2. \tag{A.16}$$

4. Comparative statics. From (A.16), derive:

$$\frac{d\bar{\kappa}_1}{d\eta} = -\frac{(y^*)^2}{2}(1+\gamma)\Big[\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta}\Big]\Big[\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta} + 2\frac{\gamma-\eta}{(1+\eta)^2}\Big(\frac{1}{2}\frac{1+\eta}{\sqrt{\gamma-\eta}} + \sqrt{1+\gamma} + \sqrt{\gamma-\eta}\Big)\Big], \tag{A.17}$$

which is unambiguously negative. Together with

$$\frac{d\eta}{d\lambda} = \frac{(\gamma\alpha)^2}{(\lambda+\gamma\alpha^2)^2} > 0 \qquad\qquad \frac{d\eta}{d\alpha} = -\frac{2\alpha\gamma^2\lambda}{(\lambda+\gamma\alpha^2)^2} < 0, \tag{A.18}$$

we get

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \qquad\qquad \frac{d\bar{\kappa}_1}{d\alpha} > 0. \tag{A.19}$$

A.3 Strategic Rule - Section 2.4.2

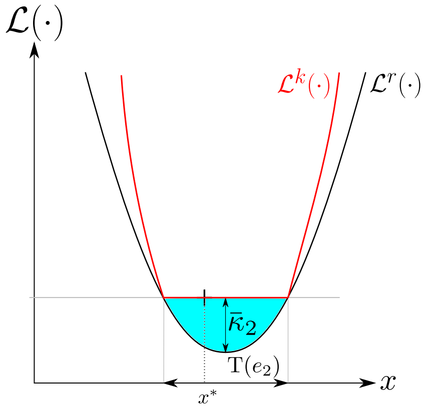
Proposition 2. The proof is supported by Figure 5.

1. Credibility cut-off. A strategic rule $\pi^k(\mathcal{S})$ implements (2.12) if credible threats satisfy (2.13). To minimize the credibility cut-off, these threats must be tailored to minimize the relative gains to reoptimize, i.e., to minimize $\mathcal{L}^{m,k}(e_2, x, \pi^k(\mathcal{S})) - \mathcal{L}^{m,r}(e_2, x, \pi^r(\mathcal{S}))$ for all $x \in \mathrm{T}(e_2)$. As illustrated in Figure 5, this is achieved if $\pi^k(\mathcal{S})$ induces an equilibrium outcome that yields the lowest loss while preserving fiscal incentives (2.13), i.e.,

$$\mathcal{L}^{m,k}(e_2, x, \pi^k(\mathcal{S})) = \mathcal{L}^{m,k}(e^*, x^*, \pi^*) \forall\ x \in \mathrm{T}(e^*). \tag{A.20}$$

To derive the credibility cut-off, one must identify the off-equilibrium path that then maximizes the relative gains to reoptimize. Given that the loss conditional on *keep* is constant for all attempts of fiscal influence, this path is given by the attempt of fiscal influence that then minimizes the loss to the monetary authority conditional on *renege*. As monetary and fiscal authorities share the same loss function, this path is one that minimizes the loss to both

Figure 5: Credibility Cut-off under Strategic Rule



This figure represents the construction of a strategic monetary rule $\pi^k(\mathcal{S})$ supporting the equilibrium outcome (2.12) with a minimum degree of commitment. To induce the fiscal authority to implement x^*, the central bank designs its rule so that the loss function for all x is higher evaluated at $\big(e_2, x, \pi^k(\mathcal{S})\big)$ than at the desirable equilibrium outcome (e_2, x^*, π^*). To minimize the degree of commitment required to support this rule, monetary interventions offset the welfare implications of fiscal deviations, within the relevant set $\mathrm{T}(e_2)$ – in blue on the figure.

authorities, i.e.,

$$\bar{\kappa}_2 = \mathcal{L}^{m,k}(e_2, x^*, \pi^*) - \min_{x,\pi} \mathcal{L}^{m,r}(e_2, x, \pi), \tag{A.21}$$

with $e_2 = e^* = x^* + \alpha\pi^*$. We get $\mathcal{L}^k(e_2, x^*, \pi^*) = \frac{\gamma(y^*)^2}{2}$.

Consider the optimization program $\min_{x,\pi} \mathcal{L}^r(e_2, x, \pi)$. The first-order conditions are:

$$(x - x^*) + \gamma\big(x - x^* + \alpha(\pi - \pi^*) - y^*\big) = 0, \tag{A.22}$$

$$\lambda(\pi - \pi^*) + \gamma\alpha\big(x - x^* + \alpha(\pi - \pi^*) - y^*\big) = 0. \tag{A.23}$$

Solving this system with unknown $x - x^*$ and $\pi - \pi^*$ leads to the following solution:

$$x - x^* = \frac{\gamma\lambda y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma} \qquad \text{and} \qquad \pi - \pi^* = \frac{\gamma\alpha y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \tag{A.24}$$

Evaluating the loss function at this policy outcome:

$$\mathcal{L}^r(\cdot) = \frac{1}{2}\frac{\lambda\gamma(y^*)^2}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \tag{A.25}$$

Altogether, we get:

$$\bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2}\frac{\gamma(\lambda + \alpha^2)}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \tag{A.26}$$

2. Comparative statics. From (A.26), derive:

$$\frac{d\bar{\kappa}_2}{d\lambda} = -\frac{\gamma(y^*)^2}{2}\frac{\gamma\alpha^2}{(\lambda + \gamma\alpha^2 + \lambda\gamma)^2} < 0 \qquad ; \qquad \frac{d\bar{\kappa}_2}{d\alpha} = \frac{\gamma(y^*)^2}{2}\frac{2\alpha\lambda\gamma}{(\lambda + \gamma\alpha^2 + \lambda\gamma)^2} > 0. \tag{A.27}$$

Proposition 3. The objective is to derive conditions on parameters (α, λ), such that $\frac{\bar{\kappa}_1}{\bar{\kappa}_2} > 1$, where $\bar{\kappa}_1$ is given by (A.16) and $\bar{\kappa}_2$ in (A.26). Note $f(\alpha, \lambda) = \frac{\bar{\kappa}_1}{\bar{\kappa}_2}$ and derive:

$$f(\alpha, \lambda) = \frac{\alpha^2(1+\gamma)(\lambda+\gamma\alpha^2)}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}\left(\sqrt{1+\gamma}+\sqrt{\gamma-\eta}\right)^2, \tag{A.28}$$

with $\eta = \frac{\gamma\lambda}{\lambda+\gamma\alpha^2}$. Expand the quadratic factor:

$$\left(\sqrt{1+\gamma}+\sqrt{\gamma-\eta}\right)^2 = \frac{\lambda+\gamma\lambda+\gamma\alpha^2}{\lambda+\gamma\alpha^2} + \frac{2(\gamma\alpha)^2}{\lambda+\gamma\alpha^2} + 2\gamma\alpha\sqrt{\frac{1+\gamma}{\lambda+\gamma\alpha^2}}, \tag{A.29}$$

and get:

$$f(\alpha, \lambda) = \frac{\alpha^2(1+\gamma)}{\lambda+\alpha^2} + \frac{2\gamma^2(1+\gamma)\alpha^4}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)} + \frac{2\gamma(1+\gamma)^{\frac{3}{2}}\alpha^3\sqrt{\lambda+\gamma\alpha^2}}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}. \tag{A.30}$$

We immediately get:

$$f(0, \lambda) = 0 \qquad \lim_{\alpha \to +\infty} f(\alpha, \lambda) > 1+\gamma \tag{A.31}$$

$$f(\alpha, 0) > 1+\gamma \qquad \lim_{\lambda \to +\infty} f(\alpha, \lambda) = 0 \tag{A.32}$$

Note $g(\alpha, \lambda) = \frac{\alpha^2}{\lambda+\alpha^2}$. This function is increasing in α and decreasing in λ. Note $h(\alpha, \lambda) = \frac{\alpha^4}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}$. This function is decreasing in λ. Further,

$$\frac{dh(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{A.33}$$

with

$$N = \alpha^4 \qquad \frac{dN}{d\alpha} = 4\alpha^3 \tag{A.34}$$

$$D = (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) \qquad \frac{dD}{d\alpha} = 2\alpha(\lambda+2\gamma\alpha^2+2\lambda\gamma) \tag{A.35}$$

$\frac{dh(\cdot)}{d\alpha}$ has the sign of H:

$$H = \frac{1}{2\alpha^3}\left(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\right) \tag{A.36}$$

$$= 2(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) - \alpha^2(\lambda+2\gamma\alpha^2+2\lambda\gamma) \tag{A.37}$$

$$= 2\lambda^2 + \lambda\alpha^2 + \lambda^2\gamma + 2\lambda\gamma\alpha^2 > 0 \tag{A.38}$$

Note $k(\alpha, \lambda) = \frac{\alpha^2\sqrt{\lambda+\gamma\alpha^2}}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}$. Deriving the monotonicity properties:

$$\frac{dk(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{A.39}$$

with

$$N = \alpha^3(\lambda+\gamma\alpha^2)^{\frac{1}{2}} \qquad \frac{dN}{d\alpha} = \alpha^2(\lambda+\gamma\alpha^2)^{-\frac{1}{2}}(3\lambda+3\gamma\alpha^2) \tag{A.40}$$

$$D = (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) \qquad \frac{dD}{d\alpha} = 2\alpha(\lambda+2\gamma\lambda+2\gamma\alpha^2) \tag{A.41}$$

$\frac{dk(\cdot)}{d\alpha}$ has the sign of K:

$$K = \frac{(\lambda+\gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^2}\Big(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\Big) \tag{A.42}$$

$$= (3\lambda+4\gamma\alpha^2)(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) - 2\alpha^2(2\gamma\lambda+2\gamma\alpha^2+\lambda)(\lambda+\gamma\alpha^2) \tag{A.43}$$

$$= \lambda^2\alpha^2 + \gamma\lambda\alpha^{+}4\lambda\gamma^2\alpha^4 + \lambda^2(3\lambda+4\gamma\alpha^2) + 3\lambda^3\gamma + 6\lambda^2\gamma\alpha^2 > 0 \tag{A.44}$$

Overall,

$$\frac{df(\cdot)}{d\alpha} = (1+\gamma)\frac{dg(\cdot)}{d\alpha} + 2\gamma^2(1+\gamma)\frac{dh(\cdot)}{d\alpha} + 2\gamma(1+\gamma)^{\frac{3}{2}}\frac{dk(\cdot)}{d\alpha} > 0, \tag{A.45}$$

which together with (A.31) gives:

$$\forall\lambda > 0\ \exists!\bar{\alpha} > 0 \text{ s.t. } \alpha > \bar{\alpha} \Leftrightarrow \bar{\kappa}_1 > \bar{\kappa}_2. \tag{A.46}$$

Similarly,

$$\frac{dk(\cdot)}{d\lambda} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{A.47}$$

with

$$N = \alpha^3(\lambda+\gamma\alpha^2)^{\frac{1}{2}} \qquad \frac{dN}{d\lambda} = \frac{\alpha^3}{2}(\lambda+\gamma\alpha^2)^{-\frac{1}{2}} \tag{A.48}$$

$$D = (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) \qquad \frac{dD}{d\lambda} = \alpha^2 + 2(\lambda+\lambda\gamma+\gamma\alpha^2). \tag{A.49}$$

$\frac{dk(\cdot)}{d\lambda}$ has the sign of K:

$$K = \frac{2(\lambda+\gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^3}\Big(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\Big) \tag{A.50}$$

$$= (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) - 2\big(\alpha^2+2(\lambda+\lambda\gamma+\gamma\alpha^2)\big)(\lambda+\gamma\alpha^2) \tag{A.51}$$

$$= -(3\lambda^2+\lambda\alpha^2+3\lambda^2\gamma+6\lambda\gamma\alpha^2+\gamma\alpha^4+4\lambda\gamma^2\alpha^2+4\gamma^2\alpha^4) < 0. \tag{A.52}$$

Overall,

$$\frac{df(\cdot)}{d\lambda} = (1+\gamma)\frac{dg(\cdot)}{d\lambda} + 2\gamma^2(1+\gamma)\frac{dh(\cdot)}{d\lambda} + 2\gamma(1+\gamma)^{\frac{3}{2}}\frac{dk(\cdot)}{d\lambda} > 0, \tag{A.53}$$

which together with (A.32) gives:

$$\forall\alpha > 0\ \exists!\bar{\lambda} > 0 \text{ s.t. } \lambda < \bar{\lambda} \Leftrightarrow \bar{\kappa}_1 > \bar{\kappa}_2. \tag{A.54}$$

A.4 Commitment-constrained strategic rule - Section 2.5

The optimal monetary rule under limited commitment is designed to trade off *intertemporal losses* against the *provision of credible sequential policy incentives.* If $\kappa \geq \bar{\kappa}_2$, then full provision of incentives: minimize intertemporal losses. If $\kappa = 0$, then no credible incentives can be provided, and the intertemporal loss is maximum, as under a regime of policy discretion. If $\kappa \in (0, \bar{\kappa}_2)$, then the central bank targets a credible equilibrium path $(\tilde{e}, \tilde{x}, \tilde{\pi})$, with $\tilde{e} = \tilde{x} + \alpha\tilde{\pi}$ that minimizes the intertemporal loss given κ. The characterization of this commitment-constrained policy rule is structured as follows:

1. Given a level of private agents' expectations e, find policy choices (x, π) that minimize the loss function s.t. $e = x + \alpha\pi$. Also, show that the intertemporal loss $\mathcal{L}(e, x(e), \pi(e))$ is increasing in expectations e.

