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Optimal Quantitative Easing in a Monetary Union

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Abstract

This paper explores the optimal allocation of government bond purchases within a monetary union, using a two-region DSGE model, where regions are asymmetric with respect to economic size and portfolio characteristics: the extent of substitutability between assets of different maturity and origin, asset home bias, and steady-state levels of government debt. An optimal quantitative easing (QE) policy under commitment does not only reflect different region sizes but is also a function of these dimensions of portfolio heterogeneity. By calibrating the model to the euro area, we show that optimal QE favors purchases from the smaller region (Periphery instead of Core), given that the former faces stronger portfolio frictions. A fully optimal policy consisting of both the short-term interest rate and QE lifts the monetary union away from the zero lower bound faster than an optimal interest rate policy alone, which entails forward guidance.

Bank topics: Monetary policy, Business fluctuations and cycles, Economic models
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1 Introduction

In practice, the design of a quantitative easing (QE) policy in a monetary union requires a framework about the allocation of government bond purchases across regions. For example, in the case of the euro area (EA), the European Central Bank (ECB) purchases government bonds from each country based on the notion of the "capital key", which determines each country's contribution to the ECB's capital and corresponds (roughly) to each country's share in the total population and gross domestic product of the EA; i.e., around 40% of bonds are purchased from Germany, 10% from Portugal, etc. The natural question that arises is whether this type of allocation is optimal. And how should QE be designed in the presence of asymmetries that go beyond the geographic and economic size of regions.

We study optimal unconventional monetary policy in a two-region world of a monetary union, where each region is asymmetric with respect to economic size and bond market characteristics. We refer to bond market characteristics as the crucial elements that affect the transmission of QE in an open economy: i) the elasticities of substitution between short-term and long-term bonds ("short-long friction"); ii) the share of short-term to long-term bonds ("short share"); iii) the share of domestic long-term bonds to foreign long-term bonds ("home bias"); and iv) the level of total government debt-to-GDP. All these asymmetries are important in dictating the effects of QE through changes in the term premium and can be interpreted as a measure of differing financial frictions between countries.

As we document in section 2, there are considerable differences in bond market characteristics between the Periphery and Core of the EA. First, there is a striking difference in the degree of home bias across regions: on average, domestic bonds make up 55% of Core bond portfolios, while they make up 97% of bond portfolios held by Periphery residents. Second, although the short share has risen steadily since the start of the ECB's QE policy in 2015 in both regions, the increase in the Core seems to be larger compared to the Periphery. This suggests a stronger portfolio rebalancing in the Core. In addition, Periphery economies have a larger debt-to-GDP ratio compared to Core economies (91% vs. 70%).[1] So, although the Periphery is smaller in population size, the absolute size of the government debt is closer to that in the Core.

Armed with these stylized facts, we build a two-region dynamic stochastic general equilibrium (DSGE) model of the EA Periphery and Core with an active role for QE through the portfolio rebalancing channel. Portfolio balancing frictions are a commonly discussed reason for the effectiveness of QE on real and financial variables. This channel crucially depends on investors' preferred habitat environment and hence their preferences over bonds with different characteristics (Vayanos and Vila, 2009). We introduce imperfect substitutability between short- and long-term, domestic and foreign bonds, in a fashion similar to Alpanda and Kabaca (2020); households hold government bonds and, in addition to interest payments, derive benefits mo-

[1] The numbers reflect the averages from 2000:Q1 to 2013:Q4. See section 6.1 for details.

tivated by these bonds' liquidity characteristics.[2] Central bank asset purchases then have effects on the real economy through the extent to which private investors are induced to re-establish the portfolio mix of their asset holdings. These purchases affect asset prices, domestic and foreign term premia, and by extension real variables.

Using this framework, we evaluate analytically the optimal monetary policy under commitment when the policy rate is subject to an occasionally-binding zero lower bound (ZLB) constraint. To do so we derive the second-order approximation of the welfare of the representative household in each region and use a weighted average of the two as a loss function for the union-wide central bank. We show that beyond the usual trade-off between relative output and relative price level stabilization, giving a role to QE in our open economy model introduces two additional trade-offs: a trade-off related to asset purchases and fluctuations in the terms of trade, and a trade-off between the terms of trade and aggregate portfolio stabilization.

Naturally, the optimal policy depends on the number of instruments available to the central bank. Optimal interest rate policy under commitment follows the standard predictions of the canonical New Keynesian model; that is, forward guidance in the face of adverse shocks, as in Eggertsson and Woodford (2003). When QE policy is also available alongside interest rate policy, the central bank is able to restore the term premium at lower levels, thus further stimulating output and inflation and providing overall more stabilization. This shortens the duration of the ZLB spell following a negative demand shock.

More importantly, optimal QE policy in an asymmetric union also determines the optimal amount of purchases across regions. For lack of a better term we call this the *"optimal fraction"*, i.e., the share of long-term bonds that the central bank purchases from the Periphery. In our model with bond market asymmetries, an optimal allocation of QE purchases across regions is not only a function of each region's size, but also reflects different bond market characteristics of regions. Being a micro-founded object, it in fact additionally depends on further dimensions of heterogeneity, such as different degrees of (nominal) rigidities, transaction costs of QE, preferences, consumption and portfolio imbalances, etc.

We show all these results both analytically and numerically. First, at the aggregate portfolio level, the optimal fraction is decreasing in the elasticity of substitution between short- and long-term sub-portfolios. A higher "short-long friction" implies a larger term premium effect for a given amount of purchases and a greater marginal benefit for the union. As a result, policy favors purchases from the region with a lower elasticity of substitution. Second, the optimal fraction is non-monotonic in the share of short-term assets in aggregate portfolios. On the one hand, a larger "short share" implies less stimulus for a given decline in the term premium, because aggregate demand depends more on the short-term rate, which is binding at zero. The

[2] Allowing for cross-border holdings of assets of different maturities has shown to play a key role in shaping the macroeconomic effects in DSGE-model-based analyses of ECB QE in the EA (see Priftis and Vogel (2017); Kolasa and Wesolowski (2018); Hohberger *et al.* (2019)). Cross-border holdings also limit the effectiveness of QE on the domestic term premium in small open economies relative to large economies (see Kabaca (2016)).

central bank therefore optimally purchases more bonds from the region where the "short share" is larger. On the other hand, a larger "short share" also implies a smaller share of available long-term debt and therefore a greater term premium effect for a given level of bond purchases. Here, the central bank optimally purchases less bonds from the region with a larger "short share". We show that the first (second) effect dominates for high (low) levels of the"short share". Third, the optimal fraction is increasing in the share of domestic long-term bonds to foreign long-term bonds. The central bank optimally favors the region with a higher degree of "home bias", because it displays a higher reliance on domestic rates in order for aggregate demand and inflation to be stimulated. Fourth, the optimal fraction is increasing in the share of government debt-to-GDP. Higher debt implies a larger market for bonds, which in turn implies a larger share of long-term bonds in private portfolios. In order to have the same portfolio switching effect (the same change in portfolio shares), the central bank optimally places more weight on those bonds that are most supplied.

Finally, we calibrate our model to the EA economy and compare the model-consistent optimal policy, comprised by QE and interest rate setting, to a proxy of the actual policy by the ECB. To calibrate the parameters related to bond market characteristics we estimate a structural vector autoregression (SVAR) of the EA Periphery and Core using bond holdings data from the Security Holding Statistics (SHS) of the ECB. The SVAR is identified with theoretical sign restrictions on the movements of these bond holdings and term premia across regions. Our calibrated model predicts that central bank purchases of Peripheral debt according to the ECB's capital key (35% of bonds purchased from the Periphery) lie below our model's optimal fraction (57% of bonds purchased from the Periphery). Furthermore, optimal QE in our model is shorter-lived than the one implied by the capital key, which results in a different speed of the lift-off upon exit from the ZLB.

Overall, the predictions of our model suggest that the central bank should favor purchases from the region that faces "stronger frictions", as these amplify the transmission of QE to the real economy. These predictions are in line with the rationale of the recently implemented Pandemic Emergency Purchase Programme (PEPP) of the ECB, which was launched in March 2020 to tackle the economic repercussions from the COVID-19 pandemic in the EA. Under the PEPP, the ECB allows for a more flexible allocation of purchases across EA jurisdictions than what the benchmark capital key would suggest, thereby boosting activity in regions that have been harder hit by the pandemic.

Related literature

Our paper lies at the intersection of two literatures: one that focuses on model-based analyses of the macroeconomic impact of QE in closed and open economies,[3] and another that relates to optimal monetary

[3] This has been summarized in e.g., Hohberger *et al.* (2019) and includes the works of Chen *et al.* (2012); Gertler and Karadi (2013); Priftis and Vogel (2016); Andrade *et al.* (2016); Kolasa and Wesolowski (2018); Mouabbi and Sahuc (2019); Cova *et al.* (2019); Alpanda and Kabaca (2020), among others. See also Bhattarai and Neely (2016) for a recent review.

policy in open economies.[4] The most relevant studies focus on optimal unconventional monetary policy; however, works in this area are limited.

Harrison (2017) analyzes optimal QE under discretion through the lens of a closed economy model with nominal rigidities and portfolio adjustment costs. In a comparable setup, Harrison (2012) analyzes optimal QE under commitment. In turn, Karadi and Nakov (2020) build on Gertler and Karadi (2011) to analyze optimal asset-purchase policies in a macroeconomic model with banks that face occasionally-binding balance-sheet constraints. Darracq Paries and Kuhl (2016) study optimal commitment policies in a closed economy banking model of the EA from a timeless perspective and for a number of predetermined loss function specifications. Our paper can be seen as an extension of these works for the case of a monetary union, with a focus on the design of the optimal monetary policy under different types of asymmetries across regions. Our choice in modeling QE using the portfolio rebalancing channel derives from the bond market asymmetries we document in the data, which are particularly important in affecting its transmission through this channel but are not necessarily equally sensitive for other frameworks.[5] Despite the differences in frameworks, these works have similar predictions to the closed economy version of our model, in that the optimal combination of QE and interest rate policy leads to a stronger stabilization and to an earlier lift-off of the policy rate compared to the case of optimal interest rate policy alone.

To the best of our knowledge we are the first to focus on the question of the optimal allocation of QE purchases across regions. Closest to us can be considered the work by Bletzinger and von Thadden (2018), who discuss the effectiveness of QE in a monetary union by building a two-country model with portfolio frictions in the banking sector and a fiscal governance structure. To establish under which circumstances QE is effective under a ZLB, they consider a range of monetary union specifications and ask whether it is possible for QE to replicate the results of an unconstrained policy rule. Our model has a more quantitative focus and instead crucially analyzes the optimal policy problem under an occasionally-binding ZLB constraint; we derive the loss function of the central bank in a model-consistent fashion and focus on asymmetric portfolio rebalancing frictions across regions.

Finally, our work is also linked to Devereux and Sutherland (2007), who explore the role of monetary policy in an open economy with endogenous portfolio choice, as well as Devereux *et al.* (2020) who analyze the tradeoffs and characterize operational policy rules in a two-country open economy DSGE model with financial frictions.

We structure the paper as follows: Section 2 presents evidence on bond market asymmetries observed in

[4] Examples (with and without cooperation) include Benigno (2004); Pappa (2004); Corsetti and Pesenti (2005); Benigno and Lopez-Salido (2006); Lombardo and Sutherland (2006); Coenen *et al.* (2007); Gali and Monacelli (2008); Corsetti *et al.* (2010); Engel (2011), among others.

[5] See also Bhattarai *et al.* (2015b) for a QE model with the signaling channel.

the data. Section 3 outlines a model for a monetary union, section 4 discusses the transmission channels of unconventional monetary policy in our setup, and section 5 presents the optimal monetary policy problem under commitment. Section 6 discusses the calibration and presents the quantitative results from the solution of the optimal policy problem. Finally, section 7 summarizes the paper and concludes.

2 Asymmetries in Portfolio Characteristics

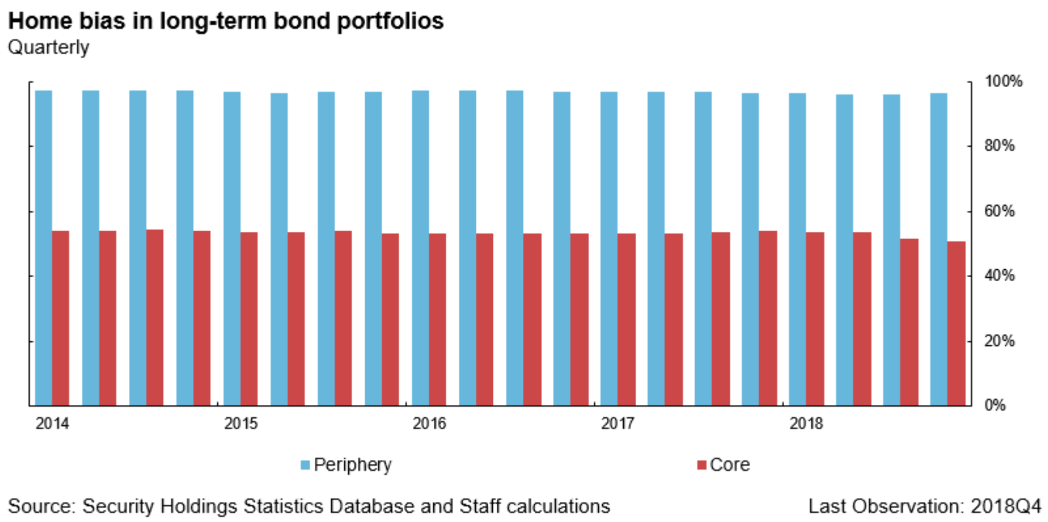
We take the euro area as being the prototypical example of a monetary union and document portfolio characteristics of Periphery and Core countries. Data on government bond holdings come from the ECB's SHS database, which reports quarterly holdings of Core and Peripheral government debt held by private agents in each region. The Periphery is represented by Italy, Portugal, and Spain, while the Core is represented by France, Germany, and the Netherlands. For each region, we retrieve data on holdings of both short- and long-term government debt securities. Long-term debt holdings represent government debt securities with maturities longer than one year, while short-term debt holdings represent government debt securities with a maturity less than one year. Following Chen *et al.* (2012) we also include monetary base holdings to the short-term portfolio holdings since i) at the ZLB, money and short-term government debt are almost perfect substitutes, and ii) the change in money holdings is an important indication of how portfolios are switching from long-term to short-term instruments following QE policy. Monetary holdings include banks' vault cash and reserves as well as currency outside of banks. Data on monetary holdings of banks are obtained from each country's central bank balance sheets and data on non-bank currency holdings are obtained from Eurostat's financial balance sheets. Lastly, the sample period is from 2013:Q4 to 2018:Q4 and reflects the availability of data from the SHS database.

Figure 1 illustrates the degree of home bias in long-term government bond portfolios for Periphery and Core residents.[6] There is a striking difference in home bias across regions: on average, domestic bonds make up 55% of Core bond portfolios, while they make up 97% of bond portfolios held by Periphery residents. Thus, Periphery residents hold on to their bonds while Core residents diversify more across the monetary union. In addition, home bias has been quite stable since the start of the ECB's QE policy in 2015. This means that while Periphery residents' portfolio return almost solely depends on their domestic bond return, Core residents enjoy returns from both regions of the area during the QE period.

Figure 2 illustrates the evolution of the share of short-term instruments in the overall portfolio for the Periphery and Core over the same period. Not surprisingly, short-term instruments steadily increase in portfolios as a result of an increased monetary base in the EA. However, the increase in the Core seems to be larger

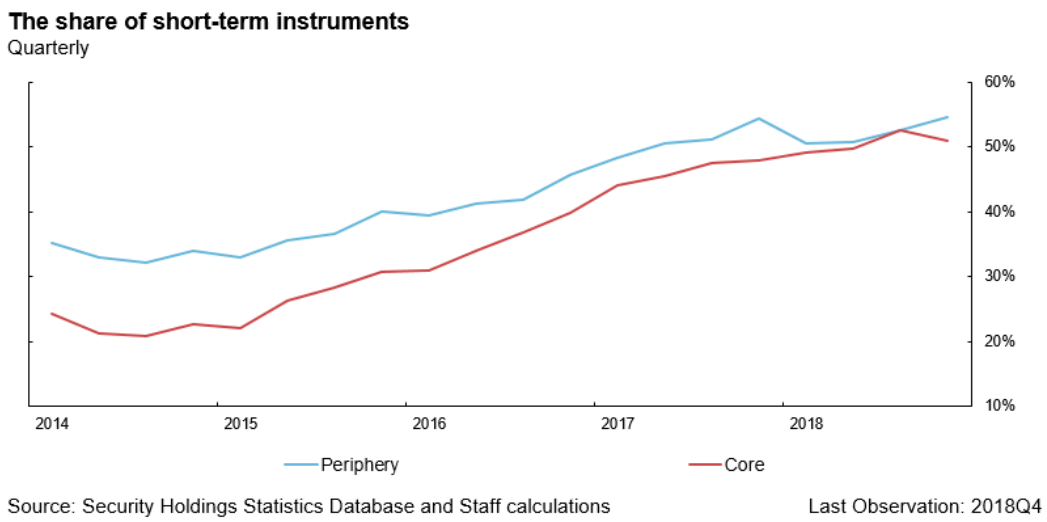
[6] Note that, for home bias, we concentrate on the long-term bond portfolio rather than short-term portfolios since about half of the short-term portfolio is composed of currency or reserve assets, and in a currency union, reserve assets are perfectly substitutable regardless of whether they are a liability to domestic or foreign governments.

Figure 1: Home Bias in the Long Share



Notes: Home bias is the share of domestic long-term bonds in the long-term sub-portfolio. The long-term sub-portfolio is the sum of domestic and foreign long-term debt holdings. Periphery consists of Spain, Italy, Portugal; Core consists of Germany, France, Netherlands.

Figure 2: Short Share



Notes: The short share is the ratio of short-term instruments to the overall portfolio. The total portfolio is defined as the sum of holdings of short- and long-term government liabilities. Short-term holdings include holdings of both short-term government debt securities and the monetary base. Periphery consists of Spain, Italy, Portugal; Core consists of Germany, France, Netherlands.

compared to the Periphery, implying a bigger shift from long- to short-term instruments for Core residents. Thus, QE led to a bigger portfolio rebalancing in the Core relative to the Periphery.

Moreover, regions differ in terms of their government debt size. From 2000:Q1 to 2013:Q4, Periphery debt averages 91% of GDP, while Core debt averages 70% of GDP. Thus, although the Periphery is smaller in economic size, the size of its government bond market in absolute terms is approximately equal to the size of the bond market of the Core.

Overall, these facts point to significant asymmetries in portfolio characteristics across regions, which we will exploit to calibrate our model with portfolio rebalancing in section 6.1. Notably, these relative differences across regions have important consequences for aggregate demand in each region and, thereby, on optimal union-wide QE policy as well.

3 A Model for a Monetary Union

The union is populated by a continuum of identical, infinitely lived households of measure 1 and consists of two regions, the home country (Periphery) and the foreign country (Core). Agents in the Periphery span the interval $[0, n]$ while agents in the Core span the interval $(n, 1]$. There is no migration. Each agent produces a single differentiated good and consumes the goods produced in both regions. Each household has access to all financial markets and can trade in assets of different maturities across borders. Monetary policy is conducted by the union-wide central bank controlling the nominal interest rate, which coincides with the rate on short-term assets. The central bank can also engage in unconventional policy through purchases of long-term government bonds. The fiscal authority in each region accumulates debt to finance lump-sum transfers. Since the regions are symmetric in terms of structure (but not in terms of parametrization), we only describe the problem of the Periphery. Variables denoted with an asterisk refer to the Core.

3.1 Households

The representative household i in the Periphery derives utility from consumption $c_t(i)$ and disutility from supplying labor $l_t(i)$ to domestic firms. The expected utility function is given by:

$$E_0 \left\{ \sum_{\tau=0}^{\infty} \beta^t \varepsilon_{d,t} \left[\log c_t(i) - \frac{l_t(i)^{1+\gamma}}{1+\gamma} \right] \right\} \tag{3.1}$$

where γ is the inverse of the Frisch elasticity of labor supply, β is the subjective discount factor at the steady state, and $\varepsilon_{d,t}$ is the discount factor shock. The representative household in the Periphery consumes home

(Peripheral) and foreign (Core) goods. The consumption bundle is a composite index described as:

$$c_t(i) = \left[\zeta^{\frac{1}{\kappa}} c_{H,t}(i)^{\frac{\kappa-1}{\kappa}} + (1-\zeta)^{\frac{1}{\kappa}} c_{F,t}(i)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{3.2}$$

where κ captures the intratemporal elasticity of substitution between Peripheral and Core goods, and ζ reflects the weights of domestic and foreign consumption goods in the aggregate bundle. ζ is a function of the relative size of the economy, n, and the degree of trade openness λ, so $1-\zeta = (1-n)\,\lambda$. $c_{H,t}(i)$ and $c_{F,t}(i)$ are the home (Peripheral) and foreign (Core) good consumption indices in the Periphery. Analogously in the Core, $c^*_{H,t}(i)$ and $c^*_{F,t}(i)$ are the home (Peripheral) and foreign (Core) consumption good indices. Consumption indices for household i in the Periphery are defined as:

$$c_{H,t}(i) = \left[\left(\frac{1}{n}\right)^{1/\theta} \int_0^n c_t(i,h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \; c_{F,t}(i) = \left[\left(\frac{1}{1-n}\right)^{1/\theta} \int_n^1 c_t(i,f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}} \tag{3.3}$$

where $c_t(i,h)$ and $c_t(i,f)$ are consumption of Periphery brand h and Core brand f by the Periphery household i at time t. θ is the elasticity of substitution of goods produced within the same region.

The real budget constraint of household i is given as:

$$\begin{aligned} & c_t\,(i) + b_{HS,t}(i) + b_{FS,t}(i) + q_{L,t} b_{HL,t}(i) + q^*_{L,t} b_{FL,t}(i) + [\Xi - \xi \alpha_t\,(i)] \\ & \leq w_t l_t\,(i) + \frac{R_{t-1} b_{HS,t-1}(i)}{\pi_t} + \frac{\psi_t R^U_{t-1} b_{FS,t-1}(i)}{\pi^*_t} + \frac{(1+\rho q_{L,t})\, b_{HL,t-1}(i)}{\pi_t} + \frac{\psi_t \left(1+\rho q^*_{L,t}\right) b_{FL,t-1}(i)}{\pi^*_t} \\ & + \frac{D_t}{\pi_t} + TR_t \end{aligned} \tag{3.4}$$

where w_t denotes the real wage, D_t are nominal profit transfers from firm ownership, TR_t are lump-sum transfers from the government, and π_t is gross inflation. Each household has access to all financial markets and can trade in assets of different maturities: domestic short-term bonds, $b_{HS,t}$; domestic long-term bonds, $b_{HL,t}$; foreign short-term bonds, $b_{FS,t}$; and foreign long-term bonds, $b_{FL,t}$.[7] The nominal returns for domestic and foreign short-term bonds are R_t and R^U_t, respectively, where R^U_t is the union-wide central bank policy rate. $\psi_t = P^*_t/P_t$ denotes relative price indices across regions, and $q_{L,t}$ and $q^*_{L,t}$ denote prices of domestic and foreign long-term bonds relative to the consumption good. Note that long-term bonds are modeled as perpetuities following Woodford (2001). Specifically, a long-term bond has a payment structure ρ^{T-t-1} for $T > t$ and $0 \leq \rho \leq 1$. Hence, the value of a long-term bond issued in period t, in any future period $t+j$, is

[7]We define: $b_{HS,t} = B_{HS,t}/P_t$, $b_{FS,t} = B_{FS,t}/P_t$, $b_{HL,t} = B_{HL,t}/P_t$, and $b_{FL,t} = B_{FL,t}/P_t$, where the capital letter, B, denotes the nominal quantities.

given by $q_{L,t+j}^{-j} = \rho^j q_{L,t+j}$, where ρ captures the maturity.[8] Nominal yields on long-term bonds can thus be expressed as $R_{L,t} = \frac{1}{q_{L,t}} + \rho$ and $R_{L,t}^* = \frac{1}{q_{L,t}^*} + \rho^*$. Finally, Ξ is assumed to be a constant cost that households incur for their transactions of goods or assets, and ξ measures the extent by which this cost can be reduced through holdings of liquid assets, α_t.

Our specification for α_t follows the approach in Alpanda and Kabaca (2020). A constant-elasticity-of-substitution (CES) aggregate of short-term and long-term bond portfolios, $\alpha_{S,t}$ and $\alpha_{L,t}$ constitutes the households aggregate portfolio α_t[9]:

$$\alpha_t\left(i\right) = \left[\zeta_\alpha^{\frac{1}{\kappa_\alpha}} \alpha_{S,t}(i)^{\frac{\kappa_\alpha - 1}{\kappa_\alpha}} + \left(1 - \zeta_\alpha\right)^{\frac{1}{\kappa_\alpha}} \alpha_{L,t}(i)^{\frac{\kappa_\alpha - 1}{\kappa_\alpha}}\right]^{\frac{\kappa_\alpha}{\kappa_\alpha - 1}} \tag{3.5}$$

where ζ_α is the share of short-term bonds in the aggregate portfolio and κ_α is the elasticity of substitution between short- and long-term bonds.

The household's short-term sub-portfolio, $\alpha_{S,t}$, is then another (nested) CES aggregate of short-term domestic bonds, $b_{HS,t}$, and short-term foreign bonds, $b_{FS,t}$:

$$\alpha_{S,t}\left(i\right) = \left[\zeta_S^{\frac{1}{\kappa_S}} b_{HS,t}(i)^{\frac{\kappa_S - 1}{\kappa_S}} + \left(1 - \zeta_S\right)^{\frac{1}{\kappa_S}} \left(\psi_t b_{FS,t}(i)\right)^{\frac{\kappa_S - 1}{\kappa_S}}\right]^{\frac{\kappa_S}{\kappa_S - 1}} \tag{3.6}$$

where $\zeta_S = 1 - \left(1 - n\right)\lambda_S$ is the share of domestic short-term bonds with λ_S being the degree of financial openness in short-term maturities, and κ_S the elasticity of substitution between domestic and foreign short-term bonds.

An additional nested CES aggregate of long-term domestic bonds, $b_{HL,t}$, and long-term foreign bonds, $b_{FL,t}$ constitutes the household's long-term portfolio $\alpha_{L,t}$:

$$\alpha_{L,t}\left(i\right) = \left[\zeta_L^{\frac{1}{\kappa_L}} \left(q_{L,t} b_{HL,t}(i)\right)^{\frac{\kappa_L - 1}{\kappa_L}} + \left(1 - \zeta_L\right)^{\frac{1}{\kappa_L}} \left(\psi_t q_{L,t}^* b_{FL,t}(i)\right)^{\frac{\kappa_L - 1}{\kappa_L}}\right]^{\frac{\kappa_L}{\kappa_L - 1}} \tag{3.7}$$

where $\zeta_L = 1 - \left(1 - n\right)\lambda_L$ is the share of domestic long-term bonds with λ_L being the degree of financial openness in short-term maturities, and κ_L the elasticity of substitution between domestic and foreign long-term bonds.

Because of the imperfect substitutability that the CES specifications introduce, changes in the relative supply of long-term bonds, both domestic and foreign (e.g., through QE) will be non-neutral. Instead, when $\kappa_S, \kappa_L, \kappa_\alpha \to \infty$, short- and long-term bonds are perfect substitutes and changes in the relative supply of long-term bonds do not matter.

[8] When $\rho = 0$ this asset collapses to a one-period bond, while for $\rho = 1$ this asset resembles a console.

[9] Alpanda and Kabaca (2020) introduce this CES bond specification into the household's utility function. As shown in their online appendix, this approach yields virtually the same quantitative results with the transaction cost approach taken here.

Household i chooses the sequences $\{c_t(i), l_t(i), b_{HS,t}(i), b_{FS,t}(i), b_{HL,t}(i), b_{FL,t}(i)\}_{t=0}^{\infty}$ to maximize expected lifetime utility (3.1) subject to the flow budget constraint (3.4) and the relevant no-Ponzi game constraints, given initial bond holdings, $b_{HS,t-1}(i), b_{FS,t-1}(i), b_{HL,t-1}(i), b_{FL,t-1}(i)$.

