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Identifying Aggregate Shocks with Micro-level Heterogeneity: Financial Shocks and Investment Fluctuation

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Abstract

This paper identifies aggregate financial shocks and quantifies their effects on business investment based on an estimated DSGE model with firm-level heterogeneity. On average, financial shocks contribute only 3% of the variation in U.S. public firms' aggregate investment. The negligible aggregate relevance of financial shocks mainly results from the interaction between firm-level heterogeneity and general equilibrium effects. Following a contractionary financial shock, financially constrained firms are directly forced to cut investment, which dampens the aggregate investment demand and lowers the price of capital good. A lower capital good price motivates the financially unconstrained firms to invest more, which largely cancels out the financial shock's direct effect on aggregate. If I abstract from firm-level heterogeneity, the implied relevance of financial shocks to aggregate investment is 15 times larger. This sharp difference indicates that representative firm models could overstate the relevance of financial shocks in driving the business cycle fluctuations, and highlights the important role played by the interaction between of micro-level heterogeneity and general equilibrium effects in shaping the transmission of aggregate shocks.

Topics: Business fluctuations and cycles; Firm dynamics

JEL codes: E12, E22, G31, G32

Résumé

L'auteur met en évidence des chocs financiers globaux et en quantifie les effets sur les investissements des entreprises à partir d'un modèle d'équilibre général dynamique et stochastique (EGDS) estimé dans lequel les entreprises sont hétérogènes. En moyenne, les chocs financiers ne sont responsables que de 3 % des variations de l'investissement global des sociétés ouvertes américaines. Ce pourcentage négligeable est principalement attribuable à l'interaction entre l'hétérogénéité des entreprises et les effets d'équilibre général. En effet, après un choc financier restrictif, les entreprises soumises à des contraintes financières sont immédiatement forcées de réduire leurs dépenses en capital, ce qui freine la demande globale d'investissement et fait baisser le prix des biens d'équipement. Une telle baisse de prix motive les autres entreprises à investir davantage, ce qui annule en grande partie l'effet direct du choc financier dans l'ensemble. Si l'on fait abstraction de l'hétérogénéité des entreprises, l'incidence implicite des chocs financiers sur les investissements globaux est 15 fois plus grande. Ce vaste écart indique que les modèles avec entreprises représentatives pourraient mener à une surestimation de l'incidence des chocs financiers sur les fluctuations du cycle économique et met en lumière l'importante influence de l'interaction entre l'hétérogénéité microéconomique et les effets d'équilibre général sur la transmission des chocs globaux.

Sujets : Cycles et fluctuations économiques; Dynamique des entreprises

Codes JEL : E12, E22, G31, G32

Non-technical Summary

Business investment is the most volatile component of GDP. Fluctuations in business investment could be driven either by shocks to firms' investment profitability, or by shocks to their financing conditions. It is important for policy makers to know which shock generates the changes in business investment to make the stabilization policies.

In previous studies, the models used for identifying financial shocks feature a representative firm whose marginal investment relies on external financing. However, cross-sectional evidence reveals that the investment decisions of a large fraction of firms do not depend on external financing. This paper contributes to the literature by incorporating this cross-sectional heterogeneity into the identification of financial shocks.

I estimate a DSGE model with firm-level heterogeneity and use it to evaluate the relevance of aggregate financial shocks for fluctuations in aggregate investment. I find that the average contribution of financial shocks to the fluctuations of U.S. public firms' aggregate investment is only 1/15 of the contribution implied by the comparable representative firm model. The results of this paper imply that the interaction between micro-level heterogeneity and general equilibrium effects plays an important role in shaping the transmission of aggregate shocks, and representative firm models could have overstated the aggregate relevance of financial shocks in driving business investment fluctuations.

1 Introduction

Investment is the most volatile component of GDP. Shocks to firms' financing cost and capacity have often been considered as a source of business investment fluctuations, but how much they matter remains an open question. The observed variation in business investment is a joint result of the unobservable shocks to firms' financing conditions and investment profitability, and identifying financial shocks is the key to quantify their relevance to investment fluctuations. In previous studies (for example, Jermann and Quadrini, 2012), the DSGE models used for identifying financial shocks are all featured with financially constrained representative firms. But as reveled by the micro-level evidence, there is a significant heterogeneity across firms in terms of how constrained they are (for example, Zetlin-Jones and Shourideh, 2017).

This paper contributes to the literature by incorporating firm-level heterogeneity into the identification of aggregate financial shocks. To characterize both the cross-sectional and cross-time variations in firms' investment and financing, I build a general equilibrium model with three components: a continuum of heterogeneous firms facing financial frictions, a group of representative agents featured with New Keynesian setups, and eight aggregate shocks.

The block of heterogeneous firms is designed to generate cross-sectional variation in firms' investment and financing. Firms are ex-ante homogeneous and their ex-post heterogeneity results from a mean-reverting idiosyncratic technology process. Firms make their choices based on the aggregate economic conditions and their idiosyncratic states. Their production technology has decreasing returns to scales (DRTS) and they face two financial frictions: a collateral constraint imposed on their debt issuance and a cost associated with their equity issuance. This combination of mean-reverting idiosyncratic technology process, DRTS technology, and financial frictions generates a "pecking-order" in firms' financing choices.

The block of New Keynesian agents is designed to capture the endogenous variation in the aggregate economic conditions faced by the firms. This block is featured with sticky prices and wages, external habit formation in consumption, and adjustment costs in capital goods production. Within the eight aggregate shocks, there are two financial shocks which separately capture the exogenous variations in the tightness of collateral constraint and the cost of equity issuance. The remaining six aggregate shocks capture the exogenous variations in aggregate productivity, price markup, wage markup, the efficiency of transforming final goods to investment goods, households' inter-temporal substitution preference, and monetary policy. The combination of these frictions and shocks help the model to match the cyclical variation in firms' investment profitability, which serves as an important control for identifying the financial shocks.

The model is quantified in two steps. First, I calibrate a group of parameters to match the life-cycle profile of firms' investment and financing in steady state with the corresponding moments from Compustat. Then I use a Bayesian likelihood method to estimate the rest of parameters to match the time-variations in both the quantities and prices at the aggregate level, and the U.S. public firms' financing choices at the disaggregate level.

With the estimated model, I quantify the effects of financial shocks on the U.S. public firms' investment. Financial shocks play an important role in driving the investment fluctuations at the disaggregate level, but not so at the aggregate level. On average, financial shocks contribute only 3% of the variance of U.S. public firms' aggregate investment. If I remove the firm-level heterogeneity, the estimated financial shocks are smaller, but much more relevant to aggregate investment fluctuations. Financial shocks contribute 46% of the variance of aggregate investment in the comparable representative firm model. This sharp difference in the implied aggregate relevance of financial shocks highlights a feature of heterogeneous firm model: the strong interaction between micro-level heterogeneity and general equilibrium effects in the transmission of financial shocks.

In the model, firms can be divided into two groups: financially constrained and unconstrained firms. In an episode with a contractionary financial shock, the shock directly hits the constrained firms and forces them to cut investment. The lower investment demand dampens the capital good price, which increases the return to investment. On average, the unconstrained firms are larger and have ample financing capacity to seize the profitable investment opportunity from the dampened capital good price. Therefore, the unconstrained firms' investment response largely cancels out the constrained firms' investment response in aggregation, leading to a negligible relevance of financial shocks to aggregate investment fluctuations. As a comparison, all firms are constrained in the representative firm model. Because constrained firms have little extra financing capacity to seize the investment opportunities from the general equilibrium effects, the structure of representative firm model largely mutes the investment responses to general equilibrium effects and implies a much larger aggregate relevance of financial shocks.

This paper contributes to the literature by incorporating firm-level heterogeneity into the identification of aggregate shocks. With the estimated model, I find that the implied relevance of financial shocks to aggregate investment fluctuations in heterogeneous firm model is much smaller than that in representative firm model. The analysis about the source of this difference highlights an important feature distinguishing the heterogeneous firm model from the representative firm model: general equilibrium effects would play a much more important role in shaping the transmission of aggregate shocks when they interact with the micro-level heterogeneity.

Related Literature This paper mainly contributes to two branches of literature.

First, this paper contributes to the literature that focuses on identifying the aggregate shocks to firms' financing conditions (see, for example, Justiniano *et al.*, 2011, Jermann and Quadrini, 2012, Christiano *et al.*, 2014, Eisfeldt and Muir, 2016). Compared with Justiniano *et al.* (2011), Jermann and Quadrini (2012), Christiano *et al.* (2014), which identify the shocks based a representative firm DSGE model and aggregate time-series, this paper incorporates firm-level heterogeneity and cross-sectional evidence into the identification. Eisfeldt and Muir (2016) also incorporated firm-level heterogeneity into the identification of aggregate financial shocks, but there is no general equilibrium feedback in their structural model. As illustrated in this paper, the interaction between firm-level heterogeneity and general equilibrium effects can lead to a significantly different implication about the aggregate relevance of financial shocks.

Second, this paper contributes to the literature that discusses how firm-level heterogeneity shapes the transmission of aggregate shocks (see, for example, Khan and Thomas, 2013, Buera and Moll, 2015, Zetlin-Jones and Shourideh, 2017, Ottonello and Winberry, 2018). Due to the shared interests in financial shocks, this paper is closely related with Khan and Thomas (2013) and Zetlin-Jones and Shourideh (2017). Without the general equilibrium feedback in the price of capital good, Khan and Thomas (2013) finds a much larger effect of financial shocks on aggregate quantities comparing with the results in this paper. Zetlin-Jones and Shourideh (2017) also finds the strong interaction between firm-level heterogeneity and general equilibrium effects, where financial shocks' impact is dampened by the general equilibrium feedback in the real interest rate.

Other than the above two groups of literature, this paper is also related to the literature studying the New Keynesian DSGE model with heterogeneous agents (see, for example, Kaplan *et al.*, 2018, Bayer and Luetticke, 2020). This paper has the similar finding with Kaplan *et al.* (2018) in terms of the transmission of aggregate shocks in heterogeneous agent model. I also find that general equilibrium effects play a much more important role in shaping the transmission of aggregate shocks in heterogeneous agent model comparing with representative agent model. This paper also shares the similar interest with Bayer and Luetticke (2020) to estimate a New Keynesian DSGE model with micro-level heterogeneity, but we focus on different types of heterogeneity.

Road Map The remainder of this paper is organized as follows. Section 2 lays out the model setup. Section 3 presents the calibration and quantification of the model. Section 4 discusses the main findings, and Section 5 concludes.

2 Model

This model consists of three blocks: a block with heterogeneous firms facing financial frictions, a block with representative agents and New Keynesian frictions, and eight aggregate shocks. The heterogeneous firm block endogenously generates the cross-sectional variations, the New Keynesian block endogenously determines the time-variations in aggregate quantities and prices faced by the heterogeneous firms, and the aggregate shocks capture the exogenous source of variations in firms' financing conditions and investment profitability over the business cycle.

2.1 Heterogeneous Firm Block

In this economy, there is a continuum of heterogeneous firms indexed by $j \in [0, 1]$. They produce homogeneous intermediate goods and sell them in a competitive market.

Idiosyncratic State For each incumbent firm j, there are three idiosyncratic state variables revealed at the beginning of each period t: $\mathcal{S}_{j,t} \equiv (d_{j,t}, k_{j,t}, a_{j,t})$. The nominal debt $d_{j,t}$ and capital stock $k_{j,t}$ and are predetermined in period $t-1$. The idiosyncratic productivity $a_{j,t}$ is exogenous and its evolution follows

$$\ln a_{j,t} = \rho_a \cdot \ln a_{j,t-1} + \sigma_a \cdot \varepsilon^a_{j,t},\ \ \varepsilon^a_{j,t} \overset{i.i.d}{\sim} \mathcal{N}(0, 1). \tag{1}$$

Exit and Entry After the realization of idiosyncratic states, firm j will receive an i.i.d. exogenous exit shock $\varepsilon^e_{j,t} \sim \texttt{Bernoulli}(\xi)$. If $\varepsilon^e_{j,t} = 1$, firm j has to be liquidated and the shareholders exit the market with liquidation value $\mathcal{LV}_t(\mathcal{S}_{j,t}) \equiv [k_{j,t} - \Phi_k(0, k_{j,t})] \cdot q_t - \frac{d_{j,t} \cdot R_{t-1}}{\pi_t}$, where q denotes the real capital good price, π denotes the gross inflation, $\Phi_k(0, k)$ is the adjustment cost when a firm fully uninstalls its capital, and R is the gross nominal interest rate. If $\varepsilon^e_{j,t} = 0$, firm j can stay in operation, producing, financing through debt and equity, and investing in physical capital. Right after the exit of incumbents, a group of entrants enter the market and operate as the same as the surviving incumbents do. The number of entrants is equal to the number of exiting incumbents, so the firm population keeps constant over time. The distribution of entrants is set as

$$\mathcal{P}^{ent}_t(d, k, a) = \phi\left(\frac{d/k - \mu^0_d}{\sigma^0_d}\right) \cdot \phi\left(\frac{\ln k - t \cdot \ln \Gamma - \mu^0_k}{\sigma^0_k}\right) \cdot \phi\left(\frac{\ln a - \mu^0_a}{\sigma^0_a}\right), \tag{2}$$

where $\phi(\cdot)$ is the p.d.f. of standard normal distribution and Γ is the average gross growth rate of entrants' capital stock.

