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# Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design

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# Abstract

Many central banks are considering issuing a central bank digital currency (CBDC). How will the CBDC affect the macroeconomy? Will its design matter? To answer these questions, we theoretically and quantitatively assess the effects of a CBDC on consumption, banking and welfare. Our model captures the competition between different means of payment and incorporates a novel general equilibrium feedback effect from transactions to deposit creation. The general equilibrium effects of a CBDC are decomposed into three channels: payment efficiency, price effects and bank funding costs. We show that a cash-like CBDC is more effective than a deposit-like CBDC in promoting consumption and welfare. Interestingly, a cash-like CBDC can also crowd in banking, even in the absence of bank market power. In a calibrated model, at the maximum, a cash-like CBDC can increase bank intermediation by 10.2% and welfare by 0.059% and capture up to 23.3% of the payment market.



*Topics: Digital currencies and fintech; Monetary policy; Monetary policy framework*

*JEL codes: E50, E58*

# 1 Introduction

Several central banks are considering issuing a central bank digital currency (CBDC) for retail payments.[1] The key motivations include domestic payments efficiency and safety, financial inclusion, and financial stability. It is commonly believed that the introduction of a CBDC can have profound implications for the efficiency and stability of the macroeconomy. In particular, one frequently raised policy concern is that a CBDC, by competing with banks for deposit funding, could crowd out banking and reduce output—a point discussed in recent reports by the International Monetary Fund (Mancini-Griffoli et al., 2018) and the report by the Committee on Payments and Market Infrastructures of the Bank for International Settlements (2018). To inform this policy discussion, this paper provides a theoretical and quantitative assessment. We show that the concern that a CBDC could crowd out banking is not warranted when an appropriate design is adopted.

We develop a model of payments and banking to evaluate the general equilibrium effects of introducing a CBDC on bank intermediation and retail transactions. In our model, banks finance investment by issuing deposits to households. Goods produced from the investment are sold to households in a frictional retail market. In the absence of perfect credit, households bring a portfolio of cash and deposits to finance these trades. As in the real world, the model features different types of transactions.[2] In type-1 transactions, cash is the only viable payment option (e.g., offline transactions). In type-2 transactions, cash is not viable and only deposits and credit are used (e.g., online transactions). In type-3 transactions, all payment instruments can be used (e.g., in most physical retail stores). As deposits are used in transactions, the implied liquidity premium lowers banks' funding costs.

Notice that our model incorporates two key features. The first feature is the direct competition between cash and deposits in type-3 transactions which generates some novel implications discussed below.[3] Another important feature of our model is that bank deposits are used to finance

[1]The Bank for International Settlements surveyed 65 central banks in 2020, covering 72% of the world population and 91% of world output. Of these central banks, 86% are engaged in work regarding a CBDC, 60% have started experiments or proofs-of-concept for a CBDC, and 14% have moved forward to development and pilot arrangements for a CBDC (see Boar and Wehrli, 2021).

[2]We follow Lucas and Stokey (1983) to model the constraints on payment options in different types of transactions. Our setup extends theirs to incorporate bank deposits that compete with cash and credit as a means of payments in type 3 transactions.

[3]Many existing models of CBDC and banking share a similar structure (e.g., Keister and Sanches (2019),

the investment in the production of goods that are traded in retail markets.[4] This new feature introduces a general equilibrium feedback effect from retail transactions to deposits creation. As discussed below, this feedback effect is quantitatively important but has so far been overlooked by the existing monetary and CBDC literature.

We investigate the effects of introducing different types of CBDCs: the cash-like, deposit-like and universal CBDC. The cash-like CBDC can be used in type-1 and type-3 transactions, while a deposit-like CBDC can be used in type-2 and type-3 transactions. The universal CBDC can be used in all transactions. We then show that the general equilibrium effects of introducing a CBDC can be decomposed into the following three channels. First, an interest-bearing CBDC lowers the opportunity costs of holding payment balances and increases payment efficiency, promoting aggregate demand for consumption and investment. We call this the *payment efficiency* channel. Second, the increase in aggregate demand raises the price of consumption goods, thereby reducing the quantity of trades in all types of transactions, including those where the CBDC is not used. We call this channel the *price effect*.[5] Third, the introduction of a CBDC may induce banks to raise the interest paid on deposits, increasing banks' funding costs and lowering consumption and investment. This is the *bank funding cost* (or disintermediation) channel.

Applying this decomposition, we demonstrate analytically that an interest-bearing CBDC can *crowd in* bank intermediation even when banks do not have any market power—a novel result in this literature. Furthermore, the effects depend crucially on the design of the CBDC. In particular, the introduction of a cash-like CBDC will promote consumption, banking and welfare. To understand this, note that an interest-bearing, cash-like CBDC lowers the opportunity costs of holding payment

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Williamson (2020a)). But, for simplicity, they often only consider type-1 and type-2 trades. Given that over 70% of trades in the US accept both cash and cards, we would argue that our setup is more consistent with the data. More importantly, this feature significantly changes the positive and normative implications of issuing a CBDC.

[4]Many firms in the retail and manufacturing industries rely on loans from deposit-taking financial institutions. According to the Survey on Financing and Growth of Small and Medium Enterprises in Canada, among those enterprises that requested debt financing in 2017, 93.4% in the manufacturing industry and 93% in the retail trade industry obtained their loans from domestic chartered banks or credit unions; i.e., deposit-taking institutions. (Source: https://bit.ly/3l21yuv).

[5]This channel is similar to that discussed in the literature on pecuniary externalities. For example, a similar price externality channel is studied in Chari and Phelan (2014). See also Kehoe and Levine (1993), Kiyotaki and Moore (1997), Lorenzoni (2008), Hart and Zingales (2011), and Chiu et al. (2018). While this channel is interesting, we show in Section 3 that this is not critical for our main findings. That is, the analytical results will not change when this channel is shut down.

balances. The direct effect is that households will buy more goods in transactions where a CBDC is used. An additional, indirect effect is that if banks are forced to raise the interest rate on deposits, then households will also buy more goods in transactions where deposits are used. In other words, a cash-like CBDC generates an interest-rate *spillover effect* from cash to non-cash transactions.[6] Through the payment efficiency channel, the higher consumption demand will induce banks to create more deposits to finance production in order to clear the goods market. This is the general equilibrium feedback effect from transactions to deposits creation that is neglected by the literature. We show that, when cash is used in type-3 transactions, the positive effect of introducing a cash-like CBDC through the payment efficiency channel can outweigh the other two channels and lead to higher consumption, intermediation and welfare.

In contrast, a deposit-like CBDC may not promote consumption and banking. First, this type of CBDC cannot be used to buy type-1 goods, so it cannot improve payment efficiency in these transactions. The payment efficiency channel is weaker here compared with a cash-like CBDC because, unlike deposits, cash is not interest bearing, so the introduction of a deposit-like CBDC cannot induce an endogenous reduction in the opportunity cost of holding cash. As a result, there are no interest-rate spillover effects from non-cash transactions to cash transactions. Second, the introduction of a deposit-like CBDC drives up the price of consumption goods and lowers type-1 consumption, further worsening the allocative efficiency. This explains why a deposit-like CBDC has smaller effect on aggregate consumption and banking.

In our quantitative exercise, we calibrate the model with the US data and assess how different types of CBDCs affect different transactions through different channels. As stated above, for a cash-like CBDC, the payment efficiency gains dominate the price and funding cost channels, leading to more bank intermediation. With a nominal interest rate of 5%, the interest rate that corresponds to the Friedman rule for CBDC, the CBDC demand is maximized, bank intermediation increases by 10.2%, retail consumption rises by 8.97%, and the CBDC captures 23.3% of the payment market. Again, the positive effect is due to the payment efficiency channel, with a positive spillover to other transactions through banks' endogenous responses. In contrast, a deposit-like CBDC leads to weaker payment efficiency gains. At the maximum, this type of CBDC can increase bank intermediation by 9.2% and consumption by 7.75%, and obtain 18.45% of the payment market.

[6] This highlights the importance of incorporating type-3 transactions because this spillover effect is overlooked in those existing models where type-3 transactions are not captured.

What is special about a cash-like CBDC? The allocative inefficiency in a monetary economy stems from the presence of liquidity constraints. It is well-known that efficiency can be restored/improved by running the Friedman rule. One common way to run the Friedman rule is to shrink the money supply to generate a deflation. This policy, however, is usually not deemed practical as central banks in reality choose to set a positive inflation target for various reasons. An alternative way to run the Friedman rule is to pay interest on money balances. The feasibility of this policy is constrained, however, by the fact that physical bank notes cannot bear interest. Hence, a cash-like CBDC can be a tool to achieve this objective: paying interest to reduce the opportunity costs of holding payment balances while the inflation target can remain positive. The unique feature of a cash-like CBDC is that (i) unlike deposits, it can be used in transactions currently only supported by cash (e.g., offline transactions), and (ii) unlike cash, it can bear an interest. The interest-bearing feature allows it to mitigate the consumption inefficiency associated with cash balances.

The discussion above takes a fixed money growth rate as given. Would CBDC still be welfare-improving when the money growth can be chosen optimally? We answer this question by extending the basic model, incorporating two realistic features suggested by the literature: demand for cash by foreigners and by the underground economy which conducts "illegal" activities (Schmitt-Grohé and Uribe (2010), Rogoff (2016)). We show that in this economy, without a CBDC, it is optimal to deviate from the Friedman rule. Moreover, even under the welfare-maximizing money growth rate, it is still optimal to introduce a CBDC with a positive interest rate to further increase welfare.

The economic insights behind the above results are completely new in the CBDC literature. When banks do not have market power, Keister and Sanches (2021) show that a deposit-like CBDC crowds out banking. Williamson (2021) finds a similar result in a slightly different setting. Andolfatto (2021) and Chiu et al. (2021) show that a deposit-like CBDC can crowd-in banking only when banks have market power. Relative to the existing literature, our paper shows that market power is not necessary for the "crowding-in" effect. In addition, we find that a cash-like CBDC is more effective in promoting banking and in increasing welfare.

More broadly, our paper complements the the fast-growing literature on CBDC. Related to our work, Agur et al. (2020) study the optimal design of a CBDC, whether a CBDC should resemble cash or bank deposits, in the presence of network effects. A deposit-like CBDC provides more security but leads to disintermediation, while a cash-like CBDC provides a higher degree of anonymity. In addition, other lines of research focus on the following aspects. Barrdear and Kumhof (2021), Davoodalhosseini (2021) and Jiang and Zhu (2021) study the monetary policy perspective; Brunnermeier and Niepelt (2019) and Niepelt (2020) derive some "equivalence" results; Chiu et al. (2020), Fernandez-Villaverde et al. (2020), Schilling et al. (2020), Keister and Monnet (2020), Monnet et al. (2020), and Williamson (2020b) examine financial stability implications; Berentsen and Schar (2018), Davoodalhosseini and Rivadenyra (2020), Mancini-Griffoli et al. (2018), Fung and Halaburda (2016), and Kumhof and Noone (2018) provide general policy discussions.[7]

This paper is organized as follows. In Section 2, we introduce a model with cash, cash-like CBDC, deposits and credit. We discuss the three channels through which a CBDC affects the economy. We also show that the cash-like CBDC can improve consumption, intermediation, and welfare. In Section 3, we study a deposit-like CBDC. Section 4 discusses policy-relevant design issues. In Section 5, we calibrate the model and evaluate the quantitative implications. Section 6 considers an extension with endogenous money growth. Section 7 discusses CBDC motivations and designs. Section 8 concludes. The omitted proofs, some details and several extensions are relegated to the appendix which is available online.

## 2 Model

Time is discrete and continues forever: $t = 0, 1, 2, ....$ Each period consists of two sub-periods with two markets that open sequentially. In the first sub-period, a frictional market opens and we call it AM. In the second sub-period, a Walrasian centralized market opens and we call it PM. The alternating market structure is based on Lagos and Wright (2005). In this economy, there is a measure one of infinitely lived households. In addition, in each PM, a measure one of new competitive bankers enter the economy and exit in the following PM. Furthermore, a measure one of new competitive retailers enter the economy in each AM and exit in the following PM. There are five goods: three consumption goods ($c_1$, $c_2$ and $c_3$) produced by retailers and consumed by households in the AM, one intermediate good produced by bankers in the AM and turned to one of the consumption goods by retailers in the AM, and a numeraire good $y$ produced by households and consumed by all agents in the PM. In the model, one should interpret a banker as a banker-

[7]There is also a related literature on private cryptocurrencies. See, for example, Biais et al. (2019), Chiu and Koeppl (2019), Cong et al. (2021), and the references therein. Other papers study currency competition with a focus on cryptocurrencies. See Ferandez-Villaverde and Sanches (2019) and Zhu and Hendry (2019).

producer pair that is engaged in both banking and intermediate-good production. One can easily introduce producers explicitly and separate these two activities.

In the PM, households work and produce the numeraire good that is subject to a constant marginal disutility of one. Households can consume the numeraire good in the PM. Alternatively, they can transfer these goods to young bankers who possess investment technology that converts $k$ units of the numeraire good into $F(k)$ units of the intermediate good in the next AM. Retailers convert the intermediate good into one of the consumption goods in the AM. The AM is frictional because perfect credit is not available. Different payment options are used to facilitate the exchange of consumption goods. Households can use credit, subject to a credit limit, $B \geq 0$, which is discussed below.[8] In addition, fiat money and deposits serve as means of payment.

Fiat money is supplied by the central bank. In our model, money represents both traditional banknotes (denoted as "cash" below) and a cash-like central bank digital currency (denoted as cash-like "CBDC" below). One difference between cash and a cash-like CBDC is that, for technological reasons, a central bank can pay interest only on the CBDC balances. We will start with a cash-like CBDC and then replace it with a deposit-like CBDC in the next section. We will elaborate on the their differences below.

A young banker in the PM issues deposits to finance the purchase of the numeraire good for investment. Bankers redeem deposits in the next PM, and their promises are backed by intermediate good sales in the AM. In the AM, deposits can be used by households as a medium of exchange. Also, retailers buy intermediate goods from bankers and convert them into consumption goods that can be sold to the households. We assume retailers trade with bankers on credit which can be enforced by the bankers. In the next PM, retailers settle their debt, consume their profit and exit.

The presence of trading frictions in the AM implies that households may not be able to trade. In particular, in the AM, households have an opportunity to consume with probability $\sigma > 0$. In the case where a household gets to consume, it buys the three consumption goods in three segmented markets. Consumption good $c_1$, or simply good 1, can be bought only with cash or a cash-like CBDC (e.g., offline transactions in which debit and credit cards cannot be used). Consumption good $c_2$, or simply good 2, cannot be purchased with cash but can be purchased only with deposits or credit (e.g., online transactions). Consumption good $c_3$, or simply good 3, can be purchased

[8]Note that the presence of credit is not critical for our theoretical results and $B$ can be simply set to zero. See related finding in Gu et al. (2016). It is useful for matching the model to payments data in our quantitative exercise.

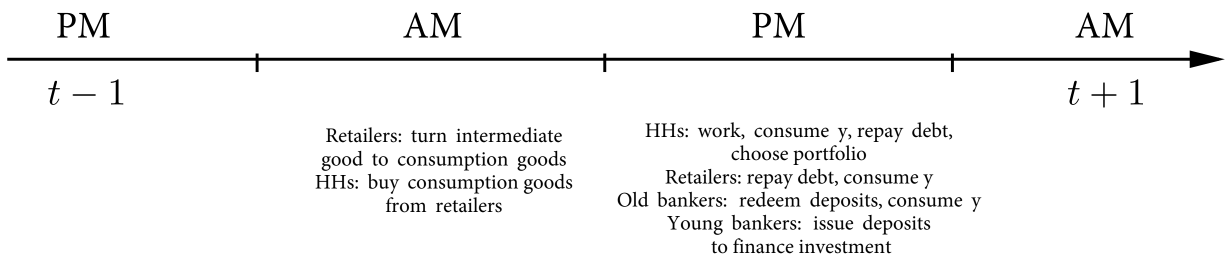


Figure 1: Time-line

using any means of payment (e.g., most physical retail stores). Anticipating their AM transactions, households acquire a portfolio of money and deposits in the PM. Figure 1 illustrates the time-line of the model.

Now let us elaborate on different ways to design a CBDC. Some typical proposals include a cash-like, deposit-like, and universal CBDC. A cash-like CBDC can be used in type-1 and type-3 transactions (e.g., a stored-value card that cannot support online transactions). A deposit-like CBDC can be used in type-2 and type-3 transactions (e.g., a payment account that is not accessible in an offline setting). A universal CBDC can be used in all transactions (See Table 1). In this section, we study the effects of a cash-like CBDC.

| | Cash | Deposits | Credit | Cash-like CBDC | Deposit-like CBDC | Universal CBDC |
|---|---|---|---|---|---|---|
| Type 1 | ✓ | | | ✓ | | ✓ |
| Type 2 | | ✓ | ✓ | | ✓ | ✓ |
| Type 3 | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Table 1: Acceptability of Payment Instruments in Different Markets

The money stocks at the beginning of period $t$ is denoted as $Z_t$ for cash and $M_t$ for a CBDC. The central bank maintains a constant money growth rate $\mu_z = Z_{t+1}/Z_t$ and $\mu_m = M_{t+1}/M_t$ by making lump-sum transfers to households and paying interest rate $i_m > 0$ to the CBDC holders in the PM.

In this paper, we focus on stationary equilibrium. It is a standard result in this literature that the inflation rate in a stationary equilibrium is equal to the inverse of the money growth rate. In the rest of the paper, we assume that the money growth rates for cash and CBDC are identical, i.e., $\mu_m = \mu_z$. This assumption is consistent with the fact that, in most discussions around CBDC, it is argued that CBDC and cash should be exchanged one for one. This assumption is innocuous in our paper, because what matters for agents is the real return rate for cash and CBDC. In the stationary equilibrium, the real rate of return on CBDC is given by $R_m = (1+i_m)/\mu_m$ and the real rate of return on cash is $R_z = 1/\mu_z$ as cash pays no interest. Since cash and a cash-like CBDC are perfect substitutes in payments, a household compares $R_m$ and $R_z$ and holds the one that offers higher real rate. If $R_m = R_z$, then households are indifferent between cash and the CBDC.

## 2.1 Bankers

We first consider the decision problem of bankers who derive utility from the numeraire good in the PM and only when they are old. Their utility function is linear. Each of these bankers possesses an investment technology. By investing $k$ units of the numeraire good in the PM, the banker produces $F(k)$ units of the intermediate good in the next AM, where $F$ is an increasing and strictly concave function. We also assume that $kF'(k)$ is increasing. The banker finances the investment by issuing deposits $k$ that bear a gross interest rate $R_d$.[9] Each banker takes $R_d$ and the real price of the intermediate good, $p$, as given and maximizes the profit (i.e., PM consumption when old):

$$\pi = \max_k pF(k) - R_d k.$$

The first-order-condition (FOC) is then given by

$$pF'(k) = R_d. \tag{1}$$

## 2.2 Retailers

Retailers use credit to buy the intermediate good from bankers in a perfectly competitive market and produce a final good. There are three types of retailers, each of which produces one type of consumption good, $c_1$, $c_2$, or $c_3$. The real price of consumption good $i$ (i.e., in terms of the numeraire good) is $p_i$. There is perfect competition in the market for each good and the production function of each consumption is linear: $c_i = a_i x_i$, where $x_i$ denotes the number of intermediate goods purchased from bankers. The maximization problem for type $i$ retailers is given by

$$\max_i p_i a_i x_i - p x_i.$$

[9] Here, we have assumed that banks rely fully on deposits. However, our results continue to hold even if banks can finance their investment partially by equity. See Appendix D.2. for an extension.

This implies that in equilibrium:

$$p_i = \frac{p}{a_i} \text{ for } i \in \{1, 2, 3\}, \tag{2}$$

and the retailers earn zero profits. From now on, we refer to the price of the intermediate good $p$ as the "AM price" in short. We call it the AM price because the prices of consumption goods are simply proportional to that and $p$ pins down all prices in the AM market.

## 2.3 Households

We now consider the households' decisions in the two markets. We use $y$ and $h$ to denote respectively the consumption and production of the numeraire good in the PM, and we use $\mathbf{c} = (c_1, c_2, c_3)$ to denote the consumption bundle in the AM. Households' period utility is given by

$$U(y) - h + \sum_{i=1}^{3} u_i(c_i),$$

where the utility from the numeraire good is $U(y)$, and the utility from the consumption good $i$ is $u_i(c_i)$. We assume $u_i(0) = 0$, $u_i'(0) = \infty$, $U'(y) > 0, u_i'(c_i) > 0$, $U''(y) \leq 0$, and $u_i''(c_i) < 0$. We assume a concave $U$ for the calibration of the model to match the ratio of money to GDP. However, assuming $U(y) = y$ will not change our analytical findings. Note also that the AM utility function is separable. In an extension in Appendix E, we obtain results for a general CES utility function, as in Piazzesi et al. (2021), which allows goods to be substitutes or complements. See a summary in Section 5.4.1.

In the PM, we use $W(Z, D, M, L)$ to denote the value function of a household that is carrying real cash balance $Z$, real deposit balance $D$, real cash-like CBDC balance $M$, and real debt $L$ carried from the last AM. The household chooses numeraire good consumption $y$, production $h$ and continuation portfolio $(\hat{Z}, \hat{D}, \hat{M})$ to solve

$$\begin{aligned} W(Z, D, M, L) &= \max_{y,h,\hat{Z},\hat{D},\hat{M}} \left\{ U(y) - h + \beta V\left(\hat{Z}, \hat{D}, \hat{M}\right) \right\} \\ \text{st. } h + Z + D + M - L + T &\geq y + R_z^{-1}\hat{Z} + R_d^{-1}\hat{D} + R_m^{-1}\hat{M}, \end{aligned}$$

where $V$ is the value function in the following AM. On the LHS of the budget constraint, a household's real income consists of production income $h$, portfolio market value $Z+D+M$, and lump-sum transfers $T$. The expenditure on the RHS consists of numeraire consumption $y$ and the costs of purchasing new portfolio $R_z^{-1}\hat{Z} + R_d^{-1}\hat{D} + R_m^{-1}\hat{M}$ for the next AM. In the PM, when a household deposits one unit of the numeraire good with the banker, the banker promises to repay $R_d$ units of the numeraire good tomorrow. The price of each unit of real deposit balances is thus $R_d^{-1}$. Similarly, $R_z^{-1}$ and $R_m^{-1}$ denote the price of a unit of the real money balances in the PM for cash and CBDC, respectively. Note that $T = Z(\mu_z - 1)/\mu_z + M(\mu_m - 1 - i_m)/\mu_m$.

Using the above budget constraint, we obtain:

$$\begin{aligned} W(Z, D, M, L) &= Z + D + M - L + T + \max_y [U(y) - y] \\ &\quad + \max_{\hat{Z},\hat{D},\hat{M}} \left\{ -R_z^{-1}\hat{Z} - R_d^{-1}\hat{D} - R_m^{-1}\hat{M} + \beta V\left(\hat{Z}, \hat{D}, \hat{M}\right) \right\}. \end{aligned}$$

Due to the linearity of the value function, we can rewrite $W$ as a function of one variable:

$$W(Z + D + M - L) \equiv W(Z, D, M, L) = Z + D + M - L + W(0).$$

The FOCs are given by

$$\begin{aligned} y &: U'(y) = 1, \\ \hat{N} &: R_N^{-1} \geq \beta V_N\left(\hat{Z}, \hat{D}, \hat{M}\right), \text{ equality if } \hat{N} > 0, \end{aligned}$$

where the subscript $N \in \{Z, D, M\}$ denotes the derivative of $V$ with respect to argument $N$.

In the AM, a household takes the real price of consumption goods $p_i$ as given, and this makes the consumption choice subject to the following payment constraints:

$$Z + M \geq c_1 p_1 + f c_3 p_3, \tag{CC}$$

$$D + L \geq c_2 p_2 + (1 - f) c_3 p_3, \tag{DC}$$

$$B \geq L. \tag{CR}$$

The first constraint denotes the constraint on the total cash or cash-like CBDC expenditure on good 1 (i.e., $c_1 p$) and good 3 (i.e., $f c_3 p$), where $f$ is the fraction of good 3 paid in cash or cash-like CBDC. We call this the *cash-payment constraint* (CC). Similarly, the second constraint denotes the constraint on the total expenditure spent on good 2 (i.e., $c_2 p$) and good 3 (i.e., $(1 - f) c_3 p$) using deposits or credit. We call this the *deposit-payment constraint* (DC). The third constraint shows that the real value of an agent's debt, $L$, is bounded by the credit limit, $B$. We call this the *credit constraint* (CR). The household's AM problem is given by

$$\begin{aligned} V(Z, D, M) &= \max_{c_1,c_2,c_3,f,L} \sigma \left[ \sum_{i=1}^{3} u_i(c_i) + W(w - L) \right] + (1 - \sigma) W(Z + D + M - 0) \\ \text{st. } w &= Z + M - (c_1 p_1 + f c_3 p_3) + D + L - (c_2 p_2 + (1 - f) c_3 p_3), \\ & \quad \text{(CC), (DC), (CR),} \end{aligned}$$

where $w$ is the unspent wealth after consumption and $L$ is the debt carried over to the next PM. The PM solution implies that the AM problem can be rewritten as

$$\begin{aligned} V(Z,D,M) &= \max_{c_1,c_2,c_3,f,L} \sigma \sum_{i=1}^{3} u_i(c_i) + Z + D + M + W(0) \\ &\quad -\sigma(c_1p_1 + c_2p_2 + c_3p_3) \\ &\quad +\sigma\lambda_c\left[Z + M - (c_1p_1 + fc_3p_3)\right] \\ &\quad +\sigma\lambda_d\left[D + L - (c_2p_2 + (1-f)c_3p_3)\right] \\ &\quad +\sigma\lambda_e\left(B - L\right), \end{aligned}$$

where $\sigma\lambda_c$, $\sigma\lambda_d$, and $\sigma\lambda_e$ are the Lagrangian multipliers associated with the CC, DC and the CR. The envelope conditions are given by

$$\begin{aligned} V_Z(Z,D,M) &= 1 + \sigma\lambda_c, \\ V_D(Z,D,M) &= 1 + \sigma\lambda_d, \\ V_M(Z,D,M) &= 1 + \sigma\lambda_c. \end{aligned}$$

Following the literature, we call $\lambda_c \geq 0$ and $\lambda_d \geq 0$ the liquidity premiums that are associated with cash and deposits. We obtain the following equations that characterize the optimal AM choices:

$$c_1 \; : \; u_1'(c_1) = (1+\lambda_c)p_1, \tag{3}$$

$$c_2 \; : \; u_2'(c_2) = (1+\lambda_d)p_2, \tag{4}$$

$$c_3 \; : \; u_3'(c_3) = (1 + f\lambda_c + (1-f)\lambda_d)p_3, \tag{5}$$

$$f \; : \; \lambda_d - \lambda_c \begin{cases} \leq 0, & \text{if } f = 0, \\ = 0, & \text{if } f \in (0,1), \\ \geq 0, & \text{if } f = 1, \end{cases} \tag{6}$$

$$L \; : \; \lambda_d = \lambda_e.$$

The first two equations equalize the marginal utility of consuming a good to the marginal cost of tightening the respective payment constraint. The third equation has a similar interpretation. The only difference is that consuming good 3 tightens both constraints, so the marginal cost is proportional to the weighted average of the Lagrangian multipliers of the two constraints. The fourth equation states that the payment choice for good 3 is related to liquidity premiums $\lambda_d$ and $\lambda_c$. An instrument is used exclusively if its liquidity premium is smaller than the other instrument's liquidity premium. Both instruments are used only when the premiums are equalized. The last equation implies that the CR is binding when the policy is away from the Friedman rule for deposits, i.e., when $R_d < 1/\beta$, and the credit is not abundant (See Lemma 1 below). Since we focus on this case, for the rest of the paper, we obtain

$$L = B.$$

Combining the FOCs in the PM and AM, we obtain the optimal portfolio choice in the PM, which is characterized by the following Euler equations:

$$\frac{1}{\beta R_z} \geq 1 + \sigma\lambda_c, \text{ equality if } Z > 0, \tag{7}$$

$$\frac{1}{\beta R_d} \geq 1 + \sigma\lambda_d, \text{ equality if } D > 0, \tag{8}$$

$$\frac{1}{\beta R_m} \geq 1 + \sigma\lambda_c, \text{ equality if } M > 0. \tag{9}$$

The LHS of these equations captures the opportunity costs of holding cash, deposits, or a CBDC. For example, for deposits, the cost depends on discounting and the interest return. The RHS captures the resale value of one plus $\sigma$ times the liquidity premium. For example, for deposits, with probability $\sigma$, the balance is spent with its marginal valuation given by $1 + \lambda_d$; with probability $1 - \sigma$, the balance is held to maturity with return 1.

Equations (7) and (9) imply that the CBDC will crowd out cash whenever $R_m > R_z$. If $R_m < R_z$, a CBDC is not used and the equilibrium is not changed compared with an economy with only cash, deposits and credit. If $R_m = R_z$, households are indifferent between cash and CBDC.