2. Consider policy cooperation under limited commitment $\kappa \in (0, \bar{\kappa}_2)$, characterize a level of expectation e s.t. the incentive constraint of the government to implement $(x(e), \pi(e))$ is just binding. Show that e is unique, and decreasing in κ.

3. Show that this constrained efficient policy can be implemented with a strategic monetary rule as the outcome of the fiscal-monetary game with asymmetric and limited commitment, where the central bank only benefits from a degree of commitment κ.

4. Finally, derive the sensitivity of equilibrium policy choices $(\tilde{x}, \tilde{\pi})$ to a change in κ, and show how variations in parameters λ and α command different adjustment of monetary and fiscal decisions $(\tilde{x}, \tilde{\pi})$ from their respective targets (x^*, π^*).

A.4.1 Expectation-adjusted optimal policy

Given $e \in (e^*, e^d)$,

$$\min_{x,\pi} \mathcal{L}(e, x, \pi) \quad \text{s.t.} \quad e = x + \alpha\pi. \tag{A.55}$$

Substituting the constraint twice

$$\mathcal{L}(e, x, \pi) = \frac{1}{2}\big[(e - \alpha\pi - x^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(y^*)^2\big]. \tag{A.56}$$

Derive the FOC w.r.t. π, and then get $x = e - \alpha\pi$. The solution is:

$$x(e) = \frac{\lambda e - \alpha\lambda\pi^* + \alpha^2 x^*}{\alpha^2 + \lambda} \geq x^* \qquad \pi(e) = \frac{\alpha e - \alpha x^* + \lambda\pi^*}{\alpha^2 + \lambda} \geq \pi^*. \tag{A.57}$$

The loss evaluated at the solution writes:

$$\mathcal{L}(e, x(e), \pi(e)) = \frac{1}{2}\big[(e - \pi(e) - x^*)^2 + \lambda(\pi(e) - \pi^*)^2 + \gamma(y^*)^2\big]. \tag{A.58}$$

and using the envelope condition $\frac{d\mathcal{L}(\cdot)}{d\pi} = 0$, get

$$\frac{d\mathcal{L}(\cdot)}{de} = x(e) - x^* \geq 0. \tag{A.59}$$

A.4.2 Constrained efficient policy

Consider the constrained optimal policy choice of a government with commitment κ. The objective is to characterize a level of expectation e and associated policy choices $(x(e), \pi(e))$

which are incentive compatible against reoptimization, i.e.,

$$\mathcal{L}(e, x(e), \pi(e)) \leq \text{argmin}_{x,\pi}\mathcal{L}(e, x, \pi) + \kappa. \tag{A.60}$$

Policy choices under reoptimization are

$$x^r(e) = \frac{(\lambda + \gamma\alpha^2)x^* - \gamma\alpha\lambda\pi^* + \lambda\gamma(e + y^*)}{\lambda + \gamma\lambda + \gamma\alpha^2}, \tag{A.61}$$

$$\pi^r(e) = \frac{\lambda(1 + \gamma)\pi^* - \gamma\alpha x^* + \gamma\alpha(e + y^*)}{\lambda + \gamma\lambda + \gamma\alpha^2}. \tag{A.62}$$

Let $\bar{\kappa}(e)$ be the credibility cut-off that implements $\{e, x(e), \pi(e)\}$:

$$\bar{\kappa}(e) = \mathcal{L}(e, x(e), \pi(e)) - \mathcal{L}(e, x^r(e), \pi^r(e)) \tag{A.63}$$

Next, show that $\frac{de}{d\kappa} = \frac{1}{\bar{\kappa}'(e)} < 0$ and verify that for all $\kappa \in (0, \bar{\kappa}_2)$, the best incentive compatible equilibrium outcome satisfies $x \geq x^*$ and $\pi \geq \pi^*$. Using envelope conditions, the derivatives of each term:

$$\frac{d\mathcal{L}(e, x(e), \pi(e))}{de} = x(e) - x^* = \frac{\lambda}{\alpha^2 + \lambda}(e - (x^* + \alpha\pi^*)), \tag{A.64}$$

$$\frac{d\mathcal{L}(e, x^d(e), \pi^d(e))}{de} = -\gamma(x^d(e) + \alpha\pi^d(e) - e - y^*) = -\frac{\gamma\lambda(x^* + \alpha\pi^* - (e + y^*))}{\lambda + \gamma\lambda + \gamma\alpha^2} \geq 0. \tag{A.65}$$

Finally get:

$$\frac{d\bar{\kappa}(e)}{de} = \frac{\lambda}{\lambda + \gamma\lambda + \gamma\alpha^2}\Big[\frac{\lambda}{\alpha^2 + \lambda}(e - (x^* + \alpha\pi^*)) - \gamma y^*\Big]. \tag{A.66}$$

Verify that $\frac{d\bar{\kappa}(e)}{de}|_{e=e^*} < 0$, $\frac{d\bar{\kappa}(e)}{de}|_{e=e^d} = 0$, $\frac{d^2\bar{\kappa}(e)}{de^2} > 0$ and $\bar{\kappa}(e^d) = 0$ to conclude that the commitment intensity required to support $(e, \tau(e), \pi(e))$ is decreasing in $e \in (e^*, e^d)$. Also, with these elements, one can rule out that a constrained equilibrium would involve $e < e^*$ and $e > e^d$.

A.4.3 Implementation as an outcome of the monetary-fiscal game

Given $\kappa < \bar{\kappa}_2$, the central bank designs a strategic monetary rule $\pi^k(\mathcal{S})$ with the objective to implement the constrained efficient policy $(\tilde{e}, \tau(\tilde{e}), \pi(\tilde{e}))$, where $\tilde{e}$ is the level of expectations associated to the degree of commitment κ derived in A.4.2.

The analysis adopts the following steps: (i) define the monetary rule, (ii) derive the equilibrium outcome of the game under the rule given κ, and (iii) verify that it is the best equilibrium that can be implemented given κ.

Monetary rule. Given $\kappa \in (0, \bar{\kappa}_2)$, suppose that expectations of private agents are anchored at $\tilde{e} \in (e^*, e^d)$, as characterized in A.4.2. First, define $\mathrm{T}(\tilde{e})$ the set of fiscal influence:

$$\mathrm{T}(\tilde{e}) = \{x \text{ s.t. } \mathcal{L}^{f,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)) \leq \mathcal{L}^{f,k}(\tilde{e}, x(\tilde{e}), \pi(\tilde{e}))\}.$$

The strategic rule prescribes that if $x \in \mathrm{T}(\tilde{e})$, then $\pi^k(\tilde{e}, x)$ satisfies:[41]

$$\mathcal{L}^{f,k}(\tilde{e}, x, \pi^k(\tilde{e}, x)) = \mathcal{L}(\tilde{e}, x(\tilde{e}), \pi(\tilde{e}))), \forall x \in \mathrm{T}(\tilde{e}).. \tag{A.67}$$

Equilibrium outcome. Private agents play $\tilde{e}$. If the fiscal authority plays $x \in \mathrm{T}(\tilde{e})$, then the central bank plays according to the rule $\pi^k(\tilde{e}, x)$ if and only if:

$$\mathcal{L}^{m,k}(\tilde{e}, x, \pi^k(\tilde{e}, x)) \leq \mathcal{L}^{m,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)) + \kappa. \tag{A.68}$$

The left-hand side is $\mathcal{L}(\tilde{e}, x(\tilde{e}), \pi(\tilde{e}))$ by (A.67). The right-hand side is larger than $\operatorname{argmin}_{x,\pi}\mathcal{L}(\tilde{e}, x, \pi)$. Since the pair $(\kappa, \tilde{e})$ is s.t. (A.60) holds, a fortiori (A.68) holds.[42]

Overall, the strategic monetary rule implements $\{\tilde{e}, \tau(\tilde{e}), \pi(\tilde{e})\}$ as the outcome of the monetary-fiscal game given central bank degree of commitment κ.

Also, the monetary-fiscal game under asymmetric commitment $\kappa \in (0, \bar{\kappa}_2)$ cannot support a better outcome. Indeed, given $\tilde{e}$, it is the best outcome as shown in A.4.1. An equilibrium with a higher e is associated to larger losses. An equilibrium with lower e would not be incentive compatible; see A.4.2.

A.4.4 Allocation of intertemporal losses

An increase in κ is associated to a decrease in e as in (A.66), which then induces the following changes in constrained policy choices:

$$\frac{d\tau}{d\kappa} = \frac{d\tau}{de}\frac{de}{d\kappa} = \frac{de}{d\kappa}\frac{\lambda}{\alpha^2 + \lambda} < 0 \qquad \frac{d\pi}{d\kappa} = \frac{d\pi}{de}\frac{de}{d\kappa} = \frac{de}{d\kappa}\frac{\alpha}{\alpha^2 + \lambda} < 0. \tag{A.69}$$

The adjustments of fiscal and monetary instruments relative to their respective targets are driven by relative cost λ vs. benefit α of adjusting π:

$$\frac{d\pi}{d\kappa} > \frac{d\tau}{d\kappa} \Leftrightarrow \frac{\alpha}{\lambda} > 1. \tag{A.70}$$

A.5 Extensions

We present in this appendix a series of extensions that investigate the sensitivity of our results to variations to the economic environment.

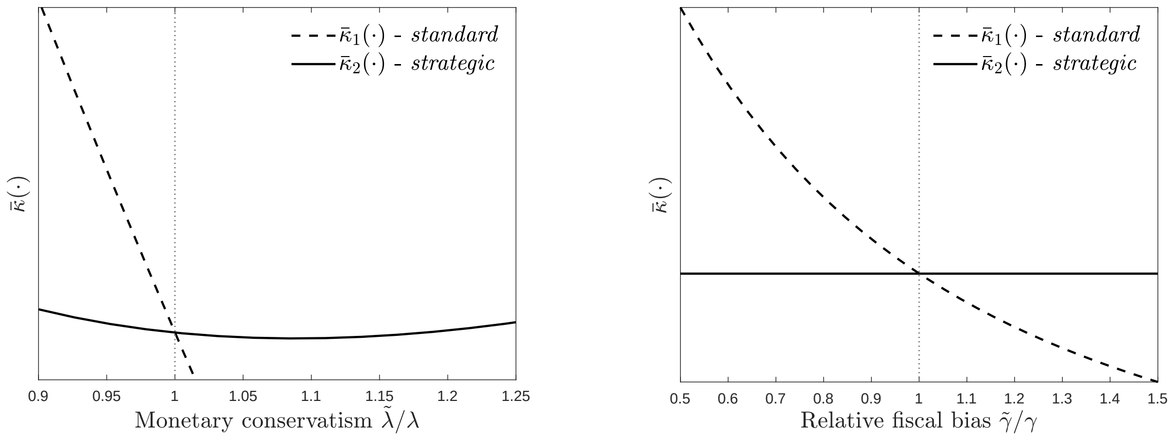
A.5.1 Credibility and Policy Preferences

Propositions 1, 2, and 3 are derived under the assumption that monetary and fiscal authorities, despite differences in their respective degree of commitment, make decisions using identical

[41] If $x \notin \mathrm{T}(\tilde{e})$ then $\pi^k(\mathcal{S})$ needs not be specified.