Technology, Capital Accumulation, and Budget Constraint Firm j in operation produces intermediate goods $\hat{y}_{j,t}$ with the technology of

$$\hat{y}_{j,t} = \Gamma^{(1-\theta)t} \cdot \exp(\eta_{z,t}) \cdot a_{j,t} \cdot \left(k^\alpha_{j,t} \cdot l^{1-\alpha}_{j,t}\right)^\theta, \ \alpha \in (0,1), \ \theta \in (0,1), \tag{3}$$

where $\eta_{z,t}$ denotes the exogenous variation in aggregate productivity and $l_{j,t}$ is the labor input of firm j. $\Gamma^{1-\theta}$ is the exogenous gross growth rate in aggregate TFP, and Γ will be the gross growth rate of aggregate output and capital stock in steady state. Firm j needs to make investment decision $i_{j,t}$ to accumulate its physical capital following:

$$k_{j,t+1} = (1-\delta) \cdot k_{j,t} + i_{j,t} - \Phi_k(k_{j,t+1}, k_{j,t}), \tag{4}$$

where $\Phi_k(k_{j,t+1}, k_{j,t})$ is the capital adjustment cost, which captures the extra managerial effort required for adjusting the scale of production. The capital adjustment cost is constructed as

$$\Phi_k\left(k', k\right) = \begin{cases} \frac{\phi_+^k}{2} \cdot \left(\frac{k'}{k} - \Gamma\right)^2 \cdot k & \text{if } k' \geq k \\ \frac{\phi_-^k}{2} \cdot \left(\frac{k'}{k} - \Gamma\right)^2 \cdot k & \text{if } k' \geq k \end{cases}. \tag{5}$$

To finance its investment expenditure, firm j can raise funding from its profits, one-period nominal debt $d_{j,t+1}$ and equity $e_{j,t}$. The budget constraint for firm j is:

$$i_{j,t}q_t = (1-\tau) \cdot \underbrace{\left[\hat{y}_{j,t}\hat{p}_t - l_{j,t}w_t\right]}_{\text{pre-tax profit}} + \underbrace{\tau \cdot \delta \cdot k_{j,t}q_t}_{\text{tax rebate for depreciation}} + \underbrace{e_{j,t}}_{\text{equity financing}} + \underbrace{d_{j,t+1} - \frac{d_{j,t} \cdot \hat{R}_{t-1}}{\pi_t}}_{\text{debt financing}}, \tag{6}$$

where $\hat{p}$ denotes the price of homogeneous intermediate good in real term, w denotes the real wage, τ denotes the corporate tax rate imposed on firms' profit, and $\hat{R}_t \equiv 1 + (1-\tau) \cdot (R_t - 1)$ denotes the post-tax gross nominal interest rate. When $e_{j,t} \leq 0$, it means that firm j is paying dividend; when $e_{j,t} > 0$, it means that firm j is issuing new equity to external shareholders.

Financial Frictions Given the quantitative purpose of this paper, I abstract from the micro-foundation and model the financial frictions in the reduced-form way following the literature (e.g., Jermann and Quadrini, 2012, Khan and Thomas, 2013, Warusawitharana and Whited, 2016): when firm j issues equity, its existing shareholders have to pay an extra cost $\phi_t^e \cdot e_{j,t}$ [1]; when firm j issues debt, the debt issuance is subject to the collateral constraint [2]

$$d_{j,t+1} \leq \phi_t^d \cdot k_{j,t} \cdot q_t. \tag{7}$$

[1]This parsimonious equity issuance cost combines both the explicit cost, i.e. floatation cost (Altinkilic and Hansen, 2000), and the implicit costs including the cost due to adverse selection premium (Myers and Majluf, 1984) and market misvaluation (Warusawitharana and Whited, 2016).

[2]The construction of the collateral constraint follows the similar setup widely used in the literature (e.g. Kiyotaki and Moore, 1997, Jermann and Quadrini, 2012, Khan and Thomas, 2013). I construct the collateral constraint based on the existing capital stock $k_{j,t}$ rather than the future capital stock $k_{j,t+1}$ mainly for a quantitative reason. If the collateral constraint is based on future capital stock $k_{j,t+1}$, there will be a stronger leverage effect, i.e. a given flow of equity financing can be levered into a much larger flow of debt financing, which contradicts with the data, where debt financing flows are generally smaller than equity financing flows among the equity issuing firms.

Both the cost of issuing equity ϕ_t^e and the tightness of collateral constraint ϕ_t^d are time-varying[3] and their time-variation are parameterized as:

$$\phi_t^x = \bar{\phi}^x \cdot \exp\left(\eta_{x,t}/\bar{\phi}^x\right), \; \forall x \in \{e, d\}, \tag{8}$$

where $\eta_{x,t}$ and $\bar{\phi}^x$ denote the time variation and steady-state level of financial frictions.

Decision Problem In each period, firms in operation have to make their decisions of labor hiring, investment, debt issuance, and equity financing to maximize the net present value of the payouts to their existing shareholders. The recursive representation of their decision problem is:

$$V_t(\mathcal{S}) = \max_{l,i,d',k',e} -e \cdot (1 + \phi_t^e \cdot \mathbb{1}_{e>0}) + \mathbb{E}_t\left[\Lambda_{t,t+1} \cdot \left[\xi \cdot \mathcal{L}\mathcal{V}_{t+1}(\mathcal{S}') + (1-\xi) \cdot V_{t+1}\left(\mathcal{S}'\right)\right] |\mathcal{S}\right] \tag{9}$$

s.t. : technology (3), budget constraint (6), capital accumulation (4), collateral constraint (7) ,

where $V_t(\mathcal{S})$ is the firms' real value in period t, and Λ denotes the real discounting factor (SDF) that will be determined by households' preference. The firms' value comes from two parts: the flow from equity payout, and the discounted future value. The future value is the weighted average of the liquidation value and the continuation value from continuing operation in the next period. The subscript t in firms' value indicates the dependence of firms' value on the aggregate economic conditions, which will be endogenously determined by the New Keynesian block.

2.2 New Keynesian Block

This block combines a set of representative agents facing the New Keynesian frictions as in Justiniano *et al.* (2011). The agents' decisions in this block play two roles in this paper. First, they model the endogenous variations of aggregate prices, which matter for the transmission of financial shocks from the qualitative perspective. Second, they provide a structure to build in both the exogenous and endogenous variations in aggregate quantities and prices, so we can match the variation in firms' investment profitability, which is important for identifying the financial shocks

[3] The evidence for time-varying equity issuance cost can be found in (Choe *et al.*, 1993) and (Baker and Wurgler, 2007). The tightness of collateral constraint reflects supply condition in debt financing market, and the time-varying supply condition in debt financing market can be found in (Becker and Ivashina, 2014).

from the quantitative perspective. Given that this block is quite standard in the New Keynesian literature, I will sketch out the setup without elaborating on the detailed decision problems (see Appendix B.1 for more details).

Retailers There is a continuum of retailers indexed by $h \in [0,1]$. Retailers produce differentiated retail goods from homogeneous intermediate goods with the technology specified as:

$$y_{h,t} = \hat{y}_{h,t}, \tag{10}$$

where $y_{h,t}$ and $\hat{y}_{h,t}$ denote the quantity of firm h's output and input. Each retailer h has monopolistic power, and following Calvo (1983), there is a probability of $1-\xi_p$ for retailer h to get the opportunity to reset its nominal prices in each period. If retailer h does not get the opportunity, it will set its nominal price following an automatic indexation rule $P_{h,t} = P_{h,t-1} \cdot \pi_t^{\iota_p} \cdot \bar{\pi}^{1-\iota_p}$, where $P_{h,t}$ denotes the nominal price for good h and $\bar{\pi}$ denotes the steady-state level of gross inflation rate. The total demand for homogeneous intermediate goods is denoted as $\hat{Y}_t$.

Final Good Producers There is a representative final good producer who produces final good Y_t by packing differentiated retail goods $\{y_{\iota,t}\}_{\iota\in[0,1]}$ through a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int y_{h,t}^{\frac{1}{\gamma_p+\eta_{p,t}}} dh \right)^{\gamma_p+\eta_{p,t}}, \tag{11}$$

where γ_p is the price markup in steady state and $\eta_{p,t}$ is the exogenous variation of the price markup. The final good market is perfectly competitive, and the nominal price of the final goods is denoted as P_t.

Households There is a representative household who consumes final good C_t, supplies labor $\hat{N}_t$, owns all the firms, and saves in one-period nominal bonds B_{t+1}. The utility function of the household is specified as

$$\sum_{t=0}^{\infty} \beta^t \cdot \exp\left(\eta_{u,t}\right) \cdot \left[\frac{\left({}^{C_t}/_{\Gamma^t} - \lambda \cdot {}^{C_{t-1}}/_{\Gamma^{t-1}}\right)^{1-\nu_c}}{1-\nu_c} - \Psi \cdot \frac{N_t^{1+\nu_l}}{1+\nu_l} \right], \tag{12}$$

where β is the discounting factor, $\eta_{u,t}$ is the exogenous variation in the households' inter-temporal substitution decision, and λ is the parameter controlling the external consumption habit formation. Here, consumption C_t enters the utility function with being detrended to ensure a balanced growth path. The budget constraint for the household is

$$C_t + B_{t+1} = \hat{N}_t \cdot \hat{w}_t + {}^{B_t \cdot R_{t-1}}/_{\pi_t} + T_t, \tag{13}$$

where $\hat{w}_t$ is the real wage to the household's labor, and T_t is the lump-sum transfer such that the bond market clears.

Labor Union There is a continuum of labor unions, indexed by $s \in [0,1]$, which purchase the homogeneous labor supply $\hat{n}_{s,t}$ from the representative household and transform it as heterogeneous intermediate labor service $n_{s,t}$ with the technology

$$n_{s,t} = \hat{n}_{s,t}. \tag{14}$$

Each union s has monopolistic power, and there is a probability of $1 - \xi_w$ for union s to get the opportunity to reset the nominal wage of its specialized labor service. If union s does not get the opportunity to reset, it will set its nominal wage following an automatic indexation rule $W_{s,t} = W_{s,t-1} \cdot \pi_t^{\iota_w} \cdot \bar{\pi}^{1-\iota_w} \cdot \Gamma$.

Labor Packer There is a representative labor packer that packages the heterogeneous types of labor supply $\{n_{s,t}\}_{s \in [0,1]}$ as the final labor service L_t with the technology:

$$L_t = \left(\int n_{s,t}^{\frac{1}{\gamma_w + \eta_{w,t}}} \right)^{\gamma_w + \eta_{w,t}},$$

where γ_w is the wage markup in the steady state and $\eta_{w,t}$ is the exogenous variation in the wage markup. The market for final labor service is perfectly competitive, and the real wage of final labor service is w_t.

Investment Good Producers There is a representative investment good producer that produces investment good $\hat{I}_t$ from final good Y_t^I with the technology

$$\hat{I}_t = \exp\left(-\eta_{q,t}\right) \cdot Y_t^I, \tag{15}$$

where $\eta_{q,t}$ is the exogenous variation in the efficiency of transforming final goods into investment goods. The investment good market is competitive, and the real investment good price is $\hat{q}_t$.

Capital Good Producers There is a representative capital good producer that produces capital good I_t from investment good $\hat{I}_t$ with the technology specified as:

$$I_t = \left[1 - S\left(\frac{\hat{I}_t}{\hat{I}_{t-1}}\right)\right] \cdot \hat{I}_t, \tag{16}$$

where $S(\cdot)$ is the function characterizing the adjustment cost, and the adjustment cost function is assumed to satisfy $S(1) = S'(1) = 0$ and $S''(1) > 0$. The capital good market is perfectly competitive and the real price of capital good is denoted as q_t.

Monetary Authority The monetary policy is assumed to follow:

$$\ln R_t - \ln \bar{R} = \lambda_R \cdot \left(\ln R_{t-1} - \ln \bar{R}\right) + \lambda_\pi \cdot \left(\ln \pi_t - \ln \bar{\pi}\right) + \eta_{m,t}, \tag{17}$$

where $\bar{R}$ denotes the level of gross nominal interest rate in steady state, the gross inflation rate $\pi_t \equiv \frac{P_t}{P_{t-1}}$, and $\eta_{m,t}$ denotes the exogenous variations in nominal interest rate.

2.3 Aggregate Shocks

There are eight exogenous variables in this model and their evolution follow the AR(1) process:

$$\eta_{x,t} = \rho_x \cdot \eta_{x,t-1} + \sigma_x \cdot \varepsilon_{x,t}, \ \forall x \in \{e, d, z, p, w, q, m, u\}, \tag{18}$$

where the independent exogenous variations $\varepsilon_{x,t} \overset{i.i.d}{\sim} \mathcal{N}(0,1)$ are the aggregate shocks to this economy. Within these eight aggregate shocks, two of them, $\varepsilon_{e,t}$ and $\varepsilon_{d,t}$, are the financial shocks capturing the exogenous variations in firms' financing conditions. The remaining six aggregate shocks directly or indirectly capture the exogenous variation in the firms' investment profitability through their impacts on the production efficiency, prices, and preferences in this economy.

