

Benefits of Macro-Prudential Policy in Low Interest Rate Environments

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European Central Bank

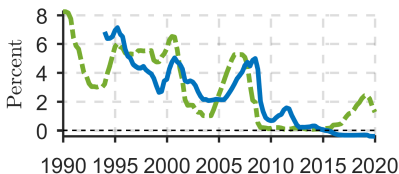
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The views expressed on this presentation are my own and do not necessarily reflect those of the ECB.

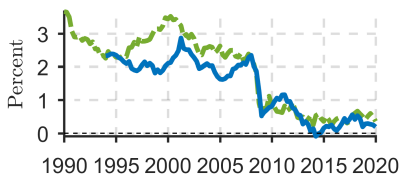
Secular Decline in Interest Rates

Panel A: Short-term Interbank Rate



— Euribor — Federal Funds Rate

Panel B: Natural Rate of Return



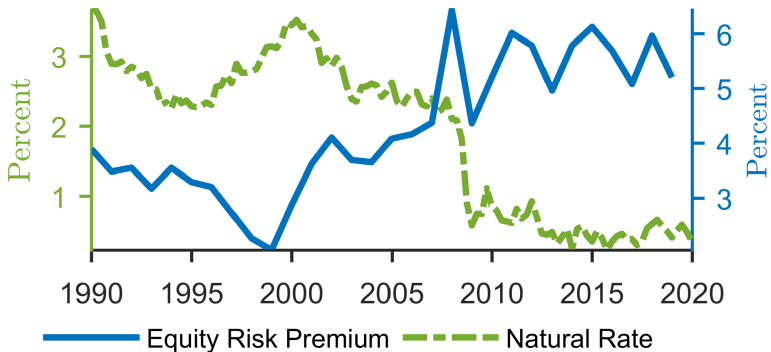
— Euro Area — United States

Source: ECB Statistical Data Warehouse (SDW) and Federal Reserve Economic Data (FRED).

Notes: Benchmark short-term nominal interest rate (panel A) and natural rate of return (panel B) for the euro area and the United States. Natural rates are estimated according to methodology in Laubach and Williams (2003) and Holston et al (2017).

Many expect the decline to continue going forward (Blanchard 2020, Jorda et al. 2020, among others).

Equity Risk Premium and the Natural Rate



Source: Damodaran (2020). Data for the United States.

This Paper – Subject of Study

Takes past, present, and expected future secular decline as given.

Focuses on the **consequences** of the decline on:

1. Cyclical relationship between systemic risk in financial markets and the natural rate.
2. Effect of macro-prudential policy (e.g., state-contingent limits on leverage) on the natural rate.
3. Benefits of macro-prudential policy on macroeconomic stabilization.

This Paper – Mechanism

Natural rate (same formula as in C-CAPM):

$$r_t dt = \rho dt + \gamma E_t [dY_t / Y_t] - \frac{1}{2} (1 + \gamma) \gamma \text{Var}_t [dY_t / Y_t].$$

ρ : time discount rate; γ : risk aversion coefficient.

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Takeaway No. 1: Systemic risk increases aggregate output risk and thus depresses the natural rate.

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Takeaway No. 1: Systemic risk increases aggregate output risk and thus depresses the natural rate.

Takeaway No. 2: Macro-prudential policy boosts the natural rate unintentionally, simply as a by-product of containing systemic risk in financial markets.

This Paper – Main Results

1. Systemic risk in financial markets depresses the natural rate.
2. A macro-prudential policy that is concerned only with financial stability boosts the natural rate unintentionally.
3. Results 1. and 2. point to a novel complementarity between financial stability and macroeconomic stabilization.
Complementarity is sufficiently strong to generate a divine coincidence if the natural rate is secularly low, but not too low.

Related Literature

Relationship between systemic risk and the natural rate:

Summers 2014, Farhi and Gourio 2018, Del Negro et al. 2019, Kuvshinov and Zimmermann 2020.

