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FINANCIAL INTERMEDIATION, LIQUIDITY, AND INFLATION

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This paper develops a search-theoretic model to study the interaction between banking and monetary policy and how this interaction affects allocation and welfare. Regarding how banking affects the welfare costs of inflation, we find that, with banking, inflation generates lower welfare costs. We also find that lowering inflation improves welfare not just by reducing consumption/production distortions, but also by avoiding financial intermediation costs. Therefore, understanding the nature of financial intermediation is critical for accurately assessing the welfare gain from lowering the inflation rate. Regarding how monetary policy affects the welfare effects of banking, we find that, when the inflation is low, banking is not active in channeling liquidity; when inflation is high, banking is active and improves welfare; and when inflation is moderate, banking is active but reduces welfare. Owing to general-equilibrium feedback, banking is supported in equilibrium even though welfare is higher without banking.

Keywords: Search, Inflation, Banking, Market for Ideas

1. INTRODUCTION

Central banks care about the welfare effects of changing the inflation rate. Existing monetary theories generally agree that inflation increases the opportunity cost of holding liquidity and thus distorts the allocation of resources that require liquidity in transaction. Many existing studies, however, abstract from financial intermediation, which plays an important role in the allocation of liquidity in modern economies. Therefore, these existing studies are not appropriate for investigating the interaction between financial intermediation and monetary policy and how this interaction affects allocation and welfare.

To study these questions, we developed a micro-founded monetary model that endogenizes the roles of liquidity and financial intermediation. In particular, we explicitly model the microeconomic frictions that generate the roles of liquid assets (e.g., *money*) and financial intermediaries (e.g., *banking*) in facilitating

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the decentralized trading of production inputs. We use the model to study the relationship between money, banking, and social welfare; that is, (i) how banking affects the welfare costs of inflation, and (ii) how monetary policy affects the welfare effects of banking.

In this paper, we model the roles of competitive banks in channelling liquidity among entrepreneurs in the decentralized trading of production projects (denoted as "ideas"). As in Berentsen et al. (2007), banks possess a record-keeping technology to keep track of financial records of borrowers and lenders, and take deposits and make loans at a competitive interest rate. This paper contrasts welfare and allocation in three different economies: an economy without banking, an economy with costless banking, and an economy with costly banking.

How does banking affect the welfare costs of inflation? First, we find that, with banking, inflation generates smaller welfare costs. Second, we show that reducing inflation improves welfare not just by lowering consumption/production distortions, but also by avoiding intermediation costs. Therefore, understanding the nature of intermediation costs is critical for accurately assessing the welfare gain from lowering the inflation rate.

How does monetary policy affect the welfare effects of banking? When inflation is low, banking is not active in channelling liquidity. When inflation is high, banking is active and improves welfare. When inflation is moderate, banking is active but reduces welfare. Owing to general-equilibrium feedback, banking is supported in equilibrium even though welfare is higher without banking.

Let us briefly describe our model and give the basic intuition behind our findings. In this paper, banking is introduced to facilitate decentralized trading of production projects that are essentially intermediate inputs for production. In particular, we build on the setup in Silveira and Wright (in press) to study the roles of banking in the market for production projects ("ideas") that are used as inputs for production. Owing to the anonymity in the decentralized market for ideas, entrepreneurs need to bring liquidity (e.g., money) to this market to purchase ideas. Because innovators (i.e., sellers of ideas) have different (random) reservation prices for their ideas, some entrepreneurs may end up with too much liquidity, whereas others may end up with too little liquidity. The tightness of this liquidity constraint depends on the real value of money, which in turn depends on the inflation rate. Inflation reduces the real value of money, and thus makes the liquidity constraint more binding. This problem can be resolved by having a financial intermediary (banks) that channels funds from entrepreneurs with excess liquidity to those lacking liquidity. The welfare costs of inflation are lower when the banking sector is better developed because, by paying interest for money deposits, banking can reduce the opportunity cost of holding liquidity (i.e., the inflation tax).

However, the use of banking might involve intermediation costs, particularly in enforcing repayment from borrowers. Naturally, at the Friedman rule, there is no need to use banking. When the inflation rate is close to the Friedman rule, money is still very valuable. Due to intermediation cost, no entrepreneurs choose to borrow. As a result, the welfare is equal to that for an economy without banking. When the inflation rate is moderate, a few entrepreneurs will borrow from banks, but the borrowing is used to finance trades of relatively low value. Because borrowing incurs intermediation costs, the welfare of a banking economy is lower than that of a no-banking economy. Finally, when the inflation rate is high, many entrepreneurs borrow from the banks, and their loans are used to finance trades of high value. As a result, the gain from trade can dominate the intermediation costs, and the welfare is higher than that in a no-banking economy.

An interesting finding is that, in an economy with moderate inflation, banking is used even though it is welfare-reducing. The intuition is that, when an entrepreneur chooses to borrow from a financial intermediary, he considers only his own net private gain from borrowing, ignoring the general-equilibrium effect. In particular, borrowing will also lower his demand for money in the money market, and thus reduce the equilibrium value of money. A lower value of money will tighten other entrepreneurs' liquidity constraints, pushing more entrepreneurs to (costly) borrow. This will lead to welfare loss for society.

Our model builds on Silveira and Wright (in press) to study the implication of introducing banking in the market for ideas. Although we choose to study banking in this specific setup, we expect that our main findings can be generalized and applied to other decentralized trading. The way banking is modeled in this paper is related to that in Berentsen et al. (2007), but with two key differences. First, Berentsen et al. study an environment in which the enforcement of repayment by borrowers is either costless or infinitely costly. In our paper, there is perfect enforcement, but it is subject to a finite fixed cost. Second, the fractions of borrowers and lenders are fixed in Berentsen et al., but in our environment they are endogenous and depend on the monetary policy. These differences generate some interesting new implications in our model. Another related paper is Bencivenga and Camera (in press), who also study the relationship between inflation and costly banking. We focus on the inefficiency of banking due to the competitive nature of the banking sector. This type of inefficiency is ruled out in Bencivenga and Camera because a bank is modeled as an optimal contract among a coalition of agents. He et al. (2005) study banking in the Lagos and Wright (2005) environment, but they focus on the safekeeping function of banking. Other related micro-founded models of money, banking, and credit include Andolfatto and Nosal (2009), Guerrieri and Lorenzoni (2009), Sun (2007), Sanches and Williamson (in press), and Dong (2009).

The remainder of this paper is organized as follows. Section 2 describes the basic setup of the model. Section 3 considers an economy without banking. Sections 4 and 5 discuss economies with costless and costly banking, respectively. Section 6 considers various extensions. Section 7 concludes the paper.

2. ENVIRONMENT

Our paper builds on the framework of the market for ideas developed by Lagos and Wright (2005) and Silveira and Wright (in press) to study the roles of money and banking. Time is discrete and denoted $t = 0, 1, 2, \ldots$. In this economy, there

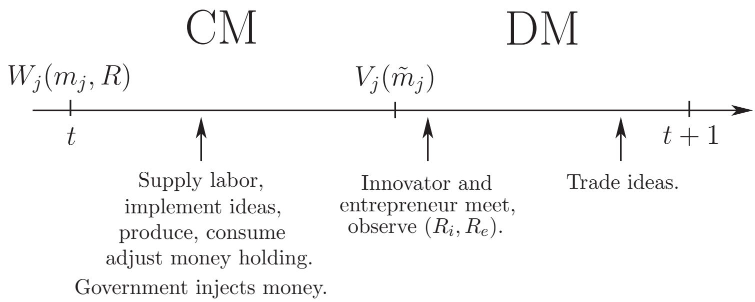

FIGURE 1. Timeline (no banking).

are two types of infinitely lived agents: measure one of innovators (who are good at coming up with ideas), and measure one of entrepreneurs (who are better at implementing ideas). There are two markets: a centralized market, denoted CM, and a decentralized market, denoted DM. In this economy, there is an additional, perfectly divisible, and costlessly storable object that cannot be produced or consumed by any private individual, called *fiat money*.[1]

The sequence of events is illustrated in Figure 1. Each period is divided into two subperiods. In the first subperiod, agents implement ideas, produce and consume, and adjust money holding in the CM. In the second subperiod, agents meet bilaterally in the DM and trade ideas, which are implemented in the next CM. When the DM opens, each innovator comes up with a new idea that can be implemented in the following CM. By implementation, we mean that the idea will be used as an input in the production of the consumption good. The value of an idea depends on who the implementor is. Entrepreneurs are good at implementing ideas. If an idea is implemented by an entrepreneur, it has value $I_e = 1$ as an input in production and generates a return $R_e = \mathcal{R}(I_e)$. Innovators are not good at implementation and cannot realize the full value of ideas. If an idea is implemented by an innovator, it has a lower value as an input, $I_i \leq I_e = 1$, and thus yields a lower return $R_i = \mathcal{R}(I_i) \leq R_e$. Here, we assume that I_i is an i.i.d. random variable with a uniform (0,1) distribution, and its value is known when one enters the DM. If an innovator meets an entrepreneur in the DM, the innovator has an idea that can generate a return R_i for her and generate a return R_e for the entrepreneur.[2] When they meet, both entrepreneurs and innovators observe (R_i, R_e). Because of the lack of information on trading history and the lack of commitment of entrepreneurs, money is required for the trading of ideas. The discount factor between one DM and the next CM is β. For simplicity, we first consider the case in which an idea is both indivisible and rivalry. We discuss more general cases in Section 6.

2.1. The Centralized Market

In the CM, agents implement ideas, produce and consume, and adjust their money holding. We consider a stationary environment. In a typical period, the utility of an agent is given by

$$U(X) - H, \tag{1}$$

where $U : \mathbf{R}_+ \to \mathbf{R}$ denotes the utility of consuming $X \geq 0$ units of the consumption good, and $H \in \mathbf{R}_+$ denotes the labor effort on production. We assume that $U(.)$ is twice continuously differentiable, strictly increasing, and strictly concave and satisfies $U(0) = 0$, $U'(\bar{X}) = 1$ for some $\bar{X} > 0$. We describe the implementation of ideas by individual $j = i, e$, where $j = i$ denotes an innovator and $j = e$ denotes an entrepreneur. Here, an idea is simply an intermediate input into production. The production technology of an individual j is

$$F(I_j, h), \tag{2}$$

where h is the employment of labor input, and $F : [0, 1] \times \mathbf{R}_+ \to \mathbf{R}_+$ denotes the production function. As a result, given the real wage rate, w, in the CM, the return from implementing an idea with a value I_j is given by

$$\mathcal{R}(I_j; w) = \max_h \{F(I_j, h) - wh\}. \tag{3}$$

That is, the return is equal to the output net of the employment cost evaluated at the profit-maximizing level of employment. For simplicity, we will consider the case in which $F(I_j, h) = I_j + h$. Under this simplifying assumption, $R_j = \mathcal{R}(I_j) = I_j$ and $w = 1$.[3] Because $R_e = I_e \geq I_i = R_i$, it is efficient to have entrepreneurs implementing all the ideas. However, due to the trading frictions and the liquidity constraint in the DM, ideas may not be allocated efficiently.

We next describe agents' money holding. Agents can hold any nonnegative amount of money $\hat{m} \in \mathbf{R}_+$. The total money stock at the beginning of the CM is M. The gross growth rate is $\mu = M/M_{-1}$, where M_{-1} denotes the money stock in the previous period. Agents receive lump-sum monetary transfers at the entrance of the CM. In what follows, we express an agent's money holding as a fraction of the beginning-of-the-period money supply: $\hat{m}/M$.[4] Let m and $\tilde{m}$ denote the normalized individual money holdings at the beginning of the CM and the DM, respectively.