The first order conditions with respect to $c_t(i)$ and $l_t(i)$ read:

$$\frac{1}{c_t} = l_t^{\gamma} w_t \tag{3.8}$$

Next, the first order conditions with respect to $b_{HS,t}(i), b_{HL,t}(i), b_{FS,t}(i), b_{FL,t}(i)$ give:

$$\frac{\varepsilon_{d,t}}{c_t} = \beta E_t\left[\frac{R_t \varepsilon_{d,t+1}}{c_{t+1}\pi_{t+1}}\right] + \frac{\varepsilon_{d,t}}{c_t}\xi\left(\zeta_\alpha \frac{\alpha_t}{\alpha_{S,t}}\right)^{\frac{1}{\kappa_\alpha}}\left(\zeta_S \frac{\alpha_{S,t}}{b_{HS,t}}\right)^{\frac{1}{\kappa_S}} \tag{3.9}$$

$$\frac{q_{L,t}\varepsilon_{d,t}}{c_t} = \beta E_t\left[\frac{R_{L,t+1} q_{L,t+1}\varepsilon_{d,t+1}}{c_{t+1}\pi_{t+1}}\right] + \frac{\varepsilon_{d,t}}{c_t}\xi\left((1-\zeta_\alpha)\frac{\alpha_t}{\alpha_{L,t}}\right)^{\frac{1}{\kappa_\alpha}} q_{L,t}\left(\zeta_L \frac{\alpha_{L,t}}{q_{L,t} b_{HL,t}}\right)^{\frac{1}{\kappa_L}} \tag{3.10}$$

$$\frac{\psi_t\varepsilon_{d,t}}{c_t} = \beta E_t\left[\frac{\psi_{t+1} R_t^{U}\varepsilon_{d,t+1}}{c_{t+1}\pi^*_{t+1}}\right] + \frac{\varepsilon_{d,t}}{c_t}\xi\left(\zeta_\alpha \frac{\alpha_t}{\alpha_{S,t}}\right)^{\frac{1}{\kappa_\alpha}}\psi_t\left((1-\zeta_S)\frac{\alpha_{S,t}}{\psi_t b_{FS,t}}\right)^{\frac{1}{\kappa_S}} \tag{3.11}$$

$$\frac{\psi_t q^*_{L,t}\varepsilon_{d,t}}{c_t} = \beta E_t\left[\frac{\psi_{t+1} R^*_{L,t+1} q^*_{L,t+1}\varepsilon_{d,t+1}}{c_{t+1}\pi^*_{t+1}}\right] + \frac{\varepsilon_{d,t}}{c_t}\xi\left((1-\zeta_\alpha)\frac{\alpha_t}{\alpha_{L,t}}\right)^{\frac{1}{\kappa_\alpha}}\psi_t q^*_{L,t}\left((1-\zeta_L)\frac{\alpha_{L,t}}{\psi_t q^*_{L,t} b_{FL,t}}\right)^{\frac{1}{\kappa_L}} \tag{3.12}$$

The last two terms in eqs. (3.9) - (3.12) capture the portfolio rebalancing effects. As we show below, these drive the term premium, which depends on the relative holdings of bonds in households' portfolios and the elasticities of substitution between maturities and location.[10]

3.2 Firms

We assume a continuum of monopolistically competitive final good firms h in each region. Each firm produces the differentiated good $y_t(h)$ using the constant returns to scale technology:

$$y_t(h) = l_t(h) \tag{3.13}$$

Each firm sets one price for its good infrequently, as in Calvo (1983). By setting one price for its good, the firm does not engage in price discrimination as it does not distinguish between the domestic and the foreign market. This means that it takes into account the total demand for its product when setting its price. At each date, each firm changes its price with probability $1-\omega$. A firm that re-optimizes at time t chooses $P^*_{h,t}(h)$ to

[10] Note that the CES specification on bond holdings excludes corner solutions. That is, there will always be a well-defined term premium between short- and long-term assets. Alternative specifications of portfolio adjustment costs can allow for a zero term premium (see Alpanda and Kabaca (2020) and Chen *et al.* (2012) and references therein).

maximize expected discounted profits:

$$\max_{P^*_{h,t}(h)} E_t \sum_{\varsigma=0}^{\infty} \omega^{\varsigma} \Lambda_{t,t+\varsigma} \left\{ P^*_{h,t}(h) y_{t+\varsigma}(h) - w_{t+\varsigma}(h) l_{t+\varsigma}(h) \right\} \tag{3.14}$$

where $\Lambda_{t,t+\varsigma} = \beta^{\varsigma} \left(C_{t+\varsigma}/C_t\right)^{-\sigma} \left(P_{t+\varsigma}/P_t\right)$ is the stochastic discount factor. The firm maximizes (3.14) subject to (3.13) and the total demand for its product, $y_{t+\varsigma}(h)$, which reads as:

$$y_{h,t}\left(h\right) = \left(\frac{P_{h,t}\left(h\right)}{P_{h,t}}\right)^{-\theta} \left(\frac{P_{h,t}}{P_t}\right)^{-\kappa} \left(c_{h,t} + \left(\frac{1}{\psi_t}\right)^{-\kappa} c^*_{h,t}\right) \tag{3.15}$$

The first order condition that determines optimal price setting is then given by:

$$\frac{\theta}{\theta-1} E_t \sum_{\varsigma=0}^{\infty} \omega^{\varsigma} \Lambda_{t,t+\varsigma} \left(\frac{P^*_{h,t}\left(h\right)}{P_{h,t+\varsigma}}\right)^{-\theta} \left(\frac{P_{h,t+\varsigma}}{P_{t+\varsigma}}\right)^{-\kappa} y_{t+\varsigma} \left[\frac{P^*_{h,t}\left(h\right)}{P_{t+\varsigma}} - \frac{w_{t+\varsigma}(h)}{P_{t+\varsigma} A_{t+\varsigma}}\right] = 0$$

We focus on a symmetric equilibrium where firms choose a common price: $P^*_{h,t}\left(h\right) = P^*_{h,t}$. This implies that the optimal price level in the Periphery evolves according to:

$$P_{h,t} = \left[\omega P^{1-\theta}_{h,t-1} + (1-\omega) P^*_{h,t}(h)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.16}$$

3.3 Policy

3.3.1 Conventional monetary policy and central bank asset purchases

The central bank has two policy instruments: the policy rate and purchases of long-term government bonds from the Periphery and Core. Both instruments are chosen optimally under commitment (see section 5.2). We let the short-term interest rate be occasionally constrained by the zero lower bound $\widehat{R}^U_t \geq \underline{R^{zlb}}$.

A quantitative easing policy for the Periphery is the purchase of long-term government bonds by the central bank, $q_{L,t} b^{CB}_{L,t}$, financed with issuance of new short-term debt, $b^{CB}_{S,t}$. The returns on long-term bond purchases, $\frac{q_{L,t} R_{L,t}}{\pi_t} b^{CB}_{L,t-1}$, are then transferred back to the government in the form of lump-sum transfers, z_t. This renders the effect of the QE policy neutral on fiscal balances, i.e., it circumvents the financing of the program via lump-sum taxes from households and isolates the portfolio rebalancing channel of QE. Net receipts from central bank purchases, z_t, which are then transferred back to the government, are given by:

$$z_t = \frac{q_{L,t} R_{L,t}}{\pi_t} b^{CB}_{L,t-1} - q_{L,t} b^{CB}_{L,t} + b^{CB}_{S,t} - \frac{R^U_{t-1}}{\pi_t} b^{CB}_{S,t-1} - \Gamma_t$$

which simplify to

$$z_t = \frac{q_{L,t} R_{L,t}}{\pi_t} b^{CB}_{L,t-1} - \frac{q_{L,t-1} R^U_{t-1}}{\pi_t} b^{CB}_{L,t-1} - \Gamma_t$$

since $q_{L,t}b_{L,t}^{CB} = b_{S,t}^{CB}$. We follow Gertler and Karadi (2011) and Karadi and Nakov (2020) and assume the central bank pays a quadratic efficiency cost τ on the square of the government bonds it purchases, so:

$$\Gamma_t = \frac{\tau}{2}\left(\frac{q_{L,t}b_{L,t}^{CB}}{q_L b_L}\right)^2$$

This cost reflects the notion that the central bank faces several distortions, such as political costs and other implementation constraints (e.g., costs of maintaining a large balance sheet or identifying preferred government sector markets) when purchasing long-term government bonds. More specifically, we express the cost in terms of the share of outstanding long-term government debt. This reflects differences in the implementation of QE in bond markets of different size. For instance, 1-billion euro purchases can be more easily performed in a larger and deeper market compared to a smaller and shallower market. This is different to Karadi and Nakov (2020) who study QE in a closed economy.

Analogously, in the Core, the central bank purchases $q_{L,t}^{*}b_{L,t}^{*,CB}$ and finances them with issuance of new foreign short-term debt $b_{S,t}^{*,CB}$. By providing these transaction services it faces a cost of intermediation, Γ_t^{*}. The central bank's balance sheet is therefore given by: $q_{L,t}^{*}b_{L,t}^{*,CB} + q_{L,t}b_{L,t}^{CB} = b_{S,t}^{CB} + b_{S,t}^{*,CB}$.

Our focus will be on the optimal responses of $q_{L,t}b_{L,t}^{CB}$ and $q_{L,t}^{*}b_{L,t}^{*,CB}$, and in particular how these evolve asymmetrically across regions. For this purpose, it is useful to define the following object, which we label the "optimal fraction" (OF_t):

$$OF_t = \frac{nq_{L,t}b_{L,t}^{CB}}{nq_{L,t}B_{L,t}^{CB} + (1-n)\,q_{L,t}^{*}B_{L,t}^{*,CB}} = \frac{nq_{L,t}b_{L,t}^{CB}}{nq_{L,t}b_{L,t}^{CB} + (1-n)\,q_{L,t}^{*}b_{L,t}^{*,CB}\psi_t} \tag{3.17}$$

where bond quantities are in real terms. In our setting, the optimal fraction is dependent on country size, but crucially also on structural parameters of the model related to portfolio frictions, steady-state debt levels, price rigidities, etc. In a purely symmetric monetary union, where structural parameters are identical across equally-sized countries, $OF_t = 0.5$ for $t \geq 0$. To the extent that regions are structurally asymmetric as well, $OF_t \neq 0.5$ for $t \geq 0$.

3.3.2 Government

The fiscal authority issues both short- and long-term bonds. Together with transfers from the central bank, z_t, they finance net transfers to households, TR_t, and interest payments on debt. The government budget constraint is then specified as follows:

$$b_{S,t} + q_{L,t}b_{L,t} + z_t = \frac{R_{t-1}b_{S,t-1}}{\pi_t} + \frac{q_{L,t}R_{L,t}b_{L,t-1}}{\pi_t} + TR_t \tag{3.18}$$

We assume that the government keeps the total amount of debt, b, and its composition fixed, which implies a constant government bond supply in both short- and long-term maturities:

$$b_{S,t} + q_{L,t} b_{L,t} = b_S + q_L b_L = b \tag{3.19}$$

$$b_{S,t} = \vartheta q_{L,t} b_{L,t} \tag{3.20}$$

3.4 Market clearing

The bond market clearing conditions for short-term and long-term debt are given by eqs. (3.21)-(3.22) for the Periphery and eqs. (3.23) - (3.24) for the Core.

$$n b_{S,t} + n b_{S,t}^{CB} = n b_{HS,t} + (1-n)\, b_{HS,t}^{*} \tag{3.21}$$

$$n b_{L,t} = n b_{HL,t} + (1-n)\, b_{HL,t}^{*} + n b_{L,t}^{CB} \tag{3.22}$$

$$(1-n)\, b_{S,t}^{*} + (1-n)\, b_{S,t}^{*,CB} = n b_{FS,t}^{*} + (1-n)\, b_{FS,t} \tag{3.23}$$

$$(1-n)\, b_{L,t}^{*} = (1-n)\, b_{FL,t}^{*} + n b_{FL,t} + (1-n)\, b_{L,t}^{*,CB} \tag{3.24}$$

Note that $b_{S,t}^{CB}$ enters the left-hand side of the equation because both the government and the central bank can issue these instruments, which would increase total short-term instruments held by private agents following a QE policy. In addition, the goods market clearing conditions for each good are given by:

$$n y_t = n c_{h,t} + (1-n) c_{h,t}^{*} + -n\left[\Xi - \xi a_t\right] + n\Gamma_t \tag{3.25}$$

$$(1-n)\, y_t^{*} = n c_{f,t} + (1-n) c_{f,t}^{*} - (1-n)\left[\Xi^{*} - \xi^{*} a_t^{*}\right] + (1-n)\,\Gamma_t^{*} \tag{3.26}$$

These conditions ensure that total output supplied is used for households' consumption, transaction services, as well as services related to the central bank's QE purchases.

Finally, we combine the market clearing conditions with the budget constraints of households and governments in both regions, as well as the zero-profit condition of firms, to obtain the law of motion for the Periphery's net foreign assets position (in real terms):

$$\begin{aligned} & n\left(\psi_t b_{FS,t} - \frac{\psi_t R_{t-1}^{U} b_{FS,t-1}}{\pi_t^{*}}\right) + n\left(\psi_t q_{L,t}^{*} b_{FL,t} - \frac{\psi_t R_{L,t}^{U} q_{L,t}^{*} b_{FL,t-1}}{\pi_t^{*}}\right) - (1-n)\left(\frac{b_{HS,t}^{*}}{\psi_t} - \frac{R_{t-1} b_{HS,t-1}^{*}}{\psi_t \pi_t}\right) \\ & - (1-n)\left(\frac{q_{L,t} b_{HL,t}^{*}}{\psi_t} - \frac{R_{L,t} q_{L,t} b_{HL,t-1}^{*}}{\psi_t \pi_t}\right) = (1-n) p_{h,t} c_{h,t}^{*} - n p_{f,t} c_{f,t} \end{aligned} \tag{3.27}$$

3.5 Equilibrium

All the equilibrium conditions can be seen in Appendix A, which, similar to the optimality conditions described here, also includes analogous optimality conditions for the Core. We repeat the definition of equilibrium here for completeness. The equilibrium consists of a block similar to the canonical New Keynesian model, in addition to a set of equations that determine bond holdings and returns as well as balance of payments and a zero lower bound constraint.

Definition 1. An imperfectly competitive equilibrium is a sequence of stochastic processes $\mathcal{X}_t \equiv \{\hat{c}_t, \hat{c}_t^*,$

$\hat{\pi}_{h,t}, \hat{\pi}^*_{f,t}, s_t, \widehat{b}_{HS,t}, \widehat{b}_{FS,t}, \widehat{qb}_{HL,t}, \widehat{qb}_{FL,t}, \widehat{b}^*_{HS,t}, \widehat{b}^*_{FS,t}, \widehat{qb}^*_{HL,t}, \widehat{qb}^*_{FL,t}\}$ that satisfy the conditions in Appendix A, given monetary policies $\mathcal{P}_t \equiv \left\{\hat{R}_t^U, \widehat{qb}^{CB}_{L,t}, \widehat{qb}^{*CB}_{L,t}\right\}$, the exogenous process r_t^n, and initial conditions $\mathcal{I}_{-1} = \{b_{HS,-1}, b_{FS,-1}, b_{HL,-1}, b_{FL,-1}, s_{t-1}\}$, for $t \geq 0$.

4 Transmission Mechanisms of Monetary Policy

The channels operating in our model can be illustrated using the optimality conditions for the Periphery, but analogous effects are also present in the Core. We solve the model by log-linearizing the private sector equilibrium around a deterministic, symmetric steady state. For any variable $\hat{X}_t = \ln\left(X_t/\bar{X}\right)$, except for the value of the central bank's QE purchases $\widehat{qb}^{CB}_{L,t} = qb^{CB}_{L,t} - q\bar{b}^{CB}_{L}$. Note that the terms of trade is given by $s_t = (1-\zeta-\zeta^*)\,\psi_t$, and its change represents relative inflation between Periphery and Core goods: $\widehat{s}_t - \widehat{s}_{t-1} = \left(\widehat{\pi}_{h,t} - \hat{\pi}^*_{f,t}\right)$.

4.1 Term premia

By combining the log-linearized first order conditions (FOCs) for domestic short-term (3.9) and domestic long-term bonds (3.10) and substituting bond prices $\widehat{q}_{L,t}$, with the long-term rate $\hat{R}_{L,t}$, we obtain the following expression:

$$\frac{R_L\widehat{R}_{L,t} - \rho E_t \widehat{R}_{L,t+1}}{R_L - \rho} = \widehat{R}_t + \left(\frac{\pi}{\beta R} - 1\right)\hat{T}_t \tag{4.1}$$

where

$$\hat{T}_t = \left(\frac{1}{\kappa_\alpha}\right)(\hat{\alpha}_{L,t} - \hat{\alpha}_{S,t}) + \left(\frac{1}{\kappa_S}\right)\left(\hat{\alpha}_{S,t} - \hat{b}_{HS,t}\right) - \left(\frac{1}{\kappa_L}\right)\left(\hat{\alpha}_{L,t} - \hat{q}_{L,t} - \hat{b}_{HL,t}\right)$$

By iterating on (4.1) and re-arranging we obtain an expression for long-term yields:

$$\hat{R}_{L,t} = \left(1 - \frac{\rho}{R_L}\right) E_t \sum_{s=0}^{\infty}\left(\frac{\rho}{R_L}\right)^s \left[\hat{R}_{t+s} + \left(\frac{\pi}{\beta R} - 1\right)\hat{T}_{t+s}\right] \tag{4.2}$$

Eq. (4.2) highlights the relationship between returns of assets of different maturities for households in the Periphery. It shows that the long-term rate can be expressed as the sum of expected future short-term rates and a term premium that agents require when switching between portfolios. The term premium arises because of imperfect substitutability between assets of different maturities within and across borders, and it is a direct consequence of the CES structure of the aggregate portfolio, $\hat{\alpha}_t$, and short-term, $\hat{\alpha}_{S,t}$, and long-term, $\hat{\alpha}_{L,t}$, sub-portfolios. Because households also have access to foreign short- and long-term bonds, the term premium consists of three components, each capturing the imperfect substitutability within each (sub-)portfolio, dictated by the strength of κ_α, κ_S, and κ_L. In the extreme case of perfect substitutability (i.e., $\kappa_\alpha, \kappa_S, \kappa_L \to \infty$), the term premium collapses to zero and the long-term rate is simply the sum of expected future short-term rates. Otherwise, the dynamic behavior of $\hat{T}_t$ and consequently $\hat{R}_{L,t}$ is determined by portfolio shares: i) the portfolio holdings of long-term bonds $\hat{\alpha}_{L,t}$ relative to short-term bonds $\hat{\alpha}_{S,t}$, ii) the portfolio holdings of short-term bonds $\hat{\alpha}_{S,t}$ relative to domestic short-term bond holdings $\hat{b}_{HS,t}$, and iii) the portfolio holdings of long-term bonds $\hat{\alpha}_{L,t}$ relative to domestic long-term bond holdings $\hat{b}_{HL,t}$.

The endogenous structure of the term premium captures the effects of changes in the supply of long-term bonds. If the central bank purchases Periphery long-term bonds, then Periphery households will face a drop in the supply of their long-term bonds, $\hat{b}_{HL,t}$. This lowers the term premium and hence the interest rate on those assets, ceteris paribus. The lower the substitutability between home and foreign long-term assets κ_α, the larger the drop in the term premium.

However, household behavior may mitigate the effects of the reduction in the supply of long-term bonds engineered by the central bank. For a given drop in the supply of Periphery long-term bonds $\hat{b}_{HL,t}$, the drop in the term premium may be partly offset by a switch to foreign long-term bonds $\hat{b}_{FL,t}$ (and hence $\hat{\alpha}_{L,t}$). In contrast, if households alter their portfolio in favor of short-term bonds $\hat{b}_{HS,t}$, the drop in the term premium is instead amplified. In sum, the nature of portfolio rebalancing taking place following a central bank intervention can work either in favor of or against the intended effects on real rates.

4.2 Arbitrage between home and foreign bonds

We now discuss the relationships between home and foreign yields at each maturity. By log-linearizing and combining the FOCs for domestic short-term bonds (3.9) and foreign short-term bonds (3.11), we obtain the no-arbitrage condition with respect to short-term bonds:

$$\widehat{R}_t - \widehat{R}_t^U = \left(\frac{\pi}{\beta R} - 1\right) \frac{1}{\kappa_S} \left(\widehat{b}_{HS,t} - \left(\widehat{\psi}_t + \widehat{b}_{FS,t}\right)\right), \tag{4.3}$$

which illustrates how, as a result of limits to arbitrage, Periphery and Core yields differ from each other. Note that when $\kappa_S \to \infty$, short-term rates are equalized across regions for every period. As we discuss in section

6.1, we calibrate κ_S to a high value, ensuring that the union-wide central bank can set the same short-term rate (i.e., the policy rate) across the union in both the Periphery and Core.

In turn, by log-linearizing and combining the FOCs for domestic long-term bonds (3.10) and foreign long-term bonds (3.12) we obtain an analogous no-arbitrage condition for long-term bonds:

$$\frac{R_L\widehat{R}_{L,t} - \rho E_t\widehat{R}_{L,t+1}}{R_L - \rho} - \frac{R_L\widehat{R}^*_{L,t} - \rho E_t\widehat{R}^*_{L,t+1}}{R_L - \rho} = \left(\frac{\pi}{\beta R} - 1\right)\frac{1}{\kappa_L}\left[(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - \widehat{\psi}_t) - (\widehat{q}^*_{L,t} + \widehat{b}_{FL,t})\right] \tag{4.4}$$

Similarly, this equation highlights the one-period holding return differentials between home and foreign long-term bonds, governed by the elasticity of substitution between Periphery and Core bonds in the long-term sub-portfolio, κ_L.

The above equations can be used to obtain a relationship between $\hat{T}_t$ and $\hat{T}^*_t$, which determines the link between domestic and foreign term premia:

$$\hat{T}_t = \hat{T}^*_t + \frac{1}{\kappa_L}\left[(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - \widehat{\psi}_t) - (\widehat{q}^*_{L,t} + \widehat{b}_{FL,t})\right]$$

assuming $\kappa_S \to \infty$. This expression shows that in order for the supply of Periphery long-term bonds to increase relative to the supply of Core long-term bonds, households must be compensated by a higher Periphery term premium. Note that when long-term bonds are perfect substitutes (i.e., $\kappa_L \to \infty$), Periphery and Core term premia are equalized.

4.3 Aggregate demand

How do the financial returns affect domestic demand? In order to illustrate the transmission of changes in term premia to the real economy, we combine the FOCs with respect to all bonds (3.9) - (3.12), as well as with the marginal utility of consumption, to obtain:

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - (PR_t - E_t\widehat{\pi}_{t+1} - r^n_t) \tag{4.5}$$

where

$$PR_t = \zeta_a\zeta_S\widehat{R}_t + (1-\zeta_a)\zeta_L\left(\widehat{R}_t + \widehat{T}_t\right) - \zeta_a(1-\zeta_S)\widehat{R}^U_t + (1-\zeta_a)(1-\zeta_L)\left(\widehat{R}^U_t + \widehat{T}^*_t\right) \tag{4.6}$$

Eq. (4.5) is the Euler equation for the Periphery and shows how consumption and inflation in the Periphery depend on the gap between portfolio returns (eq. 4.6) and the natural rate of interest r^n_t, which follows an AR(1) process $r^n_t = \rho_{rn} r^n_{t-1} + \varepsilon^{rn}_t$.[11] PR_t denotes the weighted average of returns from each bond in the

[11] Note that the discount factor shock is rescaled as $r^n_t = \widehat{\varepsilon}_{d,t} - E_t\widehat{\varepsilon}_{d,t+1}$ so that the shock can be interpreted as a deviation from

household's portfolio. If the household's portfolio is weighted more towards long-term bonds ($\zeta_\alpha < 0.5$), the impact from long-term yields becomes greater. Conversely, as households hold more short-term assets ($\zeta_\alpha > 0.5$), total portfolio returns are more heavily weighted by the policy rate set by the central bank. Similar arguments apply for the relative holdings of domestic bonds, through ζ_S and ζ_L. Thus, higher home bias in portfolios implies higher effects of domestic yields on aggregate demand.

As is standard, (4.5) shows how conventional monetary policy (i.e., a fall in the short-term rate $\hat{R}_t^U$) lowers private consumption. However, unconventional monetary policy (i.e., purchases of long-term bonds by the union-wide central bank) generate an additional effect that operates through the term premium. Namely, they lower the term premium component of long-term yields and strengthen consumption demand.

Finally, note that when all assets are perfectly substitutable, all asset classes yield the same returns. Thus, when $\kappa_\alpha, \kappa_S, \kappa_L \to \infty$, the portfolio components in the no-arbitrage conditions (eqs. 4.3 and 4.4) disappear; therefore $\widehat{R}_t = \widehat{R}_t^U = \widehat{R}_t + \widehat{T}_t = \widehat{R}_t^U + \widehat{T}_t^*$. In this frictionless setup, the model generates the same Euler condition as in the standard closed economy New Keynesian model: $\widehat{c}_t = E_t\widehat{c}_{t+1} - \left(R_t - E_t\widehat{\pi}_{t+1} - r_t^n\right)$. In the absence of these frictions, long-term government bond purchases are neutral, and monetary policy can only influence aggregate demand through changes in the short-term interest rate.

5 Optimal Monetary Policy

We use the linear-quadratic approach, as in Woodford (2003), to derive the quadratic welfare-theoretic loss function of the union-wide central bank.

5.1 Welfare loss function

The loss function of the union-wide central bank is a weighted average of the welfare losses of the two regions with weights equal to their sizes:

$$L_t = nL_t^P + (1-n)\, L_t^C \tag{5.1}$$

where L_t^P and L_t^C are the welfare losses in the Periphery and Core, respectively. The welfare loss function in each region is derived from a second order approximation to the utility function of the representative household in the Periphery and the Core. Working in this fashion yields the welfare-theoretic loss function of the union-wide central bank as summarized in proposition 1 below.

Proposition 1. *The discounted sum of the utilities of households in the union is given by:*

the natural rate of interest. Given that the level of productivity, and thus potential output, is kept fixed in the model, a shock to the discount factor is the only cause of movement in the natural rate. A relatively large negative shock to r_t^n drives the economy to the zero lower bound, $\widehat{R}_t^U = -R^{zlb}$.

$$L_t = -\frac{1}{2}\frac{U^*_{c^*}}{y+y^*}\Bigg\{\Phi_y\left(\widehat{y}_t\right)^2 + \Phi_{y^*}\left(\widehat{y}^*_t\right)^2 + \Phi_{\pi^h}\left(\widehat{\pi}_{h,t}\right)^2 + \Phi_{\pi^{f^*}}\left(\widehat{\pi}^*_{f,t}\right)^2 + \Phi_s\left(\widehat{\psi}_t\right)^2$$
$$- \Phi_{c^*}\left(\widehat{c}^*_t\right)^2 - \Phi_c\left(\widehat{c}_t\right)^2 - \Phi_a\left(\widehat{\alpha}_t\right)^2 - \Phi_{a^*}\left(\widehat{\alpha}^*_t\right)^2 + \frac{\tau n}{2}\left(\frac{\widehat{qb}^{cb}_{L,t}}{q_L b_L}\right)^2 + \frac{\tau(1-n)}{2}\left(\frac{\widehat{qb}^{*,cb}_{L,t}}{q^*_L b^*_L}\right)^2 \tag{5.2}$$
$$+ \Omega_{ij}\sigma_{ij,t}\Bigg\} + t.i.p. + O(||\xi||^3)$$

where

$\Phi_y = \frac{n}{2}\left(ny^2 + \gamma\left(y+y^*\right)\right); \quad \Phi_{y^*} = \frac{(1-n)}{2}\Big(\left(1-n\right)\left(y^*\right)^2 + \gamma^*\left(y+y^*\right)\Big)$
$\Phi_{\pi^h} = \frac{n(y+y^*)}{2}\frac{\theta\omega}{(1-\omega)(1-\omega\beta)}; \quad \Phi_{\pi^{f^*}} = \frac{(1-n)(y+y^*)}{2}\frac{\theta^*\omega^*}{(1-\omega^*)(1-\omega^*\beta)}$
$\Phi_s = \frac{(1-n)^2}{2}\Big(\left(y^*\right)^2 - \left(c^*\right)^2\Big)\left(1-\zeta-\zeta^*\right)$
$\Phi_c = \frac{n^2}{2}c^2; \quad \Phi_{c^*} = \frac{(1-n)^2}{2}c^{*2}; \quad \Phi_a = \frac{n^2}{2}\xi^2\alpha^2; \quad \Phi_{a^*} = \frac{(1-n)^2}{2}\xi^{*2}\alpha^{*2}$ *and where* Ω_{ij} *and* $\sigma_{ij,t}$ *are matrices of coefficients and covariance terms, respectively, for* $i, j = \{\hat{c}_t, \hat{c}^*_t, s_t, \hat{y}_t, \hat{y}^*_t, \widehat{a}_t, \widehat{a}^*_t\}$, *when* $i \neq j$.

Proof. In Appendix C. ■ □

The terms in the loss function illustrate the various distortions present in the model. Inflation of the two regions, $\widehat{\pi}_{h,t}$ and $\widehat{\pi}^*_{f,t}$, appear in the loss function because sticky prices cause an inefficient dispersion in prices and in the production of goods. The output gaps of both regions, $\widehat{y}_t$ and $\widehat{y}^*_t$ (defined as the gap between output and its (efficient) level in the flexible price equilibrium), appear in the loss function because sticky prices and monopolistic competition also cause an inefficiency at the aggregate level. These two terms are standard in the literature (see e.g., Woodford (2003)) and also present in the closed economy version of our model (i.e., when foreign assets are unavailable and countries are symmetric in terms of structure). Moreover, as e.g., Benigno (2004) and Ferrero (2009) have shown, the open economy formulation brings an additional, cross-country dimension into the problem described above. In a monetary union, each region faces a fixed exchange rate, resulting in sticky prices leading to a cross-country distortion in relative prices. Therefore the terms of trade, s_t, also appears in the loss function, allowing for welfare gains from its stabilization.[12]

The remaining terms that appear in the loss function are a result of the bond market imperfections. Consumption of the two regions, $\widehat{c}_t$ and $\widehat{c}^*_t$, appear in the loss function because risk-sharing is imperfect across countries, following from the imperfect substitutability between assets as well as transaction costs. These terms are also present in other open economy models with incomplete markets, such as Benigno (2009) among others. The remaining two terms are specific to our model. First, household portfolios, α_t and α^*_t, appear in the loss function because households incur transaction costs, which can be alleviated by carrying

[12]Note that we have used the definition: $\hat{\psi}_t = \left(1-\zeta-\zeta^*\right)\widehat{s}_t$.

liquid assets in their portfolio. This distortion can be mitigated by stabilizing supplies of bonds and the rate at which portfolio holdings change.[13] Second, central bank purchases of long-term government bonds, $\widehat{qb}_{L,t}^{CB}$ and $\widehat{qb}_{L,t}^{CB*}$, appear in the loss function because these purchases consume real resources. Finally, the cross-factors $\Omega_{ij}\sigma_{ij,t}$, with $i,j = \{\hat{c}_t, \hat{c}_t^*, s_t, \hat{y}_t, \hat{y}_t^*, \widehat{a}_t, \widehat{a}_t^*\}$, for $i \neq j$, represent an additional international dimension and imply further welfare effects that originate from the correlation of cross-country variables.

