BANK OF CANADA
BANQUE DU CANADA

Staff Working Paper/Document de travail du personnel — 2020-38

Last updated: September 18, 2020

Cyclicality of Schooling: New Evidence from Unobserved Components Models

by Barbara Sadaba,[1] Sunčica Vujič[2,3,4,5] and Sofia Maier[3,6]

[1]International Economic Analysis Department
Bank of Canada, Ottawa, Ontario, Canada K1A 0G9

[2]University of Bath

[3]University of Antwerp

[4]IZA, Bonn

[5]GLO, Essen

[6]EU Science Hub

bsadaba@bankofcanada.ca, suncica.vujic@uantwerpen.be, sofia.maier@ec.europa.eu

ISSN 1701-9397

Acknowledgements

We thank Diana Alessandrini, Flavio Cunha, Gabriela Galassi, Reinhard Ellwanger, Sandra McNally, Petra Todd, Subrata Sarker, and Christopher Rauh—as well as conference and seminar participants at the Bank of Canada; the 23rd International Conference on Computing in Economics and Finance; the HCEO Faculty Collaborative Seminar 2018 in Guangzhou; and the 33rd European Economic Association Meeting—for their very useful comments. We also thank Gill Wyness for kindly sharing her dataset with us. The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada.

Abstract

In this paper, we present new evidence from unobserved components time-series models on the cyclical behavior of the demand for education relative to economic cycles. We investigate the cyclical properties of schooling decisions, the time-varying exposure of these decisions to changes in the state of the macro economy, and the relative importance of shocks that drive economic fluctuations on the demand for schooling. To this end, we perform a trend-cycle decomposition of enrollment ratios for the United Kingdom over the period 1995Q1 to 2019Q4. We first establish the presence of a persistent cyclical process in the demand for schooling independent of a slow-moving trend. We then show that the direction of the effect of the economic cycle on schooling decisions (i.e., pro-cyclical, counter-cyclical, a-cyclical) is largely time-dependent, together with the degree of synchronization. Importantly, we find that changes in the demand for schooling are largely explained by economic cycles. We note, however, that the effects are different for different subsamples based on demographic characteristics..

Bank topic: Business fluctuations and cycles; Econometric and statistical methods
JEL codes: E3, I2, J2, C32

1 Introduction

The critical role played by economic fluctuations on investments in human capital has been widely recognized in both theoretical and empirical work. While no disagreement exists regarding the existence of this influence, the cyclical properties of investment in human capital decisions and how these are shaped by economic fluctuations remain largely unexplored. All is due, in fact, to the lack of a direct measure of the cycle present in human capital investments. Such a measure would allow a deeper exploration of the impact of economic cycles on human capital accumulation and provide further insights regarding the direction, timing, size and significance of this effect. This constitutes a pending question both methodologically and for policy analysis, since different estimates of the cyclical fluctuations can lead to different policy recommendations. That is the focus of this paper.

Since the seminal work of Becker (1964) and Ben-Porath (1967), human capital investment decisions are modeled as a particular type of investment taking place early in life, which can be addressed in a general framework of time and resource allocation decisions taken by utility-maximizing individuals. That is, individuals decide whether to work or to study and how much time to allocate to one or the other. In the short-run, human capital investments tend to respond rationally to benefits and costs, which vary greatly over the economic cycle (Becker, 1994). These benefits and costs are reflected in at least two channels through which economic cycles can affect the decision to invest in human capital: the income effect—the ability-to-pay for the costs of pursuing further education—and the substitution effect related to changing opportunity costs of pursuing further education during the different phases of the economic cycle (Dellas and Koubi, 2003; Ferreira and Schady, 2009).[1] This shows that there is theoretical ground to infer that skill acquisition activities are influenced by macroeconomic conditions.[2] The theory predicts

[1]The ability to purchase education is determined by the availability and cost of funds, e.g., family wealth, student (or other types) loans and/or aid and part-time job opportunities. Opportunity-cost considerations include forgone income (Siow, 1985) and direct educational costs (tuition and fees) and depend on expectations about future professional employment possibilities and future earnings (Dellas and Koubi, 2003).

[2]A related strand of literature has explored the macroeconomic implications of human capital investment decisions. These works show that the timing and level of human capital accumulation can help explain the propagation of shocks and the persistence of economic growth. Perli and Sakellaris (1998) examine the role of human capital formation in propagating shocks over the business cycle. DeJong and Ingram (2001) analyze the general equilibrium implications of a representative agent's decision to devote time to skill acquisition activities, which are modeled as boosting subsequent labor productivity by increasing the stock of human capital.

that if the channel of the income effect dominates, then the response of skills acquisition will be pro-cyclical with less individuals accumulating further human capital during recessions. On the other hand, if the substitution effect dominates over ability-to-pay considerations, we can expect a counter-cyclical response.[3] Theory provides ambiguous conclusions regarding net outcomes in these relationships, and so this constitutes an empirical question.

We use a novel empirical approach to take a closer look at the cyclicality of human capital investments in relation to economic cycles. To this end, we use the Kalman filter in an unobserved components model to obtain an estimate of the cycle present in human capital investment decisions and in the economy. We then compare these estimates to explore in depth the relationship between the two. In this way, we build extensively on previous work (Sakellaris and Spilimbergo, 2000; DeJong and Ingram, 2001; Dellas and Sakellaris, 2003; Dellas and Koubi, 2003; Méndez and Sepúlveda, 2012), but we take a different tack. Rather than looking at estimates of the average effects of economic fluctuations—proxied by some labor market indicator—on the demand for schooling, we focus on providing an actual estimate of the cycle in the demand for education. This estimate allows us to analyze the whole spectrum of the co-movement of this education cycle with the economic cycle in closer detail. This is crucial for improving our understanding of the behavior of human capital investment decisions over time.

In our empirical assessment, we view human capital as a set of skills that increase through formal schooling. The reason for this is twofold: first, schooling constitutes the most easily observable component of all human capital investments; and second, although there is more to human capital than schooling, the same forces that affect schooling are also likely to affect non-schooling investments, and so we can infer from the patterns of schooling investments what might be happening to the overall human capital accumulation (Acemoglu and Autor, 2011). More specifically, our estimations are carried out using enrollment ratios for young individuals, 16–24 years old, in full-time post-compulsory education in the United Kingdom over the period from 1995Q1 to 2019Q4. We also construct two factors that capture the income and substitution effects channels through which the economic cycle is assumed to influence

[3] Ciccone (2000) argues that a pro-cyclical behavior is possible even without credit constrains being present. He claims that if individuals form the wrong expectations regarding the nature of the current phase of the economic cycle, they may opt to not pursue further education during a crisis.

schooling decisions. Using these, we are able to explore the relative importance of each of the channels in explaining the fluctuations in the enrollment cycle. Finally, we also conduct our estimations for two subsamples based on demographic characteristics: by gender (female/male) and by age groups (16–17, those just out of high school, and 18–24, those possibly already in the labor force).

