# Expectations, Stagnation and Fiscal Policy: a Nonlinear Analysis

George W. Evans University of Oregon and University of St. Andrews

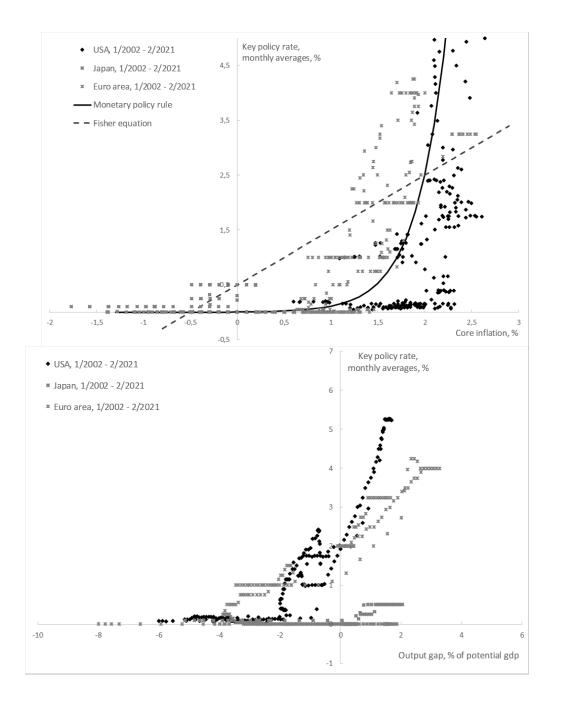
> Seppo Honkapohja, Aalto University Finland

Kaushik Mitra, University of Birmingham, UK

2021 Bank of Canada Annual Economic Conference 8 November 2021

## Introduction

- Sluggish macro performance of many economies following the Great Recession raised the possibility of a distinct **stagnation regime** associated with the interest-rate **zero lower bound** (ZLB). See Figures.
- We develop a **New Keynesian** (NK) model with a stagnation regime, associated with pessimistic expectations, with low output, below target inflation, and interest rates at the ZLB.
- We use an NK model because its **pricing friction** provides a role for expectations to affect GDP via **aggregate demand**.



- The model is **nonlinear** and has **three steady states**, two of which are stable under adaptive learning.
- The **stagnation regime** is a region of pessimistic expectations anchored by a subsistence-level stagnation steady state that acts as an attractor.
- We assume agents make forecasts using **adaptive learning** (AL) instead of rational expectations (RE).
- Using AL allows us to check which RE steady states are stable under learning and to identify a stagnation regime.

- One central policy implication: a temporary **fiscal stimulus** can be effective in avoiding stagnation.
- Exogenous shocks imply that success of fiscal policy is stochastic.
- There are many further **policy implications**:
  - forward guidance in monetary policy can supplement fiscal policy
  - policy delay can reduce probability of success
  - credit frictions shrink target steady state DOA (domain of attraction)
  - higher inflation targets expand the DOA
  - higher CB credibility of inflation target expands the DOA.

#### Background RE Literature on ZLB in NK models

- Large temporary discount/credit-spread shocks → recession at ZLB. Eggertsson and Woodford (2003), Christiano et al. (2011), Woodford (2011).
- Two RE steady states with ZLB &/or sunspot equilibria. Benhabib, Schmidt-Grohe and Uribe (2001a,b), Bullard (2010), Mertens and Ravn (2014).
- Other recent work: Policy regime switching, sentiments, OG models with downward wage rigidity, .... See paper for references.

- In contrast **our focus** is to
  - Use a standard NK set-up, but retain its global nonlinear structure.
  - Replace RE with adaptive learning (AL) and examine global expecta-

#### tional dynamics under AL

- Study what happens if there is a **pessimistic expectations overhang** after end of large adverse shocks.

• We show that existence of a stagnation regime is inherent in the standard nonlinear NK model that forms the basis of most policyoriented models used by CBs.