Given that the loss function is micro-founded, the weights on the various policy targets are functions of structural parameters and meaningful from a model-consistent perspective. First, the weights on inflation, Φ_{π^h} and $\Phi_{\pi^{f*}}$, depend on the level of price stickiness, $\{\omega, \omega^*\}$, as well as on the elasticity of substitution across varieties, $\{\theta, \theta^*\}$. The latter determines how relative price dispersion across varieties translates into relative dispersion of output. Second, the weight on the output gap, Φ_{y^h} and Φ_{y^*}, depends on the elasticity of labor, $\{\gamma, \gamma^*\}$, as this governs the disutility from supplying labor for households.[14]

Third, the weight on the terms of trade, Φ_s, depends on the share of domestic goods in the aggregate consumption basket, $\{\zeta, \zeta^*\}$, and the steady states of foreign output and consumption, c^* and y^*. The share ζ dictates openness to trade and hence determines the pass-through of dispersion of foreign prices to domestic production. Regarding c^* and y^*, note that after algebraic manipulations, they can be written as:

$$c^* = \left[1 - \Psi^* + \frac{n}{(1-n)\,\delta_2^*}\left(\Psi - \delta_1 \frac{(1-n)\,\Psi^*\delta_2^* - n\Psi\delta_1^*}{n\,(\delta_2\delta_2^* - \delta_1\delta_1^*)}\right)\right]^{\frac{\gamma^*}{\gamma^*+1}} (1-\phi^*)^{\frac{1}{\gamma^*+1}}\,; \quad y^* = \left(\frac{1-\phi^*}{c^*}\right)^{\frac{1}{\gamma^*}}$$

where $\delta_1 = \zeta_S\zeta_\alpha + \zeta_L(1-\zeta_\alpha)$; $\delta_2 = (1-\zeta_S)\,\zeta_\alpha + (1-\zeta_L)(1-\zeta_\alpha)$; $\delta_1^* = \zeta_S^*\zeta_\alpha^* + \zeta_L^*(1-\zeta_\alpha^*)$; and $\delta_2^* = (1-\zeta_S^*)\,\zeta_\alpha^* + (1-\zeta_L^*)(1-\zeta_\alpha^*)$. Through these terms, the weight Φ_s summarizes how the policy objective of terms of trade stability depends on the shares of short-to-long-term bonds, $\{\zeta_a, \zeta_a^*\}$, the shares of domestic-to-foreign short-term bonds, $\{\zeta_S, \zeta_S^*\}$, and the shares of domestic-to-foreign long-term bonds, $\{\zeta_L, \zeta_L^*\}$. It also depends on the level of government debt $\{\Psi, \Psi^*\}$, but also the steady-state net markup ϕ^*. The shares determine the effectiveness of government bond purchases by the central bank in each region, and hence the extent to which regional prices are affected. And these effects are only possible if the available government debt is high. Finally, the effect also varies with the strength of the distortion from monopolistic competition.

The weights on consumption, Φ_c and Φ_{c^*}, also depend on the steady states of consumption, both in the Periphery and in the Core, c and c^*. Note that:

$$c = \left[1 - \Psi + \frac{(1-n)\,\Psi^*\delta_2^* - n\Psi\delta_1^*}{n\,(\delta_2\delta_2^* - \delta_1\delta_1^*)}\right]^{\frac{\gamma}{\gamma+1}} (1-\phi)^{\frac{1}{\gamma+1}}\,; \quad y = \left(\frac{1-\phi}{c}\right)^{\frac{1}{\gamma}}$$

[13] These portfolio holdings are similar in spirit to the debt terms that appear in Curdia and Woodford (2010); Fiore and Tristani (2013); Bhattarai *et al.* (2015c), which capture the inefficiency caused by financial frictions and interest rate spreads.

[14] We have assumed log-consumption in the household's utility function. In a more general specification with an intertemporal elasticity of substitution $\sigma \neq 1$, σ would appear in Φ_{y^h} and Φ_{y^*}, but also in Φ_c and Φ_{c^*}.

which again summarizes how regional government bond purchases affect private consumption stabilization in each region through $\{\zeta_a, \zeta_a^*\}$, $\{\zeta_S, \zeta_S^*\}$, and $\{\zeta_L, \zeta_L^*\}$.

Next, the weights on portfolio holdings, Φ_a and Φ_{a^*}, depend on the strength of portfolio benefits in the households' problem, $\{\xi, \xi^*\}$, and on the steady-state values of portfolio holdings, a and a^*. Both terms determine the rate at which portfolio holdings change when bond supplies are altered. Note again that, following algebraic manipulations, a and a^* can be written as:

$$a = \frac{\overline{\pi}}{\overline{R} - \overline{\pi}} \left(\frac{(1-n)\,\Psi^* \delta_2^* - n\Psi\delta_1^*}{n\,(\delta_2\delta_2^* - \delta_1\delta_1^*)} \right) \left(\frac{1-\phi}{c} \right)^{\frac{1}{\gamma}}$$
$$a^* = \frac{n\overline{\pi}}{(1-n)\left(\overline{R} - \overline{\pi}\right)\delta_2^*} \left(\Psi - \delta_1 \frac{(1-n)\,\Psi^* \delta_2^* - n\Psi\delta_1^*}{n\,(\delta_2\delta_2^* - \delta_1\delta_1^*)} \right) \left(\frac{1-\phi^*}{c^*} \right)$$

Finally, the weight on central bank purchases of long-term government bonds, $\widehat{qb}_{L,t}^{CB}$ and $\widehat{qb}_{L,t}^{CB*}$, depends on the strength of the resource costs that are consumed. These are dictated by τ. When $\tau = 0$, quantitative easing is costless in the model, and the central bank can perfectly stabilize the economy, provided that the remaining distortions are mitigated (see discussion below). A variant of these terms also appear in the welfare-theoretic loss function of Harrison (2012) and Harrison (2017), but also in Darracq Paries and Kuhl (2016), who consider a form of preference-guided unconventional monetary policy.[15]

We end this subsection with comparative statics regarding policy trade-offs that the central bank is facing. The results are intuitive and follow from the discussion above.

Proposition 2. *Inflation stabilization becomes more important relative to the other policy objectives as i) the degree of price stickiness increases* $\frac{\partial \Phi_{\pi^h}}{\partial \omega} > 0$; $\frac{\partial \Phi_{\pi^{f*}}}{\partial \omega^*} > 0$, *and/or ii) the elasticity of substitution across varieties increases* $\frac{\partial \Phi_{\pi^h}}{\partial \theta} > 0$, $\frac{\partial \Phi_{\pi^{f*}}}{\partial \theta^*} > 0$. *Output stabilization becomes relatively more important as i) the inverse of the elasticity of labor supply increases* $\frac{\partial \Phi_y}{\partial \gamma} > 0$; $\frac{\partial \Phi_{y^*}}{\partial \gamma^*} > 0$, *and/or ii) the steady-state value of output increases* $\frac{\partial \Phi_y}{\partial y} > 0$; $\frac{\partial \Phi_{y^*}}{\partial y^*} > 0$. *Relative price stabilization becomes relatively more important as i) the share of domestic goods in the aggregate consumption basket decreases* $\frac{\partial \Phi_s}{\partial \zeta} < 0$; $\frac{\partial \Phi_s}{\partial \zeta^*} < 0$, *and/or ii) the steady-state value of foreign consumption falls* $\frac{\partial \Phi_s}{\partial c^*} < 0$. *Consumption stabilization becomes relatively more important as i) the steady-state values of consumption increase* $\frac{\partial \Phi_c}{\partial c} > 0$; $\frac{\partial \Phi_{c^*}}{\partial c^*} > 0$. *Portfolio stabilization becomes relatively more important as i) the measure of portfolio benefits increase* $\frac{\partial \Phi_a}{\partial \xi} > 0$; $\frac{\partial \Phi_{a^*}}{\partial \xi^*} > 0$, *and/or ii) the steady-state values of portfolio holdings increase* $\frac{\partial \Phi_a}{\partial a} > 0$; $\frac{\partial \Phi_{a^*}}{\partial a^*} > 0$. *Finally, stabilization of long-term government bond purchases become more important as i) the cost of central bank transaction services,* τ, *increases, and/or ii) as the share of outstanding debt,* $\{q_L b_L, q_L^* b_L^*\}$, *becomes smaller.*

[15] Harrison (2017) assumes quadratic portfolio adjustment costs in the budget constraint of the household. Upon aggregation to derive the resource constraint, these adjustment costs serve exactly the same purpose as our costly central bank intervention. Both assumptions serve to introduce asset purchases as one of the objectives in the central bank's welfare criterion and are necessary to pin down central bank purchases in the optimal policy problem.

Proof. In Appendix D. ■ □

Inflation stabilization becomes more important as price stickiness increases because the flattening of the Phillips curve impedes the transmission of monetary policy. As a result, the central bank needs to attach a higher weight on inflation stabilization. Importantly, this result also shows that the union central bank finds it optimal to increase the weight on the inflation of the region that is subject to higher price stickiness. When it comes to the elasticity of substitution across varieties, θ, a higher elasticity reduces the distortion due to monopolistic competition. This brings output closer to its efficient level. Consequently, and given the tradeoff between inflation and output stabilization, the central bank finds it optimal to place a higher weight on inflation stabilization. As regards output stabilization, an inelastic labor supply implies that real wages, and hence not output, adjust to the increase in labor demand following, say, an increase in aggregate demand. This may keep output away from the desired target and the central bank would need to spend more resources in stabilizing output. At the same time, an increase in steady-state output requires a higher weight on output stabilization given that the latter is now farther away from target. A similar argument applies to consumption stabilization. Lower degrees of home bias make the domestic economy more susceptible to foreign shocks and, in particular, to fluctuations in foreign prices. This necessitates a higher weight on relative price stabilization.[16]

An important aspect of our model is market incompleteness domestically as well as at the union level. This is captured by parameters ξ and ξ^*, respectively. As this distortion increases, the more agents in both regions reduce their private consumption because they need to give up a larger portion of it in order to engage in asset trading.[17] Since the central bank cannot vary ξ and ξ^*, it instead controls portfolios in order to avoid large declines in private consumption. This deems portfolio stabilization essential in order to minimize welfare losses. For the same reasons, higher steady-state portfolios trranslate to a higher weight on portfolio stabilization. Finally, the intuition behind central bank purchases as τ increases is obvious. As explained in the text above, central bank interventions are costly and consume resources.

5.2 Optimal policy under commitment

We characterize the Ramsey problem of the central bank that entails minimizing the loss function (eq. 5.2) subject to the first-order approximation of the private sector equilibrium conditions. We assume that the central bank has the enforcement technology to commit to the policy, and the monetary policy instruments consist of the policy rate, $\hat{R}_t^U$, and purchases of long-term government bonds in the Periphery, $\widehat{qb}_{L,t}^{CB}$, and Core, $\widehat{qb}_{L,t}^{*CB}$. The Lagrangian and solution to the optimal policy problem under commitment can be seen in

[16] Recall that CPI inflation in each region is a weighted average of home PPI inflation and the terms of trade (i.e., relative prices).

[17] The distortionary nature of net transaction costs, $\Xi - \xi\alpha_t$, becomes clearer when focusing on the resource constraint. Ceteris paribus, higher transaction costs consume real resources.

Appendix J.

We proceed in this section through a series of propositions that highlight the main tradeoffs that the central bank is facing, as well as illustrate optimal policies.

Proposition 3. *Prices and output trade-off away from the zero lower bound: Under optimal commitment, the central bank of the union faces a trade-off between stabilizing a weighted average of price levels in both regions and a weighted average of output in both regions.*

Proof. In Appendix E. ■ □

The first key result from the optimal policy problem is the well known trade-off between output and price level optimization. A standard result under commitment is the output/price level trade-off and not an output/inflation trade-off, as is the case under discretion. This holds true in both closed and open economy settings. In our model, the union-wide central bank faces a trade-off between stabilizing a weighted average of outputs of the two regions and stabilizing a weighted average of their price levels.[18] As Corsetti and Pesenti (2005) note, when there is no pricing-to-market, monetary policy is inward-looking, targeting domestic PPI prices $(\widehat{p}_{h,t}, \widehat{p}_{f,t})$ instead of aggregate CPI inflation, which includes the terms of trade. This is what our optimal policy also points to.[19]

Proposition 4. *QE purchases and terms of trade trade-off: For home bias in consumption, ζ, $\zeta^* > 1/2$, the union central bank faces a trade-off between terms of trade, $\widehat{s}_t$, stabilization and the purchases of long-term debt issued in the Periphery, $\widehat{qb}_{L,t}^{CB}$. Purchases of Core long-term bonds, $\widehat{qb}_{L,t}^{*,CB}$, instead are associated with an increase in the terms of trade. Under symmetry and symmetric central bank asset purchases $\left(\widehat{qb}_{L,t}^{CB} = \widehat{qb}_{L,t}^{*CB}\right)$, the terms of trade is equal to its steady state. Furthermore, the following conditions are necessary and sufficient for those two relationships to hold:*

either $\quad \kappa_S > \kappa_\alpha > \kappa_L$ *and* $\kappa_S^* > \kappa_\alpha^* > \kappa_L^*$

or

$$(1-\zeta_S)\,\tfrac{\kappa_S-\kappa_\alpha}{\kappa_S\kappa_\alpha} > (1-\zeta_L)\,\tfrac{\kappa_L-\kappa_\alpha}{\kappa_L\kappa_\alpha} \text{ and } (1-\zeta_S^*)\,\tfrac{\kappa_S^*-\kappa_\alpha^*}{\kappa_S^*\kappa_\alpha^*} > (1-\zeta_L^*)\,\tfrac{\kappa_L^*-\kappa_\alpha^*}{\kappa_L^*\kappa_\alpha^*}$$

Proof. In Appendix F. ■ □

Given the open economy structure of the model, when the central bank optimally chooses a QE policy it is faced with trade-offs that relate to variables that link the two regions. The terms of trade, $\widehat{s}_t$, plays a key role for optimal policy. Following a negative demand shock, it is optimal for the central bank to increase its purchases of Peripheral bonds if prices in the Periphery (and hence the terms of trade) decline, ceteris paribus

[18] Similarly, Engel (2011) designing optimal monetary policy under coordination in a two country model shows that there is a trade-off between the stabilization of a weighted average of outputs and the stabilization of a weighted average of price levels.

[19] This point has been analyzed extensively in the open economy literature. See also Benigno and Benigno (2006), Coenen *et al.* (2007), Corsetti *et al.* (2010), Mavromatis (2018) and the references therein.

(recall that the definition of the terms of trade is the ratio of Peripheral to Core prices, $S_t = P_{h,t}/P_{f,t}$). In this way, the central bank engineers an expansion in economic activity in order to drive inflation closer to target. Moreover, it finds it optimal to allocate relatively more asset purchases from the region where inflation drops more. We have derived proposition 4 as that region being the Periphery. The same argument applies for the Core, with the sign of the relationship reversed.[20]

An interesting observation of the discussion above is that the trade-off between asset purchases and the terms of trade depends crucially on the elasticities of substitution between assets. We discuss the calibration in section 6.1, but the near-perfect substitutability in both regions between short-term Peripheral and Core bonds, κ_S, κ_S^* , as well as stronger frictions in the market for long-term bonds, captured by κ_L, κ_L^*, guarantees that the necessary and sufficient condition in proposition 4 is satisfied. The second condition pertains to non-linear combinations of portfolio parameters. The near-perfect substitutability between Peripheral and Core short-term bonds now implies that $\kappa_S > \kappa_\alpha$ and $\kappa_S^* > \kappa_\alpha^*$, respectively. Importantly, home bias in short- and long-term portfolios, $\zeta_S, \zeta_S^*, \zeta_L, \zeta_L^*$, play a crucial role as well. Given near-perfect substitutability in short-term assets, the two necessary and sufficient conditions thus allow for some freedom as regards the relationship between the elasticity of substitution for long-term bonds, κ_L and κ_L^*, and the elasticity of substitution between short- and long-term assets in the aggregate portfolios, κ_α^* and κ_α^*.

Proposition 5. *Portfolio holdings and terms of trade tradeoff: The union central bank faces a tradeoff between terms of trade, $\widehat{s}_t$, stabilization and Peripheral aggregate portfolio stabilization, $\widehat{\alpha}_t$. Aggregate portfolio at the Core, $\widehat{\alpha}_t^*$, instead is associated with an increase in the terms of trade.*

Proof. In Appendix G. ■ □

Central bank purchases trigger portfolio rebalancing in both regions. Optimal monetary policy in our model therefore features an interaction between aggregate portfolios and the terms of trade. Following a negative demand shock, a drop in the terms of trade suppresses the purchasing power of households in the Periphery and creates a negative externality. In order to stabilize the terms of trade, the central bank has to engineer an increase in the aggregate portfolio, $\widehat{\alpha}_t$, in the Periphery. This way, the central bank increases the wealth of the representative household in the Periphery, which boosts aggregate demand.[21] Higher consumption demand can partly offset the downward pressure on prices in the Periphery and can thus stabilize the terms of trade. Form the perspective of the Core, the sign of the relationship is reversed, given the definition of the terms of trade.

[20] In a model with cross-country segmentation in bond markets and home bias in government spending, Tischbirek (2018) shows that the central bank can use government bond purchases to control the terms of trade and achieve asymmetric degrees of stimulus across the members of the currency union.

[21] We associate an increase in wealth to an increase in consumption demand because an increase in the aggregate portfolio coincides with an increase in the holdings of short-term assets, which are easily liquidated. Households increase their holdings of short-term assets since total government debt supply is fixed and the availability of long-term assets has diminished following the central bank intervention.

Proposition 6. *Optimal QE: The union central bank raises its asset purchases in the Periphery relative to the Core when private consumption in the Periphery declines more relative to the Core. This causes portfolios to increase. Moreover, the central bank has to limit its asset purchases in the Periphery relative to the Core as inflation in Periphery increases more relative to the Core. At the ZLB, asset purchases are dependent on the evolution of output, whereas above the ZLB they are not.*

Proof. In Appendix H. ■ □

A natural extension of proposition 4 is the relationship between optimal QE and other macro aggregates, such as private consumption, output, and inflation. From the discussion above and from the derivation of the loss function, it becomes evident that it is optimal for the union central bank to adjust its asset purchases according to the characteristics of each region. It becomes apparent, that when consumption and inflation in one region decline with respect to the other region, the central bank finds it optimal to increase its purchases of long-term bonds from the region that experiences a deeper contraction. In other words, the central bank may find it optimal to readjust the weights with which it allocates its asset purchases.

Proposition 7. *Optimal QE and price stickiness: The union central bank increases its asset purchases in the Periphery relative to the Core as price stickiness in the Periphery increases, ceteris paribus. The opposite holds as price stickiness in the Core increases.*

Proof. In Appendix I. ■ □

Apart from the potential asymmetries in the business cycle between the two regions, asymmetries may also arise from differences in the potency of monetary policy in the two regions. A key factor determining this is the degree of price stickiness, summarized by parameters ω and ω^*. A flatter Phillips curve thwarts a substantial part of the desired effects of QE. Therefore, the union central bank may need to increase asset purchases from the region whose prices exhibit a higher degree of stickiness.

6 Quantitative Analysis

In this section we perform a quantitative analysis of the model to investigate the effects of optimal policy in the union.

6.1 Calibration

We calibrate the model to the euro area. The region size is set to $n = 0.35$ to reflect the economic size of the Periphery countries relative to the Core.[22] We assume that parameters unrelated to portfolio rebalancing are

[22] Recall that Periphery consists of Italy, Spain, and Portugal; Core consists of Germany, France, and the Netherlands.

common across regions (e.g., preferences, price rigidities). Instead, parameters related to portfolio rebalancing in each region (e.g., portfolio shares, elasticities of substitution between assets) can vary. The values of all parameters and steady-state values can be seen in Tables 1 and 2.

Regarding the common parameters: we set the inflation targets to $\pi = \pi^* = 1.005$ (2% annualized) so that they are consistent with the mandate of the ECB in the EA. The discount factor β is set to 0.99. The inverse of the elasticity of labor supply γ is set to 1. The parameter reflecting price rigidity ω is set to 0.75, and the gross markup ϕ is set to 1.1, which implies an elasticity of substitution across good varieties $\theta = 10$. The parameter reflecting the intratemporal elasticity of substitution between domestic and foreign goods κ is set to 1.5. The coupon rate of long-term bonds ρ is calibrated to match the average duration of long-term bonds in the EA (about 25 quarters) and is set at 0.9675 in both the Periphery and Core. We calibrate the portfolio level parameter, ξ, to 0.0075 so that the Euler condition yields a 3% annualized interest rate at steady state. This implies that $R^{zlb} = -0.0075$ at a quarterly level. Finally, the resource cost associated with central bank purchases of government bonds τ is set to 1 basis point in our benchmark calibration; however, we discuss the robustness of the results to alternative values.

Regarding parameters that are different across regions: first, the shares that govern the weight of short-term bonds relative to long-term bonds in households' portfolios, ζ_a and ζ_a^*, are set to 0.34 and 0.22, consistent with the short share in the Periphery and Core presented in Figure 2 in section 2. We use 2014 averages — before the ECB started its QE policy — when calibrating these parameters. Second, the shares of domestic bonds in the long-term sub-portfolio, ζ_L and ζ_L^*, are set to 0.97 and 0.54, respectively. These values reflect the information from SHS data presented in Figure 1 in section 2.[23] This implies that the Core holds more Periphery long-term bonds than country size suggests. Third, the shares of domestic bonds in the short-term sub-portfolio, ζ_S and ζ_S^*, are set to 0.99 and 0.95, respectively. They are calibrated to match the share of domestic short-term government bonds in the short-term bond sub-portfolio obtained from the SHS dataset.[24]

Debt targets, b and b^*, are calibrated to match total government liabilities including the monetary base (as a % of annual GDP) in each region, which is equal to 91% and 70%, respectively.[25] As for the weights of domestic consumption, recall that ζ is a function of n, and the degree of home bias λ, so $1 - \zeta = (1 - n)\lambda$. We set λ to 0.38 in order to match an import-to-GDP ratio of 0.25 in the Periphery, which is close to the value found in ECB-EAGLE (Jacquinot *et al.*, 2010). This implies $\zeta = 0.75$. However, the weight of domestic

[23]Recall that these parameters are functions of country size and financial openness: $\zeta_L = 1 - (1-n)\lambda_L$ and $\zeta_L^* = 1 - n\lambda_L^*$. Given country size, our parametrization implies financial openness, λ_L and λ_L^*, to be equal to 0.04 and 1.31 in long-term maturities for the Periphery and Core respectively.

[24]We assume that monetary base holdings in the short-term portfolio exhibit the same degree of home bias as short-term government debt holdings. This implies that each country mostly holds the currency that each region's central bank is liable for. Recall that these parameters are functions of country size and financial openness: $\zeta_S = 1-(1-n)\lambda_S$ and $\zeta_S^* = 1-n\lambda_S^*$. The parametrization suggests financial openness, λ_S and λ_S^*, to be equal to 0.01 and 0.13.

[25]Total government debt is obtained from the government sector's balance sheets in the EU Financial Accounts. Monetary liabilities are included using the share of euro monetary base that each region is liable for. We use the ECB's capital key when computing monetary liabilities.

goods in the Core, ζ^*, cannot be set freely and is implied by ζ and portfolio parameters of the Periphery and Core. Based on our calibration, $\zeta^* = 0.7974$. The ratio of short-term to long-term bonds, $\vartheta = b_S/b_L$, is also determined by portfolio parameters and can also not be set freely. Portfolio parameters imply $\vartheta = 0.09$ and $\vartheta^* = 0.50$. Finally, the transaction costs in the household budget constraint, Ξ and Ξ^*, are set to 0.005 and 0.031. These have been set to match zero transaction costs (net of portfolio benefits) at the steady state, so, $\Xi = \xi\alpha$.

6.1.1 SVAR-based approach for the calibration of portfolio elasticities

Beyond the share of short-term and long-term bonds in households' portfolios, the magnitude of the effects of bond purchases by the central bank are crucially driven by the extent to which assets are imperfectly substitutable (i.e., $\kappa_a, \kappa_a^*, \kappa_S, \kappa_S^*, \kappa_L, \kappa_L^*$). First, we calibrate κ_S and κ_S^* to 100, a high enough value that, at every point in time, short-term rates are almost equal to each other across regions. This ensures that the central bank sets short-term rates across the union as a result of the monetary union assumption.

Calibrating the rest of the portfolio elasticity parameters is not an obvious task since observed changes in (relative) short and long, or domestic and foreign, bond shares over time are not uniquely driven by the quantitative easing policy of the ECB. We place empirical discipline on these elasticities by estimating an SVAR for the Periphery and Core using data on bond holdings from the ECB's SHS dataset over the period 2013:Q4-2018:Q4. We identify a quantitative easing shock in each region using theoretically-robust sign restrictions consistent with the model's predictions. The details on estimation, identification, and the results from this exercise are found in Appendix B.

Given the responses from the identified QE shock in the data, we then calibrate κ_α, κ_α^*, κ_L, and κ_L^* so that our model can capture four targets: i) an increase in the share of short-term bonds to total bonds (short-share) in the Periphery by 10%, ii) a decline in the union-wide term premium by 145 basis points, iii) an increase in the share of domestic long-term bonds to total long-term bonds (home bias) in the Periphery by 1pp., and iv) an increase in the share of domestic long-term bonds to total long-term bonds (home bias) in the Core by 4pp.

This term premium decline is slightly larger than the evidence from DSGE-based studies of QE in the EA (see Hohberger *et al.* (2019) and references therein), but in line with the evidence in Demir *et al.* (2019), who find that following an unconventional monetary policy shock, bond returns decrease by around 150-200 basis points in Spain, Italy, and Portugal, and by around 50 basis points in Germany, France, and the Netherlands. The increases in the shares of domestic long-term bonds to total long-term bonds in both the Periphery and Core are small, especially when compared to changes in the short shares, so also in line with the evidence presented in section 2. The implied parameters from this exercise are: $\kappa_\alpha = 0.1$, $\kappa_\alpha^* = 0.4$, $\kappa_L = 0.1$, and $\kappa_L^* = 0.1$.

Table 1: Parameter values

	Symbol	Periphery	Core	*Symmetry*
Country size	n	0.35	0.65	*0.5*
Inflation target (gross, qtr.)	π	1.005	1.005	*1.005*
Zero-lower bound (qtr.)	R^{zlb}	-0.0075	-0.0075	*-0.0075*
Discount factor	β	0.99	0.99	*0.99*
Inverse of the elasticity of labor supply	γ	1	1	*1*
El. subst. domestic and foreign goods	κ	1.5	1.5	*1.5*
Price rigidity	ω	0.75	0.75	*0.75*
Gross markup - domestic goods price	$\frac{\theta}{\theta-1}$	1.10	1.10	*1.10*
Coupon rate for long-term bonds	ρ	0.9674	0.9674	*0.9674*
Weight of domestic good in consumption	ζ	0.750	0.863	*0.75*
Portfolio shares - short vs. long portfolio	ζ_a	0.336	0.223	*0.5*
- short domestic vs. foreign bonds	ζ_S	0.994	0.955	*0.75*
- long domestic vs. foreign bonds	ζ_L	0.973	0.541	*0.75*
Portfolio elasticities - short vs. long portfolio	κ_α	0.1	0.4	*0.1*
- short domestic vs. foreign bonds	κ_S	100	100	*100*
- long domestic vs. foreign bonds	κ_L	0.1	0.1	*0.1*
Gov't debt short-long ratio	ϑ	0.09	0.51	*1.0*
Portfolio benefit	ξ	0.0075	0.0075	*0.01*
Transaction cost (gross)	Ξ	0.003	0.002	*0.026*
Resource cost of QE	τ	0.01	0.01	*0.01*

Table 2: Steady-state values

	Symbol	Periphery	Core	*Symmetry*
Consumption-to-GDP	c/y	0.99	1.00	*1.00*
Exports-to-GDP	c_f^*/y	0.26	0.25	*0.25*
Imports-to-GDP	c_f/y	0.25	0.14	*0.25*
Bond supply / GDP (ann.)	b/y	0.91	0.70	*0.91*
short, total	b_S/y	0.08	0.24	*0.45*
long, total	$q_L b_L/y$	0.83	0.47	*0.45*
Bond holdings / GDP (ann.)				
short, home	b_{HS}/y	0.06	0.01	*0.34*
long, home	$q_L b_{HL}/y$	0.01	0.39	*0.34*
short, foreign	b_{FS}/y	0.0001	0.23	*0.11*
long, foreign	$q_L^* b_{FL}/y$	0.0003	0.46	*0.11*
Net transfers-to-GDP	tr/y	-0.0091	-0.007	*-0.0091*

6.2 Impulse responses following a shock to the natural rate

We study impulse responses following a shock to the natural rate r_t^n.[26] The shock is of size -3% and is persistent ($\rho_{rn} = 0.7$). It is large enough so that in the absence of a policy response, output declines by around 10% in the Periphery and Core, and the policy rate remains at the zero lower bound for approximately 5 quarters.[27]

To solve the model we make two simplifying assumptions. First, before the shock to the natural interest rate arrives, the model is in steady state. This implies that the Ramsey optimal policy is equivalent to the

[26] Our exercise of subjecting the monetary union to a liquidity trap is also similar to Cook and Devereux (2013, 2019).