Effect of unconventional monetary policies on the natural rate:

Caballero and Simsek 2020, Lindé et al. 2020.

Transmission mechanism of prudential policies in low interest rate environments:

Farhi and Werning 2016, Korinek and Simsek 2016, Ferrero et al. 2018, Caballero and Simsek 2019, Fornaro and Romei 2019.

Roadmap

Proceed in three steps as follows:

1. Flexible price economy under *laissez faire*.

Examine equilibrium relationship between systemic risk and the natural rate.

2. Monetary economy without macro-prudential intervention.

Re-examine equilibrium relationship between systemic risk and the natural rate in “low” interest environments (i.e., occasionally binding **ZLB** constraint).

3. Monetary economy with the interventions.

Additional benefits of macro-prudential policy in low interest environments.

The Baseline Model

Flexible price economy under laissez faire

Model Description

Agents: Banks and households, a continuum of each. Goods: Physical capital and consumption. Time $t \geq 0$ is continuous.

Physical capital:

- (i) yields output flows per unit of time, $y_t = a_t k_t$ with $a_t = 1$ if banks hold the unit of capital and $a_t = a_h < 1$ if households instead do so;
- (ii) evolves over time stochastically according to $dk_t/k_t = \mu dt + \sigma dZ_t$, with dZ_t being a capital quality (Brownian) shock.

(Similar technology to Brunnermeier and Sannikov 2014.)

Banks — portfolio problems:

- (i) can issue short-term non-contingent debt (i.e., deposits) to take leveraged positions on physical capital;
- (ii) but are subject to an incentive-compatible (IC) leverage constraint.

(Similar frictions to Gertler and Kiyotaki 2010 and Gertler and Karadi 2011.)

Households — portfolio problems:

consume, save in deposits, can hold physical capital, and own all of the banks.

(Similar to Merton 1971.)

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Competitive Equilibrium (1/2)

The size of the economy is proportional to the aggregate capital stock.

The single relevant state is the aggregate net worth of banks as a share of total wealth. That is,

$$\eta \equiv \frac{N_b}{N_b + N_h} = \frac{N_b}{qK} \in [0, 1] \rightarrow \text{measure of financial conditions.}$$

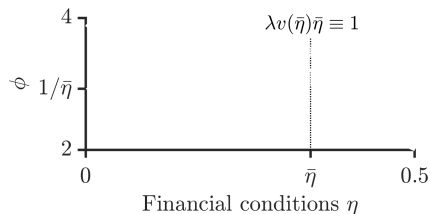
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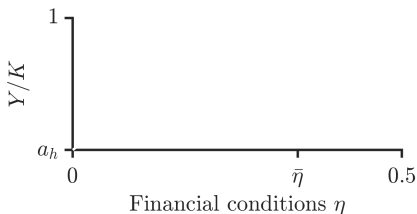
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Panel A. Leverage Multiple



Panel B. Detrended Aggregate Output



Notes:

Panel A. Leverage constraint, $\phi \leq \lambda v$. $\frac{1}{\lambda}$: portion of divertable assets. v : Tobin's Q. Limit λv is binding when $\eta < \bar{\eta}$ and it is slack otherwise.

