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# The welfare cost of inflation revisited: The role of financial innovation and household heterogeneity<sup>☆</sup>

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## ABSTRACT

The money-consumption ratio increases with age and decreases with consumption, and the recent era of low interest rates has seen a large increase in the aggregate money-consumption ratio. We estimate an overlapping generations model with money for transaction purposes for the age effects and the extent of financial innovation using aggregate and household-level money holdings. We then assess the welfare cost of a 3 percentage point increase in inflation, incorporating the cost from the redistribution of non-money nominal wealth. We find that the welfare costs are 13% of one-year consumption and are borne mostly by the poor and the old.

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## 1. Introduction

Money holdings are at their highest level in the last four decades. Hence, raising the level of long-term inflation, as it has been proposed recently in a variety of contexts,<sup>1</sup> could inflict more pain on money holders now than before. At the same time, financial innovation has altered and continues to alter how households manage their financial portfolios and pay for their consumption. Such financial innovation would likely reduce the need for money, raising a question as to the role of financial innovation in the observed high level of money holdings in the economy.

Motivated by these recent developments, we revisit the welfare cost of inflation, accounting for the distortions and costs that inflation poses on households who use money for transactions to avoid costly credit transactions and on the gains and

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<sup>1</sup> Blanchard et al. (2010) support raising the inflation target from around 2%, a commonly adopted target in most advanced countries. This issue was also debated at the 2015 Jackson Hole Economic Policy Symposium by Aruoba and Schorfheide (2016). In addition, the Bank of Canada officially reviewed a question of whether or not to raise the inflation target from 2% to 3% under its 2016 Renewal of the Inflation-Control Target. See Bank of Canada (2016).

losses associated with non-money nominal assets and debt arising from unexpected inflation and its wealth redistribution (as documented by Adam and Zhu, 2016; Doepke and Schneider, 2006a; Doepke and Schneider, 2006b; Meh et al., 2010). Inflation costs are likely to be unequal across household groups because of the very large disparities in their money holdings and non-money nominal positions.<sup>2</sup> This heterogeneity should be taken into account to assess the aggregate cost of inflation. In addition, if financial innovation were to play an important role and reduce household dependence on money for transactions over time, then the current estimates in the literature of the aggregate welfare cost of inflation without financial innovation would be overstated. This paper takes all these issues into account when providing a measure of the cost of inflation.

There is no standard model that incorporates financial innovation and the dimensions of household heterogeneities that we study. Thus, we start by building a model in which households are heterogeneous in income, age, and cohort of birth and that is capable of incorporating all these mechanisms. By confronting money holdings across time in the model and in the data and in the presence of varying nominal interest rates, we break down the higher money holdings of older people into those due to age (what we can think of as an intrinsic feature of becoming older) and those due to a secular trend over time. We can associate this trend with financial innovations that lower the cost of credit or with the adaptation of households to newly available money-saving transacting technologies. The actual nature of financial innovation as cohort or time effects cannot be ascertained and in the context of our model is irrelevant. We model it as a cohort effect, where time effects only enter through the observables of interest rate and fiscal policy variations (that is, there is no secular financial improvement in the form of a time). We could have modeled financial innovation with the same conclusions as having a secular time effect (provided we incorporate a suitable transformation of the age effect) with no cohort effects.

The model builds on the work of Erosa and Ventura (2002), who analyzed the money demand of households that were heterogeneous in income and wealth. Our model allows the money holdings per unit of consumption to vary by consumption levels of households, as well as by their date of birth (and hence their age at any point in time). It is an overlapping generations (OLG) model with age and cohort effects, both mechanisms contributing to generate larger money holdings among the elderly relative to those among their younger peers. The model is then asked to replicate aggregate money holdings over time. The large oscillations in nominal interest rates over time help identify age versus cohort effects by ensuring that the model jointly replicates the cross-sectional holdings with the variation of holdings over time. The model accommodates details of households' gains/losses from their non-money nominal positions and the windfall for the government of higher inflation (reductions in the real value of its liabilities and increases in seigniorage) and permits us to measure the costs of inflation with lower tax rates to allocate these windfall gains back to households.

Specifically, the paper makes four sets of contributions. First, using cross-sectional Canadian household survey and macroeconomic time-series data, we document three facts on the ratio of money holdings to consumption (which we label succinctly as money demand). (1) *Money demand across households increases with age.* (2) *Money demand decreases with households' consumption level.* (3) *Aggregate money demand increased by 30% between 2000 and 2010.*

Second, we document the nominal positions of households and economic sectors and calculate the extent of wealth redistribution across households and sectors under a permanent increase in inflation for the household partition that we use based on consumption. This brings additional gains to the government on top of increased seigniorage, making a larger role for the government to impact the welfare of households. A positive inflation shock induces young households to gain wealth because of their mortgage holdings and older households, especially poorer ones, to lose because of their long-term nominal assets and pension positions. The household sector as a whole loses while the government gains most of that loss (allowing for additional future fiscal adjustments) and the foreign sector gains the remaining loss of households.

Third, we provide estimates for the interest rate elasticity of money by household type. Aggregating over household types and times yields an estimate of the economy-wide elasticity of around 0.62, at the higher end of the range provided in the literature between 0.2 and 0.6. This implies that a 1% increase in the nominal interest rate decreases the transaction demand for money by 0.62%. Our findings about group-specific elasticities are that they are mostly rising with age and consumption class.

Fourth, we obtain measures of the welfare cost of inflation by introducing an unexpected permanent change in inflation in 2010 with an associated fiscal policy that reduces income tax rates to accommodate the additional revenue generated by higher inflation through seigniorage and the windfall gains from the redistribution of wealth. An increase in inflation from 2% to 5% costs 13% of one-year consumption.<sup>3</sup>

The use of a model with heterogeneous agents is important for the size and location of the welfare effects. From the point of view of age, there are large differences in who gains and who loses: the currently alive lose (they suffer from the distortion and also wealth loss with small benefits from the government windfall) while future generations gain some as their share of the government windfall is larger than the cost of the distortion (6). From the point of view of consumption class, the poor lose a lot more than the rich: 37.0% of 2010 consumption versus 5.6% for the poorest and richest quintiles. Moreover, the average money consumption ratio is 35% larger than average money holdings over average consumption. A representative agent model abstracts from age heterogeneity and therefore misses the age related asymmetry of the effects

<sup>2</sup> For instance, in the cross-section, old and poor households hold 10 times more money per unit of consumption than their young and rich counterparts; across age groups, money holdings per unit of consumption differ by a factor of 3.

<sup>3</sup> Looking at slightly different time periods with higher growth in aggregate money demand yields a slightly smaller estimate of financial innovation and higher welfare cost. See Appendix P.

of inflation. It also misses the heterogeneity in consumption, and in consumption to money holdings ratio which would result in an imputation of the aggregate cost of money holdings close to that suffered by households in the fourth quintile or even higher. The average cost of inflation across quintiles is 19% of one year of their consumption while a representative agent calculation would be below 14% (7).

A long list of studies measure the welfare cost of inflation with respect to money holdings. Lucas (2000) derives the cost of inflation using aggregate time series but points out the potential importance of distributional considerations across households. Mulligan and Sala-i-Martin (2000) highlight the importance of a fixed cost for non-money transactions by documenting the extensive margin of whether or not households have interest-bearing financial assets in addition to money. Attanasio et al. (2002)—using a unique Italian household-level data set with much richer information on cash holdings, cash transactions, and ATM usage—find that the cost of inflation varies considerably within the population but is small (0.1% of consumption or less). Alvarez and Lippi (2009) extend the Baumol-Tobin cash-inventory management model to incorporate precautionary cash holdings due to uncertainty. They find that the cost of inflation is about half of that in the Baumol-Tobin model. Our estimate of the welfare cost of inflation is generally larger than those of Attanasio et al. (2002) and Alvarez and Lippi (2009), mostly because of the definition of money. They use a narrower definition of money (e.g., currency), while, as in Lucas, we use a broader definition (e.g., currency plus chequable deposits). Since welfare costs should increase with the stock of money, broader definitions lead to larger welfare results.