### The Model

There is a continuum of identical household-producers agents i that maximize

$$E_{0,i} \sum_{t=0}^{\infty} \beta^{t} \left\{ \begin{array}{l} \log(c_{t,i} + \xi g_{t}) + \varkappa \log\left(M_{t-1,i}/P_{t}\right) - (1+\varepsilon)^{-1}h_{t,i}^{\varepsilon} - \\ \Phi\left(P_{t,i}/P_{t-1,i} - \pi^{*}\right) \end{array} \right\}$$
  
s.t.  $c_{t,i} + m_{t,i} + b_{t,i} + \Upsilon_{t,i} = m_{t-1,i}\pi_{t}^{-1} + R_{t-1}\pi_{t}^{-1}b_{t-1,i} + \frac{P_{t,i}}{P_{t}}y_{t,i}$ ,

where  $y_{t,i} = A_t h_{t,i}^{\alpha}$  and  $\Phi$  is a Rotemberg pricing friction.

-  $0 < \xi < 1$  i.e.  $c_t$  and  $g_t$  are partial substitutes. Households are Ricardian. - This is the basic NK model - no capital and no other frictions.

– Agents operate under monopolistic competition. Demand depends on  $P_{t,i}/P_t$ .

The **Temporary Equilibrium (TE) output equation** is obtained by assuming point expectations and combining the consumption Euler equation, the IBC and market clearing  $y_t = c_t + g_t$ 

$$y_{t} = (1 - \xi)g_{t} + (\beta^{-1} - 1) \left[ \sum_{s=1}^{\infty} \left( D_{t,t+s}^{e} \right)^{-1} \left( y_{t+s}^{e} - (1 - \xi)g_{t+s}^{e} \right) \right]$$

provided  $y_t > g_t$ , else  $y_t = g_t$  is a corner solution.

Here  $D_{t,t+s}^e = \prod_{j=1}^s r_{t+j}^e$ , where  $r_{t+j}^e \equiv R(\pi_{t+j}^e, y_{t+j}^e)/\pi_{t+j}^e$  is the expected discount factor, where the (known) nominal interest policy rule is

$$R_t = R(\pi_{t+1}^e, y_{t+1}^e) = 1 + (R^* - 1) \left(\pi_{t+1}^e / \pi^*\right)^{BR^* / (R^* - 1)} \left(y_{t+1}^e / y^*\right)^{\phi_y}.$$
  
Thus  $y_t$  depends on  $g_t$  and  $\{y_{t+s}^e, \pi_{t+s}^e, g_{t+s}^e\}_{s \ge 1}.$ 

The **TE inflation equation** is obtained by expressing the price-setting Euler equation as an inflation equation, solving it forward, and imposing homogeneity:

$$\Phi'(\pi_t)\pi_t = \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e, \text{ where}$$

$$\zeta_t = \nu_t \alpha^{-1} (y_t/A_t)^{(1+\varepsilon)/\alpha} - (\nu_t - 1) y_t (y_t - (1-\xi)g_t)^{-1}, \text{ and}$$

$$\zeta_{t+s}^e = (\nu_{t+s}^e/\alpha) \left( y_{t+s}^e/A_{t+s}^e \right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s} - 1) y_{t+s}^e / (y_{t+s}^e - (1-\xi)g_{t+s}^e).$$

Thus the **TE output and inflation** system has been specified: given  $\nu_t, g_t, A_t, y_{t+s}^e, \pi_{t+s}^e, g_{t+s}^e, A_{t+s}^e$ , the TE values  $y_t, \pi_t$  and  $R_t$  are determined.

Exogenous shocks  $\nu_t, g_t$ , and  $A_t$  follow AR(1) processes (with temporary mean shift of g if fiscal stimulus).

The model specification is completed by describing how expectations are formed and revised over time.

#### Steady states and learning dynamics in nonstochastic case

The key qualitative features can be seen in the nonstochastic case  $A_t = A, \nu_t = \nu, g_t = \overline{g}.$ 

Adaptive learning (AL) in this case usually assumes agents use a simple adaptive expectations rule:

$$y_{t+s}^e = y_t^e$$
, and  $\pi_{t+s}^e = \pi_t^e$  for all  $s > 0$ , where  
 $y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e)$  and  $\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e)$ ,

where  $0 < \omega < 1$  is the learning "gain" parameter, usually assumed "small."