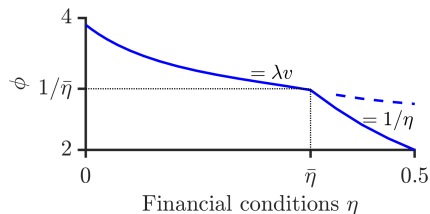
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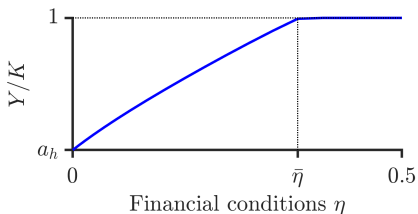
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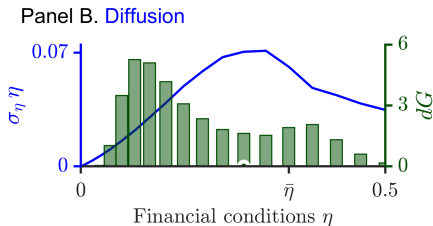
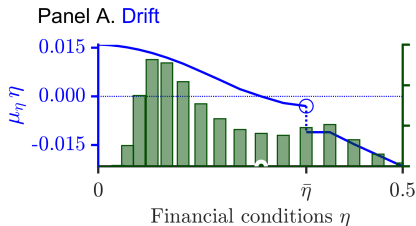


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Competitive Equilibrium (2/2)

Law of motion of the wealth share, $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$.



Notes:

Left y-axis:

Panel A. Reversion to stochastic steady state, η_{ss} .

Panel B. Nonlinear volatility, peaks around region in which λv is occasionally binding.

Right y-axis:

Ergodic distribution. Economy fluctuates continuously throughout from booms to busts.

Risk Relationships

Relationship between systemic risk in financial markets and risk in asset prices.

A. $\sigma_\eta = (\phi - 1)(\sigma_q + \sigma)$ because banks take leveraged positions on risky assets funded with risk-free debt.

B. $\sigma_q = \varepsilon_q \sigma_\eta$ with $\varepsilon_q \equiv \frac{\partial q}{\partial \eta} \frac{\eta}{q}$ because of occ. binding leverage constraint.

A. & B. \Rightarrow positive interaction between σ_η and σ_q .

Interpretation: “Fire-sales” and feedback loops in financial markets.

Amplification factor: $\frac{\sigma_\eta}{\sigma} = \frac{\phi - 1}{1 - (\phi - 1)\varepsilon_q}$. Key driver behind nonlinear volatility.

Relationship between Systemic Risk and the Natural Rate

In this economy, the natural rate of return is the short-term real interest rate (no nominal rigidities).

The formula for the natural rate is $r = \rho + \gamma\mu_Y - \frac{1}{2}\gamma(1 + \gamma)\sigma_Y^2$ (as in C-CAMP).

Mean reversion + Nonlinear volatility in financial markets \Rightarrow

with $r_E \equiv \rho + \gamma\mu - \frac{1}{2}\gamma(1 + \gamma)\sigma^2$ being the natural rate in a frictionless economy.

The natural rate tanks when systemic risk peaks.

During booms, the natural rate is high and stable; upon exiting the booms, the natural rate falls abruptly and becomes highly unstable.

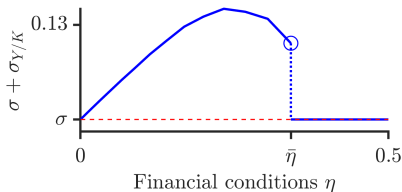
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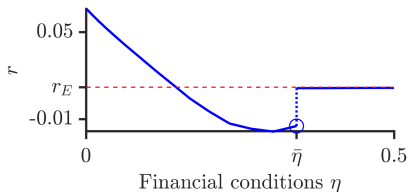
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Panel A. Aggregate Output Risk



Panel B. Natural Rate



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The Model with the ZLB Constraint

The monetary economy with an occasionally binding ZLB constraint
and without macro-prudential intervention

Nominal Rigidities and ZLB on Nominal Rates

New Keynesian economy with fully rigid nominal prices. No inflation but production can fall below installed capacity.

(Similar specification to Caballero and Simsek 2020.)

Capacity utilization rate, $u_t \equiv C_t/Y_t \leq 1$. Rate u_t isolates the output losses from nominal rigidities. Note that $Y_t = \zeta_t K_t$ already incorporates the potential losses from financial frictions.