The liquidity premiums are positive only if the following payment constraints are binding:

$$Z + M \geq c_1 p_1 + f c_3 p_3, \text{ “ = ” if } \lambda_c > 0, \tag{10}$$

$$D + B \geq c_2 p_2 + (1 - f) c_3 p_3, \text{ “ = ” if } \lambda_d > 0, \tag{11}$$

where the quantity of deposits in equilibrium is given by

$$D = R_d k. \tag{12}$$

Finally, the market clearing condition for the intermediate good is given by

$$\sigma c = F(k), \tag{13}$$

where $c \equiv \frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}$, which we call it aggregate consumption.

Let us highlight two important, distinct features of this model, relative to the existing literature that is based on the Lagos and Wright (2005) framework. First and foremost, since the investments that are used to produce consumption goods are financed by deposits issued by banks, there is a general equilibrium feedback effect from consumption demand to deposits creation. In particular, a higher consumption in the AM can lead to higher deposits because, other things being equal, $k$ is increasing in $p$ in (1). This linkage is missed in models of CBDC and banking that are based on Lagos and Wright (2005), e.g., Keister and Sanches (2019) and Chiu et al. (2021), because they typically assume that investment goods are not traded in the AM. Second, since all consumption goods are produced (based on the intermediate good which is produced) eventually by a common concave production function, our setup incorporates an inter-market linkage that endogenously links cash and non-cash transactions in different markets (i.e., (3)-(5) are linked by $p$). This linkage is missing in existing models where AM trades are bilateral and/or are subject to linear production costs. However, this channel does not affect our main results in Propositions 3-7.

Given monetary policy $(\mu_z, \mu_m, i_m)$, a steady-state monetary equilibrium consists of $\boldsymbol{c}$, $f$, $k$, $p$, $p_i$, $Z$, $D$, $M$, $R_d$, $\lambda_c$, and $\lambda_d$ that satisfy (1)-(13).

### 2.4 Efficiency

Before analyzing the equilibrium outcome, it is useful to characterize the first-best allocation, which is given by

$$\max_{c_1,c_2,c_3,k} \sigma\beta\left[u_1(c_1) + u_2(c_2) + u_3(c_3)\right] - k$$

subject to

$$\sigma\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right) = F(k).$$

That is, the planner chooses consumption and investment to maximize the sum of the period utilities of households, retailers and bankers, subject to the production technology. We denote the Lagrangian multiplier associated with the constraint as $\beta p^*$. The FOCs are given by

$$\begin{aligned} k &: \beta p^* F'(k) = 1, \\ c_i &: u_1'(c_1)a_1 = u_2'(c_2)a_2 = u_3'(c_3)a_3 = p^*. \end{aligned}$$

The solution, $k^*$ and $c_i^*$, gives us the first-best allocation. In a friction-less economy (without payment constraints), $p^*$ is the price of the intermediate good that supports $(k^*, c_i^*)$ as an equilibrium allocation. In the presence of payment constraints, the conditions under which an efficient equilibrium exists are stated by the following lemma.

**Lemma 1.** *When* $R_m = \beta^{-1}$ *and* $B \geq \max\{0, B^*\}$ *where* $B^* \equiv \frac{c_2^* p^*}{a_2} - \frac{k^*}{\beta}$, *there is a monetary equilibrium that is efficient.*[10]

All proofs are collected in Appendix A. Intuitively, efficiency is achieved when the central bank follows the Friedman rule and when credit is abundant. The former condition is required to ensure that the CC is not binding. The second condition is required to ensure that the DC is not binding. Notice that for efficient consumption, credit is needed only to make the consumption of good 2 efficient because good 3 can be consumed using cash. Moreover, when $B^* \leq 0$, credit is not needed in this economy because the real value of deposits is sufficiently high so that all type-2 trades can be financed.

When the monetary policy is away from the Friedman rule; i.e., $R_m = (1+i_m)/\mu_m < \beta^{-1}$, the CC is binding and hence the liquidity premium $\lambda_c$ is positive. The following proposition then characterizes the effects of credit limit $B$ in this situation. In this paper, we assume that $R_m < \beta^{-1}$.[11]

**Proposition 2.** *Away from the Friedman rule (*$R_z < 1/\beta$ *and* $R_m < 1/\beta$*), there exists a credit threshold,* $\overline{B}(\lambda_c)$*, such that*

*(i) if* $B \geq \overline{B}(\lambda_c)$*, then* $\lambda_d = 0, f = 0, R_d = \frac{1}{\beta}, k \leq k^*$,

*(ii) if* $B < \overline{B}(\lambda_c)$*, then* $\lambda_d > 0$ *and* $\begin{cases} \text{if } \lambda_d \leq \lambda_c, & f = 0, \\ \text{if } \lambda_d = \lambda_c, & f \in (0,1), \\ \text{if } \lambda_d \geq \lambda_c, & f = 1. \end{cases}$

This proposition states that when credit is abundant, the DC is slack. Neither Cash nor a cash-like CBDC is used to finance good-3 transactions because the CC is still binding. Under-consumption of good 1 implies that aggregate investment and consumption are below their efficient levels. If credit is scarce, then both the CC and DC are binding. Depending on the relative tightness of the constraints, cash, cash-like CBDC, or deposits may be used in type-3 transactions.

[10] Our model has a usual non-monetary equilibrium in which households believe that money is not valued and thus money is not used in transactions.

[11] The equilibrium does not exist when the rate of return on CBDC is above $1/\beta$ here. Suppose an equilibrium exists. Then households' production of the numeraire good in the PM, $h_t$, must be positive and finite. By using CBDC to store wealth across two consecutive PM periods, an individual household can deviate to produce $h_t + \Delta$ in period $t$ and produce $h_{t+1} - \Delta R_m$ in period $t+1$, for a sufficiently small $\Delta > 0$. The net utility gain from this change is $\Delta(\beta R_m - 1)$ which is strictly positive when $R_m > 1/\beta$. This is inconsistent with our initial assumption of equilibrium.

## 2.5 Characterization of Equilibrium

In the following analysis, we focus on the case where $R_m > R_z$, so that cash is entirely replaced by the cash-like CBDC. Below, we study the comparative statics with respect to the CBDC real rate $R_m$. The effects will depend on the value of $f$. Here, we focus on the case of the interior $f \in (0,1)$. We consider the boundary cases in Appendix B. We also consider an extension in Appendix G with unbanked households who do not have access to deposits and credit, so $f = 0$ for them. The interior case is the most relevant one, because the share of the type-3 consumption good is more than 70% of all consumption goods according to the payments data.[12] More details are given in the quantitative section. In addition, as mentioned earlier, we focus on the interesting case where credit is limited so that $\lambda_d > 0$.[13]

When a CBDC replaces cash and has an interior share $f \in (0,1)$, households are indifferent between using the CBDC or deposits in good-3 transactions. Equation (6) implies that their liquidity premiums are equalized; i.e., $\lambda_d = \lambda_c$. The real interest rate on the CBDC set by the central bank will then determine the real interest rate on deposits:[14]

$$R_d = \frac{1}{\beta(1+\sigma\lambda_d)} = \frac{1}{\beta(1+\sigma\lambda_c)} = R_m.$$

[12]Huynh, Schmidt-Dengler and Stix (2014) provide micro-level evidence using payment transactions data in Canada and in Austria. Table 1 of their paper considers situations where (i) the buyer had enough cash and a card, and (ii) the seller accepted both cash and card. They found that cards are used only 19% of time in Austria and 35% of time in Canada. This is consistent with the assumption of $f$ being interior.

[13]If credit is abundant, then $\lambda_d = 0$, so deposits do not serve a major role in alleviating payment frictions. Therefore, our results regarding the effectiveness of a cash-like CBDC in inducing banks to increase the deposit rates will not be obtained. For the case where credit is limited, we conduct comparative statics with respect to $B$ in Appendix C and obtain new results.

[14]Note that our model assumes that, conditional on acceptance, the rate of return is the main factor driving agents' demand for different payment instruments. As a result, the equilibrium deposit rate moves one-for-one with the CBDC rate. In other words, the pass through from the CBDC rate to the deposit rate is perfect. We study in Appendix H a variation of the model in which the pass through is imperfect. For this, we assume that households derive private value from holding CBDC balances, and that the marginal value of holding CBDC balances is decreasing (or equivalently the marginal cost is increasing) in their balances. We show that in this case an increase in the CBDC rate leads to a positive but smaller increase in the deposit rate. However, our main result that a higher interest rate on a cash-like CBDC increases consumption and intermediation remains the same.

We can then obtain the following equilibrium conditions:

$$F'(k) = \frac{1}{\beta p(1+\sigma\lambda_c)}, \tag{14}$$

$$F(k) = \sigma \sum_i \frac{u_i'^{-1}\left(\frac{(1+\lambda_c)p}{a_i}\right)}{a_i}. \tag{15}$$

The above conditions imply that

$$F\left(F'^{-1}\left(\frac{1}{\beta p(1+\sigma\lambda_c)}\right)\right) = \sigma u'^{-1}\left((1+\lambda_c)p\right), \tag{16}$$

where we used[15]

$$u'^{-1}(p) \equiv \sum_i \frac{u_i'^{-1}\left(\frac{p}{a_i}\right)}{a_i}.$$

Equation (14) determines a banker's level of investment, given the liquidity premium and the AM price. Equation (15) is the market clearing condition in the AM. Equation (16) combines the above two equations to determine the equilibrium AM price, $p$. We can then solve for consumption, investment, and the bank interest rate.

Finally, to ensure that $f$ is interior, we need $B$ to be sufficiently high so that $f < 1$ and to be sufficiently low so that $f > 0$:

$$\left[\frac{c_2}{a_2} - F'(k)k\right] p \le B \le \left[\frac{c_2}{a_2} + \frac{c_3}{a_3} - F'(k)k\right] p.$$

A higher real rate, $R_m$, on the CBDC affects the equilibrium allocation through three channels, which we will explain one by one. These channels are shown in Figure 2, which uses traditional demand and supply curves plotted in the $(c, p)$ space.

The first channel is related to *payment efficiency*. As the real rate on the CBDC rises, payments in the AM become more efficient as the opportunity costs of holding payment balances decline. This is mathematically evident from (3). While the CBDC is only used for good-1 and good-3 purchases, there is also an interest rate *spillover effect* on good-2 transactions. As the real interest rate paid on the CBDC rises, bankers are forced to raise the interest rate on deposits in order to retain households. This implies that payments in all AM transactions become more efficient. As shown in Figure 2, an increase in the CBDC rate shifts up the demand curves in all markets (market 3 is

[15]Given $f \in (0,1)$, there is at most one monetary equilibrium in which both cash and deposits are valued. If (16) has a solution, it will be unique. This is because its RHS is decreasing and the LHS is increasing in $p$, so we will have at most one solution. Given $p$, other equilibrium variables can be uniquely pinned down.

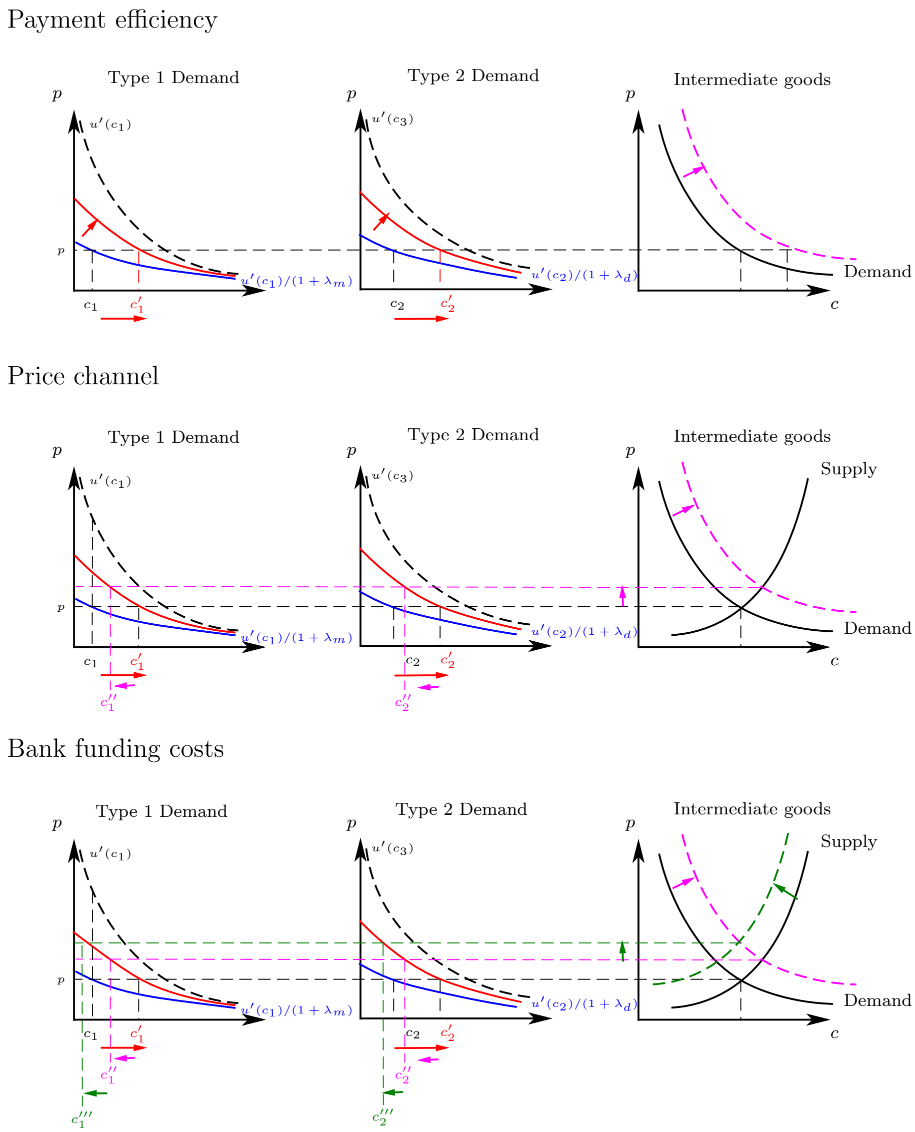


Figure 2: Effects of a Cash-like CBDC ($f \in (0, 1)$)

not shown in the diagram). For a fixed AM price, consumption in both markets increases from $c_i$ to $c_i'$.

The second channel is the *price channel*. As payments become more efficient, aggregate demand for consumption rises, pushing up the AM price and the marginal cost of production in the AM. This leads to an endogenous reduction in consumption. Because of the price linkage, there are responses in the quantities of all types of purchases. In Figure 2, a higher marginal cost of production lowers consumption from $c_i'$ to $c_i''$. Note that the magnitude of this channel depends on the shape of the production function. This channel vanishes when the production function is linear, as is assumed in many previous models.

The third channel is related to *bank funding costs*. In response to an increase in the real rate on the CBDC, bankers need to raise their deposit rates to retain clients, and this will lead to higher funding costs for investment. In Figure 2, a higher funding cost shifts up the aggregate supply curve, raising the AM price and decreasing consumption from $c_i''$ to $c_i'''$.

The above discussion implies that, while a rise in $R_m$ must drive up the AM price, the effects on the quantities of goods consumed could be ambiguous, depending on the relative strength of the supply- and demand-side effects. To determine the sign of the equilibrium effects, we examine the demand and supply conditions separately.

Equation (15) gives us the demand side of consumption,

$$F(k) = \sigma u'^{-1}((1+\lambda_c)p), \tag{17}$$

which is determined by the marginal liquidity cost of purchasing consumption goods. A lower $\lambda_c$ or a lower $p$ increases consumption demand by relaxing the liquidity constraint, raising $k$.

Similarly, (14) describes the supply side of consumption:

$$F'(k) = \frac{R_d}{p} = \frac{1}{\beta(1+\sigma\lambda_c)p}. \tag{18}$$

A lower $\lambda_c$ or a lower $p$ increases the funding cost of investment, leading to a lower $k$.

To derive the equilibrium effect of these opposite forces, note that the supply and demand equations together imply

$$F\left(F'^{-1}\left(\frac{1}{\beta(\sigma\Delta + (1-\sigma)p)}\right)\right) = \sigma u'^{-1}(\Delta), \tag{19}$$

where $\Delta \equiv (1+\lambda_c)p$. Equation (19) defines a downward-sloping curve in the $(\Delta, p)$ space. The definition of $\Delta$ gives an upward-sloping curve, $p = \Delta/(1+\lambda_c)$, in this space. As $\lambda_c$ decreases, the second curve becomes steeper, implying a higher $p$ and a lower $\Delta$ in equilibrium. Equation (15) then implies that $k$ must go up. This suggests that, in Figure 2, the demand-side effect dominates. To understand the intuition, note that for a given $k$, the impact of $\lambda_c$ on $p$ implied by the demand relationship (17) is bigger than that implied by the supply relationship (18) whenever $\sigma < 1$.[16] The reason is that a higher interest rate relaxes the liquidity constraint (i.e., lower $\lambda_c$), generating

[16]Specifically, fixing $k$, $|\frac{dp}{d\lambda_c}| = p/(1+\lambda_c)$ along the demand and $|\frac{dp}{d\lambda_c}| = p\sigma/(1+\sigma\lambda_c)$ along the supply.

| Cash-like CBDC ($f \in (0,1)$) | $c_1$ | $c_2$ | $c_3$ | $c$ |
|---|---|---|---|---|
| **- Payment efficiency** | $+$ | $+$ | $+$ | $+$ |
| **- Price effects** | $-$ | $-$ | $-$ | $-$ |
| **- Bank funding effects** | $-$ | $-$ | $-$ | $-$ |
| **Total effects** | $+$ | $+$ | $+$ | $+$ |

Table 2: Disentangling the Effects of a Cash-like CBDC

two effects: lowering the liquidity cost of consumption demand captured by (17) and raising the funding cost of consumption supply captured by (18). The latter effect, however, is dominated as $\lambda_c$ matters for the funding cost only with probability $\sigma < 1$. Hence, we have the following proposition.

**Proposition 3.** *Suppose $f \in (0,1)$ and $\sigma < 1$. A higher real rate, $R_m$, paid on the cash-like CBDC leads to higher $p$, $c_i$, $c$, $k$ and welfare.*

As discussed above, a higher rate paid on the cash-like CBDC will reduce the liquidity premium, which has a bigger impact on the demand side than on the supply side. This explains why the payment efficiency channel is stronger than the disintermediation channel.

Since the curvature of production function determines the price effect, one may wonder whether it is crucial for the above result. It turns out that the curvature of the production function only matters for the shape of the supply curve and not how much the supply curve shifts (which is determined by the disintermediation channel).[17] Hence, the above proposition holds even when the price effect is absent (i.e., the production function is linear).

Proposition 3 implies that, as long as households are indifferent between a CBDC and deposits in type-3 transactions, a higher $R_m$ will lead to higher consumption and intermediation. Notice that a key channel in the above argument is the effect of liquidity costs on deposits creation through the supply side—the missing linkage in the previous literature. Table 2 summarizes the equilibrium effects.

Obviously, the case with $R_m \leq R_z$ is equivalent to a world without a CBDC. Hence, if both cash

[17] To see why, consider the special case with a linear production function: $F(k) = Ak$. The supply curve becomes horizontal, i.e. $p = R_d/A$. The aggregate consumption is then given by $c = u'^{-1}\left(\frac{1}{\sigma A}\left(\frac{1}{\beta} - (1-\sigma)R_d\right)\right)$, which is still increasing in $R_d$. Note that, for simplicity, we have assumed $a_i = 1$. Several extensions of the supply side of the model including linear production function can be found in Appendix D.

and deposits are used in type-3 transactions ($f \in (0,1)$) in the absence of a CBDC, then the effects of introducing a cash-like CBDC with $R_m > R_z$ are captured by Proposition 3, as long as $f \in (0,1)$ remains valid. Hence we have the following corollary, which applies to cases where $R_m - R_z$ is not too big to ensure $f \in (0,1)$.

**Corollary 4.** *Suppose $\sigma < 1$. If both cash and deposits are used in type-3 transactions when $R_m \leq R_z$, then introducing a cash-like CBDC with $R_m \in (R_z, \bar{R}_m)$ increases bank intermediation and welfare for some $\bar{R}_m > R_z$.*

In other words, as long as cash and deposits are directly competing as payment options in some trades, introducing a cash-like CBDC with an appropriate rate can increase welfare and *crowd in* banking. Again, the mechanism is that the CBDC improves payment efficiency in all transactions in equilibrium. Welfare goes up as consumption gets closer to the efficient level for all goods. Despite the higher interest rate, the rise in aggregate consumption induces a higher derived demand for bank deposits, leading to more bank intermediation. Notice that this result is quite general as it is independent of the forms of the utility and production functions. This also provides a novel economic insight to the literature that ignores the general equilibrium feedback effect from transactions to deposits creation.

## 3 Deposit-like CBDC

In this section, we focus on the deposit-like CBDC, so agents have access to cash, the CBDC, deposits and credit. We first derive a new set of equilibrium conditions and then conduct the comparative statics exercise with respect to $R_m$. Again we focus on the interesting case where $R_m > R_z$. In this case, cash is dominated by the CBDC in terms of the interest rate, hence, cash is not used to purchase good 3. The real value of the CBDC chosen by households is denoted by $M$. The households' AM problem becomes

$$\begin{aligned}
V(Z,D,M) &= \max_{\mathbf{c},f,v,n,s} \sigma \left[ \sum_{i=1}^{3} u_i(c_i) + W(w) \right] + (1-\sigma) W(Z+D+M) \\
\text{st. } w &= Z + M + D - (c_1 p_1 + c_2 p_2 + c_3 p_3), \\
Z &\geq c_1 p_1, \\
D + B + M &\geq c_2 p_2 + c_3 p_3.
\end{aligned}$$

| **Deposit-like CBDC** | $c_1$ | $c_2$ | $c_3$ | $c$ |
|---|---|---|---|---|
| **- Payment efficiency** | 0 | + | + | + |
| **- Price effects** | − | − | − | − |
| **- Bank funding effects** | − | − | − | − |
| **Total effects** | − | + | + | +/− |

Table 3: Disentangling the Effects of a Deposit-like CBDC

As obtained in the previous section, households use credit up to the limit $B$, so we just removed $L$ from the problem. Note that bankers need to match the deposit rate to the CBDC rate in equilibrium. The equilibrium effects of increasing $R_m$ are given in the following proposition.

**Proposition 5.** *A higher real rate, $R_m$, paid on the deposit-like CBDC leads to a higher $p, c_2$ and $c_3$, and a lower $c_1$.*

We can again understand the equilibrium impacts that were obtained using the three channels discussed earlier: Regarding the payment efficiency channel, as $R_m$ rises, the payments for goods 2 and 3 become more efficient, but the payments for good 1 are not affected as cash is not used in good-3 transactions. Regarding the price channel, the increase in demand for goods 2 and 3 pushes up the AM price, which in turn reduces the consumption for all goods. Finally, regarding the bank funding cost channel, since banks respond to changes in the interest rate on deposit-like CBDCs, bank funding costs go up, further reducing consumption.

The effects of an increase in the interest rate on a deposit-like CBDCs on all three types of consumption goods are summarized in Table 3 and illustrated in Figure 3. Unlike in the case of a cash-like CBDC, increasing $R_m$ will reduce type-1 consumption, implying an ambiguous effect on aggregate consumption and welfare.

# 4 Different CBDC Designs

This section first compares the macroeconomic effects of cash- and deposit-like CBDCs. Then we explore the effects of a universal CBDC that can be used in all transactions.

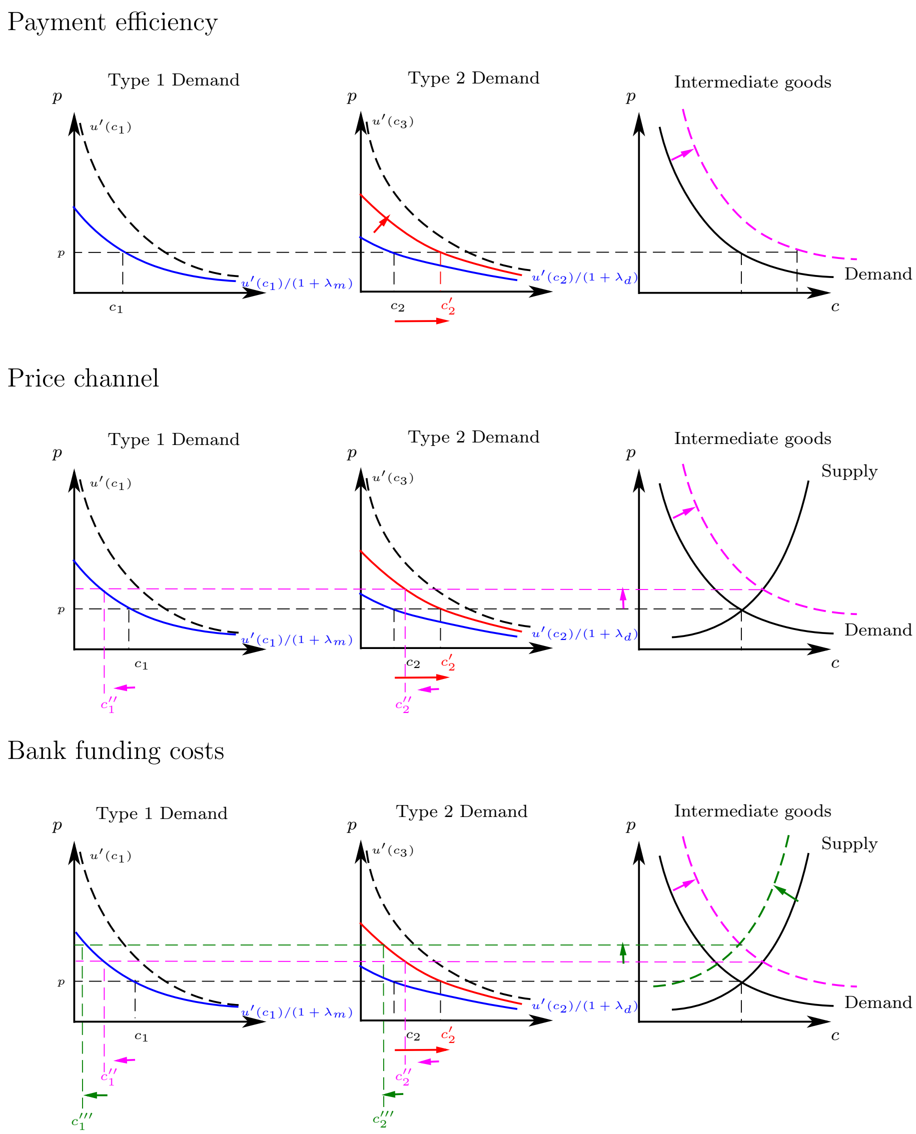


Figure 3: Effects of a Deposit-like CBDC

## 4.1 Cash-like vs. Deposit-like CBDCs

Our analysis suggests that the equilibrium effects of a CBDC depend crucially on its design. In particular, the introduction of a cash-like CBDC can promote consumption and crowd in banking. To understand this, note that an interest-bearing, cash-like CBDC lowers the opportunity costs of holding payment balances. The direct effect is that households will buy more goods in transactions where a CBDC is used. An additional, indirect effect is that if banks are forced to raise the interest rate on deposits, then households will also buy more goods in transactions where deposits are used. In other words, a cash-like CBDC generates a positive *spillover effect* from cash to non-cash transactions. Through the payment efficiency channel, the higher consumption demand will induce banks to create more deposits to finance production in order to clear the goods market. This is the positive general equilibrium feedback effect from transactions to deposits creation that is overlooked in the literature. We show that when cash is used in type-3 transactions, the positive effect through the payment efficiency channel can outweigh the other two channels and lead to higher intermediation.

In contrast, a deposit-like CBDC may not promote consumption and banking. First, a deposit-like CBDC cannot be used to purchase type-1 goods, so it cannot improve payment efficiency in these transactions. Moreover, unlike deposits, cash is not interest bearing. Hence the introduction of a deposit-like CBDC cannot induce an endogenous reduction in the opportunity costs of holding cash. As a result, there are no positive spillover effects from non-cash transactions to cash transactions. Second, the introduction of a CBDC drives up the AM price and lowers type-1 consumption, further worsening the payment efficiency in type-1 transactions. This explains why a deposit-like CBDC has lower effects on consumption and banking.

We now provide a formal comparison between the two designs. We start with an economy with a cash-like CBDC that bears interest rate $R_m > R_z$ with an interior $f$ and compare it with an economy with a deposit-like CBDC that pays the same interest rate. The equilibrium effects of these two polices are given by the following proposition.

**Proposition 6.** *Suppose $\sigma < 1$. Consider two economies: one with a cash-like CBDC with interest rate $R_m > R_z$, and one with a deposit-like CBDC with same interest rates. If agents are indifferent between a CBDC and deposits in both economies, then*

*(i) $c_1$, $c$, $k$ and $\pi$ are higher and $c_2$ and $c_3$ are lower for a cash-like CBDC compared with a deposit-*

*like CBDC,*

*(ii) welfare is higher for a cash-like CBDC compared with a deposit-like CBDC if $R_m$ is sufficiently close to $R_z$.*

The proposition suggests that for a given interest rate, introducing a cash-like CBDC is more effective in raising aggregate consumption and investment. The proposition also implies that a cash-like CBDC leads to smaller negative impacts on banks in terms of both the volume of intermediation and bank profits. The intuition is that in this economy, the main source of inefficiency is the high opportunity cost of carrying cash, which leads to a low interest rate being paid by deposits in equilibrium. Hence, offering a cash-like CBDC helps directly tackle this problem. Through the channels discussed above, a cash-like CBDC induces higher demand. As banks endogenously raise the interest rate on deposits, consumption demand rises not only in type-1 transactions but also in the other two types of transactions. As a result, the AM price increases more under the cash-like CBDC than under the deposit-like one. This implies that the consumption of type-2 and type-3 goods is lower when the CBDC is cash-like. Since aggregate output is higher, type-1 consumption must be higher under a cash-like CBDC. Under a deposit-like CBDC, banks will also raise the deposit rates, but the opportunity cost of carrying cash is not affected. This explains part (i).