Let $\phi \in \mathbf{R}_+$ be the price of money balance in terms of the consumption good in the CM. We focus on a stationary equilibrium in which the money growth rate, μ, is constant over time and the price of money is also constant over time. Let $W_j(m_j, R)$ be the value function for entrepreneurs ($j = e$) and innovators ($j = i$) entering the CM with m_j money holding and an idea with value R in hand. Then the budget constraint of agents in the CM is

$$R + H + \phi(m_j + \Delta m) \geq X + \phi \tilde{m}_j, \tag{4}$$

where $\tilde{m}_j$ is the money balance taken out of the CM, and Δm is the lump-sum money transfer from the government. For $j = i, e$, the CM problem is

$$W_j(m_j, R) = \max_{X,H,\tilde{m}_j \geq 0} U(X) - H + V_j(\tilde{m}_j), \tag{5}$$

$$\text{s.t.} \;\; X = H + R + \phi(m_j - \tilde{m}_j + \Delta m), \tag{6}$$

where $V_j(\tilde{m}_j)$ is the value function for entrepreneurs and innovators entering the DM with $\tilde{m}_j$, before meetings occur. From this point on, we will assume that the utility function U is such that $H > 0$ even for the richest agents, so that we can focus on an interior solution. Under this assumption, the budget constraint can be used to eliminate H in the objective function, simplifying the W_j to

$$\begin{aligned} W_j(m_j, R) &= \phi m_j + \phi \Delta m + R + \max_X \{U(X) - X\} + \max_{\tilde{m}_j} \{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\} \\ &= W_j(0,0) + \phi m_j + R, \end{aligned} \tag{7}$$

where $W_j(0,0) = \phi \Delta m + U(\bar{X}) - \bar{X} + \max_{\tilde{m}_j}\{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\}$. Therefore, $W_j(m_j, R)$ is linear in both m_j and R. We will use this result to derive the bargaining solution reported below.

2.2. The Decentralized Market

When an entrepreneur and an innovator meet in the DM, the value of R_i is observed by both agents. Because $R_i \leq R_e = 1$, the entrepreneur can always implement the idea at least as well as the innovator. Efficiency requires that all ideas be implemented by the entrepreneurs. Owing to liquidity constraints in the market for ideas, this efficient allocation may not be supported. Let $p \in \mathbf{R}_+$ denote the money price the entrepreneur and the innovator would agree to if there were no issues of liquidity. The liquidity constraint requires that $\tilde{m}_e \geq p$. For simplicity, we assume that the price is determined by take-it-or-leave-it offers from the entrepreneur.[5]

3. ECONOMY WITHOUT BANKING

We first examine the case without banking, and construct a monetary equilibrium with $\phi > 0$ such that (i) given ϕ, agents make an optimal choice in the DM; (ii) given ϕ, agents choose optimal money holding in the CM; and (iii) the money market in the CM is cleared.

3.1. Agents' Decision in the Decentralized Market

Consider an innovator bringing money holding $\tilde{m}_i$ and idea R_i into the DM. If an innovator keeps her idea, her payoff is

$$\beta W_i\left(\frac{\tilde{m}_i}{\mu}, R_i\right) = \beta W_i(0,0) + \phi\beta\frac{\tilde{m}_i}{\mu} + \beta R_i. \tag{8}$$

Here, the innovator does not spend her money balance but brings it forward to the next CM. The real value of this money balance (in terms of the CM good) in the next CM is $\phi\tilde{m}_i/\mu$. Note that the next period's money balance is rescaled by the money growth rate because we normalize the money balance by the total stock of money, which grows at the rate μ. Also, we make use of the result in equation (7) to evaluate the value in the next CM. If the innovator sells her idea at a price p, her payoff is

$$\beta W_i\left(\frac{\tilde{m}_i + p}{\mu}, 0\right) = \beta W_i(0,0) + \phi\beta\frac{\tilde{m}_i + p}{\mu}. \tag{9}$$

Here, the innovator's real money balance in the next CM is increased by $\phi p/\mu$. Therefore, the innovator has a reservation price of $\bar{p}(R_i) = R_i\mu/\phi$ for an idea R_i. Obviously, the reservation price is increasing in R_i.[6] Now, consider an entrepreneur with money holding $\tilde{m}_e$ meeting an innovator with idea R_i. The bargaining solution implies that, if $\tilde{m}_e \geq \bar{p}(R_i)$, then the entrepreneur will offer $\bar{p}(R_i)$ to buy the idea and get a payoff of $V_e^1(\tilde{m}_e, R_i)$, given by[7]

$$V_e^1(\tilde{m}_e, R_i) = \beta W_e\left(\frac{\tilde{m}_e - \bar{p}(R_i)}{\mu}, R_e\right) = \beta W_e(0,0) + \beta R_e + \beta\phi\frac{\tilde{m}_e - \bar{p}(R_i)}{\mu}. \tag{10}$$

Here, the entrepreneur obtains the idea and the real money balance in the next CM is reduced by $\phi p/\mu$. If $\tilde{m}_e < \bar{p}(R_i)$, the entrepreneur is liquidity-constrained and cannot afford to purchase the idea, and gets

$$V_e^0(\tilde{m}_e, R_i) = \beta W_e\left(\frac{\tilde{m}_e}{\mu}, 0\right) = \beta W_e(0,0) + \beta\phi\frac{\tilde{m}_e}{\mu}. \tag{11}$$

Therefore, whether the innovator trades or not, she gets $\beta(W_i(0,0) + R_i) + \phi\beta\tilde{m}_i/\mu$: she receives no trade surplus because she has no bargaining power.

3.2. Demand for Money in the Centralized Market

The value function of an innovator entering the DM is thus

$$V_i(\tilde{m}_i) = \int_0^{R_e} \beta(W_i(0,0) + R_i)dR_i + \phi\beta\frac{\tilde{m}_i}{\mu}. \tag{12}$$

An innovator's optimal choice of money balance taken to the DM [$\tilde{m}_i$ in equation (7)] is the solution to $\max_{\tilde{m}_i}[-\phi\tilde{m}_i + V_i(\tilde{m}_i)]$ and is given by

$$\tilde{m}_i \begin{cases} = 0, & \text{if } \mu > \beta \\ \in [0,\infty), & \text{if } \mu = \beta \\ = +\infty, & \text{if } \mu < \beta \end{cases}. \tag{13}$$

That is, an innovator chooses not to bring any money to the DM if the money growth rate is higher than β; is indifferent between any amounts of money if the money growth rate is equal to β; and brings an infinite amount if the money growth rate is lower than β. We focus on cases with $\mu \geq \beta$ and assume that, when innovators are indifferent, they choose $\tilde{m}_i = 0$. The intuition is that, because innovators do not spend money in the DM, they do not have incentives to bring any money to the DM if the opportunity cost is strictly positive (i.e., $\mu > \beta$). The value function of an entrepreneur entering the DM is

$$V_e(\tilde{m}_e) = \int_0^{\tilde{m}_e\frac{\phi}{\mu}} V_e^1(\tilde{m}_e, R_i)dR_i + \int_{\tilde{m}_e\frac{\phi}{\mu}}^1 V_e^0(\tilde{m}_e, R_i)dR_i \tag{14}$$

$$= \beta W_e(0,0) + 2\beta\phi\frac{\tilde{m}_e}{\mu} - \frac{\beta(\phi\tilde{m}_e)^2}{2\mu^2}. \tag{15}$$

The two terms on the right-hand side of the first equality capture the cases when $\tilde{m}_e \geq \bar{p}(R_i)$ and $\tilde{m}_e \leq \bar{p}(R_i)$, respectively. The second equality is derived using equations (10) and (11). An entrepreneur's optimal choice of money balance taken to the DM ($\tilde{m}_e$ in (7)) is the solution to $\max_{\tilde{m}_e}[-\phi\tilde{m}_e + V_e(\tilde{m}_e)]$. This implies that, if $\tilde{m}_e > 0$, then

$$\tilde{m}_e = \frac{2\mu\beta - \mu^2}{\beta\phi}. \tag{16}$$

3.3. Money Market Clearing in the Centralized Market

The money market equilibrium in the CM requires that

$$\tilde{m}_e + \tilde{m}_i = 1. \tag{17}$$

Denote the equilibrium price of money (without banking) as ϕ^{NB}. Under the simplifying assumption that $\tilde{m}_i = 0$ for $\mu = \beta$, we define the monetary equilibrium as follows.

DEFINITION 1. *A stationary monetary equilibrium without banking is given by ϕ^{NB} satisfying (16) and (17) with $\phi^{\text{NB}} > 0$.*

It is straightforward to show the following result.

PROPOSITION 2 (Existence of Equilibrium without Banking). *For any $\mu \in [\beta, 2\beta]$, there exists a stationary monetary equilibrium without banking.*

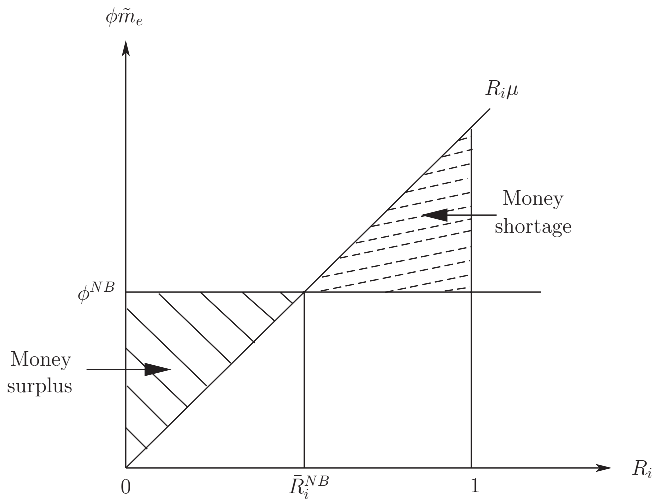

FIGURE 2. Potential gain from trading liquidity.

If $\mu > \beta$, then $\tilde{m}_i = 0$ and $\tilde{m}_e = 1$. Equation (16) then implies that $\phi^{\text{NB}} = 2\mu - \mu^2/\beta$, which is nonnegative for $\mu \leq 2\beta$.[8] When $\mu \geq 2\beta$, money has no value and there is no monetary equilibrium (i.e., no ideas are traded).[9] Let $\bar{R}_i^{\text{NB}} \in [0, 1]$ be the cutoff value of R_i such that an entrepreneur's liquidity constraint is just binding: $\tilde{m}_e = \bar{p}(R_i)$.

One can understand how the model works through the use of a simple graph (Figure 2). For a given $\tilde{m}_e$ (which is chosen in the previous CM), an entrepreneur in the DM meeting is subject to the liquidity constraint

$$R_i\mu \leq \phi^{\text{NB}}\tilde{m}_e,$$

which is represented by the upward-sloping line in Figure 2. For any given level of $\tilde{m}_e\phi$, an agent is constrained (unconstrained) when the realization of R_i is on the right (left) side of this line. Now, the money-demand decision in the CM is to pick the optimal level of real money balance ($\tilde{m}_e\phi$) on the vertical axis to trade off the cost of buying money in the CM and the likelihood of being constrained in the DM. In equilibrium, $\tilde{m}_e = 1$ and the cutoff of R_i is pinned down by the condition

$$\bar{R}_i^{\text{NB}}\mu = \phi^{\text{NB}},$$

which is represented by the intersection point. Therefore, the equilibrium amount of trade is given by

$$\bar{R}_i^{\text{NB}} = \frac{\phi^{\text{NB}}}{\mu} = 2 - \frac{\mu}{\beta}.$$

Note that money growth always reduces trade. When $\mu = \beta$, the opportunity cost of holding money is zero. As a result, no entrepreneurs are liquidity constrained and all ideas are traded (i.e., $\bar{R}_i^{\mathrm{NB}} = 1$).

To summarize our the findings in an economy without banking:

- At the Friedman rule, all ideas are traded.
- When the inflation rate is moderate, a unique monetary equilibrium exists. Inflation reduces trades.
- When the inflation rate is high, there is no monetary equilibrium.

Figure 2 shows that, in equilibrium, each entrepreneur brings $\tilde{m}_e = 1$ to the DM. An entrepreneur with $R_i \leq \bar{R}_i^{\mathrm{NB}}$ buys the idea at $p = R_i\mu$. After trade, these entrepreneurs still have money left over. The rest of the entrepreneurs are liquidity-constrained and need extra funding to purchase the idea. Therefore, there is a potential role for borrowing and lending among entrepreneurs whenever $\mu > \beta$. Owing to imperfect information and commitment, borrowing and lending between agents in the DM are not feasible. Financial intermediaries with an information and commitment advantage can help to reallocate liquidity among entrepreneurs.

4. ECONOMY WITH COSTLESS BANKING

We next introduce intermediation into the economy and study the interaction between money and banking. Suppose that, in the DM, there are competitive financial intermediaries (denoted as "banks") taking deposits at an interest rate r^D and making loans at an interest rate r^L. As in Berentsen et al. (2007), each bank has a recordkeeping technology allowing it to keep financial records of agents. Here, we assume that banks specialize in channeling funding across *entrepreneurs*.[10] In particular, entrepreneurs can commit to repay the bank in the CM and banks can commit to repay depositors in the CM. Free entry implies zero profit for banks and thus $r^D = r^L = r$ for some $r \geq 0$.

