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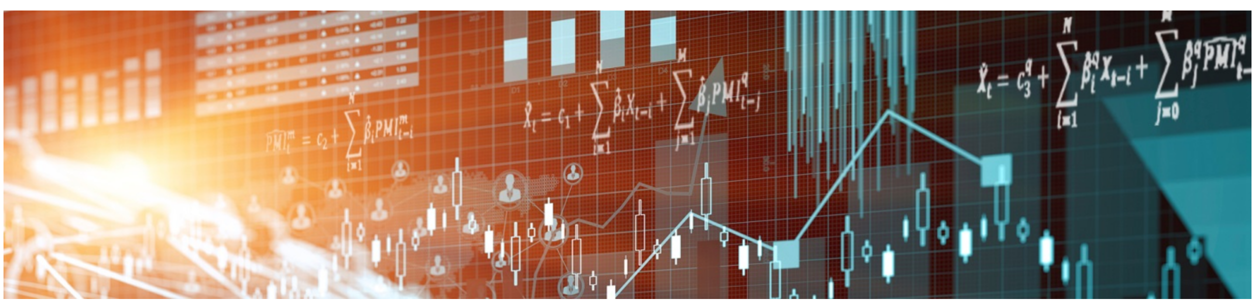
Loan Insurance, Market Liquidity, and Lending Standards



by Toni Ahnert and Martin Kuncl

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Loan Insurance, Market Liquidity, and Lending Standards

by

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Abstract

We examine loan insurance—credit risk transfer upon origination—in a model in which lenders can screen, learn loan quality over time, and can sell loans. Some lenders with low screening ability insure, benefiting from higher market liquidity of insured loans while forgoing the option to exploit future information about loan quality. Insurance also improves the quality of uninsured loans traded but lowers lending standards. We derive testable implications about loan insurance. Since lenders do not internalize its benefit for market liquidity, loan insurance is insufficient and should be subsidized. Our results can inform the design of government-sponsored mortgage guarantees.

Bank topics: Financial institutions; Financial markets; Financial system regulation and policies
JEL codes: G01, G21, G28

Résumé

Nous examinons les contrats d'assurance contre le défaut de paiement – soit le transfert du risque de crédit au moment de l'octroi des prêts – dans un modèle où les prêteurs peuvent sélectionner les emprunteurs, obtenir de l'information sur la qualité des prêts au fil du temps et céder les prêts. Certains prêteurs dont l'aptitude à sélectionner les emprunteurs n'est pas très bonne souscrivent une assurance, profitant ainsi d'une meilleure liquidité des prêts assurés sur le marché secondaire tout en renonçant à la possibilité d'exploiter les informations futures sur la qualité des prêts. L'assurance améliore aussi la qualité des prêts non assurés qui seront négociés, mais abaisse les critères d'octroi des prêts. Nous déduisons des implications vérifiables au sujet des contrats d'assurance contre le défaut de paiement. Puisque les prêteurs n'internalisent pas les effets positifs de ces contrats sur la liquidité des prêts dans le marché secondaire, la couverture d'assurance est insuffisante : elle devrait donc être subventionnée. Les résultats de notre étude permettent d'éclairer la conception des garanties hypothécaires subventionnées par l'État.

Sujets : Institutions financière; Marchés financiers; Réglementation et politiques relatives au système financier
Codes JEL : G01, G21, G28

1 Introduction

Risk in credit markets is often assumed upon origination by third parties for a fee. Typical examples are insurances and guarantees that protect the owner of a loan against borrower default and are popular in mortgage markets around the world (Blood, 2001). Governments also offer default insurance for various other loan types, including student loans, small business loans, and export loans but mortgages are the most important example. In 2018, the U.S. government insured and guaranteed 62% of outstanding residential mortgages (equal to 32% of GDP) via institutions such as the Federal Housing Administration (FHA) and Government Sponsored Enterprises (GSEs) such as Fannie Mae and Freddie Mac (Urban Institute, 2018). Similarly in Canada, 44% of mortgages are insured by a public insurer, the Canada Housing and Mortgage Corporation (CMHC), or private insurers with explicit government backing.

The widespread use of loan default insurance and repayment guarantees in credit markets leads to several important positive and normative questions: What is their impact on liquidity in secondary markets (allocative efficiency) and on lending standards in primary markets (productive efficiency)? How do changes in loan characteristics, screening technology, or the liquidity risk of lenders affect the decisions to insure a loan? How do these privately optimal choices fare when evaluated against a welfare benchmark? How should regulatory interventions (if any) be designed? What are the implications for mortgage guarantees offered by GSEs, FHA, and CMHC?

To address these questions, we study credit risk transfer via loan insurance (e.g., mortgage guarantees) in a parsimonious model of lending (Parlour and Plantin, 2008; Parlour and Winton, 2013). Lenders have three options to reduce their exposure to default risk. First, at origination each lender has access to a pool of borrowers and can screen at a heterogeneous cost. Screening improves the probability of repayment by identifying a borrower with a low default probability, raising lending standards.[1] The loan payoff is a reduced-form measure of the lending profitability or the degree of competition in lending markets, where higher competition implies a lower loan payoff.

[1] This assumption is consistent with empirical evidence. For example, Berger and Udell (2004) document a positive association between loan screening and loan quality in a sample of US banks.

The second option is to privately learn loan quality over time, perhaps because of relationship lending, and sell low-quality loans to uninformed outside financiers.[2] Such lemons can be sold above their fundamental value because some lenders are hit by a liquidity shock (e.g., an investment opportunity) and also sell. Since the realizations of loan quality and the liquidity shock are private information, the market for uninsured loans is subject to adverse selection, which reduces the gains form trade.

Our contribution is to study a third option—credit risk transfer upon origination—and its effect on lending standards and market liquidity. Loan insurance passes default risk to an insurer (outside financiers) upon origination for a fee.[3] Consistent with our principal application of mortgage guarantees (such as FHA and GSEs in the US and CMHC in Canada), whether a loan is insured is observable and the loan trades together with its insurance in a market for insured loans. An implication is that insured loans are insensitive to future private information about loan quality. Another implication is the segmentation into secondary markets for insured and uninsured loans, consistent with the existence of separate markets for agency mortgage-backed securities (agency MBS) and private-label MBS.[4] Since loan insurance is often explicitly or implicitly backed by the government, we abstract from default risk of the insurer.[5]

We start with a benchmark in which loan insurance is not available. In equilibrium, there is a screening cost threshold and only lenders with low costs choose

[2]The assumption that banks acquire private information about borrowers during the lending relationship is consistent with evidence in e.g. James (1987), Lummer and McConnell (1989), Slovin et al. (1993), Agarwal and Hauswald (2010), Norden and Weber (2010), Botsch and Vanasco (2019).

