BANK OF CANADA
BANQUE DU CANADA

Staff Working Paper/Document de travail du personnel — 2020-23

Last updated: June 4, 2020

Trading for Bailouts

by Toni Ahnert,[1] Caio Machado[2] and Ana Elisa Pereira[3]

[1]Financial Stability Department
Bank of Canada, Ottawa, Ontario, Canada K1A 0G9
tahnert@bankofcanada.ca

[2]Instituto de Economía, Pontificia Universidad Católica de Chile
caio.machado@uc.cl

[3]School of Business and Economics, Universidad de los Andes, Chile
apereira@uandes.cl

ISSN 1701-9397

Acknowledgements

We thank David Cimon, Jonathan Chiu, Qi Liu, and audiences at the Bank of Canada, the Lisbon Meetings in Game Theory, Universidad Alberto Hurtado, and University of Chile for helpful comments. All remaining errors and the views of this paper are our own.

Abstract

Government interventions such as bailouts are often implemented in times of high uncertainty. Policymakers may therefore rely on information from financial markets to guide their decisions. We propose a model in which a policymaker learns from market activity and where market participants have high stakes in the intervention. We study how the strategic behavior of informed traders affects market informativeness, the probability and efficiency of bailouts, and stock prices. We apply the model to study the liquidity support of distressed banks and derive implications for market informativeness and policy design. Commitment to a minimum liquidity support can increase market informativeness and welfare.

Topics: Financial institutions; Financial markets; Financial system regulation and policies; Lender of last resort
JEL codes: D83, G12, G14, G18

1 Introduction

A fundamental question in financial economics concerns the informativeness of market prices. When financial market participants trade on private information, prices convey information about underlying economic conditions. This fact motivates the usage of market prices to guide real decisions. As a result, prices not only reflect the fundamental value of firms, but also affect it—a feedback effect (Bond, Edmans and Goldstein, 2012). Given that government interventions, such as bailouts, are often undertaken with limited information and in highly uncertain times, policymakers are particularly likely to rely on information gleaned from market prices.

As the substantial interventions during the recent financial crisis illustrate, the stakes associated with government interventions are high. Moreover, although bailouts are meant to avoid negative spillovers to the broader economy, their benefits accrue mostly to those closely connected to the firm being bailed out (e.g., its creditors and shareholders). When the policy decision is endogenous to trading activity, investors with high stakes in an intervention may have incentives to trade based not only on their information, but also with the purpose of influencing the policy outcome. In this context, the following research questions arise: How much and under what conditions can policymakers learn from market activity? How efficient are interventions? What are the implications for stock prices? How does bailout design affect market informativeness and welfare?

To examine these issues, we propose a parsimonious model in which an informed trader has a high stake in a government intervention. By intervening, a policymaker improves the cash flow of a firm. This intervention is socially desirable when an economic fundamental (the state) is bad and the benefit of avoiding the failure of the firm and its associated spillovers exceed the cost of the intervention. The policymaker observes the activity in a market in which the shares of the firm are traded. As in Kyle (1985), there is a noise trader and a competitive market maker who meets the orders at the fair price. The key player in our setting is a large informed trader who derives a private benefit from the intervention. This benefit arises naturally when the trader is a creditor or blockholder, for instance. In the latter case, the private benefit scales with the initial block size.

We first use this model to study market informativeness, the probability of an intervention and its efficiency, stock prices, and the implications of changes in block size and cash flow risk. Next, we apply the model to study the liquidity support of banks that face costly liquidation. We relate market informativeness and welfare to market conditions and the source of uncertainty that the policymaker faces. We also derive implications for the implementation of liquidity support.

We start by characterizing how trading behavior is affected by the private benefit of intervention. Without a private benefit, the large trader trades on her private information to maximize expected trading profits, which reveals the state to the policymaker as much as possible given the presence of the noise trader. With a private benefit or a large block size, however, the large trader has incentives to trade to avoid revealing the good state in an attempt to increase the probability of an intervention. Thus, the informed trader does not trade or even sell shares of the firm in the good state if the benefit of intervention is high enough. We call this behavior *trading for bailouts*.

Informed traders, however, are not always successful in affecting the policy outcome. When the policymaker is ex ante prone to intervening, bailouts are more likely when the trader has a private benefit of intervention. When the policymaker is ex ante reluctant to intervene (because the bad state is unlikely), bailouts may actually be less likely when the trader has a private benefit. This benefit can end up shutting down an effective channel for the policymaker to learn from market activity. Whether the policymaker's reduced reliance on market activity increases the probability of intervention depends on how the policymaker would act without any additional information.

We propose a simple measure of market informativeness and show that, in equilibrium, higher informativeness increases the efficiency of real decisions (i.e., whether to bail out the firm). A general insight is that the private benefit reduces market informativeness around a government intervention and hinders the efficiency of bailouts. The loss in efficiency arising from lower informativeness can be decomposed into losses from (i) intervening less often in the bad state (higher type-I error); and (ii) intervening more often in the good state (higher type-II error). We characterize which types of mistakes the policymaker makes under different market conditions.

Our main model yields two sets of testable implications. First, we consider changes in the trader's block size, which affect the private benefit of the intervention. A larger block size reduces market informativeness around government interventions and reduces real efficiency because of stronger incentives to trade for bailouts.[1] When the policymaker is ex ante prone to intervene (e.g., in a crisis episode), the ex-ante probability of intervention increases in the block size. When the policymaker is ex ante reluctant to intervene, by contrast, the effect of block size on the ex-ante probability of intervention is non-monotonic. On the one hand, the larger private benefit incentivizes more strategic trading to manipulate the belief of the policymaker and tends to increase the chance of an intervention. On the other hand, the policymaker becomes more skeptical about market informativeness. Reducing its reliance on market activity, the policymaker places more emphasis on the prior that suggests no intervention.

Second, we consider the firm's cash flow risk. Higher risk increases the value of the trader's private information and, thus, the expected trading profit relative to any benefit of an intervention. Hence, trading reflects private information better, which increases market informativeness and real efficiency. In a slightly modified version of our model, we allow for risk choice by the firm to maximize expected shareholder value. Intriguingly, the privately optimal level of risk is inefficiently low. While the policymaker prefers high risk to support market informativeness and efficiency, the firm benefits from some trading for bailouts behavior and, therefore, chooses a lower level of risk.

We also characterize the stock price of the firm. It can be non-monotonic in the order flow, with a U-shape for an optimistic prior and an inverted U-shape for a pessimistic prior. The intuition for this price behavior comes from a tension between two forces: a higher order flow (i) increases the belief that the policymaker and the market maker form about the high state, which supports a higher price; and (ii) reduces the probability of an intervention, which lowers the price. A related non-monotonic price arises in Bond, Goldstein and Prescott (2010), who study market-based corrective action with learning from a competitive market price and a continuous fundamental. In their model, a corrective action results in a non-monotonic price in the fundamental because a small deterioration triggers an intervention and thus a discontinuous upward jump in the price.

In the final part of the paper, we apply the model to study a policymaker's decision to provide

[1] Our focus is not on informativeness in general, but on informativeness around government interventions. There are reasons why blockholders might increase price informativeness in normal times. See the discussion in Section 3.2.

liquidity support to a distressed financial institution. We consider a situation in which a bank faces a liquidity shortage and a policymaker may provide liquidity if it considers that the social gains of avoiding inefficient liquidation of assets more than compensate for the costs of intervening. We use this application to derive additional positive and normative implications.

Interestingly, an increase in intervention costs may actually improve market informativeness and welfare. When the intervention cost is large, traders anticipate that the policymaker will be reluctant to provide assistance, and this ends up facilitating learning from the market. To effectively affect the policymaker's belief when there is a selloff, the trader must buy the stock with high enough probability when observing good news. The gain in informativeness can more than compensate the higher implementation costs. We also study how the bank's asset returns, the severity of the liquidity shortage, and liquidation costs affect market informativeness and welfare.

We show that how much information can be conveyed through activity in financial markets depends critically on the type of uncertainty faced by the policymaker. If uncertainty is only about asset returns, market informativeness is not affected by the presence of traders with high stakes in the intervention. If uncertainty is solely about asset liquidity, by contrast, market informativeness is strongly affected by the strategic behavior of such speculators. Importantly, we find that the presence of a trader with an arbitrarily small private benefit of the intervention may change equilibrium outcomes dramatically in this case. The most informative equilibrium—the one in which traders buy following good news and sell following bad news—may cease to exist even when the trader's private benefit (block size) is arbitrarily small.

Finally, we investigate the consequences of commitment to a minimum liquidity support. We modify the model by allowing the policymaker to offer a minimum assistance package before observing market activity. After observing trading in financial markets, it can then decide whether to provide additional assistance. Such policy could be implemented by extending an unconditional credit line to financial institutions (e.g., the Federal Reserve discount window or liquidity assistance programs offered by other central banks) or by gradually implementing liquidity support, for instance. We show that offering a minimum support can improve informativeness and welfare. The intuition is that promising a minimum support reduces the residual benefit of additional support ex post, discouraging strategic trading and boosting informativeness. Despite part of the assistance being implemented with little information, this early decision allows the policymaker to learn more from the market and to implement any additional support more efficiently.

Literature. Market prices may contain useful information for real decision makers—an idea that goes back to Hayek (1945). Evidence that decision makers look at market activity as a source of information has been documented in different contexts (e.g., Luo, 2005; Chen, Goldstein and Jiang, 2006; Bakke and Whited, 2010; Edmans, Goldstein and Jiang, 2012). A growing body of literature has incorporated the idea that agents may look at market prices to guide a decision that ultimately affects the value of securities (for instance, Dow and Gorton, 1997; Bond, Goldstein and Prescott, 2010; Lin, Liu and Sun, 2019).

The papers most related to ours are those with feedback effects and large strategic traders, including Goldstein and Guembel (2008), Khanna and Mathews (2012), Edmans, Goldstein and Jiang (2015), and Boleslavsky, Kelly and Taylor (2017). In Goldstein and Guembel (2008), an uninformed trader has incentives to short sell a firm's security to affect a managerial decision. The manager's misguided decision leads to a decrease in the real value of the firm and ends up generating trading profits for the uninformed short seller. In contrast, our paper concerns the strategic behavior of an informed trader with high stakes in an intervention and different forces are at play (apart from the focus on policy interventions instead of managerial decisions). To manipulate the decision maker's beliefs, the trader has incentives to sell the stock when she has no information in Goldstein and Guembel (2008), while the trader has incentives not to buy even upon observing good news in our model.

Khanna and Mathews (2012) introduce an informed blockholder in the model of Goldstein and Guembel (2008) and show that the blockholder can prevent value destruction from short-selling attacks of the uninformed trader. A blockholder is one interpretation of our trader with high stakes. In Khanna and Mathews (2012), the incentives of the decision maker (a firm manager) are fully aligned with the blockholder's, conditional on the state. In our model, by contrast, the incentives of the decision maker (a policymaker) and the blockholder are fully misaligned in good states, in which the intervention is socially undesirable but profitable for the blockholder.

In Edmans, Goldstein and Jiang (2015), a firm manager uses market activity to guide an investment decision. An informed speculator trades the firm's security, and an asymmetric effect emerges: by trading on her information, the trader induces the manager to take the correct action, which always increases firm value; this increases incentives for her to buy on good news, but *decreases* incentives to sell on bad news. The main result is that there is an endogenous limit to arbitrage, and bad news is less incorporated into prices, leading to overinvestment. In the same spirit, Boleslavsky, Kelly and Taylor (2017) propose a model where an authority (e.g., a firm manager or policymaker) observes trading activity prior to deciding on an action that changes the state, thus affecting the security value. By assumption, the intervention removes the link between the initial state and firm value, and informed traders are harmed by the intervention since they lose their informational advantage. As in Edmans, Goldstein and Jiang (2015), price informativeness is also reduced, since informed investors have less incentive to sell the asset following bad news. In contrast to these papers, due to the private benefit of the policy intervention, incentives to *buy* the stock following *good news* are reduced in our model (while incentives to sell following bad news are unaffected). Moreover, differently from Boleslavsky, Kelly and Taylor (2017), the intervention does not eliminate the trader's informational advantage.

Bond and Goldstein (2015) also study policy interventions in a model of feedback. As opposed to our setting, there is a continuum of small traders that cannot move prices and, hence, cannot individually affect the policy outcome. In Bond, Goldstein and Prescott (2010), a decision maker also learns from a market price, but speculators' decisions to trade are not modeled. Finally, our paper adds to the literature on the role of blockholders (e.g., Admati and Pfleiderer, 2009; Edmans, 2009; Edmans and Manso, 2010), emphasizing how their presence affects price informativeness in the face of a potential government intervention.

The remainder of the paper is organized as follows: In Section 2 we introduce the main model. In Section 3 we present the equilibrium and main results. Section 4 presents the application to liquidity support. Section 5 concludes. We relegate all proofs to the Appendix.

2 Model

There are two dates $t = 0, 1$, no discounting, and universal risk neutrality. The cash flow v per unit of outstanding share of a firm at $t = 1$ depends on a fundamental $\theta \in \{L, H\}$, which we refer to as the bad and good state, respectively, and an intervention by a policymaker $G \in \{0, 1\}$, where $G = 1$ indicates an intervention:

$$v(\theta, G) = R_\theta + \alpha_\theta G, \tag{1}$$

where R_θ is the part of the cash flow independent of the intervention and α_θ the part caused by the intervention. Letting $\Delta_R \equiv R_H - R_L > 0$ and $\Delta_\alpha \equiv \alpha_H - \alpha_L$, we assume that the cash flow in the good state is above the cash flow in the bad state even if the policymaker intervenes, $\Delta_R + \Delta_\alpha > 0$.

The fundamental θ is drawn at $t = 0$ but unobserved by the policymaker. The good state occurs with probability $\gamma \in (0, 1)$. We assume that the intervention is socially desirable only in the bad state. That is, the social cost of intervention is $c > 0$ and the social benefit is b_θ with $b_H < c < b_L$. One interpretation is that bearing the intervention costs is only desirable in a crisis, when it is critical to avoid the failure of the firm and potential spillovers to the rest of the economy. Hence, $\overline{\gamma} \equiv \frac{b_L - c}{b_L - b_H} \in (0, 1)$ is the highest probability assigned to the good state for which the policymaker still intervenes. For simplicity, we normalize $b_H \equiv 0$ and $b_L \equiv b$ (so $\overline{\gamma} = \frac{b-c}{b}$) in the main model. (We endogenize these payoffs in Section 4.)

Before deciding whether to intervene at $t = 1$, the policymaker learns from activity in a financial market (see Table 1 for a timeline). Shares of the firm are traded by a noise trader and an informed trader at $t = 0$.[2] As in Edmans, Goldstein and Jiang (2015), traders can place three types of orders, where -1 represents a sell order, 0 represents no trade, and 1 represents a buy order.[3] The noise trader is active for exogenous reasons (e.g., liquidity shocks) and places each order $z \in \{-1, 0, 1\}$ with equal probability regardless of the state. The informed trader observes θ and places an order $s \in \{-1, 0, 1\}$ to maximize her expected payoff. The key assumption here is that the informed trader has some relevant information unknown to the policymaker. As in

[2] The assumption of a single informed trader is for expositional clarity. It captures the main economic intuition without additional technical complications that arise from multiple large informed traders.

[3] Although traders cannot buy or sell interior amounts, we allow for equilibria in mixed strategies, so traders may buy or sell with interior probability. This can be thought of as reflecting an intensive margin of trading.

Kyle (1985), there is a competitive market maker who observes the total order flow, $X = s + z$, sets the price p to the expected value of the firm at $t = 1$, and executes the order at this price. The market maker uses the information contained in the order flow and rationally anticipates the policymaker's decision when setting the price.

Government interventions—such as bailouts of financial institutions—usually have large spillovers to some agents, including large shareholders and firm creditors, who can also participate in financial markets. To capture this, we assume that the informed trader derives a (potentially state-contingent) private benefit of the intervention, β_θ.[4] That is, her payoff at $t = 1$ is

$$\pi = s\,(v - p) + \beta_\theta G. \tag{2}$$

An example where such private benefits arise naturally is in the context of outside blockholders, which are pervasive among U.S. firms (Holderness, 2009). When the trader has μ shares of the firm at $t = 0$, the profit from trading quantity s is $(s + \mu)\,v - sp = s\,(v - p) + \alpha_\theta \mu G + \mu R_\theta$. Since μR_θ is exogenous, the trader's payoff can be represented as in equation (2) by setting $\beta_\theta \equiv \alpha_\theta \mu$.

$t = 0$: Information and Trade	$t = 1$: Learning and Intervention
• State θ is realized and observed by the informed trader • Traders place orders (s, z) • Market maker sets price p at which trade occurs	• Policymaker learns from financial market • Policymaker decides on intervention • Payoffs are realized

Table 1: Timeline of events.

3 Equilibrium

We start by introducing some useful notation. A trading strategy for the informed trader is a probability distribution over orders $s \in \mathcal{S} = \{-1, 0, 1\}$ for each fundamental $\theta \in \Theta = \{L, H\}$

[4] The assumption that the large trader with a private benefit of the intervention has useful information about the firm reflects the notion that such agents also have high incentives to acquire information about the firm.

and is denoted by $l(s)$ and $h(s)$. An intervention strategy for the policymaker is a probability of intervening $g(X)$ for each total order flow $X \in \mathcal{X} = \{-2, -1, 0, 1, 2\}$. A price setting strategy for the market maker is a function $p : \mathcal{X} \to \mathbb{R}$. Moreover, $q(X)$ is the probability the policymaker and the market maker assign to the good state H upon observing the order flow X.

We study perfect Bayesian equilibrium. In our setting, such an equilibrium consists of (i) a trading strategy for the informed trader that maximizes her payoff given all other strategies and her information about the realized θ; (ii) an intervention strategy that maximizes the policymaker's payoff given all other strategies and the order flow; (iii) a price setting strategy that allows the market maker to break even in expectation given all other strategies and the order flow; and (iv) beliefs $q(X)$ consistent with Bayesian updating on the equilibrium path. Moreover, we impose that beliefs off the equilibrium path satisfy the Intuitive Criterion (Cho and Kreps, 1987).

Lemma 1. *In the bad state, the informed trader always sells,* $l(-1) = 1$.

In the bad state, the informed trader only has incentives to sell: she expects to make positive trading profits and to influence the policymaker to intervene by conveying negative information about the state. Since the informed trader always sells in the bad state, we classify possible equilibria based on the informed trader's action in the good state:

(i) Buy equilibrium (B): the informed trader always buys in the good state, $h(1) = 1$.

(ii) Inaction equilibrium (I): the informed trader does not trade in the good state, $h(0) = 1$.

(iii) Sell equilibrium (S): the informed trader always sells in the good state, $h(-1) = 1$.

(iv) Equilibria in mixed strategies are denoted by combinations of S, I, and B. For example, SB denotes an equilibrium in which $h(-1) > 0$, $h(1) > 0$, and $h(0) = 0$.

As a benchmark, we characterize the equilibrium set without a private benefit of intervention.

Proposition 1. ***Benchmark.*** *When $\beta_H = \beta_L = 0$, there is a unique equilibrium in which the informed trader always sells in the bad state and always buys in good state (B equilibrium).*

When the informed trader derives no private benefit of the intervention, the trader's orders purely reflect her private information. The trader simply trades as to fully explore her informational advantage about the firm's cash flow: she sells if the fundamental is bad and buys if the fundamental is good. The aggregate order does not reveal the state for some orders of the noise trader, so the informed trader profits from the market maker in expectation. The trading behavior of the informed trader is as different across states as possible, so the market maker and the policymaker learn as much as possible from market activity given the existence of noise traders. Total orders $X \in \{-2, -1\}$ reveal the state $\theta = L$ and the policymaker intervenes, while orders $X \in \{1, 2\}$ reveal $\theta = H$ and the policymaker does not intervene. For $X = 0$, no information is revealed and the policymaker bases its decision on the prior γ. Figure 1 illustrates.

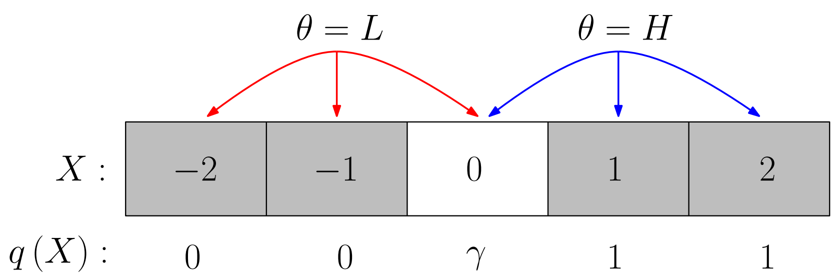

Figure 1: Benchmark without private benefit of intervention ($\beta_H = \beta_L = 0$). The aggregate order flow X given the equilibrium trading strategy of the informed trader, $l(-1) = 1$ and $h(1) = 1$, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. Order flows with updating are shaded in grey, while others are not shaded.