Figure 3 shows the sequence of events. In the DM, after meeting and observing the realization of R_i, an entrepreneur can choose to lend money to or borrow

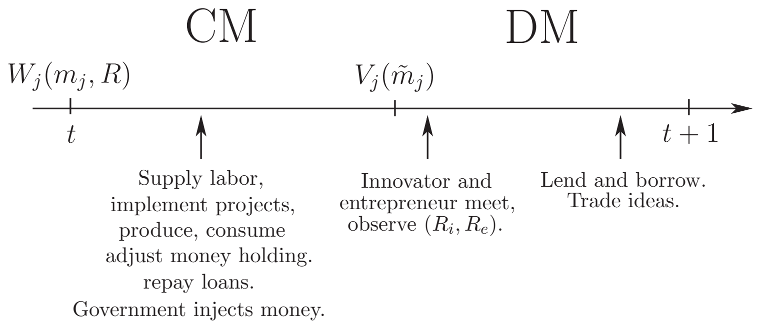

FIGURE 3. Timeline (banking).

money from a bank before trading. In the next CM, deposits and loans will be repaid. Below, we will show that an entrepreneur meeting an innovator with low R_i has excess liquidity and would like to lend his surplus money holding to a bank to earn interest income. An entrepreneur meeting an innovator with high R_i may find the surplus from trade smaller than the return from deposit, and choose instead to lend all his money holding to the bank. An entrepreneur with an intermediate level of R_i is liquidity-constrained and will choose to borrow from the bank to finance the trade. Figure 4 illustrates the flow of funds in the CM and the DM. Anonymity of entrepreneurs in the market for ideas implies that money is still needed as a medium of exchange in the DM.

In general, entrepreneurs may have to incur an additional fixed cost $\eta \geq 0$ to borrow from the bank. This section considers the case with costless banking ($\eta = 0$). We will consider the general case with costly banking ($\eta > 0$) in the next section.

We construct a monetary equilibrium with banking ($\phi > 0$ and $r \leq 0$) such that (i) given ϕ, r, banks make an optimal choice in the DM; (ii) given ϕ, r, agents make an optimal choice in the DM; (iii) given ϕ, r, agents choose the optimal money holding in the CM; (iv) the loan market in the DM is cleared; and (v) the money market in the CM is cleared.[11]

4.1. Banks' Problem in the Decentralized Market

A competitive representative bank takes r^L and r^D as given, and chooses the amount of loans (l) and deposits (d) to maximize its profit (π):

$$\max_{l,d} \pi = r^L l - r^D d, \tag{18}$$

$$\text{s.t.} \qquad d \geq l. \tag{19}$$

Here, there is a feasibility constraint restricting the amount of loans lent out to be no more than the amount of deposits taken in. In equilibrium, it cannot be the case that $r^L > r^D$; otherwise banks would choose $l = d = +\infty$, implying that $\pi = +\infty$. When $r^L < r^D$, banks choose $l = d = 0$ to earn $\pi = 0$. This cannot clear the loan market when entrepreneurs choose to save a positive amount. Therefore, whenever there is a positive saving, we must have $r^L = r^D$. The banks' optimization problem then implies that

$$\begin{cases} d = l & \text{if } r > 0 \\ d \geq l & \text{if } r = 0 \end{cases}. \tag{20}$$

In both cases, the profits of the banks are zero.

4.2. The Entrepreneur's Decision in the Decentralized Market

In a meeting in the DM, given (ϕ, r, R_i), an entrepreneur with $\tilde{m}_e$ and R_i chooses the amounts of saving [lending if positive and borrowing if negative ($s \in \mathbf{R}$)] and

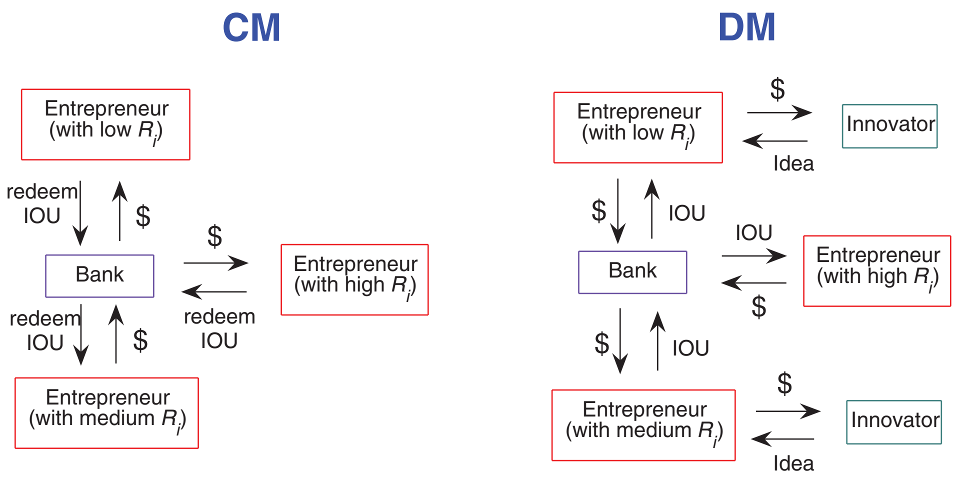

FIGURE 4. Flow of funds.

money brought to the next CM ($m_e \in \mathbf{R}_+$, as a fraction of *next*-period money stock) and whether to buy the idea ($y \in \{0, 1\}$) to maximize the expected payoff. Use $\widehat{V}_e(\tilde{m}_e, R_i)$ to denote the value function after observing R_i:

$$\widehat{V}_e(\tilde{m}_e, R_i) = \max_{s,y,m_e} \beta W_e\left((1+r)\frac{s}{\mu} + m_e, R_e y\right) \tag{21}$$

$$= \max_{s,y,m_e} \beta W_e(0,0) + \beta R_e y + \beta\phi(1+r)\frac{s}{\mu} + \beta\phi m_e, \tag{22}$$

subject to

$$m_e\mu = \tilde{m}_e - y\frac{R_i\mu}{\phi} - s \geq 0.$$

The budget constraint says that the amount of money brought to the next period is equal to the initial money holding minus the expenditure on purchasing the idea and saving. We need to adjust the left-hand side by the money growth rate because the two sides are normalized by money stocks in two different periods. Note that, by allowing borrowing (i.e., $s < 0$), banking relaxes the liquidity constraint on entrepreneurs. Substituting this budget constraint into the objective function, we have

$$\max_{y,m_e} \beta W_e(0,0) + \beta y R_e + \beta\frac{\phi}{\mu}(1+r)\left(\tilde{m}_e - \mu m_e - y\frac{R_i\mu}{\phi}\right) + \beta\phi m_e.$$

Optimization implies that $m_e = 0$ if $r > 0$ and $m_e \in \mathbf{R}_+$ if $r = 0$. Thus, an entrepreneur's value function can be simplified to

$$\widehat{V}_e(\tilde{m}_e, R_i) = \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta\max\{R_e - (1+r)R_i, 0\}. \tag{23}$$

The last term captures an entrepreneur's comparison between the value of the idea ($R_e = 1$) and the opportunity cost (including interest) of buying the idea ($(1+r)R_i$). Therefore, the value function of an entrepreneur entering the DM is

$$V_e(\tilde{m}_e) = \int_0^1 \widehat{V}_e(\tilde{m}_e, R_i)dR_i \tag{24}$$

$$= \int_0^1 \left[\beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta\max\{R_e - (1+r)R_i, 0\}\right]dR_i. \tag{25}$$

Denote the optimal saving of an entrepreneur with (m, R_i) by $s(\tilde{m}_e, R_i)$. Equation (23) implies that the cut-off value $\bar{R}_i(r)$ that makes an entrepreneur indifferent between trading and not trading is given by

$$\bar{R}_i(r) = \frac{R_e}{1+r}. \tag{26}$$

As a result, the value function of an entrepreneur entering the DM can be simplified to

$$V_e(\tilde{m}_e) = \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta\int_0^{R_e/(1+r)}(R_e-(1+r)R_i)dR_i. \quad \textbf{(27)}$$

Note that $V_e'(\tilde{m}_e) = \beta(\phi/\mu)(1+r) > 0$ and thus the value function is linear.

4.3. Money Demand in the Centralized Market

The optimal money demand of an entrepreneur, $\arg\max V_e(\tilde{m}_e) - \phi\tilde{m}_e$, is thus given by

$$\tilde{m}_e \begin{cases} = 0, & \text{if } r < \dfrac{\mu}{\beta} - 1 \\ \in [0,\infty), & \text{if } r = \dfrac{\mu}{\beta} - 1 \\ = +\infty, & \text{if } r > \dfrac{\mu}{\beta} - 1 \end{cases}. \quad \textbf{(28)}$$

To clear the money market, the equilibrium interest rate will satisfy

$$r = \frac{\mu}{\beta} - 1. \quad \textbf{(29)}$$

The use of banking relaxes the entrepreneur's liquidity constraint in purchasing ideas in the DM. Therefore, when choosing the optimal amount of money brought to the DM, an entrepreneur simply looks at whether the real rate of return of money is higher than the subjective discount rate [i.e., whether $(\phi/\mu)(1+r) - 1/\beta > 0$] across two CMs. The entrepreneur will demand $\tilde{m}_e = 0$ when the real rate of return is lower than the subjective discount rate, demand $\tilde{m}_e = +\infty$ when the real rate of return is higher than the subjective discount rate, and demand any $\tilde{m}_e \in \mathbf{R}_+$ when the rate of return is equal to the subjective discount rate. To clear the money market in the CM, the nominal interest rate, r, has to exactly compensate for the inflation and discounting.

4.4. Loan Market Clearing in the Decentralized

Equations (26) and (29) imply that the cut-off value of an idea is given by $\bar{R}_i^B = \bar{R}_i(r) = \beta/\mu$. Therefore, entrepreneurs' optimal choices of (y, s) as a function of R_i are as follows (illustrated by Figure 5):[12]

$$\begin{cases} y = 1, s = \tilde{m}_e - \dfrac{R_i\mu}{\phi} \geq 0, & \text{if } R_i \in \left[0, \dfrac{\phi}{\mu}\right] \\ y = 1, s = \tilde{m}_e - \dfrac{R_i\mu}{\phi} < 0, & \text{if } R_i \in \left(\dfrac{\phi}{\mu}, \bar{R}_i^B\right] \\ y = 0, s = \tilde{m}_e \geq 0, & \text{if } R_i \in \left(\bar{R}_i^B, 1\right] \end{cases}. \quad \textbf{(30)}$$

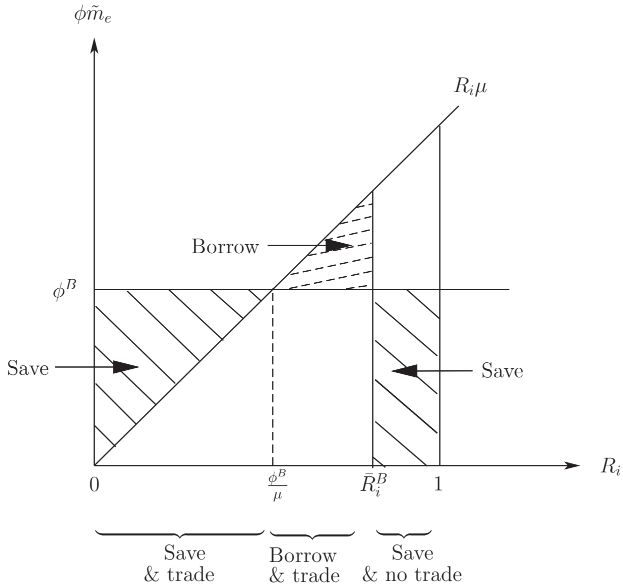

FIGURE 5. Saving and borrowing.

As discussed earlier, the entrepreneurs with low and high R_i will save, and the entrepreneurs with medium R_i will borrow. Only the entrepreneurs with low and medium R_i will trade. High-R_i entrepreneurs choose to save and not to trade because high values of R_i imply a high cost of acquiring the idea, and thus a large amount of borrowing to finance the acquisition. However, the profit for the entrepreneurs, $1 - R_i$, is small. Consequently, the interest cost of the loan exceeds the profit from acquiring the idea.

The loan market–clearing condition in the DM requires that the aggregate saving from the entrepreneurs be equal to the total deposit minus the total loans:

$$\int_0^1 s(\tilde{m}_e, R_i) dR_i = d - l. \tag{31}$$

Then condition (20) from the bank's optimization implies that

$$\begin{cases} \displaystyle\int_0^1 s(\tilde{m}_e, R_i) dR_i = 0, & \text{if } r > 0 \\ \displaystyle\int_0^1 s(\tilde{m}_e, R_i) dR_i \geq 0, & \text{if } r = 0 \end{cases}. \tag{32}$$

Substituting in the saving functions from (30), we can simplify the left-hand side to $\tilde{m}_e - \beta^2/2\mu\phi$.