Here is an outline of the proof for part (ii). This part establishes that welfare under a cash-like CBDC is higher than that under a deposit-like CBDC when the real return on CBDC is close to that of cash. Denote the welfare function by $\mathcal{W}(R_z, R_d)$, where the first (second) argument is the real return of the balances used in type-1 (type-2 and -3) transactions. To understand the welfare effects, we first focus on the case where $R_m = R_z$, which implies that the welfare under a cash-like CBDC is equal to that under a deposit-like CBDC. A slight increase in the interest on both CBDCs will change the welfare for the cash-like case to $\mathcal{W}(R_m, R_m)$ and the welfare for the deposit-like case to $\mathcal{W}(R_z, R_m)$. This is because a deposit-like CBDC does not induce an endogenous response in the return of transaction balances used in type-1 transactions. We show in the proof that the welfare function is increasing in $R_z$ when $R_z = R_m$. We then use continuity of welfare function to show that welfare is higher for a cash-like CBDC.[18]

[18]The proof requires a technical condition, which is continuity of $\mathcal{W}_1$ with respect to its second component (i.e., existence of $\mathcal{W}_{12}$).

## 4.2 Universal CBDC

Another potential policy option is to design a universal CBDC that can be used in all types of transactions. As long as $R_m > R_z$, the universal CBDC replaces cash in type-1 transactions while competing with deposits in type-2 and type-3 transactions. A natural expectation is that this would be a more powerful design than a cash- or deposit-like CBDC. We show that this design can be non-essential in the sense that it does not necessarily support more-desirable allocations relative to a cash-like CBDC.

**Proposition 7.** *Consider an economy with a cash-like CBDC and assume that an equilibrium with an interior $f$ exists. Then replacing the cash-like CBDC by a universal CBDC with the same interest rate does not affect the equilibrium allocation (consumption and investment).*

This proposition suggests that a universal CBDC does not enlarge the set of allocations achieved by a cash-like CBDC. It is true that a universal CBDC can support type-2 transactions while a cash-like CBDC cannot. With an interior $f$, however, this is not really an advantage because, by exerting competitive pressure on deposits in type-3 transactions, a cash-like CBDC can also indirectly lower the opportunity cost of trading in type-2 transactions. This conclusion relies on the assumption of an interior $f$. We argue, in Section 5, that this assumption is consistent with the empirical data.

# 5 Quantitative Analysis

We have analytically investigated the effects of different types of CBDCs and identified three channels through which a CBDC can affect the economy. In this section, we use a calibrated model to evaluate the effect of introducing various types of CBDCs and quantify the contributions of different channels to the aggregate effect.

## 5.1 Calibration

For the calibration, we use the following utility functions for the AM:

$$u_i(c_i) = \frac{\omega_i}{1-\frac{1}{\xi}} c_i^{1-\frac{1}{\xi}}.$$

We consider more general CES preferences in Section 5.4.1. As standard in the literature, the utility function in the PM is assumed to be

$$U(y) = A \log y.$$

The production function takes the form

$$F(k) = \frac{k^{1-\gamma}}{1-\gamma}.$$

To calibrate the model, we follow the literature to set $\beta = 0.97$ and $\mu_z = 2\%$ to reflect the long-term inflation rate in the U.S. Next, we construct some calibration targets by using payments and monetary statistics from three data sets: the Survey of the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; the new M1 series obtained from Lucas and Nicolini (2015); and a FRED data set. We now discuss our calibration strategy. First, as in Chiu et al. (2019), we use the DCPC and FRED to compute the targets for aggregate payment behaviour according to Table 4. Note that $c_i^{share} \equiv \frac{c_i/a_i}{\sum c_j/a_j}$ and $\alpha_1 \equiv \frac{Z}{Z+D+B}$, $\alpha_2 \equiv \frac{D}{Z+D+B}$, and $\alpha_3 \equiv \frac{B}{Z+D+B}$.[19] Second, we derive an empirical relationship between the money stock and nominal interest rates (i.e., money demand function). This is constructed by using the new M1 series from Lucas and Nicolini (2015) for the period 1982-2008.[20]

We impose $a_1 = a_2 = a_3 = 1$ as they cannot be identified from the money demand and the payments data that we use. We set parameter values for the utility functions $(\omega_i, \xi, A)$, probability of trade $(\sigma)$, the production function parameter $(\gamma)$ and the credit limit $(B)$ to match the payment and monetary targets. We follow a three-step procedure: (i) Given $(\sigma, B)$, we parameterize $\gamma$ to fit $\alpha_2$. (ii) We parameterize $A$ and $\xi$ to fit the empirical money demand. To calculate money demand, we

[19]Cash and debit shares are obtained directly from the DCPC 2018. From FRED, we obtain the share of e-commerce retail sales as a percentage of total sales, which is 9.9% at the end of 2018. According to the DCPC 2018, debit and credit cards are not accepted in 6.1% of transactions and cash is not accepted in 2.3% of transactions. Then we calculate type-1 transactions as all non-online transactions that do not accept debit and credit cards. This implies that type-1 trades account for $(1-9.9\%)6.1\% = 5.50\%$. Similarly, we calculate type-2 transactions as all online transactions plus those non-online transactions that do not accept cash. This implies that type-2 trades account for $9.9\% + (1-9.9\%)2.3\% = 11.97\%$. Type-3 transactions account for the remaining share. For the DCPC, see Greene and Stavins (2019). For the FRED data, see the entry [38] in the reference list.

[20]We exclude the post-crises period because the demand for M1 increased significantly after the crises, perhaps due to agents' store-of-value motives or foreign demand, which are not related to the transactional demand that we study in this paper. However, we do a robustness check with money demand data for period 2000-2019. The quantitative results are close to the benchmark results. See Section 5.4.2.

| | | |
|---|---|---|
| **Cash share of transactions** | $\alpha_1$ | 17.53% |
| **Debit share of transactions** | $\alpha_2$ | 38.71% |
| **Type-1 share of transactions** | $c_1^{share}$ | 5.50% |
| **Type-2 share of transactions** | $c_2^{share}$ | 11.97% |

Table 4: Calibration Targets

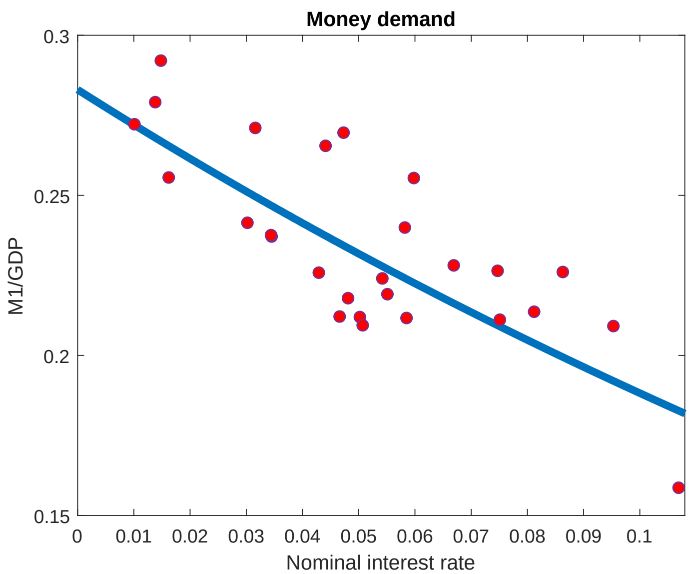


Figure 4: Money Demand Curve; Model vs. Data

use the share of good $i$ in AM transactions from payments data, $c_i^{share}$, to fix the coefficients of the utility function from the following definition:

$$c_i^{share} = \frac{\omega_i^{\xi} a_i^{\xi-1}}{\sum \omega_j^{\xi} a_j^{\xi-1}} \Rightarrow \omega_i = \left(\frac{c_i^{share}}{a_i^{\xi-1}}\right)^{\frac{1}{\xi}} \tag{20}$$

using a normalization $\sum \omega_j^{\xi} a_j^{\xi-1} = 1$. (iii) The values of $(\sigma, B)$ are then chosen to minimize a convex combination of the money demand squared errors and the distance between the payment targets and the model-implied values at 2% inflation. The details of the calibration exercise are explained in Appendix F. Table 5 summarizes the parameter values. Figure 4 plots the money demand curve predicted by the model against the data for the period 1982 to 2008.

| Parameters | Notation | Value | Notes |
|---|---|---|---|
| **Calibrated externally** | | | |
| Discount factor | $\beta$ | 0.97 | Standard |
| Productivity of good-i producers | $a_i$ | 1 | Normalization |
| Money growth rate | $\mu$ | 1.02 | 2% inflation |
| | | | |
| **Calibrated internally** | | | |
| Prob. of consumption shock | $\sigma$ | 0.457 | Payment data |
| Credit | $B$ | 0.423 | Payment data |
| Curvature of production | $\gamma$ | 0.153 | Payment data |
| Coefficient of type 1 | $\omega_1$ | 0.254 | Payment data |
| Coefficient of type 2 | $\omega_2$ | 0.367 | Payment data |
| Coefficient of type 3 | $\omega_3$ | 0.913 | Payment data |
| Coeff. of PM consumption | $A$ | 1.454 | Money demand |
| One minus inverse of consumption elasticity | $\xi$ | 2.114 | Money demand |

Table 5: Calibration Results

### 5.2 Effects of a Cash-like CBDC

We now use the calibrated model to quantify the effects of introducing a cash-like CBDC. Since the denomination of cash and the CBDC will likely be identical, we assume that the CBDC has the same growth rate of 2% but carries an interest rate of $i_{CL}$. Section 2 suggests that the equilibrium effects of introducing a CBDC depend on whether cash is used in type-3 transactions. Our calibration exercise indicates that $f$ is about 15% in the status quo with 2% inflation. Figure 5 reports the responses of consumption and investment as $i_{CL}$ increases from 0% to 5%, which is the interest rate that corresponds to the Friedman rule for CBDC. The values along the vertical axis are normalized to 1 when $i_{CL} = 0$ (i.e., the status quo). In each plot, the effects are further decomposed into the three channels discussed earlier, with the solid line combining all of them. The illustrated effects confirm our analytical result that paying a higher $i_{CL}$ improves payment efficiency for type-1 transactions, with spillovers to type 2 and type 3 through the endogenous responses of banks. The increases in consumption and investment are mitigated by the negative general equilibrium price effect and higher bank funding costs. Consistent with findings in Proposition 3, the overall effects on consumption and investment are positive. The model predicts that a cash-like CBDC crowds in banking by 10.2% at the maximum. Table 6 reports the decomposition when $i_{CL} = 5\%$ (i.e., at the Friedman rule).[21] Since this is the maximum feasible rate, we conclude that introducing a cash-like CBDC can induce a rise in retail transactions by 8.97% at the maximum. Regarding the uptake of the new CBDC relative to other payment instruments, the model predicts that the market share of a cash-like CBDC is from 17.5% to 23.3%, depending on $i_{CL}$. Cash will be completely driven out of the market and replaced by the CBDC. The welfare gain of introducing a cash-like CBDC is 0.059% at the maximum.[22]

### 5.3 Effects of a Deposit-like CBDC

Figure 6 quantifies the effects of introducing a deposit-like CBDC with its rate, $i_{DL}$, ranging from 0% to 5%. Paying interest on the CBDC improves payment efficiency for type-2 and type-3

[21]The cash-like CBDC has identical effects on different consumption goods because the preferences are separable and the elasticities of utility functions for all three goods have been assumed equal.

[22]The welfare change is measured in consumption-equivalent units according to the methodology of Lucas (2000) and Lagos and Wright (2005). That is, the welfare gain is defined to be the percentage of consumption needed to compensate the agents so that they are indifferent between the equilibrium with CBDC compared to that without CBDC but with the compensation.

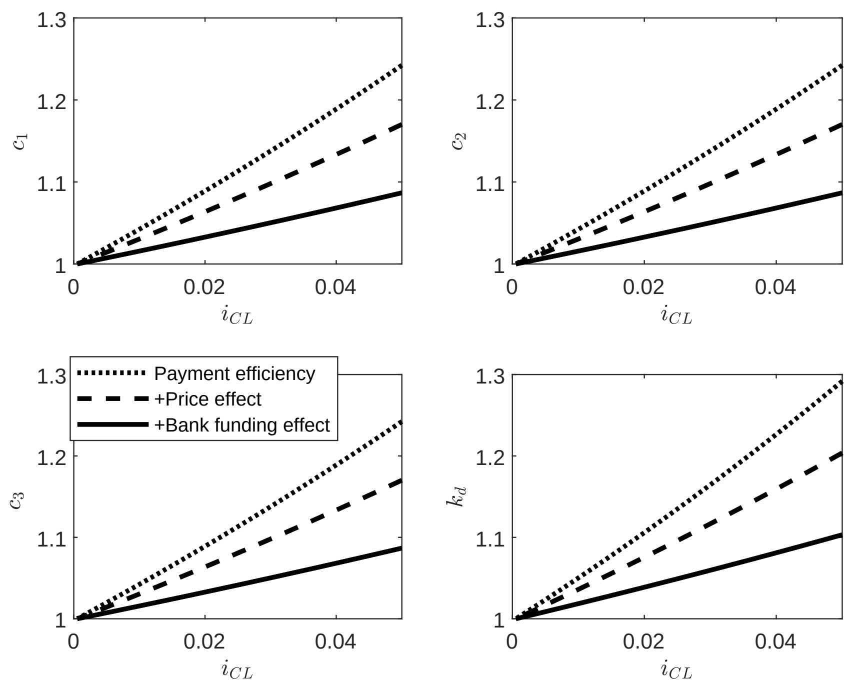


Figure 5: Effects of Interest on a Cash-like CBDC

| **Percentage change** | $c_1$ | $c_2$ | $c_3$ | $c$ |
|---|---|---|---|---|
| **- Payment efficiency** | 25.08 | 25.08 | 25.08 | 25.08 |
| **- Price effects** | $-7.39$ | $-7.39$ | $-7.39$ | $-7.39$ |
| **- Bank funding effects** | $-8.72$ | $-8.72$ | $-8.72$ | $-8.72$ |
| **Total effects** | 8.97 | 8.97 | 8.97 | 8.97 |

Table 6: Decomposition of Effects for a Cash-like CBDC with $i_{CL} = 5\%$

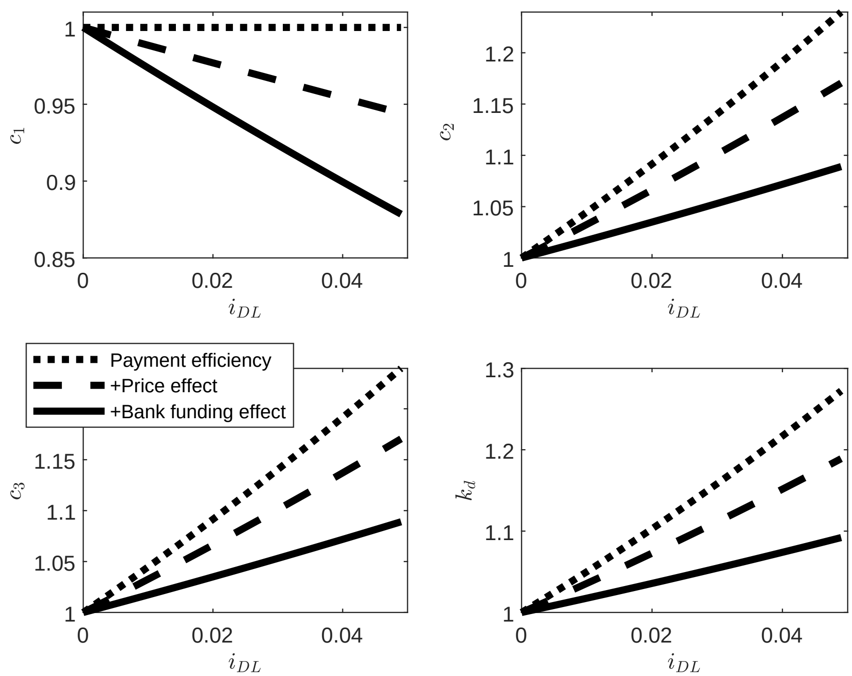


Figure 6: Effects of Interest on a Deposit-like CBDC

transactions, while there are no spillover effects to type-1 transactions since cash is dominated in the rate of return. The effects due to a higher price and higher bank funding costs are again negative. Overall, type-1 transactions decrease, while the other transactions increase. The calibrated model predicts that the effect on type-1 transactions is dominated by the others, resulting in a higher aggregate consumption and investment. At the maximum, type-1 consumption drops by 12.2%, while the other types of consumption go up by about 8.9%, as reported in Table 7. Overall, a deposit-like CBDC increases banking by 9.2% at the maximum. Regarding the uptake of a deposit-like CBDC, the model predicts that its market share is 18.45% at the maximum. Since the CBDC is not strictly dominating, cash can still maintain at least 4.48% of the payment market. The welfare gains of introducing a deposit-like CBDC is 0.046% at the maximum.

## 5.4 Sensitivity Analyses

In this subsection, we first extend AM preferences to a general CES utility function. Next, we conduct an alternative calibration using more recent data.

| Percentage change | $c_1$ | $c_2$ | $c_3$ | $c$ |
|---|---|---|---|---|
| **- payment efficiency** | 0.00 | 23.97 | 23.97 | 22.65 |
| **- price effects** | $-5.58$ | $-6.92$ | $-6.92$ | $-6.84$ |
| **- bank funding effects** | $-6.57$ | $-8.14$ | $-8.14$ | $-8.06$ |
| **Total effects** | $-12.15$ | 8.90 | 8.90 | 7.75 |

Table 7: Decomposition of Effects for a Deposit-like CBDC with $i_{DL} = 5\%$

#### 5.4.1 General Preferences

In the model, we have examined the case where the AM utility function is separable. In the numerical exercise, a CRRA preference is assumed. Here, we briefly consider a more general, non-separable utility function to study the effect of the degree of substitution among the three goods. Specifically, we assume the following CES utility function:

$$v(c_1, c_2, c_3) = \frac{1}{1-\frac{1}{\xi}} \left( \omega_1 c_1^{1-\frac{1}{\varsigma}} + \omega_2 c_2^{1-\frac{1}{\varsigma}} + \omega_3 c_3^{1-\frac{1}{\varsigma}} \right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}} \tag{21}$$

where $\xi, \varsigma > 0$, $\xi \neq 1$ and $\varsigma \neq 1$. Note that $\varsigma$ is the intra-temporal elasticity of substitution between different goods. If $\varsigma = \xi$, the utility function will be separable as in our quantitative exercise, i.e., $u_i(c_i) = \omega_i c_i^{1-\frac{1}{\xi}}/(1-\frac{1}{\xi})$. As $\varsigma$ goes to infinity, the goods become perfect substitutes, and as $\varsigma$ goes to zero, the goods become perfect complements. As $\xi$ goes to infinity, this utility converges to standard CES preferences.

Overall, our theoretical results are quite robust when we consider more general preferences. In particular, we can show analytically that, when $\varsigma \leq \xi$, our main results hold (reported in length in Appendix E). Further analytical results are not feasible. But we have conducted a quantitative analysis to check the robustness of our results with respect to changes in the elasticity of substitution. Specifically, we start with the calibrated economy (where $\varsigma = \xi$) and examine the impacts of varying the elasticity of substitution captured by $\varsigma/\xi$, keeping $\xi$ fixed. The general pattern of our results do not change. See Figures 7 and 8 in Appendix E for quantitative results.

#### 5.4.2 Alternative Calibration

To assess whether the quantitative results are sensitive to the choice of sample periods, we recalibrate the model using more recent money demand data from the period 2000-2019. This period includes recent years when the nominal interest rates have been quite lower than the longer term average. We use the Fisher equation to obtain the nominal rate on illiquid bonds as the sum of the inflation rate and the real rate, which is assumed to be 1% ($\beta = 0.99$).[23] We consider an economy with an inflation rate of 1.58% (the average level from 2009-2019), so the maximum feasible interest rate on a CBDC is 2.58%.[24]

We find that the effects of CBDCs are generally similar to those obtained in the benchmark estimation. In particular, the maximum increase in aggregate consumption and investment are 10.38% and 11.4% for a cash-like and 9.43% and 10.4% for a deposit-like CBDC. The market share of cash-like and deposit-like CBDCs will be 23.9% and 19.0%. The welfare gains for a cash-like and deposit-like CBDCs are, respectively 0.018% and 0.015%. Therefore, our results are not too sensitive to the choice of the calibration period.

## 6 Endogenous Money Growth

The benchmark model studies the effects of issuing a CBDC in an economy with an exogenous money growth rate. Since the allocative inefficiency due to monetary frictions could potentially be mitigated by adjusting the money growth rate, this section extends the benchmark model to allow for endogenous money growth. We derive conditions under which CBDC issuance can still improve social welfare in this setting. To capture the observation that central bank monetary policy deviates from the Friedman Rule, we follow the literature to formally incorporate into the model the demand for cash by foreign users and illegal activities (Schmitt-Grohé and Uribe, 2010). We also discuss other considerations.

It is well known that a significant share of U.S. currency is held abroad. For example, Porter and Judson (1996) estimate that about 53% to 66% of U.S. currency in circulation outside of banks was held abroad in 1995. Over time, the foreign demand remains significant and, according to

[23] We consider a relatively low real rate. Some scholars have argued that the real rates have declined in the past two decades possibly due to structural changes (See Bauer and Rudebusch (2020) and references in their Footnote 2). Also, we do not use directly the rate paid on T-bills because government securities seem to carry a higher liquidity premium in recent years, making it not an appropriate proxy for the “illiquid” bond rate.

[24] The average inflation rate from 2000-2019 is 2.17%, but we use the inflation rate after the global financial crisis to consider a relatively extreme scenario in terms of the nominal interest rate. Moreover, we did not include the 2020-2021 period because of the substantial increase in money supply during the pandemic. Lastly, the payments data are the same as those in the benchmark calibration.

the Federal Reserve Board's estimate, over $950 billion in U.S. dollar banknotes were held by foreigners in early 2021, accounting for about half of total U.S. dollar banknotes outstanding.[25] Similar phenomenon is observed in the Eurosystem. A recent ECB report suggests that between 30% and 50% of the value of euro banknotes were held outside the euro area in 2019 (ECB, 2021).[26]

Cash also plays a crucial role in many criminal activities. For instance, drug trade, which involves hundreds of billions in US dollars, is a highly cash-intensive industry. Rogoff (2016) provides a detailed discussion of the role of money in facilitating crimes, ranging from drug trafficking, racketeering, extortion, corruption of officials, human trafficking, to money laundering. Rogoff points out that, while large-denomination bills constitute a large share of currency (e.g., 80% of the US currency supply is in $100 bills) surveys find that only a small proportion of the cash is held by US households, suggesting that most of the large bills could be used in criminal activity. Other countries exhibit a similar pattern.

Given the importance of foreign and criminal cash demand, monetary policy can generate direct and substantial impacts on both foreign and illegal activities that are cash-intensive. In particular, adherence to the Friedman rule implies effectively making a transfer from the public sector to subsidize cash transactions in foreign and underground economies. Setting the optimal monetary policy thus involves a trade-off between the cost of subsidizing these activities and the benefit of keeping the opportunity cost of cash usage low for domestic agents with legal activities. Earlier studies confirm the relevance of these considerations for central banks in setting its optimal money growth rate. Schmitt-Grohé and Uribe (2012) show that the existence of a foreign cash demand can justify sizable deviations from the Friedman rule. Furthermore, Koreshkova (2006) and Schmitt-Grohé and Uribe (2010) show that deviating from the Friendman rule is an optimal policy to tax underground activities.[27]

[25] https://www.federalreserve.gov/econres/notes/feds-notes/the-international-role-of-the-u-s-dollar-20211006.html

[26] https://www.ecb.europa.eu/pub/pdf/ire/ecb.ire202106 a058f84c61.en.pdf

[27] Some researchers interpret central banks' decision to issue large-denomination bills as an attempt to tax foreign and underground cash holders. For example, Rogoff (1998) has suggested that "Given the apparently overwhelming preference of foreign and underground users for large-denomination bills, the European Monetary Institute's decision to issue large notes constitutes an aggressive step towards grabbing a large share of developing country demand for safe foreign currencies, which we estimate here to be in the range of $300–400 billion."

## 6.1 An Extended Model Incorporating Foreign and Illegal Activities

In this section, we add two new sets of agents. The first set of agents use cash for illegal activity (e.g., human trafficking, drug trafficking and money laundering) which imposes negative externality on all agents in that sector. The second set of agents are foreigners who use domestic cash for their transactions abroad.

Let's begin with the illegal activities (or underground economy). There is a set of measure one of buyers, called U-buyers, who use cash to trade consumption $x$ in the underground market with utility function

$$u_u(x) - e\bar{x},$$

with $\bar{x} = \int x$ and $e > 0$. The second term captures the negaitve externality that the underground consumption imposes on the buyers.

These goods are produced by sellers, called U-sellers, with a linear cost function, so one unit of the PM good produces one unit of $x$. For simplicity, we assume that buyers meet sellers with probability 1. Also, buyers make a take-it-or-leave-it offer to sellers, so the buyers capture the whole surplus and the price is 1.

The optimization problem of U-buyers gives

$$\begin{aligned} u'_u(x) &= 1 + \lambda_u, \\ \frac{1}{\beta R_z} &= 1 + \lambda_u, \end{aligned}$$

where $R_z = \frac{1}{\mu_z}$, and the derivation of these two equations is similar to that of (3) and (7). Therefore,

$$u'_u(x) = \frac{1}{\beta R_z}, \tag{22}$$

which gives $x = x(R_z)$.

We now consider the foreign demand for cash. Suppose some foreigners use the domestic cash for transactions in the foreign economy. For simplicity, without modeling the details of the foreign economy, we assume that in total they demand inelastically $x^f$ units of domestic currency (in real terms) for their transactions. It would be straightforward to incorporate more formally the foreign sector with $x^f$ being endogenous.

As in the benchmark model, we continue to assume that the central bank injects money in the PM by lump sum transfers only to domestic agents. As a result, foreigners need to acquire $\Delta m^f$

each period from the domestic PM such that in total domestic agents earn a seigniorage $\Delta m^f = m^f - \frac{m^f_{-1}}{\mu_z} = x^f(\mu_z - 1)/\mu_z$. Now the transfers to domestic households is modified to

$$T = Z(\mu_z - 1)/\mu_z + M(\mu_m - 1 - i_m)/\mu_m + x^f(\mu_z - 1)/\mu_z.$$

Notice that the transfers to the agents in the benchmark model of Section 2 was the same except the last term.

We first consider the case where the interest rate on the cash-like CBDC is so low that it is not used, i.e., $R_m < R_z$. We assume throughout this section that parameters are such that $f$ is interior. The welfare in the domestic economy in the benchmark model is denoted by $\mathcal{W}(R_z, R_d)$ as introduced in the discussion following Proposition 6. Given that $f$ is interior, we can simply write the welfare in the domestic economy in the benchmark model as $\overline{\mathcal{W}}(R_z) \equiv \mathcal{W}(R_z, R_z)$. An interior $f$ also implies that at the Friedman rule, not only the cash transactions but also the deposit transactions become efficient and the resulting allocation is socially efficient. That is, welfare is maximized when $R_z = \beta^{-1}$. See Lemma 1. As a result, $\overline{\mathcal{W}}'(R_z) = 0$ as $R_z \to 1/\beta$. The social planner maximizes social welfare which is now given by

$$\mathcal{W}^{new}(R_z) = \underbrace{\overline{\mathcal{W}}(R_z)}_{\text{benchmark economy}} + \underbrace{u_u(x(R_z)) - x(R_z) - e\bar{x}(R_z)}_{\text{underground economy}} + \underbrace{x^f(\mu_z - 1)/\mu_z}_{\text{foreign demand}}.$$

Because of symmetry, $\bar{x} = x$ in equilibrium. Therefore, we can rewrite the social planner's maximization problem as

$$\max_{R_z \leq 1/\beta} \overline{\mathcal{W}}(R_z) + u_u(x(R_z)) - x(R_z) - e.x(R_z) + x^f(1 - R_z).$$

The FOC implies that the optimal real interest rate on currency, $R_z = R_z^*$, solves:

$$\overline{\mathcal{W}}'(R_z) + [u_u'(x(R_z)) - 1 - e]x'(R_z) - x^f = 0.$$

At the Friedman rule (i.e., $R_z \to 1/\beta$), we have:

$$\frac{\partial \mathcal{W}^{new}}{\partial R_z} = \underbrace{\overline{\mathcal{W}}'(R_z)}_{=0} + [\underbrace{u_u'(x(R_z)) - 1}_{=0 \text{ by } (22)} - e]x'(R_z) - x^f = -ex'(R_z) - x^f < 0.$$

Therefore, the planner always has an incentive to deviate from the Friedman rule. By continuity of $\mathcal{W}^{new}$, the optimal policy is setting $R_z^* < 1/\beta$ or equivalently $\mu_z^* > \beta$. In other words, welfare without a CBDC in this economy is maximized away from the Friedman rule.