Given the relevance of the issue, over the last decades several works explored the average effects of different labor market measures—proxying for macroeconomic conditions—on the demand for education. Overall, their findings suggest that macroeconomic conditions are indeed a significant driver of schooling decisions and that they are so in a counter-cyclical fashion (Betts and McFarland, 1995; Sakellaris and Spilimbergo, 2000; Dellas and Koubi, 2003; Johnson, 2013; Reiling and Strom, 2015; Boffy-Ramirez, 2017). However, only a few studies attempted to explore the cyclicality of skills acquisition and the time-series dimension present in it. For the first, DeJong and Ingram (2001) showed that, in the presence of a positive total factor productivity shock, human capital is more expensive than physical capital and thus agents decrease study hours and accumulate less human capital. Dellas and Sakellaris (2003) provide evidence through model simulation using microdata that cyclical fluctuations of the aggregate economy may have been associated with significant counter-cyclical swings in enrollments. Heylen and Pozzi (2007) find that profound economic crises (i.e., extreme business cycle fluctuations) increase human capital accumulation. Lastly, Méndez and Sepúlveda (2012) exploit individual data to investigate cyclicality in both schooling and training episodes. By detrending the data, they obtain an approximation to the cycle present in education. Their findings confirm a counter-cyclical pattern in schooling, while training episodes seem to respond pro-cyclically to the economic cycle. In contrast to our paper, their estimated cyclical fluctuations do not rely on a model and it is not analyzed jointly with the cycle present in the economy. Finally, three papers look into the time-series dimension. Pissarides (1981) uses data from 1955 to 1978 for England and Wales and finds a positive relationship between participation in education and changes in relative earnings of qualified workers and real income fluctuations. Whitfield and Wilson (1991) revisit this analysis using more recent data and find that participation rates over the long-run respond to the rate of return to education, social class structure and unemployment rates. Their findings present further support to the potential for time-variation in the relationship. More recently, McVicar and Rice (2001) use cointegration analysis for England and Wales and find that the growth in the trend component in participation in education is driven in big part to improvements in the General Certificate in Secondary Education (GCSE) attainment together with an expansion in higher education. Also, they find that short-run dynamics seem to be related to fluctuations in labor demand and youth unemployment. Taken together, these works offer strong support for the impact of macroeconomic conditions on the demand for education, but they lack a robust estimate of the actual cycle in schooling that can be analyzed in depth and jointly with the economic cycle. In the present paper, we address these shortcomings, and our longer and of higher frequency dataset is better suited to undertake deep time-series analysis.

The paper makes several contributions to the existing literature. The first is methodological. Namely, we use a state-of-the-art empirical strategy to estimate the cyclical fluctuations present in the demand for post-compulsory schooling. We decompose the demand for schooling into a slow moving trend and a stationary cycle using a state-space model with the Kalman smoother providing the estimated factors. In order to focus solely on demand side effects, we control for public spending in education as a measure of supply side effects. In contrast with previous work, we obtain a time-varying measure of the cycle, separate from the trend, avoiding the use of filtering techniques, which have been shown to be problematic (Hamilton, 2018). Existing literature has largely ignored the presence of a trend in schooling series. This omission can cause structural long-run effects to be confused with transitory ones due to the lack of proper separation of long-run and short-run dynamics.[4] Using the same methodology, we obtain an estimate of the economic cycle using the real GDP series for the U.K. over our sample period. We then proceed to compare both cyclical components, which constitutes our second contribution: we explore the properties of the relationship between both cycles using the estimated cycles. Third, we estimate the cycles present in two factors that account for the *ability-to-pay* (ATP) and the *opportunity-cost* (OC) channels, respectively. In this way, we present estimates of the channels through which the economic cycle affects enrollment decisions. This allows us to provide evidence on their relative importance over time. Finally, we provide a measure of the importance of the economic cycle for explaining the observed variation in the demand for schooling. To this end, by acknowledging the endogeneity present in the

[4] Previous works have at best estimated a trend component from an assumed equilibrium condition using cointegration or used filtering techniques such as the Hodrick and Prescott (1981,1997) filter (McVicar and Rice, 2001)

joint determination of schooling decisions and economic fluctuations, we estimate a bivariate unobserved component model. This allows us to explore further the relationship between the two variables, i.e., percentage variance explained by the economic cycle of the variation in the schooling cycle.

Our findings present, first, evidence of a persistent and significant cycle in the demand for post-compulsory schooling for young individuals over our estimation period. Second, we find that the response of schooling decisions to varying economic conditions is largely time-varying. We observe that counter-cyclicality takes place only during two periods in our sample, with the longest one taking place during the Great Recession. This result is in line with several existing works that have established counter-cyclicality as the most favored finding for advanced economies such as the U.K. A counter-cyclical pattern suggests that, on average, young people are able to substitute unemployment with a longer stay in the educational system. In terms of the Great Recession, this is an important result since it can potentially enhance the skill composition of the young workforce during the recovery phase (Pissarides, 2010). For the remainder of our sample, we find both a pro-cyclical and an a-cyclical behavior in schooling decisions. In particular, the first takes place only at the beginning of our sample after the U.K. recession of the mid-90s, and the latter takes place from around 2011 to the end of our sample. Third, the degree of synchronization—the speed of adjustment of schooling decisions to changes in the economic cycle—between the demand for schooling and the economic cycle seems to be time-varying as well. We observe that for large enough economic swings the speed of adjustment is faster. Fourth, we find evidence of a large influence of the economic cycle since 40% of total variance in changes in enrollments can be attributed to the economic cycle. Fifth, we find that the schooling cycle for females is considerably more sensitive and responds quicker to changes in macroeconomic conditions than the male counterpart. Furthermore, individuals in the 18-24 age group show larger responses than the teenagers group of 16–17 year olds to the economic cycle. This could be related to more structural factors affecting teenagers driven by teenage employment dynamics and the shares of young people not in education, employment or training (NEET) in the U.K. Lastly, we find evidence that opportunity-cost considerations, relative to ability-to-pay, is the leading channel through which the economic cycle affects the demand for schooling decisions.

The outline of the paper is as follows. Section 2 gives a full description of the data. The empirical specification and the estimation method are presented in Section 3. The results are presented in Section 4, and Section 5 concludes.

2 Data

In the present paper we use quarterly data on post-compulsory education enrollment ratios available from the U.K. Labour Force Survey (LFS) provided by the Office for National Statistics. The enrollment series are defined as the percentage of individuals enrolled in formal education at each point in time over the total population in that same age group. Our sample consists of all individuals between 16 to 24 years old enrolled in full-time higher education and we consider an individual enrolled if he/she attends a full-time, 4-year college program on a regular basis. We use data from 1995Q1 to 2019Q4, which gives a total of $T = 100$ observations. We construct different subsamples using the enrollment rates based on demographic characteristics such as gender (male/female) and different age groups (16-17/18-24) to be used in the estimations. To estimate the economic cycle, we use real GDP data obtained from quarterly series provided by the Office for National Statistics. Finally, to control for supply side effects in enrollments, we use data on government spending on education taken also from the LFS, which is provided at an annual frequency. All our series are seasonally adjusted, and in the case of a contrary finding, we conduct our own seasonal adjustment to rule out any systematic seasonal effects. Furthermore, since the series for government spending in education is available annually over our estimation period, to obtain quarterly series to match the frequency of the rest of our data we use linear interpolation.

To construct the two factors that define the ability-to-pay (ATP) and opportunity-cost (OC) channels respectively, we use a set of 12 variables (six for each channel) obtained from multiple sources. For the ATP channel, we use household net wealth, net consumer credit lending, part-time jobs, household real disposable income and student loans and grants, while for the OC channel, we use real interest rates, real employees' earnings, unemployment rates for 25–69 year olds, higher education tuition fees, the inflation rate and a measure of the expected long-term real interest rate. A detailed description of all these variables is presented in Table A.2 in the Appendix together with their corresponding sources.

