BANK OF CANADA
BANQUE DU CANADA

Staff Working Paper/Document de travail du personnel — 2021-44

Last updated: September 24, 2021

A Q-Theory of Banks

by Juliane Beganau,[1] Saki Bigio,[2] Jeremy Majerovitz[3] and Matias Vieyra[4]

[1] Stanford Graduate School of Business, NBER and CEPR

[2] UCLA, Visiting Scholar Federal Reserve Bank of San Francisco and NBER

[3] Massachusetts Institute of Technology

[4] Financial Stability Department
Bank of Canada, Ottawa, Ontario, Canada K1A 0G9

begenau@stanford.edu, sbigio@econ.ucla.edu, jmajic@mit.edu, mvieyra@bankofcanada.ca

ISSN 1701-9397

Acknowledgements

We would like to thank Youngmin Kim, Adam Su, and Mengbo Zhang for research assistance. We thank Andrew Atkeson, Haelim Anderson, Patrick Bolton, Dean Corbae, Pablo D'Erasmo, Andrea Eisfeldt, Francesco Ferrante, Mark Gertler, Kinda Hachem, Valentin Haddad, Benjamin Hebert, Arvind Krishnamurthy, Anton Korinek, Hanno Lustig, Ken Miyahara, Gaston Navarro, Andrea Prestipino, Jose-Victor Rios-Rull, Pari Sastry, Jesse Schreger, Jeremy Stein, Robert Townsend and Pierre-Olivier Weill for their many useful comments. We are also grateful to Elena Loutskina, Thorsten Koeppl, David Martinez-Miera and Adrián Pardo for discussing our paper. We thank the participants of the MFM Winter 2019 Meeting, the Banco de Mexico, the Philly Fed, and the Columbia 2020 Junior Finance Conference. Previous versions of the paper circulated under the title "Data Lessons on Bank Behavior." These are our views and not necessarily those of the Bank of Canada or the Federal Reserve Bank of San Francisco.

Abstract

We document five facts about banks: (1) market and book leverage diverged during the 2008 crisis, (2) Tobin's Q predicts future profitability, (3) neither book nor market leverage appears constrained, (4) banks maintain a market-leverage target that is reached slowly, and (5) pre-crisis, leverage was predominantly adjusted by liquidating assets. After the crisis, the adjustment shifted towards retaining earnings. We present a Q-theory where notions of leverage differ because book accounting is slow to acknowledge loan losses. We estimate the model and show that it reproduces the facts. We examine counterfactuals where different accounting rules produce novel policy tradeoffs.

Topics: Financial institutions; Financial stability; Financial system regulation and policies

JEL codes: G21, G32, G33, E44

1 Introduction

Models of banks are important because they shape financial regulation and, through regulation, macroeconomic outcomes. However, developing suitable frameworks for policy analysis is a fine-tuning process in which regulators and academics alike are constantly reassessing their models. In fact, bank regulation was entirely redesigned in the aftermath of the Great Recession, only to meet a global wave of regulatory forbearance in the first major crisis that followed—the Covid-19 crisis. Hence, the continuous development of banking models is a macroeconomic priority and understanding the dynamics of bank leverage, in particular, is critical to that process.

This paper presents a novel theory of banks. Our theory is motivated by five stylized facts that inform us about the dynamics of bank leverage. A unifying theme among these facts are the cross-sectional and time-series variations in Tobin's Q; i.e., variations in the market-to-book-equity ratio. A novel aspect of our theory is the slow recognition of loan losses on banks' accounting statements. This slow loan-loss recognition causes the fundamental value of loans, which takes into account loan losses, to differ from the book value of loans, which does not account for loan losses. We label the ratio of fundamental-to-book value as little q, which differs from big Q (Tobin's Q). In our theory, variations in Q are in part driven by variations in q. Quantitatively, slow loan-loss recognition is essential to explaining this study's five stylized facts. We argue that our Q-theory improves our understanding of bank-leverage dynamics and illustrates an important policy trade-off.

The five facts that motivate our Q-theory are as follows:

1. Banks' book- and market-leverage ratios behave very differently during the 2008-2009 crisis. Market leverage rises dramatically during the crisis whereas book leverage remains constant. In particular, between 2007 Q3 and 2014 Q4, bank holding companies lose 54% of their market capitalization. Book-equity losses are only 7% and these losses are entirely made up by equity issuances.

2. Market values capture information that book values do not, in particular, information on future portfolio losses and profitability.

3. The cross-section of market leverage shows a large dispersion across banks. This dispersion is considerably smaller for book leverage and few banks are close to their regulatory constraints, even in the midst of the crisis.

4. Banks appear to operate with a target for their market-leverage ratio. The adjustment to that target is slow: In response to an unexpected negative-net-worth shock, proxied by a shock to stock returns, market leverage increases on impact and takes several years to return to its initial level. By contrast, book leverage does not respond on impact and the overall response is muted.

5. Prior to the crisis, in response to an unexpected negative-net-worth shock, banks primarily sell assets in order to return to their market-leverage target. Post-crisis, banks intensify their use of retained earnings and equity issuances and reduce the extent of their adjustment through asset sales as a means to return to their market-leverage targets.

Fact 1 emphasizes the difference between banks' book value of equity and their corresponding market value.[1] Understanding this difference is important because in models we must take a stance on whether book or market equity (or both) is the relevant state variable.[2] This stance matters for evaluating the quantitative performance of models: empirically, book-measured leverage and market-measured leverage lead to different inferences about the time-series properties of leverage and the price of risk (see the debate between Adrian et al. 2014 and He et al. 2017), but typically both measures co-move in models. Fact 2 implies that market valuations are informative about bank losses much earlier than when losses are reported on balance sheets.[3] Fact 3 points to a rich cross section of market- and book-leverage ratios. Even before the 2008 crisis, the cross-sectional dispersion of market leverage is wide. During the crisis, both the average market leverage and its cross-sectional dispersion increase dramatically.[4] In terms of book leverage, the pre-crisis distribution is more concentrated and most banks maintain a substantial equity buffer beyond the regulatory requirements. In the midst of the 2008-2009 crisis, the regulatory capital ratio of the vast majority of banks remains far above their regulatory limits, even among banks whose market valuations show significant erosion. [5] Taken together, these facts suggest that although market values of bank equity are good predictors of bank health, banks' market leverage does not appear to be constrained as it dramatically increases for many banks during the crisis. In turn, book-equity values are not timely predictors of bank health but regulatory constraints are explicitly stated in terms of book equity. Since book values take time to incorporate information on losses, regulatory constraints may not be binding even after a negative shock to banks' assets. In this paper, we argue that models should account for (i) the differences between book and market equity, and (ii) how the slow loan-loss-recognition mechanism affects the tightness of banks' regulatory constraints.

Facts 4 and 5 relate to the adjustment dynamics of banks' balance sheets after a negative

[1]This dichotomy between banks' book and market values of equity had prompted policy discussions during earlier banking crises (see post savings and loans crisis survey in Berger et al. 1995).

[2]Papers that study the asset pricing implications of intermediary net worth (e.g., He and Krishnamurthy, 2013 and Brunnermeier and Sannikov 2014) focus on market measures of equity, while papers focused on the effects of regulation focus on book measures of equity (e.g., Adrian and Boyarchenko 2013; Begenau 2020; Adrian and Shin 2013; Corbae and D'Erasmo 2019; Begenau and Landvoigt 2020).

[3]This observation is consistent with Blattner et al. (2019). This conclusion is also shared with the accounting literature (see Laux and Leuz, 2010) that explains how banks have flexibility in accounting for losses. In fact, this was an issue raised by the United States Congress after the Savings and Loans crisis (General Accounting Office, 1990).

[4]This suggests a countercyclical average leverage ratio. He et al. (2010) and He et al. (2017) document that market-based leverage for intermediaries is countercyclical. Using the book-equity definition, Adrian and Shin (2013) show that broker-dealer leverage is procyclical.

[5]An extreme example is Citibank, a bank that experienced market-based losses of up to 90% with only minor changes in its book equity.

shock to their assets. It is challenging to empirically identify such shocks: according to Fact 2, accounting measures of bank equity do not convey all available information about shocks to bank wealth. On the flip side, market valuations may capture additional information not contained in banks' books. But market valuations are also affected by variations in risk premia that are unrelated to an individual bank's health. To tackle this identification challenge, we exploit the cross-sectional variation in banks' stock returns and estimate impulse responses to innovations in individual bank stock returns. This strategy builds on the efficient-market hypothesis: The idea is that once we introduce adequate statistical controls, idiosyncratic deviations from average market returns pick up idiosyncratic information about banks' effective net worth that is not contained in their books. Thus, we contend that idiosyncratic returns shocks proxy for net-worth shocks. Once we construct a time series of returns shocks for each bank, we estimate the average impulse responses of market and book leverage, liabilities, dividends, equity, and other variables to a shock to returns. These impulse responses shed light on the adjustment process that takes place after shocks that affect a bank's net worth.

Fact 4 is obtained from this impulse-response analysis: The main observation is that the behavior of banks is consistent with a pattern where they maintain a target for market-based leverage, but the adjustment process of returning to that target is gradual. Namely, in response to a negative returns shock, which mechanically increases market leverage on impact, banks take actions to slowly reduce their market leverage to their leverage targets. This gradual response indicates that frictions prevent banks from immediately returning to these leverage targets.[6] Fact 5 describes the actual deleveraging process that takes place both before and after the 2008 financial crisis. Pre-crisis, a bank that experiences negative returns shocks relies predominantly on reducing its liabilities to lower its leverage with almost no change in its book equity.[7] During and after the financial crisis, the deleveraging process is faster but the reduction in bank debt slows down. Instead, the increase in the deleveraging speed follows from an increase in retained earnings and equity issuance.[8] Fact 5 suggests that there is a regime shift in the frictions that govern the leverage dynamics. Taken together, Facts 4 and 5 call for a theory that can account for the slow balance-sheet-adjustment process and how this process changes over time.

This paper presents a partial-equilibrium model of banks that meets the challenges brought by these facts. To reproduce these facts, the model features (1) meaningful differences between book and market values, (2) accounting valuations that are less responsive than market values to loan-default shocks, (3) a rich cross section of book- and market-leverage ratios, (4) an endogenous leverage-ratio target and frictions that lower the speed of adjustment to that target, and (5) a

[6] Gropp and Heider (2010) document that banks' capital ratios appear to follow a target leverage ratio and analyze the empirical drivers of that target capital ratio.

[7] This is consistent with Figure 2 in Adrian and Shin (2013).

[8] One interpretation of this fact is that the cost of liquidating assets increased during the crisis, presumably because the adverse selection problem got worse. This interpretation is consistent with models proposed by Gorton and Ordonez (2011) and Dang et al. (2017).

regime shift that can explain the switch in the deleveraging process.

In our model, banks are owned by diversified shareholders. Banks maximize the value of the discounted stream of dividend income, under risk neutrality. Banks issue deposits and loans, where loans are exposed to default shocks. The supply (demand) of deposits (loans) is perfectly elastic. The expected returns spread between loans and deposits is positive and constant, a feature that makes the returns on equity increasing in leverage. However, if its leverage is too high, then loan-default shocks expose a bank to liquidation risk. In particular, a bank is liquidated if it violates regulatory constraints or if it becomes insolvent. As in Leland and Pyle (1977a), there is a trade-off between levered returns and liquidation risk. This trade-off induces a notion of a target for fundamental leverage, which differs from book or market leverage.[9] This trade-off is present even though banks are owned by diversified shareholders and, thus, behave as risk-neutral firms. The model has several frictions that drive a wedge between book and market values and that produce a slow return to the market-leverage target. First, banks do not raise equity and have a preference for dividend smoothing. This friction prevents market leverage from adjusting immediately via equity finance. Furthermore, this friction drives a wedge between a dollar inside and outside the bank, one source of variation in Tobin's Q.[10] Second, the market value of loans reflects losses that book values do not reflect. Specifically, only a fraction of loan defaults are immediately recognized on the books and the full recognition of losses takes time. Delayed accounting is a second source of variation in Tobin's Q. The combination of dividend smoothing and delayed accounting already produces smooth responses of market leverage, similar to those observed in the data. Finally, the theory features a cost of reselling loans, modeled as price adjustment costs in the spirit of O'Hara (1983) or Shleifer and Vishny (1997).

We calibrate the parameters of the model except for three parameters that we estimate. Each of these three parameters corresponds to a financial friction in the model. We estimate these parameters to gauge the importance of each friction to explaining our facts. Concretely, we estimate the parameters that govern banks' dividend-smoothing motive, delayed loan-loss recognition, and loan-adjustment costs. We estimate these parameters by matching the impulse responses of market leverage, book leverage, and liabilities to the returns shocks. Identification is obtained as follows: the difference between the responses of market and book leverage is informative about the delayed recognition of losses. Once this parameter is obtained, the responses of market leverage and liabilities are informative about the degree of dividend-smoothing and the loan adjustment costs, respectively. A striking result is that to match the pre-crisis responses, the estimation requires virtually no loan-adjustment costs. That is, to match the pre-crisis impulse responses, the model

[9] In corporate finance, it is standard to explain a leverage target through either trade-off theory (a tax advantage) or a risk-return trade-off (e.g., Kraus and Litzenberger, 1973; Leland and Pyle, 1977a; Myers, 1984; Hennessy and Whited, 2005; Frank and Goyal, 2011). In an early dynamic model of banks, O'Hara (1983) shows that a leverage target follows from undiversified ownership that induces risk-averse behavior.

[10] Because of its importance, a slow adjustment to leverage was the topic of Darrell Duffie's presidential address to the American Finance Society in 2010 (Duffie, 2010).

only needs dividend smoothing and delayed loan-loss recognition. The economic intuition behind this result is that because banks do not immediately recognize losses and prefer to stay levered, they do not take immediate actions to delever when hit by a loan-default shock. This reflects in an immediate response of market leverage upon the shock but only a negligible response of book leverage. Over time, banks gradually delever as loan losses are slowly recognized on the books. As these losses are slowly recognized, the regulatory constraints tighten over time. As a result, banks need to reduce debt but only at the pace at which their loan losses are recognized. Through this channel, delayed accounting is enough to explain the slow responses of market leverage through a slow reduction of liabilities during the pre-crisis sample.

To rationalize the change in the impulse responses after the 2008 financial crisis, we feed the model with a common aggregate loan-loss shock of 2.5% and then re-estimate the model. For the post-crisis moments, the estimation needs to account for an effect that speeds up the adjustment of market leverage but slows down the reduction in liabilities. Because the market-to-book ratio (on impact) is about the same for the pre- and post-crisis samples, the estimate of the parameter that governs the recognition of loan losses remains the same. Hence, to explain the regime change in the deleveraging pattern, the estimation calls for an increase in the loanadjustment costs from a negligible to a non-negligible value. This increase in adjustment costs permits the model to explain the even slower response of liabilities in the post-crisis period.[11] In turn, the estimation of post-crisis parameters needs a reduction in the dividend-smoothing motive to explain the more-intensive use of retained earnings as a deleveraging device and the overall faster decline in leverage. These larger estimates for the adjustment costs speak to a reduction in liquidity in the secondary market for loans, which is itself consistent with systemic delayed accounting after an aggregate shock. The reduction in dividend smoothing is interpreted as banks' greater pressure to cut back on dividends upon a negative shock in the aftermath of the crisis, even for banks that are not pressured by policy. The model is not only able to reproduce the impulse responses of the data but it also goes a long way in explaining the cross-sectional variation in Tobin's Q and its predictive power, items that the estimation procedure does not target. Furthermore, with an aggregate shock of 2.5%, the model explains about 50% of the observed decline in Tobin's Q during the crisis; a decline that is exclusively attributed to changes in q, that is, excluding changes in investor risk premia.[12]

Our Q-theory allows us to study the effects of changing the degree of slow loan-loss recognition for bank-equity growth and lending. This reveals a trade-off: more-lenient accounting rules allow banks to increase their fundamental leverage beyond regulatory limits. However, more-lenient accounting rules also mitigate the effects of loan-losses as banks are no longer forced to immediately reduce their assets in order to adjust their leverage back to target. This creates a tension between riskier banks and a milder lending contraction thanks to laxer accounting rules. All in all, the

[11]The aggravation of adjustment costs is consistent with adverse selection in the secondary loans market, which is in turn consistent with the idea that banks can delay the recognition of losses.

[12]Atkeson et al. (2018) explain the reduction in the aggregate market-to-book ratio of banks, post-crisis, with a reduction in government guarantees.

mechanism of delayed loan-loss recognition acts like a counter-cyclical regulation that allows the capital requirement constraint to remain slack during a crisis with the advantage of being bank specific.

Related literature. The financial crisis of 2008 has renewed interest in banking models as banks are viewed to be critical in reallocating resources in the economy—see, e.g., Adrian and Shin 2010; Rampini and Viswanathan 2012; Jermann and Quadrini 2012; Gertler et al. 2012; Adrian and Shin 2013; He and Krishnamurthy 2012; Brunnermeier and Sannikov 2014; Gertler et al. 2016. Bank leverage is at the heart of theories that can be organized into those where markets impose constraints on leverage and others where regulation limits leverage. From a theoretical angle, more equity and lower leverage relax financial constraints and allow a bank to expand its lending. Models in this category include those of Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013); Gertler et al. (2016); Nuño and Thomas (2017).[13] For these models, a market-based equity value is the natural empirical counterpart because it measures the value of the equity that affects incentives. The second group of models takes regulation as a given institutional feature. Such models study the effects of declines in equity buffers (Adrian and Boyarchenko, 2013; Begenau, 2020; Bianchi and Bigio, 2017; Martinez-Miera and Suarez, 2011; Corbae and D'Erasmo, 2019). The present paper makes two contributions: First, it presents a set of facts that shed light on the relevant state variables and constraints that affect bank decisions. Second, it presents a Q-theory of banks that is consistent with these facts. A key contribution of this paper is to show how both market- and equity-based measures are relevant state variables. In particular, we put forth the idea that delayed accounting is important and we quantify the trade-offs involved in the design of different accounting regimes for financial stability and economic growth.

In terms of models with market-based constraints, our empirical findings suggest that such constraints operate in richer ways than conceived by many of these models. Although some of these models can generate the counter-cyclical movements we see in the data on market leverage, they typically cannot account for the cross-sectional changes in leverage: upon an aggregate shock, most of these models would predict a compression of the distribution of leverage at a higher level after a large shock as more banks get closer to their constraints.

In addition, these models cannot account for the lack of responses in book values because book and market values co-move in those models. Adrian et al. (2016) raises a similar point when they argue that "as for market leverage, we show that virtually all the cyclical variation

[13] Financial frictions on banks matter for the provision of credit —and hence economic performance. Examples of these frictions include costly verification (Townsend, 1979; Bernanke and Gertler, 1989), lack of commitment (Hart and Moore, 1994), or moral hazard (Holmstrom and Tirole, 1997, 1998). Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) were the first to model the connection between firm equity and aggregate outcomes. A different perspective is taken by Diamond and Rajan (2000) who argue that deposits through bank runs (a là Diamond and Dybvig, 1983) act as a disciplining device in the presence of agency frictions.

of market leverage is driven by fluctuations in the book-to-market ratio, reflecting the valuation changes of free cash flows generated by the bank." To meet this challenge, models based on market constraints would need to generate changes in both time *and* in the cross section of leverage ratios. In our model, due to delayed loss recognition, regulatory constraints become more slack upon an aggregate shock and book values barely move but the average market leverage and its dispersion increase.

Our model shares elements with He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) in that banks' equity accumulates slowly over time (see the survey by Gertler et al., 2016). Unlike these papers though, bankers are risk neutral despite the fact they perform dividend smoothing. Like these papers, banks face market-based constraints. Regulation that limits book leverage also connects this paper to the work of Adrian and Boyarchenko (2013). Our paper is also connected with Milbradt (2012) because we distinguish between the fundamental value of assets and book assets where losses are not registered in real time. We differ from Milbradt (2012) in that we focus on aggregate and cross-sectional bank-lending-and-leverage dynamics. Furthermore, we do not interpret delayed accounting as exclusively resulting from accounting practices but also as following from deliberate evergreening practices as described in Caballero et al. (2008).

Our evidence on the gradual adjustments in market leverage relates to two other theories on adjustment costs (see Hayashi 1982, for a neoclassical Tobin's Q theory). In finance, adjustment costs are not thought of as stemming from physical constraints. Instead, one financial literature strand rationalizes the slow adjustment of leverage with equity-issuance costs and asymmetric information. Early models of equity-issuance costs that are based on agency problems are a debt overhang model found in Myers (1977) and a private information model found in Myers and Majluf (1984). Adjustment costs on assets arise naturally when banks hold informationally sensitive assets that are typically viewed as a specialty of banks—e.g., Leland and Pyle (1977b); Diamond (1984a); Williamson (1986); Tirole (2011); Dang, Gorton, Holmström and Ordonez (2017); Shachar (2012); Hachem (2011). In a more recent strand of work, e.g.,DeMarzo and He (2016) and Gomes et al. (2016), a leverage target and slow adjustment emerge due to long-term debt and default; i.e., as the result of debt dilution. A novel feature of our Q-theory is that slow-moving leverage can exclusively result from the delayed loss-recognition mechanism. Furthermore, to explain the pre-crisis patterns, our model only needs delayed-loss accounting. However, for the post-crisis period, the model needs higher balance-sheet-adjustment costs, which are consistent with these theories.

The closest papers to ours are Corbae and D'Erasmo (2019) and Rios-Rull et al. (2020). Like these papers, we also try to match cross-sectional bank-leverage data. The novelty of our work, relative to these studies, is that our focus is on the importance of book- and market-value differences that follow from delayed accounting in shaping banks' decisions. Finally, our paper also relates to the literature on macro-prudential bank regulation. Some authors hold the view that marking assets to market can amplify a crisis by worsening financial frictions (Shleifer and Vishny, 2011; Laux and Leuz, 2010; Plantin and Tirole, 2018). In the midst of the Covid-19 crisis, questions such as how and whether to mark loans to market and how much regulatory forbearance is good for the economy are once again at the center of the discussion (Blank, Hanson, Stein and Sunderam, 2020). Our focus on delayed accounting suggests policy should also evaluate the role of evergreening practices in the formation of zombie banks (Caballero et al., 2008). In the final part of the paper, we study the effect of delayed loan recognition and also highlight a policy-relevant trade-off.

The rest of this paper is organized as follows. Section 2 presents our set of five facts while Section 3 presents the model and its results. Section 4 concludes.

2 Five Stylized Facts

Data. We use panel-level data on top-tier United States Bank Holding Companies (BHCs).[14] BHCs provide a comprehensive picture of the activities of a financial organization beyond the narrower accounts of their commercial bank subsidiaries. We take BHC accounting data (balance sheet and income statements) from the FR-Y-9C regulatory reports filed with the Federal Reserve. We merge the accounting data with market data from the Center for Research in Security Prices (CRSP). We focus on the sample period from 2000 Q1 to 2015 Q4.[15] BHCs file FR-Y-9C forms if they have assets above \$500 million.[16] This sample is highly representative of the banking sector. Appendix Section A.2 presents the time series of key balance sheet variables for all BHCs in our sample and the four largest BHCs: Bank of America; J.P. Morgan, Citigroup, and Wells Fargo.

2.1 Characteristics of bank equity

Book equity versus market equity. In most models of banks net worth is a key state variable that puts a cap on leverage. However, bank net worth can be measured in terms of accounting measures (book equity) or market value (market equity). Figure 1 presents the time series of book and market equity aggregated across all of the BHCs in our sample (left-hand panel) and the same time series for the four largest BHCs (right-hand panel).[17] The figure shows a stark discrepancy between market and book equity, particularly during the crisis. This discrepancy raises the question of which empirical counterpart best captures the economic concept of net worth. Just looking at book equity, it is hard to detect that 2008-2009 are the years of a major financial crisis! The

[14] A bank holding company is an umbrella company that holds banks and other financial institutions. A commercial bank is a single bank that provides traditional banking services such as deposits and loans. For example, Citibank, a commercial bank, is held by Citigroup, a BHC that holds Citibank and other banks, including non-commercial banks.

[15] We extend the sample when we estimate impulse-response functions. When constructing aggregate time series, we drop entrants to correct for the entry of major financial institutions, such as Goldman Sachs and Morgan Stanley. Without this correction, aggregate bank assets increase due to the reclassification of large actors, such as Morgan Stanley and Goldman Sachs, into bank holding companies.

[16] Prior to 2006 Q1, this threshold was \$100 million, and the threshold became \$1 Billion in March 2015.

[17] Book equity for publicly traded BHCs is close to book equity for all BHCs, which shows that the publicly traded sample compromises most of the banking sector (weighted by equity).

sharp drop in market equity, on the other hand, clearly indicates a crisis. The pattern for the largest banks is very similar.[18] Citigroup is an extreme example of the discrepancy between book and market values: Citigroup loses 90% of its market capitalization but its book equity remains intact.[19]

This difference between market and book equity is not the result of the composition of public equity injections. Although these injections are counted as preferred equity in accounting books and market equity is measured relative to common equity, Figure 1 shows that preferred equity cannot explain the discrepancy between market and book equity.[20]

To get a quantitative sense of how much book and market equity differ during the crisis, in Table 1 we present the percentage change in banks' market-equity valuations (top two rows) and book-equity valuations (middle two rows), together with the change in the S&P 500 stock returns index from the beginning of the crisis in 2007 Q3 to the end of each of 2008, 2009, and 2010, respectively. We report simple percentage changes in the real value (columns entitled "real change") as well as the changes in the fitted log-linear trends (columns entitled "log linear"). Between 2007 Q3 and 2008 Q4, the market capitalization of the banking sector drops by 54%, compared to a 42% drop in the S&P 500. By 2010 Q4, market equity is still down 30% from its value in 2007 Q3. Much of this rebound follows from new equity issuances. By contrast, book equity does not fall during the crisis and actually increases substantially post-crisis. In fact, recorded book-equity losses are entirely made up for by new equity issuances. This large discrepancy implies that banks' average Tobin's Q, defined as the market-to-book-equity ratio, drastically declines during the crisis and remains much lower thereafter. [21] To summarize, our first fact is as follows:

Fact 1. *Book and market values substantially diverge during the crisis.*

The divergence between book and market equity during the crisis is already known from other studies (see for example Adrian and Shin, 2010; He et al., 2017). However, this variation in Tobin's Q is a common thread in the paper and begs the question of what empirical counterpart of equity should be used in macro models. We next turn to some cross-sectional evidence that sheds some light on this question.

Information content in market and book equity. A natural question is whether differences between the market- and book-equity valuations reflect differences in the informational content of these measures. Conceptually, book measures are *backward looking* in that they register historical

[18]The discontinuities in the individual bank series reflect mergers and acquisitions; e.g., the acquisition of Wachovia by Wells Fargo during the crisis.

[19]Citigroup suffers heavy losses during the crisis and does not undergo any major mergers or acquisitions, making it a particularly clean case example.

[20]Preferred equity rises temporarily during the crisis due to the Troubled Asset Relief Program (TARP). Note that preferred equity is included in book equity but not in market equity.

[21]We are referring to the market-to-book-*equity* ratio as Tobin's Q, as opposed to the market-to-book-*assets* ratio.

losses. By contrast, market-equity measures are *forward looking* in that they price future expected cash flows. Still, this conceptual difference does not imply that both measures contain different information: In principle, we can write a model where the history of events is encoded in banks' balance sheets and the information contained in the books is enough to predict future cash flows. In such a case, the informational content of market values would be the same while the time-series and cross-sectional variation of Tobin's Q would only respond to changes in the risk premia.

However, we suspect that market- and book-value measures contain different information: one reason is that changes in the underlying market value of loans (see filing instructions for FR-Y-9C BHCs regulatory reports) reflect default expectations. These expectations are not updated in loan accounting books. During our sample period, a loan is only written off once the loss has occurred, in contrast to when the loss is expected.[22] This alone could produce differences in the informational content.[23] Another reason is the delayed acknowledgment of known losses. If banks can delay recognizing losses, or refinance non-performing loans to avoid registering losses (evergreening), then book values will be over-optimistic. If market participants can update their valuations quickly, thereby detecting these losses, then differences in informational content will emerge. A casual indication that market values contain more information can be seen from Figure 2, which shows that loan charge-offs peak in 2010, when the economy is no longer officially in a recession. The decomposition of net charge-offs shows that these losses are heavily driven by real estate, which is consistent with the nature of the crisis.[24]

Next, we formally analyze the differences in informational content. Our strategy builds on the following idea: If market-equity values contain more information about bank profitability than book-equity values do, then Tobin's Q, i.e., the market-to-book-equity ratio, should predict future profitability once it controls for book equity. In addition, Tobin's Q should be correlated with contemporaneous predictors of future performance. In Figure 3, we show binned scatter plots of (logged) outcomes on the log market-to-book ratio (market capitalization over book equity); for a pre-crisis quarter (2006 Q1) these are shown in navy and for a post-crisis quarter (2009 Q1) they are in maroon—the plots control for the log book equity.[25] The top left panel shows the log

[22] At the time of writing, a new accounting standard had been developed. According to this standard, loan losses should be calculated according to "current expected credit losses" (CECL). Publicly traded banks have been following CECL (expected loss accounting rules) since January 1st, 2020. Smaller banks were supposed begin following these rules over the next year. However, on March 27, 2020 (https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200327a.htm), the Fed moved to provide an optional extension of the regulatory capital transition for the new credit-loss accounting standard.

[23] In Appendix A.3 we survey important contributions from the accounting literature on the issue of delayed loss recognition. See also Bushman (2016) and Acharya and Ryan (2016) for useful discussions.

[24] When a bank has a loss that is estimable and probable, it first provisions for the loan loss and this show up as Provisions for Loan Losses (PLLs).. Later when a loss occurs, the asset is charged off and thus taken off the books; this shows up as a charge-off, although occasionally the bank can recover the asset later. Net charge-offs are charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLLs because the FR Y-9C does not provide information on PLLs by loan category.

[25] To control for log book equity, the left- and right-hand variables are residualized on the log book equity, and then the mean of each variable is added back to maintain the centering. It is important to control for the log book

returns on equity over the past year, plotted against the log market-to-book ratio. Banks with higher Tobin's Qs earn higher returns on equity. The top-right panel examines the log returns on equity over the next year: the higher market-to-book ratios predict higher future profits. These correlations are especially strong in the post-crisis quarter. Banks with higher market-to-book ratios also have lower shares of delinquent loans (bottom-left panel) and, in the post-crisis quarter, have lower net charge-off rates on their loans over the next quarter (bottom right).

The results suggest that market participants have some ability to predict future profitability beyond what they see in accounting books and they incorporate this into their valuations.[26] This is consistent with the view that books are slow to reflect their true conditions. Indeed, the results suggest that banks with lower profitabilities and more delinquencies have lower Tobin's Qs, and these Tobin's Qs will predict future loan writedowns and future profitability. Our second fact is the take-away from this analysis.

Fact 2. *Tobin's Q predicts future cash flows in the cross section of banks. That is, market values capture information that book values do not and book values do not fully respond to shocks.*

The time series and cross section of book and market leverage. Models of banks typically impose constraints on either market or book leverage. In this section, we show the time-series pattern of banks' market- and book-leverage ratios and their cross-sectional differences. In the left-hand panel of Figure 4, we plot the aggregate market and book leverage for the entire sample of public BHCs. In the right, we show the corresponding series for the four largest banks. A common pattern is evident: Book leverage rises only moderately pre-crisis and actually falls during the crisis. Market leverage, by contrast, dramatically spikes during the crisis and remains almost twice as high for at least four years.

In terms of the cross section, Figure 5 presents different sections of the distribution of market leverage over time. The figure plots the median market leverage (in maroon) and each 10th percentile (in blue). Market leverage increases across the board and takes a long time to return to its pre-crisis levels. Strikingly, the distribution of market leverage fans out, with a substantial 10% of banks sustaining market-leverage ratios of nearly 80. This suggests that in the midst of a deep financial crisis there is no strictly binding ceiling on market leverage.

Whereas constraints on market leverage are a theoretical possibility that have to be empirically validated, regulatory constraints are indisputable. Regulation is based on book-equity ratios. Does the cross section of book leverage show evidence of distress? How many banks are close to violating

equity to prevent spurious results that could be due to ratio bias (see Kronmal, 1993). The log book equity is part of the right-hand side, and it also appears on the left-hand side of many of these regressions; it would have a mechanical effect if it were not controlled for.

[26] Of course, an alternative explanation is that market equity overreacts to information about future profitability. However, the fact that book equity did not decline during the crisis suggests that the sluggishness of these books played a role. Also, to the extent that variation in the discount rate will similarly affect most banks, it is unlikely to drive the results in this cross-sectional regression.

their regulatory constraints during the crisis? Figure 6 presents the share of BHCs that are below different levels of regulatory capital ratios.[27] We can observe that the vast majority of banks keep a capital buffer above the regulatory minimum. The distance to the regulatory constraints shortens for a significant number of banks during the crisis but, still, only a minority of banks are close to their regulatory constraints. We summarize these observations into our third fact:

Fact 3. *Most banks keep an equity buffer above the regulatory minimum capital ratio; fewer banks do so during the crisis. Market leverage and the dispersion of market leverage substantially increase during the crisis.*

Fact 3 poses a challenge for standard models of banks. We should expect systemic loan defaults during a crisis like the one that occurrs in 2008 and this should lead to an increase in market leverage for all banks, provided that liabilities are not immediately liquidated. At the same time, we should expect substantial reallocation of assets from highly levered banks toward banks with lower leverage. This means that we should expect a compression in the dispersion of leverage in the cross section of banks. Figure 5 suggests that there is no such compression. This observation suggests there is a delay in the reallocation of assets across banks. In terms of regulatory constraints, this third fact suggests that only a minority of banks hit their regulatory constraints. In light of fact 2, we argue that this is partially due to the slow response of book values. Interestingly, the share of banks that are near the regulatory limits peak in the first quarter of 2010, at least 2 years after the first symptoms of a mortgage crisis are apparent. From the perspective of models of banks, we contend that *neither regulatory nor market constraints bind in a static way.* Of course, just because a constraint is not actively binding, we cannot conclude that a constraint does not affect banks. On the contrary, banks may worry that a constraint will bind in the future and they will take steps to avoid hitting the constraint, and this is the spirit of our Q-theory. The next section analyzes the dynamic responses of banks to shocks.

[27] Under Basel II (the regulatory standard in place during the crisis), bank holding companies are subject to regulatory minimums on their total capital ratios and their tier-1 capital ratios. These capital ratios are computed as qualifying capital/risk-weighted assets and, thus, a bank with a higher capital ratio has lower leverage. Basel II requires that banks hold a minimum tier-1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as "well capitalized," banks have to meet minimum capital ratios that are two percentage points higher (6% and 10%, respectively). Being categorized as well capitalized is desirable because banks that are not are subject to additional regulatory scrutiny (Basel Committee on Banking Supervision, 1998, 2006). After the crisis, tighter capital requirements are phased in under Basel III. The minimum total capital ratio stays at 8% throughout our sample period, but the tier-1 capital ratio rises to 4.5% in 2013, to 5.5% in 2014, and finally settles at 6% starting in 2015. Also under Basel III, additional capital ratios (e.g., tier-1 leverage and common equity capital ratios) begin being monitored (however these ratios are quite similar to the preexisting tier-1 and total capital ratios) and, starting in 2016, a "capital conservation buffer" and special requirements for systemically important financial institutions are introduced (Basel Committee on Banking Supervision, 2011). Kisin and Manela (2016) study whether banks violate different regulatory constraints and find that they do not typically violate multiple regulatory constraints.

2.2 Characterizing bank-leverage and balance sheet dynamics

This section analyzes the dynamics of leverage and the balance sheet. This analysis is informative about the constraints banks face and their process of leverage adjustment, as we will argue.

Empirical framework. We empirically investigate whether and how a target leverage ratio and adjustment costs drive the leverage dynamics of banks. To test this hypothesis, we estimate the following panel regressions:

$$\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \log(1 + \varepsilon_{i,t-h}) + \gamma_h \cdot Post_t \ \log(1 + \varepsilon_{i,t-h}) + \psi_{i,t}, \tag{1}$$

where i indexes over banks, t indexes over quarters, $y_{i,t}$ is the outcome of interest, α_t is a time fixed effect, $\varepsilon_{i,t}$ denotes our measure of a cash flow shock to net worth (i.e., the idiosyncratic excess stock returns innovations over the past quarter for bank i in quarter t; see detailed description below), and $Post_t$ is an indicator variable that is equal to one if the current quarter is post-crisis (we treat 2007 Q4 as the first quarter for which $Post_t = 1$).[28] These regressions allow us to construct impulse-response functions for liabilities, market leverage, market equity, and book equity.[29] We include time fixed effects, α_t, to absorb aggregate shocks; e.g., changes in investors' discount rates or the price of loans due to demand shocks. We thus recover a partial equilibrium supply-side impulse response, estimated from the cross-sectional variation in returns shocks. In all specifications, we use $k = 20$. Due to these many lags, we extend our data to 1990 Q3 in order to obtain precise pre-crisis estimates.[30] We cluster standard errors by bank. Finally, to report the impulse-response function, we sum the coefficients: the pre-crisis contemporaneous response is β_0, the next period is $\beta_0 + \beta_1$, and so on. For the post-crisis, we also add the corresponding γ terms.

Before we show the results, we first discuss how we obtain the shock measure $\varepsilon_{i,t}$. We follow Gandhi and Lustig (2015) to adjust bank stock returns for aggregate risk factors. That is, we regress the excess stock returns $r_{i,t} - r_t^f$ of bank i on a bank fixed effect α_i and a matrix of factors

[28] One might favor an alternative specification that includes lags of the dependent variable in addition to contemporaneous and lagged returns. This poses two issues: Nickell bias and bad control. Including the dependent variable as a lag will induce bias, as documented by Nickell (1981). Dealing with this bias is challenging and may result in poor precision. Perhaps more importantly, the lagged dependent variable is a "bad control" in that it is endogenous to the regressor. We wish to back out the effect of a returns shock in $t-3$ on the change in liabilities in t: if we condition on the liabilities in $t-1$, which is itself also affected by the past returns shock, then we will not identify our parameter of interest.