Monetary policy:

Tracks the natural rate whenever possible and remains stuck at the ZLB otherwise. Formally, $i_t = \max\{r_t, 0\}$, with r_t being the natural rate. (Nominal and real interest rates coincide because the inflation rate $\pi_t = 0$ is null.)

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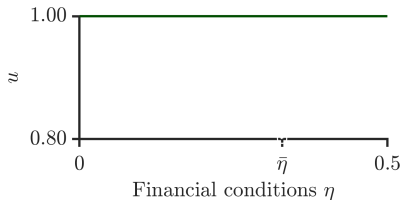
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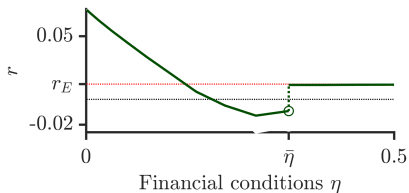
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An Economy with Liquidity Traps

Panel A. Capacity Utilization



Panel B. Natural Rate



— without ZLB Constraint

Double-whammy effect on bank profitability:

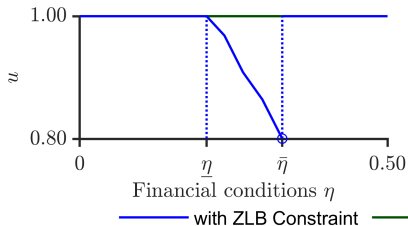
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Liquidity traps increase systemic risk and financial instability.

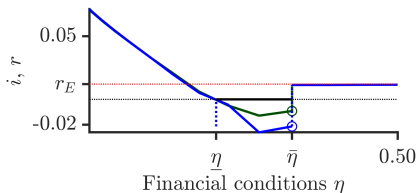
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An Economy with Liquidity Traps

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Panel B. Policy Rate, Natural Rate



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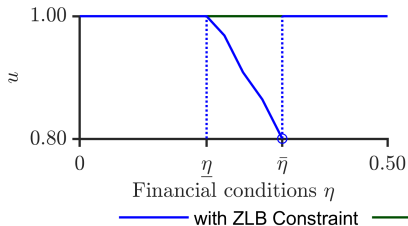
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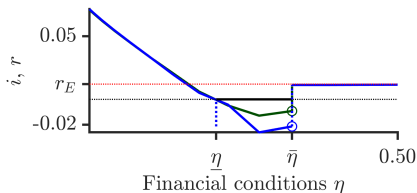
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The Model with Macro-prudential Policy

The monetary model with the occasionally binding ZLB constraint and macro-prudential intervention.

Macro-prudential Policy

Instrument:

State-contingent limit on leverage, $\Phi_t \equiv \Phi(\eta_t)$. Binding when $\Phi < \min\{\lambda\nu, \frac{1}{\eta}\}$.

Possible objectives:

Consider logarithmic preferences. Then, $W = W_M + W_F + \frac{1}{\rho} \ln K$, with

$$\rho W_M = \ln u + \frac{\partial W_M}{\partial \eta} \mu \eta \eta + \frac{1}{2} \frac{\partial^2 W_M}{(\partial \eta)^2} (\sigma \eta \eta)^2, \quad \rho W_F = \ln \zeta + \frac{\partial W_F}{\partial \eta} \mu \eta \eta + \frac{1}{2} \frac{\partial^2 W_F}{(\partial \eta)^2} (\sigma \eta \eta)^2.$$

1. Macroeconomic stabilization, $\int W_M(\eta) dG(\eta)$;
2. Financial stability, $\int W_F(\eta) dG(\eta)$;
3. Social welfare, $\int [W_M(\eta) + W_F(\eta)] dG(\eta)$.

Remark:

Improvements on the objectives over laissez faire are possible because of pecuniary externalities in financial intermediation and an aggregate demand externality.

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Optimal Macro-prudential Intervention

Consider a macro-prudential policy that is concerned only with financial stability.

Three possible cases:

Case 1. No complementarity – environments with “normal” interest rate levels

The natural rate under a policy of laissez faire is always above zero.