We turn now to the general case in which the intervention generates some private benefit for the informed trader (e.g., due to blockholding). To ease exposition, we focus on the generic case of $\gamma \neq \overline{\gamma}$.[5] Whenever there are multiple equilibria, we restrict attention to the best equilibrium from the perspective of the policymaker in the main text.[6] Figure 2 shows the equilibrium set for $\Delta_\alpha = 0$, where the left panel shows the whole equilibrium set and the right panel shows the best equilibrium.[7] For future reference, we state some bounds on parameters:

$$\begin{aligned} \underline{\beta} &= (1-\gamma)\left(\Delta_R + \Delta_\alpha\right), \qquad \overline{\beta} = (3-2\gamma)\left(\Delta_R + \Delta_\alpha\right), \qquad \utilde{\beta} = (1-\gamma)\,\Delta_R, \\ \tilde{\beta} &= \frac{(3-\gamma-\overline{\gamma})\,\Delta_R + \Delta_\alpha + \sqrt{\left[(3-\gamma-\overline{\gamma})\,\Delta_R + \Delta_\alpha\right]^2 + 4\,(1-\overline{\gamma})\,(1-\gamma)\,\Delta_R \Delta_\alpha}}{2}. \end{aligned} \tag{3}$$

[5]For $\gamma = \overline{\gamma}$, the Intuitive Criterion fails to rule out some equilibria that depend on unusual off-equilibrium beliefs.

[6]As discussed in Section 3.1, the policymaker's payoff is the relevant measure of real efficiency in this setting. Focusing on the worst equilibrium would lead to the same qualitative results and Proposition 2 would continue to hold, just with different expressions for $\overline{\beta}$ and $\tilde{\beta}$. See also Appendix A for the entire characterization of equilibrium.

[7]For $\Delta_\alpha \neq 0$, the illustration is qualitatively very similar, just with jumps at $\overline{\gamma}$.

Proposition 2. ***Equilibrium.*** *For a pessimistic prior ($\gamma < \overline{\gamma}$), the positively informed trader buys (B) if $\beta_H \leq \underline{\beta}$, does not trade ($I$) if $\underline{\beta} < \beta_H \leq \overline{\beta}$, and sells ($S$) if $\beta_H > \overline{\beta}$. For an optimistic prior ($\gamma > \overline{\gamma}$), the positively informed trader buys (B) if $\beta_H \leq \underset{\sim}{\beta}$, randomizes between buying and not trading (IB) if $\underset{\sim}{\beta} < \beta_H \leq \tilde{\beta}$, and randomizes between buying and selling (SB) if $\beta_H > \tilde{\beta}$.*

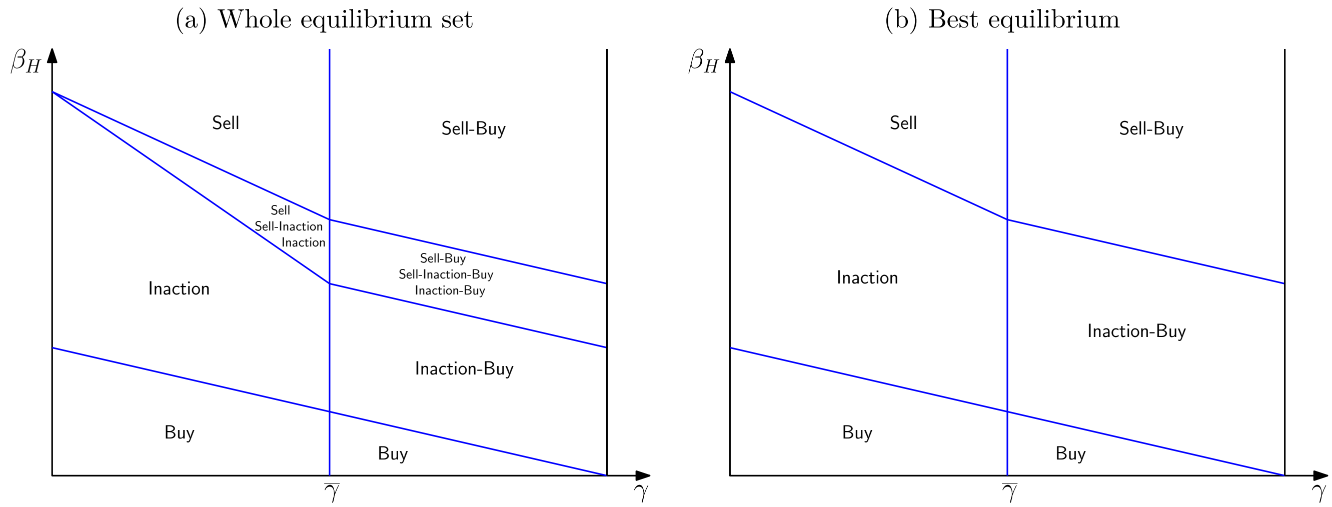

Figure 2: Equilibrium set for $\Delta_\alpha = 0$. When multiple equilibria exist, the best equilibrium is the one preferred by the policymaker.

Proposition 2 shows that the benchmark result of Proposition 1 continues to hold as long the private benefit is small enough, that is, below $\underline{\beta}$ for a low prior or below $\underset{\sim}{\beta}$ for a high prior. As β_H increases, however, the positively informed trader gains incentives to deviate from trading on her information. In particular, if β_H is high enough, the trader always sells the asset with positive probability even upon learning good news about the firm's fundamentals.

To gain some intuition, consider first the case of a low prior, $\gamma < \overline{\gamma}$. For a high private benefit, $\beta_H > \overline{\beta}$, the positively informed trader sells with probability one, $h(-1) = 1$. Since the policymaker is sufficiently pessimistic about the fundamental, an intervention takes place if activity in financial markets is absolutely uninformative. Hence, an equilibrium in which the positively informed trader perfectly mimics the behavior of the negatively informed trader can be sustained. If the private benefit of the intervention is sufficiently large, it is profitable for the positively informed trader to incur a trading loss against the market maker in order not to reveal information that could dissuade the policymaker from intervening. For an intermediate private benefit, $\underline{\beta} < \beta_H \leq \overline{\beta}$, the trader does not incur the losses of selling in the good state, but she gives up any trading profits from private information in order not to reveal too much information about the state that, in turn, could prevent the policymaker from intervening. Taken together, the informed trader opts for inaction, which is shown in Figure 3.

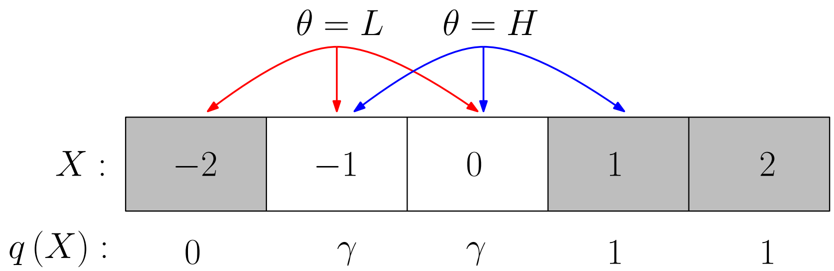

Figure 3: Inaction equilibrium: the aggregate order flow X given the equilibrium trading strategy of the informed trader, $l(-1) = 1$ and $h(0) = 1$, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. Note that the order flow $X = 2$ is off the equilibrium path, but $q(2)$ is uniquely pinned down by the Intuitive Criterion.

We turn to the high prior, $\gamma > \overline{\gamma}$. Since the policymaker is unwilling to intervene under this prior, the information from market activity must be compelling enough to revert the policymaker's prior for an intervention to occur. Thus, there is no equilibrium in pure strategies for high enough β_H. The incentives of the positively informed trader to deviate from trading on her information are high, but if she is expected to always do so, this behavior is ineffective in affecting beliefs. The equilibrium emerges from this balance. In the Inaction-Buy equilibrium (IB), for instance, both the positively informed trader and the policymaker play mixed strategies. The latter intervenes with some probability when observing an order flow of $X = -1$ such that the trader is indifferent between buying and not trading. Given the trader's randomization, the policymaker is indifferent between intervening and not intervening upon observing $X = -1$. Figure 4 shows this case.

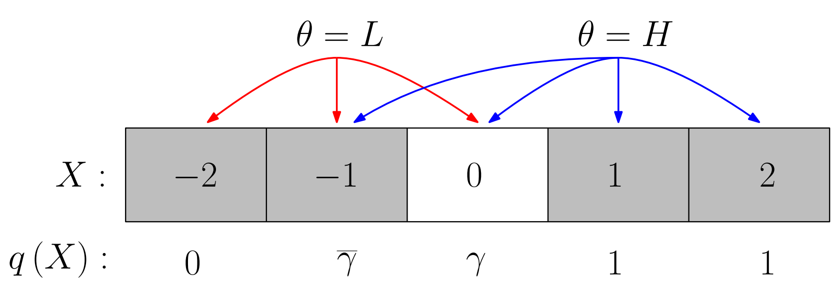

Figure 4: Inaction-Buy equilibrium: the aggregate order flow X given the equilibrium trading strategy of the informed trader, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. At $X = -1$, the beliefs are such that the policymaker is indifferent between intervening and not intervening.

For even higher values of β_H, the equilibrium similarly features mixed strategies. The policymaker randomizes between intervening or not upon observing sales ($X = -2, -1$), and the positively informed trader randomizes between buying and selling the stock.

3.1 Market informativeness and the efficiency of interventions

In models where real decision makers learn from the market, price efficiency (the extent to which the price of a security accurately predicts its future value) does not necessarily translate into real efficiency (the extent to which market information improves real decisions), as emphasized by Bond, Edmans and Goldstein (2012). To analyze the informativeness of market activity, we use the following measure.

Definition 1. *The informativeness of market activity is the expected learning rate about the state:*

$$\iota \equiv \gamma \left(\frac{\mathbb{E}\left[q(X)|\theta = H\right] - \gamma}{\gamma} \right) + (1-\gamma) \left(\frac{1 - \mathbb{E}\left[q(X)|\theta = L\right] - (1-\gamma)}{1-\gamma} \right). \tag{4}$$

Lemma 2 states properties of this informativeness measure.

Lemma 2. *Market informativeness is* $\iota = \frac{\mathbb{E}[q(X)|\theta=H]-\gamma}{1-\gamma}$ *and has the following desirable properties:*

- ι *increases in the correctness of beliefs,* $\mathbb{E}\left[q(X)|\theta = H\right]$ *and* $1 - \mathbb{E}\left[q(X)|\theta = L\right]$*;*
- $\iota = 1$ *if the state is perfectly learned (i.e.,* $\mathbb{E}\left[q(X)|\theta = H\right] = 1$ *and* $1 - \mathbb{E}\left[q(X)|\theta = L\right] = 0$*);*
- $\iota = 0$ *if nothing is learned (i.e.,* $\mathbb{E}\left[q(X)|\theta = H\right] = \gamma$ *and* $1 - \mathbb{E}\left[q(X)|\theta = L\right] = 1-\gamma$*).*

As formalized in Proposition 3 below, there is a clear mapping between market informativeness and the efficiency of real decisions. The ex-ante expected government payoff is

$$U_G = (1-\gamma)\Pr\left(G = 1|\theta = L\right)(b-c) - \gamma \Pr\left(G = 1|\theta = H\right)c, \tag{5}$$

where $\Pr(G = 1|\theta)$ denotes the probability of intervention conditional on the state. Since the intervention is the only real decision in our setting and trade in financial markets are pure transfers, we refer to U_G as a measure of real efficiency. Let U_G^E and ι^E be real efficiency and market informativeness when parameters are such that some equilibrium E arises.

Proposition 3. ***Real efficiency.*** *Fix the prior γ and the benefits and costs of the intervention (b, c). The ranking of real efficiency equals the ranking of market informativeness:*

$$U_G^{E'} > U_G^E \Leftrightarrow \iota^{E'} > \iota^E. \tag{6}$$

Market informativeness (and thus real efficiency) are ranked across equilibrium classes according to $\iota^B > \iota^I > \iota^S$ for $\gamma < \overline{\gamma}$, and $\iota^B > \iota^{IB} > \iota^{SB}$ for $\gamma > \overline{\gamma}$.

Proposition 3 shows that, given γ, b, and c, any change in parameters that induces an equilibrium with higher informativeness necessarily leads to a higher expected payoff for the government.[8] Hence, higher market informativeness increases the efficiency of real decisions in our setting. For instance, an increase in β_H associated with moving from the Buy equilibrium to the Inaction equilibrium reduces both market informativeness and the government's expected payoff.

Higher market informativeness increases real efficiency due to a reduction in the probability of two types of mistakes that the policymaker can make. A type-I error refers to the government not intervening when it should (when $\theta = L$), and a type-II error refers to intervening when it should not ($\theta = H$). The probability of those errors are $\Pr(\text{Type I}) = (1-\gamma)\Pr(G = 0|\theta = L)$ and $\Pr(\text{Type II}) = \gamma \Pr(G = 1|\theta = H)$, so the government payoff in (5) can be rewritten as

$$U_G = (1-\gamma)(b-c) - (b-c)\Pr(\text{Type I}) - c\Pr(\text{Type II}). \tag{7}$$

Equation (7) decomposes the expected payoff of the government in three terms. The first term captures the first-best payoff that would be obtained if the intervention were undertaken if and only if the bad state arises, $\theta = L$. The second term captures the expected loss due to a type-I error, when the state is bad but the government does not intervene, forgoing the net benefit of $(b - c)$. The third term captures the expected loss due to a type-II error, when the state is high

[8] Changes in γ, b, and c mechanically change the payoff of the government in addition to their impact on the equilibrium played and the level of informativeness. See also Section 4.

but the government still intervenes, incurring the cost c. An alternative expression is

$$U_G = (1-\gamma)(b-c) - b\left[\overline{\gamma}\,\Pr(\text{Type I}) + (1-\overline{\gamma})\Pr(\text{Type II})\right],$$

which has the interpretation of a weighted average of losses. The larger the relative benefit of the intervention (measured by $\overline{\gamma}$), the larger the weight given to type-I errors relative to type-II errors.

Proposition 2 implies that type-I errors never occur in equilibrium for a pessimistic policymaker ($\gamma < \overline{\gamma}$), since interventions always occur when the state is bad. In contrast, both types of errors may occur in equilibrium for an optimistic policymaker ($\gamma > \overline{\gamma}$). In what follows, we analyze the effect of block size and cash flow risk.

3.2 Block size

Blockholder sizes vary significantly across firms. In Holderness (2009), for example, 96% of the firms have at least one blockholder, with block sizes ranging from 5.4% to 85.5% of ownership.[9] Our model suggests that the blockholder size has important implications for how much a policymaker can learn from market activity, as stated in Proposition 4. Recall that the private benefit of an intervention for the positively informed trader depends on the block size, $\beta_H = \alpha_H \mu$.

Proposition 4. ***Block size.*** *The larger the block size μ: (i) the lower are both market informativeness and real efficiency; and (ii) the higher the ex-ante probability of intervention, $\mathbb{E}[g(X)]$, if $\gamma < \overline{\gamma}$. For $\gamma > \overline{\gamma}$, however, the probability of intervention is non-monotonic in the block size.*

The first part of Proposition 4 states that the larger the block size, the less able is the policymaker to learn from market activity and, hence, the less efficient is the intervention or bailout. This result arises from the positively informed trader having a higher stake in the intervention. Thus, she has higher incentives not to trade on her information, since large aggregate orders would push beliefs closer to the true state $\theta = H$, reducing the chances of an intervention.

The second part of Proposition 4 states that the effect of block size on the ex-ante probability

[9] The usual definition of a blockholder is an ownership share of at least 5%.

of intervention is positive for a pessimistic government but ambiguous for an optimistic government, as shown in Figure 5. To gain intuition, note that an increase in μ (or β_H in general) has two effects. First, the positively informed trader has more incentives to trade strategically and manipulate the belief of the policymaker. Ceteris paribus, this increases the probability of intervention. Second, and in response to the first channel, the policymaker reduces the weight given to market activity.

A pessimistic policymaker, $\gamma < \overline{\gamma}$, is willing to intervene even without additional information from the market. Hence, the *trading for bailouts* behavior is effective in increasing the probability of intervention. Both effects stated above push in the same direction: for a larger block size, the incentives to trade strategically are higher and market activity is less informative, resulting in a higher overall probability of intervention. Manipulation is quite effective in this case: as μ increases, the ex-ante probability of intervention eventually reaches 1 (see the top line in Figure 5).

In contrast, an optimistic policymaker, $\gamma > \overline{\gamma}$, requires some negative updating for an intervention to occur. Hence, no intervention occurs for uninformative market activity. As before, a marginal increase in block size can increase the probability of intervention because it encourages strategic trading to affect the policymaker's beliefs. In contrast to the previous case, the second effect opposes the first effect. For a higher block size, the policymaker is also more skeptical about the informativeness of market activity, reducing its reliance on it and ultimately reducing the probability of intervention. Taken together, the probability of intervention can be non-monotonic in the block size (see Figure 5). This result shows that manipulation can be ineffective: the presence of a blockholder can reduce market informativeness and result in a lower probability of intervention. For a large enough block size, the policymaker disregards any information from market activity and the probability of intervention approaches zero. In short, larger block sizes mitigate an effective channel of communication between the market and the policymaker: the information of the informed trader is not conveyed via market activity, and policy interventions are less efficient.

The mechanism leading to lower real efficiency as the block size increases is different for pessimistic and optimistic priors. As previously discussed, efficiency losses can arise from type-I and type-II errors. For $\gamma < \overline{\gamma}$, the policymaker always intervenes in the bad state, so the probability

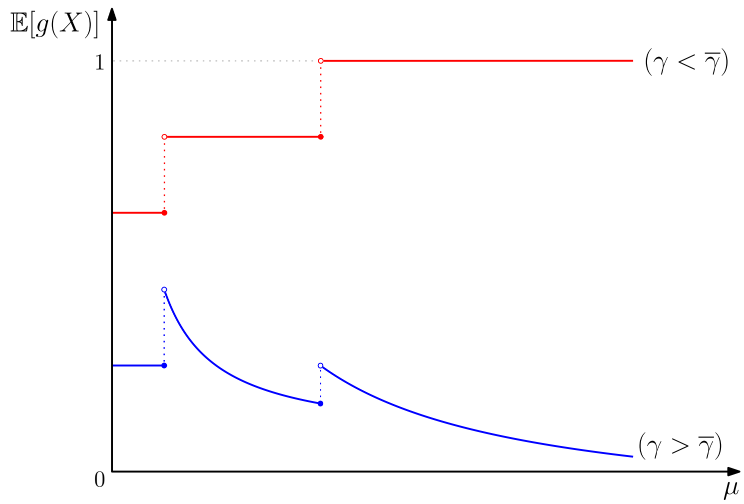

Figure 5: Expected probability of intervention as a function of block size μ: numerical example with γ slightly below or above $\overline{\gamma}$. Jumps occur at the switches between equilibrium classes—see also Figure 2.

of a type-I error is zero. The efficiency loss of larger block sizes is entirely due to an increase in type-II errors, because the policymaker often intervenes when it should not. In contrast, for an optimistic prior, $\gamma > \overline{\gamma}$, both types of errors occur in equilibrium. As the block size increases, the policymaker eventually intervenes with very low probability, and the main source of inefficiency is type-I errors: the policymaker forgoes desirable interventions too often. The probability of type-I errors increases substantially, while the probability of type-II errors vanishes.

In sum, are blockholders good or bad for market informativeness? In our model with government intervention, larger block sizes are related to lower informativeness. In practice, there are many reasons why having large blockholders may be beneficial in general. Companies with some large shareholders tend to have more informative prices (Brockman and Yan, 2009; Boehmer and Kelley, 2009; Gallagher, Gardner and Swan, 2013; Gorton, Huang and Kang, 2016), possibly due to their larger incentives to acquire information (absent in our model). Large shareholders also exert an important role in corporate governance (for an extensive review, see Edmans and Holderness, 2017). However, our focus is not on average informativeness but on informativeness *around government interventions*. We suggest that the strategic behavior of large informed blockholders can lower market informativeness around interventions and, hence, the efficiency of policy implementation. Our model also has testable implications on how the concentration of ownership, proxied by block size, affects the probability of a government intervention.