3 Empirical implementation

3.1 Univariate unobserved components model

We estimate a separate univariate unobserved components model for both the enrollment rates and GDP data. The model is also applied to extract the cycles in each of the ATP and OC channels. We assume that each series can be expressed as a combination of a non-stationary trend and a stationary cyclical component that moves symmetrically around the trend. Our dependent variable y_t in period t is assumed to be given by the following latent factor model:

$$y_t = \tau_t + c_t + \beta X_t^{sup} + \delta D^{cri} + \varepsilon_t \qquad \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2), \qquad t = 1, ..., T \tag{1}$$

where y_t is the main series of interest to be decomposed (i.e., enrollment rates/real GDP/ATP and OC factors), τ_t is the stochastic trend component, c_t is the stochastic cycle, X_t^{sup} is an exogenous variable to account for supply side effects on schooling with the corresponding coefficient β—only included in the enrollments equation—and D^{cri} is a crisis dummy to account for the effects of the Great Recession in the British economy. The dummy variable takes values equal to 0 from 1995Q1 to 2008Q2 and it is equal to 1 until the end of the sample, representing the "non-crisis" and "crisis" regime. The error term ε_t is added to the specification to account for measurement error and it is assumed to be a Gaussian white noise process with mean 0 and variance σ_ε^2.

The trend factor τ_t is assumed to follow a random walk process with a stochastic slope factor g_t. Likewise, g_t is assumed to follow a random walk process such that,

$$\tau_t = \tau_{t-1} + g_{t-1} + \nu_t \qquad \nu_t \sim iid\mathcal{N}(0, \sigma_\nu^2) \tag{2}$$

$$g_t = g_{t-1} + \omega_t \qquad \omega_t \sim iid\mathcal{N}(0, \sigma_\omega^2) \tag{3}$$

where the error terms ν_t and ω_t are Gaussian white noise processes. In turn, the cycle is modeled as a stationary AR(2) process such that:

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + e_t \qquad e_t \sim iid\mathcal{N}(0, \sigma_e^2) \tag{4}$$

where ϕ_1 and ϕ_2 are AR parameters for which $-1 < \phi_1 + \phi_2 < 1$.

3.2 Bivariate unobserved components model

On a second step, and to allow for the identification of extra parameters in our model, we estimate a bivariate unobserved components model using both series so that $y_t = [G_t\ E_t]'$ where G_t is real GDP and E_t are the enrollment ratios. For the model specification we follow Clark (1989) and extend the univariate representation given in the previous section. Here we allow the enrollment cycle to be determined both by its own cycle and by the economic cycle. We construct the following bivariate model for GDP and enrollments:

$$G_t = \tau_t + c_t + \delta_G D^{cri} + \varepsilon_t \qquad \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2) \tag{5}$$

$$\tau_t = \tau_{t-1} + g_{t-1} + \nu_t \qquad \nu_t \sim iid\mathcal{N}(0, \sigma_\nu^2) \tag{6}$$

$$g_t = g_{t-1} + \omega_t \qquad \omega_t \sim iid\mathcal{N}(0, \sigma_\omega^2) \tag{7}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + e_t \qquad e_t \sim iid\mathcal{N}(0, \sigma_e^2) \tag{8}$$

$$E_t = L_t + z_t + \beta X_t^{sup} + \delta_E D^{cri} \tag{9}$$

$$L_t = L_{t-1} + l_{t-1} + \xi_t \qquad \xi_t \sim iid\mathcal{N}(0, \sigma_\xi^2) \tag{10}$$

$$l_t = l_{t-1} + \zeta_t \qquad \zeta_t \sim iid\mathcal{N}(0, \sigma_\zeta^2) \tag{11}$$

$$z_t = \alpha_0 z_{t-1} + \alpha_1 c_{t-1} + \alpha_2 c_{t-2} + \tilde{e}_t \qquad \tilde{e}_t \sim iid\mathcal{N}(0, \sigma_{\tilde{e}}^2) \tag{12}$$

where L_t is the trend component of enrollments, l_t is the stochastic slope component of the trend and z_t is the stationary cyclical component for enrollments. The X_t^{sup} corresponds to the education supply exogenous variable, which is equivalent to the one used in the univariate specification in the previous section.

3.3 Estimation method

To obtain estimates for the unobserved components we first put both the univariate model described by eq.(1)-(4) and the bivariate model described by eqs.(5)-(12) in state-space form. In particular, we estimate a Gaussian linear state-space system. In Appendix B the state-space representation of both models is described in full. The parameters in the system are estimated via maximum likelihood. The estimates of the unobserved states are obtained with the Kalman smoother, which in contrast with the filter, it provides optimal states with respect to all the available data in our sample.

To obtain the factors for the ability-to-pay (ATP) and the opportunity-cost (OC) channels, we estimate the first principal components of two groups of variables—described in detail in Section 2—that capture both effects. The cycles are extracted using the same estimation method as for the enrollment/GDP series.

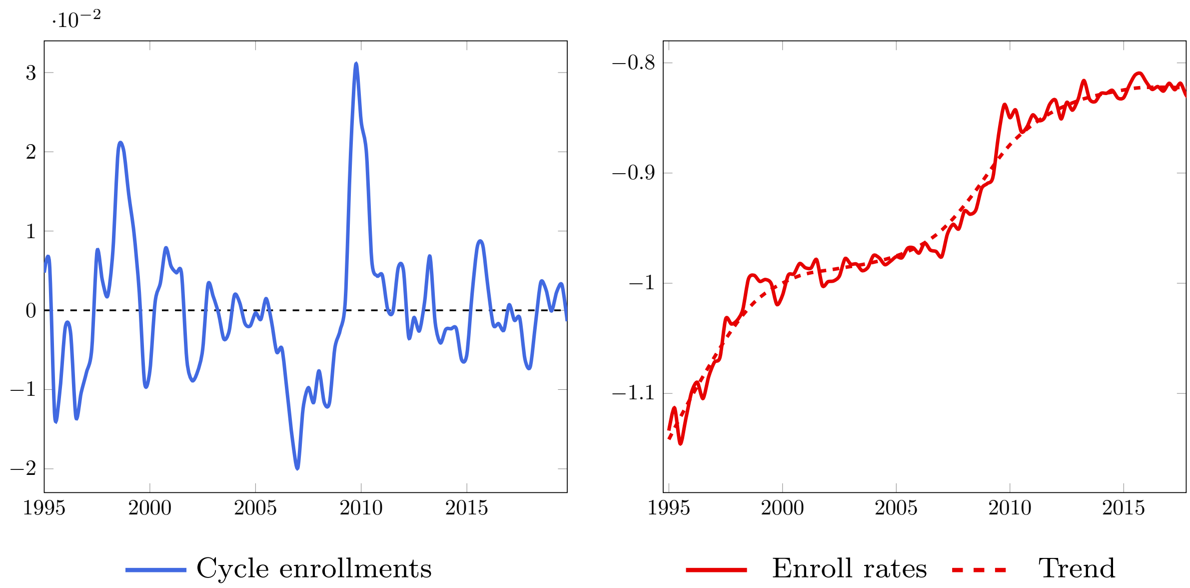
4 Results

This section presents and discusses the empirical results of both the univariate and bivariate models for the U.K. over the period from 1995Q1 to 2019Q4. These are based on post-compulsory education enrollment ratios, real GDP and ATP/OC factors using the frameworks described in Section 3. Evidence from the relative importance of the two economic channels is also discussed.