2.4 Equilibrium

An equilibrium of this model is a collection of

1. value function $V_t(\mathcal{S})$ and the associated policy functions for hiring, production, investment, debt issuance, equity financing, and capital holding: respectively $l_t(\mathcal{S})$, $\hat{y}_t(\mathcal{S})$, $i_t(\mathcal{S})$, $d'_t(\mathcal{S})$, $e_t(\mathcal{S})$, and $k'_t(\mathcal{S})$;

2. distribution of operating firms $\mathcal{P}_t(\mathcal{S})$; and

3. aggregate quantities and prices Y_t, C_t, Y_t^I, I_t, $\hat{I}_t$, N_t, L_t, $\hat{p}_t$, $\hat{Y}_t$, w_t, $\hat{w}_t$, q_t, $\hat{q}_t$, R_t, π_t, $\Lambda_{t,t+1}$

such that given the exogenous process of $\eta_{x,t}$, $\forall x \in \{e, d, z, p, w, q, m, u\}$,

1. value function $V_t(\mathcal{S})$ solves the firm's problem in (9) with the associated policy functions;

2. distribution $\mathcal{P}_t(\mathcal{S})$ evolves as

$$\begin{aligned}\mathcal{P}_t(\mathcal{S}) =&(1-\xi)\cdot\int \mathbf{1}\left\{k'_{t-1}(\mathcal{S}_-)=k, d'_{t-1}(\mathcal{S}_-)=d, \frac{\ln a-\rho_a\ln a_-}{\sigma_a}=\varepsilon^a\right\} d\Phi\left(\varepsilon^a\right) d\mathcal{P}_{t-1}(\mathcal{S}_-)\\ &+\xi\cdot\mathcal{P}_t^{ent}(\mathcal{S}),\end{aligned} \tag{19}$$

   where $\Phi(\cdot)$ is the c.d.f. of standard normal distribution;

3. the aggregate quantities and prices satisfy the monetary policy specified in (17) and the optimal decisions in the New Keynesian block (Section 2.2); and

4. the markets for final goods, intermediate goods, capital goods, and labor all clear:

$$Y_t = C_t + Y_t^I \tag{20}$$

$$\hat{Y}_t = \int \hat{y}_t(\mathcal{S}) d\mathcal{P}_t(\mathcal{S}) \tag{21}$$

$$I_t = \int i_t(\mathcal{S}) d\mathcal{P}_t(\mathcal{S}) \tag{22}$$

$$L_t = \int l_t(\mathcal{S}) d\mathcal{P}_t(\mathcal{S}). \tag{23}$$

3 Quantification

To decompose the fluctuations of aggregate investment into the contribution of different aggregate shocks, I quantify the model in two steps. First, I calibrate the model to match the average cross-sectional variations in firms' investment and financing, which discipline the model's implied heterogeneity across firms in terms of how financially constrained they are in steady state. Then I estimate the model to match the time-variations in diaggregate-level investment and financing, and aggregate quantities and prices. This will discipline the size of aggregate shocks and the model's implied correlation between aggregate quantities and prices, which are crucial to quantify the transmission of aggregate shocks and their relative contribution to aggregate investment fluctuations.

The first part of this section sketches out the algorithm used to solve this model. The second part focuses on the empirical targets used to quantify the model, i.e. how firms' investment and financing choices differ across different age subgroups and across different periods. The last two parts of this section discuss the key details of calibration and estimation.

3.1 Numerical Solution

I use a hybrid method proposed by Reiter (2009) to solve this model sufficiently fast such that estimation becomes feasible. This method combines the projection method applied on the micro-level and perturbation method applied on the aggregate level[4]. It proceeds as following two steps:

1. I solve the steady state with the aggregate shocks shut off. The solved steady state characterize the distribution of firms and the heterogeneity in their investment and financing when aggregate quantities and prices are fixed at their steady state levels.

2. I solve the first-order perturbation solution around the steady state. The solved dynamics characterize the responses of different firms' investment and financing policies, the distribution of firms, and the aggregate quantities and prices, to various types of aggregate shocks.

[4]To reduce the dimension of system, I approximate the distribution with the approach proposed by Bayer and Luetticke (2020). See Appendix C.1 for more details about the computation.

3.2 Empirical Targets

Sample The data source is 2016 Compustat North America annual dataset. Firms from financial sectors (SIC 6000-6999), regulated utility sectors (SIC 4900-4999), and quasi-governmental sectors (SIC 9000-9999) are removed from the sample. To avoid the impacts from the change of accounting rules and tax environment, the sample period started from 1989. All nominal values are converted to real values by the PPI with 2015 as the base year. Besides the standard data cleaning procedure (see Appendix A for more details), I also discard the observations with merge and acquisitions (M&A) larger than 5% of their book value assets since there could be significant changes in the capital structure of these firms.

Measurement at Firm-level There are two important firm characteristics to be measured: size and age. Size is measured by the lagged book value of capital stock, where the book value of capital stock is equal to the book value of asset minus the cash and short-term investment. To measure a firm's age, I use the first year when there was available fiscal record of the firm as its birth year. As indicated by the model in Section 2, we focus on firms' choices of investment expenditure, equity financing flow, and leverage. For each firm, investment is measured as the sum of its capital expenditure and research and development (R&D) expenditure, normalized by its size. Equity financing is measured by its net equity financing flow normalized by its size. The net equity financing flow is equal to the difference between the issuance of common and preferred stocks[5] and the sum of dividend and stock repurchase. Leverage is measured as the ratio between debt stock and the book value of capital stock, where the debt stock is measured as the sum of long-term debt and the debt in current liability, net of the cash and short-term investment[6].

Cross-section and Cross-time Variations Essentially, what I need to discipline is the model's implied heterogeneity in the degree of being financially constrained across firms. But I cannot

[5] In this paper, the issuance of common and preferred stocks is not directly measured by the item `sstk` reported in Compustat because a large part of the stock issuance reported in this item actually comes from employees exercising their stock options. These options are typically viewed as compensation, with years of delay between being granted and being exercised. To be consistent with the model where financing flows are determined by the managerial decisions in the current period, I eliminate this employee-driven equity issuance by applying the filter proposed by McKeon (2015) (see Appendix A for more details).

[6] Since saving is equivalent to negative borrowing in the model. The debt stock in the model should be interpreted as the amount of net borrowing. Therefore, cash and short-term investment, which is regarded firms' saving, is deducted when measuring firms' debt stock.

directly measure the financial constraints, so the strategy is to use the cross-section variations across different firm subgroups which are categorized by a characteristic correlated with the degree of being financially constrained as empirical targets to discipline the model. Following Haltiwanger *et al.* (2013) and Cloyne *et al.* (2018), I use age to construct the firm subgroups[7].

In the model, firms are ex-ante homogeneous and the observed heterogeneity comes from the accumulated effects from the idiosyncratic shocks they have experienced. But in the data, there could be potentially different types of ex-ante heterogeneity determining the level of firms' investment and financing choices. To ensure the consistency between model and the empirical targets to discipline the model, we need to strip out the potential ex-ante heterogeneity when extracting the cross-sectional and cross-time variations in data[8]. In this paper, I construct these empirical targets in two steps. First, I estimate the following regression:

$$\begin{aligned} Y_{j,t} =& \alpha + \sum_{s} \beta_s^{\text{Sector}} \cdot \mathbf{1}_{j,s}^{\text{Sector}} \\ &+ \sum_{g=1}^{\bar{g}-1} \beta_g^{\text{Age}} \cdot \mathbf{1}_{g,j,t}^{\text{Age}} + \sum_{\tau=\underline{t}}^{\bar{t}-1} \beta_\tau^{\text{Time}} \cdot \mathbf{1}_{\tau,t}^{\text{Time}} + \sum_{g=1}^{\bar{g}-1} \sum_{\tau=\underline{t}}^{\bar{t}-1} \beta_{g,\tau}^{\text{Age, Time}} \cdot \mathbf{1}_{g,j,t}^{\text{Age}} \cdot \mathbf{1}_{\tau,t}^{\text{Time}} + \epsilon_{j,t}. \end{aligned} \tag{24}$$

In regression (24), j indexes the firm and Y denotes the firm-level choices of interest. $\mathbf{1}_{j,s}^{\text{Sector}}$ is the dummy variable for sectors to control the potential ex-ante heterogeneity[9], where the sectors are indexed following the Fama-French 10-sector categorization. g is the index for age groups and there are $\bar{g} = 6$ age groups in total. The dummy variable $\mathbf{1}_{g,j,t}^{Age}$ equals to 1 if the age of firm j in year t falls into the interval $(\underline{A}_g, \bar{A}_g]$. The age cutoffs are constructed as $\underline{A}_1 = -\infty$, $\underline{A}_{g+1} = \bar{A}_g = 5 \times g$, $\forall g \in \{1, 2, \ldots, \bar{g} - 1\}$, and $\bar{A}_{\bar{g}} = \infty$. $\mathbf{1}_{\tau,t}^{\text{Time}}$ is the year dummy, which equals to 1 if $\tau = t$, $\forall t \in \{\underline{t}, \underline{t} + 1, \ldots, \bar{t} - 1\}$, where $\underline{t} = 1989$ and $\bar{t} = 2016$.

After obtaining the estimates $\hat{\beta}_g^{\text{Age}}$, $\hat{\beta}_t^{\text{Time}}$, and $\hat{\beta}_{g,t}^{\text{Age,Time}}$ for $g = 1, 2, \ldots, \bar{g} - 1$ and $t =$

[7] Given the limitation of Compustat data, the measured age in this paper is not measuring the number of years since the actual birth of firms. The age in this paper is closer to a measure of how long a firm has been public. But for the purpose of this paper, the essential need is that the measured age is correlated with the degree of being financially constrained. Figure A.1 provides the evidence showing that the age used in this paper is strongly correlated with the reliance of firms' investment on external financing sources.

[8] Due to this reason, I use age rather than size as the proxy characteristics for financial constraint. The evolution of size is endogenous and the observed cross-sectional variations in firms' size is largely driven by firms' ex-ante heterogeneity (e.g. Sedlek and Sterk, 2017, Pugsley *et al.*, 2020). Comparing with size, the evolution of age is fully exogenous and it works better as the proxy characteristic to construct firm subgroups.

[9] It would be ideal to also control the cohort fixed effect, but this is infeasible because we need to extract both the age and year fixed effects.

$\underline{t}, \underline{t}+1, \ldots, \bar{t}-1$, the average cross-sectional variation of Y is measured as:

$$\upsilon_g^Y = \begin{cases} \hat{\beta}_g^{\text{Age}} + \dfrac{1}{\bar{t}-\underline{t}} \cdot \sum_{\tau=\underline{t}}^{\bar{t}-1} \hat{\beta}_{g,\tau}^{\text{Age,Time}} & \text{if } g = 1, 2, \ldots, \bar{g}-1 \\ 0 & \text{if } g = \bar{g} \end{cases} . \tag{25}$$

The cross-time variation within each age group is measured as:

$$\gamma_{g,t}^Y = \begin{cases} \hat{\beta}_t^{\text{Time}} + \hat{\beta}_{g,t}^{\text{Age,Time}} & \text{if } g = 1, 2, \ldots, \bar{g}-1 \text{ and } t = \underline{t}, \underline{t}+1, \ldots, \bar{t}-1 \\ \hat{\beta}_t^{\text{Time}} & \text{if } g = \bar{g} \text{ and } t = \underline{t}, \underline{t}+1, \ldots, \bar{t}-1 \\ 0 & \text{if } g = \bar{g} \text{ and } t = \bar{t} \end{cases} . \tag{26}$$

Based on (24), (25), and (26), I extract the cross-sectional and cross-time variations in firms' growth rate of size, investment, equity financing, and change in leverage[10].

Figure 1 summarizes the variations across different age groups in terms of the average level of their investment and financing choices. On average, when firms get older, they grow slower, invest less, raise less funding through equity, and increase their leverage by less. The changes along age become much less pronouncing after 5 and almost negligible after 20, so I refer the firms younger than 5 years old as "young firms" and the ones older than 25 years old as "mature firms"[11]. I will focus on the comparison between young firms and mature firms when discussing the difference between age groups in the rest part of this paper.

Table 1 highlights the difference between different age groups in terms of the business cycle properties of their investment and financing. There are two main findings from the top panel of Table 1. First, comparing with the mature firms, the investment and financing choices of young firms are much more volatile. Second, the standard deviation at the aggregate level is smaller than those of any age groups, which indicates negative correlations between different age groups' investment and financing choices. The statistics summarized in the lower two panels of Table 1 confirm this conjecture. For young firms, investment and equity financing are pro-cyclical, and the change in leverage is counter-cyclical, which are all opposite to the cyclicality of mature firms' investment and financing choices. At the same time, the cyclicality of aggregate financial variables

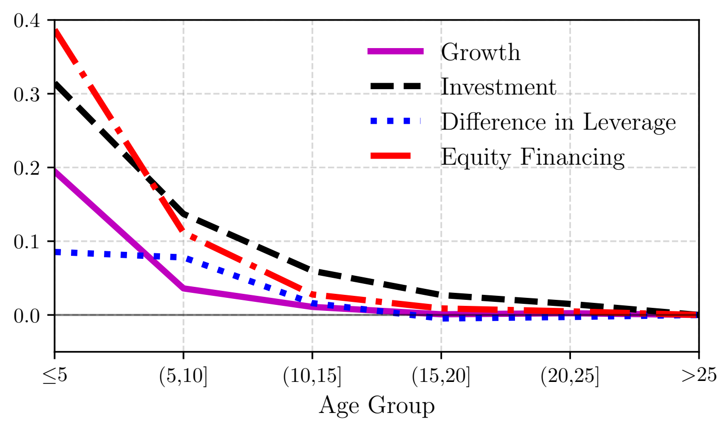
[10] The level of leverage is a stock variable and the ex-ante heterogeneity in the level of leverage is hardly removed by controlling sectors, so we investigate the life-cycle profile of the change in leverage, rather than the level of leverage.