3.3 Risk

Cash flow risk varies in the cross section of firms. The next proposition examines the effect of risk, which in our model is captured by Δ_R, the distance between returns in the good and bad states.

Proposition 5. ***Risk.*** *Market informativeness and real efficiency are increasing in risk Δ_R.*

The intuition for these results is as follows. The larger the risk Δ_R, the more valuable is the information of the trader, and the larger the profits of trading on information. This raises incentives for the positively informed trader to buy the security and raises the cost of selling for a bailout. Therefore, market activity better reflects the fundamental and the policymaker learns more from it. Overall, the implications of higher risk on real efficiency are analogous to the effects of a lower block size, which we discussed in Section 3.2.

Would the private choice of risk also maximize real efficiency? To address this question, we consider a slightly modified version of our model, in which the firm chooses between a large number of projects with varying risk at the beginning of $t = 0$ to maximize firm value, $\mathbb{E}[p(X)]$. Each project has the same expected return $\overline{R} > 0$ but different levels of risk $\zeta > 0$ (mean-preserving spreads). We parametrize the state-dependent returns as $R_L = \overline{R} - \frac{\zeta}{1-\gamma}$ and $R_H = \overline{R} + \frac{\zeta}{\gamma}$. The firm's project choice is publicly observed.[10]

Proposition 6. *Consider the model with multiple projects of varying risk and $\beta_H > -(1-\gamma)\Delta_\alpha$.*[11] *The level of risk that maximizes firm value is below the level of risk that maximizes real efficiency.*

Proposition 6 states that the risk preferences of the firm and the policymaker differ. The policymaker always prefers a large enough risk level such that the Buy equilibrium occurs and informativeness reaches its maximum (see Proposition 3). However, expected firm value is larger for a smaller ζ such that the Buy equilibrium is not played and interventions are more likely. Interestingly, we uncover a force whereby asset risk is inefficiently low.

[10]To guarantee that $R_L > 0$ and $\Delta_R + \Delta_\alpha > 0$, we assume $\zeta < (1-\gamma)\overline{R} \equiv \overline{\zeta}$ and $\zeta > -\gamma(1-\gamma)\Delta_\alpha \equiv \underline{\zeta}$ for all available projects ζ. To avoid unnecessary technical complications, we also assume that there is a large but finite number of projects evenly spread in the interval $(\underline{\zeta}, \overline{\zeta})$.

[11]This condition ensures the interesting case where the equilibrium is not the Buy equilibrium for any risk choice.

For instance, consider a pessimistic prior, $\gamma < \overline{\gamma}$. Starting at a large ζ (Buy equilibrium), a large reduction in ζ takes the economy to the Sell equilibrium, which ensures an intervention that raises the expected firm value. The driver of the firm's choice of risk is to affect the informed trader's incentives. A lower level of risk makes the informed trader (blockholder) care less about trading profits and more about the value of the firm. Therefore, low risk incentivizes *trading for bailouts*, which is effective in inducing an intervention for low γ and, hence, increases firm value.

For an optimistic prior, $\gamma > \overline{\gamma}$, trading for bailouts is effective in increasing the intervention probability for moderate risk but ineffective for low risk. Since low risk incentivizes trading bailout behavior, the policymaker reduces its reliance on learning from market activity, which eventually reduces the intervention probability for an optimistic prior γ. Thus, the firm prefers an intermediate level of risk, which is still below what the policymaker likes (high risk and Buy equilibrium).

3.4 Stock prices

Finally, we study how stock prices react to different aggregate orders. Since the market maker sets prices upon observing the aggregate order X to reflect the expected firm value, equilibrium prices are

$$p(X) = R_L + q(X)\Delta_R + g(X)\left[\alpha_L + q(X)\Delta_\alpha\right]. \tag{8}$$

For large Δ_R relative to the benefit of the intervention, the price is increasing in the order flow. Proposition 7 states the price behavior for small Δ_R and Figure 6 illustrates.

Proposition 7. *Stock price.* *For Δ_R low enough, the stock price can be non-monotonic in the order flow. On the equilibrium path, (i) $p(X)$ is flat or has an inverted-U shape for a pessimistic prior, $\gamma < \overline{\gamma}$; and (ii) $p(X)$ has a U-shape for an optimistic prior, $\gamma > \overline{\gamma}$.*

The intuition is as follows: Although the positively informed trader sometimes chooses not to buy (or to sell), the belief $q(X)$ increases in the aggregate order X because the negatively informed trader always sells. This result has two effects. On one hand, a higher X pushes prices up since returns are higher in the good state (and the more so the larger Δ_R). On the other hand, it pushes

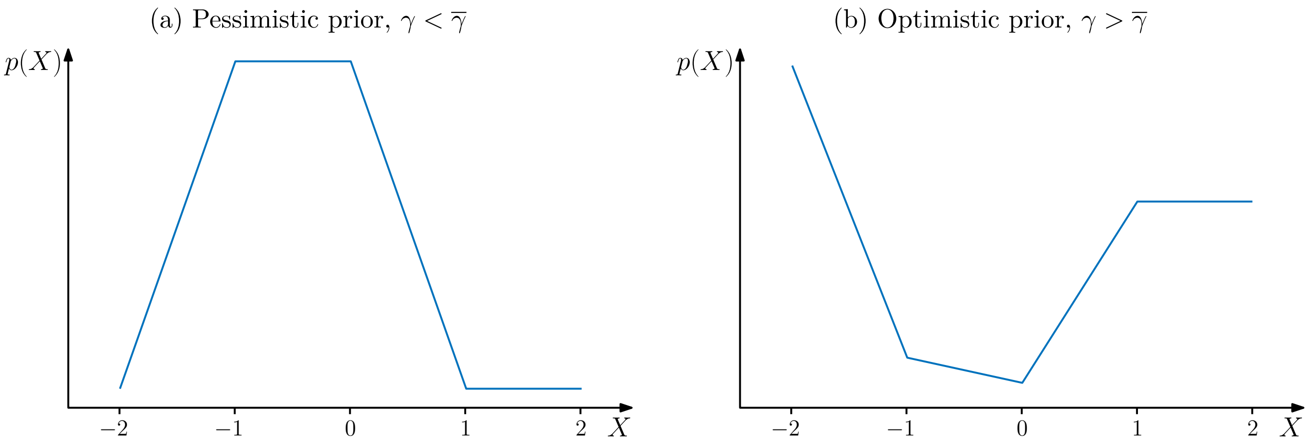

Figure 6: Stock prices and aggregate order flow: non-monotonicities can arise for small Δ_R. The figure depicts two numerical examples, with an I and an SB equilibrium on the left and right panel, respectively.

prices down since the policymaker is more likely not to intervene. For Δ_R sufficiently large, the first effect dominates and prices are increasing in X. For a low Δ_R, by contrast, the feedback effect makes the price non-monotonic in the aggregate order.

Interestingly, the form of this non-monotonicity is different for high or low priors. A pessimistic policymaker, $\gamma < \overline{\gamma}$, does not learn from market activity and intervenes for $X = -1, 0$. In the Inaction equilibrium, stock prices are larger for $X = -1, 0$ than for $X = -2$ because the intervention is undertaken in all cases, $q < \overline{\gamma}$, but the expected firm value at $X = -2$ is lower (see Figure 6). On the other hand, when market activity indicates a good state, $X = 1, 2$, stock prices fall because the intervention is no longer expected, $q = 1$, and the differential return in the good state does not compensate the loss from the policymaker not intervening (because of small Δ_R).

Instead, an optimistic policymaker, $\gamma > \overline{\gamma}$, does not intervene upon observing no market activity, $X = 0$. Hence, prices fall as the order flow moves from $X < 0$ to $X = 0$. This is because the probability of an intervention goes to zero, and even though the market maker assigns a higher probability to the good state, the differential returns are small (small Δ_R). Next, as the order flow moves from $X = 0$ to $X > 0$, prices can only go up as beliefs are more optimistic.

For comparison, the price increases in the order flow without feedback (i.e., without a policy intervention or if the policymaker did not learn from market activity). With feedback, however, we show that (i) the price can be non-monotonic in the order flow and (ii) its shape is governed by the prior about the state. A related non-monotonicity of the price arises in Bond, Goldstein and Prescott (2010), who study market-based corrective action when the decision maker learns from a competitive market price and the economic state is continuous. They show that a corrective action results in a non-monotonic price in the fundamental, because a small deterioration triggers an intervention and, therefore, a discontinuous upward jump in the market price.

4 An application to liquidity support

In this section, we consider a simple model of liquidity support to a distressed bank. We show that this setup is isomorphic to the main model but with the payoffs of the policymaker and the large informed trader (e.g., a blockholder) linked to the market conditions of the bank.

At the beginning of $t = 0$, a bank has an exogenous amount D of short-term debt not rolled over by bank creditors. The bank's assets are worth V_θ at $t = 1$ but are not perfectly liquid. When liquidated prematurely, those assets are worth only $(1 - \psi_\theta) V_\theta$, where $\psi_\theta \in (0, 1)$ is asset illiquidity (e.g., a fire-sale penalty) in state $\theta \in \{L, H\}$. We assume that assets are more valuable and are more liquid in the good state, $V_H > V_L$ and $\psi_H < \psi_L$, respectively. (Later we study the cases in which there is uncertainty only about asset value or only about asset liquidity.)

In the absence of any government assistance (explained below), the bank must sell a fraction $\tilde{y}(\theta) = \frac{D}{(1-\psi_\theta)V_\theta}$ of its assets to meet creditor withdrawals. We assume $D \leq (1 - \psi_\theta) V_\theta$ for $\theta = L, H$ in order to abstract from the possibility of bank insolvency. Without assistance, the shareholder return is $\tilde{\pi}(\theta) = [1 - \tilde{y}(\theta)] V_\theta = V_\theta - D/(1 - \psi_\theta)$. A policymaker may want to offer liquidity assistance to reduce the deadweight loss caused by the fire sale. The policymaker may purchase a fraction of the firm's debt and roll it over, but raising funds has a cost τ per dollar (due to taxation distortions, for instance). When the government buys (and rolls over) a dollar amount $A \leq D$ of firm debt, the bank only needs to liquidate a reduced fraction $y(\theta, A) = \frac{D-A}{(1-\psi_\theta)V_\theta}$ of assets. The total return for shareholders is thus $\pi(\theta, A) = V_\theta - A - \frac{D-A}{1-\psi_\theta}$.

After observing financial market activity, the (benevolent) policymaker forms the belief $q(X)$

and chooses the size of assistance A in order to maximize total expected wealth in the economy:

$$\begin{aligned} W &= \mathbb{E}\left[(1-y(\theta,A))\,V_\theta + y(\theta,A)\,(1-\psi_\theta)\,V_\theta|X\right] - \tau A \\ &= \left[q(X)\kappa_H + (1-q(X))\,\kappa_L - \tau\right]A + \Omega, \end{aligned} \tag{9}$$

where $\kappa_\theta \equiv \frac{\psi_\theta}{1-\psi_\theta}$ and $\Omega \equiv \mathbb{E}\left[V_\theta - \kappa_\theta D|X\right]$.[12] Henceforth, we refer to κ_θ as the liquidation cost, instead of ψ_θ. Welfare is the ex-ante expectation $\mathbb{E}[W]$, formed using the prior belief γ.

If τ is large enough, raising funds is too costly and the government does not intervene, regardless of its beliefs q. In contrast, if τ is low enough, the policymaker purchases all debt D regardless of its beliefs. In either case, traders trivially trade on their information in equilibrium (buying following good news and selling following bad news). Unless stated otherwise, we focus on the interesting case of $\kappa_H < \tau < \kappa_L$, in which the policymaker benefits from learning from market activity. In this case, the policymaker implements a full bailout $A = D$ if q is low enough, and does not assist otherwise. We can thus map those strategies into a binary intervention $G \in \{0,1\}$.[13] Specifically, the policymaker is willing to intervene (buying all the debt) whenever $q \leq \frac{\kappa_L-\tau}{\kappa_L-\kappa_H}$.

For the purpose of computing the equilibrium, the application is isomorphic to the main model, which can be seen by defining $R_\theta = V_\theta - (1+\kappa_\theta)D$, $\alpha_\theta = D\kappa_\theta$, $b_\theta = D\kappa_\theta$, $c = D\tau$, and $\overline{\gamma} = (\kappa_L - \tau)/\tilde{\Delta}_\kappa$, where $\tilde{\Delta}_\kappa \equiv \kappa_L - \kappa_H > 0$.

4.1 Informativeness and welfare

We turn now to studying market informativeness and welfare in the model of liquidity assistance. It is important to emphasize that we cannot directly apply the results in Proposition 3 for two reasons: (i) some parameters affect the (now endogenous) costs and benefits of the intervention b and c; and (ii) some parameters have a mechanical effect on welfare through Ω. As in the main model, the equilibrium preferred by the policymaker is selected when multiple equilibria exist.

[12] We can ignore Ω in the optimization as it does not depend on A (although it matters for comparative statics).

[13] When the policymaker is indifferent between any level of intervention, we assume that it chooses $A = D$ as a tie-break rule. This is without loss of generality because we allow for mixed strategies: choosing some $A \in (0, D)$ is analogous to choosing $A = D$ with some interior probability.

Proposition 8. ***Welfare.*** *Consider $\tau \in (\kappa_H, \kappa_L)$. A higher intervention cost (τ) increases market informativeness and can increase welfare. A higher liquidation cost in the good state (κ_H) decreases informativeness and welfare, while a higher liquidation cost in the bad state (κ_L) has an ambiguous effect on both informativeness and welfare.*

A higher intervention cost τ improves market informativeness. This effect operates through the policymaker's ex-ante willingness to intervene: higher values of τ mean that the posterior probability the policymaker must assign to the bad state for it to intervene is larger ($\overline{\gamma}$ decreases in τ). If the intervention cost is large, the policymaker is more reluctant to intervene, which facilitates learning for two reasons: (i) the positively informed trader may give up trying to convince the policymaker to intervene; (ii) if the trader is still willing to do so, for her to have any chance in affecting the policymaker's decision, she must buy the stock with larger probability so that the change in beliefs after observing low aggregate orders is more substantial.

Although the effect of τ on informativeness is positive, its effect on welfare is ambiguous. The direct effect of a higher intervention cost is to destroy value when the bailout takes place, reducing welfare. However, the indirect effect of higher τ—the gains due to larger informativeness—can overcome this direct effect and, perhaps surprisingly, welfare can be higher overall.

The explanation for the negative effect of the liquidation cost in the good state on market informativeness is threefold, paralleling the effects of κ_H on Δ_α, β_H, and $\overline{\gamma}$ in the main model. First, the trading profits of the positively informed trader are partly eroded by the bailout: if no liquidation cost is expected to be incurred, possessing information about the size of this liquidation cost is useless. To be precise, if the aggregate order is such that a bailout does not happen, the trading profit of buying the stock when $\theta = H$ is proportional to $\Delta_V + D\tilde{\Delta}_\kappa$, while if a bailout takes place, it is proportional to Δ_V only. Hence, the intervention implies an "informational tax" $D\tilde{\Delta}_\kappa$ on the trader that decreases in κ_H. Thus, the larger κ_H, the higher the incentives to trade for bailouts. Second, the effect of the intervention on the value of the block currently held by the trader increases in κ_H, since $\beta_H = \mu D \kappa_H$. That is, the larger the liquidation costs, the larger the benefits of avoiding them. Third, an increase in κ_H makes the government ex ante more prone to intervening (it increases $\overline{\gamma}$), having an effect analogous to a reduction in τ discussed above. Turning to welfare, a larger κ_H reduces welfare both due to its effect on informativeness and through the mechanical effect of increasing liquidation costs in the absence of intervention (since it reduces Ω).

The effect of the liquidation costs in the bad state on informativeness and welfare is ambiguous. On the one hand, a larger κ_L increases the informational tax of the intervention ($D\tilde{\Delta}_\kappa$) previously discussed, which reduces incentives to trade for bailouts. This pushes in the direction of higher informativeness. On the other hand, a larger κ_L hinders learning because it makes the policymaker ex ante more prone to intervening (through an increase in $\overline{\gamma}$). There is also the negative mechanical effect of higher κ_L on welfare through Ω, and the overall effect depends on parameters.

Table 2 presents additional comparative statics and summarizes those already discussed.

	Market informativeness	Welfare
Block size (μ)	$(-)$	$(-)$
Asset return in good state (V_H)	$(+)$	$(+)$
Asset return in bad state (V_L)	$(-)$	$(+/-)$
Liquidity shortage (D)	$(-)$	$(-)$
Liquidation cost in good state (κ_H)	$(-)$	$(-)$
Liquidation cost in bad state (κ_L)	$(+/-)$	$(+/-)$
Intervention cost (τ)	$(+)$	$(+/-)$

Table 2: Comparative statics in the model with liquidity support. See Appendix B.7 for a proof.

4.2 Uncertainty about asset value versus asset liquidity

Next, we compare two special cases: when the policymaker is (i) only uncertain about the value of bank assets; and (ii) only uncertain about the liquidity of bank assets. The next proposition considers the first case.

Proposition 9. ***Uncertain asset value.*** *If $\tilde{\Delta}_\kappa \to 0$, the (essentially) unique equilibrium is the Buy equilibrium.*[14] *The policymaker always intervenes if $\tau < \kappa_L$ and does not intervene if $\tau > \kappa_L$.*

[14] Up to the non-generic case of $\kappa_L = \tau$, the equilibrium is unique.

The only source of inefficiency in this economy is the early liquidation of assets. When the liquidation cost is virtually the same across states, market participants have no additional information about it, so the policymaker has no reason to learn from market activity (there is no feedback effect). The informational advantage traders possess is about asset value and is not eroded by an intervention. Hence, the incentives to trade on information are unaffected by the possibility of a bailout: negatively informed traders sell the security, and positively informed traders buy it. In sum, market activity is highly informative precisely because the policymaker does not rely on it for its intervention decision.

We now turn to the second case of uncertainty about asset liquidity.

Proposition 10. ***Uncertain asset liquidity.*** *Consider $\tau \in (\kappa_H, \kappa_L)$ and $\Delta_V \to 0$. There is an (essentially) unique equilibrium.*[15] *If $\gamma < \overline{\gamma}$, the Sell equilibrium arises for any block size $\mu > 0$.*

Figure 7 shows the entire equilibrium set when $\Delta_V \to 0$. For a pessimistic prior, $\gamma < \overline{\gamma}$, the policymaker is more prone to intervening ex ante and the positively informed trader always sells. Critically, the equilibrium outcome is drastically different from the benchmark case of $\beta_H = 0$ even if the block size is arbitrarily small. The Buy equilibrium ceases to exist for any positive μ (equivalently, β_H), with important implications for market informativeness and welfare.[16]

To understand this result, note that $\Delta_R = \Delta_V + D\tilde{\Delta}_\kappa$: without any intervention, the differential return in the good state comes from the higher asset value and higher liquidity. Also, note that $\Delta_\alpha = -D\tilde{\Delta}_\kappa < 0$: an intervention causes a relatively smaller benefit in the good state since lower liquidation costs are avoided. Hence, conditional on an intervention, the difference in the returns across states is $\Delta_R + \Delta_\alpha = \Delta_V$: if the bank is bailed out, no fire-sale costs are incurred in either state and the entire differential return is Δ_V. Therefore, if Δ_V goes to zero, the intervention completely eliminates any potential trading profit for the informed trader. Yet, for $\gamma < \overline{\gamma}$, the positively informed trader trades aggressively against her information to induce the intervention.

[15] We say the equilibrium is essentially unique for two reasons. First, multiplicity happens along the boundaries between equilibria $\mu = (1-\gamma)\tilde{\Delta}_\kappa/\kappa_H$ and $\mu = (1-\overline{\gamma})\tilde{\Delta}_\kappa/\kappa_H$. Second, in the SB equilibrium, all strategies are uniquely determined, except for $g(-1)$ and $g(-2)$. But in that case, the total probability of an intervention conditional on a sell order, given by $[g(-1)+g(-2)]/3$, is uniquely determined (and so are welfare and trader's payoffs).

[16] If $\gamma > \overline{\gamma}$, the equilibrium is the B equilibrium for $\mu < (1-\gamma)\tilde{\Delta}_\kappa/\kappa_H$, the IB equilibrium for $(1-\gamma)\tilde{\Delta}_\kappa/\kappa_H < \mu < (1-\overline{\gamma})\tilde{\Delta}_\kappa/\kappa_H$, and the SB equilibrium for $\mu > (1-\overline{\gamma})\tilde{\Delta}_\kappa/\kappa_H$.

