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(Optimal) Monetary Policy with and without Debt

by Boris Chafwehé,[1] Rigas Oikonomou,[2] Romanos Priftis[3] and Lukas Vogel[4]

[1]Université catholique de Louvain and FNRS

[2]Université catholique de Louvain

[3]Canadian Economic Analysis Department
Bank of Canada, Ottawa, Ontario, Canada K1A 0G9

[4]Directorate General for Economic and Financial Affairs
European Commission

Boris.Chafwehe@uclouvain.be, Rigas.Oikonomou@uclouvain.be, rpriftis@bankofcanada.ca, Lukas.Vogel@ec.europa.eu

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Abstract

We propose a framework of optimal monetary policy where debt sustainability may, or may not, be a relevant constraint for the central bank. We show analytically that in each environment the optimal interest rate path consists of a Taylor rule augmented with forward guidance terms. These terms arise either i) from "twisting interest rates" when the central bank ensures debt sustainability, or ii) under no debt concerns, from committing to keep interest rates low at the exit of the liquidity trap. The optimal policy is isomorphic to Leeper's (1991) "passive monetary/active fiscal policy" regime in the first instance, or "active monetary/passive fiscal policy" regime in the second. We insert our framework into a standard medium scale DSGE model calibrated to the US. Optimal passive monetary policy with debt concerns is ineffective in stabilizing inflation, whereas under no debt concerns, monetary policy is very effective in stabilizing the macroeconomy.

Bank topics: Monetary policy; Monetary policy framework; Fiscal policy; Economic models
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1 Introduction

Since the 2008–9 recession governments in developed economies have accumulated large stocks of debt. High debt levels require bold fiscal adjustments to ensure the solvency of government budgets, but in many countries it is questionable whether fiscal authorities will be able to generate the large required surpluses to finance the debt. This problem is likely to be exacerbated in the coming years; large (current and projected) deficits aimed at mitigating the effects of COVID-19 as well as anemic economic growth will likely worsen the fiscal situation in advanced economies.

At high debt levels, when fiscal authorities have little margin to adjust taxes sufficiently, debt becomes an important constraint for monetary policy. How should policy be designed under such circumstances? A sizable literature adopting the Ramsey approach to optimal policy has relied on *non-linear models* to draw conclusions about the processes of inflation and interest rates when the "planner" takes into account debt sustainability. Schmitt-Grohé and Uribe (2004), Faraglia et al. (2013), Lustig et al. (2008), Eggertsson and Woodford (2006), Jung et al. (2005), Bhattarai et al. (2015) and Adam and Billi (2006, 2008) are well-known examples of this work. Because these models are non-linear and characterize optimal policies through first order conditions they are not easy to use to draw practical conclusions about the conduct of policy. It is, for instance, not straightforward to use the optimality conditions from these models to arrive at explicit interest rate rules that could guide policy responses to macroeconomic conditions or that can be made comparable to actual policies identified in (estimated) DSGE models.

On the DSGE literature side, a widely used framework suitable to analyze the interactions between monetary and fiscal policies is that of Leeper (1991). In this framework, monetary policy is summarized in simple transparent interest rate rules (see also Bianchi and Ilut (2017), Bianchi and Melosi (2017, 2019) for recent extensions to medium scale DSGE models). However, the policy rules employed are ad hoc, so monetary policy is not designed to be optimal.

In this paper we propose a tractable framework of Ramsey optimal monetary policy in the case where debt sustainability may be a constraint for the central bank. The model gives rise to interest rates rules in closed form, which enables us to develop analytical insights and draw sharp conclusions about the conduct of policy. Moreover, our theory can be easily embedded in a medium scale DSGE model and estimated with data.

Our framework is developed in Section 2 and assumes that a Ramsey planner (the Fed) under commitment sets allocations to minimize the deviations of inflation, output and interest rates from their respective target levels, subject to the standard set of dynamic equations that define the competitive equilibrium. In the baseline version of the Ramsey policy equilibrium, in which we assume that the Fed's policies are also driven by *debt sustainability concerns*, we assume that this set also includes the *consolidated budget constraint,* which determines the value of net debt in the hands of the private sector. Taxes are distortionary and, moreover, tax policy is assumed to be exogenous to the planner's problem; it follows a simple rule that determines the tax rate as a function of lagged debt, a standard assumption in the DSGE literature (e.g., Leeper (1991)).

Our first analytical result derives the interest rate rule that emerges from optimal policy in this model. We show that interest rates follow a standard Taylor rule (respond to inflation, output growth, etc.) but also the central bank engages in "forward guidance". In our model, forward guidance is expressed as the sum of two components. The first component represents commitment to keep interest rates low at the exit from a liquidity trap (LT) episode (e.g., Eggertsson and Woodford (2003, 2006)) and is captured by the lags of the Lagrange multiplier attached to the occasionally binding zero lower bound (ZLB) constraint. The second component measures the impact of past shocks to the consolidated budget constraint, capturing the planner's commitment to "twist interest rates" in order to satisfy the intertemporal budget (e.g., Lustig, Sleet and Yeltekin (2008), Faraglia, Marcet, Oikonomou and Scott (2016), henceforth FMOS). Since debt is distortionary, ours is a model of optimal policy under incomplete markets (see e.g., Aiyagari et al. (2002), FMOS) and the Lagrange multiplier on the consolidated budget constraint is a state variable. Interest rate twisting is captured by the lags of this multiplier.

As an alternative to this benchmark model we consider a Ramsey policy where the consolidated budget does not enter into the constraint set. In this case, forward guidance emerges only from the impact of the occasionally binding ZLB and the model does not feature any interest rate twisting effects from shocks to the intertemporal budget. By switching on and off the consolidated budget from the Ramsey program in this way, we are able to contrast the properties of equilibria where monetary policy is concerned with debt sustainability with equilibria where monetary policy has no monetary policy concerns.

In Section 3 we study the properties of these two versions of our model in the neighborhood of the steady state – that is, assuming that the zero lower bound is non-binding. We first focus on the monetary/fiscal interactions and establish a link with Leeper's (1991) famous analysis. We show analytically that in the "debt concerns" model the rational expectations equilibrium is (locally) unique if taxes do not respond strongly to the deviations of government debt from its steady state level and debt becomes an explosive process. In contrast, in the case of "no debt concerns", when the consolidated budget does not enter in optimization, taxes need to strongly adjust to the debt level and debt becomes a mean reverting process. In the widely used terminology of Leeper (1991), determinacy under debt concerns requires an "active" fiscal policy whereas under no debt concerns fiscal policy is "passive".

Leeper's classification of monetary policy into active/passive hinges on the response of interest rates to inflation, so it is not always straightforward to map into our model, especially in the presence of interest rate twists. However, using a mixture of analytical results and simulations, we show that the policy rule under debt concerns is equivalent to a standard "passive money" rule, exhibiting anemic responses of interest rates to inflation. The "no debt concerns" policy is equivalent to an active monetary policy. This finding, which shows that Leeper's analysis can be made consistent with an optimal Ramsey policy framework, is to our knowledge new to the literature.

More importantly, this finding enables us to easily apprehend our optimal policy model's properties. A large body of work has been devoted to solving and estimating "passive money" models, and a standard feature of these models is that they magnify the impact of shocks on interest rates, output and inflation; the macroeconomic volatility predicted by these models tends to be higher.[1] This is precisely what happens under debt concerns: The monetary authority (partially) gives up on the goal of stabilizing interest rates, output and inflation to satisfy the intertemporal budget; this leads to higher volatility. In contrast, volatility is much lower under no debt concerns, where monetary policy focuses on stabilizing macroeconomic variables.

In Section 4 of our paper we turn to the properties of optimal policies in the liquidity trap. Our key finding is that, under debt concerns, monetary policy is ineffective in stabilizing output and inflation, and forward guidance, promising to keep interest rates low for a long period, does not stabilize inflation at the onset of the episode, and may even lead to a sharp deflation. In contrast, under no debt concerns monetary policy is effective, and promising lower future rates increases inflation.

What explains the ineffectiveness of forward guidance under debt concerns? It is now well known that under passive monetary policy an exogenous drop in interest rates (eventually) makes inflation turn negative; this is what Sims (2011) and Cochrane (2018) refer to as "stepping on a rake". Our debt concerns model has this property, and as we show, promising to lower the interest rate impinges a drop in the price level.

In Section 5 we embed our optimal policy framework into a medium scale DSGE model. We do so for robustness, to show that our results carry through in a model that provides a more accurate image of the US economy. Our quantitative model extends the baseline with preferences exhibiting habit formation, shocks to TFP, markup shocks, government transfers, realistic fiscal rules, etc. The model has a rich enough structure to match the US data; it is broadly similar to the models of Bianchi and Ilut (2017) and Bianchi and Melosi (2017). We estimate the quantitative model with standard Bayesian techniques. Our key results continue to hold: Debt concerns magnify marcoeconomic volatility, and when the economy experiences a large negative demand shock that drives interest rates to the ZLB the model predicts a worse tradeoff than under no debt concerns.

In Section 6 we present several extensions of our framework. In particular, we explore the robustness of our findings towards introducing lump sum taxation and varying the maturity of debt and also compare our results to the benchmark Ramsey equilibrium, which assumes coordinated monetary and fiscal policies.

[1]See e.g., Bianchi and Melosi (2017) and Bianchi and Ilut (2017).

More substantively, we show that our framework can be extended to allow for regime fluctuations, as recent literature has assumed (e.g., Bianchi and Ilut (2017), Bianchi and Melosi (2017)). The online appendix extends this analysis further.

Finally, Section 7 concludes.

2 Theoretical Framework

We begin with a simple setup of the Ramsey policy equilibrium, which allows us to illuminate key forces driving interest rates in our model. We consider two alternative specifications of the baseline model. In the first version of Ramsey policy we let the planner choose interest rates, inflation and output, subject to the consolidated budget constraint (of the Fed and the government). We refer to this model as the "debt concerns" (DC) model. The aim is to capture a scenario under which the planner (Fed) takes into account the sustainability of debt. We show analytically that in this case the optimal interest rate rule has three components: First, a standard Taylor rule component, which links interest rates to realized inflation, output growth and lagged values of interest rates. Second, a component that represents commitment to keep interest rates low at the exit from a liquidity trap episode (e.g., Eggertsson and Woodford (2003, 2006)). Third, a component which measures the impact of past promises made by the planner to alter interest rates in the face of shocks to the consolidated intertermporal budget constraint. The third component is an "interest twisting effect" that emerges because debt is distortionary and long term as in FMOS.

In the second setup of policy, in which the planner has "no debt concerns" (NDC), she does not account for the consolidated budget constraint in optimization. The optimal policy rule in this case is identical to debt concerns, in terms of the first two components, but features no interest rate twisting impacts.

Throughout this section we focus on monetary policy, assuming that fiscal policy is a simple tax rule of the form:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\phi^R_{\tau,b}\hat{b}_{t-1,\delta} + \epsilon_{\tau,t} \tag{2.1}$$

where $\hat{\tau}_t$ denotes the tax rate and $\hat{b}_{t-1,\delta}$ is debt (both in deviation from steady state values). $\epsilon_{\tau,t}$ is a shock to the tax rate. (2.1) is a standard rule linking taxes to lagged taxes and debt (e.g., Leeper (1991)). Coefficient $\phi^R_{\tau,b}$ measures the feedback effect of debt issued in $t-1$ on the tax rate in t. Index $R \in \{DC, NDC\}$ is used to denote that taxes will follow a different process under DC and NDC. We leave it to Section 3, where we examine the interactions between monetary and fiscal policies, to discuss what types of fiscal policies justify taking into account the consolidated budget in optimization and what types do not.

2.1 The Baseline Model

The building blocks of our model are a standard NK Phillips curve; an IS (Euler) equation, which prices a short nominal bond; and the consolidated budget constraint, which determines the value of (net) debt held by the private sector. The model is a simplified version of the New-Keynesian (NK) model that we will later employ in estimation and which we formally describe in Section 5. For brevity, we summarize here the competitive equilibrium in the linearized version of the model. For detailed derivations we refer the reader to the online appendix.

Let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The competitive equilibrium equations are as follows:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \tag{2.2}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})} > 0$, $\kappa_3 \equiv -\frac{(1+\eta}{\theta\overline{Y}}\sigma\frac{\overline{G}}{\overline{C}} > 0$,

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\overline{G}}{\overline{C}}(\hat{G}_t - \hat{G}_{t+1})\right]\right) \tag{2.3}$$

$$\hat{i}_t \geq -\frac{1}{\beta} + 1 \equiv -i^* \tag{2.4}$$

$$\begin{aligned}
&\frac{\beta\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{b}_\delta \sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\right] \\
&+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\left(\hat{G}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + \hat{\xi}_t\right) + \overline{b}_\delta\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) - \overline{b}_\delta\hat{\xi}_t \\
&= \frac{\overline{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)
\end{aligned} \tag{2.5}$$

(2.2) is the Phillips curve at the heart of our model. $\hat{Y}_t$ is the output gap, and $\hat{\tau}$ denotes a distortionary tax levied on the labor income of households. $\hat{G}_t$ denotes government spending in t. Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across differentiated products and the degree of price stickiness, respectively.[2]

(2.3) is the log-linear Euler equation, which prices a short-term nominal asset. $\hat{\xi}$ is a standard preference shock that affects the relative valuation of current vs. future utility by the household. A drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption. (2.4) is the ZLB constraint on the short-term nominal interest rate.

(2.5) is the consolidated budget constraint. We assume that debt is issued in a perpetuity bond with

[2] We assume price adjustment costs as in Rotemberg (1982). θ governs the magnitude of these costs. When θ equals zero, prices are fully flexible.

decaying coupons where δ denotes the decay factor. Short debt is in zero net supply. Finally, parameter σ denotes the inverse of the intertemporal elasticity of substitution, and γ_h is the inverse of the Frisch elasticity of labor supply. Taxes are set according to rule (2.1).

2.2 Ramsey Policy under Debt Concerns

The Ramsey policy chooses inflation, output and interest rate sequences to maximize the following objective:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_0\left\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\right\} \tag{2.6}$$

subject to equations (2.2) to (2.5). λ_Y and λ_i govern the relative weights attached to output gap and interest rate stabilization by the planner.

We further assume that tax policies cannot be controlled by the planner. This essentially means that rule (2.1) is taken as given, and therefore $\hat{\tau}_t$ is not a choice variable.[3] As is standard, we solve for optimal policies with a Lagrangian. Let $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, and $\psi_{i,t}$, $\psi_{ZLB,t}$ and $\psi_{gov,t}$ the analogous multipliers attached to the Euler equation, the ZLB constraint and the consolidated budget, respectively. The Lagrangian can be written as:

$$\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t\Bigg[-\frac{1}{2}\left\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\right\}\\
&+\psi_{gov,t}\Bigg(\frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta}+\beta\bar{b}_\delta\sum_{j=1}^{\infty}\beta^{j-1}\delta^{j-1}\Bigg[E_t\Bigg(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\Bigg)\Bigg]+\overline{S}\hat{S}_t-\frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta}\\
&-\bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^jE_t\Bigg[-\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\right)-\sum_{l=0}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\Bigg]\Bigg)+\psi_{\pi,t}\Big(\hat{\pi}_t-\kappa_1\hat{Y}_t-\kappa_2\hat{\tau}_t+\kappa_3\hat{G}_t-\beta E_t\hat{\pi}_{t+1}\Big)\\
&\quad+\psi_{i,t}\Bigg(\hat{i}_t-E_t\Bigg(\hat{\pi}_{t+1}-\hat{\xi}_{t+1}+\hat{\xi}_t-\sigma\left[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t-\hat{Y}_{t+1})-\frac{\overline{G}}{\overline{C}}(\hat{G}_t-\hat{G}_{t+1})\right]\Bigg)\Bigg)+\psi_{i,t}\Big(\hat{i}_t+i^*\Big)\Bigg]
\end{aligned} \tag{2.7}$$

where $\overline{S}\hat{S}_t\equiv\left[-\overline{G}\left(\hat{G}_t(1+\sigma\frac{\overline{G}}{\overline{C}})-\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t+\hat{\xi}_t\right)+\frac{\bar{\tau}\overline{Y}(1+\eta)}{\eta}\left((1+\gamma_h)\hat{Y}_t+\frac{\hat{\tau}_t}{1-\bar{\tau}}+\hat{\xi}_t\right)\right]$ is the surplus of the government multiplied by marginal utility.

[3]We will also not allow the planner to influence $\hat{\tau}$ through the choice of debt. This is not a strict assumption as we will later show that under DC it has to be that $\phi_{\tau,b}^{DC}$ is a small number and we will even set it to zero in simulations. To simplify, we rule it out from the outset.

The first order conditions for the optimum are given by:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} + \frac{\overline{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{2.8}$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{\overline{C}}}(\psi_{i,t} - \frac{\psi_{i,t-1}}{\beta}) + \sigma\frac{\overline{Y}}{\overline{\overline{C}}}\overline{b}_\delta \sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \omega_Y \psi_{gov,t} = 0 \tag{2.9}$$

$$-\lambda_i \hat{i}_t + \psi_{i,t} + \psi_{ZLB,t} = 0 \tag{2.10}$$

$$\frac{\beta\overline{b}_\delta}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) = 0 \tag{2.11}$$

$$\psi_{ZLB,t} \geq 0 \quad \text{and} \quad (\hat{i}_t + i^*)\psi_{ZLB,t} = 0 \tag{2.12}$$

where $\omega_Y \equiv \overline{G}\sigma\frac{\overline{Y}}{\overline{C}} + \frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)$. (2.8) is the FONC with respect to $\hat{\pi}_t$; (2.9), (2.10), (2.11) are first order conditions with respect to $\hat{Y}_t$, $\hat{i}_t$ and $\hat{b}_{t,\delta}$, respectively. (2.12) is the complementary slackness condition for the ZLB constraint.

2.2.1 Optimal Interest Rate Policy

Combining the first order conditions (2.8), (2.9) and (2.10) and following the argument of Giannoni and Woodford (2003) we can derive analytically the optimal interest rate rule from the Ramsey program. The following proposition summarizes the result:

Proposition 1. *Assume that the central bank has "debt concerns". The optimal interest rate rule is:*

$$\hat{i}_t = \max\{\mathcal{T}_t + \mathcal{D}_t + \mathcal{Z}_t, -i^*\} \tag{2.13}$$

$$\mathcal{T}_t \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta\hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta}\Delta\hat{i}_{t-1}$$

with $\phi_\pi = \frac{\kappa_1 \overline{C}}{\lambda_i \sigma \overline{Y}}$, $\phi_i = (1 + \frac{\kappa_1}{\beta\sigma}\frac{\overline{C}}{\overline{Y}})$ and $\phi_Y = \frac{\lambda_Y \overline{C}}{\sigma\lambda_i \overline{Y}}$,

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{\overline{Y}}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\overline{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} - \frac{\overline{b}_\delta}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\right) - \frac{\overline{C}\omega_Y}{\overline{\overline{Y}}\sigma\lambda_i}\Delta\psi_{gov,t} \tag{2.14}$$

and

$$\mathcal{Z}_t = -\frac{1}{\lambda_i}(1 + \frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}})\psi_{ZLB,t-1} + \frac{1}{\beta\lambda_i}\psi_{ZLB,t-2}. \tag{2.15}$$

Proof See online appendix.

As equation (2.13) shows, there are three distinct components in the interest rate rule. The first, $\mathcal{T}_t$, is a standard Taylor rule component that links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on the interest rate policy rule depends on the weights λ_i and λ_Y, which capture the output and interest rate stabilization objectives of the planner. It also depends on the structural parameters σ, κ_1 and β.