[42] The fiscal authority would not play $x \notin T(\tilde{e})$, because then even if the central bank renounces and reoptimizes, the loss to the fiscal authority is higher than if it plays $x \in \mathrm{T}(\tilde{e})$.

Figure 6: Credibility Cut-offs under Different Policy Preferences



(a) Conservative Central Bank (b) Self-interested Treasury

This figure compares credibility cut-offs $\bar{\kappa}$ for each monetary rule: *standard* and *strategic*. Panel (a) reports credibility for different level of central bank conservatism. Panel (b) compares credibility across rules for different degrees of fiscal self-interest. (Numerical values are set so that $\bar{\kappa}_1 = \bar{\kappa}_2$ when authorities have similar benevolent preferences: $\lambda = 3/2$, $\gamma = 1/2$, $\alpha = 0.952$.)

policy preferences. We relax this assumption and consider two common adjustments of policy objectives: monetary *inflation conservatism* and *self-interest* of fiscal authority.

The analysis accommodates different policy preferences in a straightforward way. Changes of monetary policy preferences affect the credibility cut-off directly as it is based on losses as evaluated by the central bank:

$$\bar{\kappa} = \max_{x \in \mathrm{T}(\tilde{e})} \; \mathcal{L}^{m,k}(\tilde{e}, x, \pi^k(\tilde{e}, \tau)) - \mathcal{L}^{m,r}(\tilde{e}, x, \pi^r(\tilde{e}, \tau)), \tag{A.71}$$

where $\pi^r(\mathcal{S}) = \mathrm{argmin}_\pi \mathcal{L}^{m,r}(e, x, \pi)$, and $\mathcal{L}^m(e, x, \pi)$ may reflect an objective function different from (2.2).

Changes of fiscal preferences affect directly the set of fiscal influence as it is based on the losses evaluated by the treasury:

$$\mathrm{T}(\tilde{e}) = \{x \;\mid \mathcal{L}^{f,r}(\tilde{e}, x, \pi^r(\tilde{e}, x)) \leq \mathcal{L}^{f,k}(\tilde{e}, \tilde{x}, \pi^k(\tilde{e}, \tilde{x}))\}, \tag{A.72}$$

where $\mathcal{L}^f(e, x, \pi)$ may reflect an objective function different from (2.2).

Monetary conservatism. *Monetary conservatism* refers to the appointment of a monetary policymaker that puts a higher weight $\tilde{\lambda} > \lambda$ on inflation stabilization.[43] The objective function of the fiscal authority is (2.2). We consider the effects of monetary conservatism on credibility cut-offs for both standard and strategic rules. A graphic illustration is presented on the left

[43] The benefits of appointing an inflation conservative central banker have been discussed first by Rogoff (1985). Recent contributions, including Adam and Billi (2008) and Niemann (2011), study the interactions of monetary conservatism with fiscal policy.

panel of Figure 6.

Under a standard rule, when the central bank is inflation conservative, $\tilde{\lambda} > \lambda$, the equilibrium outcome is (2.11). The set of fiscal influence reads:

$$\mathrm{T}(e_1) = \{x \text{ s.t. } \mathcal{L}^{f,r}(e_1, x, \pi^r(e_1, x)) \leq \mathcal{L}^{f,k}(e_1, x_1, \pi^*)\}. \tag{A.73}$$

The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{x \in \mathrm{T}(e_1)} \mathcal{L}^{m,k}(e_1, x, \pi^*) - \mathcal{L}^{m,r}(e_1, x, \pi^r(e_1, x)). \tag{A.74}$$

A similar adjustment applies to derive the credibility cut-off under strategic rule.

Under the standard rule, stronger conservatism unambiguously reduces the degree of commitment required to implement the standard rule, since upon renouncing the inflation target, a more conservative central banker would implement a lower monetary stimulus, yielding lower welfare gains. This dimension is also present under a strategic rule. However, under a strategic rule, stronger conservatism also makes off-equilibrium threats more costly to the central bank. The latter effect dominates at higher levels of conservatism. Overall, as illustrated in the left panel of Figure 6, only a moderate level of monetary conservatism relaxes the degree of commitment required to sustain a strategic rule.

Fiscal self-interest. *Treasury self-interest* refers to higher temptation of the fiscal authority to stimulate output, captured by a higher weight $\tilde{\gamma} > \gamma$ that the treasury attaches to deviations of output from the first-best level.[44] The monetary policy objective function is kept as in the baseline analysis. The right panel of Figure 6 provides a graphic illustration of the results.

Under a standard rule, the equilibrium outcome if the inflation target is credible is:

$$x_1 = x^* + \tilde{\gamma} y^* \qquad \pi_1 = \pi^* \qquad e_1 = x^* + \tilde{\gamma} y^* + \alpha \pi^* \qquad y_1 = 0, \tag{A.75}$$

where $\tilde{\gamma}$ is the preference parameter of the fiscal authority. The set of fiscal influence is:

$$\mathrm{T}(e_1) = \{x \text{ s.t. } \mathcal{L}^{f,r}(e_1, x, \pi^r(\cdot)) \leq \mathcal{L}^{f,k}(e_1, x_1, \pi^*)\}. \tag{A.76}$$

The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{x \in \mathrm{T}(e_1)} \mathcal{L}^{m,k}(e_1, x, \pi^*) - \mathcal{L}^{m,r}(e_1, x, \pi^r(e_1, x)). \tag{A.77}$$

A similar adjustment applies to derive the credibility cut-off under strategic rule.

As illustrated in the right panel of Figure 6, the credibility cut-off of a strategic monetary rule does not depend on the strength of fiscal self-interest. Importantly, contingencies in strategic rules do change with fiscal self-interest to provide appropriate incentives. Nevertheless, the

[44] The terminology fiscal *self-interest* is borrowed from Yared (2019). Political frictions can fuel this type of fiscal behavior, including electoral constraints or budgetary processes.

credibility cut-off, evaluated with the preferences of the monetary authority, is not influenced by the intensity of the fiscal temptation to stimulate output.[45]

A.5.2 Credibility and Reputation

Reputation—or trigger-type equilibria—is a common construction to support commitment. Private agents have a system of expectations that induces the government to follow a rule against the threat of reversal to a worse equilibrium outcome over subsequent periods.[46] This appendix discusses how our concepts of *degree of commitment* and *credibility cut-off* relate to these constructions within a repeated version of the game. Importantly, these equilibria emphasize strategic interactions between private agents and policymakers, without adding economic insight to our discussion of monetary-fiscal interactions. Accordingly, we choose to abstract from these elements in the main text.

Consider an infinite repetition of the game. The objective of the central bank is to implement (2.12) by following a strategic rule $\pi^k(\mathcal{S})$ of the type derived in Section 2.4.2, without the exogenous cost κ in case of renouncement to the rule. Credibility is derived from the long-run consequences to the monetary authority of renouncing the rule. We take the infinite repetition of the discretionary equilibrium outcome (2.6) as a threat point to support the rule.[47]

Let $\mathrm{T}(e^*)$ be the set of fiscal influence, as in (2.8). The central bank follows $\pi^k(e,x)$ given by (2.12) and (2.14) for all $x \in \mathrm{T}(e^*)$ if $e = e^*$. Otherwise, if $e \neq e^*$ or $x \neq \mathrm{T}(e^*)$, it implements $\pi^r(e,x)$. In turn, the process for private households' expectations keeps track of the history of the central bank's decisions:

$$\begin{aligned} e &= e^* \quad \text{if} \quad \pi_{-1} = \pi^k(e_{-1}, x_{-1}) \\ e &= e^d \quad \text{if} \quad \pi_{-1} = \pi^r(e_{-1}, x_{-1}) \end{aligned} \tag{A.78}$$

If the central bank implemented $\pi^r(\mathcal{S})$ once in the past, private agent expectations carry this information.[48] The strategic rule $\pi^k(\mathcal{S})$ implements the equilibrium outcome (2.12) if and only

[45] In contrast, fiscal self-interest reduces the degree of commitment required to deliver an unconditional inflation target, because the equilibrium *fiscal bias* reduces the relative gains for the monetary authority to renounce its rule.

[46] Various versions of this construction are discussed in Barro and Gordon (1983), Stokey (1989), or Chari and Kehoe (1990).

[47] The infinite repetition of the static Nash game is evidently subgame perfect, but it's not necessarily the most effective threat point to support other outcomes.

[48] Finite forms of punishment are also possible.

if:

$$\forall x \in \mathrm{T}(e^*)\ \Delta(x) = \underbrace{\mathcal{L}(e^*, x, \pi^k(\mathcal{S})) - \mathcal{L}(e^*, x, \pi^r(\mathcal{S}))}_{\text{Short-term Gain}} - \underbrace{\frac{\beta}{1-\beta}\Big[\mathcal{L}(e^d, x^d, \pi^d) - \mathcal{L}(e^*, x^*, \pi^*)\Big]}_{\text{Long-term Cost}} \leq 0, \tag{A.79}$$

where $\beta > 0$ is the discount factor of the monetary authority. Under this construction, deviating from the rule yields a short-term gain, which is evaluated against the cost of an infinite repetition of the discretionary equilibrium outcome (2.6). This long-term component maps precisely into κ, the cost incurred by the central bank when breaking a promise. With a discount factor β high enough, the threat to revert to the discretionary outcome is strong enough to support the rule, i.e., the long-term cost of renouncing the rule is higher than the cut-off value $\bar{\kappa}$ associated to the short-term gain in (A.79). Importantly, the long-term consequences of renouncing the rule influences only monetary decisions if the fiscal authority is myopic, i.e., the discount factor of the fiscal authority is $\beta = 0$.

A.6 Expectation-driven economic outcomes

In this Appendix, we discuss the existence of expectation-driven economic outcomes in the monetary-fiscal game presented in Section 2.[49] We find that the nature of expectation-driven equilibria is not fundamentally affected by strategic interactions between the monetary and fiscal authorities. Furthermore, we discuss how a monetary rule could be adjusted to eliminate strategic uncertainty due to possible variation in expectations of private agents. Finally, we show that without such adjustment, sunspot equilibria give rise to additional considerations for the choice of the inflation target.

Expectation-driven equilibria under a standard rule. Consider the monetary-fiscal game presented in Section 2.3, where the monetary authority commits to a standard rule $\pi^k(e, x) = \pi^*$ for all summary of expectations e and fiscal decisions x.

Suppose first that private agents expect an equilibrium path where the central bank eventually plays *keep*. This case coincides with the analysis presented in Section 2.4.1. Given e and anticipating the central bank plays π^*, the fiscal authority plays $x^k(e) = \text{argmax}_x \mathcal{L}^{f,k}(e, x, \pi^*)$, where the superscript k in $x^k(e)$ reflects the anticipation that the central bank will set policy as prescribed by the rule. In equilibrium, expectations are consistent with policy choices, i.e., $e = x^k(e) + \alpha\pi^*$. Hence, the equilibrium outcome is (e_1, x_1, π^*), as in (2.11). As in Proposition

[49] Economic environments with limited commitment ordinarily feature multiplicity of economic outcomes induced by variation in expectations of private agents. See, for instance, studies of sovereign debt pricing (Calvo (1988)), currency crises (Obstfeld (1996)), and monetary policy (Chari, Christiano and Eichenbaum (1998)).

1, a necessary condition on the degree of commitment κ to support this equilibrium path is:

$$\kappa \geq \bar{\kappa}_1 = \text{argmax}_{x \in \mathrm{T}(e_1)} \mathcal{L}^{m,k}(e_1, x, \pi^*) - \mathcal{L}^{m,r}(e_1, x, \pi^r(e_1, x)),$$

where $\mathrm{T}(e_1)$ is the set of fiscal influence under the standard rule given expectations e_1.