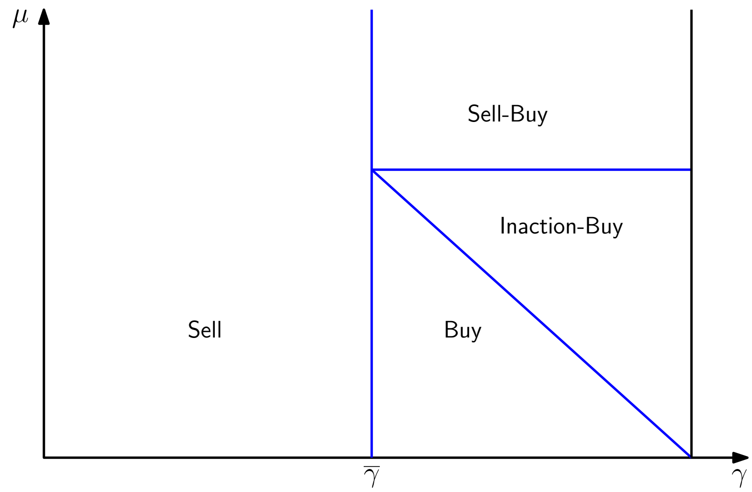

Figure 7: Equilibrium in the case of uncertainty about asset liquidity only, $\Delta_V \to 0$.

The conundrum is solved by noting that for the positively informed trader, when $\gamma < \overline{\gamma}$, avoiding the intervention implies perfectly revealing the good state. This revelation also eliminates any potential profits from her informational advantage. Hence, the trader prefers to collect the private benefit of the intervention, even if arbitrarily small. This is not costly for her precisely because the intervention eliminates not only trading profits, but also trading losses when the positively informed trader *sells* the stock.

4.3 Commitment to minimum liquidity support

We now consider the possibility of the policymaker committing to a minimum assistance package $\underline{A} > 0$ *before* observing market activity. The policymaker still reacts to market activity and chooses, ex post, whether to complement its assistance (providing additional funds). Such policy can be easily implemented by giving an unconditional credit line to banks (such as the Federal Reserve discount window), or by implementing bailouts gradually (providing a smaller assistance in a first step). The following proposition shows that such implementation can be beneficial.[17]

Proposition 11. ***Design of liquidity support.*** *Commitment to a minimum liquidity support $\underline{A} > 0$ can increase market informativeness and welfare despite the potential cost of the policy.*

[17] It is often argued that committing *not to provide* too much assistance to financial institutions could be beneficial (for instance, if there are moral hazard concerns). The main issue is that if the policymaker believes a bailout is socially desirable ex post, it has incentives to deviate and increase assistance. In our setting, limiting the size of maximum assistance can also improve welfare, but we focus on a minimum assistance that is easier to implement.

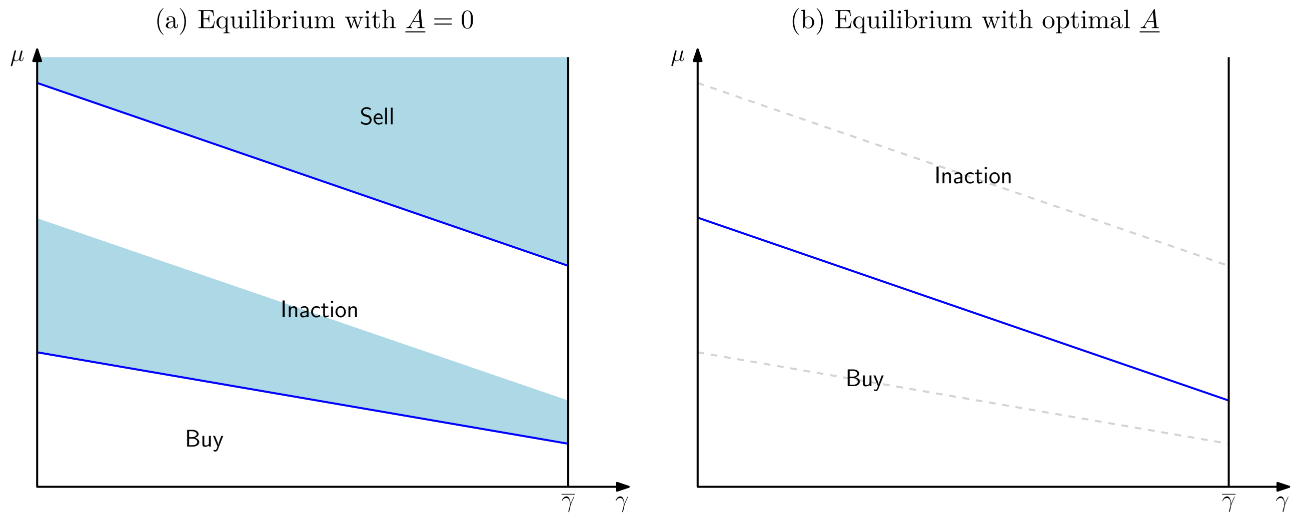

Figure 8: Incentives to commit to a minimum level of assistance: On the left, the equilibrium set with $\underline{A} = 0$ for $\gamma < \overline{\gamma}$; the shaded areas show the regions where commitment is welfare-improving. On the right, the equilibrium set under the optimal minimal assistance, $\underline{A}^*$. See Appendix B.11 for details.

If the information the government could obtain from the market were exogenous to the policy, there would be no gains from making a policy decision earlier, with less information. However, the result is different when the informational content of market activity is *endogenous* to the policy. Despite the first stage of the policy ($\underline{A}$) being undertaken with little information, this early decision boosts the informativeness of market activity on which the policymaker can rely. As a result, the uncertainty regarding the desirability of additional liquidity support is reduced.

The intuition is that committing to providing a minimum assistance reduces the residual benefit of an (ex-post) additional intervention. That is, offering $\underline{A} > 0$ *ex ante* reduces the stakes of the trader on the policy decision to provide additional assistance *ex post*. Therefore, incentives to trade for bailouts are reduced, allowing the policymaker to learn more from market activity and to implement additional assistance (beyond $\underline{A}$) more efficiently.

However, committing to $\underline{A} > 0$ can be costly. For instance, even if market activity perfectly reveals the good state, the government has to incur the cost of the minimum support ($\tau\underline{A}$), which in this case is smaller than the social benefit ($\kappa_H\underline{A}$). Still, committing to a minimal assistance is often welfare-improving, with gains in market informativeness more than compensating for the additional implementation cost.

Figure 8 illustrates this result for a low prior, $\gamma < \overline{\gamma}$. If a Sell equilibrium is played under no commitment, the policymaker always benefits from committing to the lowest $\underline{A}$ that triggers the Inaction equilibrium instead. The reason is that commitment in the Sell equilibrium is cheap. In its absence, the policymaker cannot learn from the market and would always give a full bailout D, so committing to assist with at least $\underline{A} < D$ means committing to something it would do with probability one if there was no commitment. Committing to a large enough minimum assistance $\underline{A}$ actually *reduces* the expected bailout size. By triggering the Inaction equilibrium, the policymaker ensures higher market informativeness and no longer implements the full bailout with certainty.

In contrast, if for $\underline{A} = 0$ the Inaction equilibrium is played, commitment is not always beneficial. The reason is that commitment is costly in this case. In the Inaction equilibrium, market activity reveals the good state with some probability, in which case the policymaker (correctly) refrains from providing any assistance. Committing to some $\underline{A} > 0$ then implies additional costs. As a result, the policymaker only commits if the amount $\underline{A}$ needed to trigger the Buy equilibrium is low enough (shaded area in Figure 8), in which case the gain in informativeness achieved with a switch to the Buy equilibrium more than compensates the additional implementation costs.

As a result, under the optimal minimum assistance $\underline{A}^*$, the equilibrium set features an enlarged Buy equilibrium region, and the least informative equilibrium (Sell equilibrium) disappears altogether (see right panel of Figure 8).

5 Conclusion

We study the extent to which policymakers can learn from market activity when large informed traders have high stakes in the policy outcome. Such high stakes naturally arise when the trader is a blockholder or a creditor of the firm targeted by the government intervention. Market informativeness and the efficiency of bailouts are particularly compromised when the firm's cash flow risk is low or the private benefits of the intervention are large (such as for a large block size). We characterize conditions under which *trading for bailouts* behavior effectively alters policy outcomes.

In the context of liquidity support to distressed banks, the source of uncertainty facing the policymaker is key to determining market informativeness. Even an arbitrarily small private benefit can change equilibrium outcomes substantially relative to a benchmark without private benefits. Moreover, a larger cost of implementing bailouts boosts informativeness and can increase welfare. We also discuss implications for bailout design, offering a rationale for the gradual implementation of liquidity support.

References

Admati, A.R. and Pfleiderer, P. (2009): The "Wall Street walk" and shareholder activism: Exit as a form of voice. *Review of Financial Studies*, 22, 7, 2645–2685.

Bakke, T.E. and Whited, T.M. (2010): Which firms follow the market? An analysis of corporate investment decisions. *Review of Financial Studies*, 23, 5, 1941–1980.

Boehmer, E. and Kelley, E.K. (2009): Institutional investors and the informational efficiency of prices. *Review of Financial Studies*, 22, 9, 3563–3594.

Boleslavsky, R., Kelly, D. and Taylor, C. (2017): Selloffs, bailouts, and feedback: Can asset markets inform policy? *Journal of Economic Theory*, 169, 294–343.

Bond, P., Edmans, A. and Goldstein, I. (2012): The real effects of financial markets. *Annual Review of Financial Economics*, 4, 1, 339–360.

Bond, P. and Goldstein, I. (2015): Government intervention and information aggregation by prices. *Journal of Finance*, 70, 6, 2777–2812.

Bond, P., Goldstein, I. and Prescott, E.S. (2010): Market-based corrective actions. *Review of Financial Studies*, 23, 2, 781–820.

Brockman, P. and Yan, X.S. (2009): Block ownership and firm-specific information. *Journal of Banking & Finance*, 33, 2, 308–316.

Chen, Q., Goldstein, I. and Jiang, W. (2006): Price informativeness and investment sensitivity to stock price. *Review of Financial Studies*, 20, 3, 619–650.

Cho, I.K. and Kreps, D.M. (1987): Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102, 2, 179–221.

Dow, J. and Gorton, G. (1997): Stock market efficiency and economic efficiency: Is there a connection? *Journal of Finance*, 52, 3, 1087–1129.

Edmans, A. (2009): Blockholder trading, market efficiency, and managerial myopia. *Journal of Finance*, 64, 6, 2481–2513.

Edmans, A., Goldstein, I. and Jiang, W. (2012): The real effects of financial markets: The impact of prices on takeovers. *Journal of Finance*, 67, 3, 933–971.

Edmans, A., Goldstein, I. and Jiang, W. (2015): Feedback effects, asymmetric trading, and the limits to arbitrage. *American Economic Review*, 105, 12, 3766–97.

Edmans, A. and Holderness, C.G. (2017): Blockholders: A survey of theory and evidence. In *Handbook of the Economics of Corporate Governance*, vol. 1, 541–636. Elsevier.

Edmans, A. and Manso, G. (2010): Governance through trading and intervention: A theory of multiple blockholders. *Review of Financial Studies*, 24, 7, 2395–2428.

Gallagher, D.R., Gardner, P.A. and Swan, P.L. (2013): Governance through trading: Institutional swing trades and subsequent firm performance. *Journal of Financial and Quantitative Analysis*, 48, 2, 427–458.

Goldstein, I. and Guembel, A. (2008): Manipulation and the allocational role of prices. *The Review of Economic Studies*, 75, 1, 133–164.

Gorton, G.B., Huang, L. and Kang, Q. (2016): The limitations of stock market efficiency: Price informativeness and CEO turnover. *Review of Finance*, 21, 1, 153–200.

Hayek, F.A. (1945): The use of knowledge in society. *American Economic Review*, 35, 4, 519–30.

Holderness, C.G. (2009): The myth of diffuse ownership in the United States. *Review of Financial Studies*, 22, 4, 1377–1408.

Khanna, N. and Mathews, R.D. (2012): Doing battle with short sellers: The conflicted role of blockholders in bear raids. *Journal of Financial Economics*, 106, 2, 229–246.

Kyle, A. (1985): Continuous auctions and insider trading. *Econometrica*, 53, 6, 1315–36.

Lin, T.C., Liu, Q. and Sun, B. (2019): Contractual managerial incentives with stock price feedback. *American Economic Review*, 109, 7, 2446–68.

Luo, Y. (2005): Do insiders learn from outsiders? Evidence from mergers and acquisitions. *Journal of Finance*, 60, 4, 1951–1982.

A Equilibrium characterization

Before characterizing the equilibrium, we introduce some notation and establish some basic results. In the appendix, we often refer to the positively (negatively) informed trader as the high (low) type. Using (8), the payoff of the high type can be written as $\pi^H(s) = \Pi_T^H(s) + \Pi_G^H(s)$, where $\Pi_T^H(s) = \mathbb{E}_s\left[s\left(1-q(X)\right)\left(\Delta_R + g(X)\Delta_\alpha\right)\right]$ is the expected trading profit and $\Pi_G^H(s) = \mathbb{E}_s\left[g(X)\beta_H\right]$ is the expected benefit of the intervention. Similarly, the payoff of the low type can be written as $\pi^L(s) = \Pi_T^L(s) + \Pi_G^L(s)$, where $\Pi_T^L(s) = \mathbb{E}_s\left[-sq(X)\left(\Delta_R + g(X)\Delta_\alpha\right)\right]$ and $\Pi_G^L(s) = \mathbb{E}_s\left[g(X)\beta_L\right]$. Note that $\Pi_G^H(s) = \frac{\beta_H}{\beta_L}\Pi_G^L(s)$. Also note that the market maker never learns the true state when observing $X = 0$, regardless of the trader's strategies. Therefore, we assume $q(0) \in (0,1)$ hereafter. This implies that $\Pi_T^L(-1) > 0$, $\Pi_T^L(1) < 0$, $\Pi_T^H(-1) < 0$, and $\Pi_T^L(1) > 0$ in any possible equilibrium: in expectation, the high (low) type always makes trading profit when she buys (sells) and a trading loss when she sells (buys).

A.1 Equilibrium strategies for the low type

In this section we prove Lemma 1. We begin by showing that there can be no equilibrium with $l(-1) < 1$ and $h(-1) > 0$. First, suppose $l(-1) < 1$ and $l(0) > 0$. We must then have $\Pi_G^L(0) \geq \Pi_T^L(-1) + \Pi_G^L(-1)$ and therefore $\Pi_G^L(0) > \Pi_G^L(-1)$. But then $\pi^H(0) = \frac{\beta_H}{\beta_L}\Pi_G^L(0) > \Pi_T^H(-1) + \frac{\beta_H}{\beta_L}\Pi_G^L(-1) = \pi^H(-1)$. Hence, $h(-1) = 0$. Second, suppose $l(-1) < 1$ and $l(1) > 0$. We must then have $\Pi_T^L(1) + \Pi_G^L(1) \geq \Pi_T^L(-1) + \Pi_G^L(-1)$, which implies $\Pi_G^L(1) > \Pi_G^L(-1)$. But then $\pi^H(1) = \Pi_T^H(1) + \frac{\beta_H}{\beta_L}\Pi_G^L(1) > \Pi_T^H(-1) + \frac{\beta_H}{\beta_L}\Pi_G^L(-1) = \pi^H(-1)$. Hence, $h(-1) = 0$. To summarize, we have shown that if there exists an equilibrium with $l(-1) < 1$ we must have $h(-1) = 0$. In what follows, we look for an equilibrium with $h(-1) = 0$ and $l(-1) < 1$ and show that it cannot be constructed. We divide the remainder of this proof into three cases.

Case 1: $\boldsymbol{l(-1) \in (0,1)}$. Suppose $l(-1) \in (0,1)$. Since $h(-1) = 0$, we have $g(-2) = 1$. First, assume $l(0) > 0$. Since $\pi^L(-1) - \pi^L(0) = \Pi_T^L(-1) + \frac{1}{3}\beta_L\left[g(-2) - g(1)\right] > 0$, there can be no equilibrium with $l(0) > 0$ and $l(-1) \in (0,1)$. Second, assume $l(0) = 0$. Then $l(1) > 0$ and $g(2) + g(1) > g(-2) + g(-1)$ (otherwise the low type would not play $s = 1$ with positive probability). Since $g(-2) = 1$, $g(2) - g(-1) > 1 - g(1) \geq 0$, so

$$\pi^H(1) - \pi^H(0) = \Pi_T^H(1) + \frac{1}{3}\beta_H\left[g(2) - g\left(-1\right)\right] > 0. \tag{A.1}$$

Hence, the high type plays $h(1) = 1$. But this implies that $g(-1) = 1$, which together with $g(-2) = 1$ shows that $g(2) + g(1) > g(-2) + g(-1)$ cannot be satisfied, a contradiction.

Case 2: $\boldsymbol{l(-1) = 0}$ and $\boldsymbol{l(1) > 0}$. Suppose $l(-1) = 0$ and $l(1) > 0$. We must have $\pi^L(1) - \pi^L(0) = \Pi_T^L(1) + \frac{1}{3}\beta_L\left[g(2) - g\left(-1\right)\right] \geq 0$, and since $\Pi_T^L(1) < 0$, $g(2) > g\left(-1\right)$. But then $h(1) = 1$ (see (A.1)) and $g(-1) = 1$, which implies that $g(2) > g(-1) = 1$ is violated, a contradiction.

Case 3: $\boldsymbol{l(0) = 1}$. Suppose $l(0) = 1$. We must then have $\pi^L(0) - \pi^L(-1) = -\Pi_T^L(-1) + \frac{1}{3}\beta_L\left[g(1) - g(-2)\right] \geq 0$, and since $\Pi_T^L(-1) > 0$, $g(1) > g(-2)$. Since $h(-1) = 0$, $q(1) = \gamma$. But if $\gamma > \overline{\gamma}$, we should have $g(1) = 0$, and hence $g(1) > g(-2)$ cannot be satisfied. Thus, it cannot be an equilibrium when $\gamma > \overline{\gamma}$. Hence, in the remainder of this proof we assume $\gamma < \overline{\gamma}$.

First, suppose $h(1) = 1$. Then, $g(-1) = g(0) = g(1) = 1$ and $g(2) = 0$. Moreover, $q(-1) = 1$, $q(2) = 0$ and $q(0) = q(1) = \gamma$. For the high type not to deviate to $s = 0$ we must have $\pi^H(1) - \pi^H(0) = \Pi_T^H(1) + \frac{1}{3}\beta_H [g(2) - g(-1)] \geq 0$, which using $g(2) = 0$ and $g(-1) = 1$ becomes $\beta_H \leq 3\Pi_T^H(1)$. Note that $\Pi_T^H(1) = \frac{2}{3}(1-\gamma)(\Delta_R + \Delta_\alpha)$, and therefore the high type will not deviate to zero if

$$\beta_H \leq 2(1-\gamma)(\Delta_R + \Delta_\alpha). \tag{A.2}$$

Hence, when the condition above is violated it cannot be an equilibrium. Suppose now (A.2) is satisfied. In that case, the Intuitive Criterion implies $q(-2) = 0$. To see that, note that a strict upper bound on the high type gain from deviating from $s = 1$ to $s = -1$ is $\overline{\Delta}_{DEV} = \beta_H - \pi^H(1) = \beta_H - \frac{2}{3}[(1-\gamma)(\Delta_R + \Delta_\alpha) + \beta_H]$ (the gain if an intervention happens for sure and she does not incur any trading loss). The actual payoff of deviating (under the best case scenario) is strictly smaller than $\overline{\Delta}_{DEV}$ since the high type incurs a trading loss when $X = 0$. One can verify that $\overline{\Delta}_{DEV} \leq 0$ when $\beta_H \leq 2(1-\gamma)(\Delta_R + \Delta_\alpha)$, and therefore the deviation is strictly dominated by the equilibrium strategy for the high type. The low type clearly has incentives to deviate if she believes $g(-2) = 1$ (under that scenario she still gets the intervention with probability 1, but could make trading profits when $X = 0$). Hence, the Intuitive Criterion imposes $q(-2) = 0$, which implies $g(-2) = 1$. But then the low type deviates to -1 and it cannot be an equilibrium.