[27] The shock is calibrated in the model without optimal policy, where the interest rate is determined by a Taylor-type rule and QE is exogenous and set to zero (see section 6.2.5).

optimal policy viewed from a timeless perspective. Moreover, we assume that the Ramsey planner has perfect foresight, so after the shock hits, its future path is known with certainty. This permits employing the "piecewise linear" solution approach of Guerrieri and Iacoviello (2015) for dealing with the occasionally-binding zero lower bound constraint.

6.2.1 Special case: symmetric union

Before presenting the results for the more general case of an asymmetric union, we start off with optimal policy in a symmetric union. The parameterization is standard and can be seen under the column labelled "Symmetry" in Table 1. This exercise allows us to clearly show the effectiveness of each instrument and compare our results to the other standard frameworks in the literature. Note that in this environment, there are no welfare gains from stabilization of the terms of trade.

We begin the analysis by showing how optimal policy under commitment affects the economy away from the ZLB. Figure 3 plots impulse responses following a shock to the natural rate for three cases: i) optimal interest rate policy, ii) optimal QE policy, and iii) fully optimal policy where both the interest rate and asset purchases are chosen optimally.[28]

When the interest rate is the only instrument available to the central bank (dotted red lines), the predictions of the model are similar to those from the canonical New Keynesian model with perfect asset substitutability. The optimal policy is to commit to setting the policy rate to perfectly track movements in the natural rate. Given that the policy rate is unconstrained by the ZLB, this completely stabilizes the output gap and inflation, and insulates the economy from the negative demand shock. In addition, given that the ratio of short-to-long-term bonds in portfolios is unchanged, the term premium is constant.

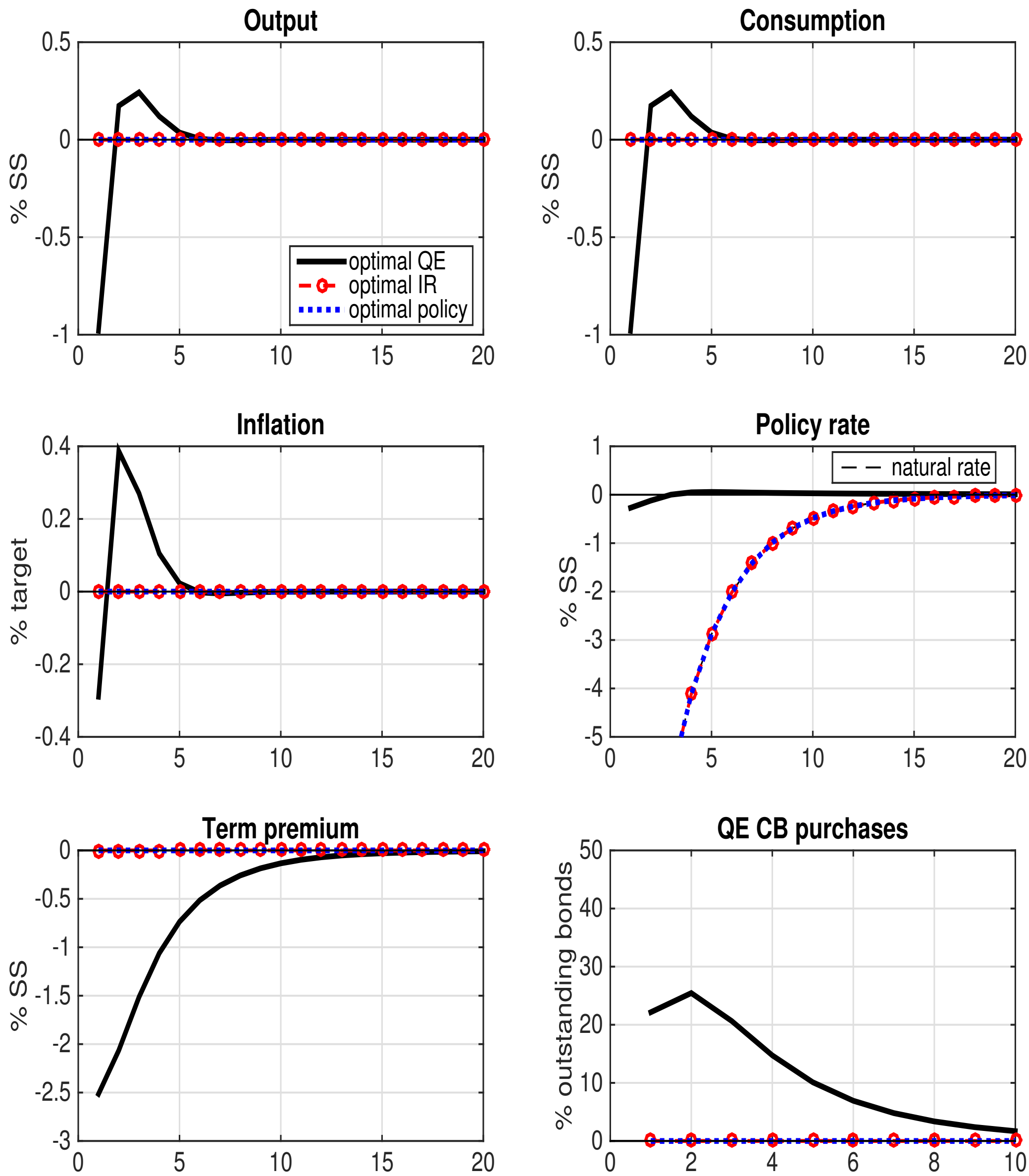
The optimal policy is starkly different when QE is the only instrument available to the central bank (solid black lines). Purchases of long-term government bonds lower the term premium due to the imperfect substitution between short- and long-term bonds. However, QE policy alone is ineffective and cannot stabilize output and inflation. This is because QE consumes real resources, and as a result, the central bank faces a trade-off between QE purchases and macroeconomic stabilization. This tradeoff leaves the economy prone to the negative demand shock, which reduces output and inflation on impact by about 1% and 0.5%, respectively.

Finally, the fully optimal policy (dashed blue lines) in the absence of the ZLB is identical to interest rate policy. The central bank sets the policy rate to perfectly track the natural rate, as explained above, and QE purchases are zero because they are costly.

Figure 4 considers the same set of policies but when the policy rate is instead constrained by the ZLB. In

[28]In the experiment with only QE as the available instrument (case ii), we assume that the policy rate is determined according to a Taylor-type rule targeting average inflation in the union $R_t^U = \rho_i R_{t-1}^U + (1-\rho_i)\left(r_\pi \widehat{\pi}_t^U + r_y \widehat{y}_t^U\right) + \varepsilon_{r,t}$, where $\widehat{\pi}_t^U = n\widehat{\pi}_t + (1-n)\,\widehat{\pi}_t^*$ and $\widehat{y}_t^U = n\widehat{y}_t + (1-n)\,\widehat{y}_t^*$, with $\rho_i = 0.894$, $r_\pi = 0.5$, and $r_y = 2.038$, in line with the calibration from the EAGLE model. For an explicit comparison between optimal policy and rules-based policy see section 6.2.5.

Figure 3: Optimal QE and Optimal Interest Rate Policy *Away* from the ZLB



Notes: Optimal QE policy (black solid line), optimal interest rate policy (dotted red line), and fully optimal policy (dashed blue line) following a -3% discount factor shock. Policy rate is not constrained by the ZLB.

this setting, optimal interest rate policy under commitment (dotted red lines) follows the standard predictions of the canonical New Keynesian model; that is, forward guidance as in Eggertsson and Woodford (2003). The central bank commits to keep the policy rate low at the exit from the liquidity trap (i.e., past 5 periods), affecting expectations of future short-term rates and engineering a boom in the short run. However, given that the interest rate cannot sufficiently decline because of the ZLB constraint, the effects of the shock are meaningful and the output gap and inflation fall by about 3% and 1%, respectively, on impact.

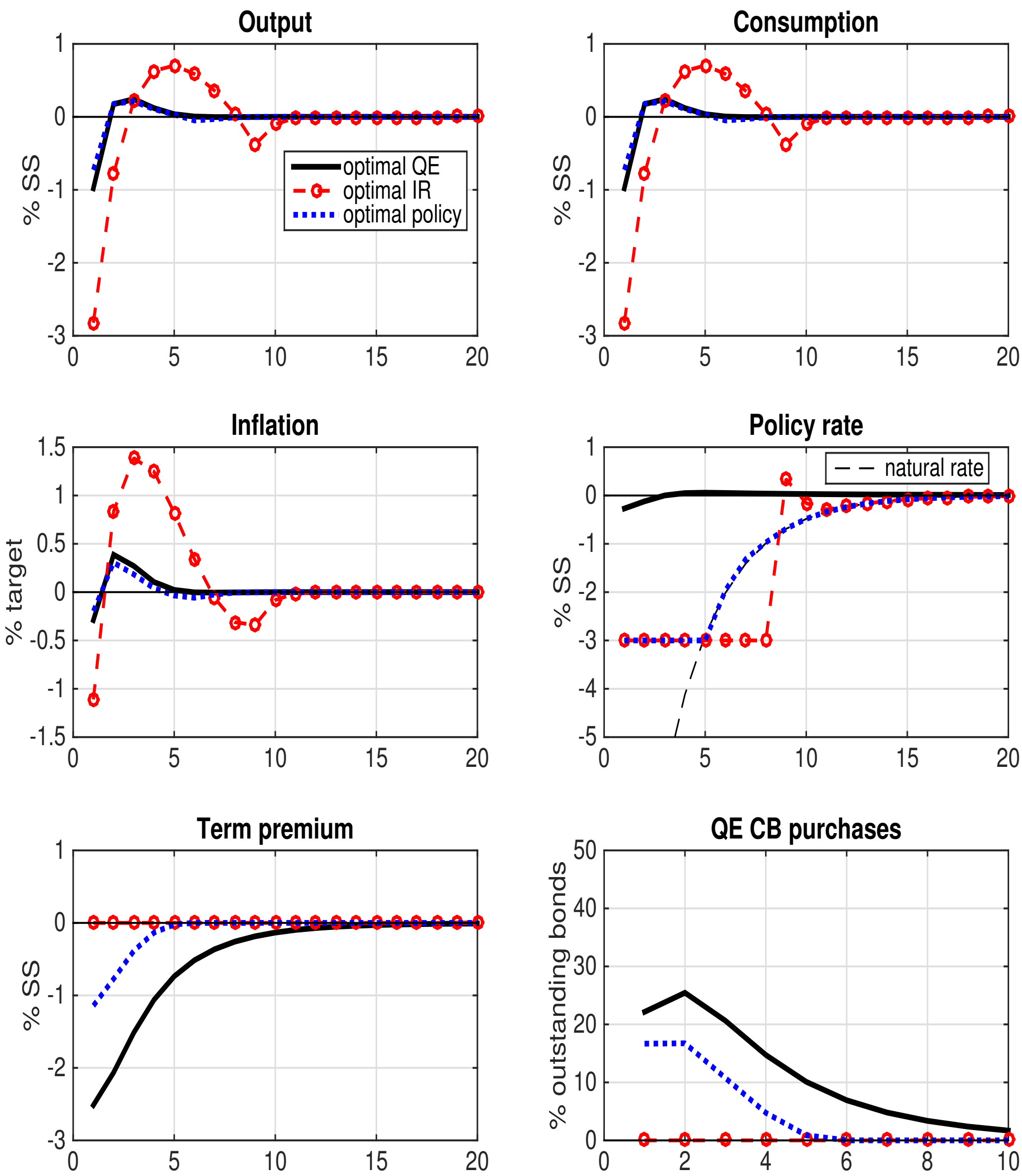
Optimal QE at the ZLB (solid black lines) is now relatively more effective than optimal interest rate policy. But, the relative gains of QE are not due to the change in effectiveness of QE; rather, they derive from interest rate policy being less effective at the ZLB. As can be seen, the central bank purchases the same amount of long-term bonds as when away from the ZLB (22% of outstanding bonds on impact), causing the term premium to decline and the output gap and inflation to fall by 1% and 0.3%, respectively, on impact, as in Figure 3.

When both instruments are available to the central bank (dashed blue lines) the fully optimal policy at the ZLB no longer resembles optimal interest rate policy. By using QE alongside interest rate policy, the planner is able to restore the term premium at lower levels, thus stimulating output and inflation. Given that both instruments are available, the amount of purchases required to stabilize the economy from the negative demand shock is less than with just QE (17% vs. 22% of outstanding bonds). At the same time, the policy rate is lifted from the ZLB earlier than with using interest rate policy alone (at 5 periods). However, given that QE is costly, the effect of the shock is still significant, with the output gap and inflation still declining, but by less than without QE (i.e., less than 3% and 1%, respectively, on impact).

How do the results change if we consider a different degree of QE costs? In Figure 5, we plot the impulse responses of the fully optimal policy for varying degrees of central bank transaction services, τ. For τ low (triangle, green lines) the central bank incurs negligible resource costs, which does not materially impact welfare when engaging in QE. In this way, purchases of long-term government bonds are optimally increased (30% of outstanding bonds on impact), which allow for the term premium to decline further. In parallel, setting the policy rate to perfectly track the natural rate, subject to the ZLB constraint, is optimal to (almost completely) stabilize the economy.

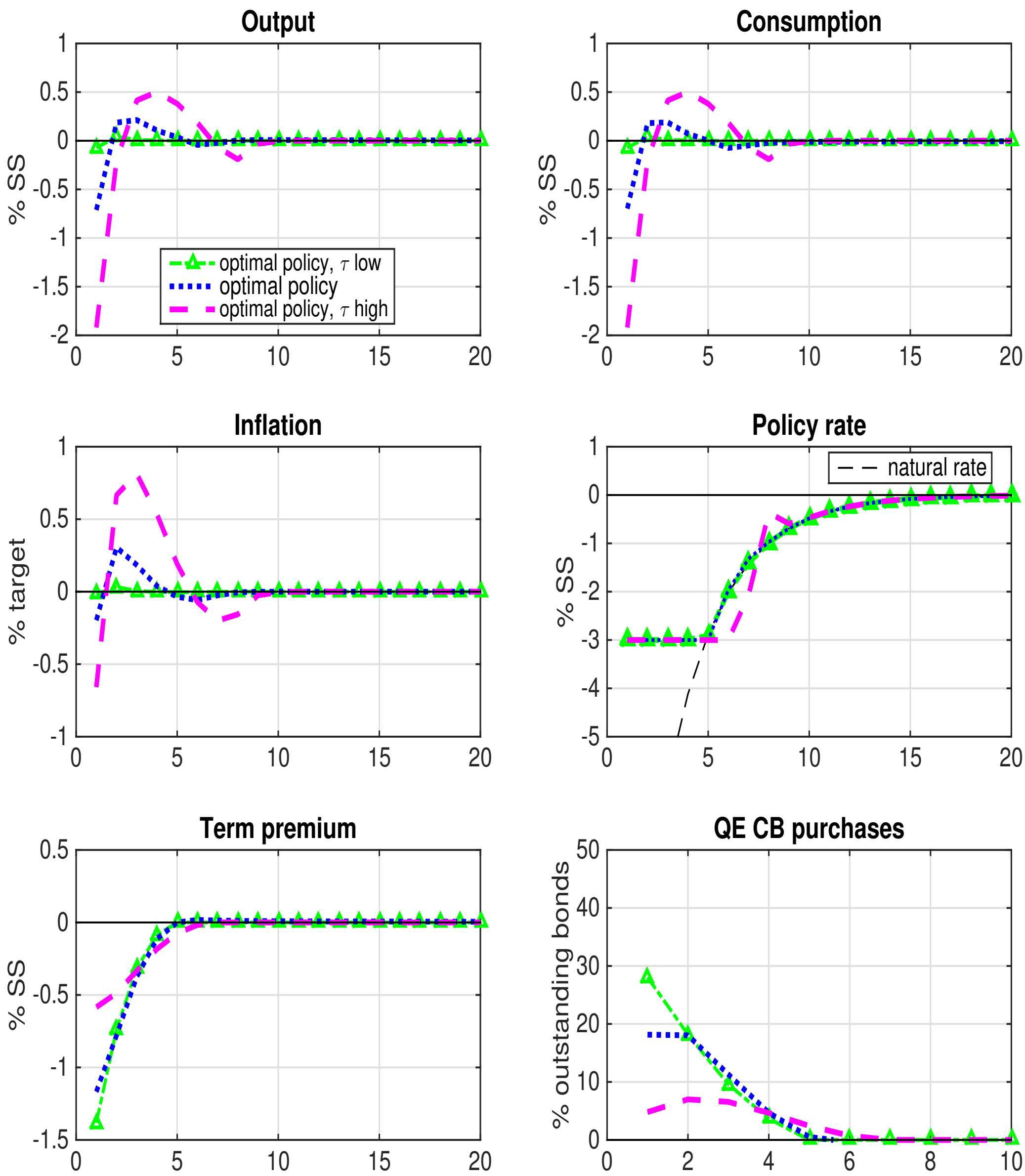
Instead, for τ high (dashed purple line), purchases of long-term government bonds are particularly costly and reduce aggregate welfare meaningfully. The central bank purchases lower amounts of long-term bonds (5% on impact), which does not sufficiently lower the term premium. The insufficient expansion of activity has to be then compensated by keeping the interest rate low for longer (past 5 periods). Overall, this leaves the economy prone to the negative demand shock, where the output gap and inflation decline by about 2% and 0.7%, respectively, on impact.

Figure 4: Optimal QE and Optimal Interest Rate Policy at the ZLB



Notes: Optimal QE policy (black solid line), optimal interest rate policy (dotted red line), and fully optimal policy (dashed blue line) following a -3% discount factor shock. Policy rate is constrained by the ZLB, $R^{zlb} = -3$ percentage points (annualized).

Figure 5: Optimal Policy at the ZLB for Varying Degrees of τ



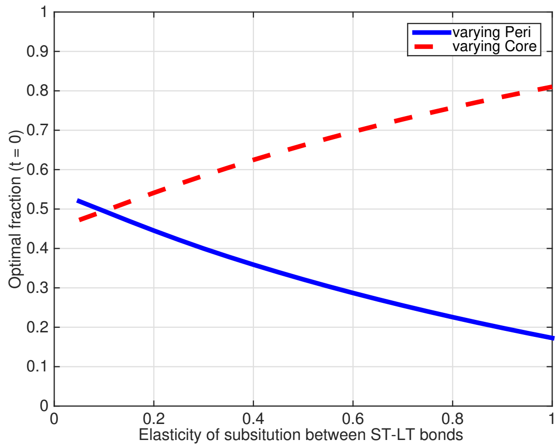
Notes: Fully optimal policy for varying degrees of central bank transaction service cost, τ, following a -3% discount factor shock. $\tau = 0.0005$ (triangle, green line), $\tau = 0.01$ (dotted blue line), $\tau = 0.1$ (dashed purple line). Policy rate is constrained by the ZLB, $R^{zlb} = -3$ percentage points (annualized).

6.2.2 The role of bond market asymmetries

We now discuss how bond market characteristics affect the optimal fraction of QE. The exercise involves deviating from perfect symmetry in only one dimension each time, while keeping all other remaining parameters symmetric across regions. The solid blue (dotted red) lines in figures 6 - 10 represent how the optimal fraction varies for different values of each parameter under consideration in the Periphery (Core) for a given value of that parameter in the Core (Periphery). Since the union is otherwise symmetric, we only discuss the sensitivity in the Periphery; the inverse intuition will apply to the Core.

The optimal fraction is decreasing in κ_α and increasing in κ_α^*: Figure 6 shows the role of the elasticity of substitution between short- and long-term sub-portfolios, κ_α, on the optimal fraction of Periphery purchases. The optimal fraction decreases as κ_α increases, i.e., the central bank chooses to purchase less Periphery bonds as the short-long friction decreases. Intuitively, higher κ_α (weaker short-long friction) generates a smaller term premium effect from a given amount of Periphery purchases and a smaller marginal benefit for the union. Therefore, the central bank chooses to purchase less of those bonds. Conversely, lower κ_α (a higher short-long friction) increases the fraction of Periphery purchases.

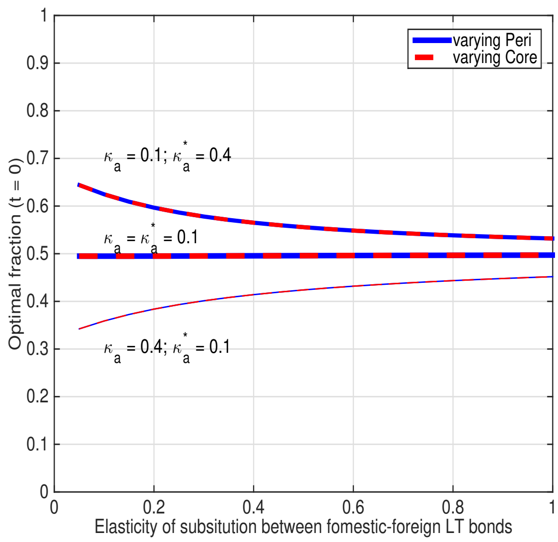
Figure 6: Optimal Fraction and κ_α



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of the elasticity of substitution between short- and long-term bonds in the Periphery and Core. Solid blue line: varying κ_α with $\kappa_\alpha^* = 0.1$. Dotted red line: varying κ_α^* with $\kappa_\alpha = 0.1$.

For $\kappa_\alpha = \kappa_\alpha^*$, the optimal fraction is constant in κ_L and κ_L^*. For $\kappa_\alpha < \kappa_\alpha^*$, the optimal fraction is decreasing in κ_L and κ_L^*. For $\kappa_\alpha > \kappa_\alpha^*$, the optimal fraction is increasing in κ_L and κ_L^*: Figure 7 shows the role of substitution between home and foreign long-term bonds, κ_L and κ_L^*, on the optimal fraction of Periphery purchases. The impact of this home-foreign friction, however, crucially depends on whether the short-long substitution, κ_α, is symmetric, or not, across regions. When $\kappa_\alpha = \kappa_\alpha^*$ (thick lines), a change in κ_L or κ_L^* does not move the optimal fraction. This is because the optimal policy does not depend on the level of home-foreign substitution. When $\kappa_\alpha < \kappa_\alpha^*$ (medium lines), we know from the previous discussion that the optimal fraction has to be higher, which implies an upward shift in the optimal fraction nexus. In this case, increasing κ_L and making Periphery and Core bonds more substitutable causes the central bank to become more indifferent between which bonds to purchase. Hence, it chooses to lower the optimal fraction of Periphery purchases. Conversely, in the case of $\kappa_\alpha > \kappa_\alpha^*$ (thin lines), the optimal fraction is less than 0.5. By increasing κ_L, the marginal benefit from higher Core purchases falls and the optimal fraction increases.

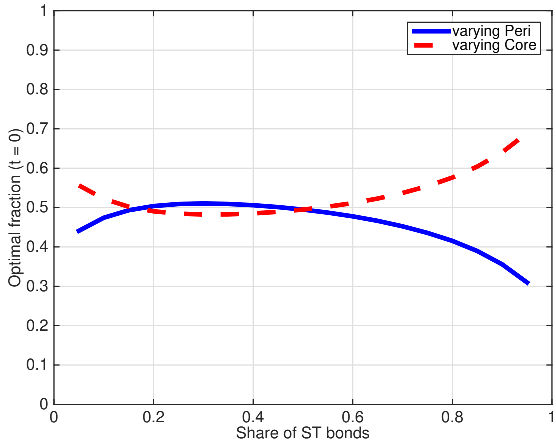
Figure 7: Optimal Fraction and κ_L



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of the elasticity of substitution between domestic and foreign long-term bonds in the Periphery and Core. Solid blue line: varying κ_L with κ_L^* fixed at value in chart. Dotted red line: varying κ_L^* with κ_L fixed at value in chart.

The optimal fraction is not monotonic in ζ_α and ζ_α^*: Figure 8 shows the role of short bias, ζ_α and ζ_α^*, on the optimal fraction of Periphery purchases. The impact on the optimal fraction is not monotonic in the short bias, which suggests different trade-offs operating in the optimal policy problem. Note that higher short bias has crucial implications for both the term premium and aggregate demand in the model. On the one hand, higher short bias implies less stimulus in the Periphery for a given amount of decline in the term premium. This is because aggregate demand in the Periphery now depends more on the short-term rate, which is binding at zero. On the other hand, higher short bias also implies a lower share of Periphery long-term debt. Here, the effect of the Periphery's long-term rate on the term premium is greater, ceteris paribus. The first effect dominates for $\zeta_\alpha > 0.30$, which makes the central bank more reluctant to purchase Periphery bonds as the marginal benefit is falling. For $\zeta_\alpha < 0.30$, however, the second effect dominates. Thus, by increasing purchases of Periphery bonds, the central bank can stabilize the union more due to the greater drop in term premia as ζ_α increases.

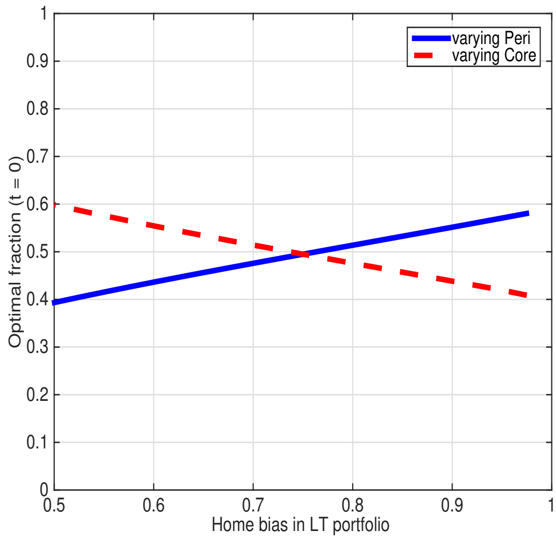
Figure 8: Optimal Fraction and ζ_α



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of the share of short-term bonds in the aggregate portfolio in the Periphery and Core. Solid blue line: varying ζ_α with $\zeta_\alpha^* = 0.5$. Dotted red line: varying ζ_α^* with $\zeta_\alpha = 0.5$.

The optimal fraction is increasing in ζ_L and decreasing in ζ_L^*: Figure 9 shows the role of home bias, ζ_L and ζ_L^*, on the optimal fraction of Periphery purchases. Higher home bias, ζ_L, increases the optimal purchases from the Periphery. Intuitively, higher home bias implies that the Periphery places a higher reliance on domestic rates. So, in order to stabilize the Periphery, the central bank now has to place a greater weight on that region. Otherwise, with the same amount of purchases, the Periphery would be disproportionately hit by the negative demand shock and such a policy would destabilize Periphery inflation and terms of trade.

Figure 9: Optimal Fraction and ζ_L

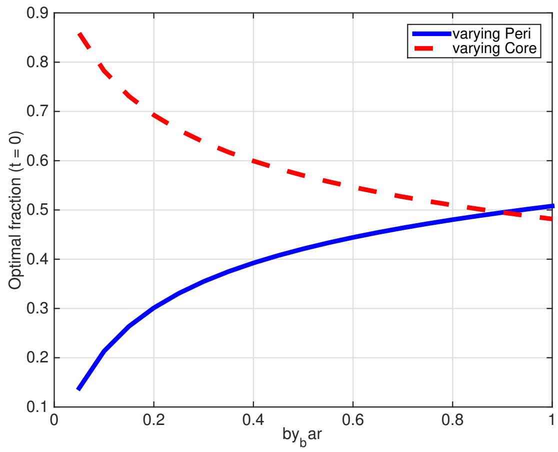


Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of the share of domestic bonds in the long-term sub-portfolio in the Periphery and Core. Solid blue line: varying ζ_L with $\zeta_L^* = 0.5$. Dotted red line: varying ζ_L^* with $\zeta_L = 0.5$.

The optimal fraction is increasing in the share of Periphery government debt-to-GDP and de-

creasing in the share of Core government debt-to-GDP: Figure 10 shows the role of steady-state government debt-to-GDP (annualized), b/y, on the optimal fraction of Periphery purchases. Higher debt in the Periphery increases the optimal fraction. Higher debt implies a bigger market for Periphery bonds, which implies a larger share of Periphery bonds in private portfolios of each region. In order to achieve the same portfolio switching effect (the same change in portfolio shares), the central bank has to optimally place more weight on these bonds. At the same time, it is also cost-efficient to purchase more from a larger market since QE costs are scaled down by the size of the bond market. This result is also consistent with closed economy versions of portfolio rebalancing models in the literature: as debt size increases, the central bank has to increase its purchases to obtain the same financial and aggregate demand effects.

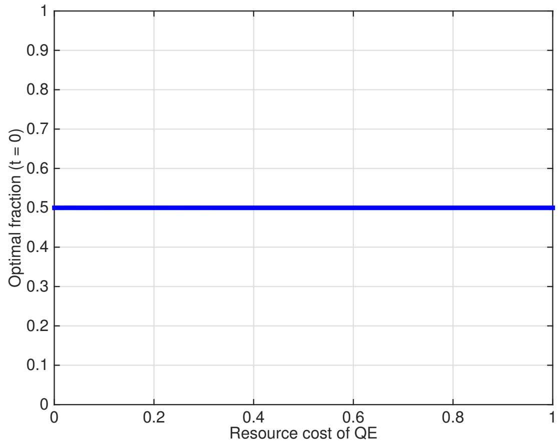
Figure 10: Optimal Fraction and Steady-State Government Debt-to-GDP (%)



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of debt-to-GDP (%) in the Periphery and Core. Solid blue line: varying debt-to-GDP in the Periphery with debt-to-GDP in the Core equal to 95%. Dotted red line: varying debt-to-GDP in the Core with debt-to-GDP in the Periphery set to 95%.