The TE equations then simplify to a nonlinear system

$$y_t = G_2(\pi_t^e, y_t^e), \ \pi_t = G_1(y_t, y_t^e).$$

The system has three steady states:

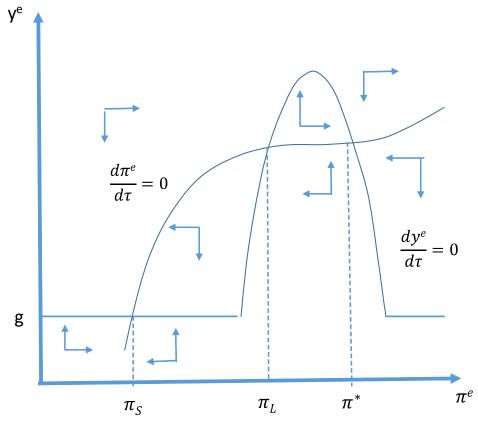
- the targeted steady state  $y_t = y^*, \pi_t = \pi^*,$
- the low inflation unintended steady state  $y_t = y_L, \pi_t = \pi_L$ , and
- the stagnation steady state  $y_S = g, \pi_t = \pi_S \ll 1$  with c = 0.

The Fisher equation  $r = R/\pi = \beta^{-1}$  holds at the targeted and unintended steady states. In the stagnation steady state  $r \approx 1/\pi_s > \beta^{-1}$ .

Stability results under AL can be analyzed using E-stability based on an associated differential equations: see figure.

– the targeted steady state  $(y^*, \pi^*)$  and the stagnation steady state  $(y_S, \pi_S)$  are both locally stable under AL

- the unintended steady state  $(y_L, \pi_L)$  is unstable under AL.



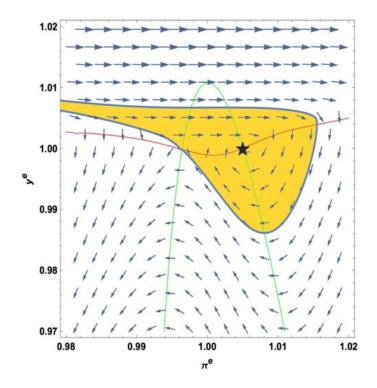
Global E-stability dynamics.

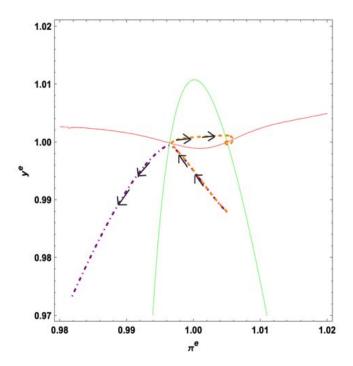
The next figure uses a calibrated model to show the dynamics more completely in a region that includes  $(\pi^*, y^*)$  and  $(\pi_L, y_L)$ .

The left panel shows the domain of attraction (DOA) of  $(\pi^*, y^*)$ , shaded in yellow.

The right panel illustrates sensitive dependence of dynamics on initial conditions. Arrows show the direction of movement.

From two nearby initial pessimistic expectations the paths are initially close together, heading toward  $(\pi_L, y_L)$ , before veering off in different directions towards  $(\pi^*, y^*)$  or  $(y_S, \pi_S)$ .





#### Extension to stochastic economy

Including now observable AR(1) exogenous shocks  $\ln(A_t/\bar{A})$  and  $\ln(\nu_t/\bar{\nu})$ , under AL agents use (constant gain) RLS to estimate

$$\begin{aligned} \ln(y_t) &= f_y + d_{yA} \ln(\tilde{A}_t) + d_{y\nu} \ln(\tilde{\nu}_t) + \eta_{yt} \\ \ln(\pi_t) &= f_\pi + d_{\pi A} \ln(\tilde{A}_t) + d_{\pi\nu} \ln(\tilde{\nu}_t) + \eta_{\pi t}, \end{aligned}$$

where  $\tilde{A}_t \equiv A_t/\bar{A}$  and  $\tilde{\nu}_t = \nu_t/\bar{\nu}$ . To form  $y_{t+s}^e$  and  $\pi_{t+s}^e$  at t agents use current parameter estimates and AR parameters to iterate forward to t+s.