Suppose next that private agents expect an equilibrium path where the central bank eventually plays *renege*. Given e and anticipating the central bank plays $\pi^r(e, x)$, the fiscal authority plays $x \in \mathrm{T}(e)$ that minimizes its losses and induces the central bank to *renege* the rule. Formally, the fiscal authority plays:

$$x^r(e) = \text{argmin}_{x \in \mathrm{T}(e)} \mathcal{L}^{f,r}(e, x, \pi^r(e, x))$$

subject to

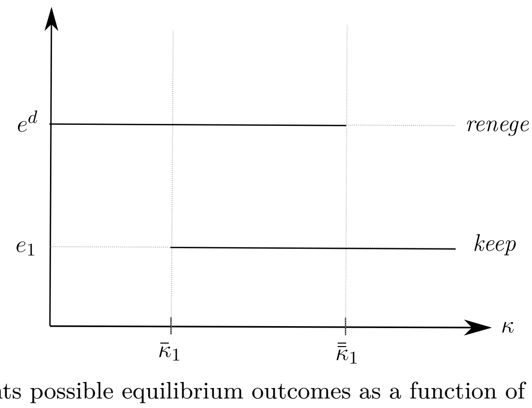
$$\mathcal{L}^{m,k}(e, x, \pi^*) - \mathcal{L}^{m,r}(e, x, \pi^r(e, x)) - \kappa \geq 0 \tag{A.80}$$

The only equilibrium path consistent with the equilibrium requirement $e = x^r(e) + \alpha\pi^r(e, x)$ is (e^d, x^d, π^d), i.e., it coincides with the cooperative outcome without commitment; see (2.6). Hence, a necessary condition for this equilibrium path is:

$$\kappa \leq \bar{\bar{\kappa}}_1 = \mathcal{L}^{m,k}(e^d, x^d, \pi^*) - \mathcal{L}^{m,r}(e^d, x^d, \pi^d).$$

Note that, conditional on *renege*, there is no equilibrium path where expectations and policy decisions are different from (e^d, x^d, π^d), even with a binding constraint (A.80).[50]

Figure 7: Expectation-Driven Equilibrium Outcomes



This figure represents possible equilibrium outcomes as a function of the monetary degree of commitment κ. For low $\kappa \leq \bar{\kappa}_1$, the standard rule is not credible and the equilibrium is (e^d, x^d, π^d). For high $\kappa \geq \bar{\bar{\kappa}}_1$, the monetary rule is credible and the only equilibrium outcome is (e_1, x_1, π^*). For intermediate values $\kappa \in [\bar{\kappa}_1, \bar{\bar{\kappa}}_1]$, the outcome is sensitive to private agents' expectations.

Naturally, $\bar{\kappa}_1 \leq \bar{\bar{\kappa}}_1$ since at least one of the paths *keep* or *renege* is an equilibrium for every degree of monetary commitment $\kappa \geq 0$.[51] For relatively low (high) value of κ, there is a unique

[50]The only path $\big(e, x, \pi^r(e, x)\big)$ associated with the binding constraint (A.80) and consistent expectations in equilibrium is (e^d, x^d, π^d).

[51]Also, in general, $\bar{\kappa}_1 < \bar{\bar{\kappa}}_1$ because $\bar{\kappa}_1$ and $\bar{\bar{\kappa}}_1$ are different objects.

equilibrium path associated to *renege* (*keep*). For intermediate values of $\kappa \in [\bar{\kappa}_1, \bar{\bar{\kappa}}_1]$, there are two possible equilibria, whose occurrence depends on expectations of private agents. Figure 7 provides a graphic illustration.

Importantly, expectation-driven outcomes reflect strategic interaction between private agents and the overall public sector with two authorities. Specific strategic interactions across policy-makers do not have any fundamental relevance: multiplicity of the same nature would arise in the environment with a single policy authority subject to a limited degree of commitment.

Expectation-driven equilibria under a strategic rule. Similarly, if the central bank adopts a strategic rule as in Section 2.4.2, expectation-driven outcomes can arise for intermediate values of $\kappa \in [\bar{\kappa}_2, \bar{\bar{\kappa}}_2]$. The cut-off value $\bar{\bar{\kappa}}_2$ satisfies:

$$\bar{\bar{\kappa}}_2 = \mathcal{L}^{m,k}(e^d, x^d, \pi^k(e^d, x^d)) - \mathcal{L}^{m,r}(e^d, x^d, \pi^r(e^d, x^d)),$$

where $\pi^k(e, x)$ is designed to induce the fiscal authority to play x^* and the central bank to play π^* for all e.[52] The equilibrium path associated to *renege* is not sensitive to the degree of commitment κ. Overall, the qualitative implications of introducing expectation-driven equilibria under the strategic rule remain the same as under the standard rule.

Eliminating strategic uncertainty. The analysis above shows how different equilibria could arise for the intermediate degrees of commitment depending on private agents' expectations. Can a central bank shield against this source of strategic uncertainty? Suppose the central bank commits to the following expectation-augmented standard rule:[53]

$$\pi^k(e, x) = \pi^* + \delta(\pi^e - \pi^*). \tag{A.81}$$

If private agents anticipate the central bank to play *keep*, then the equilibrium outcome coincides with (e_1, x_1, π^*), i.e., the equilibrium characterized under a credible standard rule.

Alternatively, consider the case when private agents expect the central bank to *renege* on the rule and the fiscal authority to choose $x^r(e) = \text{argmin}_x \mathcal{L}^{f,r}(e, x, \pi^r(e, x))$. Given (e, x) the central bank could play according to rule (A.81), with an appropriate δ such that $\pi^k(e, x)$ is very close but not equal to $\pi^r(e, x)$. Since the policy choice is very close to the choice under reoptimization, it is credible for the central bank to play *keep*. Note that, as long as $\delta \neq 1$, the only expectation consistent with rule (A.81) is $\pi^e = \pi^* + \delta(\pi^e - \pi^*)$, i.e., $\pi^e = \pi^*$. Hence,

[52] Formally, for all e, $\mathcal{L}^{m,k}(e^d, x, \pi^k(e^d, x)) = \mathcal{L}(e^d, x^*, \pi^*)$. In particular, under a strategic rule, if the central bank can rule out the path that yields the lowest loss to the fiscal authority, then it can rule out *renege* for all attempt of fiscal influence $x \in \mathrm{T}(e)$.

[53] This rule is standard because it is not conditional on fiscal policy. A similar analysis can be conducted for a strategic rule.

there is a contradiction with the initial assumption that agents would expect the central bank to renounce the rule.

Overall, by adopting the expectation-augmented standard rule, the central bank can eliminate expectation-driven equilibrium outcomes.

Adjusting the monetary target to strategic uncertainty. Augmenting monetary rules as discussed above is not the only policy adjustment that could address the issue of expectation-driven equilibria. The central bank could adjust its inflation target $\hat{\pi} \in [\pi^*, \pi^d]$ to mitigate the possibility of the expectation-driven outcome and the associated welfare loss.

Consider the central bank that follows an unconditional standard rule $\pi^k(e,x) = \hat{\pi}$ with a limited degree of commitment κ. For a given inflation target $\hat{\pi}$, there are two credibility cut-offs $\bar{\kappa}(\hat{\pi}) \leq \bar{\bar{\kappa}}(\hat{\pi})$ such that the path *keep* can arise if $\kappa \geq \bar{\kappa}(\hat{\pi})$ and the path *renege* if $\kappa \leq \bar{\bar{\kappa}}(\hat{\pi})$.

Let $s \in \{s_k, s_r\}$ be a sunspot variable that reflects private agents' expectations (e_k, e_r), where k stands for *keep* and r for *renege*. The probability of sunspots are $p(s_k) = p$ and $p(s_r) = 1 - p$.

The timing of the game is as follows. First, the central bank picks the inflation target: for all (e,x), $\pi^k(e,x) = \hat{\pi}$. Then the sunspot shock $s \in \{s_k, s_r\}$ realizes and agents form expectations $e \in \{e_k, e_l\}$ accordingly. Then the fiscal authority chooses x. Finally, given (e,x), the monetary authority either *keeps* and implements $\hat{\pi}$ or *reneges* at cost $\kappa \geq 0$ and plays $\pi^r(e,x)$.

At the constitutional stage, when choosing the inflation target, the central bank considers the following program:

$$\min_{\hat{\pi} \geq \pi^*} \begin{cases} \mathcal{L}^{m,r}(e_r, x^r(e_r), \pi^r(e_r, x^r(e_r))) & \text{if } \kappa \leq \bar{\kappa}(\hat{\pi}) \text{, i.e., } \textit{renege} \text{ zone} \\ p\mathcal{L}^{m,k}(e_k, x^k(e_k), \hat{\pi}) + (1-p)\mathcal{L}^{m,r}(e_r, x^r(e_r), \pi^r(e_r, x^r)) & \text{if } \bar{\kappa}(\hat{\pi}) \leq \kappa \leq \bar{\bar{\kappa}}(\hat{\pi}) \text{, i.e., } \textit{sunspot} \text{ zone} \\ \mathcal{L}^{m,k}(e_k, x^k(e_k), \hat{\pi}) & \text{if } \kappa \geq \bar{\bar{\kappa}}(\hat{\pi}) \text{, i.e., } \textit{keep} \text{ zone} \end{cases}$$

s.t. $e_k = x^k(e_k, \hat{\pi}) + \alpha\hat{\pi}$ and $e_r = x^r(e_r) + \alpha\pi^r(e_r, x^r(e_r))$.

The left panel of Figure 8 shows credibility cut-offs $\bar{\kappa}$ and $\bar{\bar{\kappa}}$ as functions of the inflation target $\hat{\pi} \in [\pi^*, \pi^d]$. Given degree of monetary commitment κ,

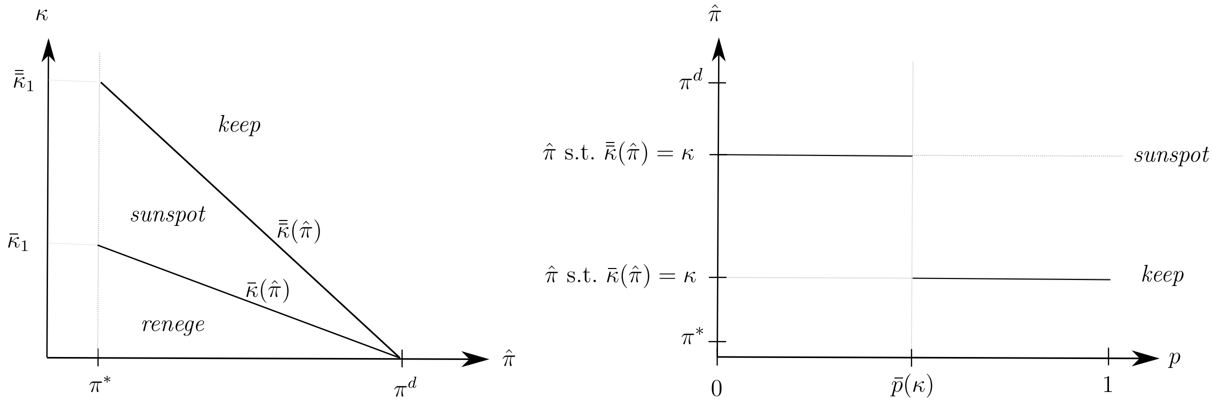
– conditional on being in the *renege* zone, the choice of $\hat{\pi}$ is irrelevant for welfare.

– conditional on being in the *keep* zone, the choice of $\hat{\pi}$ that minimizes the loss to the monetary authority solves $\kappa = \bar{\bar{\kappa}}(\hat{\pi})$.

– conditional on being in the *sunspot* zone, the choice of $\hat{\pi}$ that minimizes the loss to the monetary authority solves $\kappa = \bar{\kappa}(\hat{\pi})$.

Overall, the solution to the program of the central bank at the constitutional stage given κ is to choose an inflation target $\hat{\pi}$ such that (i) it satisfies $\kappa = \bar{\kappa}(\hat{\pi})$ or $\kappa = \bar{\bar{\kappa}}(\hat{\pi})$ and (ii) it yields

Figure 8: Sunspot Equilibria and Inflation Target



(a) Equilibrium Regimes (b) Optimal Target

Panel (a) reports equilibrium regimes as a function of the inflation target $\hat{\pi}$ and monetary degree of commitment κ. Panel (b) represents the optimal choice of the inflation target given κ, as a function of the probability p: for $p \leq \bar{p}(\hat{\pi})$, the central bank chooses the lowest $\hat{\pi}$ consistent with *keep*, while for higher p, it chooses a lower target $\hat{\pi}$ in the sunspot region.

the lowest loss to the monetary authority. If $p = p(e_k) \approx 1$, then the solution is to choose $\hat{\pi}$ such that $\bar{\kappa}(\hat{\pi}) = \kappa$. If $p = p(e_k) \approx 0$, then the choice is $\hat{\pi}$ such that $\bar{\bar{\kappa}}(\hat{\pi}) = \kappa$. As illustrated on the right panel of Figure 8, for every $\kappa \in [0, \bar{\bar{\kappa}}_1]$ there is a cut-off probability $\bar{p}(\kappa) \in [0, 1]$ s.t.:

- if $p \leq \bar{p}(\kappa)$, then the solution to the central bank program is to choose the lowest $\hat{\pi}$ consistent with the *keep* zone, i.e., s.t. $\bar{\bar{\kappa}}(\hat{\pi}) = \kappa$. Indeed, if the probability of expectation e_r is high, the central bank picks a higher inflation target to eliminate the possibility of such outcome.
- if $p \geq \bar{p}(\kappa)$, then the solution to the program of the central bank is to choose the lowest $\hat{\pi}$ consistent with the *sunspot* zone. If the probability of expectation e_r is low, then the central bank decides on a lower inflation target to anchor expectations, at the risk of expectation-induced equilibrium path.