[11] On average, the fractions of "young firms" and "mature firms" are 24.2% and 25.4%.

are closer to the cyclicality of mature firms' financing choices, which shows the dominance of mature firms in the aggregate data because they are much larger.

Even though above evidences are all descriptive, they still show us why and how we should incorporate firm-level heterogeneity into the identification of aggregate financial shocks. On average, young firms have higher investment demand, increase their leverage by more, and issue more equity. Based on "pecking order" theory, they are more likely to be financially constrained[12]. But the time-variation of aggregate investment and financing variables is dominated by mature firms' choices, which have much less direct exposure to financial shocks. Therefore, it would be more reasonable and effective to identify financial shocks using the time-variation of young firms' choices.

Figure 1: Average Investment and Financing Choices, Relative to Mature Firms



Note: This figure collects the average cross-sectional variations in firms' investment and financing choices, which are constructed following (24) and (25). The growth is measured by the log difference in firms' book value of capital stock. Change in leverage is measured by the difference in firms' leverage rate. Investment and equity financing are measured by the firms' investment expenditure and net equity financing flows normalized by their size.

3.3 Calibration and Steady State

Fixed Parameters The model is quantified at annual frequency. I first calibrate the parameters collected in Table 2 based on the literature or the directly corresponding empirical moments. In the first group of parameters, I set the returns to scale at 0.85 and capital share at 0.3. The corporate tax rate τ is set at 35%, which is the median tax rate as reported in Graham (2000). The exogenous exit rate is set at 5.4% to match the average fraction of young firms. The persistence

[12] This is confirmed by the evidence in Figure A.1, where firms' investment depend less on external financing when firms get older.

Table 1: Cyclicality of Firms' Investment and Financing

	Aggregate	Dis-Aggregate by Age Groups					
		≤ 5	(5,10]	(10,15]	(15,20]	(20,25]	> 25
Panel 1. Standard Deviation (%)							
Investment	1.2	9.2	4.4	4.8	3.7	4	2.6
Equity financing	1.1	16.7	7.5	5.3	6.7	7.4	5.1
Change in leverage	1.7	14.6	9.1	7.2	6.2	6.3	5.4
Panel 2. Correlation with the Aggregate Investment Rate							
Investment	1	0.55	0.29	-0.34	-0.38	-0.01	-0.1
Equity financing	-0.36	0.31	-0.06	-0.22	-0.44	-0.33	-0.31
Change in leverage	0.44	-0.16	0.51	0.38	0.6	0.59	0.41
Panel 3. Correlation with the Growth Rate of Value Added, Non-financial Corporate Sector							
Investment	0.44	0.49	0.34	-0.21	-0.29	-0.2	-0.28
Equity Financing	-0.4	0.37	0.16	-0.04	-0.06	-0.24	-0.14
Change in Leverage	-0.02	-0.22	0.62	0.34	0.45	0.46	0.42

Note: On the aggregate level, the investment and equity financing are measured by the aggregate flows of investment and equity financing normalized by the lagged aggregate capital stock. The leverage is measured by the ratio between total net debt and total capital stock. These aggregate investment and financing choices are calculated over the sample of non-financial U.S. public firms. All time-series are linearly detrended before computing their standard deviation and correlation with the aggregate investment rate and growth rate of value added.

of idiosyncratic TFP process is set at 0.7 following the estimate in Imrohoroglu and Tuzel (2014). To be consistent with the fact in Compustat that most firms hold little debt at IPO during the sample period, the average leverage of entrants is fixed at 0. The standard deviation of entrants' distribution σ_x^0, σ_k^0 and σ_a^0 are all set at 0.01, which is small enough to have negligible impacts on the numerical results, but can rule out the mass points in distribution and ensure the accuracy of numerical approximation.

In the second group of parameters, I set the steady-state level of aggregate output gross growth rate to target the average GDP growth rate at 2%. The steady-state level of inflation rate is fixed at 2% and the discounting factor is calibrated to target the real interest rate at 2% in steady state. The remaining parameters of households preference and average markups are calibrated to the values commonly used in the literature.

Table 2: Fixed Parameters

Parameter		Value	Target
Panel 1: Heterogeneous Firms Block			
θ	Return to scale	0.85	Standard
α	Capital share	0.3	Standard
τ	Corporate tax rate	0.35	Graham (2000)
ξ	Exogenous exit rate	0.054	Average fraction of young firms, 24.2%
ρ_a	Persistence of idiosyncratic TFP	0.7	Imrohoroglu and Tuzel (2014)
μ_d^0	Average leverage of entrants	0	Firms hold little debt when going public
Panel 2: New Keynesian Block			
Γ	Output growth rate in steady state	1.02	Average aggregate output growth rate, 2%
β	Discount factor	$0.98\times\Gamma$	Average real interest rate, 2%
π_{ss}	Inflation rate in steady state	1.02	Average inflation rate, 2%
ν_c ν_l	Curvature of utility function	2 1	Standard
h	External consumption habit	0.5	Standard
γ_p	Price markup in steady state	1.1	Standard
γ_w	Wage markup in steady state	1.1	Standard

Fitted Parameters I calibrate the parameters in the upper part of Table 3 to match the two groups of target moments collected in the lower part of Table 3. By targeting at these two groups of moments, the model matches both the level of firms' investment and financing, and the heterogeneity in these choices across different age groups. In the first group of moments, I include the dispersion of mature firms' choices besides the level of their choices, which places important discipline over firms' idiosyncratic risk and precautionary motives.

Table 3: Fitted Parameters and Target Moments

Parameter		Value	
δ	Depreciation rate	0.080	
ϕ_K^+	Capital (upward) adjustment cost	0.017	
ϕ_K^-	Capital (downward) adjustment cost	0.091	
ϕ_{ss}^e	Equity issuance cost in steady state	0.078	
ϕ_{ss}^d	Collateral constraint in steady state	0.379	
σ_a	Standard deviation of idiosyncratic TFP shock	0.163	
μ_a^0	Mean of log idiosyncratic TFP, entrant distribution	-5	
μ_k^0	Mean of log size, entrant distribution	1.884	
Target Moment		**Data**	**Model**
Group 1: Investment and Financing Choices of Matured Firms (Age $\geq$ 25)			
Investment, weighted average		0.099	0.103
Equity financing, weighted average		-0.055	-0.052
Leverage, weighted average		0.216	0.211
Investment, standard deviation		0.250	0.250
Equity financing, frequency of positive flow		0.051	0.024
Group 2: Difference between Young Firms (Age $<$ 5) and Mature Firms (Age$\geq$ 25)			
Investment		0.315	0.288
Equity financing		0.387	0.148
Change in leverage		0.085	0.071

Note: In the first group of moments in the lower panel, investment and equity financing choices are measured by the flows of investment expenditure and equity financing normalized by firms' size. When calculating the weighted average level of mature firms' choices, firms' size is used as the weight. In the second group of target moments, the differences in data are constructed following the procedures specified by (24) and (25), i.e. the v_1^Y for $Y \in$ {Investment, Equity financing, Change in leverage}. The differences in model are calculated as the difference between the average choices of young firms and mature firms in steady state.

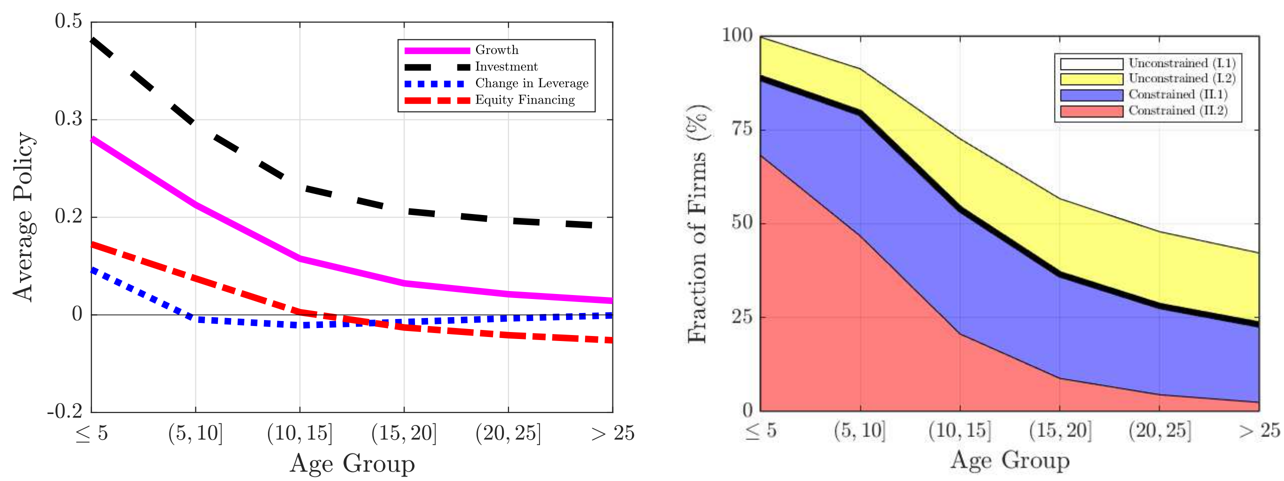
Firms' Life-cycle Dynamics in Steady State Based on the calibrated parameters, I can solve the steady state. Figure 2a plots the average level of firms' investment and financing choices within each age group, which are consistent with the empirical evidence in Figure 1. When firms get older, their growth gradually slows down and converges to the steady-state level determined by the exogenous growth rate in aggregate TFP. Their investment rate also gradually decreases and converges to the level which is just enough for firms to replenish their depreciated capital and keep up with the steady-state level of growth rate. Because of the decreasing investment demand, firms slow down the pace of levering up and their leverage slowly converge to the long-term target level. At the same time, firms raise less funding through equity and start paying dividend.

In this model, firms' financing policy follows the "pecking order" and there are four types of firms based on their financing policies: the ones paying dividends, the ones neither paying dividend nor binding the collateral constraint, the ones binding the collateral constraint but not issuing equity, and the ones issuing equity. The first two types of firms are referred as "unconstrained firms" because their marginal financing cost are not directly subject to the variations in financial frictions. The rest two types of firms are referred as "constrained firms". Figure 2b presents the fractions of these four types of firms within each age groups in the steady state. A clear message from Figure 2b is that the fraction of constrained firms in young firms is much higher than that in mature firms, which highlights the importance of young firms' financing choices in identifying financial shocks.

Figure 2: Heterogeneity of Investment and Financing across Different Age Groups in Steady State

(a) Investment and Financing Choices

(b) Fraction of Constrained Firms



Note: In Figure 2a, the average investment and financing choices are calculated as the mean of the individual firms' policies within each age group in steady state. In Figure 2b, Unconstrained (I.1) refers to the firms paying dividends, Unconstrained (I.2) refers to the ones neither paying dividend nor binding the collateral constraint, Constrained (II.1) refers to the ones binding the collateral constraint but not issuing equity, and Constrained (II.2) refers to the ones issuing equity.

3.4 Estimation

Observable Time-series for Estimation I use the Bayesian method (An and Schorfheide (2007)) to estimate the remaining parameters. There are eight aggregate shocks in this model, and I choose the following eight observable time-series as the input for estimation:

$$\left\{\widetilde{I_t \cdot q_t / K_t},\ \tilde{\gamma}_{1,t}^{\text{Change in leverage}},\ \tilde{\gamma}_{1,t}^{\text{Equity financing}},\ \widetilde{\ln \pi_t},\ \widetilde{\ln R_t},\ \widetilde{\Delta \ln C_t},\ \widetilde{\Delta \ln w_t},\ \widetilde{\Delta \ln \hat{q}_t}\right\}. \tag{27}$$

Here, $\tilde{X}_t$ denotes the cyclical variation of X_t after being linearly detrended. $\frac{I_t \cdot q_t}{K_t}$ denotes the aggregate investment rate, where $I_t \cdot q_t \equiv \sum_j i_{j,t} \cdot q_t$ and $K_t \equiv \sum_j k_{j,t}$ are the aggregate investment expenditure and aggregate size of the non-financial U.S. public firms. $\gamma_{1,t}^{\text{Change in leverage}}$ and $\gamma_{1,t}^{\text{Equity financing}}$ denote the time-variations in young firms' change of leverage and net equity financing flow. To measure inflation $\pi_t \equiv \frac{P_t}{P_{t-1}}$, I measure the nominal price level P_t by the weighted average deflator for non-durable goods and services[13]. Consumption C_t is the consumption of non-durable goods and services in real terms, where the measured P_t is used to the convert the nominal values to real values. The nominal wage W_t is measured as the hourly earnings in the U.S. manufacturing sectors and the real wage w_t is calculated as W_t / P_t. The nominal price of investment good $\hat{Q}_t$ is measured by the weighted average deflator of durable-good and private investment, and the real investment good price $\hat{q}_t$ equals to $\hat{Q}_t / P_t$. Nominal interest rate $\ln R_t$ is measured by the average federal funds rate in each year.