[29] Since market returns are changes in market equity valuations, taking the first differences in logs provides a tight conceptual link between the outcome and the regressor. Using levels would mean that the outcome is highly correlated with bank size. This would raise concerns about stationarity. Using levels could also result in a regression that is heavily influenced by a few large banks, given the highly skewed distribution of bank size. For the same reason, we do not weight our regressions: the distribution of bank size is highly skewed and so a weighted regression would be equivalent to a regression with only a handful of the largest banks. If the variance of the residuals were lower for larger banks, then using weights would yield a more-efficient estimator. Empirically, however, the variance of the residuals does not appear to vary substantially by bank size.

[30] This is the first quarter in which we can identify which banks are top-tier BHCs from the FR Y-9C.

X_t as follows:

$$\underbrace{r_{it}}_{\text{Raw returns}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t \beta_i}_{\text{factor loadings}} + \underbrace{\varepsilon_{i,t}}_{\text{Idiosyncratic Component}}. \tag{2}$$

The vector of factor loadings β has dimension $K \times 1$ and the matrix of factors X_t has dimension $T \times K$. We include the same factors as in Gandhi and Lustig (2015), namely the three Fama-French factors (Fama and French, 1993), a credit factor calculated as the excess returns on an index of investment-grade corporate bonds and an interest rate factor calculated as the excess returns on an index of 10-year U.S. Treasury bonds.[31] (See Appendix Section B.1 for further details on the risk-adjustment process.) The idea behind risk-adjusting returns and using their innovations is that we want to isolate information about banks' cash flows as opposed to discount rates, the latter which are driven by aggregate movements in the factors.[32] Crucially, we rely on the efficient-markets hypothesis according to which excess returns variations should be unpredictable *ex ante* after adjusting for the risk premium. By stripping out the predictable components of returns, the innovations, $\varepsilon_{i,t}$, to the risk-adjusted returns are ex-ante unpredictable across banks. This forms the basis of our identification strategy: we treat the cross-sectional variation in $\varepsilon_{i,t}$ as unanticipated shocks that perturb bank equity. In the Appendix (see Figure 4), we show for the largest four banks that the time series of $\varepsilon_{i,t}$ indeed resembles white noise. In Section 2.2, we conduct a variety of robustness checks to validate our identification strategy and interpretation of $\varepsilon_{i,t}$ as cash flow shocks. For the rest of the paper, we refer to these innovations, $\varepsilon_{i,t}$, as returns shocks.

Impulse responses. How do shocks affect banks' balance sheets, financing, and payout choices? We estimate impulse-response functions for liabilities, market capitalization, book equity, market leverage, and the common dividend rates and show the results in Figure 7. To normalize the effect, we report the response to a negative one percent returns shock. The y-axis of our plots shows the contemporaneous response ($-\beta_0$ for pre-crisis and $-\beta_0 - \gamma_0$ for post-crisis) as quarter 1, the cumulative response one quarter later ($-\beta_0 - \beta_1$ and $-\beta_0 - \beta_1 - \gamma_0 - \gamma_1$) as quarter 2, and so on. If banks maintain a target market-leverage ratio, then we would expect banks to respond to a negative wealth shock (which mechanically increases market leverage) by moving back towards their target leverage. As we can see from the impulse-response function of market leverage in Panel a of Figure 7, the data is consistent with an apparently slow adjustment back to the target, presumably due to adjustment costs. As discussed before, adjustment costs are a key ingredient of our Q-theory.

[31] The three Fama-French factors are downloaded from Ken French's website. The credit factor is the excess returns on the Dow Jones Corporate Bond Index that we download from Global Financial Data. The interest rate factor is the U.S. 10-year Treasury Bond Total Return Index (ltg) that we also download from Global Financial Data. We use the one-month risk-free rate from Ken French's website to calculate excess returns.

[32] The results for simply adjusting returns with a time fixed effect are qualitatively and quantitatively similar and are also reported in the Appendix Section B.2.

The impulse response of the log market leverage, defined here as log(liabilities/market capitalization), is simply the difference between the response of the log market capitalization (Panel b) and the log liabilities (Panel c).[33] The impulse-response function for the log market capitalization reveals that in the quarter of the impact of a returns shock, a mechanical effect on the denominator dominates and explains the jump in market leverage. The adjustment on the numerator is slow and the effect of returns shocks on market leverage does not vanish, even after five years. This yields our fourth fact:

Fact 4. *Banks appear to operate with a target leverage ratio to which they only return slowly after shocks, suggesting adjustment costs that are consistent with the Q-theory.*

A few noteworthy differences in the pre- and post-crisis impulse responses emerge from Figure 7. First, banks adjust their leverage ratios more quickly during the post-crisis years compared to the pre-crisis. As a way to delever and return to the target leverage ratio, banks can increase their equity and/or decrease their liabilities. Second, Panel b shows that, during the pre-crisis period, market equity does not change after a shock decreases it (a returns shock mechanically lowers equity one-for-one). Third, by contrast, in the post-crisis period, half of the impact is reversed over 5 years. In particular, Panel c shows that banks decrease liabilities by around 0.6% during the pre-crisis period and but only by 0.2% in the post-crisis in response to a 1% returns shock. Panel d, in turn, shows that book equity values adjust slowly in the pre- and post-crisis periods. This is consistent with Section 2.1, where we show that book values respond only slowly to losses that are more quickly reflected in market returns. The final observation regards the behavior of dividends: In Panel e of Figure 7, we estimate the impulse response of the common dividend rate.[34] The response of the common dividend rate to a negative returns shock (top left panel) is surprisingly positive pre-crisis. This is driven by the initial mechanical effect on the denominator. Market equity falls in response to a negative returns shock. Post-crisis, the initial positive mechanical effect is overtaken by the negative effect on dividends. All in all, these impulse responses show that banks switch from responding exclusively by decreasing their assets and liabilities in the pre-crisis period to a combination of balance sheet and equity adjustments in the post-crisis period, with equity adjustments being more important. To summarize, our final stylized fact is as follows:

Fact 5. *Prior to the crisis, banks adjust leverage primarily by reducing debt and keeping equity unchanged. Post-crisis leverage adjustments appear to be more gradual. During this period, banks also adjust their leverage by raising equity.*

Identification and robustness. Our interpretation of the estimates relies on the assumption that bank-specific variations in risk-adjusted bank stock returns identify cash flow shocks on ex-

[33] The impulse responses of leverage instead defined as log(liabilities + market capitalization)/market capitalization) are nearly identical.

[34] For this ratio, we use $\Delta \log(1 + y_{i,t})$ as the outcome variable in our specification 1, since $\log(1 + y_{i,t})$ provides an approximation to percentage points and flow variables such as dividends can be equal to zero.

isting portfolios, such as specific default shocks, as opposed to shocks to the profitability of future business opportunities. We conduct various analyses to alleviate identification concerns, including a narrative approach to validate our interpretation of the returns shocks as being unanticipated and specific cash flow shocks.

As stated above, one could be concerned that the returns shocks capture idiosyncratic information about the relative profitability of a bank's future portfolio (e.g., the default rate on this bank's future mortgages) and, thus, affect the bank's problem through channels other than perturbing equity. If a bank's expected returns on its future assets fall, then this bank would want to reduce its equity or lower its scale for a reason that would be unrelated to a target leverage ratio and adjustment costs.

To investigate this concern, we study how banks' liquid asset ratios respond to a negative returns shock. If negative returns shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolios into liquid assets. We test this notion by looking at the impulse-response function of banks' liquidity ratios, calculated as (cash + treasury bills) / total assets. The impulse-response function in the Appendix, Figure 9, shows no statistically significant response pre-crisis. There is a small temporary response post-crisis that is reversed within a few quarters (recall that we show cumulative responses).[35] In sum, banks do not tilt their portfolios towards safe and liquid assets in response to our returns shock, which pushes against a story of worsening investment opportunities.

The lack of response of the liquid-asset ratio is suggestive for our interpretation of the returns shocks. However, we cannot fully rule out that the shock picks up information about the profitability of future assets. To provide additional corroborating evidence for our identification strategy, we use a narrative approach, which is detailed in Appendix B.3. To this end, we take the largest positive and negative values of the returns shocks, $\varepsilon_{i,t}$, over the sample period for each of the four largest banks (J.P. Morgan Chase, Bank of America, Citigroup, and Wells Fargo). We then search various newspapers for articles that mention the names of any of the four banks in the quarter for which the absolute value of $\varepsilon_{i,t}$ is high. Table 5 in the Appendix lists the results of our newspaper article search. In most cases, we can find supporting evidence for our $\varepsilon_{i,t}$ estimates. For example, in the second quarter of 2009, the Bank of America has a high and positive value of $\varepsilon_{i,t}$. Our article search reveals that this bank fares better in the stress test and exceeds expectations. In 1999 Q1, Citigroup has a large positive $\varepsilon_{i,t}$. This coincides with a Wall Street Journal article that states that Citigroup exceeds its profit expectations even though its profits fall. In 2001 Q1, a negative shock at Wells Fargo coincides with news reports that state that Wells Fargo's venture capital portfolios are incurring significant losses.

In Appendix Section B.3, we provide additional robustness checks. We verify that our results

[35] This is not a perfect test as perhaps banks would also want to raise liquidity in response to a cash flow shock on their current portfolio. A hypothetical example is the following: Suppose the bank is caught in unfair lending practices that cause a lawsuit. Banks might respond by increasing their cash holdings to prepare for the upcoming lawsuit.

are not driven by mergers, or by specific events during the crisis, by excluding mergers and the crisis years 2008 and 2009 from our sample. For more details refer to Appendix Section B.3.[36]

2.3 Taking stock

In this section we presented five facts about bank leverage, a key variable in models of banks. The denominator of leverage, net worth, can have two different empirical counterparts: book or market equity. However, these two measures diverge during the Great Recession (fact 1) and in ways related to their informational content (fact 2). Importantly, leverage constraints based on market equity measures do not seem to put a cap on market leverage during the crisis (fact 3). Furthermore, regulatory constraints, which depend on book leverage, seem to be circumvented by delaying losses. We also argue that banks operate as if they have a market-leverage target, but the adjustment to that target is slow (fact 4). Furthermore, pre-crisis, banks would predominantly reduce liabilities as a means to delever, but the deleveraging pattern changes in the post-crisis period and retained earnings gain a predominant role to the point that leverage adjusts faster (fact 5).

What do these facts mean for macro-finance-oriented banking models? They call for a theory that can explain book- and market-value differences and that can account for slow responses of leverage. They also call for thinking of leverage constraints as not directly binding but rather as affecting banks in a dynamic sense. In the neo-classical Q-theory of investment (Hayashi, 1982), the discrepancy between market and book measures follows physical adjustment costs that create a wedge between the average and the marginal cost of capital. Since bank assets consist mainly of financial contracts, it is difficult to interpret this difference as the result of decreasing returns to technology. However, there is a tradition in finance that explains financial adjustment costs through various financial frictions. For example, the illiquidity of loans can result from asymmetric information—Leland and Pyle, 1977b; Diamond, 1984b; Dang, Gorton, Holmström and Ordonez, 2017— or from fire-sale costs as in Shleifer and Vishny (1997). Thus, taking the insights from the neoclassical Q-theory, one route is to explain the slow adjustments and variation in Q through asset adjustment costs.

A model with only loan adjustment costs would not explain why book equity reacts so little during the crisis (upon a large shock) and would not explain the predictive power of Tobin's Q. For that reason, the Q-theory we develop in the next section is motivated by the idea of delayed accounting. We introduce loan adjustment costs for comparison and also because they are needed to explain the fifth fact, but we will show that delayed accounting can explain the discrepancies between book and market values, the predictability of Tobin's Q, the lack of binding constraints and also the delayed responses in market leverage. In particular, delayed accounting can explain

[36] We also test for heterogeneous impulse responses by type of bank. We do not find evidence of sizable heterogeneity, but we have limited statistical power to detect these differences.

the slow adjustment of market leverage because, upon a shock, leverage does not have to be adjusted immediately to satisfy regulatory constraints. Rather, liabilities are gradually reduced as losses are slowly recognized.

There is substantial work on delayed accounting in the accounting literature. In particular, like us, Laux and Leuz (2010) also argue that delayed accounting is a prevalent phenomenon during the Great Recession. In Appendix A.3, we further detail the bank accounting literature (Bushman, 2016; and Acharya and Ryan, 2016 also offer useful discussions). In macro-finance, delayed accounting is a topic that is being overlooked, with some exceptions. Among these, in Milbradt (2012) distorted incentives are brought about by Level 3 fair-value accounting and Caballero et al. (2008) note that regulatory constraints may be a factor contributing to evergreening. Motivated by the facts presented in this paper, we argue that delayed accounting is furthermore important to explaininig the dynamics of book and market leverage. We also argue that delayed accounting has important policy implications. We will argue that with delayed accounting, banks can weather loan losses and hope to recover some of the loans. On the flip side, delaying expected losses allows banks to maintain greater leverage or continue to extend loans to businesses that should fail (Caballero et al., 2008; Blattner et al., 2019).

3 Q-Theory

We now present our Q-theory of banks. The theory is inspired by the facts presented in Section 2. The main innovation is that the dynamics of leverage are affected by accounting rules. The model has three frictions: first, equity-financing frictions prevent banks from offsetting loan losses with equity issuances. Second, delayed loan-loss recognition induces a dynamic tradeoff between loan origination and the tightening of future regulatory constraints. Finally, we introduce loan adjustment costs that, although not central to our theory, allow us to contrast our Q-theory with models of adjustment costs and to fit the post-crisis responses. Later in the section, we match the model with the data and discuss the model's ability to replicate the facts we highlight above. Proofs and derivations are provided in Appendix C.

3.1 The Model

Environment. Time is continuous, infinite, and indexed by t.[37] There is a continuum of banks. Loan defaults are the only source of risk. Each bank maximizes the expected discounted value of dividends, C_t. Banks are risk neutral but prefer to smooth dividends across time. Their objective function is defined recursively as

[37] We choose a continuous-time setup for computational reasons: An earlier version of this paper presented the same model in discrete time. Whereas the cross-sectional properties of both models are quantitatively practically identical, the speed of computation is substantially faster in the continuous-time setup, something that facilitates the estimation step.

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f(C_s, V_s)\, ds \right],$$

where

$$f(C,V) \equiv \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi}}}{\{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi}-1}} \right].$$

C_t denotes dividend payouts at time t and f is a Duffie-Epstein aggregator with a time discount rate $\rho > 0$, an intertemporal elasticity of substitution (IES) of $1/\theta$, and a risk aversion of ψ.[38] The Duffie-Epstein aggregator is the continuous-time counterpart of the Epstein-Zin preferences. The recursive formulation allows us to characterize banks as risk neutral (assuming $\psi \to 0$), as is standard in the theory of the firm.[39] At the same time, we retain flexibility to introduce dividend-smoothing motives that deliver smooth dividend payout patterns that are consistent with the empirical evidence; e.g., Lintner (1956); Dickens et al. (2002); Leary and Michaely (2011).

Bank balance sheet. Banks hold long–term loans that are funded with deposits and equity. At each instant, a fraction δ of loans matures. Maturity is innocuous but is introduced to distinguish between loan flows and stocks. Loan-default shocks are governed by a Poisson process N_t: a default event occurs with instantaneous probability σ and a constant fraction ε of loans defaults during said event.

Book accounting and fundamental values differ.[40] Defaults are slowly recognized in the accounting values, whereas they are immediately captured in the fundamental values. We denote the fundamental value of loans by L_t and the book value of loans by $\bar{L}_t$. Deposits, denoted D_t, are risk free and, thus, their accounting and fundamental values coincide. The banks' state variables are $\{L, \bar{L}, D\}$.

Financing frictions. Banks face an equity-financing friction: banks cannot issue equity and must rely on retained earnings to grow equity. As we noted, the banks' utility embeds a dividend-smoothing motive that violates the assumptions of the Modigliani-Miller theorem. The assumption of no equity finance can be relaxed by defining utility over net dividends.

[38] With other forms of dividend adjustment costs, dividend smoothing and risk aversion are coupled together. In the model, however, the bank can be risk averse (due to its franchise value) or risk loving (due to the option to default), as we explain later.

[39] Similar objective functions for banks are found in Bianchi and Bigio (2017) and Di Tella and Kurlat (Forthcoming). We are the first to model dividend smoothing in isolation from risk aversion.

[40] This is a novel feature of our model and is critical to capturing banks' ability to engage in evergreening and to avoid the immediate recognition of losses (see empirical evidence in Blattner, Farinha and Rebelo (2019)). Evergreening, as described in Caballero, Hoshi and Kashyap (2008), occurs when banks roll over a loan that will not be paid. The objective is to avoid registering losses. Since rolling over a loan does not require new funds, evergreening allows the bank to reduce its accounting equity without incurring a cost.

Laws of motion. The law of motion for the fundamental value of loans is

$$dL = \left(-\delta L + I\right) dt - \varepsilon L dN. \tag{3}$$

The change in the fundamental value equals new issuance I, net of maturing loans δL, minus the defaulting fraction of loans εL, in the event of default $dN = 1$.

The law of motion for book loans is

$$d\bar{L} = \left(-\delta L + I\right) dt - \alpha \left(\bar{L} - L\right) dt - \tau \varepsilon L dN. \tag{4}$$

This law of motion is similar to Equation (3). It fully captures the flow of repaid principal and new loans, $\left(-\delta L + I\right) dt$. Differently from Equation (3), the loan-default shock affects the book value with a delay. When the fraction ε of loans L defaults, only a fraction $\tau \in [0,1]$ is recognized immediately.[41] The term $\alpha \left(\bar{L} - L\right)$ reflects the speed of the loss recognition: α is the rate at which the gap between $\bar{L}$ and L closes. This partial recognition of losses in the model is motivated by the above discussion of evergreening and internal-valuation accounting.

The law of motion for deposits is

$$dD = \left[r^D D - \left(r^L + \delta\right) L + \Phi\left(I, L\right) + C\right] dt. \tag{5}$$

The bank issues deposits to pay out dividends, C, to fund the cost of new lending, $\Phi\left(I, L\right)$, and to pay the interest, r^D, on deposits. The bank receives an inflow of deposits from the interest, r^L, earned on loans and from the repayment of maturing loans, δL. The interest rates, r^L and r^D, are exogenous and constant.[42]

Loan market friction. The loan market is simplified for tractability. Each instant, banks choose a flow of new loans, I_t. The cost of issuing loans (in deposits) is

$$\Phi\left(I, L\right) = I + \frac{\gamma}{2}\left(I/L - \delta\right)^2 L,$$

where $\gamma \geq 0$. Given L, $\Phi\left(I, L\right)$ determines the increase (or decrease) in bank funds that stem from issuing (selling) new loans.[43]

[41] If $\tau = 0$, then books do not acknowledge any fraction of the loss on impact. If $\tau = 1$, then the book and fundamental values are equal since the losses are fully accounted for immediately. An initial condition for any bank is that $\bar{L}_0 \geq L_0$, which guarantees that $\bar{L}_t \geq L_t$.

[42] A constant interest margin is consistent with the empirical evidence in Atkeson, d'Avernas, Eisfeldt and Weill (2018) or Wang (2018). It is important to note that constant net-interest-rate margins are not informative about banks' interest rate risk exposure, as shown by Begenau et al. (2015) and Begenau and Stafford (2019). Gomez et al. (2020) and Haddad and Sraer (2019) present empirical evaluations of banks' interest rate exposures.

[43] This function has the following properties: First, $\Phi\left(\delta L, L\right) = \delta L$; that is, the bank does not incur a higher issuance cost when it replaces maturing loans. Second, $\Phi\left(I, L\right)$ is increasing in I ($\Phi_I\left(I, L\right) > 0$) for any $I > \bar{I} \equiv \left(\delta - 1/\gamma\right) L$. Note that the bank would never choose to sell assets (negative issuance) below $\bar{I}$ as the bank would

The function Φ induces an exogenous portfolio readjustment cost. For $I > \delta L$, the convexity in I in $\Phi(I, L)$ can be interpreted as representing decreasing returns to lending activities. For $I < \delta L$, the marginal cost of selling can represent fire-sale costs, as in Shleifer and Vishny (2011), different areas of lending expertise, or adverse selection in the secondary loans market.

Notation. Before we proceed, we define the ratio of the fundamental value to the book value of loans by $q_t \equiv L_t/\bar{L}_t \in [0, 1]$. The fundamental value of bank equity is $W_t \equiv L_t - D_t$. In addition, we define leverage as $\lambda_t \equiv D_t/W_t$. Our notion of q_t is purposely chosen to relate to Tobin's Q, which measures the ratio of market-to-book values: In our model, market values differ from fundamental values due to the external funding frictions and the accounting rules. Hence, q_t and Tobin's Q_t are different but related concepts.

Since the laws of motion feature drifts and jumps produced by defaults, we introduce the following notation: We use $\mu^x W$ to denote the drift of a variable, x; and μ^x is the drift scaled by wealth, W. Similarly, we use $J^x W$ to refer to the jump in x in proportion to W.

Liquidation. Banks continue to operate as long as they satisfy two constraints. First, banks are subject to a regulatory requirement that stipulates that deposits cannot exceed a fraction ξ of their book loans $D \leq \xi\bar{L}$, for $\xi < 1$. We can express this regulatory constraint more conveniently:

$$\lambda \leq \xi/(q - \xi). \tag{6}$$

Notice that the higher the q, the tighter the constraint. This is the main constraint of interest in our theory. Second, banks are subject to a market-based leverage constraint. This constraint requires bank leverage to be below $\bar{\lambda}$:

$$\lambda \leq \bar{\lambda} \equiv (1 - \varepsilon)/\varepsilon. \tag{7}$$

The value of $\bar{\lambda}$ guarantees solvency in all states, $(1 - \varepsilon) L - D > 0$.

If either the market or the regulatory conditions are violated, then the bank is liquidated. The value after liquidation is an exogenous value that is proportional to equity $V_o \equiv v_o W^{1-\psi}$. Because of its franchise value, the bank has incentives to avoid defaults: when active, it earns a positive intermediation spread. The problem is that a bank cannot fully control its leverage since loans are subject to random default shocks and adjusting leverage takes time. Let Γ^r be the state where regulatory liquidations occur and Γ^m the state where market liquidations occur. The overall states that trigger liquidations are $\Gamma \equiv \Gamma^r \cup \Gamma^m$.

actually have to pay another bank to take its loans. Third, $\Phi(I, L)$ is convex in I, $\Phi_{II}(I, L) \geq 0$. The latter property captures that the greater the loan issuance, the costlier each additional unit of I becomes; i.e., the bank has to issue more deposits on the margin. When a loan is sold, the bank receives fewer deposits in return for each additional unit $I < \delta L$ that is sold.

Bank problem. At each t, banks choose a dividend payout C and a flow of new loans I to solve the following problem:

Problem 1 *[Bank's Problem] The bank's policy functions are the solutions to the following:*

$$
\begin{aligned}
0 &= \max_{\{C,I\}} f\left(C, V\left(L,\bar{L},D\right)\right) + V_L\left(L,\bar{L},D\right)\mu^L W + V_{\bar{L}}\left(L,\bar{L},D\right)\mu^{\bar{L}} W + V_D\left(L,\bar{L},D\right)\mu^D W \\
&\quad +\sigma\left[V\left(\left(1-\varepsilon\right)L,\left(1-\tau\varepsilon\right)\bar{L},D\right) - V\left(L,\bar{L},D\right)\right]
\end{aligned}
$$

subject to(3), (4), (5) and liquidation $V\left(L,\bar{L},D\right) = \upsilon_o W^{1-\psi}$ *if* $\left\{L,\bar{L},D\right\} \in \Gamma$.

This problem is a standard Hamilton-Jacobi-Bellman (HJB) equation, associated with the Duffie-Epstein preferences. The last term is the change in value that results from liquidation.

Market value of equity. To construct market returns, we need an asset pricing model of banks. For that, we assume that banks are owned by outside investors. Investors can hold bank shares but they cannot directly lend or issue deposits. Because of this friction, the market value of equity diverges from W, the fundamental value of bank equity. Hence, we distinguish between three concepts of bank equity: the market value, the accounting value, and the fundamental value. Investors price bank shares according to the net present value of discounted dividends. We assume that investors are diversified so that a bank's idiosyncratic risk does not affect their discount factor. Because we are interested in the cross-sectional behavior of banks, we abstract from the expected aggregate shocks and the shocks to the investor's risk premia. For that reason, we endow the investor with a constant discount rate ρ^I.[44]

We construct a pricing equation for the bank's equity and map the underlying default shocks to the returns shocks, to be able to build the analogue impulse responses to those presented in Section 2. The market value of a bank, $S\left(L,\bar{L},D\right)$, satisfies the following recursive representation:

$$
\begin{aligned}
\rho^I S\left(L,\bar{L},D\right) &= C\left(L,\bar{L},D\right) + S_L\left(L,\bar{L},D\right)\mu^L W + S_{\bar{L}}\left(L,\bar{L},D\right)\mu^{\bar{L}} W + S_D\left(L,\bar{L},D\right)\mu^D W \\
&\quad +\sigma\left[S\left(\left(1-\varepsilon\right)L,\left(1-\tau\varepsilon\right)\bar{L},D\right) - S\left(L,\bar{L},D\right)\right].
\end{aligned}
\tag{8}
$$

This captures the dividend flow $C\left(L,\bar{L},D\right)$ determined by the payout policy. Naturally, this recursive representation reflects the law of motion of the bank's state variables. The valuation takes into account how changes in the state variables will affect future valuations, considering the effect of loan defaults. We assume that, upon liquidation, the investor receives zero for their equity.

An important point is that, implicitly, Equation (8) assumes that investors observe dN. Thus, information about losses is contained in market prices but not in book values. This is important

[44] In principle, we could allow discount rates to vary with time, ρ^I_t, which would not change the cross-sectional implications of the model. For simplicity, we set investors' discount rate to a constant.

for our analysis because it implies that market values are indeed informative about loan-default shocks beyond the information content in banks' books, as we argue in Section 2.

The ratio of market-to-accounting book values, Tobin's Q, is given by

$$Q\left(L,\bar{L},D\right)\equiv\frac{S\left(L,\bar{L},D\right)}{\bar{L}-D}.$$

We exploit this expression in Section 3.4. In the model, variations in Q will result from changes in the value of loans that are not immediately recognized on the books.

The optimization problem of banks. The bank's problem is scale invariant, as we show formally in Appendix 1. Banks with the same λ and q but different Ws are scaled replicas. We express the dividend rate as $c\equiv C/W$ and the loan growth rate by $\iota\equiv I/L-\delta$. The growth rate of bank equity satisfies the following stochastic differential equation:

$$\frac{dW}{W}=\underbrace{\left[\underbrace{r^{L}\left(\lambda+1\right)-r^{D}\lambda}_{\text{net interest income}}-\underbrace{\frac{\gamma}{2}\iota^{2}\left(\lambda+1\right)}_{\text{capital loss from adjustment}}-\underbrace{c}_{\text{dividend rate}}\right]}_{\equiv\mu^{W}}dt+\underbrace{\underbrace{\left(-\varepsilon\left(\lambda+1\right)\right)}_{\text{loss after default}}}_{\equiv J^{W}}dN,\quad(9)$$

where μ^{W} denotes the drift and J^{W} the jump in wealth after a default. The first term of Equation (9) captures the interest income on loans per unit of wealth, $\lambda+1$, net of the interest on leverage, λ. The second term is the net capital loss from adjusting the loan portfolio: when the issuance rate equals the maturing fraction of loans, loan issuances do not eat up bank capital. The greater the deviation, the greater the cost. The third term is the dividend rate. The final term is the jump in wealth after a default: the jump scales with leverage, λ. Hence, the expression captures how leverage increases risk. In turn, the law of motion for leverage is

$$d\lambda=\underbrace{\left(\iota-\mu^{W}\right)\left(\lambda+1\right)}_{\equiv\mu^{\lambda}}dt+\underbrace{\frac{\varepsilon\left(\lambda+1\right)}{1-\varepsilon\left(\lambda+1\right)}}_{\equiv J^{\lambda}}\lambda\,dN.\quad(10)$$

The drift in leverage is the difference between the growth rate of loans, ι, and the growth rate of equity, μ^{W}, scaled by leverage $\lambda+1$. The jump in leverage J^{λ} reveals how leverage changes with defaults. The jump in leverage increases with leverage. Finally, q has the following law of motion:

$$dq=\underbrace{\left(\iota+\alpha\right)\left(1-q\right)q}_{\equiv\mu^{q}}dt+\underbrace{\left(-\left(\frac{\varepsilon-\tau\varepsilon q}{1-\tau\varepsilon q}\right)q\right)}_{\equiv J^{q}}dN,\quad(11)$$

where μ^{q} denotes the drift and J^{q} its jump. Notice that q drifts to one with the loan growth and loss recognition rates, $\{\iota,\alpha\}$. The term $(1-q)\,q$ is intuitive: when q is close to zero, the ratio does not move because the numerator is very small relative to the denominator; when q is 1, accounting loans already reflect the fundamental value and the issuances do not change this aspect. The following proposition is a characterization of the bank's problem and the market values:

Proposition 1 *[Bank's Problem] Given* $\{\lambda, q\}$, $V\left(L, \bar{L}, D\right) = (1 + v(\lambda, q)) W - 1$, $C\left(L, \bar{L}, D\right) = c(\lambda, q) \cdot W$ *and* $I\left(L, \bar{L}, D\right) = (\iota(\lambda, q) + \delta) \cdot L$, *where* $\{v, c, \iota\}$ *solve*

$$0 = \max_{\{c,\iota\}} f(c, v) + \underbrace{v_\lambda \mu^\lambda + v_q \mu^q}_{\text{change in financial ratios}} + \underbrace{(1+v)\,\mu^W}_{\text{equity growth}} \tag{12}$$
$$+\, \sigma \underbrace{\left[\left(1 + v\left(\lambda + J^\lambda, q + J^q\right)\right)\left(1 + J^W\right) - (1 + v)\right]}_{\text{default jump in wealth}} \ \text{in} \ (\lambda, q) \notin \Gamma$$

and $v = v_o$ *for* $\{\lambda, q\} \in \Gamma$. *The bank's market value is* $S\left(L, \bar{L}, D\right) \equiv s(\lambda, q) \cdot W$, *where* s *solves a version of (12) where* $f(c(\lambda, q), s)$ *is substituted with* $c(\lambda, q) - \rho^I s$. *Finally, Tobin's* Q *is* $Q(\lambda, q) = s(\lambda, q) \times \left(\left(q^{-1} - 1\right)\lambda + 1\right)^{-1}$.

A takeaway from this proposition is that the model is scale invariant. Also, a key object of interest is the equity multiplier function, v: the term $1 + v$ represents how a unit of bank net worth is transformed into a unit of the certainty-equivalent net present value of dividends. By itself, v is the solution to (HJB) Equation (39). The novelty of this HJB is that it takes into account the growth in equity and the evolution of bank ratios. Notice that although the model is not stationary in levels, it is stationary in ratios. For common $\{\lambda, q\}$, the dividend and lending policies scale with W. Since the problem is scale invariant, the market capitalization is also proportional to W, where the price per unit of wealth s depends only on $\{\lambda, q\}$. Below, we describe how v governs the dynamics of leverage through its influence on the dividend and also through its issuance policies, given q.

3.2 Inspecting the Mechanism

We next proceed to explain the model's mechanics, turning on one friction at a time. Formal results are relegated to Appendix C.

Immediate accounting without loan adjustment costs. To shed light on the mechanics of our Q-theory, we first solve for the case where the default shocks are instantaneously recognized ($\tau = 1$) and there are no adjustment costs ($\gamma = 0$). In this case, $q = 1$ at all times so α plays no role. Since $q = 1$, only the regulatory liquidation matters and the liquidation boundary is given by a threshold leverage, $\Gamma = \{\lambda | \lambda > \xi / (1 - \xi)\}$. Also, since $\gamma = 0$, the bank can choose a jump in the stock of loans and deposits and, hence, controls a jump in λ.[45] By $\bar{J}^x$, we refer to the controlled (endogenous) jump for any variable x.

[45] In this version, the path of loans does not have to be continuous even in the absence of shocks.

A key object for the dynamics is the shadow liquidation boundary, Λ. The shadow boundary is the leverage rate such that upon a loan-default shock, leverage jumps exactly to the boundary of the liquidation set. Thus, Λ solves

$$\Lambda + J^{\lambda} = \xi / (1-\xi),$$

where J^{λ} is the jump in leverage after a default shock, starting from leverage Λ.

For this limiting case of the model, the multiplier v is a constant and leverage solves

$$\max_{\lambda \in [0,\xi/(1-\xi)]} \underbrace{(1+v)\left(r^L - r^D\right)}_{\text{value of levered returns}} \lambda + \sigma \left\{ (1+v) \underbrace{\left(1+J^W\right) \mathbb{I}[\lambda \leq \Lambda]}_{\text{wealth upon default shock}} + \underbrace{v_o \mathbb{I}[\lambda > \Lambda]}_{\text{liquidation value}} \right\}. \tag{13}$$

This problem clarifies that leverage results from a trade-off between intermediation profits and liquidation risk. The first term in the objective describes how leverage increases levered returns: intermediation profits $\left(r^L - r^D\right)$ increase equity at the margin. In turn, the marginal value of equity is the multiplier $1+v$. The second term is the value after a loan default. Defaults occur with intensity σ and lead to two possibilities. The bank can avoid liquidation if $\lambda \leq \Lambda$ and, in that case, the cost of default is only the reduction in the bank's scale by $1 + J^W$. Otherwise, the bank is liquidated and recovers v_o per unit of equity. Because the objective is piece-wise linear, the target leverage is a corner solution. The interesting parameter combination satisfies $\left(r^L - r^D\right) \in [\epsilon\sigma(1+v), \sigma v_o]$.[46] Under this combination, the expected returns from increasing leverage are positive up to $\lambda = \Lambda$. Past the shadow boundary, the hazard of default makes the benefit of increasing leverage negative. Hence, under those parameters, the bank sets its target leverage to $\lambda^* = \Lambda$.[47]

To guarantee that $\lambda^* = \Lambda$ always, the endogenous jump must neutralize the jump caused by defaults $\bar{J}^{\lambda} = -J^{\lambda}(\Lambda)$. With neither delayed accounting nor loan adjustment costs the bank can do that adjustment at no cost. When $dN = 0$, the dividend and loan issuance rates are constant. In particular, c^* is given by a formula that captures the wealth and substitution effects[48]

$$c^*(v) = \rho^{1/\theta}(1+v)^{1-1/\theta}. \tag{14}$$

[46] Otherwise, leverage is set to zero or the bank is liquidated upon its first default: If $\left(r^L - r^D\right) < \epsilon\sigma[1+\bar{v}]$, then the expected returns of increasing leverage are negative and $\lambda^* = 0$. If $\left(r^L - r^D\right) > \epsilon\sigma[1+\bar{v}]$, then the bank would like to lever up as much as possible. If $v_o > \left(r^L - r^D\right) > \sigma$, then the returns spread exceeds the bank's expected liquidation value σv_o. In that case, the bank operates at the fringe of solvency, setting λ^* to its permissible limit. As soon as it suffers a loan default, the bank is liquidated. Generically, leverage can be set to either of three values, depending on the parameters: to zero, to the shadow boundary value, or to the liquidation boundary value.

[47] Critical to this result is that the risk aversion is zero. Otherwise, leverage would involve an additional trade-off between the returns and the equity growth risk.

[48] Here, $1+v$ acts like a total return on wealth. When $\theta > 1$ ($\theta < 1$), the substitution (wealth) effect dominates and the bank retains (pays out) more dividends as v increases.

Given c^*, the loan issuance rate ι^* is such that leverage is constant, $\mu_\lambda = 0$. Finally, the equity multiplier solves

$$0 = f\left(c^*\left(v\right), v\right) + \left(1+v\right)\left(\mu^W + \sigma\left(J^W - 1\right)\right).$$

Turning back to our motivating facts, this limit case produces a leverage target, as desired. However, the impulse response of leverage to a returns shock looks like a blip with no persistence. In turn, the response of total liabilities looks like a one-time shock. Clearly, other aspects of the model are needed to match all of the facts highlighted above.

Delayed accounting without loan adjustment costs. We now study the case with only delayed accounting ($\tau < 1$ and $\gamma = 0$). This case explains most of the intuition of the general model. In this case, since losses are not immediately recognized , in general $q < 1$. The liquidation region therefore depends on q, and $\Gamma(q) = \left\{\lambda | \lambda > \min\left\{\xi / \left(q - \xi\right), \bar{\lambda}\right\}\right\}$. As a result, the shadow boundary is no longer a scalar but a function of q, which we now label $\Lambda\left(q\right)$. To construct the shadow boundary, we must consider the jumps J^q and J^λ. The shadow boundary $\Lambda\left(q\right)$ solves

$$\Lambda\left(q\right) + J^\lambda\left(\Lambda\left(q\right)\right) = \min\left\{\frac{\xi}{q + J^q\left(q\right) - \xi}, \bar{\lambda}\right\}. \tag{15}$$

As with immediate loss-recognition accounting, a loan default takes the bank from a point $\{\lambda, q\}$ in the shadow boundary to a point in the boundary of the liquidation set Γ. Again, for suitable parameter conditions, the dynamics are such that a bank will immediately delever until it reaches another point in the shadow liquidation boundary $\Lambda\left(q\right)$. The necessary loan sale to return to the shadow boundary induces a jump $\bar{J}^\lambda$, but it also alters q, inducing a jump $\bar{J}^q$. Hence, the bank no longer returns to the same leverage but rather a loan default takes the bank from a point $\{q, \lambda\}$ in the shadow boundary to another point, $\{q', \lambda'\} = \left\{q + J^q + \bar{J}^q, \lambda + J^\lambda + \bar{J}^\lambda\right\}$, in the shadow boundary.

It is important to understand the choices of $\{\lambda, q\}$ and $\{c, \iota\}$ along the boundary to understand the ability to match the impulse responses in the data. Figure 1 allows us to sketch these dynamics: the y-axis represents values of λ and the x-axis values of q. Starting from an arbitrary point $\{q, \lambda\}$ in the shadow boundary, a loan-default event takes the bank to a point on the boundary of Γ. Since the bank survives but wishes to avoid a future liquidation, it immediately sells assets to delever. This takes the bank to another point in the shadow boundary, $\Lambda\left(q'\right)$. Absent defaults, when $dN = 0$, the state variable $\{\lambda, q\}$ drifts along the shadow boundary. This is because to the right of the boundary, liquidation risk is positive. To the left of the boundary, the bank prefers to lever up to increase its equity returns. Hence, the choice of issuances and dividends must guarantee a smooth drift along the shadow boundary, slowly taking the bank to $q = 1$, as shown in the figure. Because the shadow boundary has a negative slope, $\Lambda_q\left(q\right)$, the bank must guarantee a decline in leverage as q increases with the pace of the loan-loss recognition.