The optimal fraction is constant in τ: Figure 11 shows the role of the resource cost of QE on the optimal fraction of Periphery purchases. The optimal fraction does not depend on the resource cost. This is because the same cost applies to both regions, which are otherwise symmetric by construction.

Figure 11: Optimal Fraction and τ



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of central bank transaction service cost, τ.

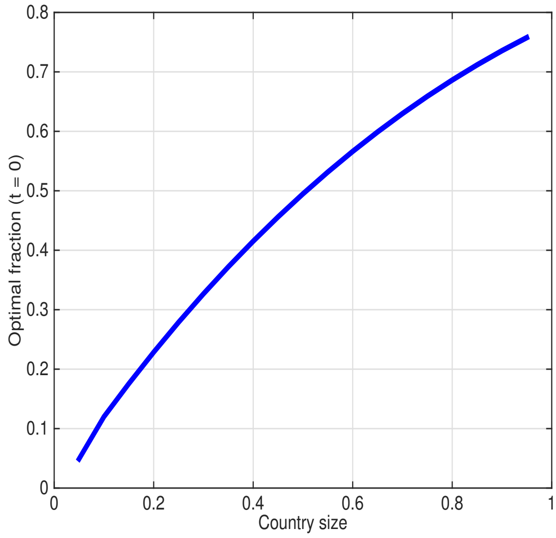
The optimal fraction is increasing in country size n: Figure 12 shows the role of country size, n, on the optimal fraction of Periphery purchases. Similar to the debt size, a higher country size also implies a larger market size for Periphery bonds and thus a larger fraction of these bonds in private portfolios. Therefore, the central bank can only stabilize union-wide macroeconomic variables by purchasing more from the larger region. However, also note that the optimal fraction is increasing non-linearly in country size. Specifically, for $n < 0.5$ the optimal fraction increases more than proportionately to n. Instead, for $n > 0.5$ the optimal fraction increases less than proportionately to n.[29] This non-linearity arises because asset purchases exhibit decreasing marginal returns; in particular, as asset purchases increase, the marginal utility of consumption drops.[30] From a welfare point of view, the central bank therefore finds it optimal to raise the optimal fraction at a decreasing rate, following the pattern imposed by the decreasing marginal utility of consumption.

Finally, it is interesting to note that in the absence of additional asymmetries, if we set $n = 0.35$, which represents the size of the Periphery relative to the Core, the optimal fraction in our model is close to the ECB capital key, which recall, allocates QE purchases according to country size only.

[29] For $n = 0.5$, the optimal fraction coincides with the country size.

[30] To see this more clearly, consider the combined interest rate effect on private consumption given by equation (4.6). A higher domestic country size increases the market size of domestic long-term bonds. As a result, the central bank needs to buy more of those bonds to achieve a given outcome. This means that private consumption increases further owing to a larger drop in PR_t.

Figure 12: Optimal Fraction and n



Notes: Optimal fraction at $t = 0$ (as defined in eq. 3.17) for varying degrees of country size, n.

6.2.3 General case: asymmetric union

We now relax the assumption of a symmetric union and calibrate each region according to the regional values in Table 1. Figure 13 plots impulse responses in the Periphery (solid blue lines) and Core (dashed red lines) following a shock to the natural rate for the fully optimal policy, where both the interest rate and QE are available as instruments to the central bank. Besides the variables discussed before, the figure also illustrates the evolution of the terms of trade, as well as of the optimal fraction. Given the size of the Periphery (recall that $n = 0.35$) if only region sizes were asymmetric but structural parameters remained identical, $OF_t = 0.35$ for $t \geq 0$. To the extent that regions are structurally asymmetric, as is the case here, the optimal fraction does not necessarily have to be equal to the country size.

On aggregate, the fully optimal policy in the asymmetric union resembles the fully optimal policy in the symmetric union. The planner purchases long-term government bonds, thereby lowering the term premium and stimulating activity. In parallel, the policy rate reflects the principles of forward guidance, being lowered at the level implied by the ZLB and gradually lifted as the economy exits from the liquidity trap. On average, the union-wide output gap and inflation decline by about 1.1% and 0.3%, respectively, overshoot, and return to their baseline values by period 8.

Looking at the effects in each region in more detail yields a number of important observations. First, *absolute* QE purchases are higher in the Periphery than what country size implies. This can be seen by the response of the optimal fraction, which is around 0.57 for about 4 periods before converging to 0.35 in period 5. However, because steady-state government debt (% of annual GDP) is 70% in the Core while 91% in the

Periphery, QE purchases, as a share of outstanding bonds, are only slightly higher in the Periphery relative to the Core (20% vs. 12% in period 2). Second, despite the larger QE purchases that the planner allocates to the Periphery, the term premium falls by more in the Core, albeit by a small amount. This also leads to the Core becoming more stabilized from the negative demand shock.

To understand these results consider the hypothetical case of a QE policy that follows country size (i.e., purchases of 35% of bonds from the Periphery). Such a policy implies relatively lower (higher) purchases of Periphery (Core) bonds. Since term premium spillovers are not perfect (because $\kappa_L < \infty$), a smaller purchase from the Periphery would imply a smaller fall in its term premium. In addition, Periphery demand depends more on its domestic rates because of higher home bias in portfolios (recall that $\zeta_S > \zeta_S^*$ and $\zeta_L > \zeta_L^*$). A given amount of QE would therefore stabilize less (more) the Periphery (Core). If inflation falls by more in the Periphery, the terms of trade, defined as $s_t = P_{h,t}/P_{f,t}$, would also decline more on impact. Given that the welfare-theoretic loss function also attaches a positive weight to terms of trade stabilization, overall this policy is clearly suboptimal and the central bank instead chooses to increase the optimal fraction so that $OF_t > 0.35$. In fact, our calibration suggests that $OF_t > 0.5$ for $0 \leq t \leq 4$.

6.2.4 Contribution of bond market asymmetries on the optimal fraction of QE

Taking as given the asymmetric calibration, we now discuss in more detail how each bond market characteristic contributes in shaping the optimal fraction. Specifically, we shut down key asymmetries one by one and see how the optimal fraction varies. Table 3 shows the optimal fraction using different parametrizations of the model starting with the baseline case of section 6.2.3 in column 2.

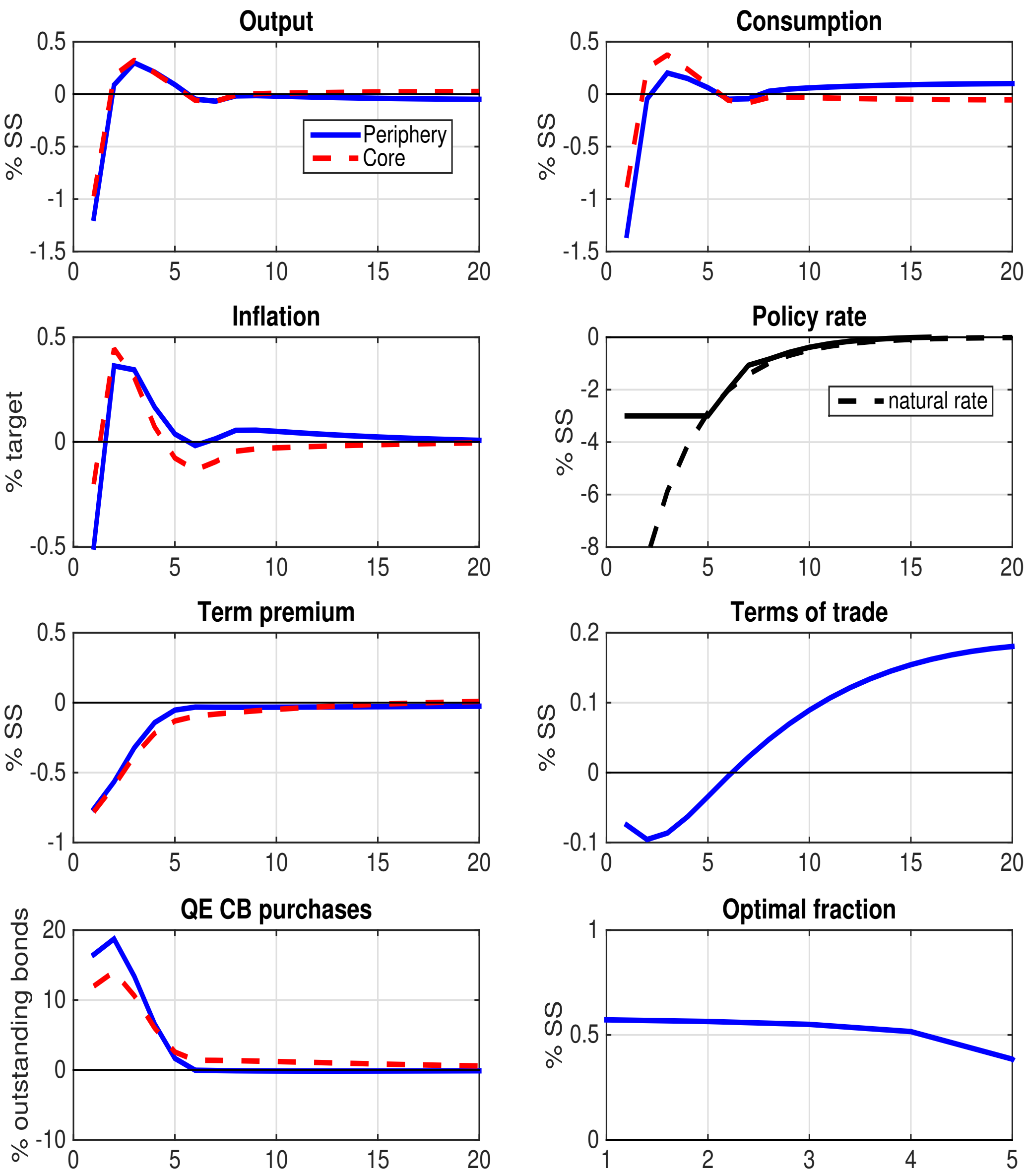
Column 3 shows the contribution of the asymmetry in the short-long friction, κ_α, to the optimal fraction of Periphery purchases. If this parameter were equal in both the Periphery and the Core (i.e., $\kappa_a = \kappa_a^* = 0.40$), the optimal fraction would be slightly lower (0.55) than in the baseline case (0.57, where $\kappa_a = 0.1$). This result is consistent with our discussion in section 6.2.2, which implies a lower fraction when the short-long friction is decreased: a lower friction in the Periphery implies relatively less purchases from this region. Equivalently, a higher short-long friction for the Periphery in the baseline case contributes to a slightly higher fraction of Periphery bond purchases.

Table 3: Contribution of each asymmetry to the optimal fraction of QE

	Baseline	$\kappa_\alpha = 0.40$	$\zeta_\alpha = 0.22$	$\zeta_L = 0.54$	$\frac{b}{y} = 0.7$	Implied by Capital key
Optimal fraction of Periphery bonds	0.57	0.55	0.58	0.40	0.60	0.35

Notes: Baseline calibration, $\kappa_\alpha = 0.10$, $\kappa_L = 0.15$, $\zeta_\alpha = 0.34$, $\zeta_L = 0.97$, and $\frac{b}{y} = 0.91$. The last column indicates the capital key, which is the ratio of Periphery bonds purchased by the ECB under a QE policy.

Figure 13: Optimal Policy in the Periphery and Core



Notes: Fully optimal policy in the Periphery (solid blue lines) and Core (dashed red lines) following a -3% discount factor shock. Policy rate is constrained by the ZLB, $R^{zlb} = -3$ percentage points (annualized). Terms of trade are defined as $S_t = P_{h,t}/P_{f,t}$. Optimal fraction is defined as in eq. 3.17

Column 4 shows the contribution of the asymmetry in the short bias, ζ_α, to the optimal fraction of Periphery purchases. When short bias is equalized across the union ($\zeta_\alpha = \zeta_\alpha^* = 0.22$), the optimal fraction increases to 0.58. Hence, asymmetries in short bias contribute to slightly lower the optimal fraction. Note that, in the baseline case, we have a higher short share for Periphery portfolios (0.34 vs. 0.22), implying that Periphery demand depends relatively more on short-term rates than does Core demand. A higher reliance on short-term rates in the Periphery lowers the marginal benefit of purchasing Periphery bonds, as discussed in the previous subsection; thereby, it implies a smaller fraction of Periphery bonds in the baseline case than in the case of symmetric short bias (0.57 vs. 0.58).

Column 5 shows the contribution of the asymmetry in the home bias, ζ_L, to the optimal fraction of Periphery purchases. In the baseline calibration, we have a significant asymmetry between Periphery and Core home bias (0.97 vs. 0.54). By lowering home bias in the Periphery to its Core levels ($\zeta_L = \zeta_L^* = 0.54$), the central bank reduces purchases from the Periphery and the optimal fraction declines from 0.57 to 0.40. As discussed previously, when home bias is lower, Periphery demand relies less on domestic rates and, thereby, the central bank can stimulate Periphery demand even with Core bond purchases, which results in a significantly smaller fraction of Periphery purchases. This result highlights the importance of home bias asymmetry on the optimal fraction in our model.

Column 6 shows the impact of government debt as a share of GDP on the optimal fraction. Note that in the baseline calibration, Periphery debt is much higher than Core debt (90% vs. 71%). When we equalize debt ratios across regions ($\frac{b}{y} = \frac{b^*}{y^*} = 0.71$), the optimal fraction increases to 0.60. This result is different than in section 6.2.2 and the symmetric parametrization, where a larger debt size implied a higher fraction of Periphery purchases. The difference here lies in the fact that the Periphery portfolio is characterized by stronger home bias relative to the more balanced Core portfolio. Given a significant asymmetry in portfolio home bias, a fall in Periphery steady-state government debt lowers the Periphery steady-state portfolio holdings significantly, which implies a smaller weight on Periphery portfolio stabilization relative to the Core (see the weights in the loss function under proposition 1). The central bank therefore optimally places a greater weight on Core portfolio stabilization and purchases less Core bonds and more Periphery bonds. This result highlights that under full asymmetry, the optimal fraction is not driven by the larger debt-to-GDP size for the Periphery but rather the very high home bias in Periphery portfolios.

Finally, column 7 reports the ECB's capital key for the fraction of Periphery bond purchases, which is 0.35. Clearly, this value is significantly lower than in our baseline model (0.57). Therefore, our analysis suggests that portfolio asymmetries are important in driving the optimal fraction in the EA away from the capital key, which is primarily determined by economic size and population.

6.2.5 Optimal fraction vs. rule-based QE policy

We end this section by comparing the impulse responses under optimal policy to a model where only the interest rate is chosen optimally by the central bank. Instead, QE follows a predetermined rule based on the ECB's capital key. This exercise serves to illustrate more explicitly how different ways to allocate bond purchases across regions determine the effectiveness of QE.

Under a rule-based policy we assume that the central bank performs QE by allocating its purchases of bonds across countries according to the fixed share ϖ, which reflects the fraction of long-term bonds purchased from the Periphery. Conversely, the fraction $1-\varpi$ reflects the share of long-term bonds purchased from the Core. Consistent with the ECB's capital key, we set $\varpi = 0.35$.

$$
\begin{aligned}
q_{L,t} b_{L,t}^{CB} &= \varpi \upsilon_t (q_{L,t} b_{L,t} + q_{L,t}^{*} b_{L,t}^{*}) \\
q_{L,t}^{*} b_{L,t}^{*CB} &= (1-\varpi) \upsilon_t (q_{L,t} b_{L,t} + \psi_t q_{L,t}^{*} b_{L,t}^{*})
\end{aligned} \tag{6.1}
$$

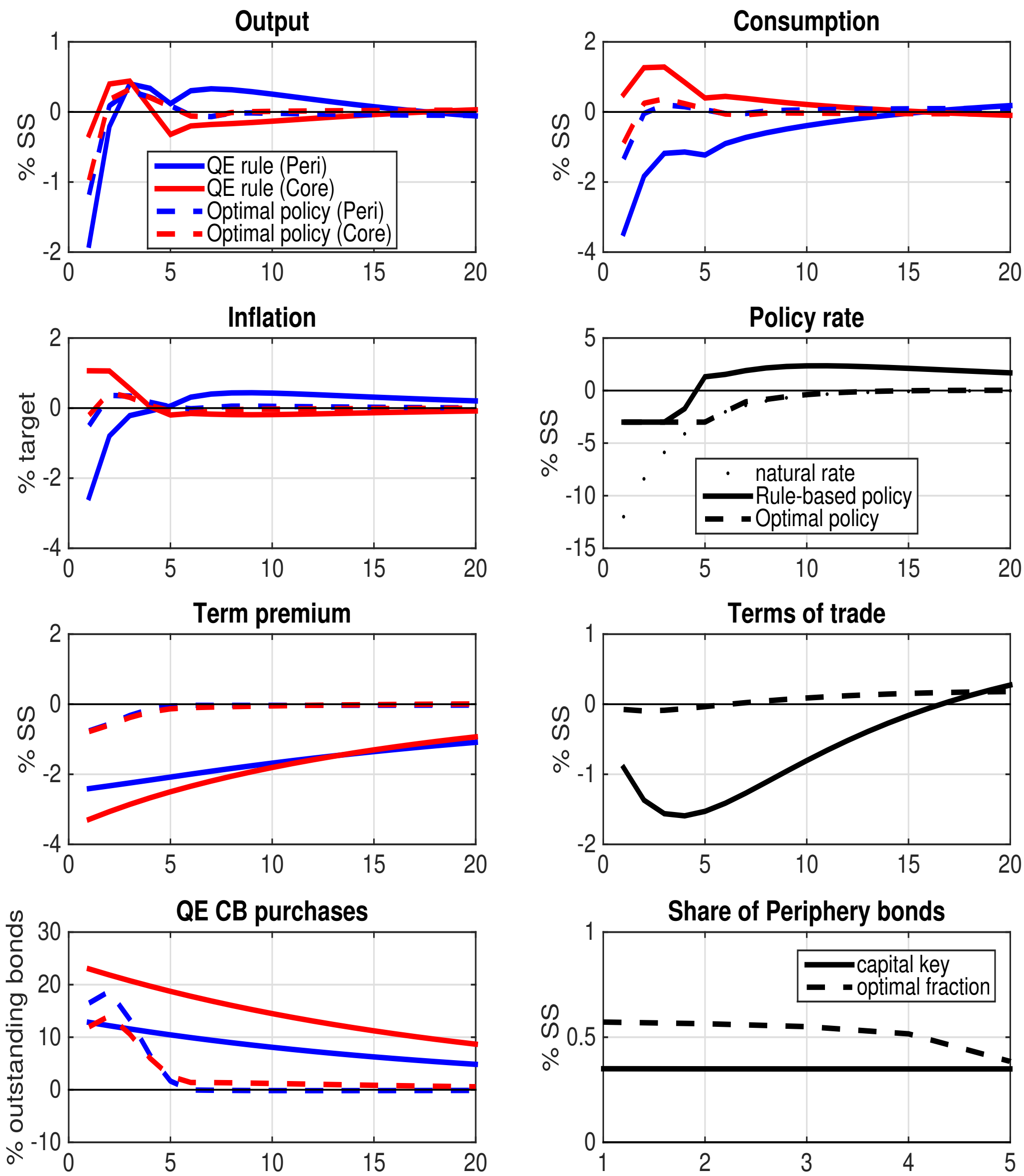
υ_t represents total long-term bonds issued in the union and follows an $AR(1)$ stationary process, $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_{\upsilon,t}$. A positive shock to υ_t represents the QE shock. We calibrate its size to be 10% of EA GDP and assume it is persistent with ρ_υ equal to 0.95. This size is in line with the announcement of the ECB in January 2015, when the program was originally implemented.[31]

Figure 14 compares the rule-based QE policy with the fully optimal policy. Not surprisingly, the rule-based policy implies less stabilization for the union. Particularly, Periphery output, consumption, and inflation fall by more as a result of purchases less than optimal from this region. On the other hand, it generates a smaller drop in Core output, but bigger consumption and inflation booms in this region. Higher macroeconomic volatility for the Core also results in the terms of trade falling more than under optimal policy.

Notably, even though rule-based policy is sub-optimal, it succeeds in lowering term premia in both regions by more than the optimal policy. This is because a bigger drop in term premia generates a bigger consumption boom and higher inflation rates in the Core, which causes an earlier lift-off in the union policy rate from the ZLB. However, note that our calibration suggests a higher reliance on short-term rates for the Periphery than the Core (recall that $\zeta_\alpha > \zeta_\alpha^*$). An earlier lift-off in short-term rates, therefore, implies less stabilization for the Periphery. This suggests that it is not only the extent of term premium declines that matter for stabilization policies in monetary unions, but also to what extent each region is sensitive to changes in the policy rate.

[31] The qualitative predictions are robust to assuming an AR(2) specification in line with Andrade *et al.*, 2016; Carlstrom *et al.*, 2017; Hohberger *et al.*, 2019.

Figure 14: Rule-Based QE vs. Optimal Policy in the Periphery and Core



Notes: Optimal interest rate policy with rule-based QE (as in eq. 6.1) (solid lines) and fully optimal policy (dashed lines) in the Periphery (blue lines) and Core (red lines) following a -3% discount factor shock. Policy rate is constrained by the ZLB. Terms of trade are defined as $S_t = P_{h,t}/P_{f,t}$. Optimal fraction is defined as in eq. 3.17.

7 Conclusion

This paper is motivated by two considerations. First, bond market characteristics are the crucial elements that affect the transmission of QE in an open economy, and we observe significant differences in these characteristics across the Periphery and Core of the EA: i) a striking difference in the degree of home bias across regions, ii) a steeper rise in the short share in the Core over time, and iii) a larger debt-to-GDP ratio in the Periphery. Second, in practice, the design of a QE policy in a monetary union requires a framework about the allocation of government bond purchases across regions. The current framework that is employed in the EA relies on allocating bond purchases according to each region's economic size and population. In light of these additional dimensions of heterogeneity, is this allocation optimally designed? And what does optimal monetary policy look like?

We have tackled these questions by building a two-region DSGE model of a monetary union where regions are asymmetric with respect to bond market characteristics. We characterized the optimal monetary policy as well as the optimal allocation of government bond purchases across regions. In our asymmetric model of a monetary union, an optimal QE policy under commitment does not only reflect different region sizes, but is also a function of those dimensions of portfolio heterogeneity. By calibrating our model to the euro area, we show that optimal QE favors purchases from the smaller region (Periphery instead of Core), given that the former faces stronger portfolio frictions. A fully optimal policy, consisting of both the short-term interest rate and QE, lifts the monetary union away from the zero lower bound faster than an optimal interest rate policy alone, which entails forward guidance.

Finally, we compare the model-consistent optimal policy, comprised by QE and interest rate setting, to a proxy of the actual policy in the EA. Our calibrated model predicts that central bank purchases of Peripheral debt according to the ECB's capital key (35%) lie below our model's optimal fraction (57%).

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A Private Sector Equilibrium

For each region, we first obtain the first order conditions from each agent's problem, find the steady state, and then log-linearize those conditions around the steady state. After some manipulations, we arrive at the following set of conditions, which characterize the private sector's equilibrium.

Euler equation (IS curve):

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - (PR_t - E_t\widehat{\pi}_{t+1} - r_t^n)$$
$$\widehat{c}_t^* = E_t\widehat{c}_{t+1}^* - \left(PR_t^* - E_t\widehat{\pi}_{t+1}^* - r_{n,t}^*\right)$$

where

$$PR_t = \zeta_a\zeta_S\widehat{R}_t + (1-\zeta_a)\zeta_L\left(\widehat{R}_t + \widehat{T}_t\right) + \zeta_a(1-\zeta_S)\widehat{R}_t^U + (1-\zeta_a)(1-\zeta_L)\left(\widehat{R}_t^U + \widehat{T}_t^*\right)$$
$$PR_t^* = \zeta_a^*\zeta_S^*\widehat{R}_t^U + (1-\zeta_a^*)\zeta_L^*\left(\widehat{R}_t^U + \widehat{T}_t^*\right) + \zeta_a^*(1-\zeta_S^*)\widehat{R}_t + (1-\zeta_a^*)(1-\zeta_L^*)\left(\widehat{R}_t + \widehat{T}_t\right)$$
$$\widehat{T}_t = \frac{R_L}{R_L-\rho}\widehat{R}_{L,t} - \frac{\rho}{R_L-\rho}E_t\widehat{R}_{L,t+1} - \widehat{R}_t$$
$$\widehat{T}_t^* = \frac{R_L}{R_L-\rho^*}\widehat{R}_{L,t}^* - \frac{\rho^*}{R_L-\rho^*}E_t\widehat{R}_{L,t+1}^* - \widehat{R}_t^U$$

Note that we rescale the discount factor shock so that the shock is represented in deviations from the natural rate of interest. Given that productivity is constant in the model, this shock is the only factor that moves the natural rate.

New-Keynesian Phillips Curve:

$$\widehat{\pi}_{h,t} = \beta E_t\widehat{\pi}_{h,t+1} + \frac{(1-\omega)(1-\beta\omega)}{\omega}\left[\gamma\widehat{y}_t + \widehat{c}_t - (1-\zeta)\,\widehat{s}_t\right]$$
$$\widehat{\pi}_{f,t}^* = \beta E_t\widehat{\pi}_{f,t+1}^* + \frac{(1-\omega^*)(1-\beta\omega^*)}{\omega^*}\left[\gamma^*\widehat{y}_t^* + \widehat{c}_t^* + (1-\zeta^*)\,\widehat{s}_t\right]$$

Terms of trade:

$$\widehat{s}_t - \widehat{s}_{t-1} = \left(\widehat{\pi}_{h,t} - \hat{\pi}_{f,t}^*\right)$$

Optimal allocation in short-term sub-portfolio:

$$\widehat{R}_t - \widehat{R}_t^U = \left(\frac{\pi}{\beta R^U} - 1\right)\frac{1}{\kappa_S}\left[\widehat{b}_{HS,t} - \left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right)\right]$$
$$\frac{1}{\kappa_S^*}\left[(\widehat{b}_{HS,t}^* - (1-\zeta-\zeta^*)\widehat{s}_t) - \widehat{b}_{FS,t}^*\right] = \frac{1}{\kappa_S}\left[\widehat{b}_{HS,t} - \left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right)\right]$$

Optimal allocation in long-term sub-portfolio:

$$\begin{aligned}
\left(\widehat{R}_t+\widehat{T}_t\right)-\left(\widehat{R}^U_t+\widehat{T}^*_t\right) &= \left(\frac{\pi}{\beta R^U}-1\right)\frac{1}{\kappa_L}\left[\widehat{qb}_{HL,t}-\left((1-\zeta-\zeta^*)\widehat{s}_t+\widehat{qb}_{FL,t}\right)\right]\\
\frac{1}{\kappa^*_L}\left[(\widehat{qb}^*_{HL,t}-(1-\zeta-\zeta^*)\widehat{s}_t)-\widehat{qb}^*_{FL,t}\right] &= \frac{1}{\kappa_L}\left[\widehat{qb}_{HL,t}-\left((1-\zeta-\zeta^*)\widehat{s}_t+\widehat{qb}_{FL,t}\right)\right]
\end{aligned}$$

Optimal allocation between short- vs. long-term bonds:

$$\begin{aligned}
\widehat{T}_t &= \left(\frac{\pi}{\beta R^U}-1\right)\left[-\frac{1}{\kappa_a}\left(\widehat{a}_{S,t}-\widehat{a}_{L,t}\right)-\frac{1}{\kappa_L}\left(\widehat{a}_{L,t}-\widehat{qb}_{HL,t}\right)+\frac{1}{\kappa_S}\left(\widehat{a}_{S,t}-\widehat{b}_{HS,t}\right)\right]\\
\widehat{T}^*_t &= \left(\frac{\pi}{\beta R^U}-1\right)\left[-\frac{1}{\kappa^*_a}\left(\widehat{a}^*_{S,t}-\widehat{a}^*_{L,t}\right)-\frac{1}{\kappa^*_L}\left(\widehat{a}^*_{L,t}-\widehat{qb}^*_{FL,t}\right)+\frac{1}{\kappa^*_S}\left(\widehat{a}^*_{S,t}-\widehat{b}^*_{FS,t}\right)\right]
\end{aligned}$$

Bonds market clearing for short-term bonds:

$$\begin{aligned}
\frac{b_{HS}/y}{b_S/y}\widehat{b}_{HS,t}+(1-\frac{b_{HS}/y}{b_S/y})\widehat{b}^*_{HS,t} &= \widehat{qb}^{CB}_{L,t}\frac{1}{b_S}\\
\frac{b^*_{FS}/y^*}{b^*_S/y^*}\widehat{b}^*_{FS,t}+(1-\frac{b^*_{FS}/y^*}{b^*_S/y^*})\widehat{b}_{FS,t} &= \widehat{qb}^{*CB}_{L,t}\frac{1}{b^*_S}
\end{aligned}$$

Bonds market clearing for long-term bonds:

$$\begin{aligned}
-\frac{b_{HL}/y}{b_L/y}\widehat{qb}_{HL,t}-(1-\frac{b_{HL}/y}{b_L/y})\widehat{qb}^*_{HL,t} &= \frac{1}{q_L b_L}\widehat{qb}^{CB}_{L,t}\\
-\frac{b^*_{FL}/y^*}{b^*_L/y^*}\widehat{qb}^*_{FL,t}-(1-\frac{b^*_{FL}/y^*}{b^*_L/y^*})\widehat{qb}_{FL,t} &= \frac{1}{q^*_L b^*_L}\widehat{qb}^{*CB}_{L,t}
\end{aligned}$$