Estimates The priors and posteriors of the estimated parameters are summarized in Table 4. As a comparison, I also estimate a similar representative firm model with aggregate time-series. In terms of the setup, the representative firm model (RFM) can be simply regarded as a version of the heterogeneous firm model (HFM) without idiosyncratic technology shocks[14]. In terms of the input time-series, I use the change in aggregate leverage and aggregate equity financing flow in the estimation of RFM. When comparing the estimates in HFM and RFM, most of the parameters end up with similar estimates, except for the parameters governing the process of debt financing shock.

[13] The deflator is the weighted average of the deflator for non-durable good and the deflator for services. The quantity of non-durable good and services consumption are used as the weight. A similar construction also applies to the measure of the investment good price. Details about the underlying data source can be found in Appendix A.

[14] See Appendix B.2 for more details.

The estimated unconditional standard deviation of debt financing shock in HFM ($\frac{0.193}{\sqrt{1-0.225^2}} \approx 0.198$) is around 8.5 times as large as the one in RFM ($\frac{0.015}{\sqrt{1-0.0.751^2}} \approx 0.023$). This is not very surprising given that the time-series of young firms' financing choices are much more volatile than their counterpart at the aggregate level.

Table 4: Priors and Posteriors in Bayesian Estimation

Parameter		Prior			Posterior, HFM			RFM
		Type	Mean	Std	Mode	5%	95%	Mode
Panel 1. New Keynesian Block								
ξ_p	1-Prob. of wage adjustment	B	0.66	0.1	0.706	0.597	0.869	0.825
ξ_w	1-Prob. of price adjustment	B	0.66	0.1	0.878	0.826	0.945	0.898
ι_p	Price indexation	B	0.5	0.15	0.304	0.117	0.527	0.342
ι_w	Wage indexation	B	0.5	0.15	0.277	0.077	0.372	0.275
λ_π	Taylor rule, inflation	N	1.7	0.3	1.942	1.541	2.354	1.141
λ_R	Taylor rule, lagged interest rate	B	0.6	0.2	0.264	0.046	0.370	0.109
$S''(1)$	Elasticity of capital good price	G	0.1	0.3	0.585	0.421	1.104	0.290
Panel 2. Persistence of Aggregate Shocks								
ρ_z	TFP	B	0.7	0.1	0.364	0.265	0.673	0.450
ρ_d	Debt financing	B	0.7	0.1	0.225	0.215	0.432	0.751
ρ_e	Equity financing	B	0.7	0.1	0.822	0.619	0.880	0.756
ρ_u	Preference	B	0.7	0.1	0.763	0.625	0.885	0.760
ρ_q	Investment good price	B	0.7	0.1	0.822	0.722	0.910	0.828
ρ_m	Monetary policy	B	0.7	0.1	0.419	0.278	0.492	0.381
ρ_p	Price markup	B	0.7	0.1	0.856	0.766	0.921	0.544
ρ_w	Wage markup	B	0.7	0.1	0.747	0.573	0.861	0.772
Panel 3. Standard Deviation of Aggregate Shocks								
σ_z	TFP	IG	0.001	0.01	0.049	0.029	0.126	0.046
σ_d	Debt financing	IG	0.001	0.01	0.193	0.161	0.323	0.015
σ_e	Equity financing	IG	0.001	0.01	0.020	0.016	0.025	0.036
σ_u	Preference	IG	0.001	0.01	0.074	0.049	0.099	0.044
σ_q	Investment good price	IG	0.001	0.01	0.008	0.007	0.011	0.008
σ_m	Monetary policy	IG	0.001	0.01	0.017	0.013	0.022	0.012
σ_p	Price markup	IG	0.001	0.01	0.062	0.048	0.153	0.148
σ_w	Wage markup	IG	0.001	0.01	0.184	0.135	0.674	0.149

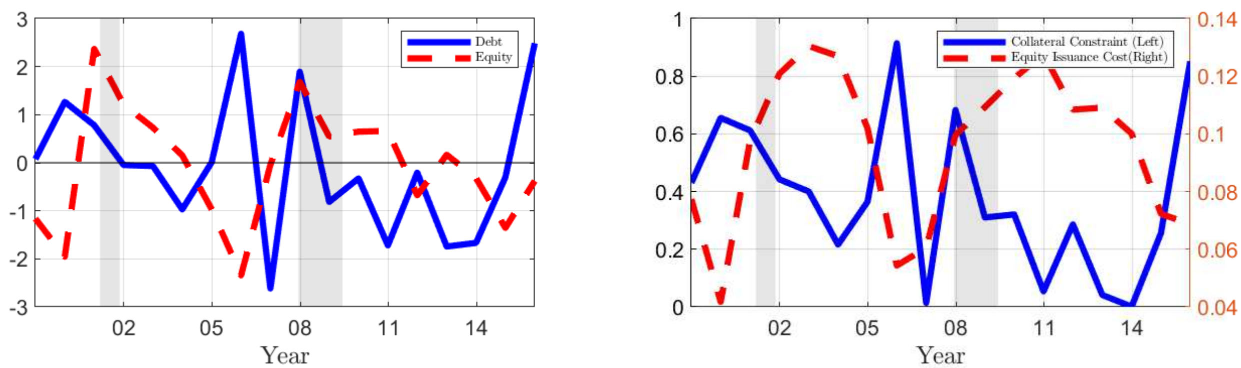
Note: B, N, G, IG refer to Beta, Normal, Gamma and Inverse Gamma distribution respectively. The posterior distribution is obtained using the Metropolis-Hastings algorithm with 25,000 draws.

History of Financial Shocks Based on the estimates in Table 4, the histories of smoothed financial shocks and the corresponding levels of financial frictions are summarized in Figure 3.

During the two recession episodes within the sample period, there were both jumps in equity financing cost and tightening of collateral constraint. But the relative magnitude of these two types of financial shocks is different in these two recession episodes. During the 2001 recession, the increase in equity financing cost was much more pronouncing than the tightening in collateral constraint, which is consistent with the narrative about the burst of technology bubble in stock market. During the 2008 recession, the tightening of collateral constraint was much more severe than the increase in equity financing cost, which echoes to the narrative that this recession was featured with significant disruption in debt financing market.

Figure 3: History of Financial Shocks and Frictions

(a) Financial Shocks **(b)** Financial Frictions



4 Results and Analysis

4.1 The Relevance of Financial Shocks to Investment Fluctuations

To evaluate the relative importance of financial shocks in driving the business investment fluctuations, I decompose the variance of investment rates at both aggregate and disaggregate levels[15] into the contribution of different aggregate shocks. Besides the results from the HFM, Table 5 also present the decomposition results from on the comparable RFM as the benchmark for analysis. There are two main findings about the relevance of financial shocks in HFM:

[15]In this paper, I only report the results for young and mature firm groups for two reasons. First, they are the largest two groups and they account for more than half of the firm population. Second, the variations in young and mature firms' investment and financing are sufficient to represent the variations in different firm subgroups' policies because firms' policies are changing monotonically with age as shown in 1.

1. On the aggregate level, financial shocks make almost negligible contribution to the investment fluctuations. Financial shocks contribute only 2.8% of the variance of the aggregate investment rate of U.S. non-financial public firms. This is much lower than the aggregate relevance implied by RFM, where financial shocks contribute 45.8% of the variance.

2. On the disaggregate level, financial shocks make large contributions to investment fluctuations. Financial shocks cause 48.4% of the variations in young firms' investment rate, and 67.6% of the variations in mature firms' investment rate.

Table 5: Decomposition of the Variance of Investment Rate (%)

	RFM	**HFM**		
	Aggregate	Aggregate	Young	Mature
Financial Shocks	45.8	2.8	48.4	67.6
Equity	40.3	2.3	46.0	9.9
Debt	5.5	0.5	2.4	57.7
Non-Financial Shocks	54.2	97.2	51.6	32.4

Note: The decomposition is based on the modes of posterior. The aggregate investment rate refers to the aggregate investment expenditure normalized by aggregate size. The young (mature) firms' investment rate refers to the average investment rate within young (mature) firms. The decomposition illustrates the fraction of the unconditional variance of investment rates around steady state contributed by different types of aggregate shocks.

A natural question is: why do financial shocks matter much less to the aggregate investment fluctuations in HFM? Given that the estimated size of financial shocks is much larger in HFM, the answer to this question will base on the difference in the transmission of financial shocks between HFM and RFM. At the first glance, the composition difference could be a candidate explanation: there are less financially constrained firms in HFM comparing with RFM [16], so financial shocks are less important to the aggregate investment in HFM. But the results in Table 5 will reject this explanation for two reasons. First, if the composition difference can explain the huge difference in the implied aggregate relevance of financial shocks between HFM and RFM, we should expect the relevance of financial shocks for mature firms' investment to be much smaller than that for young firms given that mature firms are much less likely to be constrained as shown in Figure 2b. But financial shocks matter even more for mature firms' investment in Table 5. Second,

[16] Precisely, all firms in RFM are financially constrained and 55.5% of the firm population are constrained in the steady-state of HFM.

if the transmission of financial shocks in HFM and RFM differs solely due to the composition difference, we should expect the importance of financial shocks for aggregate investment to be a weighted average of their importance for the investment of different firm subgroups in HFM. But the aggregate relevance of financial shocks is much smaller than their relevance to both young and mature firm gorups as shown in Table 5.

4.2 The Transmission of Financial Shocks in HFM

To better understand the transmission of financial shocks in HFM, I plot the impulse responses of investment rates, at both aggregate and disaggregate levels, to contractionary financial shocks in Figure 4. The analysis about the impulse responses explains why financial shocks are very relevant at the disaggregate level but not so at the aggregate level.

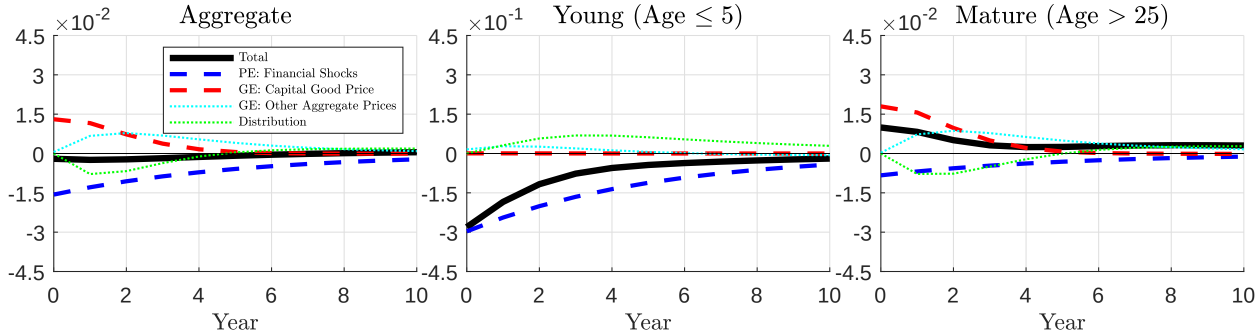
Negatively Correlated Responses The magnitudes of both young and mature firms' investment responses are pronouncing, but the directions of their responses are to the opposite. Because the investment responses of different firm groups largely cancel each other in aggregation, the response of aggregate investment is tiny. The impulse responses in Figure 4 are consistent with the findings in Table 5 that financial shocks make sizable contributions to the investment fluctuations at disaggregate level but not so at aggregate level. The negative correlation between the investment responses of different firm groups explains the negligible aggregate relevance of financial shocks from a statistical perspective.

Partial Equilibrium Effects vs. General Equilibrium Effects To understand the economic mechanism behind the negatively correlated responses of different firm subgroups triggered by financial shocks, I decompose the total impulse response in each plot of Figure 4 into three components[17]. The first component is the response triggered by the partial equilibrium (PE) effect, i.e. the responses of firms' investment policy to the variations in financial frictions. The second component is the responses triggered by the general equilibrium (GE) effects, i.e. the responses of firms' investment policy to the endogenous variations in the aggregate prices including the capital good

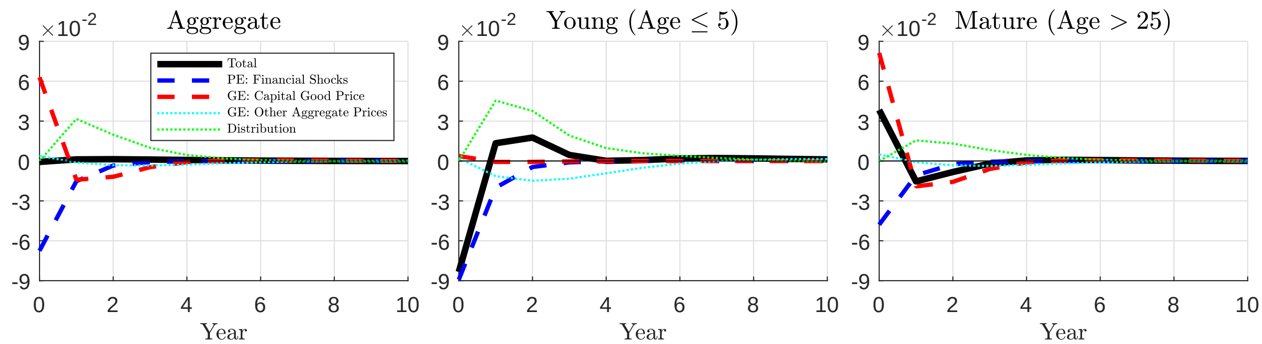
[17] The procedure to compute the decomposition is similar with the one used in Kaplan *et al.* (2018). I first solve the equilibrium path of the levels of financial frictions, aggregate prices, and distribution following the financial shocks. To compute the response due to the variation of a certain variable, I fit the equilibrium path of this variable into the model and fix all the others at the steady-state level, then solve the impulse responses using backward induction.