The intuition is straight forward. Without underground economy and foreign demand for domestic cash, the planner would like the real return on domestic currency to be equal to $1/\beta$. However, activities in the underground economy impose externality on others, so the planner wants to reduce these cash-intensive underground activities by imposing an inflation tax on them. Moreover, foreign demand for domestic currency means that the planner can transfer resources from abroad to domestic agents. The higher the inflation, the higher the size of this transfer.

We now study the effects of introducing a cash-like CBDC. We assume that the CBDC cannot be used for underground activities and by foreign users. The result below holds true as long as CBDC cannot be used at least in one of these sectors, not necessarily both. When CBDC pays a positive interest rate and with the same inflation rate for cash and CBDC, CBDC dominates cash, so welfare in the benchmark economy is given by $\overline{\mathcal{W}}(R_m)$. We showed in Proposition 3 that welfare in the benchmark economy is increasing in the interest rate on CBDC away from the Friedman rule:

$$\overline{\mathcal{W}}'(R_m) > 0 \text{ when } R_m < 1/\beta$$

for $R_m > R_z$, with $R_m$ sufficiently close to $R_z$.

The modified welfare function with CBDC and underground and foreign activities can be written as:

$$\mathcal{W}^{new,CBDC}(R_z, R_m) = \overline{\mathcal{W}}(R_m) + u_u(x(R_z)) - x(R_z) - e.x(R_z) + x^f(1 - R_z).$$

Therefore:

$$\frac{\partial \mathcal{W}^{new,CBDC}(R_z, R_m)}{\partial R_m} = \overline{\mathcal{W}}'(R_m) \text{ at } R_m = R_z < \frac{1}{\beta}.$$

The intuition is as follows. Start with an economy without a CBDC. Introducing a CBDC with a positive interest rate implies that it will dominate cash in the domestic economy. Since the underground and foreign users cannot use CBDC, only the welfare in the domestic, regular economy rises, without affecting the underground and foreign demand. Altogether, we can summarize these results in the following proposition.

**Proposition 8.** *(i) Without a CBDC (i.e., if $R_m < R_z$), it is optimal to set $\mu_z > \beta$.*

*(ii) Given the assumptions specified in proposition 3, introducing a cash-like CBDC can improve welfare. That is, for a given $R_z < 1/\beta$, there exists $R_m > R_z$, with $R_m$ sufficiently close to $R_z$, such that welfare is higher compared with the economy with $R_m = R_z$.*

A corollary of part (ii) is that, if we begin with the welfare-maximizing inflation rate on cash in an economy without a CBDC, it is still optimal to introduce a CBDC with a positive interest rate so that the real rate of return on CBDC is higher than that of cash. This level of welfare is not attainable with cash, because by assumption we began with the welfare-maximizing inflation rate on cash.

**Corollary 9.** *It is always feasible to find $\mu_z > \beta$ and $i_m > 0$ such that the welfare with a cash-like CBDC dominates the highest welfare attainable without the CBDC.*

### 6.2 Other Considerations

We finish this section by briefly discussing other considerations that could explain why deviating from the Friedman rule is socially optimal. One argument is related to nominal rigidity. When price adjustment is subject to a menu cost, marginally deviating from the Friedman rule can lead to a first-order benefit of reducing the menu cost but only generate a second-order loss due to inflation distortion. As a result, it is socially optimal to deviate from the Friedman rule. See, for example, Goodfriend and King (1997) and Schmitt-Grohé and Uribe (2010). In Appendix I.1, we present a simple extension of our model that generates this finding.

Another explanation proposed by the literature is related to credit frictions. In an environment with limited commitment, higher money growth can reduce the value of the outside option in autarky for borrowers and hence relaxes their endogenous credit limits. See, for example, Aiyagari and Williamson (2000), Antinolfi, Azariadis and Bullard (2014) and Berentsen, Camera and Waller (2007). We also present a simple model that reproduces this result in Appendix I.2.

## 7 Discussion of Policy and Design Issues

In this section, we discuss some important issues related to CBDCs, including the motivations for their issuance and practical design options.

### 7.1 Motivations for Issuance

There are multiple arguments for issuing a CBDC. According to the BIS survey on CBDC (Boar and Wehrli, 2021), domestic payments efficiency is a key motivation for issuing a retail CBDC in both advanced and emerging market economies. Our paper explicitly models this argument and studies its positive and normative implications.

Our theory suggests that the unique features of a cash-like CBDC are that, unlike deposits, it can be used in transactions currently only supported by cash (e.g., offline transactions), and, unlike cash, the CBDC can bear interest (or other perks) to reduce the opportunity costs of holding payment balances. This interest-bearing feature allows it to mitigate consumption efficiency and improve welfare when the money growth rate deviates from the Friedman rule, as shown in Section 6.

Another important feature of the welfare-improving role of a CBDC is that it can exclude certain agents from participation (e.g., criminals and foreigners). As suggested by Chiu and Wong (2015), CBDC is very different from cash which permits non-exclusive participation: anyone can freely participate in the monetary system to hold cash without other prerequisites. How can a CBDC system restrict participation? One possible arrangement is that only consumers and merchants who have obtained particular devices (e.g., apps, cards, readers/writers) from the issuer can participate in the system to conduct payments. Another possible arrangement is that only individual and business users who have already signed up for an account can hold, send and receive CBDC balances. Non-compliance leads to exclusion from the system.

According to the BIS survey, other motivations suggested by central banks include safety and resiliency of the payment system, monetary policy sovereignty, financial inclusion, and data privacy. Our paper does not explicitly model these features. However, regardless of the motivation, central banks still need to understand the potential impact of a CBDC on banking. The positive implications of our model remain relevant.

### 7.2 Practical Implementation Options

We want to briefly discuss a few practical issues related to CBDC designs. A CBDC can be based on different architectures. If the central bank intends to issue the CBDC directly to end users, then new payment technologies, such as blockchains, may help reduce the costs of operating an independent payment system. For example, Riksbank's e-krona pilot considers running an independent payment system based on Distributed Ledger Technology. However, in such a system, it may be challenging to provide comprehensive customer service. Also, a blockchain-based CBDC may not fully replicate a cash-like design as it cannot be used in an offline setting. An alternative option supporting offline transactions is card-based or mobile-based payment products that allow only close-range value transfers (e.g., Octopus cards or Mastercard Mondex). These can be a "token based" systems in the sense that balances are stored in an anonymous device without requiring a registered account.[28]

Instead of running the CBDC system directly, a central bank can also consider a public-private partnership. An "indirect or synthetic CBDC" is basically a narrow bank system where users hold claims on intermediaries which offer all retail payment services. These intermediaries need to keep central bank reserves to fully back the value of the underlying CBDC balances (Adrian 2019; Adrian, Mancini-Griffoli, 2019). This arrangement helps reduce the operational burden on the central bank. However, the CBDC is not a direct claim on the central bank, and thus there may be concerns regarding its financial and operational risks.

A final question is whether privately issued stablecoins would be sufficient to replicate the role of a CBDC. The literature casts doubts on the stability of stablecoins, arguing that they are susceptible to runs given that they are lightly regulated (Carapella et al. 2022; Gorton and Zhang, forthcoming). Others worry that their widespread acceptance could lead to disintermediation and potential loss of monetary sovereignty for central banks (Brunnermeier, James, and Landau 2019).

In addition, the objective function and feasibility constraints for a private issuer of stablecoin are very different from those of a central bank. The development and operation of the CBDC may incur a loss. A private operator who maximizes profits may not have the incentive to offer a stablecoin to replicate a CBDC solution, while a public entity may find it socially optimal to do so.

One may argue that central banks have an advantage in the provision of transactional balances. First, given their historical involvement, central banks have accumulated some tangible and intangible capital, such as payment infrastructures and social trust, that could facilitate their entry into the digital payments market. Second, as argued by Holmstrom and Tirole (1998), taxation power gives central banks an advantage in providing safe, liquid balances, relative to the private sector.

[28]In such a token-based system, interest can be paid when the device is connected directly or indirectly to the central system. For example, each time when the device is synced, the amount of interest can be calculated depending on the transaction history recorded in the device. Interest can then be paid to the device. Unlike cash, the digital records stored in the system and in the device ensure that a holder cannot claim the interest twice. For example, the Octopus card system in Hong Kong set up kiosks for users to collect transaction bonuses and subsidies.

# 8 Conclusion

Our study extends a standard monetary model to assess different CBDC designs by incorporating two realistic features. First, cash competes with deposits as payment instruments in some transactions. Second, deposit-taking banks help fund the production of consumption. We found some policy-relevant results that are both interesting and novel. Specifically, a cash-like CBDC can promote consumption and welfare, thereby outperforming a deposit-like CBDC. In addition, a cash-like CBDC can crowd in banking, which suggests that the worry about disintermediation is not warranted. More importantly, a CBDC generates these benefits only when it bears interest—a result that casts doubt on the optimality of not paying interest on CBDC balances. Overall, our results show that ignoring the general equilibrium effects results in misleading qualitative and quantitative predictions.

Our novel results are due to the general equilibrium feedback effect (from transactions to deposit creation) incorporated in our model. In the existing CBDC literature, banks generally play two roles: (i) on the asset side of their balance sheets, banks offer loans to finance production; (ii) on the liability side, banks issue deposits (backed by loans) to facilitate goods transactions. In the existing models, these two sides are "disconnected" in the sense that the goods produced by using bank loans (role i) and the goods consumed by using bank deposits (role ii) are assumed to be different. As a result, CBDC issuance tends to generate a negative effect on banking: CBDC competes with deposits as a means of payments and lowers the demand for deposits, crowding out bank intermediation. In contrast, our model connects the two sides by assuming that the goods produced by using bank loans are those traded using bank deposits. As a result, the introduction of a CBDC will generate an additional, positive impact on banking: by facilitating goods transactions, CBDC leads to a higher demand for goods, inducing more loans. As loans are used to back the creation of deposits, CBDC can raise the supply of deposits. This is the novel feedback effect mentioned above that is omitted in the existing CBDC model. We show that this new, positive channel can mitigate the negative channel mentioned above, and, under appropriate conditions, will crowd in banking even when banks do not have market power. Note that, our model also contributes to the broader monetary and banking literature. Existing monetary models with banks (e.g., Berentsen, Camera, Waller (2007), Cavalcanti and Wallace (1998)) often do not focus on the role of banks in financing investment. Those that consider this role (e.g., Andolfatto, Berentsen and Waller, 2016) do not study the the competition between cash and deposits as alternative means of payment.

In this paper, we abstract from other frictions in the banking sector such as bank market power and pledgeability constraints, both of which have already been explored in the literature. While it would be interesting to introduce these features into our model, this extension is left for future research.

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# Appendix (For Online Publication Only)

This online appendix includes sections A to I. The numbering of equations, propositions and others follows the numbering of the main text.

## A Proofs and Derivations

Proof of Lemma 1

*Proof.* From the first-best problem, we obtain $c_i^*$ and $k^*$, with the price $p^*$ solving

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{\beta p^*}\right)\right) = \frac{u_1'^{-1}(\frac{p^*}{a_1})}{a_1} + \frac{u_2'^{-1}(\frac{p^*}{a_2})}{a_2} + \frac{u_3'^{-1}(\frac{p^*}{a_3})}{a_3}.$$

To ensure that the CC is not binding, we must have

$$R_z = \frac{1}{\beta}.$$

To ensure that the DC is not binding, we must have

$$R_d k + B \geq \left(\frac{c_2}{a_2} + (1-f)\frac{c_3}{a_3}\right)p.$$

This gives a threshold for $B$ such that an efficient equilibrium exists when

$$B \geq \max\{0, B^*\},$$

where $B^* \equiv \frac{c_2^* p^*}{a_2} - \frac{k^*}{\beta}$. ∎

Proof of Proposition 2

*Proof.* Assume that cash is not used for the purchase of good $c_3$ and that the DC is slack (i.e., $f = 0$ and $\lambda_d = 0$). Using equilibrium conditions together with the market clearing condition, we obtain

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{\beta p}\right)\right) = \frac{u_1'^{-1}(\frac{(1+\lambda_c)p}{a_1})}{a_1} + \frac{u_2'^{-1}(\frac{p}{a_2})}{a_2} + \frac{u_3'^{-1}(\frac{p}{a_3})}{a_3}.$$

This equation implicitly defines $p = \overline{p}(\lambda_c)$ as a decreasing function of $\lambda_c$.

Notice that at the first best, the AM price is given by $\overline{p}(0)$; i.e., $p^* = \overline{p}(0)$. In this case, we need to verify that the DC is not binding; i.e., $R_d k + B \geq \left(\frac{c_2}{a_2} + \frac{c_3}{a_3}\right) p$, which is equivalent to

$$B \geq \overline{B}(\lambda_c) \equiv p \left[ \frac{u_2'^{-1}(\frac{p}{a_2})}{a_2} + \frac{u_3'^{-1}(\frac{p}{a_3})}{a_3} - \frac{1}{\beta p} F'^{-1}\left(\frac{1}{\beta p}\right) \right].$$

Note that $p = \overline{p}(\lambda_c)$, but we have simplified the exposition. In this case, cash is not used in good-3 transactions because it is more costly than deposits, so $f = 0$ is verified. Moreover, from $k = F'^{-1}\left(\frac{1}{\beta p}\right)$, we know that $k$ is also decreasing in $\lambda_c$. At $\lambda_c = 0$, $k = k^*$, so $k \leq k^*$ for $\lambda_c \geq 0$.

For a given cash inflation rate, if credit is scarce; i.e., $B < \overline{B}(\lambda_c)$, then the DC is binding, the liquidity premium on deposits is positive, $\lambda_d > 0$, and cash may or may not be used for good 3, depending on whether $\lambda_d$ is greater than $\lambda_c$. ∎

*Proof of Proposition 3*

*Proof.* We separate the proof into two parts.

[1] The effects of $R_m$ on AM price and quantity

Suppose $\sigma < 1$. First, we want to know how $p$ changes with $R_m$. Equivalently, we study how $p$ changes with $\lambda_c$, as we know that $\lambda_c$ and $R_m$ move in opposite directions because $R_m = 1/(\beta(1 + \sigma\lambda_c))$. We use this trick in most proofs.

As $R_m$ increases, or equivalently as $\lambda_c$ decreases, the RHS and LHS of (16) both shift upwards, so $p$ increases.

Now we show that $(1 + \lambda_c)p$ must decrease. Suppose by way of contradiction that $(1 + \lambda_c)p$ increases, then the RHS of (15) decreases, so the LHS should decrease as well. Then, (14) implies that $\sigma(1 + \lambda_c)p + (1 - \sigma)p$ should decrease. But $p$ increases, so $(1 + \lambda_c)p$ should decrease. This is a contradiction.

Hence, $(1 + \lambda_c)p$ must decrease. As a result, all $c_i$s and consequently $c$ and $k$ increase.[29]

[2] Effects of $R_m$ on welfare

[29]For completeness, suppose $\sigma = 1$. Note that the RHS of (16) is a strictly decreasing function in $(1 + \lambda_c)p$ and the LHS is strictly increasing in $(1 + \lambda_c)p$. Therefore, if a solution exists, there exists a unique one. Hence, $(1 + \lambda_c)p$ is constant and the values of the LHS and the RHS, which are each equal to $\sigma c$, is kept constant as $\lambda_c$ increases. Consequently, $k$ is kept constant too.

To calculate welfare, we first need to calculate the derivative of $p$ with respect to $R_m$. Equivalently, we calculate the derivative of $p$ with respect to $\lambda_c$, as we know that $\lambda_c$ and $R_m$ move in opposite directions because $R_m = 1/(\beta(1+\sigma\lambda_c))$. Denote $p' \equiv \frac{\partial p}{\partial \lambda_c}$. We take the derivative of (16) with respect to $\lambda_c$:

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta(1+\sigma\lambda_c)}\right)\right) = \frac{u_1'^{-1}(\frac{(1+\lambda_c)p}{a_1})}{a_1} + \frac{u_2'^{-1}(\frac{(1+\lambda_c)p}{a_2})}{a_2} + \frac{u_3'^{-1}(\frac{(1+\lambda_c)p}{a_3})}{a_3}$$

Therefore:

$$\frac{dk}{d\lambda_c} = \frac{\sigma}{F'(k)}\left[\frac{(1+\lambda_c)p'+p}{a_1^2 u_1''} + \frac{(1+\lambda_c)p'+p}{a_2^2 u_2''} + \frac{(1+\lambda_c)p'+p}{a_3^2 u_3''}\right]$$

Note that we drop the arguments of the functions (e.g., $F' \equiv F'(k)$ and $u_1'' \equiv u_1''(c_1)$).

Welfare is given by

$$\mathcal{W} = U(Y^*) - Y^* + \beta\sigma\left(u_1(c_1) + u_2(c_2) + u_3(c_3)\right) - k,$$

where $U'(Y^*) = 1$. The change in welfare is given by (note that $c_i = u_i'^{-1}(\frac{(1+\lambda_c)p}{a_i})$)

$$\begin{aligned}
\frac{d\mathcal{W}}{d\lambda_c} &= u_1'dc_1 + u_2'dc_2 + u_3'dc_3 - \frac{1}{\beta\sigma}\frac{dk}{d\lambda_m} \\
&= u_1'a_1\frac{(1+\lambda_c)p'+p}{a_1^2u_1''} + u_2'a_2\frac{(1+\lambda_m)p'+p}{a_2^2u_2''} + u_3'a_3\frac{(1+\lambda_c)p'+p}{a_3^2u_3''} \\
&\quad -\frac{1}{\beta F'(k)}\left[\frac{(1+\lambda_m)p'+p}{a_1^2u_1''} + \frac{(1+\lambda_m)p'+p}{a_2^2u_2''} + \frac{(1+\lambda_m)p'+p}{a_3^2u_3''}\right] \\
&= \underbrace{\left((1+\lambda_c)p'+p\right)}_{>0 \text{ since } c_i \text{ is decreasing in } \lambda_c} \underbrace{\left[(1+\lambda_m)p - \frac{1}{\beta F'(k)}\right]}_{=(1-\sigma)\lambda_c p \geq 0} \sum_i \frac{1}{a_i^2 u_i''} \leq 0,
\end{aligned}$$

with strict inequality if $\sigma < 1$. We have used $u' \equiv a_1u_1' = a_2u_2' = a_3u_3' = (1+\lambda_m)p$, and

$$(1+\lambda_c)p - \frac{1}{\beta F'(k)} = (1+\lambda_m)p - \frac{p}{\beta R_d} = (1+\lambda_c)p - (1+\sigma\lambda_m)p = (1-\sigma)\lambda_c p.$$

∎

*Solving the Deposit-like CBDC Equilibrium*

We first collect the equations for the deposit-like CBDC and then prove the result.

$$\begin{aligned}
V(Z,D,M) &= \max_{\mathbf{c},f} \sigma\sum_{i=1}^{3} u_i(c_i) + Z + D + M + W(0) \\
&\quad -\sigma(c_1p_1 + c_2p_2 + c_3p_3) \\
&\quad +\sigma\lambda_c\left[Z - (c_1p_1 + fc_3p_3)\right] \\
&\quad +\sigma\lambda_d\left[D + B + M - (c_2p_2 + (1-f)\,c_3p_3)\right].
\end{aligned}$$

The equilibrium conditions are given by

$$
\begin{aligned}
k &: pF'(k) = R_d \\
c_1 &: u_1'(c_1) = (1+\lambda_c)p_1 \\
c_2 &: u_2'(c_2) = (1+\lambda_d)p_2 \\
c_3 &: u_3'(c_3) = \left[f(1+\lambda_c) + (1-f)\,(1+\lambda_d)\right] p_3 \\
\text{AM good market} &: \sigma\left(\frac{c_1}{a_1} + \frac{c_1}{a_2} + \frac{c_1}{a_3}\right) = F(k)
\end{aligned}
$$

$$
f \ : -\lambda_c + \lambda_d \begin{cases} \leq 0, & \text{if } f = 0, \\ = 0, & \text{if } f \in (0,1), \\ \geq 0, & \text{if } f = 1, \end{cases}
$$

$$
\begin{aligned}
Z &: \frac{1}{\beta R_z} \geq \sigma\lambda_c + 1, \text{ equality if } Z > 0, \\
D &: \frac{1}{\beta R_d} \geq \sigma\lambda_d + 1, \text{ equality if } D > 0, \\
M &: \frac{1}{\beta R_m} \geq \sigma\lambda_d + 1, \text{ equality if } M > 0, \\
ZZ &: Z \geq c_1 p_1 + f c_3 p_3, \text{ ''$=$'' if } \lambda_c > 0 \\
DC &: R_d k + B + M \geq c_2 p_2 + (1-f) c_3 p_3, \text{ ''$=$'' if } \lambda_d > 0
\end{aligned}
$$

In addition, if both deposits and CBDC are used in equilibrium, then bankers need to match the deposit rate to the CBDC rate in equilibrium:

$$
R_d = R_m = \frac{1}{\beta\left(1+\sigma\lambda_d\right)}.
$$

The market clearing condition can then be written as

$$
F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right) = \sigma\frac{u_1'^{-1}\left(\frac{(1+\lambda_c)p}{a_1}\right)}{a_1} + \sigma u_{23}'^{-1}\left((1+\lambda_d)p\right), \tag{23}
$$

where $u_{23}'^{-1}(p) \equiv \frac{1}{a_2}u_2'^{-1}(\frac{p}{a_2}) + \frac{1}{a_3}u_3'^{-1}(\frac{p}{a_3})$.

*Proof of Proposition 5*

*Proof.* Given $R_m > R_z$, cash is only used for good 1 transactions. We also have $\lambda_d = \frac{1}{\sigma}\left[\frac{1}{\beta R_m} - 1\right]$ as discussed.

As $R_m$ increases, the LHS of (23) shifts downward and the RHS shifts upward, so $p$ increases.

Suppose by way of contradiction that $(1+\lambda_d)p$ increases, then the RHS of (23) decreases, so the LHS should decrease as well. As a result, $\sigma(1+\lambda_d)p+(1-\sigma)p$ should decrease. But $(1-\sigma)p$ weakly increases (it is constant when $\sigma=1$), implying that $(1+\lambda_d)p$ should decrease. This is a contradiction!

Therefore, $(1+\lambda_d)p$ should decrease. As a result, $c_1$ decreases, and $c_2$ and $c_3$ increase. ∎

As mentioned in the text, the effects of a deposit-like CBDC on aggregate consumption is ambiguous. However, we can show that aggregate consumption decreases when $\sigma$ is close to 1.

We know that $\beta F'(k) = \frac{1}{\sigma p(1+\lambda_d)+(1-\sigma)p}$. We take the derivative with respect to $p$:

$$\beta F''(k)\frac{dk}{d\lambda_d} = -\beta^2 F'^2(k)\left(\sigma\frac{d(p(1+\lambda_d))}{d\lambda_d}+(1-\sigma)\frac{dp}{d\lambda_d}\right)$$

We showed above that $p(1+\lambda_d)$ is strictly increasing in $\lambda_d$. We also show below that $\frac{dp}{d\lambda_d}$ is bounded at $\sigma=1$. In this case, (23) gives $p$ and $F'(k)=1/(\beta p(1+\lambda_d))$ gives $k$. Both $p$ and $k$ are in $(0,\infty)$ and are well defined. As a result, $\frac{\partial k}{\partial p}$ is also well defined and is in $(-\infty,0)$ . If $\sigma$ is sufficiently close to 1, then $\frac{dk}{d\lambda_d}$ continues to be strictly positive. Therefore, $k$ and consequently $F(k)$ should decrease with an increase in $R_m$ for $\sigma$ sufficiently close to 1. But $F(k)$ is proportional to aggregate consumption, so aggregate consumption goes down too.

For $\frac{dp}{d\lambda_d}$, we need to calculate the derivative of $p$ with respect to $\lambda_d$ :

$$\begin{aligned}&\frac{1}{\sigma}F'\left(\frac{-1}{F''p^2\beta(1+\sigma\lambda_d)}\right)p'+\frac{1}{\sigma}F'\left(\frac{-p\sigma}{F''p^2\beta(1+\sigma\lambda_d)^2}\right)\\ =\;&\frac{(1+\lambda_c)p'}{a_1^2u_1''}+\frac{(1+\lambda_d)p'+p}{a_2^2u_2''}+\frac{(1+\lambda_d)p'+p}{a_3^2u_3''}.\end{aligned}$$

Therefore,

$$p' = \frac{-\left(\frac{F'}{F''p^2\beta(1+\sigma\lambda_d)^2}+\frac{1}{a_2^2u_2''}+\frac{1}{a_3^2u_3''}\right)p}{\frac{F'}{\sigma F''p^2\beta(1+\sigma\lambda_d)}+\frac{1+\lambda_c}{a_1^2u_1''}+\frac{1+\lambda_d}{a_2^2u_2''}+\frac{1+\lambda_d}{a_3^2u_3''}}.$$

The denominator is equal to $\frac{F'^3}{\sigma F'' R_d^2\beta(1+\sigma\lambda_d)}+\frac{1+\lambda_d}{a_1^2u_1''}+\frac{1+\lambda_d}{a_2^2u_2''}+\frac{1+\lambda_d}{a_3^2u_3''}$. This is strictly negative around $\sigma=1$, thus it is away from zero. Therefore, $|p'|$ is bounded.

Proof of Proposition 6

*Proof.* The following equations characterize the equilibrium AM price for the cash-like and deposit-like CBDCs, respectively:

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right)=\frac{u_1'^{-1}\left(\frac{(1+\lambda_d)p}{a_1}\right)}{a_1}+\sum_{j\in\{2,3\}}\frac{u_j'^{-1}\left(\frac{(1+\lambda_d)p}{a_j}\right)}{a_j}, \tag{24}$$

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right)=\frac{u_1'^{-1}\left(\frac{(1+\lambda_c)p}{a_1}\right)}{a_1}+\sum_{j\in\{2,3\}}\frac{u_j'^{-1}\left(\frac{(1+\lambda_d)p}{a_j}\right)}{a_j}. \tag{25}$$

Consider the RHS and LHS in the $(p, y)$ space for both equations. The curve on the LHS is the same for both cash- and deposit-like CBDCs. The curve on the RHS; i.e., aggregate demand, is higher for the cash-like CBDC and, as a result, the AM price is higher for that. Also, aggregate consumption and investment are higher. A higher AM price for the cash-like CBDC implies lower consumption of goods 2 and 3. Since aggregate consumption is higher for the cash-like CBDC, the consumption of good 1 should also be higher for this type of CBDC.

Note that, given $R_d$, the profit function is increasing in $p$ by the envelope theorem. Hence, the equilibrium profit is higher under the cash-like CBDC.

Now we compare the welfare level, for which we need to calculate the derivative of $p$ with respect to $\lambda_c$ from (25):

$$\frac{1}{\sigma}F'\frac{-p'}{F''p^2\beta(1+\sigma\lambda_d)}=\frac{(1+\lambda_c)p'+p}{a_1^2u_1''}+\frac{(1+\lambda_d)p'}{a_2^2u_2''}+\frac{(1+\lambda_d)p'}{a_3^2u_3''},$$

where $p'\equiv\frac{\partial p}{\partial\lambda_c}$. We also drop the arguments of the functions (e.g., $F'\equiv F'(k)$ and $u_1''\equiv u_1''(c_1)$) to simplify the exposition. Therefore,

$$p'=\frac{-\frac{p}{a_1^2u_1''}}{\frac{F'}{\sigma F''p^2\beta(1+\sigma\lambda_d)}+\frac{1+\lambda_c}{a_1^2u_1''}+\frac{1+\lambda_d}{a_2^2u_2''}+\frac{1+\lambda_d}{a_3^2u_3''}}. \tag{26}$$

Welfare is given by

$$\mathcal{W}=U(Y^*)-Y^*+\beta\sigma\left(u_1(c_1)+u_2(c_2)+u_3(c_3)\right)-k.$$

Now we calculate the change in welfare:

$$\begin{aligned}
\frac{1}{\beta\sigma}\frac{d\mathcal{W}}{d\lambda_c} &= u_1' dc_1 + u_2' dc_2 + u_3' dc_3 - \frac{1}{\beta\sigma}\frac{dk}{d\lambda_c} \\
&= u_1' a_1 \frac{(1+\lambda_c)p' + p}{a_1^2 u_1''} + u_2' a_2 \frac{(1+\lambda_d)p'}{a_2^2 u_2''} + u_3' a_3 \frac{(1+\lambda_d)p'}{a_3^2 u_3''} \\
&\quad - \frac{1}{\beta F'(k)}\left[\frac{(1+\lambda_c)p' + p}{a_1^2 u_1''} + \frac{(1+\lambda_d)p'}{a_2^2 u_2''} + \frac{(1+\lambda_d)p'}{a_3^2 u_3''}\right]
\end{aligned}$$

At $\lambda_c = \lambda_d$, we have $u_1' a_1 = u_2' a_2 = u_3' a_3 = (1+\lambda_d)p$ and $F' = \frac{1}{p\beta(1+\sigma\lambda_d)}$. Given $\sigma < 1$, we have

$$\frac{1}{\beta\sigma}\frac{d\mathcal{W}}{d\lambda_c} = \underbrace{\left[(1+\lambda_d)p - \frac{1}{\beta F'(k)}\right]}_{=(1-\sigma)\lambda_d p \geq 0} \underbrace{\left[\frac{p}{a_1^2 u_1''} + (1+\lambda_d)p' \sum_i \frac{1}{a_i^2 u_i''}\right]}_{= -\frac{F'}{\sigma F'' p^2 \beta(1+\sigma\lambda_d)} p' < 0} < 0,$$

where we used the following for the last step (at $\lambda_c = \lambda_d$):

$$\begin{aligned}
\frac{p}{a_1^2 u_1''} + (1+\lambda_d)p' \sum_i \frac{1}{a_i^2 u_i''} &= \frac{p}{a_1^2 u_1''} + \frac{-\frac{p}{a_1^2 u_1''}(1+\lambda_m)\sum_i \frac{1}{a_i^2 u_i''}}{\frac{F'}{\sigma F'' p^2 \beta(1+\sigma\lambda_d)} + (1+\lambda_d)\sum_i \frac{1}{a_i^2 u_i''}} \\
&= \frac{\frac{p}{a_1^2 u_1''}\frac{F'}{\sigma F'' p^2 \beta(1+\sigma\lambda_d)}}{\frac{F'}{\sigma F'' p^2 \beta(1+\sigma\lambda_d)} + (1+\lambda_d)\sum_i \frac{1}{a_i^2 u_i''}} \\
&= -\frac{F'}{\sigma F'' p^2 \beta(1+\sigma\lambda_d)} p'.
\end{aligned}$$

Note that $p'$ is obtained from (26) and $p' < 0$ is obtained from the first part of the proposition.

