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Monetary Policy Implementation and Payment System Modernization

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Abstract

24/7 payment settlement may impact the demand for central bank reserves and thus could have an effect on monetary policy implementation. Using the standard workhorse model of monetary policy implementation (Poole, 1968), we show that 24/7 payment settlement induces a precautionary demand for central bank balances. Absent any changes or response by the central bank, this will put upward pressure on the overnight interest rate in a standard corridor system of monetary policy implementation. A floor system is much less sensitive to this change, as long as excess balances are large enough.

Topics: Monetary policy implementation; Payment clearing and settlement systems
JEL codes: E, E4, E40, E42, E43

1. Introduction

Payment, clearing and settlement systems have undergone drastic changes since banks began accepting claims on each other (Norman et al. (2011)). Technological change and a regulatory interest in systemic risk oversight over the last decade or so has accelerated the pace of these changes. Now, several countries have retail payment systems that provide settlement in real time or near real time 24 hours a day, 7 days a week (Tompkins and Olivares (2016)). Other countries also plan on adopting such systems. Canada has planned for such a system, and the United States' FedNow system that will also offer 24/7 payment clearing services to retail customers is anticipated to be available in 2023 or 2024.

Payment systems are inextricably linked to the implementation of monetary policy — i.e., how the central bank sets overnight interbank rates. The demand for reserves in a standard model of monetary policy implementation (e.g., Poole (1968)) is generated by how uncertainty over interbank payment flows affects the use of central bank borrowing and lending facilities. 24/7 payment settlement has the potential to change both these factors and alter the demand for central bank reserves. For example, demand for reserves could be a function of whether the central bank provides access to its lending facilities (i.e., intraday credit) only during standard operational hours or always.[1] This paper aims to understand how demand for reserves is a function of those hours. If the central bank does not provide access to an after-hours central bank lending facility, a bank needs to have positive reserve balances greater than the payment amount to process a given payment. While a bank could establish a credit line to borrow from another bank to meet the payment, that other bank would also be worried about an inability to process payments. In either case, an extra dollar of reserves in the after-hours market provides a benefit in that it helps banks avoid having insufficient funds to process payments in the after-hours market. Does this matter for overnight interbank interest rates? Under what conditions will this matter, and is the impact different for different implementation frameworks? To answer these questions, we develop a model of monetary policy implementation with after-hours payment shocks.

To our knowledge, such a model has not been developed in the literature. Traditional models (e.g., Poole (1968) and Bech and Keister (2013)) have a payment shock that occurs while banks have access to central bank facilities. Thus, in these models, the cost and probability of accessing these facilities influences the interbank rate. If a payment shock occurs when banks do not have access to this facility, then banks need to factor the cost of having

[1] The Federal Reserve is considering whether to provide intraday credit on a 24/7 basis with the implementation of its FedNow system. For more information, please refer to the following press release: https://www.federalreserve.gov/newsevents/pressreleases/files/other20190805a1.pdf

insufficient funds in the after-hours market into their marginal benefit of an extra dollar of reserves. After-hours shocks have not been modeled in this framework before because these shocks only matter should payments be settled 24/7.[2] Once we start thinking about 24/7 settlement, it opens up several questions related to monetary policy implementation, which this paper attempts to answer.

Other models consider how differential access to central bank facilities and segmentation in the overnight market impact the interbank interest rate (Williamson (2015), Bech and Klee (2011), Armenter and Lester (2015), Martin et al. (2013)). In these papers, access to central bank facilities is segmented by participant. In our model, all participants have the same access, but that access is segmented by time. Like these other models, segmentation impacts interbank interest rates.

We provide the conditions under which after-hours payments can have an effect on interbank interest rates. When after-hours payment volatility is material relative to intraday payment volatility, banks will have an increased demand for reserves. This increased demand increases with the volatility of the after-hours payment shock and is precautionary in that banks want to hold extra reserves to avoid having insufficient funds in the after-hours session. How this affects inter-bank rates depends on the monetary policy implementation framework. Interbank rates in monetary policy frameworks that naturally have large reserves will not be affected much by such a change. This includes a floor operating framework as well as corridor frameworks with a large amount of required reserves. On the other hand, there will be upward pressure on interbank rates in corridor frameworks with zero or low levels of required reserves.

While the central bank can intervene by providing more aggregate reserves to offset this upward pressure, this could be more challenging if the volatility of the after-hours payment shock fluctuates. This could happen, for instance, if the after-hours period is longer in certain periods such as weekends or holidays. We therefore investigate how a change in the volatility of the after-hours payment shock impacts the volatility of the overnight rate. When reserves are sufficiently large, changes in after-hours payment volatility do not matter. When reserves are smaller, changes in after-hours payment volatility result in volatility in the overnight interbank rate, absent a central bank response. We also show that the central bank could respond in such a situation to keep the overnight interbank rate stable, but determining the optimal response requires knowledge of the demand for overnight funds as well as knowledge about the volatility of the after-hours payment shock.

[2] For example, in the U.S., Fedwire Funds Service operates until 6:30 pm and the National Settlement Service (NSS) operates until 5:30 pm. The Federal Reserve is considering expanding their hours to 24/7 to provide a liquidity management tool to support a 24/7 RTGS service.

Finally, we examine the impact of having payments spread across two payment systems on the interbank overnight rate — an intraday system and a 24/7 system. Parallel payment systems exist in several jurisdictions. If the funds are transferable between the two systems, there will be one overnight rate. This overnight rate could be different from the overnight rate in one single payment system. First, with two payment systems, it is possible that some funds are left in the intraday system at the end of the day and hence cannot be deployed to reduce the expected reputation cost of having insufficient funds in the after-hours market. This would put upward pressure on the overnight rate relative to a single payment system. It is also possible that there could be downward pressure on the overnight rate in multiple systems. This would happen if it is possible to borrow more in multiple systems, and if the net cost of this additional borrowing is less than the expected reputation cost. In contrast, if funds are not transferable between the two systems, the after-hours payment shock will not have an impact on the overnight interbank rate in the intraday system. However, in this case, the overnight rate in the 24/7 system will diverge from the overnight rate in the intraday system.