[3]Consistent with this timing, the insurance of individual mortgages by the FHA or CMHC requires that loan insurance occurs upon origination. Similarly, a popular business model is to specialize in origination of conforming loans, followed by the immediate sale to GSEs (Hurst et al., 2016; Buchak et al., 2018). We show a formal equivalence between the two forms of credit risk transfer (loan insurance and outright sale) in Appendix A.1. Critical to our results is that credit risk is transferred before the lender learns private information about loan quality, e.g. because of relationship lending.

[4]FHA loans are fully backed by the FHA and trade in a separate market enabled by securitization, mainly through Ginnie Mae bonds. For Fannie Mae and Freddie Mac, the guarantee is provided when newly originated loans are sold by lenders to these GSEs, who in return for a fee provide the guarantee and securitize loans into agency MBS. Agency MBS are traded separately from private-label MBS. Insured loans in Canada are traded separately in secondary markets, typically in the form of National Housing Act Mortgage Backed Securities (NHA MBS) (Crawford et al., 2013).

[5]See Appendix A.2 for analysis of partial insurance that can be interpreted as insurer default risk in reduced form. For comprehensive analyses of counterparty risk in financial insurance contracts, see, for example, Thompson (2010) and Stephens and Thompson (2014).

to screen. This threshold affects productive efficiency—the average quality of loans originated net of screening costs. Private learning of loan quality generates asymmetric information between lenders and financiers and thus adverse selection. A liquid equilibrium exists when high-quality loans are sold in the market for uninsured loans upon a liquidity shock, which arises for a large liquidity shock. An illiquid equilibrium always exists because a low price and no trade of high-quality loans are mutually consistent. Allocative efficiency refers to the social gains from trade and depends on both the type of equilibrium and the price at which uninsured loans are traded.

When loan insurance is available, low-cost lenders screen and do not insure, while high-cost lenders do not screen and some of them insure. Consistent with this self-selection result, some lenders (e.g., non-banks, monoline lenders) specialize in the issuance of FHA loans or conforming loans sold to GSEs upon origination. For example, Buchak et al. (2018) document that shadow banks rely on such credit risk transfer and serve riskier, less creditworthy borrowers. Insurance reveals that a lender does not screen, so the competitive fee reflects the expected cost of guaranteeing the repayment of non-screened loans. Without insurer default risk, insured loans are safe and always trade in a liquid market. Hence, insured loans fetch a higher price than uninsured loans because of adverse selection in the latter market. Consistent with this differential pricing implication, agency MBS maintained robust issuance and trading volumes as well as low spreads compared to private-label MBS even during the recent financial crisis (e.g., Vickery and Wright 2013; Loutskina and Strahan 2009).

We characterize loan insurance in the liquid equilibrium. For high-cost lenders, the private benefit of insurance is to sell the loan upon a liquidity shock for a higher price in the market for insured loans. The private cost is to lose the option to sell lemons in the market for uninsured loans without a liquidity shock. In equilibrium, both effects equalize, and high-cost lenders are indifferent about loan insurance.

Testable implications are that credit risk transfer upon origination occurs for a (i) low default probability (such as for borrowers with higher credit scores and in regions with lower predictable default risk; Hurst et al., 2016), (ii) low loan profitability (which is consistent with evidence in Loutskina and Strahan, 2009) or high competition in the lending market (such as in the US as opposed to Canada), (iii) large liquidity shock (such as times or countries with high vulnerabilities in the financial sector), and (iv) high screening costs. Hence, the recent rise in Fintech has an ambiguous effect on the occurrence of loan insurance because Fintechs increase competition (e.g., via online lenders) but also lower screening costs (e.g., via extensive data analysis or big-data approaches (Fuster et al., 2019; Buchak et al., 2018)).

Credit risk transfer upon origination improves market liquidity and welfare. Apart from creating a liquid market for insured loans, it also improves the quality of uninsured loans traded because of selection and commitment. First, insured lenders (none of whom screen) have a lower average loan quality than uninsured lenders (some of whom screen). Second, some lemons that lenders without liquidity shock would have sold are removed from the uninsured loans market. The credit risk transfer upon origination commits such lenders to not exploiting future private information about loan quality. The resulting higher price of uninsured loans improves allocative efficiency: (i) the market for uninsured loans liquifies for some parameters and (ii) higher social gains from trade are realized within the liquid equilibrium. While screening incentives are lower, overall welfare improves.

Turning to normative implications, we consider the benchmark of a planner who chooses loan insurance for all lenders and can select the equilibrium (liquid or illiquid). In contrast to the unregulated equilibrium, the planner internalizes the positive effect of insurance on the price of uninsured loans. Hence, the planner insures more loans and for a larger set of parameters in the liquid equilibrium. The planner also insures loans to liquify the market: creating a liquid equilibrium (when it does not exists in the unregulated economy) improves market liquidity and allocative efficiency. For some parameters, however, liquifying the market is feasible but would reduce screening incentives so severely that the planner prefers to keep the market illiquid.

We show that a regulator subject to a balanced budget and with no information advantage over financiers can achieve the planner's allocation. The regulator promises a minimum price of uninsured loans to eliminate the illiquid equilibrium when it is inferior. This policy can be credibly implemented via a subsidy to sales of uninsured loans, as originally envisioned by TARP programs in the U.S., for example. Once the liquid equilibrium is the unique equilibrium in the regulated economy, the planner's allocation is implemented via a subsidy on loan insurance. It induces lenders to internalize the full benefit of their individual insurance choice for the price of uninsured loans. We interpret this loan insurance subsidy as governmental support to mortgage guarantees (by e.g. FHA, GSEs, CHMC). The sale subsidy for uninsured loans is not used in the liquid equilibrium because it does not benefit from this positive externality and is therefore more expensive to implement. When the illiquid equilibrium is superior, all high-cost lenders insure and there is no role for any regulation.

Our results contribute to a debate about the design of mortgage guarantees. Subsidies on mortgage guarantees should occur for higher-quality loans, e.g. borrowers with sufficiently high credit scores—consistent with the the practices of FHA and GSEs in the US and CMHC in Canada—or in regions with lower predictable default risk—inconsistent with current practice (Hurst et al., 2016). These subsidies should also occur for loans with lower payoffs or when lending markets are more competitive. In the cross-section, this arises in countries with a less concentrated lending market, e.g. more in the U.S. than in Canada, while it occurs in the time series via higher recent competition from specialized online lenders (e.g., by FinTechs). Loan insurance subsidies should occur when lenders face larger liquidity needs or when screening costs are higher. The latter result suggests that recent technological advances and extensive data analysis of borrowers (e.g., by FinTechs) reduce the benefits of insurance. We also derive implications for the optimal size of mortgage guarantee subsidies.