Goods market clearing

$$\begin{aligned}
y_t &= \zeta\frac{c_h}{y}\widehat{c}_t+\left(1-\zeta\frac{c_h}{y}\right)\widehat{c}^*_t-\left[\zeta\frac{c_h}{y}\kappa\left(1-\zeta\right)+\left(1-\zeta\frac{c_h}{y}\right)\kappa^*\zeta^*\right]\widehat{s}_t\\
y^*_t &= \zeta^*\frac{c^*_f}{y^*}\widehat{c}^*_t+\left(1-\zeta^*\frac{c^*_f}{y^*}\right)\widehat{c}_t+\left[\zeta^*\frac{c^*_f}{y^*}\kappa^*\left(1-\zeta^*\right)+\left(1-\zeta^*\frac{c^*_f}{y^*}\right)\kappa\zeta\right]\widehat{s}_t
\end{aligned}$$

Consumer price index (CPI) inflation:

$$\widehat{\pi}_t=\zeta\widehat{\pi}_{h,t}+\left(1-\zeta\right)\widehat{\pi}^*_{f,t}$$

Portfolio aggregates:

$$\widehat{a}_t = \zeta_a \widehat{a}_{S,t} + (1-\zeta_a)\, \widehat{a}_{L,t}$$
$$\widehat{a}^*_t = \zeta^*_a \widehat{a}^*_{S,t} + (1-\zeta^*_a)\, \widehat{a}^*_{L,t}$$
$$\widehat{a}_{S,t} = \zeta_S \widehat{b}_{HS,t} + (1-\zeta_S)\left[(1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right]$$
$$\widehat{a}^*_{S,t} = \zeta^*_S \widehat{b}^*_{FS,t} + (1-\zeta^*_S)\left[\widehat{b}^*_{HS,t} - (1-\zeta-\zeta^*)\widehat{s}_t\right]$$
$$\widehat{a}_{L,t} = \zeta_L \widehat{qb}_{HL,t} + (1-\zeta_L)\left[(1-\zeta-\zeta^*)\widehat{s}_t + \widehat{qb}_{FL,t}\right]$$
$$\widehat{a}^*_{L,t} = \zeta^*_L \widehat{qb}^*_{FL,t} + (1-\zeta^*_L)\left[\widehat{qb}^*_{HL,t} - (1-\zeta-\zeta^*)\widehat{s}_t\right]$$

Bond prices:

$$\widehat{q}^*_{L,t} = -\frac{\widehat{R}^*_{L,t}}{\left(1-\frac{\rho^*}{R_L}\right)}$$
$$\widehat{q}_{L,t} = -\frac{\widehat{R}_{L,t}}{\left(1-\frac{\rho}{R_L}\right)}$$

Balance of payments:

$$\begin{aligned}
&\frac{b_{FS}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right) - \frac{R}{\pi}\frac{b_{FS}}{y}((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{R}^U_{t-1} + \widehat{b}_{FS,t-1} - \widehat{\pi}^*_t) \\
&+ \frac{q^*_L b_{FL}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{qb}_{FL,t}\right) - \frac{R}{\pi}\frac{q^*_L b_{FL}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{R}^*_{L,t} + \widehat{q}^*_{L,t} + \widehat{qb}_{FL,t-1} - \widehat{q}^*_{L,t-1} - \widehat{\pi}^*_t\right) \\
&- \frac{b^*_{HS}}{y^*}\frac{1-n}{n}\left(\widehat{b}^*_{HS,t}\right) + \frac{R}{\pi}\frac{b^*_{HS}}{y^*}\frac{1-n}{n}\left(R_{t-1} + \widehat{b}^*_{HS,t-1} - \widehat{\pi}_t\right) \\
&- \frac{q_L b^*_{HL}}{y^*}\frac{1-n}{n}\left(\widehat{qb}^*_{HL,t}\right) + \frac{R}{\pi}\frac{q_L b^*_{HL}}{y^*}\frac{1-n}{n}\left(\widehat{R}_{L,t} + \widehat{q}_{L,t} + \widehat{qb}^*_{HL,t-1} - \widehat{q}_{L,t-1} - \widehat{\pi}_t\right) \\
&= \left(1-\zeta\frac{c}{y}\right)((1-\zeta)\,\widehat{s}_t - \kappa^*\zeta^*\widehat{s}_t + \widehat{c}^*_t) + \left(1-\zeta^*\frac{c^*}{y^*}\right)\frac{1-n}{n}\left(-\zeta\widehat{s}_t + \kappa\zeta\widehat{s}_t + \widehat{c}_t\right)
\end{aligned}$$

Lower bound for policy rate:

$$\widehat{R}^U_t = R^{zlb}$$

B Calibration of Elasticities

SVAR

For each region (Periphery = IT, ES, PT; Core = DE, FR, NL), the objective is to estimate the following system of equations:

$$AY_t = \sum_{k=1}^{K} C_k Y_{t-k} + Bu_t \tag{B.1}$$

where Y_t is a vector of endogenous variables for a given quarter t and C_k is a matrix of the own- and cross-effects of the k^{th} lag of the variables on their current observations. B is a diagonal matrix so that u_t is a vector of orthogonal i.i.d. shocks to government expenditures such that $Eu_t = 0$ and $E\left[u_t u_t^{'}\right] = I$. A is a matrix that allows for contemporaneous effects between the endogenous variables in Y_t. The specification is estimated using an OLS regression. OLS provides an estimate for the matrices $A^{-1}C$, but additional identification assumptions are necessary to estimate the coefficients in A and B.

Identifying a quantitative easing shock

Y_t contains these variables: *GDP, CPI deflator, short share, home bias LT, term premium. Short share* is defined as the share of short-term bond holdings to total bond holdings. *Home bias LT* is defined as the share of domestic long-term bond holdings to total long-term holdings. Data for GDP and the CPI deflator are obtained from Eurostat, while data for bond holdings is collected from the ECB's SHS. The term premium is defined as the yield on bonds of 10-year maturity and is available from IMF-IFS. The sample runs from 2013:Q4 - 2018:Q4 and is constrained by the availability of bond holdings data in the ECB SHS.

The specification is estimated using an OLS regression where GDP, the CPI deflator, *short share,* and *home bias LT* are in logs, while the term premium is in levels. We employ 2 lags of the endogenous variables given that our data is short.

Our focus is on the macroeconomic effects of QE using exogenous variation in portfolio holdings, on which we impose non-recursive short-run restrictions. This methodology has been widely used in the literature by e.g., Gambacorta *et al.* (2014) and Gambetti and Musso (2017), among others. The approach we take here is closest to Bhattarai *et al.* (2015a).

Table 4 describes the identifying restrictions. Supply and demand shocks affect the real economy, determining variables like output and prices. The financial shock includes restrictions that the long-term interest rate adjusts contemporaneously to changes in output and prices. The QE shock is identified as an unanticipated exogenous disturbance, which increases the *short share* (i.e., the central bank purchases long-term bonds) and lowers the term premium. At the same time, prices are assumed to increase. All restrictions are intuitive and consistent with our model's predictions following an exogenous QE shock, as shown in section

6.2.5.

Table 4: Identification restrictions

	Supply shock	Demand shock	Financial shock	QE shock
GDP	+	+	+	
CPI deflator		+	+	+
Short share				+
Home bias LT				
Term premium			-	-

Notes: Rows denote the variables in the VAR. Columns denote the identified shocks. Sign restrictions (+) are imposed for 1 quarter. *Short share* = share of short-term bond holdings to total bond holdings. *Home bias LT* = share of domestic long-term bond holdings to total long-term bond holdings. Term premium = yield on 10-year maturity bonds.

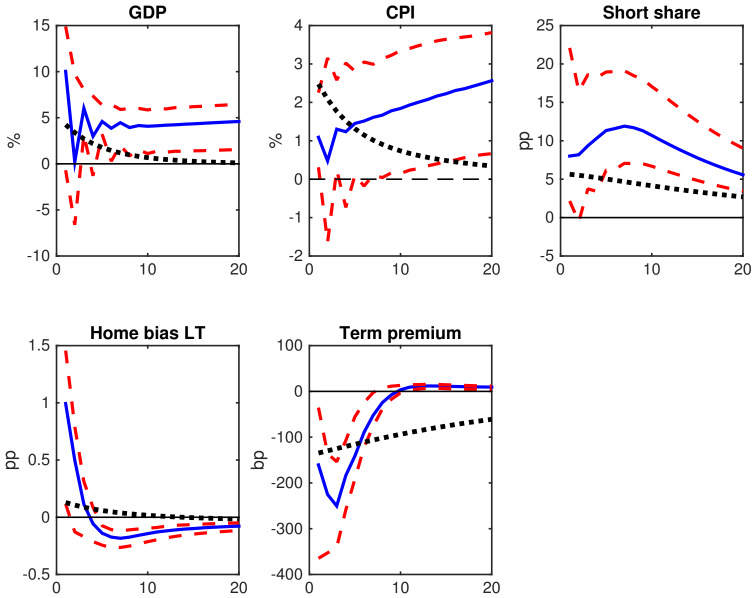
Estimation of the SVAR for each region yields the following results. Figure 15 plots the impulse responses for the Periphery while Figure 16 for the Core. We have scaled the responses of the variables to be consistent with a QE shock of size 5% of EA GDP. In both regions, GDP and inflation increase following a QE shock. The median effect on GDP is larger in the Periphery (10% vs. 4% on impact), but the median effect on prices is higher in the Core (1% vs. 2% on impact). The stronger effect on GDP in the Periphery follows from a larger decline in the term premium (200 bp vs. 100 bp). At the same time, the *short share* and *home bias LT* increase by less in the Periphery. Given that for an equal size of QE shock across regions, Periphery portfolios rebalance to a lesser degree than in Core, we can conclude that portfolio frictions are higher, and hence elasticities of substitution are lower, in the Periphery. This then translates to a stronger tranmission of QE in the Periphery.

Finally, Figure 16 plots a weighted average of the impulse response following a QE shock for the EA as a whole, where the weights reflect the size of each region in the EA GDP and population; 35% of long-term bonds are purchased from the Periphery, and the remaining 65% from the Core. This is also the practice of the ECB ("capital key") when allocating long-term government bond purchases across countries.

Matching Impulse Responses in the Quantitative Model and in the Data

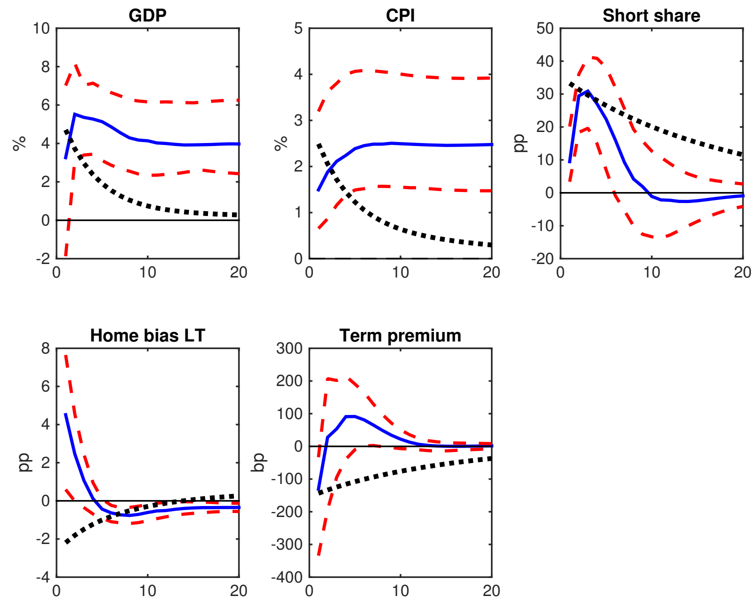
Given these responses from the QE shock in the data we then perform the following exercise. We set a fine grid for parameters κ_α, κ_α^*, κ_L, κ_L^* and select the parameterization that minimizes the distance between the impulse response functions in the model and in the data, on impact, for a QE shock of size 5% of EA GDP. We calibrate κ_α, κ_α^*, κ_L, and κ_L^* so that our model can capture four moments: i) an increase in the share of short-term bonds to total bonds (short-share) in the Periphery by 10%, ii) a decline in the union-wide term premium by 145 basis points, iii) an increase in the share of domestic long-term bonds to total long-term bonds (home bias) in the Periphery by 1 pp., and iv) an increase in the share of domestic long-term bonds to total long-term bonds (home bias) in the Core by 4 pp. This term premium decline is slightly larger than the evidence from

Figure 15: SVAR: QE Shock in the Periphery



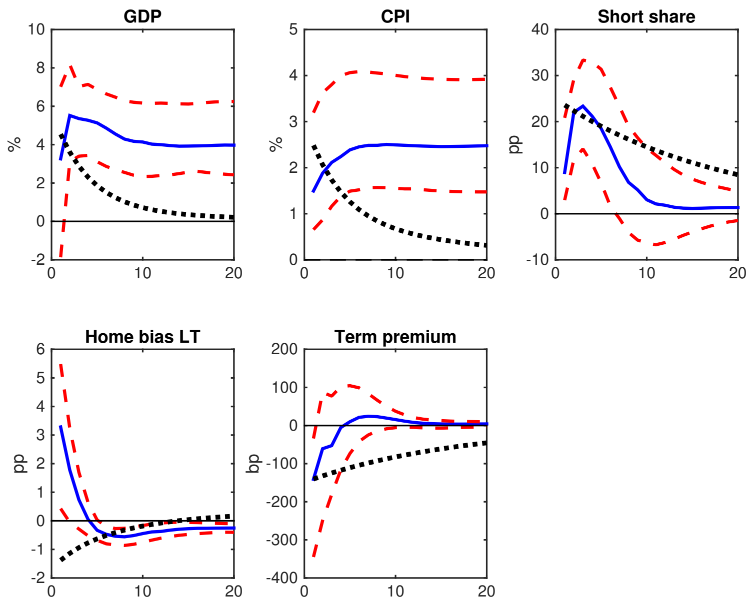
Notes: SVAR identified with sign restrictions. (Scaled) impulse response functions in the Periphery to a quantitative easing shock of size 5% of EA GDP. Solid blue lines are median estimates. Dashed red lines correspond to error bands of one standard deviation. Black dotted lines are impulse responses from the calibrated theoretical model following a QE shock of size 5% of EA GDP, where the interest rate follows a Taylor rule.

Figure 16: SVAR: QE Shock in the Core



Notes: SVAR identified with sign restrictions. (Scaled) impulse response functions in the Core to a quantitative easing shock of size 5% of EA GDP. Solid blue lines are median estimates. Dashed red lines correspond to error bands of one standard deviation. Black dotted lines are impulse responses from the calibrated theoretical model following a QE shock of size 5% of EA GDP, where the interest rate follows a Taylor rule.

Figure 17: SVAR: QE Shock in the EA



Notes: SVAR identified with sign restrictions. (Scaled) impulse response functions in the EA (weighted responses of Periphery and Core using $n = 0.35$) to a quantitative easing shock of size 5% of EA GDP. Solid blue lines are median estimates. Dashed red lines correspond to error bands of one standard deviation. Black dotted lines are impulse responses from the calibrated theoretical model following a QE shock of size 5% of EA GDP, where the interest rate follows a Taylor rule.

DSGE-based studies of QE in the EA (see Hohberger *et al.* (2019) and references therein), but in line with the evidence in Demir *et al.* (2019), who find that following an unconventional monetary policy shock, bond returns decrease by around 150-200 basis points in Spain, Italy, and Portugal, and by around 50 basis points in Germany, France, and the Netherlands. The increases in the shares of domestic long-term bonds to total long-term bonds in both the Periphery and Core are small, especially when compared to changes in the short shares, so also in line with the evidence presented in section 2. The implied parameters from this exercise are as follows: $\kappa_\alpha = 0.1$, $\kappa_\alpha^* = 0.4$, $\kappa_L = 0.1$, and $\kappa_L^* = 0.1$.

C Proof of Proposition 1 (Welfare-Theoretic Loss Function)

We derive the welfare criterion for each country using a second order Taylor series expansion of the representative household's utility function (eq. 3.1) around the efficient steady state. We make use of the assumption that the planner has access to labor subsidies, making the steady state efficient and exhausting the distortions from monopolistic competition. The welfare measure is expressed in deviations from the flexible price equilibrium, which is also efficient, given the labor subsidy. Since the derivation steps are the same for both the Periphery and the Core, here we describe only the second order approximation of household utility in the Periphery.

The second order approximation of the welfare of the representative optimizing household receives the following form:

$$W_t = U + U_c c\left(\widehat{c}_t + \frac{1}{2}(1+\frac{U_{cc}c}{U_c})\widehat{c}_t^{\,2}\right) - U_l l\left(\widehat{l}_t + \frac{1}{2}(1+\frac{U_{ll}l}{U_l})\widehat{l}_t^2\right) \tag{C.1}$$

where $U_c = 1/c$, $U_{cc} = c^{-2}$, $U_l = l^{\gamma}$, and $U_{ll} = \gamma l^{\gamma-1}$. Using the fact that $\hat{y}_t(h) = \widehat{z}_t + \widehat{l}_t$ and approximating it up to a second order we receive the following expression for labor:

$$\hat{l}_t = 1 + \frac{y(h)}{L}E_t(\hat{y}_t(h)) + a_t + \frac{y(h)}{2L}var(\hat{y}_t(h)) + a_t^2 - \frac{1}{2}\hat{L}_t^2 \tag{C.2}$$

The variance of $\hat{y}_t(h)$ is related to the variance of the prices that producers face through the individual demand curve for each product in the following way:

$$var(\hat{y}(h)) = \theta^2 var\left(\tilde{p}_t(h)\right) \tag{C.3}$$

and in turn, the variance of prices is related to inflation by:

$$\sum_{t=0}^{\infty}\beta^t var(log(\tilde{p}_t(h)) = \frac{1}{(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega}{1-\omega}\widehat{\pi}_{h,t}^2\right] + t.i.p. + O(||\xi||^3) \tag{C.4}$$

Additionally, note that for the home output the following relationship holds (and similarly for foreign output):

$$\widehat{y}_t = E_t(\widehat{y}_t(h)) + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)var(\widehat{y}_t(h)) + O(||\xi||^3) \tag{C.5}$$

Using the above expression to substitute for $E_t(\hat{y}_t(i))$ in equation (C.2), we receive the following expression for $\hat{L}_t$:

$$\hat{l}_t \approx 1 + \frac{y}{l}\widehat{y}_t - \frac{1}{2\theta}\frac{y}{l}var(\hat{y}_t(h)) - \frac{1}{2}\widehat{l}_t^2 + t.i.p. \tag{C.6}$$

The market clearing condition for home goods writes as follows:

$$\frac{c_h}{y}\widehat{c}_{h,t} + \left(1 - \frac{c_h}{y}\right)\widehat{c}_{h,t}^* = \widehat{y}_t \tag{C.7}$$

At the same time we know that $\widehat{c}_{h,t}$ and $\widehat{c}_{h,t}^*$ are specified as:

$$\widehat{c}_{h,t} = -\kappa\,(1-\zeta)\,\widehat{s}_t + \widehat{c}_t, \qquad \widehat{c}^*_{h,t} = -\kappa^*\zeta^*\widehat{s}_t + \widehat{c}^*_t \tag{C.8}$$

Solving for $E_t(\widehat{y}_t(h))$ in (C.5) and substituting in (C.6), we receive:

$$\hat{l}_t = 1 + \frac{y}{l}\hat{y}_t + \frac{1}{2}\frac{1}{\theta}\frac{y}{l}var(\hat{y}_t(h)) - \frac{1}{2}\hat{l}^2_t + \widehat{z}_t + \widehat{z}^2_t + t.i.p. \tag{C.9}$$

Normalizing steady-state real wages, $\bar{w}$, to 1 and, from the FOC, the fact that $U_c = U_l$ we may rewrite the loss function (C.1) as follows:

$$W_t = U + U_c\left\{ c(\widehat{c}_t + \frac{1}{2}c(1 + \frac{U_{cc}c}{U_c})\widehat{c}^2_t) - l\widehat{l}_t - \frac{1}{2}l(1 + \frac{U_{ll}l}{U_l})\widehat{l}^2_t \right\} \tag{C.10}$$

Substituting (C.9) into (C.10), and gathering the productivity terms in $t.i.p.$ we obtain:

$$\begin{aligned} W_t = U + U_c &\left\{ (c\widehat{c}_t + \frac{1}{2}c(1 + \frac{U_{cc}c}{U_c})\widehat{c}^2_t) \right. \\ &- \left[1 + y\hat{y}_t + \frac{1}{2}\frac{1}{\theta}y var(\hat{y}_t(h)) - \frac{1}{2}y\hat{l}^2_t \right] \\ &\left. - \frac{1}{2}y(1 + \frac{U_{ll}l}{U_l})\widehat{l}^2_t \right\} + t.i.p. \end{aligned} \tag{C.11}$$

Using (C.3) and (C.4) to substitute $var(\hat{y}_t(h))$ in (C.11), we receive:

$$\begin{aligned} W_t = U + U_c &\left\{ (c\widehat{c}_t + \frac{1}{2}c(1 + \frac{U_{cc}C}{U_c})\widehat{c}^2_t) \right. \\ &- \left[1 + y\hat{y}_t + \frac{1}{2}\frac{1}{\theta}y var(\hat{y}_t(h)) - \frac{1}{2}y\hat{l}^2_t \right] \\ &\left. - \frac{1}{2}y(1 + \frac{U_{ll}l}{U_l})\widehat{l}^2_t \right\} + t.i.p. \end{aligned} \tag{C.12}$$

At the steady state $y = l$, the expression above simplifies to: to:

$$\begin{aligned} W_t = U + U_c &\left\{ (c\widehat{c}_t + \frac{1}{2}c(1 + \frac{U_{cc}C}{U_c})\widehat{c}^2_t) \right. \\ &- \left[1 + y\hat{y}_t + \frac{1}{2}y\frac{1}{\theta}var(\hat{y}_t(h)) - \frac{1}{2}y\hat{l}^2_t \right] \\ &\left. - \frac{1}{2}y(1 + \frac{U_{ll}l}{U_l})\widehat{l}^2_t \right\} + t.i.p. \end{aligned} \tag{C.13}$$

From equations (C.3) and (C.4) we have:

$$var(log(\widehat{y}_t(h)) = \theta^2 var(log(\tilde{p}_t(h)) = \frac{\omega}{(1-\omega)(1-\omega\beta)}\widehat{\pi}^2_{h,t} + t.i.p. + O(||\xi||^3) \tag{C.14}$$

Substituting (C.14) into (C.13), we end up with:

$$
\begin{aligned}
W_t = U + U_c \Bigg\{ & (c\widehat{c}_t + \frac{1}{2}c(1+\frac{U_{cc}C}{U_c})\widehat{c}_t^2) \\
& - \left[1 + y\hat{y}_t - \frac{1}{2}y\theta\frac{\omega}{(1-\omega)(1-\omega\beta)}\widehat{\pi}_{h,t}^2 - \frac{1}{2}y\widehat{l}_t^2\right] \\
& - \frac{1}{2}y(1+\frac{U_{ll}l}{U_l})\widehat{l}_t^2 \Bigg\} + t.i.p. + O(||\xi||^3)
\end{aligned} \tag{C.15}
$$

Gathering terms and exploiting the production function to substitute out for labor $\widehat{l}_t$, we receive:

$$
\begin{aligned}
W_t = U + U_c \Bigg\{ & (c\widehat{c}_t + \frac{1}{2}c(1+\frac{U_{cc}C}{U_c})\widehat{c}_t^2) \\
& - y\hat{y}_t - \frac{1}{2}y\theta\frac{\omega}{(1-\omega)(1-\omega\beta)}\widehat{\pi}_{h,t}^2 \\
& - \frac{1}{2}y(\frac{U_{ll}l}{U_l})\widehat{y}_t^2 \Bigg\} + t.i.p. + O(||\xi||^3)
\end{aligned} \tag{C.16}
$$

Note that:

$$
\begin{aligned}
\frac{U_{cc}c}{U_c} &= -1 \\
\frac{U_{ll}l}{U_l} &= \gamma
\end{aligned} \tag{C.17}
$$

Putting the above relationships into (C.16), the latter simplifies further to:

$$
W_t = U - U_c \left\{ y\hat{y}_t - c\widehat{c}_t + \frac{1}{2}y\theta\frac{\omega}{(1-\omega)(1-\omega\beta)}\widehat{\pi}_{h,t}^2 + \frac{1}{2}y\gamma\widehat{y}_t^2 \right\} + t.i.p. + O(||\xi||^3) \tag{C.18}
$$

A similar expression holds for the Core:

$$
W_t^* = U - U_{c^*}^* \left\{ y^*\hat{y}_t^* - c^*\widehat{c}_t^* + \frac{1}{2}y^*\theta^*\frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)}\left(\widehat{\pi}_{f,t}^*\right)^2 + \frac{1}{2}y^*\gamma\left(\widehat{y}_t^*\right)^2 \right\} + t.i.p. + O(||\xi||^3) \tag{C.19}
$$

We assume that the social planner takes the sum of the welfares of the two regions where each welfare is weighed by the region size, n and $1-n$, respectively:

$$
W_t^u = W_t + W_t^* \tag{C.20}
$$

which leads to the aggregate welfare function:

$$
\begin{aligned}
W_t^u = &-\frac{1}{2}U^*_{c^*}\left(y+y^*\right)\Bigg\{\frac{1}{2}\left(1-n\right)^2\left(\widehat{y}^*_t\right)^2+\frac{1}{2}n^2\widehat{y}_t^2+\frac{1}{2}\left(1-n\right)^2\left(1-\frac{c^{*2}}{y^{*2}}-\frac{\left(\Xi^*-\xi^*\alpha^*\right)^2}{y^{*2}}\right)\left(1-\zeta-\zeta^*\right)^2\widehat{s}_t^2 \\
&-\frac{1}{2}\frac{c^{*2}}{y^{*2}}\left(1-n\right)^2\left(\widehat{c}^*_t\right)^2-\frac{1}{2}\frac{c^2}{y^2}n^2\widehat{c}_t^2-\frac{1}{2}\frac{\xi^2\alpha^2}{y^2}n^2\widehat{\alpha}_t^2-\frac{1}{2}\frac{\xi^{*2}\alpha^{*2}}{y^{*2}}\left(1-n\right)^2\widehat{\alpha}_t^{*2} \\
&+n\left(1-n\right)\widehat{y}_t\widehat{y}^*_t-\left(\frac{c^*-\Xi^*+\xi^*\alpha^*}{y^*}\right)\frac{c^*}{y^*}\left(1-n\right)^2\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{c}^*_t \\
&-\left(\frac{c^*-\Xi^*+\xi^*\alpha^*}{y^*}\right)\frac{c}{y}n\left(1-n\right)\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{c}_t+\left(\frac{c^*-\Xi^*+\xi^*\alpha^*}{y^*}\right)\frac{\xi\alpha}{y}n\left(1-n\right)\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{\alpha}_t \\
&+\left(\frac{c^*-\Xi^*+\xi^*\alpha^*}{y^*}\right)\frac{\xi^*\alpha^*}{y^*}\left(1-n\right)^2\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{\alpha}^*_t-\frac{c}{y^*}\frac{c^*}{y^*}n\left(1-n\right)\widehat{c}_t\widehat{c}^*_t \\
&-\frac{c^*}{y^*}\frac{\xi\alpha}{y}n\left(1-n\right)\widehat{c}^*_t\widehat{\alpha}_t-\frac{c^*}{y^*}\frac{\xi^*\alpha^*}{y^*}\left(1-n\right)^2\widehat{c}^*_t\widehat{\alpha}^*_t-\frac{c}{y}\frac{\xi\alpha}{y}n^2\widehat{c}_t\widehat{\alpha}_t-\frac{c}{y}\frac{\xi^*\alpha^*}{y^*}n\left(1-n\right)\widehat{c}_t\widehat{\alpha}^*_t \\
&-\frac{\xi\alpha}{y}\frac{\xi^*\alpha^*}{y^*}n\left(1-n\right)\widehat{\alpha}_t\widehat{\alpha}^*_t+\left(1-n\right)^2\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{y}^*_t+n\left(1-n\right)\left(1-\zeta-\zeta^*\right)\widehat{s}_t\widehat{y}_t \\
&+\frac{\tau}{2}\left(n\frac{1}{y}\left(\frac{\widehat{qb}^{cb}_{L,t}}{q_Lb_L}\right)^2+\left(1-n\right)\frac{1}{y^*}\left(\frac{\widehat{qb}^{*,cb}_{L,t}}{q^*_Lb^*_L}\right)^2\right) \\
&+\frac{1}{2}n\theta\frac{\omega}{(1-\omega)(1-\omega\beta)}\widehat{\pi}^2_{h,t}+\frac{1}{2}\left(1-n\right)\theta^*\frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)}\left(\widehat{\pi}^*_{f,t}\right)^2 \\
&+\frac{1}{2}n\gamma\widehat{y}_t^2+\frac{1}{2}\left(1-n\right)\gamma^*\left(\widehat{y}^*_t\right)^2\Bigg\}+t.i.p.+O(||\xi||^3)
\end{aligned}
\tag{C.21}
$$

Finally, letting $\psi_t=\left(1-\zeta-\zeta^*\right)\widehat{s}_t$, and imposing $\Xi^*-\xi^*\alpha^*=0$, the aggregate welfare function can be simplified to:

$$
\begin{aligned}
L_t = &-\frac{1}{2}\frac{U^*_{c^*}}{y+y^*}\Bigg\{\Phi_y\left(\widehat{y}_t\right)^2+\Phi_{y^*}\left(\widehat{y}^*_t\right)^2+\Phi_{\pi^h}\left(\widehat{\pi}_{h,t}\right)^2+\Phi_{\pi^{f*}}\left(\widehat{\pi}^*_{f,t}\right)^2+\Phi_s\left(\widehat{\psi}_t\right)^2 \\
&-\Phi_{c^*}\left(\widehat{c}^*_t\right)^2-\Phi_c\left(\widehat{c}_t\right)^2-\Phi_a\left(\widehat{\alpha}_t\right)^2-\Phi_{a^*}\left(\widehat{\alpha}^*_t\right)^2+\frac{\tau n}{2}\left(\frac{\widehat{qb}^{cb}_{L,t}}{q_Lb_L}\right)^2+\frac{\tau\left(1-n\right)}{2}\left(\frac{\widehat{qb}^{*,cb}_{L,t}}{q^*_Lb^*_L}\right)^2 \\
&+\Omega_{ij}\sigma_{ij,t}\Bigg\}+t.i.p.+O(||\xi||^3)
\end{aligned}
\tag{C.22}
$$

where,

$\Phi_y = \frac{n}{2}\left(ny^2 + \gamma\left(y + y^*\right)\right); \quad \Phi_{y^*} = \frac{(1-n)}{2}\left(\left(1-n\right)\left(y^*\right)^2 + \gamma^*\left(y + y^*\right)\right)$

$\Phi_{\pi^h} = \frac{n(y+y^*)}{2}\frac{\theta\omega}{(1-\omega)(1-\omega\beta)}; \quad \Phi_{\pi^{f*}} = \frac{(1-n)(y+y^*)}{2}\frac{\theta^*\omega^*}{(1-\omega^*)(1-\omega^*\beta)}$

$\Phi_s = \frac{(1-n)^2}{2}\left(\left(y^*\right)^2 - \left(c^*\right)^2\right)\left(1 - \zeta - \zeta^*\right)$

$\Phi_c = \frac{n^2}{2}c^2; \quad \Phi_{c^*} = \frac{(1-n)^2}{2}c^{*2}; \quad \Phi_a = \frac{n^2}{2}\xi^2\alpha^2; \quad \Phi_{a^*} = \frac{(1-n)^2}{2}\xi^{*2}\alpha^{*2}$ and where Ω_{ij} and $\sigma_{ij,t}$ are matrices of coefficients and covariance terms, respectively, for $i, j = \{\hat{c}_t, \hat{c}_t^*, s_t, \hat{y}_t, \hat{y}_t^*, \widehat{a}_t, \widehat{a}_t^*\}$, when $i \neq j$.