4.5. Money-Market Clearing in the Centralized Market

Finally, imposing the money market–clearing condition in the CM (i.e., $\tilde{m}_e = 1$), we can solve for the equilibrium price of money:

$$\begin{cases} \phi = \dfrac{\beta^2}{2\mu}, & \text{if } r > 0 \\ \phi \geq \dfrac{\beta^2}{2\mu}, & \text{if } r = 0 \end{cases}. \tag{33}$$

DEFINITION 3. *A stationary monetary equilibrium with costless banking is a pair* (ϕ^B, r) *satisfying (29) and (33) with* $\phi^B > 0$, $r \geq 0$.

Equations (26), (29), and (33) imply that, when $\mu > \beta$, there exists a unique stationary monetary equilibrium with costless banking where $\phi^B = \beta^2/2\mu$, $r = \mu/\beta - 1 > 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$, and $\bar{R}_i^B = \beta/\mu$. A fraction, $\bar{R}^B - \phi^B/\mu = (\beta/\mu)(1-\beta/2\mu)$, of entrepreneurs are borrowers and the rest are lenders. Because the interest rate in the loan market is positive, the excess supply of loans is zero.

When $\mu = \beta$, we have multiple equilibria: $\phi^B \in [\beta/2, \infty)$, $r = 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$, and $\bar{R}_i^B = 1$. A fraction $\max\{1 - \phi^B/\beta, 0\}$ of entrepreneurs are borrowers and the rest are lenders. All these equilibria are equivalent in terms of real allocations and payoffs. They differ only in terms of the real value of money and the borrowing–lending decision in the DM. From equation (29), we know that borrowing is positive when $\phi^B \in [\beta/2, \beta]$ and that borrowing is zero when $\phi^B > \beta$. At the lower bound where $\phi^B = \beta/2$, half of the set of entrepreneurs are liquidity-constrained and need to borrow. The excess supply of loans is zero. As the value of money (ϕ^B) goes up, fewer entrepreneurs are liquidity-constrained and there are fewer borrowers. There is excess supply of loans in the loan market, but it is consistent with the interest rate being zero. For $\phi^B \geq \beta$, no entrepreneurs are liquidity constrained and there are no borrowers. Again, there is excess supply of loans. Also, at the Friedman rule, a banking equilibrium with $\phi^B = \beta$ is identical to an equilibrium without banking.

4.6. Inflation, Banking, and Welfare

Note that the measure of trade ($\bar{R}_i^B = \beta/\mu$) is decreasing in inflation. The maximum amount of trade ($\bar{R}_i^B = 1$) is achieved when $\mu = \beta$. Measuring welfare by the average utility of all agents, we have the welfare for $k = \text{NB}, B$ given by

$$\bar{W}^k = 2(U(\bar{X}) - \bar{X}) + \int_0^{\bar{R}_i^k} R_e dR_i + \int_{\bar{R}_i^k}^1 R_i dR_i \tag{34}$$

$$= 2(U(\bar{X}) - \bar{X}) + \bar{R}_i^k + \frac{1}{2} - \frac{\left(\bar{R}_i^k\right)^2}{2}. \tag{35}$$

The three terms on the right-hand side of the first equality capture, respectively, the surplus in the CM, the value of ideas implemented by entrepreneurs, and the value of ideas implemented by innovators. In particular, we have

$$\bar{W}^B = \bar{W}^{\text{NB}} + \int_{\bar{R}_i^{\text{NB}}}^{\bar{R}_i^B} (R_e - R_i) dR_i, \tag{36}$$

where the second term captures the welfare gain from a better allocation of ideas. Therefore, to contrast the welfare in the two economies, we need to compare the cut-off values of idea trading with and without banking. Note that, without banking, the cut-off value, $\bar{R}_i^{\text{NB}}$, is pinned down by the money-demand decision. In equilibrium, the first-order condition (16) implies that

$$\begin{aligned} \frac{\phi}{\mu}\left(1 - \bar{R}_i^{\text{NB}}\right) &= \frac{\phi}{\mu}\left(\frac{\mu}{\beta} - 1\right) \\ \Rightarrow 1 - \bar{R}_i^{\text{NB}} &= \frac{\mu}{\beta} - 1. \end{aligned} \tag{37}$$

The left- and right-hand sides capture, respectively, the benefit and cost of bringing the marginal dollar to the DM. Bringing one extra dollar to the DM relaxes the liquidity constraint and allows ϕ/μ more extra trades, each of which generates $1 - \bar{R}_i^{\text{NB}}$ in terms of next-period utility. At the same time, bringing one extra dollar incurs a (net) opportunity cost of $(\mu/\beta - 1)$ in terms of next-period dollars. In terms of next-period utility, the cost is $(\phi/\mu)(\mu/\beta - 1)$.

With banking, the cut-off value, $\bar{R}_i^B$, is pinned down by the borrowing decision. In equilibrium, condition (26) implies that

$$1 - \bar{R}_i^B = \left(\frac{\mu}{\beta} - 1\right) \bar{R}_i^B. \tag{38}$$

The left- and right-hand sides capture, respectively, the benefit and cost of borrowing to finance the payment of $\bar{R}_i^B$ for the marginal entrepreneur in the DM. Comparing the right-hand sides of (38) and (38), we see that banking reduces entrepreneurs' cost of buying ideas: by lending out the money balance unused for trade, the excess balance is no longer subject to inflation and discounting. Therefore, when $\mu > \beta$, we have $\bar{R}_i^B > \bar{R}_i^{\text{NB}}$ and thus $\bar{W}^B > \bar{W}^{\text{NB}}$ (Figure 6). It is straightforward to show the following result.

PROPOSITION 4 (Inflation and welfare with costless banking).

(i) *When* $\mu = \beta$, $\bar{R}_i^B = \bar{R}_i^{\text{NB}} = 1$ *and* $\bar{W}^B = \bar{W}^{\text{NB}}$.

(ii) *When* $\mu \in (\beta, 2\beta]$, $1 > \bar{R}_i^B > \bar{R}_i^{\text{NB}} > 0$, $\bar{W}^B > \bar{W}^{\text{NB}}$, $0 > d\bar{R}_i^B/d\mu > d\bar{R}_i^{\text{NB}}/d\mu$, *and* $0 > d\bar{W}^B/d\mu > d\bar{W}^{\text{NB}}/d\mu$.

(iii) *When* $\mu > 2\beta$, $\bar{R}_i^B > \bar{R}_i^{\text{NB}} = 0$ *and* $\bar{W}^B > \bar{W}^{\text{NB}}$.

When $\mu = \beta$, all ideas are traded and welfare is maximized with or without banking. In this case, the existence of banking cannot improve welfare.

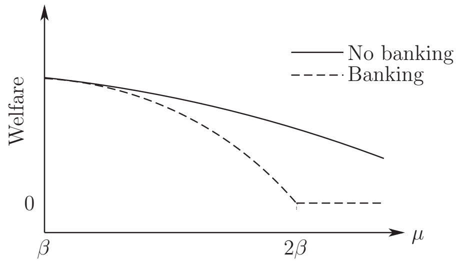

FIGURE 6. Welfare: no banking and costless banking.

When $\mu \in (\beta, 2\beta]$, banking allows more ideas to be traded and thus implies higher welfare. The marginal effect of inflation is higher in magnitude when there is no banking, for two reasons. First, the marginal effect of inflation on the number of trades is larger without banking [i.e., $|d\bar{R}_i^{\mathrm{NB}}/d\mu| > |d\bar{R}_i^B/d\mu|$]. Condition (37) suggests that, without banking, higher inflation raises the opportunity cost of holding money, and thus fewer ideas are traded (i.e., lower $\bar{R}_i^{\mathrm{NB}}$). Condition (38) suggests that, with banking, the impact of inflation on $\bar{R}_i^{\mathrm{NB}}$ is smaller because unspent money can now be saved and thus is not subject to the inflation tax. Second, the gain from the marginal trade is higher without banking ($1 - \bar{R}_i^{\mathrm{NB}} > 1 - \bar{R}_i^B$), because the marginal value of trades is diminishing and because the number of trades is higher with banking. Therefore, inflation is less harmful in the presence of banking.

When $\mu > 2\beta$, monetary equilibrium does not exist without banking, but exists with costless banking. Without banking, the only reason to bring money to the DM is to buy ideas. Very high inflation will make the cost of holding money so high that no ideas are traded, implying zero value of money. With banking, there is an additional motive to bring money to the DM to lend to the banks. Therefore, when the money growth rate is higher than 2β, banking is needed to support a monetary equilibrium (Figure 6).

Next, we compare the price of money in economies with banking (ϕ^B) and without banking (ϕ^{NB}). Mathematically, considering the price ϕ as a function of μ [i.e., $\phi^{\mathrm{NB}}(\mu) = 2\mu - \mu^2/\beta$ and $\phi^B(\mu) = \beta^2/2\mu$], we have $\phi^{\mathrm{NB}}(\beta) > \phi^B(\beta)$ and $\phi^B(2\beta) > \phi^{\mathrm{NB}}(2\beta) = 0$. Because $\phi^B(.)$ and $\phi^{\mathrm{NB}}(.)$ are strictly decreasing and continuous in μ, we have the following result.

PROPOSITION 5 (Value of Money with Costless Banking). *There exists a unique $\mu^* \in (\beta, 2\beta)$ such that $\phi^{\mathrm{NB}}(\mu) \gtreqless \phi^B(\mu)$ for $\mu \lesseqgtr \mu^*$.*

To see the intuition as to why banking reduces ϕ when μ is low and increases ϕ when μ is high, let us consider the two cases illustrated in Figure 7. Start with an economy without banking. At the Friedman rule, every entrepreneur is liquidity-unconstrained and has excess liquidity after trades. Banking is not needed. Close to the Friedman rule (i.e., with low μ), the price of money is relatively high and most ideas are traded. Now, suppose we introduce banking into this economy.

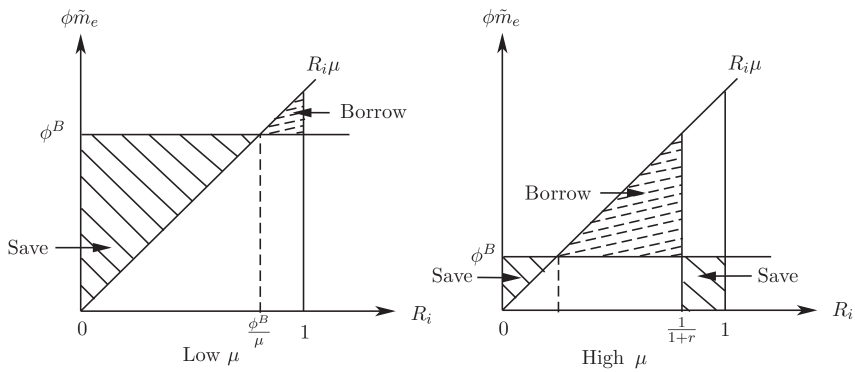

FIGURE 7. Borrowing and saving in the DM for different μ.

Let us first look at the *partial* equilibrium in the DM by keeping the original ϕ unchanged. As illustrated in the figure, there is an excess supply of loans in the DM, driving the interest rate r to 0. Next, we consider the determination of ϕ in the general equilibrium. Anticipating $r = 0$ in the DM, entrepreneurs have a lower incentive to demand money in the CM (because they can always borrow at $r = 0$ in the DM). As a result, the equilibrium price of money in the CM has to go down. Because the real money demand is $\phi\tilde{m} = \phi$, the use of banking comes with a lower demand for real money balances. In a sense, banking is a substitute for real money balances when the money growth rate is low.

Now, consider an economy without banking and where μ is high. The equilibrium price of money is relatively low and most ideas are not traded. Suppose we introduce banking into this economy and again look at the *partial* equilibrium in the DM by keeping the original ϕ unchanged. As illustrated in Figure 7, there is excess demand for loans in the DM, driving up the interest rate r. But, in the general equilibrium, anticipating a high r in the DM, entrepreneurs have a stronger incentive to demand money in the CM (because they do not want to borrow at a high rate and can always save at a high r in the DM). As a result, the equilibrium price of money in the CM has to go up. In this case, banking is a complement for real money balances when the money growth rate is high.