Second, suppose $h(0), h(1) > 0$. Beliefs on the equilibrium path are $q(0) = q(1) = \gamma$, $q(2) = 1$, and $q(-1) < \gamma$. Thus, $g(-1) = g(0) = g(1) = 1$ and $g(2) = 0$. Indifference between 0 and 1 for the high type implies $\pi^H(1) - \pi^H(0) = \frac{2}{3}[(1-\gamma)(\Delta_R + \Delta_\alpha) + \beta_H] - \beta_H = 0$. Thus, unless $\beta_H = 2(1-\gamma)(\Delta_R + \Delta_\alpha)$, that cannot be an equilibrium. If $\beta_H = 2(1-\gamma)(\Delta_R + \Delta_\alpha)$, the high type's payoff in such an equilibrium would be β_H. If the high type deviates to -1 she is worse off in any scenario: she gets at most β_H from the intervention but incurs a trading loss when $X = 0$. The low type could have a profitable deviation in a scenario where $g(-2) = 1$. Hence, when $\beta_H = 2(1-\gamma)(\Delta_R + \Delta_\alpha)$, the Intuitive Criterion requires $g(-2) = 1$. But then, the low type has incentives to deviate to -1 and it cannot be an equilibrium.

Third, suppose $h(0) = 1$. Then $q(-1) = q(0) = q(1) = \gamma$, which implies $g(-1) = g(0) = g(1) = 1$. The Intuitive Criterion requires $q(-2) = 0$. To see that, note the high type payoff under the presumed equilibrium is β_H. If she deviates to -1 she incurs a trading loss when $X = 0$ and, at best, still gets the intervention with probability 1, so the deviation cannot be profitable. The low type can have a profitable deviation if $g(-2) = 1$. Hence, the Intuitive Criterion imposes $q(-2) = 0$, and thus $g(-2) = 1$. But then, the low type deviates. We have then ruled out any possibility other than $l(-1) = 1$ in equilibrium. □

A.2 Equilibrium characterization for $\gamma < \overline{\gamma}$

A.2.1 Beliefs and equilibrium strategies for policymaker

The next lemma reduces the set of possible strategies for the policymaker and beliefs.

Lemma A.1. *Assume $\gamma < \overline{\gamma}$. Then, in any equilibrium: $q(-2) < \gamma$, $q(-1) < \gamma$, $q(0) = \gamma$, $q(1) = q(2) = 1$, $g(-2) = g(-1) = g(0) = 1$, and $g(1) = g(2) = 0$.*

Proof. Notice that conditional on any state and trader's strategies, $X = 0$ with probability 1/3. Hence $q(0) = \gamma$ and $g(0) = 1$ in any equilibrium, and from Lemma 1, $l(-1) = 1$. The probability that $X = x$ for $x \in \{-2, -1, 0\}$ conditional on $\theta = H$ is never higher than the probability of $X = x$ conditional on $\theta = L$. Bayes' rule then implies that the policymaker never updates the probability of $\theta = H$ upwards upon observing $X \in \{-2, -1\}$. Hence, $q(-2) < \gamma$, $q(-1) < \gamma$, and thus $g(-2) = 1$ and $g(-1) = 1$. In the remainder of this proof we assume that in any equilibrium $g(-2) = g(-1) = g(0) = 1$, $q(-2) < \gamma$, $q(-1) < \gamma$, $q(0) = \gamma$, and $l(-1) = 1$. It remains to show that $g(1) = g(2) = 0$ and $q(1) = q(2) = 1$ in any equilibrium.

First, suppose $h(-1) = 1$. In that case, $X = 1$ or $X = 2$ are off the equilibrium path. The low type has no incentives to deviate, since under her equilibrium strategy $l(-1) = 1$ the intervention happens for sure and she makes trading profits (any deviation implies she does not make a trading profit). For the high type, a deviation could be profitable, for instance, if the policymaker would intervene with probability 1 after the deviation (she would eliminate the trading loss and still get the intervention for sure). Thus, by the Intuitive Criterion, agents must believe that this deviation comes from the high type and therefore $q(1) = q(2) = 1$, implying $g(1) = g(2) = 0$. Second, suppose $h(0) > 0$. Since $l(-1) = 1$, observing $X = 1$ reveals that $\theta = H$, so $q(1) = 1$ and $g(1) = 0$. When $X = 2$, by the same reasons as in the previous paragraph, the Intuitive Criterion implies $q(2) = 1$ and therefore $g(2) = 0$. Third, suppose $h(1) > 0$. Since $l(-1) = 1$, observing $X = 1, 2$ reveals that $\theta = H$ and thus $q(1) = q(2) = 1$ and $g(1) = g(2) = 0$. $\square$

In the remainder of Section A.2 we assume that $l(-1) = 1$ (Lemma 1) and that the functions $q(\cdot)$ and $g(\cdot)$ satisfy the conditions in Lemma A.1. Notice that, in any equilibrium candidate satisfying those lemmas, the low type has no incentives to deviate from $l(-1) = 1$. Hence, to verify if a strategy profile and beliefs are an equilibrium we only need to check if the high type has incentives to deviate.

A.2.2 High type equilibrium payoffs

Since $g(1) = g(2) = 0$ and $q(1) = q(2) = 1$ in equilibrium, the high type obtains zero payoff when $X = 1, 2$. Moreover, using $q(0) = \gamma$ and $g(0) = 1$, her expected payoffs of playing $s = 1, 0, -1$ are $\pi^H(1) = \frac{1}{3}\left[(1-\gamma)(\Delta_R + \Delta_\alpha) + \beta_H\right]$, $\pi^H(0) = \frac{2}{3}\beta_H$, and $\pi^H(-1) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)\left[3 - \gamma - q(-1) - q(-2)\right] + \beta_H$. We define the following upper bound on the high type payoff of selling: $\pi^H_{UB}(-1) = -(1-\gamma)(\Delta_R + \Delta_\alpha) + \beta_H$. It is constructed assuming that when playing $s = -1$ the high type gets the intervention for sure and is faced with a market maker as optimistic as possible given the restrictions on $q(\cdot)$ imposed by Lemma A.1 (that is, assuming beliefs $q(-2) = q(-1) = q(0) = \gamma$ for the market maker).

A.2.3 Buy equilibrium (B)

Here we look for an equilibrium with $h(1) = 1$. A necessary condition to sustain such an equilibrium is

$$\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)(\Delta_R + \Delta_\alpha) - \frac{1}{3}\beta_H \geq 0, \tag{A.3}$$

which is equivalent to $\beta_H \leq (1-\gamma)(\Delta_R + \Delta_\alpha)$. It remains to check that the high type has no incentive to deviate to -1. A sufficient condition for this is $\pi^H(1) - \pi^H_{UB}(-1) = \frac{4}{3}(1-\gamma)(\Delta_R + \Delta_\alpha) - \frac{2}{3}\beta_H \geq 0$, which is automatically satisfied if (A.3) holds. Therefore, a B equilibrium exists if and only if (henceforth abbreviated as iff) $\beta_H \leq (1-\gamma)(\Delta_R + \Delta_\alpha)$.

Uniqueness of Buy equilibrium. When $\beta_H < (1-\gamma)(\Delta_R + \Delta_\alpha)$, we have $\pi^H(1) - \pi^H(0) > 0$ and $\pi^H(1) - \pi^H(-1) > 0$, and thus B is the unique equilibrium. When $\beta = (1-\gamma)(\Delta_R + \Delta_\alpha)$, $\pi^H(1) - \pi^H(-1) > 0$, but $\pi^H(1) = \pi^H(0)$. Therefore, in this case there are also IB equilibria in which $h(1) = m \in [0,1)$ and $h(0) = 1-m$. Given that we have fully characterized the equilibrium set for $\beta_H \leq (1-\gamma)(\Delta_R + \Delta_\alpha)$ and $\gamma < \overline{\gamma}$, in the remainder of Section A.2 we assume $\beta_H > (1-\gamma)(\Delta_R + \Delta_\alpha)$.

A.2.4 Sell equilibrium (S)

Since $\beta_H > (1-\gamma)(\Delta_R + \Delta_\alpha)$, we have that $\pi^H(0) > \pi^H(1)$ (see equation (A.3)). Hence, an S equilibrium exists iff $\pi^H(-1) - \pi^H(0) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)[3 - \gamma - q(-1) - q(-2)] + \frac{1}{3}\beta_H \geq 0$, which in an S equilibrium is equivalent to $\beta_H \geq 3(1-\gamma)(\Delta_R + \Delta_\alpha)$ since $q(-2) = q(-1) = \gamma$. Therefore, an S equilibrium exists iff $\beta_H \geq 3(1-\gamma)(\Delta_R + \Delta_\alpha)$.

A.2.5 Inaction equilibrium (I)

Since $\beta_H > (1-\gamma)(\Delta_R + \Delta_\alpha)$, we have $\pi^H(0) > \pi^H(1)$. Hence, an I equilibrium exists iff $\pi^H(0) - \pi^H(-1) = \frac{1}{3}(\Delta_R + \Delta_\alpha)[3 - \gamma - q(-1) - q(-2)] - \frac{1}{3}\beta_H \geq 0$. Note that in an I equilibrium $q(-2) = 0$ and $q(-1) = \gamma$. Therefore the previous inequality becomes $\beta_H \leq (\Delta_R + \Delta_\alpha)(3 - 2\gamma)$. Hence, an I equilibrium exists iff $(1-\gamma)(\Delta_R + \Delta_\alpha) \leq \beta_H \leq (\Delta_R + \Delta_\alpha)(3 - 2\gamma)$. (Recall that the I equilibrium exists for $\beta_H = (1-\gamma)(\Delta_R + \Delta_\alpha)$; see Section A.2.3).

A.2.6 Mixed strategies equilibria

We now check if there are other mixed strategies equilibria besides the one when $\beta_H = (1-\gamma)(\Delta_R + \Delta_\alpha)$ (see Section A.2.3). Since we are focusing on the case with $\beta_H > (1-\gamma)(\Delta_R + \Delta_\alpha)$, we already know that $\pi^H(0) > \pi^H(1)$ (see equation (A.3)) and therefore $h(1) = 0$. Hence, we only need to look for equilibria in which only $h(0)$ and $h(-1)$ are interior. Indifference between 0 and -1 implies

$$\pi^H(-1) - \pi^H(0) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)[3 - \gamma - q(-1) - q(-2)] + \frac{1}{3}\beta_H = 0. \tag{A.4}$$

By Bayes' rule:

$$q(-1) = \gamma \quad \text{and} \quad q(-2) = \frac{h(-1)\gamma}{\gamma h(-1) + (1-\gamma)}. \tag{A.5}$$

Replacing (A.5) in (A.4) and solving for $h(-1)$ we get

$$h(-1) = -\frac{1-\gamma}{\gamma}\left(\frac{3 - 2\gamma - \frac{\beta_H}{\Delta_R + \Delta_\alpha}}{2 - 2\gamma - \frac{\beta_H}{\Delta_R + \Delta_\alpha}}\right) \equiv \xi. \tag{A.6}$$

If $\xi \in (0,1)$ we have found an SI equilibrium with $h(-1) = 1 - h(0) = \xi$. For $\xi > 0$ we need $3 - 2\gamma - \frac{\beta_H}{\Delta_R + \Delta_\alpha} > 0$ and $2 - 2\gamma - \frac{\beta_H}{\Delta_R+\Delta_\alpha} < 0$, which requires $2(1-\gamma)(\Delta_R + \Delta_\alpha) < \beta_H < (3-2\gamma)(\Delta_R + \Delta_\alpha)$. One can verify that $\xi < 1$ whenever $\beta_H > 3(1-\gamma)(\Delta_R + \Delta_\alpha)$. Therefore, there is an SI equilibrium iff $3(1-\gamma)(\Delta_R + \Delta_\alpha) < \beta_H < (3-2\gamma)(\Delta_R + \Delta_\alpha)$. In that case, $h(-1)$ is given by (A.6).

A.2.7 Summary of equilibrium set for $\gamma < \overline{\gamma}$

The next proposition summarizes the results of Section A.2. Define the following boundaries: $\delta_1 \equiv (1-\gamma)(\Delta_R + \Delta_\alpha)$, $\delta_2 \equiv 3(1-\gamma)(\Delta_R + \Delta_\alpha)$, $\delta_3 \equiv (3-2\gamma)(\Delta_R + \Delta_\alpha)$. Note that $\delta_3 > \delta_2 > \delta_1$.

Proposition A.1. *Suppose $\gamma < \overline{\gamma}$. In any equilibrium, $l(-1) = 1$, $g(-2) = g(-1) = g(0) = 1$, $g(1) = g(2) = 0$, $q(0) = \gamma$, and $q(1) = q(2) = 1$. The positively informed trader's strategy is as follows:*

- *If $\beta_H < \delta_1$ there is a B equilibrium and it is the unique equilibrium.*
- *If $\beta_H = \delta_1$ the equilibrium set consists of a B equilibrium, an I equilibrium, and a continuum of IB equilibria with any $h(0) \in (0,1)$ and $h(1) = 1 - h(0)$.*
- *If $\delta_1 < \beta_H < \delta_2$ there is an I equilibrium and it is the unique equilibrium.*
- *If $\beta_H = \delta_2$ the equilibrium set consists of an I equilibrium and as S equilibrium.*
- *If $\delta_2 < \beta_H < \delta_3$ the equilibrium set consists of an SI equilibrium in which $h(-1)$ is given by (A.6), an I equilibrium, and an S equilibrium.*
- *If $\beta_H = \delta_3$ the equilibrium set consists of an I equilibrium and an S equilibrium.*
- *If $\beta_H > \delta_3$ there is an S equilibrium and it is the unique equilibrium.*

Beliefs $q(-2)$ and $q(-1)$ are omitted in Proposition A.1, but can be easily computed by Bayes' rule.

A.3 Equilibrium characterization for $\gamma > \overline{\gamma}$

We start establishing the following result that reduces the set of possible equilibria for $\gamma > \overline{\gamma}$.

Lemma A.2. *Assume $\gamma > \overline{\gamma}$. Then, there is no S, I, nor SI equilibrium.*

Proof. By Lemma 1, $l(-1) = 1$ in any equilibrium. First, suppose $h(-1) = 1$. Then, $q(-2) = q(-1) = q(0) = \gamma$ and there is no intervention on the equilibrium path. But then, since the high type is making a trading loss, she would deviate to $s = 0$ or $s = 1$. Second, suppose $h(0) = 1$. Then, $q(-2) = 0$, $q(-1) = q(0) = \gamma$, and $q(1) = 1$, and there is no intervention when $X \in \{-1, 0, 1\}$, so the high type has zero payoff. Therefore, she would deviate to $s = 1$ and make a trading profit when $X = 0$ at least. Third, suppose $h(0) = 1 - h(-1) > 0$. In this case, $q(-1) = q(0) = \gamma > \overline{\gamma}$ and $q(1) = 1$, which implies that $g(-1) = g(0) = g(1) = 0$. But then, the high type would deviate to a strategy with $h(0) = 0$ and $h(1) > 0$, since $s = 0$ yields zero payoff, while $s = 1$ yields some trading profit when $X = 0$. $\square$

A.3.1 Beliefs and equilibrium strategies for policymaker

The next lemma is analogous to Lemma A.1 for the case with $\gamma > \overline{\gamma}$.

Lemma A.3. *Assume $\gamma > \overline{\gamma}$. In any equilibrium, $q(-2) < \gamma$, $q(-1) < \gamma$, $q(0) = \gamma$, $q(1) = q(2) = 1$, and $g(0) = g(1) = g(2) = 0$.*

Proof. The proof that $q(-2) < \gamma$, $q(-1) < \gamma$, $q(0) = \gamma$ is identical to the proof in Lemma A.1, since we only use $l(-1)$ to show those relations. The fact that $g(0) = 0$ follows from $q(0) = \gamma > \overline{\gamma}$. Finally, by Lemma A.2, $h(1) > 0$ in any equilibrium. Hence, $q(1) = q(2) = 1$ and $g(1) = g(2) = 0$. $\square$

In the remainder of Section A.3, we assume that $l(-1) = 1$ (by Lemma 1) and that $q(\cdot)$ and $g(\cdot)$ satisfy the conditions in Lemma A.3. Note that in any equilibrium candidate that satisfies the conditions in Lemma A.3 the low type has no incentives to deviate from $l(-1) = 1$. Hence, to verify if a strategy profile and beliefs satisfying Lemma A.3 constitute an equilibrium we only need to check if the high type has no incentives to deviate.

A.3.2 Buy equilibrium (B)

Suppose $h(1) = 1$. Then, since $l(-1) = 1$, $q(-2) = q(-1) = 0$ and $g(-2) = g(-1) = 1$. Hence, $\pi^H(1) = \frac{1}{3}(1-\gamma)\Delta_R$, $\pi^H(0) = \frac{1}{3}\beta_H$, and $\pi^H(-1) = -\frac{2}{3}(\Delta_R + \Delta_\alpha) - \frac{1}{3}(1-\gamma)\Delta_R + \frac{2}{3}\beta_H$. For the high type not to deviate to $s = 0$, we need $\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)\Delta_R - \frac{1}{3}\beta_H \geq 0$, which is equivalent to $\beta_H \leq (1-\gamma)\Delta_R$. A deviation to $s = -1$ is not profitable if $\pi^H(1) - \pi^H(-1) = \frac{1}{3}(1-\gamma)\Delta_R + \frac{2}{3}(\Delta_R + \Delta_\alpha) + \frac{1}{3}(1-\gamma)\Delta_R - \frac{2}{3}\beta_H \geq 0$, which is equivalent to $\beta_H \leq (\Delta_\alpha + \Delta_R) + (1-\gamma)\Delta_R$. Hence, since $\Delta_\alpha + \Delta_R > 0$, a B equilibrium exists iff $\beta_H \leq (1-\gamma)\Delta_R$.

A.3.3 Inaction-Buy equilibrium (IB)

Suppose $h(0) = 1 - h(1) > 0$. Since $l(-1) = 1$, $q(-2) = 0$, $g(-2) = 1$, and

$$q(-1) = \frac{\gamma h(0)}{\gamma h(0) + (1-\gamma)}. \tag{A.7}$$

Indifference between $s = 0$ and $s = 1$ for the high type requires $\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)\Delta_R - \frac{1}{3}g(-1)\beta_H = 0$, which implies that

$$g(-1) = \frac{(1-\gamma)\Delta_R}{\beta_H}. \tag{A.8}$$

Therefore, a necessary condition for such an equilibria to exist is $\beta \geq (1-\gamma)\Delta_R$.

First, consider $\beta_H > (1-\gamma)\Delta_R$, in which case $g(-1) \in (0,1)$. Indifference for the policymaker requires $q(-1) = \frac{\gamma h(0)}{\gamma h(0) + (1-\gamma)} = \overline{\gamma}$, and therefore

$$h(0) = \frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}} < 1. \tag{A.9}$$

Therefore, the payoff for the high type of deviating to $s=-1$ is

$$\pi^{H}(-1)=-\frac{1}{3}\left(\Delta_{R}+\Delta_{\alpha}\right)-\frac{1}{3}\left(1-\overline{\gamma}\right)\left[\Delta_{R}+\frac{\left(1-\gamma\right)\Delta_{R}}{\beta_{H}}\Delta_{\alpha}\right]-\frac{1}{3}\left(1-\gamma\right)\Delta_{R}+\left(\frac{1}{3}+\frac{\left(1-\gamma\right)\Delta_{R}}{3\beta_{H}}\right)\beta_{H}.$$

Also note that $\pi^{H}(0)=\pi^{H}(1)=\frac{1}{3}\left(1-\gamma\right)\Delta_{R}$. For the high type not to deviate to $s=-1$ we need $\pi^{H}(1)\geq\pi^{H}(-1)$, which yields

$$\beta_{H}^{2}-\left[\left(3-\gamma-\overline{\gamma}\right)\Delta_{R}+\Delta_{\alpha}\right]\beta_{H}-\left(1-\overline{\gamma}\right)\left(1-\gamma\right)\Delta_{R}\Delta_{\alpha}\leq 0. \tag{A.10}$$

Using $\Delta_{R}+\Delta_{\alpha}>0$ and doing some algebra, one can verify that (A.10) is satisfied with strict inequality for $\beta_{H}=\left(1-\gamma\right)\Delta_{R}$. Hence, we only need to ensure that $\beta_{H}\leq r_{1}$, where r_{1} is the largest root of the LHS of (A.10), given by

$$r_{1}=\frac{\left(3-\gamma-\overline{\gamma}\right)\Delta_{R}+\Delta_{\alpha}+\sqrt{\left[\left(3-\gamma-\overline{\gamma}\right)\Delta_{R}+\Delta_{\alpha}\right]^{2}+4\left(1-\overline{\gamma}\right)\left(1-\gamma\right)\Delta_{R}\Delta_{\alpha}}}{2}. \tag{A.11}$$

Note that r_{1} is a positive real number and $r_{1}>\left(1-\gamma\right)\Delta_{R}$, since (A.10) is satisfied with inequality for $\beta_{H}=\left(1-\gamma\right)\Delta_{R}$. Hence, when $\left(1-\gamma\right)\Delta_{R}<\beta_{H}\leq r_{1}$ there is an IB equilibrium, with $h(0)$ given by (A.9), $g(-1)$ given by (A.8), $q(-1)=\overline{\gamma}$, $q(-2)=0$, and $g(-2)=1$.