4.1 Univariate cyclical evidence

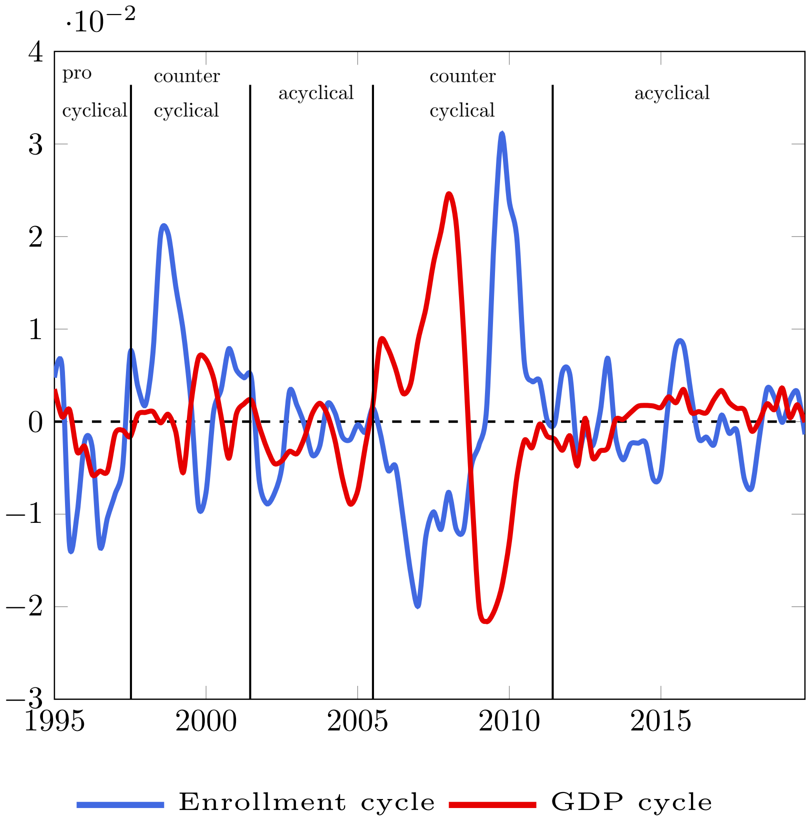
Our first result provides evidence of the presence of a significant cycle in the demand for schooling. This is shown in Figure 1 together with the estimated stochastic trend. These are obtained by estimating the univariate model given by eqs.(1)-(4) using enrollment ratios controlling for supply side effects. One first notable feature is that our decomposition produces a persistent cycle for the enrollment series. This is confirmed by the sum of the estimated AR coefficients, $\phi_1 + \phi_1$, for the transitory component, which is equal to 0.82. Furthermore, to test for the relevance of the cycle component in our model, we conduct a likelihood ratio test that compares our trend-cycle decomposition model with an alternative restricted model in which there is no cyclical component. This restricted model is obtained from our model by setting the AR(2) coefficients ϕ_1 and ϕ_2 equal to zero. The result of the test strongly rejects the null hypothesis of a restricted model with no cycle and thus we confirm the presence of a stationary cyclical component in the enrollment ratios. Finally, statistical tests are conducted to determine the adequacy of the specification and the dependency structure of the series. A Ljung-Box test for autocorrelation at 4 and 12 lags is conducted on the estimated one-step-ahead standardized prediction errors of each equation. These are reported in Table A-1, which shows that the null hypothesis of no autocorrelation is never rejected at the 5% level of significance. This supports our choice to model the stationary cyclical component as an AR(2) process. Furthermore, we test for heteroskedasticity conducting the same Ljung-Box test but on the squared prediction errors. We find that the null is never rejected, so there is no evidence of heteroskedasticity. The estimated variances σ_i^2 for $i = \nu, e, \omega$ for the system are in general of small magnitude and in most cases significant. Therefore, we conclude that our chosen model specification is appropriate to capture the dynamics present in the data.

Figure 1: Cyclical and trend components of enrollment ratios for 16-24 years old



In a second step, we compare the estimated cycle on the demand for schooling with the estimated economic cycle obtained from real GDP data. Both series are plotted together in Figure 2. It is clear from the graph that the direction and size of the relationship between the demand for schooling and the economic cycle cannot be summarized in one average measure but that it is largely time-dependent. This is to say that the response of the demand for schooling to economic fluctuations can be counter-, pro- or a-cyclical depending on the time period we consider. This highlights the need to move beyond a unique average measure to make conclusions regarding the cyclicality of schooling decisions. This time dependency is also reflected in the fact that schooling responses to economic cycles are largely dependent on the size of the macroeconomic shock. This becomes evident during the last Great Recession where both series become much better synchronized.

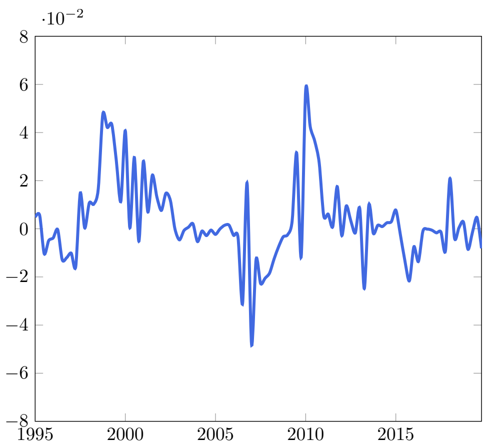
Figure 2: Cyclical components of schooling for 16-24 year olds and the economic cycle



On closer inspection, we observe that the standard result of a counter-cyclical pattern in the demand for schooling present in most existing literature takes place only during two periods in our sample, with the longest one taking place during the Great Recession. For the rest of the sample period we find that the demand for schooling presents either a pro-cyclical pattern or simply a null response to changes in macroeconomic conditions, a.k.a. an a-cyclical pattern. The latter is present for the period between the start of 2001 until the end of 2015 and for the last part of our sample until the end of 2019. The pro-cyclical pattern is observed only for a brief period at the beginning of the sample in the second half of the 90s. The graph also shows that the degree of synchronization—the speed of adjustment of schooling decisions to changes in the economic cycle—between the series seems to be largely time-dependent. Overall, the economic cycle seems to lead the cyclical movements in the demand for schooling. This is expected since people are willing to substitute away from work towards further schooling only after observing economic fluctuations that can impact their short-term job prospects relative to their expectations for the future.

Economic cycle channels. Theory shows that the economic cycle affects the demand for education through two main channels that vary considerably along the economic cycle. These are the ability-to-pay for education and the opportunity costs implied by choosing to spend time on schooling rather than working, for instance. In Figure 3 we show the relative impact of the opportunity-costs channel versus the ability-to-pay channel over time. More specifically, this series is the difference between the estimated cycle present in the OC channel and the cycle in the ATP channel. In this way, we are able to present time-varying evidence on the relative effect of the OC channel—which is the expected leading channel in developed countries such as the U.K. according to existing literature. We observe that for most periods, OC considerations lead the influence of the economic cycle, with large peaks around the 2000s and 2010. We note that these periods coincide with the periods of counter-cyclicality found in Figure 1. Also, for the pro-cyclical behavior found at the beginning of our sample we find that the ATP channel dominates. It is important to note that the Higher Education Reform Act that raised the cap on fees to 3000 GBP (from 1000 GBP in 1998/99) was made effective for the academic year 2006/07. A further fee increase was introduced in 2012/13, which raised the fees to 6000-9000 GBP (Bradley and Migali, 2019). During this time, we observe our series turn negative, indicating that the ATP channel dominates. In all, these results present support to the existing empirical findings regarding pro-cyclicality being driven by funding issues that prevent counter-cyclicality from taking place.

Figure 3: Relative cyclical impact of opportunity-cost channel



(a) Differential impact of OC channel

We find potential explanations to these outcomes. Individual expectations regarding the nature and duration of economic fluctuations and perceptions of labor market risks may have played a role. For the beginning of our sample until 1997Q4, we find a pro-cyclical behavior turning into a counter-cyclical one later on. One possible interpretation is related to the findings of Burgess and Turon (2005) and individual perception of labor market risks. These authors explore the dynamics of the British labor market during different phases of the economic cycle and show that for different time periods, unemployment dynamics can be influenced either by inflow rates—the probability of becoming unemployed—or outflow rates—duration in unemployment—and these will determine an individual's perception of risk. More specifically, they document that during the 90s in the U.K. unemployment dynamics were highly influenced by inflow rates, rather than outflow rates. This implies that despite being faced with high probabilities of becoming unemployed, individuals expected the duration of the unemployment spell to be rather short and thus the decision to educate could be delayed. This could explain a pro-cyclical behavior at first followed by a change to a counter-cyclical pattern later. Further evidence of this is found in our results of the ATP and OC channels affecting education decisions shown in Figure 3. For this time period, we observe that the differential impact of opportunity-cost considerations over the ability to pay for further education is negative. This means that funding issues were more important than outside employment options. This could contribute to the pro-cyclicality observed in the series.