Figure 4: Impulse Responses of Investment Rates to Contractionary Financial Shocks

(a) Impulse Response to Equity Financing Shock



(b) Impulse Response to Debt Financing Shock



Note: The quantitative results are based on the mode of posteriors. The contractionary financial shocks refer to the exogenous increase in equity financing cost and the exogenous tightening of collateral constraint. The size of the input shock is equal to one standard deviation of the corresponding shock. The aggregate investment rate is computed as the total investment expenditure normalized by the total size. The young (mature) firms' investment rate is calculated as the average investment rate within young (mature) firms.

price, price of homogeneous intermediate good, wage, nominal interest rate, and inflation. The third component is the responses due to the evolution of distribution, i.e. the effects of distribution change on the average investment rate of the firm population or subgroups.

Comparing with the other two component effects, distributional effect plays a relatively minor role in determining the aggregate investment dynamics in Figure 4, so the PE and GE effects will be the focus to understand the transmission of financial shocks. Among the different types of GE effects, the response to capital price variations dominates the others, so the discussion of GE effects will focus on the response to capital good price variations. To evaluate the relative importance of PE and GE effects, I decompose the on-impact responses in Figure 4 and summarize the relative magnitude of each component effect in Table 6.

Following contractionary financial shocks, there are negative PE responses and positive GE

Table 6: Decomposition of Investment Responses to Financial Shocks, PE vs. GE

	Equity			Debt		
	Aggregate	Young	Mature	Aggregate	Young	Mature
PE: Financial Shocks	-1	-18.96	-0.53	-1	-1.32	-0.71
GE: Capital Good Price	0.83	0.03	1.15	0.93	0.06	1.20
GE: Other Aggregate Prices	0.04	1.00	0.02	0.06	0.03	0.07
Total	-0.13	-17.92	0.63	-0.01	-1.24	0.57

Note: This table summarize the decomposition of the initial responses of investment rates, at both aggregate and disaggregate levels, to a standard deviation of contractionary financial shocks around steady state. Since distribution responds with a lag, there is no contribution from distribution evolution in the initial response. For each given type of financial shock, the responses are normalized by the size of aggregate investment rate's PE response.

responses at both the aggregate and disaggregate levels. At the disaggregate level, the relative magnitudes of PE and GE responses are quite different between young and mature firms. Comparing with mature firms, young firms' PE responses are much larger and their GE responses are much weaker. When adding up the GE and PE responses, GE effects only slightly dampen the PE responses of young firms, but totally revert the direction of mature firms' PE responses. GE effects play a key role in generating the negative correlation between the investment responses of different firm groups.

At the aggregate level, GE responses are the key reason for the negligible aggregate relevance of financial shocks. Because mature firms are much larger than young firms, the pronouncing GE effects on mature firms' investment also emerge in the decomposition of aggregate investment responses. For equity financing shocks, GE responses cancel out 87% of the PE response. This dampening effect of GE response is even more significant in the case of debt financing shocks: the GE responses cancel out 99% of the PE response.

Constrained Firms vs. Unconstrained Firms A key difference between young and mature firms is the fraction of financially constrained firms. To explain the different relative magnitudes of GE and PE effects on different firm groups, I first categorize the firms into two groups based on their policies in steady state: constrained and unconstrained firms. Then I separately decompose the investment responses of financially constrained and unconstrained firms within each firm groups. As shown in Table 7, GE effects dominate PE effects on constrained firms and the vice on unconstrained

Table 7: Transmission of Financial Shocks, Constrained vs. Unconstrained Firms

	Aggregate	Full Sample		Young		Mature	
		I	II	I	II	I	II
Panel 1. Responses to Equity Financing Shocks							
PE: Financial Shocks	-1	-8.18	-0.85	-21.09	-1.96	0.00	-0.69
GE: Capital Good Price	0.83	-0.12	1.52	-0.13	1.32	-0.08	1.51
GE: Other Aggregate Prices	0.04	0.36	0.07	1.05	0.68	-0.08	0.02
Total	-0.13	-7.95	0.75	-20.17	0.04	-0.16	0.84
Panel 2. Responses to Debt Financing Shocks							
PE: Financial Shocks	-1	-1.98	-0.13	-1.47	-0.16	-2.79	-0.08
GE: Capital Good Price	0.93	-0.04	1.53	-0.04	0.84	-0.03	1.57
GE: Other Aggregate Prices	0.06	0.01	0.08	0.04	-0.01	-0.02	0.10
Total	-0.01	-2.01	1.48	-1.47	0.67	-2.83	1.59

Note: This table summarize the decomposition of the initial responses of investment rates, at both aggregate and disaggregate levels, to a standard deviation of contractionary financial shocks around steady state. Since distribution will respond with a lag, there is no contribution from distribution evolution in the initial response. For a given type of financial shock, the responses are normalized by the size of aggregate investment rate's PE response. Within each firm group, firms are categorized into two types: I and II. Type I labels the constrained firms, i.e. the firms who are either issuing equity or hitting the collateral constraint, and the other firms are labeled as Type II.

firms, no matter within the full sample or within each age groups. These differences provide a comprehensive picture about the transmission of financial shocks in HFM.

Following a contractionary financial shock, constrained firms cut their investment because their marginal financing cost is directly affected. Unconstrained firms also cut their investment due to precautionary motives, but by a much smaller magnitude. The PE responses lower the aggregate demand for capital good and depress the price of capital good. The lower price of capital good increases the profitability of investment and motives firms to increase their investment. Because the constrained firms are facing much steeper marginal financing cost curve comparing with the unconstrained firms, their GE responses are much weaker than the unconstrained firms'. As a result, the GE effects are relatively smaller than the PE effects on constrained firms, but it is the opposite on the unconstrained firms. Because the fraction of constrained firms in young firms is much higher than that in mature firms, this difference in the relative magnitude of GE and PE effects between constrained and unconstrained firms also shows up in the comparison between young and mature firms.

4.3 The Interaction between Micro-level Heterogeneity and General Equilibrium Effects

The results in Table 7 highlight a distinguishing feature of HFM about the transmission of financial shocks: the strong interaction between firm-level heterogeneity and general equilibrium effects. If there is only the firm-level heterogeneity but no general equilibrium effects, financial shocks would matter less for aggregate investment fluctuations in HFM than they do in RFM because there are less financially constrained firms in HFM. But the aggregate relevance of financial shocks in HFM would not be as small as shown in Table 5. It is the strong dampening responses triggered by the GE effects significantly reducing the aggregate relevance of financial shocks in HFM.

Then the first follow-up question is: why don't GE effects generate similarly strong dampening responses in RFM? It turns out that the setup of RFM automatically limits the GE effects. To study the effects of financial shocks within a DSGE model with a representative firm, we have to model the representative firm as a financially-constrained firm. But as shown in Table 7, constrained firms are "constrained" to respond to the GE effects due to their limited financing capacities, so GE responses are automatically muted in RFM. This difference between HFM and RFM echos to the findings in Kaplan *et al.* (2018) about the difference between HANK and RANK. In HANK, the transmission of monetary policy works mainly through the responses of households, especially the "constrained households" (i.e. the hand-to-mouth households), to the GE effects on their liquidity. In RANK, the representative household is unconstrained and respond little to the temporary changes in liquidity, so the GE responses are much weaker in RANK. Together with the findings from Kaplan *et al.* (2018), the finding in this paper highlights the important role played by the interaction between micro-level heterogeneity and general equilibrium effects in studying the transmission of aggregate shocks.

Another follow-up question is: why are GE effects so strong in HFM such that they can cancel most of the PE effects at aggregate level? A specific feature of the cross-sectional distribution answers this question. Mostly due to the decreasing returns to scale in firms' technology, there is a strong negative correlation between firms' size and how much they are financially constrained in steady state. Because the unconstrained firms are much larger than the constrained firms, GE effects become pronouncing at aggregate level. The answer to this question points to a more general implication from this paper: the transmission of aggregate shocks depends on the micro-level distribution. The correlation between agents' size and their sensitivity to both PE and GE effects in the cross-sectional distribution is very important in quantifying the effects of aggregate shocks.

5 Concluding Remarks

This paper incorporates firm-level heterogeneity into the identification of aggregate financial shocks. The identification is based on an estimated DSGE model that includes a continuum of heterogeneous firms to match the cross-sectional evidence about firms' investment and financing, and eight aggregate shocks to separately capture the exogenous variation in firms' financing conditions and investment profitability. Based on the estimated model, I quantify the effects of financial shocks on the investment of U.S. non-financial public firms. Due to the strong interaction between firm-level heterogeneity and general equilibrium effects, financial shocks explain a significant part of investment fluctuations at disaggregate level, but not so at aggregate level. On average, financial shocks contribute only about 3% of the variance of U.S. non-financial public firms' aggregate investment. In a comparable DSGE model with representative firms, the aggregate relevance of financial shocks is about 15 times larger.

There are two major takeaways from the quantitative results and analyses in this paper. First, representative firm models could overstate the aggregate relevance of financial shocks. And more generally, incorporating micro-level heterogeneity into DSGE model could bring us new insights about the source of business cycle fluctuations. Second, the effects of aggregate shocks depend on the micro-level distribution. Because the interaction between micro-level heterogeneity and general equilibrium effects plays an important role in shaping the transmission of aggregate shocks, a good discipline of the correlations between agents' size and their sensitivity to both PE and GE effects will be crucial to quantify the aggregate effects of an aggregate shock. Empirically measuring these correlations is beyond the scope of this paper and it will be left for the future researcha.

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Appendix

A Empirics

A.1 Sample Construction

Firm-level Data Our sample is constructed based on the Compustat annual data spanning from 1989 to 2016. The sample is cleaned by following procedures:

1. Only keep U.S. firms, i.e., `fic`=‘USA’;
2. Discard the observations from financial, utility, and quasi-governmental sectors, i.e., the observations with `sic` in 6000 $\sim$ 6999, 4900 $\sim$ 4999, and 9000 $\sim$ 9999;
3. Only keep the records with standard format, i.e., `datafmt`=‘STD’;
4. Only keep the records with SCF format code of 7, i.e., `scf`=7;
5. Only keep only the records listed in U.S. stock exchanges, i.e., `exchg` in 0 $\sim$ 4 and 11 $\sim$ 20;
6. Discard the records with M&A (`aqc`) larger than 5% of its book value assets (`at`);
7. Discard the observations with missing values for book-value assets (`at`);
8. Discard observations with real book value assets smaller than \$1 million (2015 \$);
9. For each firm, its flow and stock variables reported based on fiscal year are converted to the measures based on calendar year by linear interpolation.

The measurement of the key variables in this paper are summarized in Table A.1.

Table A.1: Firm-level Data: Data Source and Measurement

Variable	Measurement
Panel 1. Compustat Variables	
M&A	`aqc`
Book value asset	`at`
Long-term debt	`dltt`
Current debt	`dlc`
Cash and short-term investment	`che`
Capital expenditure	`capx`
R&D expenditure	`xrd`
Stock issuance	refined based on `sstkq`
Dividend	`dv`
Stock repurchase	`prstkc`
Issuance of long-term debt	`dltis`−`dltr`
Issuance of current debt	`dlcch`
Panel 2. Measurement of the Key Variables in Model	
Capital k'	book value asset - cash and short-term investment
Debt stock d'	long-term debt + current debt - cash and short-term investment
Investment expenditure $i \cdot q$	capital expenditure + R&D expenditure
Equity financing e	stock issuance - stock repurchase - dividend payment
Leverage $\frac{d'}{k'}$	debt stock/capital
Debt financing $d' - \frac{d}{\pi}$	issuance of long-term debt + issuance of current debt - liquidity accumulation

Note: Since a significant part of the reported stock issuance comes from the exercises of employees' options, I measure the stock issuance following two steps. First, I use the approach proposed by McKeon (2015) to refine the stock issuance in quarterly data: if a firm's reported stock issuance is lower than 3% of the total market value of its outstanding shares, its stock issuance will be counted as 0; otherwise, the firm's stock issuance will be measured as reported. Then I aggregate each firms' quarterly stock issuance to annual stock issuance.

Aggregate Data The aggregate variables are extracted from FRED and the details about the data source and measurement are listed in Table A.2.