Now consider $\beta_{H}=\left(1-\gamma\right)\Delta_{R}$. In an IB equilibrium we must have $g(-1)=1$ (see (A.8)) and hence $q(-1)=\frac{\gamma h(0)}{\gamma h(0)+(1-\gamma)}\leq\overline{\gamma}$, which implies that $h(0)\leq\frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}$. Therefore, if $\beta_{H}=\left(1-\gamma\right)\Delta_{R}$, any $h(0)\in\left(0,\frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}\right]$ and $h(1)=1-h(0)$ constitute an IB equilibrium. To conclude, we have shown that an IB equilibrium exists iff $\left(1-\gamma\right)\Delta_{R}\leq\beta_{H}\leq r_{1}$.

A.3.4 Sell-Buy equilibrium (SB)

Suppose $h(-1)=1-h(1)>0$. Since $l(-1)=1$, we have

$$q(-1)=q(-2)=\frac{\gamma h(-1)}{\gamma h(-1)+(1-\gamma)}. \tag{A.12}$$

Note we cannot have $g(-1)=g(-2)=0$, since that would be inconsistent with the choice of $h(-1)>0$. Hence, it must be that $q(-1)=q(-2)\leq\overline{\gamma}$ (so that $g(-2)>0$ and/or $g(-1)>0$). In what follows we first search for SB equilibria with $q(-1)=q(-2)=\overline{\gamma}$ and then we consider equilibria with $q(-1)=q(-2)<\gamma$.

Case 1. Suppose $q(-1)=q(-2)=\overline{\gamma}$. Then, (A.12) implies $h(-1)=\frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}<1$. The indifference condition between $s=1$ and $s=-1$ for the high type, after some rearranging, implies

$$g\left(-1\right)+g\left(-2\right)=\frac{2(2-\gamma-\overline{\gamma})\Delta_{R}}{\beta_{H}-\left(1-\overline{\gamma}\right)\Delta_{\alpha}}. \tag{A.13}$$

For the high type not to be willing to deviate to $s = 0$ we need $\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)\Delta_R - \frac{1}{3}g(-1)\beta_H \geq 0$, implying

$$g(-1) \leq \frac{(1-\gamma)\Delta_R}{\beta_H} \equiv \mathcal{M}_1. \tag{A.14}$$

We need to guarantee that there exists a $g(-2) \in [0,1]$ implied by (A.13), for a given $g(-1) \leq \mathcal{M}_1$. Using (A.13), we need to check that $g(-2) = \frac{2(2-\gamma-\overline{\gamma})\Delta_R}{\beta_H-(1-\overline{\gamma})\Delta_\alpha} - g(-1) \in [0,1]$. This imposes the following additional bounds on $g(-1)$:

$$g(-1) \leq \frac{2(2-\gamma-\overline{\gamma})\Delta_R}{\beta_H - (1-\overline{\gamma})\Delta_\alpha} \equiv \mathcal{M}_2 \quad \text{and} \quad g(-1) \geq \mathcal{M}_2 - 1 \equiv \mathcal{L}_1. \tag{A.15}$$

For a $g(-1) \in [0,1]$ satisfying (A.14) and (A.15) to exist, we need $\mathcal{M}_1 \geq 0$, $\mathcal{M}_2 \geq 0$, $\mathcal{L}_1 \leq 1$ and $\mathcal{L}_1 \leq \min\{\mathcal{M}_1, \mathcal{M}_2\}$. $\mathcal{M}_1$ is clearly larger than zero. $\mathcal{M}_2 \geq 0$ whenever $\beta_H \geq (1-\overline{\gamma})\Delta_\alpha$, so we assume that is the case from now on in Case 1. One can verify that $\mathcal{L}_1 \leq 1$ whenever

$$\beta_H \geq (1-\gamma)\Delta_R + (1-\overline{\gamma})(\Delta_R + \Delta_\alpha). \tag{A.16}$$

Note that $\mathcal{M}_2 \geq \mathcal{M}_1$ whenever (A.16) holds. Thus, it remains to check whether $\mathcal{L}_1 \leq \mathcal{M}_1$, which is equivalent to

$$\beta_H^2 - [(3-\gamma-2\overline{\gamma})\Delta_R + (1-\overline{\gamma})\Delta_\alpha]\beta_H - (1-\gamma)(1-\overline{\gamma})\Delta_\alpha\Delta_R \geq 0. \tag{A.17}$$

Using $\Delta_R + \Delta_\alpha > 0$, with some algebra one can see that (A.17) is violated for $\beta_H = (1-\gamma)\Delta_R + (1-\overline{\gamma})(\Delta_R + \Delta_\alpha)$. This implies that the equilibria we have been looking for exists iff $\beta_H \geq r_2$, where r_2 is the largest root of the LHS of (A.17), which is given by

$$r_2 = \frac{(3-\gamma-2\overline{\gamma})\Delta_R + (1-\overline{\gamma})\Delta_\alpha + \sqrt{[(3-\gamma-2\overline{\gamma})\Delta_R + (1-\overline{\gamma})\Delta_\alpha]^2 + 4(1-\gamma)(1-\overline{\gamma})\Delta_R\Delta_\alpha}}{2}. \tag{A.18}$$

Note that r_2 is a positive real number. We have then shown that an SB equilibrium with $q(-1) = q(-2) = \overline{\gamma}$ exists iff $\beta_H \geq r_2$. In such an equilibrium, $h(-1) = \frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}$. Any combination of $g(-2)$ and $g(-1)$ satisfying $g(-1) \in [\mathcal{L}_1, \mathcal{M}_1]$ and (A.13) is consistent with such an equilibrium.

Case 2. Now suppose $q(-1) = q(-2) < \overline{\gamma}$. Then, $g(-1) = g(-2) = 1$. Using $q(-1) = q(-2)$, the indifference condition between -1 and 1 for the high type, after some rearranging, yields $q(-1) = 1 - \frac{\beta_H-(1-\gamma)\Delta_R}{\Delta_R+\Delta_\alpha}$. A necessary condition for $q(-1) < \overline{\gamma} < 1$ is $\beta_H > (1-\gamma)\Delta_R$. For the high type not to deviate to $s = 0$, it must be that $\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)\Delta_R - \frac{1}{3}\beta_H \geq 0$, which is equivalent to $\beta_H \leq (1-\gamma)\Delta_R$, contradicting $\beta_H > (1-\gamma)\Delta_R$. Hence, we have shown that an SB equilibrium with $q(-1) = q(-2) < \overline{\gamma}$ does not exist. Hence, an SB equilibrium exists iff $\beta_H \geq r_2$.

A.3.5 Sell-inaction-Buy equilibrium (SIB)

Suppose that $h(-1), h(0), h(1) > 0$. Since $l(-1) = 1$, we have

$$q(-1) = \frac{[h(0) + h(-1)]\,\gamma}{[h(0) + h(-1)]\,\gamma + (1-\gamma)} \quad \text{and} \quad q(-2) = \frac{h(-1)\gamma}{h(-1)\gamma + (1-\gamma)}. \tag{A.19}$$

After some rearranging, indifference between $s = -1$ and $s = 0$ for the high type implies

$$[3 - q(-2) - q(-1) - \gamma]\,\Delta_R + [(1 - q(-2))\,g(-2) + (1 - q(-1))\,g(-1)]\,\Delta_\alpha = g(-2)\beta_H. \tag{A.20}$$

If $\Delta_\alpha \geq 0$ the LHS of (A.20) is clearly strictly larger than zero. If $\Delta_\alpha < 0$ one can see that it is also strictly larger than zero since $\Delta_R + \Delta_\alpha \geq 0$. Hence, if $g(-2) = 0$, (A.20) cannot be satisfied and therefore if there is an SIB equilibrium it must be that $g(-2) > 0$ and $q(-2) \leq \overline{\gamma}$. Indifference between $s = 0$ and $s = 1$ for the high type implies $\pi^H(1) - \pi^H(0) = \frac{1}{3}(1-\gamma)\,\Delta_R - \frac{1}{3}g(-1)\beta_H = 0$. Therefore:

$$g(-1) = \frac{(1-\gamma)\,\Delta_R}{\beta_H}. \tag{A.21}$$

Hence, a necessary condition for such an equilibrium to exist is $\beta_H \geq (1-\gamma)\,\Delta_R$. Suppose such an SIB equilibrium exists for $\beta_H = (1-\gamma)\,\Delta_R$. Then $g(-1) = 1$, $q(-1) \leq \overline{\gamma}$, $q(-2) < \overline{\gamma}$ (since by (A.19) $q(-2) < q(-1)$), and $g(-2) = 1$. But then, indifference condition (A.20) gives us the following contradiction: $\beta_H = (1-\gamma)\,\Delta_R + [2 - q(-2) - q(-1)]\,(\Delta_\alpha + \Delta_R) > (1-\gamma)\,\Delta_R$. Hence, another necessary condition for an SIB equilibrium to exist is $\beta > (1-\gamma)\,\Delta_R$.

Suppose then $\beta_H > (1-\gamma)\,\Delta_R$. In this case, (A.21) implies $g(-1)$ is interior and therefore $q(-1) = \overline{\gamma}$. Since $q(-2) < q(-1) = \overline{\gamma}$, we must have $g(-2) = 1$. Replacing those equalities, (A.19) and (A.21) in (A.20) we get

$$\frac{\beta_H\,(1-\gamma)\,(\Delta_R + \Delta_\alpha)}{h(-1)\gamma + (1-\gamma)} = \beta_H^2 - (2 - \gamma - \overline{\gamma})\,\Delta_R\beta_H - (1-\overline{\gamma})\,(1-\gamma)\,\Delta_R\Delta_\alpha. \tag{A.22}$$

Note that the LHS is strictly positive. Hence, we need the RHS to be positive. One can verify that when $\beta_H = (1-\gamma)\,\Delta_R$, the RHS is negative. Hence, we assume in what follows that $\beta_H > r_3$ where r_3 is the largest root of the RHS of (A.22), given by

$$r_3 = \frac{(2 - \overline{\gamma} - \gamma)\,\Delta_R + \sqrt{[(2 - \overline{\gamma} - \gamma)\,\Delta_R]^2 + 4\,(1-\overline{\gamma})\,(1-\gamma)\,\Delta_R\Delta_\alpha}}{2}.$$

Solving (A.22) for $h(-1)$:

$$h(-1) = -\frac{(1-\gamma)}{\gamma}\left\{\frac{\beta_H^2 - [(2-\gamma-\overline{\gamma})\,\Delta_R + (\Delta_R + \Delta_\alpha)]\,\beta_H - (1-\overline{\gamma})\,(1-\gamma)\,\Delta_R\Delta_\alpha}{\beta_H^2 - (2-\gamma-\overline{\gamma})\,\Delta_R\beta_H - (1-\overline{\gamma})\,(1-\gamma)\,\Delta_R\Delta_\alpha}\right\}. \tag{A.23}$$

The numerator of the term in braces is positive since $\beta_H > r_3$. Using $q(-1) = \overline{\gamma}$ and (A.19) we can solve for $h(0)$ as a function of $h(-1)$:

$$h(0) = \frac{\overline{\gamma}}{1-\overline{\gamma}} \frac{1-\gamma}{\gamma} - h(-1). \tag{A.24}$$

Note that (A.24) implies $h(0) + h(-1) < 1$ and then $h(1) > 0$. Since we need $h(0) > 0$, (A.24) requires that $h(-1) < \frac{\overline{\gamma}}{1-\overline{\gamma}} \frac{1-\gamma}{\gamma}$. Therefore, we need to check under which parameters $h(-1)$ given by (A.23) is contained in $\left(0, \frac{\overline{\gamma}}{1-\overline{\gamma}} \frac{1-\gamma}{\gamma}\right)$. After some rearranging $h(-1) < \frac{\overline{\gamma}}{1-\overline{\gamma}} \frac{1-\gamma}{\gamma}$ implies

$$\beta_H^2 - \left[(2-\gamma-\overline{\gamma})\,\Delta_R + (1-\overline{\gamma})\,(\Delta_R + \Delta_\alpha)\right]\beta_H - (1-\overline{\gamma})\,(1-\gamma)\,\Delta_R \Delta_\alpha > 0. \tag{A.25}$$

Notice that the LHS of (A.25) is the same as the LHS of (A.17). Also note that when $\beta_H = r_3$, (A.25) is violated. Then, we only need to check if $\beta_H > r_2$, where r_2 is the largest root of the LHS of (A.25), given by (A.18). Thus, we further limit β_H, assuming $\beta_H > r_2$ in what follows. Since $\beta_H > r_2 > r_3$, $h(-1) > 0$ iff the numerator of the term in braces in (A.23) is strictly smaller than zero:

$$\beta_H^2 - \left[(2-\gamma-\overline{\gamma})\,\Delta_R + (\Delta_R + \Delta_\alpha)\right]\beta_H - (1-\overline{\gamma})\,(1-\gamma)\,\Delta_R \Delta_\alpha < 0. \tag{A.26}$$

Note that the LHS of (A.26) is the same as the LHS of (A.10). Also, notice that when $\beta_H = r_2$, (A.26) is satisfied. Hence, we need to ensure that $\beta_H < r_1$, where r_1 is the largest root of (A.26), given by (A.11). Note that $r_1 > r_2$, since (A.26) is satisfied for $\beta_H = r_2$.

Therefore, we have shown that there is an SIB equilibrium iff $r_2 < \beta_H < r_1$ (where r_2 and r_1 are given by (A.18) and (A.11)). In this equilibrium, $h(-1)$ is given by (A.23) and $h(0)$ is given by (A.24). The government plays $g(-2) = 1$, and $g(-1)$ is interior and given by (A.21). Moreover, $q(-1) = \gamma$ and $q(-2)$ is obtained by combining (A.19) and (A.23).

A.3.6 Summary of equilibrium set when $\gamma > \overline{\gamma}$

Before we summarize the results, it useful to define $\varphi_1 = (1-\gamma)\Delta_R$, $\varphi_2 = r_2$ (where r_2 is given by (A.18)), and $\varphi_3 = r_1$ (where r_1 is given by (A.11)). The next lemma establishes some relations between those variables.

Lemma A.4. *For any parameters we have $\varphi_3 > \varphi_2 > \varphi_1$.*

Proof. That $\varphi_3 > \varphi_2$ was established in Section A.3.5. To check that $\varphi_2 > \varphi_1$, we replace $_H = \varphi_1$ in (A.17), which after some rearranging yields $\Delta_R + \Delta \leq 0$, which is violated by the assumption. Since φ_2 is by the definition the largest root of the LHS of (A.17), it must be that $\varphi_2 > \varphi_1$. ∎

Note that the lemma above implies the B equilibrium is unique when $_H < \varphi_1$. The next proposition summarizes the characterization for $\gamma > \overline{\gamma}$.

Proposition A.2. *Suppose $\gamma > \overline{\gamma}$. Then in any equilibrium we have $l(-1) = 1$, and $g(0) = g(1) = g(2) = 0$, $q(0) = \gamma$, $q(1) = q(2) = 1$. Regarding the strategy of the positively informed trader, there is no S, I, nor SI equilibria. Moreover:*

- *A B equilibrium exists iff $\beta_H \le \varphi_1$, and it is the unique equilibrium when $\beta_H < \varphi_1$. In that equilibrium, $g(-2) = g(-1) = 1$ and $q(-2) = q(-1) = 0$.*
- *An IB equilibrium exists iff $\varphi_1 \le \beta_H \le \varphi_3$. In those equilibria, $g(-2) = 1$, $g(-1) = \varphi_1/\beta_H$, $q(-2) = 0$. If $\beta_H > \varphi_1$ then we have $h(0)$ given by (A.9) and $q(-1) = \overline{\gamma}$. If $\beta_H = \varphi_1$ then any $h(0) \in \left(0, \frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}\right]$ is consistent with such an equilibrium and $q(-1)$ is given by (A.7).*
- *An SB equilibrium exists iff $\beta_H > \varphi_2$. In that case, $q(-1) = q(-2) = \overline{\gamma}$, $h(-1) = \frac{\overline{\gamma}}{\gamma}\frac{1-\gamma}{1-\overline{\gamma}}$ and any combination of $g(-1)$ and $g(-2)$ satisfying $g(-1) \in [\mathcal{L}_1, \mathcal{M}_1]$ and (A.13) are consistent with equilibrium (with $\mathcal{L}_1$ and $\mathcal{M}_1$ given by (A.14) and (A.15)).*
- *An SIB equilibrium exists iff $\varphi_2 < \beta_H < \varphi_3$. In this equilibrium, $h(-1)$ is given by (A.23) and $h(0)$ is given by (A.24). Moreover, $q(-1) = \overline{\gamma}$, $q(-2)$ is given by (A.19), $g(-2) = 1$, and $g(-1) = \varphi_1/\beta_H$.*

A.4 Efficiency of intervention

The next proposition characterizes the ranking of equilibrium according to the government payoff.

Proposition A.3. *Fix b, c, and γ. For a given equilibrium $E \in \{B, I, S, IS, IB, SB, SIB\}$ described in A.1 and A.2, let U_G^E denote the ex-ante government payoff in equilibrium E for any arbitrary set of parameters such that equilibrium E exists. Suppose $\gamma < \overline{\gamma}$. Then, $U_G^B > U_G^I > U_G^{IS} > U_G^S$. Now suppose $\gamma > \overline{\gamma}$. Then, $U_G^B > U_G^{IB} > U_G^{SIB} > U_G^{SB}$.*

Proof. Let $\Pr(\cdot)$ denote the probability of a given event taking as given some (equilibrium) strategy profile and agents' prior belief about the state. To ease the notation, we omit the strategy profile as an argument of $\Pr(\cdot)$. The ex-ante government payoff is given by (5).

Part 1. We start by analyzing the case with $\gamma < \overline{\gamma}$. In that case, from Proposition A.1, in any equilibrium $\Pr(G = 1|\theta = L) = 1$. Hence, when there are multiple equilibria, the best one for the government is the one with the lowest $\Pr(G = 1|\theta = H)$. Since $g(1) = g(2) = 0$ and $g(-1) = g(-2) = g(0) = 1$ in any equilibrium, we have that $\Pr(G = 1|\theta = H) = \Pr(X = -2|\theta = H) + \Pr(X = -1|\theta = H) + \Pr(X = 0|\theta = H)$. Therefore, under an I equilibrium, $\Pr(G = 1|\theta = H) = 2/3$. Under an S equilibrium we have $\Pr(G = 1|\theta = H) = 1$. Under an IS equilibrium, $\Pr(G = 1|\theta = H) \in (2/3, 1)$. Under the B equilibrium $\Pr(G = 1|\theta = H) = 1/3$, which yields the desired result.

Part 2. Now assume $\gamma > \overline{\gamma}$. From Proposition A.2, $g(0) = g(1) = g(2) = 0$ in any equilibrium. Hence, under any equilibrium strategy profile we can write:

$$U_G = \Pr(X = -2)\{g(-2)[(1 - q(-2))b - c]\} + \Pr(X = -1)\{g(-1)[(1 - q(-1))b - c]\}. \tag{A.27}$$

Note that the first (second) term in braces denotes the government expected payoff conditional on observing $X = -2$ ($X = -1$). Hence, whenever the government is indifferent between intervening or not for a given $X \in \{-1, -2\}$, the associated term in braces is equal to zero.

First, we show that the government always prefers the B equilibrium over any other equilibria. Under the B equilibrium $\Pr(G = 1|\theta = H) = 0$ and $\Pr(G = 1|\theta = L) = 2/3$ (which is the maximum possible value given that in all equilibria $g(0) = 0$). Since in all equilibria with $h(0) > 0$ we have $g(-1) > 0$, those equilibria have $\Pr(G = 1|\theta = H) > 0$. In any SB equilibrium we have $g(-1)+g(-2) > 0$ and therefore $\Pr(G = 1|\theta = H) > 0$ as well. Using (5), we have then shown that the government *strictly* prefers the B equilibrium to any other. Therefore, in what follows we focus on the case of parameters where the B equilibrium does not exist, assuming $\beta_H > \varphi_1$ hereafter.