The dynamic path under RLS adaptive learning is specified recursively as before.

Remark: In our global nonlinear set-up we impose  $r_{t+s}^e = \beta^{-1}$  for  $s \ge T_1 = 20$  to ensure consumption is well-defined.

The central stability results are qualitatively unchanged in the stochastic model, but there are two differences.

Consider initial pessimistic output expectations  $y^e$  modeled as a decrease in estimated intercept  $f_y$ .

1. Whether the economy converges back to  $(\pi^*, y^*)$  or instead to  $(\pi_S, y_S)$  is stochastic near DOA border.

2. The DOA of  $(\pi^*, y^*)$  is smaller than in the nonstochastic case.

For example, if  $y_0^e/y^* = 0.99745$  (equivalent to a 2-year recession of 3.3% of GDP) with unchanged policy P(target) = 15% vs P(stagn.) = 85%.

#### **Fiscal Policy**

We study the efficacy of a fiscal stimulus in the face of pessimistic output expectations  $y_0^e = \lambda y^*$  for  $0 < \lambda < 1$ .

For  $\lambda = 0.997$ , which corresponds to expected 2-year recession of 3.9% of GDP, this would (almost) always lead to stagnation with unchanged policy.

Consider an announced fiscal stimulus increasing  $\bar{g}$  to  $\bar{g}' > \bar{g}$  for  $T_p$  quarters. A 4 quarter stimulus of the right size is (almost) always successful.

$T_p \setminus \overline{g'}$	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4
4	0	95	100	100	100	100	100	67	1

Table 2: Percentage successful.  $y^e_0 = 0.997 \times y^*$ . 100 simulations each cell.

For very pessimistic expectations fiscal policy success is stochastic and highest with a very large 4- to 6-quarter stimulus.

$T_p \setminus \overline{g}'$	0.4	0.45	0.5	0.55	0.575	0.6	0.625	0.65	0.675	0.7
3	0	0	0	0	0	0	39	75	80	91
4	0	0	0	83	89	94	90	83	40	11
5	0	9	92	65	22	3	1	0	0	0
6	0	86	29	1	0	0	0	0	0	0

Table 3: Percentage of simulations in which fiscal policy successfully results in convergence to the targeted steady state starting from very pessimistic output expectations  $y_0^e = 0.991 \times y^*$ . Based on 100 replications in each cell.

 $y_0^e = 0.991 \times y^*$  corresponds to expected 2-year recession of 11.7% of GDP.

#### Discussion

– The nonlinear NK model has within it a stagnation regime, a region in which expectations can become trapped, preventing a return to  $(\pi^*, y^*)$ .

- A large temporary fiscal stimulus has the potential to break the economy out of the stagnation regime and return it to the targeted steady state.

– At the ZLB  $R \approx 1$  the mechanism is:

 $\uparrow g \to \uparrow \text{ demand} \to \uparrow y, \pi \to \uparrow y^e, \pi^e \text{ under AL and } \downarrow R/\pi^e \to \uparrow y, \pi, \dots$ 

- The stimulus needs to be of the right size and duration

Extensions: forward guidance, policy delay, credit frictions, higher inflation targets, CB credibility.

#### Including forward guidance in monetary policy

For even more pessimistic  $y^e$  the probability of fiscal policy success becomes lower. Adding monetary policy forward guidance – a promise of  $R \approx 1$  for  $T_m$  periods – can help.

 $y^e = 0.985 \times y^*$  corresponds to expected 2-year recession of 19.5% of GDP. - Using **only** fiscal policy the probability of success is 60% or less.

– Using **both** fiscal policy and forward guidance with  $T_m = 6$ , can increases the success probability to 73%.

- Using **only** forward guidance the success probability is 43% or less.

Conclusion: For very pessimistic expectations, combining fiscal policy with forward guidance is most effective.