The second component, $\mathcal{Z}_t$, is a pull factor that relates interest rates to the "bindness" of the ZLB constraint. If for example $\hat{i}_{t-1} = i^*$, then (from complementary slackness) we have $\psi_{ZLB,t-1} > 0$. In this case $\mathcal{Z}_t$ becomes negative and so the planner keeps the interest rate in t lower than the value implied by the Taylor rule component (and partially reverses the effect in $t+1$). The impact of this channel on the level of interest rates depends on λ_i, with higher values of λ_i leading to a lower impact, since the planner's objective to minimize the deviation of the interest rate from its target becomes stronger.

Finally, the term $\mathcal{D}_t$ measures the impact of changes in the value of the multiplier $\psi_{gov,t}$ on the optimal interest rate path. To analyze this term, use equation (2.11). We have:

$$\psi_{gov,t+1} = \psi_{gov,t} + \epsilon_{t+1,G} \tag{2.16}$$

In other words, the multiplier $\psi_{gov,t}$ is a random walk, and $\epsilon_{t+1,G}$ denotes a mean zero, i.i.d shock to the value of the multiplier. We can now write (2.14) as:

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{\overline{Y}}} \frac{\kappa_1}{\lambda_i \sigma} \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \epsilon_{t-l,G} - \frac{\bar{b}_\delta}{\lambda_i} \sum_{l=0}^{\infty} \delta^l \left(\epsilon_{t-l,G} - \epsilon_{t-l-1,G} \right) - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i} \epsilon_{t,G} \tag{2.17}$$

which relates $\mathcal{D}_t$ to current and past shocks to the value of the multiplier.

What do these shocks capture? Notice that since (real) debt can either be financed through distortionary taxes or through distortionary inflation, ours is a standard model of optimal policy under incomplete markets (e.g., Aiyagari et al. (2002), Schmitt-Grohé and Uribe (2004), Lustig et al. (2008), Faraglia et al. (2013), FMOS among others). As is well known, in these models shocks to the economy translate to changes in the excess burden of distortions and the multiplier ψ_{gov} that measures the magnitude of these distortions behaves like a random walk, since the planner wants to spread evenly the costs across periods. In the presence of long-term debt ($\delta > 0$), the sequence of shocks $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ influences interest rates because all the lags of these variables enter into the state vector, as the FONC reveal.

To clarify further the role played by the sequence $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ we iterate forward on constraint (2.5) to get:

$$E_t \sum_{j=0}^{\infty} \beta^j \overline{S} \hat{S}_{t+j} = \frac{\overline{b}_\delta}{1-\beta\delta} \hat{b}_{t-1,\delta} + \overline{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j E_t \left[-\sigma \left(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j} \right) - \sum_{l=0}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right] \tag{2.18}$$

(2.18) is the *intertemporal consolidated budget constraint* linking the present discounted value of the fiscal surplus to the real value of debt outstanding in t. Notice also that (2.18) is equivalent to (2.5) in terms of the Ramsey policy.[4] Consider the impact of a shock that lowers the LHS of (2.18) relative to the RHS. This may, for example, occur following a shock that lowers taxes. In response to this shock the constraint tightens and the value of the multiplier ψ_{gov} increases. Therefore, $\epsilon_{t,G} > 0$. To satisfy the constraint, the monetary authority needs to engineer a drop in the real payout of debt (the RHS of (2.18)) either through increasing future inflation and/or increasing future output when $\sigma > 0$. Note that under commitment it is feasible to make such promises about the future course of economic variables. The terms that enter in $\mathcal{D}_t$ in equation (2.17) are essentially the promises made by the planner to manipulate inflation and output, and hence also manipulate the interest rate, in response to shocks to the consolidated budget that have occurred in the past. Following FMOS, we label this impact an "interest rate twisting" effect of optimal policy.

2.3 Assuming No Debt Concerns

We now consider an alternative setup in which the planner maximizes (2.6) subject to (2.2), (2.3) and (2.4) and leaves the consolidated budget (2.5) outside the optimal policy program. (2.5) continues to hold, and as we shall show later, it will be satisfied in equilibrium given optimal policies and the tax rule (2.1) (more on this below). In this version of the model the multiplier $\psi_{gov,t}$ is obviously dropped from the list of model variables. The Lagrangian for this program is otherwise similar to (2.7), and for the sake of brevity we omit it.

Letting superscript NDC denote the equilibrium under "no debt concerns", the first order conditions[5] become:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta} = 0$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}^{NDC} \kappa_1 + \sigma \frac{\overline{Y}}{\overline{C}} (\psi_{i,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta}) = 0$$

$$-\lambda_i \hat{i}_t + \psi_{i,t}^{NDC} + \psi_{ZLB,t}^{NDC} = 0$$

The above equations give rise to the following interest rate rule:

Proposition 2. *Assume that the planner does not account for the consolidated budget in optimization. The*

[4] See, for example, Aiyagari et al. (2002).

[5] See online appendix for a more detailed description of the planner's program.

optimal interest rate rule is given by

$$\hat{i}_t = \max\{\mathcal{T}_t^{NDC} + \mathcal{Z}_t^{NDC}, -i^*\} \tag{2.19}$$

$$\mathcal{T}_t^{NDC} \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta} \Delta \hat{i}_{t-1}$$

and

$$\mathcal{Z}_t^{NDC} = -\frac{1}{\lambda_i}(1 + \frac{1}{\beta} + \frac{\kappa_1 \overline{C}}{\sigma \beta \overline{Y}}) \psi_{ZLB,t-1}^{NDC} + \frac{1}{\beta \lambda_i} \psi_{ZLB,t-2}^{NDC}.$$

Proof. See online appendix.

Several comments are in order. First, notice that the expressions for components $\mathcal{T}_t^{NDC}$ and $\mathcal{Z}_t^{NDC}$ in (2.19) are essentially the same as the expressions for $\mathcal{T}_t$ and $\mathcal{Z}_t$ in the optimal policy (2.13) of the DC model. The difference between the two policies is that (2.19) omits the term $\mathcal{D}_t$, which appears in the debt concerns version of optimal policy. This may seem to imply that the two policies are similar, at least in the case where the shocks to the consolidated budget are not large, so that $\mathcal{D}_t$ is close to zero. In Section 3 we will show that this is not the case. Even in the absence of (large) shocks, the monetary policy under DC has considerably different properties and effects than the policy under NDC.

Second, as we have seen, both $\mathcal{Z}_t$ ($\mathcal{Z}_t^{NDC}$) and $\mathcal{D}_t$ are functions of lagged state variables (ψ_{ZLB} and ψgov), and these lagged variables summarize the effects of shocks that have occurred in the past. Since their values have been revealed in past periods, and apply to policy in t, we interpret them as forward guidance in optimal monetary policy.

A recent literature using DSGE models to analyze the behavior of macroeconomic variables in the Great Recession has utilized monetary policy rules with forward guidance, similar to the rules in Propositions 1 and 2. In Laseen and Svensson (2011), Campbell et al. (2012), Campbell et al. (2016), Del Negro and Sims (2015), De Graeve et al. (2016) and others, interest rate policy is modelled in the following form:

$$\hat{i}_t^{DSGE} = \max\{\mathcal{T}_t^{DSGE} + \sum_{l=0}^{M} \epsilon_{t-l}^{l,DSGE}, -\hat{i}^*\} \tag{2.20}$$

where $M > 0$. $\mathcal{T}_t^{DSGE}$ is a Taylor rule, a function of inflation, output growth and lags of the interest rate. ϵ_t^0 is a standard monetary policy shock, and $\left(\epsilon_{t-1}^1, ..., \epsilon_{t-M}^M\right)$ are "forward guidance shocks". The policy rules in Propositions 1 and 2 are indeed similar to equation (2.20); the difference, however, is that the shocks $\left(\epsilon_{t-1}^1, ..., \epsilon_{t-M}^M\right)$ are exogenous, whereas the state variables that enter in $\mathcal{Z}_t$ ($\mathcal{Z}_t^{NDC}$) and $\mathcal{D}_t$ are endogenous functions of fundamental shocks to preferences, taxes, spending, etc. They are *endogenous forward guidance.*

[6]

2.4 Discussion

Thus far we have derived optimal interest rate rules in the two environments: Under debt concerns the planner takes into account the consolidated budget constraint, whereas under no debt concerns she does not have to satisfy the constraint. We now provide some more context behind our modelling assumptions.

First, the reader may be asking herself if in the NDC case the optimal program is not complete if the full structure of the macroeconomy in not taken into account. As was discussed previously, coefficient $\phi^R_{\tau,b}$ in the fiscal policy rule will be different in the two scenarios; we will later show (in Section 3) that DC corresponds to the case where fiscal policy does not generate sufficient surpluses to finance debt, and therefore monetary policy has to satisfy the consolidated budget constraint, and NDC corresponds to the case where fiscal surpluses are sufficient to satisfy the intertemporal budget. Thus, from the point of view of the planner, under NDC the consolidated budget constraint is slack. It then holds that $\psi_{gov} = 0$.

Next, one may think that if a central bank is concerned with stabilizing debt, then the real value of debt should appear in the objective of the planner rather than as part of the constraint set. We claim that these alternatives yield similar policy outcomes: If the central bank's loss function invoked a penalty in the case where the RHS of (2.5) is not equal to the LHS (i.e., the real value of debt is not compensated by the surpluses), then monetary policy will adjust inflation and output to satisfy the intertemporal budget just like in our DC program. The reason that we do not pursue such modelling here is that adding the intertemporal budget as a constraint seems to us a much more credible setup of policy. Cochrane (2001) solves a similar problem assuming that a Ramsey planner that minimizes the volatility of inflation has to satisfy the constraint. Thus our assumptions are in line with previous work in the literature. Moreover, we will later show that, in the case where fiscal policy does not generate surpluses to finance debt and when the planner fails to take into account the intertemporal budget, the rational expectations equilibrium is not unique and therefore solutions that lead to explosive paths of inflation or deflation cannot be ruled out. Therefore, a planner that is concerned about stabilizing inflation and output should also be concerned about satisfying the constraint.

The reader may also be asking whether objective (2.6) is meant to represent a second order approximation to the household's utility. Though our analysis below will extend to various calibrations of parameters λ_Y

[6] As discussed previously, several papers have studied optimal monetary policies using frameworks broadly similar to ours. See, for example, Eggertsson and Woodford (2003, 2006), Lustig et al. (2008) and Faraglia et al. (2013) among others. These papers also give rise to endogenous forward guidance. For example in Faraglia et al. (2013) and Lustig et al. (2008) the presence of long-term debt implies that an element such as $\mathcal{D}_t$ is part of optimal policy. Moreover, optimal policy in models with occasionally binding zero lower bound constraints should include a term of the form $\mathcal{Z}_t$.

However, the focus of these papers is mainly theoretical and, building on non-linear models, they do not derive simple interest rate rules comparable to the recent DSGE literature, as we do here. Moreover, since many of these models bring together monetary and fiscal policies under one authority, "forward guidance" is not only promises made to manipulate future inflation and interest rates, but also promises to manipulate taxes. This makes it difficult to evaluate which of the recommendations of these models are relevant for monetary policy and which are not.

and λ_i (including $\lambda_i = 0$, which would result from a second order approximation in a cashless economy like ours), we do not consider that the monetary authority's objective has to be identical to the objective of the household. If it were, then minimizing the volatility of distortionary taxes would also be part of the welfare criterion and our policy problem would look like a joint optimal monetary and fiscal policy problem (as, for example, in Schmitt-Grohé and Uribe (2004) and Siu (2004) and numerous others). We thus assume that the monetary authority has its own objective, which focuses on stabilizing inflation, output and the interest rate, and this loss function will not, in general, coincide with the loss function of the household. [7]

3 Monetary and Fiscal Policy Interactions

In this section we study the properties of the models in the neighborhood of the steady state, when the ZLB does not bind. We first ask, what types of fiscal rules are compatible with an equilibrium where the central bank has debt concerns, and what types of rules are compatible with no debt concerns? We establish a link between the DC/NDC models and Leeper's (1991) seminal analysis on monetary/fiscal policy interactions. Our key result is that in the case of debt concerns, monetary policy becomes subservient to fiscal policy and behaves similarly to the passive monetary policy model defined in Leeper's work. Fiscal policy is "active" and does not respond to government debt. The opposite holds under no debt concerns.

We then study the effects of disturbances on macroeconomic variables. The main finding of this analysis is that under debt concerns, macroeconomic volatility increases, and inflation, output and interest rates are more exposed to both demand and supply disturbances. This property is a standard feature of "passive money models". In our optimal policy framework under debt concerns, it arises because the planner needs to partially give up on the goal of stabilizing inflation, output and interest rates to satisfy the consolidated budget constraint.

3.1 Determinacy under Optimal Policies

3.1.1 Fiscal Policy

Consider the case where the ZLB constraint is non-binding. Since ours is a linear model approximated around a non-stochastic steady state with a positive nominal interest rate, equilibria with a non-binding ZLB constraint can occur if shocks are not too big to drive the economy far away from steady state. We have: $\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t$ and $\hat{i}_t^{NDC} = \mathcal{T}_t^{NDC}$. To derive analytical results we first consider a simplistic setup letting $\lambda_Y = \sigma = \delta = \rho_\mathcal{T} = \overline{G} = 0$. We further assume that tax shocks are the only source of uncertainty in the

[7] Analogously, implicit in rule (2.1) is the assumption that the fiscal authority aims to smooth taxes. The objective of the fiscal authority is, however, not explicitly modelled here.

economy.[8] Under these assumptions and using equations (2.9) and (2.10) to substitute out $\psi_{\pi,t}$ and $\psi_{i,t}$ in the debt concerns model we get:

$$-\hat{\pi}_t - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}_\delta \widetilde{\eta} \Delta\psi_{gov,t} = 0 \tag{3.1}$$

where $\widetilde{\eta} = \left(1 + \frac{(1-\beta)(1+\gamma_h)}{\kappa_1}\right)$.[9] $\psi_{gov,t}$ is again a martingale and so $E_t \Delta\psi_{gov,t+1} = 0$. From the Euler equation we have $\hat{i}_t = E_t \hat{\pi}_{t+1}$.

Using the Phillips curve to substitute $\hat{Y}_t = \frac{\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \kappa_2 \hat{\tau}_t}{\kappa_1}$, the consolidated budget constraint can be written as follows:

$$\beta \bar{b}_\delta \hat{b}_{t,\delta} - \beta \bar{b}_\delta \widetilde{\eta} E_t \hat{\pi}_{t+1} + (1-\beta)\bar{b}_\delta \left(\frac{1}{1-\overline{\tau}} - \frac{\kappa_2}{\kappa_1}(1+\gamma_h)\right)\hat{\tau}_t = \bar{b}_\delta \hat{b}_{t-1,\delta} - \bar{b}_\delta \widetilde{\eta} \hat{\pi}_t \tag{3.2}$$

Finally, using $\frac{\kappa_2}{\kappa_1} = \frac{\overline{\tau}}{(1-\overline{\tau})\gamma_h}$ and the tax rule (2.1) we have:

$$\hat{b}_{t,\delta} - \widetilde{\eta} E_t \hat{\pi}_{t+1} = \underbrace{\frac{1}{\beta}\left[1 - \frac{(1-\beta)\phi_{\tau,b}^{DC}}{(1-\overline{\tau})\gamma_h}\left(\gamma_h - \overline{\tau}(1+\gamma_h)\right)\right]}_{\widetilde{\epsilon}_{DC}} \hat{b}_{t-1,\delta} - \frac{\widetilde{\eta}}{\beta}\hat{\pi}_t - \underbrace{\frac{1}{\beta}\frac{(1-\beta)}{(1-\overline{\tau})\gamma_h}\left(\gamma_h - \overline{\tau}(1+\gamma_h)\right)}_{A} \epsilon_{\tau,t} \tag{3.3}$$

Coefficient $\widetilde{\epsilon}_{DC}$ is a key object in determining the dynamics of government debt. In the case where $\widetilde{\epsilon}_{DC} > 1$, government debt becomes an explosive process, whereas if $\widetilde{\epsilon}_{DC} < 1$, debt is a stationary process. Notice that the magnitude of $\widetilde{\epsilon}_{DC}$ hinges on the size of the feedback effect of lagged debt on taxes, $\phi_{\tau,b}^{DC}$, and also on the steady state level of taxes and the Frisch elasticity $\frac{1}{\gamma_h}$, since these quantities influence the responsiveness of aggregate hours and tax revenues to shocks when taxes are distortionary.

To identify which values of $\widetilde{\epsilon}_{DC}$ give rise to a unique rational expectations equilibrium, use the Euler equation together with (3.1) to write

$$\frac{\lambda_i \hat{i}_t}{\beta} = -E_t \hat{\pi}_{t+1} + \bar{b}_\delta \widetilde{\eta} E_t \Delta\psi_{gov,t+1} = -\hat{i}_t, \tag{3.4}$$

where the last equality makes use of the martingale property of $\psi_{gov,t+1}$. From (3.4) we get $\hat{i}_t = 0$ when $\lambda_i \geq 0$. Therefore, we have $\hat{\pi}_t = \bar{b}_\delta \widetilde{\eta} \Delta\psi_{gov,t}$ and with this we can write (3.3) as

$$\hat{b}_{t,\delta} = \widetilde{\epsilon}_{DC} \hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta \widetilde{\eta}^2}{\beta} \Delta\psi_{gov,t} - A\epsilon_{\tau,t} \tag{3.5}$$

[8] Our findings in this section do not hinge on the nature of shocks that hit the economy. We assume only tax shocks and set $\overline{G} = 0$ to simplify the algebra; otherwise we could assume more than one disturbance in the model and all results described below would carry through.

[9] Under $\sigma = \overline{G} = 0$ we have $\omega_Y = \frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)$. From the budget constraint we get $\frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y} = (1-\beta)\bar{b}_\delta$.

Equation (3.5) together with the martingale property, $E_t\Delta\psi_{gov,t+1} = 0$, form the system of equations that needs to be resolved to find the values of $\hat{b}_{t,\delta}$ and $\Delta\psi_{gov,t}$.

It is now easy to show that the solution to this system is unique only when $\widetilde{\epsilon}_{DC} > 1$. To see this, assume that $\widetilde{\epsilon}_{DC}$ is less than one and solve equation (3.5) backwards to obtain:

$$\hat{b}_{t,\delta} = -\sum_{j=0}^{\infty} \widetilde{\epsilon}_{DC}^{j} \left(\frac{\overline{b}_\delta \widetilde{\eta}^2}{\beta} \Delta\psi_{gov,t-j} + A\epsilon_{\tau,t-j} \right) \tag{3.6}$$

which determines the debt level at t as a function of the lagged shocks $\Delta\psi_{gov,t-j}$ and $\epsilon_{\tau,t-j}$. Notice that the value of $\Delta\psi_{gov,t-j}$ is not pinned down in this model; the martingale property $E_{t-j-1}\Delta\psi_{gov,t-j} = 0$ does not determine a unique value for this object.