Second, we show that the IB equilibrium is preferred over the SIB equilibrium. Note that under the IB equilibrium with $\beta_H > \varphi_1$ the government is indifferent between intervening or not when $X = -1$. Hence, using (A.27) we can write $U_G^{IB} = \frac{1}{3}(1-\gamma)(b-c) > 0$. In the SIB equilibrium, the government is indifferent between intervening or not when $X = -1$. Also, $g(-2) = 1$ and $q(-2) = \frac{h(-1)\gamma}{h(-1)\gamma+(1-\gamma)}$, which implies that we can write $U_G^{SIB} = \frac{1}{3}[(1-\gamma)(b-c) - \gamma h(-1)c]$, and therefore $U_G^{SIB} < U_G^{IB}$.

Finally, as shown in Section A.3.5, in the SIB equilibrium $h(-1) < \frac{\overline{\gamma}}{1-\overline{\gamma}}\frac{1-\gamma}{\gamma}$, which implies that $U_G^{SIB} > 0$. From Proposition A.1, whenever an SB equilibrium exists, the government is indifferent between intervening or not when $X \in \{-1, -2\}$, which yields a payoff $U_G^{SB} = 0 < U_G^{SIB}$. □

B Proofs

Proposition 1 follows immediately from Propositions A.1 and A.2, and Proposition 2 follows from Propositions A.1, A.2, and A.3 in Appendix A if one defines $\underline{\beta} = \delta_1$, $\overline{\beta} = \delta_3$, $\underset{\sim}{\beta} = \varphi_1$, and $\tilde{\beta} = \varphi_3$. Also, Lemma 1 is proved in Appendix A.1. The remainder results are proved in this section.

B.1 Proof of Lemma 2

Fix any strategy profile. Note that $\mathbb{E}[q(X)|\theta = L] = \sum_{X\in\mathcal{X}} \Pr(X|\theta = L)q(X)$ and $\mathbb{E}[q(X)|\theta = H] = \sum_{X\in\mathcal{X}} \Pr(X|\theta = H)q(X)$. Using Bayes' rule, we have that $\Pr(X|\theta = L) = \frac{\Pr(\theta=L|X)\Pr(X)}{\Pr(\theta=L)}$ and $\Pr(X|\theta = H) = \frac{\Pr(\theta=H|X)\Pr(X)}{\Pr(\theta=H)}$. Moreover, we know that $\Pr(\theta = L) = 1-\gamma$, $\Pr(\theta = H) = \gamma$, and $\Pr(\theta = L|X) = 1 - \Pr(\theta = H|X)$. Hence, $\mathbb{E}[q(X)|\theta = L] = \sum_{X\in\mathcal{X}} \frac{[1-\Pr(\theta=H|X)]\Pr(X)}{1-\gamma} q(X)$, which can be written as

$$\mathbb{E}[q(X)|\theta = L] = \frac{1}{1-\gamma}\sum_{X\in\mathcal{X}} \Pr(X)q(X) - \frac{1}{1-\gamma}\sum_{X\in\mathcal{X}} \Pr(\theta = H|X)\Pr(X)q(X),$$

where the last sum is equal to $\gamma\mathbb{E}[q(X)|\theta = H]$. Finally, using the law of iterated expectations, it must be that $\mathbb{E}[q(X)] = \sum_{X\in\mathcal{X}} \Pr(X)q(X) = \gamma$. Thus, $\mathbb{E}[q(X)|\theta = L] = \frac{\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma}\mathbb{E}[q(X)|\theta = H]$, which after some rearranging yields

$$\frac{\mathbb{E}[q(X)|\theta = H] - \gamma}{\gamma} = \rho\frac{1 - \mathbb{E}[q(X)|\theta = L] - (1-\gamma)}{1-\gamma} \tag{B.1}$$

where $\rho = (1-\gamma)^2/\gamma^2 > 0$. The definition of informativeness in (4) and (B.1) yield $\iota = \frac{\mathbb{E}(q(X)|\theta=H)-\gamma}{1-\gamma}$. The listed properties follow immediately from (4). □

B.2 Proof of Proposition 3

From the equilibrium ranking established in Proposition A.3 in Appendix A (only focusing on the best equilibrium whenever there is multiplicity), we have that $U_G^B > U_G^I > U_G^S$ and $U_G^B > U_G^{IB} > U_G^{SB}$. We then need to show that equilibria are ranked in the same way in terms of informativeness. First consider $\gamma < \overline{\gamma}$. Given the equilibrium characterization in Proposition A.1 one can easily check that in the B equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \frac{1}{3}\left(2+\gamma\right)$; in the I equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \frac{1}{3}\left(1+2\gamma\right)$; and in the S equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \gamma$. Hence, using the expression for ι in Lemma 2,

$$\iota^B = \frac{2}{3} > \iota^I = \frac{1}{3} > \iota^S = 0. \tag{B.2}$$

Now consider $\gamma > \overline{\gamma}$. One can verify that in the B equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \frac{1}{3}\left(2+\gamma\right)$; in the IB equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \frac{1}{3}\left[2+\gamma - \frac{\overline{\gamma}}{\gamma}\left(1-\gamma\right)\right]$; and in the SB equilibrium, $\mathbb{E}\left(q(X)|\theta = H\right) = \frac{1}{3}\left[2+\gamma - 2\frac{\overline{\gamma}}{\gamma}\left(1-\gamma\right)\right]$. Hence,

$$\iota^B = \frac{2}{3} > \iota^{IB} = \frac{2}{3} - \frac{\overline{\gamma}}{3\gamma} > \iota^{SB} = \frac{2}{3} - \frac{2\overline{\gamma}}{3\gamma}. \tag{B.3}$$

□

B.3 Proof of Proposition 4

Using (5) and the equilibrium characterization in Propositions A.1 and A.2, the government payoffs in the best equilibria are $U_G^B = (1-\gamma)(b-c) - \gamma c/3$, $U_G^I = (1-\gamma)(b-c) - 2\gamma c/3$, and $U_G^S = (1-\gamma)b - c$ if $\gamma < \overline{\gamma}$, and $U_G^B = 2(1-\gamma)\left(b-c\right)/3$, $U_G^{IB} = \left(1-\gamma\right)\left(b-c\right)/3$ and $U_G^{SB} = 0$ if $\gamma > \overline{\gamma}$. This, combined with equations (B.2) and (B.3), implies that β_H only affects the speculator's strategy, market informativeness, and the government payoff through the determination of which equilibrium will be played—that is, through the relative position of β_H with respect to $\underline{\beta}$, $\overline{\beta}$, $\underset{\sim}{\beta}$, and $\tilde{\beta}$—but within each equilibrium class strategies $h(-1)$, $h(0)$, $h(1)$, informativeness and government payoffs do not depend on β_H (and thus on μ). Denote with the superscript E the value of each variable under equilibrium E.

Consider $\gamma < \overline{\gamma}$. Given our focus on the best equilibrium, as μ increases, the equilibrium eventually switches from a B equilibrium ($h(1) = 1$) to an I equilibrium ($h(0) = 1$), and then to an S equilibrium ($h(-1) = 1$). Also, from Proposition 3 we know that $\iota^B > \iota^I > \iota^S$ and $U_G^B > U_G^I > U_G^S$. Hence, informativeness and government payoffs are decreasing in μ. Regarding the ex-ante probability of intervention, we have that $\mathbb{E}\left[g(X)\right]^B = 1 - \frac{2\gamma}{3} < \mathbb{E}\left[g(X)\right]^I = 1 - \frac{\gamma}{3} < \mathbb{E}\left[g(X)\right]^S = 1$.

Now consider $\gamma > \overline{\gamma}$. As μ (and β_H) increases, we move from a B equilibrium, to an IB equilibrium and then to an SB equilibrium. Hence, informativeness and the government payoff are decreasing in β_H, as Proposition 3 states that $\iota^B > \iota^{IB} > \iota^{SB}$ and $U_G^B > U_G^{IB} > U_G^{SB}$. Regarding the ex-ante probability of intervention, one can verify that $\mathbb{E}\left[g(X)\right]^B = (1-\gamma)\frac{2}{3}$,

$$\mathbb{E}\left[g(X)\right]^{IB} = \frac{1-\gamma}{3}\left[1 + \frac{\left(1-\gamma\right)\Delta_R}{\left(1-\overline{\gamma}\right)\beta_H}\right] \quad \text{and} \quad \mathbb{E}\left[g(X)\right]^{SB} = \frac{\left(1-\gamma\right)}{3\left(1-\overline{\gamma}\right)}\frac{2\left(2-\gamma-\overline{\gamma}\right)\Delta_R}{\beta_H - (1-\overline{\gamma})\Delta_\alpha}.$$

Notice that as $\beta_H \to \underset{\sim}{\beta} = (1-\gamma)\,\Delta_R$, $\mathbb{E}\left[g(X)\right]^{IB} > \mathbb{E}\left[g(X)\right]^{B}$, but $\mathbb{E}\left[g(X)\right]^{IB}$ is strictly decreasing in β_H. This shows the non-monotonicity of the expected probability of intervention with respect to β_H: for some $\varepsilon > 0$, when β_H increases from $\underset{\sim}{\beta} - \varepsilon$ to $\underset{\sim}{\beta} + \varepsilon$, $\mathbb{E}\left[g(X)\right]$ increases, but as β_H continues to grow in the range where the IB equilibrium is played, $\mathbb{E}\left[g(X)\right]$ decreases. Moreover, $\mathbb{E}\left[g(X)\right]^{SB}$ can be larger or smaller than $\mathbb{E}\left[g(X)\right]^{IB}$ depending on parameters, but whenever we are in the parameter range where SB is played, the probability of intervention decreases in β_H (and thus in μ). $\square$

B.4 Proof of Proposition 5

Within each equilibrium class we have been analyzing (the best equilibria from the perspective of the policymaker), market informativeness and the government payoff do not depend on Δ_R—see the first paragraph of Section B.3. Hence, the effect of Δ_R on informativeness and efficiency operates only through switches across equilibria. From Proposition 3 we know that $\iota^B > \iota^I > \iota^S$ and $\iota^B > \iota^{IB} > \iota^{SB}$, and the same rankings hold for U_G. It is easy to see that the boundaries $\underline{\beta}$, $\overline{\beta}$, and $\underset{\sim}{\beta}$ of Proposition 2 are increasing in Δ_R. It remains to show that $\tilde{\beta}$ is also increasing in Δ_R. Let J denote the expression inside the square root in the definition of $\tilde{\beta}$ (see equation (3)). We know that $J > 0$. The derivative of $\tilde{\beta}$ with respect to Δ_R has the same sign as

$$3 - \gamma - \overline{\gamma} + J^{-\frac{1}{2}} \left\{\left[(3-\gamma-\overline{\gamma})\,\Delta_R + \Delta_\alpha\right](3-\gamma-\overline{\gamma}) + 2\,(1-\overline{\gamma})\,(1-\gamma)\,\Delta_\alpha\right\}. \tag{B.4}$$

The term in braces is increasing in Δ_α, and by assumption $\Delta_\alpha > -\Delta_R$. Hence (B.4) is larger than

$$3 - \gamma - \overline{\gamma} + J^{-\frac{1}{2}} \left\{\left[(3-\gamma-\overline{\gamma})\,\Delta_R - \Delta_R\right](3-\gamma-\overline{\gamma}) - 2\,(1-\overline{\gamma})\,(1-\gamma)\,\Delta_R\right\},$$

where the term in braces simplifies to $\Delta_R\left[4 + \gamma^2 + \overline{\gamma}^2 - 3\gamma - 3\overline{\gamma}\right]$, which is larger than zero for all $\gamma, \overline{\gamma} \in (0,1)$. Hence, $\tilde{\beta}$ is increasing in Δ_R, and so are market informativeness and real efficiency. $\square$

B.5 Proof of Proposition 6

Since $\Delta_R = \frac{\zeta}{\gamma(1-\gamma)}$, for ζ sufficiently close to $\overline{\zeta} = (1-\gamma)\overline{R}$ the B equilibrium occurs, while for ζ sufficiently close to $\underline{\zeta} = -\gamma(1-\gamma)\Delta_\alpha$ it does not occur (see Proposition 2). It follows from Proposition 3 that real efficiency is maximized when ζ is sufficiently close to $\overline{\zeta}$ (so that the B equilibrium is played). Note that expected firm value can be written as $\mathbb{E}\left[p(X)\right] = \gamma\left[R_H + \Pr\left(G=1|\theta=H\right)\alpha_H\right] + (1-\gamma)\left[R_L + \Pr\left(G=1|\theta=L\right)\alpha_L\right]$. Consider $\gamma < \overline{\gamma}$. In the I equilibrium $\Pr\left(G=1|\theta=H\right)$ is larger and $\Pr\left(G=1|\theta=L\right)$ is the same as in the B equilibrium (see Proposition A.1). Hence, any sufficiently low ζ that leads to B is dominated for the firm. Now consider $\gamma > \overline{\gamma}$. In this case, $g(0) = g(1) = g(2) = 0$ in any equilibrium (see Proposition A.2). Note that if $\Delta_R \geq \frac{\beta_H}{1-\gamma}$ the B equilibrium is played, and if $\Delta_R < \frac{\beta_H}{1-\gamma}$ the IB equilibrium is played, with $h(0)$ bounded away from zero. Since in the IB equilibrium $g(-1) = \frac{\beta_H}{(1-\gamma)\Delta_R}$ and $g(-2) = 1$, at $\Delta_R = \frac{\beta_H}{1-\gamma}$ a marginal decrease in Δ_R causes an upward jump in $\Pr\left(G=1|\theta=H\right)$ and a negligible decrease in $\Pr\left(G=1|\theta=L\right)$, and hence increases firm value. Therefore, choosing ζ slightly below $\gamma\beta_H$ dominates any $\zeta \geq \gamma\beta_H$ (which would lead to the B equilibrium). $\square$

B.6 Proof of Proposition 7

To compute stock prices, we use the expressions for $q(X)$ and $g(X)$ in each equilibrium given by the equilibrium characterization in Appendix A. Since in any equilibrium $q(1) = q(2) = 1$, we have that $p(1) = p(2) = R_H$.

First, consider $\gamma < \overline{\gamma}$. If $\beta_H \leq \underline{\beta}$, the B equilibrium is played and $p(0) = R_L + \alpha_L + \gamma\,(\Delta_R + \Delta_\alpha) > R_L + \alpha_L = p(-2) = p(-1)$. Hence, $p(X)$ is (weakly) increasing in X iff $p(1) \geq p(0)$, or equivalently,

$$\Delta_R \geq \frac{\alpha_L + \gamma\Delta_\alpha}{(1-\gamma)}. \tag{B.5}$$

If instead $(1-\gamma)\,\Delta_R < \alpha_L + \gamma\Delta_\alpha$, we have that $p(-2) = p(-1) < p(0)$ and $p(1) < p(0)$, so prices have an inverted-U shape. If $\underline{\beta} < \beta_H \leq \overline{\beta}$, the I equilibrium is played and $p(0) = p(-1) = R_L + \alpha_L + \gamma\,(\Delta_R + \Delta_\alpha) > R_L + \alpha_L = p(-2)$. Once again prices are increasing in X if (B.5) holds, and is an inverted-U otherwise. If $\beta_H > \overline{\beta}$, the S equilibrium is played and $p(-2) = p(-1) = p(0) = R_L + \alpha_L + \gamma\,(\Delta_R + \Delta_\alpha)$, that is, prices are flat on the (on-the-equilibrium-path) aggregate orders (orders $X = 1, 2$ are off-equilibrium). If $\Delta_\alpha > 0$, as $\Delta_R \to 0$, the economy can be on the B, I, or S equilibrium, depending on parameters. In the first two cases, $p(X)$ would have an inverted-U shape since (B.5) is violated as $\Delta_R \to 0$, and in the latter, $p(X)$ would be flat. If instead $\Delta_\alpha \leq 0$, as Δ_R reaches its lower limit ($-\Delta_\alpha$ by assumption), the economy is in the S equilibrium and $p(X)$ is flat on the equilibrium path.

Now, consider $\gamma > \overline{\gamma}$. In any equilibrium, $q(0) = \gamma$ and $g(0) = 0$, so $p(0) = R_L + \gamma\Delta_R < p(1) = p(2) = R_H$. If $\beta_H \leq \underset{\sim}{\beta}$, the B equilibrium is played and we have $p(-2) = p(-1) = R_L + \alpha_L$. Prices are weakly increasing in X whenever $p(0) \geq p(-1)$, which is equivalent to

$$\Delta_R \geq \alpha_L/\gamma. \tag{B.6}$$

If instead $\Delta_R < \alpha_L/\gamma$, $p(-2) = p(-1) > p(0)$ and $p(0) < p(1)$, and hence $p(X)$ is U-shaped. If $\underset{\sim}{\beta} < \beta_H \leq \tilde{\beta}$, IB is played and we have that $p(-1) = R_L + \overline{\gamma}\Delta_R + \frac{(1-\gamma)\Delta_R}{\beta_H}\,[\alpha_L + \overline{\gamma}\Delta_\alpha]$ and $p(-2) = R_L + \alpha_L$. Note that $p(-2) > p(-1)$ iff

$$\Delta_R < \frac{\alpha_L}{\overline{\gamma} + \frac{(1-\gamma)}{\beta_H}\,(\alpha_L + \overline{\gamma}\Delta_\alpha)}. \tag{B.7}$$

Hence, if IB is played and (B.7) is satisfied, since $p(0) < p(1) = p(2)$, $p(X)$ is U-shaped. If $\beta_H > \tilde{\beta}$, SB is played and we have $p(-2) = R_L + \overline{\gamma}\Delta_R + g(-2)\,[\alpha_L + \overline{\gamma}\Delta_\alpha]$ and $p(-1) = R_L + \overline{\gamma}\Delta_R + g(-1)\,[\alpha_L + \overline{\gamma}\Delta_\alpha]$. Proposition A.2 imposes restrictions on $g(-2)$ and $g(-1)$ in the SB equilibrium—see (A.13) and (A.14). For us to have $p(-2) > p(-1)$, it must be that $g(-2) > g(-1)$, which given (A.13), is the case whenever $g(-2) = \frac{2(2-\gamma-\overline{\gamma})\Delta_R}{\beta_H - (1-\overline{\gamma})\Delta_\alpha} - g(-1) > g(-1)$. Given (A.14), a sufficient condition for the former inequality is that $\beta_H > -\,(1-\gamma)\,\Delta_\alpha$, which is always satisfied since $\Delta_R > -\Delta_\alpha$ and $\beta_H > \tilde{\beta} > \underset{\sim}{\beta} = (1-\gamma)\,\Delta_R$. Hence, in any SB equilibrium $p(-2) > p(-1)$, and given that $p(0) < p(1) = p(2)$, prices are U-shaped.

