## Falling Interest Rates and Credit Misallocation: Lessons from General Equilibrium

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- Conventional wisdom: declining interest rates stimulate economic activity.
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- Some suggestive evidence:
  - Recent credit booms characterized by low productivity growth Gopinath et al. 2017; Garcia-Santana et al 2020.
  - Low-interest rate environments characterized by "zombie" lending Banerjee and Hofmann 2018; Schivardi et al. 2020.
- Questions
  - Do low interest rates foster (socially) unproductive activities?
  - Under what conditions?
  - Can this effect be strong enough to hamper economic activity and growth?
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- Key ingredients of the framework:
  - entrepreneurs borrow to invest in capital,
  - heterogeneous productivity,
  - financial constraints.

#### Main insight: falling interest rates...

- Prompt investment by less productive entrepreneurs,
- Raise price of capital and crowd out more productive entrepreneurs.
- Induced reallocation weakens expansionary effect:
  - Can be strong enough to reduce aggregate output!
  - Is inefficient due to excessive investment by less productive entrepreneurs.
  - Dynamically interacts with balance sheet channel ightarrow boom-bust dynamics of output.
- Empirical evidence in support of the mechanism (in progress).

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#### Related literature

- Credit booms and low productivity growth:
  - Reis (2013), Gopinath et al. (2017), Doerr (2018), Garcia-Santana et al. (2020), Benigno et al. (2020), Caggese and Perez-Olive (2020), Gorton and Ordonez (2020), Asriyan et al. (2021).

#### • Zombie lending:

- Caballero et al. (2006), Adalet-McGowan et al. (2018), Banerjee and Hofmann (2018), Tracey (2019), Schivardi et al. (2020).
- Heterogeneity and response to monetary policy shocks:
  - Cloyne et al. (2018), Jeenas (2019), Manea (2020), Anderson and Cesa-Bianchi (2020), Ottonello and Winberry (2020), Leahy and Thapar (2021).
- Negative side-effects of low and declining interest rates:
  - Rajan (2015), Dell'Ariccia and Marquez (2015), Martinez-Miera and Repullo (2017), Brunnermeier and Koby (2018), Liu et al. (2019), Bolton et al. (2021).
- Factor competition and financial frictions:
  - ► Ventura and Voth (2015), Martin et al. (2018), Asriyan et al. (2021).

The Model

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- Two time periods: t = 0, 1.
- Two goods: consumption (c) and capital (k).
- All agents have preferences:

$$U^i = E_0\{C_1^i\},$$

where  $C_1^i$  is individual *i*'s consumption at t = 1.

- Entrepreneurs (unit mass):
  - Endowed with w > 0 consumption goods at t = 0,
  - Can install k units of capital at t = 0 and receive A · k consumption goods at t = 1, where A ~<sup>iid</sup> G with pdf g that has full support on [0, 1].

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  - ► (For today) SOE: agents can borrow and lend at world interest rate *R*.
  - Friction: entrepreneur can walk away with a fraction  $1 \lambda$  of her output.
  - Endogenous borrowing limit:

$$R \cdot b \leq \lambda \cdot A \cdot k.$$

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• Capitalists' optimization implies weakly increasing capital supply  $K^{S}(q)$ .

Entrepreneurs' optimization implies

$$k_{A}(q; R) \begin{cases} = 0 & \text{if } \frac{A}{R} < q \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } q = \frac{A}{R} \\ = \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{R} < q < \frac{A}{R} \\ = \infty & \text{if } q \leq \frac{\lambda \cdot A}{R} \end{cases}$$

Capital market clearing: q is such that

$$K^{S}(q) = K = K^{D}(q; R) \equiv \int k_{A}(q; R) \cdot dG(A)$$

Aggregate output of the economy at t = 1:

$$Y = \int A \cdot k_A(q, R) \cdot dG(A), \text{ where TFP} = \frac{Y}{K}.$$

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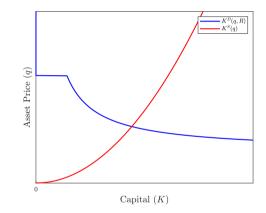
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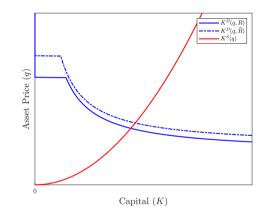
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#### Benchmark: homogeneous productivity All entrepreneurs have productivity A



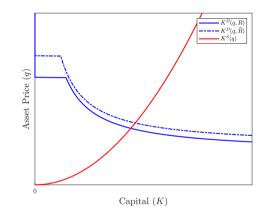
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• Expansionary effect of a fall in *R*: *K* and *Y* increase (no change in TFP).