In the unique rational expectations equilibrium, $\widetilde{\epsilon}_{DC} > 1$, and (3.5) is solved forward to give

$$\hat{b}_{t-1,\delta} = E_t \sum_{j=0}^{\infty} \frac{1}{\widetilde{\epsilon}_{DC}^{j+1}} \left(\frac{\overline{b}_\delta \widetilde{\eta}^2}{\beta} \Delta\psi_{gov,t+j} + A\epsilon_{\tau,t+j} \right) = \frac{1}{\widetilde{\epsilon}_{DC}} \left(\frac{\overline{b}_\delta \widetilde{\eta}^2}{\beta} \Delta\psi_{gov,t} + A\epsilon_{\tau,t} \right) \tag{3.7}$$

From (3.7) the equilibrium satisfies $\Delta\psi_{gov,t} = -\frac{\beta}{\overline{b}_\delta \widetilde{\eta}^2} A\epsilon_{\tau,t}$ and therefore $\hat{b}_{t,\delta} = 0$ for all t.[10]

Now consider the case of "no debt concerns". We have $\psi_{gov,t} = 0$ for all t, and the consolidated budget constraint can be written as:

$$\hat{b}_{t,\delta}^{NDC} = \widetilde{\epsilon}_{NDC} \hat{b}_{t-1,\delta}^{NDC} - A\epsilon_{\tau,t} \tag{3.8}$$

(where $\widetilde{\epsilon}_{NDC}$ is essentially given by the same expression as $\widetilde{\epsilon}_{DC}$, but $\phi_{\tau,b}^{DC}$ is replaced with $\phi_{\tau,b}^{NDC}$). A unique equilibrium can be found when $\widetilde{\epsilon}_{NDC} < 1$ so that (3.8) is solved backwards. We summarize the above findings in the following proposition:

Proposition 3. *Assume that parameters λ_Y, $\overline{G}$, δ and σ equal zero. Define*

$$\widetilde{\epsilon}_R \equiv \frac{1}{\beta} \left[1 - \frac{(1-\beta)\phi_{\tau,b}^R}{(1-\overline{\tau})\gamma_h} \left(\gamma_h - \overline{\tau}(1+\gamma_h) \right) \right]$$

for $R \in \{DC, NDC\}$. Assume that the planner takes into account the consolidated budget constraint. Determinacy of the equilibrium requires $\widetilde{\epsilon}_{DC} > 1$. In contrast, in the "no debt concerns" case determinacy requires $\widetilde{\epsilon}_{NDC} < 1$.

The above condition extends the analysis of Leeper (1991) to the case of distortionary taxes and in a

[10]In other words, debt does not explode if it equals zero for all t.

Notice that if this fails we have: $\Delta\psi_{gov,t} = \frac{\beta}{\overline{b}_\delta \widetilde{\eta}^2} \left(\widetilde{\epsilon}_{DC} \hat{b}_{t-1,\delta} - A\epsilon_{\tau,t} \right)$ and therefore, $E_{t-1}\Delta\psi_{gov,t} = \frac{\beta}{\overline{b}_\delta \widetilde{\eta}^2} \widetilde{\epsilon} DC \hat{b}_{t-1,\delta} \neq 0$. Thus $\hat{b}_{t,\delta} = 0$ is the only path consistent with the random walk.

model where monetary policy is optimal. When $\widetilde{\epsilon}_R < 1$ (i.e. in the no debt concerns model), fiscal policy is sufficiently responsive to debt levels, so taxes adjust to guarantee the sustainability of debt. In the sense of Leeper (1991), fiscal policy is "passive". In contrast, in the model with debt concerns, fiscal policy needs to be "active" for the rational expectations equilibrium to be unique or, to put it differently, taxes should not respond aggressively to deviations of debt from its steady state value.

The above findings – that determinacy requires an explosive debt process in the debt concerns model and a mean reverting process under no debt concerns – do not hinge on the assumptions made in Proposition 3. They hold more generally for positive values of λ_Y, σ, δ and $\overline{G}$ and, therefore, also hold when monetary policy follows the rules derived in Propositions 1 and 2.

3.1.2 Monetary Policy: An Example Affirming Leeper

What do these optimal interest rate rules tell us about whether monetary policy is active/passive? Leeper's classification hinges on the response of interest rates to inflation. When interest rates respond strongly to inflation, monetary policy is active, and conversely when they do not respond strongly, it is passive. Admittedly, Leeper's analysis is not easy to map into our optimal policy framework: Even if we can show that $\mathcal{T}^{NDC}$ defines an active monetary policy, it is not obvious how we could then show that $\hat{i}_t = \mathcal{T} + \mathcal{D}$ defines a passive policy, especially when $\mathcal{T}^{NDC} = \mathcal{T}$, as was previously illustrated. If anything, since $\mathcal{D}$ is a moving average of mean zero innovations to the Lagrange multiplier it would seem that interest rates in the debt concerns model will on average be equal to $\mathcal{T}$ and therefore also equal to $\mathcal{T}^{NDC}$.

This is, however, not the case. The innovations in $\mathcal{D}$ are not orthogonal to inflation; they are functions of the same fundamental shocks (in preferences, spending, taxes, etc.) that drive inflation, so they are correlated with inflation (and with the remaining variables in $\mathcal{T}$). In principle we can express $\mathcal{D}$ as a function of inflation, lagged values of interest rates, etc. We can thus map the optimal policy into Leeper's analysis.

In the next subsection we will use the numerical solution of the model to approximate $\mathcal{D}$ as a function of the variables in $\mathcal{T}$. To derive an analytical solution we assume $\lambda_Y = \sigma$ as in the previous paragraph, however now let $\delta \geq 0$ and $\lambda_i = 0$. Note that under these assumptions we cannot derive an optimal policy rule directly from the first order conditions; however, we can find a Taylor rule that implements the allocation under optimal policy.

Assume that the policy rule is of the form:

$$\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t \tag{3.9}$$

Consider first the no debt concerns case; we can show that in the presence of only tax shocks in this model, we will have $\hat{i}_t = \hat{\pi}_t = 0$.[11] Combining (3.9) with the Euler equation $\hat{i}_t = E_t\hat{\pi}_{t+1}$ we get the following difference equation in inflation:

$$\hat{\pi}_t\widetilde{\phi}_\pi = E_t\hat{\pi}_{t+1}$$

Standard results yield that the equilibrium is unique if and only if $\widetilde{\phi}_\pi > 1$.

Now consider the case of debt concerns. The equilibrium under optimal policy satisfies:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \qquad \hat{i}_t = \frac{\bar{b}_\delta\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \tag{3.10}$$

Assuming that parameter values are such that $\frac{\omega_Y}{\kappa_1} \approx 0$ and considering a rule of the form (3.9) that can implement (3.10) we have:

$$\widetilde{\phi}_\pi\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = \delta\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \tag{3.11}$$

Hence $\widetilde{\phi}_\pi = \delta < 1$.

This simple example shows that whereas the no debt concerns model requires a coefficient $\widetilde{\phi}_\pi$ that exceeds unity (the standard condition for "active" monetary policy), the debt concerns model features "passive" monetary policy under an interest rate rule of the form (3.9).

We summarize the above in the following proposition:

Proposition 4. *Assume $\lambda_Y = \sigma = \lambda_i = 0$. The optimal policy is a rule of the form (3.9). In the case of no debt concerns $\widetilde{\phi}_\pi > 1$ and monetary policy is "active". Under debt concerns, $\widetilde{\phi}_\pi = \delta < 1$ and monetary policy is "passive".*

3.1.3 Monetary Policy: Numerical Examples

For more plausible calibrations of the model the above properties can be demonstrated using numerical simulations. In Table 1 we show the coefficients we obtain from Taylor rule approximations of optimal policy in the debt concerns model under various model specifications.[12] More precisely, we approximate the optimal policies using rules of the form:

$$\hat{i}_t = \widetilde{\phi}_\pi\hat{\pi}_t + \widetilde{\phi}_Y\hat{Y}_t + \widetilde{\phi}_i\hat{i}_{t-1} + \widetilde{\phi}_{\Delta i}\Delta\hat{i}_{t-1} \tag{3.12}$$

[11] See below in subsection 3.2 where we look at the effect of tax shocks.

[12] The calibration of model parameters $\eta, \theta, \ldots$ follows Table 2 (see subsection 3.2). However, we consider here a wider range of values for $\sigma, \lambda_i, \lambda_Y$ than what is reported in this table. The values we consider are reported in the text.

which do not include object $\mathcal{D}_t$.[13] The coefficients $\widetilde{\phi}$ are such that the model with rule (3.12) produces impulse responses to shocks close to the analogous objects in the optimal policy model.[14]

In the top panel of Table 1 we assume $\lambda_Y = 0$ and thus set $\widetilde{\phi}_Y = 0$ to isolate our focus on coefficient $\widetilde{\phi}_\pi$. Each of the columns of the table corresponds to a different calibration of parameters λ_i and σ. Consider the first of these columns, which sets $\lambda_i = 0.5$ and $\sigma = 1$. For each parameter ($\widetilde{\phi}_\pi$, $\widetilde{\phi}_i$, etc.) we report two sets of numbers: The top numbers correspond to the estimates of (3.12). The bottom numbers (reported in parenthesis) correspond to the coefficients in $\mathcal{T}$ derived in Proposition 1. Notice that the estimates of (3.12) yield a much weaker response of interest rates to inflation ($\widetilde{\phi}_\pi = 0.104$ vs. 0.537 in the analytic solution) and also the coefficients on $\hat{i}_{t-1}$ and $\Delta\hat{i}_{t-1}$ are much lower than their analytic solution counterparts. (3.12) is basically a "passive money" rule.

The same pattern emerges in the remaining model specifications. For each of the calibrations considered in columns 2 to 4, the estimates of $\widetilde{\phi}_\pi$, $\widetilde{\phi}_i$ and $\widetilde{\phi}_{\Delta i}$ are low. The bottom panel of the table considers the case $\lambda_Y > 0$ and thus also reports the estimates of coefficient $\widetilde{\phi}_Y$. The response to inflation in (3.12) continues to be weaker than in the exact solution of the model. The estimated output coefficient $\widetilde{\phi}_Y$ is also weaker.

The above patterns hold for many alternative calibrations of the model, which for brevity we leave outside the table.

[Table 1 About Here]

3.2 The Effects of Shocks

We have seen that in the debt concerns model fiscal policy is "active" and debt becomes a non-stationary process. Monetary policy is subservient to fiscal policy and it can be represented (in reduced form) with a Taylor rule that has the standard features of passive money models. The opposite holds under no debt concerns: The policy mix becomes active monetary/passive fiscal.

These findings are crucial to understand how the model works and the effects of economic shocks, which we now study. A standard prediction of passive money models is that they magnify macroeconomic volatility. Shocks that hit the economy lead to larger fluctuations in inflation and output (see, for example, Bianchi and Ilut (2017)). In our optimal policy model this will also be the case. When optimization is subject to the consolidated budget, the planner needs to partially give up on the goal of stabilizing inflation, output and interest rates to satisfy the constraint.

[13] Equivalently we approximate $\mathcal{D}_t$ as a function of the RHS variables in (3.12).

[14] Computing coefficients through OLS regressions (that is, fitting (3.12) to simulated data from the debt concerns model) does not (generally) provide a good fit. It may also give us coefficients that are not consistent with a unique equilibrium. This pattern is consistent with the findings of Cochrane (2011) and Schmitt-Grohé and Uribe (2004).

Obviously, the impulse responses of the model under (3.12) do not perfectly match the responses under optimal policy. In the case where $\delta > 0$ we need several lags of interest rates (and also lags of inflation and output) to produce a perfect fit. We minimize the distance between the two models. We obtain a perfect fit only when we set $\delta = 0$.

To show this clearly, we first derive the impulse responses analytically in a simplified setup assuming, as previously, $\lambda_Y = \lambda_i = \sigma = 0$ but now let $\overline{G}, \rho_\tau > 0$. We consider shocks at date 0, which change the values of parameters $\{G_0, \xi_0, \tau_0\}$ assuming that after $t = 0$ there are no further shocks to the economy. Under these assumptions we derive in the online appendix the optimal paths of inflation and interest rates and the path of the multiplier $\psi_{gov,t}$.

The online appendix arrives at the following expressions for $\hat{\pi}_t$ and $\hat{i}_t$ under DC:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \frac{\overline{b}_\delta \delta^t}{1-\beta\delta}\Delta\psi_{gov,0} \tag{3.13}$$

$$\hat{i}_t = -\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0 + \frac{\overline{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0} \tag{3.14}$$

for $t = 0, 1, 2...$ and where $\mathcal{I}_{t=0}$ takes the value 1 at $t = 0$ and 0 otherwise. Moreover, $\Delta\psi_{gov,0}$ is a linear function of $\{\hat{G}_0, \hat{\xi}_0, \hat{\tau}_0\}$. We have:

$$\upsilon_{gov}\Delta\psi_{gov,0} = \upsilon_G \hat{G}_0 + \upsilon_\tau \hat{\tau}_0 + \upsilon_\xi \hat{\xi}_0 \tag{3.15}$$

where $\upsilon_{gov}, \upsilon_G, \upsilon_\xi > 0$ and $\upsilon_\tau < 0$ and therefore partial derivatives satisfy $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{G}_0} > 0$, $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{\tau}_0} < 0$ and $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{\xi}_0} > 0$.

We also show in the online appendix that in the no debt concerns model $\hat{i}_t = -\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0$ and $\hat{\pi}_t = 0$ for all t.

3.2.1 Preference Shocks

We consider first the impact of a preference shock, which lowers the value $\hat{\xi}_0$. The previous derivations show that in the no debt concerns equilibrium this shock has no effect on inflation. This result is standard: Since the planner does not want to smooth the nominal interest rate, i.e., $\lambda_i = 0$, $\hat{i}_t$ drops one for one with the real rate, thus fully stabilizing inflation.

Now consider the case of the "debt concerns" model. Since $\frac{\partial\Delta\psi_{gov,0}}{\partial\hat{\xi}_0} > 0$, a drop in $\hat{\xi}_0$ lowers $\Delta\psi_{gov,0}$. From (3.13) and (3.14) we see that inflation turns negative after the shock and the nominal interest rate drops below the real rate, $-\rho_\xi^t(\rho_\xi - 1)\hat{\xi}_0$.

What is going on? Notice that a preference shock has two impacts on the consolidated budget: First, it increases real bond prices and hence increases the real payout of government debt (the RHS of equation (2.18)); second, it increases the present value of surpluses (LHS of (2.18)) that finance the debt. Since the planner has to satisfy the intertemporal constraint she will either use inflation or use deflation to adjust the real payout of debt so that the constraint is satisfied with equality. If the first effect is stronger, and debt increases more than the surpluses, then inflation is optimal. In contrast, if the second effect dominates, then it is optimal to make inflation negative. In our analytic solution the second effect is stronger for all values $0 \leq \delta < 1$, so inflation becomes negative in response to the preference shock.[15]

Figure 1 shows the above responses over 40 periods.[16] The left column of the figure shows the case of the preference shock; the solid line represents the case of the debt concerns model and the dashed line shows no debt concerns. The top two panels show the responses of inflation and interest rates, and the bottom two panels show output and the market value of debt.

[Figure 1 About Here]

Equilibrium output under debt concerns is given by:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left(\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,0}\mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0\right)$$

(see the online appendix). Output drops in response to the preference shock because $\Delta\psi_{gov,0}$ turns negative. Under no debt concerns, output remains roughly constant after the shock.[17]

3.2.2 Fiscal Shocks

The middle and right columns of Figure 1 study the responses to a positive spending shock and a negative tax shock, respectively. In both cases the consolidated budget constraint tightens because the present value of the government's surplus falls, so $\Delta\psi_{gov,0} > 0$. From (3.13) and (3.14), interest rates rise in response to the shocks in the debt concerns model and inflation rises above its steady state value. Positive inflation ensures the solvency of the consolidated budget. In contrast, under no debt concerns inflation does not respond to these shocks because debt rises and taxes eventually adjust to make the surplus positive.

[15]When $\delta = 1$, long bonds are consols, and preference shocks have zero impact on the consolidated budget. To see this, simply use (2.18) assuming that, in equilibrium, under the preference shock $\hat{Y}_t = \hat{b}_{t,\delta} = \hat{\pi}_t = \hat{G}_t = \hat{\tau}_t = 0$ for all t. Then (2.18) becomes:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta} - \overline{G}\right)\hat{\xi}_{t+j} = \overline{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j E_t \hat{\xi}_{t+j}$$

Since $\frac{\overline{\tau}\overline{Y}(1+\eta)}{\eta} - \overline{G} = (1-\beta)\overline{b}_\delta$ the above equation is satisfied when $\delta = 1$.

This result, that preference shocks are neutral when long bonds are consols, is consistent with recent papers on optimal government debt management (e.g., Debortoli et al. (2017) and Bhandari et al. (2017)). See Section 6 where we briefly discuss optimal portfolios in our setup.

[16]The values we assign to the model's parameters are presented in Table 2. The top panel of the table reports values for parameters that are common across the two versions of the model, and the bottom panel reports the value that we assign to $\phi_{\tau,b}$ in each version separately. In the debt concerns model we set $\phi_{\tau,b} = 0.0$, and in the case where the Fed has no debt concerns we set $\phi_{\tau,b} = 0.07$. $\phi_{\tau,b} = 0.07$ is chosen so that debt has a near unit root, consistent with the empirical evidence on US government debt presented in Marcet and Scott (2009). The cutoff for determinacy is 0.065.

[17]In the NDC scenario output changes are driven by taxes. From the Phillips curve we get:

$$\hat{Y}_t^{NDC} = -\frac{\kappa_2}{\kappa_1}\hat{\tau}_t^{NDC} \tag{3.16}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\gamma_h$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})}$.

Under no debt concerns a negative shock in preferences makes taxes drop. Even though the market value to GDP initially increases (due to the increase in bond prices), fewer bonds need to be issued and so $\hat{b}_{t,\delta}$ is lower along the optimal path.

In the debt concerns case, taxes do not adjust to debt, since we have assumed $\phi_{\tau,b} = 0$. The real market value of debt increases due to the negative inflation.

[Table 2 About Here]

3.2.3 Responses under $\lambda_i, \lambda_Y, \sigma > 0$

In Figure 2 we assume $\lambda_i = \lambda_Y = 0.5$ and $\sigma = 1$. Now inflation and output change in response to the shocks in the no debt concerns model: Because the planner wants to smooth interest rates, following a preference shock, $\hat{i}_t$ does not drop one for one with the real rate to fully stabilize inflation; moreover, since the planner also wants to smooth output, she optimally adjusts inflation in order to partially offset fiscal shocks.

Contrasting the responses of the two models, it is clear that the qualitative patterns outlined previously remain. In the debt concerns model inflation becomes negative in response to a preference shock, and positive in response to spending and tax shocks. The (numerical) partial derivatives $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{G}_0}, \frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\tau}_0}, \frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\xi}_0}$ have the same sign as before. This again implies that interest rates drop more in the debt concerns model following a negative shock to preferences and they rise (relative to the no debt concerns model) in response to shocks to the fiscal variables.

[Figure 2 About Here]

The responses shown in Figures 1 and 2 reveal that shocks have a larger impact on macroeconomic variables under debt concerns than in the case of no debt concerns. As discussed previously, higher volatility is a standard prediction of models where monetary policy is passive. In our optimal policy model, higher volatility derives from the fact that the planner needs to partially give up on the objective to stabilize macroeconomic variables in order to satisfy the consolidated budget.

4 Optimal Policies at the ZLB

The previous section investigated the properties of optimal policy in the case where interest rates are not constrained by the ZLB. We now turn to the dynamics of the economy when interest rates hit the ZLB. As it is common in the literature, we consider a shock that lowers the value of $\hat{\xi}_0$ sufficiently so that the ZLB binds. Standard results imply that this shock puts downward pressure on prices and output, leading to deflation and to a recession during the LT. Monetary policy can then commit to keep interest rates low for a long period and stabilize the macroeconomy (e.g., see Eggertsson and Woodford (2003)). We show in this section that in the case of debt concerns, the above well-known result does not apply; forward guidance – committing to keep interest rates low – does not stabilize inflation during the liquidity trap and may even lead to deflation. We explain that this model property is due to what Sims (2011) and Cochrane (2018) label "stepping on a rake": In an equilibrium with passive monetary policy, lowering the nominal interest rate leads (eventually) to a drop in the price level.