To summarize our findings for an economy with costless banking:

- At the Friedman rule, banking is not used.
- Above the Friedman rule, banking is used and is welfare-improving.
- When the inflation rate is low, banking reduces the price of money. Banking and real money balances are substitutes. When the inflation rate is high, banking increases the price of money. Banking and real money balances are complements.
- When banking is used, inflation is less harmful.

5. ECONOMY WITH COSTLY BANKING

This section considers the case with costly banking. Suppose that entrepreneurs have to incur a fixed effort/utility cost, η, to borrow, but no cost to deposit. One can consider Section 3 as analyzing the case when the fixed cost is infinite, and Section 4 as analyzing the case when the fixed cost is zero. In this section, we consider the general case. We can think of the intermediation costs in our model as those involved in writing debt contracts that are enforceable.[13,14]

We aim to construct a monetary equilibrium with banking ($\phi > 0$ and $r \leq 0$) such that (i) given ϕ, r, banks' choices maximize their profit; (ii) given ϕ, r, agents make an optimal choice in the DM; (iii) given ϕ, r, agents choose optimal money holding in the CM; (iv) the loan market in the DM is cleared; and (v) the money market in the CM is cleared. We are particularly interested in finding the conditions for banks to be active in channeling liquidity from lenders to borrowers.

5.1. Banks' Problem in the DM

The banks' problem is exactly the same as that in the preceding section, and the solution is given by equation (20).

5.2. Entrepreneur's Decision in the DM

Given ϕ, r, R_i, an entrepreneur in a DM meeting chooses saving (s), money brought to the CM (m_e), and whether to buy the idea ($y \in \{0, 1\}$):

$$\widehat{V}_e(\tilde{m}_e, R_e) = \max_{s,y,m_e} \beta W_e\left((1+r)\frac{s}{\mu} + m_e, yR_e\right) - \iota(s)\eta \tag{39}$$

$$= \max_{s,y,m_e} \beta W_e(0,0) + \beta y R_e + \beta\frac{\phi}{\mu}(1+r)s + \beta\phi m_e - \iota(s)\eta, \tag{40}$$

subject to $m_e\mu = \tilde{m}_e - yR_i\mu/\phi - s \geq 0$ and an indicator function

$$\iota(s) = \begin{cases} 1, & \text{if } s < 0 \\ 0, & \text{if } s \geq 0 \end{cases}.$$

Again, the nonnegativity constraint for m_e requires that $rm_e = 0$. Also, there is no reason to pay the fixed cost and borrow unless an entrepreneur is liquidity-constrained. Therefore, $\iota(s) = 0$ when $\tilde{m}_e - yR_i\mu/\phi \geq 0$ and thus the value function becomes

$$\widehat{V}_e(\tilde{m}_e, R_e) = \begin{cases} \beta W_e(0,0) + \beta \max\left\{1 + \frac{\phi}{\mu}(1+r) \right. \\ \qquad \left. \times \left(\tilde{m}_e - \frac{R_i\mu}{\phi}\right), \frac{\phi}{\mu}(1+r)\tilde{m}_e\right\}, & \text{if } \tilde{m}_e \geq \frac{R_i\mu}{\phi} \\ \beta W_e(0,0) + \beta \max\left\{1 + \frac{\phi}{\mu}(1+r) \right. \\ \qquad \left. \times \left(\tilde{m}_e - \frac{R_i\mu}{\phi}\right) - \frac{\eta}{\beta}, \frac{\phi}{\mu}(1+r)\tilde{m}_e\right\}, & \text{if } \tilde{m}_e < \frac{R_i\mu}{\phi} \end{cases}. \tag{41}$$

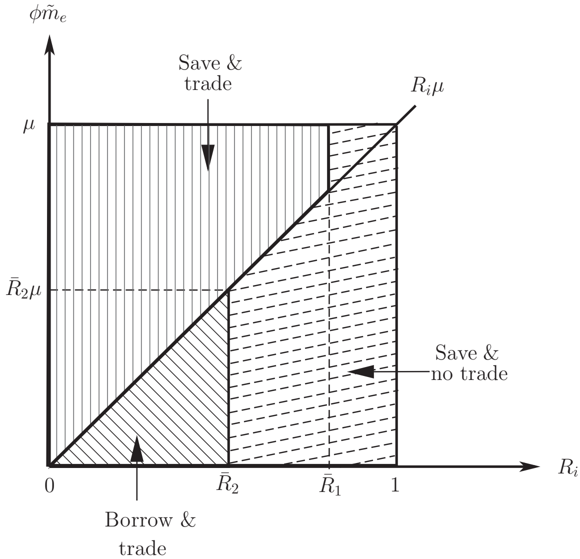

FIGURE 8. Optimal choice of entrepreneur.

Here, one can solve for two critical values of R_i that characterize the trading and saving decisions of entrepreneurs. If $\tilde{m}_e \geq \bar{p}(R_i) = R_i\mu/\phi$, an entrepreneur is not liquidity constrained and will choose to save and trade if and only if

$$R_i \leq \bar{R}_1 \equiv \frac{1}{1+r}.$$

If $\tilde{m}_e < p = R_i\mu/\phi$, an entrepreneur is liquidity-constrained and will choose to borrow and trade if and only if

$$R_i \leq \bar{R}_2 \equiv \frac{1}{1+r} - \frac{\eta}{\beta(1+r)}. \tag{42}$$

Figure 8 illustrates the optimal choice of an entrepreneur for any level of the real money balance ($\phi\tilde{m}_e$) and any realization of R_i. Again, the upward-sloping line indicates the liquidity constraint $\phi\tilde{m}_e \geq R_i\mu$. Given any level of $\phi\tilde{m}_e$, if R_i is low (i.e., on the left side of the line), then the entrepreneur is not liquidity-constrained. In this case, the entrepreneur chooses to trade whenever $R_i \leq \bar{R}_1$. If R_i is high (i.e., on the right side of the line), then the entrepreneur is liquidity-constrained. In this case, he chooses to trade whenever $R_i \leq \bar{R}_2$.

We will first construct the equilibrium in which banks actively channel liquidity from the savers to the borrowers, and then consider the other cases. To generate positive borrowing in equilibrium, two conditions have to be satisfied: (i) a positive mass of entrepreneurs are constrained, and (ii) a positive mass of the constrained entrepreneurs want to borrow. That is, the equilibrium money holding of entrepreneurs, $\phi\tilde{m}_e$, has to be at a level such that there exists some R_i satisfying

$$\phi\tilde{m}_e \leq R_i\mu, \quad (43)$$

$$R_i \leq \bar{R}_2, \quad (44)$$

or simply

$$\phi\tilde{m}_e \leq \bar{R}_2\mu. \quad (45)$$

As illustrated in Figure 8, when this condition is satisfied, there is a positive amount of saving and borrowing.

To construct such an equilibrium, we will first assume that condition (45) is satisfied, and characterize this equilibrium. Then we will derive the conditions for the existence of this equilibrium. In this equilibrium, an entrepreneur brings $\tilde{m}_e$ to the DM and an entrepreneur chooses to trade and save if $R_i \in [0, \phi\tilde{m}_e/\mu]$, chooses to trade and borrow if $R_i \in (\phi\tilde{m}_e/\mu, \bar{R}_2]$, and chooses to save and not trade if $R_i \in (\bar{R}_2, 1]$. As shown in the Appendix, the value function over the relevant region (which is $[0, \bar{R}_2\mu/\phi]$) is given by

$$V_e(\tilde{m}_e) = \beta(W_e(0,0) + R_e) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2)$$
$$- \eta + \frac{\phi}{\mu}[\beta(1+r) + \eta]\tilde{m}_e. \quad (46)$$

Therefore, $V_e'(\tilde{m}_e) = (\phi/\mu)[\beta(1+r) + \eta]$. The idea is that the marginal value of bringing an extra dollar to the DM consists of two components. The first part is the real return of money [i.e., $\beta(\phi/\mu)(1+r)$]. The second part is that it helps to reduce the likelihood of being liquidity constrained and thus avoiding the expected fixed cost of borrowing [i.e., $(\phi/\mu)\eta$]. Note that V_e is linear in $\tilde{m}_e$ in this region.

5.3. Money Demand in the Centralized Market

The optimal money demand of an entrepreneur, arg max $V_e(\tilde{m}_e) - \phi\tilde{m}_e$, is positive and finite only when the first-order condition with respect to the money demand is satisfied with equality [i.e., $V_e'(\tilde{m}_e) = \phi$]. Therefore,

$$r = \frac{\mu - \eta}{\beta} - 1. \quad (47)$$

When equation (47) is satisfied, entrepreneurs are indifferent between any $\tilde{m}_e \in [0, \bar{R}_2\mu/\phi]$.[15,16]

5.4. Loan Market Clearing in the Decentralized Market

Let $\bar{R}_i^{\text{CB}}$ denote the cut-off value of an idea such that an entrepreneur is indifferent regarding trading or not trading. Condition (42) implies that $\bar{R}_i^{\text{CB}} = \bar{R}_2 =$

$(\beta-\eta)/(\mu-\eta)$. As in condition (33), we can use the banks' optimal decision to derive equilibrium conditions relating the excess supply of loans to the interest rate. The loan market–clearing condition in the DM implies that

$$\begin{cases} \tilde{m}_e - \dfrac{\mu(\beta-\eta)^2}{2\phi(\mu-\eta)^2} = 0, & \text{if } r > 0 \\ \tilde{m}_e - \dfrac{\mu(\beta-\eta)^2}{2\phi(\mu-\eta)^2} \geq 0, & \text{if } r = 0 \end{cases}. \tag{48}$$

5.5. Money-Market Clearing in the Centralized Market

Finally, imposing the money market–clearing condition in the CM ($\tilde{m}_e = 1$), we have

$$\begin{cases} \phi = \dfrac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}, & \text{if } r > 0 \\ \phi \geq \dfrac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}, & \text{if } r = 0 \end{cases}. \tag{49}$$

The money market–clearing condition and (45) imply that

$$\phi \leq \bar{R}_2\mu. \tag{50}$$

5.6. Equilibrium

DEFINITION 6. *A stationary monetary equilibrium with costly banking is a pair (ϕ^{CB}, r) satisfying (47), (49), and (50) with $\phi^{\mathrm{CB}} > 0$, $r \geq 0$.*

We have the following result (derived in the Appendix).

PROPOSITION 7 (Existence of Equilibrium with Costly Banking). *If $\eta \leq \min\{\beta, \mu-\beta\}$, there exists an equilibrium with costly banking.*

When $\eta < \mu - \beta$, we have a unique equilibrium with $\phi^{\mathrm{CB}} = \mu(\beta-\eta)^2/2(\mu-\eta)^2$, $r = (\mu-\eta)/\beta - 1 > 0$, and $\bar{R}_i^{\mathrm{CB}} = (\beta-\eta)/(\mu-\eta)$. A fraction $(\beta-\eta)(2\mu-\beta-\eta)/2(\mu-\eta)^2 > 0$ of entrepreneurs are borrowers. Note that the equilibrium with costless banking is a special case when $\eta = 0$.

When $\mu = \beta + \eta$, we have a continuum of equilibria with any $\phi^{\mathrm{CB}} \in [(\beta+\eta)(\beta-\eta)^2/2\beta^2, (\beta-\eta)(\beta+\eta)/\beta]$, $r = 0$, and $\bar{R}_i^{\mathrm{CB}} = 1 - \eta/\beta$. Corresponding to these equilibria, the equilibrium fraction of borrowers is $\max\{\bar{R}^{\mathrm{CB}} - \phi^{\mathrm{CB}}/\beta, 0\} \in [0, (\beta-\eta)(\beta+\eta)/2\beta^2]$. These equilibria are equivalent in terms of the allocation of ideas, but are *not* payoff-equivalent, due to the fixed cost of borrowing. With the highest value of money [i.e., $\phi^{\mathrm{CB}} = (\beta-\eta)(\beta+\eta)/\beta$], there are no borrowers and thus no fixed cost is incurred. As the value of money goes down, there are more and more borrowers and thus a higher total amount of fixed cost is incurred.

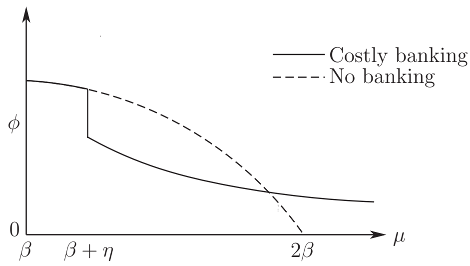

FIGURE 9. Inflation and the price of money.