Erosa and Ventura (2002) made an important contribution accommodating differences in cross-sectional money holdings when assessing the welfare cost of inflation. They extend the Aiyagari model to include cash-in-advance constraints and study the welfare distribution of changing inflation rates. They find that the distributional effects of inflation are large with low-income households disproportionately hurt by inflation. They abstract from cohort and age effects and from the inflation-induced windfall gains/losses associated with nominal positions of households and the government.

Our paper abstracts from some key frictions that could affect the cost of inflation. First, our analysis is on long-term inflation and abstracts from frictions linked to business cycles, such as nominal price rigidities and zero lower bound on nominal interest rates. The former could imply higher welfare costs of inflation due to inefficiencies associated with increased price dispersion and the latter lower costs due to the reduced frequency of hitting the zero lower bound. Second, we model money for transaction purposes but abstract from its role as a store of value in environments with uninsurable risk. Higher welfare costs would be the result of this additional incentive to hold money. Finally, we abstract from the additional cost of higher inflation that comes from increasing marginal tax rates on nominal income.

In Section 2, we use Canadian data to document how money holdings vary with age and consumption. Section 3 develops the life-cycle model of demand for money, and Section 4 maps it to the data. Section 5 evaluates welfare from changing inflation and Section 6 provides some sensitivity analyses around the baseline welfare findings. Section 7 concludes.

## 2. Money demand of Canadian households

The ratio of money holdings to consumption (MCR) is the centerpiece of our analysis. MCRs at the household level are drawn from the Canadian Financial Monitor (CFM) by Ipsos Reid, an annual survey data set containing information on household income, expenditure, and balance sheets that is available with consistency for the period since 2008. In order to construct household-level MCR across household groups, we pool CFM information over the 2008–2013 period and smooth out time variations.<sup>4</sup> In addition, we group households into six age-of-the-household-head groups,  $\{\leq 35, 36-45, 46-55, 56-65, 66-75, 76-85\}$  (which we refer to as ages 30, 40, 50, 60, 70 and 80). Within each age group, we further make five sub-groups based on their within-age-group consumption quintile. MCR are calculated as the ratio of the average money holdings and the average consumption of a given group.

Our notion of money includes cash and low-interest bank account balances, i.e., chequing, chequing/savings, and business accounts. This is a measure of the liquid assets that are used for transactions and whose real values are sensitive to inflation, as their nominal rates of return are typically lower than the nominal interest rates and are not adjusted for inflation. Thus, our definition of money is broader than cash in the wallet but narrower than a popular aggregate measure such as M1+ in Canada (equivalent to M1 in the United States) and close to the definition of aggregate money held by the household sector, which includes currency circulated outside banks and personal chequable deposits at banks. Consumption is defined as the household's sum of gross (annualized) monthly spending on non-durable goods, services, and durable goods but excludes expenses on housing services. Aggregate data on money and consumption from Statistics Canada complement these micro data. We apply a similar definition to construct aggregate consumption and use non-housing consumption by excluding actual and imputed rental fees for housing services from final consumption expenditures.

Fig. 1 displays the cross-sectional relation of consumption and MCR by age group. Two main facts arise: the MCR increases with age conditional on consumption, and this ratio decreases with consumption. The literature has documented the second fact (see Erosa and Ventura, 2002), suggesting that as household consumption increases, the fraction of consumption purchased with money becomes smaller or that of non-cash payment methods become more important. What has not been extensively studied is the age aspect of money demand, i.e., the first fact.

<sup>4</sup> Details on the CFM and a table of the money holdings are in Appendix A.

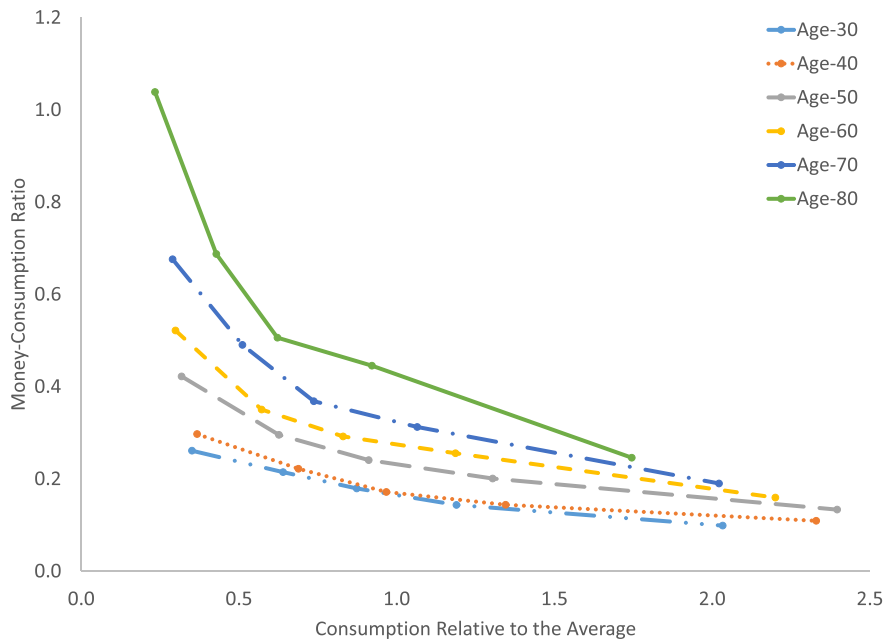


Fig. 1. Money-consumption ratio by age (Data source: CFM 2008–2013).

Why do older households have a higher money-to-consumption ratio than younger households? Is it just because they are older (age effect),<sup>5</sup> or is it because they were born at an earlier time, in a world with fewer financial instruments, and this shaped their ability to save in non-money financial instruments (cohort effect)? The answer matters. If it is an age effect, the current young households will increase money holdings as they age and no persistent change in aggregate money holdings will occur over time. But if it is a cohort effect, there will be a persistent decrease in aggregate money holdings over time without an increase with age.

Aggregate money holdings over time can shed some light on this question: if it is mostly the age effect, money holdings will not have come down over time, while if it is mostly a cohort or a time effect, then money holdings will have shrunk.<sup>6</sup> Using the consistent definition of money for transaction purpose (i.e., currency and personal chequable deposits) and annual non-housing consumption as from CFM, aggregate MCR has been in an upward trend rising from 0.22 in early 1980s to almost 0.5 in 2017. At the same time, the prime rate (i.e., a nominal interest rate that banks give to their best customers for credit and hence the opportunity cost of holding money) has been declining from around 20% to 3% over the same period. This indicates that, even in the presence of financial innovation, the interest rate elasticity of money demand has played a central role in shaping the aggregate quantity of money.<sup>7</sup>

The financial innovation that we model here as a cohort effect could be the result of a pure time effect, that is, changes that affect everybody at each point in time. As discussed in more detail in Section 4, cohort and time effects cannot be separately identified as secular time trends (provided a suitable adjustment of age effects), so we circumscribe time effects as the result of observable time specific variation while all financial innovation is modeled as a cohort effect.

Our objective of separating age and cohort effects in the determination of the higher money holdings of older households is compounded by the substitution elasticity. Fortunately, to disentangle these effects, we can use a structural model capable of making predictions simultaneously about the age distribution of money holdings and its evolution over time when nominal interest rates, inflation and fiscal variables also change over time. In the next section, we describe such a model.

### 3. The model

Equal-sized overlapping generations of agents use money and credit to purchase consumption. Within each cohort there are types (i.e., consumption classes) that determine the endowments and the age profile of the desired timing of con-

<sup>5</sup> It could be that older households face more difficulty processing complex financial information associated with the use of credit technology. The literature contains some empirical evidence that older households pay fewer visits to bank branches (see Mulligan and Sala-i-Martin, 2000) and use cash more and credit cards less for grocery transactions (see Klee, 2008) than younger households.

<sup>6</sup> Appendix B discusses the macro data series used in this section in detail.

<sup>7</sup> Appendix C explores the possibility that the increase in personal chequable deposits is due to regulatory changes without a change in the nature of money demand. We find that regulatory changes did not cause an increase in the deposits. In addition, Appendix H provides supporting evidence of financial innovation using CFM.

sumption. Households can purchase consumption using some combination of a cash-in-advance constraint and a credit-transaction technology that depends on the age and cohort of the household. We now turn to the details.