To probe the robustness of our results, we study an extension with asymmetric information in the loan insurance market. We modify the model to let screening lenders learn loan quality directly upon origination. The resulting adverse selection in the insurance market lowers the private benefits of insurance and also creates an equilibrium with an illiquid insurance market. Due to lower insurance, the gains from trade are lower, screening is higher, and welfare is reduced. A new feature of this equilibrium is strategic complementarity in insurance because more insurance reduces its fee as the cross-subsidization to low-cost lenders who selectively insure lemons is spread more widely. The case for insurance subsidies becomes even stronger: loan insurance is more beneficial for market liquidity, including eliminating the welfare-dominated equilibrium with an illiquid insurance market. Other extensions in Appendix A consider partial insurance coverage, upfront payment of the insurance fee, and partial loan sales and we show that our results are qualitatively unchanged.

Literature. Our paper is related to four strands of literature. The first literature concerns the screening choices of lenders and the resulting lending standards in loan origination (Broecker 1990, Thakor 1996, Hauswald and Marquez 2003, Ruckes 2004, Dell'Ariccia and Marquez 2006, and Hu 2018, among others). Our contribution is to study a two-way feedback between screening incentives and the incentives to transfer credit risk upon loan origination as well as the implications for market liquidity.

Second, our paper is related to work on adverse selection (Akerlof, 1970) and policies to unfreeze markets. Using mechanism design, Tirole (2012) shows that market freezes due to adverse selection can be tackled by removing lemons from the market.[6] Similarly, the credible option of a direct intervention in the market for uninsured loans (e.g., as intended by TARP) is useful in our paper to coordinate lenders on the liquid equilibrium when it is welfare-superior. However, adverse selection is also mitigated via market segmentation in our model. Insuring loans upon origination reduces the degree of asymmetric information in secondary markets. Studying the mix of policy tools, we show that in the liquid equilibrium subsidizing loan insurance is preferred to direct intervention in the market for uninsured loans.

Third, our paper is related to work on credit risk transfer. Perhaps closest to our paper is Parlour and Winton (2013), who study the effect of credit default swaps (CDS) as an alternative to loan sales in secondary markets. Both CDS and loan sales affect a lender's incentive to monitor its borrower but the lender retains control rights only with CDS. There are several differences to our paper. First, we study the incentives to screen borrowers at origination as opposed to monitoring them during the lending relationship. Second, credit risk transfer occurs before the lender

[6]See also e.g. Philippon and Skreta (2012), Camargo et al. (2016), and Chiu and Koeppl (2016).

learns private information about the borrower in our model, while credit risk transfer occurs after private information is received in Parlour and Winton (2013). Third, insurance is observable and inseparable from loans in the secondary market in our model. Our different but complementary modeling approach is motivated by our principal application to mortgage loans and their guarantees.

A related literature studies the interaction between productive efficiency and allocative efficiency. The market liquidity of a loan affects the lender's incentive to screen or monitor borrowers (e.g., Pennacchi 1988; Gorton and Pennacchi 1995; Parlour and Plantin 2008; Chemla and Hennessy 2014; Vanasco 2017; Daley et al. 2020). We share with this literature the trade-off between productive and allocative efficiency. Our contribution is to examine the implications of credit risk transfer upon origination and its impact on both productive efficiency and allocative efficiency.

Finally, our paper is related to a literature on the role of the government in the consumer mortgage market (e.g., Campbell 2012). The financial crisis of 2007-09 sparked interest in the reform of housing finance and government-sponsored mortgage guarantees (e.g., Jeske et al. 2013; Elenev et al. 2016; Hurst et al. 2016; Gete and Zecchetto 2018). While the recent literature has emphasized mostly the negative effects of mortgage guarantees, we uncover a complementary and positive effect: the benefits of mortgage guarantees for secondary market liquidity. This channel suggests a role for policy in subsidizing mortgage guarantees.

2 Model

There are three dates $t = 0, 1, 2$ and one good for consumption and investment. Two groups of risk-neutral agents are protected by limited liability.[7] Outside financiers are competitive, deep-pocketed at $t = 1, 2$, and require a return normalized to one. Each lender $i \in [0, 1]$ has one unit of funds to originate a loan at $t = 0$ and access to an individual pool of borrowers. Without screening at $t = 0$, $s_i = 0$, lender i

[7] Loan insurance would clearly be desirable if lenders were risk-averse. We deliberately focus on risk-neutrality throughout in order to highlight the effects of loan insurance beyond risk-sharing.

finds an average borrower and receives A (repayment) with probability $\mu \in (0,1)$ or 0 (default) at $t = 2$. The loan payoff $A_i \in \{0, A\}$ is independently and identically distributed across lenders and publicly observable at $t = 2$. A higher payoff A reflects more profitable lending opportunities, a less competitive lending market, or a lower bargaining power of borrowers. The repayment probability μ reflects a credit score.

Screening, $s_i = 1$, improves the repayment probability to $\psi \in (\mu, 1)$, as shown in Figure 1. The heterogeneous non-pecuniary cost of screening, η_i, reflects difference in lender types (e.g., traditional versus online lenders) or in screening ability (e.g., because of pre-existing relationships with a borrower). It is distributed according to a density function $f(\eta) > 0$ with support $[0, \overline{\eta}]$ and cumulative distribution $F(\eta)$.[8] The cost and the choice of screening, η_i and s_i, are private information to lender $i \in [0, 1]$.

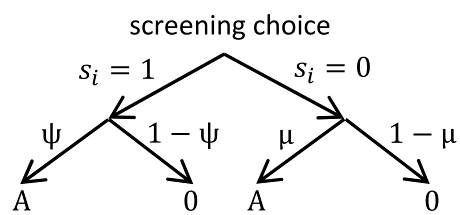


Figure 1: Screening and loan payoffs: screening improves the probability of loan repayment.

At $t = 1$, lenders receive two pieces of private information. First, each lender learns the future loan payoff A_i. This assumption is consistent with (i) relationship lending and (ii) learning-by-holding an asset (Plantin, 2009). The assumption implies that screening at $t = 0$ does not increase the degree of asymmetric information at $t = 1$. Second, each lender learns the realization of an idiosyncratic liquidity shock λ_i, whereby the preference for interim consumption is $\lambda_i \in \{1, \lambda\}$ with $\lambda > 1$.[9] The liquidity shock is independently and identically distributed across lenders, independent of the loan payoff, and arises with probability $Pr\{\lambda_i = \lambda\} \equiv \nu \in (0, 1)$. The utility of lender i is $u_i = \lambda_i c_{i1} + c_{i2} - \eta_i s_i$, where c_{it} is her consumption at date t.

[8]If screening costs were homogeneous, all lenders are indifferent about screening in equilibrium. All our qualitatively results carry over to this alternative setup as long as lenders share a common pool of borrowers and the screening cost increases in the share of lenders who screen.