Finally, note that if $\Delta_\alpha \geq 0$, the lower limit of Δ_R is zero, and one can easily verify that as $\Delta_R \to 0$, (B.6) is violated and (B.7) is satisfied. Hence $p(X)$ is U-shaped in that case (regardless of the equilibrium being played). If $\Delta_\alpha < 0$, by assumption $\Delta_R > -\Delta_\alpha$. One can verify that as $\Delta_R \to -\Delta_\alpha$, (B.6) is violated. Moreover, we must check if (B.7) holds when an IB equilibrium is being played in this limit, which only happens when $\beta_H > (1-\gamma)\Delta_R$. Hence, since the RHS of (B.7) is increasing in β_H, it suffices to check that $\Delta_R \leq \frac{\alpha_L}{\overline{\gamma}+\frac{1}{\Delta_R}(\alpha_L+\overline{\gamma}\Delta_\alpha)}$ holds in the limit as $\Delta_R \to -\Delta_\alpha$. Taking the limit of both sides of the previous inequality as $\Delta_R \to -\Delta_\alpha$ yields the desired result. $\square$

B.7 Proof of Proposition 8 and comparative statics of Table 2

Using the definition of the boundaries $\underline{\beta}$, $\overline{\beta}$, $\utilde{\beta}$, and $\tilde{\beta}$ in (3), define $\underline{\mu} \equiv \underline{\beta}/\alpha_H$, $\overline{\mu} \equiv \overline{\beta}/\alpha_H$, $\utilde{\mu} \equiv \utilde{\beta}/\alpha_H$, and $\tilde{\mu} \equiv \tilde{\beta}/\alpha_H$. Then, in the liquidity crises model we have $\underline{\mu} = (1-\gamma)\frac{\Delta_V}{D\kappa_H}$, $\overline{\mu} = (3-2\gamma)\frac{\Delta_V}{D\kappa_H}$, $\utilde{\mu} = (1-\gamma)\left[\frac{\Delta_V}{D\kappa_H} + \frac{\tilde{\Delta}_\kappa}{\kappa_H}\right]$, and

$$\tilde{\mu} = \frac{\frac{\Delta_V}{D\kappa_H} + (2-\gamma-\overline{\gamma})\left(\frac{\Delta_V}{D\kappa_H} + \frac{\tilde{\Delta}_\kappa}{\kappa_H}\right) + \sqrt{\left((3-\gamma-\overline{\gamma})\frac{\Delta_V}{D\kappa_H} + (\gamma-\overline{\gamma})\frac{\tilde{\Delta}_\kappa}{\kappa_H}\right)^2 + 4(1-\gamma)(2-\gamma)\frac{\Delta_V}{D\kappa_H}\frac{\tilde{\Delta}_\kappa}{\kappa_H}}}{2}, \tag{B.8}$$

where $\overline{\gamma} \equiv \frac{\kappa_L - \tau}{\kappa_L - \kappa_H}$. Suppose $\gamma < \overline{\gamma}$. Then, the (best) equilibrium is (i) the B equilibrium if $\mu \leq \underline{\mu}$; (ii) the I equilibrium if $\underline{\mu} < \mu \leq \overline{\mu}$; (iii) the S equilibrium if $\mu > \overline{\mu}$. Similarly, if $\gamma > \overline{\gamma}$, then the (best) equilibrium is (i) the B equilibrium if $\mu \leq \utilde{\mu}$; (ii) the IB equilibrium if $\utilde{\mu} < \mu \leq \tilde{\mu}$; (iii) the SB equilibrium if $\mu > \tilde{\mu}$. (See Propositions A.1, A.2, and A.3). We cannot directly apply the results in Proposition 3 for two reasons. First, the government payoff in the liquidity crises model only maps into the general model once we subtract Ω in equation (9)—which we can do without loss of generality for the purpose of computing and ranking the equilibria for a *given* set of parameters, but not to assess how the government payoff responds to changes in parameters that affect Ω. Second, that proposition was stated fixing b_θ and c, which here are endogenous to the variables we are analyzing. Let $\underline{\mathcal{W}}^E$ and $\underline{\iota}^E$ denote the government ex-ante expected payoff and market informativeness, respectively, when parameters are such that equilibrium E is played and $\gamma < \overline{\gamma}$. $\overline{\mathcal{W}}^E$ and $\overline{\iota}^E$ are defined in an analogous manner for $\gamma > \overline{\gamma}$. Using the results in Propositions A.1 and A.2:

$$\underline{\mathcal{W}}^B = \tilde{V} - D\left[\frac{2\gamma}{3}\kappa_H + \left(1 - \frac{2\gamma}{3}\right)\tau\right], \quad \underline{\mathcal{W}}^I = \tilde{V} - D\left[\frac{\gamma}{3}\kappa_H + \left(1 - \frac{\gamma}{3}\right)\tau\right], \quad \underline{\mathcal{W}}^S = \tilde{V} - \tau D, \tag{B.9}$$

$$\overline{\mathcal{W}}^B = \tilde{V} - D\left[\gamma\kappa_H + (1-\gamma)\left(\frac{1}{3}\kappa_L + \frac{2}{3}\tau\right)\right], \quad \overline{\mathcal{W}}^{IB} = \tilde{V} - D\left[\gamma\kappa_H + (1-\gamma)\left(\frac{2}{3}\kappa_L + \frac{1}{3}\tau\right)\right], \tag{B.10}$$

$$\overline{\mathcal{W}}^{SB} = \tilde{V} - D\left[\gamma\kappa_H + (1-\gamma)\kappa_L\right], \tag{B.11}$$

where $\tilde{V} \equiv \gamma V_H + (1-\gamma)V_L$. From (B.2) and (B.3) we have:

$$\underline{\iota}^B = 2/3, \quad \underline{\iota}^I = 1/3, \quad \underline{\iota}^S = 0, \tag{B.12}$$

$$\bar{\iota}^{B}=\frac{2}{3},\quad \bar{\iota}^{IB}=\frac{2}{3}-\frac{1}{3\gamma}\left(\frac{\kappa_L-\tau}{\kappa_L-\kappa_H}\right),\quad \bar{\iota}^{SB}=\frac{2}{3}-\frac{2}{3\gamma}\left(\frac{\kappa_L-\tau}{\kappa_L-\kappa_H}\right). \tag{B.13}$$

Note that $\underline{\mathcal{W}}^B > \underline{\mathcal{W}}^I > \underline{\mathcal{W}}^S$, $\overline{\mathcal{W}}^B > \overline{\mathcal{W}}^{IB} > \overline{\mathcal{W}}^{SB}$, $\underline{\iota}^B > \underline{\iota}^I > \underline{\iota}^S$, and $\bar{\iota}^B > \bar{\iota}^{IB} > \bar{\iota}^{SB}$.

Comparative statics wrt $\boldsymbol{\mu}$. Within a given equilibrium class, neither the government payoff nor informativeness depend on μ. Moreover, it is easy to see that an increase in μ can only induce a switch to an equilibrium with lower informativeness and lower welfare.

Comparative statics wrt $\boldsymbol{V_H}$. It is straightforward to check that $\partial\underline{\mu}/\partial\Delta_V > 0$, $\partial\overline{\mu}/\partial\Delta_V > 0$, and $\partial\underset{\sim}{\mu}/\partial\Delta_V > 0$. Now note that $\Delta_R = \Delta_V + D\left(\kappa_L - \kappa_H\right)$ and that Δ_α, α_H, and $\overline{\gamma}$ do not depend on Δ_V. Hence $\frac{\partial\tilde{\mu}}{\partial\Delta_V} = \frac{1}{\alpha_H}\frac{\partial\tilde{\beta}}{\partial\Delta_R} > 0$, since in the proof of Proposition 5 we have shown that $\tilde{\beta}$ is increasing in Δ_R. Moreover, (B.9) to (B.13) imply that, within an equilibrium class, welfare is increasing in V_H and V_H does not affect informativeness. Hence (i) if an increase in V_H does not change the equilibrium being played, welfare increases and informativeness remains constant; (ii) if an increase in V_H induces a change in the class of equilibrium, it changes for an equilibrium with higher informativeness and higher welfare.

Comparative statics wrt $\boldsymbol{V_L}$. Regarding V_L, the comparative statics for informativeness follow immediately from the comparative statics with respect to V_H, since $\Delta_V = V_H - V_L$ and V_L also does not affect informativeness within an equilibrium class. Moreover, notice that $\partial\underline{\mu}/\partial V_L < 0$, $\partial\overline{\mu}/\partial V_L < 0$, $\partial\underset{\sim}{\mu}/\partial V_L < 0$, and $\partial\tilde{\mu}/\partial V_L < 0$. Suppose $\gamma < \overline{\gamma}$ and initially the economy is in the B equilibrium with μ slightly below $\underline{\mu}$. Fix $\epsilon > 0$ such that an increase of $m \cdot \epsilon$ units in V_L induces a change to the I equilibrium when $m = 1$. For m small enough, the economy remains in B and, from (B.9), welfare increases. Moreover, welfare as a function of m has a discrete downward jump when m is such that $\mu = \underline{\mu}$, since $\underline{\mathcal{W}}^B > \underline{\mathcal{W}}^I$.

Comparative statics wrt $\boldsymbol{D}$. It is straightforward to check that $\partial\underline{\mu}/\partial D < 0$, $\partial\overline{\mu}/\partial D < 0$, and $\partial\underset{\sim}{\mu}/\partial D < 0$. Also, inspecting (B.8) one can see that an increase in Δ_V is equivalent to a decrease in D and therefore $\frac{\partial\tilde{\mu}}{\partial D} < 0$. Moreover, within an equilibrium class, welfare is decreasing in D and does not affect informativeness. Hence, using (B.9) to (B.13) it is easy to see that (i) if an increase in D does not change the equilibrium being played, welfare falls and informativeness remains constant; (ii) if an increase in D induces a change in the class of equilibrium being played, it changes for an equilibrium with lower informativeness and lower welfare.

Comparative statics wrt $\boldsymbol{\kappa_H}$. Note that a sufficiently large increase in κ_H can change whether γ is smaller or larger than $\overline{\gamma}$. First, consider the effect of an increase in κ_H that does not change the ranking between γ and $\overline{\gamma}$. Note that $\partial\underline{\mu}/\partial\kappa_H < 0$, $\partial\overline{\mu}/\partial\kappa_H < 0$, $\partial\underset{\sim}{\mu}/\partial\kappa_H < 0$, and $\partial\tilde{\mu}/\partial\kappa_H < 0$, and that welfare is strictly decreasing and informativeness is either constant or strictly decreasing in κ_H within an equilibrium class. Hence, we have that the increase in κ_H in some cases reduces informativeness and welfare, but never increases them. Now consider an increase in κ_H that changes the ranking between γ and $\overline{\gamma}$. Since $\overline{\gamma}$ is increasing in κ_H it must be that before the increase we had $\gamma > \overline{\gamma}$. One can verify that when $\gamma = \overline{\gamma}$ either we have

$$\tilde{\mu} > \underset{\sim}{\mu} \geq \overline{\mu} > \underline{\mu} \quad \text{or} \quad \tilde{\mu} > \overline{\mu} > \underset{\sim}{\mu} > \underline{\mu}. \tag{B.14}$$

This condition implies that equilibrium switches due to this increase in κ_H can only happen (i) from B to B; (ii) from B to I; (iii) from B to S; (iv) from IB to I; (v) from IB to S; (vi) from SB to S. Since we have shown that any increase in κ_H that does not change the ranking between γ and $\overline{\gamma}$ does not increase informativeness and welfare, to get the desired result it then suffices to show that

$$\lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^B, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^I, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^S, \tag{B.15}$$

$$\lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^{IB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^I, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^{IB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^S, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\mathcal{W}}^{SB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\mathcal{W}}^S, \tag{B.16}$$

$$\lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^B, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^I, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^B \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^S, \tag{B.17}$$

$$\lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^{IB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^I, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^{IB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^S, \qquad \lim_{\gamma\to\overline{\gamma}^+} \overline{\iota}^{SB} \geq \lim_{\gamma\to\overline{\gamma}^-} \underline{\iota}^S, \tag{B.18}$$

which can be easily verified to hold using equations (B.9) to (B.13).

Comparative statics wrt $\boldsymbol{\tau}$. Note that $\overline{\gamma}$ is decreasing in τ, $\partial\underline{\mu}/\partial\tau = \partial\overline{\mu}/\tau = \partial\underset{\sim}{\mu}/\partial\tau = 0$, and $\partial\tilde{\mu}/\partial\tau > 0$. From (B.12) and (B.13) one can verify that an increase in τ that does not change the equilibrium class leads either to an increase or no change in informativeness. If the increase in τ induces a move from SB to IB, informativeness increases. Condition (B.14), (B.17), and (B.18) imply that any other change in equilibrium induced by an increase in τ cannot reduce informativeness. Regarding welfare, suppose $\gamma > \overline{\gamma}$ and initially the economy is in the SB equilibrium with μ slightly above $\tilde{\mu}$. Fix $\epsilon > 0$ such that an increase of $m \cdot \epsilon$ units in τ induces a change to IB when $m = 1$. For m small enough, the economy remains in SB and, from (B.11), welfare falls. However, welfare as a function m has a discrete upward jump when m is such that $\mu = \tilde{\mu}$, since $\overline{\mathcal{W}}^{IB} > \overline{\mathcal{W}}^{SB}$.

Comparative statics wrt $\boldsymbol{\kappa_L}$. First note that $\partial\underset{\sim}{\mu}/\partial\kappa_L > 0$. Assume $\gamma > \overline{\gamma}$ and that initially the economy is in IB with μ slightly above $\underset{\sim}{\mu}$. Fix $\epsilon > 0$ such that an increase of $m \cdot \epsilon$ units in κ_L induces a change to B when $m = 1$. For m small enough, the economy remains in IB and, from (B.10) and (B.13), welfare and informativeness fall. Moreover, welfare and informativeness as a function m has a discrete upward jump when m is such that $\mu = \underset{\sim}{\mu}$, since $\overline{\mathcal{W}}^B > \overline{\mathcal{W}}^{IB}$. □

B.8 Proof of Proposition 9

Note that (9) can be rewritten as $W = \left[k_L - q\tilde{\Delta}_\kappa - \tau\right] I + \Omega$. Hence the policymaker intervenes if $\tau < \kappa_L - q\tilde{\Delta}_\kappa$ and does not intervene if $\tau > \kappa_L - q\tilde{\Delta}_\kappa$. Suppose $\kappa_L \neq \tau$. When $\tilde{\Delta}_\kappa$ is sufficiently close to zero, the policymaker either intervenes or does not intervene regardless of its belief q, so the action played by the trader does not affect the intervention decision. Therefore, the speculator either receives β_θ or does not receive it regardless of her action, and the equilibrium can be computed as in the benchmark case of Proposition 1, only with a constant β_θ being added to speculators' payoffs in each state θ. □

B.9 Proof of Proposition 10

Consider $\gamma < \overline{\gamma}$ and recall the definitions of δ_1, δ_2, and δ_3 used in Proposition A.1. Since in the liquidity support model we have that $\Delta_R + \Delta_\alpha = \Delta_V$, $\lim_{\Delta_V \to 0} \delta_1 = \lim_{\Delta_V \to 0} \delta_2 = \lim_{\Delta_V \to 0} \delta_3 = 0$, and thus for any $\mu > 0$ we have that $\beta_H = \mu D \kappa_H > \delta_3$. Hence, from Proposition A.1, the unique equilibrium when $\gamma < \overline{\gamma}$ is the S equilibrium. Now consider $\gamma > \overline{\gamma}$ and recall the definitions of φ_1, φ_2, and φ_3 used in Proposition A.2. One can verify that $\lim_{\Delta_V \to 0} \varphi_1 = (1-\gamma) D \tilde{\Delta}_\kappa$ and $\lim_{\Delta_V \to 0} \varphi_2 = \lim_{\Delta_V \to 0} \varphi_3 = (1-\overline{\gamma}) D \tilde{\Delta}_\kappa$. The equilibrium characterization in this case follows from Proposition A.2 using these limit cutoffs and the fact that $\beta_H = \mu \alpha_H = \mu D \kappa_H$. □

B.10 Proof of Proposition 11

Suppose the policymaker commits to a minimum assistance of $\underline{A}$ dollars. Define $\hat{D} \equiv D - \underline{A}$ and denote by $a \in \left[0, \hat{D}\right]$ the additional assistance provided after the observation of market activity. We can write the shareholder return (given $\underline{A}$) as $\pi(\theta, a) = V_\theta - a - \frac{D-a}{1-\psi_\theta} + \frac{\psi_\theta}{1-\psi_\theta}\underline{A}$, and the policymaker's payoff given beliefs q as $W = [q\kappa_H + (1-q)\kappa_L - \tau](a + \underline{A}) + \Omega$. Note the policymaker's decision is to set $a = \hat{D}$ if $q < \overline{\gamma} = \frac{\kappa_L - \tau}{\tilde{\Delta}_\kappa}$ and 0 if $q > \overline{\gamma}$. Again, we can map this into a binary decision $G \in \{0, 1\}$. With no (additional) assistance, shareholder return is $R_\theta = V_\theta - \underline{A} - (1 + \kappa_\theta)\hat{D}$, and her additional benefit when $G = 1$ is $\alpha_\theta = \kappa_\theta \hat{D}$. Hence, for the purpose of computing the equilibrium, we have the same game as in Section 4.3, only replacing D with $\hat{D} < D$ in the expressions for Δ_R, Δ_α, and β_H ($\overline{\gamma}$ is unchanged). It then follows immediately from the proof of the comparative statics with respect to D (Appendix B.7) that commitment to a minimum assistance $\underline{A}$ increases informativeness whenever it drives a switch across equilibrium classes (informativeness remains constant otherwise).

We now show that commitment to some $\underline{A} > 0$ can increase welfare with an example. Consider $\gamma < \overline{\gamma}$ and $\underline{\mu} = \frac{(1-\gamma)\Delta_V}{D\kappa_H} < \mu < \frac{2(1-\gamma)\Delta_V}{D\kappa_H} \equiv \mu_{\max}$: with $\underline{A} = 0$, the equilibrium is the I equilibrium and welfare is given by $\underline{\mathcal{W}}^I$ in (B.9). If the policymaker commits to some $\underline{A} > 0$ that triggers the B equilibrium, welfare is given by

$$\mathcal{W}^B_{\text{com}}(\underline{A}) \equiv \gamma V_H + (1-\gamma)V_L - D\left[\frac{2\gamma}{3}\kappa_H + \left(1 - \frac{2\gamma}{3}\right)\tau\right] - \frac{2\gamma}{3}\underline{A}(\tau - \kappa_H). \tag{B.19}$$

The first two terms correspond to what welfare would be in the B equilibrium with $\underline{A} = 0$ (i.e., $\underline{\mathcal{W}}^B$ in (B.9)); the last term captures the additional cost of commitment: with probability $\frac{2\gamma}{3}$, the policymaker spends resources $\underline{A}\tau$ to avoid a fire sale with social cost $\underline{A}\kappa_H < \underline{A}\tau$. Note $\mathcal{W}^B_{\text{com}}(\underline{A})$ decreases in $\underline{A}$. The smallest $\underline{A}$ that triggers the B equilibrium is $\underline{A}^B_{\min} \equiv D - \frac{(1-\gamma)\Delta_V}{\mu\kappa_H}$. One can verify that the welfare gain from committing to $\underline{A}^B_{\min}$ (versus setting $\underline{A} = 0$ and staying in the I equilibrium) is $\frac{\gamma}{3}(\tau - \kappa_H)\left(\frac{2(1-\gamma)\Delta_V}{\mu\kappa_H} - D\right)$, which is strictly positive since $\mu < \mu_{\max}$. □

B.11 Optimal minimal liquidity support (Figure 8)

In this appendix we characterize the optimal minimal assistance for $\gamma < \overline{\gamma}$. Consider $\gamma < \overline{\gamma}$. It follows from the proof of Proposition 11 that for $\mu \in \left(\underline{\mu}, \mu_{\max}\right)$ the policymaker commits to $\underline{A}^B_{\min} = D - \frac{(1-\gamma)\Delta_V}{\mu\kappa_H}$. Note that for $\mu \in (\mu_{\max}, \overline{\mu}]$ committing to $\underline{A}^B_{\min}$ decreases welfare, and therefore the optimal minimal assistance is $\underline{A} = 0$. (At $\mu = \mu_{\max}$, both $\underline{A} = 0$ and $A = \underline{A}^B_{\min}$ yield the same government payoff.) Also, for $\mu \leq \underline{\mu}$, the optimal minimal assistance is 0 since $\mathcal{W}^B_{\text{com}}(\underline{A})$ in (B.19) is decreasing in $\underline{A}$. Consider now $\mu > \overline{\mu}$. For $\underline{A} = 0$, the S equilibrium is played and welfare is given by $\underline{\mathcal{W}}^S$ in (B.9). If the policymaker commits to some $\underline{A} > 0$ that triggers the B equilibrium, welfare is given by $\mathcal{W}^B_{\text{com}}(\underline{A})$ in (B.19), and the smallest $\underline{A}$ that triggers the B equilibrium is $\underline{A}^B_{\min}$. If the policymaker commits to some $\underline{A} > 0$ that triggers the I equilibrium, welfare is given by

$$\mathcal{W}^I_{\text{com}}(\underline{A}) \equiv \gamma V_H + (1-\gamma)V_L - D\left[\frac{\gamma}{3}\kappa_H + \left(1 - \frac{\gamma}{3}\right)\tau\right] - \frac{\gamma}{3}\underline{A}(\tau - \kappa_H),$$

and the minimum $\underline{A}$ that leads to I is $\underline{A}^I_{\min} \equiv D - \frac{(3-2\gamma)\Delta_V}{\mu\kappa_H}$. One can verify that $\mathcal{W}^I_{\text{com}}\left(\underline{A}^I_{\min}\right) > \mathcal{W}^B_{\text{com}}\left(\underline{A}^B_{\min}\right) > \underline{\mathcal{W}}^S$ for $\mu > \overline{\mu}$, and thus the optimal minimal support for $\mu > \overline{\mu}$ is $\underline{A}^I_{\min}$. □