### 3.1. Model description and equilibrium

Each household, of which there is a continuum, is indexed by its age  $i \in \{0, \dots, I\}$ , its type  $j \in \{1, \dots, J\}$  and its cohort  $h$ , or period of birth. Households supply labour exogenously and have a fixed labour endowment. The fraction of consumption purchased with money is subject to a cash-in-advance constraint. Purchasing by credit involves transaction costs. This generates a trade off between using money and using credit for purchases because holding money precludes gaining interest, which is particularly taxing in periods of high inflation or high interest rates. The credit transaction technology is a function of the fraction of consumption purchased with credit,  $s$ , and is given by<sup>8</sup>

$$\xi_{hi}(s) = \int_0^s \gamma_i \cdot \eta^h \cdot \left(\frac{x}{1-x}\right)^{\theta_i} dx, \tag{1}$$

where  $\gamma_i > 0$ ,  $\theta_i \geq 1$  and  $\eta > 0$ . This function is convex, strictly increasing in  $s$  for all  $s \in [0, 1)$  and independent of the level of consumption. Thus, the credit technology exhibits increasing returns to scale: the credit transaction cost per unit of consumption decreases with consumption given  $s$ . This assumption helps generate the second fact: the MCR decreases with consumption. Both  $\gamma_i$  and  $\theta_i$  vary with age to replicate both the different levels and the different slopes displayed in Fig. 1 and are responsible for the age effects. Cohort effects are captured by  $\eta$  in Eq. (1). A value of  $\eta < 1$  implies, ceteris paribus, that credit becomes less costly over time (as  $h$  increases) and hence that the demand for money declines. This implies that cohort effects incorporate secular changes over time, i.e., financial innovation. The crucial difference between age and cohort effects is that the former predicts that the young households from any cohort will use more money and less credit as they age, while the latter will make the money-credit choice differ across cohorts for any given age. An important property of the transaction cost in 1 is the independence of age and cohort parameters. The next section discusses the identification of these parameters.

Cohort  $h$ , age  $i$  and class  $j$  households choose consumption,  $c_{hij}$ , non-money assets,  $a_{hij}$  and real money holdings  $m_{hij}$ . The sum of cohort and age indices,  $h + i$ , defines the time index  $t$  such that  $t = h + i + \kappa$ , where  $\kappa$  is a reference time period. The cohort  $h$  and class  $j$  household solves

$$\max_{\substack{(c_{hij}, a_{hij}, m_{hij}) \\ a_{h,i+1,j}, m_{h,i+1,j}}} \sum_{i=0}^I \beta_{ij} \frac{c_{hij}^{1-\sigma} - 1}{1-\sigma} \quad s.t. \tag{2}$$

$$c_{hij} (1 - s_{hij}) \leq m_{hij}, \tag{3}$$

$$c_{hij} + q_t \xi_{hi}(s_{hij}) + a_{h,i+1,j} + (1 + \pi_{t+1}) m_{h,i+1,j} \leq [1 + r_t(1 - \tau_t)] a_{hij} + m_{hij} + (1 - \tau_t) w_t z_{ij} \quad \forall i < I, \tag{4}$$

$$c_{hlj} + q_t \xi_{hi}(s_{hlj}) \leq [1 + r_t(1 - \tau_t)] a_{hlj} + m_{hlj} + (1 - \tau_t) w_t z_{lj}, \tag{5}$$

$$m_{h,0,j} = \underline{m}, \quad m_{hij} \geq 0 \quad \text{and} \quad t = h + i + \kappa, \tag{6}$$

where  $q_t$  is the price per unit of credit-transaction service,  $\pi_{t+1}$  the inflation rate from time  $t$  to  $t + 1$ ,  $w_t$  the wage rate,  $r_t$  the interest rate, and  $\tau_t$  the tax rate on income, all at time  $t$ .<sup>9</sup> The labour endowment,  $z_{ij}$ , is assumed to be independent of cohorts or time. To have an interior solution for money holdings even for the youngest agents, we assume that newborns are endowed with a small amount of initial money holdings,  $\underline{m}$ .

Condition (3) is the cash-in-advance constraint. Given its current money holdings, a household chooses total consumption and associated amount of credit. Conditions (4) and (5) are the budget constraints for households aged  $i < I$  and  $i = I$ , respectively. We pose age- and class-specific discount factors or utility weights to capture the age profile of consumption for each class of households without attempting to understand the origins of such consumption patterns.<sup>10</sup> Assets  $a$  are a composite of five categories: real assets and four nominal positions. These are described in detail in Section 5.

<sup>8</sup> This specification is an extension of that used in Dotsey and Ireland (1996) and Erosa and Ventura (2002), with age-specific parameters and a parameter capturing cohort effects.

<sup>9</sup> Following Erosa and Ventura (2002), we assume  $q_t = w_t$ , implying that credit-transaction costs are specified in terms of time costs.

<sup>10</sup> Age- and class-specific discount factors allow us to precisely replicate the relative consumption levels across households to match the data. It could have also been accomplished with a form of equivalence scales.

There is also a government that, every period, spends, has assets and debts, supplies money, collects tax and seigniorage revenues, and faces the following budget constraint:

$$G_t = -A_{t+1}^G + (1 + r_t)A_t^G + B_{t+1} - (1 + r_t)B_t + (1 + \pi_{t+1})M_{t+1} - M_t + \tau_t(r_t K_t + w_t Z), \tag{7}$$

where  $G_t$  is government spending,  $A_t^G$  are government assets,  $M_t$  aggregate money supply,  $K_t$  aggregate productive capital,  $Z(\equiv \sum_{ij} z_{ij})$  aggregate labour endowments,  $r_t$  the real interest rate,  $\pi_t$  the inflation rate,  $w_t$  the wage rate, and  $\tau_t$  the income tax rate.  $B_t$  indicates the government’s nominal positions in the same categories that we use for households. All variables are expressed in real terms. Seigniorage is  $(1 + \pi_{t+1})M_{t+1} - M_t$ , a specification motivated by our assumptions that all money is held by households and that the central bank perfectly controls inflation. The behaviour of the government is exogenous and it replicates the observed one. When we consider permanent inflation increases, the government receives more seigniorage as well as one-time windfall gains on their non-money net nominal positions. In the baseline, the government policy permanently reduces the income tax rates to allocate the seigniorage and the windfall gains.

Regarding the nominal positions of the sectors, the foreign sector has nominal positions with respect to assets and liabilities in Canada. The behaviour of the foreign sector is taken as exogenous and its nominal positions over the four categories are given by  $a_t^F$ , where the following accounting identity holds by each of the four categories at  $t$ :  $\sum_{h(t)i(t)j} a_{hij}^N = B_t + a_t^F$ .<sup>11</sup>

The assumption of a small open economy closes the model. We pose a Cobb-Douglas production technology,  $F(K_t, Z) = K_t^\alpha Z^{1-\alpha}$ , that uses capital and labour with a capital depreciation rate of  $\delta$  with the exogenous time-varying real interest rate,  $r_t$ , determined in the global capital market to determine wages given the local labour input, is assumed to be non-tradable and only domestically supplied. In equilibrium, these assumptions imply time-varying wage rates that are consistent with the rest of the economy. The quantity of money and the tax rate are also determined in equilibrium. Specifically, the government supplies whatever quantity of money households demand, and the level of taxation is determined by the government budget constraint. The nominal interest rate is implied by the Fisher equation. The exogenous macroeconomic variables in this economy are  $G_t$ ,  $A_t^G$ ,  $B_t$ ,  $\pi_t$  and  $r_t$ .<sup>12</sup> Finally, there is no growth, because the demand for money is relative to consumption units.

### 3.2. Characterization

The solution of the household problem implies the following relation<sup>13</sup>:

$$\frac{m_{hij}}{c_{hij}} = 1 - s_{hij} = \frac{1}{1 + [\tilde{R}_t c_{hij} / (q_t \gamma_i \eta^h)]^{1/\theta_i}}, \tag{8}$$

where  $\tilde{R}_t \equiv (1 + \pi_t)[1 + r_t(1 - \tau_t)] - 1$  denotes the after-tax nominal interest rate. Note that money demand increases with age if  $\gamma_i$  also increases with age. The impact of  $\theta_i$  on MCR depends on the relative importance of money and credit in purchasing consumption. If the share of money in purchasing consumption is less than that of credit, the MCR goes up with  $\theta_i$ . In addition, money holdings are lower with higher financial innovation (e.g., lower  $\eta$  or a larger cohort effect). Furthermore, money demand goes down over time, i.e., as new cohorts come into the economy.<sup>14</sup> Hence, the model qualitatively allows both the age and the cohort effects to account for the increase in money holdings with age, i.e., the first fact discussed in Section 2.