[9]Our reduced-form modelling of the gains from the loan sale before maturity captures investment opportunities, consumption needs, capital constraints, bank runs, or risk management and is common in the literature (e.g., Aghion et al. (2004), Holmstrom and Tirole (2011), Vanasco (2017)).

At $t = 0$, each lender chooses whether to insure the loan against default, $\ell_i \in \{0, 1\}$. We focus on a full transfer of credit risk to financiers without loss of generality.[10] This insurance contract guarantees the payoff A to the loan owner for a fee k. Both the insurance payoff and the fee are charged at $t = 2$, resulting in a safe payoff $\pi = A - k$.[11] Insurance swaps a loan's risky payoff A_i for a safe payoff π.[12]

At $t = 1$, each lender can sell the loan in secondary markets to outside financiers who are uninformed about the screening cost η_i and choice s_i, liquidity shock λ_i, and loan quality A_i. Consistent with our principal application of mortgage guarantees (see also introduction), financiers observe whether a loan is insured, ℓ_i, and insured loans are sold together with their insurance. Thus, segmented markets for insured (I) and uninsured (U) loans exist with respective prices p_I and p_U and sale choices $q_i^I \in \{0, \ell_i\}$ and $q_i^U \in \{0, 1 - \ell_i\}$.[13] Figure 2 shows the timeline of events.

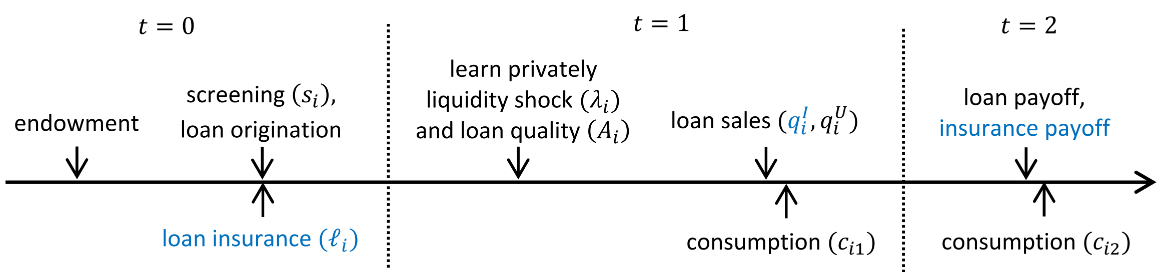


Figure 2: Timeline. The actions and payoffs in blue highlight loan insurance.

2.1 Benchmark without credit risk transfer upon origination

We start with a benchmark in which loan insurance is not available. We sketch the key parts of the analysis and economic forces at play here but relegate most of the formal derivations, comparative statics, and proofs to Appendix B.1.

[10]We show in Appendix A.2 that partial insurance is neither privately nor socially optimal.

[11]This approach parallels the non-pecuniary screening cost in that it does not affect lending volume at $t = 0$. It is feasible as contracts can be written on the observable loan payoff at $t = 2$. For an extension with an insurance fee that must be paid up front (i.e. at $t = 0$), see Appendix A.3.

[12]Our approach is equivalent to outright sales of the loan to financiers after origination at $t = 0$ if the source of asymmetric information at $t = 1$ is relationship lending. See Appendix A.1 for details.

[13]We allow for partial sales in Appendix A.4 and show that our results are qualitatively unchanged.

Without loss of generality, lenders use a threshold strategy for their screening choice. Each lender with a screening cost below a threshold η chooses to screen:

$$s_i^* = \mathbf{1}\Big\{\eta_i \leq \eta\Big\}, \tag{1}$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. The screening threshold affects productive efficiency—the average quality of loans originated net of screening costs. We refer to lenders with screening costs below (above) the threshold as low-cost (high-cost).

Multiple stable equilibria may exist, where high-quality loans (worth A) are traded in a liquid equilibrium but not in an illiquid equilibrium.[14] Due to asymmetric information between lenders and financiers at $t = 1$, all lenders sell low-quality loans (worth 0) and the competitive price is $p_U < A$. Thus, lenders do not sell high-quality loans without a liquidity shock. A defining feature of equilibrium is whether lenders sell them upon a shock:

$$\lambda p_U \geq A. \tag{2}$$

If condition (2) holds, the market for loans is liquid, which we call a liquid equilibrium. We consider the liquid equilibrium first, so the sales of lender i at $t = 1$ are:[15]

$$q_i^U(A_i, \lambda_i) = \mathbf{1}\Big\{\lambda_i = \lambda\Big\} + \mathbf{1}\Big\{\lambda_i = 1\Big\}\mathbf{1}\Big\{A_i = 0\Big\}. \tag{3}$$

Allocative efficiency refers to the distribution of funds across lenders and financiers at $t = 1$ and the social gains from trade, $\nu(\lambda - 1)p_U$. Allocative efficiency is affected by whether trade of high-quality loans takes place (quantity dimension), that is whether the equilibrium is liquid, and the price of loans traded (price dimension). Due to adverse selection, high-quality loans are sold at a discount $A - p_U$, which redistributes resources from lenders selling high-quality loans to lenders selling low-quality loans (lemons). High-quality loans are only sold by liquidity-shocked lenders (with a high utility of interim consumption, λ), while lemons are sold by a mix of lenders with an average utility, $\kappa \equiv \nu\lambda + 1 - \nu \in (1, \lambda)$, as shown in panel (a) of Figure 4.

[14] We exclude the unstable equilibrium in which some shocked lenders trade high-quality loans.

[15] Similar to Parlour and Plantin (2008), the binary choice of loan sales and limited liability preclude signaling in this market. See Appendix A.4 for an analysis of partial loan sales.

The competitive price is the value of high-quality loans sold divided by total loans sold, where a share $1-\mu$ of high-cost and $1-\psi$ of low-cost lenders sell lemons:

$$p_U = \nu A \frac{\psi F(\eta) + \mu(1-F(\eta))}{\nu + (1-\nu)\left[(1-\psi)F(\eta) + (1-\mu)(1-F(\eta))\right]}. \tag{4}$$

Screening supports the price, $\frac{dp_U}{d\eta} > 0$. First, more screening leads to fewer low-quality loans originated at $t=0$, which improves the quality of loans traded. Second, screening at $t=0$ does not increase the asymmetric information between lenders and financiers at $t=1$ because all lenders privately learn loan quality A_i at $t=1$.

The marginal lender, $\eta_i = \eta$, is indifferent between screening with expected payoff $\nu\lambda p_U + (1-\nu)(\psi A + (1-\psi)p_U) - \eta$ and not screening with expected payoff $\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)$. Equating these payoffs yields the screening cost threshold:

$$\eta = (1-\nu)(\psi-\mu)(A-p_U). \tag{5}$$

The benefit of screening arises without a liquidity shock because all liquidity-shocked lenders sell their loans in the liquid equilibrium. Without a shock, that is with probability $1-\nu$, the screening benefit is the higher probability of originating a high-quality loan, $\psi-\mu$, and keeping it until maturity instead of selling a lemon, $A-p_U$. A higher price reduces the screening benefit, $\frac{d\eta}{dp_U} < 0$, due to the option to sell lemons.