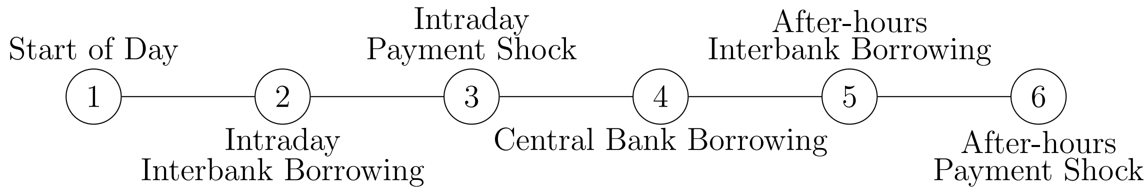
In practice, several central banks have already implemented payment systems with 24/7 retail payments, but overnight interbank rates still trade close to target in their jurisdictions. Our model would imply that either: i) uncertainty about retail payment flows in the after-hours session is small in these jurisdictions, or ii) the level of reserves in these jurisdictions is large enough such that there is little chance that banks will have insufficient funds to process payment flows in the after-hours session. However, should more payment flows migrate to the 24/7 system, our model would suggest that this could lead to deviations from the target interest rate. Further, the implementation of a 24/7 payment system in countries that operate a near-zero-reserve corridor (like systems traditionally operated by Australia and Canada) or plan to return to a low-reserve implementation framework could be very different than the experience thus far.

2. Model

2.1. Model Timing

Our model extends Boutros and Witmer (2019) and Bech and Keister (2013) and consists of six stages. We assume a continuum of perfectly competitive banks indexed by $i \in [0, 1]$. The first four stages presented in figure (1) are standard in the literature. Banks borrow from (and lend to) each other during the day. After this borrowing and lending window is over, banks are subject to a payment shock. If they are short reserve balances after this payment shock, they must borrow from the central bank at rate r_X to make up the shortfall. If they have excess reserves, they get deposited with the central bank and earn interest r_D.

Figure 1: Model Timing



We depart from the standard models by introducing an after-hours payment shock in stage 6. What distinguishes this after-hours payment shock from the intraday payment shock is that we assume that banks do not have recourse to the central bank borrowing facility in the after-hours market.

Banks begin the day with bond holdings, B^i, reserves, R^i, deposits, D^i, and equity, E^i. Bond holdings and equity are exogenous and fixed throughout the day. Aggregate reserves are defined as $R = \int_i R^i \mathrm{d}i$.

In the standard model, a bank becomes a net lender ($\Delta^i_{intra} < 0$) or net borrower ($\Delta^i_{intra} > 0$) in stage 2 to position itself for the intraday payment shock it experiences in stage 3. In our model with after-hours payments shocks (i.e., shocks that happen after clearing and settlement of intraday balances), the banks' decisions are going to change. Specifically, a bank must also consider the effect of the reputation cost of the after-hours payment shock on its profitability. That is, it is not only minimizing the penal borrowing and lending rates associated with the central bank facilities, but is also minimizing reputation costs of having insufficient funds to process payments in the after-hours market.

In stage 3, after the trading session is closed, each bank experiences an intraday payment shock, $\epsilon^i_{intraday}$. This payment shock is independent and normally distributed with mean zero and standard deviation σ_G. We denote the cumulative distribution function of this shock $G(\epsilon^i_{intraday})$. This payment shock lowers the bank's reserves on the asset side of its balance sheet and correspondingly lowers its deposits on its liabilities side.

Table 1: Bank i's End-of-Day Balance Sheet

Assets	Liabilities
B^i Bonds	$D^i - \epsilon^i_{intraday}$ Deposits
$R^i + \Delta^i_{intra} - T^i - \epsilon^i_{intraday} + \ X^i$ Reserves	E^i Equity
	Δ^i_{intra} Interbank Borrowing
	X^i Central Bank Borrowing

After the intraday payment shock, each bank borrows X_i from the central bank if its reserves after the payment shock are less than its required level of reserves $K \geq 0$. We assume that each bank has the same required level of reserves, and that this required level of reserves is positive. The aggregate reserve requirement is defined as $K = \int_i K \mathrm{d}i$. Banks that must borrow from the central bank do so at a rate of r_X. That is, each bank will borrow

$$X^i = \max\{0, K - (R^i + \Delta^i_{intra} - \epsilon^i_{intraday})\} \tag{1}$$

At this point, the bank earns r_K on its required reserves and $r_R < r_X$ on any reserves in excess of its required reserves. Table 1 illustrates bank i's balance sheet at the end of the day, before the after-hours payment shock.

In the after-hours session, banks are freely able to borrow from one another before the realization of the after-hours payment shock (similar to how they can borrow from each other during the day). A bank can be either a net lender ($\Delta^i_{after} < 0$) or a net borrower ($\Delta^i_{after} > 0$) in the after-hours market.

In the final stage, each bank receives a payment shock in the after-hours market, $\epsilon^i_{overnight}$. The overnight payment shock is independent and normally distributed with mean zero and standard deviation σ_F.[3] We denote the cumulative distribution function of this shock $F(\epsilon^i_F)$. The bank cannot meet its payment if

$$\epsilon^i_{overnight} \geq R^i + \Delta^i_{intra} + \Delta^i_{after} - \epsilon^i_{intraday} + X^i \tag{2}$$

If the bank cannot meet its payment, it suffers a reputation cost s on Z^i, each dollar of payment it is unable to make.

$$Z^i = \max\{0, \epsilon^i_{overnight} - (R^i + \Delta^i_{intra} + \Delta^i_{after} - \epsilon^i_{intraday} + X^i)\} \tag{3}$$

2.2. Bank Profits

Banks earn r_B on their bond holdings and pay r_D on their deposit holdings, both of which are exogenously determined. After the after-hours payment shock, bank i's realized