The implications of sunspot shocks for the selection of a policy target is reminiscent of the analysis conducted by Cole and Kehoe (2000) in the context of self-fulfilling debt crises. In their dynamic environment, a government facing the risk of self-fulfilling market shutdown is motivated to reduce its outstanding debt level to exit the "crisis zone," i.e., the sunspot zone. The speed of deleveraging depends on the probability of a bad expectation shock in a way that is similar to the inflation target $\hat{\pi}$ in our environment.

B Cash-Credit Economy

B.1 Competitive Equilibrium

Definition 6. *A competitive equilibrium consists of a price system* $\{P_t, q_t\}_{t=0}^{\infty}$*, a private sector allocation* $\{c_t, d_t, l_t, M_t^h, B_t^h\}_{t=0}^{\infty}$*, and a government policy* $\{M_t, B_t, \tau_t\}_{t=0}^{\infty}$ *s.t.:*

– *Given initial asset positions* $\{M_{-1}^h, B_{-1}^h\}$ *as well as price system and policy, the allocation solves the maximization program of the representative household* (3.4) *subject to the sequence of household budget constraints* (3.2)*, the cash-in-advance constraints* (3.3)*, and exogenous debt limits;*

– *Given initial liabilities* $\{M_{-1}, B_{-1}\}$ *as well as allocation and price system, the policy satisfies the sequence of government budget constraints* (3.5)*;*

– *All markets clear, hence at all times* $M_t^h = M_t$*,* $B_t^h = B_t$*, and the resource constraint* (3.1) *holds.*

The following expressions characterize household choice given government policy:

$$u_{l,t} = u_{c,t}(1-\tau_t) \quad \Leftrightarrow \quad (1-\tau_t) = \frac{\gamma}{\alpha}c_t, \tag{B.1}$$

$$u_{c,t} = \beta\frac{P_t}{P_{t+1}}u_{d,t+1} \quad \Leftrightarrow \quad \alpha P_{t+1}d_{t+1} = \beta(1-\alpha)P_t c_t, \tag{B.2}$$

$$u_{c,t} = \beta\frac{1}{q_t}\frac{P_t}{P_{t+1}}u_{c,t+1} \quad \Leftrightarrow \quad P_{t+1}c_{t+1} = \beta\frac{1}{q_t}P_t c_t. \tag{B.3}$$

A positive tax rate drives a wedge on the consumption-leisure choice as described by (B.1). Equation (B.2) maps current consumption of credit good c_t with next-period consumption of cash good d_{t+1}: the wedge is driven by variations in the price level $\frac{P_t}{P_{t+1}}$, which is the real return to holding money. Equation (B.3) is a standard Euler equation, where the intertemporal allocation of credit good is driven by the inverse real interest rate $\tilde{q}_t$:

$$\tilde{q}_t \equiv q_t\frac{P_{t+1}}{P_t} = \beta\frac{u_{c,t+1}}{u_{c,t}}. \tag{B.4}$$

Finally, inequality $(1-\alpha)c_t \geq \alpha d_t$ is a complementary slackness condition due to the cash-in-advance constraint (3.3).

Proof of Lemma 1. Combine (B.1) and (B.2) with the binding cash-in-advance constraint to get $M_t = \frac{\beta(1-\alpha)}{\gamma}P_t(1-\tau_t)$.

Proof of Lemma 2. Start by proving necessity. First, combine the resource constraint (3.1), binding cash-in-advance constraint and market clearing conditions to rewrite the household's budget constraint (3.2) in real terms, after dividing it by M_{t-1}:

$$z_t(1+\sigma_t)q_t + (1+\sigma_t) - \Big(\frac{1-\tau_t}{d_t}\Big)g + \tau_t\Big(1+\frac{c_t}{d_t}\Big) - 1 = z_{t-1},$$

where $z_t \equiv B_t/M_t$. Second, using household's optimality conditions (B.1) to (B.3):

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma_{t+1}}\right]z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right]z_{t-1},$$

where $\Phi \equiv \big(\beta(1-\alpha)+\alpha-\gamma g\big)$.

To prove sufficiency, consider a sequence $\{\tau_t,\sigma_t\}_{t=0}^{\infty}$ of policy instruments that satisfies implementability constraints (3.7) and let $\{B_t, M_t\}_{t=0}^{\infty}$ be the associated paths of government nominal liabilities. One can then derive a sequence of quantities and prices that satisfy Definition (6) as follows. Let $B_t^h = B_t$ and $M_t^h = M_t$ for all $t \geq 0$, cash good consumption sequence $\{d_t\}_{t=0}^{\infty}$ given by

$$d_t = \frac{\beta(1-\alpha)}{\gamma}\frac{(1-\tau_t)}{(1+\sigma_t)}, \tag{B.5}$$

credit good consumption $\{c_t\}_{t=0}^{\infty}$ and leisure $\{l_t\}_{t=0}^{\infty}$ be given by (B.1) and (3.1). Also, let bond prices $\{q_t\}_{t=0}^{\infty}$ be given by $q_t = \beta/(1+\sigma_{t+1})$. With this construction, (B.2) and (B.3) are satisfied and the sequence of implementability constraints (3.7) implies that (3.2) and (3.5) are satisfied. Hence, all conditions of a competitive equilibrium are met by these sequences.

Proof of Lemma 3. Use the resource constraint (3.1) to substitute leisure into the utility function:

$$u(c_t, d_t, l_t) = \alpha\log(c_t) - \gamma c_t + (1-\alpha)\log(d_t) - \gamma d_t,$$

where constant terms independent of policy are discarded, without loss of generality. Next, substitute c_t and d_t with policy instrument choices τ_t and σ_t with (B.1) and (B.5):

$$U(\tau_t,\sigma_t) = \alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right].$$

B.2 Ramsey Equilibrium: Proof of Proposition 5.

The Ramsey policy problem determines a whole sequence of policy instruments to maximize households' welfare subject to the implementability constraint, given $z_{-1} > 0$:

$$\begin{aligned}
&\max_{\{\tau_t,\sigma_t\}} && \sum_{t=0}^{\infty}\beta^t\left\{\alpha\big[\log(1-\tau_t)-(1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right]\right\}\\
&\text{subject to} && \sum_{t=0}^{\infty}\beta^t\left\{\Phi - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right\} = \left[\frac{(1-\alpha)\beta}{(1+\sigma_0)}\right]z_{-1}.
\end{aligned} \tag{B.6}$$

The first-order conditions read as follows:

$$1 = (1+\lambda)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right], \qquad \forall t \geq 0, \tag{B.7}$$

$$1 = (1+\lambda)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}, \qquad \forall t \geq 1, \tag{B.8}$$

$$1 = (1+\lambda)\beta\frac{(1-\tau_0)}{(1+\sigma_0)} + \lambda\frac{\beta}{(1+\sigma_0)}z_{-1}, \tag{B.9}$$

where $\lambda \geq 0$ is the Lagrange multiplier attached to (B.6). (B.7) and (B.8) imply $\tau_t = \bar{\tau} \equiv \lambda/(1+\lambda)$ and $\sigma_t = \bar{\sigma} \equiv \beta - 1$ for all $t \geq 1$. Using (B.1), (B.5), and (3.1), it is straightforward to get that consumption of both goods, leisure, and debt-to-money ratio are constant for all $t \geq 1$, with

$$z_t = \bar{z} \equiv \frac{1}{(1-\alpha)(1-\beta)}\left[\Phi - \frac{1}{1+\lambda}\right], \quad t \geq 0. \tag{B.10}$$

Contrasting optimality conditions for $t = 0$ and $t \geq 1$ yields $\tau_0 < \bar{\tau}$ and $\sigma_0 > \bar{\sigma}$. Finally, substituting (B.7) into (B.6) gives $\bar{z} = \frac{\beta}{1+\sigma_0}z_{-1}$, i.e., if $z_{-1} > 0$ then $\bar{z} < z_{-1}$.

B.3 Cooperative Optimal Policy without Commitment

Consider the case when both cooperating authorities do not have a technology to commit to future policy choices. Instead, they act discretionary, tempted to reap short-term welfare benefits in every period of time. One can effectively think of this environment as a dynamic game between successive selves of the consolidated government, which consists of the central bank and the treasury, as if these were separate policymakers in every period of time. In what follows, we characterize a stationary Markov-perfect equilibrium of this game defined along the lines of Klein, Krusell and Ríos-Rull (2008).

In the Markov-perfect equilibrium, choices of σ_t and τ_t depend the level of outstanding debt z_{t-1}. In each period t, the government maximizes welfare from its incumbent period onward. When choosing period-t policy, the government internalizes its effect on decisions of the private sector given anticipated future policy. Formally, the Markov optimization problem of the discretionary government in any period t can be written as choosing σ_t, τ_t, and z_t that maximize

$$\alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right] + \beta\mathcal{V}(z_t)$$

subject to

$$\beta\left[\frac{(1-\alpha)\beta}{1+\Sigma(z_t)}\right]z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right]z_{t-1}, \tag{B.11}$$

given z_{t-1} and anticipated future choices described by $(\Sigma, \mathcal{T}, \mathcal{Z})$ together with implied private-sector equilibrium that provide continuation utility $\mathcal{V}$. For optimal policy to be time-consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov-perfect equilibrium.

Definition 7. *The Markov-perfect equilibrium is a function* $\mathcal{V}$ *and decision rules* $(\Sigma, \mathcal{T}, \mathcal{Z})$*, each being a function of* z_{t-1}*, such that for all* z_{t-1}*:*

- *Given* $\mathcal{V}$*, the decisions rules solve the Markov problem of the government;*
- $\mathcal{V}$ *is the value function of the government*

$$\mathcal{V}(z_{t-1}) = \alpha\Big[\log\left(1-\mathcal{T}(z_{t-1})\right) - \left(1-\mathcal{T}(z_{t-1})\right)\Big]$$
$$+ (1-\alpha)\left[\log\left(\beta\frac{(1-\mathcal{T}(z_{t-1}))}{(1+\Sigma_t(z_{t-1}))}\right) - \beta\frac{(1-\mathcal{T}(z_{t-1}))}{(1+\Sigma_t(z_{t-1}))}\right] + \beta\mathcal{V}(\mathcal{Z}(z_{t-1})).$$

We restrict analysis to equilibria with differentiable value functions and decision rules, as is common in the literature; see, for instance, Debortoli and Nunes (2013) and Martin (2009).[54] Assuming that such equilibrium exists, it can be characterized by the following first-order conditions of the policy problem:

$$1 = (1+\theta_t)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{1+\sigma_t}\right], \tag{B.12}$$

$$1 = (1+\theta_t)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \theta_t\frac{\beta}{1+\sigma_t}z_{t-1}, \tag{B.13}$$

$$0 = \theta_{t+1} - \theta_t\left[1 - \frac{z_t}{(1+\sigma_{t+1})}\frac{d\sigma(z_t)}{dz_t}\right], \tag{B.14}$$

where $\theta_t \geq 0$ is the Lagrange multiplier attached to (B.11).