Lemma 1 describes the liquid equilibrium and Figure 3 shows when it exists.

Lemma 1. ***Liquid equilibrium when loan insurance is unavailable.*** *If $\lambda \geq \underline{\lambda}_L$ and screening costs are heterogeneous enough, $\bar{\eta} > \frac{(1-\nu)(\psi-\mu)(1-\psi)}{\nu+(1-\nu)(1-\psi)}A$, then there exists a unique and interior liquid equilibrium. Its cost threshold, $\eta^* \in (0, \bar{\eta})$, is implicitly given by*

$$\eta^* = \frac{(1-\nu)(\psi-\mu)\left[1-\mu-(\psi-\mu)F(\eta^*)\right]}{\nu + (1-\nu)\left[1-\mu-(\psi-\mu)F(\eta^*)\right]}A \tag{6}$$

and the price of uninsured loans is $p_U^* = A - \frac{\eta^*}{(1-\nu)(\psi-\mu)} \in \left[\frac{A}{\lambda}, A\right)$. The lower bound on the size of the liquidity shock is $\underline{\lambda}_L = \frac{A}{p_U^*} \in (1, \infty)$.

Proof. See Appendix B.1 (which defines $\underline{\lambda}_L$ and derives comparative statics). ■

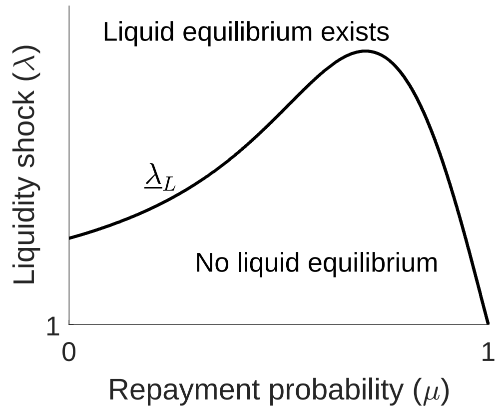


Figure 3: Existence of a liquid equilibrium (when loan insurance is unavailable).

We turn to the illiquid equilibrium, which is characterized in Lemma 2.

Lemma 2. ***Illiquid equilibrium when loan insurance is unavailable.*** *There always exists an illiquid equilibrium,* $p_U^* = 0$*. The screening threshold is* $\eta^* = (\psi - \mu)A$*.*

An illiquid equilibrium always exists. If the price is zero, lenders do not sell high-quality loans and only low-quality loans are traded, $q_i^U = \mathbf{1}\Big\{A_i = 0\Big\}$—consistent with the zero price. The screening threshold again arises from an indifference of the marginal lender: since loans are kept until maturity, the expected payoff is $\psi A - \eta_i$ with screening and μA without screening. Since lenders cannot sell loans in the illiquid equilibrium, they have higher incentives to screen than in the liquid equilibrium.[16]

3 Equilibrium

We define the equilibrium with credit risk transfer upon origination via loan insurance.

Definition 1. *An equilibrium comprises screening* $\{s_i\}$*, insurance* $\{\ell_i\}$*, the sales of insured and uninsured loans* $\Big\{q_i^I, q_i^U\Big\}$*, prices* p_I *and* p_U*, and a fee* k *such that:*

1. *At* $t = 1$*, for each liquidity shock* $\lambda_i \in \{1, \lambda\}$ *and loan quality* $A_i \in \{0, A\}$*, each lender* i *optimally chooses the sales of insured and uninsured loans,* q_i^I *and* q_i^U*.*

[16] Evidence consistent with this implication includes Mian and Sufi (2009) and Keys et al. (2010).

2. *At $t = 0$, each lender i chooses screening s_i and loan insurance ℓ_i to solve*

$$\begin{aligned}
\max_{s_i,\ell_i,c_{i1},c_{i2}} \quad & \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to} \\
c_{i1} &= q_i^I p_I + q_i^U p_U, \\
c_{i2} &= (\ell_i - q_i^I)\pi + (1 - \ell_i - q_i^U)A_i, \quad \Pr\{A_i = A\} = \psi s_i + \mu(1 - s_i).
\end{aligned}$$

3. *The insurance fee k at $t = 0$ and the prices of loans p_I and p_U at $t = 1$ are set for outside financiers to break even in expectation.*

Our first result describes which lenders transfer credit risk upon loan origination.

Proposition 1. ***Loan insurance.*** *Low-cost lenders screen and do not insure: $s_i^* = 1$ and $\ell_i^* = 0$ if $\eta_i \leq \eta^*$. The insurance fee is $k^* = (1 - \mu)A$, resulting in the safe payoff and price of insured loans $\pi^* = \mu A = p_I^*$. The share of high-cost lenders who insure is $m^* \in [0, 1)$ in a liquid equilibrium for $\psi > \underline{\psi}$ and $m^* = 1$ in an illiquid equilibrium.*

Proof. See Appendix B.2. ■

Insurance converts the risky loan payoff into a safe payoff π independent of the screening choice. Since screening is costly, lenders who insure loans do not screen them because they cannot be rewarded for their unobservable action. Hence, the competitive insurance fee is the expected cost of guaranteeing the repayment of non-screened loans, $(1 - \mu)A$. For such a fee, no lender who screens chooses to buy insurance. In equilibrium, insuring a loan thus reveals that the lender did not screen. Consistent with this result, Buchak et al. (2018) document that shadow banks rely on credit risk transfer upon origination and serve riskier, less creditworthy borrowers.

Since an insured loan is sold together with its insurance at $t = 1$, insured loans are risk-free and its secondary market is free from adverse selection and always liquid. These results arise because lenders do not yet know loan quality A_i when insuring at $t = 0$. The competitive price of insured loans at $t = 1$ equals its payoff at $t = 2$:

$$p_I^* = \pi^* = A - k^* = \mu A. \tag{7}$$

In the liquid equilibrium[17] with insurance, $m^* \in (0,1)$, high-cost lenders are indifferent about insurance for sufficiently productive screening technology, $\psi > \underline{\psi}$, which we focus on henceforth.[18] With insurance, a high-cost lender prefers selling the insured loan after the liquidity shock at $t = 1$ at price p_I^* and is indifferent after no shock because $p_I^* = \pi^*$. Thus, the expected payoff from insuring is κp_I^*. Without insurance, a high-cost lender sells the uninsured loan after a liquidity shock at price p_U^*. Without a shock, the loan is also sold if it is of low quality, else it is kept until maturity. Thus, the expected payoff from not insuring is $\nu\lambda p_U^* + (1-\nu)[\mu A + (1-\mu)p_U^*]$. Equating both expected payoffs yields the indifference condition for loan insurance:

$$\nu\lambda\left(p_I^* - p_U^*\right) = (1-\nu)\,(1-\mu)\,p_U^*. \tag{8}$$

This condition has an intuitive interpretation. Its left-hand side (LHS) is the benefit of insurance, which is the gain of selling the loan, after a liquidity shock, at a higher price in the insured market than in the uninsured market, $p_I^* > p_U^*$. The right-hand side (RHS) of equation (8) is the (private) cost of insurance, that is losing the option to sell low-quality loans (lemons) in the uninsured market without a liquidity shock.