Turning to the impulse responses: on impact, q falls whereas λ increases. As a result, the market leverage features a jump by more than the book leverage, a feature of our impulse responses. After the period of impact, λ will fall along the boundary whereas q approaches 1. This is consistent with a mean reversion in the market leverage and only a modest response in the book leverage, as in the data. We can also describe in detail the dynamics of the liabilities in the model. With delayed accounting, each point in the boundary is associated with a value $v(q)$. The dividend policy is modified to

$$c^* = \rho^{1/\theta} \left[\frac{(1+v(q))}{\left(1+v(q)+v_q \frac{(1+\Lambda(q))}{\Lambda_q(q)}\right)^{1/\theta}} \right]. \tag{16}$$

The expression reveals that even under delayed accounting, the dividend choice is still governed by a race between wealth (in the numerator) and the substitution effects (in the denominator). The substitution effect is modified because on the margin, the choice of dividends indirectly affects q. [49] Once the dividends are determined, to stay at the shadow boundary, issuances $\{\iota\}$ must satisfy the restriction that $\Lambda_q(q) = \mu^{\lambda}/\mu^{q}$. We can obtain a formula for issuances that guarantees that the bank stays in the shadow boundary:

$$\iota(q) = \mu^{W}(q)\,\omega(q) - \alpha(1-\omega(q)) \tag{17}$$

with the weight given by

$$\omega(q) \equiv \frac{(1+\Lambda(q))}{q(1-q)} \Big/ \left(\frac{(1+\Lambda(q))}{q(1-q)} - \Lambda_q(q) \right).$$

The intuition for the slow adjustment of liabilities that what we find in the impulse responses is also found in the formula. Upon a default shock, losses are slowly recognized on the books, at the rate α. As the losses are recognized, and q increases, banks must take gradual actions to delever because their fundamental leverage is too high. The expression banks use for their issuances captures how they chose to use issuances to reduce their leverage. Leverage has a tendency to fall because retained earnings increase equity, $\mu^{W}(q)$; recall that $\Lambda_q(q)$ has a negative slope. Hence, the higher the growth in equity, the less the need to reduce loan issuances to reduce leverage. By contrast, a higher α implies that the losses are recognized faster. Hence, with a higher α, the bank requires lower (and possibly negative) issuances. Since liabilities increase one-to-one with issuances, the expression captures how liabilities fall after a default shock. This formula captures the pattern for issuances, which fall with the rate of loan-loss recognition. However, notice that the issuance rate only affects liabilities slowly which, unlike the version without delayed accounting, is capable of explaining the slow response of the liabilities.

To summarize, the model with delayed accounting is consistent with dynamics where, upon a

[49] Because leverage must stay in the boundary, $(1+\Lambda)/\Lambda_q(q)$ measures the change in q corresponding to a change in leverage that keeps the bank along the boundary. The change in q effectively has a price of v_q.

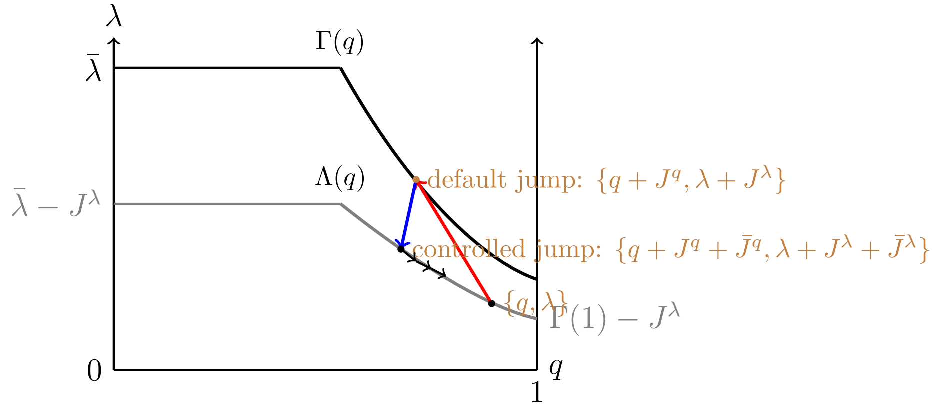


Figure 1: Illustration of $\{\lambda, q\}$ dynamics.

default shock, the bank takes quick action to return to the shadow boundary. Once at the shadow boundary, the bank continues a slow deleveraging, which induces a slow response of market leverage and total liabilities, as we observed in the data. Thus, even though there are no loan adjustment costs, the leverage adjustment is gradual along the shadow boundary as the losses are slowly recognized on the balance sheet. This slow adjustment forms the basis of our Q-theory.

Delayed accounting and loan adjustment costs. We now discuss the dynamics that allow for loan adjustment costs ($\tau < 1$ and $\gamma > 0$). These only introduce inertia to any attempt to deviate from $\iota = 0$, but the mechanics are very similar to those of the earlier case. From the previous limit cases, we know that banks have a target for leverage that trades off returns for liquidation risk. We also know that without adjustment costs, upon receiving a default shock, banks rapidly delever to return to the shadow boundary and then slowly drift along this boundary. Loan adjustment costs slow down the endogenous response on impact and slightly perturb the dynamics along the shadow boundary.

Unfortunately, this case must be solved numerically. Figure 8 presents a numerical solution to v, the drift of λ and the policy functions $\{\iota, c\}$ for different values of λ in the x-axis and for various cross sections of q. The figure is constructed under the calibration detailed in the following section. Panel (a) depicts v. The trade-off between returns and liquidation risk is evident from the shape of v: for a fixed q, the value v is first increasing but becomes flatter as leverage approaches the shadow liquidation boundary. Past the liquidation region, the value function jumps to a value that is equal to its liquidation value. From the figures, we can also observe that v is decreasing in q, a reflection that a bank that seems better capitalized in its books faces fewer regulatory constraints.

Panel (b) presents the drift of leverage for various values of q. The figure shows how, in this complete version of the model, leverage is again mean reverting. The target leverage is the point where the expected change in λ is zero: when leverage is low the drift is very high, reflecting the effort to increase leverage. To the right of the shadow boundary, the drift is negative and, furthermore, very steep, reflecting a slightly smoother version of the jumps that occur without adjustment costs.

As before, the behavior of leverage is governed by the bank's payout and lending policies. The dividend rate in this case solves

$$c^{*}=\rho^{1/\theta}\left[\frac{(1+v)}{\left((1+v)-v_{\lambda}\left(\lambda+1\right)\right)^{1/\theta}}\right]. \tag{18}$$

Relative to the dividend policy in Equation (14), the substitution effect is also corrected by the change in the value function as a result of the change in leverage produced by dividends.

The loan growth policy solves

$$\underbrace{\gamma\left((1+v)-v_{\lambda}\right)\cdot\iota^{*}}_{\text{marginal cost of change in assets}} \quad = \quad \underbrace{v_{q}\frac{(1-q)\,q}{(\lambda+1)}+v_{\lambda}}_{\text{marginal benefit of change in financial ratios}}. \tag{19}$$

The left-hand side is the marginal cost of changing loans. These deviations cost γ in equity and have a marginal value of $(1+v)$ but carry an effect by increasing leverage v_{λ}. The right-hand side is the marginal benefit of changing the bank's ratios. Panels (c) and (d) of Figure 8 show how the desire to delever is reflected in the bank's loan growth and payout policies. When leverage is low, the bank has incentives to lever up; it cuts back on dividends and lending. Close to the liquidation boundary, the bank is eager to delever. It cuts back its loans growth and slashes dividends. The effects are more dramatic the more realistic its books.

The desire to remain in the neighborhood of the shadow boundary is still present. This is evident from the invariant distribution depicted in Figure 10 but not from the Equations (18) and (19) because this desire is encoded in the shape of v, which is not evident from the formula. This is why it is important to describe the mechanics of the model in layers. For completeness, Figure 9 is the analogue of Figure 8 along the q-dimension.

3.3 Calibration and Estimation

We now describe the calibration and estimation procedures and then investigate the model's ability to reproduce the five facts. We use quarterly data from 1990 Q3 to 2015 Q4, as described in Section 2, to produce the target moments. Thus, all corresponding model moments are also at the quarterly frequency. To keep the parametrization tractable, we calibrate $\left\{r^{L},r^{D},\delta,\xi,\rho,\rho^{I},\varepsilon,\sigma,\alpha\right\}$ independently, matching model moments to target moments in the data. Then, conditional on these calibrated parameters, we jointly estimate $\{\gamma,\theta,\tau\}$, the parameters that govern the delay in the balance sheet responses. As in Section 2, we break the sample into two periods, corresponding to the pre- and post-crisis periods, and estimate the pre- and post-crisis values for $\{\gamma,\theta,\tau\}$ to match the impulse-response functions to returns shocks, as discussed in Section 2. The parameter values are listed in Table 2. Table 3 presents both the targeted and untargeted moments in the data and the corresponding model moment.

Calibrated parameters. The exogenous returns on loans and deposits, r^L and r^D, are respectively set to 1.01% and 0.51%, consistent with the quarterly yield on loans (total interest income on loans divided by total loans) and the rate banks pay on their debt (total interest expenses divided by interest-bearing liabilities) in bank call reports. These values are consistent with the calibration in Corbae and D'Erasmo (2019).

We set the capital requirement parameter ξ to 92.6% in order to reflect a tier-1 risk-based capital ratio requirement of 8%, a value for which a bank is considered well capitalized.[50] This means that banks' book-leverage ratios (debt-to-book equity) cannot exceed 14.

The parameters $\{\alpha, \tau\}$ directly speak to our Q-theory. Whereas τ accounts for loan-loss recognition on impact, α governs the speed of loan-loss recognition over time. We estimate τ but calibrate α because we have a direct counterpart for α. Namely, we set α to 4%. With this value for α, 65% of unrecognized losses are recognized within 10 quarters.[51] This delay is consistent with Figure 2, Panel (d), where the net charge-offs taper off by the end of 2010, about two and a half years after the trough in bank market values.

In our model, banks choose a book-leverage ratio on the shadow boundary. The distance between the shadow boundary and the liquidation set is determined by the size of the idiosyncratic loan-default shock ε. Thus, once ξ is fixed, we pick $\varepsilon = 0.25\%$ to match the ergodic mean of banks' book-leverage ratio to the average pre-crisis book-leverage ratio of 10.6. We choose the pre-crisis period to calibrate this idiosyncratic shock parameter as the post-crisis sample is more likely to capture the effects of the financial crisis, which is an aggregate event.

Market leverage is a function of the market price of equity, which in turn is increasing in the bank's discount rate, ρ. Therefore, given a value for book leverage that is pinned by the regulatory parameter, ρ, affects market leverage by moving dividends and, therefore, the market price of equity, we set ρ to 0.25% to target an average market leverage of 6.8 for the pre-crisis period. Note that while most of the parameters discussed in this section are calibrated as if they influence only their target moment, ρ and ε alone do not pin down book leverage and market leverage. We would ideally estimate ρ and ε jointly with τ, θ, and γ, if we were not limited by computational tractability. For this reason, the targets are not matched exactly.

Finally, given the value for the default shocks ε, the default intensity σ is set to match the mean net charge-off rate of 0.48% per year in the full sample. We use the full sample of the net charge-off rates to allow for a larger time series as credit events are rare. Also, because investors are risk neutral, their discount factor ρ^I approximately equals the average market returns on bank shares. Thus, we set ρ^I to 3.5% to approximately produce that value for banks' market-equity

[50] See the publication by the Federal Reserve.

[51] See Appendix C.7 for the derivation.

returns.

Estimated parameters. We estimate $\{\tau, \theta, \gamma\}$, the parameters that govern the speed of the responses of the bank's balance sheet, using the simulated method of moments. Each parameter in this subset is directly associated with a different friction that affects the balance sheet adjustment process after a loan-default shock. Thus, these parameters determine the model's ability to replicate facts 4 and 5.

We estimate these parameters to match the impulse-response functions of market leverage, book leverage, and bank liabilities to a returns shock in the data. These impulse responses render a transparent identification: the sensitivity of loan adjustment costs, γ, governs the cost of deleveraging through asset sales. Thus, γ affects the speed at which banks can delever by selling liabilities along the shadow boundary. A higher γ translates into a slower response of liabilities to a returns shock. In turn, the desire to smooth dividends is governed by θ—see Equation (18). Thus, θ governs the speed at which leverage falls through the effect of dividends on retained earnings. A lower value of θ turns the bank's objective closer to linear and makes the bank deleverage faster by cutting back dividends and increasing retained earnings. Hence, the response of market leverage is informative about θ. Finally, recall that τ governs how much of a loan-default shock is recognized on impact. Therefore, τ directly maps into the response of book leverage, on impact. For that reason, because τ governs the response only on impact, we only use the first period response of book leverage as a target to identify τ.

To produce the analogue of the estimated impulse responses to the returns shocks in the model, we solve and simulate the model. We run the same specification for the impulse responses of Section (2). Given that the impulse responses of liabilities and market leverage differ pre– and post-crisis, we estimate values for θ and γ for both periods; we label these values as $\{\theta^{pre}, \gamma^{pre}\}$ and $\{\theta^{post}, \gamma^{post}\}$, correspondingly for these estimates. We construct excess returns shocks by first calculating the realized returns on equity between the adjacent quarters and the cross-sectional equity returns. The bank-specific returns shock is then just the difference between a bank's individual realized returns on equity and the cross-sectional average of these returns. The latter absorbs any potentially time varying aggregate effects on banks' equity returns, similar to only using a time fixed-effect specification. Formally, the model is over identified because each impulse response in the data effectively contains 21 moments, one for each β_h in Equation (1). However, model-generated moments such as these are highly correlated so the effective degree of over-identification is lower.[52]

To generate a panel of banks as in the data, we first compute the stationary distribution by simulating 10,000 banks until the cross-sectional mean and standard deviation of λ and q are approximately constant. We then take this cross section as the initial condition and simulate 66 quarters that represent the pre-crisis period. With this sample, we run the same pre-crisis cross-

[52]Each impulse response is well approximated by only two moments, a jump on impact and a persistence.

sectional regressions on the model-simulated data, as we did with the actual data, to estimate θ^{pre} and γ^{pre}. To simulate the post-crisis, we hit the stationary pre-crisis economy with an aggregate shock and continue simulating for an additional 33 quarters. That is, to estimate the post-crisis parameters, θ^{post} and γ^{post}, we start banks from their pre-crisis stationary distribution and hit all banks at once with a 2.5% aggregate loan-default shock in one quarter. We chose a 2.5% default shock as this number is in line with the accumulated loan-default shocks from 2008 through 2010, which is consistent with our assumed length of loan-default recognition. Once again, we estimate the same regression and construct impulse-response functions on the model-simulated data as we did with the actual bank data. We discuss the values from this estimation and the resulting model fit in the next subsection.

Estimated values, model fit, and interpretation. To match the initial impulse response of book leverage, τ needs to be small, about 1%. The impact response of book leverage to the returns shocks does not change across periods. For this reason, we estimate τ only for the pre-crisis period. The pre-crisis period estimates of the other parameters are $\theta^{pre} = 2.3$ and $\gamma^{pre} = 0.01$. For this low adjustment cost value, the effect of γ on the model is negligible in the pre-crisis period in the sense that the dynamics of the model are almost identical to not having adjustment costs. This is interesting because it means that delayed accounting—and dividend smoothing—can account for all of the slow balance sheet adjustments after the shocks.

The post-crisis impulse responses of market leverage and bank liabilities produce values of $\theta^{post} = 1.7$ and $\gamma^{post} = 4.0$. This means that, unlike in the pre-crisis period, the post-crisis period model estimation requires loan adjustment costs to fit the data. The estimation thus reflects some of the narratives around the financial crisis: a higher value of γ implies that banks face higher costs to selling loans. This feature fits the narrative of aggravated fire-sale externalities during the post-crisis period. In turn, a lower value for θ makes the banks' objectives closer to being linear. This means that banks accept larger adjustments to dividend payouts and, therefore, can rely more on internal equity accumulation as a mechanism to delever. This estimate is in line with the fact that banks cut dividends during the period.[53] We return to these results in Section 3.4, when we discuss the model's ability to reproduce the impulse responses again.

To conclude the evaluation of the calibration, Table 3 compares the moments generated by the model and those obtained from the data: our model fits most data moments well, with the exception of pre-crisis leverage and (untargeted) dividends. The model fits the charge-off rates both in the pre- and post-crisis. Average market-equity returns in the model match the data during the pre-crisis sample but differ for the crisis sample. This is because, in both cases, we calibrate the parameters to their pre-crisis values. With a value of 9.4, the model overshoots its 6.8 data target for market leverage during the pre-crisis period. As noted above, market leverage depends on more parameters than just ρ so it is more difficult to closely match market leverage

[53] See for example, https://www.ft.com/content/7a4aa5bc-0a85-11de-95ed-0000779fd2ac.

without a computationally intensive joint estimation. The model perfectly fits the untargeted market leverage moment during the post-crisis period. The model hits the book leverage in the targeted pre-crisis period but it does not generate the same reduction in the book leverage as in the data. This is not surprising because our model abstracts from the increase in bank capital requirements that occurred during the post-crisis period and forced banks to decrease their book leverage. Finally, our model generates a similar level in the market-to-book-equity ratio during the pre-crisis (1.53 in the data vs. 1.32 in the model) and a reduction in the market-to-book equity ratio in the post-crisis (0.90 in the data vs. 1.108 in the model). Thus, our simple model explains about one-third of the reduction in the market-to-book ratio of banks from the pre- to the post-crisis period.

The bottom panel of Table 3 shows the cross-sectional average and standard deviations of key model variables. It also includes the two unobservable state variables of our model: fundamental leverage λ, and q, the discrepancy between banks' fundamental value of assets and their corresponding accounting values. The fundamental leverage is 18 in the pre-crisis period and jumps to 22 in response to the 2.5% aggregate default shock. The average value for q is 0.977, implying that the fundamental values and the accounting values differ by 2.3%. The post-crisis q is slightly lower compared to the pre-crisis q. The loan growth rate is about 2.4% and the growth rate of equity is around 1.2%. Our model also produces values of equity (equity price) of 1.91 in the pre-crisis and 1.97 in the post-crisis. The valuation is roughly constant across periods. While bank dividends fall slightly during the post-crisis period, banks operate with higher leverage (and therefore higher equity growth).

The distribution of the state variables and market leverage are reported in Figure 10. Panel (a) shows how the distribution of $\{\lambda, q\}$ traces out a shadow boundary. Panel (b) contrasts the distribution of market leverage with the distribution in the data. Although book leverage has virtually no dispersion, market leverage captures the substantial variation that is driven entirely by variations in q.

3.4 Taking Stock: Matching Facts 1 to 5

Fact 1. Market- and book-equity value divergence. The first fact we highlight in the paper is the divergence between book and market bank-equity values during the financial crisis of 2008. We now discuss the extent to which variations in Tobin's Q can be explained with our model. In our Q-theory, Tobin's Q can diverge through changes in the price per share, s, and the discrepancy between the book and fundamental values, captured by q. To that end, we produce an approximation of the law of motion of q in our model (see Equation 11). We denote by $\boldsymbol{x}$ the aggregate version of a variable x. The law of motion for aggregate $\boldsymbol{q}$ is approximately

$$\frac{d\boldsymbol{q}}{dt} \approx (\boldsymbol{\iota} + \alpha)(1 - \boldsymbol{q})\,\boldsymbol{q} - \sigma\varepsilon\,(1 - \tau\boldsymbol{q})\,\boldsymbol{q}.$$

Setting $d\boldsymbol{q}/dt = 0$, we obtain an approximate value of $\boldsymbol{q} = 0.977$, using our pre-crisis model values (see Tables 2 and 3). This approximated value is identical to the average simulated value of q reported in Table 3. With an aggregate loan-default shock ϵ of 2.5%, which is common to all banks, the jump in the aggregate $\boldsymbol{q}$ is approximately

$$J^{\boldsymbol{q}} = -\epsilon \frac{(1-\tau\boldsymbol{q})}{(1-\tau\epsilon\boldsymbol{q})}\boldsymbol{q} \approx -2.4\%.$$

Thus, since the initial q is close to one and τ is close to zero, the jump in q is almost equal to the size of the aggregate default shock.

We can decompose that jump in Tobin's Q into the effect that is exclusively attributed to the jump in q as opposed to a change in banks' stock price. This approximate response is given by

$$\Delta Q \text{ due to } q \approx J^q \cdot (\lambda + 1) = -45\%.$$

The approximation states that the jump in Tobin's Q attributed to q is the jump in J^q times the fundamental leverage. The value of this approximation is rather large. A 2.5% shock in loan losses gets amplified almost twenty times. Recall that although book and market leverage are around 10, fundamental leverage is about 20, due to the initial accounting differences. Hence, although q is close to 98% in normal times, implying that banks' books are an almost accurate description of their reality, small unrecognized losses become magnified by fundamental leverage, accounting for a decline of almost 45% in Tobin's Q.

The quality of this approximation is good and telling of the contribution of delayed accounting to rationalizing the changes in Tobin's Q in the data. In the simulations used to construct Table 3, we feed the model with a 2.5% loan-default shock. With this shock, on impact the induced reduction in the average Tobin's Q at the moment of the shock is 28%. In the simulations, the isolated effect of little q on the drop in big Q is 53%, which is close to the analytic approximation above. The overall effect is offset by an increase in the market value per unit of fundamental wealth, s, which increases by 25%; recall that the marginal value of s increases with the jump in λ. Thus, the 28% drop in Q that our model can generate accounts for two-thirds of the change in the Tobin's Q during the 2008 financial crisis, which falls by 42%. Our Q-theory explains this large portion of the decline without changes in aggregate risk premia, any changes in the regulatory landscape, or any decline in implicit government guarantees.

Fact 2. Predictive power. Our second fact of interest is the predictive power of Tobin's Q in terms of book-equity returns and loan charge-off rates even up to two years. Our model captures this effect because market values capture losses that are unrecognized in the accounting books. We replicate Figure 3 with data generated by the model (see Figure 13 in the Appendix). This figure shows that market equity contains predictive power for returns on equity and loan losses over and above the information contained in book equity. To see why, recall that without considering the changes in the price per unit of real equity, upon a default shock, returns fall by approximately J^W. A bank's charge-off rate per unit of equity is approximately $\alpha \cdot (1/q - 1)(\lambda + 1)$. Since the jump J^q is correlated with the jump J^W, the predictability follows by construction.

Fact 3. Equity buffer. The third fact highlights that banks keep a buffer of book equity away from the maximal leverage allowed by regulation. Earlier we discussed that liquidation is not desirable because of liquidation costs. For this reason, banks stay at a shadow boundary, which guarantees solvency in case of a default shock event. Note that in our model, while the default event is random, the size of the default shock is not. In our model, average book leverage is 12.4, almost 2 points below the maximal admissible value, given the regulatory constraint. The model also predicts that in a recession, the fraction of banks near the regulatory constraint increases. While our model features little cross-sectional variation in book leverage, it captures some of the cross-sectional dispersion in market leverage, as can be seen in Figure 11b. All cross-sectional dispersion in market leverage must come from dispersion in equity valuation and cross-sectional differences in fundamental value to the accounting value of bank assets. This is because our model features negligible dispersion in book leverage. Thus, even though we abstract from features in the data that could generate dispersion in book leverage, for example, different business models, the accounting mechanism alone generates dispersion in market leverage.

Fact 4. Target leverage and slow adjustments. Banks appear to have a target leverage ratio but only slowly respond to deviations from it (see the impulse-response estimations described in Section 2). Figure 12a compares the pre-crisis impulse responses to returns shocks of total liabilities (left panel) and market leverage (right panel) of the model with the data. The black lines represent the average estimated response from the data—the shaded area represents their 95% confidence intervals. The blue lines represent the analogue impulse response in the model. The red line represents the model responses that correspond to the model without balance sheet adjustment costs and accounting frictions.

The figure shows that the model generates an initial (mechanical) jump and a slow adjustment of market leverage as well as a slow adjustment of liabilities, as in the data. Note that in the pre-crisis period, our Q–theory does not rely on loan adjustment costs to reproduce the data as γ is near zero. Once we allow for accounting values to differ from fundamental values, i.e., $\tau < 1$, our model does not require further adjustment costs to generate slow adjustments in the liabilities. The red lines in Figure 12a also show that a dividend-smoothing motive cannot account for the impulse-response function of the data without accounting frictions. In fact, without accounting or balance sheet frictions (red lines), banks immediately reduce assets in response to a negative wealth shock while leaving leverage unchanged.

Fact 5. Post-crisis leverage adjustment. Bank leverage adjusted faster during the crisis and immediate post-crisis period, compared to the pre-crisis period.. Unlike the pre-crisis, adjustments in leverage were driven by increases in equity. The response of assets, as captured by the response in liabilities, was slower during this period. To capture these features, our model needs higher balance- sheet- adjustment costs and a lower dividend-smoothing motive as captured by a higher estimated value for γ and a lower estimated value for θ, respectively. A higher estimated value for γ in the post-crisis means that selling assets is costlier. These post-crisis estimates are consistent with several narratives regarding the aggravation of frictions during the crisis. The reduction in θ during the crisis/post-crisis period means that the model responds to shocks by increasing retained earnings (and therefore reducing dividends), which leads to more equity. The estimated lower value for θ could reflect pressures by regulators to cut dividends during the crisis to bolster banks' equity positions. All in all, our Q-theory generates all five facts fairly well.

3.5 Effects of Accounting Rules

In the previous sections, we argued that delayed accounting is key to explaining the five facts this paper highlights. But why should we care about accounting rules? We argue that a reform toward more-transparent accounting rules, accounting that leads to faster loss recognition, involves a trade-off that policy makers should be aware off. Namely, faster accounting involves a trade-off between the scale of bank losses and the speed of adjustment after these losses.

To highlight that trade-off, we solve the model for different values of α. Recall that a lower α means that losses are more slowly recognized. Lower values of α have the interpretation of accounting rules that are less transparent. In the model, a lowering α has effects on the distribution of state variables: on one hand, it lowers the average q and, on the other, it increases the fundamental leverage, λ. Indeed, lower values of α provide banks with more slack with which to circumvent regulatory constraints and this manifests in a higher fundamental leverage and a greater discrepancy between the fundamental value and the book value of loans. Figure 13a reports cross-sectional means of q and λ, as we vary α, keeping other parameters in their pre-crisis values. The negative relationship between q and λ is evident from the figure. Strikingly, although the fundamental leverage ratio λ differs for different accounting rules, the average book leverage is constant in each of these equilibria. According to the model, to a regulator that uses accounting leverage to gauge the health of the financial system, all of these economies will look the same. Yet, laxer accounting rules increase the potential equity losses and will produce greater assets sales in response to those losses. This feature highlights one aspect of the trade-off: laxer accounting rules can lead to greater potential losses.

The other aspect of the trade-off is that laxer accounting rules allow for a smoother adjustment process to those equity losses. To see this, Figure 13b reports how bank loans respond to a negative-net-worth shock, depending on α. The lower the α, the smaller the decline in bank loans in response to a negative wealth shock. Given a 10% negative wealth shock, our baseline calibration of $\alpha = 4\%$ implies a 6% decline in loans after 5 years. Lowering α to 3% reduces the decline to 3.7%, while with a value of α of 5%, loans fall by 7%. The reason for this pattern is that by delaying losses, banks no longer need to sell additional loans to keep their book leverage constant. In that sense, delayed accounting acts like an automatic bank-specific countercyclical buffer.

We can transparently represent this trade-off from the model's equilibrium equations. One side of the trade-off, the increased effect on risk (equity losses), can be read from how a lower α leads to a higher λ that increases $-J^W$ in Equation 9. The other side of the trade-off, the slower adjustment of liabilities after a default shock, can be read from how a lower α leads to a smaller reduction in ι in Equation 17.

The result connects our model with the debate on whether accounting rules should incorporate market-based information to improve macro-prudential regulation. In this paper, we argued that market values incorporate information on losses much faster than book values. Thus we can envision how incorporating market values to accounting would increase α. We also discussed how delayed accounting can follow from either the backward-looking nature of book values and or evergreening, which is the continuous rollover of non-performing loans in order to avoid booking equity losses. The literature has identified another trade-off: On the one hand, marking assets to market can exacerbate fire-sale dynamics (Laux and Leuz, 2010; Ellul et al., 2011; Shleifer and Vishny, 2011). If particular market values are driven by risk premia, then regulation should insulate banks from changes in risk premia that are not germane to the health of the banking industry. On the other hand, the discretion in accounting rules opens the door to evergreening, which contributes to the creation of zombie loans, which are drags on economic efficiency (Caballero, Hoshi and Kashyap (2008); Huizinga and Laeven (2012); Blattner, Farinha and Rebelo (2019)). While a welfare analysis that quantifies this additional trade-off is beyond the scope of this paper, we believe that the race between the size and smoothing out of the equity losses we illustrate here should also be part of that active trade-off.

4 Conclusion

This paper summarizes five empirical facts about the dynamics of bank leverage. We use these facts to explore the features banking models need to explain the data. Our empirical findings suggest a theory wherein banks target their market leverage but where adjustments to that target are gradual. A comparison between pre- and post-crisis responses suggests that, in contrast to the pre-crisis period, in the post-crisis banks rely more on retained earnings than on asset sales to readjust market leverage back to target.

This paper presents a heterogeneous bank model that distinguishes book from market values. In our model, both measures of equity matter for banking decisions. A novel feature is that banks have the ability to delay the recognition of losses on their books. The model produces an endogenous target for leverage- and features adjustment costs to the resale of assets. The model reproduces the impulse responses that we estimated from the cross-sectional data. Strikingly, the estimation highlights the exclusive role of delayed loss accounting to explain the data and puts little weight on standard adjustment costs. We also demonstrate that regulatory reforms designed to accelerate loss recognition introduce a trade-off between the scale of equity losses and the posterior adjustment process.

The model highlights the essential frictions that are necessary to reproduce our five empirical facts. As part of the continuous fine-tuning process of banking models, future work could use our model as a building block to study these frictions in general equilibrium.

References

Acharya, Viral V. and Stephen G. Ryan, "Banks' Financial Reporting and Financial System Stability," *Journal of Accounting Research*, May 2016, *54* (2). 23, 2.3, A.3

Adrian, Tobias and Hyun Song Shin, "Liquidity and leverage," *Journal of financial intermediation*, 2010, *19* (3), 418–437. 1, 2.1

_ and _ , "Procyclical leverage and value-at-risk," *The Review of Financial Studies*, 2013, *27* (2), 373–403. 2, 4, 7, 1

_ and Nina Boyarchenko, "Intermediary Leverage Cycles and Financial Stability," 2013. New York Federal Reserve, Staff Report No. 567. 2, 1

_ , Erkko Etula, and Tyler Muir, "Financial intermediaries and the cross-section of asset returns," *The Journal of Finance*, 2014, *69* (6), 2557–2596. 1

_ , Nina Boyarchenko, and Hyun Song Shin, "On the Scale of Financial Intermediaries," 2016. New York Federal Reserve, Staff Report No. 743. 1

Atkeson, Andrew G, Adrien d'Avernas, Andrea L Eisfeldt, and Pierre-Olivier Weill, "Government Guarantees and the Valuation of American Banks," in "NBER Macroeconomics Annual 2018, volume 33," University of Chicago Press, 2018. 12, 42

Basel Committee on Banking Supervision, "International Convergence of Capital Measurement and Capital Standards," Technical Report, Bank for International Settlements July 1998. 27

_ , "International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version," Technical Report, Bank for International Settlements 2006. 27

_ , "Basel III: A global regulatory framework for more resilient banks and banking systems," Technical Report, Bank for International Settlements June 2011. 27

Begenau, J. and T. Landvoigt, "Financial Regulation in a Quantitative Model of the Modern Banking System," *Unpublished working paper. Stanford University and University of Pennsylvania*, 2020. 2

Begenau, Juliane, "Capital requirements, risk choice, and liquidity provision in a business-cycle model," *Journal of Financial Economics*, 2020, *136* (2), 355–378. 2, 1

_ and Erik Stafford, "Do Banks have an Edge?," *Available at SSRN 3095550*, 2019. 42

_ , Monika Piazzesi, and Martin Schneider, "Banks' risk exposures," Technical Report, National Bureau of Economic Research 2015. 42

Berger, Allen N., Richard J. Herring, and Giorgio P. Szegï¿œ, "The role of capital in financial institutions," *Journal of Banking & Finance*, 1995, *19* (3), 393 – 430. The Role of Capital in Financial Institutions. 1

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth and Business Fluctuations," *The American Economic Review*, March 1989, *79* (1), 14–31. 13

Bianchi, Javier and Saki Bigio, "Banks, Liquidity Management and Monetary Policy," 2017. Unpublished Manuscript. 1, 39

Blank, Michael, Samuel G Hanson, Jeremy C Stein, and Aditya Vikram Sunderam, "How Should US Bank Regulators Respond to the COVID-19 Crisis?," 2020. 1

Blattner, Laura, Luisa Farinha, and Francisca Rebelo, "When losses turn into loans: The cost of undercapitalized banks," 2019. 3, 2.3, 40, 3.5

Brunnermeier, Markus K. and Yuliy Sannikov, "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 2014, *104* (2), 379–421. 2, 1

Bushman, Robert M., "Transparency, Accounting Discretion, and Bank Stability," *FRBNY Economic Policy Review*, August 2016, pp. 129–149. 23, 2.3, A.3

Caballero, Ricardo J., Takeo Hoshi, and Anil K. Kashyap, "Zombie Lending and Depressed Restructuring in Japan," *The American Economic Review*, 2008, *98* (5), 1943–1977. 1, 2.3, 40, 3.5

Corbae, Dean and Pablo D'Erasmo, "Capital requirements in a quantitative model of banking industry dynamics," Technical Report, National Bureau of Economic Research 2019. 2, 1, 3.3

Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordonez, "Banks as secret keepers," *The American Economic Review*, 2017, *107* (4), 1005–1029. 8, 1, 2.3

DeMarzo, Peter and Zhiguo He, "Leverage Dynamics without Commitment," 2016. NBER Working Paper No. 22799. 1

Diamond, Douglas W., "Financial Intermediation and Delegated Monitoring," *The Review of Economic Studies*, 1984, *51* (3), pp. 393–414. 1

_ , "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 1984, *51*, 393–414. 2.3

_ **and Philip H. Dybvig**, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 1983, *91*, 401–417. 13

_ **and Raghuram G. Rajan**, "A Theory of Bank Capital," *The Journal of Finance*, 2000, *55* (6), pp. 2431–2465. 13

Dickens, Ross N, K Michael Casey, and Joseph A Newman, "Bank dividend policy: explanatory factors," *Quarterly journal of Business and Economics*, 2002, pp. 3–12. 3.1

Duffie, Darrell, "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," *The Journal of Finance*, 2010, *LXV* (4). 10

Ellul, Andrew, Chotibhak Jotikasthira, and Christian T Lundblad, "Regulatory pressure and fire sales in the corporate bond market," *Journal of Financial Economics*, 2011, *101* (3), 596–620. 3.5

Fama, Eugene F. and Kenneth R. French, "Common Risk Factors in the Returns on Stock and Bonds," *Journal of Financial Economics*, 1993, *33*, 131–156. 2.2

Francis, Jennifer, J. Douglas Hanna, and Linda Vincent, "Causes and Effects of Discretionary Asset Write-Offs," *Journal of Accounting Research*, 1996. A.3

Frank, Murrayz and Vidhan K Goyal, "Trade-off and Pecking Order Theories of Debt," *Handbook of Empirical Corporate Finance: Empirical Corporate Finance*, 2011, *2*, 135. 9

Gandhi, Priyank and Hanno Lustig, "Size anomalies in US bank stock returns," *The Journal of Finance*, 2015, *70* (2), 733–768. 2.2, 2.2

General Accounting Office, "Report to the Congress on the Bank Insurance Fund," Technical Report, General Accounting Office September 1990. 3

Gertler, M., N. Kiyotaki, and A. Prestipino, "Chapter 16 - Wholesale Banking and Bank Runs in Macroeconomic Modeling of Financial Crises," in John B. Taylor and Harald Uhlig, eds., *Handbook of Macroeconomics*, Vol. 2, Elsevier, 2016, pp. 1345 – 1425. 1

Gertler, Mark and Nobuhiro Kiyotaki, "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 2010, *3*, 547. 1

_ , _ , and Albert Queralto, "Financial crises, bank risk exposure and government financial policy," *Journal of Monetary Economics*, 2012, *59*, S17–S34. 1

Goh, Beng Wee, Dan Li, Jeffrey Ng, and Kevin Ow Yong, "Market pricing of banks' fair value assets reported under SFAS 157 since the 2008 financial crisis," *Journal of Accounting and Public Policy*, jan 2015. 58

Gomes, João, Urban Jermann, and Lukas Schmid, "Sticky Leverage," *American Economic Review*, 2016, *106* (12), 3800–3828. 1

Gomez, Matthieu, Augustin Landier, David Sraer, and David Thesmar, "Banks? exposure to interest rate risk and the transmission of monetary policy," *Journal of Monetary Economics*, 2020. 42

Gorton, Gary and Guillermo Ordonez, "Collateral Crises," 2011. 8

Gropp, Reint and Florian Heider, "The determinants of bank capital structure," *Review of Finance*, 2010, *14*, 587–622. 6

Hachem, Kinda, "Relationship lending and the transmission of monetary policy," *Journal of Monetary Economics*, 2011, *58* (6), 590 – 600. 1

Haddad, Valentin and David A Sraer, "The banking view of bond risk premia," Technical Report, National Bureau of Economic Research 2019. 42

Harris, Trevor, Urooj Khan, and Doron Nissim, "The Expected Rate of Credit Losses on Banks' Loan Portfolios," 2013. A.3

Hart, Oliver and John Moore, "A Theory of Debt Based on the Inalienability of Human Capital," *The Quarterly Journal of Economics*, 1994, *109* (4), pp. 841–879. 13

Hayashi, Fumio, "Tobin's marginal q and average q: A neoclassical interpretation," *Econometrica: Journal of the Econometric Society*, 1982, pp. 213–224. 1, 2.3

He, Zhiguo and Arvind Krishnamurthy, "A Model of Capital and Crises," *Review of Economic Studies*, 2012, *79(2)*, 735–777. 1