In a steady state, the generalized Euler equation (B.14) becomes:

$$0 = \bar{\theta}\frac{\bar{z}}{(1+\bar{\sigma})}\frac{d\sigma(\bar{z})}{dz}, \tag{B.15}$$

which indicates possible existence of three different steady states. Steady-state properties and transition dynamics depend on the derivative $d\sigma(z_t)/dz_t$. Our numerical simulations display equilibria that feature gradual convergence to the zero-debt steady state, $\bar{z} = 0$.[55]

B.4 Standard Monetary Rule

Proof of Lemma 4. Let the monetary authority with full commitment follow a constant money growth rate rule: $\tilde{\sigma}_t = \sigma \geq \beta - 1$. The Markov-Perfect equilibrium of the game is associated with a solution of the following dynamic programming problem of the fiscal authority:

$$V^F(z_{t-1}) = \max_{\tau_t, z_t}\left\{\alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma)}\right] + \beta V^F(z_t)\right\},$$

[54] As discussed in Martin (2009), various models with lack of commitment have been found to feature co-existence of differentiable and non-differentiable equilibria. The latter are characterized by having discontinuous decision rules that arise as an artifact of the infinite horizon and mimic behavior under trigger-type strategies.

[55] This finding echoes the analysis of fiscal policy under discretion with real debt in Debortoli and Nunes (2013). Debortoli and Nunes (2013) highlighted the importance of endogenous choice of government spending in generating smooth debt dynamics. A similar effect in our case is instead due to the adjustment of cash good consumption.

subject to

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma}\right]z_t-\alpha(1-\tau_t)-(1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma)}+\Phi=\left[\frac{(1-\alpha)\beta}{1+\sigma}\right]z_{t-1}. \tag{B.16}$$

The first-order envelope conditions result in the following expressions for all $t\geq 0$:

$$1=(1+\theta_t)\left[\alpha(1-\tau_t)+(1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma)}\right], \tag{B.17}$$

$$0=\theta_{t+1}-\theta_t, \tag{B.18}$$

where $\theta_t\geq 0$ is the Lagrange multiplier associated with (B.16). (B.18) implies $\theta_t=\bar{\theta}$ for all $t\geq 0$. Hence, starting from $t=0$, $\tau_t=\bar{\tau}$ as can be seen from (B.17). Importantly, using the implementability constraint (B.16) we get that the debt-to-money ratio is equal to its initial outstanding value, i.e., $\bar{z}=z_{-1}$.

When the constant money growth rate specified by the rule is $\sigma=\beta-1$, conditions (B.17) and (B.18) imply $\tau_t=\bar{\tau}\equiv\bar{\theta}/(1+\bar{\theta})$. The Lagrange multiplier $\bar{\theta}\geq 0$, in turn, is implicitly determined by the following equation derived from (B.16):

$$z_{-1}=\frac{1}{(1-\alpha)(1-\beta)}\left[\Phi-\frac{1}{1+\bar{\theta}}\right]. \tag{B.19}$$

This characterization coincides with the tail policy induced by the choice of a consolidated government under full commitment for $t\geq 1$, as described in Proposition 5. In particular, comparing (B.19) and (B.10) shows that $\bar{\theta}=\lambda$ when z_{-1} is equal to the long-run debt-to-money ratio z^* in the Ramsey plan. The corresponding tax rate and money growth rates also match. Therefore, the equilibrium of the game described in Definition 3 coincides with the Ramsey tail that has a stationary level of debt $z^*=z_{-1}$.

Proof of Proposition 6. Let the monetary authority with full commitment follow the standard rule such that: $\tilde{\sigma}_0=\sigma_0^*$ and $\tilde{\sigma}_t=\beta-1$ for all $t\geq 1$.

Lemma 4 implies that the fiscal choice in period 0 and the associated level of debt pin down a stationary tail outcome that prevails starting from period 1 onward. It is straightforward to see that as long as the fiscal choice in period 0 coincides with τ_0^*, the entire outcome coincides with the Ramsey plan. Recall that the Ramsey plan is the best equilibrium that one could attain when designing policy in period 0. Hence, fiscal authority would indeed pick $\tilde{\tau}_0=\tau_0^*$.

B.5 Strategic Monetary Rule

Proof of Lemma 5. Recall design of the strategic rule. For all $z_{t-1}\geq 0$,

– if $\tau_t=\tau^*(z_{t-1})$, then $\varrho^{Mk}(z_{t-1},\tau_t)=\beta-1$,

– if $\tau_t \neq \tau^*(z_{t-1})$ and $\tau_t \in \mathrm{T}(z_{t-1})$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \sigma$, such that:

$$V^{Fk}(z_{t-1}, \tau_t) = W^{ta}(z_{t-1}), \tag{B.20}$$

where

$$\mathrm{T}(z_{t-1}) = \{\tau \ \mid V^{Fr}(z_{t-1}, \tau) \geq V^{Fk}(z_{t-1}, \tau^*(z_{t-1})) = W^{ta}(z_{t-1})\}. \tag{B.21}$$

Given $z_{-1} > 0$, if credible, the strategic monetary rule is designed to implement a stationary equilibrium outcome with a constant growth rate of money, a constant tax rate, and debt sustained at the initial level:

$$\tau_t = \tau^*(z_{-1}) \qquad \sigma_t = \beta - 1 \qquad z_t = z_{-1}, \quad \forall t \geq 0, \tag{B.22}$$

The value of both the monetary and fiscal authorities equals the welfare associated with the stationary equilibrium, $W^{ta}(z_{-1})$.

To prove that off-equilibrium threats are well-defined, consider the following sequential policy decision: the fiscal authority chooses $\tau \in \mathrm{T}(z_{-1})$, then the central bank can:

– implement the equilibrium money growth rate equal to $\beta - 1$, in which case the welfare to the fiscal authority is lower than at the desired equilibrium path by virtue of Lemma 4:

$$V^{Fk}(\tau, z_{-1}) \leq W^{ta}(z_{-1}),$$

– or, it can implement $\sigma^d = \sigma^r(\tau, z_{-1}) \geq \beta - 1$, in which case by definition of $\mathrm{T}(z_{-1})$:[56]

$$V^{Fk}(\tau, z_{-1}) \geq W^{ta}(z_{-1}).$$

By continuity, there is $\sigma \in [\sigma^*, \sigma^d]$ s.t.: $V^{Fk}(\tau, z_{-1}) = W^{ta}(z_{-1})$. This expression implicitly defines $\varrho^{Mk}(z_{-1}, \tau)$ for $\tau \in \mathrm{T}(z_{-1})$.

We now follow Section 3.3.1 to characterize the credibility cut-off given z_{-1}. By design, the value of the monetary authority (on and off equilibrium) conditional on *keeping* is:

$$V^{Mk}(z_{t-1}, \tau_t) = W^{ta}(z_{t-1}), \tag{B.23}$$

and the value (off equilibrium) conditional on *reneging* is:

$$V^{Mr}(z_{t-1}, \tau_t) = \max_{\sigma, z} U(\tau, \sigma) + \beta V^M(z, \varrho^F(\cdot)). \tag{B.24}$$

The rule is credible at (z_{t-1}, τ_t) if and only if

$$\kappa \geq \Delta(z_{t-1}, \tau_t) = V^{Mr}(z_{t-1}, \tau_t) - V^{Mk}(z_{t-1}, \tau_t) = V^{Mr}(z_{t-1}, \tau_t) - W^{ta}(z_{t-1}), \tag{B.25}$$

[56] Formally, $\sigma^r(\cdot) = \mathrm{argmax}_\sigma V^M(\cdot)$.

and at z_{t-1} if and only if

$$\kappa \geq \max_{\tau_t \in \mathrm{T}(z_{t-1})} \Delta(z_{t-1}, \tau_t) \tag{B.26}$$

In particular,

$$\max_{\tau_t} V^{Mr}(z_{t-1}, \tau_t) = \max_{\tau_t} \max_{\sigma_t, z_t} U(\tau_t, \sigma_t) + \beta V^M(z_t, \varrho^F(\cdot)) \tag{B.27}$$

$$\leq W^{rp}(z_t) = \max_{\tau_t, \sigma_t, z_t} U(\tau_t, \sigma_t) + \beta W^{ta}(z_t), \tag{B.28}$$

where the inequality comes from the fact that the highest welfare that can be reached given z_{t-1} is induced by a Ramsey plan; see Proposition 5.

We form and then verify the following conjecture: if ϱ^{Mk} is credible at $z_{-1} \geq 0$, then it is credible at $0 \leq z'_{-1} \leq z_{-1}$. Under the conjecture, and if the continuation debt level z_t of a Ramsey plan given z_{t-1} satisfies $z_t \leq z_{t-1}$,

$$\max_{\tau_t} V^{Mr}(z_{t-1}, \tau_t) = W^{rp}(z_{t-1}). \tag{B.29}$$

Indeed, (i) $\tau^{ra}(z_{t-1})$ the "reset" Ramsey deviation belongs to $\mathrm{T}(z_{t-1})$. (ii) given $\tau^{ra}(z_{t-1})$, the maximum welfare to the monetary authority is reached if the central bank reoptimizes, implements $\sigma^{ra}(z_{t-1})$, and induces $z_t = z^{ra}(z_{t-1})$ with continuation welfare $W^{ta}(z_t)$. (iii) if the rule is credible today, then the tail allocation is credible at z_t if $z_t < z_{t-1}$, in which case the continuation welfare in (B.27) is $V^M(z_t, \varrho^F(z_t)) = W^{ta}(z_t)$ since under these conditions $\varrho^F(z_t) = \tau^{ta}(z_t)$. Overall, the strategic rule is credible at z_{t-1} if and only if:

$$\kappa \geq \tilde{\kappa}(z_{-1}) = W^{rp}(z_{t-1}) - W^{ta}(z_{t-1}), \tag{B.30}$$

and the continuation level of debt under the Ramsey plan is $z_t \leq z_{t-1}$. The credibility cut-off writes then:

$$\bar{\kappa}^{ta}(z_{t-1}) = \tilde{\kappa}(z_{-1}) = W^{rp}(z_{t-1}) - W^{ta}(z_{t-1}). \tag{B.31}$$

It is left to verify the conjecture above. Consider the following program:

$$W(z_{-1}) = \max_{\tau, \sigma, z} U(\tau, \sigma) + \beta W^{ta}(z) \tag{B.32}$$

subject to the implementability constraint (3.7) and possibly an additional constraint $z = z_{-1}$. Note $\lambda > 0$ and $\mu > 0$ the respective Lagrange multipliers. With both constraints, $W(z_{-1}) = W^{ta}(z_{-1})$ and with the implementability constraint only, $W(z_{-1}) = W^{rp}(z_{-1})$. Then:

$$\frac{d\bar{\kappa}^{ta}(z_{-1})}{dz_{-1}} = \frac{dW^{rp}(z_{-1})}{dz_{-1}} - \frac{dW^{ta}(z_{-1})}{dz_{-1}}, \tag{B.33}$$

$$= -\lambda^{ra} \frac{(1-\alpha)\beta}{1+\sigma^{ra}} - \big(- \lambda^{ta}(1-\alpha) - \mu^{ta} \big), \tag{B.34}$$

where the second equality comes from the envelope conditions of each program. Reorganizing,

$$\frac{d\bar{\kappa}^{ta}(z_{-1})}{dz_{-1}} = (1-\alpha)\Big[\lambda^{ta} - \lambda^{ra}\frac{\beta}{1+\sigma^{ra}}\Big] + \mu^{ta}. \tag{B.35}$$

Since $\lambda^{ta} \geq \lambda^{ra}$ and $\sigma^{ra} > \beta - 1$, one gets $\frac{d\bar{\kappa}^{ta}(z_{-1})}{dz_{-1}} \geq 0$.

Proof of Proposition 7. Strategic rule credibly implements the Ramsey plan as an outcome of the game if and only if it credibly implements the Ramsey tail. In particular, note that provided the Ramsey tail is credible, and strategic rule in period t is specified according to (p.1'), there is no better choice for the fiscal authority in period 0 than to pick $\tau_0 = \tau_0^*$. Hence, the credibility cut-off of the strategic rule that implements the Ramsey plan is tied to the credibility of supporting the associated tail:

$$\bar{\kappa}_2(z_{-1}) = \bar{\kappa}^{ta}(z_0^*),$$

where $z_0^*(z_{-1})$ is the tail level of debt of the Ramsey plan. Given Lemma 5 and the fact that z^* is increasing in z_{-1}, one can see that $\bar{\kappa}_2$ is also increasing in z_{-1}.

C Credibility of Strategic Rule: Additional Factors

C.1 Debt characteristics

Before proceeding with a proof of Proposition 8, this appendix describes model changes resulting from extending the cash-credit model by introducing real government debt with long maturity.