[3]Some banks may be able to predict their after-hours payment shock. We can relax the assumption that the after-hours payment shock is mean zero and obtain similar results (assuming that, across banks, the mean of the mean after-hours payment shock equals zero). Given the fact that banks can trade with each other in the after-hours market, differences in the mean payment shock for different banks will have no impact on equilibrium rates, much the same way that differences in initial reserves position across banks have no impact because banks can trade with each other.

profits are therefore:

$$\begin{aligned}\pi^i &= r_B B^i - r_D(D^i - \epsilon^i_{intraday}) + r_K K^i - r_\Delta \Delta^i_{intra} \\ &\quad - r_X X^i + r_R(R^i + X^i + \Delta^i - \epsilon^i_{intraday} - K^i) \\ &\quad - r_{after}\Delta^i_{after} - s * Z^i \end{aligned} \tag{4}$$

In stage 5, banks will choose their net interbank after-hours borrowing Δ^i_{after} to maximize their expected profits in the after-hours market (the last two terms in the above equation):

$$\mathrm{E}[\pi^i_{after}] = -r_{after}\Delta^i_{after} - s\int_{\epsilon^i_Z}^{\infty}(\epsilon^i_{overnight} - \epsilon^i_Z)\mathrm{dF}(\epsilon^i_{overnight}) \tag{5}$$

The threshold before the bank is expected to experience the reputation cost, $\epsilon^i_Z \equiv R^i + \Delta^i_{intra} + \Delta^i_{after} + \int_{\epsilon^i_K}^{\infty}(\epsilon^i_{intraday} - \epsilon^i_K)\mathrm{dG}(\epsilon^i_{intraday})$, is equal to the amount of reserves before the intraday payment shock, plus the expected amount of central bank borrowing after the intraday payment shock and the amount borrowed in the intraday and after-hours markets. In this expected profit equation, the threshold for central bank borrowing $\epsilon^i_K \equiv R^i + \Delta^i_{intra} - K$ is equal to the amount of excess reserves before the intraday payment shock.

The value of Δ^i_{after} that maximizes the expected after-hours profits in equation (5) is given by the following first order condition:

$$r_{after} = s(1 - F(\epsilon^i_Z)) \tag{6}$$

Given that banks borrow and lend from each other at the same rate in the after-hours market (r_{after}), it follows from equation (6) that banks will trade with each other such that they have the same $\epsilon^i_Z \equiv \epsilon_Z$. The bank's expected trading (before the intraday payment shock) in the after-hours market can thus be written as:

$$E[\Delta^i_{after}] = \epsilon_Z - (R^i + \Delta^i_{intra} + \int_{\epsilon^i_K}^{\infty}(\epsilon^i_{intraday} - \epsilon^i_K)\mathrm{dG}(\epsilon^i_{intraday})) \tag{7}$$

In stage 2, banks will take into account that they can trade in the after-hours market, and will choose their net interbank intraday borrowing Δ^i_{intra} to maximize the expected value of their profits:

$$
\begin{aligned}
\mathrm{E}[\pi^i] = {} & r_B B^i - r_D D^i + r_K K^i - r_\Delta \Delta^i_{intra} \\
& + r_R \epsilon^i_K + (r_R - r_X) \int_{\epsilon^i_K}^{\infty} (\epsilon^i_{intraday} - \epsilon^i_K) \mathrm{dG}(\epsilon^i_{intraday}) \\
& - r_{after} E[\Delta^i_{after}] - s \int_{\epsilon_Z}^{\infty} (\epsilon^i_{overnight} - \epsilon_Z) \mathrm{dF}(\epsilon^i_{overnight})
\end{aligned}
\tag{8}
$$

Relative to a standard Poole (1968) model, the bank's expected profit in (8) includes two extra terms. The first term accounts for the expected borrowing and lending in the after-hours market. Thus, the bank can reduce its expected borrowing and lending costs by holding more reserves heading into the after-hours session. Relative to its initial level of reserves, it can reduce this expected trading cost either by borrowing funds in the intraday interbank market, or by borrowing from the central bank before the after-hours session. The second term accounts for the reputation cost, s, of falling short of funds in the overnight session. This second term has the same threshold for each bank since their after-hours trading will make their after-hours reserves position the same. As the integral suggests, the bank only pays this cost if the overnight payment shock, $\epsilon^i_{overnight}$, exceeds the threshold ϵ_Z. This second term is not affected by the bank's intraday interbank trading.

Since these expectations are taken in stage 2, the expected amount of borrowing in the after-hours interbank market is based on the expected borrowing from the central bank, which itself is a function of the bank's level of excess reserves after the intraday trading session is complete (ϵ^i_K). This suggests that borrowing funds in the intraday interbank market does not increase after-hours trading one-for-one, since borrowing an extra dollar of reserves in the interbank market makes it less likely that the bank will need to borrow from the central bank at the end of the day. This can be seen by taking the derivative of equation (7) with respect to Δ^i_{intra}. Borrowing an additional dollar in the intraday market will reduce expected after-hours borrowing by $G(\epsilon^i_K) = 1 - (1 - G(\epsilon^i_K))$.

More formally, banks will choose Δ^i_{intra} to maximize their expected profits in equation (8), resulting in the following first order condition:

$$
r_\Delta = r_R + (r_X - r_R)(1 - G(\epsilon^i_K)) + r_{after} G(\epsilon^i_K) \tag{9}
$$

Because G and F are cumulative normal distributions, this first order condition above can be further reduced to:

$$r_\Delta = r_R + (r_X - r_R)(1 - \Phi(\frac{\epsilon_K^i}{\sigma_G})) + r_{after}\Phi(\frac{\epsilon_K^i}{\sigma_G}) \tag{10}$$

Further, the threshold for the reputation cost can be expressed as:

$$\epsilon_Z^i = K + \epsilon_K^i \Phi(\frac{\epsilon_K^i}{\sigma_G}) + \sigma_G \phi(\frac{\epsilon_K^i}{\sigma_G}) \tag{11}$$

2.3. Equilibrium

Definition. *An equilibrium consists of interest rates r_Δ and r_{after} and individual bank net borrowing decisions (Δ_{intra}^i) and (Δ_{after}^i) such that:*

(i) Banks choose Δ_{after}^i to maximize expected profits in the after-hours market, as in (5).