We shall take a deeper look at the conditions for the existence of an equilibrium with costly banking [i.e., $\eta \leq \min\{\beta, \mu-\beta\}$]. First, if $\beta < \eta$, the payoff of getting an idea ($\beta R_e = \beta$) is lower than the fixed cost (η), and thus no entrepreneurs want to borrow even when the price of the idea is almost zero. Second, note that the *net* real rate of return of buying money in the CM is

$$\frac{\beta(1+r)+\eta}{\mu} - 1 \tag{51}$$

$$= \frac{\beta r + (\beta+\eta-\mu)}{\mu}. \tag{52}$$

Because $r \geq 0$, if $\eta > \mu - \beta$, the net real rate of return is always positive, implying that entrepreneurs would choose to hold so much money balance that no one would borrow in the DM.

Next, we consider the alternative case when $\phi\tilde{m}_e > \bar{R}_2\mu$. As shown in Figure 8, there is no borrowing in this equilibrium, implying that $r = 0$, and, accordingly, $\bar{R}_1 = 1$. The equilibrium allocation is exactly the same as in an equilibrium without banking: entrepreneurs with $R_i \leq \phi/\mu$ will trade, and others will save in the bank at a zero interest rate. In this case, the equilibrium price of money is ϕ^{NB}, which is derived in Section 3. As shown in the Appendix, this equilibrium exists when

$$\eta > \min\{\beta, \mu-\beta\}. \tag{53}$$

5.7. Inflation, Banking, and Welfare

PROPOSITION 8 (Banking and Trading). *If $\eta \leq \min\{\beta, \mu-\beta\}$, then $\bar{R}_i^{\mathrm{NB}} \leq \bar{R}_i^{\mathrm{CB}} \leq \bar{R}_i^{B}$.*

Fewer ideas are traded with costly banking than with costless banking. More ideas are traded in an equilibrium with banking than in an equilibrium without banking. Also, as shown in the last section, costly banking is a substitute for real money balances when the inflation rate is low, and is a complement when the inflation rate is high.[17] As shown in Figure 9:

PROPOSITION 9 (Value of Money with Costly Banking). *There exists a unique $\mu^* \in (\beta+\eta, 2\beta)$ such that $\phi^{\mathrm{NB}}(\mu) \gtreqless \phi^{\mathrm{CB}}(\mu)$ for $\mu \lesseqgtr \mu^*$.*

Although banking can increase trades, it also incurs a fixed cost. We can measure the welfare by the average utility of entrepreneurs and innovators. As before, when there is no banking, the welfare is

$$\bar{W}^{\mathrm{NB}}(\mu) = 2(U(\bar{X}) - \bar{X}) + \int_0^{\bar{R}_i^k} R_e dR_i + \int_{\bar{R}_i^k}^1 R_i dR_i \tag{54}$$

$$= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\left(1 - \bar{R}_i^{\mathrm{NB}}\right)^2}{2}. \tag{55}$$

When there is costly banking (i.e., $\mu \geq \eta + \beta$), the welfare is

$$\bar{W}^{\mathrm{CB}}(\mu) = 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\left(1 - \bar{R}_i^{\mathrm{CB}}\right)^2}{2} - \eta\left[\bar{R}_i^{\mathrm{CB}} - \frac{\phi^{\mathrm{CB}}}{\mu}\right] \tag{56}$$

$$= \bar{W}^{\mathrm{NB}} + \int_{\bar{R}_i^{\mathrm{NB}}}^{\bar{R}_i^{\mathrm{CB}}} (R_e - R_i) dR_i - \eta\left[\bar{R}_i^{\mathrm{CB}} - \frac{\phi^{\mathrm{CB}}}{\mu}\right]. \tag{57}$$

Therefore, the effects of banking on welfare can be decomposed into two parts: the welfare gain from a better allocation of ideas, and the welfare cost due to the fixed cost, which is the product of fixed cost (η) and the number of borrowers ($\bar{R}_i^{\mathrm{CB}} - \phi^{\mathrm{CB}}/\mu$).

Next, we compare steady-state welfare between economies with different money growth rates, μ. We have shown that, for $\mu \in [\beta, \beta + \eta)$, banking is not viable and thus the welfare is given by $\bar{W}^{\mathrm{NB}}$. As discussed above, when $\mu = \beta + \eta$, there is a continuum of banking equilibria with different welfare levels. All these equilibria support the same amounts of trade ($\bar{R}^{\mathrm{CB}} = \bar{R}^{\mathrm{NB}}$), but they incur different amounts of total fixed cost. There is a "high-welfare equilibrium," associated with a high value of money and zero fixed cost. There is also a continuum of "low-welfare equilibria," associated with lower values of money and positive amounts of fixed cost incurred. The lowest welfare level among these "low-welfare equilibria" is given by $\bar{W}^{\mathrm{CB}}(\beta + \eta)$. It is easy to show that $\bar{W}^{\mathrm{CB}}(\beta + \eta) < \bar{W}^{\mathrm{NB}}(\beta + \eta)$. By the continuity of $\bar{W}^{\mathrm{NB}}$ and $\bar{W}^{\mathrm{CB}}$, for sufficiently small $\Delta > 0$, we still have $\bar{W}^{\mathrm{CB}}(\beta + \eta + \Delta) < \bar{W}^{\mathrm{NB}}(\beta + \eta + \Delta)$. Therefore, for moderate inflation, even though banking is used, it does not improve welfare. An economy without banking can achieve a higher welfare.

The welfare ranking is reversed when the inflation rate is high. In particular, when $\mu = 2\beta$, $\bar{R}_i^{\mathrm{NB}} = 0$ and $\bar{W}^{\mathrm{NB}}(2\beta) = 2(U(\bar{X}) - \bar{X}) + 1/2$. The welfare level in a banking equilibrium is[18]

$$\bar{W}^{\mathrm{CB}}(2\beta) = 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\left(1 - \bar{R}_i^{\mathrm{CB}}\right)^2}{2} - \eta\left(\bar{R}_i^{\mathrm{CB}} - \frac{\phi^{\mathrm{CB}}}{\mu}\right)$$

$$> \bar{W}^{\mathrm{CB}}(2\beta).$$

Therefore, we have the following result.

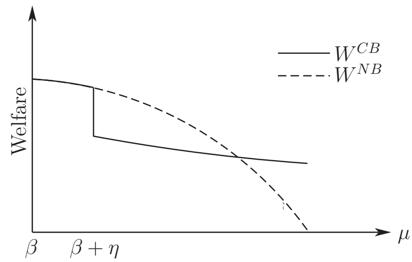

FIGURE 10. Welfare, inflation, and banking.

PROPOSITION 10 (Inflation and Welfare). *For any* $\eta \in (0, \min\{\beta, \mu - \beta\})$, *there exists* $\overline{\Delta}_1, \overline{\Delta}_2 > 0$ *such that (i)* $\bar{W}^{\mathrm{CB}} < \bar{W}^{\mathrm{NB}}$ *for any* $\mu \in [\beta + \eta, \beta + \eta + \overline{\Delta}_1]$, *and (ii)* $\bar{W}^{\mathrm{CB}} > \bar{W}^{\mathrm{NB}}$ *for any* $\mu \in [2\beta - \overline{\Delta}_2, +\infty]$.

This is illustrated in Figure 10. The idea is that banking has both positive and negative effects on welfare. On the negative side, the use of banking incurs a fixed cost. On the positive side, banking improves the allocation of ideas. Note that the improvement of welfare depends on the inflation rate. When the inflation rate is low, most of the ideas are efficiently allocated even without banking; thus, the gain from trading those remaining ideas is small. In this case, the welfare improvement from better allocation of ideas is outweighed by the fixed cost. When the inflation rate is high, most of the ideas are not traded without banking. In this case, the welfare improvement from better allocation of ideas outweighs the fixed cost.

Banking is used even though it is welfare-reducing because, when an entrepreneur decides whether to borrow, he takes into account only his own private cost and benefit. An entrepreneur chooses to borrow whenever the net private gain from borrowing is larger than the fixed cost, ignoring the general-equilibrium feedback effect of his action. In particular, a borrower neglects the fact that his borrowing in the DM reduces his demand for money in the CM. As a result, the price of money in the CM goes down. At equilibrium, a drop in the price of money will tighten other entrepreneurs' liquidity constraints, pushing more entrepreneurs toward costly borrowing from banks. Therefore, an individual's choice to borrow from a bank can lead to welfare loss to society.

Next, we study the welfare effect of changing the fixed cost.

PROPOSITION 11 (Fixed Cost and Welfare). *For any* $\mu \in (\beta, 2\beta)$, *there exist* $\widetilde{\Delta}_1, \widetilde{\Delta}_2 > 0$ *such that*

(i) $\bar{W}^{\mathrm{CB}} > \bar{W}^{\mathrm{NB}}$ *for any* $\eta \in [0, \widetilde{\Delta}_1]$,
(ii) $\bar{W}^{\mathrm{CB}} < \bar{W}^{\mathrm{NB}}$ *for any* $\eta \in [\mu - \beta - \widetilde{\Delta}_2, \mu - \beta]$,
(iii) *For* $\mu = 1$, *there exists an* $\bar{\eta} \in (0, 1 - \beta)$ *such that*

$$\begin{cases} \bar{W}^{\mathrm{CB}} > \bar{W}^{\mathrm{NB}} & \text{if } \eta < \bar{\eta} \\ \bar{W}^{\mathrm{CB}} = \bar{W}^{\mathrm{NB}} & \text{if } \eta = \bar{\eta}\,. \\ \bar{W}^{\mathrm{CB}} < \bar{W}^{\mathrm{NB}} & \text{if } \eta > \bar{\eta} \end{cases}$$

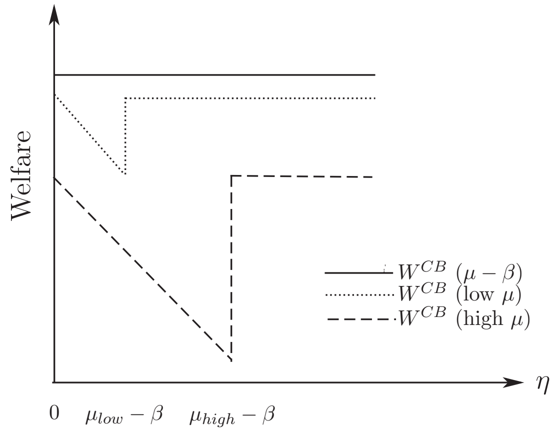

FIGURE 11. Effect of fixed cost on welfare.

Proposition 4 implies that, when the fixed cost is zero, banking can always improve welfare. By continuity, banking can improve welfare for small fixed costs. Moreover, for a fixed cost sufficiently large relative to the inflation rate, the fixed cost of using banks outweighs the gain from a better allocation of ideas. Figure 11 plots various $\bar{W}^{\mathrm{CB}}$ for different sizes of the inflation rate. When $\mu = \beta$, the welfare is independent of the fixed cost, because banking is never used. For $\mu > \beta$, banking is used whenever $\eta < \mu - \beta$. Below this threshold, welfare is decreasing in the fixed cost. Above this threshold, banking is not used. Note that, due to the general-equilibrium feedback mentioned above, the welfare can be higher when the fixed cost goes up. The implication is that the range of fixed costs that can support banking is increasing in the inflation rate.

Figure 12 shows the distribution of outcomes for different combinations of the fixed cost and money growth rate. In Figure 12, we can see that banking is used only

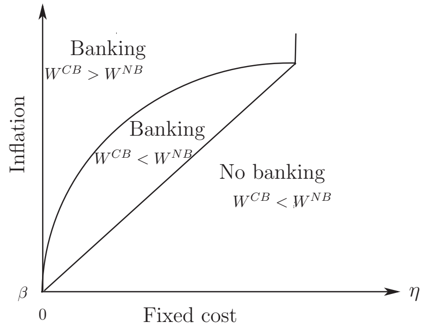

FIGURE 12. Distribution of equilibrium.

when two conditions are satisfied: (i) $\eta < \beta$ (indicated by the vertical line) and (ii) $\eta < \mu - \beta$ (indicated by the upward-sloping straight line). The curve indicating $\bar{W}^{\mathrm{CB}} = \bar{W}^{\mathrm{NB}}$ is concave because, when there is no banking, the marginal distortion of inflation is increasing in money growth rate. It is interesting to compare this result with Bencivenga and Camera (in press). In their model, banking can also potentially reduce welfare, but this suboptimal outcome cannot be supported in equilibrium because banking is modeled as an optimal contract.

Even when banking is costly, inflation is less harmful whenever banking is used.