Denote by $\mathcal{W}(\lambda_c, \lambda_d)$ the welfare level when the liquidity premium for cash is $\lambda_c$ and for deposits is $\lambda_d$.[30]

Denote by $\mathcal{W}^{CashLike}$ the welfare level with a cash-like CBDC with real interest rate $R_m$ and when $f$ is interior. Therefore, $\mathcal{W}^{CashLike} = \mathcal{W}(\lambda_d, \lambda_d)$.

Denote by $\mathcal{W}^{Deposit-like}$ the welfare level with a deposit-like CBDC with real interest rate $R_m$ and when $f$ is interior and the liquidity premium for cash is $\lambda_c$. Therefore, $\mathcal{W}^{Deposit-like} = W(\lambda_c, \lambda_d)$.

We have shown above that $\mathcal{W}_1(\lambda_c, \lambda_c) < 0$ for all $\lambda_c$. $\mathcal{W}_1$ is continuous in both components. Therefore, for a given $\lambda_c$, we can find a $\Delta > 0$ such that $\mathcal{W}_1(\lambda_c, \lambda) < -\frac{|\mathcal{W}_1(\lambda_c,\lambda_c)|}{2}$ for $\lambda \in (\lambda_c - \Delta, \lambda_c + \Delta)$.

Note that $\mathcal{W}^{CashLike} - \mathcal{W}^{DepositLike} = \mathcal{W}(\lambda_d, \lambda_d) - \mathcal{W}(\lambda_c, \lambda_d)$. **We want to show that for $\lambda_d$ sufficiently close to $\lambda_c$, this difference is positive.** (Note that $\Delta$ is a function of $\lambda_c$ not a function of $\lambda_d$).

[30]Please note that in the main body of the paper, we considered welfare as a function of real return, i.e., we used $\mathcal{W}(R_z, R_d)$ instead of $\mathcal{W}(\lambda_c, \lambda_d)$, only to make the explanation more intuitive.

By definition of $\mathcal{W}_1(\lambda_c, \lambda)$, we have the following: For any $\varepsilon > 0$, there exists $\delta > 0$ such that for $\lambda_n \in (\lambda_c - \delta, \lambda_c)$:

$$\left| -\frac{\mathcal{W}(\lambda_n, \lambda) - \mathcal{W}(\lambda_c, \lambda)}{\lambda_c - \lambda_n} - \mathcal{W}_1(\lambda_c, \lambda) \right| < \varepsilon.$$

Suppose $\lambda \in (\lambda_c - \Delta, \lambda_c)$. Set $\varepsilon = -\frac{\mathcal{W}_1(\lambda_c,\lambda)}{3}$. By definition of $\mathcal{W}_1(\lambda_c, \lambda)$, there exists $\delta > 0$ such that for $\lambda_n \in (\lambda_c - \delta, \lambda_c)$, we have

$$\begin{aligned} \mathcal{W}(\lambda_n, \lambda) - \mathcal{W}(\lambda_c, \lambda) &= (\lambda_c - \lambda_n) \frac{\mathcal{W}(\lambda_n, \lambda) - \mathcal{W}(\lambda_c, \lambda)}{\lambda_c - \lambda_n} \\ &\geq -(\lambda_c - \lambda_n)(\mathcal{W}_1(\lambda_c, \lambda) + \varepsilon) = -(\lambda_c - \lambda_n) \frac{2\mathcal{W}_1(\lambda_c, \lambda)}{3} > 0. \end{aligned}$$

That is, if $\lambda_d$ satisfies $\max(\lambda_c - \Delta, \lambda_c - \delta) < \lambda_d < \lambda_c$, then $\mathcal{W}(\lambda_d, \lambda_d) - \mathcal{W}(\lambda_c, \lambda_d) > 0$. This concludes the proof. Intuitively, as we change $\lambda_{DL} = \lambda_c$ to $\lambda_{CL} = \lambda_d$ where $\lambda_d < \lambda_c$, welfare increases as long as $\lambda_d$ is close to $\lambda_c$. In other words, when we change the type of CBDC from the deposit-like to the cash-like CBDC, welfare increases. ∎

*Proof of Proposition 7*

*Proof.* Fix the interest rate on a CBDC: $R_m > R_z$. Also assume $R_m < 1/\beta$. Take an equilibrium with a universal CBDC, which produces an allocation (consumption bundle $c_i$ and $k$). We want to show that we can always find an equilibrium with a cash-like CBDC that replicates that allocation. To show this, consider that in the original allocation, we have the following logical possibilities:

- Case 1: the Universal CBDC is not used for purchasing good 2 and 3
- Case 2: the Universal CBDC is used for purchasing good 2 or 3
  - Case 2a: $R_d k + B \geq c_2 p_2$
  - Case 2b: $R_d k + B < c_2 p_2$

We prove this proposition in the following steps. In Lemma 10, we show that in case 1, an equilibrium with a cash-like CBDC and $f = 0$ that replicates the same allocation exists. Similarly for case 2a, we construct an equilibrium with a cash-like CBDC in Lemma 11. Condition $R_d k + B \geq c_2 p_2$ ensures that deposits and credit are sufficient to finance all good 2 transactions. If this condition fails, then a cash-like CBDC (which cannot be used to buy good 2) may fail to replicate the role of a universal CBDC.[31] Then comes Lemma 12 in which we use the assumption that an equilibrium with a cash-like CBDC with an interior $f$ exists to show that $R_d k + B \geq c_2 p_2$ cannot be the case. ∎

**Lemma 10.** *Suppose a universal CBDC is not used in type-2 and type-3 transactions. Then there exists an equilibrium with a cash-like CBDC that can support the same equilibrium allocation.*

*Proof.* Take the equilibrium with a universal CBDC in which the CBDC is not used in type-2 and type-3 transactions. Consider now an economy with a cash-like CBDC with the same price $p$ (taken from the equilibrium with a universal CBDC) in the AM. Generally, if universal CBDC is not used in type-2 and type-3 transactions, then agents would not be worse off by not using the cash-like CBDC, because the cash-like CBDC allows only a subset of transactions that the universal CBDC allows.[32] Therefore, the same allocation can be replicated with a cash-like CBDC, and in the equilibrium, $f$ is zero because the CBDC is not used for type-3 transactions. ∎

**Lemma 11.** *Suppose a universal CBDC is used in type-2 or type-3 transactions. If $R_d k + B \geq c_2 p_2$ in this equilibrium, then one can support the same equilibrium allocation with a cash-like CBDC.*

*Proof.* The AM value function with a universal CBDC can be written as:

$$
\begin{aligned}
V(Z, D, M) &= \max_{\mathbf{c}, f, v, n, s} \sigma \left[ \sum_{i=1}^{3} u_i(c_i) + W(w) \right] + (1-\sigma) W(Z + D + M) \\
\text{st. } w &= Z + M + D - (c_1 + c_2 + c_3), \\
Z &\geq (1-s) c_1 p_1 \\
D + B &\geq (1-v) c_2 p_2 + (1-n) c_3 p_3, \\
M &\geq s c_1 p_1 + v c_2 p_2 + n c_3 p_3,
\end{aligned}
$$

where $s$, $v$, and $n$ denote the share of goods 1, 2, and 3 consumption purchased using the universal

[31] If $R_d k + B < c_2 p_2$, there is shortage of good assets to back deposits. Agents want to consume $c_2$ units of good 2, but the real value of deposits together with credit is not enough. In this case, a universal CBDC can help.

[32] More specifically, for type 2 transactions, agents would make the same choice, because the universal CBDC was available but agents did not choose it, so they are not worse off by not using the CBDC in type 2 transactions. For type 3 transactions, given that the universal CBDC is weakly dominated by deposits or credit for type-2 and type-3 transactions, the cash-like CBDC with the same interest rate should be weakly dominated too.

CBDC. Therefore,

$$
\begin{aligned}
V(Z,D,M) \quad = \quad & \max_{\mathbf{c},f} \sigma \sum_{i=1}^{3} u_i(c_i) + Z + D + M + W(0) \\
& -\sigma(c_1 p_1 + c_2 p_2 + c_3 p_3) \\
& +\sigma\lambda_c \left[Z - (1-s)c_1 p_1\right] \\
& +\sigma\lambda_d \left[D + B - ((1-v)c_2 p_2 + (1-n)c_3 p_3)\right] \\
& +\sigma\lambda_m \left[M - (sc_1 p_1 + vc_2 p_2 + nc_3 p_3)\right]
\end{aligned}
$$

The equilibrium conditions are then given by

$$
\begin{aligned}
k \quad &: \quad pF'(k) = R_d \\
c_1 \quad &: \quad u_1'(c_1) = \left[s(1+\lambda_m) + (1-s)(1+\lambda_c)\right] p_1 \\
c_2 \quad &: \quad u_2'(c_2) = \left[v(1+\lambda_m) + (1-v)(1+\lambda_d)\right] p_2 \\
c_3 \quad &: \quad u_3'(c_3) = \left[n(1+\lambda_m) + (1-n)(1+\lambda_d)\right] p_3
\end{aligned}
$$

$$
n \ : \lambda_d - \lambda_m \begin{cases} \leq 0, & \text{if } n = 0, \\ = 0, & \text{if } n \in (0,1), \\ \geq 0, & \text{if } n = 1, \end{cases}
$$

$$
v \ : \lambda_d - \lambda_m \begin{cases} \leq 0, & \text{if } v = 0, \\ = 0, & \text{if } v \in (0,1), \\ \geq 0, & \text{if } v = 1, \end{cases}
$$

$$
s \ : \lambda_c - \lambda_m \begin{cases} \leq 0, & \text{if } s = 0, \\ = 0, & \text{if } s \in (0,1), \\ \geq 0, & \text{if } s = 1, \end{cases}
$$

$$
\begin{aligned}
Z \quad &: \quad \frac{1}{\beta R_z} \geq \sigma\lambda_c + 1, \ \text{equality if } Z > 0, \\
D \quad &: \quad \frac{1}{\beta R_d} \geq \sigma\lambda_d + 1, \ \text{equality if } D > 0, \\
M \quad &: \quad \frac{1}{\beta R_m} \geq \sigma\lambda_m + 1, \ \text{equality if } M > 0.
\end{aligned}
$$

Assuming $R_z < R_m$, we have $Z = 0$, $s = 1$. Since the CBDC is used in type-2 or type-3 transactions, we should have $M > 0$, implying $\frac{1}{\beta R_m} = \sigma\lambda_m + 1$, and we should have $v > 0$ or $n > 0$. If $v$ or $n$ is in $(0,1)$, then $\lambda_d = \lambda_m$. If $v = 1$ and $n = 0$, then $\lambda_d \geq \lambda_m$ and $\lambda_d \leq \lambda_m$, so $\lambda_d = \lambda_m$. Similarly, if $v = 0$ and $n = 1$, then $\lambda_d = \lambda_m$. In any case, $\lambda_d = \lambda_m$, so $R_d = R_m$. Therefore:

$$\begin{aligned} D + B &= (1-v)c_2p_2 + (1-n)c_3p_3, \\ M &= c_1p_1 + vc_2p_2 + nc_3p_3, \\ \frac{1}{\beta R_d} = \frac{1}{\beta R_m} &= \sigma\lambda_d + 1 = \sigma\lambda_m + 1. \end{aligned}$$

$$k \; : \; pF'(k) = R_m \tag{27}$$

$$c_i \; : \; u_1'(c_1)a_1 = u_2'(c_2)a_2 = u_3'(c_3)a_3 = (1+\lambda_m)p_1 \tag{28}$$

$$\text{AM good market} \; : \; \sigma\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right) = F(k) \tag{29}$$

These equations uniquely pin down the allocation.

If $D + B \geq c_2p_2$, then construct $f^C$ and $M^C$ as follows:

$$\begin{aligned} D + B &= c_2p_2 + (1-f^C)c_3p_3 \\ M^C &= c_1p_1 + f^C c_3p_3. \end{aligned}$$

Then all $c_i$, $k, f^C$ and $p$ construct an equilibrium with a cash-like CBDC. ∎

**Lemma 12.** *Take an equilibrium with a universal CBDC in which the CBDC is used in type-2 or type-3 transactions. Assume also that an equilibrium with an interior $f$ with a cash-like CBDC exists. In the equilibrium with a universal CBDC, $R_dk + B < c_2p_2$ cannot be the case.*

*Proof.* If such an equilibrium exists, then the equilibrium conditions in the cash-like CBDC are the same as (27)-(29). These equations determine a unique allocation as established in Footnote 17. But in that equilibrium, $R_dk + B = c_2p_2 + (1-f)c_3p_3 \geq c_2p_2$. This concludes the proof. ∎

*Proofs of Section 6.1*

*Proof.* The proof of Proposition 8 came in the text preceding the proposition. We just prove Corollary 9. Set $\mu_z = 1/R_z^*$ (i.e., $R_z = R_z^*$), and set $i_m > 0$ sufficiently small such that $\mathcal{W}'(R_m)$ remains positive. Since $R_m = \frac{1+i_m}{\mu_z}$ (remember $\mu_z = \mu_m$), we have $\frac{\partial \mathcal{W}^{new,CBDC}(R_z,R_m)}{\partial R_m} > 0$ around $(R_z, R_z)$. As $\mathcal{W}^{new,CBDC}(R_z, R_z)$ is the highest welfare attainable in the economy without a CBDC, the welfare goes up further with a CBDC paying $i_m$. ∎

# B Boundary Cases of $f = 0$ and $f = 1$ with a Cash-like CBDC

We focus on two special cases. We assume that the DC constraint is binding (i.e.,$\lambda_d > 0$) for both.

**Case 2: A CBDC is not used in good-3 transactions ($f = 0$)**

The following proposition characterizes the equilibrium effects.

**Proposition 13.** *Suppose $f = 0$ and $\sigma = 1$. With a higher real rate, $R_m$, paid on the cash-like CBDC,*

*(i) $p$, $k$, $c_1$ and $y$ increase, and*

*(ii) $c_2$ and $c_3$ decrease.*

*Proof.* A combination of $\lambda_d = \frac{1}{\sigma}\left[\frac{1}{\beta R_d} - 1\right] = \frac{1}{\sigma}\left[\frac{1}{\beta p F'(k)} - 1\right]$, $c_2 = u_2'^{-1}[(1+\lambda_d)p]$, $c_3 = u_3'^{-1}[(1+\lambda_d)p]$, and (52) gives

$$F'(k)k + \frac{B}{p} = \sum_{j\in\{2,3\}} \frac{u_j'^{-1}\left(\frac{\frac{1}{\sigma\beta F'(k)} - \frac{1-\sigma}{\sigma}p}{a_j}\right)}{a_j}. \tag{30}$$

From (30), the RHS is decreasing and the LHS is increasing in $k$, given that $kF'(k)$ is increasing. Therefore, for a given $p$, there exists at most one $k$. Also, the RHS is increasing in $p$ and the LHS is decreasing in $p$. Therefore, $k$ is increasing in $p$.

We show that, as $R_m$ increases (or equivalently as $\lambda_c$ decreases), $p$ increases. By way of contradiction, assume $p$ decreases, then $c_1$ increases. Also, $k$ is increasing in $p$, so $k$ falls. Given that $\sigma = 1$, $c_2$ and $c_3$ rise. But the fact that $c_1$, $c_2$ and $c_3$ all increase and $k$ decreases is inconsistent with the market clearing condition:

$$\frac{F(k)}{\sigma} = \frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}. \tag{31}$$

Therefore, $p$ increases with $R_m$. From (30), $k$ should increase too. Hence, $y = F(k)$ increases too. As a result, $c_2$ and $c_3$ decrease as $\lambda_d$ decreases. Market clearing condition then implies that $c_1$ must increase, so should $(1+\lambda_c)p$ because $c_1 = u_1'^{-1}\left((1+\lambda_c)p\right)$. ∎

**Case 3: Deposits are not used in good-3 transactions ($f = 1$)**

**Proposition 14.** *Suppose $f = 1$ and $\sigma = 1$. With a higher real rate $R_m$ paid on the cash-like CBDC,*

*(i) $p$, $k$, $c_1$, $c_3$ and $y$ increase, and*

*(ii) $c_2$ decreases.*

*Proof.* Again, note that $\sigma\lambda_d + 1 = \frac{1}{\beta R_d} = \frac{1}{\beta p F'(k)}$. This, together with (52), can be written as

$$pF'(k)k + B = \frac{u_2'^{-1}\left(\frac{1}{a_2 \beta F'(k)}\right)}{a_2},$$

given that $\sigma = 1$. This equation implies that $k$ is increasing in $p$ because the LHS is increasing in $k$, the RHS is decreasing in $k$, and the LHS shifts downwards with an increase in $p$. Since $k$ goes up, $\frac{1}{\beta F'(k)}$ also goes up, so does $(1 + \lambda_d)p$. Therefore, $c_2$ goes down.

As $R_m$ goes up, $\lambda_c$ goes down. Here, we argue that $p$ must also go up. By way of contradiction, suppose $p$ decreases. Then $c_1 = u_1'^{-1}((1 + \lambda_c)p)$ and $c_3 = u_3'^{-1}((1 + \lambda_c)p)$ increase. Also, as shown above, $c_2$ is decreasing in $p$, so $c_2$ also increases. As a result, aggregate consumption should increase. Consequently, $k_d$ should go up. However, as shown above, $k_d$ is increasing in $p$, so a lower $p$ implies a lower $k_d$. This is a contradiction!

Now we have shown that $p$ should go up as $\lambda_c$ decreases. A higher $p$ implies a higher $k$. Therefore, $y$ goes up. Since $c_2$ goes down, $c_1$ and $c_3$ should go up too. ∎

# C Comparative Static Exercises with Respect to the Credit Limit $B$

What are the effects of changing credit limit $B$? When cash and deposits are both used in type-3 transactions, relaxing the credit limit has no effects on the real economy except for changing the composition of the means of payment. This is because the return on deposits is pinned down by monetary policy. When cash is not used in type-3 transactions, relaxing the credit limit reduces the consumption of good 1 as well as aggregate consumption. This result is similar to that of Chiu et al. (2018), who show that, due to price externality, improving credit arrangements can have a negative impact on money users' consumption.

To support these claims, we conduct comparative statics with respect to $B$ in this appendix. We focus on two sub-cases: one with $f \in (0,1)$ and one with $f = 0$. The case with $f = 1$ does not give us new insights.

**Case 1: Cash and deposits are used in good-3 transactions ($f \in (0,1)$)**

**Proposition 15.** *As long as $0 < f < 1$, as $B$ increases, $p$, $R_d$, $k$, $c_1$, $c_2$ and $c_3$ are all constant but $f$ increases and $Z$ decreases.*

*Proof.* Because $0 < f < 1$, $\lambda_d$ and consequently $R_d$ are fixed by monetary policy because $\lambda_d = \lambda_c$. Then, (16) implies that $p$ cannot change. Given $\lambda_d = \lambda_c$ and that $\lambda_c$ and $p$ are all constant, $c_i$ should be constant. Given that $R_d$ and $p$ are fixed, $k$ should be constant too. As $B$ increases, only $f$ changes to satisfy the DC. Consequently, $Z$ changes to have the CC satisfied. ∎

This result simply states that as long as agents are indifferent between using cash or deposits, extending credit conditions has no effect on the real economy. This is because the return on deposits is pinned down by monetary policy and extending credit conditions only changes the composition of the means of payment. Credit crowds out cash, but the quantity of deposits remains unchanged.

**Case 2: Cash is not used for good 3, $f = 0$**

Characterization in general is not easy, so we focus on the case in which $\sigma = 1$.

**Proposition 16.** *Assume $f = 0$ and $\sigma = 1$. As $B$ increases, $p$, $c_2$ and $c_3$ increase and $c_1$, $y$, and $k$ decrease.*

*Proof.* We solved for $f = 0$ in Appendix B. The following two equations characterize the equilibrium:

$$F'(k)k + \frac{B}{p} = \sum_{j\in\{2,3\}} \frac{u_j'^{-1}\left(\frac{\frac{1}{\sigma\beta F'(k)} - \frac{1-\sigma}{\sigma}p}{a_j}\right)}{a_j},$$

$$\frac{F(k)}{\sigma} = \frac{u_1'^{-1}\left(\frac{(1+\lambda_c)p}{a_1}\right)}{a_1} + \sum_{j\in\{2,3\}} \frac{u_j'^{-1}\left(\frac{\frac{1}{\sigma\beta F'(k)} - \frac{1-\sigma}{\sigma}p}{a_j}\right)}{a_j}.$$

Consider the $(k, p)$ space. The first equation illustrates an increasing function of $p$ in terms of $k$. If $\sigma = 1$, the second equation illustrates a decreasing function of $p$ in terms of $k$. Therefore, there exists at most one solution. With an increase in $B$, the first curve shifts upwards, so $p$ increases and $k$ decreases. These together imply that $c_2$ and $c_3$ increase. A higher $p$ implies a lower $c_1$. A lower $k$ implies lower aggregate consumption. ∎

According to this result, when cash is used only in good-1 transactions and credit is scarce, improving credit conditions reduces the consumption of non-credit goods as well as aggregate consumption.

There are spillovers in this environment. As $B$ increases, the DC becomes more relaxed, increasing aggregate demand and consequently the AM price. This, in turn, tightens the CC, so $c_1$ decreases. To determine the overall effect on investment, note that the increase in $B$ means that agents would need smaller deposits to make their payments. This decreases the demand for deposits, thus increasing $R_d$. The DC tells us that as $\sigma$ gets close to 1, the second effect is dominant. That is, the overall effect is that credit crowds out bank intermediation and leads to less production in the special case where $\sigma = 1$. Yet, the comparative statics may change when $\sigma$ is small.

**Discussion of comparative statics**

We learn that the effect of credit on bank intermediation and real variables will be different across the two above cases. If agents are indifferent between cash and deposits, extending credit conditions does not have real effects on the economy but crowds out cash. Yet, monetary policy is quite effective as it determines the interest rate on deposits and the cost of funding for banks. If agents prefer deposits to cash, then extending credit conditions crowds out bank intermediation and has negative spillovers on cash transactions.

# D Alternative Setups of the Supply Side of the Model

In this appendix, we consider several alternative setups of the supply side of our model and consider a cash-like CBDC with an interior $f$.

## D.1 Linear Production Technology

Here, we assume $F(k) = Ak$ and investigate the effects of a cash-like CBDC. The equilibrium conditions turn into:

$$Ak = \sigma c = \sigma u'^{-1}((1+\lambda_d)p).$$

The bankers' maximization problem can be written as:

$$F'(k) = \frac{R_d}{p}$$

Therefore:

$$\begin{aligned}
p &= \frac{R_d}{A} \\
&\rightarrow \quad c = u'^{-1}((1+\lambda_d)p) = u'^{-1}\left((1+\lambda_d)\frac{R_d}{A}\right) = u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma} - 1\right)R_d\right)\right) \\
&\rightarrow \quad k = \frac{\sigma}{A}c
\end{aligned}$$

If $\sigma < 1$ : As $R_m$ goes up, an interior $f$ implies $R_d = R_m$, so $c$ and $k$ go up, similar to the benchmark model.

If $\sigma = 1$ : $c$ and $k$ are fixed, independent of $R_d$.

## D.2 Bank Financial Constraint

In the model, the banks finance all their investment by issuing deposits. Here, we assume that banks can only finance a fraction $\theta$ of their investment by deposits and should work in the PM to finance the rest. The maximization problem of a bank can be written as:

$$\begin{aligned}
\pi &= \max_k pF(k) - R_d d - \frac{e}{\beta} \\
\text{subject to} &: \quad k = d + e, \\
d &\leq \theta k,
\end{aligned}$$

where $d$ denotes deposits and $e$ denotes equity. The first constraint is the balance sheet identity and the second one is the financial constraint. As long as $R_d < 1/\beta$, the problem turns into:

$$\pi = \max_k pF(k) - R_d \theta k - (1-\theta)\frac{k}{\beta}.$$

The first-order-condition (FOC) is then given by

$$pF'(k) = \theta R_d + \frac{1-\theta}{\beta} \to k = F'^{-1}\left(\frac{\theta R_d + \frac{1-\theta}{\beta}}{p}\right).$$

Market clearing condition:

$$F(k) = \sigma c = \sigma u'^{-1}((1+\lambda_c)p),$$

and liquidity premium:

$$1 + \lambda_c = \frac{1}{\sigma \beta R_m} - \left(\frac{1}{\sigma} - 1\right),$$

where we used $R_d = R_m$ for an interior $f$. Therefore:

$$F\left(F'^{-1}\left(\frac{\theta R_m + \frac{1-\theta}{\beta}}{p}\right)\right) = \sigma u'^{-1}\left(\left(\frac{1}{\sigma\beta R_m} - \left(\frac{1}{\sigma} - 1\right)\right)p\right).$$

The LHS is increasing and the RHS is decreasing in $p$. As $R_m$ goes up, the LHS shifts downwards and the RHS shifts upwards, so $p$ increases.

In an extreme case of $\theta = 0$, where banks cannot use deposits, the LHS does not move, which means that we don't have disintermediation effect and we only have the payments efficiency effect. Therefore, $p$ and $c$ both increase. In another extreme case, banks can use deposits for all of their financing. This is the case in our benchmark model and we know that $k$ and $c$ are increasing in $R_m$. In any case, the results are similar to the benchmark model.

### D.3 Some Production Not Financed by Banks

We consider two extensions together in this part. First, Suppose the cost of funding for banks is $\Lambda R_m$, where $\Lambda > 0$ and $\Lambda$ can be even greater than 1. $\Lambda - 1$ represent additional resources banks need to spend to raise deposits. Second, suppose there is another sector in this economy that produces the same intermediate good. This sector has an increasing supply curve $G(p)$, i.e., the amount of production is $G(p)$ given price $p$. Importantly, this sector does not rely on the banking system for production. Here is the market clearing condition:

$$\frac{\theta}{\sigma}F\left(F'^{-1}\left(\frac{\Lambda}{p\beta(1+\sigma\lambda_c)}\right)\right) + \frac{1-\theta}{\sigma}G(p) = \frac{u_1'^{-1}(\frac{(1+\lambda_c)p}{a_1})}{a_1} + \frac{u_2'^{-1}(\frac{(1+\lambda_c)p}{a_2})}{a_2} + \frac{u_3'^{-1}(\frac{(1+\lambda_c)p}{a_3})}{a_3}.$$

Note that we drop the arguments of the functions (e.g., $F' \equiv F'(k)$ and $u_1'' \equiv u_1''(c_1)$). To evaluate effects of a CBDC, we take the derivative with respect to $\lambda_c$:

$$\frac{\theta}{\sigma}F'\frac{-\Lambda p'}{F''p^2\beta(1+\sigma\lambda_c)} + \frac{\theta}{\sigma}F'\frac{-\Lambda p\sigma}{F''p^2\beta(1+\sigma\lambda_c)^2} + \frac{1-\theta}{\sigma}G'(p)p' = \left((1+\lambda_c)p'+p\right)\sum\frac{1}{a_i^2u_i''},$$

so $p'$ is given by

$$p' = \frac{-\left(\frac{\Lambda\theta F'}{F''p^2\beta(1+\sigma\lambda_c)^2}+\sum\frac{1}{a_i^2u_i''}\right)p}{-\frac{1-\theta}{\sigma}G'(p)+\frac{\Lambda\theta F'}{\sigma F''p^2\beta(1+\sigma\lambda_c)}+\sum\frac{1+\lambda_c}{a_i^2u_i''}} < 0,$$

thus,

$$(1+\lambda_c)p'+p \quad = \quad \frac{-\frac{1-\theta}{\sigma}G'(p)+\frac{\Lambda\theta F'(1-\sigma)}{\sigma F''p^2\beta(1+\sigma\lambda_c)^2}}{-\frac{1-\theta}{\sigma}G'(p)+\frac{\theta F'}{\sigma F''p^2\beta(1+\sigma\lambda_c)}+\sum\frac{1+\lambda_c}{a_i^2u_i''}}p < 0.$$

Note that $p'$ is always negative.