## 4. Mapping the model to data

Money demand depends on the cost of credit, which may depend on age, cohort and time, i.e.,  $t = h + i + \kappa$ . Our model is subject to the classic identification issue over age, cohort and time effects, and hence we do not identify these three effects. Instead, we embed all financial innovation as a cohort effect and pose time effects only in the form of the response to observables (interest rates, fiscal policy). Specifically, our calibration relies on the following two key assumptions to identify  $\eta$ ,  $\gamma_i$ 's and  $\theta_i$ 's: (1) a time trend in money demand comes from the secular change in the credit cost 1, dictated by the term  $\eta^h$ , and (2) the cost non-linearly depends on age by age-indexed parameters  $\gamma_i$ 's and  $\theta_i$ 's. In the first assumption, as the cohort index  $h$  increases for a younger generation,  $\eta < 1$  implies a secular decline in credit costs, i.e., financial innovation. With this assumption, financial innovation is realized across cohorts and is fixed within each cohort. We call these ‘‘cohort effects.’’ Appendix G elaborates the identification issue between the cohort and time effects in our model. It provides a proposition that there is an alternative parametrization of the secular change in credit cost as a function of the time index that leads to the same answers (household decisions and welfare costs) as in the cohort-effect specification. Hence, what we call the cohort effect can be more generally interpreted as the secular effects of financial innovation via either cohort or time effects. The second assumption allows the model to replicate the observed money-consumption ratios (MCRs) across

<sup>11</sup>  $h(t)$  and  $i(t)$  indicate the cohort and age, respectively, active at  $t$  based on  $t = h + i + \kappa$ .

<sup>12</sup> Appendix D provides the formal definition of the equilibrium.

<sup>13</sup> See Appendix E for the derivation details.

<sup>14</sup> Appendix F has details on this point.

household groups. We call the impact of these age-indexed parameters on MCRs “age effects.” Finally, the fluctuations in macroeconomic conditions (e.g., nominal interest rates) directly impact money demand, which we call “time effects.”

These assumptions allow the identification of the credit cost parameters  $\gamma_i$ ,  $\eta$  and  $\theta_i$  as follow.  $\gamma_i \eta^h$  (i.e.,  $\gamma_i$  and  $\eta$  together) and  $\theta_i$  vary to match the cross-sectional data on MCRs by household age and class in 1 to their respective counterparts from the model in 2010 (both level and slope), while  $\eta$  varies to jointly match the change in the aggregate MCR between 2000 and 2010. We think that the latter helps identify financial innovation,  $\eta$ , while the cross-sectional patterns, conditional on financial innovation modeled as a cohort effect, determine the age effects. The parameter estimates give us the secular trend in the credit cost and hence that in money demand. The deviations from the trend—what we call time effects—are due to the dynamics of the macroeconomic environment. This includes the after-tax nominal interest rates ( $\bar{R}_t$ ) and the wage rates ( $w_t$ ), which are equilibrium objects and do not have free parameters directly controlling them.

The actual change in the aggregate MCR from 2000 to 2010, is a 30% increase,<sup>15</sup> but because of the large reduction in nominal interest rates, such an increase together with the cross-sectional patterns in money holdings point to substantial financial innovation.<sup>16</sup> The actual estimation of the parameters involve solving a fixed point with standard iterative methods (the parameters that yield the desired moments have to be found along with the tax rate that balances the budget of the government).

#### 4.1. Calibration details

A model period is 10 years. There are five equal-sized income groups (i.e., the classes),  $J = 5$ . Households live seven periods, indexed by  $i = 0, 1, \dots, 6$ , which correspond to households aged 25 or younger, 26–35, 36–45, 46–55, 56–65, 66–75 and 76 or older, respectively. The age  $i = 0$  households receive an exogenous amount of money,  $\underline{m}$ , which we set to 0.01% of the average consumption of all households or 0.024% of the average consumption of the age  $i = 0$  households. Our calibration and welfare analysis are based on the six oldest age groups ( $i = 1, \dots, 6$ ), as we require endogenous MCRs when the model is brought to the data.

The households' (inverse of the) inter-temporal elasticity of substitution,  $\sigma$ , is 2. Labour endowments of households are determined to replicate the age profile in labour earnings from the 2008–2013 CFM with a normalization that their present value of life-cycle endowments is the same as that of consumption for the period 2000 in the model.<sup>17</sup> To get wages, we pose an annual capital depreciation rate of 0.07 and a labour share of 0.65.

The rest of the parameters are jointly determined in equilibrium by solving the model and matching a set of moments. In total, there are 42 parameters and 42 moments that are calibrated by solving the equilibrium of the model: 29  $\beta_{ij}$ 's, 6  $\gamma_i$ 's, 6  $\theta_i$ 's and  $\eta$ . A discussion to associate a specific parameter with a particular target is helpful in better understanding the potential link between parameters and data for the identification. The job of replicating the hump-shaped consumption profiles over the life cycle of the various income groups is handled by households' type-discount factors,  $\beta_{ij}$ , perhaps better thought of as consumption-age weights. There are 30  $\beta_{ij}$ 's in total, one of which is a normalization.

The credit transaction technology has three sets of parameters,  $\gamma_i$ 's,  $\theta_i$ 's and  $\eta$ . The first two capture age effects, including the steepness at which money and credit substitute each other over consumption, and  $\eta$  captures the cohort effects. Average MCRs for each age group  $i$  (i.e.,  $\frac{1}{5} \sum_{j=1}^5 \frac{m_{ij}}{c_{ij}}$ ) in 2010, are mainly responsible for the corresponding  $\gamma_i$ . The  $\theta_i$ 's are mostly

responsible for the slopes of the MCR curve over consumption for each age group  $i$  (i.e.,  $\frac{1}{4} \sum_{j=2}^5 \frac{m_{ij}/c_{ij} - m_{i,j-1}/c_{i,j-1}}{c_{ij} - c_{i,j-1}}$ ) from Fig. 1.

Finally, the role of making aggregate changes in the MCR of all households between 2000 and 2010 is what helps identify  $\eta^h$  separately from the  $\gamma_i$ 's. We target a 30% increase in aggregate MCR observed between the two years. The aggregate change in money demand between 2000 and 2010 is also determined by the changing macroeconomic conditions, i.e., what we call the time effects. Overall, our model simulation for calibration spans the experience of Canada over the last 80 years, starting from a steady state in 1940 and facing the realized interest rates, inflation and taxes since then. Post-2010 we maintain the 2010 inflation and nominal interest rates with taxes, balancing the budget period by period all the way up to 2170, well past the time when all the living generations in 2010 will have disappeared. The model is closed by assuming it was in a stationary equilibrium prior to 1940 and goes to another after 2170, without any cohort effects after 2070, the exit time of the youngest cohort (these details are not quantitatively important).<sup>18</sup>

The values of the exogenous macroeconomic variables are in the upper panel of Table 1. Inflation rates are the 10-year average of annual changes in the CPI in Canada. Nominal interest rates are the prime rates. The table also shows the wage rate consistent with the exogenous real interest rates. Government variables are expressed as a percent of annual GDP and obtained from the national accounts. Government assets correspond to  $A_t^G$  and debts to  $B_t$ . Its revenues and expenditures are  $(1 + \pi_{t+1})M_{t+1} - M_t + \tau_t(r_t K_t + w_t Z) + r_t^G A_t^G$  and  $G_t + r_t B_t$ , respectively.

<sup>15</sup> The data are from Statistics Canada. See Appendix B for details. Furthermore, the period of this increase coincides with that of financial innovation. Appendix I documents several measures of financial innovations on credit during this period, including the declining cost of credit cards.