(ii) Banks choose Δ_{intra}^i to maximize expected profit, as in (8).

(iii) The interbank markets are closed systems that clear, that is, $\Delta_{intra} = \int_i \Delta_{intra}^i \mathrm{d}i = 0$ and $\Delta_{after} = \int_i \Delta_{after}^i \mathrm{d}i = 0$.

We assume a regularity condition that s is not too large relative to $r_X - r_R$. Specifically, s should be small enough such that the first derivative of equation (9) with respect to ϵ_K^i should always be negative. For this to be the case, s must be small enough such that $r_{after} <= r_X - r_R$. If this were not the case, those with fewer reserves will want to lend reserves to other banks (and borrow more from the central bank), and those with more reserves will want to borrow more (and deposit more with the central bank). They will end up in two camps. There will be those that are borrowing from the central bank, and lending in the interbank market at $r_\Delta > r_X$. There will be those that are depositing with the central bank and borrowing at $r_\Delta < r_R + r_{after}$. Thus, although individual banks cannot borrow precautionary balances, (and they would want to do so if s is large enough such that $r_R + r_{after} > r_X$), they would get around this in the aggregate. They will want to do this until r_{after} decreases due to aggregate borrowing from the central bank increasing reserves in the after-hours market so that $r_R + r_{after} = r_X$.

By our regularity condition and the first order condition in (9), and by the fact that banks are all subject to the same reserve requirement K, they will have the same ϵ_K^i in equilibrium. That is, since r_Δ is the same for all banks, there is only one value of ϵ_K^i that will solve (9).

In aggregation, given that $\Delta = 0$, it follows that

$$\epsilon_K = \int_i \epsilon_K^i \mathrm{d}i = R - K \tag{12}$$

$$\epsilon_Z = \int_i \epsilon_Z^i \mathrm{d}i = K + (R - K)\Phi(\frac{R-K}{\sigma_G}) + \sigma_G \phi(\frac{R-K}{\sigma_G}) \tag{13}$$

Given that each bank trades to hold the same threshold amounts before the intraday payment shock, it follows from the equilibrium definition that $\epsilon_K = \epsilon_K^i$ and $\epsilon_Z = \epsilon_Z^i$. Thus, equation (9) can be written as a function of these aggregate threshold amounts, which themselves depend on the aggregate bank's balance sheet (i.e., as in equations (12) and (13)).

$$r_\Delta = (r_R + r_{after})\Phi(\frac{\epsilon_K}{\sigma_G}) + r_X(1 - \Phi(\frac{\epsilon_K}{\sigma_G})) \tag{14}$$

Equation (14) shows that the overnight rate with an overnight payment shock is equal to the overnight rate in a standard model, $r_{Poole} \equiv r_R \Phi(\frac{\epsilon_K}{\sigma_G}) + r_X(1 - \Phi(\frac{\epsilon_K}{\sigma_G}))$, plus an additional term to account for the expected reputation cost the bank would experience from being short of funds in the overnight market. Put another way, the overnight rate in the intraday market is essentially the same as the Poole rate, except that the benefit of additional funds at the end of the intraday period is not only the deposit rate, but also includes an extra term to account for the benefit of additional funds in the after-hours market in avoiding the reputation shock.

Proposition 1. *The overnight rate in the presence of an overnight payment shock will be weakly greater than the overnight rate in the absence of one:*

$$r_\Delta = r_{Poole} + s(1 - \Phi(\frac{\epsilon_Z}{\sigma_F}))\Phi(\frac{\epsilon_K}{\sigma_G}) \tag{15}$$

Thus, there is upward pressure on the overnight rate (relative to the Poole rate) when there is an overnight payment shock. This pressure occurs because banks demand extra precautionary reserves to reduce the expected reputation cost associated with the overnight shock. Figure 2 illustrates how the demand for overnight reserves changes in the presence of an overnight payment shock. In the figure on the left-hand side, where there are no required reserves, the demand for reserves in the presence of an overnight payment shock (dashed line) is higher than the demand for reserves in the absence of this shock (solid line). This will, if anything, put upward pressure on the overnight interbank rate.

Interestingly, the demand for reserves is unaffected if aggregate reserves are very large or very small. When aggregate reserves are very large, there is almost zero probability that a bank would experience a payment shock that fully drains its reserves, so the expected reputation cost associated with having insufficient reserves is negligible. On the other hand, when aggregate reserves are very small (large, negative value), banks will almost surely borrow from the central bank at the end of the day. Therefore, trading away an additional dollar in the interbank market will have no effect on the probability of having insufficient funds in the overnight market, since the bank would borrow an extra dollar from the central bank at the end of the day before the overnight payment shock. As such, its interbank trading would have no effect on its reserve position before experiencing the overnight payment shock.

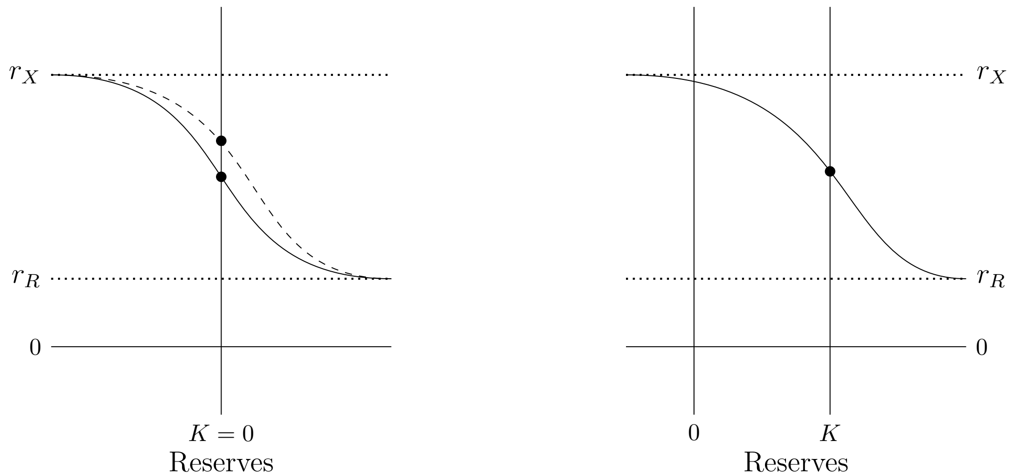
The following corollaries highlight how the supply of central bank reserves interacts with this change in demand for central bank reserves to affect the overnight interbank rate.