A requirement for loan insurance to occur in equilibrium is that price of insured loans exceeds the price of uninsured loans, $p_I^* > p_U^*$. This differential price is consistent with evidence from the US mortgage market, whereby agency MBS had lower spreads than private-label MBS (Vickery and Wright, 2013; Loutskina and Strahan, 2009).

In the illiquid equilibrium, uninsured loans must be kept to maturity. The payoff of high-cost lenders without insurance is μA. However, insurance allows such lenders to sell an insured loan upon a liquidity shock. Thus, the payoff with insurance is $\kappa\mu A > \mu A$ and all high-cost lenders insure, $m^* = 1$. In contrast to the liquid

[17]We continue to refer to the equilibrium in which high-quality loans are sold in the market for uninsured loans as the liquid equilibrium (because the market for insured loans is always liquid).

[18]For a sufficiently productive screening technology, $\psi > \underline{\psi}$, there does not exist a liquid equilibrium in which all high-cost lenders insure, $m^* < 1$. If $m = 1$, no lemons would be sold by high-cost lenders, implying a price $p_U = \frac{\nu\psi}{\nu+(1-\nu)(1-\psi)}A$. However, for $\psi > \underline{\psi}$ this price contradicts the condition for high-cost lenders preferring insurance (with payoff $\kappa\mu A$) over no insurance (with payoff $\nu\lambda p_U + (1-\nu)[\mu A + (1-\mu)p_U]$), which is required for the supposed insurance of all high-cost lenders. For an analysis of low screening productivity and the corner solution $m = 1$, see Appendix A.5.

equilibrium, insurance has no trade-off: its benefits is a higher price, $p_I > p_U$, but its cost—forgoing the option to sell a lemon—does not apply in the illiquid equilibrium.

We turn to characterizing the liquid equilibrium with loan insurance.

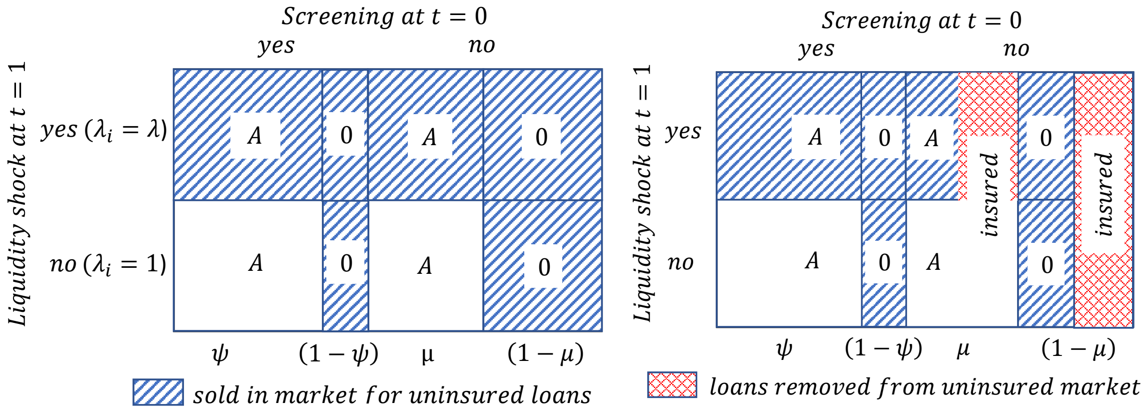
Proposition 2. ***Liquid equilibrium when loan insurance is available.*** *There exist unique bounds $\tilde{\mu}_I$ and $\tilde{\lambda}_L \equiv \frac{1}{2\mu} + \sqrt{\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}}$. If $\mu > \tilde{\mu}_I$ and $\lambda \geq \tilde{\lambda}_L$, there exist a liquid equilibrium in which loans are insured, $m^* > 0$. In this equilibrium:*

1. *Loan insurance increases the price of uninsured loans and lowers screening.*
2. *The screening threshold is $\eta^* = \frac{(1-\nu)(1-\mu)(\psi-\mu)\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}$, the price is $p_U^* = \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)} \in \left[\frac{A}{\lambda}, p_I^*\right)$, and the share of insured loans is $m^* = 1 - \frac{(\kappa(1-\mu)\psi-(1-\psi)\lambda\mu)F(\eta^*)}{\mu(\lambda-1)(1-\nu)(1-\mu)(1-F(\eta^*))}$.*
3. *The proportion of high-cost lenders who insure m^* increases in μ and λ, decreases in A and upon a FOSD reduction in F and can be non-monotonic in ν. The screening threshold η^* increases in A, decreases in μ, ν and λ. The price p_U^* increases in A, μ, ν and λ. The bound $\tilde{\lambda}_L$ decreases in μ and ν. The bound $\tilde{\mu}_I$ decreases in λ and increases in A and upon a FOSD reduction in F.*

Proof. See Appendix B.3 (which also defines the bound $\tilde{\mu}_I$.) ■

A critical mechanism is how loan insurance affects the quality of uninsured loans traded (Figure 4). Panel (a) shows loan sales in the liquid equilibrium without insurance. The loan payoff A_i at $t = 2$ depends on the screening choice s_i at $t = 0$. The area shaded in blue lines shows uninsured loans traded at $t = 1$, which depends on the liquidity shock λ_i and the private information about A_i. Panel (b) shows the impact of credit risk transfer at $t = 0$ that segments secondary markets at $t = 1$. A share of high-quality and low-quality loans are removed from the uninsured market—shaded in red crosses—and trade in a separate market for insured loans.

Loan insurance improves the quality of uninsured loans traded because of selection and commitment. First, liquidity-shocked lenders who insured have an average loan quality (μA), which is worse than shocked lenders who did not insure and some of



(a) Insurance unavailable

(b) Insurance available

Figure 4: Loan insurance improves the quality of uninsured loans traded.

them screen $(F\psi A+(1-F)\mu A)$. Second, some low-quality loans $(1-\nu)(1-\mu)(1-F)m$ that would have been traded for informational reasons are removed from the uninsured loans market. These are lemons owned by high-cost lenders without a shock, who commit themselves to not exploiting future private information about loan quality.