_ and _ , "Intermediary asset pricing," *The American Economic Review*, 2013, *103* (2), 732–770. 2, 1

_ , Bryan Kelly, and Asaf Manela, "Intermediary Asset Pricing: New Evidence From Many Asset Classes," *Journal of Financial Economics*, 2017, *126* (1), 1–35. 1, 4, 2.1

_ , In Gu Khang, and Arvind Krishnamurthy, "Balance sheet adjustments during the 2008 crisis," *IMF Economic Review*, 2010, *58* (1), 118–156. 4

Hennessy, Christopher A. and Toni M. Whited, "Debt Dynamics," *The Journal of Finance*, 2005, *60* (3), 1129–1165. 9

Holmstrom, Bengt and Jean Tirole, "Financial intermediation, loanable funds, and the real sector," *the Quarterly Journal of economics*, 1997, *112* (3), 663–691. 13

_ and _ , "Private and Public Supply of Liquidity," *Journal of Political Economy*, August 1998, *106* (1). 13

Huizinga, Harry and Luc Laeven, "Bank valuation and accounting discretion during a financial crisis," *Journal of Financial Economics*, December 2012, *106* (3), 614–634. 3.5, A.3

Jermann, Urban and Vincenzo Quadrini, "Macroeconomic Effects of Financial Shocks," *American Economic Review*, February 2012, *102* (1), 238–71. 1

Kisin, Roni and Asaf Manela, "The shadow cost of bank capital requirements," *The Review of Financial Studies*, 2016, *29* (7), 1780–1820. 27

Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy*, 1997, *105*, 211–248. 13

Kolev, Kalin, "Do Investors Perceive Marking-to-Model as Marking-to-Myth? Early Evidence from FAS 157 Disclosure," 2009. 58

Kraus, Alan and Robert H Litzenberger, "A state-preference model of optimal financial leverage," *The Journal of Finance*, 1973, *28* (4), 911–922. 9

Kronmal, Richard A., "Spurious Correlation and the Fallacy of the Ratio Standard Revisited," *Journal of the Royal Statistical Society. Series A (Statistics in Society)*, 1993, *156* (3), 379–392. 25

Laux, Christian and Christian Leuz, "Did fair-value accounting contribute to the financial crisis?," *The Journal of Economic Perspectives*, 2010, *24* (1), 93–118. 3, 1, 2.3, 3.5, A.3, 58

Leary, Mark T. and Roni Michaely, "Determinants of Dividend Smoothing: Empirical Evidence," *The Review of Financial Studies*, 10 2011, *24* (10), 3197–3249. 3.1

Leland, Hayne E. and David H. Pyle, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *The Journal of Finance*, 1977, *32* (2), pp. 371–387. 1, 9

— and —, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *Journal of Finance*, 1977, *32*, 371–387. 1, 2.3

Lintner, John, "Distribution of incomes of corporations among dividends, retained earnings, and taxes," *The American economic review*, 1956, *46* (2), 97–113. 3.1

Martinez-Miera, David and Javier Suarez, "A macroeconomic model of endogenous systemic risk taking," 2011. Unpublished Manuscript. 1

Milbradt, Konstantin, "Level 3 Assets: Booking Profits and Concealing Losses," *The Review of Financial Studies*, 2012, *25* (1), 55–95. 1, 2.3

Myers, Stewart C., "Determinants of corporate borrowing," *Journal of Financial Economics*, 1977, *5* (2), 147 – 175. 1

Myers, Stewart C, "The capital structure puzzle," *The journal of finance*, 1984, *39* (3), 574–592. 9

Myers, Stewart C. and Nicholas S. Majluf, "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, 1984, *13* (2), 187 – 221. 1

Nickell, Stephen, "Biases in dynamic models with fixed effects," *Econometrica: Journal of the Econometric Society*, 1981, pp. 1417–1426. 28

Nuño, Galo and Carlos Thomas, "Bank Leverage Cycles," *American Economic Journal: Macroeconomics*, April 2017, *9* (2), 32–72. 1

O'Hara, Maureen, "A Dynamic Theory of the Banking Firm," *The Journal of Finance*, 1983, *38* (1), pp. 127–140. 1, 9

Plantin, Guillaume and Jean Tirole, "Marking to market versus taking to market," *American Economic Review*, 2018, *108* (8), 2246–76. 1

Rampini, Adriano A. and S. Viswanathan, "Financial Intermediary Capital," 2012. Working paper, Duke. 1

Rios-Rull, Jose Victor, Tamon Takamura, and Yaz Terajima, "Banking Dynamics, Market Discipline and Capital," March 2020. 1

Shachar, Or, "Exposing The Exposed: Intermediation Capacity in the Credit Default Swap Market," 2012. Working paper, NYU. 1

Shleifer, Andrei and Robert Vishny, "Fire Sales in Finance and Macroeconomics," *The Journal of Economic Perspectives*, 2011, *25* (1), pp. 29–48. 1, 3.1, 3.5

— **and Robert W. Vishny**, "The Limits of Arbitrage," *The Journal of Finance*, 1997, *52* (1), pp. 35–55. 1, 2.3

Song, Chang Joon, Wayne B. Thomas, and Han Yi, "Value relevance of FAS No. 157 Fair Value hierarchy information and the impact of corporate governance mechanisms," *Accounting Review*, 2010, *85*, 1375–1410. 58

Tella, Sebastian Di and Pablo Kurlat, "Why are banks exposed to monetary policy?," *American Economic Journal: Macroeconomics*, Forthcoming. 39

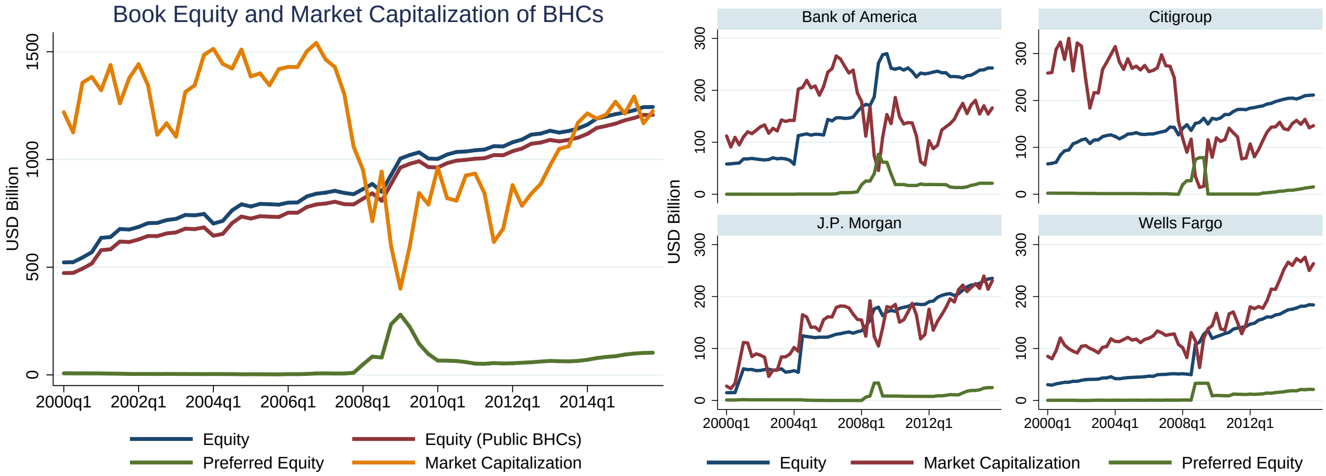
Tirole, Jean, "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning," January 2011. American Economic Review, forthcoming. 1

Townsend, Robert, "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 1979, *22*, 265–293. 13

Wang, Olivier, "Banks, low interest rates, and monetary policy transmission," *Available at SSRN 3520134*, 2018. 42

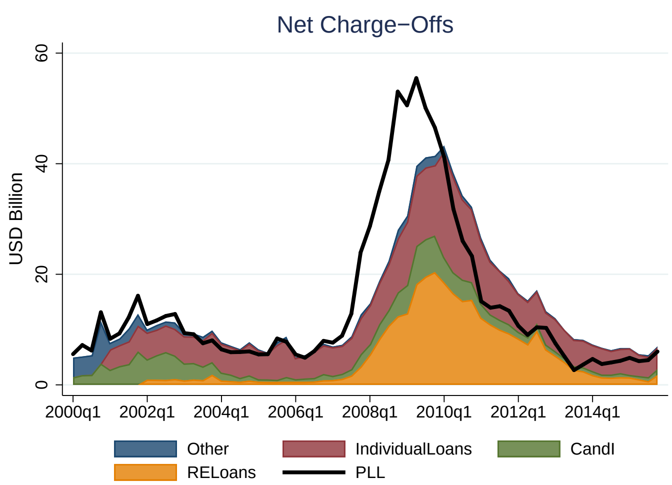
Williamson, Stephen D., "Costly monitoring, financial intermediation, and equilibrium credit rationing," *Journal of Monetary Economics*, 1986, *18* (2), 159 – 179. 1

Figure 1: Book and Market Equity for Bank Holding Companies.



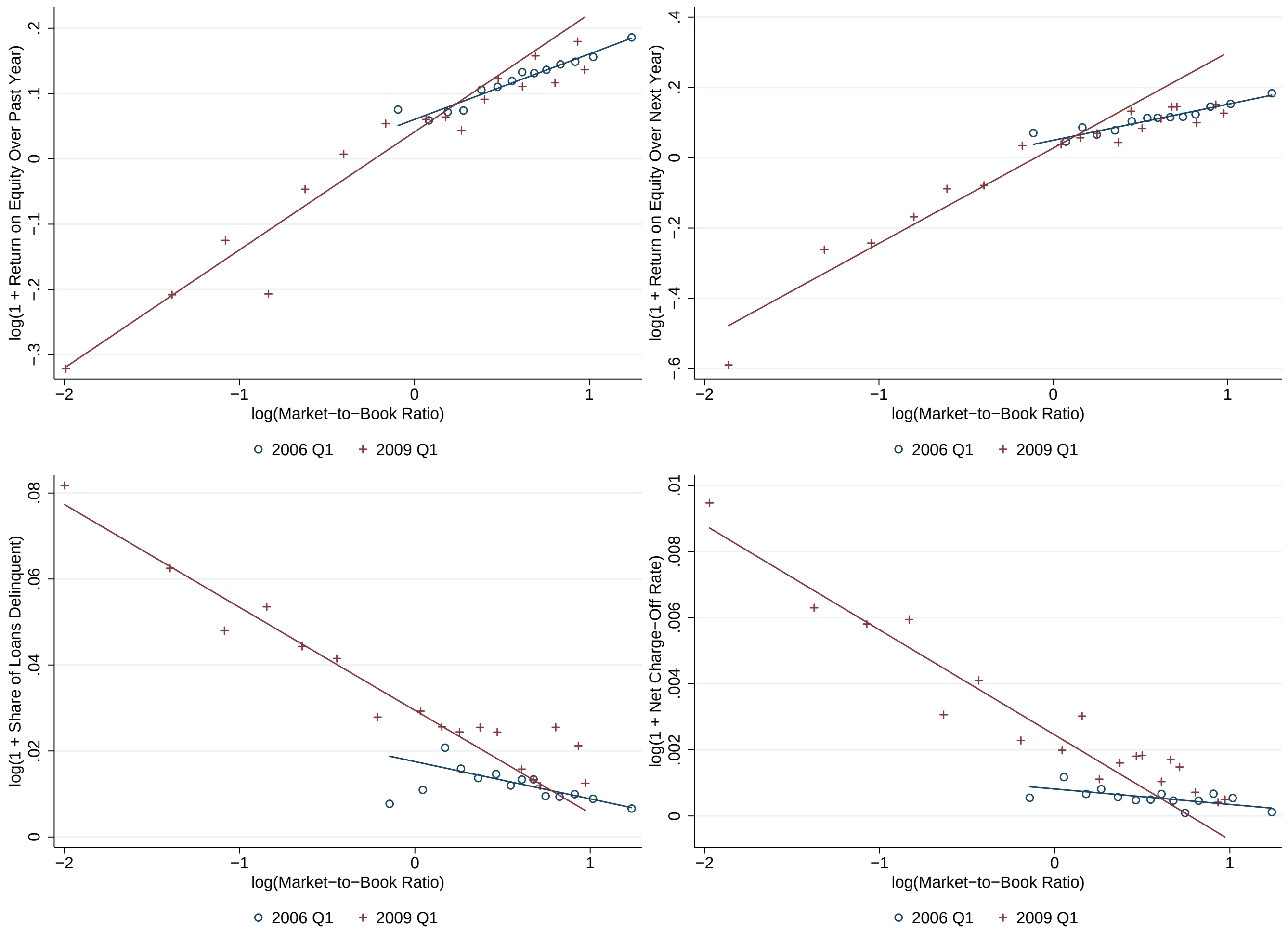
Notes: These figures show data on book equity, market capitalization, and preferred equity for BHCs. Book equity and preferred equity data come from the FR Y-9C, and market capitalization data is based on Center for Research in Security Prices (CRSP) data. All variables are converted to 2012 Q1 dollars, using the seasonally adjusted GDP deflator. The left-hand panel shows aggregate series, excluding new entrants such as Goldman Sachs and Morgan Stanley. "Equity" refers to book equity for all BHCs in the sample and "Equity (Public BHCs)" refers to only publicly traded BHCs that we can match to the CRSP data. We also show preferred equity for all banks and aggregate market capitalization; i.e., shares outstanding times the share price of the publically traded BHCs in our sample. The right-hand panel shows the equivalent time series for the four largest BHCs.

Figure 2: Decomposition of Net Charge-offs



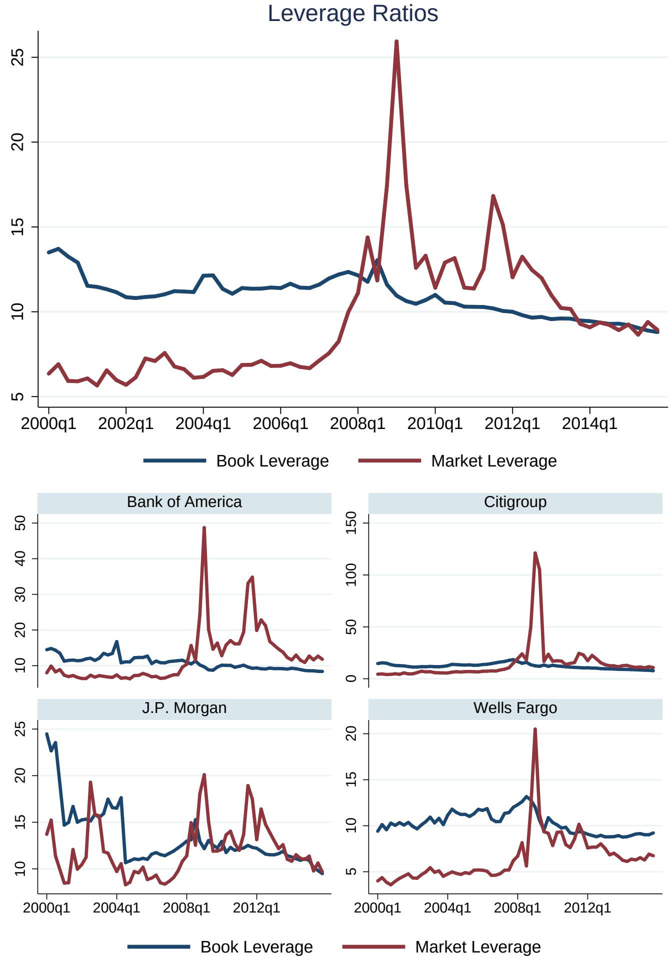
Notes: This figure shows aggregate net charge-offs for different categories (area chart) and aggregate loan-loss provisions (solid black line). The data source are FR Y-9C reports. Net charge-offs for loans are defined as charge-offs minus recoveries. We decompose the net charge-offs into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (e.g., such as credit card loans), and all other loans (e.g., interbank loans, agricultural loans, and loans to foreign governments).

Figure 3: Market equity contains more cash-flow relevant information than book equity



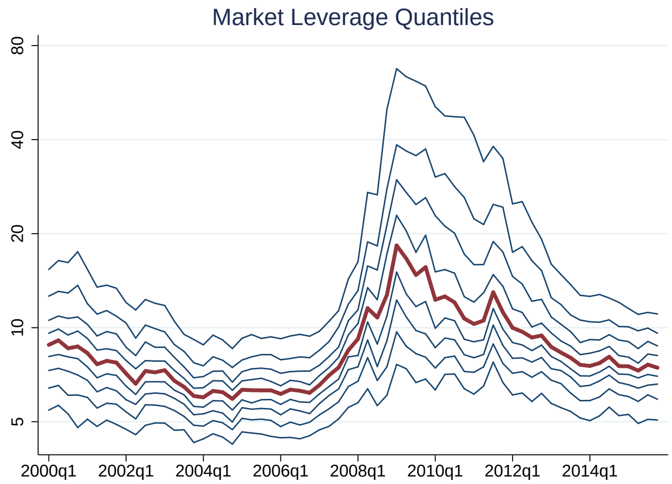
Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log market-to-book equity ratio for BHCs, in 2006 Q1 and 2009 Q1. All plots control for the log book equity by residualizing the variables on the log book equity and then adding back the mean of each variable to maintain centering. Data on market capitalization are obtained from the CRSP, and all other data are from the FR Y-9C. ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; ROE over the next year is defined as the one lead of this variable (i.e., profits over the next four quarters divided by current book equity). The share of delinquent loans is the ratio of loans 30 days or more past due plus loans in non-accrual over total loans. The net charge-off rate is the loan charge-offs over the next quarter minus the loan recoveries over the next quarter divided by the total loans this quarter. In all plots, the Irwin Financial Corporation is dropped from the sample in both time periods because it is an extreme outlier for 2009 Q1 (a few months before its failure in September 2009).

Figure 4: Book and Market Leverage of Bank Holding Companies



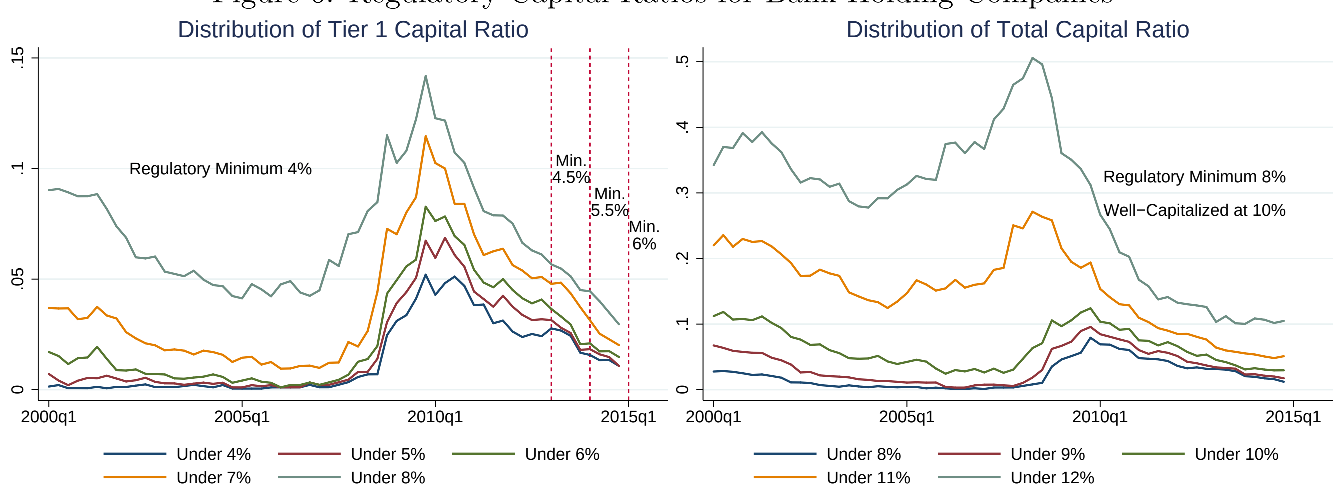
Notes: These figures show data on book and market leverage for BHCs. Book data (book equity and liabilities) come from the FR Y-9C, and market-equity data is from the CRSP data. The left-hand panel shows aggregates from BHC balance sheets (excluding new entrants such as Goldman Sachs and Morgan Stanley). The right-hand panel shows data for the four largest BHCs. Book leverage is computed as assets/book equity, and market leverage is computed as (liabilities + market equity)/market equity. The aggregate leverage ratios are computed as (aggregate liabilities + aggregate book equity)/aggregate book equity.

Figure 5: Quantiles of Bank Holding Companies' Market Leverage



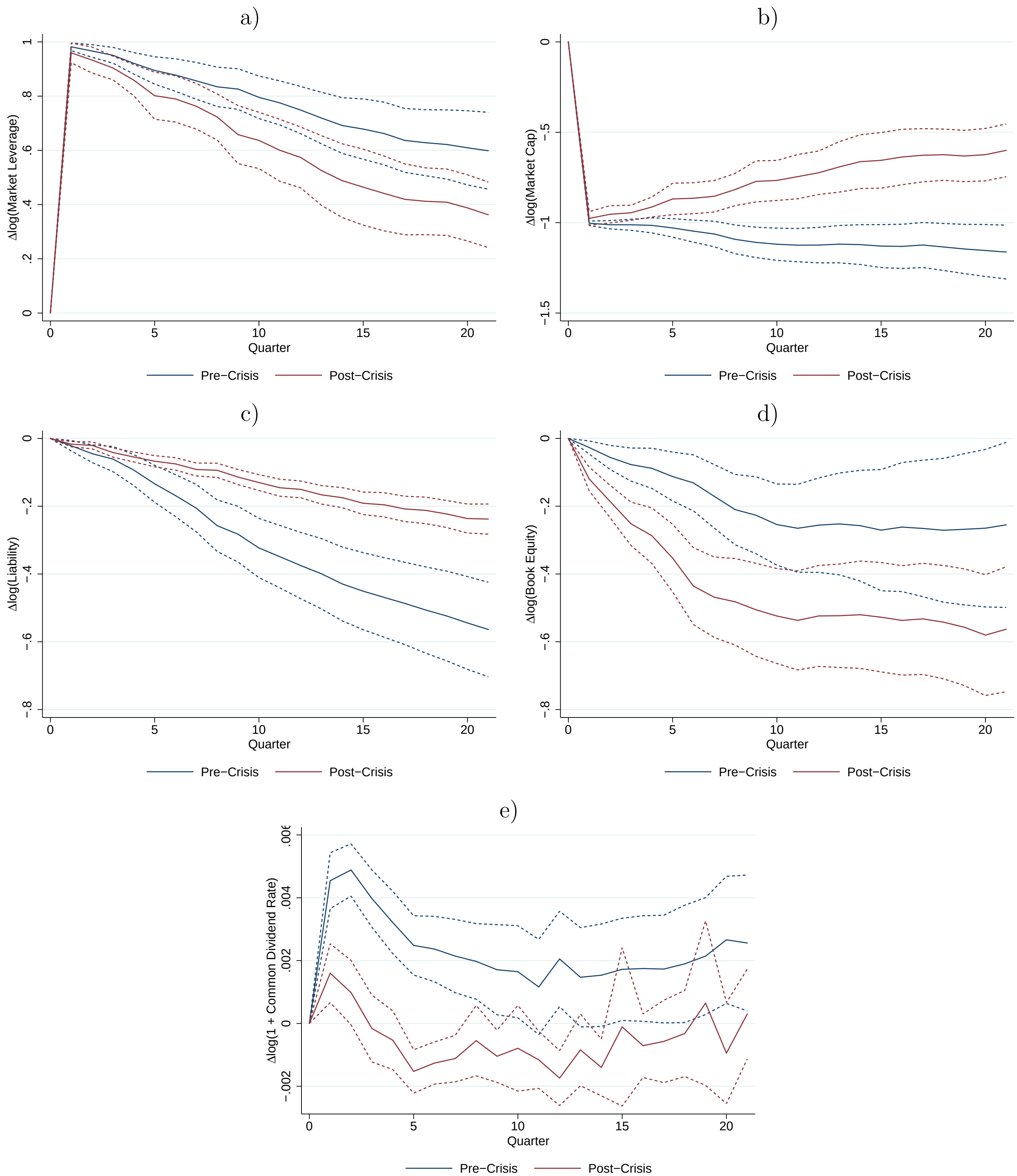
Notes: This figure shows data on the quantiles of market leverage for BHCs. Book data (liabilities) comes from the FR Y-9C, and market-equity data is from the CRSP data. Market leverage is computed as (liabilities + market equity)/market equity. The median market leverage is plotted in maroon and each tenth percentile is plotted in blue. To improve visibility, the vertical axis uses a log scale.

Figure 6: Regulatory Capital Ratios for Bank Holding Companies



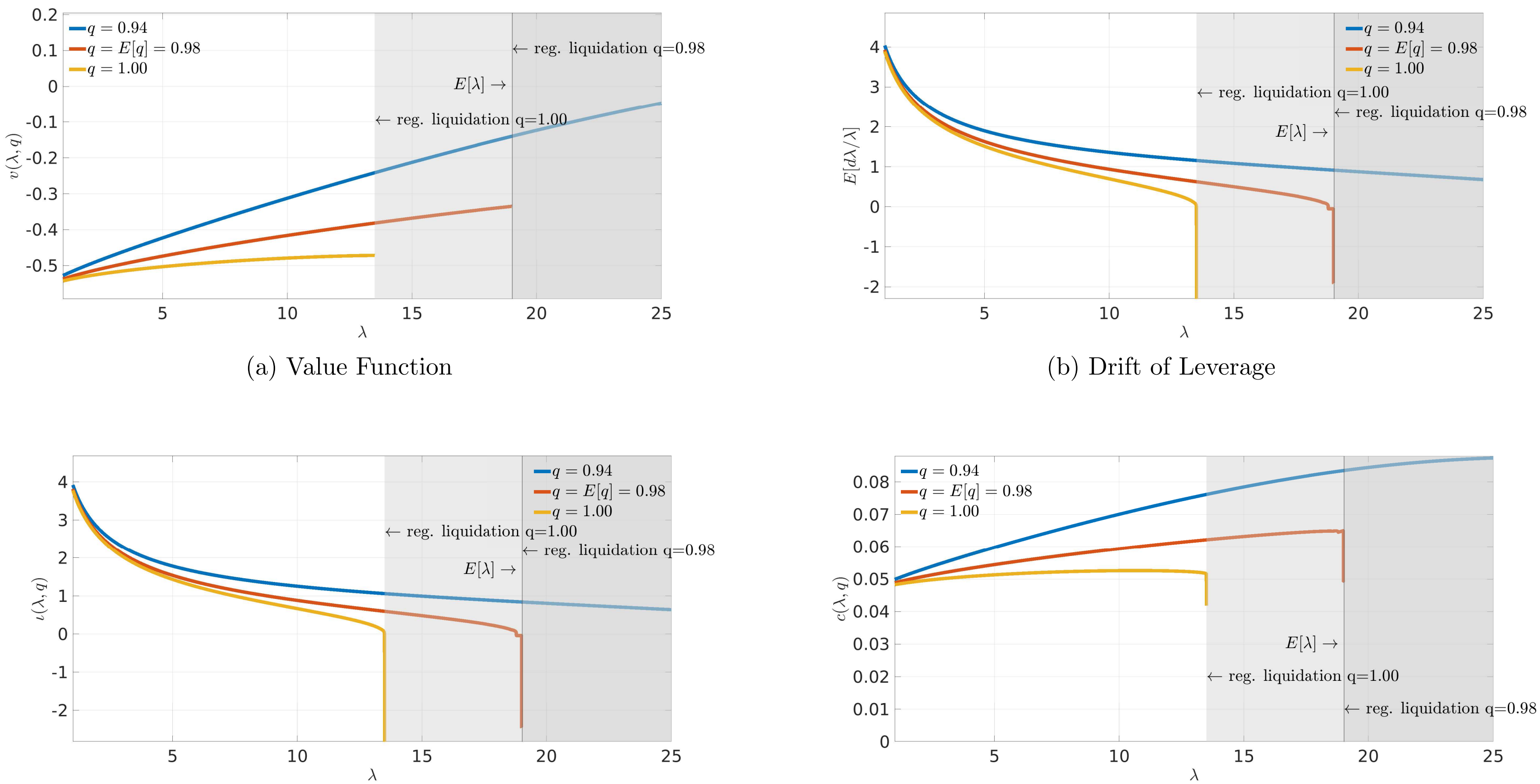
Notes: These figures show data on regulatory capital ratios for BHCs from the FR Y-9C. The left-hand panel shows data on the distribution of the tier-1 capital ratio, defined as (tier-1 capital)/(risk-weighted assets), and the right-hand panel shows data on the distribution of the total capital ratio defined as (total capital / risk weighted assets). The figures plot the share of banks whose regulatory capital ratio falls below a given level, computed using the full unweighted sample. The regulatory capital requirements are shown on the graph and described in the text.

Figure 7: Estimated Impulse Responses



Notes: These figures show estimated impulse-response functions for BHCs. The figures show the estimated percent impulse response to a 1% negative returns shock. For example, in Panel b we show that market capitalization decreases by roughly 1% in response to a 1% negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are obtained from the CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the log market leverage (Panel a), log market capitalization (Panel b), log liabilities (Panel c), log book equity (Panel d), and the common dividend rate (Panel e). Market leverage is defined as (liabilities/market capitalization). The logged common dividend rate is defined as log(1 + common dividends/market capitalization).

Figure 8: Value and Policy Functions for given q's



(a) Value Function

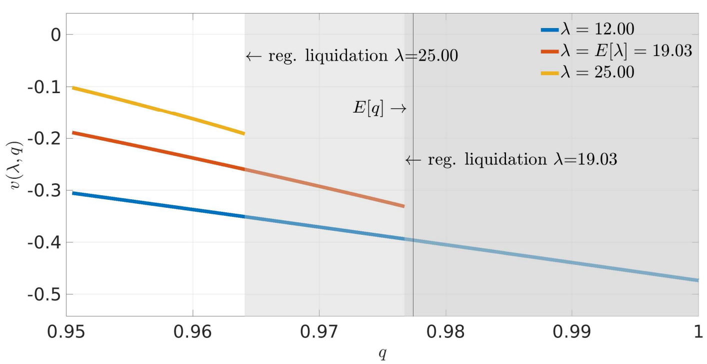
(b) Drift of Leverage

(c) Issuance Rate

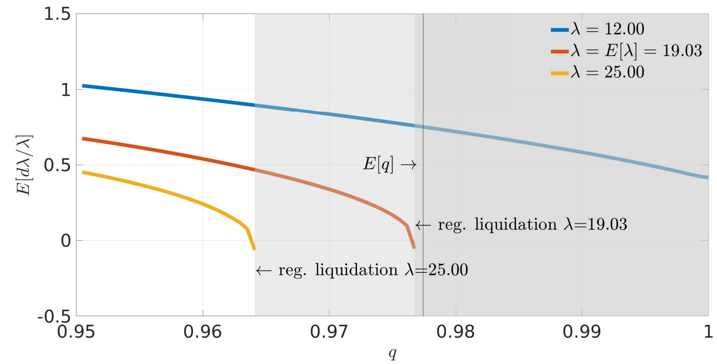
(d) Dividend Rate

Notes: These figures show the value and policy functions that are generated under the pre-crisis parametrization of the model, across the λ dimension, for three particular values of q.

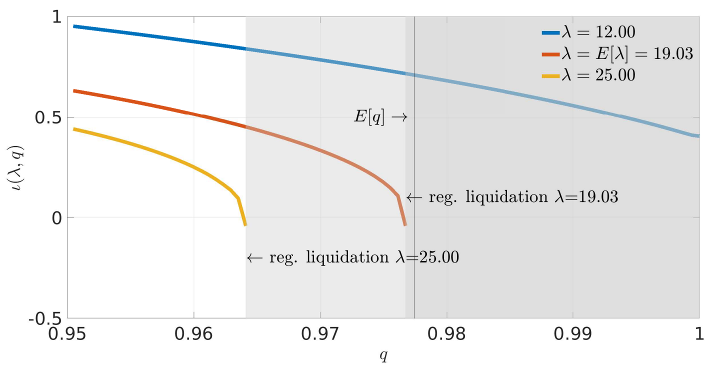
Figure 9: Value and Policy Functions for given λ's



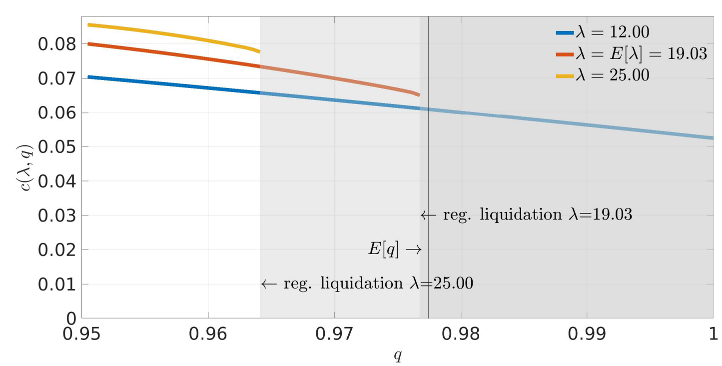
(a) Value Function



(b) Drift of Leverage



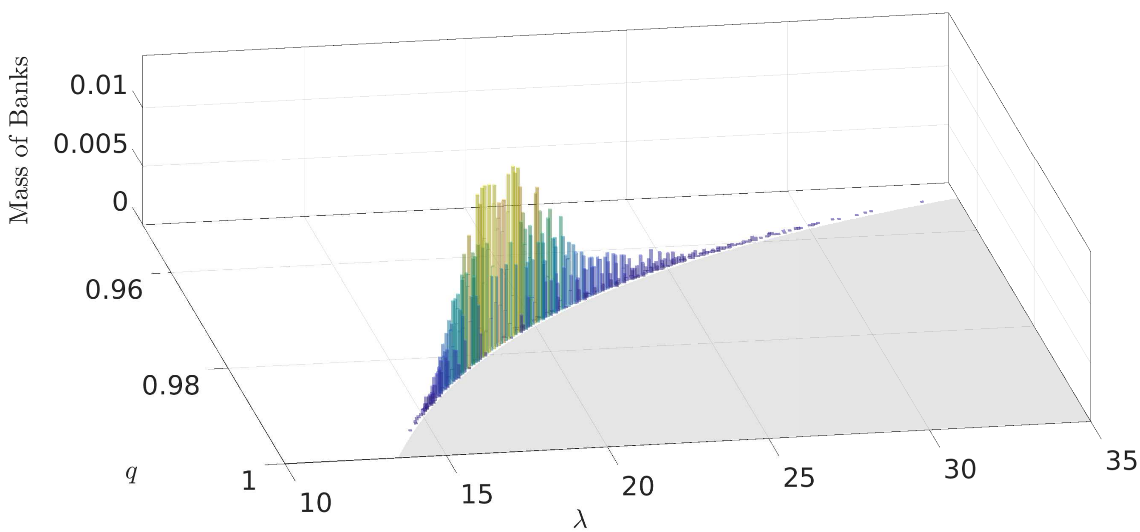
(c) Issuance Rate



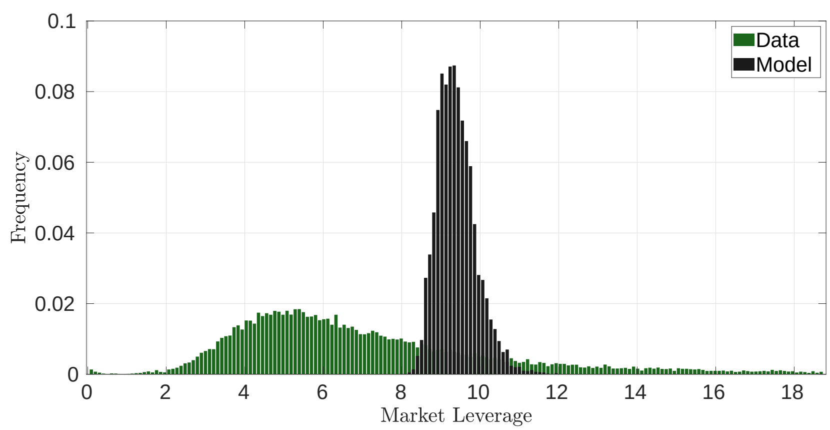
(d) Dividend Rate

Notes: These figures show the value and policy functions that are generated under the pre-crisis parametrization of the model, across the q dimension, for three particular values of λ.