On the other hand, our findings of counter-cyclicality relate more closely to the findings of Ciccone (2000). He shows that the direction of cyclicality can depend on individuals' expectations regarding whether an economic downturn is permanent or transitory. If individuals believe a bad economic period is only transitory they might behave in a pro-cyclical manner even though the opportunity cost of pursuing further education has decreased. Expectations might have played a big role during the Great Recession and the counter-cyclical pattern we observe during that time. Given the size of the downturn, individuals' expectations became very well aligned with the knowledge that this was not a minor transitory shock but rather a permanent and lasting event. This could explain the decision to pursue further schooling observed during those years and the increased synchronization between the series. We seem to find a confirmation of this on a closer look at our series. Although the Great Recession didn't hit Europe until around 2008, the contractionary effect started to be felt early on in the U.K., especially in the youth labor market (Petrongolo and Van Reenen, 2011). We identify this effect of worsening economic conditions in the upward tick in our economic cycle at around mid-2006. This coincides with a change in the direction of the schooling cycle, which shows that the share of individuals pursuing further education was increasing. This shows that faced with this signal of worsening conditions, individuals rightly evaluated it as the early warning of a more permanent and deeper effect that justified an increased demand for further education.

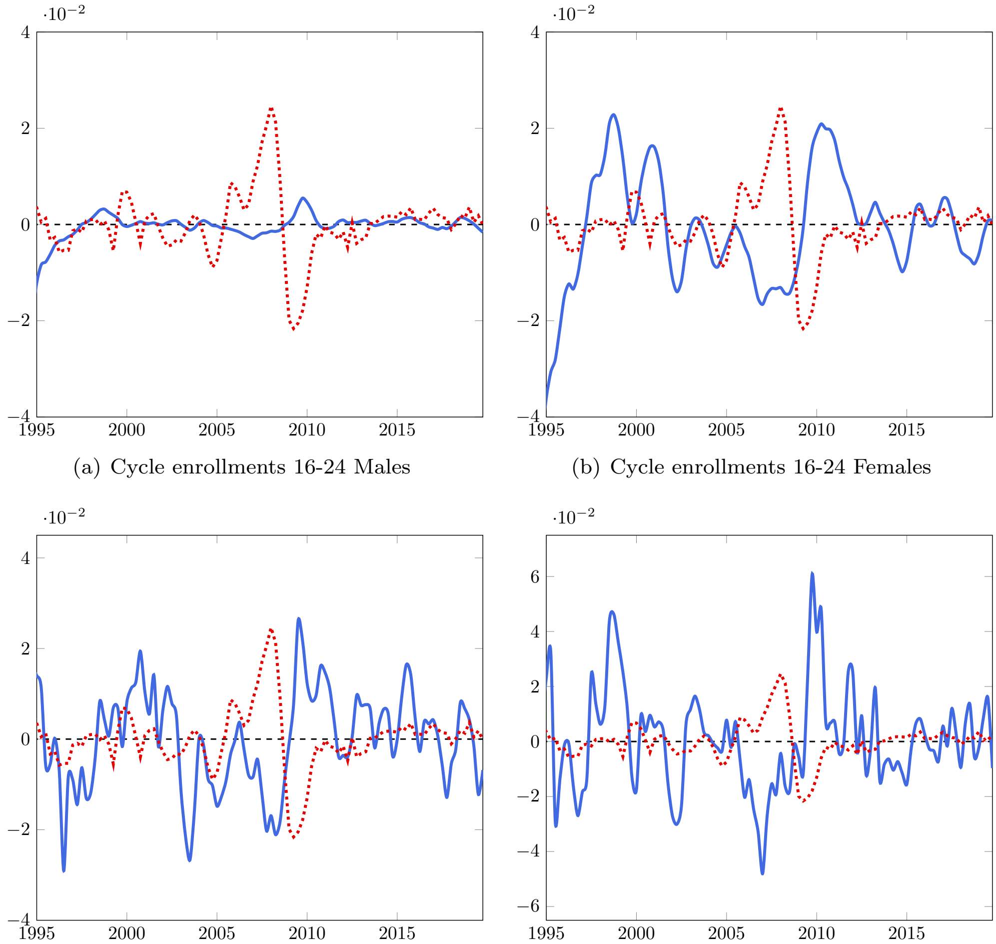
Lastly, a different situation arises in the most recent part of our sample from 2011Q4 until the end of 2019. The schooling cycle seems largely disconnected from changes in macroeconomic conditions. One might speculate that individuals became less sensitive to economic signals due to both the large degree of uncertainty surrounding the recovery after the recession and the many changes occurring in the labor market, which accelerated during the crisis—i.e., the surge of the gig-economy, part-time jobs, etc.

4.1.1 Subsample evidence

In a last step, we follow existing literature and explore the dynamics of the demand for schooling for different subsamples. Existing works show that for different demographic subsamples, the reaction to changing economic conditions regarding education decisions can be substantially different. In Figure 4, we show the estimated schooling cycle for four different subsamples of enrollments, i.e., male/female and teenagers (16-17)/young (18-24). A first very interesting result comes from comparing the charts of males versus females. The male cycle is much less volatile and smaller in size than the one for females, and this becomes especially clear during the Great Recession, when females reacted in a very timely and synchronized fashion to changing economic conditions while males present a much more muted effect. Additionally, females seem to present much more volatility over the whole sample period, which is in line with existing literature on the female elasticity of labor supply. Also, evidence has been found that males are more sensitive to structural factors such as increases in education fees (Bradley and Migali, 2019). This would suggest that males react less to cyclical fluctuations and more to structural changes. This is an important result since female participation in the labor market has been the principal driver of labor force participation rates over the last decades. Therefore, the positive effect of continuing education during bad economic swings increases the chances of rejoining the labor market when it picks up since it helps to avoid the discouraged worker effect and the loss of skills due to long unemployment spells.

Moving to panels (c) and (d), we observe that the cycles for the different age groups appear to be somewhat similar. One difference we find, however, is the size of the reactions over time of the education cycle in the different groups. The 18-24 group presents larger peaks, showing that they react more intensively than 16-17 year olds to changes in the economic cycle. This finding could be related to teenagers' employment dynamics in the U.K. together with the share of young people not in education, employment or training (NEET). Unemployment for the 16-17 fraction has been since the 90s much higher than for the 18-24. This is unsurprising since employers will be reluctant to lose more experienced workers who have firm-specific skills and greater redundancy costs. So the burden of adjustment typically falls on low-wage workers such as young people. Teenagers have not experienced the same recovery in employment after the 1990s recession. This could be explained by concealed "selection effects" since an increasing number of teenagers without jobs stay in education. These dynamics are reinforced by the fact that the share of NEET for the 16-17 group has been in a downward trend since the 1990s while that for the 18-24 group has not. Overall, this points to more structural factors driving the participation of teenagers in education and thus their sensitivity to the economic cycle.