The competitive price of uninsured loans when loan insurance is available is

$$p_U = \nu A \frac{\psi F(\eta) + \mu \left(1 - F(\eta)\right)(1-m)}{\nu \left(1 - (1-F)\,m\right) + (1-\nu)\left[(1-\psi)F + (1-\mu)\,(1-F)\,(1-m)\right]}, \qquad (9)$$

where the price equals the value of uninsured loans sold by liquidity-shocked lenders divided by the amount of uninsured loans from shocked lenders, $\nu\left(1-(1-F)\,m\right)$, and lenders with lemons and no shock, $(1-\nu)\left[(1-\psi)F + (1-\mu)\,(1-F)\,(1-m)\right]$.

Figure 5 shows the areas for which a liquid equilibrium exists and for which loan insurance occurs. The liquid equilibrium exists for $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, where $\tilde{\lambda}_L \equiv \frac{A}{p_U}$ when loan insurance is used, $m^* > 0$. Due to the higher quality of uninsured loans traded at $t=1$, loan insurance improves the price p_U^*. Hence, the liquid equilibrium can be sustained for a larger range of parameters as the respective thresholds on λ decrease in p_U, resulting in $\tilde{\lambda}_L < \underline{\lambda}_L$ when insurance is used. Conditional on the existence of liquid equilibrium, insurance is used if $\mu > \tilde{\mu}_I$.[19] Combining these two conditional expressions defines the parameter space where there exists a liquid

[19]The condition $\mu > \tilde{\mu}_I$ can also be expressed as $A < \tilde{A}_I$ or an expensive enough screening technology, $F(\cdot)$. If $\nu \leq \frac{2\mu}{1+2\mu}$ (a high degree of adverse selection), then $\lambda > \tilde{\lambda}_I$ is equivalent.

equilibrium in which insurance is used. Since these bounds can also be expressed as $\tilde{\mu}_I$ and $\tilde{\mu}_L$, loan insurance generically occurs in the liquid equilibrium if $\mu \geq \max\{\tilde{\mu}_I, \tilde{\mu}_L\}$.

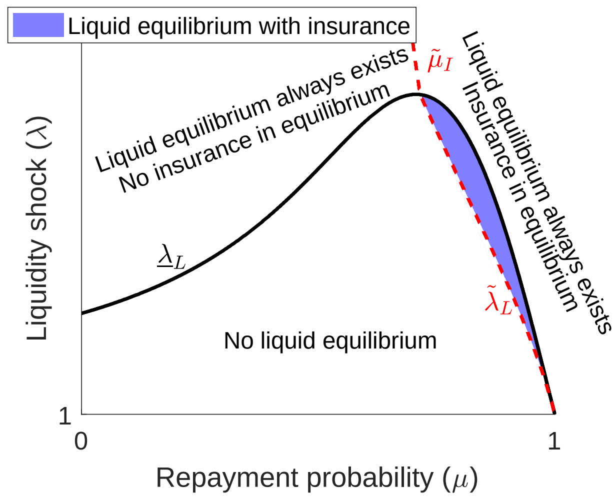


Figure 5: Existence of a liquid equilibrium (when loan insurance is available). A liquid equilibrium with loan insurance arises when $\mu > \tilde{\mu}_I$ and $\lambda \geq \tilde{\lambda}_L$. Insurance allows for the existence of a liquid equilibrium for $\tilde{\lambda}_L \leq \lambda < \underline{\lambda}_L$ (liquifying the uninsured loans market). Below $\underline{\lambda}_L$, the more restrictive condition is the sale of high-quality uninsured loans, $\lambda \geq \tilde{\lambda}_L$, while above $\underline{\lambda}_L$, the more restrictive condition is the usage of insurance, $\mu > \tilde{\mu}_I$.

Loan insurance lowers screening. First, it raises the price of uninsured loans, lowering screening in the liquid equilibrium. Consistent with this implication, Gete and Reher (2020) document that exogenously higher market prices of FHA loans (due to a change in liquidity regulation) reduces lending standards. Second, insurance widens the parameter range for which the liquid equilibrium exists. Screening is lower in the liquid equilibrium with insurance than in the illiquid one without insurance.

Testable implications. We turn to the comparative statics of the liquid equilibrium and focus on $m^* \in (0, 1)$.[20] A larger liquidity shock λ—that may be interpreted as countries or times with high vulnerabilities in the financial sector such as high leverage or large liquidity and maturity mismatches on lender balance sheets—increases the insurance benefits (LHS of equation 8). More insurance improves the price of uninsured loans traded, which lowers screening incentives (equation 5).

Higher loan profitability A—which may also proxy for lower lending market competition or lower borrower bargaining power—directly scales prices p_U and p_I, with

[20] For $m^* = 0$, the comparative statics of the benchmark model stated in Appendix B.1 apply.

no net effect on the incentives to insure. But higher profitability also indirectly increases screening incentives, improving the average quality of uninsured loans traded, so the price of uninsured loans tends to increase more than the price of insured loans (equation 9). Hence, the relative costs of insurance—the option to sell lemons—rises and reduces the share of insured loans m^*. Our model thus implies that loan insurance (i) occurs less in countries with more concentrated lending markets (such as Canada in contrast to the US) and (ii) is more prevalent due to the recent competition from Fintech (e.g. specialized online lenders). Interpreted for a cross-section of banks, more profitable banks should screen more and insure fewer loans. Consistent with this implication, Loutskina and Strahan (2009) document that banks with lower deposit costs originate more non-conforming loans relative to conforming loans.

A first-order stochastic dominance (FOSD) reduction in screening costs $F(\cdot)$—which may proxy for a more productive screening technology, e.g. recent technological advances and better data processing and analysis by Fintechs (e.g., Fuster et al. 2019)—directly increases the incentives to screen, which tends to improve the average quality of uninsured loans traded. Hence, the relative cost of loan insurance increases and reduces the share of insured loans. The equilibrium effect is a reduction in insurance without a change in either the screening threshold or the price of uninsured loans. Taking the results on competition and screening costs together, the overall impact of FinTechs on loan insurance and mortgage guarantees is ambiguous.

A higher probability of repayment μ and of the liquidity shock ν directly increase the insurance benefit and reduce its cost. These changes also lower screening incentives, putting downward pressure on the average quality of uninsured loans traded and its price p_U, further increasing the incentives to insure. However, higher μ and ν also directly tend to increase the price because of a lower share of lemons and a higher share of liquidity sellers in the uninsured loans market, respectively, which tends to increase the cost of insurance. For a higher repayment probability μ—e.g. a high credit score of borrowers or regions with lower predictable default risk (Hurst et al., 2016)—the direct insurance and indirect screening effects dominate the direct price effect, raising the share of insured loans. For the probability of liquidity shock ν—

which may be linked to systemic vulnerabilities of lender balance sheets—the effect on insurance is ambiguous: the direct price effect dominates $\left(\frac{dm^*}{d\nu} < 0\right)$ for high ν, otherwise the direct insurance effect and indirect screening effect dominate under a sufficient condition stated in Appendix B.3 $\left(\frac{dm^*}{d\nu} > 0\right)$. Since a higher μ or ν support the price, it is also easier to sustain the liquid equilibrium, lowering $\tilde{\lambda}_L$.