Figure 10: Invariant Distributions



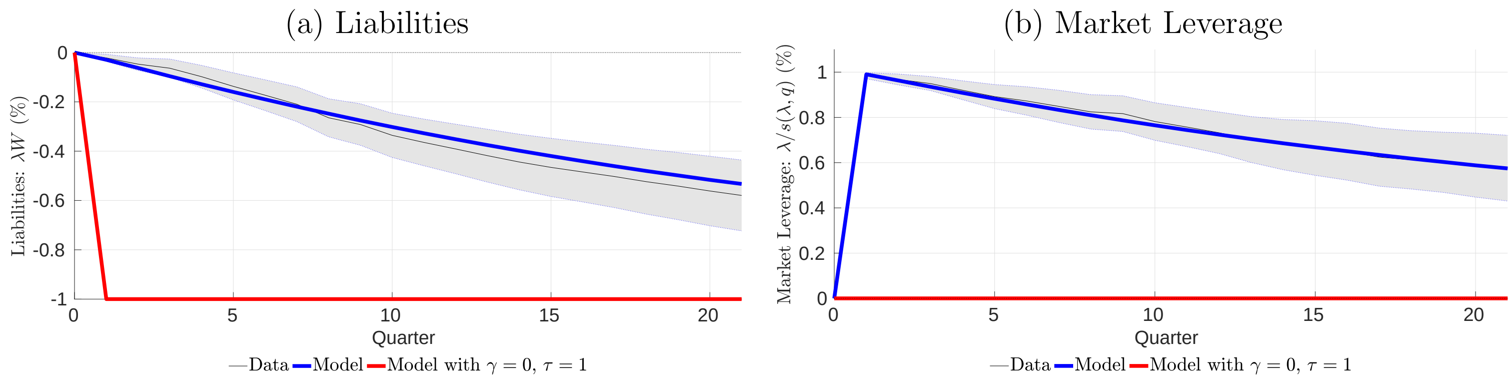
(a) Model Stationary Distribution of Banks Across the q and λ State Space



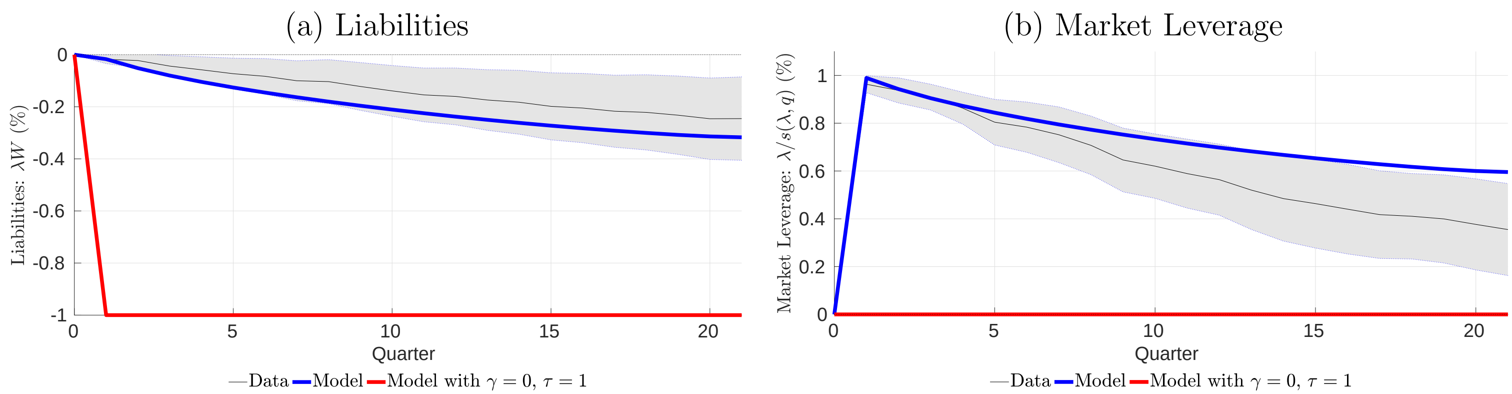
(b) Histogram for Market Leverage

Notes: Panel a shows a two-dimensional histogram of the stationary distribution of banks across the (λ, q) space. The grey area shows the regulatory liquidation region. Panel b compares the distribution of the market leverage generated by the model against the data.

Figure 11: Model and Data Impulse Responses to a Returns Shock



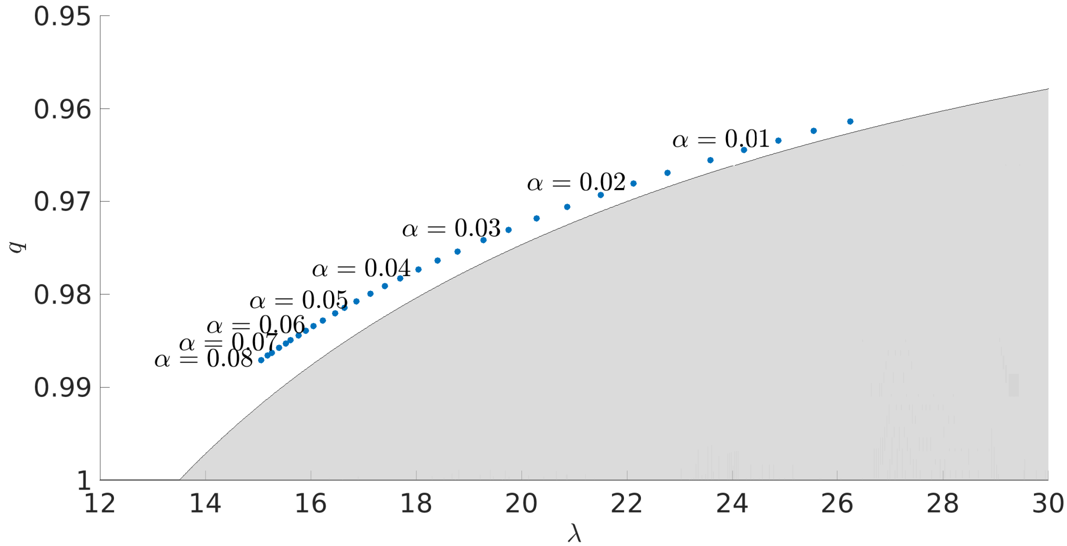
(a) Pre-crisis IRFs



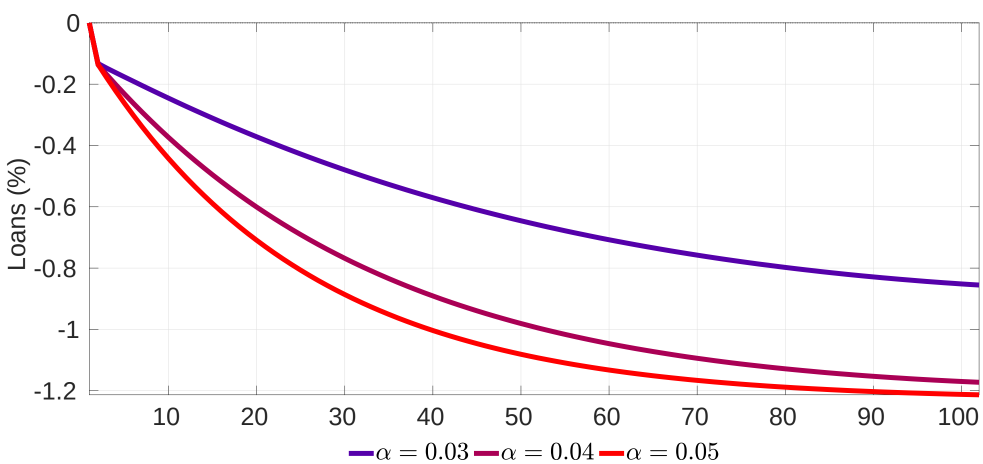
(b) Post-crisis IRFs

Notes: The figure shows the impulse-response functions (IRFs) of market leverage and liabilities to returns shocks. The blue line corresponds to the model-generated IRFs, whereas the black line is from the empirical section, with the shaded region corresponding to the confidence band for the estimated responses. The shock occurs in quarter 1.

Figure 12: Counterfactual Exercise



(a) Effect of Delayed Loss Recognition on q and λ



(b) Impulse-Response Function of Loans

Notes: The figures show the results from our counterfactual exercise. Each dot in the top figure is a pair of cross-sectional means of λ and q in the pre-crisis stationary allocation for a given value of α. The gray area represents the regulatory liquidation set. Each of these stationary allocations is characterized by approximately the same mean book leverage of 12.4. The bottom panel shows the IRF to a 1% returns shock for aggregate loans for three different values of α.

Table 1: Aggregate Descriptive Statistics

	Real Change			Log-Linear		
	2008	2009	2010	2008	2009	2010
Market Equity	-54.08% (-\$705B)	-39.35% (-\$513B)	-29.03% (-\$378B)	-61.21% (-\$945B)	-49.98% (-\$790B)	-42.86% (-\$694B)
Book Equity	11.83% (\$94B)	21.70% (\$172B)	25.97% (\$206B)	-3.46% (-\$32B)	-1.50% (-\$15B)	-4.41% (-\$46B)
S&P 500	-42.08%	-28.83%	-21.20%	-25.55%	-7.01%	4.63%

Notes: The columns headed with the label "Real Change" show the percentage change from the raw variables. The columns headed with the label "Log-Linear" show the cyclical deviations from a log-linear trend in percentage points since 2007 Q3. Market Equity refers to shares outstanding times the share price aggregated across all publicly traded BHCs. Book equity is the book equity of publicly traded BHCs. All variables are deflated using the seasonally adjusted GDP deflator and converted to 2012 Q1 dollars. The dollar values are obtained by multiplying the cumulative percentage point deviation by the real market capitalization and real book equity at the end of 2007 Q3, respectively. The last row shows the percentage change in the returns on the S&P 500 in the first three columns, while the last three columns show the change relative to a linear log-linear trend.

Table 2: Parametrization

Parameter	**Description**	**Target**
Independently calibrated		
$r^L = 1.01\%$	Loan yield	BHC data: interest income / loans
$r^D = 0.51\%$	Bank debt yield	BHC data: interest expense / debt
$\delta = 7.69\%$	Loan maturity	FFIEC 031/041: average maturity of loans and securities
$\xi = 0.926$	Capital requirement	Capital requirement of 8% to be well capitalized
$\varepsilon = 0.25\%$	Average default shocks	Accumulated bank losses
$\sigma = 0.4791$	Arrival rate of Poisson process	Match loan charge-off rate
$\alpha = 4\%$	Recognition rate of books	Peak of charge-off rate after financial crisis
$\rho = 0.25\%$	Banker's discount rate	CRSP: Mean market leverage
$\rho^I = 3.51\%$	Investor's discount rate	CRSP: Bank equity returns
Jointly calibrated		
$\tau = 1\%$	Initial fraction of loan-loss recognition	Initial jump of book leverage IRF
$\theta^{pre} = 2.30$	Inverse IES pre-crisis	Match market leverage IRF pre-crisis
$\theta^{post} = 1.71$	Inverse IES post-crisis	Match market leverage IRF post-crisis
$\gamma^{pre} = 0.01$	Balance sheet adj. costs pre-crisis	Match liabilities IRF pre-crisis
$\gamma^{post} = 3.96$	Balance sheet adj. costs post-crisis	Match liabilities IRF post-crisis

Table 3: Model and Data Moments

	Pre-crisis		Post-crisis	
	Data	Model	Data	Model
Log Market Returns	0.035	0.045	-0.012	0.046
	(0.149)	(0.015)	(0.230)	(0.017)
Market Leverage	6.799	9.388	10.025	11.056
	(0.598)	(0.052)	(0.815)	(0.070)
Book Leverage	10.619	12.409	9.870	11.967
	(0.366)	(0.001)	(0.502)	(0.010)
Market to Book Equity	1.532	1.322	0.897	1.082
	(0.469)	(0.052)	(0.584)	(0.076)
Log Common Dividend Rate	0.006	0.033	0.005	0.028
	(0.006)	(0.000)	(0.006)	(0.002)
Log Net Charge-Off Rate	0.001	0.001	0.002	0.001
	(0.003)	(0.002)	(0.004)	(0.002)

	Pre-crisis	Post-crisis
λ	18.048	22.029
	(2.114)	(4.142)
q	0.977	0.966
	(0.006)	(0.008)
c	0.065	0.055
	(0.005)	(0.005)
ι	0.024	0.024
	(0.002)	(0.004)
dW/W	0.012	0.029
	(0.033)	(0.042)
$s(\lambda, q)$	1.914	1.972
	(0.119)	(0.218)

Notes: The columns labeled "Pre-crisis Data" refer to the period 1990 Q3 to 2007 Q3, whereas "Post-crisis Data" refers to the period 2007 Q4 to 2015 Q4. The moments from the model are generated from a panel of 10,000 banks with the same number of quarters as in the respective periods for the data. For the column "Pre-crisis model," we calculate the moments using the stationary distribution of banks. We compute the stationary distribution by first simulating enough quarters so that the means and standard deviations of the state variables (λ, q) are approximately constant and then we keep the last one as the initial quarter of the simulated sample. For the post-crisis model moments, banks suffer an aggregate default shock of 2.5% in the first quarter. The first row shows the mean for each variable. The second row shows standard deviations in parenthesis. For market leverage, book leverage and market-to-book equity, the mean and standard deviations are computed on the logs, but when reporting the mean we apply exponentials to show the means in levels.

For Online Publication

Online Appendix for
"A Q-Theory of Banks"

Juliane Begenau Saki Bigio

Jeremy Majerovitz Matias Vieyra

A Data Appendix

A.1 Sample Selection

We analyze bank holding companies (BHCs), drawing data from multiple sources. We focus on top-tier bank holding companies that are headquartered in the 50 US states or in Washington D.C. In most of our analyses, we analyze data from 2000 Q1 to 2015 Q4. For the analysis of impulse-response functions, we extend the sample back to 1990 Q3 (this is the first year for which we can identify whether a BHC is top tier). For book variables, we use data from the FR Y-9C, downloaded through Wharton Research Data Services (WRDS). We match this to data on market capitalization and returns, which we obtain from the Center for Research in Securities Prices (CRSP) by using the PERMCO-RSSD links dataset provided by the New York Fed (https://www.newyorkfed.org/research/banking_research/datasets.html). For analyses that use solely book data, we use data for those BHCs that we find in our sample in the FR Y-9C; for analyses that use market data, we use only the observations that we observe in both FR Y-9C and the CRSP. In one robustness check, we use information on the dates of and the participants in bank mergers and acquistions; we obtain data on bank mergers from the Chicago Fed (https://www.chicagofed.org/banking/financial-institution-reports/merger-data). In an additional robustness check, we drop all banks that were ever stress tested (CCAR and DFAST). We obtain information from the Federal Reserve, on whether banks were ever stress tested (The main website is https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm, and the specific data sets can be found at https://www.federalreserve.gov/supervisionreg/ccar.htm and https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm).

A.2 Evolution of Main Balance Sheet Variables

To get a sense of how the 2008 crisis affects banks, we report the evolution of key balance sheet components in Figure 1. This figure shows total assets, liabilities, and loans—not netting out the allowance for loan losses—or the aggregate banking sector (left-hand panel) and the four largest BHCs in terms of assets. The banking industry is highly concentrated: The "Big Four" is used in reference to accounting firms. The largest BHCs account for roughly 50% of aggregate assets. At the onset of the crisis, the growth of bank assets, loans, and liabilities slow down but never drop as dramatically as market valuations for bank equity (see below). The amount of outstanding loans, the largest component of bank assets, stagnate during the crisis and eventually falls. By 2009 Q4, loans net of the allowance for loan losses fall by \$361 billion, a drop of only 6.84%.[54] This number is driven only in part by losses as banks also slow down their issuances of new loans.

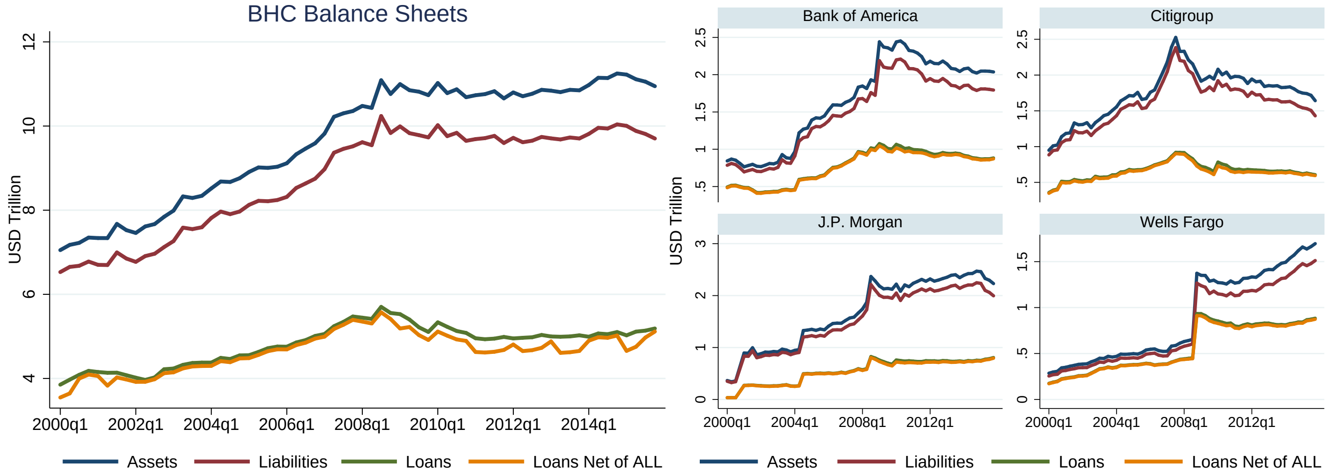
A.3 Difference between Market and Book Data

Difference between market and book data. To get a sense about how the crisis affects banks, we report the changes in select aggregate balance-sheet components and aggregate bank-equity-return data since the beginning of the Great Recession in 2007 Q3 in Table 4. We do so in two ways. We first fit a linear trend to the logged real series and we report deviations from that trend in the first three columns of the table.[55] We estimate the trend, using the data through 2007

[54] The allowance for loan losses is an estimate of likely loan losses for the outstanding loans on the balance sheet. The next subsection provides more detail on how bank accountants calculate this number.

[55] We use the seasonally-adjusted GDP deflator to adjust for inflation, and report all values in 2012 Q1 dollars.

Figure 1: Balance Sheets of BHCs



>

Notes: These figures show data on assets, liabilities, loans, and loans net of ALL for BHCs. Data come from the FR Y-9C. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out "probable and estimable" future losses on the current stock of loans). All variables are converted to 2012 Q1 dollars, using the seasonally adjusted GDP deflator. The left-hand panel shows aggregate time series, excluding new entrants to the sample BHC such as Goldman Sachs. The right-hand panel shows the same data for the four largest BHCs. Note that the spike in the data on Wells Fargo is due to its acquisition of Wachovia. Likewise, JP Morgan acquires Bear Stearns and WaMu, while Bank of America takes on Merrill Lynch and what was left of CountryWide.

Q3 and report the changes since that trend.[56] Second, we report simply the real changes since 2007 Q3 in the last three columns of the table. Each column computes the change until the fourth quarter of the year indicated by the column. For aggregate bank balance sheet quantities, we focus our attention on the aggregate series of loans and different measures of equity since these are the quantities that are at the head of macro-finance models. We also report the changes in bank-equity-returns data to provide a summary of shareholder losses and, for comparison, we report the changes in the S&P stock market index.

Bank accounting practices. The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g., Laux and Leuz 2010) because they are not required to mark to market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring the success of a bank are its book returns on equity (ROE).[57] Given that ROE is a measure of success, manager compensation is linked to book-value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet

[56] Since market returns and book ROE are flows rather than levels, we detrend by simply subtracting the pre-crisis average. Also, since flows can be negative, we use $\log(1+r)$ instead of $\log(r)$. A concern with log-linear detrending is that it could be based on an unsustainable boom, yielding an overestimate of the size of the cyclical deviation. Simply looking at raw changes in this series sidesteps these concerns but only by not dealing with the trend altogether. We report both estimates for completeness, but we acknowledge that each of these estimates is imperfect. We also compute (available upon request) estimates from HP-filtered data. The HP-filtered residuals were typically of substantially smaller magnitude than the residuals estimated with a log-linear trend. The HP-filter seems to be overfitting the data and treating as a trend what is really just a persistent cyclical component.

[57] For example, JP Morgan's 2016 annual report states "the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured" (on page 83 of the report).

Table 4: Aggregate Descriptive Statistics

	Log-Linear			Real Change		
	2008	2009	2010	2008	2009	2010
Market Cap.	-61.21%	-49.98%	-42.86%	-54.08%	-39.35%	-29.03%
	(-$945B)	(-$790B)	(-$694B)	(-$705B)	(-$513B)	(-$378B)
Book Equity	-3.46%	-1.50%	-4.41%	11.83%	21.70%	25.97%
	(-$32B)	(-$15B)	(-$46B)	($94B)	($172B)	($206B)
Common Equity	-28.44%	-11.69%	-10.42%	-17.35%	8.29%	16.64%
	(-$275B)	(-$120B)	(-$114B)	(-$145B)	($69B)	($139B)
Loans Net of ALL	2.68%	-10.41%	-14.33%	2.58%	-6.84%	-7.27%
	($141B)	(-$571B)	(-$819B)	($136B)	(-$361B)	(-$384B)
S&P 500	-25.55%	-7.01%	4.63%	-42.08%	-28.83%	-21.20%
Bank Market Return	-57.87%	-61.42%	-60.23%	-54.26%	-55.28%	-50.78%
	(-$755B)	(-$801B)	(-$785B)	(-$708B)	(-$721B)	(-$662B)
Book Return on Equity	-20.30%	-27.89%	-33.58%	-7.84%	-6.34%	-3.11%
	(-$171B)	(-$236B)	(-$284B)	(-$66B)	(-$54B)	(-$26B)

Notes: Top row shows cyclical deviations in percentage points since 2007 Q3; bottom row shows deviations converted into raw values. Book equity refers to book equity of publicly traded BHCs. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out "probable and estimable" future losses on the current stock of loans). All variables are deflated using the seasonally adjusted GDP deflator. Level variables are converted to 2012 Q1 dollars, flow variables are deflated by subtracting inflation. Bank market returns deviations and book returns on equity are cumulated since the end of 2007 Q3, and dollar values are obtained by multiplying the cumulative percentage point deviation by real market capitalization and real book equity at the end of 2007 Q3, respectively.

capital standards that are based on book values.

Banks' accounting flexibility is studied extensively in the accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 review the literature on this issue, Francis et al., 1996 study the same issue for non-financial firms). In practice, banks can use two methodologies to record securities on the books: either amortized historical cost (the security is worth what it cost the bank to buy it, along with appropriate amortization) or fair value accounting.[58] In addition to mispricing securities, another degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost if they deem those impairments as temporary (i.e., if they believe the asset will return to its previous price). This gives banks substantial leeway and leads banks to overvalue assets on their books during the crisis. Huizinga and Laeven (2012) find that banks use discretion to hold real-estate-related assets at values higher than their market value. (Laux and Leuz, 2010) point to some notable cases of inflated books that occurr during the crisis: Merrill Lynch record sales of $30.6 billion dollars of collateralized debt obligations (CDOs) for 22 cents on the dollar while the book value is 65% higher than its sale price. Similarly, Lehman Brothers writes down its portfolio of commercial mortgage-backed securities (MBS) by only 3%, even when an index of commercial MBS is falling by 10% in the first quarter of 2008. Laux and Leuz (2010) also document substantial underestimation of loan losses in comparison to external estimates.

This shows up in our own analysis as well: Figure 2 shows that provisions for loan losses and net charge-offs only reach their peak in 2009 and 2010, respectively, and remain quite elevated at least through 2011, well after the recession ends. The decomposition of net charge-offs shows that these losses are heavily driven by real estate, suggesting they are associated with the housing crisis.[59] In 2011, banks' books only acknowledge losses that the market had already predicted when the crisis hit.

Harris et al. (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than does the allowance for loan losses.[60] This implies that the allowance for loan losses is not capturing all of the available information to estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the "model" that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is "estimable and probable" (Harris et al., 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they are not supposed to update their books until the loss is imminent.

[58] Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g., a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh et al. 2015; Laux and Leuz 2010). Recent work shows that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to misprice assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets are valued substantially below one (Goh et al. 2015; Kolev 2009; Song et al. 2010). Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moves $53 billion into Level 3 between the fourth quarter of 2007 and the first quarter of 2008 and reclassifies $60 billion in securities as held-to-maturity, an accounting measure that enables Citi to use historical costs.

[59] When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as a PLL. Later, when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset. Net charge-offs are charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for the PLL because the FR Y-9C does not provide information on the PLL by loan category.

[60] The ALL is the stock variable that corresponds to the PLL.

Information content We test whether book equity captures all available information about bank cash flows by using cross-sectional regressions of market equity on book equity and several other profitability measures. We are motivated by the efficient-markets hypothesis that suggests that market values reflect all available information about future dividends and, by extension, about banks' future profits and present net worth: If market equity indeed contains additional information about bank profitability that is not captured by their book values, then market equity will be correlated with variables that capture profitability, even after conditioning on book equity. To help us visualize the additional information content of market values over and above book values, consider the following cross-sectional regression:

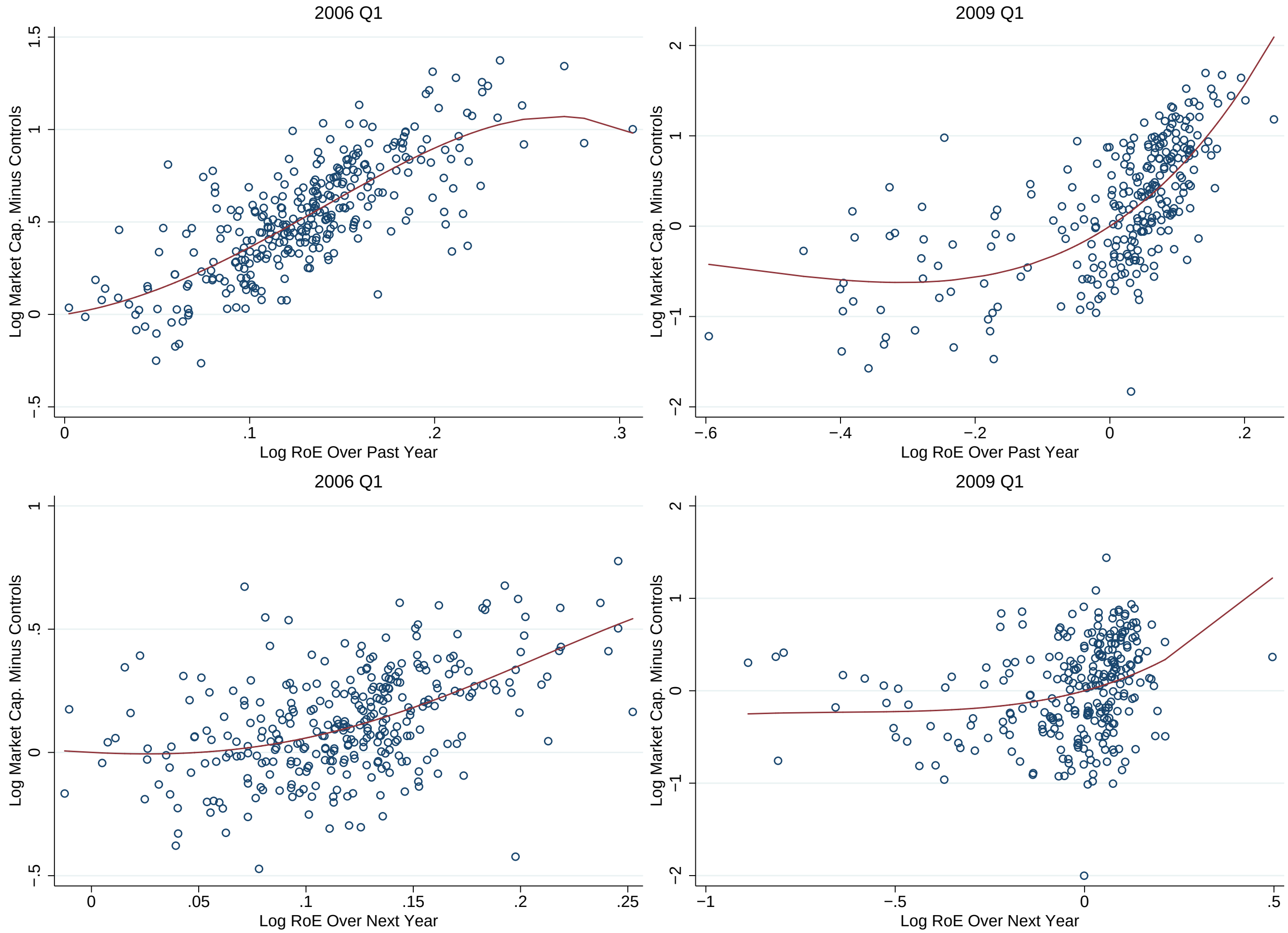
$$\log(\text{Market Equity}_i) = \alpha + \beta \log(\text{Book Equity}_i) + f(X_i) + \epsilon_i,$$

where $f(X)$ represents the polynomials in our variables of interest and i indexes banks.[61] We then construct the partial residual $\log(\text{market equity}) - \alpha - f(X)$ and plot this on the vertical axis of Figure 2. We plot the regressor of interest, X, on the horizontal axis. By construction, the polynomial $f(X)$ that best fits the outcome variable log (market equity) will also be the polynomial that best fits the partial residual. Thus, Figure 2 allows us to plot $f(X)$ and assess the goodness of its fit. We consider a quartic in log ROE over the past year (controlling for the log book equity) and a quartic in the log ROE over the next year (controlling for the log book equity and a quartic in the log ROE over the past year) as $f(X)$.[62] These graphs confirm that market capitalization, controlling for book equity, is increasing in both ROE over the past year and ROE over the next year. Hence, even after controlling for book equity, market capitalization captures the information content of net income from the past and for the upcoming year. Note that the non-linear regression specification is important. For example, in the post-crisis period, there is a left tail of banks with very negative ROEs. In this region, the marginal effect of the ROE on market capitalization is much smaller.

[61] Appendix Section A.3 shows the partial R^2 for this regression for a range of variables capturing profitability measures such as loan charge-offs and ROE over various time horizons.

[62] For improved visibility, we exclude outliers from the graph window by limiting the graph's horizontal axis to values within ± 3 standard deviations from the mean.

Figure 2: Information Content in Books



Notes: These figures show the results from a cross-sectional regression of the log market equity on assorted variables. The top row shows the results from a regression of the log market capitalization on the log book equity and a quartic in the log ROE over the past year. The bottom row shows the results from a regression of the log market capitalization on the log book equity, a quartic in the log ROE over the past year, and a quartic in the log ROE for the next year. The horizontal axis shows the regressor of interest, and the vertical axis shows the outcome minus the effect of the controls (for the top row, the controls are a constant and the log book equity; for the bottom row, the controls are a constant, the log book equity, and a quartic in the log ROE over the past year). The left-hand column shows the results for 2006 Q1, the right-hand column shows the results for 2009 Q1. Regressions are run on the cross section of banks with all variables available, but to improve visibility the horizaontal axis of the graph window is restricted to ± 3 standard deviations from the mean. Data on market capitalization and returns are obtained from the CRSP, and all other data are from the FR Y-9C. The log RoE is defined as $\log(1 + \text{ROE})$. The ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; The ROE over the next year is defined as being a one year lead of this variable.

B Additional Impulse Responses

B.1 Risk Adjustment

For our main impulse-response results, we wish to use risk-adjusted returns rather than raw returns. More formally, we assume that the market returns of bank i at time t are given by

$$\underbrace{r_{it}}_{\text{Raw Returns}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t}_{\text{Factors}} \underbrace{\beta_i}_{\text{Loadings}} + \underbrace{\varepsilon_{i,t}}_{\text{Idiosyncratic Component}}$$

All returns are logged; e.g., r_{it} refers to $\log(1 + \text{raw bank returns})$. We wish to isolate the variation in the idiosyncratic shocks, $\varepsilon_{i,t}$, and we use this variation to estimate the impulse responses.

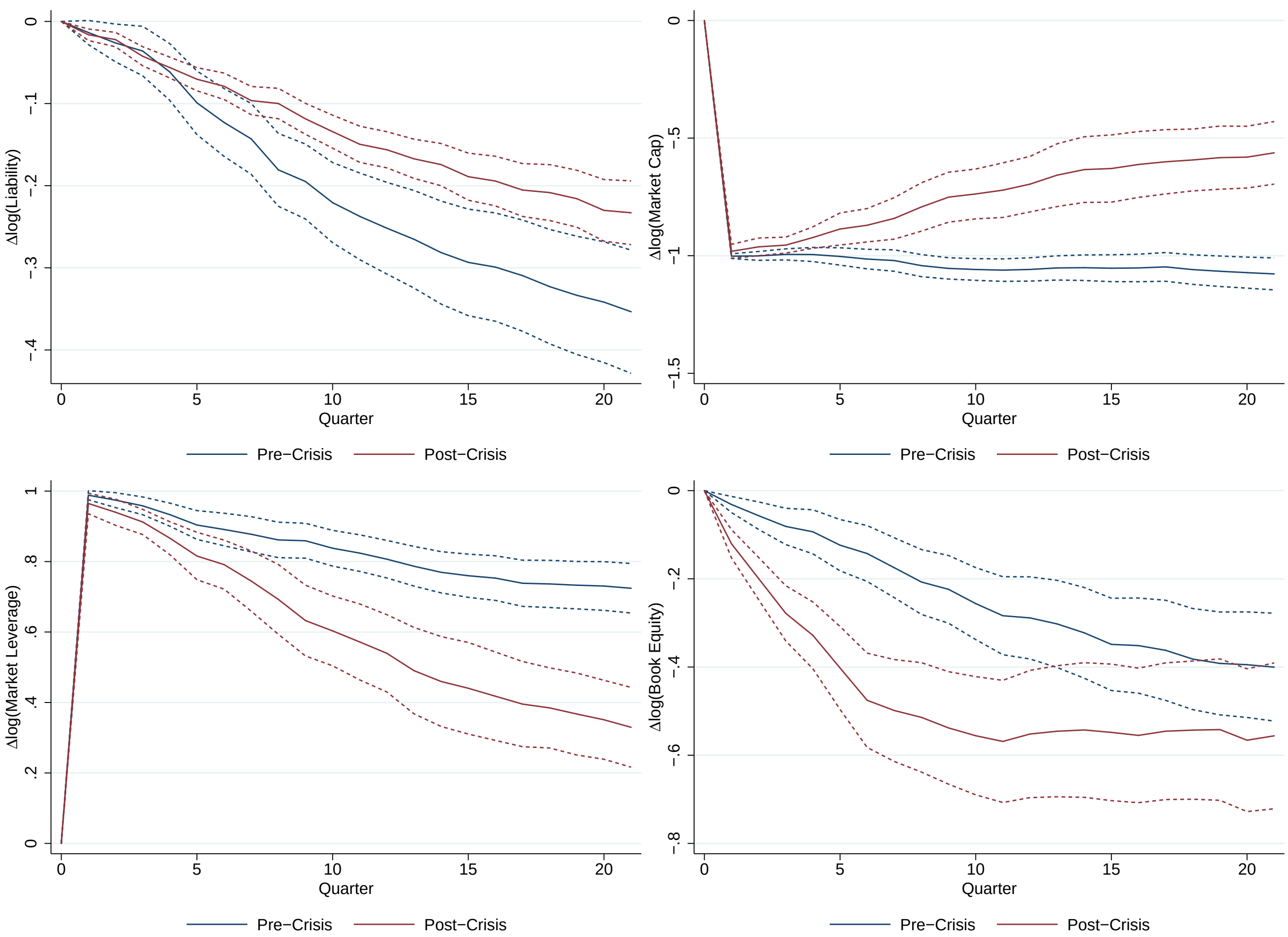
A natural but naive approach would be to estimate the above model for each bank i, using OLS, and then to use the estimated residuals, $\hat{\varepsilon}_{it}$, as the regressors in the impulse-response estimation. The problem here is that it induces bias: $\hat{\varepsilon}_{it}$ is a noisy measure of the true regressor ε_{it}, which leads to bias as long as T is finite (the bias will shrink as T grows large because $\hat{\varepsilon}_{it}$ will converge to the true ε_{it}).

Fortunately, there is a simple solution: we estimate $\hat{\varepsilon}_{it}$ using OLS, and then we use $\hat{\varepsilon}_{it}$ as an instrument for the unadjusted returns. Since our main regressions use contemporaneous returns, twenty lags, and their interaction with a post-crisis dummy, this means we use as instruments contemporaneous $\hat{\varepsilon}_{it}$, twenty lags of $\hat{\varepsilon}_{it}$, and their interaction with a post-crisis dummy. Instrumental variables do not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the "good variation," ε_{it}, and uncorrelated with the "bad variation," $\alpha_i + X_t\beta_i$. This is mechanically what we are doing when we run OLS at the bank level, and if the assumed model for returns is correct, then we have $\mathbb{E}[\hat{\eta}_{it}(\alpha_i + X_t\beta_i)] = \alpha_i\mathbb{E}[\hat{\eta}_{it}] + \mathbb{E}[\hat{\eta}_{it}X_t]\beta_i = 0 + 0$. Thus, our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS regression estimating $\hat{\varepsilon}_{it}$ is conducted at the bank level, we cluster our standard errors at the bank level (clustering at the bank level is already a good idea).

B.2 Results Without Factor Risk Adjustment

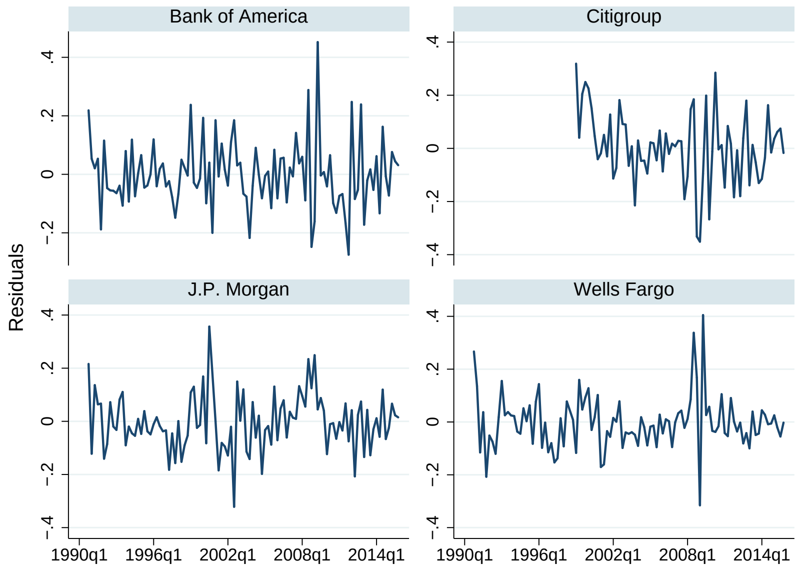
While we favor the risk-adjusted results, we also compute "unadjusted results" for the impulse responses, which we report here for completeness. The results are qualitatively and quantitatively similar across these methods. Compared to the risk-adjusted results, however, the unadjusted results suggest a smaller response of liabilities in the pre-crisis period and, thus, they also suggest a slower pre-crisis leverage adjustment.

Figure 3: Estimated Impulse Responses for Stock Variables (No Risk Adjustment)



Notes: These figures show estimated impulse-response functions for BHCs. The figures show the estimated impulse responses to one–unit negative-returns shocks. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and log market capitalization (results using log(liabilities + market capitalization)/market capitalization) are extremely similar).

Figure 4: Idiosyncratic Shock Series of Big Four Bank Holding Companies



Notes: This figure plots the idiosyncratic shocks (for the four largest BHCs) used to estimate the impulse-response functions. First, we isolate the idiosyncratic component of returns using the factor model, and then we residualize this on time fixed effects.

B.3 Robustness and Validity of the Identification Strategy

In this section, we conduct various tests to check the validity of our identification strategy and the robustness of our results.