D Proof of Proposition 2

It is straightforward to obtain the derivatives.

E Proof of Proposition 3

When the short-term interest rate is away from the zero lower bound, $\lambda_t^{zlb} = 0$. Combining the first order conditions with respect to inflation in the Periphery, $\widehat{\pi}_{h,t}$, and the Core, $\widehat{\pi}_{f,t}$, the FOCs with respect to outputs, $\widehat{y}_t$, $\widehat{y}_t^*$, and solving for the Lagrange multipliers λ_t^3, λ_t^4, and λ_t^5 we receive the following expression:

$$\begin{aligned} &\varpi_y \widehat{y}_t + \varpi_y^* \widehat{y}_t^* = \\ &\quad - \varpi_{\pi_h} \widehat{\pi}_{h,t} - \varpi_{\pi_f} \widehat{\pi}_{f,t}^* + \varpi_y \widehat{y}_{t-1} + \varpi_y^* \widehat{y}_{t-1}^* + \Omega_t \end{aligned} \tag{E.1}$$

where Ω_t captures terms irrelevant to the trade-off between prices and outputs. Iterating (E.1) backwards, we end up with:

$$\begin{aligned} &\varpi_y \widehat{y}_t + \varpi_y^* \widehat{y}_t^* = \\ &\quad - \varpi_{\pi_h} \widehat{p}_{h,t} - \varpi_{\pi_f} \widehat{p}_{f,t}^* + \Omega_t \end{aligned} \tag{E.2}$$

where

$$\varpi_y = \left(\frac{U^*_{c^*}}{y+y^*}\right)\left(\frac{\omega}{(1-\omega)(1-\omega\beta)\gamma}\Phi_y + yy^*\frac{\omega^* n(1-n)}{(1-\omega^*)(1-\omega^*\beta)\gamma^*}\right)$$

$$\varpi^*_y = \left(\frac{U^*_{c^*}}{y+y^*}\right)\left(\frac{\omega^*}{(1-\omega^*)(1-\omega^*\beta)\gamma^*}\Phi_{y^*} + yy^*\frac{\omega n(1-n)}{(1-\omega)(1-\omega\beta)\gamma}\right)$$

$$\varpi_{\pi_{h,t}} = \left(\frac{U^*_{c^*}}{y+y^*}\right)\left(\Phi_{\pi_h} + yy^* n(1-n)(1-\zeta-\zeta^*)\left(\frac{\omega n}{(1-\omega)(1-\omega\beta)\gamma} + \frac{\omega^*(1-n)}{(1-\omega^*)(1-\omega^*\beta)\gamma^*}\right)\right)$$

$$\varpi_{\pi_{f,t}} = \left(\frac{U^*_{c^*}}{y+y^*}\right)\left(\Phi_{\pi_f} + yy^*(1-n)^2(1-\zeta-\zeta^*)\left(\frac{\omega n}{(1-\omega)(1-\omega\beta)\gamma} + \frac{\omega^*(1-n)}{(1-\omega^*)(1-\omega^*\beta)\gamma^*}\right)\right)$$

F Proof of Proposition 4

Solving the FOC with respect to central bank purchases of Peripheral and Core long-term bonds, $\widehat{qb}^{CB}_{L,t}$ and $\widehat{qb}^{*,CB}_{L,t}$, for λ^8_t and λ^9_t, respectively, we receive:

$$\lambda^8_t = -q_L b_L \frac{U^*_{c^*}}{2(y+y^*)}\tau n\widehat{qb}^{CB}_{L,t} + \frac{q_L b_L}{b_S}\lambda^6_t \tag{F.1}$$

$$\lambda^9_t = -q^*_L b^*_L \frac{U^*_{c^*}}{2(y+y^*)}\tau(1-n)\widehat{qb}^{*,CB}_{L,t} + \frac{q^*_L b^*_L}{b^*_S}\lambda^6_t \tag{F.2}$$

Solving the FOCs with respect to $\alpha_{S,t}$, $\alpha^*_{S,t}$, $\alpha_{L,t}$ and $\alpha^*_{S,t}$ for λ^{25}_t, λ^{26}_t, λ^{27}_t, and λ^{28}_t, plugging them into the FOC with respect to $\widehat{s}_t$ and solving for $\widehat{s}_t$, we receive the following expression:

$$\begin{aligned}\widehat{s}_t = &\\ &-\frac{\varphi}{\tilde{\Phi}_s}\frac{b_{HL}/y}{b_L/y}\kappa_L\tau n\widehat{qb}^{CB}_{L,t} + \frac{\varphi^*}{\tilde{\Phi}_s}\frac{b^*_{FL}/y^*}{b^*_L/y^*}\kappa^*_L\tau(1-n)\widehat{qb}^{*CB}_{L,t}\\ &-\frac{1}{\tilde{\Phi}_s}\left[(1-\zeta_S)\zeta_\alpha + (1-\zeta_L)(1-\zeta_\alpha)\right]\lambda^{23}_t + \frac{1}{\tilde{\Phi}_s}\left[(1-\zeta^*_S)\zeta^*_\alpha + (1-\zeta^*_L)(1-\zeta^*_\alpha)\right]\lambda^{24}_t\\ &+\frac{1}{\tilde{\Phi}_s}(\varphi^*-\varphi)\Omega_t\end{aligned} \tag{F.3}$$

where

$$\varphi = (1-\zeta_S)\,\frac{\kappa_S-\kappa_\alpha}{\kappa_S\kappa_\alpha} - (1-\zeta_L)\,\frac{\kappa_L-\kappa_\alpha}{\kappa_L\kappa_\alpha} \tag{F.4}$$

$$\varphi^* = (1-\zeta_S^*)\,\frac{\kappa_S^*-\kappa_\alpha^*}{\kappa_S^*\kappa_\alpha^*} - (1-\zeta_L^*)\,\frac{\kappa_L^*-\kappa_\alpha^*}{\kappa_L^*\kappa_\alpha^*} \tag{F.5}$$

$$\tilde{\Phi}_s = (\zeta+\zeta^*-1)\,\frac{(1-n)^2}{2}\left((y^*)^2-(c^*)^2\right) \tag{F.6}$$

where Ω_t stands for the rest of the terms, which are irrelevant to the trade-off between terms of trade and union central bank purchases of Peripheral long-term bonds.

Clearly, for $\kappa_S > \kappa_\alpha > \kappa_L$ and $\kappa_S^* > \kappa_\alpha^* > \kappa_L^*$ or $(1-\zeta_S)\,\frac{\kappa_S-\kappa_\alpha}{\kappa_S\kappa_\alpha} > (1-\zeta_L)\,\frac{\kappa_L-\kappa_\alpha}{\kappa_L\kappa_\alpha}$, $(1-\zeta_S^*)\,\frac{\kappa_S^*-\kappa_\alpha^*}{\kappa_S^*\kappa_\alpha^*} > (1-\zeta_L^*)\,\frac{\kappa_L^*-\kappa_\alpha^*}{\kappa_L^*\kappa_\alpha^*}$ and for home bias in consumption, $\zeta, \zeta^* > 1/2$

$$\frac{\partial \widehat{s}_t}{\partial \widehat{qb}_{L,t}^{CB}} < 0 \qquad \frac{\partial \widehat{s}_t}{\partial \widehat{qb}_{L,t}^{*,CB}} > 0$$

However, under symmetry and symmetric central bank asset purchases $\left(\widehat{qb}_{L,t}^{CB} = \widehat{qb}_{L,t}^{*CB}\right)$, and $\varphi = \varphi^*$, the terms of trade is equal to its steady state. As a result, there is no trade-off between central asset purchases in the Periphery and the terms of trade.

G Proof of Proposition 5

From equation (F.3), we have that the terms of trade $\widehat{s}_t$ depends on Lagrange multipliers λ_t^{23} and λ_t^{24}, which are the multipliers on the aggregate portfolios in the Periphery and the Core, respectively, $\widehat{\alpha}_t$ and $\widehat{\alpha}_t^*$. Substituting the FOCs with respect to $\widehat{\alpha}_t$ and $\widehat{\alpha}_t^*$ from the optimal monetary policy problem we receive the following expression:

$$
\begin{aligned}
\widehat{s}_t = \\
& -\frac{\varphi}{\tilde{\Phi}_s}\frac{b_{HL}/y}{b_L/y}\tau n\widehat{qb}_{L,t}^{CB} + \frac{\varphi^*}{\tilde{\Phi}_s}\frac{b^*_{FL}/y^*}{b^*_L/y^*}\tau\left(1-n\right)\widehat{qb}_{L,t}^{*CB} \\
& -\frac{1}{\tilde{\Phi}_s}\left\{\left[\left(1-\zeta_S\right)\zeta_\alpha + \left(1-\zeta_L\right)\left(1-\zeta_\alpha\right)\right]\Phi_\alpha + \left[\left(1-\zeta^*_S\right)\zeta^*_\alpha + \left(1-\zeta^*_L\right)\left(1-\zeta^*_\alpha\right)\right]n\left(1-n\right)\xi\alpha\xi^*\alpha^*\right\}\widehat{\alpha}_t \\
& +\frac{1}{\tilde{\Phi}_s}\left\{\left[\left(1-\zeta^*_S\right)\zeta^*_\alpha + \left(1-\zeta^*_L\right)\left(1-\zeta^*_\alpha\right)\right]\Phi^*_\alpha + \left[\left(1-\zeta_S\right)\zeta_\alpha + \left(1-\zeta_L\right)\left(1-\zeta_\alpha\right)\right]n\left(1-n\right)\xi\alpha\xi^*\alpha^*\right\}\widehat{\alpha}^*_t \\
& +\frac{1}{\tilde{\Phi}_s}\left(\varphi^*-\varphi\right)\tilde{\Omega}_t \qquad \text{(G.1)}
\end{aligned}
$$

where the $\tilde{\Omega}_t$ captures terms unrelated to the relation between portfolios and the terms of trade or QE purchases.

H Proof of Proposition 6

Combining the FOCs with respect to $\widehat{qb}_{L,t}^{CB}$, $\widehat{qb}_{L,t}^{*CB}$, $\widehat{c}_t, \widehat{c}^*_t, \widehat{\pi}_{h,t}, \widehat{\pi}_{f,t}, \widehat{\alpha}_t, \widehat{\alpha}^*_t, \widehat{\alpha}_{L,t}, \widehat{\alpha}^*_{L,t}, \widehat{R}_{L,t}, \widehat{R}^*_{L,t}$ and solving for $\widehat{qb}_{L,t}^{CB}$, $\widehat{qb}_{L,t}^{*CB}$, we receive the following expression:

$$
\begin{aligned}
&\left(\frac{b_{HL}/y}{b_L/y}q_L b_L \frac{U^*_{c^*}}{2\left(y+y^*\right)}\right)\tau n q \widehat{b}^{CB}_{L,t} - \left(1-\frac{\kappa_L}{\kappa_\alpha}\right)\kappa^*_L \Upsilon \left(\frac{b^*_{FL}/y^*}{b^*_L/y^*}q^*_L b^*_L \frac{U^*_{c^*}}{2\left(y+y^*\right)}\tau\right)\left(1-n\right) q \widehat{b}^{*CB}_{L,t} \\
&= -\gamma \frac{U^*_{c^*}}{2\left(y+y^*\right)}\Bigg\{ -2\Phi_c \hat{c}_t - n\left(1-n\right) c^* c \left(1-\zeta-\zeta^*\right)\widehat{s}_t - n\left(1-n\right) c^* c \widehat{c}^*_t - n^2 c \xi \alpha \widehat{\alpha}_t \\
&\quad - n\left(1-n\right) c \xi^* \alpha^* \widehat{\alpha}^*_t \Bigg\} \\
&\quad + \kappa_L\left(1-\zeta_\alpha\right)\frac{U^*_{c^*}}{2\left(y+y^*\right)}\Bigg\{ -2\Phi_a \widehat{\alpha}_t + n\left(1-n\right) c^* \xi \alpha \left(1-\zeta-\zeta^*\right)\widehat{s}_t - n\left(1-n\right) c^* \xi \alpha \widehat{c}^*_t - n^2 c \xi \alpha \widehat{c}_t \\
&- n\left(1-n\right)\xi \alpha \xi^* \alpha^* \widehat{\alpha}^*_t \Bigg\} \\
&\quad - \gamma \frac{c}{y}\frac{\left(1-\omega\right)\left(1-\omega\beta\right)}{\omega}\frac{U^*_{c^*}}{\left(y+y^*\right)}\Phi_{\pi^h}\widehat{\pi}_{h,t} - \gamma \frac{c}{y}\left(\lambda^1_t + \frac{\zeta}{1-\zeta}\lambda^{19}_t + \frac{y}{c}\left(1-\zeta^*\frac{c^*}{y^*}\right)\lambda^{20}_t\right) \\
&\quad - \gamma \frac{c}{y}\frac{\left(1-\omega\right)\left(1-\omega\beta\right)}{\omega}\left(\lambda^5_t + \zeta\lambda^{21}_t + \left(1-\zeta^*\right)\lambda^{22}_t\right) \\
&\quad + \gamma^* \frac{U^*_{c^*}}{2\left(y+y^*\right)}\Bigg\{ -2\Phi_{c^*}\hat{c^*}_t - \left(1-n\right)^2 c^{*2}\left(1-\zeta-\zeta^*\right)\widehat{s}_t - n\left(1-n\right) c^* c \widehat{c}_t \\
&\quad - n\left(1-n\right) c^* \xi \alpha \widehat{\alpha}_t - \left(1-n\right)^2 c^* \xi^* \alpha^* \widehat{\alpha}^*_t \Bigg\} \\
&\quad + \gamma^* \left(\frac{1}{1-\zeta\frac{c}{y}}\lambda^2_t - \lambda^{19}_t - \zeta^* \frac{c^*}{y^*}\left(\frac{1}{1-\zeta\frac{c}{y}}\right)\lambda^{20}_t\right) \\
&\quad + \gamma^* \frac{\left(1-\omega^*\right)\left(1-\omega^*\beta\right)}{\omega^*}\left(\frac{1}{1-\zeta\frac{c}{y}}\right)\frac{U^*_{c^*}}{\left(y+y^*\right)}\Phi_{\pi^{f*}}\widehat{\pi}_{f,t} \\
&\quad + \gamma^* \frac{\left(1-\omega^*\right)\left(1-\omega^*\beta\right)}{\omega^*}\left(\frac{1}{1-\zeta\frac{c}{y}}\right)\left(-\lambda^5_t + \zeta\lambda^{21}_t + \left(1-\zeta^*\right)\lambda^{22}_t\right) \\
&\quad - \left(1-\frac{\kappa_L}{\kappa_\alpha}\right)\Upsilon\left(\frac{\Pi}{\beta R^U}-1\right)\left(\lambda^{29}_t - \frac{\rho^*}{\beta R_L}\lambda^{32}_t\right) \\
&\quad + \zeta^*_L\left(1-\frac{\kappa_L}{\kappa_\alpha}\right)\left(1-\zeta^*_\alpha\right)\frac{U^*_{c^*}}{2\left(y+y^*\right)}\Upsilon\Bigg\{ -2\Phi_{a^*}\widehat{\alpha}^*_t + \left(1-n\right)^2\left(1-\zeta-\zeta^*\right) c^* \xi^* \alpha^* \widehat{s}_t - \left(1-n\right)^2 c^* \xi^* \alpha^* \widehat{c}^*_t \\
&\quad - n\left(1-n\right) c \xi^* \alpha^* \widehat{c}_t - n\left(1-n\right)\xi \alpha \xi^* \alpha^* \widehat{\alpha}_t \Bigg\}
\end{aligned} \tag{H.1}
$$

In the expression above, the multipliers $\lambda^1_t, \lambda^2_t, \lambda^{19}_t, \lambda^{20}_t = 0$, and hence optimal QE, are independent of output fluctuations outside the ZLB. At the ZLB, we substitute λ^{19}_t and λ^{20}_t for the FOCs with respect to Peripheral and Core output, respectively. In this case, a trade-off between Peripheral (Core) asset purchases and Peripheral (Core) output stabilization arises.

Using the definition for Φ_{π^h} and $\Phi_{\pi^{f*}}$, the coefficients on $\widehat{\pi}_{h,t}$ and $\widehat{\pi}_{f,t}$, respectively, are:

$$n\gamma \frac{c}{y}\frac{U^*_{c^*}}{2} > 0$$

$$\left(1-n\right)\gamma \frac{1}{1-\zeta \frac{c}{y}}\frac{U^*_{c^*}}{2} > 0$$

which indicate that purchases of long-term Peripheral bonds relative to purchases of Core long-term bonds depend negatively on inflation in the Periphery, $\widehat{\pi}_{h,t}$, and positively on inflation at the Core, $\widehat{\pi}_{f,t}$.

Finally, gathering terms we receive the following coefficient next to Peripheral consumption:

$$-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\left[-\gamma n^2 c^2 + \kappa_L \left(1-\zeta_\alpha\right) c\xi\alpha n^2 + \gamma^* c c^* n \left(1-n\right) + \zeta^*_L \left(1-\frac{\kappa_L}{\kappa_\alpha}\right)\left(1-\zeta_\alpha\right)\Upsilon c\xi^*\alpha^* n\left(1-n\right)\right] < 0$$

which is negative. As a result, a rise in private consumption in the Periphery, ceteris paribus, requires a drop in purchases of Peripheral long-term bonds relative to Core long-term bonds.

I Proof of Proposition 7

It is straightforward to derive the first order derivatives with respect to ω and ω^* from Proposition 6.

J Optimal Quantitative Easing Under Commitment

J.1 Optimal Policy

Note that, we assume: $\Xi^* - \xi^* \alpha^* = 0$. Also, for the purposes of the welfare-theoretic loss function definition, recall that $\psi_t = (1 - \zeta - \zeta^*)\, \widehat{s}_t$.

$$L_t = -\frac{1}{2}\frac{U^*_{c^*}}{y+y^*}\Bigg\{\Phi_y\left(\widehat{y}_t\right)^2 + \Phi_{y^*}\left(\widehat{y}^*_t\right)^2 + \Phi_{\pi^h}\left(\widehat{\pi}_{h,t}\right)^2 + \Phi_{\pi^{f^*}}\left(\widehat{\pi}^*_{f,t}\right)^2 + \Phi_s\left(\psi_t\right)^2$$

$$-\Phi_{c^*}\left(\widehat{c}^*_t\right)^2 - \Phi_c\left(\widehat{c}_t\right)^2 - \Phi_a\left(\widehat{\alpha}_t\right)^2 - \Phi_{a^*}\left(\widehat{\alpha}^*_t\right)^2 + \frac{\tau n}{2}\left(\frac{\widehat{qb}^{cb}_{L,t}}{q_L b_L}\right)^2 + \frac{\tau(1-n)}{2}\left(\frac{\widehat{qb}^{*,cb}_{L,t}}{q^*_L b^*_L}\right)^2$$

$$+n(1-n)\Big[yy^*\left(\widehat{y}_t\widehat{y}^*_t\right) - c^*c\left(\psi_t\widehat{c}_t\right) + c^*\xi\alpha\left(\psi_t\widehat{\alpha}_t\right) - cc^*\left(\widehat{c}_t\widehat{c}^*_t\right) - c^*\xi\alpha\left(\widehat{c}^*_t\widehat{\alpha}_t\right)$$

$$-c\xi^*\alpha^*\left(\widehat{c}_t\widehat{\alpha}^*_t\right) - \xi\alpha\xi^*\alpha^*\left(\widehat{\alpha}_t\widehat{\alpha}^*_t\right) + yy^*\left(\psi_t\widehat{y}_t\right)\Big]$$

$$+(1-n)^2\Big[c^*\xi^*\alpha^*\left(\psi_t\widehat{\alpha}^*_t\right) - (c^*)^2\left(\psi_t\widehat{c}^*_t\right) - c^*\xi^*\alpha^*\left(\widehat{c}^*_t\widehat{\alpha}^*_t\right) + (y^*)^2\left(\psi_t\widehat{y}^*_t\right)\Big]$$

$$-n^2c\xi\alpha\left(\widehat{c}_t\widehat{\alpha}_t\right)\Bigg\} + t.i.p. + O(||\xi||^3)$$

where

$$\Phi_y = \frac{n}{2}\left(ny^2 + \gamma\left(y+y^*\right)\right); \qquad \Phi_{y^*} = \frac{(1-n)}{2}\left((1-n)\,y^{*,2} + \gamma^*\left(y+y^*\right)\right)$$
$$\Phi_{\pi^h} = \frac{n\left(y+y^*\right)}{2}\frac{\theta\omega}{(1-\omega)(1-\omega\beta)}; \qquad \Phi_{\pi^{f^*}} = \frac{(1-n)\left(y+y^*\right)}{2}\frac{\theta^*\omega^*}{(1-\omega^*)(1-\omega^*\beta)};$$
$$\Phi_s = \frac{(1-n)^2}{2}\left((y^*)^2 - (c^*)^2\right)(1-\zeta-\zeta^*)$$
$$\Phi_c = \frac{n^2}{2}c^2; \qquad \Phi_{c^*} = \frac{(1-n)^2}{2}c^{*2}$$
$$\Phi_a = \frac{n^2}{2}\xi^2\alpha^2; \qquad \Phi_{a^*} = \frac{(1-n)^2}{2}\xi^{*2}\alpha^{*2}$$

Note that the steady-state shares of consumption, output, and portfolio holdings are:

$$c = \left[1 - \Psi + \frac{(1-n)\Psi^*\delta^*_2 - n\Psi\delta^*_1}{n\left(\delta_2\delta^*_2 - \delta_1\delta^*_1\right)}\right]^{\frac{\gamma}{\gamma+1}}(1-\phi)^{\frac{1}{\gamma+1}}$$

$$c^* = 1\left[-\Psi^* + \frac{n}{(1-n)\delta^*_2}\left(\Psi - \delta_1\frac{(1-n)\Psi^*\delta^*_2 - n\Psi\delta^*_1}{n\left(\delta_2\delta^*_2 - \delta_1\delta^*_1\right)}\right)\right]^{\frac{\gamma^*}{\gamma^*+1}}(1-\phi^*)^{\frac{1}{\gamma^*+1}}$$

$y=\left(\frac{1-\phi}{c}\right)^{\frac{1}{\gamma}}$ and $y^{*}=\left(\frac{1-\phi^{*}}{c^{*}}\right)^{\frac{1}{\gamma^{*}}}$

$a=\frac{\overline{\pi}}{\overline{R}-\overline{\pi}}\left(\frac{(1-n)\Psi^{*}\delta_{2}^{*}-n\Psi\delta_{1}^{*}}{n\left(\delta_{2}\delta_{2}^{*}-\delta_{1}\delta_{1}^{*}\right)}\right)\left(\frac{1-\phi}{c}\right)^{\frac{1}{\gamma}}$

$a^{*}=\frac{n\overline{\pi}}{(1-n)\left(\overline{R}-\overline{\pi}\right)\delta_{2}^{*}}\left(\Psi-\delta_{1}\frac{(1-n)\Psi^{*}\delta_{2}^{*}-n\Psi\delta_{1}^{*}}{n\left(\delta_{2}\delta_{2}^{*}-\delta_{1}\delta_{1}^{*}\right)}\right)\left(\frac{1-\phi^{*}}{c^{*}}\right);$

where

$\delta_{1}=\zeta_{S}\zeta_{\alpha}+\zeta_{L}\left(1-\zeta_{\alpha}\right)$

$\delta_{2}=\left(1-\zeta_{S}\right)\zeta_{\alpha}+\left(1-\zeta_{L}\right)\left(1-\zeta_{\alpha}\right)$

$\delta_{1}^{*}=\zeta_{S}^{*}\zeta_{\alpha}^{*}+\zeta_{L}^{*}\left(1-\zeta_{\alpha}^{*}\right)$

$\delta_{2}^{*}=\left(1-\zeta_{S}^{*}\right)\zeta_{\alpha}^{*}+\left(1-\zeta_{L}^{*}\right)\left(1-\zeta_{\alpha}^{*}\right)$

The private sector equilibrium conditions enter as constraints into the policymakers' problem. The Lagrangian of the optimal policy problem takes the form:

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^1 \left\{ \widehat{c}_t - \widehat{c}_{t+1} + (PR_t - E_t \widehat{\pi}_{t+1} - r_t^n) \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^2 \left\{ \widehat{c}_t^* - \widehat{c}_{t+1}^* + \left(PR_t^* - E_t \widehat{\pi}_{t+1}^* - r_{n,t}^* \right) \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^3 \left\{ \widehat{\pi}_{h,t} - \beta E_t \widehat{\pi}_{h,t+1} - \frac{(1-\omega)(1-\beta\omega)}{\omega} \left[\gamma \widehat{y}_t + \widehat{c}_t - (1-\zeta)\, \widehat{s}_t \right] \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^4 \left\{ \widehat{\pi}_{f,t}^* - \beta E_t \widehat{\pi}_{f,t+1}^* - \frac{(1-\omega^*)(1-\beta\omega^*)}{\omega^*} \left[\gamma^* \widehat{y}_t^* + \widehat{c}_t^* + (1-\zeta^*)\, \widehat{s}_t \right] \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^5 \left\{ \widehat{s}_t - \widehat{s}_{t-1} - \left(\widehat{\pi}_{h,t} - \hat{\pi}_{f,t}^* \right) \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^6 \left\{ \frac{b_{HS}/y}{b_S/y} \widehat{b}_{HS,t} + (1 - \frac{b_{HS}/y}{b_S/y}) \widehat{b}_{HS,t}^* - \widehat{qb}_{L,t}^{CB} \frac{1}{b_S} \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^7 \left\{ \frac{b_{FS}^*/y^*}{b_S^*/y^*} \widehat{b}_{FS,t}^* + (1 - \frac{b_{FS}^*/y^*}{b_S^*/y^*}) \widehat{b}_{FS,t} - \widehat{qb}_{L,t}^{*CB} \frac{1}{b_S^*} \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^8 \left\{ \frac{b_{HL}/y}{b_L/y} \widehat{qb}_{HL,t} + (1 - \frac{b_{HL}/y}{b_L/y}) \widehat{qb}_{HL,t}^* + \frac{1}{q_L b_L} \widehat{qb}_{L,t}^{CB} \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^9 \left\{ \frac{b_{FL}^*/y^*}{b_L^*/y^*} \widehat{qb}_{FL,t}^* + (1 - \frac{b_{FL}^*/y^*}{b_L^*/y^*}) \widehat{qb}_{FL,t} + \frac{1}{q_L^* b_L^*} \widehat{qb}_{L,t}^{*CB} \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{10} \left\{ \frac{1}{\kappa_S} \left[\widehat{b}_{HS,t} - \left((1-\zeta-\zeta^*) \widehat{s}_t + \widehat{b}_{FS,t} \right) \right] - \frac{1}{\kappa_S^*} \left[(\widehat{b}_{HS,t}^* - (1-\zeta-\zeta^*) \widehat{s}_t) - \widehat{b}_{FS,t}^* \right] \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{11} \left\{ \frac{1}{\kappa_L} \left[\widehat{qb}_{HL,t} - \left((1-\zeta-\zeta^*) \widehat{s}_t + \widehat{qb}_{FL,t} \right) \right] - \frac{1}{\kappa_L^*} \left[(\widehat{qb}_{HL,t}^* - (1-\zeta-\zeta^*) \widehat{s}_t) - \widehat{qb}_{FL,t}^* \right] \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{12} \left\{ \Xi_1 \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{13} \left\{ \Xi_2 \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{14} \left\{ PR_t - \zeta_a \zeta_S \widehat{R}_t - (1-\zeta_a)\zeta_L \left(\widehat{R}_t + \widehat{T}_t \right) - \zeta_a (1-\zeta_S) \widehat{R}_t^U - (1-\zeta_a)(1-\zeta_L) \left(\widehat{R}_t^U + \widehat{T}_t^* \right) \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{15} \left\{ PR_t^* - \zeta_a^* \zeta_S^* \widehat{R}_t^U - (1-\zeta_a^*)\zeta_L^* \left(\widehat{R}_t^U + \widehat{T}_t^* \right) - \zeta_a^* (1-\zeta_S^*) \widehat{R}_t - (1-\zeta_a^*)(1-\zeta_L^*) \left(\widehat{R}_t + \widehat{T}_t \right) \right\} \\
& - E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^{16} \left\{ \widehat{R}_t - \widehat{R}_t^U - \left(\frac{\pi}{\beta R^U} - 1 \right) \frac{1}{\kappa_S} \left[\widehat{b}_{HS,t} - \left((1-\zeta-\zeta^*) \widehat{s}_t + \widehat{b}_{FS,t} \right) \right] \right\}
\end{aligned}
$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{17}\left\{\widehat{T}_t-\left(\frac{\pi}{\beta R^U}-1\right)\begin{bmatrix}-\frac{1}{\kappa_a}\left(\widehat{a}_{S,t}-\widehat{a}_{L,t}\right)\\ -\frac{1}{\kappa_L}\left(\widehat{a}_{L,t}-q\widehat{b}_{HL,t}\right)\\ +\frac{1}{\kappa_S}\left(\widehat{a}_{S,t}-\widehat{b}_{HS,t}\right)\end{bmatrix}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{18}\left\{\widehat{T}_t^*-\left(\frac{\pi}{\beta R^U}-1\right)\begin{bmatrix}-\frac{1}{\kappa_a^*}\left(\widehat{a}_{S,t}^*-\widehat{a}_{L,t}^*\right)\\ -\frac{1}{\kappa_L^*}\left(\widehat{a}_{L,t}^*-q\widehat{b}_{FL,t}^*\right)\\ +\frac{1}{\kappa_S^*}\left(\widehat{a}_{S,t}^*-\widehat{b}_{FS,t}^*\right)\end{bmatrix}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{19}\left\{y_t-\zeta\frac{c}{y}\widehat{c}_t-\left(1-\zeta\frac{c}{y}\right)\widehat{c}_t^*+\left[\zeta\frac{c}{y}\kappa\left(1-\zeta\right)+\left(1-\zeta\frac{c}{y}\right)\kappa^*\zeta^*\right]\widehat{s}_t\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{20}\left\{y_t^*-\zeta^*\frac{c^*}{y^*}\widehat{c}_t^*-\left(1-\zeta^*\frac{c^*}{y^*}\right)\widehat{c}_t-\left[\zeta^*\frac{c^*}{y^*}\kappa^*\left(1-\zeta^*\right)+\left(1-\zeta^*\frac{c^*}{y^*}\right)\kappa\zeta\right]\widehat{s}_t\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{21}\left\{\widehat{\pi}_t-\zeta\widehat{\pi}_{h,t}-\left(1-\zeta\right)\widehat{\pi}_{f,t}^*\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{22}\left\{\widehat{\pi}_t^*-\zeta^*\widehat{\pi}_{f,t}^*-\left(1-\zeta^*\right)\widehat{\pi}_{h,t}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{23}\left\{\widehat{a}_t-\zeta_a\widehat{a}_{S,t}-\left(1-\zeta_a\right)\widehat{a}_{L,t}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{24}\left\{\widehat{a}_t^*-\zeta_a^*\widehat{a}_{S,t}^*-\left(1-\zeta_a^*\right)\widehat{a}_{L,t}^*\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{25}\left\{\widehat{a}_{S,t}-\zeta_S\widehat{b}_{HS,t}-\left(1-\zeta_S\right)\left[(1-\zeta-\zeta^*)\widehat{s}_t+\widehat{b}_{FS,t}\right]\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{26}\left\{\widehat{a}_{S,t}^*-\zeta_S^*\widehat{b}_{FS,t}^*-\left(1-\zeta_S^*\right)\left[\widehat{b}_{HS,t}^*-(1-\zeta-\zeta^*)\widehat{s}_t\right]\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{27}\left\{\widehat{a}_{L,t}-\zeta_Lq\widehat{b}_{HL,t}-\left(1-\zeta_L\right)\left[(1-\zeta-\zeta^*)\widehat{s}_t+q\widehat{b}_{FL,t}\right]\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{28}\left\{\widehat{a}_{L,t}^*-\zeta_L^*q\widehat{b}_{FL,t}^*-\left(1-\zeta_L^*\right)\left[q\widehat{b}_{HL,t}^*-(1-\zeta-\zeta^*)\widehat{s}_t\right]\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{29}\left\{\widehat{q}_{L,t}^*+\frac{\widehat{R}_{L,t}^*}{\left(1-\frac{\rho^*}{R_L}\right)}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{30}\left\{\widehat{q}_{L,t}+\frac{\widehat{R}_{L,t}}{\left(1-\frac{\rho}{R_L}\right)}\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{31}\left\{\frac{R_L}{R_L-\rho}\widehat{R}_{L,t}-\frac{\rho}{R_L-\rho}E_t\widehat{R}_{L,t+1}-\widehat{R}_t+\widehat{T}_t\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{32}\left\{\frac{R_L}{R_L-\rho^*}\widehat{R}_{L,t}^*-\frac{\rho^*}{R_L-\rho^*}E_t\widehat{R}_{L,t+1}^*-\widehat{R}_t^U+\widehat{T}_t^*\right\}$$

$$-E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{33}\left\{\widehat{R}_t^U+\underline{R^{zlb}}\right\}$$

where $\left\{\lambda_t^1\right\}, \left\{\lambda_t^2\right\}, \ldots \left\{\lambda_t^{33}\right\}$ are the sequence of Lagrange multipliers, and where

$$\begin{aligned}
\Xi_1 = & \begin{bmatrix} -\frac{1}{\kappa_a}\left(\widehat{a}_{S,t} - \widehat{a}_{L,t}\right) \\ -\frac{1}{\kappa_L}\left(\widehat{a}_{L,t} - \widehat{qb}_{HL,t}\right) \\ +\frac{1}{\kappa_S}\left(\widehat{a}_{S,t} - \widehat{b}_{HS,t}\right) \end{bmatrix} - \begin{bmatrix} -\frac{1}{\kappa_a^*}\left(\widehat{a}_{S,t}^* - \widehat{a}_{L,t}^*\right) \\ -\frac{1}{\kappa_L^*}\left(\widehat{a}_{L,t}^* - \widehat{qb}_{FL,t}^*\right) \\ +\frac{1}{\kappa_S^*}\left(\widehat{a}_{S,t}^* - \widehat{b}_{FS,t}^*\right) \end{bmatrix} \\
& - \frac{1}{\kappa_L}\left[\widehat{qb}_{HL,t} - \left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{qb}_{FL,t}\right)\right] \\
& + \frac{1}{\kappa_S}\left(\widehat{b}_{HS,t} - \left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right)\right)
\end{aligned}$$

and

$$\begin{aligned}
\Xi_2 = & \frac{b_{FS}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{b}_{FS,t}\right) - \frac{R}{\pi}\frac{b_{FS}}{y}((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{R}_{t-1}^U + \widehat{b}_{FS,t-1} - \widehat{\pi}_t^*) \\
& + \frac{q_L^* b_{FL}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{qb}_{FL,t}\right) - \frac{R}{\pi}\frac{q_L^* b_{FL}}{y}\left((1-\zeta-\zeta^*)\widehat{s}_t + \widehat{R}_{L,t}^* + \widehat{q}_{L,t}^* + \widehat{qb}_{FL,t-1} - \widehat{q}_{L,t-1}^* - \widehat{\pi}_t^*\right) \\
& - \frac{y^*}{y}\frac{b_{HS}^*}{y^*}\frac{1-n}{n}\left(\widehat{b}_{HS,t}^*\right) + \frac{R}{\pi}\frac{y^*}{y}\frac{b_{HS}^*}{y^*}\frac{1-n}{n}\left(R_{t-1} + \widehat{b}_{HS,t-1}^* - \widehat{\pi}_t\right) \\
& - \frac{y^*}{y}\frac{q_L b_{HL}^*}{y^*}\frac{1-n}{n}\left(\widehat{qb}_{HL,t}^*\right) + \frac{R}{\pi}\frac{y^*}{y}\frac{q_L b_{HL}^*}{y^*}\frac{1-n}{n}\left(\widehat{R}_{L,t} + \widehat{q}_{L,t} + \widehat{qb}_{HL,t-1}^* - \widehat{q}_{L,t-1} - \widehat{\pi}_t\right) \\
& - \left(1 - \zeta\frac{c}{y}\right)\left((1-\zeta)\,\widehat{s}_t - \kappa^*\zeta^*\widehat{s}_t + \widehat{c}_t^*\right) + (1-\zeta)\,\frac{c}{y}\left(-\zeta\widehat{s}_t + \kappa\zeta\widehat{s}_t + \widehat{c}_t\right)
\end{aligned}$$

J.2 FOCs

First order conditions for the optimal policy problem are given as:

$$\partial\hat{c}_t: \quad 0=-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\left\{-2\Phi_c\hat{c}_t+n\left(1-n\right)\left[-c^*c\psi_t-cc^*\widehat{c}^*_t-c\xi^*\alpha^*\widehat{\alpha}^*_t\right]-n^2c\xi\alpha\widehat{\alpha}_t\right\}$$
$$-\lambda^1_t+\frac{1}{\beta}\lambda^1_{t-1}+\lambda^3_t\frac{(1-\omega)(1-\beta\omega)}{\omega}+\lambda^{13}_t\left(1-\zeta\right)\frac{c}{y}+\lambda^{19}_t\zeta\frac{c}{y}+\lambda^{20}_t\left(1-\zeta^*\frac{c^*}{y^*}\right)$$

$$\partial\hat{c}^*_t: \quad 0=-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\left\{-2\Phi_{c^*}\widehat{c}^*_t+n\left(1-n\right)\left[-cc^*\widehat{c}_t-c^*\xi\alpha\widehat{\alpha}_t\right]+\left(1-n\right)^2\left[-\left(c^*\right)^2\psi_t-c^*\xi^*\alpha^*\widehat{\alpha}^*_t\right]\right\}$$
$$-\lambda^2_t+\frac{1}{\beta}\lambda^2_{t-1}+\lambda^4_t\frac{(1-\omega^*)(1-\beta\omega^*)}{\omega^*}+\lambda^{13}_t\left(1-\zeta\frac{c}{y}\right)+\lambda^{19}_t\left(1-\zeta\frac{c}{y}\right)+\lambda^{20}_t\zeta^*\frac{c^*}{y^*}$$

$$\partial\hat{\pi}_{h,t}: \quad 0=-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\left\{2\Phi_{\pi^h}\hat{\pi}_{h,t}\right\}-\lambda^3_t+\lambda^3_{t-1}+\lambda^5_t+\zeta\lambda^{21}_t+\lambda^{22}_t\left(1-\zeta^*\right)$$

$$\partial\hat{\pi}_{f,t}: \quad 0=-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\left\{2\Phi_{\pi^{f^*}}\hat{\pi}_{f,t}\right\}-\lambda^4_t+\lambda^4_{t-1}-\lambda^5_t+\left(1-\zeta\right)\lambda^{21}_t+\zeta^*\lambda^{22}_t$$

$$\partial s_t: \quad 0=-\frac{U^*_{c^*}}{2\left(y+y^*\right)}\Bigg\{\left(1-\zeta-\zeta^*\right)\Big[2\Phi_s\hat{s}_t+n\left(1-n\right)\left(-cc^*\widehat{c}_t+c^*\xi\alpha\widehat{\alpha}_t+yy^*\widehat{y}_t\right)$$
$$+\left(1-n\right)^2\left(c^*\xi^*\alpha^*\widehat{\alpha}^*_t-\left(c^*\right)^2\widehat{c}^*_t+\left(y^*\right)^2\widehat{y}^*_t\right)\Big]\Bigg\}-\lambda^3_t\frac{(1-\omega)(1-\beta\omega)}{\omega}\left(1-\zeta\right)$$
$$-\lambda^4_t\frac{(1-\omega^*)(1-\beta\omega^*)}{\omega^*}\left(1-\zeta^*\right)-\lambda^5_t+\beta\lambda^5_{t+1}+\lambda^{10}_t\frac{1}{\kappa_S}\left(1-\zeta-\zeta^*\right)$$
$$-\lambda^{10}_t\frac{1}{\kappa^*_S}\left(1-\zeta-\zeta^*\right)+\lambda^{11}_t\frac{1}{\kappa_L}\left(1-\zeta-\zeta^*\right)-\lambda^{11}_t\frac{1}{\kappa^*_L}\left(1-\zeta-\zeta^*\right)$$
$$-\lambda^{12}_t\frac{1}{\kappa_L}\left(1-\zeta-\zeta^*\right)+\lambda^{12}_t\frac{1}{\kappa_S}\left(1-\zeta-\zeta^*\right)-\lambda^{13}_t\frac{b_{FS}}{y}\left(1-\zeta-\zeta^*\right)+\lambda^{13}_t\frac{R}{\pi}\frac{b_{FS}}{y}\left(1-\zeta-\zeta^*\right)$$
$$-\lambda^{13}_t\frac{q^*_Lb_{FL}}{y}\left(1-\zeta-\zeta^*\right)+\lambda^{13}_t\frac{R}{\pi}\frac{q^*_Lb_{FL}}{y}\left(1-\zeta-\zeta^*\right)+\lambda^{13}_t\left(1-\zeta\frac{c}{y}\right)\left(1-\zeta\right)-\lambda^{13}_t\left(1-\zeta\frac{c}{y}\right)\kappa^*\zeta^*$$
$$+\lambda^{13}_t\left(1-\zeta\frac{c}{y}\right)\zeta-\lambda^{13}_t\left(1-\zeta\right)\frac{c}{y}\kappa\zeta$$
$$-\lambda^{16}_t\left(1-\zeta-\zeta^*\right)-\lambda^{19}_t\left[\zeta\frac{c}{y}\kappa\left(1-\zeta\right)+\left(1-\zeta\frac{c}{y}\right)\kappa^*\zeta^*\right]$$
$$+\lambda^{20}_t\left[\zeta^*\frac{c^*}{y^*}\kappa^*\left(1-\zeta^*\right)+\left(1-\zeta^*\frac{c^*}{y^*}\right)\kappa\zeta\right]+\lambda^{25}_t\left(1-\zeta_S\right)\left(1-\zeta-\zeta^*\right)$$
$$-\lambda^{26}_t\left(1-\zeta^*_S\right)\left(1-\zeta-\zeta^*\right)+\lambda^{27}_t\left(1-\zeta_L\right)\left(1-\zeta-\zeta^*\right)-\lambda^{28}_t\left(1-\zeta^*_L\right)\left(1-\zeta-\zeta^*\right)$$

$$\partial \hat{R}_t^U: \qquad 0 = \beta \lambda_{t+1}^{13} \frac{R}{\pi} \frac{b_{FS}}{y} + \lambda_t^{14} \left[\zeta_a (1-\zeta_S) + (1-\zeta_a)(1-\zeta_L) \right] + \lambda_t^{15} \left[\zeta_a^* \zeta_S^* + (1-\zeta_a^*)\zeta_L^* \right] + \lambda_t^{16} + \lambda_t^{32} - \lambda_t^{33}$$

$$\partial \widehat{qb}_{L,t}^{CB}: \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ \frac{\tau n \widehat{qb}_{L,t}^{cb}}{q_L b_L} \right\} + \frac{\lambda_t^6}{b_S} - \frac{\lambda_t^8}{q_L b_L}$$

$$\partial \widehat{qb}_{L,t}^{*CB}: \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ \frac{\tau\,(1-n)\,\widehat{qb}_{L,t}^{*,cb}}{q_L^* b_L^*} \right\} + \frac{\lambda_t^7}{b_S^*} - \frac{\lambda_t^9}{q_L^* b_L^*}$$

$$\partial \widehat{b}_{HS,t}: \qquad 0 = -\lambda_t^6 \frac{b_{HS}/y}{b_S/y} - \frac{\lambda_t^{10}}{\kappa_S} + \frac{\lambda_t^{12}}{\kappa_S} - \frac{\lambda_t^{12}}{\kappa_S} + \left(\frac{\pi}{\beta R^U} - 1 \right) \frac{\lambda_t^{16}}{\kappa_S} - \left(\frac{\pi}{\beta R^U} - 1 \right) \frac{\lambda_t^{17}}{\kappa_S} + \lambda_t^{25} \zeta_S$$

$$\partial \widehat{b}_{FS,t}: \qquad 0 = -\lambda_t^7 (1 - \frac{b_{FS}^*/y^*}{b_S^*/y^*}) + \frac{\lambda_t^{10}}{\kappa_S} + \frac{\lambda_t^{12}}{\kappa_S} - \lambda_t^{13} \frac{b_{FS}}{y} + \beta \lambda_{t+1}^{13} \frac{R}{\pi} \frac{b_{FS}}{y} - \left(\frac{\pi}{\beta R^U} - 1 \right) \frac{\lambda_t^{16}}{\kappa_S} + \lambda_t^{25}\,(1-\zeta_S)$$

$$\partial \widehat{qb}_{HL,t}: \qquad 0 = -\lambda_t^8 \frac{b_{HL}/y}{b_L/y} - \frac{\lambda_t^{11}}{\kappa_L} - \frac{\lambda_t^{12}}{\kappa_L} + \frac{\lambda_t^{12}}{\kappa_L} + \left(\frac{\pi}{\beta R^U} - 1 \right) \frac{\lambda_t^{17}}{\kappa_L} + \lambda_t^{27} \zeta_L$$

$$\partial \widehat{qb}_{FL,t}: \qquad 0 = -\lambda_t^9 (1 - \frac{b_{FL}^*/y^*}{b_L^*/y^*}) + \frac{\lambda_t^{11}}{\kappa_L} - \frac{\lambda_t^{12}}{\kappa_L} - \frac{q_L^* b_{FL}}{y}\lambda_t^{13} + \beta \frac{R}{\pi}\frac{q_L^* b_{FL}}{y}\lambda_{t+1}^{13} + \lambda_t^{27}(1-\zeta_L)$$

$$\partial \widehat{b}_{HS,t}^*: \qquad 0 = -\lambda_t^6 (1 - \frac{b_{HS}/y}{b_S/y}) + \frac{\lambda_t^{10}}{\kappa_S^*} + \frac{y^*}{y}\frac{b_{HS}^*}{y^*}\frac{1-n}{n}\lambda_t^{13} - \beta \frac{R}{\pi}\frac{y^*}{y}\frac{b_{HS}^*}{y^*}\frac{1-n}{n}\lambda_{t+1}^{13} + \lambda_t^{26}(1-\zeta_S^*)$$

$$\partial \widehat{b}_{FS,t}^*: \qquad 0 = -\lambda_t^7 \frac{b_{FS}^*/y^*}{b_S^*/y^*} - \frac{\lambda_t^{10}}{\kappa_S^*} - \frac{\lambda_t^{12}}{\kappa_S^*} - \left(\frac{\pi}{\beta R^U} - 1\right)\frac{\lambda_t^{18}}{\kappa_S^*} + \lambda_t^{26}\zeta_S^*$$

$$\partial \widehat{qb}_{HL,t}^*: \qquad 0 = -\lambda_t^8 (1 - \frac{b_{HL}/y}{b_L/y}) + \frac{\lambda_t^{11}}{\kappa_L^*} + \frac{y^*}{y}\frac{q_L b_{HL}^*}{y^*}\frac{1-n}{n}\lambda_t^{13} - \beta \frac{R}{\pi}\frac{y^*}{y}\frac{q_L b_{HL}^*}{y^*}\frac{1-n}{n}\lambda_{t+1}^{13} + \lambda_t^{28}(1-\zeta_L^*)$$

$$\partial \widehat{qb}_{FL,t}^*: \qquad 0 = -\lambda_t^9 \frac{b_{FL}^*/y^*}{b_L^*/y^*} - \frac{\lambda_t^{11}}{\kappa_L^*} + \frac{\lambda_t^{12}}{\kappa_L^*} + \left(\frac{\pi}{\beta R^U} - 1\right)\frac{\lambda_t^{18}}{\kappa_L^*} + \lambda_t^{28}\zeta_L^*$$

$$\partial PR_t: \qquad 0 = -\lambda_t^1 - \lambda_t^{14}$$

$$\partial PR_t^*: \qquad 0 = -\lambda_t^2 - \lambda_t^{15}$$

$$\partial \hat{R}_t: \qquad 0 = -\beta \frac{R}{\pi}\frac{y^*}{y}\frac{b_{HS}^*}{y^*}\frac{1-n}{n}\lambda_{t+1}^{13} + \lambda_t^{14}\left[\zeta_a\zeta_S + (1-\zeta_a)\zeta_L\right] + \lambda_t^{15}\left[\zeta_a^*(1-\zeta_S^*) + (1-\zeta_a^*)(1-\zeta_L^*)\right]$$
$$- \lambda_t^{16} + \lambda_t^{31}$$

$$\partial \hat{T}_t : \qquad 0 = \lambda_t^{14}(1-\zeta_a)\zeta_L + \lambda_t^{15}(1-\zeta_a^*)(1-\zeta_L^*) - \lambda_t^{17} - \lambda_t^{31}$$

$$\partial \hat{T}_t^* : \qquad 0 = \lambda_t^{14}(1-\zeta_a)(1-\zeta_L) + \lambda_t^{15}(1-\zeta_a^*)\zeta_L^* - \lambda_t^{18} - \lambda_t^{32}$$

$$\partial \hat{y}_t : \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ 2\Phi_y \widehat{y}_t + n\,(1-n) \left[yy^* \widehat{y}_t^* + yy^* \widehat{\psi}_t \right] \right\} + \lambda_t^3 \frac{(1-\omega)(1-\beta\omega)}{\omega}\gamma - \lambda_t^{19}$$

$$\partial \hat{y}_t^* : \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ 2\Phi_{y^*} \widehat{y}_t^* + n\,(1-n)\,[yy^* \widehat{y}_t] + (1-n)^2\, yy^* \psi_t \right\} + \lambda_t^4 \frac{(1-\omega^*)(1-\beta\omega^*)}{\omega^*}\gamma^* - \lambda_t^{20}$$

$$\partial \hat{\pi}_t : \qquad 0 = \frac{\lambda_t^1}{\beta} + \lambda_t^{13} \left[\frac{R}{\pi} \frac{y^*}{y} \frac{b_{HS}^*}{y^*} \frac{1-n}{n} + \frac{R}{\pi} \frac{y^*}{y} \frac{q_L b_{HL}^*}{y^*} \frac{1-n}{n} \right] - \lambda_t^{21}$$

$$\partial \hat{\pi}_t^* : \qquad 0 = \frac{\lambda_t^2}{\beta} - \lambda_t^{13} \left[\frac{R}{\pi} \frac{b_{FS}}{y} + \frac{R}{\pi} \frac{q_L^* b_{FL}}{y} \right] - \lambda_t^{22}$$

$$\partial \widehat{a}_t : \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ -2\Phi_a \widehat{\alpha}_t + n\,(1-n) \left[c^*\xi\alpha\widehat{\psi}_t - c^*\xi\alpha\widehat{c}_t^* - \xi\alpha\xi^*\alpha^*\widehat{\alpha}_t^* \right] - n^2 c\xi\alpha\widehat{c}_t \right\}$$
$$- \lambda_t^{23}$$

$$\partial \widehat{a}_t^* : \qquad 0 = -\frac{U_{c^*}^*}{2\,(y+y^*)} \left\{ -2\Phi_{a^*} \widehat{\alpha}_t^* + n\,(1-n)\,[-c\xi^*\alpha^*\widehat{c}_t - \xi\alpha\xi^*\alpha^*\widehat{\alpha}_t] + (1-n)^2 \left[c^*\xi^*\alpha^*\widehat{\psi}_t - c^*\xi^*\alpha^*\widehat{c}_t^* \right] \right\}$$
$$- \lambda_t^{24}$$

$$\partial\widehat{a}_{S,t}: \qquad 0=\lambda_t^{12}\left[\frac{1}{\kappa_a}-\frac{1}{\kappa_S}\right]-\lambda_t^{17}\left(\frac{\pi}{\beta R^U}-1\right)\left[\frac{1}{\kappa_a}-\frac{1}{\kappa_S}\right]+\lambda_t^{23}\zeta_a-\lambda_t^{25}$$

$$\partial\widehat{a}^*_{S,t}: \qquad 0=-\lambda_t^{12}\left[\frac{1}{\kappa^*_a}-\frac{1}{\kappa^*_S}\right]-\lambda_t^{18}\left(\frac{\pi}{\beta R^U}-1\right)\left[\frac{1}{\kappa^*_a}-\frac{1}{\kappa^*_S}\right]+\lambda_t^{24}\zeta^*_a-\lambda_t^{26}$$

$$\partial\widehat{a}_{L,t}: \qquad 0=-\lambda_t^{12}\left[\frac{1}{\kappa_a}-\frac{1}{\kappa_L}\right]+\lambda_t^{17}\left(\frac{\pi}{\beta R^U}-1\right)\left[\frac{1}{\kappa_a}-\frac{1}{\kappa_L}\right]+\lambda_t^{23}\left(1-\zeta_a\right)-\lambda_t^{27}$$

$$\partial\widehat{a}^*_{L,t}: \qquad 0=\lambda_t^{12}\left[\frac{1}{\kappa^*_a}-\frac{1}{\kappa^*_L}\right]+\lambda_t^{18}\left(\frac{\pi}{\beta R^U}-1\right)\left[\frac{1}{\kappa^*_a}-\frac{1}{\kappa^*_L}\right]+\lambda_t^{24}\left(1-\zeta^*_a\right)-\lambda_t^{28}$$

$$\partial\widehat{q}_{L,t}: \qquad 0=-\frac{R}{\pi}\frac{y^*}{y}\frac{q_L b^*_{HL}}{y^*}\frac{1-n}{n}\lambda_t^{13}+\beta\frac{R}{\pi}\frac{y^*}{y}\frac{q_L b^*_{HL}}{y^*}\frac{1-n}{n}\lambda_{t+1}^{13}-\lambda_t^{30}$$

$$\partial\widehat{q}^*_{L,t}: \qquad 0=\frac{R}{\pi}\frac{q^*_L b_{FL}}{y}\lambda_t^{13}-\beta\frac{R}{\pi}\frac{q^*_L b_{FL}}{y}\lambda_{t+1}^{13}-\lambda_t^{29}$$

$$\partial\widehat{R}_{L,t}: \qquad 0=-\frac{R}{\pi}\frac{y^*}{y}\frac{q_L b^*_{HL}}{y^*}\frac{1-n}{n}\lambda_t^{13}-\frac{\lambda_t^{30}}{\left(1-\frac{\rho}{R_L}\right)}-\frac{R_L}{R_L-\rho}\lambda_t^{31}+\frac{\rho}{\beta\left(R_L-\rho\right)}\lambda_{t-1}^{31}$$

$$\partial\widehat{R}^*_{L,t}: \qquad 0=\frac{R}{\pi}\frac{q^*_L b_{FL}}{y}\lambda_t^{13}-\frac{\lambda_t^{29}}{\left(1-\frac{\rho^*}{R_L}\right)}-\frac{R_L}{R_L-\rho^*}\lambda_t^{32}+\frac{\rho^*}{\beta\left(R_L-\rho^*\right)}\lambda_{t-1}^{32}$$

Consequently, the equilibrium time path of

$\left\{\hat{R}^U_t,\widehat{qb}^{CB}_{L,t},\widehat{qb}^{*CB}_{L,t},\hat{c}_t,\hat{c}^*_t,\hat{\pi}_{h,t},\hat{\pi}^*_{f,t},s_t,\widehat{b}_{HS,t},\widehat{b}_{FS,t},\widehat{qb}_{HL,t},\widehat{qb}_{FL,t},\widehat{b}^*_{HS,t},\widehat{b}^*_{FS,t},\widehat{qb}^*_{HL,t},\widehat{qb}^*_{FL,t}\right\}_{t=0}^{\infty}$

$\left\{PR_t,PR^*_t,\hat{R}_t,\hat{T}_t,\hat{T}^*_t,\hat{y}_t,\hat{y}^*_t,\hat{\pi}_t,\hat{\pi}^*_t,\widehat{a}_t,\widehat{a}^*_t,\widehat{a}_{S,t},\widehat{a}^*_{S,t},\widehat{a}_{L,t},\widehat{a}^*_{L,t},\widehat{q}_{L,t},\widehat{q}^*_{L,t},\widehat{R}_{L,t},\widehat{R}^*_{L,t}\right\}_{t=0}^{\infty}$

$\left\{\lambda_t^1,\lambda_t^2,...,\lambda_t^{33}\right\}_{t=0}^{\infty}$

is characterized by 68 equations: 33 constraints + 35 FOCs (plus any exogenous shocks).