Table A.2: Aggregate Data: Data Source and Measurement

Variable	Measurement
Panel 1. FRED Variables	
Consumption, non-durable good	`PCNDA`
Consumption, service	`PCESVA`
Consumption, durable good	`PCDGA`
Investment, private	`GPDIA`
Deflator, non-durable good	`DNDGRD3A086NBEA`
Deflator, service	`DSERRD3A086NBEA`
Deflator, durable good	`DDURRD3A086NBEA`
Deflator, private investment	`A006RD3A086NBEA`
Hourly earnings (manufacturing)	`USAHOUREAAISMEI`
Federal fund rate (monthly)	`FEDFUNDS`
Panel 2. Measurement of the Key Variables in Model	
Nominal interest rate $\ln R$	Annual rate aggregated from monthly federal fund rate
Aggregate price level P	Weighted average deflator of non-durable good and service
Inflation π	Log difference of P
Investment good price $\hat{Q}$	Weighted average deflator of durable good and private investment
Wage W	Hourly earnings (manufacturing)

A.2 Extra Details about the Heterogeneity in Firms' Investment and Financing

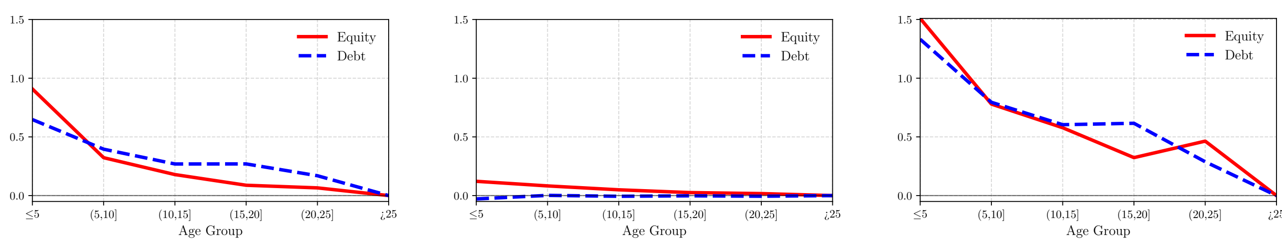
Reliance of Investment on External Financing I measure the reliance of firms' investment on different sources of external financing following Zetlin-Jones and Shourideh (2017). Figure A.1 summarizes the heterogeneity in the reliance of investment on different source of external financing across different age groups. There are two findings from Figure A.1:

1. When firms get older, their investment becomes less dependent on external financing.
2. The decrease in reliance of investment on external financing sources along age is mainly driven by the variation through intensive margin but not extensive margin.

The empirical evidence in Figure A.1 indicates that young firms are more financially constrained than mature firms.

Figure A.1: Age Profiles: Dependency of Investment on Different External Financing Sources

(a) Overall **(b)** Extensive Margin **(c)** Intensive Margin



Note: For each firm j in year t, the dependency of its investment expenditure $\mathtt{Inv}_{j,t} \equiv i_{j,t} \cdot q_t$ on financing flow $\mathtt{F}_{j,t}$ is measured as:

$$I^{\mathtt{F}}_{j,t} \equiv \begin{cases} \frac{\mathtt{F}_{j,t}}{\mathtt{Inv}_{j,t}} & \text{if } \mathtt{F}_{j,t} > 0 \text{ and } \mathtt{Inv}_{j,t} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{A.1}$$

where $\mathtt{F}_{j,t}$ denotes the equity or debt financing flow of firm j in year t. Here, (a) summarize the age profile if we run regression (24) with $I^{\mathtt{F}}_{j,t}$ over the full sample; (b) summarize the age profile if we run regression (24) with dummy variable $\mathbb{1}_{I^{\mathtt{F}}_{j,t}>0}$ over the full sample; and (c) summarize the age profile if we run regression (24) with $I^{\mathtt{F}}_{j,t}$ over the sample of firms with $I^{\mathtt{F}}_{j,t} > 0$.

B Model

B.1 Decisions in New Keynesian Block of Heterogeneous Firm Model

I directly present the decisions in log-linearized forms in this section, where $\tilde{X}_t$ denotes the deviation of X_t from its detrended steady-state level.

Final Goods Supply and Inflation Dynamics The final good producers maximize their expected total discounted profits by choosing their input of retailed goods. Given the demand from final good producers, retailers maximize their expected total discounted profits by setting the nominal price of their goods. Following Calvo (1983), it is assumed that only a randomly chosen fraction $(1-\xi_p)$ of the retailers can reset their price in each period. The decisions of final good producers and retailers jointly determine the aggregate supply of final goods and inflation dynamics:

$$\tilde{Y}_t = \tilde{\hat{Y}}_t, \tag{B.2}$$

$$\begin{aligned}\tilde{\pi}_t =& \frac{\xi_p \cdot \iota_p}{\xi_p \cdot (1 + \iota_p \cdot \Gamma \cdot \Lambda_{ss})} \cdot \tilde{\pi}_{t-1} \\ &+ \frac{(1-\xi_p) \cdot (1 - \xi_p \cdot \Gamma \cdot \Lambda_{ss})}{\xi_p \cdot (1 + \iota_p \cdot \Gamma \cdot \Lambda_{ss})} \cdot (\tilde{\hat{p}}_t + \eta_{p,t}) + \frac{\xi_p \cdot \Gamma \cdot \Lambda_{ss}}{\xi_p \cdot (1 + \iota_p \cdot \Gamma \cdot \Lambda_{ss})} \cdot \mathbb{E}_t \left[\tilde{\pi}_{t+1}\right],\end{aligned} \tag{B.3}$$

where Y_t is the total output of final goods, $\hat{Y}_t$ is the total output of homogeneous intermediate goods, and $\hat{p}_t$ is the intermediate good price in real terms.

Labor Demand and Wage Dynamics The labor packer maximizes their expected total discounted profits by choosing their input of differentiated labor services. Given the demand from the labor packer, labor unions maximize their expected total discounted profits by setting the nominal wage of their differentiated labor service. It is also assumed that only a randomly chosen fraction $(1-\xi_w)$ of the labor unions can reset their wages in each period. The decisions of the labor packer and labor unions jointly determine the aggregate demand for the households' labor and wage dynamics:

$$
\begin{aligned}
\tilde{\hat{N}}_t =& \tilde{L}_t, && \text{(B.4)} \\
\tilde{w}_t =& \frac{(1-\xi_w)\cdot(1-\xi_w\cdot\Gamma\cdot\Lambda_{ss})}{1+\xi_w^2\cdot\Gamma\cdot\Lambda_{ss}}\cdot\left(\tilde{\hat{w}}_t+\eta_{w,t}\right)-\frac{\xi_w\cdot(1+\Gamma\cdot\Lambda_{ss}\cdot\iota_w)}{1+\xi_w^2\cdot\Gamma\cdot\Lambda_{ss}}\cdot\tilde{\pi}_t \\
&+\frac{\xi_w}{1+\xi_w^2\cdot\Gamma\cdot\Lambda_{ss}}\cdot(\iota_w\cdot\tilde{\pi}_{t-1}+\tilde{w}_{t-1})+\frac{\xi_w\cdot\Gamma\cdot\Lambda_{ss}}{1+\xi_w^2\cdot\Gamma\cdot\Lambda_{ss}}\cdot\mathbb{E}_t\left[\tilde{w}_{t+1}+\tilde{\pi}_{t+1}\right], && \text{(B.5)}
\end{aligned}
$$

where $\hat{N}_t$ denotes the quantity of households' labor, L_t denotes the total final labor service used by the intermediate good firms, and w_t and $\hat{w}_t$ are the wage of final labor service and household labor in real terms.

Capital Good Supply and Capital Good Price Dynamics The investment good producer and the capital good producer maximize their expected total discounted profits by choosing their inputs Y_t^I and output $\hat{I}_t$. Based on their optimal choice, the total supply of investment good and the price of investment good satisfies:

$$
\begin{aligned}
\tilde{\hat{I}}_t =& \tilde{Y}_t^I-\eta_{q,t}, && \text{(B.6)} \\
\tilde{\hat{q}}_t =& \eta_{q,t}. && \text{(B.7)}
\end{aligned}
$$

The total supply of the capital good and the capital good price dynamics are:

$$
\begin{aligned}
\tilde{I}_t =& \tilde{\hat{I}}_t && \text{(B.8)} \\
\tilde{q}_t =& \tilde{\hat{q}}_t+S''(1)\cdot\left[\left[\tilde{\hat{I}}_t-\tilde{\hat{I}}_{t-1}\right]-\Lambda_{ss}\cdot\mathbb{E}_t\left[\tilde{\hat{I}}_{t+1}-\tilde{\hat{I}}_t\right]\right], && \text{(B.9)}
\end{aligned}
$$

where $\hat{q}_t$ and q_t denote the prices of the investment good and capital good in real terms.

Labor Supply and Stochastic Discounting Factor Dynamics The representative household maximizes its utility specified in (12) subject to their budget constraint in (13). The consumption

Euler equation and labor supply are:

$$0 = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} + \tilde{R}_t - \tilde{\pi}_{t+1} \right] \tag{B.10}$$

$$\tilde{\hat{w}}_t = \eta_{u,t} + \nu_L \cdot \tilde{\hat{N}}_t - \widetilde{MU}_t. \tag{B.11}$$

The dynamics of the real stochastic discounting factor (SDF) is disciplined by:

$$\widetilde{MU}_t = \eta_{u,t} - \left[\frac{1}{1-h} \cdot \tilde{C}_t - \frac{h}{1-h} \cdot \tilde{C}_{t-1} \right] \tag{B.12}$$

$$\tilde{\Lambda}_{t,t+1} = \widetilde{MU}_{t+1} - \widetilde{MU}_t. \tag{B.13}$$

B.2 The Comparable Representative Firm Model

The comparable RFM shares the same setup of New Keynesian block and the aggregate shocks with HFM. The RFM differs from HFM only in two aspects. First, in terms of the setup about firms' life-cycle, there is no idiosyncratic technology shock and the entrants fully inherit the capital and debt of the exiting firms in RFM. Second, in terms of the frictions on investment and financing, the equity financing cost is constructed as $\Phi^e_t(e,k) \equiv \phi^e_t \cdot (e - e_{ss}) + \frac{\bar{\phi}^e_2}{2} \cdot (\frac{e}{k} - \frac{e_{ss}}{k_{ss}})^2 \cdot k$ and the capital adjustment cost is symmetric, i.e. $\Phi^k(k',k) \equiv \frac{\phi^k}{2} \cdot \left(\frac{k'}{k} - \Gamma \right)^2 \cdot k$. The Bellman Equation for the representative firm is:

$$\begin{aligned}
V_t(d,k) = & \max_{l,e,i,d',k'} - e - \Phi^e_t(e,k) + \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \left[(1-\xi) \cdot V_{t+1}\left(d',k'\right) + \xi \cdot \mathbf{LV}_{t+1}(d',k') \right] \right] \\
\text{s.t.} \quad & i \cdot q_t = (1-\tau) \cdot \left(\Gamma^t \cdot \exp(\eta_{z,t}) \cdot (k^\alpha \cdot l^\alpha)^\theta - l \cdot w_t \right) + \tau \cdot \delta \cdot k \cdot q_t + d' - \frac{d \cdot \hat{R}_{t-1}}{\pi_t} + e \\
& k' = (1-\delta) \cdot k + i - \Phi^k(k',k) \\
& d' \leq \phi^d_t \cdot k \cdot q_t
\end{aligned}$$

C Quantification

C.1 Key Details of Computation

Value Function Approximation The value function in computation is defined over the transformed state space $(\frac{d}{k}, k, a)$. The function is approximated over a tensor space with $(22 \times 25 \times 5)$ grid points. The process of a is approximated by Rouwenhorst (1995). The boundary of $\frac{d}{k}$ and k are chosen such that they can cover the ergodic distribution in steady state. The grid point allocations on the dimension of $\frac{d}{k}$ and k follow $\ln\left[\frac{\frac{d}{k}_{i+1} - \frac{d}{k}_{i}}{\frac{d}{k}_{i} - \frac{d}{k}_{i-1}}\right] = -0.5$ and $\ln\left[\frac{k_{i+1} - k_i}{k_i - k_{i-1}}\right] = 0.8$ to accommodate the value function's feature that it has relatively higher curvature in the areas with higher leverage and smaller capital.

Distribution Approximation The distribution in computation is defined over the transformed state space $(\frac{d}{k}, \ln k, a)$. The distribution is approximated over a tensor space with $(30 \times 50 \times 5)$ grid points. The grid points of a are the same with the ones used in the value function approximation. On the dimension of $\frac{d}{k}$ and $\ln k$, the grid points are allocated with equal distance. When approximating the distribution evolution, the transition probability matrix is constructed by the approach proposed by Young (2010). When solving the dynamics of the model, I use the approach proposed by Bayer and Luetticke (2020), i.e. tracking the responses of marginal distribution but keeping the copula at its steady-state level.

C.2 Quantification of HFM: Extra Details

Constrained and Unconstrained Firms in Steady State The characteristics of constrained and unconstrained firms in steady state are summarized in Table C.3. There are three differences between constrained and unconstrained firms worth to be emphasized:

1. Even though there are more constrained firms in the economy, their total capital accounts for a smaller fraction of the aggregate capital. This indicates that unconstrained firms are much larger than constrained firms. On average, unconstrained firms are 78% larger than constrained firms.