Corollary 1. *In a floor system with abundant excess reserves ($\Phi(\frac{\epsilon_K}{\sigma_G}) \approx 1$), the overnight rate is equal to the Poole interest rate so long as reserves are also abundant relative to the overnight payment shock ($\Phi(\frac{\epsilon_Z}{\sigma_F}) \approx 1$).*

Proof. Substituting the two conditions in Proposition (1) into Equation (15) yields $r_\Delta = r_{Poole}$. □

The result is intuitive. When reserves are abundant, there is close to zero probability that the after-hours payment shock will reduce the bank's reserves below zero and lead to the reputation cost. Therefore, the expected value of the reputation cost is near zero as well, leaving the result that the overnight rate is the Poole rate. More reserves may be required

Figure 2: Demand for Overnight Reserves



(a) $K = 0$ (b) $K > 0$

Notes: Panel (a) illustrates the case where the required level of reserves, K, is equal to zero. The solid line in this figure illustrates the demand for reserves when there is no after-hours payment shock (i.e., the traditional Poole model). The dashed line represents the demand for reserves when there is an after-hours payment shock. The dots represent the equilibrium allocation and rates when the level of reserves is also equal to zero, showing that the interbank rate could trade above the middle of the corridor when there is an after-hours payment shock. In panel (b), the required level of reserves, K, is a large positive number. In this figure, the demand for reserves is unaffected by the presence of an after-hours payment shock and the dashed line and the solid line coincide. The dot represents the equilibrium allocation and rate when the level of reserves is equal to the required level of reserves, showing that, with large required reserves, the interbank rate could still trade in the middle of the corridor when there is an after-hours payment shock.

than in a typical floor system, given that the second condition in Corollary (1) may be more restrictive than the condition defining a floor system. Nonetheless, the overnight rate in a floor system should always equal the Poole rate when the volatility of the overnight payment shock is less than that of the intraday payment shock ($\sigma_F < \sigma_G$). In this case, then the second condition in Corollary (1) is met whenever the first condition (i.e. the definition of a floor system) is met.

Visually, this can be illustrated with the central bank supplying a large quantity of reserves in Figure 2 (a), such that the supply of reserves is a vertical line that intersects with the demand for reserves at r_R. At this point, the expected reputation cost is zero and the demand for reserves when reserves are that abundant is the same as it would be in the absence of an after-hours payment shock.

Corollary 2. *In a zero-reserve requirement corridor system ($R = K = 0$), the overnight rate will equal the Poole rate only when the volatility of the overnight payment shock is relatively small (i.e., $\Phi(\frac{\sigma_G}{\sigma_F}\phi(0)) \approx 1$).*

Proof. Substituting $R = K = 0$ into equation (15) yields the desired result, $r_\Delta = r_{Poole} + \frac{s}{2}(1 - \Phi(\frac{\sigma_G}{\sigma_F}\phi(0)))$. □

This suggests that the overnight rate will generally not trade at the midpoint of the corridor in a zero-reserve corridor system. It will only trade at the midpoint in the case where the overnight payment shock is immaterial. When the payment shock is material, the central bank would have to supply more reserves, $R > K$, to target the midpoint of the operating band.

Thus, determining the optimal level of aggregate reserves to target the midpoint of the zero-reserves corridor is more challenging in the presence of material overnight payment shocks. In the absence of an overnight payment shock, the central bank simply needs to target aggregate reserves equal to the aggregate reserve requirement, $R = K$. With a material overnight payment shock, the central bank needs to understand the demand for reserves to determine the amount of aggregate reserves to supply to the market. This will depend on, among other things, the size of the reputation cost s and the magnitude of overnight payment shocks, σ_F. This can be seen in Figure 2 (a). With $R = K = 0$, the equilibrium interest rate (the intersection of the supply and the dashed demand curve) will be higher than the equilibrium interest rate in the absence of an overnight payment shock (the intersection of the supply and the solid demand curve).

An alternative for the central bank could be to establish a higher required reserves amount. This leads to our next proposition.

Corollary 3. *In a positive-reserve requirement corridor system ($R = K > 0$), the overnight rate will equal the Poole rate when the aggregate reserve requirement is sufficiently large (e.g., $\Phi(\frac{K}{\sigma_F}) \approx 1$).*

Proof. Substituting $R = K$ and $\Phi(\frac{K}{\sigma_F}) \approx 1$ into equation (15) yields $r_\Delta = r_{Poole}$. □

The intuition for this result is the same as the intuition for Corollary (1). When K is sufficiently large, all banks will hold enough reserves such that there is a near zero probability that the overnight payment shock will bring the bank's level of reserves to zero, where it will begin to experience the reputation cost.The higher amount of required reserves shifts the demand curve for reserves to the right, as seen in Figure 2 (b). Since aggregate required reserves are sufficiently far away from zero, there is zero probability that a bank would experience a large enough overnight payment shock to decrease its reserves below zero. Thus, the demand for overnight reserves is the same as in the standard case.

2.3.1. Effect of after-hours payment shock volatility on the equilibrium

A good monetary policy implementation framework should be characterized by low volatility in the overnight rate (e.g., Bindseil (2016)). In our model, we can examine the volatility of overnight rates by examining the effect of an increase in payment uncertainty on the overnight rate. All else equal, an overnight rate that is more responsive to after-hours payment shock uncertainty will be more volatile.