A narrative approach to corroborate the idiosyncratic shocks To provide corroberating evidence of the validity of our identification strategy, we first show that the estimated returns shocks do indeed look like idiosyncratic shocks for the four largest banks (Bank of America, J.P. Morgan Chase, Wells Fargo, Citigroup). To construct the idiosyncratic shocks, we regress each bank's market returns on the Fama-French three-factor returns and regress the residual further on the time fixed effects. The residuals from this regression represent our idiosyncratic shocks.[63] Figure 4 presents our estimates of the idiosyncratic shocks. They indeed look like white noise and do not seem to be substantially autocorrelated. Note that the time series for Citigroup starts a little later because Citigroup did not exist until 1998 when Traveler's merged with Citicorp.

We also provide narrative support for the idiosyncratic nature of our estimated shocks by using an extensive search of newspaper articles for large idiosyncratic shock value estimates.

[63]We are controlling for the time fixed effects because they are included in the regression we actually run to get the impulse-response function.

Table 5: Narrative Support for Idiosyncratic Shocks

Bank Name	Year-Qtr	Idiosyncratic shock	Bank-specific events
Bank of America	2000q4	-0.200	Sunbeam (which BofA lended to) posts an $86M loss. BofA said net charge-offs in Q4 will double. BofA issues warning on $1B uncollectible debt, may miss the December quarter profit forecast by as much as 27%.
	2003q4	-0.218	BofA agrees to pay $47 to buy FIeet Boston Financial "hefty premium" & "could dilute earnings."
	2008q3	0.288	BofA to buy Merrill for $50B (Sept 15).
	2009q2	0.452	Stress test: BofA needs to address $34B capital shortfall, better than expectation.
	2011q4	-0.275	Merrill Lynch agrees to pay $315 million to end a mortgage-securities lawsuit (Dec 7).
	2012q4	0.248	BofA considered better buy after increase in house prices that (given its portfolio composition) particularly benefited BofA.
Citigroup	1999q1	0.319	Citigroup profit falls 53% in 4th period but still topps analysts' expectations.
	1999q3	0.205	Citigroup posts an unexpected increase of 9.3% in net income for second quarter (July 20).
	1999q4	0.250	Citigroup's citibank unit is marketing credit card for the internet to millions.
	2000q1	0.226	Citi Intelligent Technology receives investment; dividends increase from $1.05 to $1.20.
	2003q4	-0.215	Citi to repay certain funds $16 M plus interest; Citigroup Asset Management faces federal probe.
	2009q1	-0.351	Citigroup has $2B in direct gross exposure to LyondellBasell Industries, which filed for bankruptcy protection last week. Fitch cuts Citi preferred to junk.
	2009q3	0.199	Citi reports profit after gain from Smith Barney. Citigroup's mortgage mitigation rises by 29% in second quarter.
	2009q4	-0.267	Citi fined in tax crackdown. Abu Dhabi's sovereign wealth fund is demanding that Citigroup scraps a deal that would see the fund make a heavy loss on a $7.5 billion investment in the bank.
	2010q2	0.285	Citi reportes quarterly earnings of $4.4B exceeding expectations.
J.P. Morgan Chase	1997q2	-0.182	J.P. Morgan particularly large exposure to 1997 Asian Financial Crisis. https://www.imf.org/external/pubs/ft/wp/1999/wp99138.pdf
	2000q1	0.169	J.P. Morgan told investors on Monday that January and February had topped performance levels seen in the fourth quarter. Dividends increase from $0.2733 to $0.3200 on March 21.
	2000q3	0.357	Chase buying J.P. Morgan.
	2001q2	-0.185	J.P. Morgan Chase disclosed this week that their venture capital portfolios had incurred significant losses.
	2002q3	-0.322	JPMorgan Partners reports $165M operating loss for Q2. J.P. Morgan sees third-quarter shortfall.
	2004q4	-0.198	JPMorgan Chase profit falls 13%.
	2008q3	0.234	J.P. Morgan profit falls 53% but tops Wall Street target.
	2009q1	0.249	J.P. Morgan net falls sharply but tops Wall Street view. J.P. Morgan to sell Bear Wagner to Barclays Capital: WSJ.
	2012q2	-0.207	J.P. Morgan: London Whales $2 Billion Losses. Two shareholder suits filed against J.P. Morgan.
Wells Fargo	2001q2	-0.161	Wells Fargo discloses that their venture capital portfolios incurred significant losses. Wells Fargo to take $1.1 billion charge
	2008q3	0.338	Wells Fargo's net dropps 21% as it sets aside $3 billion for loan losses, better than expected. Earnings decline but beat estimates.
	2009q1	-0.315	Wells Fargo posts a surprise $2.55B Q1 loss, later revised to $2.77B. Wells Fargo adds a pretax $328.4M impairment of perpetual preferred securities to its fourth-quarter loss.
	2009q2	0.405	Wells Fargo sees record Q1 profit, projections easily exceed expectations (expects earnings of $3 billion).

Table 5 shows that large absolute idiosyncratic shock values are consistent with good or bad bank-specific events, such as "Wells Fargo sees record Q1 profit, projections easily exceed expectations," or "Citi fined in tax crackdown." The table shows that large positive or negative idiosyncratic shocks can be corroborated with specific events that appear bank specific, which supports the validity of our identification strategy.

Placebo Tests To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags, as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends. We are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo tests are encouraging and suggest that our results are not driven by prior differences in the behavior of banks that experience returns shocks.

Identification robustness We provide a few additional pieces of evidence that corroborate the validity and robustness of our identification strategy.

First, we verify that our results are robust to excluding the crisis years 2008 and 2009 from our sample. The idea is to rule out a lot of stories related to specific events during the crisis (e.g., the realization that the government might not guarantee that a bank would not fail, or that this was somehow about exposure to Lehman). The results are shown below (for our main outcomes: liabilities, market cap, and market leverage). It makes no noticeable difference to the results.

Second, we check whether bank mergers drive the results. To this end, we drop the quarter of the merger as well as the quarter before and after the merger. The results for our main outcomes: liabilities, market equity, and market leverage, are provided in Figure 7. Again, it makes no noticeable difference to the results.

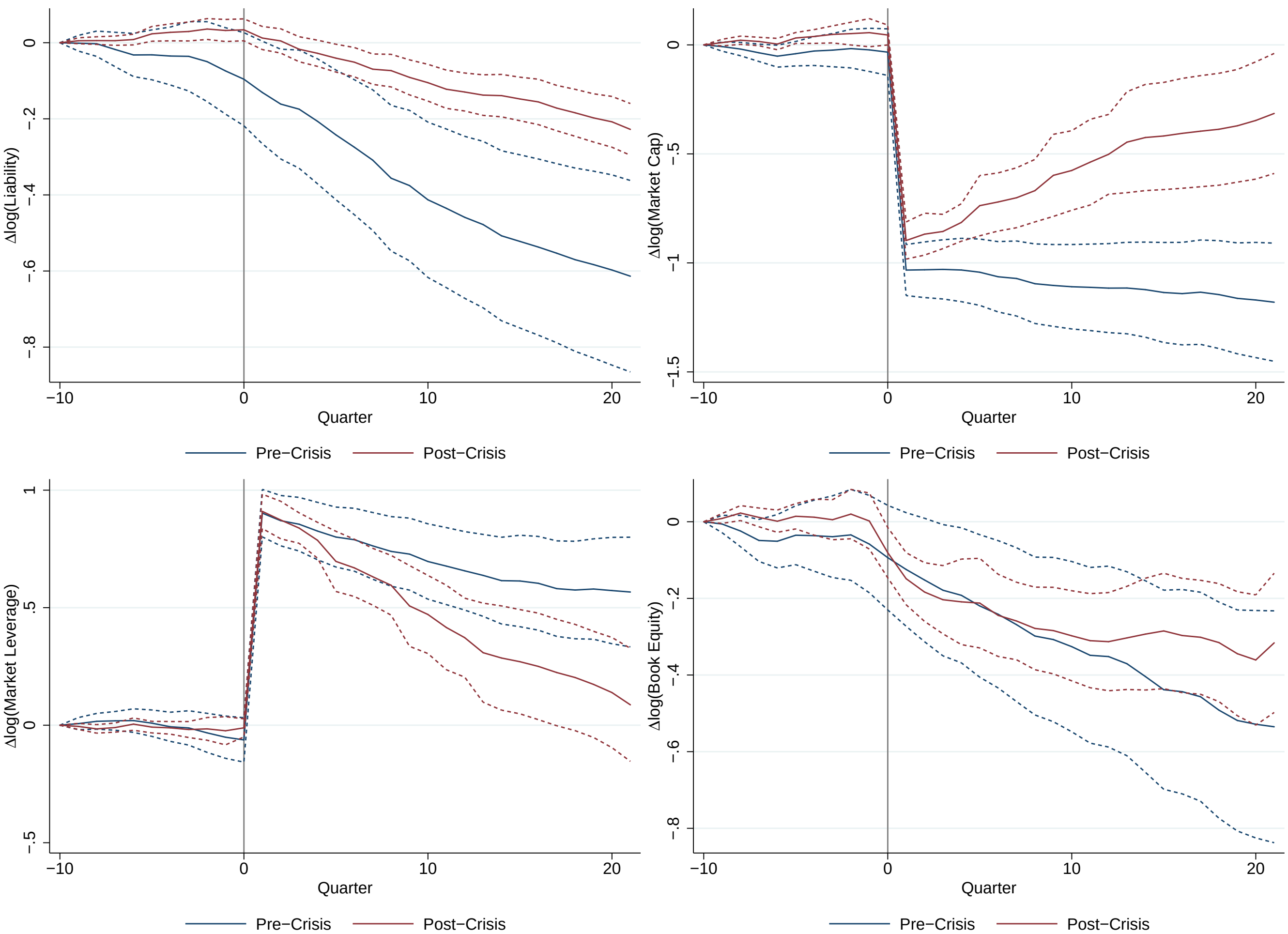
Similarly, we check whether the results are driven by the stress tests performed by banks: these stress tests were implemented after the onset of the crisis and they encouraged or mandated that banks raise additional capital. To show that the stress tests do not drive the results, we drop all banks that ever participated in a stress test (e.g., Bank of America participated in stress tests, and so we drop Bank of America from our sample in all periods). The results for our main outcomes are shown in Figure 8. Again, it makes no noticeable difference to the results.

Another potential concern is that the returns shocks, rather than the default shocks, could be picking up shocks to future investment opportunities. To test this concern, we check the response of the liquid assets ratio: if negative returns shocks indeed predict lower future investment opportunities rather than current cash flows, then we would expect banks to respond to these shocks by moving their portfolios into liquid assets. The results, shown in Figure 9, show no statistically significant response of liquid assets, pre-crisis, and a small temporary response post-crisis that is reversed within a few quarters. We take this as evidence against the hypothesis that returns shocks reflect shocks to investment opportunities.

An alternative, broader version of the liquidity ratio test calculates the liquidity ratio as the ratio of (Cash + Federal Funds Sold + Securities Purchased Under Agreement to Resell + Securities)/Total Assets. Figure 10 shows the impulse-response function for this version of the liquidity ratio. . The impulse-response function has no significant response pre-crisis and a significant but quantitatively small response post-crisis.

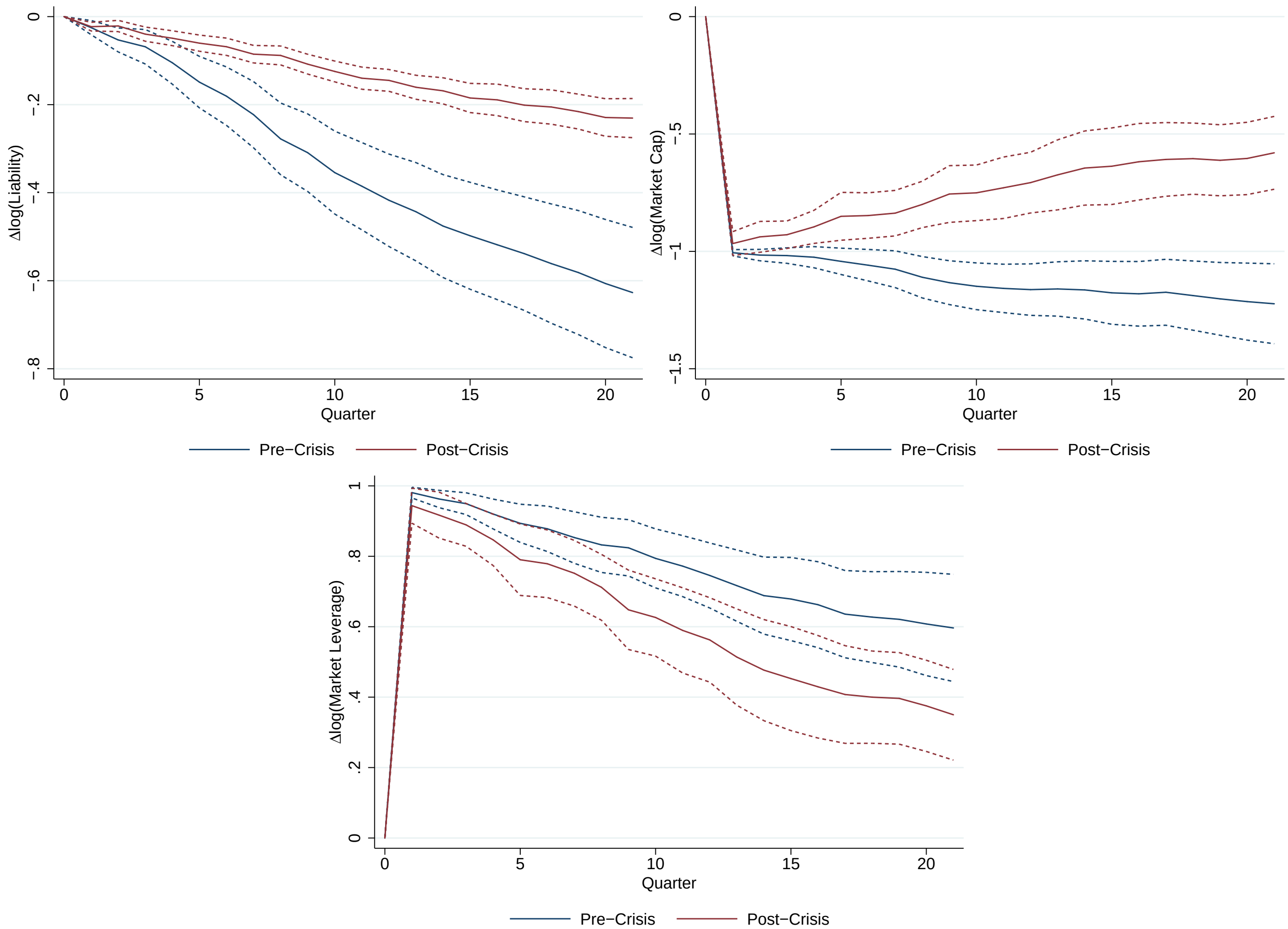
To put the size of the post-crisis response in perspective, the graph is saying that if there is a 10% negative shock to market returns, then the liquid assets ratio would rise by 0.02 over the

Figure 5: Estimated Impulse Responses for Stock Variables (Risk Adjusted, with Placebo)



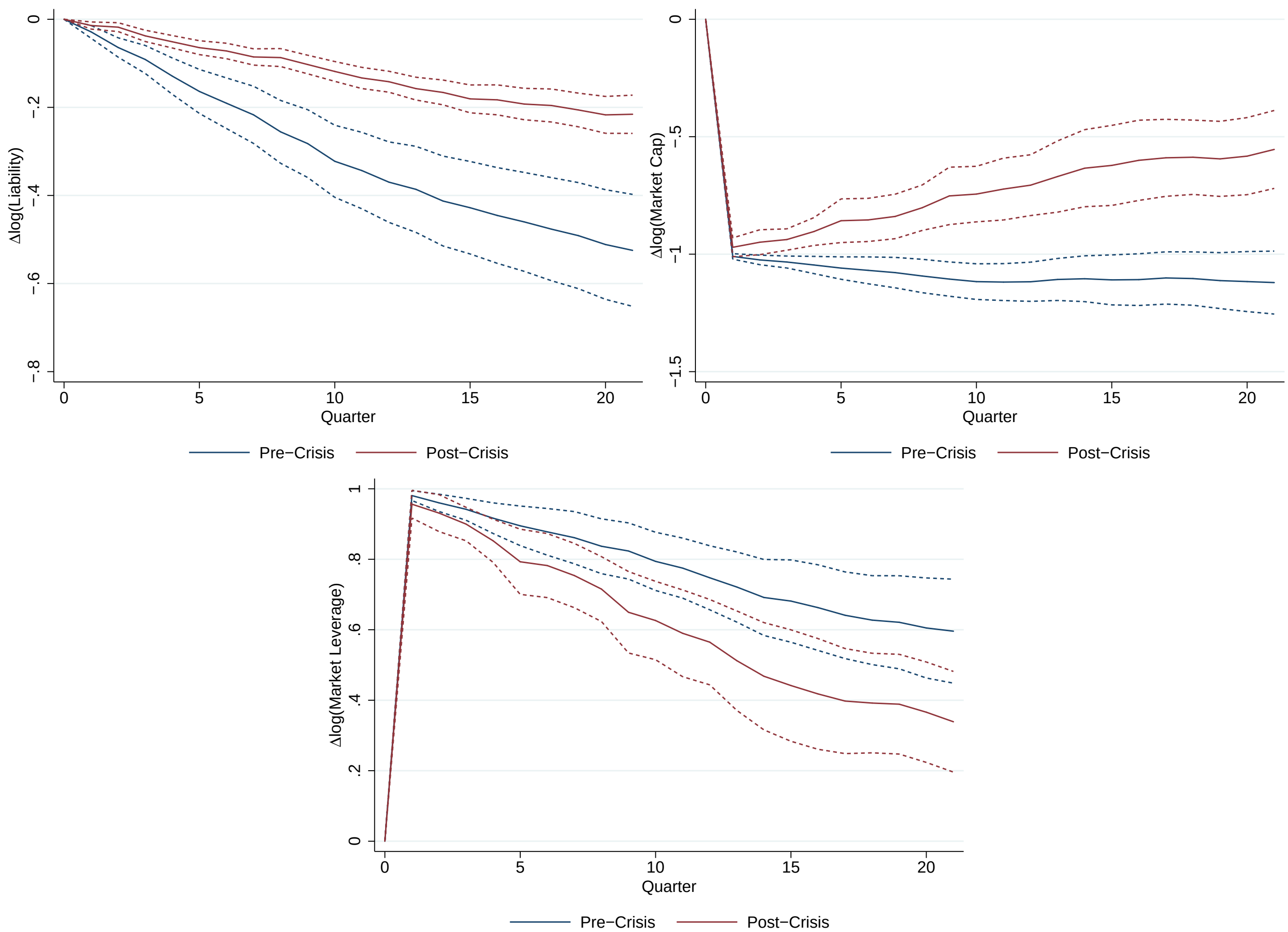
Notes: These figures show estimated impulse-response functions for BHCs. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalizations and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and log market capitalization (results using log(liabilities + market capitalization)/market capitalization) are extremely similar).

Figure 6: Estimated Impulse Responses: Dropping 2007 and 2008



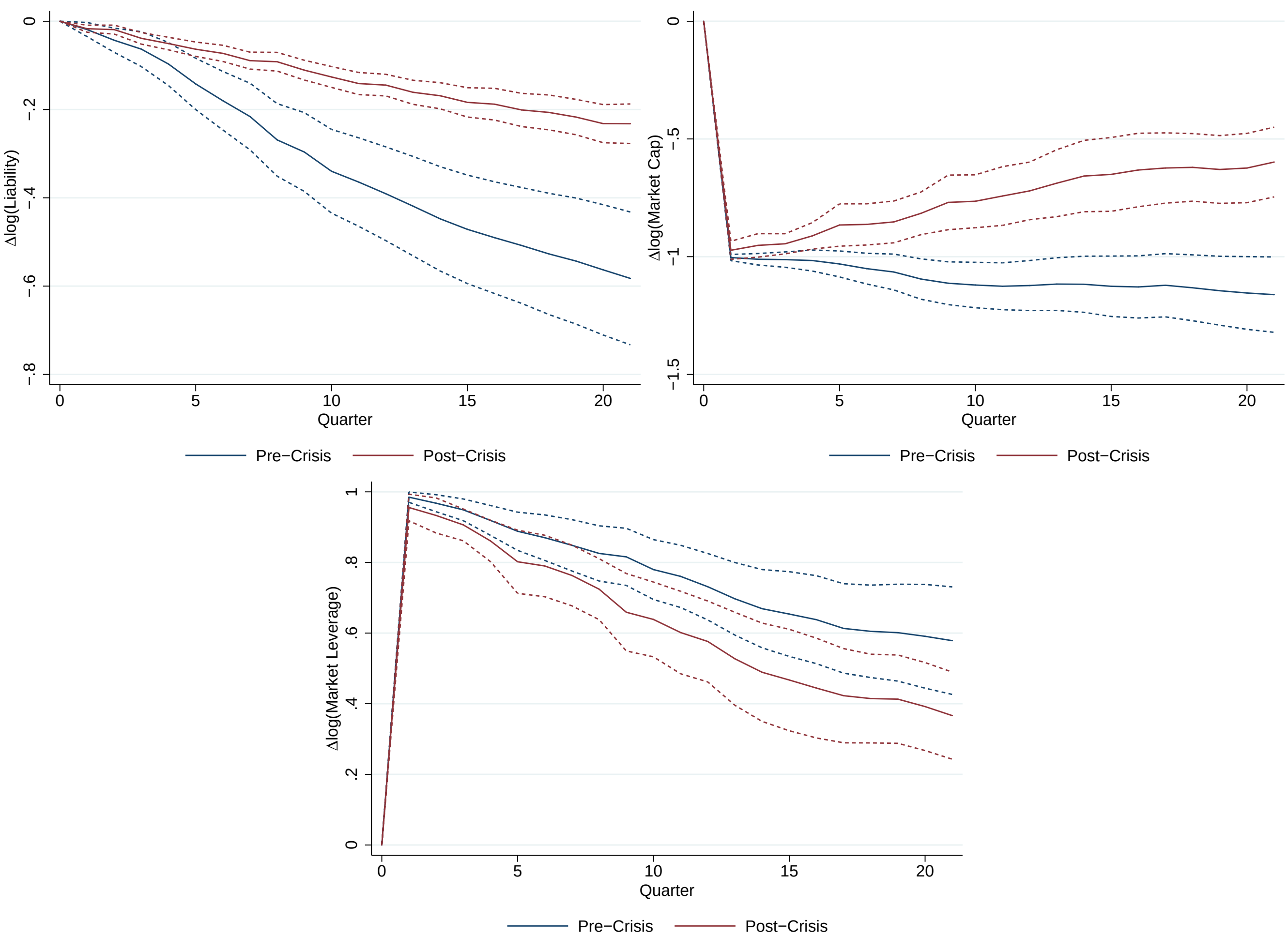
Notes: These figures show estimated impulse-response functions for BHCs, dropping observations from the years 2007 and 2008. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalizations and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and the log market capitalization.

Figure 7: Estimated Impulse Responses: Excluding Mergers



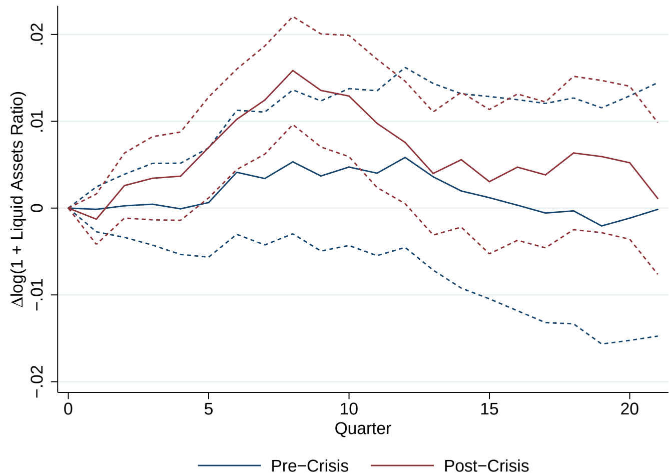
Notes: These figures show estimated impulse-response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after the merger. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and the log market capitalization.

Figure 8: Estimated Impulse Responses: Excluding Mergers



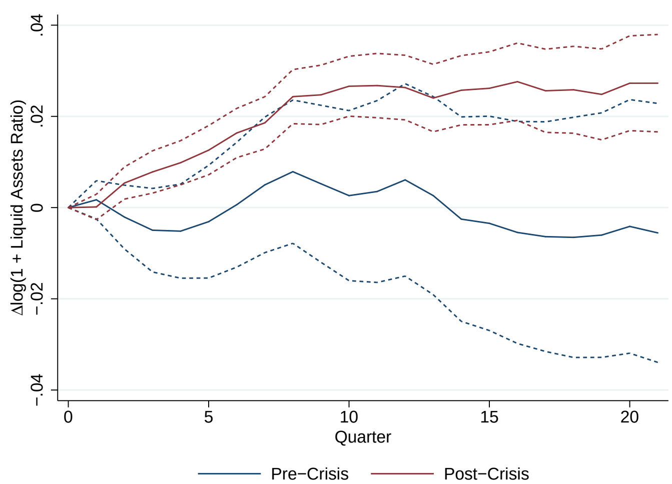
Notes: These figures show estimated impulse-response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after the merger. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and the log market capitalization.

Figure 9: Estimated Impulse Responses of the Liquidity Ratio



Notes: This figure shows the estimated impulse-response function for BHCs to a 1% negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The liquid assets ratio is defined as log((cash + treasury bills) /total assets). Within the regression sample, the average liquid assets ratio is 0.057.

Figure 10: Estimated Impulse Responses of Liquidity Ratios (Alternative Formula)



Notes: This figure shows estimated impulse-response functions for BHCs. The figure shows the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The liquid assets ratio is defined as log((cash + fed funds sold + securities purchased under agreement to resell + securities) /total assets).

course of two years. This is off a base of 0.25-0.30, depending on whether we are taking the mean of the log(1+ratio) or of the raw liquid assets ratio.

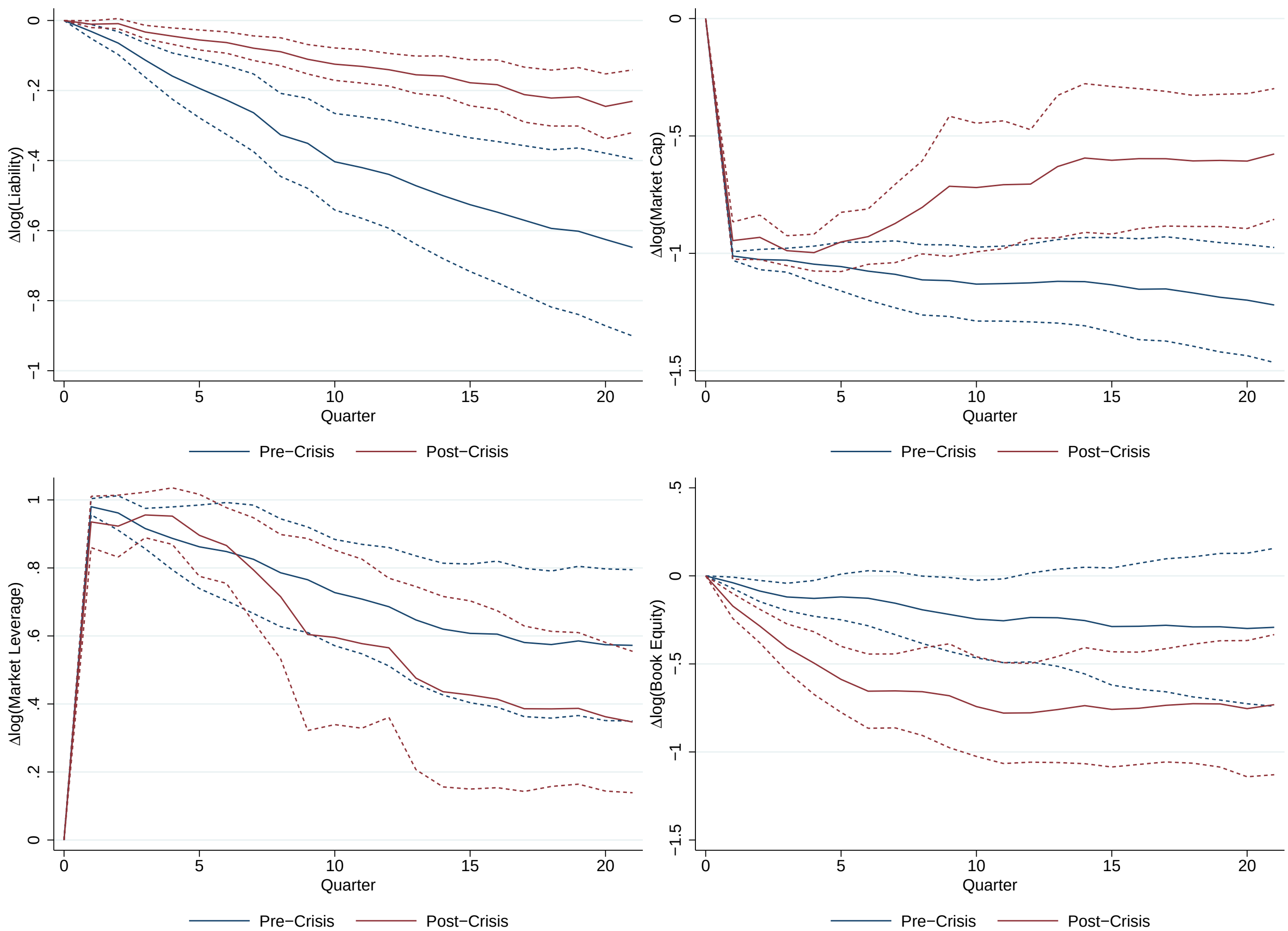
B.4 Heterogeneity

We explore heterogeneity in the impulse-response functions by dividing banks into two groups, based on a variable, and estimating the impulse responses separately for each group. We divide banks by size (total assets), trading assets ratio (trading assets as a share of total assets), the risk-weighted assets ratio (risk-weighted assets as a share of total assets), and the mortgage ratio (real-estate loans as a share of total assets). We use the value of the variable in 2000 Q1 to sort banks into two groups: above-median and below-median banks. We report the results in this section. Broadly, we do not find strong evidence of differential responses, but we lack statistical power to rule out some meaningful differences.

Since bank size is among the most important differences across different banks, we begin by discussing the results for heterogeneity by size. The results are shown in Figures 11 and 12. Visually, these impulse responses look remarkably similar to each other. However, the standard errors are sufficiently large that we cannot rule out meaningful differences in the impulse responses.

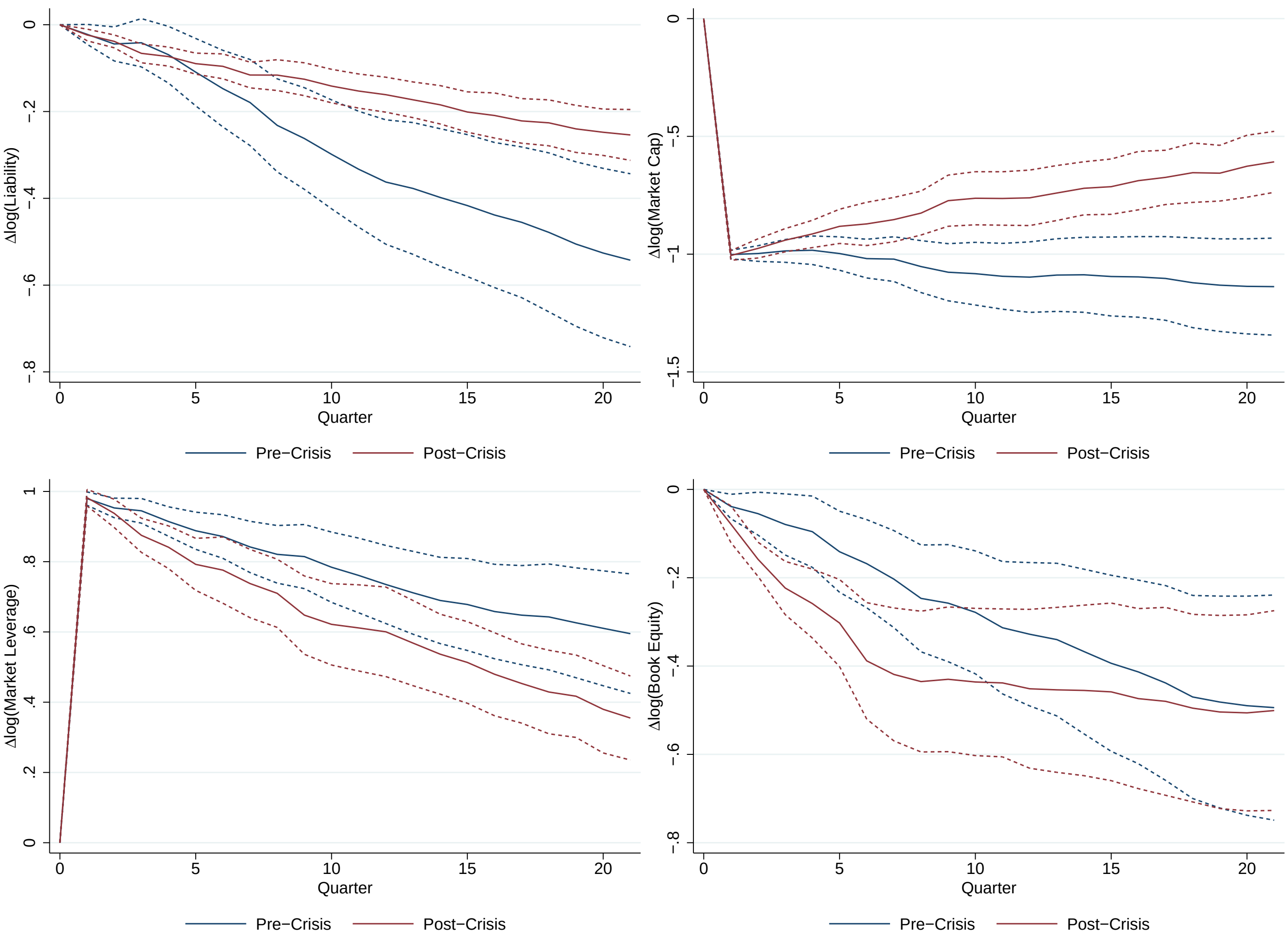
We summarize the results of these impulse responses as well as those of the other potential groupings (by trading assets ratio, risk-weighted assets ratio, and mortgage ratio) in Tables 6, 7, 8, and 9 below. For each grouping, we report the cumulative impulse response for the high- and low-asset groups after 10 quarters and after 20 quarters, and we also report the p-values of a test of equality between the impulse responses of the two groups. In a table of 64 tests, only one test rejects the null at the 5% level. As before, we take this to suggest that there is not strong evidence in favor of sizable heterogeneity, but we caution that the standard errors are too large to rule out meaningful heterogeneity.

Figure 11: Impulse Responses for Small Banks



Notes: These figures show estimated impulse-response functions for BHCs. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and log market capitalization (results using log(liabilities + market capitalization)/market capitalization) are very similar).

Figure 12: Impulse Responses for Large Banks



Notes: These figures show estimated impulse-response functions for BHCs. The figures show the estimated impulse response to a one-unit negative-returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from the CRSP and all other data are from the FR Y-9C. The panels display the impulse responses of the log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as the log(liabilities/market capitalization), so that it represents the difference between the response of the log liabilities and log market capitalization (results using log(liabilities + market capitalization)/market capitalization) are every similar).

Table 6: Heterogeneity in Impulse Responses: Small vs. Large Banks

		Response After 10 Quarters			Response After 20 Quarters		
		Small	Large	p-value on Equality	Small	Large	p-value on Equality
Market Equity	Pre-Crisis	-1.13	-1.09	0.75	-1.22	-1.14	0.61
		(0.08)	(0.07)		(0.12)	(0.11)	
	Post-Crisis	-0.71	-0.76	0.71	-0.58	-0.61	0.84
		(0.14)	(0.06)		(0.14)	(0.07)	
Liabilities	Pre-Crisis	-0.42	-0.33	0.39	-0.65	-0.54	0.52
		(0.07)	(0.07)		(0.13)	(0.10)	
	Post-Crisis	-0.13	-0.15	0.49	-0.23	-0.25	0.67
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.71	0.76	0.59	0.57	0.60	0.87
		(0.08)	(0.05)		(0.11)	(0.09)	
	Post-Crisis	0.58	0.61	0.81	0.35	0.35	0.95
		(0.13)	(0.06)		(0.11)	(0.06)	
Book Equity	Pre-Crisis	-0.25	-0.31	0.68	-0.29	-0.49	0.44
		(0.12)	(0.08)		(0.23)	(0.13)	
	Post-Crisis	-0.78	-0.44	0.04	-0.73	-0.50	0.32
		(0.15)	(0.09)		(0.20)	(0.12)	

Notes: The table compares impulse responses of small vs. large BHCs. BHCs are categorized into the small vs. large group, based on their total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative-impulse response after 10 quarters of each variable, pre- and post-crisis, to a one-unit negative-return shock, for small banks. The second column shows the same results but for large banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for small vs. large banks. The fourth through sixth columns mirror the first three columns but examine the cumulative impulse response after 20 quarters.

Table 7: Heterogeneity in Impulse Responses: Low vs. High Trading Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.10	-1.20	0.60	-1.15	-1.32	0.54
		(0.05)	(0.19)		(0.07)	(0.28)	
	Post-Crisis	-0.76	-0.57	0.11	-0.60	-0.47	0.33
		(0.09)	(0.08)		(0.10)	(0.09)	
Liabilities	Pre-Crisis	-0.36	-0.36	0.99	-0.56	-0.63	0.78
		(0.05)	(0.14)		(0.08)	(0.22)	
	Post-Crisis	-0.14	-0.15	0.73	-0.24	-0.25	0.91
		(0.02)	(0.04)		(0.03)	(0.05)	
Market Leverage	Pre-Crisis	0.74	0.84	0.35	0.59	0.70	0.50
		(0.05)	(0.10)		(0.07)	(0.14)	
	Post-Crisis	0.63	0.42	0.08	0.37	0.22	0.25
		(0.08)	(0.09)		(0.08)	(0.09)	
Book Equity	Pre-Crisis	-0.25	-0.42	0.35	-0.28	-0.75	0.17
		(0.07)	(0.17)		(0.13)	(0.32)	
	Post-Crisis	-0.62	-0.45	0.39	-0.66	-0.54	0.64
		(0.10)	(0.16)		(0.13)	(0.20)	

Notes: The table compares impulse responses of low vs. high trading asset ratio BHCs. BHCs are categorized into the low vs. high group, based on their trading assets as a share of total assets in 2000 Q1 relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre- and post-crisis, to a one-unit negative-return shock, for low banks. The second column shows the same results but for high-trading banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-values of a test of equality between the impulse response for low vs. high-trading banks. The fourth through sixth columns mirror the first three columns but examine the cumulative impulse response after 20 quarters.