Figure 4: Estimated cycles τ_t for enrollment subsamples and the inverse GDP cycle



(a) Cycle enrollments 16-24 Males

(b) Cycle enrollments 16-24 Females

(c) Cycle enrollments 16-17

(d) Cycle enrollments 18-24

4.2 Evidence from joint estimation of real GDP and enrollment ratios

This section presents and discusses the results of the joint estimation of GDP and enrollment ratios in the bivariate model given by eqs.(5)-(12). Here, we extend the results of the univariate model in two respects. First, the independent models estimated before depend on the assumption that trend and cycle are orthogonal to each other. While not abandoning this assumption, within the bivariate setup the correlation between the cyclical and trend disturbances in enrollments (i.e., the shocks to the trend and the cycle) can be allowed to vary and the system remains identified. In the case of enrollments, a surge in demand for further schooling could generate a cyclical upturn as well as improving longer-run attachment to further education. Second, we allow for the cycle in GDP to enter directly in the dynamic equation of the unobserved enrollments cycle, as in eq.(12). These two extensions allow us to provide additional evidence on the effect of the economic cycle on the demand for schooling.

The maximum likelihood estimates are shown in Table A-1. We note first that the values for the coefficients α_1 and α_2 that capture the effect of the first and second lag of the economic cycle in the enrollment cycle equation are -0.344 and -0.060 respectively. The negative signs imply that on average an economic upturn will decrease the demand for further schooling, i.e., it is counter-cyclical. This is no surprise since it confirms the standard result in the literature of average counter-cyclical effects. What is more novel, though, is that the first lag presents a much larger impact on the cyclicality of education than the second. This seems to suggest that the speed of adjustment to changing economic conditions is rather fast. This is in line with our findings in Figure 2, where co-movement between the series appears to be well synchronized in most periods. Second, the covariance of cycle and trend of enrollments, $\sigma_{\tilde{e}_t,\xi_t}$, is equal to 0.010 and significant. This means that as far as the stationary component of enrollment is driven by its own innovations (the GDP cycle is also included in the equation), these innovations are not independent of the trend innovations in enrollments. Evidently, cycle-driven educational patterns do not cancel out when averaged over the business cycle, meaning they may represent more than pure timing and are more structural in nature.

Finally, we estimate the relative importance of the economic cycle for the demand for schooling. To this end, we conduct an unconditional variance decomposition exercise applied to enrollment ratios given by eq.(9). We derive a simple statistic from the expression for the change in enrollments:

$$\Delta E_t = \xi_t + \zeta_t + \alpha_0 \Delta z_{t-1} + \alpha_1 \Delta c_{t-1} + \alpha_2 \Delta c_{t-2} + \beta \Delta X_t^{sup} + \Delta \tilde{e}_t$$

which implies that the variance of ΔE_t can be expressed as

$$\sigma^2_{\Delta E} = \sigma^2_\xi + \sigma^2_\zeta + \alpha_0^2 \theta_0 + \alpha_1^2 \rho_0 + \alpha_2^2 \rho_0 + \beta^2 V(\Delta X_t^{sup}) + \sigma^2_{\tilde{e}}$$

where θ_0 and ρ_0 are the unconditional variances of the first difference of the cycle process of enrollment and output respectively.[5] We then construct the corresponding ratios to calculate the fraction of $\sigma^2_{\Delta E}$ due to the economic cycle, $\frac{\alpha_1^2\rho_0+\alpha_2^2\rho_0}{\sigma^2_{\Delta E}}$, and to the enrollments own cycle, $\frac{\alpha_0^2\theta_0+}{\sigma^2_{\Delta E}}$. The higher this ratio, the higher the importance of the corresponding cycle to explain the movements in the demand for schooling. Interestingly, we find that around 20% of total variance is explained by schooling own cycle, while 44%

[5]By taking the first differences we drop the dummy variable in eq.(9).

is given by the economic cycle. This shows a particularly large influence of the economic cycle but may not be as large as expected. At the same time, it shows a very persistent effect of schooling cyclical fluctuations over time. We should note, however, that this is an average measure that could be subjected to fluctuations over time.

5 Conclusions

Human capital models state that education is a particular type of investment that can be addressed in a general framework of time and resource allocation decisions taken by utility-maximizing individuals (Becker, 1964; Ben-Porath, 1967). Individuals decide whether to work or study and how much time to allocate to one or the other. These decisions are largely influenced by the state of the macro economy through at least two channels: the income effect that leads to pro-cyclical behavior of the demand for education and the substitution effect that leads to counter-cyclical responses due to reduced opportunity costs of an extra year of education during recessions. The net outcome of these effects and the dynamic properties of such influences are in theory unresolved, and therefore they constitute an empirical question.

This paper aims to fill this gap by providing a systematic empirical investigation on the cyclical properties of the demand for post-compulsory education and its interaction with the economic cycle. To this end, we estimate univariate and bivariate trend-cycle decomposition models for post-compulsory education enrollment ratios and real GDP for the U.K. over the period from 1995Q1 to 2019Q4. Our estimations identify the presence of a persistent stationary cyclical component in the enrollment ratios after controlling for the supply of education. Upon comparison, the estimated education cycle shows a closer connection to the estimated economic cycle. This is further confirmed by the results provided by the bivariate model regarding the relative importance of the economic cycle in explaining variations in enrollment rates, which is shown to be 44%. Furthermore, we find that the education cycle is indeed sensitive to fluctuations in the state of the macro economy, but the direction of this effect is largely time-dependent. We find that for some periods the behavior is counter-cyclical, as found in most of existing literature, but for others it is pro-cyclical and even a-cyclical for the most recent period. Also, our results suggest that the degree of synchronization between the cycles in the demand for education and the economic cycle changes over time. Finally, we present evidence that the importance of the ability-to-pay and opportunity-cost channels is largely time varying.

References

Acemoglu, D. and Autor, D. (2011). *Lectures in Labor Economics.* MIT.

Becker, G. (1964). Human capital theory. *Columbia, New York.*

Becker, G. (1994). Human capital revisited. In *Human Capital: a theoretical and empirical analysis with special reference to education (3rd Edition)*, pages 15–28. The University of Chicago Press.

Ben-Porath, Y. (1967). The production of human capital and the life-cycle of earnings. *Journal of Political Economy*, 75(4):352–365.

Betts, J. and McFarland, L. (1995). Safe port in a storm: the impact of labor market conditions on community college enrollments. *Journal of Human Resources*, 30(4):741–765.

Boffy-Ramirez, E. (2017). The heterogeneous impacts of business cycles on educational attainment. *Education Economics*, 25(6):554–561.

Bradley, S. and Migali, G. (2019). The effects of the 2006 tuition fee reform and the great recession on university student dropout behaviour in the U.K. *Journal of Economic Behavior and Organization*, 164:331–356.

Burgess, S. and Turon, H. (2005). Unemployment dynamics in Britain. *The Economic Journal*, 115(503):423–448.

Ciccone, A. (2000). Business cycle and investment in human capital: international evidence on higher education: a comment. In *Carnegie-Rochester Conference Series on Public Policy*, volume 52, pages 257–262. Elsevier.

Clark, P. (1989). Trend reversion in real output and unemployment. *Journal of Econometrics*, 40(1):15–32.

Dearden, L., Fitzsimons, E., and Wyness, G. (2014). Money for nothing: estimating the impact of student aid on participation in higher education. *Economics of Education Review*, 43:66–78.

DeJong, D. and Ingram, B. (2001). The cyclical behavior of skill acquisition. *Review of Economic Dynamics*, 4(3):536–561.

Dellas, H. and Koubi, V. (2003). Business cycles and schooling. *European Journal of Political Economy*, 19(4):843–859.

Dellas, H. and Sakellaris, P. (2003). On the cyclicality of schooling: theory and evidence. *Oxford Economic Papers*, 55(1):148–172.

Ferreira, F. and Schady, N. (2009). Aggregate economic shocks, child schooling, and child health. *The World Bank Research Observer*, 24(2):147–181.