Now we calculate $\frac{dk}{d\lambda_c}$:

$$\begin{aligned}\frac{\theta}{\sigma p}F'(k)\frac{dk}{d\lambda_c} &= -\frac{1-\theta}{\sigma p}G'(p)p' + \frac{(1+\lambda_m)p'+p}{p}\sum\frac{1}{a_i^2u_i''} \\ &= \frac{\frac{\Lambda\theta F'}{F''p^2\beta(1+\sigma\lambda_c)^2}\left[\frac{1-\theta}{\sigma}G'(p)+\frac{1-\sigma}{\sigma}\sum\frac{1}{a_i^2u_i''}\right]}{-\frac{1-\theta}{\sigma}G'(p)+\frac{\Lambda\theta F'}{\sigma F''p^2\beta(1+\sigma\lambda_c)}+\sum\frac{1+\lambda_c}{a_i^2u_i''}}\end{aligned}$$

If $G'(p) = 0$, which is consistent with our benchmark model, then $(1+\lambda_c)p'+p < 0$ and $\frac{dk}{d\lambda_c} < 0$, and we obtain our main results.

However, in general if $G'(p) > 0$, the term $\frac{1}{\sigma}G'(p)$ in the numerator counteracts the other term and can make it less negative or even positive. Therefore, if $G'(p)$ is sufficiently large, then $\frac{dk}{d\lambda_c}$ will be positive and the effect on investment can be overturned. However, notice that the aggregate consumption increases because of this term: $(1+\lambda_c)p'+p$. That is, still the aggregate demand goes up, but the amount of production that is financed by banks may go down.

In summary, if there is another supply source that is not affected by the change in the policy rate and responds only to the AM price, then the change in aggregate demand may increase the AM price so much as to induce a big increase in the production by the other sector. This may lead to the case in which the amount of production financed by banks may fall. In this case, we have disintermediation, although the aggregate consumption still goes up.

Here is the intuition. Paying higher interest on CBDC increases demand. For the sake of argument, suppose the AM price is constant. An increase in demand should be satisfied by banks and the other sector. Since the AM price is fixed, other sector's production does not change, so all the excess demand should be supplied by banks. As AM price goes up, both banks and the other sector produce more. However, if the response of the other sector is sufficiently strong, the banks' supply may go down because the other sector crowds out banking as the banks' intermediation cost is now higher.

Consider one **special case** where $\Lambda = 1$ and $G(p) = F(F'^{-1}(\frac{1}{p\beta}))$. The interpretation is that firms in this sector can work in the PM and can consume in the next PM. That is, the firms in this sector solve: $\max_k \beta p F(k) - k$, which gives $k = F'^{-1}(\frac{1}{p\beta})$. Then

$$G'(p) = -\frac{F'}{F'' p^2 \beta},$$

so

$$\frac{1}{\sigma} F'(k) \frac{dk}{d\lambda_c} = \frac{\frac{\theta F'}{F'' p \beta (1+\sigma \lambda_m)^2} \left[ -(1-\theta) \frac{F'}{\sigma F'' p^2 \beta} + \frac{1-\sigma}{\sigma} \sum \frac{1}{a_i^2 u_i''} \right]}{(1-\theta) \frac{F'}{\sigma F'' p^2 \beta} + \theta \frac{F'}{\sigma F'' p^2 \beta (1+\sigma \lambda_c)} + \sum \frac{1+\lambda_c}{a_i^2 u_i''}}.$$

For a given $\theta \in (0,1)$, if $\sigma$ is close to 1, $\frac{dk}{d\lambda_c}$ will be positive, so investment decreases with a higher interest rate on CBDC.[33]

[33] Note that $D = R_d k = pkF'(k)$ and $kF'(k)$ is increasing in $k$ by assumption. However, $p$ is decreasing in $\lambda_c$. Therefore, the effect on $D$ is generally ambiguous in this extension.

# E General CES Utility Function

In this section, we modify the AM utility function but everything else remains the same as in the benchmark model. The AM utility function is given by

$$v(c_1, c_2, c_3) = \frac{1}{1-\frac{1}{\xi}}\left(\omega_1 c_1^{1-\frac{1}{\varsigma}} + \omega_2 c_2^{1-\frac{1}{\varsigma}} + \omega_3 c_3^{1-\frac{1}{\varsigma}}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}}.$$

When $\varsigma = \xi$, the utility function is separable, which is a special case of the benchmark model. The derivatives are given by $v_i(.) = \omega_i c_i^{-\frac{1}{\varsigma}} Q$, where

$$Q \equiv \left(c_1^{1-\frac{1}{\varsigma}} + \omega_2 c_2^{1-\frac{1}{\varsigma}} + \omega_3 c_3^{1-\frac{1}{\varsigma}}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}-1}.$$

Like in the main body of the paper, we start by having cash, cash-like CBDC, deposits and credit. The FOCs are given by:

$$\begin{aligned}
c_1 &: v_1(.) = (1+\lambda_c)p_1, \\
c_2 &: v_2(.) = (1+\lambda_d)p_2, \\
c_3 &: v_3(.) = (1+f\lambda_c + (1-f)\lambda_d)p_3.
\end{aligned}$$

Therefore, the equilibrium conditions are:

$$\begin{aligned}
c_1 &: c_1 = \left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma} \\
c_2 &: c_2 = \left(\frac{\omega_2 a_2 Q}{(1+\lambda_d)p}\right)^{\varsigma} \\
c_3 &: c_3 = \left(\frac{\omega_3 a_3 Q}{(1+f\lambda_c + (1-f)\lambda_d)p}\right)^{\varsigma} \\
f &: \lambda_d - \lambda_c \begin{cases} \leq 0, & \text{if } f = 0, \\ = 0, & \text{if } f \in (0,1), \\ \geq 0, & \text{if } f = 1, \end{cases}
\end{aligned}$$

As before:

$$p_i = \frac{p}{a_i}.$$

As long as $f \in (0,1)$, we have $\lambda_d = \lambda_c$. We now solve for $Q$ using its definition as well as the FOCs:

$$\begin{aligned}
Q &= \left(\omega_1 c_1^{1-\frac{1}{\varsigma}} + \omega_2 c_2^{1-\frac{1}{\varsigma}} + \omega_3 c_1^{1-\frac{1}{\varsigma}}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}-1} \\
&= \left(\omega_1\left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma-1} + \omega_2\left(\frac{\omega_2 a_2 Q}{(1+\lambda_c)p}\right)^{\varsigma-1} + \omega_3\left(\frac{\omega_3 a_3 Q}{(1+\lambda_c)p}\right)^{\varsigma-1}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}-1}
\end{aligned}$$

$$\rightarrow Q = ((1+\lambda_c)p)^{1-\frac{\xi}{\varsigma}} \left(\omega_1^{\varsigma} a_1^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}} = \omega_0^{-1} ((1+\lambda_c)p)^{1-\frac{\xi}{\varsigma}},$$

where

$$\omega_0 \equiv \left(\omega_1^{\varsigma} a_1^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}}.$$

When $\varsigma = \xi$, the utility function is separable and $Q = 1$. However, when $\varsigma \neq \xi$, $Q$ is generally a function of $p$. Now, we can rewrite the FOCs for the case of interior $f$ as follows:

$$c_i = \left( \frac{\left(\frac{\omega_i a_i}{\omega_0}\right)^{\frac{\varsigma}{\xi}}}{(1+\lambda_c)p} \right)^{\xi} \text{ for } i \in \{1,2,3\}. \tag{32}$$

This equation suggests that all results in the benchmark model with separable utility continue to hold in the model with a non-separable, CES utility function with appropriate change of variables:

$$\omega_i^{new} \equiv \left(\frac{\omega_i}{\omega_0}\right)^{\frac{\varsigma}{\xi}} a_i^{\frac{\varsigma}{\xi}-1} \text{for } i \in \{1,2,3\}.$$

When $\varsigma$ goes up, the goods become better substitutes. In the extreme case of $\varsigma = \infty$, the goods are perfect substitutes.

Market clearing condition implies:

$$\sigma \left[\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right] = F(k). \tag{33}$$

The LHS is the total demand for intermediate good and the RHS is the supply of intermediate good. The LHS is a decreasing function of $p$ as shown above, and the RHS is an increasing function of $p$ obtained from bankers' maximization problem: $F'(k) = \frac{R_d}{p}$.

**Efficiency**

We characterize the first-best allocation, which is given by

$$\max_{c_1,c_2,c_3,k} \sigma\beta v(c_1,c_2,c_3) - k$$

subject to (33). As before, we denote the Lagrangian multiplier associated with the constraint as $\beta p^*$. The Lagrangian and FOCs are given by

$$\begin{aligned} L &= \sigma\beta v(c_1,c_2,c_3) - k - \sigma\beta p^* \left[\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right] + \beta p^* F(k), \\ k &: \beta p^* F'(k) = 1 \rightarrow k^* = F'^{-1}\left(\frac{1}{\beta p^*}\right), \\ c_i &: v_1 a_1 = v_2 a_2 = v_3 a_3 = p^*. \end{aligned}$$

But $v_i(.) = \omega_i c_i^{-\frac{1}{\varsigma}} Q$, so FOCs give

$$c_i^* = \left(\frac{\omega_i a_i Q^*}{p^*}\right)^{\varsigma}.$$

We now obtain $Q^*$ after doing some algebra:

$$Q^* = \omega_0^{-1} p^{*\left(1-\frac{\xi}{\varsigma}\right)}.$$

Hence:

$$c_i^* = \left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma} p^{*-\xi}.$$

The resource constraint, $\sum \frac{c_i^*}{a_i} = \frac{1}{\sigma} F(k^*)$, gives

$$\sum \left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma} a_i^{-1} p^{*-\xi} \quad = \quad \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}} p^{*-\xi} = \frac{1}{\sigma} F\left(F'^{-1}\left(\frac{1}{\beta p^*}\right)\right).$$

The following lemma is equivalent to Lemma 1, stating the conditions under which an efficient equilibrium exists.

**Lemma 17.** *The equilibrium allocation is efficient when* $R_z = \beta^{-1}$ *and* $B \geq \max\{0, B^*\}$*, where*

$$B^* \equiv \frac{c_2^*}{a_2} p^* - \frac{k^*}{\beta}.$$

The intuition and proof are the same as those of Lemma 1.

Since we use special production function $F(k) = \frac{k^{1-\gamma}}{1-\gamma}$ for our calibration, we solve for the efficient allocation for that:

$$\begin{aligned}
\sigma \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}} p^{*-\xi} \quad &= \quad \frac{k^{1-\gamma}}{1-\gamma} = \frac{\beta^{\frac{1-\gamma}{\gamma}} p^{*\frac{1-\gamma}{\gamma}}}{1-\gamma} \to p^* = \left[\sigma\left(1-\gamma\right)\beta^{-\frac{1-\gamma}{\gamma}} \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{1}{\xi+\frac{1-\gamma}{\gamma}}}, \\
k^* \quad &= \quad F'^{-1}\left(\frac{1}{\beta p^*}\right) = \beta^{\frac{1}{\gamma}} p^{*\frac{1}{\gamma}} = \beta^{\frac{1}{\gamma}} \left[\sigma\left(1-\gamma\right)\beta^{-\frac{1-\gamma}{\gamma}} \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{1}{\gamma\xi+1-\gamma}} = \beta^{\frac{\xi}{\gamma\xi+1-\gamma}} \left[\sigma\left(1-\gamma\right)\omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{1}{\gamma\xi+1-\gamma}}, \\
c_i^* \quad &= \quad \left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma} \left[\sigma\left(1-\gamma\right)\beta^{-\frac{1-\gamma}{\gamma}} \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{-\frac{\xi}{\xi+\frac{1-\gamma}{\gamma}}} \\
&= \quad \omega_i^{\varsigma} a_i^{\varsigma} \left(1-\gamma\right)^{-\frac{\xi}{\xi+\frac{1-\gamma}{\gamma}}} \sigma^{-\frac{\xi}{\xi+\frac{1-\gamma}{\gamma}}} \beta^{\frac{1-\gamma}{\gamma}\frac{\xi}{\xi+\frac{1-\gamma}{\gamma}}} \omega_0^{-\frac{1-\xi}{\frac{\xi}{\varsigma}-1}\frac{\xi}{\xi+\frac{1-\gamma}{\gamma}}-\varsigma}, \\
B^* \quad &\equiv \quad \frac{c_2^*}{a_2} p^* - \frac{k^*}{\beta} \\
&= \quad \frac{\omega_2^{\varsigma} a_2^{\varsigma-1}}{\omega_0^{\varsigma}} \left[\sigma\left(1-\gamma\right)\beta^{-\frac{1-\gamma}{\gamma}} \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{1-\xi}{\xi+\frac{1-\gamma}{\gamma}}} - \beta^{\frac{(\xi-1)(1-\gamma)}{\gamma\xi+1-\gamma}} \left[\sigma\left(1-\gamma\right)\omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{1}{\gamma\xi+1-\gamma}} \\
&= \quad \beta^{\frac{(\xi-1)(1-\gamma)}{\gamma\xi+1-\gamma}} \left[\sigma\left(1-\gamma\right)\omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right]^{\frac{\gamma-\gamma\xi}{\gamma\xi+1-\gamma}} \left[\omega_2^{\varsigma} a_2^{\varsigma-1} \omega_0^{-\varsigma} - \sigma\left(1-\gamma\right)\omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}\right].
\end{aligned}$$

With separable utility, $\omega_0 = 1$, so:

$$B^* = \beta^{\frac{(\xi-1)(1-\gamma)}{\gamma\xi+1-\gamma}} \left[\sigma\left(1-\gamma\right)\right]^{\frac{\gamma-\gamma\xi}{\gamma\xi+1-\gamma}} \left[\omega_2^{\xi} a_2^{\xi-1} - \sigma\left(1-\gamma\right)\right].$$

**Summary of results with a cash-like CBDC and interior $f$**

In this case, $R_d = R_m$ and $\sigma\lambda_c + 1 = \frac{1}{\beta R_m} = \sigma\lambda_d + 1$. We combine supply and demand equations to obtain:

$$F\left(F'^{-1}\left(\frac{R_m}{p}\right)\right) = \sigma\left((1+\lambda_c)p\right)^{-\xi} \sum \frac{1}{a_i}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}. \tag{34}$$

The following result is an extension of Proposition 3 for the general CES case.

**Proposition 18.** *Suppose $f \in (0,1)$ and $\sigma < 1$. A higher real rate, $R_m$, paid on the cash-like CBDC leads to higher $p$, $c_i$, $c$, $k$ and welfare.*

*Proof.* For the effects on $p$, $c_i$, $c$, $k$, we follow exactly the proof of Proposition 3. This is possible because $c_i$ is simply a decreasing function of $(1+\lambda_c)p$ according to (32).

Consider (34). The RHS is decreasing and the LHS is increasing in $p$. As $R_m$ increases, the LHS decreases and the RHS increases. Therefore, $p$ increases. If $(1+\lambda_c)p$ increases, then the RHS of (34) decreases, so the LHS should decrease as well. This means that $p/R_m$, which is equal to $\beta\sigma(1+\lambda_m)p + \beta(1-\sigma)p$, should decrease. But $p$ increases, so $(1+\lambda_c)p$ should decrease, which is a contradiction. Therefore, $(1+\lambda_c)p$ decreases with an increase in $R_m$. As a result, $k$ increases, and (32) implies that $c_i$ increases. Consequently, aggregate $c$ increases.

Now we **calculate the effect of $R_m$ on welfare.** Denote $p' \equiv \frac{\partial p}{\partial \lambda_c}$. We also drop the arguments of the functions (e.g., $F' \equiv F'(k)$). Welfare is given by

$$\mathcal{W} = U(Y^*) - Y^* + \beta\sigma v(c_1, c_2, c_3) - k,$$

so

$$\frac{1}{\beta\sigma}\frac{d\mathcal{W}}{d\lambda_c} = \sum v_i \frac{dc_i}{d\lambda_c} - \frac{1}{\beta\sigma}\frac{dk}{d\lambda_c}.$$

But $v_i(.) = \omega_i c_i^{-\frac{1}{\varsigma}} Q$, so

$$v_i(.) \quad = \quad \omega_i \left[\frac{\left(\frac{\omega_i a_i}{\omega_0}\right)^{\frac{\varsigma}{\xi}}}{(1+\lambda_c)p}\right]^{-\frac{\xi}{\varsigma}} \omega_0^{-1}\left((1+\lambda_c)p\right)^{1-\frac{\xi}{\varsigma}} = \frac{(1+\lambda_c)p}{a_i}.$$

Therefore,

$$\begin{aligned}\sum v_i \frac{dc_i}{d\lambda_c} &= -\sum \frac{(1+\lambda_c)p}{a_i}\xi\left[(1+\lambda_c)p' + p\right]((1+\lambda_m)p)^{-\xi-1}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}\\ &= -\sum \frac{\xi}{a_i}\left[(1+\lambda_c)p' + p\right]((1+\lambda_c)p)^{-\xi}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}.\end{aligned}$$

We also have $F'(k) = \frac{R_m}{p} = \frac{1}{p\beta(1+\sigma\lambda_c)}$, so

$$\frac{1}{\beta\sigma}\frac{dk}{d\lambda_c} = -\frac{p'(1+\sigma\lambda_c) + p\sigma}{\beta\sigma F'' p^2 \beta(1+\sigma\lambda_c)^2}.$$

Now we calculate $p'(1+\sigma\lambda_c) + p\sigma$. For that, we take the derivative of (34) with respect to $\lambda_c$:

$$-F'\frac{p'(1+\sigma\lambda_c) + p\sigma}{F'' p^2\beta(1+\sigma\lambda_c)^2} = -\xi\left[(1+\lambda_m)p' + p\right]\sigma\left((1+\lambda_c)p\right)^{-\xi-1}\sum\frac{1}{a_i}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}.$$

Altogether:

$$\begin{aligned}\frac{1}{\beta\sigma}\frac{d\mathcal{W}}{d\lambda_c} &= -\sum \frac{\xi}{a_i}\left[(1+\lambda_c)p' + p\right]((1+\lambda_c)p)^{-\xi}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma} + \frac{p'(1+\sigma\lambda_c) + p\sigma}{\beta\sigma F'' p^2\beta(1+\sigma\lambda_c)^2}\\ &= -\sum \frac{\xi}{a_i}\left[(1+\lambda_c)p' + p\right]((1+\lambda_c)p)^{-\xi}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}\\ &\quad + \frac{\xi}{\beta\sigma F'}\left[(1+\lambda_c)p' + p\right]\sigma\left((1+\lambda_c)p\right)^{-\xi-1}\sum\frac{1}{a_i}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}\\ &= -\underbrace{\left((1+\lambda_c)p' + p\right)}_{>0}\left((1+\lambda_c)p\right)^{-\xi-1}\xi\left[\underbrace{(1+\lambda_c)p - \frac{1}{\beta F'}}_{=\beta(1+\lambda_c)p - \frac{p}{R_m} = p[\beta(1+\lambda_c) - \beta(1+\sigma\lambda_c)]\geq 0}\right]\sum\frac{1}{a_i}\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}.\end{aligned}$$

Note that $\frac{d[p(1+\lambda_c)]}{d\lambda_m}$ is positive because $c_i$ decreases with $\lambda_c$, as shown above. Therefore, $d\mathcal{W}/d\lambda_c$ is strictly negative for $\sigma < 1$. ∎

As a side-product, this proof shows that: $\frac{1}{\beta\sigma}\frac{dk}{d\lambda_m} = \frac{1+\sigma\lambda_m}{1+\lambda_m}\sum v_i \frac{dc_i}{d\lambda_m}$.

**Summary of results with a deposit-like CBDC**

We have $R_d = R_m$. Consumption of different goods are given by

$$\begin{aligned} c_1 &: \; c_1 = \left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma}\\ c_j &: \; c_j = \left(\frac{\omega_j a_j Q}{(1+\lambda_d)p}\right)^{\varsigma}\end{aligned}$$

for $j \in \{2,3\}$. We now solve for $Q$ using its definition as well as the FOCs:

$$\begin{aligned}
Q(\lambda_c,\lambda_d) &= \left(\omega_1 c_1^{1-\frac{1}{\varsigma}} + \omega_2 c_2^{1-\frac{1}{\varsigma}} + \omega_3 c_1^{1-\frac{1}{\varsigma}}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}-1} \\
&= \left(\omega_1\left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma-1} + \omega_2\left(\frac{\omega_2 a_2 Q}{(1+\lambda_d)p}\right)^{\varsigma-1} + \omega_3\left(\frac{\omega_3 a_3 Q}{(1+\lambda_d)p}\right)^{\varsigma-1}\right)^{\frac{1-\frac{1}{\xi}}{1-\frac{1}{\varsigma}}-1} \to \\
Q &= \left(\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}} (1+\lambda_d)^{1-\frac{\xi}{\varsigma}} p^{1-\frac{\xi}{\varsigma}}, \\
\left(\frac{Q}{p}\right)^{\varsigma} &= [\omega(\lambda_c,\lambda_d)]^{-\varsigma}(1+\lambda_d)^{\varsigma-\xi} p^{-\xi}, \qquad (35)
\end{aligned}$$

where

$$\omega(\lambda_c,\lambda_d) \equiv \left(\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}}. \qquad (36)$$

Therefore:

$$\begin{aligned}
c_1 &= \left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma} = (\omega_1 a_1)^{\varsigma}\left[\omega(\lambda_c,\lambda_d)(1+\lambda_d)^{\frac{\xi-\varsigma}{\varsigma}}(1+\lambda_c)\right]^{-\varsigma} p^{-\xi} \\
&= \left(\frac{\omega_1 a_1}{\omega(\lambda_c,\lambda_d)}\right)^{\varsigma}(1+\lambda_d)^{-\xi}\left(\frac{1+\lambda_c}{1+\lambda_d}\right)^{-\varsigma} p^{-\xi}, \\
c_j &= \left(\frac{\omega_j a_j Q}{(1+\lambda_d)p}\right)^{\varsigma} = (\omega_j a_j)^{\varsigma}\left[\omega(\lambda_c,\lambda_d)(1+\lambda_d)^{\frac{\xi-\varsigma}{\varsigma}}(1+\lambda_d)\right]^{-\varsigma} p^{-\xi} \\
&= \left(\frac{\omega_j a_j}{\omega(\lambda_c,\lambda_d)}\right)^{\varsigma}(1+\lambda_d)^{-\xi} p^{-\xi} \text{ where } j \in \{2,3\},
\end{aligned}$$

All $c_i$s are decreasing in $p$. Here, we show the following:

**Lemma 19.** *$c_1$ is decreasing in $1+\lambda_c$. Also, if $\xi \geq \varsigma$,* ***then all $c_i$s are decreasing in*** *$1+\lambda_d$.*

*Proof.* Take derivative with respect to $\lambda_c$:

$$\frac{\partial\omega(\lambda_c,\lambda_d)}{\partial\lambda_c} = \left(\frac{\xi}{\varsigma}-1\right)\left(\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1} \omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1}{1+\lambda_z}\right)^{\varsigma}(1+\lambda_d)^{\varsigma-1}.$$

For $X \in \mathbb{R}$, we have

$$\frac{\frac{\partial}{\partial\lambda_c}\left[\omega(1+\lambda_z)^X\right]}{(1+\lambda_c)^{X-1}} = \frac{\partial\omega}{\partial\lambda_c}(1+\lambda_c) + X\omega$$

$$= \left(\frac{\xi}{\varsigma}-1\right)\left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1}{1+\lambda_c}\right)^{\varsigma-1}(1+\lambda_d)^{\varsigma-1}$$

$$+X\left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}}$$

$$= \left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_m}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\left[\begin{array}{c}\left(\frac{\xi}{\varsigma}-1+X\right)\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}\\ +X\omega_2^{\varsigma}a_2^{\varsigma-1}+X\omega_3^{\varsigma}a_3^{\varsigma-1}\end{array}\right].$$

Hence,

$$\frac{\partial}{\partial\lambda_c}\left[\omega\left(1+\lambda_c\right)\right] = \left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\left[\begin{array}{c}\frac{\xi}{\varsigma}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}\\ +\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\end{array}\right]$$

$$= \left[\omega\left(\lambda_c,\lambda_d\right)\right]^{1+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}\left[\frac{\xi}{\varsigma}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right]>0.$$

But $c_1$ is proportional to $\left[\omega\left(1+\lambda_c\right)\right]^{-1}$; therefore, $c_1$ is decreasing in $1+\lambda_c$.

Show that $c_i$ is a decreasing function of $(1+\lambda_d)$. First, note that:

$$\frac{\partial\omega\left(\lambda_c,\lambda_d\right)}{\partial\lambda_d} = \left(1-\frac{\xi}{\varsigma}\right)\left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\omega_1^{\varsigma}a_1^{\varsigma-1}\frac{(1+\lambda_m)^{\varsigma-2}}{(1+\lambda_z)^{\varsigma-1}}.$$

Hence,

$$\frac{\frac{\partial}{\partial\lambda_d}\left[\omega\left(1+\lambda_d\right)^X\right]}{(1+\lambda_d)^{X-1}} = \frac{\partial\omega}{\partial\lambda_d}\left(1+\lambda_d\right)+X\omega$$

$$= \left(1-\frac{\xi}{\varsigma}\right)\left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\omega_1^{\varsigma}a_1^{\varsigma-1}\frac{(1+\lambda_d)^{\varsigma-1}}{(1+\lambda_z)^{\varsigma-1}}$$

$$+X\left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}}$$

$$= \left(\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_m}{1+\lambda_c}\right)^{\varsigma-1}+\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1}\left[\begin{array}{c}\left(1-\frac{\xi}{\varsigma}+X\right)\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1}\\ +X\omega_2^{\varsigma}a_2^{\varsigma-1}+X\omega_3^{\varsigma}a_3^{\varsigma-1}\end{array}\right]$$

Therefore:

$$\text{For } c_1: \quad \frac{\frac{\partial}{\partial \lambda_d}\left[\omega\left(1+\lambda_d\right)^{\frac{\xi}{\varsigma}}\right]}{\left(1+\lambda_d\right)^{\frac{\xi}{\varsigma}-1}} = \left(\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_z}\right)^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1} \times$$

$$\left[\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} + \frac{\xi}{\varsigma}\omega_2^{\varsigma} a_2^{\varsigma-1} + \frac{\xi}{\varsigma}\omega_3^{\varsigma} a_3^{\varsigma-1}\right] > 0,$$

$$\text{For } c_2 \& c_3: \quad \frac{\frac{\partial}{\partial \lambda_d}\left[\omega\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]}{\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}-1}}$$

$$= \frac{\xi-\varsigma}{\varsigma}\left(\omega_1^{\varsigma} a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_z}\right)^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}-1} \left[\omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right].$$

If $\xi \geq \varsigma$, then all $c_i$s are decreasing in $1+\lambda_d$. ∎

**Comparison between cash-like and deposit-like CBDC**

**Proposition 20** (Extension of Proposition 6 for general CES utility function). *Assume $\xi \geq \varsigma$. Suppose $\sigma < 1$. Consider two economies: one with a cash-like CBDC with interest rate $R_m > R_z$, and one with a deposit-like CBDC with same interest rates. If agents are indifferent between a CBDC and deposits in both economies, then*

*(i) a cash-like CBDC leads to higher $c_1$, $c$, $k$, $\pi$,*

*(ii) overall, the cash-like CBDC leads to higher welfare if $R_m$ is sufficiently close to $R_z$.*

This is similar to Proposition 6 except that we do not have part (ii) (that a deposit-like CBDC leads to higher $c_2$, $c_3$).

*Proof.* Similar to the separable case, we have:

$$\text{Cash-like CBDC:} \quad \frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right)$$

$$= \left[\omega\left(\lambda_d,\lambda_d\right)\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma} p^{-\xi}\begin{pmatrix} \omega_1^{\varsigma}a_1^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma} \\ +\omega_2^{\varsigma}a_2^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma} \\ +\omega_3^{\varsigma}a_3^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\end{pmatrix}, \tag{37}$$

$$\text{Deposit-like CBDC:} \quad \frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right)$$

$$= \left[\omega\left(\lambda_c,\lambda_d\right)\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma} p^{-\xi}\begin{pmatrix} \omega_1^{\varsigma}a_1^{\varsigma-1}\left[1+\lambda_c\right]^{-\varsigma} \\ +\omega_2^{\varsigma}a_2^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma} \\ +\omega_3^{\varsigma}a_3^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\end{pmatrix}. \tag{38}$$

The following proof works somehow similar to the proof of Proposition 6 (i-ii). However, we cannot determine how $c_2$ and $c_3$ change for a deposit-like.

**Proof of part (i)**

**Step 1:** Consider the RHS and LHS in the $(p, y)$ space for both (37) and (38). The curve on the LHS is the same for both cash- and deposit-like CBDCs. Since $\lambda_d \leq \lambda_c$, $\frac{\partial\omega(\lambda_c,\lambda_d)}{\partial\lambda_c} > 0$ and $\frac{\partial\omega(\lambda_c,\lambda_d)(1+\lambda_z)}{\partial\lambda_c} > 0$, the curve on the RHS is higher for the cash-like CBDC and, as a result, the AM price is higher for that. Also, aggregate consumption and investment are higher. That is, $c^{CashLike} > c^{DepositLike}$ and $k^{CashLike} > k^{DepositLike}$.