Economic Environment

We consider economic environment that is otherwise identical to the one described in Section 3 but with long-term government debt indexed to inflation. Let b_t be a perpetual bond with payoffs decaying at the exponential rate $\rho \in [0,1]$, as, e.g., in Woodford (2001). Moreover, the payoffs are indexed to inflation, which effectively makes them fixed in real terms. The consolidated government budget constraint (3.5) becomes:

$$q_t P_{t+1} b_t + M_t + P_t \tau_t (1 - l_t) = P_t g + (P_t + \rho q_t P_{t+1}) b_{t-1} + M_{t-1}. \tag{C.1}$$

Adjusting the household budget constraint (3.2) accordingly results in the following Euler equation instead of (B.3):

$$u_{c,t} = \beta \underbrace{\frac{(P_{t+1} + \rho q_{t+1} P_{t+2})}{q_t P_{t+1}} \frac{P_t}{P_{t+1}}}_{\text{one-period real rate}} u_{c,t+1}. \tag{C.2}$$

The maturity of long-term debt is conveniently characterized by the Macaulay duration. In particular, we compute the weighted average term to maturity of bond payoffs. The steady-state duration of debt can be shown to be equal to $x = (1-\beta\rho)^{-1}$. Note that one-period debt is a special case with $\rho = 0$. More generally, the higher is ρ, the longer is the duration of debt. The case with $\rho = 1$ corresponds to consol bonds that have equal payoffs throughout time.

The changes of structural equations described above lead to a change of the implementability constraint that characterizes competitive equilibria in terms of policy instruments. The implementability constraint (3.7) becomes:

$$\beta\left[\frac{\gamma(P_{t+1}+\rho q_{t+1}P_{t+2})}{(1-\tau_{t+1})P_{t+1}}\right]b_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{\gamma(P_t+\rho q_t P_{t+1})}{(1-\tau_t)P_t}\right]b_{t-1}, \quad \text{(C.3)}$$

with the following additional recursive equation:

$$q_t = \beta\frac{(1-\tau_t)}{(1-\tau_{t+1})}\frac{P_t}{P_{t+1}}\frac{(P_{t+1}+\rho q_{t+1}P_{t+2})}{P_{t+1}}. \quad \text{(C.4)}$$

Iterating forward (C.3) and (C.4) starting from $t = 0$, we get the following intertemporal implementability constraint:

$$\sum_{t=0}^{\infty}\beta^t\left\{\Phi - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right\} = \gamma b_{-1}\sum_{t=0}^{\infty}\left\{\frac{(\rho\beta)^t}{1-\tau_t}\right\} \quad \text{(C.5)}$$

Proof of Proposition 8

The Ramsey policy problem is defined as in Appendix B.2 with (C.5) as a constraint instead of (B.6). The first-order conditions are:

$$1 = (1+\lambda)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right] - \lambda\frac{\gamma\rho^t}{1-\tau_t}b_{-1}, \qquad \forall t \geq 0, \quad \text{(C.6)}$$

$$1 = (1+\lambda)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}, \qquad \forall t \geq 0, \quad \text{(C.7)}$$

and the implementability constraint (C.5) can be rewritten as:

$$\frac{(1+\lambda)}{(1+2\lambda)(1-\beta)}\left[\Phi - \frac{1}{1+\lambda}\right] = \gamma b_{-1}\sum_{t=0}^{\infty}\left\{\frac{(\rho\beta)^t}{1-\tau_t}\right\}. \quad \text{(C.8)}$$

In general, the solution is constructed as follows. Guess λ and generate recursively the corresponding sequence $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ using (C.6) and (C.7). Stop if the implementability constraint (C.8) is satisfied. Otherwise adjust λ and repeat the procedure.

Consider a special case with consol bonds, $\rho = 1$. (C.6) and (C.7) imply $\tau_t = \bar{\tau} < \lambda/(1+\lambda)$ and $\sigma_t = \bar{\sigma} > \beta - 1$ for all $t \geq 0$. Furthermore, the implementability constraint implies that debt stays at its initial level, $b_{t-1} = b_{-1}$, for all $t \geq 1$. Hence, the Ramsey plan is time consistent when government bonds are in the form of inflation-indexed consol bonds.

C.2 Fiscal Hedging

Before proceeding with a proof of Proposition 9, this appendix describes model changes that arise when public consumption is no longer constant but changes over time.

Economic Environment

Public consumption g is assumed to be exogenous and follow a stochastic process with realizations of g at each period drawn from a discrete set with finite number N of possible states. These states are indexed with n and denoted as $\hat{g}_n$.

Preferences of the representative household instead of (3.4) are represented by the expected discounted value of utility flow from private consumption and leisure:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, d_t, l_t) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big(\alpha \log c_t + (1-\alpha) \log d_t + \gamma l_t\big), \tag{C.9}$$

where $\mathbb{E}_0$ is the mathematical expectation conditioned on information in period 0.

In every period, the representative household trades with the government a complete set of one-period nominal state-contingent bonds. Let $B_t(\hat{g}_n)$ denote the amount of bonds issued by the government in period t that pay one unit of account in the next period only if $g_{t+1} = \hat{g}_n$. The price of these bonds is denoted as $q_t(\hat{g}_n)$. Each period t there are markets for a complete set of N distinct bonds contingent on every realization of g in the next period. The market value of government debt portfolio at issuance is $\sum_{n=1}^{N} q_t(\hat{g}_n) B_t(\hat{g}_n)$. Hence, the government budget constraint (3.5) becomes:

$$\sum_{n=1}^{N} q_t(\hat{g}_n) B_t(\hat{g}_n) + M_t + P_t \tau_t (1 - l_t) = P_t g_t + B_{t-1}(g_t) + M_{t-1}. \tag{C.10}$$

Adjusting the household budget constraint (3.2) accordingly results in the following Euler equations instead of (B.2) and (B.3):

$$u_{c,t} = \beta \mathbb{E}_t \left\{ \frac{P_t}{P_{t+1}} u_{d,t+1} \right\} \tag{C.11}$$

$$u_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{q_t(g_{t+1})} \frac{P_t}{P_{t+1}} u_{c,t+1} \right\}. \tag{C.12}$$

Given changes of the economic environment described above, the implementability constraint (3.7) becomes:

$$\beta \mathbb{E}_t \left\{ \left[\frac{(1-\alpha)\beta}{1+\sigma_{t+1}} \right] z_t(g_{t+1}) \right\} - \alpha(1-\tau_t) - (1-\alpha)\beta \frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi_t = \left[\frac{(1-\alpha)\beta}{1+\sigma_t} \right] z_{t-1}(g_t), \tag{C.13}$$

where the bond-to-money ratio z is now a state-contingent variable and the composite variable Φ is now time-varying due to changes in g. Iterating forward (C.13) starting from $t = 0$, we get the intertemporal implementability constraint:

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\Phi_t-\alpha(1-\tau_t)-(1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right]\right\}=\frac{(1-\alpha)\beta}{(1+\sigma_0)}z_{-1}(g_0). \tag{C.14}$$

Ramsey Equilibrium

Consider a consolidated government with a commitment technology to pursue a dynamic policy plan defined at $t=0$ for all possible future paths (histories) of public consumption.

Proposition 10. *Given positive initial government debt $z_{-1}(g_0)>0$, the Ramsey plan has the following characteristics:*

- *"tail policy and allocation" for all $t\geq 1$:*
  - *Constant path of consumption of both cash and credit goods. Supporting this allocation is a perfectly smooth path of tax rates that stay constant across time and states. Similarly, the money growth rate follows the Friedman rule: $\sigma_t^*=\beta-1$.*
  - *Leisure varies to ensure production output matches changes in public consumption:*

$$1-l_t^*=\bar{c}+\bar{d}+g_t. \tag{C.15}$$

  - *State-contingent bonds are issued to perfectly hedge government budget against tax revenue variation, which results from changing labor income that shifts the tax base:*

$$z_{t-1}^*(g_t\mid g^{t-1})=\frac{1}{1-\alpha}\left[\frac{-1}{(1+\lambda)(1-\beta)}+\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\beta^{t+k}\Phi_{t+k}\right\}\right], \tag{C.16}$$

  where $g^{t-1}\equiv\{g_0,g_1,\dots,g_{t-1}\}$ is the history of all the past realizations up to and including period $t-1$, and λ is a Lagrange multiplier on equation (C.14)*.*

- *"initial policy and allocation": at $t=0$, policy choices (and allocation) differ from tail levels. In particular, $\tau_0^*<\tau_1^*$ and $\sigma_0^*>\sigma_1^*$; the resulting debt dynamics is such that*

$$\mathbb{E}_0 z_0^*(g_1\mid g_0)<z_{-1}(g_0). \tag{C.17}$$

Proof. The Ramsey policy problem determines a whole sequence of policy instruments to maximize households' welfare subject to the implementability constraint, given $z_{-1}(g_0)>0$:

$$\max_{\{\tau_t,\sigma_t\}}\quad \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\alpha\left[\log(1-\tau_t)-(1-\tau_t)\right]+(1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right)-\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right]\right\}$$

subject to (C.14)

The first-order conditions for every history g^t are identical to their counterparts (B.7), (B.8), and (B.9) from the deterministic economy.

Hence, $\tau_t = \bar{\tau} \equiv \lambda/(1+\lambda)$ and $\sigma_t = \bar{\sigma} \equiv \beta - 1$ for all $t \geq 1$, where $\lambda \geq 0$ is the Lagrange multiplier attached to (C.14). Using (B.1), (B.5), it is straightforward to get that private consumption of both goods is constant for all $t \geq 1$. This result coupled with (3.1) gives (C.15). Additionally, iterating (C.13) forward starting from any history g^t and using (B.7) gives (C.16)

Contrasting first-order conditions for $t = 0$ and $t \geq 1$ yields $\tau_0 < \bar{\tau}$ and $\sigma_0 > \bar{\sigma}$. Finally, substituting (B.7) into (C.14) implies (C.17). ∎

The characterization in Proposition 10 illustrates well-established properties of optimal policy in a stochastic economy with complete markets. First, as implied by equation (C.15), the optimal allocation of consumption and leisure only depends on the contemporaneous realization of public consumption g, whereas a specific history of past realizations of g leading to the current outcome is irrelevant.

Second, as implied by equation (C.16), the amount of issued government debt lacks any intrinsic history dependence. The stream of past government consumption g^{t-1} affects $z^*_{t-1}(g_t)$ only to the extent of its effect on the evolution of future realizations of g. Hence, when government spending evolves as a Markov process, the portfolio of state-contingent debt issuance stays constant over time: $z^*_{t-1}(\hat{g}_n \mid g^{t-1}) = z^*(\hat{g}_n)$ for all $n \in \{1, \ldots, N\}$ and $t \geq 1$.

Third, given that Φ depends negatively on g, equation (C.16) shows that debt issuance is structured so as to make outstanding liabilities of the government low during times with high public spending. This is an illustration of fiscal hedging embedded into the Ramsey plan with complete markets: debt management is what allows the government to insulate its budget from the need to adjust the tax rate and the growth rate of money.

Using the characterization above, we can establish a partial isomorphism between the described stochastic economy and a deterministic doppelgänger. To this end, define a (rescaled) conditional expectation at period t of the discounted stream of public consumption as G_t:

$$G_t \equiv (1-\beta)\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s g_{t+s}; \tag{C.18}$$

that is, G_t is the Chisini mean of the expected discounted sum of public consumption starting from t onward.

Corollary 1. *Consider an economy with stochastic public spending and an identical economy where the path of public consumption stays at the constant level $g_t = G_0$ for all t.*

- *The Ramsey plan for policy choices and private consumption in these economies is the same.*
- *Lifetime welfare of the households corresponding to these Ramsey plans is the same.*

Proof. The (expected) path of public consumption affects optimal policy decisions characterized in Proposition 10 through its role in the intertemporal budget constraint (C.14). Note that each expected period-t value of public consumption enters (C.14) additively (as a component of Φ_t). One can then see that what matters for policy choices is the overall expected discounted sum G_0.