To see the effect of an increase in after-hours payment shock uncertainty on overnight rates, we take the derivative of Equation (14) with respect to after-hours payment shock volatility, σ_F. Since some central banks may also adjust aggregate reserves to deal with after-hours payment shock volatility, we also consider that aggregate reserves are a function of after-hours payment shock volatility (e.g., $R = R(\sigma_F)$). That is, the central bank can offset the effect of volatility by changing aggregate reserves:

$$\frac{\partial r_\Delta}{\partial R} = \frac{1}{\sigma_G}[(r_R + s(1 - \Phi(\frac{\epsilon_Z}{\sigma_F})) - r_X)\phi(\frac{\epsilon_K}{\sigma_G}) + s(-\phi(\frac{\epsilon_Z}{\sigma_F}))\frac{\partial \epsilon_Z}{\partial R}\Phi(\frac{\epsilon_K}{\sigma_G})] \tag{16}$$

Equation (16) shows that increasing aggregate reserves decreases the overnight rate not only because it reduces the probability of central bank borrowing, but also because it reduces the probability of suffering the reputation cost of having insufficient funds in the overnight period. Given this, the derivative of the overnight rate with respect to after-hours payment uncertainty can be written as

$$\frac{\partial r_\Delta}{\partial \sigma_F} = s\Phi(\frac{\epsilon_K}{\sigma_G})\phi(\frac{\epsilon_Z}{\sigma_F})\frac{\epsilon_Z}{\sigma_F^2} + \frac{\partial R}{\partial \sigma_F}\frac{\partial r_\Delta}{\partial R} \tag{17}$$

In the absence of a central bank response ($\frac{\partial R}{\partial \sigma_F} = 0$), an increase in after-hours payment uncertainty will weakly increase the overnight rate for typical central bank operating frameworks (e.g., $\epsilon_Z \geq 0$). In an abundant reserves system where $\Phi(\frac{\epsilon_Z}{\sigma_F}) = 1$, after-hours payment certainty has no effect on the overnight rate. In this case reserves are sufficiently large such that a change in after-hours payment volatility does not change the probability of running out of funds in the overnight session.

In a corridor system, on the other hand, an increase in after-hours payment volatility will put upward pressure on the overnight rate. Payment volatility could increase, for instance, if the after-hours session is longer, i.e., over a weekend. Thus, when after-hours payment volatility fluctuates, it can induce fluctuations in the overnight rate.

Of course, the central bank can offset this pressure by adjusting aggregate reserves. But, it may be difficult for the central bank to determine the necessary adjustment to maintain stable overnight rates in a corridor system since it requires knowledge of the demand for overnight funds as well as knowledge about the volatility of the after-hours payment shock. To see this, we can solve equation (18) for the optimal central bank response to keep the overnight stable with $\frac{\partial r_{\Delta}}{\partial \sigma_F} = 0$:

$$\frac{\partial R}{\partial \sigma_F} = -s \frac{\Phi(\frac{\epsilon_K}{\sigma_G})\phi(\frac{\epsilon_Z}{\sigma_F})\frac{\epsilon_Z}{\sigma_F^2}}{\frac{\partial r_{\Delta}}{\partial R}} \tag{18}$$

2.4. Voluntary Reserve Targets in a Corridor System

The previous subsection suggests that the central bank can maintain a corridor system if it chooses required reserves that are sufficiently high. With a high reserve requirement, the expected reputation cost of having insufficient funds in the after-hours market is negligible.

However, it is likely that commercial banks have better knowledge of their payment flows and may be in a better position to set an optimal reserve requirement that minimizes the expected reputation cost of having insufficient funds in the overnight market. Prior to the crisis, the Bank of England, for example, allowed banks to set their own required reserves. Baughman and Carapella (2018) develop a model of voluntary reserve targets and show the potential advantages of such a model over other models. What are the conditions under which self-determination of required reserves is optimal when there is an after-hours payment shock?

To answer this question, we allow banks to choose their required relative reserves before the start of intraday trading, under the assumption that the central bank will supply aggregate reserves equal to the aggregate reserve requirement ($R = K$). This is a slight departure from Baughman and Carapella (2018), given that in their model targets adjust to supply of reserves rather than the other way around. That is, voluntary reserve targets adjust to anticipated central bank liquidity injections.

We assume that bank bond holdings remain exogenous and that deposits are endogenously determined. Given the balance sheet identity and the fact that $R = K$, aggregate deposits are equal to $B + K$. We also assume that a bank now faces a balance sheet cost that is an increasing function of balance sheet size, $c(B + K) \geq 0$, $c'(B + K) \geq 0$ and

$c''(B+K) \geq 0$. Then, the aggregate bank will choose K to maximize expected profits:

$$\begin{aligned}
\mathrm{E}[\pi] = & \; r_B B - r_D(B+K) - c(B+K) + r_K K \\
& + (r_R - r_X) \int_0^{\infty} \epsilon^i_{intraday} \mathrm{dG}(\epsilon^i_{intraday}) \\
& - s \int_{\epsilon^i_Z}^{\infty} (\epsilon^i_{overnight} - \epsilon_Z) \mathrm{dF}(\epsilon^i_{overnight})
\end{aligned} \tag{19}$$

First order condition is

$$r_D + c'(B+K) = r_K + s(1 - \Phi(\frac{K + \sigma_G \phi(0)}{\sigma_F})) \tag{20}$$

The left-hand side of the first order condition represents the marginal cost of an additional dollar of required reserves. This is the bank's marginal funding cost, and it includes the deposit rate, as well as the cost associated with increasing the bank's balance sheet. The right-hand side of the equation is the marginal benefit of an additional dollar of required reserves. It consists of two components. The first is r_K, the rate at which the central bank compensates required reserves. The second is the reduction in the expected reputation costs associated with having insufficient funds to process overnight transactions.

Whether this produces a social optimum depends on the social planner's objective function as well as the rates r_D and r_K. If, for instance, the social planner was interested in minimizing the sum of balance sheet costs and expected reputation costs, it could set the rate on required reserves equal to the deposit rate ($r_D = r_K$), and banks would choose the social optimum. If, on the other hand, the social planner was only interested in minimizing reputation costs (and hence the deviation of the overnight rate from target), it could set $r_K = r_D + c'(B+K)$, and banks would select the social optimum in that case.