<sup>16</sup> Appendix H provides evidence supporting the presence of financial innovation using the CFM data.

<sup>17</sup> This ensures that the model will be able to capture the observed consumption dispersion in the data. The resulting endowment profiles display a hump-shape over the life cycle with the level of the richer households higher than their poorer peers. They are shown and discussed in Appendix J.

<sup>18</sup> Solving the model for such a long period is necessary to capture the potential impact of cohort and time effects on household decisions over the life cycle and the resulting implications for the equilibrium of the economy.

**Table 1**  
Macroeconomic variables.

	1940	1950	1960	1970	1980	1990	2000	2010
<i>Exogenous variables:</i>								
Annual inflation (%)	1.55	4.64	2.22	2.74	8.07	5.99	1.99	2.02
Annual nom. int. rate (%)	5.12	4.69	4.98	6.36	9.57	12.72	7.02	4.48
Wage rate	0.384	0.525	0.411	0.384	0.461	0.312	0.345	0.421
<i>Government variables (% of annual GDP)</i>								
Expenditure	9.13	23.91	15.74	15.28	18.53	21.85	19.85	15.52
Revenue	7.32	17.86	15.98	15.24	16.31	16.81	17.92	15.47
Debt	76.34	102.65	62.02	50.33	41.82	66.39	70.69	55.17
Asset	27.36	59.78	51.61	44.53	24.30	14.18	14.88	21.81
<i>Endogenous variables:</i>								
Tax rate (%)	6.73	26.56	18.53	16.75	21.53	16.34	19.47	19.28
Aggregate MC ratio (%)	30.36	43.65	37.21	28.55	22.07	11.96	18.23	23.88

**Table 2**  
Calibration results.

Parameter	Value	Target	Data	Model
$\gamma_1$	0.0021	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{1,j}$	0.1796	0.1786
$\gamma_2$	0.0042	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{2,j}$	0.1889	0.1885
$\gamma_3$	0.0046	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{3,j}$	0.2586	0.2577
$\gamma_4$	0.0063	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{4,j}$	0.3160	0.3146
$\gamma_5$	0.0107	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{5,j}$	0.4076	0.4058
$\gamma_6$	0.0184	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{6,j}$	0.5849	0.5838
$\theta_1$	1.9619	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{1,j} / \Delta c_{1,j}$	-0.1195	-0.1194
$\theta_2$	1.7650	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{2,j} / \Delta c_{2,j}$	-0.1308	-0.1307
$\theta_3$	1.7978	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{3,j} / \Delta c_{3,j}$	-0.1917	-0.1908
$\theta_4$	1.7055	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{4,j} / \Delta c_{4,j}$	-0.2626	-0.2632
$\theta_5$	1.5263	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{5,j} / \Delta c_{5,j}$	-0.4193	-0.4198
$\theta_6$	1.3763	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{6,j} / \Delta c_{6,j}$	-0.7968	-0.7912
$\eta$	0.7612	$\frac{m_{2010} / c_{2010}}{m_{2000} / c_{2000}}$	1.30	1.31

#### 4.2. Calibration results

The lower panel of Table 1 presents the two main endogenous macroeconomic variables, the income tax rates that balance the government budget each period, and the aggregate MCRs, obtained from the household decisions and aggregating them. Note the negative relation between the MCRs and the nominal interest rates, driven by the negative nominal interest rate elasticity of money demand.

Table 2 shows the parameter values for the  $\gamma_i$ 's,  $\theta_i$ 's and  $\eta$ . The cohort effect is  $\eta = 0.7612$ , indicating that the cost of credit transactions decline by 24% for each 10-year cohort (about 2% yearly). The age effect parameters,  $\gamma_i$ , monotonically increases with age, as should be expected, while the elasticity parameters,  $\theta_i$ , almost monotonically decline. Overall, it implies that the dependence on money for consumption is increasing with age even after accounting for the cohort effect. The table also reports the average MCs and their slopes in both model and data.<sup>19</sup> shows the resulting MCRs over class separately for each household age group against those from the data. As required, the model matches the targets.<sup>20</sup>

#### 4.3. Nominal interest rate elasticity of money demand

The model generates an analytical expression for the nominal interest rate elasticity of money demand as follows, for each  $\{h, i, j\}$  type:

$$\frac{\partial (m_{hij}/c_{hij})}{\partial R_t} \frac{R_t}{m_{hij}/c_{hij}} = -\frac{1}{\theta_i} \cdot \left(\frac{\tilde{R}_t c_{hij}}{q_t \gamma_i \eta^h}\right)^{1/\theta_i} \cdot \left[1 + \left(\frac{\tilde{R}_t c_{hij}}{q_t \gamma_i \eta^h}\right)^{1/\theta_i}\right]^{-1} \cdot \frac{1 + \tilde{R}_t}{\tilde{R}_t} \cdot \frac{R_t}{1 + R_t},$$

<sup>19</sup> Appendix J provides a table with the values of calibrated  $\beta_{ij}$ 's. Appendix K displays the life cycle dynamics of household decisions by the middle-class households active in 2010.

<sup>20</sup> Appendix L disentangles the across-age variation of MCRs in Fig. 1 between the age and cohort effects. A comparative-statics analysis that sets  $h = 1$  for all cohorts, shutting down the cohort effects, implies that the cohort effects account for 53.1% of the variation between the average MCR of the youngest and that of the oldest, while the age effects account for the rest. The appendix also discusses the contribution of the income effects.



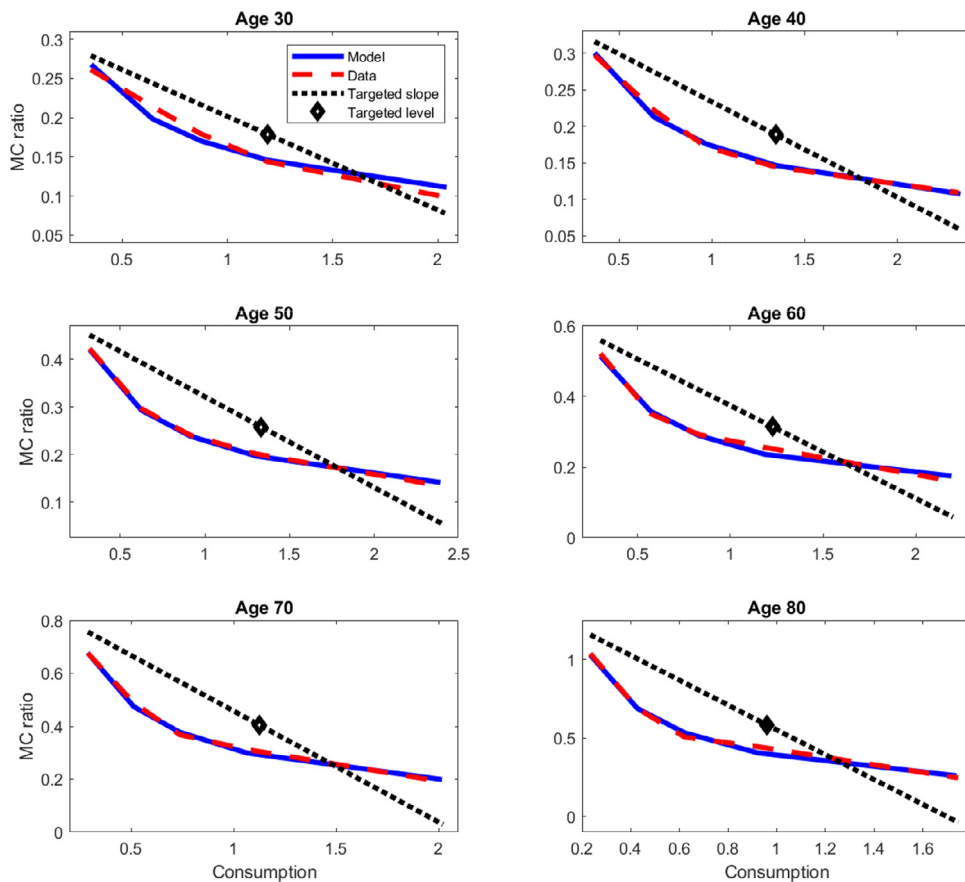


Fig. 2. Comparison of data and calibrated money-consumption ratios.

where  $R_t$  is the nominal interest rate, i.e.,  $R_t = (1 + \pi_t)(1 + r_t) - 1$ .<sup>21</sup> The model simulation with calibrated parameter values directly provides the estimates of the elasticity by household type and time. The average interest rate elasticity of money demand across household types and time is around 0.62 and lies at the higher end of the estimates in the literature. Across household types, our finding is that the elasticity increases mostly with age and class. To put it differently, the older or richer the household is, the more sensitive it is to changes in nominal interest rates. For instance, the time-averaged nominal interest rate elasticity facing old households is about 0.76 compared with 0.54 for young households, a difference of more than 0.2.