Hamilton, J. (2018). Why you should never use the Hodrick-Prescott filter. *Review of Economics and Statistics*, 100(5):831–843.

Heylen, F. and Pozzi, L. (2007). Crises and human capital accumulation. *Canadian Journal of Economics*, 40(4):1261–1285.

Johnson, M. (2013). The impact of business cycle fluctuations on graduate school enrollment. *Economics of Education Review*, 34:122–134.

McVicar, D. and Rice, P. (2001). Participation in further education in England and Wales: an analysis of post-war trends. *Oxford Economic Papers*, 53(1):47–66.

Méndez, F. and Sepúlveda, F. (2012). The cyclicality of skill acquisition: evidence from panel data. *American Economic Journal: Macroeconomics*, 4(3):128–52.

Perli, R. and Sakellaris, P. (1998). Human capital formation and business cycle persistence. *Journal of Monetary Economics*, 42(1):67–92.

Petrongolo, B. and Van Reenen, J. (2011). Youth unemployment. Technical report, Center for Economic Performance LSE.

Pissarides, C. (1981). Staying-on at school in England and Wales. *Economica*, 48(192):345–363.

Pissarides, C. (2010). Regular education as a tool of countercyclical employment policy. *LSE*.

Reiling, R. and Strom, B. (2015). Upper secondary school completion and the business cycle. *Scandinavian Journal of Economics*, 117(1):195–219.

Sakellaris, P. and Spilimbergo, A. (2000). Business cycles and investment in human capital: international evidence on higher education. In *Carnegie-Rochester Conference Series on Public Policy*, volume 52, pages 221–256. Elsevier.

Siow, A. (1985). Occupational choice under uncertainty. *Econometrica*, 52(3):631–645.

Whitfield, K. and Wilson, R. (1991). Staying on in full-time education: the educational participation rate of 16-year-olds. *Economica*, 58(231):391–404.

Appendix A. Tables and Figures

Table A-1: ML estimation results

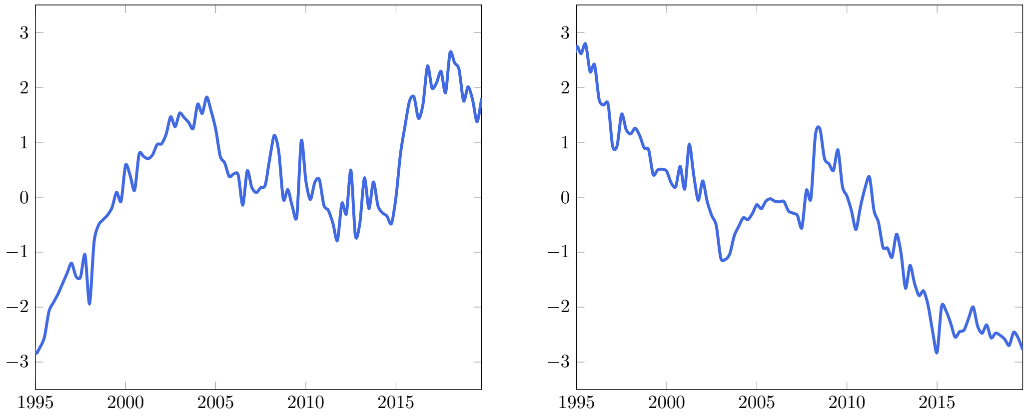
	Univariate model[a]						Bivariate model[a]	
	Enrol 16-24	Enrol 16-24 Fem	Enrol 16-24 Male	Enrol 16-17	Enrol 18-24	GDP	Enrol 16-24	GDP
σ_ε	4E-05	9E-05	7E-06	2E-05	3E-06	1E-09		7E-06
	(2E-05)	(2E-05)	(9E-05)	(2E-05)	(4E-05)	(1E-06)		(1E-05)
σ_ν	5E-09	4E-09	7E-09	5E-09	8E-06	3E-06		1E-03
	(9E-08)	(9E-08)	(7E-06)	(1E-06)	(6E-05)	(3E-06)		(3E-03)
σ_ω	1E-06	7E-07	3E-09	5E-07	2E-06	2E-06		0.011
	(1E-06)	(1E-06)	(1E-06)	(1E-06)	(3E-06)	(2E-06)		(0.003)
σ_e	7E-05	3E-05	7E-06	8E-05	3E-04	1E-05		0.040
	(4E-05)	(2E-05)	(3E-05)	(3E-05)	(2E-04)	(4E-06)		(0.004)
ϕ_1	0.841	1.333	1.285	0.847	0.742	1.374		1.386
	(0.063)	(0.151)	(0.643)	(0.068)	(0.358)	(0.005)		(0.098)
ϕ_2	-0.019	-0.436	-0.412	-0.007	0.019	-0.454		-0.481
	(0.005)	(0.086)	(0.926)	(0.009)	(0.005)	(0.006)		(0.166)
β	-0.012	0.002	-0.023	0.003	-0.024		-0.017	
	(0.011)	(0.013)	(0.012)	(0.010)	(0.016)		(0.040)	
δ_E	7E-04	0.008	-0.001	-0.001	0.009	-0.001	-0.010	
	(0.013)	(0.015)	(0.014)	(0.011)	(0.021)	(0.004)	(0.013)	
δ_G								-0.009
								(0.130)
σ_ξ							0.002	
							(0.005)	
σ_ζ							0.025	
							(0.011)	
$\sigma_{\tilde{e}}$							0.032	
							(0.004)	
α_0							0.397	
							(0.104)	
α_1							-0.344	
							(0.313)	
α_2							-0.060	
							(0.093)	
$\sigma_{\tilde{e}_t,\xi_t}$							0.010	
							(3E-04)	
Test for Autocorrelation[b],[c]								
lag 4	1.59	3.25	1.22	3.60	0.33	8.86	10.36	25.94
	[0.620]	[0.484]	[0.126]	[0.537]	[0.012]	[0.935]	[0.965]	[0.999]
lag 12	12.85	17.63	7.15	17.72	12.49	18.35	19.47	26.27
	[0.189]	[0.872]	[0.153]	[0.876]	[0.594]	[0.895]	[0.922]	[0.990]
Test for Heteroskedasticity[b],[d]								
lag 4	12.38	5.21	3.89	4.54	16.26	18.71	22.62	4.48
	[0.985]	[0.734]	[0.578]	[0.662]	[0.997]	[0.999]	[0.999]	[0.655]
lag 12	17.32	14.00	11.40	11.06	23.77	22.79	35.88	4.49
	[0.862]	[0.699]	[0.505]	[0.476]	[0.978]	[0.970]	[0.999]	[0.027]

Notes: The models include: trend, slope, cycle, exogenous controls and a dummy variable as shown in eq.(1). (a) Standard errors in parentheses. (b) p-values in square brackets. (c) Box-Ljung statistic with H_0: no autocorrelation in the one-step-ahead prediction errors. (d) Test for equal variances H_0: homoscedasticity in the one-step-ahead prediction errors.