#### **Delays in policy**

Following a large pessimistic expectations shock, it may be important to implement a fiscal stimulus quickly.

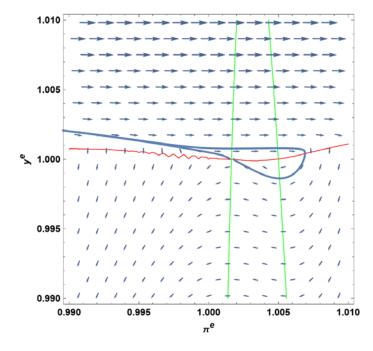
Consider again the large expectations shock  $y_0^e = 0.991 \times y^*$  (expected 2-year recession of 11.7% of GDP), for which the highest probability of success was 94%.

If there is a 4 quarter delay in implementing a fiscal stimulus, the highest probability of success falls to 53%.

#### Credit frictions and the discount factor

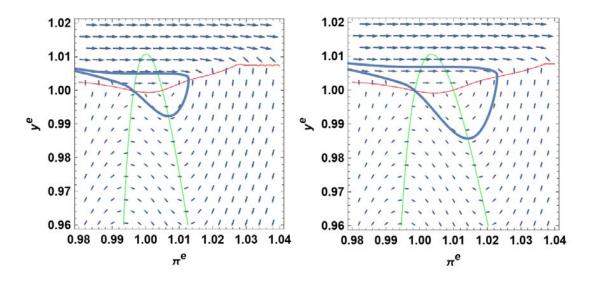
- Credit frictions  $\varphi = R - i > 0$  create a wedge the between the interest rate for household decision-making R and the policy rate i. (Curdia & Woodford).

- DOA size depends negatively on  $\pi_L$ , and hence **negatively** on  $\beta$  and  $\varphi$ .
- For  $\varphi = 0.0025$  (CW) and  $\beta = 0.9975$  the DOA is greatly reduced.



#### Higher inflation target

- The size of the DOA is **positively** related to inflation target  $\pi^*$ .
- $\pi^* = 4\%$  annual instead of  $\pi^* = 2\%$  gives a 4-fold increase in DOA area.

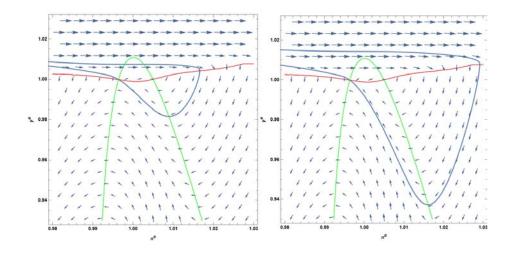


DOA for  $\pi^* = 1.005$  quarterly (left) vs  $\pi^* = 1.01$  (right panel).

Central Bank  $\pi^*$  credibility: anchoring and blended expectations A credible inflation target  $\pi^*$  can "anchor"  $\pi^e$  and increase DOA. We model this as

$$\pi^e_t = arpi ilde{\pi}^e_t + (1 - arpi) \pi^*, ext{ for } 0 < arpi < 1,$$

where  $\tilde{\pi}_t^e$  is the AL forecast, with weight  $\varpi$ , and  $\pi^*$  has weight  $1 - \varpi$ . Qualification: With reinforcement learning, the weight  $\varpi_t$  would also adjust to data.



Left: DOA for 1-arpi=0.2. Right: DOA for 1-arpi=0.5.

#### Conclusions

- We use a standard NK framework except that (i) we retain its nonlinear structure, and (ii) agents form expectations using AL to update forecast parameters.

- Globally the model has 3 steady states, two of which are locally stable under learning: the targeted steady state, and a stagnation steady state anchoring a stagnation regime.

- Pessimistic expectations, e.g. in the wake of large adverse shocks, can, under normal policy, trap the economy in the stagnation regime.

- A fiscal stimulus can be used to shift expectations into the domain of attraction of the targeted steady state.

- There are policy implications for forward guidance, delay in policy implementation, credit frictions, level of the inflation target, and CB credibility.