In order to demonstrate transparently the effects of forward guidance (FG), we first use a model where FG is exogenous, as in the recent DSGE papers cited previously. This allows us to demonstrate how FG shocks impact inflation and output under passive money and active money rules, keeping the timing and duration of the shocks constant across the two regimes. We show that in the case of passive monetary policy, not only can inflation turn negative in response to a FG shock, but also FG is much less powerful than when policy is active.

Under optimal policy the timing and the duration of FG are endogenous and generally are different between the debt concerns and no debt concerns models. In the analytical example we construct, FG can last for much longer in the case of debt concerns, and the duration of FG hinges on the magnitude of the effect of the preference shock to the intertemporal budget. We show that in spite of keeping interest rates at zero for a long time, when monetary policy becomes subservient to fiscal policy, FG can lead to negative inflation.

Lastly, we contrast the properties of optimal policy with commitment with models where the monetary authority has no commitment. In the case of no commitment, passive monetary policy improves the tradeoff of the planner in the LT.

4.1 FG under Passive and Active Monetary Policies

We first study the effect of FG under passive/active policies in a model where monetary policy is not optimal. Consider an economy where the Phillips curve, the Euler equation and the budget constraint are given by (2.2), (2.3) and (2.5) respectively and fiscal policy follows (2.1). Also assume no preference shocks, for the moment.

Monetary policy follows an interest rate rule of the form

$$\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t + \mathcal{Z}_t \tag{4.1}$$

where $\mathcal{Z}_t$ is a standard (exogenous) FG shock, as in the DGSE literature discussed previously, which is revealed to private agents in period 0.

We study the impact of a shock in period $\bar{t}$, $\mathcal{Z}_{\bar{t}} < 0$, on equilibrium inflation, output and interest rates in the case where monetary policy is active ($\widetilde{\phi}_\pi > 1$) and in the case where it is passive ($\widetilde{\phi}_\pi < 1$).

4.1.1 "Stepping on a Rake": An Analytic Example

Consider first the case where $\widetilde{\phi}_\pi > 1$ and assume further that $\sigma = 0$. Combining (4.1) with the Euler equation and iterating forward on the resulting difference equation gives:

$$\hat{\pi}_t = -\sum_{j\geq 0} \left(\frac{1}{\widetilde{\phi}_\pi}\right)^{j+1} \mathcal{Z}_{t+j}. \tag{4.2}$$

According to (4.2) a shock $\mathcal{Z}_{\bar{t}} < 0$ lowering the nominal interest rate increases the value of inflation in periods $0, 1, ..., \bar{t}$. (The effect is zero after period $\bar{t}$.)

Consider now the case of passive policy, assuming wlog that $\widetilde{\phi}_\pi = \delta < 1$. Equilibrium inflation satisfies:

$$\hat{\pi}_t = \sum_{j\geq 0}^{t-1} \delta^j \mathcal{Z}_{t-j-1} + \delta^t \hat{\pi}_0. \tag{4.3}$$

which implies that a shock $\mathcal{Z}_{\bar{t}} < 0$ will lower the value of $\hat{\pi}_{\bar{t}-1}, \hat{\pi}_{\bar{t}}, \hat{\pi}_{\bar{t}+1}, \ldots$. Under passive policy, therefore, the FG shock seems to have the opposite impact: Promising to lower the interest rate reduces inflation, starting one period before the shock arrives.

What is going on? Though (4.2) fully characterizes the path of inflation in response to an anticipated FG shock in $\bar{t}$ (and this conforms with the common intuition that lower rates produce higher inflation), under passive policy (4.3) does not fully reveal how inflation will behave, because we do not have the initial condition $\hat{\pi}_0$. To fully characterize inflation we need to make use of the intertemporal consolidated budget constraint at $t = 0$.[18] Under the assumptions made in this section we can show that government surpluses are constant in the absence of shocks. The date zero constraint is:

$$-\frac{\bar{b}_\delta}{1-\beta\delta} \sum_{t\geq 0} (\beta\delta)^t \hat{\pi}_t \approx 0 \tag{4.4}$$

which basically says that the drop in inflation in $\bar{t} - 1$ and onwards needs to be compensated by a rise in inflation between periods 0 and $\bar{t} - 2$. When the monetary authority announces $\mathcal{Z}_{\bar{t}} < 0$, $\hat{\pi}_0$ increases to satisfy (4.4), and since $\hat{i}_0 = \delta\hat{\pi}_0$, the nominal interest rate will also increase. This will lead to a further increase in prices in period 1 and positive interest rates subsequently, until inflation switches sign in $\bar{t} - 1$.

This pattern is essentially the stepping on a rake of Sims (2011) and Cochrane (2018). When monetary policy is passive and debt is long term, a drop in the interest rate leads to a rise in inflation first (the conventional effect), only to reverse the sign of inflation after a few periods.

[18] Since we assume perfect foresight, the date 0 constraint is sufficient. The intertemporal constraints at $t \geq 1$ will be satisfied since $\hat{b}_{t\geq 0}$ can be chosen as a residual (see FMOS).

4.1.2 Effects of FG under $\sigma > 0$

The top graphs of panels a) and b) of Figure 3 demonstrate the responses of inflation and interest rates in the case where $\sigma = 1$. The solid lines correspond to the case where $\bar{t} = 4$. The dashed lines set $\bar{t} = 8$. The left plots show the behavior of interest rates, and the right plots trace the responses of inflation.

[Figure 3 About Here]

Notice that just like in our analytical example with $\sigma = 0$, in both models and under both timings of the shock inflation rises on impact when the shock is announced and remains above steady state value until the shock occurs. However, after period $\bar{t}$ inflation turns negative in the passive money model, whereas under active money, it returns to steady state.

We also see from the figure that interest rates rise between the announcement of the shock and date $\bar{t}$. Clearly, monetary policy attempts to ward off the inflationary effects of the shock. Under passive policy, the rise in interest rates is somewhat weaker and, eventually, when inflation turns negative, interest rates do so as well.

We now ask: What would be the impact of the shock if the planner were to eliminate the rise in interest rates between 0 and $\bar{t} - 1$? In the bottom graphs of panels a) and b) of Figure 3 the planner announces $\hat{i}_0, \hat{i}_1, .., \hat{i}_{\bar{t}-1} = 0$ along with $\mathcal{Z}_{\bar{t}} < 0$. Notice that now inflation rises considerably under active policy (bottom right of panel a)). Moreover, note that the initial rise in inflation is larger the higher $\bar{t}$ is. This is the so called "FG puzzle" (see for example Del Negro et al. (2015) and McKay et al. (2015)).

In contrast, under passive policy the rise in inflation is moderate, and when $\bar{t} = 8$ inflation rises (slightly) less initially than when $\bar{t} = 4$. Moreover, since inflation becomes negative after a few periods, the cumulative response of the price level to FG is weak. Overall, FG exerts a much weaker effect in the passive money model.[19]

To reiterate, we showed that FG in a passive money model has moderate effects. First, because of the stepping on a rake, inflation at some point turns negative following a promise to lower interest rates. Second, because the power of FG is limited, a longer horizon $\bar{t}$ does not imply a larger initial impact on inflation. These results are drawn from a model, where monetary policy is not optimal, which enabled us to vary exogenously the path of interest rates. We next study optimal policy where interest rates respond endogenously to a shock to preferences that drives the economy to a LT.

[19] A recent literature has explored alternative mechanisms to reduce the power of FG in New Keynesian models (see, for example, McKay et al. (2015) and references therein). Our result that the impact of FG under passive money is limited should be of interest.

4.2 Optimal FG Policies

4.2.1 An Analytical Example

We now turn to optimal policy to determine the impact of forward guidance. Assume that the preference shock occurs in period 0, lowering the value of $\hat{\xi}_0$. The shock is i.i.d. and moreover, for simplicity, assume that after period 0 there are no further shocks to the economy. We set $\hat{\xi}_0 < -\hat{i}^*$. The shock is large enough for the ZLB to bind.

Consider first the case where $\sigma = \lambda_Y = \lambda_i = 0$. Under these assumptions a rule of the form (4.1) implements optimal policy (now the sequence $\mathcal{Z}$ is, however, endogenous). The online appendix shows that the optimal paths of inflation and interest rates are given by:

$$\hat{\pi}_t = \begin{cases} \frac{\bar{b}_\delta}{1-\beta\delta}\Delta\psi_{gov,0} & t=0 \\ -\hat{i}^* - \hat{\xi}_0 & t=1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^t}{1-\beta\delta}\Delta\psi_{gov,0}\} & t \geq 2 \end{cases} \qquad \hat{i}_t = \begin{cases} -\hat{i}^* & t=0 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} & t \geq 1 \end{cases} \tag{4.5}$$

in the case of the debt concerns model and

$$\hat{\pi}_t^{NDC} = \begin{cases} 0 & t = 0 \; and \; t \geq 2 \\ -\hat{i}^* - \hat{\xi}_0 & t=1 \end{cases} \qquad \hat{i}_t^{NDC} = \begin{cases} -\hat{i}^* & t=0 \\ 0 & t \geq 1 \end{cases} \tag{4.6}$$

in the no debt concerns model. Notice that according to (4.5) and (4.6) the two models predict positive inflation $t = 1$ (both equal to $-\hat{i}^* - \hat{\xi}_0$, trivially, otherwise the ZLB would be violated) and different inflation levels at $t = 0$ and $t \geq 2$. In the case of no debt concerns, inflation returns to its steady state value from period 2 onwards. Under debt concerns, inflation and interest rates continue being different from zero and the sign of the responses hinges on the sign of the term $\Delta\psi_{gov,0}$, which measures the impact of the liquidity trap shock on the consolidated budget. In the case where $\Delta\psi_{gov,0} < 0$, following a negative shock to preferences, the inflation rate becomes negative. In contrast, if $\Delta\psi_{gov,0} > 0$, inflation turns positive after the shock.

What does FG do in the optimal policy model? Using (4.6) and the rule (4.1) we can show that the optimal path for $\mathcal{Z}^{NDC}$ is:

$$\mathcal{Z}_1^{NDC} = \widetilde{\phi}_\pi(\hat{i}^* + \hat{\xi}_0) < 0 \quad \text{and} \quad \mathcal{Z}_t^{NDC} = 0, \quad t \neq 1.$$

In other words, FG keeps the interest rate low in period 1, which enables inflation to rise in that period. This is the standard result of Eggertsson and Woodford (2003).

Now consider the case of DC. We can show that:

$$\mathcal{Z}_t = \begin{cases} \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} + \delta(\hat{\xi}_0 + \hat{i}^*) < 0 & t = 1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}\} + \delta\hat{i}^* < 0 & t > 1, i_{t-1} = -\hat{i}^* \\ 0 & \text{othws} \end{cases} \tag{4.7}$$

Let us consider $\Delta\psi_{gov,0} < 0$ as in the previous section. For the sake of the exposition let us also (momentarily) assume $\frac{\bar{b}_\delta \delta}{1-\beta\delta}\Delta\psi_{gov,0} < -\hat{i}^* < \frac{\bar{b}_\delta \delta^2}{1-\beta\delta}\Delta\psi_{gov,0}$. Under this condition the shock is large enough to make the ZLB bind in period 0, but from period 1 onwards the interest rate is given by $\frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta}\Delta\psi_{gov,0}$.

From (4.7) it is clear that $\mathcal{Z}_1$ is negative. Moreover, it holds that $\mathcal{Z}_{\geq 2} = 0$. Therefore, FG lowers the interest rate in period 1 only. We can show that $\hat{\pi}_t$ is given by:

$$\hat{\pi}_t = \delta^{t-1}\underbrace{(-\hat{i}^* - \hat{\xi}_0)}_{\hat{\pi}_1} + \delta^{t-2}\mathcal{Z}_1, \qquad t = 2, 3, , ...$$

Since $\mathcal{Z}_1 < 0$, this equation suggests that the impact of $\mathcal{Z}_1$ is to lower inflation after $t = 1$. Thus, in this example FG under debt concerns has exactly the opposite effect than under NDC: It reduces inflation.

Finally, let us consider a sufficiently large shock in preferences so that $\Delta\psi_{gov,0}$ becomes sufficiently negative to make the ZLB bind for several periods. From (4.5) and (4.7) it is evident that the longer the duration of the trap, the more persistent deflation is and the longer is the commitment to keep interest rates low. Thus, in the case of larger shocks the planner announces a path that keeps interest rates at the ZLB over a longer horizon; however, this does not stabilize inflation – it leads to a sharp drop in the price level.

Finally, note that according to (4.5), in the debt concerns model there is a tendency for interest rates to be kept at the lower bound for longer, and consequently for inflation to be negative over longer time periods, since shocks exert an impact on the intertemporal constraint. In simulations (not shown) we also found that the ZLB is hit more frequently in this model. This adds another channel via which the DC model translates to higher macroeconomic volatility.

4.2.2 A Numerical Example

The example of subsection 4.2.1 is of course a very particular case. Assuming $\sigma = 0$ means that output growth exerts no influence on the real rate. Inflation in $t = 1$ has to equal $-(\hat{i}^* + \hat{\xi}_0)$ under both debt concerns and no debt concerns, otherwise the ZLB is violated. In the case where $\sigma > 0$, we can have deflation at the onset of the liquidity trap: Since output drops, positive expected output growth will increase the real rate, "allowing" inflation to turn negative without violating the ZLB. It is thus important to contrast the properties of the two models under more plausible calibrations, when $\sigma, \lambda_Y, \lambda_i > 0$, to see whether the findings of the previous paragraph generalize.

This is done in Figure 4, which shows the responses of inflation, output, interest rates and debt under the two versions of the model assuming the parameter values of Table 2. The solid (blue) line shows responses in the debt concerns model. The dashed line shows responses under no debt concerns.

Qualitatively, the pattern of adjustment of inflation and interest rates resembles the analytical paths derived previously (liquidity traps last longer now in both models because we assume a persistent shock.) Under no debt concerns inflation turns positive during the period when the interest rate is at the ZLB and gradually goes back to zero after the economy escapes from the liquidity trap, as interest rates also gradually return to their steady state value.[20] When the shock hits, aggregate output drops, and the planner promises to gradually increase output and ultimately engineer a boom as the economy escapes from the liquidity trap. These are standard properties of the New Keynesian model's response to the liquidity trap shock (see Eggertsson and Woodford (2003)).

[Figure 4 About Here]

In the case of debt concerns, we see that following the preference shock in period 0, inflation drops sharply. It turns positive for a few periods, but subsequently becomes negative again. As the bottom left panel shows, interest rates remain at the ZLB for longer. This leads to a drop in the inflation rate below its steady state value, rather than a rise, consistent with our results in the previous subsection.

4.3 Discussion

The key result we obtain in this section is that when monetary policy is subservient to fiscal policy, the tradeoff facing the planner in a liquidity trap worsens. We now place this finding within the literature on optimal policy in a LT.

Several papers (see e.g., Eggertsson (2006), Bhattarai et al. (2015), Burgert and Schmidt (2014) amongst numerous others) have considered models of jointly optimal monetary and fiscal policies in a liquidity trap assuming that the planner cannot commit to allocations. The findings of these papers suggest that in response to a liquidity trap shock, the planner will find it optimal to reduce taxes (or increase spending in the case where it is not exogenous to the program) and increase government debt. Since debt is a state variable in these

[20] The fact that the adjustment of inflation is gradual is due to the persistent shock and also due to the assumption $\sigma > 0, \lambda_Y > 0$. In the analytical example of the previous subsection the planner lowered interest rates only in $t = 1$ in the no debt concerns model, and this led to a rise in inflation in that period. This path was optimal, as opposed to extending lower rates beyond period 1, because inflation beyond period 1 is not useful to satisfy the ZLB. The optimal path implies a sharp rise in $\hat{Y}_1$ and then $\hat{Y}_{t\geq 2} = 0$.
When the planner wants to smooth output she commits to keep interest rates at the ZLB for longer, generating high inflation today through expectations of high inflation tomorrow. This mitigates the output rise in period 1.

models and higher debt levels lead to higher inflation, this increases inflation at the exit from the liquidity trap episode. Through the Phillips curve, higher future inflation translates into higher inflation in the LT. Thus in these models when monetary policy takes into account the consolidated budget constraint, the planner's tradeoff improves.

There are two main differences here. First, we assume that taxes (and spending) are exogenous to the planner's program, and second we assume commitment. Had we assumed that the planner can control the tax rate, the optimal policy response would be to lower taxes temporarily, and this would tighten the intertemporal budget. [21] Equation (4.5) then says that instead of deflation a liquidity trap could lead to inflation when $\Delta\psi_{gov,0} > 0$.[22] In our model where fiscal variables are exogenous this could only happen by chance, if a negative tax shock (equivalently a positive spending shock) hits the economy in period 0.

To show the effect of removing the full commitment assumption we now simulate the behavior of macroeconomic variables in a model where the central bank cannot commit to future allocations. The optimal program is similar to the one described in Section 2, however, we now let the planner reoptimize in every period and set the lagged values of the multipliers in equations (2.8) to (2.11) equal to zero in every period (see e.g., Debortoli and Nunes (2010)). The optimal policy rule that emerges from this model is

$$\hat{i}_t^{nc} = \underbrace{\phi_\pi \hat{\pi}_t^{nc} + \phi_Y \hat{Y}_t^{nc}}_{\mathcal{T}^{nc}} + \omega_{gov} \psi_{gov,t}^{nc} \tag{4.8}$$

where nc denotes "no commitment" and the last term in (4.8) is relevant only under debt concerns (ψ_{gov}^{nc} will continue to follow a random walk). There is no forward guidance in this model.

[Figure 5 About Here]

In Figure 5 we plot the response of the economy to the liquidity trap shock under DC and NDC. Notice that now inflation drops more in the NDC scenario at the onset of the LT, whereas the drop is mild under DC. Analogously, output losses are less in the DC model. The planner's tradeoff improves. We conclude that absent commitment, our framework produces a similar result to the models of optimal monetary/fiscal policies cited previously.

5 A Medium Scale DSGE Model

In this section we show that our findings generalize to a medium scale DSGE model. We fit an augmented version of the theoretical framework developed in the previous sections to US data. We augment the pre-

[21]Note that this would then be similar to the response of optimal policy in Eggertsson and Woodford (2006) who consider jointly optimal policies under commitment.

[22]Note that in the case of the jointly optimal monetary/fiscal program the equations we derived in this section remain. See Section 6 where we describe this model.

vious setup with habit formation, a trend in TFP, and shocks to markups, TFP and the consolidated budget. Moreover, we enrich the fiscal block following recent DSGE modelling (e.g., Bianchi and Ilut (2017)); we assume that besides spending, governments have to finance transfers to the private sector, which we model as a stochastic process with two components (a trend and a cyclical component). Also, we augment the tax rule so that taxes now adjust to other macroeconomic variables, besides debt, in order to capture the endogenous response of taxes to the cycle.

Demonstrating robustness of our findings towards introducing these additional model features is important. The previous sections highlighted that in the debt concerns model the response of the intertemporal constraint of the government to shocks is key to understanding the behavior of monetary policy. Yet, our model thus far has relied on simple fiscal rules for tractability. Here we use the data to identify more realistic fiscal rules and study optimal policy in a model that provides a more accurate image of the US economy.