2. On average, the productivity of unconstrained firms is 24.2% lower than that of constrained firms.

3. On average, unconstrained firms' leverage is far away from binding collateral constraint and their reserved financing capacity is equivalent to 24.3% of their capital.

To summarize, unconstrained firms are a groups of large, less productive firms with much lower investment demand and amble financing capacity.

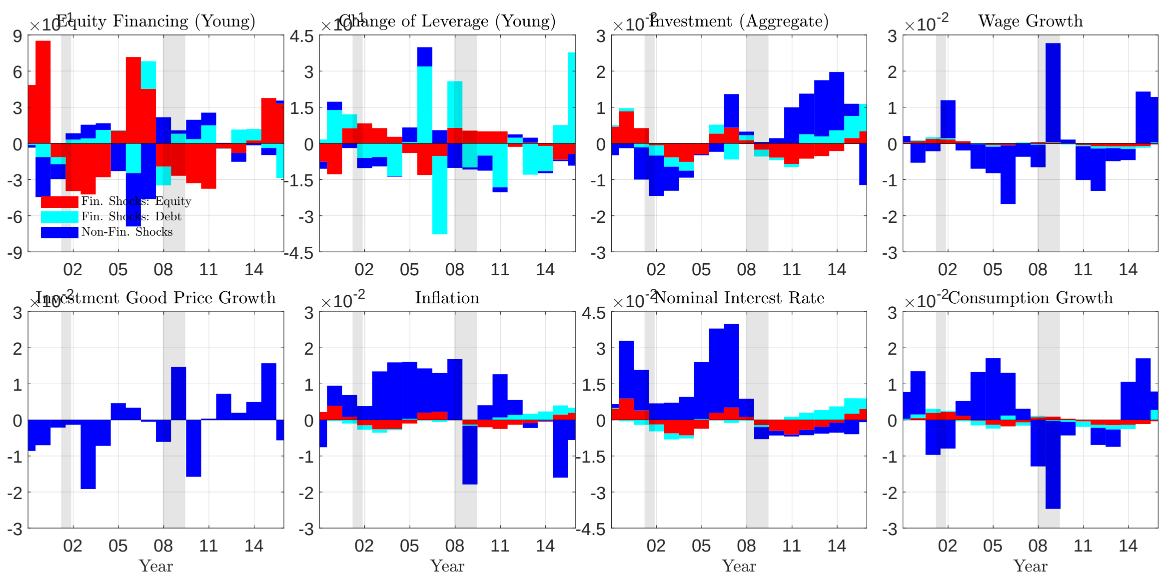
Table C.3: Difference between Constrained and Unconstrained Firms in Steady State

	Constrained	Unconstrained	Difference
Panel 1: Fraction of the Aggregate			
Frequency	0.555	0.445	-0.110
Capital	0.397	0.603	0.206
Investment	0.878	0.122	-0.756
Panel 2: Average States			
Log of size	4.530	5.310	0.780
Log of idiosyncratic TFP	-0.240	-0.482	-0.242
Panel 3: Average Policies			
Growth rate	0.233	-0.018	-0.251
Investment rate	0.366	0.097	-0.269
Equity financing	0.069	-0.035	-0.104
Frequency of equity issuance	0.534	0.000	-0.534
Leverage	0.324	0.082	-0.243

Note: The constrained firms refer to the firms who are either issuing equity or hitting the collateral constraint. The other firms are categorized as unconstrained firms.

Decomposition of the History of Input Time-series Figure C.2 summarize the contribution of different aggregate shocks to the variations of input time-series over the sample period. This decomposition results illustrate the key intuitions about the identification of aggregate shocks. Between financial and non-financial shocks, financial shocks are mainly pined down by the young firms' financing choices and the non-financial shocks are mainly identified by the other time-series about the aggregate quantities and prices. Within the financial shocks, the equity financing shocks are mainly identified by the young firms' equity financing flows and the debt financing shocks are mainly identified by the change in their leverage.

Figure C.2: Decomposition of the History of Input Time-series in Estimation



C.3 Quantification of RFM: Key Details

Calibration and Estimation All the parameters shown up in both HFM and RFM are calibrated at the same values as shown in Table 2 and 3. The special parameters in RFM, i.e. $\bar{\phi}_2^e$ and ϕ_k, are estimated together with the other parameters, and the estimation results are summarized in Table C.4.

Table C.4: Estimation of Representative Firm Model

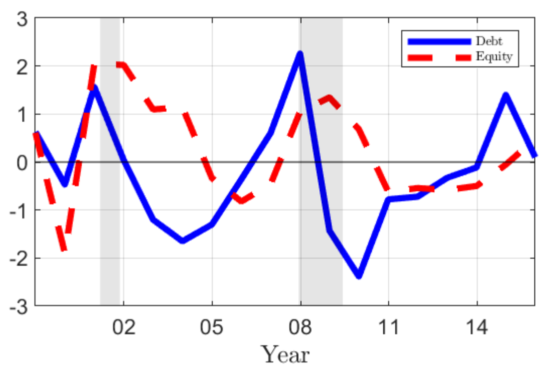
Parameter		Prior			Posterior		
		Type	Mean	Std	Mode	5%	95%
$\bar{\phi}_2^e$	Elasticity of equity issuance cost	IG	0.8	0.5	0.805	0.301	2.193
ϕ_k	Capital adjustment cost	IG	0.1	0.1	0.058	0.037	0.172
ξ_p	1-Prob. of wage adjustment	B	0.66	0.1	0.825	0.783	0.886
ξ_w	1-Prob. of price adjustment	B	0.66	0.1	0.898	0.866	0.929
ι_p	Price indexation	B	0.5	0.15	0.342	0.157	0.535
ι_w	Wage indexation	B	0.5	0.15	0.275	0.113	0.392
λ_π	Taylor rule, inflation	N	1.7	0.3	1.141	1.100	1.355
λ_R	Taylor rule, lagged interest rate	B	0.6	0.2	0.109	0.018	0.187
$S''(1)$	Elasticity of capital good price	G	0.1	0.3	0.290	0.033	0.695
ρ_z	TFP	B	0.7	0.1	0.450	0.307	0.578
ρ_d	Debt financing	B	0.7	0.1	0.751	0.614	0.877
ρ_e	Equity financing	B	0.7	0.1	0.756	0.646	0.872
ρ_u	Preference	B	0.7	0.1	0.760	0.665	0.857
ρ_q	Investment good price	B	0.7	0.1	0.828	0.726	0.916
ρ_m	Monetary policy	B	0.7	0.1	0.381	0.284	0.494
ρ_p	Price markup	B	0.7	0.1	0.544	0.369	0.696
ρ_w	Wage markup	B	0.7	0.1	0.772	0.594	0.888
σ_z	TFP	IG	0.001	0.01	0.046	0.038	0.058
σ_d	Debt financing	IG	0.001	0.01	0.015	0.012	0.020
σ_e	Equity financing	IG	0.001	0.01	0.036	0.028	0.053
σ_u	Preference	IG	0.001	0.01	0.044	0.037	0.056
σ_q	Investment good price	IG	0.001	0.01	0.008	0.007	0.010
σ_m	Monetary policy	IG	0.001	0.01	0.012	0.010	0.015
σ_p	Price markup	IG	0.001	0.01	0.148	0.088	0.280
σ_w	Wage markup	IG	0.001	0.01	0.149	0.102	0.292

Note: B, N, G, IG refer to Beta, Normal, Gamma and Inverse Gamma distribution respectively. The posterior distribution is obtained using the Metropolis-Hastings algorithm with 25,000 draws.

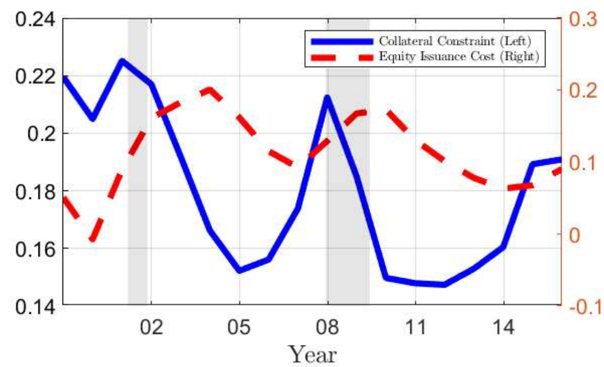
History of Financial Shocks The smoothed history of financial shocks based on the modes of posterior are summarized in Figure C.3.

Figure C.3: History of Financial Shocks and Frictions

(a) Financial Shocks, RFM



(b) Financial Frictions, RFM

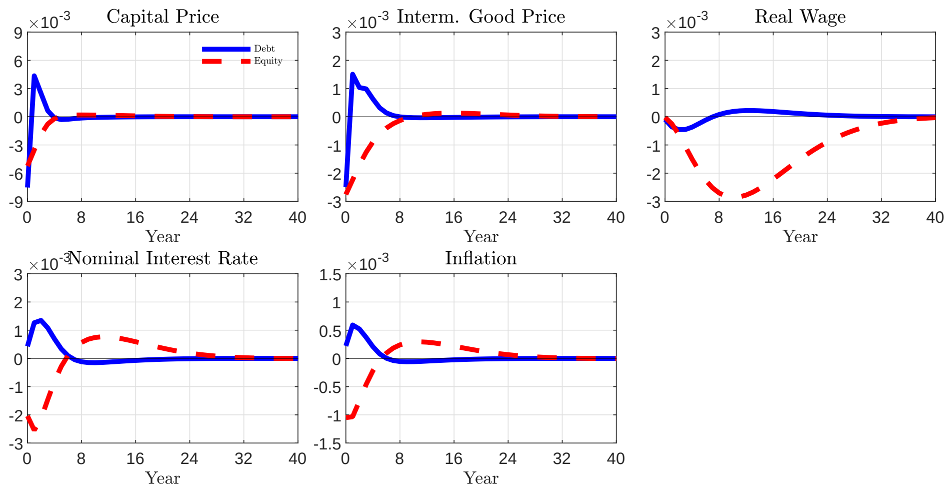


D Results

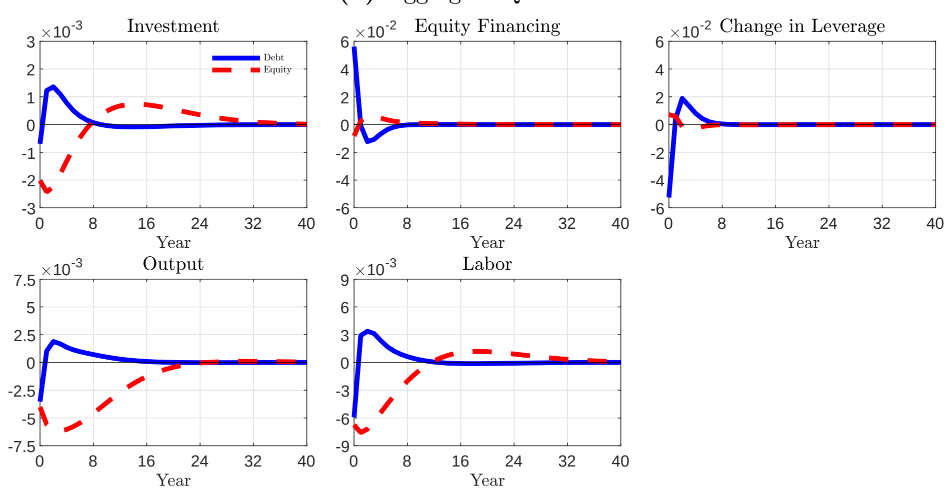
Impulse Responses of Aggregate Prices and Quantities The responses of aggregate prices and quantities to contractionary financial shocks are summarized in Figure D.4.

Figure D.4: Impulse Responses to Contractionary Financial Shocks

(a) Aggregate Prices



(b) Aggregate Quantities



Note: The quantitative results are based on the mode of posteriors. The contractionary financial shocks refer to the exogenous increase in equity financing cost and the exogenous tightening of collateral constraint. The size of the input shock is equal to one standard deviation of the corresponding shock.

Detailed Variance Decomposition of Investment Rates As summarized in Table D.5, the most relevant shock to aggregate investment fluctuations is the price markup shocks.

Table D.5: Detailed Variance Decomposition of Investment Rates

	RFM	HFM		
	Aggregate	Aggregate	Young	Mature
Financial	45.8	2.8	48.4	67.6
Equity	40.3	2.3	46	9.9
Debt	5.5	0.5	2.4	57.7
Non-Financial	54.2	97.1	51.6	32.4
TFP	1.8	8.8	20.8	4
Wage Markup	1.8	4.6	0.6	1.7
Price Markup	31.6	69.6	11	19.2
Monetary Policy	14.5	8	16.9	6.1
Investment Good Price	3.2	0.3	0.1	0.6
Preference	1.5	5.8	2.2	0.8

Note: The decomposition is based on the modes of posterior. The aggregate investment rate refers to the aggregate investment expenditure normalized by aggregate size. The young (mature) firms' investment rate refers to the average investment rate within young (mature) firms. The decomposition illustrates the fraction of the unconditional variance of investment rates around steady state contributed by different types of aggregate shocks.