The ZLB constraint is irrelevant. Only the pecuniary externalities matter.

$\Phi < \min\{\lambda\nu, \frac{1}{\eta}\}$ only when $\eta \approx \bar{\eta}$. This $\downarrow \sigma_{\eta}\eta$ and $\uparrow r$.

Case 2. Divine coincidence (complementarity is sufficiently strong)

The natural rate under laissez faire is occasionally below zero, but it is always above zero under the optimal intervention.

Same intervention as in the economy without the ZLB constraint (as in Case 1.).

The ZLB constraint is also irrelevant but because of the intervention.

Case 3. Complementarity but no coincidence

None of the above two conditions holds.

Tighter intervention than in the economy without the ZLB constraint.

Further $\downarrow \sigma_{\eta}\eta$ and further $\uparrow r$.

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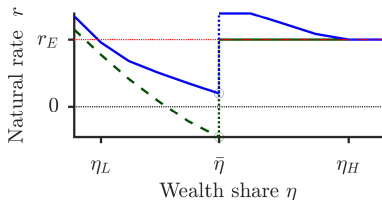
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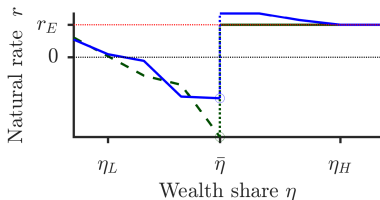
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Macro-prudential Policy in Low Interest Rate Environment

Panel A. Divine Coincidence (Case 2.)



Panel B. No Divine Coincidence (Case 3.)



— with Intervention - - - without Intervention

Notes:

Limit $\Phi < \min\{\lambda v, \frac{1}{\eta}\}$ is binding only when $\eta \in [\eta_L, \eta_H] \supset \bar{\eta}$.

Panel A. Macro-prudential policy renders the ZLB constraint irrelevant. No liquidity trap nor aggregate demand externality in equilibrium.

Panel B. Macro-prudential policy only softens the constraint. Liquidity traps do occur and the aggregate demand externality exists.

Conclusion

- Systemic risk in financial markets depresses the natural rate.
- Macro-prudential policy (i.e., state-contingent limits on leverage) boosts the natural rate unintentionally, simply as a by-product of containing systemic risk.
- New complementarity between financial stability and macroeconomic stabilization in low interest rate environments.
- Complementarity is sufficiently strong to generate a divine coincidence if the natural rate is low, but not too low.

Additional Material

Portfolio Problem of Banks

Banks maximize the present discounted value of their dividend payout

$$V_t \equiv \max_{k_{b,t} \geq 0} E_t \int_t^{\infty} \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} n_{b,s} ds ,$$

subject to solvency constraint $n_{b,t} \geq 0$; the law of motion of net worth,

$$dn_{b,t} = dR_{b,t} q_t k_{b,t} - (q_t k_{b,t} - n_{b,t}) r_t dt ;$$

and portfolio constraint,

$$q_t k_{b,t} \leq \lambda V_t ,$$

with

$$dR_{b,t} \equiv \frac{1}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t} \quad \text{and} \quad \frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t} dZ_t .$$

[Back](#)

Portfolio Problem of Households

Households maximize the present discounted value of their utility flows

$$\max_{k_{h,t}, c_t \geq 0} E_t \int_t^\infty e^{-\rho(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} ds ,$$

subject to solvency constraint $n_{h,t} \geq 0$, and the law of motion of their net worth,

$$dn_{h,t} = dR_{h,t} q_t k_{h,t} + (n_{h,t} - q_t k_{h,t}) r_t dt - \tau_t dt - c_t dt ,$$

with

$$dR_{h,t} \equiv \frac{a_h}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t} .$$

Note that $\Lambda_t \equiv e^{-\rho t} c_t^{-\gamma}$.

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