Table 8: Heterogeneity in Impulse Responses: Low vs. High Risk-Weighted Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.09	-1.15	0.59	-1.10	-1.22	0.43
		(0.07)	(0.08)		(0.11)	(0.12)	
	Post-Crisis	-0.72	-0.79	0.64	-0.52	-0.66	0.44
		(0.12)	(0.09)		(0.14)	(0.11)	
Liabilities	Pre-Crisis	-0.29	-0.41	0.21	-0.47	-0.66	0.20
		(0.06)	(0.07)		(0.10)	(0.11)	
	Post-Crisis	-0.13	-0.17	0.17	-0.25	-0.25	0.97
		(0.03)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.80	0.73	0.47	0.63	0.56	0.59
		(0.07)	(0.06)		(0.09)	(0.10)	
	Post-Crisis	0.59	0.62	0.82	0.27	0.41	0.31
		(0.11)	(0.09)		(0.11)	(0.09)	
Book Equity	Pre-Crisis	-0.19	-0.35	0.23	-0.24	-0.45	0.40
		(0.10)	(0.09)		(0.16)	(0.18)	
	Post-Crisis	-0.49	-0.74	0.17	-0.51	-0.81	0.23
		(0.09)	(0.16)		(0.11)	(0.23)	

Notes: This table compares impulse responses of low vs. high risk-weighted asset ratio BHCs. BHCs are categorized into the low vs. high group based on their risk-weighted assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre- and post-crisis, to a one-unit negative-returns shock, for low risk-weighted banks. The second column shows the same results but for high risk-weighted banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-values of a test of equality between the impulse response for low vs. high risk-weighted banks. The fourth through sixth columns mirror the first three columns but examine the cumulative impulse response after 20 quarters.

Table 9: Heterogeneity in Impulse Responses: Low vs. High Mortgage Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.04	-1.21	0.17	-1.07	-1.27	0.26
		(0.05)	(0.11)		(0.07)	(0.17)	
	Post-Crisis	-0.75	-0.75	0.98	-0.61	-0.56	0.79
		(0.13)	(0.08)		(0.17)	(0.09)	
Liabilities	Pre-Crisis	-0.28	-0.46	0.11	-0.45	-0.73	0.09
		(0.05)	(0.10)		(0.08)	(0.15)	
	Post-Crisis	-0.17	-0.11	0.09	-0.28	-0.19	0.13
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.76	0.75	0.92	0.62	0.54	0.55
		(0.06)	(0.07)		(0.08)	(0.11)	
	Post-Crisis	0.59	0.64	0.72	0.34	0.37	0.81
		(0.11)	(0.09)		(0.13)	(0.07)	
Book Equity	Pre-Crisis	-0.20	-0.36	0.32	-0.27	-0.42	0.63
		(0.07)	(0.15)		(0.09)	(0.30)	
	Post-Crisis	-0.66	-0.56	0.57	-0.70	-0.59	0.65
		(0.10)	(0.14)		(0.13)	(0.19)	

Notes: The table compares impulse responses of low vs. high mortgage ratio BHCs. BHCs are categorized into the low vs. high group based on their real-estate loans as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre- and post-crisis, to a one-unit negative-returns shock, for low mortgage-ratio banks. The second column shows the same results but for high mortgage-ratio banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-values of a test of equality between the impulse response for low vs. high mortgage-ratio banks. The fourth through sixth columns mirror the first three columns but examine the cumulative impulse response after 20 quarters.

C Model Appendix: Derivations and Proofs

C.1 Derivation of Laws of Motion

Summary table. Table 10 provides a summary table of the drift and jump objects as functions of the ratios $\{\lambda, q\}$.

Table 10: DRIFT AND JUMP VARIABLES IN TERMS OF $\{\lambda, q\}$

	Formula	Interpretation
μ^W	$r^L(\lambda+1) - r^D\lambda + (\iota - \Phi(\iota,1))(\lambda+1) - c$	Levered returns
μ^L	$\iota(\lambda+1)$	Drift of loans
$\mu^{\bar{L}}$	$\left(\iota - \alpha\left(\frac{1}{q} - 1\right)\right)(\lambda+1)$	Drift of book loans
μ^D	$\left(r^D\lambda - \left(r^L+\delta\right)(\lambda+1) + (\Phi(\iota,1)+\delta)(\lambda+1) + c\right)$	Drift of deposits
μ^q	$(\iota+\alpha)(1-q)\text{q}$	Drift of q
μ^λ	$\left(\iota - \mu^W\right)(\lambda+1)$	Drift of leverage
μ_c^W	-1	Dividend effect on wealth growth
μ_ι^W	$(1-\Phi_\iota(\iota,1))(\lambda+1)$	Issuance effect on wealth growth
μ_c^λ	$(\lambda+1)$	Dividend effect on leverage growth
μ_ι^λ	$(1-(1-\Phi_\iota(\iota,1))(\lambda+1))(\lambda+1)$	Issuance effect on leverage growth
μ_ι^q	$(1-q)\,q$	Issuance effect on q growth
J^W	$-\varepsilon(\lambda+1)$	Jump in wealth
J^L	$-\varepsilon(\lambda+1)$	Jump in loans
$J^{\bar{L}}$	$-\tau\varepsilon\frac{1}{q}(\lambda+1)$	Jump in book loans
J^D	0	Jump in deposits
J^q	$-\frac{(\varepsilon-\tau\varepsilon q)}{(1-\tau\varepsilon q)}q$	Jump in q
J^λ	$\frac{\varepsilon(\lambda+1)}{1-\varepsilon(\lambda+1)}\lambda$	Jump in leverage

The rest of the appendix derives the terms of this table and the proofs of the theoretical results.

Notation and definitions. We begin by presenting some definitions and deriving the laws of motion of the state variables. We use μ^x and J^x to refer to the drift and jump components, respectively, of the path of a variable x scaled by the variable for wealth, W.

We define the net investment rate of the bank as

$$\iota \equiv I/L - \delta$$

and express the dividend-to-equity ratio as

$$c \equiv C/W.$$

Note that the following identities allow us to recover the original state variables $\left\{L, \bar{L}, D\right\}$ from the triplet $\{\lambda, q, W\}$:

$$L = (\lambda + 1) W \quad (20)$$
$$D = \lambda W \quad (21)$$
$$\bar{L} = q^{-1} (\lambda + 1) W. \quad (22)$$

We present some observations that aid the proof of the proposition.

Observation 1: Homogeneity of Φ in W. We prove the results for a more general class of adjustment costs,

$$\Phi (I, L) = I + \frac{\gamma}{2} \left| \frac{I}{L} - \delta \right|^{\kappa} L.$$

Recall that in the body of the paper $\kappa = 2$. To avoid cluttering the notation, we use $\Phi (I, L)$ when we refer to $\Phi (I, L, 0)$.

We can factor out L and employ the definition of ι to obtain

$$\begin{aligned} \Phi (I, L) &= \left(\iota + \delta + \frac{\gamma}{2} |\iota|^{\kappa} \right) L \\ &= \Phi (\iota, 1) L + \delta L. \end{aligned}$$

Thus, we can express the funding cost relative to equity as

$$\Phi (I, L) / W = (\Phi (\iota, 1) + \delta) (\lambda + 1), \quad (23)$$

which is a function independent of the bank's size and depends on leverage and the investment rate.

Observation 2: Homogeneity of the regulatory constraint in W. We want to express the regulatory capital requirement in terms of the end-of-period choices (λ, q). The regulatory constraint is

$$D \leq \xi \bar{L}, \quad (24)$$

as we noted in the main body of the text. By dividing both sides by bank net worth, we obtain

$$\frac{D}{W} \leq \xi \frac{\bar{L}}{L} \frac{L}{W}$$

Using the definitions of λ and q

$$\lambda \leq \xi \frac{1}{q} (\lambda + 1)$$

and clearing out λ, we obtain

$$\lambda \leq \frac{1}{\frac{q}{\xi} - 1}. \quad (25)$$

Note that the constraint is independent of W and only depends on (λ, q). The solvency constraint is expressed in terms of leverage

$$\lambda \leq \bar{\lambda} \equiv (1 - \varepsilon)/\varepsilon. \quad (26)$$

Hence, we summarize the set of states where the bank is not liquidated by

$$\lambda = \min\left\{\frac{1}{\frac{q}{\xi} - 1}, \bar{\lambda}\right\}. \tag{27}$$

Observation 3: Derivations of the laws of motion. With probability σ over interval Δ, the bank receives deterministic default shock $\varepsilon < 1$. Let

$$dN = \begin{cases} 0 & \text{with prob } 1 - \sigma dt \\ 1 & \text{with prob } \sigma dt \end{cases}$$

denote a default event process. Recall that dN is a Poisson process.

Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies

$$L_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \varepsilon L_t (N_{t+\Delta} - N_t),$$

with the interpretation that the first term is the non-maturing fraction of loans, the second represents loan issuances and the third represents losses in a time interval. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dL = (I - \delta L)\, dt - \varepsilon L dN.$$

We express this law of motion in terms of net worth, replacing (20), to obtain

$$dL = \iota(\lambda + 1) W dt - \varepsilon(\lambda + 1) W dN. \tag{28}$$

To ease the notation, we define the growth rate of fundamental loans and the jump relative to net worth:

$$\mu^L \equiv \iota(\lambda + 1) \text{ and } J^L \equiv -\varepsilon(\lambda + 1).$$

Similarly, for deposits we have that

$$D_{t+\Delta} = \left(1 + r^D\Delta\right) D_t - \left(r^L\Delta + \delta\Delta\right) L_t + \Phi(I_t, L_t)\,\Delta + C_t\Delta$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + \Phi(I, L) + C\right] dt.$$

We express this law of motion in terms of wealth, by using (21), to obtain

$$dD = \left[r^D\lambda - \left(r^L + \delta\right)(\lambda + 1) + (\Phi(\iota, 1) + \delta)(\lambda + 1) + c\right] W dt. \tag{29}$$

We define the growth rate of deposits relative to net worth as

$$\mu^D \equiv r^D\lambda - \left(r^L + \delta\right)(\lambda + 1) + (\Phi(\iota, 1) + \delta)(\lambda + 1) + c.$$

Finally, the law of motion for book loans satisfies

$$\bar{L}_{t+\Delta} = (1-\delta\Delta) L_t + I_t\Delta - \alpha\Delta\left(\bar{L}_t - L_t\right) - \tau\varepsilon L_t \left(N_{t+\Delta} - N_t\right),$$

with the interpretation that the first term represents how book loans fall as the principal amounts of the fundamental loans are repaid; the second term increases book loans through newly issued loans; the third term decreases book loans at the speed of loan-loss recognition α times the gap in the book versus fundamental loans; and the final term is the fraction of losses recognized in books upon receiving a default shock. Taking $\Delta \to 0$, we obtain the following law of motion:

$$d\bar{L} = (-\delta L + I)\,dt - \alpha\left(\bar{L} - L\right) dt - \tau\varepsilon L dN.$$

We express this law of motion by using Equation (22) in terms of wealth to obtain

$$d\bar{L} = \left[\iota - \alpha\left(\frac{1}{q} - 1\right)\right](\lambda+1)\,W dt - \tau\varepsilon\frac{1}{q}(\lambda+1)\,W dN. \tag{30}$$

We define the growth rate of the book loans and the jump relative to net worth accordingly

$$\mu^{\bar{L}} \equiv \left[\iota - \alpha\left(\frac{1}{q} - 1\right)\right](\lambda+1) \text{ and } J^{\bar{L}} \equiv -\tau\varepsilon\frac{1}{q}(\lambda+1).$$

Observation 4: Growth independence. Next, we present the evolution of net worth, which evolves according to

$$\begin{aligned} dW &= dL - dD \\ &= \Bigg[\underbrace{\left(r^L+\delta\right)(\lambda+1) - r^D\lambda}_{\text{levered returns}} + \underbrace{(\iota - (\Phi(\iota,1)+\delta))(\lambda+1)}_{\text{capital loss from adjustment}} - \underbrace{c}_{\text{dividend rate}}\Bigg] W dt \\ &= \underbrace{(-\varepsilon(\lambda+1))}_{\text{loss rate}} W dN. \end{aligned} \tag{31}$$

where the second line uses the laws of motion in Equations (28) and (29) and those employed in observation 1. The interpretation of this expression is natural: the terms' multiplying rates represent the net interest margin on the bank, which are the banks' levered returns; the second term represents the capital gains that are accounted immediately as the bank creates an asset that can be worth more or less than a liability; the third term is the bank's dividend rate; and the final term is the loss rate, which scales with leverage. To aid the calculations, we define the drift of the growth rate of the bank's equity as

$$\mu^W \equiv \left[r^L(\lambda+1) - r^D\lambda + (\iota - \Phi(\iota,1))(\lambda+1) - c\right] W$$

and denote the jump component of wealth as

$$J^W \equiv -\varepsilon(\lambda+1)\,W = J^L W.$$

We also note that

$$\mu^W = \mu^L - \mu^D.$$

Observation 5: Law of motion for leverage. Next, we derive the law of motion for leverage λ, given any choice of ι and c. Employing the formula for the differential of a ratio we get

$$\begin{aligned}\mu^{\lambda} &= \left(\mu^{D}W-\frac{D}{W}\mu^{W}W\right)\frac{1}{W} \\ &= \mu^{D}-\lambda\mu^{W} \\ &= \mu^{L}-(\lambda+1)\,\mu^{W}.\end{aligned} \tag{32}$$

Upon a default shock, the discontinuous jump in leverage is given by

$$J^{\lambda}=\frac{D}{(W-\varepsilon\,(\lambda+1)\,W)}-\frac{D}{W}=\left(\frac{1}{1-\varepsilon\,(\lambda+1)}-1\right)\lambda.$$

Therefore, combining the drift and jump portions of the law of motion, we obtain

$$d\lambda=\left(\iota-\mu^{W}\right)(\lambda+1)\,dt+\frac{\varepsilon\,(\lambda+1)}{1-\varepsilon\,(\lambda+1)}\lambda dN. \tag{33}$$

The interpretation of this law of motion is that leverage increases with the issuance rate, falls as loans mature and falls as the bank earns income on its current portfolio, μ^{W}. We thus have

$$\mu^{\lambda}=\left(\iota-\mu^{W}\right)(\lambda+1)\,.$$

Naturally, leverage jumps with defaults and more so the more levered the bank is.

Observation 6: Law of motion for q. Next, we produce the law of motion for leverage λ, given any choice of ι and c. We first describe the continuous portion of the law of motion dq^{c}. Employing the formula for the ratio

$$dq^{c}=\left(\frac{dL^{c}}{L}-\frac{d\bar{L}^{c}}{\bar{L}}\right)qdt.$$

The first term is

$$\frac{dL^{c}}{L}=\iota$$

and the second term is

$$\frac{d\bar{L}^{c}}{\bar{L}}=\frac{\left(\iota-\alpha\left(\frac{1}{q}-1\right)\right)(\lambda+1)\,W}{\frac{1}{q}\,(\lambda+1)\,W}=\iota q-\alpha\,(1-q)\,.$$

Consequently,

$$dq^{c}=(\iota+\alpha)\,(1-q)\,qdt. \tag{34}$$

Upon a default shock, the discontinuous jump in leverage is given by

$$J^{q}=\frac{L-\varepsilon L}{\bar{L}-\tau\varepsilon L}-q=\frac{(1-\varepsilon)\,L}{(1-\tau\varepsilon q)\,\bar{L}}-q=-\frac{(\varepsilon-\tau\varepsilon q)}{(1-\tau\varepsilon q)}q.$$

Therefore, combining the continuous and discrete portions of the law of motion, we obtain

$$dq = (\iota + \alpha)(1 - q)\, q dt - \left(\frac{\varepsilon - \tau\varepsilon q}{1 - \tau\varepsilon q}\right) q dt. \tag{35}$$

Finally, we note the relationship

$$dq^c = \mu^q = \left[\mu^L - \mu^{\bar{L}} q\right] \frac{q}{(\lambda + 1)} dt.$$

Duffie-Epstein. The value function of the Duffie-Epstein satisfies

$$V_t = E_t \int_t^\infty f(C_s, V_s)\, ds,$$

where the f is given by

$$\begin{aligned} f(C, V) &\equiv \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi}}}{\{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi} - 1}}\right] \\ &= \frac{\rho}{1-\theta} \{1 + (1-\psi) V\} \left[\frac{C^{1-\theta}}{\{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi}}} - 1\right]. \end{aligned}$$

A useful calculation is the derivative with respect to dividends:

$$f_c(C, V) = \rho \frac{C^{-\theta}}{\{1 + (1-\psi) V\}^{\frac{1-\theta}{1-\psi} - 1}}.$$

We have some limits of interest. First, the risk-aversion limit vanishes:

$$\lim_{\psi \to 0} f(C, V) = \frac{\rho}{1-\theta} (1 + V) \left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right].$$

and

$$\lim_{\psi \to 0} f_c(C, V) = \rho C^{-\theta} (1 + V)^{\theta}.$$

second, the IES limit goes to 1:

$$\lim_{\theta \to 1} \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right] = \lim_{\theta \to 0} \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right] - \rho V = \log C - \rho V.$$

and

$$\lim_{\theta \to 1} = f_c(C, V) = \rho C^{-\theta} (1 + V)^{\theta} = \rho \frac{(1+V)}{C}.$$

C.2 Proof of Proposition 1

In this appendix, we prove the following detailed version of Proposition 1:

Proposition 2 *[Bank's Problem] Given* $\{\lambda, q\}$, $V\left(L, \bar{L}, D\right) = \left(1 + v\left(\lambda, q\right)\right) W - 1$, *where* v *is the solution to the following HJB equation:*

$$0 = \max_{\{c,\iota\}} f\left(c, v\right) + \underbrace{v_{\lambda}\mu^{\lambda} + v_{q}\mu^{q}}_{\text{change in financial ratios}} + \underbrace{\left(1+v\right)\mu^{W}}_{\text{equity growth}} \ldots$$
$$+ \sigma \underbrace{\left[\left(1 + v\left(\lambda + J^{\lambda}, q + J^{q}\right)\right)\left(1 + J^{W}\right) - \left(1 + v\right)\right]}_{\text{default jump in wealth}} \text{ in } \left(\lambda, q\right) \notin \Gamma \tag{36}$$

and $v = v_o$ *for* $\{\lambda, q\} \in \Gamma$. *Policy functions can be recovered through the following relationships:* $C\left(L, \bar{L}, D\right) = c\left(\lambda, q\right) \cdot W$ *and* $I\left(L, \bar{L}, D\right) = \left(\iota\left(\lambda, q\right) + \delta\right) \cdot L$. *The bank's market value satisfies* $S\left(L, \bar{L}, D\right) \equiv s\left(\lambda, q\right) \cdot W$, *where* s *solves*

$$\rho^{I} s = c\left(\lambda, q\right) + s_{\lambda}\mu^{\lambda} + s_{q}\mu^{q} + s\mu^{W} + \sigma\left[s\left(\lambda + J^{\lambda}, q + J^{q}\right)\left(1 + J^{W}\right) - s\right], \tag{37}$$

and $s = 0$ *for* $\{\lambda, q\} \in \Gamma$. *Finally, Tobin's* Q *is given by*

$$Q\left(\lambda, q\right) = s\left(\lambda, q\right) \times \left(\left(q^{-1} - 1\right)\lambda + 1\right)^{-1}. \tag{38}$$

Formulation. We next prove Proposition 2. The primitive bank value HJB equation is given by

$$0 = \max_{\{C,I\}} f\left(C, V\left(L, \bar{L}, D\right)\right) + \frac{E\left[dV\left(L, \bar{L}, D\right)\right]}{dt} \tag{39}$$

subject to the laws of motion in Equations (28), (29), (30) and the boundary $V = V_o$ when Equations (24) and (26) are not satisfied. In the objective, the differential form is

$$\begin{aligned}\frac{E\left[dV\left(L, \bar{L}, D\right)\right]}{dt} &= V_{L}\left(L, \bar{L}, D\right)\mu^{L}W + V_{\bar{L}}\left(L, \bar{L}, D\right)\mu^{\bar{L}}W + V_{D}\left(L, \bar{L}, D\right)\mu^{D}W \\ &+ \sigma\left[V\left(\left(1-\varepsilon\right)L, \left(1 - \tau\varepsilon\right)\bar{L}, D\right) - V\left(L, \bar{L}, D\right)\right].\end{aligned}$$

Conjecture. We conjecture a solution to the value function and verify that it satisfies the HJB equation. The conjecture is

$$V\left(L, \bar{L}, D\right) = \frac{\left[1 + \left(1-\psi\right)v\left(\lambda, q\right)\right]W^{1-\psi} - 1}{1-\psi}, \tag{40}$$

for a suitable candidate $v\left(\lambda, q\right)$. Under this conjecture, we verify that $C\left(L, \bar{L}, D\right) = c\left(\lambda, q\right)W$ and $I = \left(\iota\left(\lambda, q\right) + \delta\right)\left(\lambda + 1\right)W$.

Factorization. We perform some useful calculations on the guess in Equation (40). In particular, we factorize equity from every term in the HJB equation. Under the conjecture, we have that

$$\left(1 + \left(1-\psi\right)V\right) = \left[1 + \left(1-\psi\right)v\right]W^{1-\psi}.$$

Therefore,

$$\begin{aligned}
f(C,V) &= f\left(c(\lambda,q)W, \frac{[1+(1-\psi)v(\lambda,q)]W^{1-\psi}-1}{1-\psi}\right) \\
&= \frac{\rho}{1-\theta}[1+(1-\psi)v]W^{1-\psi}\left[\frac{c(\lambda,q)^{1-\theta}W^{1-\theta}}{([1+(1-\psi)v]W^{1-\psi})^{\frac{1-\theta}{1-\psi}}}-1\right] \\
&= \frac{\rho}{1-\theta}[1+(1-\psi)v]W^{1-\psi}\left[\frac{c(\lambda,q)^{1-\theta}}{([1+(1-\psi)v])^{\frac{1-\theta}{1-\psi}}}-1\right] \\
&= f(c(\lambda,q),v)W^{1-\psi}. \qquad (41)
\end{aligned}$$

The change in the value function with respect to loans is

$$\begin{aligned}
V_L &= \partial\left[\frac{\left[1+(1-\psi)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right](L-D)^{1-\psi}\left[\frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\gamma\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right]^{\frac{1}{\kappa-1}}-1}{1-\psi}\right]/\partial L \\
&= -v_\lambda\frac{D}{(L-D)^2}W^{1-\psi}+v_q\frac{1}{\bar{L}}W^{1-\psi}+[1+(1-\psi)v]W^{-\psi} \\
&= -v_\lambda\frac{\lambda W}{W^2}W^{1-\psi}+v_q\frac{1}{\frac{1}{q}(\lambda+1)W}W^{1-\psi}+[1+(1-\psi)v]W^{-\psi} \\
&= \left(-v_\lambda\lambda+v_q\frac{q}{(\lambda+1)}+[1+(1-\psi)v]\right)W^{-\psi}. \qquad (42)
\end{aligned}$$

The change in the value function with respect to deposits is

$$\begin{aligned}
V_D &= \partial\left[\frac{\left[1+(1-\psi)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right](L-D)^{1-\psi}-1}{1-\psi}\right]/\partial D \\
&= v_\lambda\frac{1}{W}W^{1-\psi}+v_\lambda\frac{D}{(L-D)^2}W^{1-\psi}-[1+(1-\psi)v]W^{-\psi} \\
&= v_\lambda\frac{1}{W^2}W^{1-\psi}+v_\lambda\frac{\lambda W}{W^2}W^{1-\psi}-[1+(1-\psi)v]W^{-\psi} \\
&= (v_\lambda(1+\lambda)-[1+(1-\psi)v])W^{-\psi}. \qquad (43)
\end{aligned}$$

The derivative of the value function with respect to $\bar{L}$ is given by

$$\begin{aligned}
V_{\bar{L}} &= \partial\left[\frac{\left[1+(1-\psi)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right](L-D)^{1-\psi}-1}{1-\psi}\right]/\partial\bar{L} \\
&= -v_q\frac{L}{\bar{L}^2}W^{1-\psi} \\
&= -v_q\frac{(\lambda+1)W}{\left(\frac{1}{q}(\lambda+1)W\right)^2}W^{1-\psi} \\
&= -v_q\frac{q^2}{(\lambda+1)}W^{-\psi}. \qquad (44)
\end{aligned}$$

Finally, the jump in the value function even after a default is

$$J^{V}=\left[V\left(\left(1-\varepsilon\right)L,\left(1-\tau\varepsilon\right)\bar{L},D\right)-V\left(L,\bar{L},D\right)\right]$$

which according to our guess can be written as

$$\begin{aligned}J^{V} &= \left[\frac{\left[1+\left(1-\psi\right)v\left(\lambda+J^{\lambda},q+J^{q}\right)\right]\left(\left(1+J^{W}\right)W\right)^{1-\psi}-1}{1-\psi}-\frac{\left[1+\left(1-\psi\right)v\left(\lambda,q\right)\right]W^{1-\psi}-1}{1-\psi}\right]\\ &= \left[\frac{\left[1+\left(1-\psi\right)v\left(\lambda+J^{\lambda},q+J^{q}\right)\right]}{1-\psi}\left(1+J^{W}\right)^{1-\psi}-\frac{\left[1+\left(1-\psi\right)v\left(\lambda,q\right)\right]}{1-\psi}\right]W^{1-\psi}\\ &= \left[\frac{\left[1+\left(1-\psi\right)v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)},\left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right]}{1-\psi}\left(1+J^{W}\right)^{1-\psi}-\frac{\left[1+\left(1-\psi\right)v\left(\lambda,q\right)\right]}{1-\psi}\right]W^{1-\psi}. \qquad (45)\end{aligned}$$

Verification. We verify that the conjecture satisfies its HJB equation. With the factorizations above, Equations (41-45), we have that Equation (39) can be written as

$$\begin{aligned}0 &= \max_{\{c,\iota\}} f\left(c,v\right)W^{1-\psi}\ldots\\ &\quad \underbrace{+\left[\begin{array}{ccc} v_{\lambda} & v_{q} & \left(1+\left(1-\psi\right)v\right)\end{array}\right]\times\left[\begin{array}{ccc}-\lambda & \left(1+\lambda\right) & 0\\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^{2}}{(\lambda+1)}\\ 1 & -1 & 0\end{array}\right]\times\left[\begin{array}{c}\mu^{L}\\ \mu^{D}\\ \mu^{\bar{L}}\end{array}\right]}_{\equiv\mu^{V}}W^{1-\psi}\ldots\\ &\quad +\sigma\underbrace{\left[\frac{\left[1+\left(1-\psi\right)v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)},\left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)\right)\right]}{1-\psi}\left(1+J^{W}\right)^{1-\psi}-\frac{\left[1+\left(1-\psi\right)v\left(\lambda,q\right)\right]}{1-\psi}\right]}_{\equiv J^{v}}W^{1-\psi},\end{aligned}$$

where we used the fact that any choice of C and I can be expressed as a choice of $c\left(\lambda,q\right)W$ as there is a one-to-one map from the $\{\lambda,q,W\}$ space to the original space—by a change in coordinates. Then, we can factor wealth from this HJB equation to express it as

$$0=W^{1-\psi}\left[\max_{\{c,\iota\}} f\left(c,v\right)+\mu^{V}+J^{V}\right],$$

and since the maximization is independent of net worth, this verifies the linearity of the controls. To verify the conjecture, we need to express the drifts and jumps, $\left\{\mu^{V},J^{V}\right\}$ exclusively in terms of $\{\lambda,q\}$. To do so, observe that

$$\mu^V = \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^D \\ \mu^{\bar{L}} \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^L - \mu^W \\ \mu^{\bar{L}} \end{bmatrix} \dots$$

$$= \begin{bmatrix} -\lambda\mu^L + (1+\lambda)\left(\mu^L - \mu^W\right) \\ \frac{q}{(\lambda+1)}\mu^L - \frac{q}{(\lambda+1)}q\mu^{\bar{L}} \\ \mu^L - \left(\mu^L - \mu^W\right) \end{bmatrix}$$

$$= \begin{bmatrix} \mu^\lambda \\ \mu^q \\ \mu^W \end{bmatrix}.$$

Thus, we have that

$$\mu^V(\lambda, q) = v_\lambda \mu^\lambda + v_q \mu^q + (1 + (1-\psi)\, v)\, \mu^W,$$

where all of the terms are functions of the state variables $\{\lambda, q\}$.

Consequently, the HJB solution to the HJB equation is

$$0 = \left[\max_{\{c,\iota\}} f(c, v) + v_\lambda \mu^\lambda + v_q \mu^q + (1 + (1-\psi)\, v)\, \mu^W + \sigma J^V\right], \tag{46}$$

subject to the solvency conditions in Equations (25) and (26), the liquidiation value v_o, and the laws of motion in Equations (33-35). Since the choice is independent of wealth and only depends on λ and q, this verifies that $\bar{v}$ is only a function of $\{\lambda, q\}$ and is not indexed by W. Thus, we verify the conjecture that the formula in Equation (40) satisfies the HJB in Equation (36) for v.

Applying the result to the special case with $\psi = 0$, yields

$$\begin{aligned} 0 &= \max_{\{c,\iota\}} f(c, v) + v_\lambda \mu^\lambda + v_q \mu^q + (1+v)\, \mu^W \\ &+\sigma\left[\left(1 + v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right) q\right)\right)(1-\varepsilon(\lambda+1)) - (1 + v(\lambda, q))\right] \end{aligned}$$

subject to the boundary conditions given by Equations (25) and (26)—taking the value v_o and the laws of motion shown in Equations (33), (35).

Limits of interest. We now let $\psi \to 0$ to obtain

$$\begin{aligned} 0 &= \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^D \\ \mu^{\bar{L}} \end{bmatrix} \\ &= \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^L - \mu^W \\ \mu^{\bar{L}} \end{bmatrix} \dots \\ &= \begin{bmatrix} -\lambda\mu^L + (1+\lambda)\left(\mu^L - \mu^W\right) \\ \frac{q}{(\lambda+1)}\mu^L - \frac{q}{(\lambda+1)} q\mu^{\bar{L}} \\ \mu^L - \left(\mu^L - \mu^W\right) \end{bmatrix} \\ &= \begin{bmatrix} \mu^\lambda \\ \mu^q \\ \mu^W \end{bmatrix}. \end{aligned}$$

Hence, we obtain

$$\begin{aligned} 0 &= \max_{\{c,\iota\}} \frac{\rho}{1-\theta}(1+v)\left[\frac{c^{1-\theta}}{(1+v)^{1-\theta}} - 1\right] + v_\lambda\mu^\lambda + v_q\mu^q + (1+v)\,\mu^W \\ &\quad + \sigma\left[\left(1 + v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right)(1-\varepsilon(\lambda+1)) - (1+v(\lambda,q))\right]. \end{aligned}$$

Now consider the limite where $\theta \to 1$. We recover

$$\begin{aligned} \rho v &= \max_{\{c,\iota\}} \rho\log c + v_\lambda\mu^\lambda + v_q\mu^q + (1+v)\,\mu^W \\ &\quad + \sigma\left[\left(1 + v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right)(1-\varepsilon(\lambda+1)) - (1+v(\lambda,q))\right]. \end{aligned}$$

C.3 Policy Functions

We derive the first-order conditions of this problem.

Optimal dividend. The first-order condition for dividends is given by

$$f_c(c,v) + v_\lambda\mu_c^\lambda + (1+(1-\psi)v)\,\mu_c^W = 0,$$

and arranging terms, we obtain

$$f_c(c,v) + v_\lambda(\lambda+1) = (1+(1-\psi)v)\,. \tag{47}$$

In the special case of risk neutrality, we have

$$\rho\frac{c^{-\theta}}{(1+v)^{-\theta}} + v_\lambda(\lambda+1) = (1+v)\,,$$

which we can solve to obtain

$$c=\rho^{1/\theta}\left[\frac{(1+(1-\psi)\,v)}{(((1+(1-\psi)\,v)-v_{\lambda}\,(\lambda+1)))^{1/\theta}}\right]. \tag{48}$$

Take the risk-neutral limit, $\psi\to 0$ and we obtain

$$c=\rho^{1/\theta}\left[\frac{(1+v)}{(((1+v)-v_{\lambda}\,(\lambda+1)))^{1/\theta}}\right].$$

In the special case of $v_{\lambda}=0$ we obtain

$$c=\rho^{1/\theta}\,(1+v)^{1-1/\theta}\,.$$

In the special case of $\theta\to 1$, we have that

$$c=\frac{\rho}{\left(1-v_{\lambda}\frac{(\lambda+1)}{(1+v)}\right)}. \tag{49}$$

This solution closely resembles the solution to a portfolio problem where the dividend rate is exactly the discount rate, ρ. However, in this problem, because leverage is a slow-moving object, there's a correction term given by $v_{\lambda}\frac{(\lambda+1)}{(1+\bar{v})}$, which measures the additional advantage of affecting leverage through the dividend decision. If leverage is too high, such that it reaches beyond the point of zero, then the dividend rate is distorted downwards.

The elasticity of dividends with respect to leverage in this special case is given by

$$\begin{aligned} dc &= \left(\frac{v_{\lambda\lambda}\frac{(\lambda+1)}{(1+v)}-\frac{(v_{\lambda})^{2}}{(1+v)}\frac{(\lambda+1)}{(1+v)}}{1-v_{\lambda}\frac{(\lambda+1)}{(1+v)}}\right)cd\lambda \\ &= \left(\frac{\left(\frac{v_{\lambda\lambda}}{v_{\lambda}}\,(\lambda+1)-\frac{v_{\lambda}}{(1+v)}\,(\lambda+1)\right)v_{\lambda}}{(1+v)-v_{\lambda}\,(\lambda+1)}\right)cd\lambda \end{aligned}$$

Optimal issuance. Next, we discuss the first-order condition in issuances, which yields

$$v_{\lambda}\mu_{\iota}^{\lambda}+v_{q}\mu_{\iota}^{q}+(1+(1-\psi)\,v)\,\mu_{\iota}^{W}=0.$$

Using the expressions in Table 10, we obtain

$$v_{q}\,(1-q)\,q+v_{\lambda}\,(1-(1-\Phi_{\iota}\,(\iota,1))\,(\lambda+1))\,(\lambda+1)+(1+(1-\psi)\,v)\,(1-\Phi_{\iota}\,(\iota,1))\,(\lambda+1)=0,$$

and collecting terms yields

$$v_{q}\,(1-q)\,q+v_{\lambda}\,(\lambda+1)+((1+(1-\psi)\,v)-v_{\lambda}\,(\lambda+1))\,(1-\Phi_{\iota}\,(\iota,1))\,(\lambda+1)=0. \tag{50}$$

Using the first-order condition yields

$$(1-\Phi_{\iota}\,(\iota,1))=\frac{v_{q}\,(1-q)\,q+v_{\lambda}\,(\lambda+1)}{(v_{\lambda}\,(\lambda+1)-(1+(1-\psi)\,v))\,(\lambda+1)}.$$

The right-hand side equals

$$(1-\Phi_{\iota}(\iota,1)) = sign(\iota)\,\frac{\gamma}{2}\kappa\,(sign(\iota)(\iota))^{\kappa-1}$$

and thus

$$(sign((\iota))\,\iota) = \left[sign(\iota)\,\frac{v_q(1-q)\,q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa\,(v_\lambda(\lambda+1)-(1+(1-\psi)\,v))\,(\lambda+1)}\right]^{\frac{1}{\kappa-1}}.$$

Thus, we have that issuances are given by

$$\iota = \begin{cases} \left(\frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)} > 0 \\ \left(-\frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)} < 0. \end{cases}$$

In the limit as $\psi \to 0$, we obtain

$$\iota = \begin{cases} \left(\frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)} > 0 \\ \left(-\frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \frac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)} < 0. \end{cases}$$

In the special case considered in the paper, $\kappa = 2$ and

$$1-\Phi_{\iota}(\iota,1) = \gamma\iota.$$

Thus,

$$\iota = \frac{1}{\gamma}\cdot\left(\frac{v_q(1-q)\,q+v_\lambda(\lambda+1)}{(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right).$$

C.4 Proofs for $\gamma = 0$ and $\tau = 1$ limit

Before we proceed to derive the main result, we first derive a some preliminary observations.

No adjustment costs. Without adjustment costs on ι, the bank can reset leverage at will. It can do so through an issuance intensity ι that controls the drift of λ; it can also do so through the endogenous jump $\bar{J}^{\lambda}$. In particular, the bank can choose a discrete jump in loans, $\bar{J}^{L}$. Thus, the process for loans is given by

$$dL = (\iota - \delta)(\lambda + 1) W dt + J^{L} W dN + \bar{J}^{L} W d\tilde{N},$$

where $d\tilde{N}$ is the event of a controlled jump. Likewise, the evolution of deposits satisfies

$$dD = \left(c - \mu^{W}\right)(\iota - \delta)(\lambda + 1) W dt + J^{L} W dN + \bar{J}^{L} W d\tilde{N}.$$

Next, we can define the controlled jump in leverage which, given $\bar{J}^{L}$, yields

$$\bar{J}^{\lambda} = \frac{D + \bar{J}^{L} W}{L + \bar{J}^{L} W - D - \bar{J}^{L} W} - \lambda = \left(\lambda + \bar{J}^{L} - \lambda\right) = \bar{J}^{L}.$$

Shadow boundary. Let Λ be the value of leverage such that a loan–default shock takes leverage to the regulatory limit:

$$\Lambda = \frac{(1-\varepsilon)\,\Xi}{1 + \varepsilon \Xi} = \frac{(1-\varepsilon)\,\xi}{1 - (1-\varepsilon)\,\xi},$$

As in the text, we label this leverage the shadow liquidation boundary.