Furthermore, as can be seen from Propositions 5 and 10, optimal policy choices depend on G_0 in exactly the same way as optimal choices in the deterministic economy depend on the underlying constant value of public spending. Hence, if G_0 and g in the two economies coincide, then so do policy choices (and consumption levels of both goods; see Lemma 2). In turn, identical sequences of policy choices imply the identical sequences of indirect utility flows. Therefore, expected lifetime welfare starting from period 0 in the two economies is the same.[57] ∎

Welfare under the Ramsey plan in both economies can be characterized as an outcome of the following problem of maximizing the initial and tail components:

$$W^{rp}(z_{-1}, G_0) = \max_{\tau_0, \sigma_0, z_0} U(\tau_0, \sigma_0) + \beta W^{ta}(z_0, G_0), \tag{C.19}$$

where maximization over τ_0, σ_0, and z_0 is subject to the implementability constraint (3.7) at $t = 0$ with the tail growth rate of money $\sigma_1 = \beta - 1$ and $g = G_0$. Furthermore, $W^{ta}(z, G)$ is the continuation welfare under the tail policy at $z_0 = z$:

$$W^{ta}(z_0, G_0) = \frac{1}{1-\beta} U(\tau^*(z_0, G_0), \sigma^*),$$

where $\tau^*(z, G)$ is the tax rate required to sustain debt at z permanently when monetary policy is set according to the Friedman rule $\sigma^* = \beta - 1$ and public consumption equals $G_0 = G$.

Next, we define the highest possible level of debt embedded in the Ramsey plan:

$$z^*_{\max} \equiv \max_{\{g^t\}_{t=1}^{\infty}} z^*_{t-1}(g^t), \tag{C.20}$$

and, analogously, the lowest possible level of debt embedded in the Ramsey plan:

$$z^*_{\min} \equiv \min_{\{g^t\}_{t=1}^{\infty}} z^*_{t-1}(g^t). \tag{C.21}$$

One can also find histories g^t with the highest, $G_{\max}$, and lowest, $G_{\min}$, values of G_t. Importantly, (C.16) implies that the histories behind $G_{\max}$ are the same as the ones behind $z^*_{\min}$ and vice versa. More generally, ($z^*_{t-1}(g^t)$,$G_t(g^t)$) lie on a straight line between ($z^*_{\min}$,$G_{\max}$) and ($z^*_{\max}$,$G_{\min}$).

Analysis of policy isomorphism and underlying debt structure described above are key in adjusting design of the strategic rule and characterizing the associated credibility cut-off.

[57] Note that welfare of the representative household in a given period of time also depends on the particular realization of public consumption. The indirect utility function omits the corresponding additive terms because they do not affect policy choice. Importantly, the expected discounted sum of these omitted terms starting from period 0 onward—which is what matters for the lifetime welfare—is equal in the two economies when $g = G_0$.

Strategic Rule

As in the case with constant public consumption, the promise to follow the Friedman rule conditional on tax smoothing and the associated threats are imposed starting from period 1. Given any z_{t-1} and G_t, as long as fiscal policy is set to smooth taxes, the monetary rule prescribes to set the money growth rate according to the Friedman rule:

(p.1) if $\tau_t = \tau^*(z_{t-1}, G_t)$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \beta - 1, \quad \forall t \geq 1$.

The associated values to both the fiscal authority, $V^{Fk}(z_{t-1}, G_t, \tau^*(z_{t-1}, G_t))$, and monetary authority, $V^{Mk}(z_{t-1}, G_t, \tau^*(z_{t-1}, G_t))$, coincide with the welfare under the tail segment of a Ramsey plan, $W^{ta}(z_{t-1}, G_t)$.

The central bank calibrates its off-equilibrium reaction to make the fiscal incentive constraint binding for all deviations from $\tau^*(z, G)$:

(p.2) if $\tau_t \neq \tau^*(z_{t-1}, G_t)$ and $\tau_t \in \mathrm{T}(z_{t-1}, G_t)$, then $\varrho^{Mk}(z_{t-1}, G_t, \tau_t) = \sigma$, such that:

$$V^{Fk}(z_{t-1}, G_t, \tau_t) = W^{ta}(z_{t-1}, G_t), \tag{C.22}$$

where the set of fiscal influence $\mathrm{T}(z_{t-1}, G_t)$ is formally defined as:

$$\mathrm{T}(z_{t-1}, G_t) = \{\tau_t \; | V^{Fr}(z_{t-1}, G_t, \tau_t) \geq V^{Fk}(z_{t-1}, G_t, \tau^*(z_{t-1}, G_t)) = W^{ta}(z_{t-1}, G_t)\}. \tag{C.23}$$

In period $t = 0$, given initial outstanding debt z_{-1}, the rule prescribes to set the money growth rate at the initial value prescribed by the Ramsey plan when the fiscal policy is set accordingly:

(p.1') if $\tau_0 = \tau_0^*$, then $\varrho_0^{Mk}(z_{-1}, \tau_0) = \sigma_0^*$.

Proof of Proposition 9

Just like in the case with constant public consumption, the fiscal authority finds it optimal to set the initial tax rate at τ_0^* if the degree of monetary commitment is high enough to support the Ramsey tail as the continuation path. However, the credibility cut-off is no longer pinned down by comparing welfare levels associated with the Ramsey tail and a "reset" plan starting from the constant tail level of debt at which the Ramsey plan settles in period 1.

[**1**] Indeed, variation in public spending makes the tail level of debt vary according to (C.16). Hence, the attractiveness of the "reset" plan changes over time. Importantly, histories g^t associated with the highest outstanding debt and the lowest expected discounted flow of public consumption are associated with the most tempting "reset" Ramsey plan:

$$(z^*_{\max}, G_{\min}) = \arg \max_{\{g^t\}_{t=1}^{\infty}} W^{rp}(z^*_{t-1}(g^t), G_t(g^t)).$$

To see this, recall that each history g^t is positioned on a straight line in the space $(z^*_{t-1}(g^t),G_t(g^t))$ with $(z^*_{\min},G_{\max})$ and $(z^*_{\max},G_{\min})$ as end points. Therefore, it suffices to show that for any pair of points on this line, $W^{rp}(z^*_{t-1}(g^t), G_t(g^t))$ is always higher at the point that has a higher outstanding debt (and lower expected discounted flow of public consumption).

Let there be two histories g^k and g^s such that $z^*_{k-1}(g^k) < z^*_{s-1}(g^s)$. Then,

$$\begin{aligned}
&W^{rp}(z^*_{k-1}(g^k), G_k(g^k)) - W^{rp}(z^*_{s-1}(g^s), G_s(g^s)) \leq \\
&\qquad L^{rp}(\mathbf{x}^r(g^k), \lambda^r(g^k), z^*_{k-1}(g^k), G_k(g^k)) - L^{rp}(\mathbf{x}^r(g^s), \lambda^r(g^s), z^*_{s-1}(g^s), G_s(g^s)) = \\
&\qquad\qquad - \underbrace{\lambda^r(g^s)}_{>0}\, \underbrace{g(\mathbf{x}^r(g^k), z^*_{s-1}(g^s), G_s(g^s))}_{<>0?}, \qquad \text{(C.24)}
\end{aligned}$$

where the inequality follow from an envelope-type argument (see, e.g., Theorem 1 in Chavas (2001)), L^{rp} denotes the Lagrangian corresponding to the Ramsey policy problem, $\mathbf{x}^r$ denotes sequences of optimal reset policy choices, λ^r denotes the optimal reset value of Lagrange multiplier corresponding to the intertemporal implementability condition, and g is the function that represents the intertemporal implementability constraint: $g(\mathbf{x}(g^t), z_{t-1}(g^t), G_t(g^t)) = 0$.

Next, using characterization in Proposition 10 to capture the effect of the optimal reset policies and the link (C.16) between $z^*_{t-1}(g^t)$ and $G_t(g^t)$ built into the desired plan, we get:

$$\begin{aligned}
0 =& g(\mathbf{x}^r(g^t), z^*_{t-1}(g^t), G_t(g^t)) \\
=& \tilde{g}(\mathbf{x}^r(g^t), z^*_{t-1}(g^t)) \\
=& z^*_{t-1}(g^t) \underbrace{\left[1 - \frac{\beta}{(1+\sigma^r_t(g^t))}\right]}_{>0} - z_{-1}(g_0)\left[\frac{\beta}{(1+\sigma^*_0)}\right] + \frac{1}{1-\alpha}\left[\mathbb{E}_0\Big\{\sum_{l=0}^{\infty}\beta^l \Phi_l\Big\} - \frac{1}{(1+\lambda^r(g^t))(1-\beta)}\right],
\end{aligned}$$

where σ^r_t denotes the initial reset growth rate of money and λ^r captures the remaining effect of x^r in a concise way. Hence, $g(\mathbf{x}^r(g^k), z^*_{s-1}(g^s), G_s(g^s)) > 0$. In turn, (C.24) implies

$$W^{rp}(z^*_{k-1}(g^k), G_k(g^k)) - W^{rp}(z^*_{s-1}(g^s), G_s(g^s)) < 0.$$

[2] At the same time, the tail levels of policy choices remain unchanged across histories. Hence, by design, the value of keeping the strategic rule does not vary with public consumption and coincides with $W^{ta}(\bar{z}^*_0, G_0)$, where $\bar{z}^*_0$ is the tail debt level in the deterministic doppelgänger economy that solves the problem (C.19) given z_{-1} and public consumption equal G_0 at all times.

[3] The credibility cut-off is pinned down by the histories that are associated with the largest relative gains from renouncing the rule. Therefore, it is pinned down by comparing welfare levels associated with the tail and the "reset" plan at the histories g^t where outstanding debt is at its highest, $z^*_{\max}$, and the expected flow of public consumption is at its lowest, $G_{\min}$:

$$\bar{\kappa}_2(z_{-1}) = W^{rp}(z^*_{\max}, G_{\min}) - W^{ta}(\bar{z}^*_0, G_0),$$

where both $z^*_{\max}$ and $\bar{z}^*_0$ depend on z_{-1}.

C.3 Money Growth Rate

This appendix describes model changes brought by considering the adjusted strategic rule that targets the tail money growth rate different from the one prescribed by the Friedman rule.

Desired Equilibrium

When the targeted money growth rate is different from the Friedman rule prescription, the corresponding stationary allocation is different from the tail optimal allocation. We use $\tilde{\tau}(z, \tilde{\sigma})$ to denote the tax rate required to sustain debt at the level z indefinitely when monetary authority sets the tail money growth rate equal to $\tilde{\sigma} \geq \beta - 1$. Let W^s be the welfare of the representative household sustained by this policy mix:

$$W^s(z, \tilde{\sigma}) = \frac{1}{1-\beta} U\big(\tilde{\tau}(z, \tilde{\sigma}), \tilde{\sigma}\big), \tag{C.25}$$

where the superscript s stands for *stationary*. Note that W^s is a generalization of W^{ta} defined in (3.11): $W^s(z, \beta - 1) = W^{ta}(z)$.

Strategic Rule

The strategic rule is adjusted as follows. First, the promise to keep money growth at its targeted level is conditional on smoothing taxes at a different level. Specifically, the first property reads:

(p.1) if $\tau = \tilde{\tau}(z, \tilde{\sigma})$, then $\varrho^{Mk}(z, \tau) = \tilde{\sigma}$.

Second, the associated off-equilibrium threats are calibrated based on the different welfare level. In particular, the second property reads:

(p.2) if $\tau \in \mathrm{T}(z, \tilde{\sigma})$ and $\tau \neq \tilde{\tau}(z, \tilde{\sigma})$, then $\varrho^{Mk}(z, \tau) = \sigma$ such that:

$$V^{Fk}(z, \tau) = W^s(z, \tilde{\sigma}), \tag{C.26}$$

where the fiscal authority considers the following set of deviations from $\tilde{\tau}(z, \tilde{\sigma})$ at z:

$$\mathrm{T}(z, \tilde{\sigma}) = \big\{\tau \mid V^{Fr}(z, \tau) \geq W^s(z, \tilde{\sigma})\big\}. \tag{C.27}$$

Credibility Cut-off

Finally, characterization of the credibility cut-off, $\bar{\kappa}^{ta}(z, \tilde{\sigma})$, is affected by the change of the monetary rule as follows:

$$\bar{\kappa}^{ta}(z, \tilde{\sigma}) \leq \max_{\tau, \sigma, z'} U(\tau, \sigma) + \beta W^s(z', \tilde{\sigma}) - W^s(z, \tilde{\sigma}), \tag{C.28}$$

where the optimization program is subject to (3.7) with the outstanding level of debt z and continuation growth rate of money $\tilde{\sigma}$. The relation in (C.28) holds with equality if the choice of z' is such that $z' \leq z$.