We first provide a brief overview of the model and describe our estimation approach and the output we get from this exercise. We then use the estimated output to study the effects of shocks on macroeconomic variables and optimal policy.

5.1 The Model

5.1.1 Households

We assume that household preferences are of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \bigg(\log(C_t - \Omega C_{t-1}^a) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \bigg)$$

where C_t denotes the consumption of the household, ΩC_{t-1}^a is an external habit stock, where $0 < \Omega < 1$ and C_{t-1}^a denotes the average level of consumption in $t-1$. The household derives disutility from exerting labor effort h_t. Parameters χ and γ_h govern the household's preferences over leisure. As previously, ξ_t is a preference shifter that impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_t C_t + P_{t,L} B_{t,\delta} + P_{t,S} B_{t,S} = (1-\tau_t) W_t h_t + P_t Tr_t + B_{t-1,S} + (1 + \delta P_{t,L}) B_{t,\delta} + P_t Div_t$$

$B_{t,\delta}$ is a long government bond. $P_{t,L}$ is the price of the asset. $B_{t,S}$ denotes the quantity of short-term (one-period) debt, which is in zero net supply. W_t denotes the nominal wage, and P_t is the price level. Finally, Div_t is real dividends paid by monopolistically competitive firms, and Tr_t denotes lump sum transfers given to the household by the fiscal authority. The household maximizes utility subject to the flow budget constraint. For brevity the first order conditions are stated in the online appendix.

5.1.2 Firms

We assume that output is produced by a continuum of monopolistically competitive firms that operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer that aggregates the intermediate products of firms according to $Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj \right)^{\frac{\eta_t}{1+\eta_t}}$. η_t is a (time varying) parameter that governs the elasticity of substitution across differentiated products. Profit maximization for final-good producers gives the following demand of intermediate goods: $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\eta_t} Y_t$.

The production function of the generic intermediate-good firm j is $Y_t(j) = A_t h_t(j)^{1-\alpha}$, where A_t denotes the level of TFP in the economy.

We further assume that intermediate goods firms face price adjustment costs as in Rotemberg (1982). The cost function of firm j is the following: $AC_t(j) = \frac{\theta}{2}(\frac{P_t(j)}{P_{t-1}(j)} - \pi)^2 Y_t$. $\theta \geq 0$ again governs the degree of price stickiness. π is the steady state level of gross inflation.

Intermediate-good producers seek to maximize profits subject to the constraints imposed by the above equations. The first order conditions (stated formally in the online appendix) give us the (non-linear) New-Keynesian Phillips curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = (1 + \eta_t)(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C_{t-1}^a}{C_{t+1} - \Omega C_t^a} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \overline{\pi})\pi_{t+1}$$

where MC_t denotes marginal costs of production.

Finally, we assume that the log growth rate of TFP evolves according to the following stochastic process:

$$\ln\left(\frac{A_t}{A_{t-1}}\right) \equiv a_t = (1 - \rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

The parameter γ denotes the steady state growth rate of the economy.

5.1.3 Government

The government levies distortionary taxes to finance spending G_t and transfers T_t. Imposing that short debt is in zero net supply we write the flow budget constraint as:

$$P_{t,L}B_{t,\delta} = (1 + \delta P_{t,L})B_{t-1,\delta} + P_t(G_t + Tr_t) - \tau_t W_t h_t + \Lambda_t$$

We augment the flow budget with an exogenous shock variable Λ_t, capturing features that we have left outside the model. These could derive from changes in the maturity of debt or the term premium, but also (more crucially) from variation in revenues and spending from sources that we do not model explicitly here (tax revenues from capital income or consumption taxation, public investment, etc.).

As in Section 2 we assume that taxes follow an exogenous rule that relates the current tax rate to the lagged value of debt. In the next subsection we define this rule in the log-linear version of the model.

5.1.4 Log-Linear Model

Since productivity grows over time in our model, we rescale model variables and linearize the model equations around the deterministic steady state. For brevity we relegate all derivations and the description of the log-linear equations characterizing the competitive equilibrium of the economy to the online appendix. Here we describe the functional forms we adopt for the fiscal policy variables (taxes and transfers) and the stochastic processes for the exogenous shocks.

We assume the following feedback rule for taxes:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\left[\phi_{\tau,b}\hat{b}_{\delta,t-1} + \phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t^*)\right] + \epsilon_{\tau,t} \tag{5.1}$$

Notice that we now allow aggregate output (in deviation from its natural level $\hat{Y}_t^n$)[23] and the level of government expenditures (composed of government spending and transfers) to impact directly the tax rate in period t. The variable $\hat{tr}_t^*$ is defined as the long-term component in government transfers, and as in Bianchi and Ilut (2017) evolves exogenously following an AR(1) process:

$$\hat{tr}_t^* = \rho_{tr^*}\hat{tr}_{t-1}^* + \epsilon_{tr^*,t} \tag{5.2}$$

The deviation of transfers $\hat{tr}_t$ from their long-run trend $\hat{tr}_t^*$ responds to its first order lag and to the output gap. We have:

$$\hat{tr}_t - \hat{tr}_t^* = \rho_{tr}(\hat{tr}_{t-1} - \hat{tr}_{t-1}^*) + (1-\rho_{tr})\phi_{tr,y}(\hat{Y}_t - \hat{Y}_t^n) + \epsilon_{tr,t} \tag{5.3}$$

We assume that $\epsilon_{tr,t}$ and $\epsilon_{tr^*,t}$ are i.i.d. The stochastic processes of the remaining exogenous variables are assumed to follow:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \epsilon_{x,t}$$

for $\hat{x} \in \{\hat{G}, \hat{a}, \hat{\Lambda}, \hat{\xi}, \hat{\eta}\}$ and where $\epsilon_{x,t}$ is an i.i.d shock to variable x. Thus, exogenous shocks to spending, TFP, $\hat{\Lambda}$, markups, and preferences follow first order autoregressive processes.

[23]We define the natural level of output as the flexible price level. When the planner chooses optimal policies we assume she does not account for their impact on $\hat{Y}_t^n$. For brevity, we discuss the details in the online appendix.

5.1.5 The Planner's Objective

We maintain in estimation the assumption that monetary policy is optimal; this allows us to recover directly from the data parameters λ_i, λ_Y along with other model parameters. Moreover, for the quantitative model of this section, we adopt the following objective function:

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\hat{\pi}_t^2+\lambda_Y\left(\hat{Y}_t-\hat{Y}_t^n\right)^2+\lambda_i\left(\hat{i}_t-\hat{i}_{t-1}\right)^2\right] \tag{5.4}$$

assuming that the planner seeks to minimize the deviation of inflation in t from the steady state level $\overline{\pi}$, the deviation of output from its natural level $\hat{Y}_t^n$, and the change in value of the nominal interest rate relative to the value in the previous period. Note that (5.4) is commonly used in estimated DSGE models with optimal monetary policy (see, for example, Debortoli and Lakdawala (2016) among others). We thus follow the recent literature in our choice of the objective, however we note that our results below do not hinge crucially on this choice (we get similar findings when we assume a loss function as in (2.6)). Under (5.4) the optimal policy model continues to admit a closed form expression for the interest rate rule which, as in Section 2, features components $\mathcal{T}$, $\mathcal{D}$ and $\mathcal{Z}$. For brevity we do not derive the interest rate rule here.

5.2 Estimation

We now turn to the estimation of the model. We fit the model to US observations using data from the period 1980Q1–2008Q4. We stop before the first quarter of 2009 because, as is well known, the short-term interest rate in that quarter was at the ZLB. Accounting for this in estimation implies that standard estimation techniques (e.g., Smets and Wouters (2007)) do not apply.

The macroeconomic aggregates that we employ in estimation are output, inflation, the federal funds rate, tax revenues, total government expenditures, government spending, and the market value of government debt. We express the last four series as a fraction of GDP. The details on the sources and construction of these variables, together with the measurement equations we employ to link the series to model variables, are spelled out in the online appendix.

Notice that the task of estimating the model is complicated because we have two model versions we can choose from: the version where monetary policy takes into account the consolidated budget and the version where monetary policy has no debt concerns. To choose which version we want to estimate we have to decide whether assuming no debt concerns describes more accurately US monetary policy since the 1980s when our sample begins, or if over this period monetary policy was subservient to fiscal policy as in the DC model. Bianchi and Ilut (2017) estimate a DSGE model allowing for the monetary/fiscal policy mix to vary through time. They find that from the 1980s until 2008 monetary policy was active and fiscal policy passive. Since this corresponds to the no debt concerns model, we estimate our structural optimal policy framework assuming that the Fed chooses allocations without taking into account the consolidated budget.

5.2.1 Priors and Posterior Distributions

As is typical, we proceed with estimation by first selecting prior distributions for the parameters we wish to estimate and picking values for parameters that we want to fix in estimation. Table 3 summarizes the calibrated values of the parameters that we fix, and the right side of Table 4 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g., Bianchi and Ilut (2017)) and are relatively loose.

[Tables 3 and 4 About Here]

We fix the values of the labor share, α, the elasticity of labor supply, $\frac{1}{\gamma_h}$, the demand elasticity parameter, η, and the decay factor of the long-term bond, δ. We assume $\alpha = 0.66$ and $\gamma_h = 1$. The decay factor δ is set to 0.95, which gives us an average maturity of 5 years, consistent with US data. The steady state value of η is such that markups are 15 percent. Finally, we normalize the steady state value of output to unity.

The left side of Table 4 reports the posterior estimates of the model parameter distributions. According to the values reported in the table, the Phillips curve is relatively flat (the mean estimate of κ is 0.017) and the distribution of the habit parameter Ω is centered at roughly 0.5. Moreover, the estimated response of taxes to the lagged value of debt is low ($\phi_{\tau,b} = 0.068$ at the mean). These values are very close to the analogous objects reported in Bianchi and Ilut (2017).

Finally, notice that the estimates of the coefficients λ_i and λ_Y are in range of recent estimates of optimal Ramsey models with US data (see e.g., Debortoli and Lakdawala (2016) and references therein).

5.3 Optimal Policies in Response to Shocks

We now use the estimation output to investigate the impact of shocks on macroeconomic variables. We do this, first, by assuming that the ZLB does not bind, then separately consider the case where the ZLB constraint binds.

5.3.1 The Effects of Shocks away from the ZLB

Figures 6 and 7 show the responses of inflation, interest rates, output and debt-to-GDP to each of the shock processes considered in the model. We assume a one standard deviation innovation to each process, based on the estimates reported in Table 4. Figure 6 considers the case of a preference shock (left panel), a spending shock (middle left panel), a tax shock (middle right panel) and a markup shock (right panel). Figure 7 shows the responses to a shock to the government budget, a shock to TFP and shocks to the business cycle and long-term components of transfers (respectively, from left to right). Notice that the impact effects of each of the shocks on the model variables are now measured in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase, etc.

[Figures 6 and 7 About Here]

From the figures we see that under no debt concerns, inflation is basically unaffected by shocks to preferences, spending, taxes, transfers and shocks to the budget. Shocks to TFP exert a small influence on inflation, but mainly markup shocks are the key driving force behind inflation variability. This model property can be explained as follows: First, preference and spending shocks exert only a minor influence because these shocks are persistent and also because monetary policy is optimal. Due to high persistence, these shocks do not provoke large movements to the real rate. Given welfare losses derive from the volatility of interest rate growth in (5.4), the planner can adjust permanently the nominal interest rate by a few basis points in response to the shocks, without impinging substantial welfare losses. Tax shocks on the other hand, under no debt concerns, affect the inflation output tradeoff through their influence on the Phillips curve, but the estimates in Table 4 suggest that this effect is not large. This also applies to shocks to the consolidated budget, which can lead to changes in taxes.

The finding that markup shocks are a key driving force behind inflation dynamics is not out of line with the rest of the literature. Several studies have reached a similar conclusion (e.g., Fratto and Uhlig (2014), Hall (2011), Michallat and Saez (2014)). Some authors have suggested that this property hints at a failure of the NK model in (endogenously) explaining inflation. Our optimal policy model offers a different perspective: Inflation is driven by shocks to markups only, because monetary policy is very effective in stabilizing inflation against other types of shocks.

In contrast, as can be seen from Figures 6 and 7, under debt concerns inflation responds strongly to all types of shocks, including shocks in fiscal variables and also the effects of markup shocks, and TFP shocks are now larger. The reason is that now shocks exert an influence on the consolidated budget, and since the planner has to satisfy the constraint, she will use inflation (or deflation) to adjust real debt accordingly. For example, following a negative TFP shock, tax revenues will drop (since real wages decrease) and this will tighten the intertemporal constraint. Analogously, a markup shock increases firm profits (not taxed) and lowers real wages, thus reducing overall government revenues. This induces the planner to engineer a larger rise in inflation to ensure debt sustainability. Finally, transfer shocks and the shock to Λ affect the consolidated budget in the same manner as a shock to spending or taxes. Consistent with our previous findings, debt concerns introduce a new channel (the consolidated budget) via which shocks can affect macroeconomic variables and it magnifies macroeconomic volatility.

5.3.2 Optimal Policies at the ZLB

We now turn to study optimal policies in response to a shock that drives the economy in a LT. Rather than simply asking the model to draw a large enough preference shock from the posterior distribution, we ask the model to recover the shocks that, in the first quarter of 2009, brought the US economy to the LT. Thus we first use the data to recover smoothed shocks so that our (NDC) model can fit the macroeconomic time series in 2009Q1, and then analyze the effect of the preference shock separately in the DC and NDC models.[24]

Figure 8 plots the paths of inflation (top left), interest rate (top right) and debt and output growth (bottom left and right, respectively). Since the responses are generated using the preference shock we recovered from the data, the horizonal axis is dated. The solid lines show again the case of debt concerns, and the dashed lines show the no debt concerns model. For comparison we also show in the dotted (black) lines the (filtered) data observations.

[Figure 8 About Here]

Consider first the responses in the no debt concerns model. Notice that the model generates a positive inflation rate in response to the preference shock; the inflation rate drops to around 1 percent in the beginning of 2009 and very quickly reaches the steady state level (around 2 percent). Thus inflation is stable in the LT. Second, the model predicts that interest rates remain at the ZLB for several quarters after 2009. In particular, interest rates are at the ZLB until 2011Q1 and then gradually return to steady state. Third, as can be seen from the middle right panel, the model predicts that output growth recovers rapidly in 2009 and subsequently is stabilized close to the steady state rate of TFP growth.

In contrast, the DC model predicts that inflation turns negative in 2009 and continues being negative until the final quarter of 2010. Interest rates remain at the ZLB until the end of 2013, and finally, output growth is more negative at the onset of the liquidity trap episode than in the NDC model.

These predictions are clearly in line with our previous findings: Promising to keep interest rates at the ZLB for a long period when monetary policy is subservient to fiscal policy does not improve the planner's tradeoff. FG under debt concerns leads to a sharp drop in the price level, and as we saw, output growth is less stable than in the case of no debt concerns. Moreover, the duration of the liquidity trap is longer. We conclude that the theoretical results in Section 4 hold also in the larger scale model we employ in this section.

[24]For brevity we describe in detail in the online appendix the procedure that we follow to recover the shocks. Basically we use a standard Kalman filter augmented to account for piecewise linear solutions, allowing us to deal with an occasionally binding ZLB constraint.

6 Robustness and Extensions

In this section we explore the robustness of our findings when we vary several model features. First, we consider the case where taxes are lump sum. Second, we consider a jointly optimal monetary and fiscal policy program whereby we assume the planner can set the tax rate along with inflation, output and the interest rate. Third, we investigate how varying the maturity of debt affects our findings. Lastly, we briefly explore an extension of our model that allows for regime fluctuations.

6.1 Lump Sum Taxes

Our analysis throughout Sections 2 to 5 assumes that the government levies distortionary taxes on labour income. This assumption is realistic and also standard in models of optimal policy under incomplete markets. In numerous DGSE models, however, taxes are lump sum. We show that our findings are robust towards this assumption.

Note that assuming lump sum taxes essentially amounts to setting $\kappa_2 = 0$ in equation (2.2). Moreover, the surplus becomes $\overline{S}\hat{S}_t = \left[(\overline{\tau} - \overline{G})\left(\hat{\xi}_t - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \hat{\xi}_t\right) + \overline{\tau}\hat{\tau}_t - \overline{G}\hat{G}_t\right]$. Notice that these changes have basically no impact on the monetary policies derived in Propositions 1 and 2. Thus, the categorization of monetary policy into passive/active is the same when we introduce lump sum taxation.

In contrast, the conditions under which fiscal policy is active/passive do change. For instance when we use the assumptions of subsection 3.1.1 and assume lump sum taxation we get:

Proposition 5. Determinacy under lump sum taxes *Assume that taxes are lump sum and follow a rule of the form (2.1). Also assume that parameters λ_Y, $\overline{G}$, δ, σ and ρ_τ have values as in subsection 3.1.1 (Proposition 3). The equilibrium under DC (NDC) is uniquely determined if*

$$\frac{1}{\beta}\left[1 - (1-\beta)\phi^R_{\tau,b}\right] > 1 \ (< 1)$$

Proof: See online appendix.

Clearly the regions in which fiscal policy is active/passive defined in Proposition 5 differ from the analogous objects in Proposition 3. However, the principle that fiscal policy is active when taxes strongly respond to debt and passive otherwise continues to hold.

Given this finding, it is possible to rederive all results of Section 3 with the assumption that taxes are lump sum. Rather than showing additional derivations in the online appendix we resort to simulations to study the responses to shocks and compare them with the case where taxes are distortionary. We find that the responses are very similar in both the debt concerns and no debt concerns models. We thus conclude that our findings continue to hold when we introduce lump sum taxation in our framework.

6.2 Jointly Optimal Policies

We now extend our analysis to consider the case where the planner can set taxes along with inflation, interest rates and output. As discussed previously, many papers have studied jointly optimal monetary and fiscal policies using models that are broadly similar to ours but mainly resorting to global solution methods to approximate equilibria numerically. We investigate whether in our linear model, an optimal allocation with coordinated policies is close to the debt concerns equilibrium or to the no debt concerns outcome.

To allow taxes to be set optimally we abandon fiscal rule (2.1). We further assume that distortionary taxation enters into the loss function (2.6) so that the planner seeks to minimize tax volatility. We assign a weight $\lambda_\tau \geq 0$ to this objective. The optimal policy is otherwise similar to the debt concerns model; the planner optimizes the loss function subject to constraints (2.2) to (2.5). It is easy to show that the first order condition with respect to $\hat{\tau}_t$ gives

$$-\lambda_\tau \hat{\tau}_t + \psi_{gov,t} \frac{(1+\eta)\overline{\tau}\overline{Y}}{\eta(1-\overline{\tau})} - \psi_{\pi,t}\kappa_2 = 0$$

The remaining FONC (for $\hat{\pi}_t, \hat{Y}_t, ...$) for this program are unchanged relative to Section 2.

Consider the assumption $\sigma = \lambda_Y = 0$. We can show (see the online appendix) that optimal taxes are given by

$$\hat{\tau}_t = \psi_{gov,t} \frac{1}{\lambda_\tau} \frac{(1+\eta)\overline{\tau}\overline{Y}}{\eta(1-\overline{\tau})} [1 - \frac{\overline{\tau}\gamma_h}{1+\gamma_h}] \tag{6.1}$$

Moreover, note that $[1 - \frac{\overline{\tau}\gamma_h}{1+\gamma_h}]$ exceeds 0, otherwise the economy is on the wrong side of the Laffer curve. Thus, from (6.1) taxes adjust upwards whenever the consolidated budget tightens and vice versa. According to (2.11) taxes follow a random walk.