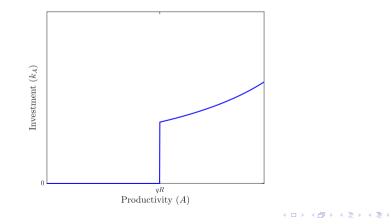
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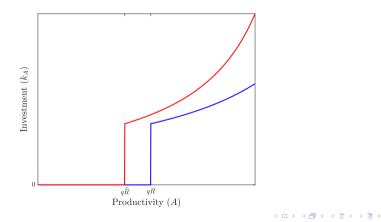
Heterogeneous productivity

• Now, we care about the distribution of  $k_A$ . Given  $\{q, R\}$ :



#### Heterogeneous productivity

- Given q, a fall in the interest rate:
  - generates investment by some infra-marginal entrepreneurs,
  - increases investment by supra-marginal entrepreneurs.



### General-equilibrium effects

- Higher capital demand  $\rightarrow q$  must rise to ensure market clearing.
- Hence, investment of supra-marginal entrepreneurs must change:

$$rac{dk_A}{dR} = rac{\left|rac{dq}{dR}
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PE effect: a fall in R raises \$\frac{\lambda A}{R}\$ and reduces the required "down payment".
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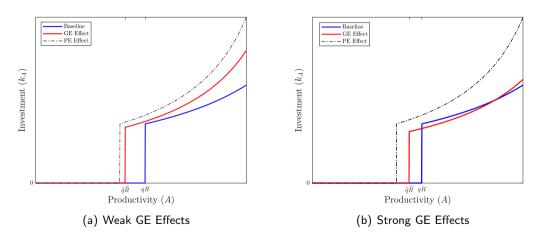
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#### General-equilibrium effects



• Effect of changes in *R* on aggregate output:

$$\frac{dY}{dR} = \underbrace{q \cdot R \cdot \frac{dK^{S}(q)}{dR}}_{\equiv \mathcal{K}} + \underbrace{\int_{q \cdot R}^{1} (A - q \cdot R) \cdot \frac{dk_{A}}{dR} \cdot dG(A)}_{\equiv \mathcal{R}}$$

- K captures a capital-supply effect:
  - always (weakly) negative;
  - a fall in R raises q and thus the aggregate stock of capital.
- R captures a capital-reallocation effect:
  - can be positive or negative, depending on strength of GE effects;
  - a fall in R raises q and redistributes K among entrepreneurs;
  - equals zero absent heterogeneous productivity or financial frictions.
- R is strong if it is more positive, whereas K is strong if it is more negative

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## Main result

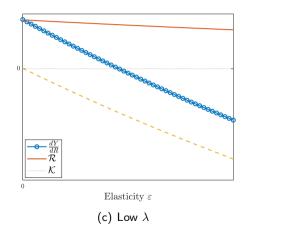
#### Proposition

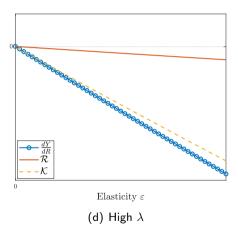
Fix an equilibrium, and let  $\varepsilon$  denote the elasticity of capital supply with respect to the price of capital q at equilibrium. All else equal, as  $\varepsilon$  decreases:

- *K* gets weaker,
- ${\cal R}$  gets stronger,
- dY/dR increases.

Moreover, for low enough  $\varepsilon$ , dY/dR becomes positive if  $\lambda$  is below a threshold.

## Main result





Normative properties

#### Constrained planning problem

- Consider a planner who dictates how much each entrepreneur invests:
  - subject to competitive markets, budget and financial constraints.
- The social planner maximizes aggregate consumption (welfare):

$$\max_{\{b_A,k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot (\chi(K^S(q)) - w)$$
  
subject to:  
 $q \cdot k_A = w + b_A; \quad R \cdot b_A \le \lambda \cdot A \cdot k_A; \quad K^S(q) = \chi^{-1}(q) = \int k_A \cdot dG(A).$ 

The planner's optimality condition for k<sub>A</sub>:

$$A - q \cdot R - \chi''\left(K^{S}(q)\right) \cdot \int \gamma_{\widehat{A}} \cdot \left| \frac{\partial}{\partial q} \left( \frac{1}{q - \frac{\lambda \cdot \widehat{A}}{R}} \cdot w \right) \right| \cdot dG(\widehat{A}) \lessapprox 0,$$

where  $\gamma_A > 0$  if the financial constraint of entrepreneur with prod. A binds.