PROPOSITION 12 (Banking and Welfare Costs of Inflation). *For any* $\eta \in (0, \min\{\beta, \mu - \beta\})$,

$$\left|\frac{d}{d\mu}\bar{W}^{\mathrm{CB}}\right| < \left|\frac{d}{d\mu}\bar{W}^{\mathrm{NB}}\right|.$$

One implication of this finding is that, in measuring the welfare gain of reducing inflation by extrapolating observable data points from the high-inflation region with banking (far from the Friedman rule) to the unobservable low-inflation region without banking (close to the Friedman rule), we may underestimate the actual welfare gain of following the Friedman rule, because this approach ignores the intermediation costs involved in using banking to solve the liquidity problem. Mathematically, denote $\bar{W}^{\mathrm{CB}}(\mu)$ as the level of welfare in an economy with costly banking. The actual welfare gain of moving from an inflation rate μ to the Friedman rule is $G(\mu) = \bar{W}^{\mathrm{CB}}(\beta) - \bar{W}^{\mathrm{CB}}(\mu)$. A first-order approximation of this welfare gain is

$$\hat{G}(\mu) = \frac{d\bar{W}^{\mathrm{CB}}(\mu)}{d\mu}(\beta - \mu).$$

It is shown in the Appendix that

PROPOSITION 13 (Underestimation of Welfare Gain). *For any* μ, β, *there exists an* $\bar{\eta}(\mu, \beta) < \beta$ *such that* $G(\mu) > \hat{G}(\mu)$ *for all* $\eta \in (\bar{\eta}, \beta)$.

The idea is that banking is used and the fixed intermediation costs are incurred only when the inflation rate is sufficiently high. As a result, measuring the welfare change by extrapolating from the high-inflation region (where banking is used) to the Friedman rule (where banking is not used) does not take into account the potential saving of the fixed costs as the money growth rate drops to β.

To summarize our findings for an economy with costly banking:

- Banking improves the allocation of ideas.
- When the inflation rate is low, banking is not used.
- When the inflation rate is high, banking is used and is welfare-improving. Banking increases the price of money and the real money demand.
- When the inflation rate is moderate, banking is used but is welfare-reducing. Banking reduces the price of money and the real money demand.
- When banking is used, inflation is less harmful.

6. EXTENSIONS

This section briefly discusses the robustness of our findings when some of the simplifying assumptions are relaxed. First, we relax the assumption that "ideas" are pure private goods. In particular, we assume that, after trading an idea, the entrepreneur receives the implementation value R_e, whereas the innovator retains a fraction λ of her implementation value (i.e., λR_i), with $\lambda \in (0, 1)$.[19] We can show that there exists a $\bar{\lambda}$, such that for all $\lambda \leq \bar{\lambda}$, all of our main findings still hold true.[20]

Another simplifying assumption of our model is that ideas are indivisible and agents are not allowed to use lotteries to convexify their bargaining problems. Next, we consider the case in which it is feasible for agents to use lotteries. In particular, an offer from an entrepreneur to an innovator consists of a pair (p, θ), where p is the price paid by the entrepreneur and $\theta \in [0, 1]$ is the probability of transferring the idea. One can show that there are parameter values under which all the main findings still hold true. The only important difference from the benchmark case is that, in an economy without banking, a monetary equilibrium always exists for any monetary growth rate.[21]

One interesting extension is to consider a general nonseparable production function. In the Appendix, we discuss the case in which $F(I, h) = If(h)$ with $f' > 0$ and $f'' < 0$. In this case, the equilibrium prices (r^*, w^*, ϕ^*) and the fraction of ideas traded I^* are jointly determined. For example, an improved allocation of ideas in the DM will lead to a higher labor demand, which tends to drive up the wage rate in the CM. A high wage rate will then affect the entrepreneurs' implementation returns relative to the innovators' returns, and thus has feedback effects on the allocation of ideas in the DM.

Finally, in this model, entrepreneurs can always successfully implement the ideas. In an environment in which idea implementation is risky, banks may suffer losses associated with loan defaults. One may modify the current framework to study banks' willingness to take risk under different monetary policy regime. We will leave this extension for future research.

7. CONCLUSION

This paper develops a search-theoretical model to study how money and banking interact to affect allocation and welfare. We highlight that banking and monetary models need to be studied together to properly assess the welfare effects of banking and the welfare costs of inflation. An interesting implication of our model is that, due to general-equilibrium feedback, banking can exist in equilibrium even when it is welfare-reducing. Moreover, the nonlinear welfare effect of inflation implies that measuring welfare costs of inflation by extrapolating historical data may underestimate the actual cost.

This paper considers the case in which ideas are not durable. It would be interesting to study how the accumulation of ideas can affect long-term economic performance. Chiu et al. (2009) extend this model to study the relationship between the market for ideas and economic growth. In particular, they consider a general production function $y = zf(h)$ where the level of productivity, z, can be improved over time by accumulating ideas. They use this model to study the relationship between monetary policy, financial intermediation, and economic growth.

NOTES

1. Here, the only liquid asset is fiat money. Silveira and Wright (in press) consider the general case with an interest-bearing liquid asset.

2. Here, I_e being 1 is a normalization. Also, Silveira and Wright (in press) consider a more general case in which both innovators and entrepreneurs have random valuations.

3. We will discuss the general case in Section 6.

4. Alternatively, one can normalize by dividing the nominal variables by the price level. Obviously, this does not affect our results.

5. Silveira and Wright (in press) consider a more general case in which the price is determined by generalized Nash bargaining. Also, in their model, the innovator and the entrepreneur have an option to meet again in the next CM, where the entrepreneur can raise more money. We abstract from these interesting extensions to focus on the effects of banking on the market for ideas in the simplest possible case.

6. For simplicity, we first consider the case in which a lottery is not allowed. We will discuss the case where a lottery is available in Section 6.

7. Given the fact that $R_e \geq R_i$, the entrepreneur will always trade when he can afford to.

8. The upper bound 2β is a result of the assumptions $R_e = 1$ and $R_i \sim U(0, 1)$.

9. Note that the nonexistence of monetary equilibrium for high money growth rates is related to the assumptions that ideas are indivisible and that lotteries are not allowed. Please see Section 6.

10. Note that, at equilibrium, innovators do not have incentives to use banking even if they have access to banking. Also, we assume that banks cannot issue inside money in this economy. Equivalently, one can also allow banks to issue inside money, but subject to a 100% reserve requirement.

11. Banks are owned by the agents and distribute the profit to the shareholders. In equilibrium, banks have zero profit.

12. Here we assume that, when entrepreneurs are indifferent between saving and not saving (which happens when $r = 0$), they choose to save.

13. In our model, agents are anonymous and therefore in order to borrow from the banks they must incur these costs in writing enforceable debt contracts. The costs could include time costs (e.g., time and effort to collect documents) and possibly resource costs. Given that we interpret the intermediation costs as those involved in writing enforceable debt contracts and because banks are assumed to be able to commit in our model, depositors do not need to face these costs when lending to/depositing at the bank.

14. In an unpublished note, we have also considered different cost structures (e.g., symmetric fixed costs for borrowers and depositors, proportional costs for borrowers) and found that the specific cost structure we assumed is not crucial for the main results of the paper. This note is available upon request.

15. Note that r is the interest rate of a loan between one DM and the next CM. If agents make a loan from one CM to the next CM, the interest rate will still satisfy the Fisher equation; otherwise, it is not arbitrage-free.

16. Looking at equation (47), one may wonder whether the interest rate depends on the (arbitrary) unit of measuring utility. To show that this is not true, one can consider a more general setting in which agents have the CM utility given by $U(X) - AH$. In an unpublished technical appendix (which is available upon request), we show that, in this case, the equilibrium interest rate becomes $r = (\mu - \eta/A)/\beta - 1$. So what matters is the *relative fixed cost* η/A (i.e., the disutility of borrowing

relative to the marginal disutility of labor), which does not depend on the unit of measuring utility. In the benchmark case, we have equation (47) because the marginal disutility of labor is normalized to one. The authors would like to thank Christopher Waller for pointing this out in his discussion of our paper at the joint conference of the Federal Reserve Bank of Philadelphia and the University of Pennsylvania in 2008.

17. Obviously, $\phi^B > \phi^{\mathrm{CB}}$ because $d\phi^{\mathrm{CB}}/d\eta < 0$. Considering the price ϕ as a function of μ [i.e., $\phi^{\mathrm{NB}}(\mu) = 2\mu - \mu^2/\beta$ and $\phi^{\mathrm{CB}}(\mu) = \mu(\beta-\eta)^2/2(\mu-\eta)^2$], we have $\phi^{\mathrm{NB}}(\beta+\eta) > \phi^B(\beta+\eta)$ and $\phi^B(2\beta) > \phi^{\mathrm{NB}}(2\beta) = 0$, because $\phi^{\mathrm{CB}}(.)$ and $\phi^{\mathrm{NB}}(.)$ are strictly decreasing and continuous in μ.

18. Specifically, the welfare level in an economy with costly banking is

$$\begin{aligned}\bar{W}^{\mathrm{CB}}(2\beta) &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\left(1-\bar{R}_i^{\mathrm{CB}}\right)^2}{2} - \eta\left(\bar{R}_i^{\mathrm{CB}} - \frac{\phi^{\mathrm{CB}}}{\mu}\right)\\ &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta-\eta)^2} - \eta\frac{(\beta-\eta)(2\beta-\eta)}{2(2\beta-\eta)^2}\\ &> 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta-\eta)^2} - \frac{(\beta-\eta)(2\beta-\eta)}{2(2\beta-\eta)^2}\\ &= \bar{W}^{\mathrm{CB}}(2\beta).\end{aligned}$$

19. When $\lambda = 0$, it is the benchmark case when ideas are pure private goods. When $\lambda = 1$, ideas are pure public goods. In this case, innovators will sell all their ideas at a zero price. Efficiency is always achieved. Therefore, we will focus on the interesting case with $\lambda \in (0, 1)$.

20. In particular, banking can improve the allocation of ideas. When μ is low, banking is not used. When μ is moderate, banking is used but is welfare-reducing. When μ is high, banking is used and is welfare-improving. Also, banking is needed to support a monetary equilibrium when μ is high.

21. The analysis and implications are the same if we assume that ideas are divisible, instead of allowing for lotteries. This unpublished note is available upon request.

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APPENDIX A

A.1. PROOF OF PROPOSITION 7

We first derive the value function $V_e(m)$ by evaluating its value over three regions: $m \in [0, \bar{R}_2\mu/\phi]$, $(\bar{R}_2\mu/\phi, \bar{R}_1\mu/\phi]$, and $(\bar{R}_1\mu/\phi, \infty)$. We will use $V_e^1(m)$, $V_e^2(m)$, and $V_e^3(m)$ to denote the value function over these three regions, respectively:

(1) For $m \in [0, \bar{R}_2\mu/\phi]$:

$$V_e^1(m) = \int_0^1 V_e(m, R_i)dR_i \tag{A.1}$$

$$= \beta W_e(0,0) + \beta\int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)\left(m - \frac{R_i\mu}{\phi}\right)dR_i$$

$$+\beta\int_{\frac{\phi}{\mu}m}^{\bar{R}_2} 1 + \frac{\phi}{\mu}(1+r)\left(m - \frac{R_i\mu}{\phi}\right) \tag{A.2}$$

$$-\frac{\eta}{\beta}dR_i + \beta\int_{\bar{R}_2}^1 \frac{\phi}{\mu}(1+r)m\,dR_i \tag{A.3}$$

$$= \beta W_e(0,0) + \beta - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2)$$

$$-\eta + \left[\beta\frac{\phi}{\mu}(1+r) + \eta\frac{\phi}{\mu}\right]m. \tag{A.4}$$

(2) For $m \in (\bar{R}_2\mu/\phi, \bar{R}_1\mu/\phi]$:

$$V_e^2(m) = \int_0^1 V_e(m, R_i)dR_i \tag{A.5}$$

$$= \beta W_e(0,0) + \beta\int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)\left(m - \frac{R_i\mu}{\phi}\right)dR_i$$

$$+\beta\int_{\frac{\phi}{\mu}m}^1 \frac{\phi}{\mu}(1+r)m\,dR_i \tag{A.6}$$

$$= \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)m + \beta\frac{\phi}{\mu}m - \beta(1+r)\frac{\left(\frac{\phi}{\mu}m\right)^2}{2}. \tag{A.7}$$

(3) For $m \in (\bar{R}_1\mu/\phi, \infty)$:

$$V_e^3(m) = \int_0^1 V_e(m, R_i)dR_i \tag{A.8}$$

$$= \beta W_e(0,0) + \beta \int_0^{\bar{R}_1} 1 + \frac{\phi}{\mu}(1+r)\left(m - \frac{R_i}{\frac{\phi}{\mu}}\right)dR_i$$

$$+ \beta \int_{\bar{R}_1}^1 \frac{\phi}{\mu}(1+r)m dR_i \tag{A.9}$$

$$= \beta W_e(0,0) + \beta + \beta\frac{\phi}{\mu}(1+r)m - \beta(1+r)\frac{\bar{R}_1^2}{2}$$

$$- \beta(1+r)\bar{R}_1(1-\bar{R}_1). \tag{A.10}$$

LEMMA. *In an equilibrium with costly banking, the optimal money holding stays in the interval* $[0, \bar{R}_2\mu/\phi]$ *if* $\eta \leq \min\{\beta, \mu - \beta\}$.