Is fiscal/monetary policy active/passive? Optimal monetary policy follows a rule of the form:

$$\hat{i}_t = (1+\delta)\hat{\pi}_t - \frac{\overline{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} \tag{6.2}$$

The impact of the last (interest rate twisting) term on the RHS is key. When this term is zero, then monetary policy is active, otherwise it is passive. Therefore, ultimately, the behavior of policy depends on whether shocks to the consolidated budget exert a significant influence, so that the multiplier $\psi_{gov,t}$ displays volatility. In the case where $\lambda_\tau = 0$ taxes will adjust to fully absorb shocks to the consolidated budget and we can show that $\psi_{gov,t} = 0$ for all t. In this case fiscal policy is passive and monetary policy is active. In contrast, when λ_τ approaches infinity, taxes are constant and equal to the steady state value. Under this scenario, fiscal policy is active and monetary policy is passive.

Intermediate cases cannot be characterized analytically. It is evident that the model has both forces, passive and active. Notice that this is consistent with previous findings in the literature (e.g., Leeper and Leith (2016), Schmitt-Grohé and Uribe (2004)). To find out which of the two regimes is closer to equilibrium policies in the coordinated policies outcome we need to resort to numerical simulations. In the online appendix, we consider alternative values of λ_τ and study impulse responses to shocks. We find that even moderate values for the weight λ_τ brings us close to the debt concerns outcome.

Lastly, consider the case where taxes are lump sum. Since in this case taxes are essentially a slack variable in the government budget constraint we have $\psi_{gov,t} = 0$ at the optimum. The optimal mix of monetary/fiscal policies is thus "active/passive".

6.3 The Role of Debt Maturity

We provided several analytical expressions in previous sections, showing how monetary and fiscal policy rules vary with parameter δ. The maturity of debt is an important variable here since long-term bonds allow the planner to spread inflation over more periods. Hence, the policy responses to shocks depend on whether the government issues short debt ($\delta = 0$) or long-term bonds ($\delta > 0$). This property is well known (see, for example, Lustig et al. (2008)) and for brevity we leave it to the online appendix to show impulse responses of key model variables under alternative calibrations of δ. It is, however, important to highlight here that none of our previous results regarding the characterization of monetary and fiscal policies hinges on the exact value of δ. Thus, broadly speaking, our analysis is robust towards changing the value of this parameter.

The assumption we made in this paper, that governments issue debt in the form of a single bond that pays decaying coupons, is standard in DSGE models. A recent literature, however, studying optimal debt management in macroeconomic models (see, for example, Angeletos (2002), Buera and Nicolini (2004), Debortoli et al. (2017), Bhandari et al. (2017), Faraglia et al. (2018) among others) allows governments to issue multiple bonds simultaneously and investigates how the maturity structure of debt can be targeted to make debt sustainable. If we included in the model an optimizing debt management authority, as recent papers in debt management do, would our conclusions be affected?

In subsection 3.2.1 (footnote 15) we gave an example where issuing a flat maturity structure ($\delta = 1$) completely eliminates the impact of preference shocks on the consolidated budget. We had $\psi_{gov} = 0$ and therefore the debt concerns outcome became identical to the no debt concerns outcome. This provided an example where debt management overrides fiscal/monetary policies and the distinction between active and passive policies becomes less meaningful. Analogously, the work of Angeletos (2002) and Buera and Nicolini (2004) showed that governments that want to hedge against spending shocks can issue long-term debt and accumulate assets in short maturity. Presumably, if an optimizing debt management authority followed this strategy in our model, the impact of spending shocks to the consolidated budget would also be zero.

In both these cases debt management can "complete the market", so that intertemporal consolidated budgets become irrelevant to the planner's program. Recent papers in the debt management literature have argued, however, that such an outcome is practically unattainable, since governments face frictions when issuing debt. Lustig et al. (2009) find it is implausible to assume that governments invest in private assets, Faraglia et al. (2018) argue that transaction costs associated with repurchasing long-term bonds limit the scope of hedging against fiscal shocks and finally Bhandari et al. (2020) argue that debt management has too few instruments to deal with a large numbers of shocks. In all of these papers markets are "incomplete" (as in our model) and thus intertemporal budget constraints matter in equilibrium. Under incomplete markets, we have $\psi_{gov} \neq 0$ and the results of this paper continue to hold.

Clearly, the interplay between debt management, fiscal and monetary policies is interesting and worthwhile exploring in future research.

6.4 Switching across Regimes

In this final section of the paper we discuss an extension of our framework that allows for regime fluctuations. A recent stream of papers (e.g., Davig and Leeper (2007), Bianchi and Ilut (2017), Bianchi and Melosi (2017, 2019)) considers models where the policy mix can oscillate between "active/passive" and "passive/active", randomly through time. To complete our analysis, we discuss here how to model shifts across the DC and NDC equilibria when monetary policy is optimal. Since a full theory of recurrent shifts is beyond the scope of this paper, we assume that they are one-off events. Thus we will consider the case where, starting from the DC model, there is a sudden and unexpected shift to the NDC equilibrium and vice versa. This is sufficient to demonstrate that our framework can encompass regime fluctuations, but also allows us to show transparently key aspects of the transition between the two equilibria. In the online appendix we provide a more complete treatment allowing for recurrent shifts across DC and NDC equilibria and derive optimal interest rate rules in this case.

As we have seen, the optimal rules in Propositions 1 and 2 differ when $\mathcal{D}_t \neq 0$. When the fiscal authority does not guarantee solvency of the intertemporal budget, the multiplier ψ_{gov} differs from zero and, conversely, when $\phi^R_{\tau,b}$ is sufficiently high, entailing a strong response of taxes to debt, then $\psi_{gov} = 0$ in the NDC regime. Thus, switching from the DC to the NDC equilibrium, requires changing the value of $\phi^R_{\tau,b}$, and setting $\psi_{gov} = 0$ when the switch occurs.

An important aspect of this transition (from DC to NDC) has to do with the promises $\mathcal{D}$ that the planner made in past periods. These interest rate twists are time inconsistent, in the sense that if the planner is allowed to reoptimize when the switch occurs, she will set $\mathcal{D} = 0$.

Formally, suppose that in period $\bar{t}$ the feedback tax rule coefficient changes from $\phi_{\tau,b}^{DC}$ to $\phi_{\tau,b}^{NDC}$. Since now fiscal policy generates sufficient surpluses to finance debt, the consolidated budget constraint is, from the point of view of the planner, slack. The continuation Ramsey program solves:

$$\max -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_{\bar{t}}\left\{\hat{\pi}_{t+\bar{t}}^2 + \lambda_Y \hat{Y}_{t+\bar{t}}^2 + \lambda_i \hat{i}_{t+\bar{t}}^2\right\} \tag{6.3}$$

subject to the Phillips curve and the Euler equation (assuming we are away from the ZLB). It can be easily shown that the solution to this program is a policy rule of the form $\hat{i}_{t+\bar{t}} = \mathcal{T}_{t+\bar{t}}^{NDC}$ for $t = 0, 1, 2....$ The planner sets $\mathcal{D}_{\bar{t}} = 0$ when reoptimization occurs.

An alternative setup in which the planner "remembers" promises made in the past and in which the solution to the continuation problem in $\bar{t}$ does not set $\mathcal{D}_{\bar{t}} = 0$ is the following: The planner maximizes

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_{\bar{t}}\left\{\hat{\pi}_{t+\bar{t}}^2 + \lambda_Y \hat{Y}_{t+\bar{t}}^2 + \lambda_i \hat{i}_{t+\bar{t}}^2\right\} + \underbrace{\left(-\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^l \Delta\psi_{gov,\bar{t}-l}\right)\sum_{t=0}^{\infty}(\delta^t\beta^t)E_{\bar{t}}\left(\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_{t+\bar{t}} + \sigma\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta\hat{Y}_{\bar{t}+t}\right)}_{P}$$

subject to (2.2) and (2.3). Adding the second order term P in the continuation program forces the planner to keep with past promises.[25] It can be easily checked that the FONC with respect to $\hat{Y}_{\bar{t}+t}$ and $\hat{\pi}_{\bar{t}+t}$ are essentially equations (2.8) and (2.9) when $\psi_{gov,t+\bar{t}} = 0, t > 0$. The solution to the above program is a policy rule of the form:

$$\begin{aligned}\hat{i}_{t+\bar{t}} = \mathcal{T}_{t+\bar{t}} - \frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i\sigma}\frac{\bar{b}_\delta}{1-\beta\delta}\left(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{l+t}\Delta\psi_{gov,\bar{t}-l}\right)\\ -\frac{\bar{b}_\delta}{\lambda_i}\left(-\delta^t\psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty}\delta^{l+t}\left(\Delta\psi_{gov,\bar{t}-l} - \Delta\psi_{gov,\bar{t}-l-1}\right)\right) + \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\psi_{gov,\bar{t}-1}\mathcal{I}_{t=0}\end{aligned} \tag{6.4}$$

which is basically the interest rate rule in Proposition 1, with multipliers $\psi_{gov,\bar{t}} = \psi_{gov,\bar{t}+1}$ set to zero. Since the economy was in the DC regime before $\bar{t}$, the multipliers $\psi_{gov,\bar{t}-l}, l > 0$ will generally be different from zero.

Optimal policy under rule (6.4) and optimal policy when the planner "forgets" past promises will in general have different effects on the macroeconomy. First, obviously, because interest rate commitments made between period 0 and $\bar{t}-1$ will have a bearing on the path of interest rates from date $\bar{t}$ onwards and thus impact macroeconomic variables. Second, because of these extra terms that capture previous commitments, monetary policy may "partially" be in the passive regime even after date $\bar{t}$, for a while, until the influence of these terms fades out.

To evaluate this we turn to a numerical example. We consider an "optimal disinflation scenario": We

[25] See, e.g., FMOS. Alternatively, we could assume optimal policy from a "timeless perspective" to force the planner to keep with past promises.

assume that at date 0 the planner inherits a high debt level, and for the first $\bar{t}-1$ periods the economy is in the DC regime. Fiscal policy is active, unable to generate sufficient surpluses to ensure debt sustainability, and inflation rises to make debt sustainable. In period $\bar{t}$, unexpectedly, fiscal policy becomes passive, and monetary policy switches to NDC. We consider the impact of the switch in policy on macroeconomic variables in the case where the planner remembers past commitments and in the case where she reneges on them.

6.4.1 An Optimal Disinflation

Consider for simplicity the calibration $\sigma = \lambda_Y = \lambda_i = 0$. Under these assumptions, rule (6.4) becomes

$$\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}} - \frac{\bar{b}_\delta}{1-\beta\delta}\left(-\delta^t \psi_{gov,\bar{t}-1} + \sum_{l=1}^{\infty} \delta^{t+l} \Delta\psi_{gov,\bar{t}-l}\right) \tag{6.5}$$

In the case where monetary policy abandons past promises at $\bar{t}$, the interest rate rule becomes $\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}}$. Between period 0 and $\bar{t}-1$, optimal policy is of the form $\hat{i}_t = \delta\hat{\pi}_t$.

The top panels of Figure 9 plot the responses of inflation, output interest rates and debt when we assume that initially debt is 30 percent above the steady state and set $\bar{t} = 10$. Between periods 1 and 9 (in the DC regime) inflation turns positive; this reduces the real payout of debt and makes the intertemporal budget solvent. Nominal interest rates rise, since inflation and interest rates are positively correlated in the DC model. The dashed red line shows the case where the planner remembers past commitments in the NDC regime and the solid (blue) line the case where she sets $\mathcal{D}_{\bar{t}} = 0$. To make the effects transparent, we show the behavior of macroeconomic variables over 20 model periods.

[Figure 9 About Here]

Notice first that the two scenarios give the same outcome between 0 and $\bar{t}-1$. This is so because the switch in period $\bar{t}$ is fully unanticipated. After period $\bar{t}$, the two models behave quite differently, with the solid blue lines suggesting that debt, inflation, output and interest rates are all stabilized when the planner reneges on past commitments. In contrast, the dashed red lines show that inflation turns negative, the real value of debt grows and gradually reverts back to the pre-switch level and, finally, interest rates drop below their steady state value.

To understand these results, notice that when fiscal policy becomes active, the government's future surpluses increase. Standard arguments imply that inflation can turn negative in order to stabilize the intertemporal budget (so that debt does not continue decreasing rapidly). In the case where the policy rule is of the form $\hat{i}_{t+\bar{t}} = (1+\delta)\hat{\pi}_{t+\bar{t}}$, the planner would lower interest rates aggressively in response to deflation. This accomplishes stabilizing prices. Debt continues to be higher than its steady state level, since taxes remain low, when $\rho_\tau > 0$ as is assumed in the figure.

In contrast, if monetary policy is of the form (6.5) it resembles a passive money policy. The term $-\frac{\bar{b}_\delta}{1-\beta\delta}\Big(-\delta^t\psi_{gov,\bar{t}-1}+\sum_{l=1}^{\infty}\delta^{t+l}\Delta\psi_{gov,\bar{t}-l}\Big)$ exceeds 0 and this makes interest rates less responsive to deflation. In equilibrium, prices drop and interest rates turn negative because the leading term $(1+\delta)\hat{\pi}_{t+\bar{t}}$ in (6.5) dominates.

The bottom panels of Figure 9 perform the above experiment under the calibration $\sigma = \lambda_Y = 1$ and $\lambda_i = 0.5$. The results are essentially the same. The above results suggest that in the case where regime-switching is not accompanied by reoptimization, an optimal policy that aims at stabilizing inflation has the opposite impact and instead destabilizes inflation.

Note that these findings are partly driven by the assumption that regime changes are unanticipated one-off events: If regime changes were recurrent events, then setting lagged multipliers to zero when there is a switch towards NDC would introduce an element of lack of commitment that would possibly make policy less effective in stabilizing debt during a DC regime, since the private sector would expect interest rate commitments not to be kept. On the other hand, keeping with past commitments leads to deflation in the NDC regime. This will also affect the planner's ability to reduce debt. The interplay between these forces is left to explore in future work.

In the online appendix we extend this analysis further. We first derive optimal interest rate rules in the case of recurrent Markov switches across regimes. We then show how in this model, in which the history of regimes needs to remembered by the planner, the state vector can be condensed so that standard solution techniques are utilized.

7 Conclusion

A large literature on optimal monetary and fiscal policies has relied on non-linear models to characterize optimal policies. These models cannot be mapped to DSGE models on monetary/fiscal interactions. We offer a novel framework that brings together these strands of literature and analyze *optimal monetary policy with and without debt.*

In our framework a Ramsey planner (the central bank) sets allocations under commitment to minimize the deviations of inflation, output and interest rates from their respective target levels. When monetary policy exhibits "debt concerns" then optimization is subject to the consolidated budget constraint; otherwise it is not.

Our model is tractable and admits an analytical solution for interest rate rules. In the case where debt is taken into account in optimization, monetary policy becomes subservient to fiscal policy and resembles a "passive money" policy. Fiscal policy is "active" and taxes do not finance debt. In contrast, under "no debt concerns", monetary policy is "active" and fiscal policy is "passive" and ensures debt sustainability.

We analyze the effectiveness of monetary policy in stabilizing the macroeconomy under each of the two setups considered. We find that under debt concerns, policy is much less effective in dealing with shocks, and macroeconomic volatility increases. During liquidity trap episodes, the tradeoff facing the planner worsens. These findings continue to hold in a medium scale version of our model that we estimate with US data.

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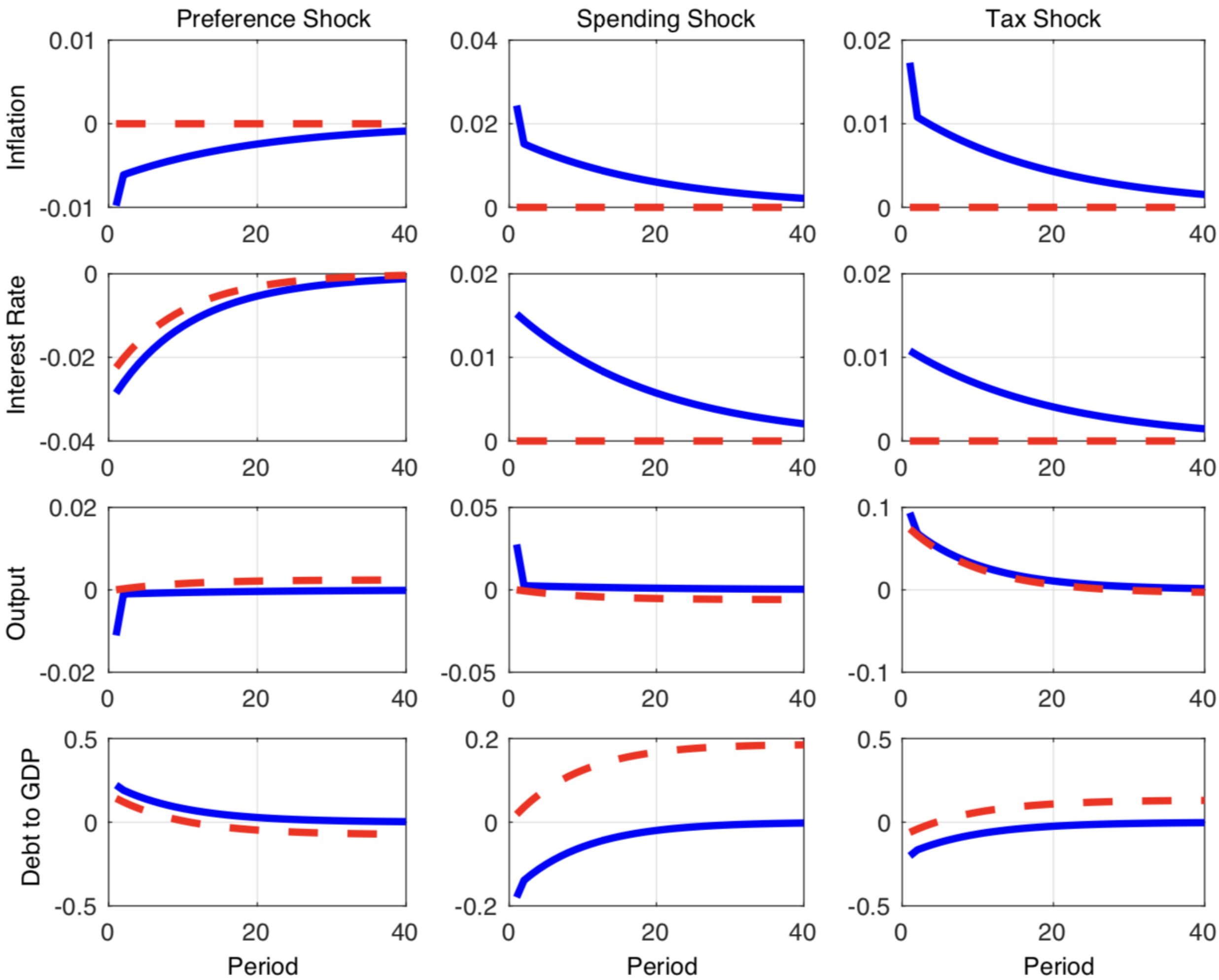
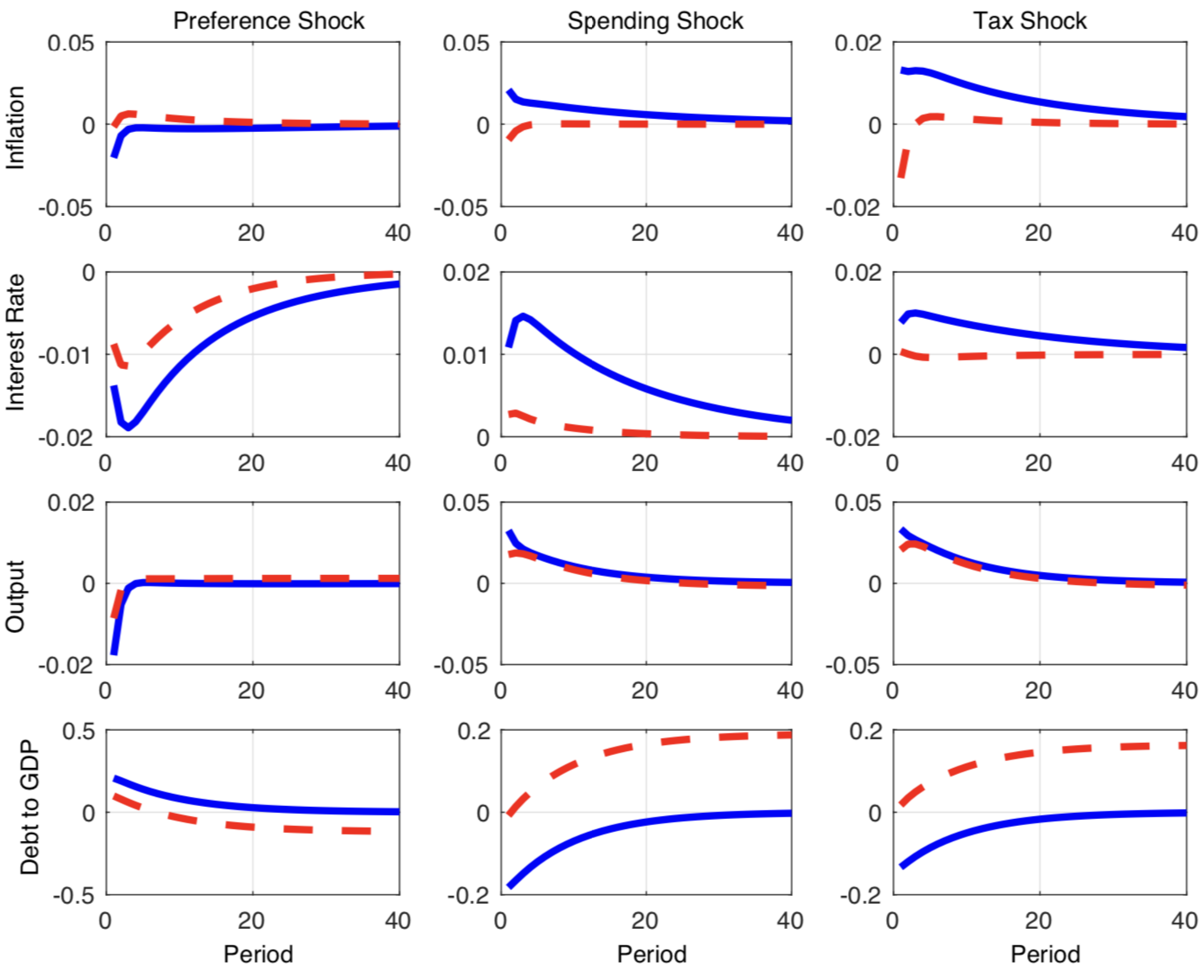
Figure 1: **Optimal Policies away from the ZLB:** $\lambda_Y = \lambda_i = 0$



Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0$ in the planner's objective. See Section 3.2 for details.

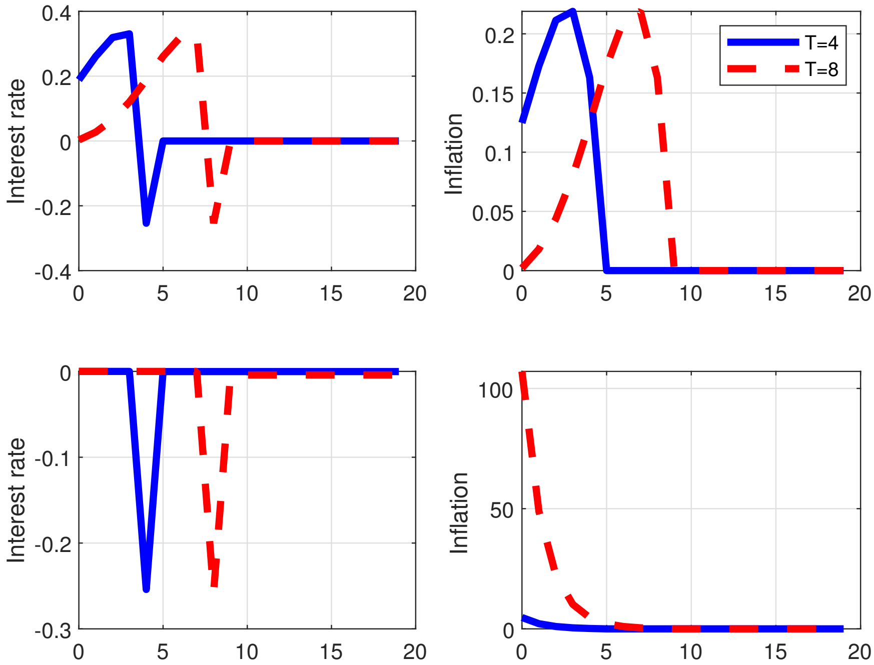
Figure 2: **Optimal Policies away from the ZLB:** $\lambda_Y = \lambda_i = 0.5$, $\sigma = 1$



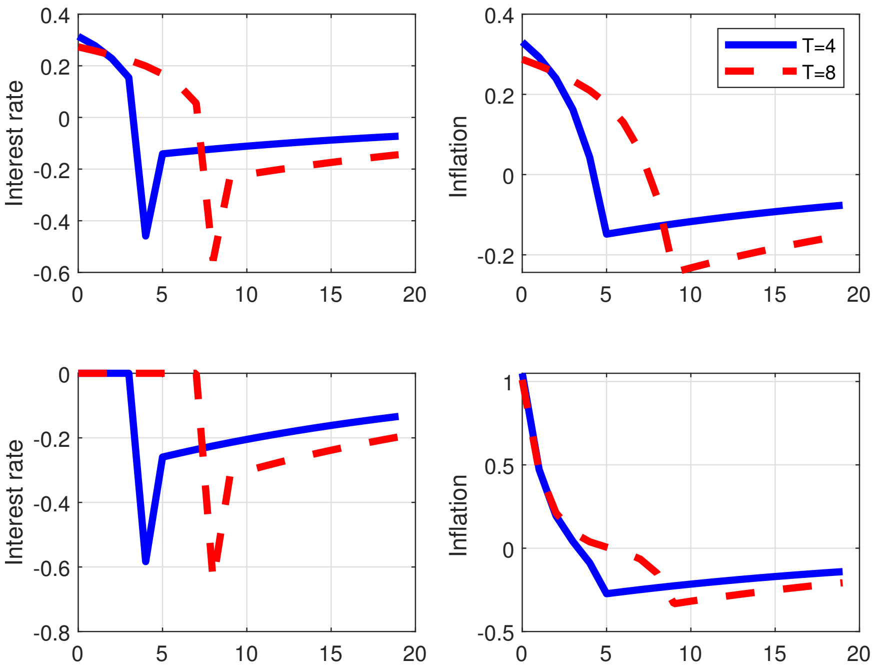
Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective. See Section 3.2 for details.

Figure 3: **Forward Guidance in the Model with Taylor Rules**

a) Active Money

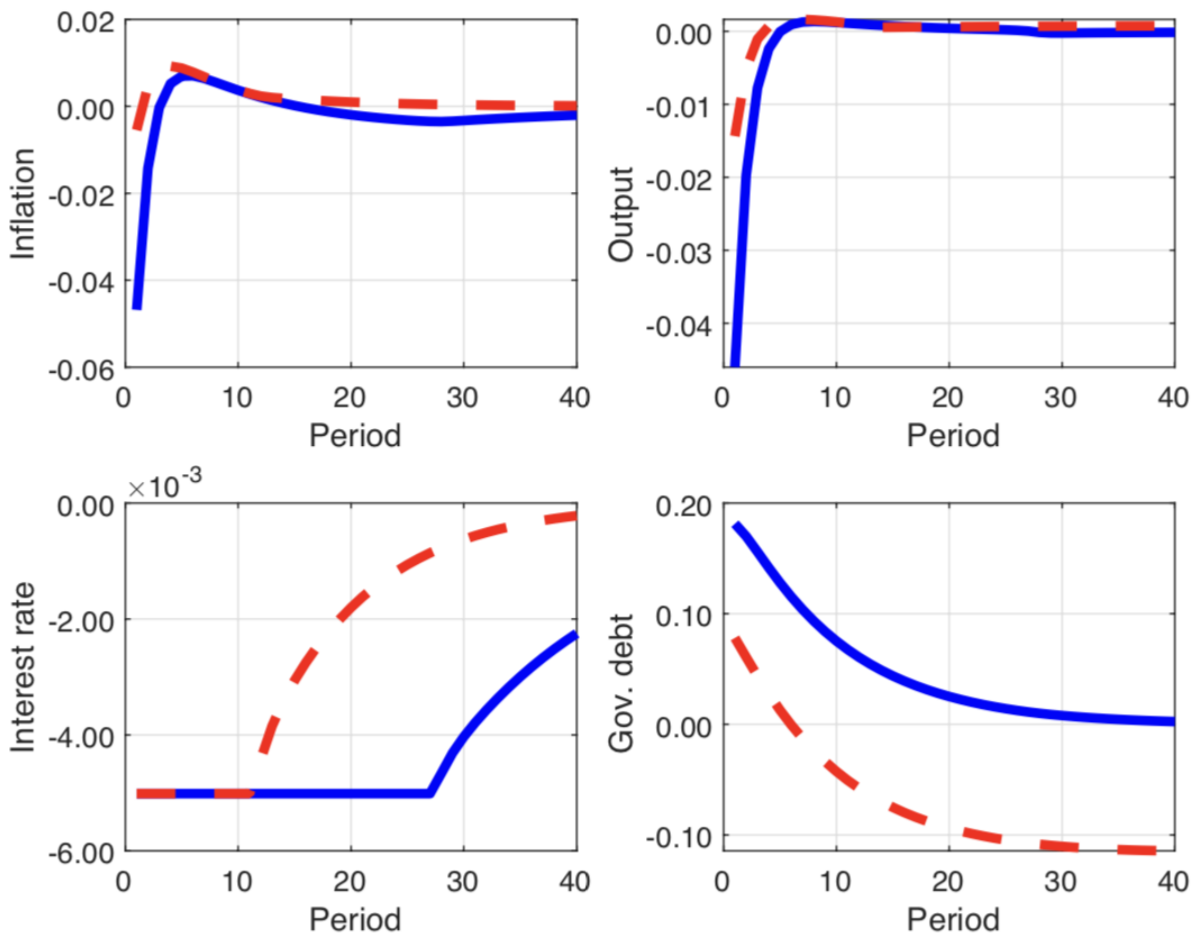


b) Passive Money



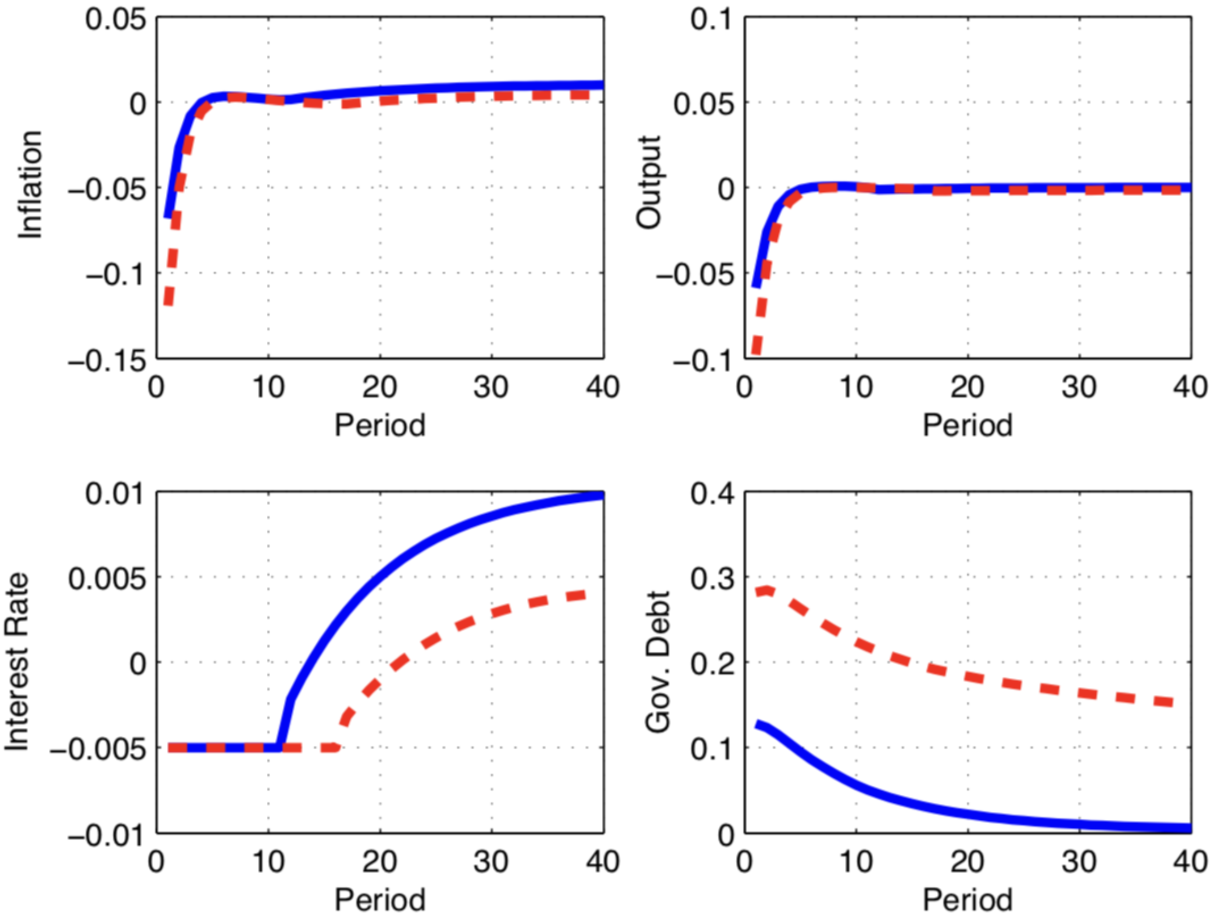
Notes: The figure plots the response of model variables to a FG shock. We assume that monetary policy follows rule (4.1) and $\sigma = 1$. The solid lines assume that the planner commits to lower interest rates in period $\bar{t} = 4$. The dashed lines set $\bar{t} = 8$. In the bottom graphs of panels a) and b) of the figure the planner announces that interest rates will be kept at zero between periods 0 and $\bar{t} - 1$ and also announces lower rates in period $\bar{t}$.

Figure 4: **Optimal Policies at the ZLB**



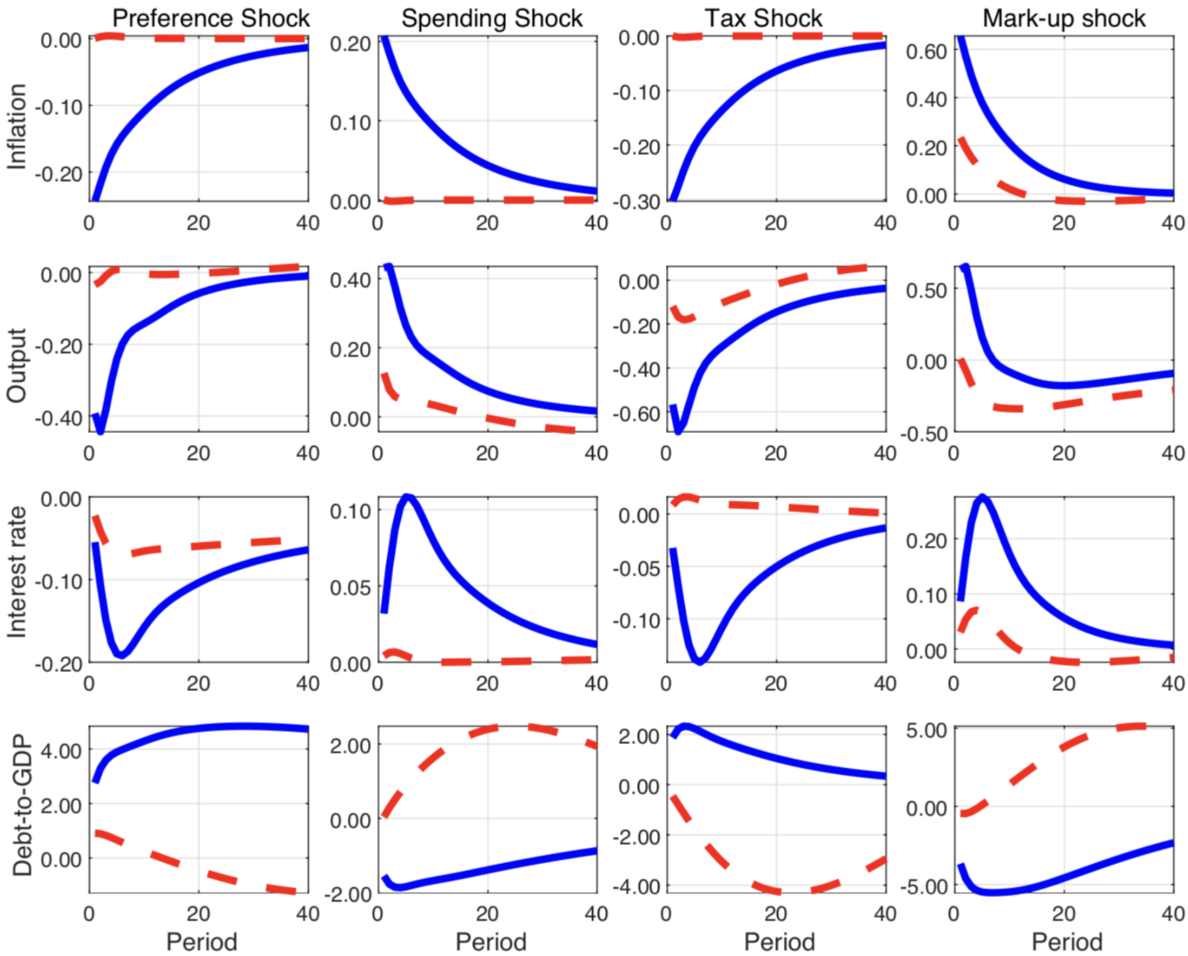
Notes: The figure plots the response of model variables to a preference shock that drives the economy to the LT. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective.