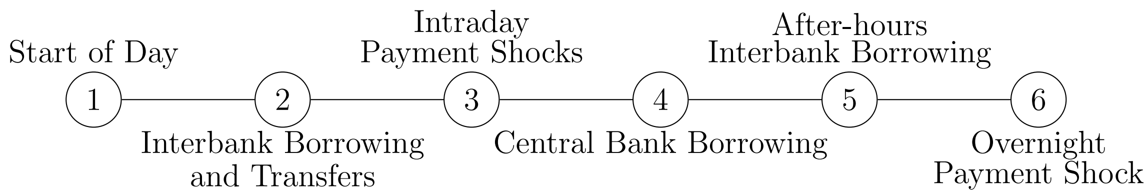
Thus, as long as the central bank correctly sets r_K, banks' self-determination of their required reserves could be socially optimal.

3. Multiple Payment Systems

How does the presence of multiple payment systems affect monetary policy implementation? In this section, we analyze how the results are affected by the operation of two interlinked payment systems: one which operates only during the day and has access to the central bank borrowing and lending facilities, and one which operates 24/7 and has access to the central bank deposit facility but not the lending facility.

To analyze this setup, we need to modify the baseline model to account for additional choices that the commercial banks can make in stage 2. Figure (3) illustrates how the setup

Figure 3: Model Timing with Two Payment Systems



is modified.

For ease of notation, we assume that banks begin the day with zero reserves in the 24/7 payment system. This does not impact the results since, in Stage 2, in addition to borrowing in the interbank market, we assume banks can also transfer funds between the two payment systems. We denote the net transfer from the intraday payment system to the 24/7 payment system $T^i \geq 0$.

In stage 3, after the trading session is closed, each bank experiences an intraday payment shock, $\epsilon^i_{intraday}$, in the intraday payment system. This is the same payment shock as in the baseline setup, with cumulative distribution function $G(\epsilon^i_{intraday})$. For simplicity and comparison with the earlier results, we assume there is no intraday payment shock in the 24/7 payment system.

After experiencing the intraday payment shock, like before banks can borrow from the central bank in stage 4 if they are in a negative excess reserve position.[4] Then, the bank earns r_R on positive balances they hold in either system. They also pay r_X on their borrowing from the central bank.

In stage 5, banks will still choose their net interbank after-hours borrowing Δ^i_{after} to maximize their expected profits in the after-hours market, given the payment shock it is exposed to in stage 6 in the 24/7 payment system:

$$\mathrm{E}[\pi^i_{after}] = -r_{after}\Delta^i_{after} - s\int_{\epsilon^i_{Z,T}}^{\infty}(\epsilon^i_{overnight} - \epsilon^i_{Z,T})\mathrm{dF}(\epsilon^i_{overnight}) \tag{21}$$

where the threshold, $\epsilon^i_{Z,T} \equiv T^i + \Delta^i_{after}$, is a little different because it accounts for the effect of transfers between the two systems. A transfer from the intraday system to the 24/7 system increases this threshold because it directly increases the amount of reserves in the 24/7 system. Since central bank borrowing only impacts the reserve position in the intraday

[4]If we assume banks can transfer between systems in stage 4 and reserve requirements apply to balances in the 24/7 system, results will be identical to the baseline setup as long as banks are not able to be short in the intraday system and long in the 24/7 system at the end of the day. Each bank will make inter-system transfers such that a net positive balance across the two systems will be held in the 24/7 payment system and a net negative balance across the two systems will be held in the intraday payment system.

system, it does not impact the threshold for experiencing the reputation cost in the 24/7 system.

Like before, the first order condition from equation (21) provides the after-hours interbank rate that maximizes the expected after-hours profits:

$$r_{after} = s(1 - F(\epsilon^i_{Z,T})) \tag{22}$$

Equation (22) implies that banks will trade with each other until they have the same $\epsilon^i_{Z,T} \equiv \epsilon_{Z,T}$

Each bank will choose their borrowing, lending, and transfer activity in stage 2 to maximize expected profits:

$$\begin{aligned} \mathrm{E}[\pi^i] &= r_B B^i - r_D D^i + r_K K^i - r_\Delta \Delta^i \\ &+ r_R(\epsilon^i_K - T^i) + (r_R - r_X)\int_{\epsilon^i_K - T^i}^{\infty} (\epsilon^i_{intraday} - (\epsilon^i_K - T^i))\mathrm{dG}(\epsilon^i_{intraday}) \\ &- r_{after}(T^i + \Delta^i_{after}) - s\int_{\epsilon_{Z,T}}^{\infty} (\epsilon^i_{overnight} - \epsilon_{Z,T})\mathrm{dF}(\epsilon^i_{overnight}) \end{aligned} \tag{23}$$

Maximizing expected profits produces two first order conditions. After combining these two conditions and aggregating across all banks, the optimality conditions can be written as:

$$\begin{gathered} r_\Delta = r_R + (r_X - r_R)(1 - \Phi(\tfrac{R-K-T}{\sigma_G})) \\ r_\Delta = r_R + s(1 - \Phi(\tfrac{T}{\sigma_F})) \end{gathered}$$