Main result. In this Appendix we prove the following result:

Proposition 3 *[Bank's Problem] Let $\tau = 1$ and $\gamma = 0$. Then, leverage λ^{*} is constant. For suitable parameter conditions, $\lambda^{*} = \Lambda$. In that case, the equity multiplier is the constant that solves*

$$0 = \max_{\{c,\lambda\}} f(c, v) + (1+v)\left[r^{L} + \left(r^{L} - r^{D}\right)\Lambda - c\right] + \sigma (1+v)\left(\left(1 + J^{W}\right) - 1\right).$$

Then, for $dN = 0$ the dividend rate is the constant, c^{*}, that solves

$$c^{*} = \rho^{1/\theta} (1+v)^{1-1/\theta}, \tag{51}$$

the issuance rate is ι^{*} such that $\mu^{\lambda} = 0$.

Furthermore, $d\tilde{N} = dN$ and for $dN = 1$, and λ is reflected back to λ^{*} ($\bar{J}^{\lambda} = -J^{\lambda}$). Finally, the multiplier v is the constant that solves (36), given the constant values c and λ. The necessary parameter condition for this dynamics is

$$\frac{\left(r^{L} - r^{D}\right)}{\sigma} \in [v_0, 1+v].$$

Derivation of the main result. For this proof we work directly with the risk-neutral case as we already showed the effect of risk aversion in the earlier proof. Consider the case where $\gamma = 0$

such that $\Phi(\iota,1)=\iota$ and $\tau=1$. In this case, we have that each issued loan increases deposits. Hence, loan issuances do not affect wealth. The evolution of net worth evolves according to

$$\begin{aligned} dW &= dL-dD \\ &= \left[r^L+\underbrace{\left(r^L-r^D\right)}_{\text{levered returns}}\lambda-\underbrace{c}_{\text{dividend rate}}\right]Wdt-\underbrace{\varepsilon\left(\lambda+1\right)}_{\text{loss rate}}WdN, \end{aligned} \tag{52}$$

where the second line uses the laws of motion in Equations (28) and (29) and also uses observation 1. Since there are no adjustment costs, leverage can be immediately adjusted to any level. This is done using a combination of an issuance intensity ι and an endogenous discrete jump in issuances that produces a jump in leverage, $\bar{J}^{\lambda}$, as we show next.

The jump in wealth upon a default shock is

$$J^W\equiv-\varepsilon\left(\lambda+1\right),$$

and the drift in wealth is given by levered returns minus dividends:

$$\mu^W\equiv\left[r^L+\underbrace{\left(r^L-r^D\right)}_{\text{levered returns}}\lambda-\underbrace{c}_{\text{dividend rate}}\right].$$

Recall that the regulatory constraint (6) is

$$1=\xi\left(1+\lambda\right)/\lambda\rightarrow\lambda\leq\frac{\xi}{1-\xi}.$$

In this case, λ is chosen every period and ι is defined to be consistent with the drift in that choice. Since $q=1$, the relevant constraint is the regulatory constraint and no longer the market-based constraint. Then, the HJB equation in (46) becomes

$$0=\left[\max_{\{c,\lambda\}}f\left(c,v\right)+v_{\lambda}\mu_{\lambda}+\left(1+v\right)\mu^W v+\sigma J^V\right]. \tag{53}$$

Different from Equation (46), we conjecture that, in this case, $\bar{v}$ is a scalar rather than a function of λ—or q, which in this case is constant. Under this guess, the jump term upon a default event is

$$J^V=\left(1+v\right)\left(\left(1+J^W\right)^{1-\psi}\mathbb{I}\left[\lambda+J^{\lambda}<\frac{1}{\frac{1}{\rho}-1}\right]+v_o\mathbb{I}\left[\lambda+J^{\lambda}>\frac{1}{\frac{1}{\rho}-1}\right]-1\right),$$

whereas before

$$J^{\lambda}=\frac{\lambda}{1-\varepsilon\left(\lambda+1\right)}.$$

Substituting for the drift μ^W, $v_{\lambda}=0$, and the jump in wealth J^W in (53) we obtain

$$0=\max_{\{c,\lambda\}} f\left(c,v\right)+\left(1+v\right)\left[r^{L}+\left(r^{L}-r^{D}\right)\lambda-c\right]$$
$$+\sigma\left(1+v\right)\left(\left(1+J^{W}\right)\left[\lambda+J^{\lambda}<\frac{1}{\frac{1}{\rho}-1}\right]+v_{o}\mathbb{I}\left[\lambda+J^{\lambda}>\frac{1}{\frac{1}{\rho}-1}\right]-1\right).$$

Let us first solve for consumption. In this case, the first-order condition for consumption is

$$\rho\frac{c^{-\theta}}{\left(1+\bar{v}\right)^{-\theta}}=\left(1+v\right).$$

We rearrange terms to obtain

$$c=\rho^{1/\theta}\left(1+v\right)^{1-1/\theta}. \tag{54}$$

Next, we obtain the solution for leverage. Then, the leverage choice maximizes

$$\max_{\lambda\in\left[0,\frac{\xi}{1-\xi}\right]}\left(1+v\right)\left(r^{L}-r^{D}\right)\lambda$$
$$+\sigma\left(1+v\right)\left(1-\varepsilon\left(\lambda+1\right)\right)\mathbb{I}\left[\frac{\lambda}{1-\varepsilon\left(\lambda+1\right)}<\frac{\xi}{1-\xi}\right]+\sigma\frac{v_{0}}{\left(1+\bar{v}\right)}\mathbb{I}\left[\frac{\lambda}{1-\varepsilon\left(\lambda+1\right)}>\frac{\xi}{1-\xi}\right].$$

This is a linear program. The solution thus generically falls into a corner

$$\lambda^{*}\in\begin{cases}0 & \text{if }\left(r^{L}-r^{D}\right)<\epsilon\sigma\left[1+\bar{v}\right]\\ \frac{1}{\frac{1}{\xi}-1} & \text{if }\left(r^{L}-r^{D}\right)>\sigma U\left(\eta\right)\\ \Lambda & \text{otherwise.}\end{cases} \tag{55}$$

Considering the interesting case where $\lambda=\Lambda$, substituting Equation (51) we have that $\bar{v}$ solves

$$\begin{aligned}0 &= \frac{\rho}{1-\theta}\left[\frac{\left(\rho^{1/\theta}\left(1+\bar{v}\right)^{1-1/\theta}\right)^{1-\theta}}{\left(1+\bar{v}\right)^{1-\theta}}-\left(1+\bar{v}\right)\right]\\ &+ \left(1+\bar{v}\right)\left[r^{L}+\left(r^{L}-r^{D}\right)\Lambda+\left(1+J^{W}\left(\Lambda\right)\right)-\rho^{1/\theta}\left(1+\bar{v}\right)^{1-1/\theta}\right].\end{aligned} \tag{56}$$

This equation verifies that indeed the value function is

$$\iota=\mu^{W}=r^{L}+\left(r^{L}-r^{D}\right)\Lambda-\rho^{1/\theta}\left(1+\bar{v}\right)^{1-1/\theta}. \tag{57}$$

Finally, $\bar{J}^{\lambda}=-J^{\lambda}\left(\Lambda\right)$ whenever $dN=1$.

Log limit. Consider now the limit $\theta\to1$. Therefore, this case produces the usual formula

$$c^{*}=\rho.$$

Thus, the value per unit of wealth is

$$\bar{v} = \log \rho + \frac{\left[r^L + \left(r^L - r^D\right)\lambda^* - \rho\right] + \sigma\left(1 - \varepsilon\left(\lambda^* + 1\right)\right)}{\rho}.$$

C.5 Proofs for $\gamma = 0$ and $\tau \neq 1$ limit

We begin with a preliminary set of observations.

Endogenous jump in q. As in the previous case, in this version, banks can immediately control leverage through a combination of an issuance intensity, ι, and a jump in leverage. The novelty is that with delayed accounting the jump in loans produces a controlled jump in q. We can define the jump in q given $\bar{J}^{\lambda}$ as

$$\bar{J}^{q}\left(q,\lambda,\bar{J}^{\lambda}\right) = \frac{L+\bar{J}^{L}W}{\bar{L}+\bar{J}^{L}W} - q = \frac{(\lambda+1)+\bar{J}^{L}}{q^{-1}(\lambda+1)+\bar{J}^{L}} - q = q\left(\frac{(\lambda+1)+\bar{J}^{\lambda}}{(\lambda+1)+q\bar{J}^{\lambda}} - 1\right).$$

Thus, we can think of the bank as controlling a jump in leverage that produces a corresponding jump in $\bar{J}^{q}$.

Liquidation and shadow boundary. With abuse of notation, we now define the following function that characterizes the liquidation boundary $\partial\Gamma$ by the function

$$\Gamma(q) \equiv \min\left\{\frac{\xi}{q-\xi}, \bar{\lambda}\right\},$$

and a corresponding "shadow boundary" as given by the following function:

$$\Lambda(q) \equiv \Gamma(q+J^{q}) - J^{\lambda} = \min\left\{\frac{\xi}{\frac{q-\varepsilon q}{1-\tau\varepsilon q}-\xi}, \bar{\lambda}\right\} - \frac{\varepsilon\lambda(\lambda+1)}{1-\varepsilon(\lambda+1)}.$$

The graph of the shadow boundary is the set of points were $\{q,\lambda\}$ such that, given a default event, the bank ends at the boundary of the liquidation region. Finally, note that

$$\Gamma(q+J^{q}) = \min\left\{\frac{\xi}{\frac{q-\varepsilon q}{1-\tau\varepsilon q}-\xi}, \bar{\lambda}\right\}.$$

From liquidation to shadow boundary. We want to solve for the position where the bank ends if it jumps from liquidation to the shadow boundary. We solve for the jump size $\bar{J}^{L}$ that solves

$$\Lambda\left(q+\bar{J}^{q}\right) \equiv \Gamma(q) + \bar{J}^{L}.$$

Then, considering an initial position $\{q,\lambda\}$ in the shadow boundary, the terminal jump is given by

$$\Lambda\left(q+J^{q}+\bar{J}^{q}\left(q+J^{q},\bar{J}^{L}\right)\right) = \Gamma(q+J^{q}) - J^{\lambda} + \bar{J}^{L}.$$

The critical value of q^{m} below which market liquidation occurs is

$$\frac{\xi}{q^{m}-\xi} = \bar{\lambda}.$$

Hence, the critical point is

$$q^m = \xi \frac{1+\bar{\lambda}}{\bar{\lambda}}.$$

Thus, the slope of the liquidation boundary is

$$\Gamma_q(q) = \begin{cases} 0 & \text{if } q < q^m \\ -\frac{\xi}{(q-\xi)^2} & \text{if } q \geq q^m. \end{cases}$$

Likewise, the slope of the shadow boundary is

$$\Lambda_q(q) = \begin{cases} 0 & \text{if } \frac{1-\varepsilon}{1/q-\tau\varepsilon q} < q^m + \xi \\ -\frac{\xi}{\left(\frac{1-\varepsilon}{1/q-\tau\varepsilon}-\xi\right)^2} \cdot \frac{1}{(1/q-\tau\varepsilon)^2} \cdot \frac{1}{q} & \text{otherwise.} \end{cases}$$

The observation that the slope of the shadow boundary is negative is key for the proof.

Characteristic curves. Consider any pair $\{q, \lambda\}$. Then, consider two jumps, $\bar{J}^\lambda$ and $-\bar{J}^\lambda$, such that leverage remains constant. After the first jump, λ jumps to $\lambda' = \lambda + J^\lambda$. Next, observe that after these jumps, q remains the same. After the first jump

$$q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right) = q\frac{(\lambda+1)+\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda} \equiv q'.$$

Then, after the second jump, we obtain that

$$\begin{aligned} q' + \bar{J}^q\left(q', \lambda', -\bar{J}^\lambda\right) &= q'\frac{(\lambda'+1)-\bar{J}^\lambda}{(\lambda'+1)-q'\bar{J}^\lambda} \\ &= \left[q\frac{(\lambda+1)+\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}\right] \cdot \left[\left(\lambda+\bar{J}^\lambda+1\right) - \frac{q(\lambda+1)+q\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}\bar{J}^\lambda\right] \\ &= q. \end{aligned}$$

Consequently, for any $\{q, \lambda\}$, we define a characteristic curve as the set of points,

$$\left\{q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right), \lambda + \bar{J}^\lambda\right\},$$

for any $\bar{J}^\lambda \in [-\lambda, \infty]$. A characteristic curve is a set of points for λ and q that are connected through a controlled jump in leverage. Thus, we can think of a characteristic curve as a function $q^c(\lambda; \lambda_o)$ such that a given parameter, q_o, maps a value of leverage to a value of q—with one particular point given by $(\lambda; q_o)$.

Now consider an infinitesimal increase in leverage, $d\lambda$, then we obtain the following limit:

$$\frac{dq}{d\lambda} \equiv \lim_{\bar{J}^\lambda \to 0} \frac{J^q}{\bar{J}^\lambda} = \lim_{\bar{J}^\lambda \to 0} \frac{\left(\frac{q(\lambda+1)+q\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda} - q\right)}{\bar{J}^\lambda} = \frac{\frac{q(1-q)\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}}{\bar{J}^\lambda} = \frac{\mu_\iota^q}{\mu_\iota^\lambda}\frac{q(1-q)}{(\lambda+1)} = \frac{\mu_\iota^q}{\mu_\iota^\lambda}.$$

This is the slope of the characteristic curve at q and λ. The slope is positive and strictly so for any $q \in [0,1]$. Since the curves of the characteristics are positive, but the slope of the shadow boundaries are negative, each characteristic crosses one point of the shadow boundary for any q_o. Thus, for each $q^0 \in [0,1]$ in the shadow boundary, we can associate one characteristic curve. This definition is important for the main result.

Main result. We prove the following result:

Proposition 4 *[Bank's Problem] Let $\tau \neq 1$ and $\gamma = 0$. Then, for any q, given suitable parameter conditions, leverage is set to the shadow boundary $\lambda = \Lambda(q)$. Then, the equity multiplier v is a function of q that solves*

$$0 = \left[\max_{\{c\}} f(c,v) + v_q \mu^q + (1+v)\mu^W + \sigma\left(\left(1 + v\left(q + J^q + \bar{J}^q\right)\right)\left[1 + J^W\right] - (1 + v(q))\right)\right],$$

subject to

$$\Lambda_q(q) = \frac{\mu^\lambda}{\mu^q}$$

For $dN = 0$ the (constant) dividend rate, c^*, is given by

$$c^* = \rho^{1/\theta} \frac{(1 + v(q))}{\left(1 + v(q) - v_q \frac{(1+\lambda)}{\Lambda_q(q)}\right)}, \tag{58}$$

and the issuance rate $\iota^*(q)$ is such that given for $\{c^*(q), \lambda(q)\}$

$$\Lambda_q(q) = \frac{\mu^\lambda}{\mu^q}. \tag{59}$$

Thus,

$$\iota(q) = \mu^W(q)\frac{1}{1 - R(q)} - \alpha\frac{1}{1 - R(q)^{-1}},$$

where

$$R(q) = \Lambda_q(q)\frac{q(1-q)}{(1 + \Lambda(q))} < 0$$

Hence, for $dN = 0$, the state $\{\lambda, q\}$ moves continuously along the shadow boundary. Furthermore, $d\tilde{N} = dN$. When $dN = 1$, q jumps by $J^q + \bar{J}^q\left(q, \bar{J}^\lambda\right)$ where

$$\bar{J}^q\left(q, \bar{J}^\lambda\right) \equiv q\left(\frac{(\lambda+1) + \bar{J}^\lambda}{(\lambda+1) + q\bar{J}^\lambda} - 1\right)$$

and λ jumps by $J^\lambda + \bar{J}^\lambda$ where $\bar{J}^\lambda$ solves

$$\Lambda\left(q + J^q + \bar{J}^q\left(q + J^q, \bar{J}^\lambda\right)\right) = \Gamma\left(q + J^q\right) - J^\lambda + \bar{J}^\lambda.$$

The necessary parameter condition for these dynamics are

$$\frac{\left(r^L - r^D\right)}{\sigma} \in \left[U(\eta), 1 + v(q)\right] \text{ for any } q.$$

Derivation of the main result. To derive the main result, we proceed through a sequence of observations. The proof follows similar steps as the proof with immediate accounting but now considers the changes in q. We lever on the method of characteristics for the proof. Let the value function be given by some $v(\lambda, q)$, as derived earlier, for the general case with convex adjustment costs:

$$0 = \max_{\{c,\lambda\}} f(c, v) + v_\lambda \mu^\lambda + v_q \mu^q + (1 + v)\mu^W + \sigma J^v \tag{60}$$

where the jump in the value after the shock is given by

$$\begin{aligned} J^V &= \left[1 + v\left(\lambda + J^\lambda, q + J^q\right)(1 - \varepsilon(\lambda + 1)) - (1 + v(\lambda, q))\right] \mathbb{I}\left[\lambda + J^\lambda \leq \Gamma(q + J^q)\right] \\ &\quad + (v_o - (1 + v(\lambda, q))) \mathbb{I}\left[\frac{\lambda}{1 - \varepsilon(\lambda + 1)} > \frac{1}{\frac{1}{\rho} - 1}\right], \end{aligned}$$

subject to the boundary conditions given by Equations (25) and (26), the laws of motion in Equations (33), (35), and the definitions of the exogenous jumps J^q and J^λ.

Conjecture: Constant value along characteristics. Next we guess and verify that $v(\lambda, q)$ is constant along the characteristic curve where (q, λ) also belongs. That is,

$$v(\lambda, q) = v\left(\lambda + \bar{J}^\lambda, q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right)\right) \text{ for any } \bar{J}^\lambda.$$

Under this conjecture, the value will equal $v(\lambda, q) = v(\Lambda(q_o), q_o)$, where q_o is the parameter of the characteristic curve of $\{q, \lambda\}$. Under this guess, taking total differentials with respect to λ, we obtain that

$$v_\lambda - v_q \frac{dq}{d\lambda} = v_\lambda + v_q \frac{q(1-q)}{\lambda + 1} = v_\lambda + v_q \frac{\mu_\iota^q}{\mu_\iota^\lambda} = 0.$$

This expression is the PDE representation of the condition that the value function is linear along the characteristic function. Thus, we obtain a relationship between the derivatives of the original value function

$$v_\lambda = -v_q \frac{\mu_\iota^q}{\mu_\iota^\lambda}. \tag{61}$$

Verification: Constant value along characteristics. Consider the optimal choice of λ. Under our guess, we have that the value does not change along the characteristic curve of $\{q, \lambda\}$. Hence, leverage can be chosen without considering the change in the marginal value of equity $\bar{v}(\lambda, q)$ in (60). Thus, leverage solves

$$\begin{aligned} &\max_{\lambda \in \left[0, \frac{\xi}{1-\xi}\right]} (1 + v)\left(r^L - r^D\right)\lambda \\ &\qquad + \sigma(1 - \varepsilon(\lambda + 1))\mathbb{I}\left[\lambda + J^\lambda \leq \Gamma\left(q + J^q\left(q, \lambda, J^\lambda\right)\right)\right] + \sigma v_o \mathbb{I}\left[\frac{\lambda}{1 - \varepsilon(\lambda + 1)} > \frac{1}{\frac{1}{\rho} - 1}\right]. \end{aligned}$$

This again is clearly a linear program. The solution is

$$\lambda \in \begin{cases} 0 & \text{if } \left(r^L - r^D\right) < \sigma\epsilon\left(1 + v\left(q, \lambda\right)\right) \\ \Gamma\left(q\right) & \text{if } \left(r^L - r^D\right) > \sigma v_o \\ \Lambda\left(q\right) & \text{otherwise.} \end{cases} \tag{62}$$

In the paper, we consider only interesting cases where leverage is neither zero nor at the liquidation boundary, rather, we consider cases where leverage is set to the shadow boundary. Then, clearly when $dN = 1$, after any shock, λ must return to the shadow boundary

$$\Lambda\left(q + J^q + \bar{J}^q\left(q + J^q, \bar{J}^\lambda\right)\right) = \Gamma\left(q + J^q\right) - J^\lambda + \bar{J}^\lambda.$$

In this case, we verify that leverage remains in the shadow boundary. Thus, for any $\{\lambda, q\}$ outside of the shadow boundary, the state variables jump to a point in the shadow boundary. Hence, under this guess, we obtain that $\bar{v}$ is constant along a characteristic curve and is equal to the value at the shadow boundary.

Whenever there is no shock, it must be the case that

$$\mu^q \Lambda_q\left(q\right) = \mu^\lambda, \tag{63}$$

which guarantees that leverage and q remain at the shadow boundary.

Auxiliary value function. For any q, we can define a function q to its value at the point in the shadow boundary corresponding to q:

$$\tilde{v}\left(q\right) = v\left(\Lambda\left(q\right), q\right).$$

Therefore, if we differentiate this expression with respect to q, then we obtain

$$\tilde{v}_q\left(q\right) = \left(v_\lambda \Lambda_q + v_q\right). \tag{64}$$

Multiplying both sides by μ^q,and using that Equation (63) must hold along the optimal path, we obtain that

$$\tilde{v}_q\left(q\right)\mu^q = \left(v_\lambda \Lambda_q + v_q\right)\mu^q = v_\lambda \mu^\lambda + v_q \mu^q.$$

Then, substituting v for $\tilde{v}$ in (60), using the above result and the optimal policies for $\left\{\bar{J}^\lambda, \bar{J}^q\right\}$, we obtain an auxiliary HJB representation:

$$\begin{aligned} 0 \quad = \quad & \max_{\{c\}} f\left(c, \tilde{v}\right) + \tilde{v}_q\left(q\right)\mu^q + \left(1 + \tilde{v}\right)\left(r^L + \left(r^L - r^D\right)\Lambda\left(q\right) - c - \sigma\right) \\ & + \sigma\left(1 + \tilde{v}\left(q + \bar{J}^q\left(q, \Lambda\left(q\right), \bar{J}^\lambda\left(\Lambda\left(q\right)\right)\right) + J^q\right)\right)\left(1 - \varepsilon\left(\lambda + 1\right)\right), \end{aligned}$$

subject to

$$\mu^q \Lambda_q\left(q\right) = \mu^\lambda.$$

Substituting the constraint, we have that

$$
\begin{aligned}
0 &= \max_{\{c\}} f(c,\tilde{v}) + \tilde{v}_q \frac{\mu^\lambda}{\Lambda_q} + (1+\tilde{v})\left(r^L + \left(r^L - r^D\right)\Lambda(q) - c - \sigma\right) \\
&\quad + \sigma\left(1 + \tilde{v}\left(q + \bar{J}^q\left(q, \Lambda(q), \bar{J}^\lambda(\Lambda(q))\right) + J^q\right)\right)\left(1 - \varepsilon(\lambda+1)\right).
\end{aligned}
$$

Optimal dividend. We take the first-order condition with respect to dividends and obtain

$$
f_c(c,\tilde{v}) = \left(1 + \tilde{v} - \tilde{v}_q \frac{\mu_c^\lambda}{\Lambda_q}\right),
$$

and therefore

$$
c^* = \rho^{1/\theta} \frac{(1+\tilde{v}(q))}{\left(1 + \tilde{v}(q) - \tilde{v}_q(q)\frac{\mu_c^\lambda}{\Lambda_q}\right)^{-1/\theta}}.
$$

Solution to the value function. In this case, using the expression for c^*, we obtain that $v(q)$ solves the equation

$$
\begin{aligned}
0 &= f\left(\rho^{1/\theta}\frac{(1+\tilde{v}(q))}{\left(1+\tilde{v}(q)+\tilde{v}_q(q)\frac{1+\Lambda(q)}{\Lambda_q}\right)^{-1/\theta}}, \tilde{v}(q)\right) \\
&+ (1+\tilde{v})\left(r^L + \left(r^L - r^D\right)\Lambda(q) - \rho^{1/\theta}\frac{(1+\tilde{v}(q))}{\left(1+\tilde{v}(q)+\tilde{v}_q(q)\frac{\mu_c^\lambda}{\Lambda_q}\right)^{-1/\theta}} - c\right) \\
&+ \sigma\left(1+\tilde{v}\left(q+\bar{J}^q(\lambda)+J^q\right)\right)\left(1-\varepsilon(\lambda+1)\right).
\end{aligned}
$$

With this value, we obtain

$$
c(q) = \frac{(1+\tilde{v}(q))}{\left(1+\tilde{v}(q)+\tilde{v}_q(q)\frac{1+\Lambda(q)}{\Lambda_q}\right)^{-1/\theta}}
$$

and a drift in wealth of

$$
\mu^W(q) = r^L + \left(r^L - r^D\right)\Lambda(q) - \rho^{1/\theta}(1+\tilde{v}(q))^{1-1/\theta}.
$$

Finally, we obtain $\iota(q)$ from the following condition:

$$
\Lambda_q(q) = \frac{\left(\iota(q) - \mu^W(q)\right)(\lambda+1)}{(\iota(q)+\alpha)(1-q)q}.
$$

We can obtain an interpretation for this formula by using an expression for the ratio of the slope of the shadow boundary and the direction of the change of the vector $\{q,\lambda\}$ along an infinitesimal issuance ι:

$$
R(q) = \frac{\Lambda_q(q)}{\mu_\iota^\lambda/\mu_\iota^q} < 0.
$$

With the aid of this expression, the rule for issuances is given by

$$\iota\left(q\right)=\mu^{W}\left(q\right)\frac{1}{1-R\left(q\right)}-\alpha\frac{1}{1-R\left(q\right)^{-1}}.$$

Since the ratio of slopes $R\left(q\right)$ is negative,the issuances are increasing in the returns on equity. This is because as equity increases, leverage falls so issuances can remain high to keep leverage at the shadow boundary. In turn, the second term states that the faster the accounting of past losses, the more the issuances must be cut back.

C.6 Approximate Jump in Q after an Aggregate Shock

Approximation to the law of motion for aggregate q. To obtain an approximation to the aggregate behavior of q and to analyze the impact of an aggregate shock, we approximate the law of motion of aggregate loans and book loans around unconditional means for λ and q. The approximation is exact for a representative bank. We denote by $\bar{\lambda}$, that $\bar{\iota}$ and $\bar{c}$ are the population averages of λ and q, and we consider that every bank has approximately the same issuance rate ι and the same leverage λ. Then, the law of motion of the aggregate volume of loans, $\mathcal{L}$, will be given by

$$\frac{d\mathcal{L}}{\mathcal{L}}=\bar{\iota}-\chi,$$

where $\chi=\sigma\varepsilon$ is the the unconditional expectation of bank defaults per instant of time. Then, let $\boldsymbol{q}$ denote the aggregate version of q.

In the law of motion for book loans, the law of motion would be approximately

$$\frac{d\bar{\mathcal{L}}^{c}}{\bar{\mathcal{L}}}=\frac{\left(\bar{\iota}-\alpha\left(\frac{1}{\boldsymbol{q}}-1\right)\right)\left(\bar{\lambda}+1\right)W-\tau\chi\left(\bar{\lambda}+1\right)W}{\frac{1}{\boldsymbol{q}}\left(\bar{\lambda}+1\right)W}=\bar{\iota}\boldsymbol{q}-\alpha\left(1-\boldsymbol{q}\right)-\tau\chi\boldsymbol{q}.$$

By the differential of the ratios, we have that

$$d\boldsymbol{q}=\left(\frac{d\mathcal{L}}{\mathcal{L}}-\frac{d\bar{\mathcal{L}}^{c}}{\bar{\mathcal{L}}}\right)\boldsymbol{q}\approx\left(\iota+\alpha\right)\left(1-\boldsymbol{q}\right)\boldsymbol{q}-\chi\left(1-\tau\boldsymbol{q}\right)\boldsymbol{q}.$$

In a stationary equilibrium, $\mathbb{E}\left[dq\right]$, hence

$$\left(\iota+\alpha\right)\left(1-\boldsymbol{q}\right)\approx\chi\left(1-\tau\boldsymbol{q}\right)$$

and thus

$$\left(\iota+\alpha-\chi\right)\approx\left(\iota+\alpha-\tau\chi\right)\boldsymbol{q}.$$

As a result

$$\mathbb{E}\left[q\right]\approx\boldsymbol{q}=\frac{\left(\iota+\alpha-\chi\right)}{\left(\iota+\alpha-\tau\chi\right)}<1.$$

Now consider an aggregate shock ϵ. We obtain aggregate jump

$$J^{\boldsymbol{q}}=-\epsilon\frac{\left(1-\tau\boldsymbol{q}\right)}{\left(1-\tau\epsilon\boldsymbol{q}\right)}\boldsymbol{q}.$$

The jump in

$$J^{\lambda} = \frac{\epsilon\left(\lambda+1\right)}{1-\epsilon\left(\lambda+1\right)}.$$

Now consider the aggregate Tobin's Q, call it $\boldsymbol{Q}$. We have that

$$\boldsymbol{Q} = s\left(\boldsymbol{q},\lambda\right)\frac{\lambda}{\boldsymbol{q}\left(1+\lambda\right)-\lambda}.$$

Therefore, the jump in $\boldsymbol{Q}$ is given by

$$d\boldsymbol{Q} = s\left(\boldsymbol{q}+J^{\boldsymbol{q}},\lambda+J^{\lambda}\right)\frac{\lambda+J^{\lambda}}{\left(\boldsymbol{q}+J^{\boldsymbol{q}}\right)\left(1+\lambda+J^{\lambda}\right)-\lambda+J^{\lambda}} - s\left(\boldsymbol{q},\lambda\right)\frac{\lambda}{\boldsymbol{q}\left(1+\lambda\right)-\lambda}$$

C.7 Calibration of α

We derive the fraction of losses that are recognized in the books after T quarters as a default shock of size ε. To this end, consider a sequence of default shocks such that $dN_0 = 1$ and $dN_{t>0} = 0$. Normalize $L_0 = 1$ and set $\bar{L}_0 = 1/q_0$. Then,

$$L_t = 1-\varepsilon \qquad t>0$$

$$d\bar{L}_t = -\alpha\left(\bar{L}_t - L_t\right)dt \qquad t>0,$$

with $\bar{L}_0 = 1/q_0 - \tau\varepsilon$. Fundamental loans immediately drop from the initial value 1 to the after-default value $1-\varepsilon$. Book loans, on the other hand, only fall by $\tau\varepsilon$, on impact, and recognize the remaining fraction $(1-\tau)\varepsilon$ at a rate governed by α and the gap in book and fundamental loans. We can guess and verify that

$$\bar{L}_t = (1-\varepsilon) + \left[\left(\frac{1}{q_0}-1\right) + (1-\tau)\,\varepsilon\right]e^{-\alpha t}$$

We define the fraction of the loss generated by this single default shock that is recognized by time t as the fall in book loans over the size of the loss

$$f_t \equiv \frac{1/q_0 - \bar{L}_t}{\varepsilon}$$

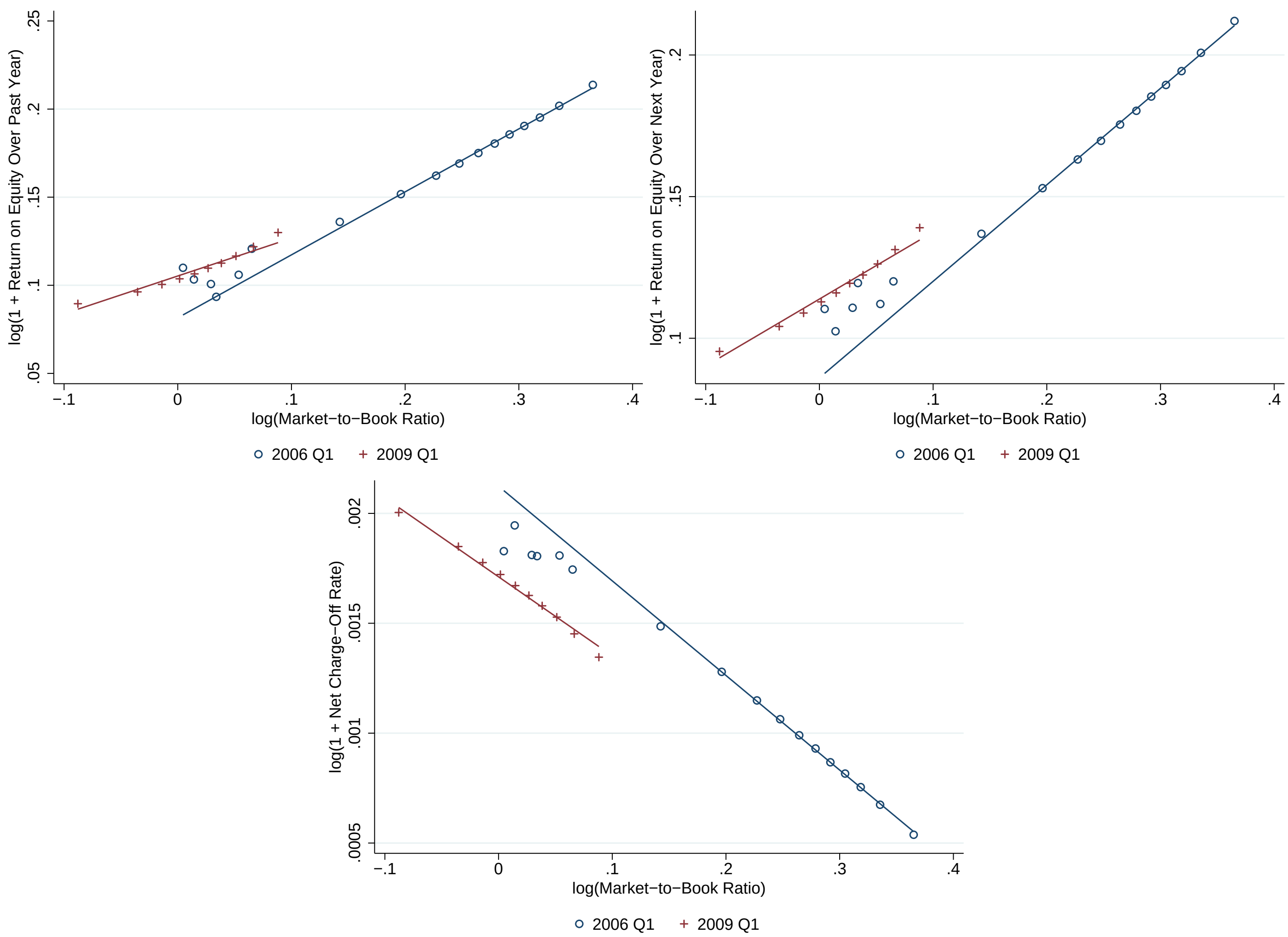
Substituting in the formula for $\bar{L}_t$ and the parameters in the calibration and starting from $q_0 = 0.977$, as in the pre-crisis mean value of q, and using a default shock of $\varepsilon = 2.5\%$, like the aggregate shock we use in the estimation of $\{\gamma,\theta\}$ for the post-crisis period, yields that after 10 quarters the fraction of losses recognized is 64.68%.

C.8 Replication of Fact 2

To demonstrate that the model delivers Fact 2, we take a straightforward approach and replicate Figure 3, using simulated data from the model. We construct the variables in exactly the same way as in the original figure; we do not construct a panel relating the loan delinquency rate to the market-to-book ratio because we do not have the concept of "delinquent" loans in the model. We construct the quarters such that 2007 Q3 is the last pre-crisis quarter and, thus, 2007 Q4 is the start of the post-crisis period.

Figure 13 shows the results. Astute readers will note that the magnitudes are somewhat different from the original figure. However, the model-generated figure matches the qualitative results of the original. Thus, the model delivers the following fact: Tobin's Q predicts future cash flows in the cross section and market values capture information that book values do not.

Figure 13: Model Replication: Market equity contains more cash-flow-relevant information than book equity does



Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log market-to-book-equity ratio for 2006 Q1 and 2009 Q1. All plots control for the log book equity by residualizing the variables on the log book equity, and then adding back the mean of each variable to maintain centering. Data are generated from 10,000 simulated banks and by using the model at estimated parameters. The ROE over the past year is defined as the book net income over the last four quarters divided by the book equity four quarters ago; the ROE over the next year is defined as the one lead of this variable (i.e., profits over the next four quarters divided by current book equity). The net charge-off rate is the loan charge-offs over the next quarter minus the loan recoveries over the next quarter divided by the total loans this quarter.

D Model Appendix: Numerical Solution

We solve the model by using the finite-differences method with an upwind scheme for the choice of forward or backward differences. Specifically, we compute the numerical derivatives of the value function $v(\lambda, q)$ by using finite differences and we use the first-order conditions to solve for policies (c, ι), and iterate on the HJB equation. A detailed description of this algorithm for a general class of models known as mean-field games can be found in Achdou et al. (2020).

Our model, which belongs to this class, is simpler to solve because we keep prices constant; however, it presents an added complication in that the size of the jump depends on the endogenous state variables. In particular, starting from a point (λ, q), upon receiving a Poisson shock the bank jumps to $\left(\lambda + J^\lambda, q + J^q\right) = \left(\lambda + \frac{\epsilon(\lambda+1)}{1-\epsilon(\lambda+1)}, q - \epsilon\frac{(1-\tau q)}{(1-\tau\epsilon q)}q\right)$. To avoid having to interpolate the value function, when constructing the grid we take advantage of the fact that the size of the default shock ε is constant.

We build a non-uniform grid iteratively; starting from an initial grid point (λ_0, q_0), with $\lambda_0 \approx 0$ and $q_0 = 1$, we pick the following points in the grid by using the recursion $(\lambda_n, q_n) = \left(\lambda_{n-1} + J^{\lambda_{n-1}}, q_{n-1} + J^{q_{n-1}}\right)$. This way, upon receiving the default shock the bank always jumps to a point that belongs to the grid. We depart from this grid construction scheme in two regions:

1. When λ is large, the size of the jump is also large, so the grid may become too coarse. We add points to the grid wherever we have $\lambda_n - \lambda_{n-1}$ above a certain threshold, by setting $\lambda_0 = (\lambda_n + \lambda_{n-1})/2$ and adding points to the grid, following the previous recursion;

2. When λ is close to but below $\xi/(1-\xi)$, or when $q < \xi\left(1+\lambda^h\right)/\lambda^h$ for some high value λ^h, the stationary distribution features are close to zero mass and the value-function features show less curvature. In these two regions of the state space we use a uniform grid and interpolate $\left(\lambda + J^\lambda, q + J^q\right)$ using the closest point.

To compute the stationary distribution, we simulate the model for enough periods such that the mean and standard deviations of λ and q are approximately constant. Finally, to aggregate the variables to a quarterly frequency, we set time steps $dt = 1/90$ and, for every 90 time steps, we use the last value for stocks and the mean for flows.