Figure 5: **Optimal Policies at the ZLB under No Commitment**



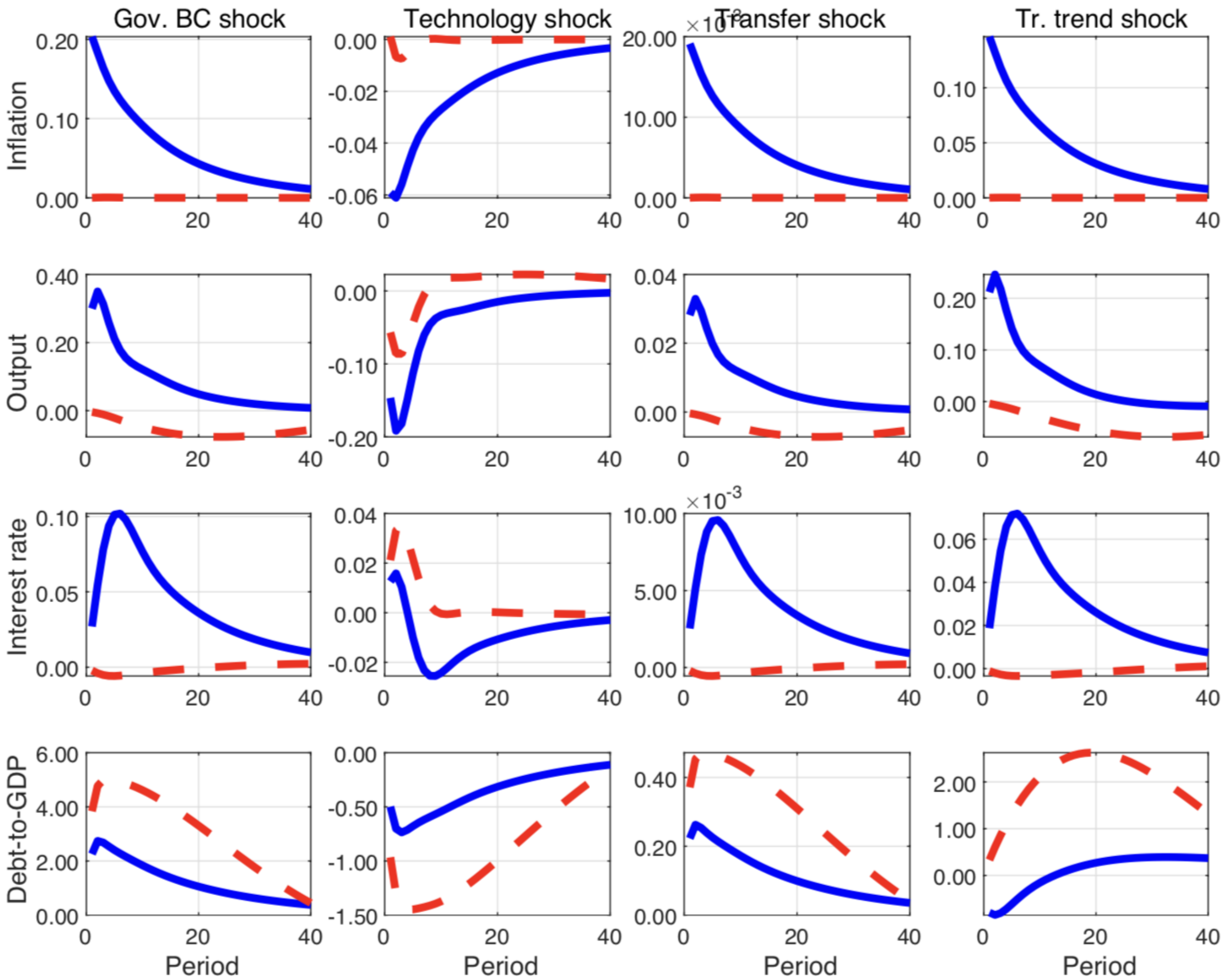
Notes: The figure plots the response of model variables to a preference shock that drives the economy to the LT. We assume that the planner cannot commit to future allocations. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 6: **Responses to Shocks: Quantitative Model**



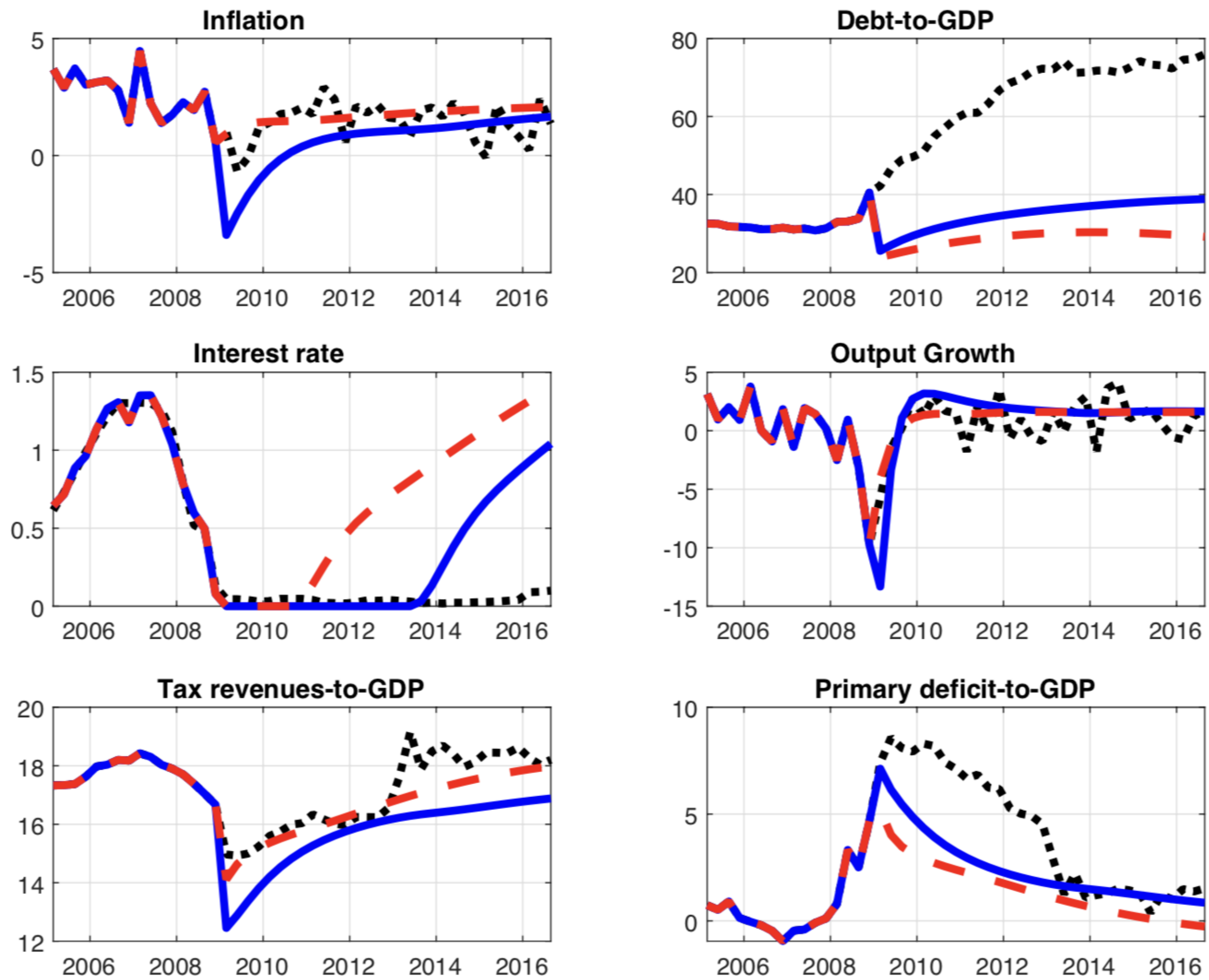
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. To construct the impulse response functions we apply the estimates of the model reported in that section. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a preference shock, the middle left panels the responses to a spending shock, the middle right panels the responses to a tax shock and the right panels the responses to a markup shock. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 7: **Responses to Shocks: Quantitative Model**



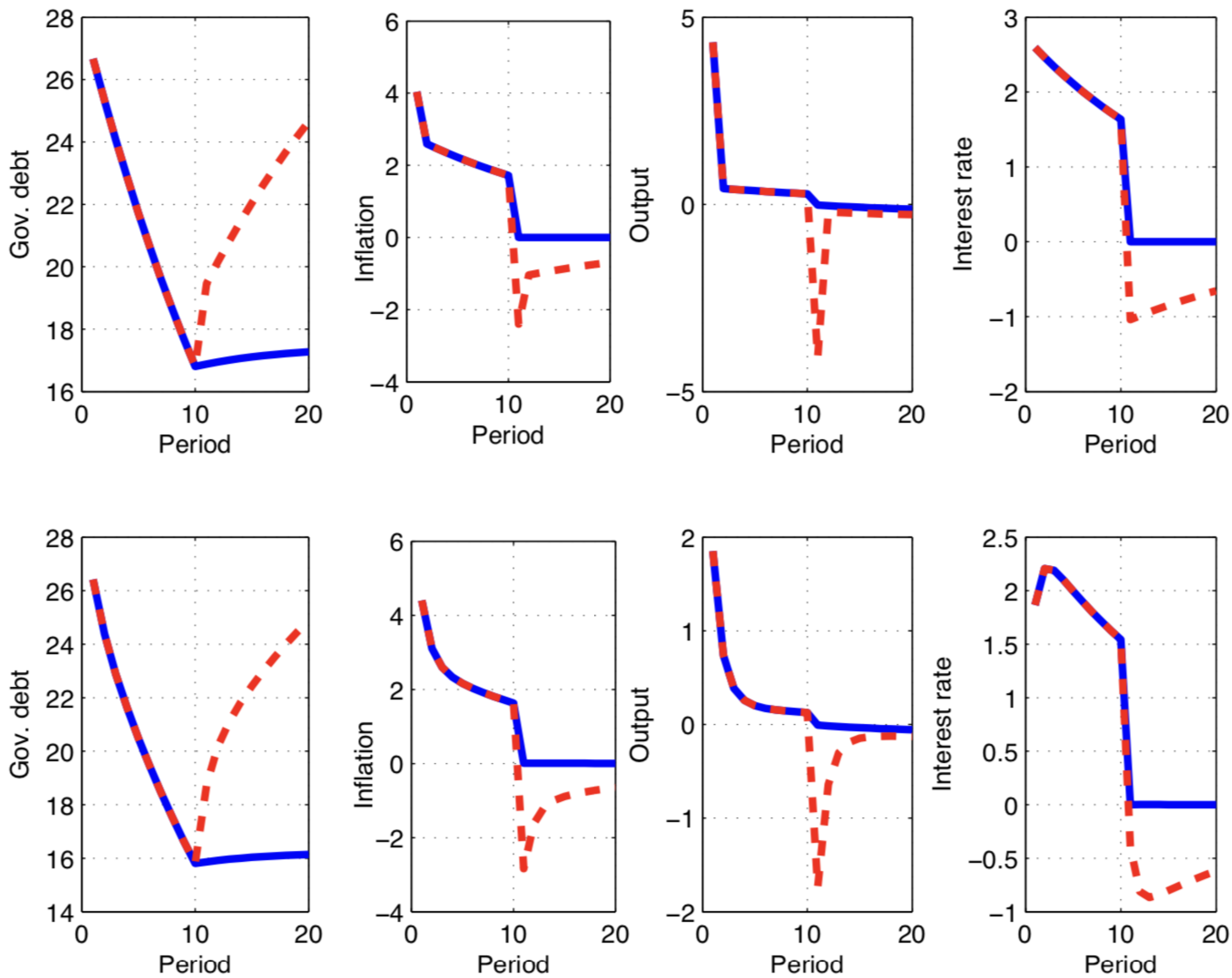
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. The responses are constructed in the same fashion as those shown in Figure 6. The left panels show the responses to a shock to the government budget constraint, the middle left panels the responses to a TFP shock, the middle right panels the responses to a transfer shock and the right panels the responses to the long-run components of transfers. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 8: **Optimal Policy in Response to a LT Shock**



Notes: The figure plots forecasts of inflation (top left), market value of debt-to-GDP (top right), interest rates (middle left), output growth (middle right), tax revenues-to-GDP (bottom left) and the primary deficit-to-GDP ratio (bottom right) in the models, in the counter-factual case where all fiscal variables (government debt, government spending, taxes and transfers) are set to their steady state value at the end of period 2009Q1. The solid (blue) line represents the debt concerns model. The dashed (red) line is the no debt concerns model. The data are the same as in Figure 7.

Figure 9: **Switching from DC to NDC**



Notes: The figure plots the behavior of macroeconomic variables when there is an unanticipated change from DC to NDC. The top panels simulate the economy when $\sigma = \lambda_Y = 0$. The bottom panels assume $\sigma = \lambda_Y = 1$. The solid lines represent the case where the planner reneges on promises $\mathcal{D}$ when the shift to the NDC regime occurs. The dashed lines assume the planner "remembers" $\mathcal{D}$ after the switch.

Table 1: **Model Implied Taylor Rules**

Parameter	**Coefficients of Taylor Rule**			
	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
$\widetilde{\phi}_\pi$	0.104	0.070	0.080	0.055
	(0.537)	(0.268)	(0.268)	(0.134)
$\widetilde{\phi}_i$	0.870	0.905	0.897	0.923
	(1.270)	(1.270)	(1.135)	(1.135)
$\widetilde{\phi}_{\Delta i}$	0.509	0.528	0.585	0.599
	(1.005)	(1.005)	(1.005)	(1.005)
$\widetilde{\phi}_\pi$	0.271	0.209	-0.039	-0.024
	(0.537)	(0.268)	(0.268)	(0.134)
$\widetilde{\phi}_Y$	0.052	0.003	0.2305	0.156
	(0.800)	(0.800)	(0.200)	(0.200)
$\widetilde{\phi}_i$	0.629	0.750	0.705	0.765
	(1.270)	(1.270)	(1.135)	(1.135)
$\widetilde{\phi}_{\Delta i}$	0.0904	0.252	-0.272	-0.232
	(1.005)	(1.005)	(1.005)	(1.005)

Notes: The table reports model implied Taylor rules $\hat{i}_t = \widetilde{\phi}_\pi \hat{\pi}_t + \widetilde{\phi}_Y \Delta \hat{Y}_t + \widetilde{\phi}_i \Delta \hat{i}_{t-1} + \widetilde{\phi}_{\Delta i} \Delta \hat{i}_{t-1}$ under debt concerns (see Section 3). The top panel assumes $\lambda_Y = 0$ and the bottom panel sets $\lambda_Y = 0.5$. Each of the columns reports the values of parameters $(\widetilde{\phi}_\pi, \widetilde{\phi}_Y, \widetilde{\phi}_i, \widetilde{\phi}_{\Delta i})$ under alternative calibrations of λ_i and σ. The numbers in parentheses report the values we obtain from the analytical solution of the model (object $\mathcal{T}$).

Table 2: **Calibration**

Parameters Common Across Models		
Parameter	Value	Label
β	0.995	Discount factor
λ_Y	$\{0, 0.5\}$	Loss function - weight on output
λ_i	$\{0, 0.5\}$	Loss function - weight on interest rate
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
σ	$\{0, 1\}$	Inverse of IES
γ_h	1	Inverse of Frisch Elasticity
ρ_τ	0.9	Persistence of Taxes
ρ_ξ	0.9	Persistence of ξ
ρ_G	0.9	Persistence of Spending
$\overline{b}_\delta$	0.1321	Debt Level
$\overline{\tau}$	0.2545	Tax Rate
$\overline{Y}$	1	Output
$\overline{G}$	0.1	Spending

Parameters Not Common Across Models		
Parameter	Value	Label
$\phi_{\tau,b}$	0.00 0.07	Tax rule coefficients

Notes: The table reports the values of model parameters assumed in the numerical experiments in Section 3. β notes the discount factor chosen to target a steady state real interest rate of 2 percent. λ_Y and λ_i are the weights on output and interest rates in the objective of the planner. Parameter η is calibrated to target markups of 17 percent in steady state. θ is calibrated as in Schmitt-Grohé and Uribe (2004). Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output, which is normalized to unity in the steady state. The bottom panel of the table reports the value of the coefficient $\phi_{\tau,b}$ in the tax policy rule (2.1). As discussed in the text, we set $\phi_{\tau,b} = 0.07$ in the no debt concerns model to have a determinate equilibrium. In the debt concerns case we set $\phi_{\tau,b} = 0.00$ to find a unique equilibrium. See text for further details.

Table 3: **Calibrated Parameters**

	Parameter	Value
y	steady state output (normalization)	1
$1-\alpha$	labor share	0.66
δ	decaying rate of coupon bonds	0.95
η	demand elasticity	-7.66
γ_h	inverse of Frisch elasticity	1

Notes: The table reports model parameters whose values we fix in estimation. See the text for details.

Table 4: **Estimated Parameters**

	Parameter	Posterior		Prior		
		mean	90 % interval	distrib	par A	par B
Quarterly trends						
100γ	growth rate	0.397	[0.318 ; 0.469]	G	0.4	0.05
$100(\beta^{-1}-1)$	discount rate	0.226	[0.089 ; 0.363]	G	0.25	0.1
Households and firms						
$100\log\pi$	inflation	0.586	[0.517 ; 0.656]	G	0.5	0.05
g	g-to-GDP	1.066	[1.055 ; 1.077]	N	1.06	0.04
$b_L/4$	debt-to-GDP	0.241	[0.177 ; 0.301]	N	0.25	0.05
tax	taxes-to-GDP	0.044	[0.042 ; 0.047]	N	0.045	0.0025
Ω	habits	0.456	[0.359 ; 0.554]	B	0.7	0.1
κ	slope NKPC	0.017	[0.003 ; 0.028]	G	0.3	0.15
Central bank preferences						
λ_i	i.r smoothing	0.167	[0.08 ; 0.253]	G	0.25	0.1
λ_Y	y smoothing	1.218	[0.875 ; 1.557]	G	1	0.2
Fiscal rules						
$\phi_{\tau,b}$	τ response to b	0.068	[0.039 ; 0.097]	G	0.07	0.02
$\phi_{\tau,y}$	τ response to y	0.283	[-0.033 ; 0.622]	N	0.4	0.2
$\phi_{\tau,g}$	τ response to g	0.458	[0.037 ; 0.856]	N	0.5	0.25
$\phi_{tr,y}$	tr response to y	-0.141	[-0.356 ; 0.078]	N	-0.4	0.2
Shocks, persistence						
ρ_ξ	preference	0.993	[0.989 ; 0.996]	B	0.5	0.2
ρ_η	markup	0.95	[0.902 ; 0.996]	B	0.5	0.2
ρ_a	tfp	0.477	[0.4 ; 0.552]	B	0.5	0.2
ρ_g	gov. spending	0.976	[0.96 ; 0.993]	B	0.5	0.2
ρ_τ	tax rate	0.948	[0.914 ; 0.983]	B	0.5	0.2
ρ_{tr}	transfers	0.224	[0.133 ; 0.311]	B	0.2	0.05
ρ_{tr^*}	transfers trend	0.95	[0.912 ; 0.988]	B	0.5	0.2
ρ_λ	government b.c	0.247	[0.096 ; 0.389]	B	0.5	0.2
Shocks, standard deviations						
σ_τ	tax rate	9.365	[6.826 ; 11.963]	IG	10	2
σ_g	gov. spending	0.045	[0.031 ; 0.058]	IG	0.1	2
σ_η	markup	0.825	[0.703 ; 0.944]	IG	1	1
σ_ξ	preference	0.036	[0.03 ; 0.041]	IG	0.1	2
σ_a	tfp	0.226	[0.202 ; 0.251]	IG	1	1
σ_m	int. rate, m.e	0.484	[0.429 ; 0.535]	IG	2	2
σ_λ	government b.c	0.367	[0.318 ; 0.413]	IG	2	2
σ_{tr}	transfers	0.303	[0.248 ; 0.356]	IG	1	1
σ_{tr^*}	transfers trend	3.793	[3.328 ; 4.228]	IG	1	1

Notes: The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90 percent HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.