#### 4.4. Validation of the model: non-targeted moments

The model-generated interest rate elasticity of money demand that is in line with the literature from the previous section serves as one validation of the model through non-targeted moments. We also compare other non-targeted moments in the model against their data counterparts to check the validity of the model. The model only targeted for each age group the MC averages and slopes across consumption groups. In Fig. 2, all the MC-ratio curves are plotted against the model to see how well these are captured by the model. The diamond and the slope of the black dotted line in each panel display these moments.

We also present a measure of financial innovation using synthetic 5-year cohorts of households in CFM. Average MCR across these cohorts in 2013 declined by 0.67% relative to that in 2008 despite a large decrease in the nominal interest rate by 2.75 percentage points during the period. In addition, using the information on consumer credit and a panel sub-sample in CFM covering the period 2000–2013, we estimate how the secular financial innovation (working through the cohort or the time effect) impacts the use of consumer credit. The finding supports that the use of consumer credit increases by 0.9% as the cohort becomes younger by one year. As the use of credit and money demand would negatively correlate, this finding also suggests a decline in money holdings. All of this is independent evidence of large financial innovation.<sup>22</sup>

<sup>21</sup> See Appendix M for more details.

<sup>22</sup> Appendix H provides more discussion. In addition, Appendix I discusses other evidence regarding the decrease of credit card cost.

**Table 3**  
Nominal positions by sector, % of GDP.

Nominal category	Household	Government	Foreign
Short	11.09	−7.11	−3.98
Mortgage	−17.52	3.48	13.46
Long	16.58	−26.11	9.53
Pension	7.66	−2.79	−4.87

**Table 4**  
Nominal positions by household type, % of own net worth.

Nominal category	Households by age group					
	80	70	60	50	40	30
	Poor					
Short	0.14	0.13	0.52	−0.31	−1.12	1.56
Mortgage	6.17	3.31	−1.86	−7.05	−28.03	−62.97
Long	22.11	15.88	12.42	10.08	5.31	−4.01
Pension	0.09	2.09	1.88	5.66	3.66	11.09
	Poor-Middle					
Short	−0.41	0.34	−0.01	0.10	−0.84	1.02
Mortgage	6.39	2.58	−1.43	−8.75	−31.09	−79.06
Long	21.60	16.00	12.36	9.26	4.20	−3.74
Pension	−0.56	0.47	4.66	4.89	4.50	4.58
	Middle					
Short	−0.29	−0.84	−0.73	−0.14	−1.54	0.25
Mortgage	6.04	3.18	−0.08	−8.3	−30.29	−73.71
Long	17.76	14.81	11.69	8.26	4.68	−3.03
Pension	−0.87	0.27	4.52	5.45	4.77	6.17
	Middle-Rich					
Short	−1.13	−0.60	−0.74	−0.58	−1.79	−0.39
Mortgage	7.42	4.78	1.82	−6.75	−23.34	−79.28
Long	18.62	15.17	12.21	8.21	3.91	−5.88
Pension	−0.69	−0.13	4.41	5.01	3.55	6.41
	Rich					
Short	−1.79	−1.19	−1.12	−0.90	−1.13	−0.77
Mortgage	8.81	7.05	2.98	−3.15	−19.59	−54.70
Long	17.44	15.84	12.76	8.13	4.41	−2.20
Pension	−1.00	−0.79	1.91	3.71	1.60	3.46

## 5. Welfare implications of a permanent switch to 5% inflation

To find the welfare implications of a surprise permanent increase in inflation from 2% to 5%, we have to answer three questions. First, what is the size of the windfall to the government of the higher inflation due to the capital loss associated with the reduced value of the nominal positions that households have (Section 5.1)?<sup>23</sup> Second, how will the government use such a windfall? We consider a policy where there is a permanent income tax cut while maintaining the government budget balance in the present value from 2010 and onward.<sup>24</sup> Third, what is the response of households to the increase in the costs of transacting for consumption (and the associated change in the relative costs of the available transaction technologies) (Section 5.2)? The inflation shock leads not only to increased transaction costs of using money but also induces changes in the seigniorage revenue to the government, implying that the windfall and the tax rate cut the government sets have to be determined simultaneously with the effects of that policy on households.

### 5.1. Changes in the real value of non-money nominal positions

In addition to money, households hold, directly and indirectly, other nominal assets and debts. The welfare costs of inflation also depend on the change of the real value of these nominal positions. Our welfare analysis incorporates the inflation-induced redistribution by calculating the change in the aggregate value of nominal positions, then allocating the change into the sectors by their share of holdings and, finally, assigning this change across household groups according to their ownership shares.<sup>25</sup>

Tables 3 and 4 show the nominal positions of economic sectors and household groups in four categories: short-term (less than a year), mortgages, long-term (more than a year) and pensions. Short-term nominal assets exclude money since

<sup>23</sup> See Doepke and Schneider (2006b), Meh and Terajima (2011) and Adam and Zhu (2016) for similar studies for the United States, Canada and Euro area countries, respectively.

<sup>24</sup> Appendix N discusses the results of other policies.

<sup>25</sup> Appendix O discusses the steps followed in this analysis in more detail.

**Table 5**

Redistribution of wealth from 3 p.p. increase in inflation, % of own net worth.

Class	Households by age group					
	80	70	60	50	40	30
Poor	-3.82	-3.08	-2.22	-3.48	1.02	-0.08
Poor-Middle	-3.68	-2.76	-2.94	-2.59	0.48	6.83
Middle	-2.92	-2.48	-2.89	-2.69	0.17	5.08
Middle-Rich	-3.15	-2.61	-3.15	-2.68	0.08	6.09
Rich	-2.95	-2.76	-2.73	-2.64	0.44	4.15

**Table 6**Welfare cost in percent of own consumption ( $\lambda_{hj}^c \cdot 100$ ) at 5% inflation.

Class	Age in 2010					
	80 ( $h = 1$ )	70 ( $h = 2$ )	60 ( $h = 3$ )	50 ( $h = 4$ )	40 ( $h = 5$ )	30 ( $h = 6$ )
	Baseline Calibration					
Poor	2.89	2.45	1.97	2.01	0.49	0.51
Poor-Middle	2.31	1.67	1.94	1.38	0.25	-0.05
Middle	1.64	1.39	1.61	1.14	0.16	-0.09
Middle-Rich	1.64	1.24	1.46	0.92	0.08	-0.17
Rich	1.07	0.85	0.81	0.58	-0.08	-0.19
	Age-Specific $\beta_i$ 's					
Poor	2.80	2.47	1.89	1.90	0.51	0.51
Poor-Middle	2.20	1.63	1.85	1.33	0.26	-0.03
Middle	1.64	1.36	1.58	1.13	0.16	-0.09
Middle-Rich	1.71	1.26	1.47	0.92	0.08	-0.16
Rich	1.22	0.93	0.84	0.61	-0.09	-0.20
	Common $\theta$					
Poor	3.09	2.53	2.06	2.10	0.50	0.52
Poor-Middle	2.44	1.82	2.04	1.44	0.26	-0.04
Middle	1.74	1.53	1.72	1.20	0.17	-0.09
Middle-Poor	1.69	1.36	1.60	0.98	0.09	-0.17
Rich	1.08	0.96	0.89	0.61	-0.08	-0.20
	Survival risk with accidental bequests					
Poor	2.74	2.39	1.96	2.00	0.52	0.52
Poor-Middle	2.20	1.65	1.91	1.37	0.28	-0.02
Middle	1.58	1.36	1.59	1.13	0.18	-0.06
Middle-Poor	1.58	1.22	1.44	0.91	0.10	-0.14
Rich	1.03	0.84	0.80	0.58	-0.06	-0.17
	Constant wage					
Poor	2.84	2.56	2.19	2.22	0.68	0.70
Poor-Middle	2.27	1.74	2.07	1.51	0.37	0.05
Middle	1.61	1.42	1.71	1.24	0.25	-0.02
Middle-Rich	1.60	1.26	1.54	0.99	0.15	-0.12
Rich	1.04	0.85	0.83	0.61	-0.04	-0.18