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## Planner-optimal allocations

Let superscripts CE and SP denote the competitive equilibrium and the social planner's allocations, respectively.

#### Proposition

If  $\widetilde{A}$  denotes the productivity of the marginal entrepreneur in the social planner's allocation, then:

$$\widetilde{A} > q^{CE} \cdot R > q^{SP} \cdot R.$$

Moreover, in the planner's allocation, a fall in R is always expansionary:

$$\frac{dY^{SP}}{dR} \le 0,$$

with strict inequality if the capital supply is not perfectly inelastic.

#### Robustness and extensions

- Unconstrained firms, in addition to constrained entrepreneurs.
- Diminishing returns at entrepreneur level.
- Closed economy: fall in R is a result of a savings' glut.
- Dynamics of net worth accumulation + balance sheet effects.

# **Dynamics**

20 / 25

## Dynamic setup

- Time is continuous,  $t \ge 0$ .
- Entrepreneurs: log-preferences with discount rate ho>r,
  - ▶ allocate net worth w between capital  $k \ge 0$  and risk-free debt b:

$$q \cdot k - b = w$$
.

produce:

$$y = A \cdot k.$$

heterogeneous productivity A (exogenous) and wealth w (endogenous).

- each instant fraction  $\theta$  of entrepreneurs draws new productivity from G.
- evolution of net worth:

$$\dot{w} = y + \dot{q} \cdot k - r \cdot b - c.$$

- Capital stock is fixed at  $\overline{K}$  in aggregate and traded at price q.
- Friction: entrepreneurs can walk away with fraction  $1-\lambda$  of capital,

 $b \leq \lambda \cdot q \cdot k.$ 

• Optimization:

consumption:

$$c = \rho \cdot w.$$

net worth evolves according to:

$$\dot{w} = (A + \dot{q} - r \cdot q) \cdot k + (r - \rho) \cdot w.$$

investment:

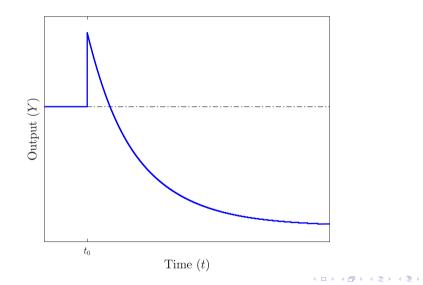
$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \ge r \cdot q \\ 0 & \text{otherwise} \end{cases}$$

• Market clearing:

$$\int_{A\geq q\cdot r-\dot{q}}\frac{1}{1-\lambda}\cdot\frac{W_A}{q}\cdot dA=\bar{K},$$

where  $W_A = \int w \cdot f(A, w) \cdot dw$  and  $f(A, w) \ge 0$  is the share of entrepreneurs with productivity A and net worth w.

## Boom-bust dynamics of output



23 / 25

3

# Empirical evidence (in progress)

- Main insight: a fall in the interest rate reallocates credit/investment towards relatively unproductive activities.
- Diff-in-diff analysis at sector-region level: when the interest rate falls...
  - 1. output expands less in regions with lower capital supply elasticity/sectors that are more capital intensive;
  - 2. productivity grows less be lower in regions with lower capital supply elasticity/sectors that are more capital intensive.
- Data from the US and Spain:
  - 1. real-world counterpart to capital supply:
    - interpret capital as real-estate,
    - use real-estate supply elasticity measures,
    - compute real-estate intensity using sectoral data.
  - 2. changes in interest rates:
    - alternative measures of interest rate changes (e.g. monetary policy shocks).

#### What have we learned?

- Stylized model with three key features:
  - entrepreneurs borrow to invest in capital,
  - heterogeneous productivity,
  - financial constraints.
- Main insight: falling interest rates...
  - Prompt investment by less productive entrepreneurs,
  - Raise price of capital and crowd out more productive entrepreneurs.
  - Induced reallocation weakens expansionary effect:
    - Can be strong enough to reduce aggregate output!
    - Is inefficient due to excessive investment by less productive entrepreneurs.
    - Dynamically interacts with balance sheet channel  $\rightarrow$  boom-bust dynamics of output.
- Empirical evidence in support of the mechanism (in progress).