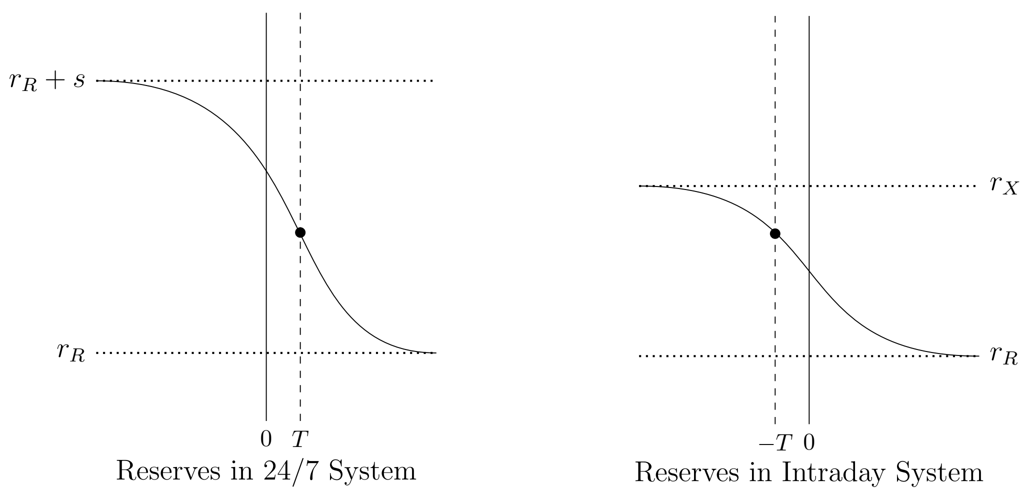
The first equation represents the marginal value of reserves in the intraday system, and the second equation represents the marginal value of reserves in the 24/7 system. Since both equations have the overnight interbank rate on the left-hand side, it suggests that banks will transfer funds between the two systems until the marginal value of reserves is equal across both systems.

The marginal value of reserves suggested by the first equation is a small departure from the standard model, in that it includes an additional term for transfers to the 24/7 system. This, in turn, will reduce reserves in the intraday system and increase the probability that the bank will have to borrow from the central bank. Hence, it will put upward pressure on the overnight rate.

The marginal value of reserves in the 24/7 system is a function of two factors. First, banks will earn interest on reserves on funds held in the 24/7 system. Second, an extra dollar of reserves in the 24/7 system lowers the likelihood that the bank will be short of funds in the overnight market, thus reducing the expected reputation cost of being short of funds.

Overall, the interbank rate in the two-system environment may be higher or lower than the interbank rate in a single system. On the one hand, banks will borrow more in an intraday system than they do in a single system, since transfers to the 24/7 system reduce reserves in the intraday system and hence increase the amount of interbank borrowing. Also, some banks will end up with positive balances at the end of the day in the intraday system that cannot be used to reduce the probability of experiencing the reputation cost in the 24/7 system (since post-intraday shock transfers are not allowed). This will put upward pressure on the interbank rate in a dual system, relative to a single system. On the other hand, in a single system, the marginal benefit of an additional dollar of reserves includes both the marginal benefit of reducing the expected cost of central bank borrowing, as well as the marginal benefit of reducing the expected reputation cost. Because it contains both of these marginal benefits, and the interbank rate reflects these marginal benefits, the interbank rate in a single system could be higher. Depending on which of these two competing effects dominates, the rate could be higher in a single system or in a dual system.

Figure 4: Overnight Rate in Current System



(a) 24/7 System

(b) Intraday System

Notes: The panels represent overnight trading in the two systems when the required level of reserves, K, and aggregate level of reserves, R, are both equal to zero. Panel (a) illustrates the equilibrium allocation and rate in the 24/7 system, while Panel (b) illustrates the equilibrium allocation and rates in the intraday system. Assuming that both systems begin the day with zero aggregate reserves, T represents the transfers between the two systems that occur in stage 2. Specifically, transfers occur until the overnight rates in the two different systems are equal.

Figure 4 illustrates how these transfers affect the overnight interbank rate. When both the intraday system and the 24/7 system begin the day with zero reserves, the marginal value of funds in the 24/7 system (Figure 4 (a)) is greater than the marginal value of funds in the intraday system (Figure 4 (b)). Since the marginal value of funds is higher in the overnight system, participants will have incentives to move reserves into that system. They continue doing so until the marginal value of funds in the two systems is equal.

3.1. Restrictions on transfers

The effects of having multiple payment systems depends critically on the ability to transfer between systems. We assumed that these transfers can occur during the interbank trading period, and showed that, under certain assumptions, allowing transfers after the resolution of the intraday payment shock equates the problem to that of a single payment system.

At the other extreme, the two payment systems could be completely segregated, with transfers restricted between the two systems.[5] Instead, in stage 2, banks could either trade funds in the intraday system with each other, or funds in the 24/7 system with each other. In this case, there will be an overnight interbank rate for the intraday system, and an overnight interbank rate for the 24/7 system:

$$r_{\Delta,intraday} = r_R + (r_X - r_R)(1 - \Phi(\tfrac{R-K}{\sigma_G}))$$
$$r_{\Delta,24/7} = r_R + s(1 - \Phi(0))$$

Thus, a central bank could insulate the intraday trading market from effects of the after-hours payment shock by restricting transfers between the two systems. For example, it could provide reserves equal to required reserves in the intraday payment system, and thus implement the Poole rate in that system. However, the implied rate in the interbank system would be much higher.

4. Conclusion

Our paper shows how changes in the payment system could have implications for overnight interest rates. Specifically, monetary policy implementation frameworks that naturally have a large amount of settlement balances are less impacted by a move to 24/7 payment settlement in our model. More broadly, while our model focuses on the effect of 24/7 payment settlement on interbank rates, it can also be applied to other factors that increase the benefits of reserves. For example, the reputation costs of having insufficient funds after hours could

[5] This assumes an unsecured interbank market. In a secured interbank market, the secured rate also reflects the shadow value of the collateral when the collateral constraint is binding. In this case, the rates in the two markets may not be completely segregated if the markets share the same pool of collateral. Analyzing this is beyond the scope of this model.

be interpreted as a cost of having insufficient reserves relative to a target level of reserves that could be driven by regulation or other factors.

Further, our model is derived in a centralized market so does not say anything about trading volumes or dispersion of traded rates. While we believe our model delivers the important implications of 24/7 settlement, a search model with a decentralized market (e.g., Afonso and Lagos (2015)) could provide additional implications for trading activity.

Finally, we assume that the intraday shock process and other underlying features of the system, such as the number and characteristics of participants, are invariant to payment system modernization. In practice, these features may adjust to a lengthening of the payment period. We leave a more detailed exploration of these drivers to future work.

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