the model explicitly captures it through household decisions. These categories are chosen because the term of the underlying asset is crucial in the calculation of the inflation-induced redistribution (determined by the difference between the price level today and that at the end of the term). The positions reported are net (assets minus liabilities). Those of the business sector are allocated to other sectors and to household groups based on the ownership of shares of businesses. The household sector is a net debtor in mortgages and it has a positive asset position in the other categories. The government is a net debtor in all but mortgages. The foreign sector has a positive position in mortgages and long-term assets and a negative position in the rest, many of which are from the indirect positions held through businesses. Household groups have mortgage debt during young and middle-aged periods that later turn positive via their indirect holding of those assets through businesses. Nominal pension assets, especially for the young, have the longest term and hence are most exposed to inflation. The magnitude of the gains/losses is the difference in the price levels between the period of the inflation shock and the period in which the nominal payment occurs, so the longer the term of the position (e.g., pension for the young), the higher the gains or losses.

A permanent inflation shock of 3 percentage points reduces the wealth of the household sector by 6.3% of 2010 GDP and increases the government wealth holdings by 5.8% and those of the foreign sector by 0.5%. The foreign sector makes a small gain despite its loss in the values of mortgage and long-term assets. This is because of its large exposure to pension liabilities and the decline in their value due to this category's long-term exposure to higher inflation. Table 5 summarizes the results of the wealth redistribution across households. Young households gain, except the young and poor ones, due to

**Table 7**  
Welfare cost in aggregate ( $W$ ) and by class ( $W_j$ ), in percent of 2010 annual consumption of the respective group.

$W_j$					$W$
Poor	Poor-Middle	Middle	Middle-Rich	Rich	
Baseline					
37.00	22.94	17.37	13.50	5.58	13.33
Age-specific discount factor					
36.21	22.59	15.98	13.53	5.73	13.28
No age effects in $\theta$ , $\theta_i = \theta$					
37.96	23.15	17.27	7.82	4.47	11.52
Survival risk with accidental bequests					
39.44	24.72	18.76	14.64	6.37	14.52
Constant wage rates					
43.79	27.37	20.68	16.04	7.23	16.11

their mortgage asset holdings, and the middle-aged households lose the most due to their holdings of long-term assets and pensions. Across classes, the poor tend to lose more than the rich.

### 5.2. Welfare measures of inflation

The consumption equivalent variation indexed by cohort  $h$  and class  $j$  measures the welfare loss and is expressed as a constant proportion  $\lambda_{hj}$  of life-cycle consumption. Let  $V_{hj}^0 = \sum_{i=1}^6 \beta_{ij} u(c_{hij}^0)$  be the lifetime utility of  $\{h, j\}$  households, where  $c_{hij}^0$  is the consumption obtained in the baseline. Let the values under policy  $\ell$  be  $V_{hj}^\ell = \sum_{i=1}^6 \beta_{ij} u(c_{hij}^\ell)$ , where  $c_{hij}^\ell$  is consumption under policy  $\ell$ .<sup>26</sup> Then,  $\lambda_{hj}^\ell$  solves  $\sum_{i=1}^6 \beta_{ij} u\left[\left(1 + \lambda_{hj}^\ell\right) c_{hij}^\ell\right] = V_{hj}^0$ .<sup>27</sup>

We also calculate aggregate welfare measures. We change how we report this measure shifting to a stock measure. The reason is that aggregation is not straightforward since the  $\lambda_{hj}$ 's are expressed relative to each group's consumption. Hence, the aggregate welfare measure sums the changes in the consumption of individual households implied by the respective  $\lambda_{hj}$  and discounts them using real interest rates to their 2010 values. They are expressed as a fraction of 2010 consumption (a one-year loss, not a recurrent one). The measure includes households that are active in 2010 to those born in 2060. To see the details, define the changes in units of 2010 consumption as  $\Delta c_{hij}^\ell \equiv \lambda_{hj}^\ell \cdot c_{hij}^\ell / (1 + r_{2010})^{h+i-1-t_{2010}} \quad \forall h, i \in \{1, \dots, 6\}, h + i - 1 \geq t_{2010}$ , where the last inequality restricts the relevant consumption to be that in 2010 or later and  $t_{2010} = 6$ . Here,  $r_{2010}$  is the 10-year real interest rate in 2010, which is held constant afterwards. We define three measures:

$$W_h \equiv \left( \sum_{ij} \Delta c_{hij}^\ell \right) / C_{2010}^h, \quad W_j \equiv \left( \sum_{hi} \Delta c_{hij}^\ell \right) / C_{2010}^j, \tag{9}$$

$$W \equiv \left( \sum_{hij} \Delta c_{hij}^\ell \right) / C_{2010}, \text{ where} \tag{10}$$

$$C_{2010}^h \equiv \sum_{ij} [c_{hij}^0 / (1 + r_{2010})^{h+i-1-t_{2010}}], \quad C_{2010}^j \equiv \sum_{hi} c_{hij}^0 \quad \forall h, i \in \{1, \dots, 6\}$$

such that  $h + i - 1 = t_{2010}$ , and

$$C_{2010} \equiv \sum_{hij} c_{hij}^0 \quad \forall h, i \in \{1, \dots, 6\} \text{ and } j \in \{1, \dots, 5\}, h + i - 1 = t_{2010}.$$

$C_{2010}^h$  is the sum of the life-cycle consumption of cohort  $h$  discounted to 2010, and  $C_{2010}^j$  that of class  $j$  and  $C_{2010}$  of all groups in annual units.  $W_h$ ,  $W_j$ , and  $W$  are welfare measures for cohort  $h$ , class  $j$  and for all households.

*Results of individual welfare:*  $\lambda_{hj}^\ell$  Table 6 displays the resulting  $\lambda_{hj}^\ell$ , the welfare costs of the increase in permanent inflation, expressed as a constant proportion of per-period consumption for each household type active in 2010. As the government uses its windfall to cut tax rates permanently, the income tax rate declines from 19.3% to 19% from 2010. This tax cut benefits all households but not enough to reduce the welfare cost to zero because: (1) there is a net loss from the redistribution of non-money nominal positions for Canada because the gain by the foreign sector is not redistributed back

<sup>26</sup> Appendix N describes and analyzes other policies.

<sup>27</sup> For households who are alive in 2010,  $1 + \lambda_{hj}^\ell$  is multiplied only by their consumption from 2010, since pre-2010 consumption is not affected in this exercise. Hence, for households who are alive in both 2010 and earlier periods,  $\lambda_{hj}^\ell$  is set to zero in the pre-2010 periods and accordingly their welfare measure reflects the consumption equivalence since 2010.