**Step 2: Show that $c_1^{CashLike} \geq c_1^{DepositLike}$**. By way of contradiction suppose not, i.e., suppose $c_1^{CashLike} < c_1^{DepositLike}$; therefore,

$$\left[\omega\left(\lambda_d,\lambda_d\right)\left(1+\lambda_m\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma}\left(p^{CashLike}\right)^{-\xi}\left[1+\lambda_d\right]^{-\varsigma} < \left[\omega\left(\lambda_c,\lambda_d\right)\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma}\left(p^{DepositLike}\right)^{-\xi}\left[1+\lambda_c\right]^{-\varsigma}$$

But $\lambda_d \leq \lambda_c$, so $[1+\lambda_d]^{-\varsigma} \geq [1+\lambda_c]^{-\varsigma}$; therefore,

$$\left[\omega\left(\lambda_d,\lambda_d\right)\left(1+\lambda_m\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma}\left(p^{CashLike}\right)^{-\xi}\left[1+\lambda_d\right]^{-\varsigma} < \left[\omega\left(\lambda_c,\lambda_d\right)\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma}\left(p^{DepositLike}\right)^{-\xi}\left[1+\lambda_d\right]^{-\varsigma}$$

implying that $c_j^{CashLike} < c_j^{DepositLike}$ for $j = 2, 3$. Hence, $\sum_i \frac{c_i^{CashLike}}{a_i} < \sum_i \frac{c_i^{DepositLike}}{a_i}$. However, this is a contradiction with the fact that aggregate consumption with a cash-like CBDC is higher, as indicated in the first step.

**Step 3:** Note that, given $R_d$, the profit function is increasing in $p$ by the envelope theorem. Hence, the equilibrium profit is higher under the cash-like CBDC.

**Proof of part (ii): Comparison of welfare for cash like and deposit like CBDC**

Welfare is given by:

$$\mathcal{W} = U(Y^*) - Y^* + \beta\sigma v(c_1, c_2, c_3) - k.$$

Note that $\mathcal{W}$ is a function of $(\lambda_c, \lambda_d)$. Therefore,

$$\begin{aligned}
\frac{1}{\beta\sigma}\frac{d\mathcal{W}}{d\lambda_c} &= \sum v_i \frac{dc_i}{d\lambda_c} - \frac{1}{\beta\sigma}\frac{dk}{d\lambda_c} \\
&= \sum v_i \frac{dc_i}{d\lambda_c} - \frac{1}{\beta\sigma}\frac{\sigma}{F'}\sum \frac{1}{a_i}\frac{dc_i}{d\lambda_c} \\
&= \sum \left(v_i a_i - \frac{1}{\beta F'}\right)\frac{1}{a_i}\left[\frac{\partial c_i}{\partial\lambda_c} + \frac{\partial c_i}{\partial p}p'\right] \\
&= \left[(1+\lambda_c)p - \frac{1}{\beta F'}\right]\frac{1}{a_1}\left[\frac{\partial c_1}{\partial\lambda_c} - \frac{\xi}{p}c_1 p'\right] + \left[(1+\lambda_d)p - \frac{1}{\beta F'}\right]\sum_{j=2,3}\frac{1}{a_j}\left[\frac{\partial c_j}{\partial\lambda_c} - \frac{\xi}{p}c_j p'\right] \\
&= \left[(1+\lambda_c)p - (1+\lambda_d)p\right]\frac{1}{a_1}\frac{\partial c_1}{\partial\lambda_c} + \left[(1+\lambda_d)p - \frac{1}{\beta F'}\right]\sum\frac{1}{a_i}\frac{\partial c_j}{\partial\lambda_c} \\
&\quad - \left[\lambda_c - \lambda_d\right]\frac{\xi}{a_1}c_1 p' - \left[(1+\lambda_d)p - \frac{1}{\beta F'}\right]\frac{\xi}{p}p'\sum\frac{c_j}{a_j} \\
&= \left[p\frac{\partial c_1}{\partial\lambda_c} - \xi c_1 p'\right] + \left[1+\lambda_d - \frac{1}{\beta R_m}\right]p\left[\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c} - \frac{\xi}{p}p'\frac{F}{\sigma}\right] \\
&= \frac{\lambda_c - \lambda_d}{a_1}\left[p\frac{\partial c_1}{\partial\lambda_c} - \xi c_1 p'\right] + (1-\sigma)\,\lambda_d\left[\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c} - \frac{\xi}{p}p'\frac{F}{\sigma}\right] \\
&= \frac{\lambda_c - \lambda_d}{a_1}\left[p\frac{\partial c_1}{\partial\lambda_c} - \xi c_1 p'\right] + (1-\sigma)\,\lambda_d\left[\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c} - \frac{\sum\frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c}}{1-\frac{pF'}{\xi FF''p^2\beta(1+\sigma\lambda_d)}}\right] \\
&= \frac{\lambda_c - \lambda_d}{a_1}\left[p\frac{\partial c_1}{\partial\lambda_c} - \xi c_1 p'\right] - (1-\sigma)\,\lambda_d\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_z}\left[\frac{\frac{pF'}{\xi FF''p^2\beta(1+\sigma\lambda_d)}}{1-\frac{pF'}{\xi FF''p^2\beta(1+\sigma\lambda_d)}}\right] \\
&= \left[\lambda_c - \lambda_d\right]\frac{1}{a_1}\left[p\frac{\partial c_1}{\partial\lambda_c} - \xi c_1 p'\right] - (1-\sigma)\,\lambda_d\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c}\left[\frac{F'}{\xi FF''p\beta(1+\sigma\lambda_d) - F'}\right]
\end{aligned}$$

Note that $\frac{1}{\beta R_m} = \sigma(1+\lambda_d) + 1 - \sigma$, so $1 + \lambda_d - \frac{1}{\beta R_m} = 1 + \lambda_d - \sigma(1+\lambda_m) - 1 + \sigma = (1-\sigma)\,\lambda_d$.

We have used the derivation of $v_i$, $p'$, $\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c}$, and $\frac{dk}{d\lambda_c}$ from the next few parts (see below).

When $\lambda_c = \lambda_d$, then $\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial\lambda_c} < 0$ and $\xi FF''p\beta(1+\sigma\lambda_d) - F' < 0$. Therefore, $\frac{1}{\beta\sigma}\frac{dW}{d\lambda_c}$ is strictly negative at $(\lambda_c, \lambda_d)$. The rest of the proof follows directly from the proof of Proposition 6 thus omitted from here. ∎

**Calculate $\frac{\partial c_i}{\partial p}$ and $\frac{\partial c_i}{\partial \lambda_c}$**

$$\begin{aligned} c_1 &: \; c_1 = \left(\frac{\omega_1 a_1 Q}{(1+\lambda_c)p}\right)^{\varsigma} = (\omega_1 a_1)^{\varsigma}\left[\omega\left(\lambda_c,\lambda_d\right)(1+\lambda_d)^{\frac{\xi-\varsigma}{\varsigma}}(1+\lambda_c)\right]^{-\varsigma} p^{-\xi}, \\ c_j &: \; c_j = \left(\frac{\omega_j a_j Q}{(1+\lambda_d)p}\right)^{\varsigma} = (\omega_j a_j)^{\varsigma}\left[\omega\left(\lambda_c,\lambda_d\right)(1+\lambda_d)^{\frac{\xi-\varsigma}{\varsigma}}(1+\lambda_d)\right]^{-\varsigma} p^{-\xi}. \end{aligned}$$

First, it's easy to see that

$$\frac{\partial c_i}{\partial p} = -\frac{\xi}{p} c_i.$$

Now, we take derivative with respect to $\lambda_c$:

$$\begin{aligned} \frac{\partial c_1}{\partial \lambda_c} &= -\varsigma\left(\omega_1 a_1\right)^{\varsigma}(1+\lambda_d)^{\varsigma-\xi}\left[\omega\left(\lambda_c,\lambda_d\right)(1+\lambda_z)\right]^{-\varsigma-1}\frac{\partial\left[\omega\left(\lambda_c,\lambda_d\right)(1+\lambda_c)\right]}{\partial \lambda_c}p^{-\xi} \\ &= -\varsigma\left(\omega_1 a_1\right)^{\varsigma}(1+\lambda_d)^{\varsigma-\xi}(1+\lambda_c)^{-\varsigma-1}\left[\omega\left(\lambda_c,\lambda_d\right)\right]^{-\varsigma+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}\begin{bmatrix} \frac{\xi}{\varsigma}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} \\ +\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\end{bmatrix}p^{-\xi}, \end{aligned}$$

$$\begin{aligned} \frac{\partial c_j}{\partial \lambda_c} &= -\varsigma\left(\omega_j a_j\right)^{\varsigma}\omega\left(\lambda_c,\lambda_d\right)^{-\varsigma-1}\frac{\partial \omega\left(\lambda_c,\lambda_d\right)}{\partial \lambda_c}(1+\lambda_d)^{-\xi}p^{-\xi} \\ &= -\varsigma\left(\omega_j a_j\right)^{\varsigma}\omega\left(\lambda_c,\lambda_d\right)^{-\varsigma-1}.\left(\frac{\xi}{\varsigma}-1\right)\left[\omega\left(\lambda_c,\lambda_d\right)\right]^{1+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}\omega_1^{\varsigma}a_1^{\varsigma-1}(1+\lambda_c)^{-\varsigma}(1+\lambda_d)^{\varsigma-1}.(1+\lambda_m)^{-\xi}p^{-\xi} \\ &= -\varsigma\left(\omega_j a_j\right)^{\varsigma}\left(\frac{\xi}{\varsigma}-1\right)\left[\omega\left(\lambda_c,\lambda_d\right)\right]^{-\varsigma+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}\omega_1^{\varsigma}a_1^{\varsigma-1}(1+\lambda_c)^{-\varsigma}(1+\lambda_d)^{\varsigma-1-\xi}p^{-\xi}, \end{aligned}$$

where we calculated $\frac{\partial[\omega(\lambda_z,\lambda_d)(1+\lambda_c)]}{\partial \lambda_c}$ and $\frac{\partial \omega(\lambda_c,\lambda_d)}{\partial \lambda_c}$ from the proof of Lemma 19. We calculated $v_i$ as follows:

$$\begin{aligned} v_1(.) &= \omega_1 c_1^{-\frac{1}{\varsigma}}Q = \frac{(1+\lambda_c)p}{a_1}, \\ v_j(.) &= \omega_j c_j^{-\frac{1}{\varsigma}}Q = \frac{(1+\lambda_d)p}{a_j}, \end{aligned}$$

for $j=2,3$, and we used the definition of Q from (35).

**Calculate $\sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial \lambda_c}$**

$$\begin{aligned} \sum_i \frac{1}{a_i}\frac{\partial c_i}{\partial \lambda_c} &= \begin{bmatrix} -a_1^{-1}\varsigma\left(\omega_1 a_1\right)^{\varsigma}\left[1+\lambda_c\right]^{-\varsigma-1}\begin{bmatrix} \frac{\xi}{\varsigma}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} \\ +\omega_2^{\varsigma}a_2^{\varsigma-1}+\omega_3^{\varsigma}a_3^{\varsigma-1}\end{bmatrix} \\ -\sum_j a_j^{-1}\varsigma\left(\omega_j a_j\right)^{\varsigma}\left(\frac{\xi}{\varsigma}-1\right)\omega_1^{\varsigma}a_1^{\varsigma-1}(1+\lambda_c)^{-\varsigma}(1+\lambda_d)^{-1}\end{bmatrix}(1+\lambda_d)^{\varsigma-\xi}\left[\omega\left(\lambda_c,\lambda_d\right)\right]^{-\varsigma+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}p^{-\xi} \\ &= -\begin{bmatrix} \frac{\xi}{\varsigma}\omega_1^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma-1} \\ +\sum_j \omega_j^{\varsigma}a_j^{\varsigma-1}\left[1+\left(\frac{\xi}{\varsigma}-1\right)\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{-1}\right]\end{bmatrix}\varsigma\omega_1^{\varsigma}a_1^{\varsigma-1}\frac{(1+\lambda_d)^{\varsigma-\xi}}{(1+\lambda_c)^{\varsigma+1}}\left[\omega\left(\lambda_c,\lambda_d\right)\right]^{-\varsigma+\frac{\varsigma-1}{\frac{\xi}{\varsigma}-1}}p^{-\xi}<0. \end{aligned}$$

**Calculate $p' \equiv \frac{\partial p}{\partial \lambda_c}$ and $\frac{\partial k}{\partial \lambda_z}$** We calculate the derivative of $p$ with respect to $\lambda_c$ from (38):

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right) = \frac{c_1}{a_1}+\frac{c_2}{a_2}+\frac{c_3}{a_3} \rightarrow$$

$$\begin{aligned}
\frac{1}{\sigma}F'\frac{-p'}{F''p^2\beta(1+\sigma\lambda_d)} &= \sum \frac{1}{a_i}\left[\frac{\partial c_i}{\partial \lambda_c}+\frac{\partial c_i}{\partial p}p'\right] \rightarrow \\
p' &= -\frac{\sum \frac{1}{a_i}\frac{\partial c_i}{\partial \lambda_c}}{\frac{1}{\sigma}\frac{F'}{F''p^2\beta(1+\sigma\lambda_d)}+\sum \frac{1}{a_i}\frac{\partial c_i}{\partial p}} = -\frac{\sigma\sum \frac{1}{a_i}\frac{\partial c_i}{\partial \lambda_z}}{\frac{F'}{F''p^2\beta(1+\sigma\lambda_m)}-\frac{\xi}{p}F} \rightarrow \\
\frac{\xi}{p}\frac{F}{\sigma}p' &= \frac{\sum \frac{1}{a_i}\frac{\partial c_i}{\partial \lambda_c}}{1-\frac{pF'}{\xi F F''p^2\beta(1+\sigma\lambda_d)}},
\end{aligned}$$

where $p' \equiv \frac{\partial p}{\partial \lambda_c}$. We also drop the arguments of the functions (e.g., $F' \equiv F'(k)$ and $u_1'' \equiv u_1''(c_1)$) to simplify the exposition. Therefore,

$$\frac{dk}{d\lambda_c} = \frac{\sigma}{F'}\sum \frac{1}{a_i}\frac{dc_i}{d\lambda_c}.$$

## E.1 Special Production Function, $F(k) = \frac{k^{1-\gamma}}{1-\gamma}$

In this subsection, we collect equations for the special case of $F(k) = \frac{k^{1-\gamma}}{1-\gamma}$. This will be useful for the calibration.

From the bank's FOC, we have $k = \left(\frac{p}{R_m}\right)^{1/\gamma}$ and $F(k) = \frac{k^{1-\gamma}}{1-\gamma} = \frac{\left(\frac{p}{R_m}\right)^{\frac{1-\gamma}{\gamma}}}{1-\gamma}$.

**Cash-like CBDC** We have:

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{R_m}{p}\right)\right) = \left(1+\frac{1}{\beta\sigma R_m}-\frac{1}{\sigma}\right)^{-\xi}\frac{\sum \omega_i^{\varsigma}a_i^{\varsigma-1}}{\omega_0^{\varsigma}}p^{-\xi}.$$

Using the functional form, we obtain

$$\frac{1}{\sigma}\frac{1}{1-\gamma}\left(\frac{p}{R_m}\right)^{\frac{1-\gamma}{\gamma}} = \left(1+\frac{1}{\beta\sigma R_m}-\frac{1}{\sigma}\right)^{-\xi}\frac{\sum \omega_i^{\varsigma}a_i^{\varsigma-1}}{\omega_0^{\varsigma}}p^{-\xi},$$

so

$$p^{\frac{1-\gamma}{\gamma}+\xi} = \left(\frac{1}{\beta\sigma R_m}-\left(\frac{1}{\sigma}-1\right)\right)^{-\xi}R_m^{\frac{1-\gamma}{\gamma}}\sigma(1-\gamma)\sum\left(\frac{\omega_i}{\omega_0}\right)^{\varsigma}a_i^{\varsigma-1}, \tag{39}$$

where we have used $\frac{1}{\beta R_d} = \sigma\lambda_d + 1 = \sigma\lambda_c + 1$.

**Deposit-like CBDC** We have

$$\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{p\beta}\frac{1}{\sigma(1+\lambda_d)+1-\sigma}\right)\right) = \left[\omega\left(\lambda_c,\lambda_d\right)\left(1+\lambda_d\right)^{\frac{\xi-\varsigma}{\varsigma}}\right]^{-\varsigma}p^{-\xi}\begin{pmatrix}\omega_1^{\varsigma}a_1^{\varsigma-1}\left[1+\lambda_c\right]^{-\varsigma}\\ +\omega_2^{\varsigma}a_2^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\\ +\omega_3^{\varsigma}a_3^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\end{pmatrix}.$$

Using the functional form, we obtain

$$\frac{1}{\sigma}\frac{1}{1-\gamma}\left(\frac{p}{R_m}\right)^{\frac{1-\gamma}{\gamma}} = \left[\omega\left(\lambda_c,\lambda_m\right)\right]^{-\varsigma}\left(1+\lambda_d\right)^{\varsigma-\xi}p^{-\xi}\begin{pmatrix}\omega_1^{\varsigma}a_1^{\varsigma-1}\left[1+\lambda_c\right]^{-\varsigma}\\ +\omega_2^{\varsigma}a_2^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\\ +\omega_3^{\varsigma}a_3^{\varsigma-1}\left[1+\lambda_d\right]^{-\varsigma}\end{pmatrix},$$

so

$$p^{\frac{1-\gamma}{\gamma}+\xi} = \left(1+\lambda_d\right)^{-\xi}R_m^{\frac{1-\gamma}{\gamma}}\sigma\left(1-\gamma\right)\begin{pmatrix}\left(\frac{\omega_1}{\omega(\lambda_c,\lambda_d)}\right)^{\varsigma}a_1^{\varsigma-1}\left(\frac{1+\lambda_d}{1+\lambda_c}\right)^{\varsigma}\\ +\sum_j\left(\frac{\omega_j}{\omega(\lambda_c,\lambda_d)}\right)^{\varsigma}a_j^{\varsigma-1}\end{pmatrix}, \quad (40)$$

where $\frac{1}{\beta R_m}=\sigma\lambda_d+1$. Remember that $\omega\left(\lambda_c,\lambda_d\right)$ has been defined in (36).

## E.2 Quantitative Exercise

In this part, we report the quantitative results for the non-separable utility function.

Figure 7 considers a cash-like CBDC. The first row plots the shares of different means of payments $\alpha_i$ (i.e., the cash-like CBDC, deposits and credit) for different values of $\varsigma/\xi$. The second row plots the responses of the shares of different consumption goods, as well as investment $k$. Different curves in a graph denote different interest rates paid on the CBDC. Similarly, Figure 8 reports the case for a deposit-like CBDC with an additional fourth graph in the top row depicting the market share of the CBDC, $\alpha_4 \equiv M/(Z+D+B+M)$. For different degrees of substitutability, we obtain the same qualitative effects on investment (and banking accordingly): paying a higher rate expands investment, with a cash-like CBDC generating a bigger impact than a deposit like CBDC. Similarly, the impacts on the market shares of different payment instruments and goods are qualitatively robust.

Finally, we can also learn some interesting results regarding the effects of the elasticity of substitution on payment shares, consumption shares and investment. In general, for a given interest rate, a higher $\varsigma/\xi$ leads to a higher (lower) market share of credit (cash and CBDC), a higher (lower) share of type 3 (1 and 2) goods and a lower investment.

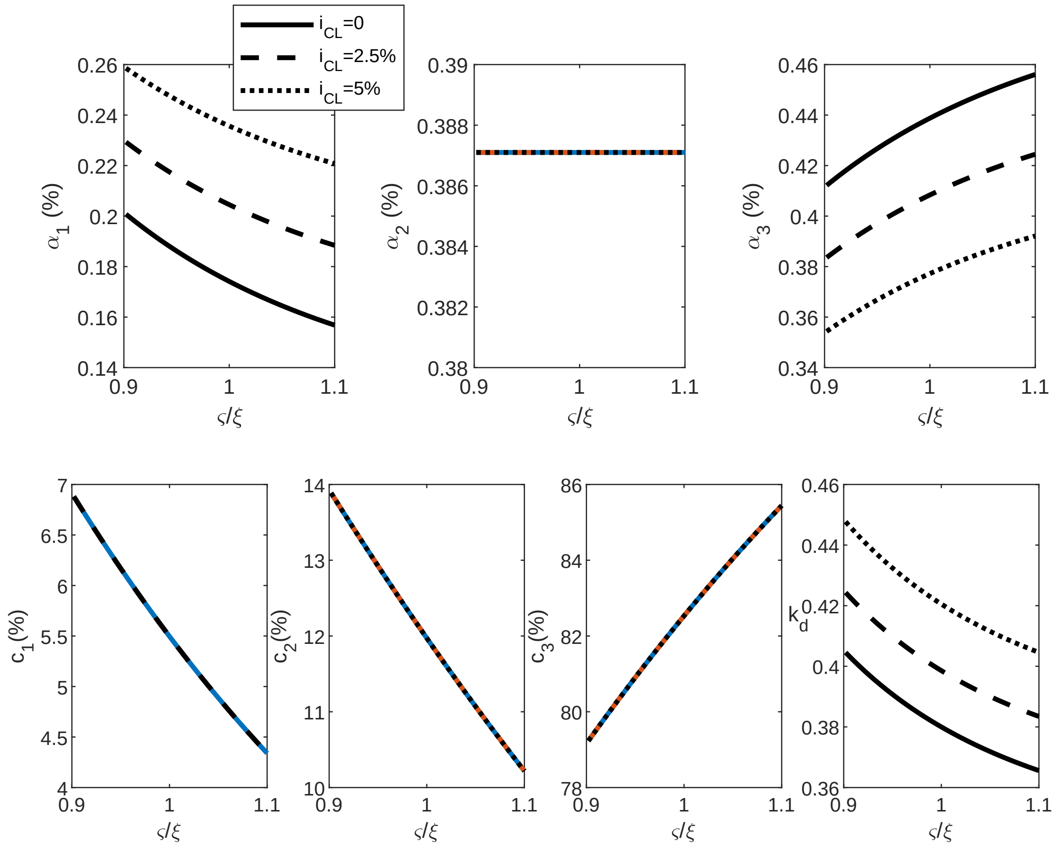


Figure 7: Cash-like CBDC: Effects of Changing the Elasticity of Substitution

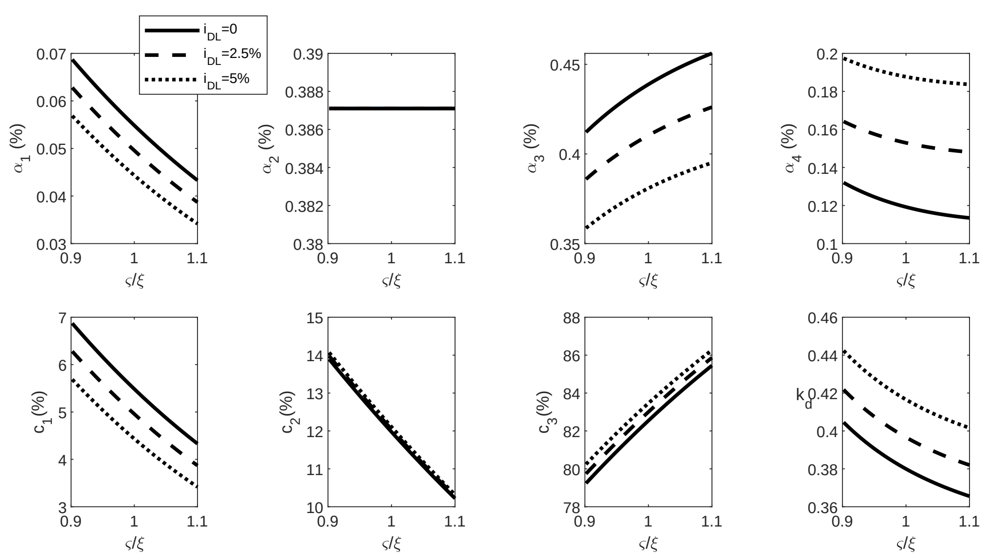


Figure 8: Deposit-like CBDC: Effects of Changing the Elasticity of Substitution

# F Calibration

This appendix gives the details of our calibration exercise. First, it is useful to collect all equations necessary for the calibration in one place:

$$\begin{aligned} c_i &= \frac{\left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma}}{(1+\lambda_c)^{\xi} p^{\xi}} \to c_i^{share} = \frac{p_i c_i}{\sum p_j c_j} = \frac{\frac{c_i}{a_i}}{\sum \frac{c_j}{a_j}} = \frac{\omega_i^{\varsigma} a_i^{\varsigma-1}}{\sum \omega_j^{\varsigma} a_j^{\varsigma-1}} \\ &\to \omega_i^{\varsigma} = \frac{c_i^{share}}{a_i^{\varsigma-1}} \sum \omega_j^{\varsigma} a_j^{\varsigma-1}. \end{aligned}$$

Since $\sum c_i^{share} = 1$, we need one more equation to pin down all three $\omega_i$s. For simplicity, we normalize

$$\sum \omega_j^{\varsigma} a_j^{\varsigma-1} = 1.$$

Also,

$$\sigma(Z+D+B) = \sigma\left[\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right] p = pF(k) = \frac{F(k)}{kF'(k)} R_d k = \frac{D}{\eta_F},$$

$$\alpha_1 = \frac{Z}{Z+D+B} = \frac{\frac{c_1}{a_1} + f\frac{c_3}{a_3}}{\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}} = c_1^{share} + f c_3^{share}, \tag{41}$$

$$\alpha_2 = \frac{D}{Z+D+B} = \sigma\eta_F \to \eta_F = \frac{\alpha_2}{\sigma}, \tag{42}$$

where we used $\eta_F = kF'/F$. Output is calculated by summing up household's consumption in the AM and PM, $\sigma\left[c_1 p_1 + c_2 p_2 + c_3 p_3\right] + A$, old bankers' consumption, $pF(k) - Rk$, new bankers' investment, $k$, and retailer's consumption, 0, so

$$GDP_{\text{model}} = \sigma\left[c_1 p_1 + c_2 p_2 + c_3 p_3\right] + A + pF(k) - (R_d - 1)k \tag{43}$$

$$M1_{\text{model}} = Z + D. \tag{44}$$

Welfare is given by

$$\sigma\beta\left[u_1(c_1) + u_2(c_2) + u_3(c_3)\right] + \log A - A - k$$

and the resource constraint is

$$\sigma\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right) = F(k).$$

**Calibration strategy.** Here is our numerical procedure to estimate the parameters. We have 12 parameters: $\omega_i$ (3), $a_i$ (3), $\varsigma, \xi, \sigma, \gamma, A, B$. We impose some structure on the parameters.

Since $\sum c_i^{share} = 1$, we need one more equation to be able to pin down all three $\omega_i$s from $c_i^{share}$s. We normalize

$$\sum \omega_j^{\varsigma} a_j^{\varsigma-1} = 1,$$

so

$$\omega_i^{\varsigma} = \frac{c_i^{share}}{a_i^{\varsigma-1}}.$$

The code provided allows general $a_i$s. But since they cannot be identified from the money demand or by the payments data that we use; we continue with the following assumption:

$$a_1 = a_2 = a_3 = 1.$$

Moreover, we impose separability of the AM utility function:

$$\varsigma = \xi.$$

Therefore, the utility function will be

$$v(c_1, c_2, c_3) = \frac{\omega_1}{1-\frac{1}{\xi}} c_1^{1-\frac{1}{\xi}} + \frac{\omega_2}{1-\frac{1}{\xi}} c_2^{1-\frac{1}{\xi}} + \frac{\omega_3}{1-\frac{1}{\xi}} c_3^{1-\frac{1}{\xi}}.$$

With the special production function, $F(k) = \frac{k^{1-\gamma}}{1-\gamma}$, we have $\eta_F = kF'/F = 1-\gamma$, so we can pin down $\gamma$ from (42):

$$\gamma = 1 - \frac{\alpha_2}{\sigma}. \tag{45}$$

Given these assumptions, we follow the steps below to calibrate the model:

- For a given $\sigma$ and $B$, we calculate $\gamma$ from (45) and then minimize the distance between money demand derived from the model and that from data given the constraints obtained from payments data. More specifically:
  - Estimate $\xi$ and $A$ by minimizing

    $$MSE = \sum_{interest\ rate} \left( \frac{\frac{M1_{\text{model}}}{GDP_{\text{model}}}}{\frac{M1}{GDP}} - 1 \right)^2$$

    For this we need to calculate $\frac{M1_{\text{model}}}{GDP_{\text{model}}}$ from (43) and (44) for different interest rates, for which we need to calculate $p$, $k$,, $c_i$, $Z$ and $D$. First, we assume $f$ is interior. In this case:

* Given separability ($\varsigma = \xi$), we calculate $\omega_i$'s from (20).
* Then calculate $p^{\frac{1-\gamma}{\gamma}+\xi} = \left(\frac{1}{\beta\sigma R_m} - \left(\frac{1}{\sigma} - 1\right)\right)^{-\xi} R_m^{\frac{1-\gamma}{\gamma}} \sigma\left(1-\gamma\right) \sum \left(\frac{\omega_i}{\omega_0}\right)^{\varsigma} a_i^{\varsigma-1}$.
* Next, $k = \left(\frac{p}{R_m}\right)^{\frac{1}{\gamma}}$, so

$$k^{1-\gamma+\gamma\xi} = \left(\frac{p}{R_m}\right)^{\frac{1-\gamma+\gamma\xi}{\gamma}} = \sigma^{\xi}\left(\frac{1}{\beta} - \left(1-\sigma\right) R_m\right)^{-\xi} \sigma\left(1-\gamma\right) \sum \left(\frac{\omega_i}{\omega_0}\right)^{\varsigma} a_i^{\varsigma-1},$$

where $\omega_0 \equiv \left(\omega_1^{\varsigma} a_1^{\varsigma-1} + \omega_2^{\varsigma} a_2^{\varsigma-1} + \omega_3^{\varsigma} a_3^{\varsigma-1}\right)^{-\frac{\frac{\xi}{\varsigma}-1}{\varsigma-1}}$. We can also use $\sum \left(\frac{\omega_i}{\omega_0}\right)^{\varsigma} a_i^{\varsigma-1} = \omega_0^{\frac{1-\xi}{\frac{\xi}{\varsigma}-1}}$.
* Finally, $c_i = \left(\frac{\omega_i a_i}{\omega_0}\right)^{\varsigma} (1+\lambda_c)^{-\xi} p^{-\xi}$ where $1+\lambda_c = 1 + \frac{1}{\beta\sigma R_m} - \frac{1}{\sigma}$.