Proof. We aim to show the following:

(1) $V_e^1(m) - \phi m = k$ for $m \in [0, \bar{R}_2\mu/\phi]$, where k is a positive constant.
(2) $V_e^2(m) - \phi m < k$ for $m \in (\bar{R}_2\mu/\phi, \bar{R}_1\mu/\phi]$.
(3) $V_e^3(m) - \phi m < k$ for $m \in (\bar{R}_1\mu/\phi, \infty)$.

Case 1: $r > 0$.

First, consider the equilibrium with $r > 0$. Using the result that $\bar{R}_1 = \beta/(\mu - \eta)$, $\bar{R}_2 = (\beta - \eta)/(\mu - \eta)$, $\phi = \mu(\beta - \eta)^2/2(\mu - \eta)^2$, and $1+r = \mu - \eta/\beta$, we can simplify the V_e derived above to get the following:

(1)

$$V_e^1(m) - \phi m$$

$$= \beta(W_e(0,0) + 1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta$$

$$+ \left[\beta\frac{\phi}{\mu}(1+r) + \eta\frac{\phi}{\mu}\right]m - \phi m \tag{A.11}$$

$$= \beta W_e(0,0) + \frac{(\beta-\eta)^2}{2(\mu-\eta)} = k > 0. \tag{A.12}$$

(2) Similarly, we get $V_e^2(m) - \phi m = \beta W_e(0,0) + [(\beta-\eta)^2/2(\mu-\eta)^2]$ $[(\beta-\eta)m - (\beta-\eta)^2m^2/4(\mu-\eta)]$. We can show that $V_e^2 - \phi m$ is strictly concave and attains its maximum at $m = 2(\mu-\eta)/(\beta-\eta)$ (which is the lower bound of region 2), with the maximum equal to k. Therefore, $V_e^2(m) - \phi m < k$ for all $m \in (\bar{R}_2\mu/\phi, \bar{R}_1\mu/\phi]$.
(3) First, note that $V_e^3(m) - \phi m$ is linear and strictly decreasing with $d/dm[V_e^3(m) - \phi m] = -(\phi/\mu)\eta < 0$. Therefore, for any $m \in (\bar{R}_1\mu/\phi, \infty)$, $V_e^3(m) - \phi m$ is lower

than $V_e^3(\bar{R}_1\mu/\phi) - \phi(\bar{R}_1\mu/\phi) = W_e(0,0) + (-2\beta\eta + \beta^2/2(\mu-\eta))$, which is lower than k if $\eta^2/2(\mu-\eta) > 0$.

Case 2: $r = 0$.

Next, we consider the equilibrium with $r = 0$. Condition (47) implies that $\mu = \beta + \eta$. We can follow the same analysis as above to show that $V_e^2(m) - \phi m$ is maximized at $m = \bar{R}_2\mu/\phi$, which is equal to $V_e^1(m) - \phi m$ for any $m \in [0, \bar{R}_2\mu/\phi]$. Also, $r = 0$ implies $\bar{R}_1 = 1$, and so the third region vanishes.

We have therefore proved that $\max_m V_e(m) - \phi m \in [0, \bar{R}_2\mu/\phi]$; indeed, in equilibrium, an entrepreneur is indifferent between any m in this interval. We need to check that this is not an empty set, that is, $\bar{R}_2 \geq 0$, which requires $\eta \leq \beta$. Finally, $r \geq 0$ requires $\eta \leq \mu - \beta$.

To prove Proposition 7, we need to derive the conditions in Proposition 4. First, consider the case with $r > 0$. Condition (47) implies that $\mu - \eta - \beta > 0$. Condition (49) then implies that $\phi^{\mathrm{CB}} = \mu(\beta-\eta)^2/2(\mu-\eta)^2$. Then condition (50) is satisfied if and only if $\phi^{\mathrm{CB}} \leq \bar{R}_2\mu$, which is equivalent to $\eta \leq \beta$. Second, consider the case with $r = 0$. Condition (47) implies that $\mu - \eta - \beta = 0$. Condition (49) then implies that $\phi^{\mathrm{CB}} \geq \mu(\beta-\eta)^2/2(\mu-\eta)^2$. Then conditions (49) and (50) are satisfied if and only if $\eta \leq \beta$. ■

A.2. PROOF OF CONDITION (53)

We want to show that $\arg\max_m V_e(m) - \phi m > \bar{R}_2\mu/\phi$ when $r = 0$ and condition (53) is satisfied. First, if $\eta > \beta$, then $\bar{R}_2 < 0$, and the optimal choice of money is obviously above $\bar{R}_2\mu/\phi$. Now suppose $\eta < \beta$. It is easy to show that, when $r = 0$, $V_e^2(m) - \phi m$ in the previous proof attains its global maximum at $m = \bar{R}_2(\beta+\eta)/\phi$. Then, to show that choosing $m \leq \bar{R}_2\mu/\phi$ is not optimal (where V_e^1 is the corresponding value function), note that $V_e^2(\bar{R}_2\mu/\phi) - \phi\bar{R}_2\mu/\phi = V_e^1(m) - \phi m$ for all $m \leq \bar{R}_2\mu/\phi$. Therefore, we just need to show that $\bar{R}_2(\beta+\eta)/\phi > \bar{R}_2\mu/\phi$, which requires $\eta > \mu - \beta$. ■

A.3. PROOF OF PROPOSITION 8

$\bar{R}^B = \beta/\mu \geq \bar{R}^{\mathrm{CB}} = (\beta-\eta)/(\mu-\eta)$ is obvious. Also, $\bar{R}^{\mathrm{CB}} = (\beta-\eta)/(\mu-\eta) \geq \bar{R}^{\mathrm{NB}} = 2 - \mu/\beta$ if $(\mu-\beta)(\beta-\mu+\eta) \leq 0$. ■

A.4. PROOF OF PROPOSITION 11

First, $\bar{W}^{\mathrm{NB}}(\eta) < \bar{W}^{\mathrm{CB}}(\eta)$ for $\eta = 0$. Second, $\bar{W}^{\mathrm{NB}}(\eta) > \bar{W}^{\mathrm{CB}}(\eta)$ for $\eta = \mu - \beta$. Finally, for $\mu = 1$, $\mathrm{sign}(\bar{W}^{\mathrm{CB}} - \bar{W}^{\mathrm{NB}}) = \mathrm{sign}(D(\eta))$ where $D(\eta) = (1-\beta)^2((1-\eta)^2 - \beta^2) - \eta(\beta-\eta)\beta^2(2-\eta-\beta)$. From above, we know already that $D(0) > 0$ and $D(1-\beta) < 0$. Also, we can show that $dD/d\eta(1-\beta) < 0$ and $d^2D/d\eta^2 > 0$, implying that $dD/d\eta < 0$ for $\eta \in (0, 1-\beta)$. Therefore, there exists a cut-off $\bar{\eta}$ such that $\bar{W}^{\mathrm{NB}} = \bar{W}^{\mathrm{CB}}$. ■

A.5. PROOF OF PROPOSITION 12

For any $\eta \in (0, \min\{\beta, \mu-\beta\})$, $|\frac{d}{d\mu}\bar{W}^{\mathrm{CB}}| < |\frac{d}{d\mu}\bar{W}^{\mathrm{NB}}|$.

The welfare effects of inflation when there is costly banking are given by

$$\frac{d}{d\mu}\bar{W}^{\mathrm{CB}} = -\frac{d}{d\mu}\left[\frac{\left(1-\bar{R}_i^{\mathrm{CB}}\right)^2}{2}\right] - \frac{d}{d\mu}\left[\eta\left(\bar{R}_i^{\mathrm{CB}} - \frac{\phi^{\mathrm{CB}}}{\mu}\right)\right] \tag{A.13}$$

$$= -\frac{(1-\eta)(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3}. \tag{A.14}$$

The welfare effects of inflation when there is no banking are given by

$$\frac{d}{d\mu}\bar{W}^{\mathrm{NB}} = -\frac{d}{d\mu}\left[\frac{\left(1-\bar{R}_i^{\mathrm{NB}}\right)^2}{2}\right] \tag{A.15}$$

$$= -\frac{\mu-\beta}{\beta^2}. \tag{A.16}$$

Therefore, the welfare effects of inflation are smaller with banking when

$$\left|\frac{d}{d\mu}W^{CB}\right| < \left|\frac{d}{d\mu}\bar{W}^{\mathrm{NB}}\right| \tag{A.17}$$

$$\Leftrightarrow (1-\eta)(\beta-\eta) < \frac{(\mu-\eta)^2}{\beta^2}(\mu-\eta), \tag{A.18}$$

which is true, because $(1-\eta)(\beta-\eta) < \beta < [(\mu-\eta)^2/\beta^2](\mu-\eta)$. ■

A.6. PROOF OF PROPOSITION 13

First, note that

$$G(\mu) = W^{\mathrm{CB}}(\beta) - W^{\mathrm{CB}}(\mu) \tag{A.19}$$

$$= \frac{(\mu-\beta)^2}{2(\mu-\eta)^2} + \frac{\eta(\beta-\eta)(2\mu-\beta-\eta)}{2(\mu-\eta)^2}. \tag{A.20}$$

$$\hat{G}(\mu) = \frac{d}{d\mu}W^{\mathrm{CB}}(\mu)(\beta-\mu) \tag{A.21}$$

$$= \frac{(1-\eta)(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3}. \tag{A.22}$$

Therefore, $G(\mu)|_{\eta=\beta} > \hat{G}(\mu)|_{\eta=\beta}$. Moreover, $\frac{d}{d\eta}\hat{G}(\mu;\eta) < 0$ and $\frac{d}{d\eta}G(\mu;\eta) > 0$ for all $\eta > \bar{\eta}$, for some sufficiently large $\bar{\eta} < \beta$. ■

A.7. GENERAL PRODUCTION FUNCTION

Suppose that $F(I,h) = If(h)$ with $f' > 0$ and $f'' < 0$. Also, we assume that the utility of agents is given by $X - D(Y)$, with $D' > 0$ and $D'' > 0$. The labor demand is then given by the first-order condition $f_h(I,h) = w$, which implies a labor demand function $h(I,w)$. The labor supply is characterized by $D'(H) = w$, which implies a labor supply function $H(w)$. We first consider the equilibrium with banking. Here, the equilibrium prices (ϕ^*, w^*) and allocation of ideas I^* are jointly determined:

(i) Money market–clearing condition:

$$\frac{\mu}{\beta} = \frac{\pi(1, w^*) - \phi^*/\mu}{f(h^*)},$$

where $h^* = h(I^*, w^*)$ and $\pi(I, w^*) = If(h(I, w^*)) - h(I, w^*)$.

(ii) The fraction of ideas traded:

$$\frac{\phi^*}{\mu} = \pi(I^*, w^*).$$

(iii) The labor market–clearing condition:

$$(D')^{-1}(w^*) = \int_{I^*}^{1} h(I, w^*)dI + h(1, w^*)I^*.$$

Similarly, in an equilibrium with costless banking, the prices (ϕ^*, w^*, r^*) and allocation I^* are given by

(i) Money market–clearing condition:

$$r^* = \frac{\mu}{\beta} - 1.$$

(ii) The fraction of ideas traded:

$$1 + r^* = \frac{\pi(1, w^*)}{\pi(I^*, w^*)}.$$

(iii) The labor market–clearing condition:

$$(D')^{-1}(w^*) = \int_{I^*}^{1} h(I, w^*)dI + h(1, w^*)I^*.$$

(iv) The banking sector equilibrium:

$$\frac{\phi^*}{\mu} = \int_{0}^{I^*} \pi(I, w^*)dI.$$