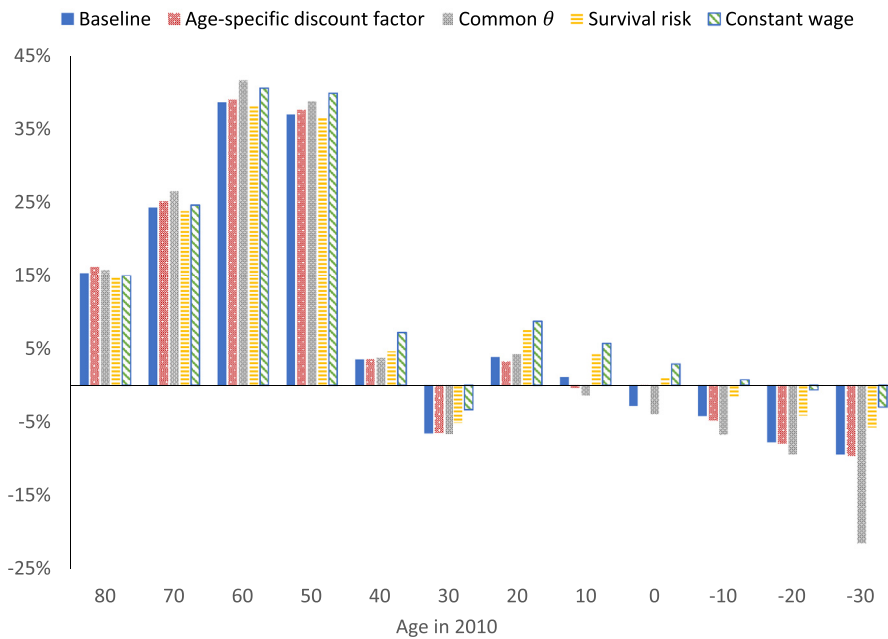


Fig. 3. Welfare cost of inflation by cohort ( $W_h$ ), in percent of own annual consumption in 2010 value.

to the households; and (2) the benefits from the permanent tax cut are shared between the current cohorts, who mostly suffer wealth losses, and future cohorts, who do not. Overall, the young cohorts in 2010 win, except for the young-poor, because of the positive redistribution of wealth and the lower tax rates throughout their lives. Older cohorts lose, especially the poor, reflecting the loss from both money and non-money nominal positions. The young-rich win and the older-rich do not lose much given their sizeable gains from lower income taxation.

*Aggregate measures of welfare:  $W_h$ ,  $W_j$  and  $W$*

Fig. 3 displays them for all relevant cohorts. The youngest cohort in 2010 (aged 30) and future cohorts (aged 0 or younger in 2010) gain from inflation because of the long-lasting effects of the lower tax rates arising from the initial price increase. The losers across cohorts are the middle-aged and the old in 2010, due to both their high money holdings and their severe wealth loss from the nominal positions that are not offset by lower taxes.

Table 7 displays the welfare costs by class,  $W_j$ , aggregating over various cohorts using Eq. (9). The poor lose a lot (37% of their one-year consumption) and the loss declines as households become richer (5.6% of the one-year consumption for the rich), the average loss being 19.6%. The bulk of the difference arises from three sources: the higher dependency on money for transactions of the poor; the larger losses associated with non-money nominal positions; and the tax cut helping the poor less than the rich. The measure of loss  $W$  from (10) adds all consumption needed and is only 13% showing how aggregate consumption measures misrepresent the losses of actual households.

Finally, the average of individual MCRs is 35% larger than average money holdings over average consumption. A representative agent model abstracts from age and therefore misses the age related asymmetry of the effects of inflation. It also misses the heterogeneity in consumption, and in MCRs. Ignoring all these sources of heterogeneity would result in an imputation of the aggregate cost of money holdings much lower than what we have found. A summary assessment of these differences tells us that accounting for age and consumption differences yield a cost of inflation that is almost 50% higher than those based on a representative agent framework.

*Comparison to the literature* We find our aggregate welfare cost lower than that of Lucas (2000) and Erosa and Ventura (2002). We calculate that the consumption losses of similar inflation increases are 25% and 17% respectively. After the adjustment for the government policy in these studies, our model gives 18% and 12%. We argue that the remaining difference is due to financial innovation dampening the welfare cost of future generations.<sup>28</sup>

*Gradual reduction of tax rates* Instead of a constant tax cut, an alternative policy gradually reducing tax rates over time would hurt the current cohorts and benefit their future peers more than the constant tax cut considered in the paper. Since the welfare of the latter have lower weights due to discounting, the aggregate welfare cost would be higher.

<sup>28</sup> Appendix Q provides the details of these comparisons. In addition, the Working Paper version of the paper measures the contribution of financial innovation to the aggregate welfare cost without the revaluation channel of nominal claims and the beneficial use of government gains. The cost is 47% higher when abstracting from financial innovation. See Section 7.4 of the working paper at <https://www.bankofcanada.ca/2018/08/staff-working-paper-2018-40/>.

## 6. Sensitivity analysis

We eliminate consumption group differences in terms of the timing of consumption and age differences in the elasticity of the MCR with respect to consumption. We also consider early mortality with accidental bequests, and a constant wage rate for all periods. All economies are re-calibrated with the calibration details presented in Appendix P. A lower target of financial innovation, a lower intertemporal elasticity of substitution, an economy with shorter periods and a common discount factor are also relegated to the same appendix.

*Age-specific discount factors* We posed utility weights by age and class. We now make all classes share the same preferences so only age consumption profiles are replicated. The implied discount factors are now higher for poorer households and lower for richer households relative to the baseline. Table 7 and Fig. 3 show that the aggregate cost of inflation is similar to the baseline. However, some difference arises in the distribution of welfare costs across household groups. Table 6 summarizes these differences. Younger and richer households now experience a higher welfare cost of inflation, while older and poorer households face lower welfare cost, an implication of the changes imposed in the relative consumption across households.

*Common MCR consumption elasticity ( $\theta$ )* We now target the average slope of all MCR curves in Fig. 1, so the slope of the MCR curve becomes flatter for the old and steeper for the young. The implied changes in welfare depend on the consumption level at which this deviation from the baseline occurs for each age group. The deviation is towards the lower end of consumption for the youngest age group, meaning that money holdings of this age group are now lower than in the baseline, while older households hold more money. Table 6 and Fig. 3 show that the aggregate welfare cost is lower than in the baseline. This is due to the large gains of future young cohorts which partially offset the welfare loss of the old in 2010.

*Survival risk* If households in the oldest two groups can die with their assets becoming accidental bequests for all, higher inflation shrinks bequests, given the large money holdings of dying households. This accounts for the higher aggregate welfare cost relative to the baseline, which is mostly borne by the youngest in 2010 and the newborns in 2020, as shown in Fig. 3. In the model, newborns enter with virtually zero amount of money and without any non-money asset; bequests are therefore more important for very young households. When inflation rises, the value of bequests shrinks, harming young households. As a result of this additional layer of cost through bequests, Table 7 indicates a slightly higher aggregate welfare cost relative to the baseline.

*Constant wage* In the baseline calibration, the wage rate grew by 22% from 2000 to 2010, compared with the 11% growth rate from the data.<sup>29</sup> Wage growth could lower MCRs through Eq. (8). This in turn could have contributed to the model's high interest rate elasticities of money demand to match the observed increase in the MCR during this period. To examine its quantitative impact, we keep the wage rates fixed in all periods at 0.42, the level in 2010 implied by the baseline calibration, and re-calibrate the model. The results indicate a smaller cohort effect leading to a slightly larger aggregate welfare cost than in the baseline (see Table 7). The interest rate elasticity of money demand became slightly smaller at 0.630 in 2010 in comparison to 0.638 from the baseline, suggesting that the high elasticity in the baseline calibration is not the result of the flexible-wage assumption.<sup>30</sup>

## 7. Conclusion

In the cross-section, money holdings per unit of consumption increase with age and decrease with consumption. We pose a heterogeneous agent OLG model where households use of money and credit for consumption accounts for these observations. We map this model to Canadian data inferring the amount of financial innovation that has been happening. We conduct welfare analysis of a surprise permanent increase in inflation by 3 percentage points using our model. Taking into account the increased cost of holding money and the redistribution of non-money wealth from this shock, the aggregate welfare cost is 13% of one-year consumption under the policy that lowers income tax rates for the increased seigniorage and the windfall gains that the government receives. Across households, the poor and the middle-aged active in 2010 lose the most while the current young and future cohorts who benefit from lower income taxes for a longer period than their older peers win. The aggregate welfare cost is lower than those from the previous literature, reflecting the role of financial innovation lowering money demand and dampening the cost of younger and future cohorts.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.jmoneco.2020.11.004](https://doi.org/10.1016/j.jmoneco.2020.11.004).

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<sup>29</sup> The calculation based on the Labour Force Survey, Statistics Canada.

<sup>30</sup> The small change in the elasticity is because the credit transaction cost moves with the wage rate given the assumption of  $q_t = w_t$ . Higher wage growth directly increases consumption but also leads to higher credit costs that induce a negative income effect on consumption. This counters the direct increase in consumption, limiting the decrease in MCRs and the corresponding need for the higher elasticity.

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