– We check whether $f \geq 0$ is binding or not. If binding, then we calculate money demand from the equilibrium conditions specified in Appendix B, Equations (30) and (31).

- Find $\sigma$ and $B$ that solve

$$\min \left(\frac{\frac{Z}{Z+D+B}}{\alpha_1} - 1\right)^2 + \left(\frac{\frac{D}{Z+D+B}}{\alpha_2} - 1\right)^2 + \left(\frac{\frac{B}{Z+D+B}}{\alpha_3} - 1\right)^2 + \psi MSE$$

for some constant $\psi > 0$. We use $\psi = 1$ in our exercise.

# G Introducing Unbanked Households

In this appendix, we consider an extension of the model in which a fraction $\kappa$ of households are banked and the rest are unbanked. The banked agents are exactly like the agents in the benchmark model. The **unbanked** agents, however, have access only to cash. We study effects of a cash-like CBDC to check whether our main results hold. We also assume for simplicity of notation that $R_m \geq R_z$ and that cash is not used. This is without loss of generality, as this covers $R_m = R_z$ in which case cash and cash-like CBDC are identical.

We show that Proposition 3 still holds as long as $f$ (for banked people) becomes interior when the interest rate paid on the CBDC is sufficiently high (equal to the prevailing deposit rate). If $f = 0$, and under certain conditions, then the effects on aggregate consumption and investment remain as in the benchmark model, but the effects on $c_2$ and $c_3$ are opposite, because the spillover effect from cash to non-cash transactions is absent.

## G.1 Unbanked Household's Problem

An unbanked household's problem in the AM can be summarized as follows:

$$\begin{aligned} V^u\left(M^u\right) &= \max_{c_1^u, c_3^u} \sigma \sum_{i=1}^{3} u_i(c_i^u) + M^u + W(0) \\ &\quad -\sigma(c_1^u p_1 + c_3^u p_3) + \sigma\lambda_c\left[M^u - (c_1^u p_1 + c_3^u p_3)\right], \end{aligned}$$

where $\sigma\lambda_c$ is the Lagrangian multiplier associated with CC, and superscript $u$ shows the variables for unbanked households. Notice that $c_2^u = 0$ because they have only cash. The envelope condition is given by

$$V_1^u\left(M^u\right) = \sigma\lambda_c + 1.$$

The optimal AM choices for unbanked households are

$$\begin{aligned} c_1^u &: \ u_1'(c_1^u) = (1+\lambda_c)\frac{p}{a_1}, \\ c_3^u &: \ u_3'(c_3^u) = (1+\lambda_c)\frac{p}{a_3}, \end{aligned}$$

and the payment constraint is given by

$$M^u \geq c_1^u \frac{p}{a_1} + c_3^u \frac{p}{a_3}, \ \text{“} = \text{”} \text{ if } \lambda_c > 0.$$

We consider two cases as in the benchmark model: (i) $f = 0$ for banked households, (ii) $f$ is interior for banked households.

## G.2 Case 1: $f = 0$ for Banked Households

Since $f = 0$ for **banked** households, we must have $\lambda_d \leq \lambda_c$. The equilibrium conditions can then be summarized into:

$$\begin{aligned} c_1 &: \; u_1'(c_1) = (1+\lambda_c)\frac{p}{a_1} \\ c_2 &: \; u_2'(c_2) = (1+\lambda_d)\frac{p}{a_2} \\ c_3 &: \; u_3'(c_3) = (1+\lambda_d)\frac{p}{a_3} \end{aligned}$$

$$\begin{aligned} \frac{1}{\beta R_m} &= \sigma\lambda_c + 1, \\ \frac{1}{\beta R_d} &= \sigma\lambda_d + 1, \end{aligned}$$

$$\begin{aligned} M &\geq c_1\frac{p}{a_1}, \;\; \text{“}=\text{” if } \lambda_c > 0, \\ D + B &\geq c_2\frac{p}{a_2} + c_3\frac{p}{a_3}, \;\; \text{“}=\text{” if } \lambda_d > 0, \end{aligned}$$

The quantity of deposits in equilibrium is given by

$$\kappa D = R_d k_d.$$

Finally, the market clearing condition for the DM is given by

$$(1-\kappa)\left(\frac{c_1^u}{a_1} + \frac{c_3^u}{a_3}\right) + \kappa\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right) = \frac{F(k_d)}{\sigma}.$$

Using consumption equations for both types of households, we can write:

$$\frac{1}{a_1}u_1'^{-1}\left((1+\lambda_c)\frac{p}{a_1}\right) + \frac{\kappa}{a_2}u_2'^{-1}\left((1+\lambda_d)\frac{p}{a_2}\right) + \frac{\kappa}{a_3}u_3'^{-1}\left((1+\lambda_d)\frac{p}{a_3}\right) + \frac{1-\kappa}{a_3}u_3'^{-1}\left((1+\lambda_c)\frac{p}{a_3}\right) = \frac{F(k)}{\sigma}. \tag{46}$$

We also have the following conditions as before:

$$\begin{aligned} pF'(k) &= R_d, \\ R_d &= \frac{1}{\beta\left(1+\sigma\lambda_d\right)}. \end{aligned}$$

As long as $\lambda_d > 0$ and $B$ is small, the DC for banked households should be binding. This fact together with $\kappa D = R_d k_d$ implies:

$$\frac{\frac{R_d k_d}{\kappa} + B}{p} = \frac{F'(k)k}{\kappa} + \frac{B}{p} = \frac{1}{a_2} u_2'^{-1}\left((1+\lambda_d)\frac{p}{a_2}\right) + \frac{1}{a_3} u_3'^{-1}\left((1+\lambda_d)\frac{p}{a_3}\right)$$

As long as $f = 0$, the last four equations determine $p$, $k_d$, $R_d$ and $\lambda_d$. More specifically, the last three equations give:

$$\frac{F'(k)k}{\kappa} + \frac{B}{p} = \frac{1}{a_2} u_2'^{-1}\left(\frac{1}{a_2}\left(\frac{1}{\sigma\beta F'(k)} - \frac{1-\sigma}{\sigma}p\right)\right) + \frac{1}{a_3} u_3'^{-1}\left(\frac{1}{a_3}\left(\frac{1}{\sigma\beta F'(k)} - \frac{1-\sigma}{\sigma}p\right)\right). \quad (47)$$

In the following proposition, we obtain the same result as in Proposition 13.

**Proposition 21** (Extension of Proposition 9)**.** *Suppose $f = 0$ and $\sigma = 1$. With a higher real rate, $R_m$, paid on the cash-like CBDC,*

*(i) $c_2$ and $c_3$ decrease, and*

*(ii) $p$, $k$, $c$, $c_1^u$, $c_1$ and $c_3^u$ increase.*

*Proof.* From (47), we know that the RHS is decreasing and the LHS is increasing in $k$, given that $kF'(k)$ is increasing. Therefore, for a given $p$, there exists at most one $k$. Also, the RHS is increasing in $p$ and the LHS is decreasing in $p$. Therefore, $k$ is increasing in $p$.

We show that, as $R_m$ increases, $p$ increases. By way of contradiction, assume $p$ decreases, then $c_1^u$, $c_3^u$ and $c_1$ increase. Also, $k$ is increasing in $p$, so $k_d$ falls. Given that $\sigma$ is equal to 1, $c_2$ and $c_3$ rise. But the facts that $c_1^u$ and $c_3^u$, and $c_1$, $c_2$ and $c_3$ all increase and that $k$ decreases are inconsistent with (46). Therefore, $p$ increases with $R_m$.

As a result, $c_2$ and $c_3$ decrease as $\lambda_m$ decreases. According to (46), the increase in $k$ and decreases in $c_2$ and $c_3$ imply that $(1+\lambda_m)p$ decreases. As a result, $c_1^u$, $c_1$ and $c_3^u$ increase. Finally, $c$ is simply proportional to $F(k)$, so $c$ increases too. ∎

### G.3 Case 2: $f$ Is Interior for Banked Households

Because $f$ is interior, we must have $\lambda_d = \lambda_c$. For simplicity assume that $\lambda_c > 0$. The equilibrium conditions for banked and unbanked households can then be summarized into:

$$u_1'(c_1^u)a_1 = u_1'(c_1)a_1 = u_2'(c_2)a_2 = u_3'(c_3^u)a_3 = u_3'(c_3)a_3 = (1+\lambda_d)p,$$

$$
\begin{aligned}
M^u &= c_1^u \frac{p}{a_1} + c_3^u \frac{p}{a_3}, \\
M &= c_1 \frac{p}{a_1} + f c_3 \frac{p}{a_3}, \\
D + B &= c_2 \frac{p}{a_2} + (1-f) c_3 \frac{p}{a_3},
\end{aligned}
$$

$$\kappa D = R_d k,$$

$$(1-\kappa)\left(\frac{c_1^u}{a_1} + \frac{c_3^u}{a_3}\right) + \kappa\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3}\right) = \frac{F(k)}{\sigma},$$

$$
\begin{aligned}
pF'(k) &= R_d, \\
R_d &= \frac{1}{\beta\left(1+\sigma\lambda_d\right)}.
\end{aligned}
$$

Because the interest rate that banked and unbanked households face are the same (although unbanked agents cannot use deposits or credit), their consumption of goods 1 and 3 are the same. Therefore, the market clearing condition can be simplified to

$$\frac{c_1}{a_1} + \kappa\frac{c_2}{a_2} + \frac{c_3}{a_3} = \frac{F(k)}{\sigma}.$$

Therefore:

$$\frac{1}{a_1} u_1'^{-1}\left((1+\lambda_d)\frac{p}{a_1}\right) + \frac{\kappa}{a_2} u_2'^{-1}\left((1+\lambda_d)\frac{p}{a_2}\right) + \frac{1}{a_3} u_3'^{-1}\left((1+\lambda_d)\frac{p}{a_3}\right) = \frac{1}{\sigma} F\left(F'^{-1}\left(\frac{1}{\beta\left(1+\sigma\lambda_d\right)p}\right)\right),$$

where $k$ has been replaced from the bankers' FOC. This equation is similar to (24). The only difference is that there is $\kappa$ in the LHS, but this is inconsequential for the results that we obtained in Proposition 3. The key is that the demand side (LHS) is a function of $(1+\lambda_d)p$ for both banked and unbanked households, and the supply side (RHS) is a function of $(1+\sigma\lambda_d)\,p$. Therefore, we obtain the following result immediately.

**Proposition 22** (Extension of Proposition 3)**.** *Suppose $f \in (0,1)$ and $\sigma < 1$. A higher real rate, $R_m$, paid on the cash-like CBDC leads to higher $p$, $c_i^u$, $c_i$, $c$, $k$ and welfare.*

## G.4 Summary

Comparing these results against the benchmark model, which does not have unbanked households, we draw the following conclusions.

When $f = 0$, first, the spillover from cash to non-cash transactions is absent, so a higher interest rate on a cash-like CBDC decreases $c_2$ and $c_3$ in this model in contrast to the benchmark model.

Second, a cash-like CBDC still increases the investment and consumption in cash transactions. Since $k$ increases with the CBDC rate, the aggregate consumption increases too.

When $f$ is interior, we have exactly the same results as in the benchmark model regarding the effects of a cash-like CBDC. This is because, with an interior $f$, an increase in the interest rate of a cash-like CBDC leads to an increase in the interest on deposits, so both banked and unbanked households face the same interest rates although they might use different means of payments.

# H Imperfect Pass-through

One may argue that a one-to-one effect of the CBDC rate on the deposit rate in our benchmark model, i.e., perfect pass-through, is extreme. In this section, we explore a variation of our model in which the pass through from the CBDC rate to the deposit rate is imperfect. For that, we assume that carrying $M$ units of real balances cost $C(M)$ units of real balances. We assume that this cost is increasing and convex and $C(0) = 0$. We again focus on the effects of a cash-like CBDC and the case in which $f$ is interior. We also assume that $R_m > R_z$.

Equilibrium conditions in this case can be written as

$$\frac{1}{\beta\left(R_m - C'(M)\right)} \geq \sigma\lambda_m + 1, \text{ equality if } M > 0, \tag{48}$$

$$\frac{1}{\beta R_d} \geq \sigma\lambda_d + 1, \text{ equality if } D > 0. \tag{49}$$

An interior $f$ implies

$$R_d = \frac{1}{\beta(\sigma\lambda_d + 1)} = \frac{1}{\beta(\sigma\lambda_m + 1)} = R_m - C'(M). \tag{50}$$

Again, we assume that both payment constraints are binding, so

$$M = c_1 p_1 + f c_3 p_3, \tag{51}$$

$$D + B = c_2 p_2 + (1 - f) c_3 p_3. \tag{52}$$

The rest of equilibrium conditions are

$$D = R_d k, \tag{53}$$

$$\sigma\left(\frac{c_1}{b_1} + \frac{c_2}{b_2} + \frac{c_3}{b_3}\right) = F(k). \tag{54}$$

It is difficult in general to solve for equilibrium analytically, because $R_d$ is not equal to $R_m$ and is a function of $M$. We restrict our attention to a linear production technology so that we can characterize the equilibrium analytically as much as possible. The main insights go through even without the linearity assumption.

## H.1 Linear Production Technology

Here, we assume $F(k) = Ak$ as in Appendix D.1, so we have

$$F'(k) = \frac{R_d}{p} \rightarrow p = \frac{R_d}{A}.$$

Therefore,

$$c = u'^{-1}((1+\lambda_d)p) = u'^{-1}\left((1+\lambda_d)\frac{R_d}{A}\right) = u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right). \tag{55}$$

But $Ak = \sigma c = \sigma u'^{-1}((1+\lambda_d)p)$, so

$$k = \frac{\sigma}{A}c = \frac{\sigma}{A}u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right).$$

Given that both payment constraints are binding, we have:

$$\begin{aligned} M &= c_1p_1 + c_2p_2 + c_3p_3 - D - B \\ &= \left(\frac{c_1}{b_1} + \frac{c_2}{b_2} + \frac{c_3}{b_3}\right)p - R_dk - B \\ &= \frac{Ak}{\sigma}\frac{R_d}{A} - R_dk - B \\ &= \left(\frac{1}{\sigma}-1\right)R_dk - B. \end{aligned}$$

Therefore,[34]

$$M = \frac{1-\sigma}{A}u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right)R_d - B. \tag{56}$$

Given (50), we have

$$M = C'^{-1}(R_m - R_d). \tag{57}$$

The last two equations determine $M$ and $R_d$. The first one is increasing and the second one is decreasing in $R_d$. The maximum value possible for $R_d$ is $R_m$. Define the real value of balances at $R_d = R_m$:

$$M_m \equiv \frac{1-\sigma}{A}u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_m\right)\right)R_m - B.$$

Therefore, if $M_m > 0$, then the last two numbered equations determine a unique solution:

$$C'^{-1}(R_m - R_d) \quad = \quad \frac{1-\sigma}{A}u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right)R_d - B. \tag{58}$$

As $R_m$ increases, $R_d$ and $M$ both increase. Now we calculate the derivative of $R_d$ with respect to $R_m$:

$$R_d' = \left[1 + C''(M)\left(\frac{1-\sigma}{A}u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right) - \frac{1-\sigma}{A}\frac{A^{-1}\left(\frac{1}{\sigma}-1\right)R_d}{u''\left(u'^{-1}\left(A^{-1}\left(\frac{1}{\sigma\beta} - \left(\frac{1}{\sigma}-1\right)R_d\right)\right)\right)}\right)\right]^{-1}$$

[34]Remember, $\sigma$ should be sufficiently low. Otherwise, banks issue too many deposits, then $\sigma\lambda_d = 0$, so agents are no longer indifferent between CBDC and deposits: $R_d$ would be equal to $1/\beta$ and $M$ would be too little.

where $R'_d \equiv \frac{dR_d}{dR_m}$. The term within large brackets is positive, therefore, $R'_d < 1$. Of course, $R'_d > 0$. This, together with (55), implies that $c$ and consequently $k$ increase as $R_m$ rises. That is, the main result that a higher interest rate on a cash-like CBDC increases consumption and intermediation remains the same although the pass-through is not one to one.

The intuition is that, as a higher interest rate is paid on a cash-like CBDC, there is more output and higher demand for CBDC. This implies that marginal cost of CBDC increases (as $C$ is convex), so the banks do not need to respond one-to-one to an increase in the CBDC rate.

# I Other Reasons to Support Monetary Policy Away from the Friedman Rule

## I.1 Nominal Rigidity

We now add a sector with nominal rigidity to the benchmark model. Suppose in the PM, there is a measure one of sellers, $i \in [0,1]$, who produce an intermediate, differentiated output using the PM good according to a linear technology. Note that this intermediate good is different from that in the benchmark model. Each unit of the differentiated good requires $c_o$ units of the PM good. Sellers need to quote their prices in nominal terms, $P_{i,t}$, subject to a real **price adjustment cost** $\frac{c_a}{2}\Delta_{i,t}^2$, where

$$\Delta_{i,t} \equiv \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}.$$

The price of the PM good in terms of money is $1/\phi_t$. The intermediate good sellers sell their goods to final good producers in a perfectly competitive market. The problem for sellers of the differentiated goods is given by

$$\max_{\{P_{i,t}\}} E \sum_{t=0}^{\infty} \beta^t \left[ \left(P_{i,t}\phi_t + \tau - c_o\right) q_{it} - \frac{c_a}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right)^2 \right]$$

in terms of the PM good, where $\tau$ is the subsidy to producers per unit of their production, and $q_{it}$ is the demand that these producers face.[35] Given the prices of intermediate goods and the final good, the final producers solve:

$$\max_{q_{i,t}} P_t \phi_t q_t - \int_0^1 P_{i,t}\phi_t q_{i,t} di$$

$$s.t. \quad : \quad \left(\int_0^1 q_{i,t}^{\frac{\mathcal{X}-1}{\mathcal{X}}} di\right)^{\frac{\mathcal{X}}{\mathcal{X}-1}} = q_t,$$

where $\mathcal{X}$ denotes the intratemporal elasticity of substitution across various intermediate goods and is assumed to be greater than 1 following the literature. This generates the demand function for good $i$:

$$q_{i,t} = q_t \left(\frac{P_{i,t}}{P_t}\right)^{-\mathcal{X}}.$$

Buyers can work. One hour of work gives $c_I^{-1}$ units of the PM good. They then sell the PM good

[35] Note that $\tau$ is needed to correct inefficiency coming from the differentiated goods sellers' market power.

for money to buy the final good. The buyer's utility maximization problem is simply given by:

$$\begin{aligned} & \max_{y_t,q_t} -y_t + \overline{U}(q_t) \\ st \quad : \quad & q_t = \frac{y_t}{P_t \phi_t c_I}. \end{aligned}$$

This implies that:

$$\overline{U}'(q_t) = P_t \phi_t c_I. \tag{59}$$

Given the demand function, the FOC for $P_{i,t}$ is

$$\phi_t \left(\frac{P_{i,t}}{P_t}\right)^{-\mathcal{X}} q_t - \mathcal{X}\left(P_{i,t}\phi_t + \tau - c_o\right)\frac{1}{P_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\mathcal{X}-1} q_t - \frac{c_a}{P_{i,t-1}}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right) + \beta c E\left[\left(\frac{P_{i,t+1}}{P_{i,t}} - 1\right)\frac{P_{i,t+1}}{P_{i,t}^2}\right] = 0.$$

In the symmetric equilibrium:

$$\left[\frac{\mathcal{X}-1}{\mathcal{X}}\phi_t P_t + \tau - c_o\right]\frac{\mathcal{X}q_t}{c_a} + \left(\frac{P_t}{P_{t-1}} - 1\right)\frac{P_t}{P_{t-1}} = \beta E\left[\left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t}\right].$$

This is similar to the standard Philips curve obtained in NK models.

In the steady state equilibrium with inflation rate $\mu_z$, we have $\phi_t P_t = \overline{p}$, $\frac{\phi_t}{\phi_{t+1}} = \frac{P_{t+1}}{P_t} = \mu_z$, and $q_t = q$; so:

$$\left[\frac{\mathcal{X}-1}{\mathcal{X}}\overline{p} + \tau - c_o\right]\frac{\mathcal{X}q}{c_a} = (\beta - 1)\,\mu_z\left(\mu_z - 1\right). \tag{60}$$

In line with the literature, we assume that the planner can make lump sum subsidies to firms to undo the effects of market power. With these subsidies, the only friction left in this sector is price stickiness. The social planner maximizes the welfare of all agents:

$$W^{StickyP} = \overline{U}\left(q\right) - c_o c_I q - \frac{c_a}{2}\left(\mu_z - 1\right)^2 c_I.$$

where $q$ and $\overline{p}$ are given by (59) and (60) for a given $\mu_z$. It is obvious that the welfare in this sector is maximized when $\overline{U}'\left(q\right) = c_o c_I$ and $\mu_z = 1$.

Considering only this sector of the the economy, the following solution ensures that the competitive equilibrium allocation is the same as the planner's one:

$$\begin{aligned} \tau &= \frac{1}{\mathcal{X}}c_o, \\ \overline{p} &= c_o, \\ \mu_z &= 1. \end{aligned}$$

We show below that $\frac{\partial W^{StickyP}}{\partial \mu_z}$ is strictly positive as $\mu_z \to \beta$. Given that and considering both sectors together, **we conclude that the inflation rate that maximizes total welfare satisfies** $\mu_z = \mu_z^* > \beta$, because a small increase in inflation from $\mu_z = \beta$ leads to a second order loss in welfare in the benchmark sector but leads to a first-order gain in welfare in the sticky-price sector.

Therefore, we have shown that $\mu_z^* > \beta$ in this economy without a CBDC. That is, Proposition 8(i) continues to hold. If CBDC is added to this economy but is not used in the sticky-price sector, we can make exactly the same argument as before to obtain the result in Proposition 8(ii), so **Proposition 8 continues to hold for the model of this section too**.

Finally, note that another argument in the literature for why a positive inflation rate is optimal is the existence of **Zero Lower Bound**. Consider the same economy as in this section but suppose that interest rates cannot go below zero. Facing with sever downturns, the planner may want to lower the nominal rate substantially, possibly below zero, but the planner is bound by the ZLB. Therefore, the welfare would recover to the optimal level more slowly. The more frequent the economy is hit by the ZLB, the higher the welfare costs are. Therefore, the planner may want to increase the inflation target so that the nominal interest rate rises, which in turn leading to less frequency of hitting the ZLB. Smith-Grohe and Uribe (2010) evaluate this argument quantitatively and show that this force generates less than 1% inflation and cannot fuly explain the 2% inflation target that we see in many advanced economies.[36] This concludes the discussion of sticky prices.

**Proof of** $\frac{\partial W^{StickyP}}{\partial \mu_z} > 0$ **as** $\mu_z \to \beta$**:**

For simplicity, we assume that production subsidies are fixed at $\tau = \frac{1}{x} c_o$.

$$W^{StickyP} = \overline{U}(q) - c_o c_I q - \frac{c_a}{2}(\mu_z - 1)^2 c_I.$$

$$\frac{\partial W^{StickyP}}{\partial \mu_z} = \underbrace{\left[\overline{U}'(q) - c_o c_I\right]}_{>0} \frac{dq}{d\mu_z} - c_a c_I (\mu_z - 1).$$

We show below that, as $\mu_z \to \beta$, we have $\frac{dq}{d\mu_z} > 0$ and $\overline{U}'(q) - c_o c_I > 0$. Therefore, $\frac{\partial W^{StickyP}}{\partial \mu_z} \geq c_a c_I (1 - \beta)$ as $\mu_z \to \beta$.

**Proof of** $\frac{dq}{d\mu_z} > 0$ and $\overline{U}'(q) - c_o c_I > 0$ as $\mu_z \to \beta$ :

[36] Another argument is about quality adjusted prices. Pleas see Smith-Grohe and Uribe (2010) for more details.

According to (60), we have:

$$\left[\frac{\mathcal{X}-1}{\mathcal{X}}\frac{\overline{U}'(q)}{c_I}+\tau-c_o\right]\frac{\mathcal{X}q}{c_a}=(\beta-1)\,\mu_z\,(\mu_z-1)\,.$$

Taking derivative with respect to $\mu_z$ :

$$\left[\ \frac{\mathcal{X}-1}{\mathcal{X}}\frac{\overline{U}''(q)}{c_I}\frac{\mathcal{X}q}{c_a}+\left[\frac{\mathcal{X}-1}{\mathcal{X}}\frac{\overline{U}'(q)}{c_I}+\tau-c_o\right]\frac{\mathcal{X}}{c_a}\ \right]\frac{\partial q}{\partial \mu_z}=(\beta-1)\,(2\mu_z-1)\,. \tag{61}$$

Using (60) again, we have:

$$\left[\ \frac{\mathcal{X}-1}{\mathcal{X}}\frac{\overline{U}''(q)}{c_I}\frac{\mathcal{X}q}{c_a}+\beta\mu_z\,(\mu_z-1)\,\frac{1}{q}\ \right]\frac{\partial q}{\partial \mu_z}=(\beta-1)\,(2\mu_z-1)\,.$$

We know $\mathcal{X}>1$ and $\mu_z\to\beta$, so $\frac{dq}{d\mu_z}>0$. Also, $q$ is increasing in $\mu_z$, so $q$ at $\mu_z=\beta$ is less than $q$ at $\mu_z=1$. So, $\overline{U}'\,(q(\beta))>\overline{U}'\,(q(1))$. Furthermore, at $\mu_z=1$, we have $p=c_o$, and $\overline{U}'\,(q(1))=c_oc_I$. Therefore, $\lim_{\mu_z\to\beta}\left[\overline{U}'\,(q)-c_oc_I\right]>0$.

### I.2 Credit Enforcement

In this section, we study another force that gives rise to the optimal inflation away from the Friedman rule.

Again we add a separate market in which there is measure $s\in\mathbb{R}_+$ of agents trading indivisible goods with money or credit. Buyers can buy goods from sellers in the DM and derive ultility $\hat{U}$ while the seller incurs production cost $\hat{C}$. We assume for simplicity that the buyer makes a take-it-or-leave-it offer to the seller so that the price is fixed at $\hat{C}$. The seller can offer trade credit such that the buyer pays in the next PM. After defaulting, however, the buyer will not be allowed to use credit in future and will need to use only cash to trade. The life-time payoff of using credit is $\beta\frac{\hat{U}-\hat{C}}{1-\beta}$. The life-time payoff of using cash is $\frac{-\mu_z\hat{C}+\beta\hat{U}}{1-\beta}$, because the buyer needs to acquire $\mu_z\hat{C}$ units of cash in the current period to have the purchasing power of $\hat{C}$ in the next period.

There is repayment iff

$$-\hat{C}+\beta\frac{\hat{U}-\hat{C}}{1-\beta}\geq\frac{-\mu_z\hat{C}+\beta\hat{U}}{1-\beta},$$

or

$$\mu_z\geq 1.$$

The welfare in this sector is given by:

$$W^{CreditEcon}=s(\hat{U}-\hat{C})\mathbb{I}(\mu_z),$$

where $\mathbb{I}(\mu_z)$ is an indicator function such that $\mathbb{I}(\mu_z) = 1$ when $\mu_z \geq 1$, and $\mathbb{I}(\mu_z) = 0$ otherwise. Hence, considering both sectors, the benchamrk sector studied in the main text and the credit sector in this subsection, we conclude that it is optimal to set $\mu_z > \beta$.[37] This means that Proposition 8(i) continues to hold in this environment. If the agents pushed to ataurky cannot use CBDC, then exactly the same argument can be used to show Proposition 8 (ii) holds too.

[37]Moreover, for sufficiently large $s(\hat{U} - \hat{C})$, it is optimal to set $\mu = 1$.