Table A.2 : Description of variables and data sources

Variable	Description	Primary Source
	Opportunity cost	
Real IR	Quarterly real interest rate given by personal sector real rate. Seasonally adjusted and expressed in percentage points.	Oxford Economics
Real wage	Quarterly change of total real earnings per-employee for the whole U.K. economy minus inflation. Seasonally adjusted.	Oxford Economics
UN rate 25-69	Quarterly unemployment rate for the population between 25 and 69 year olds. Seasonally adjusted and expressed in percentage points.	U.K. Office for National Statistics
Tuition fees	Annual data on tuition fees for higher education institutions. Expressed in British pounds per year at 2006 prices.	Dearden et al. (2014)
Inflation	Quarterly growth rates of consumer price index, all items, not seasonally adjusted.	IMF
Exp(LT IR)	Expected long-term real interest rate constructed as the difference between the nominal yield on new home mortgages and the expected inflation rate calculated as the average of the inflation rates in the last 12 quarters, not seasonally adjusted.	IMF, Oxford Economics
	Ability-to-pay	
Wealth	Quarterly household sector net wealth. Seasonally adjusted and expressed in percentage points.	Oxford Economics
Credit	Quarterly changes in net lending in consumer credit. Seasonally adjusted.	Bank of England
Part-time jobs	Quarterly data on the number of part-time workers who are students or at school. Seasonally adjusted.	LFS - U.K. Office for National Statistics
Disposable income	Quarterly data on households' real disposable income. Constant prices, seasonally adjusted.	U.K. Office for National Statistics
Maintenance loans	Annual data on student loans repayable as a percentage of earnings when the graduate is in employment and earning over a certain threshold. Expressed in British pounds per year at 2006 prices.	Dearden et al. (2014)
Maintenance grants	Annual data on student grants which are a non-repayable form of support for higher education. Expressed in British pounds per year at 2006 prices.	Dearden et al. (2014)

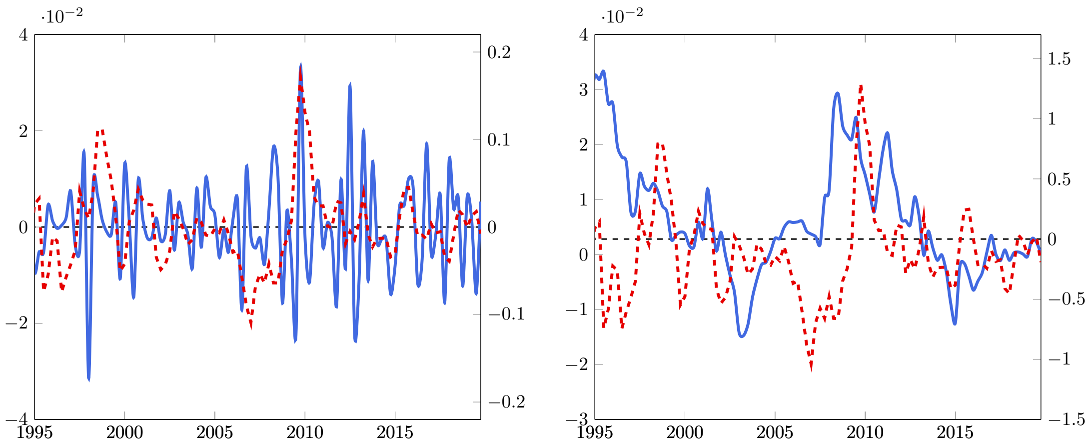
Figure A-1: Estimated ability-to-pay and opportunity-cost factors



(a) Ability-to-pay factor (b) Opportunity-cost factor

Notes: The factors are the first principal component of each group of variables described in Table A.2. These capture the ability-to-pay and opportunity-cost channels that affect the decision to pursue further education over the economic cycle.

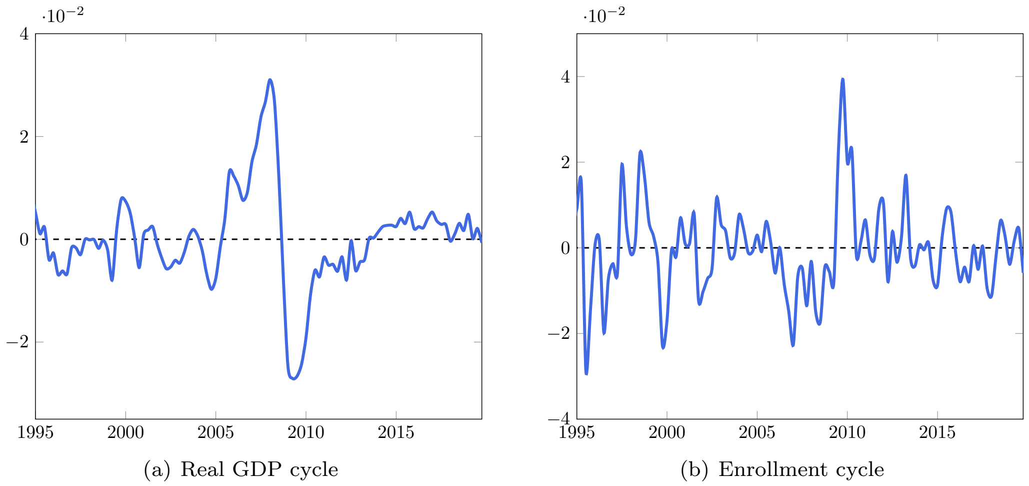
Figure A-2: Estimated ability-to-pay and opportunity-cost cycles



(a) Enrollment and ability-to-pay cycles (b) Enrollment and opportunity-cost cycles

Notes: In panel (a) the solid lines (right axis) correspond to the estimated cycle of the ability-to-pay factor and the opportunity-cost factor in panel (b). These are obtained from a state-space univariate trend-cycle decomposition factor model such as that described in Appendix B. The dashed lines (left axis) show the estimated cycle of enrollment rates for 16-24 year olds individuals (same cycle shown in Figure 1).

Figure A-3: Bivariate model cycles



Notes: The cycles correspond to the stationary components c_t—panel (a)—and z_t—panel (b)—estimated using the bivariate model given by eqs.(5)-(12).

Appendix B. State-space representation of the univariate and bivariate models

The state-space system with state vector Ω_t is given by

$$Y_t = Z_t\Omega_t + \varepsilon_t \tag{A.1}$$

$$\Omega_t = T_t\Omega_{t-1} + \eta_t \tag{A.2}$$

with,

$$\begin{aligned} \varepsilon_{t|t-1} &\sim N(0, H) \\ \eta_{t|t-1} &\sim N(0, Q_t) \end{aligned}$$

The distribution of initial state vector $\Omega_1 \sim \mathcal{N}(a_1, P_1)$ is assumed to have a diffuse prior density initialized with $a_1 = 0$ and $P_1 \to \infty$.

For the univariate case we have,

$$Y_t = y_t$$

$$\Omega_t = [\tau_t\ c_t\ c_{t-1}\ g_t]'$$

$$\varepsilon_t = \varepsilon_t$$

$$\eta_t = [\nu_t\ e_t\ \omega_t]'$$

$$Z_t = [1\ 1\ 0\ 0]$$

$$H = \sigma_\varepsilon^2$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_\nu^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\omega^2 \end{bmatrix}$$

$$pars = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\nu^2 & \sigma_\omega^2 & \sigma_e^2 & \phi_1 & \phi_2 & \beta & \delta \end{bmatrix}$$

For the bivariate case we have,

$$Y_t = [y_{1t}\ y_{2t}]'$$

$$\Omega_t = [\tau_t\ c_t\ c_{t-1}\ z_t\ L_t\ l_t\ g_t]'$$

$$\varepsilon_t = [\varepsilon_t\ 0]'$$

$$\eta_t = [\nu_t\ e_t\ 0\ \tilde{e}_t\ \xi_t\ \zeta_t\ \omega_t]'$$

$$Z = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha 1 & \alpha 2 & \alpha 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_\nu^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\tilde{e}}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\xi^2 & \sigma_{\tilde{e}_t,\xi_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_\omega^2 \end{bmatrix}$$

$$pars = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\nu^2 & \sigma_\omega^2 & \sigma_e^2 & \phi_1 & \phi_2 & \sigma_\xi^2 & \sigma_\zeta^2 & \sigma_{\tilde{e}}^2 & \alpha_0 & \alpha_1 & \alpha_2 & \sigma_{\tilde{e}_t,\xi_t} & \beta & \delta_E & \delta_G \end{bmatrix}$$