We turn to the illiquid equilibrium in the market for uninsured loans.

Proposition 3. ***Illiquid equilibrium when loan insurance is available.*** *There always exists an illiquid equilibrium, $p_U^* = 0$, with a screening cost threshold $\eta^* = (\psi - \kappa\mu)A$. If $\lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the illiquid equilibrium is the unique equilibrium.*

Proof. See Appendix B.4. ■

Loan insurance creates a liquid market for insured loans, so high-cost lenders can sell upon a liquidity shock (even in the illiquid equilibrium). Thus, loan insurance increases the payoff without screening, lowering incentives to screen compared to the illiquid equilibrium in the benchmark without loan insurance. Compared to the liquid equilibrium, screening incentives are higher when the uninsured loan market is illiquid. This implication is consistent with evidence provided in Choi and Kim (2018).

4 Welfare benchmark and regulation

We turn to normative implications of credit risk transfer upon loan origination. We first define and characterize a welfare benchmark and then examine whether an uninformed regulator subject to a balanced budget constraint can achieve this benchmark.

4.1 Welfare benchmark

We consider a constrained planner P who maximizes utilitarian welfare W. To highlight the effects of credit risk transfer upon loan origination, we let the planner choose loan insurance for all lenders, based on observing lender screening costs and subject to the privately optimal choices of screening and loan sales. In contrast to the unregulated economy in section 3, the planner internalizes the beneficial impact of loan insurance on secondary market liquidity (shown in Figure 4). The planner can also select the preferred equilibrium (liquid or illiquid). Thus, the planner solves

$$\max\{W^L, W^{NL}\}, \tag{10}$$

where welfare in the liquid (L) and not liquid (NL) equilibrium, respectively, solve[21]

$$\begin{aligned} W^L \quad &\equiv \quad \max_m W \equiv \max_m \overbrace{\nu(\lambda-1)\left[p_U + (p_I - p_U)(1-F(\eta))\,m\right]}^{\text{Social gains from trade}} \\ &\quad + \underbrace{\left[\psi F(\eta) + \mu(1-F(\eta))\right] A}_{\text{Fundamental value}} - \underbrace{\int_0^\eta \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} \qquad (11) \\ \text{s.t.} \quad & (5), \quad (9), \quad \lambda\, p_U \geq A, \quad \text{and} \quad p_I = \mu A. \\ W^{NL} \quad &= \quad \max_m W \text{ s.t. } \eta = (\psi - \kappa\mu)A, \quad \lambda\, p_U = 0 < A, \quad \text{and} \quad p_I = \mu A. \qquad (12) \end{aligned}$$

Welfare is the sum of expected payoffs of lenders (up to a constant representing the expected payoff of financiers) and is derived in Appendix B.5. It comprises terms associated with productive efficiency and allocative efficiency. Productive efficiency refers to the average quality of loans originated (the fundamental value) net of total screening costs. Allocative efficiency refers to the social gains from trade. These are proportional to the difference in marginal utilities, $\lambda - 1$, and the market value of uninsured and insured loans sold by the share of liquidity-shocked lenders, ν. Regarding this market value, equation (11) highlights the price differential of insured and uninsured loans, $p_I - p_U$, received by lenders who sell an insured loan, $m(1-F)$.[22]

[21] Choosing the proportion of high-cost lenders who insure, m, is equivalent to choosing loan insurance for each lender, $\{\ell_i\}$. A lender required by the planner to insure chooses not to screen. Since the benefit of insuring any given loan in terms of market liquidity is the same across lenders but the cost of screening is heterogeneous, the regulator targets insurance to high-cost lenders.

[22] The social gains from trade in the liquid equilibrium can be decomposed into

$$(\lambda-1)\left\{\nu\left[\psi F + \mu(1-F)\right] A - p_U(1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)(1-m)\right]\right\}, \tag{13}$$

which reflects the fundamental value of loans sold by liquidity-shocked lenders in both markets (the first term of equation 13) and the funds diverted from liquidity-shocked lenders by sellers of lemons without a liquidity shock who exploit private information about loan quality (second term).

Proposition 4 summarizes the planner's allocation and Figure 6 illustrates it.

Proposition 4. ***Welfare benchmark.*** *There exists a unique* $\lambda_L^P < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$.

1. *For* $\lambda \leq \lambda_L^P$*, the planner chooses the illiquid equilibrium,* $p_U^P = 0$*, with* $m^P = 1$.

2. *For* $\lambda > \lambda_L^P$*, the planner chooses the liquid equilibrium:*

   a. *For* $\lambda_L^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$*, the planner "liquifies the market" by choosing a share of insured loans* m^P *high enough to ensure* $p_U^P \geq A/\lambda$.

   b. *For* $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$*, the planner insures more loans,* $m^P \geq m^*$*, which raises the price of uninsured loans,* $p_U^P \geq p_U^*$*, and lowers screening incentives,* $\eta^P \leq \eta^*$*. All inequalities are strict for* $\mu > \mu_I^P$*, where* $\mu_I^P < \tilde{\mu}_I$.

Proof. See Appendix B.5 (which also defines the bounds λ_L^P). ∎

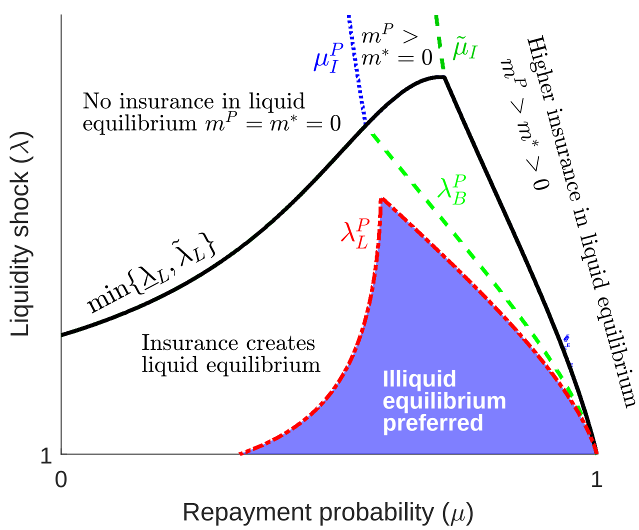


Figure 6: Equilibrium chosen by planner (plotted for $\psi \to 1$). For $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$ and $\mu > \mu_I^P$, the planner improves the price dimension of allocative efficiency by insuring more loans than in the unregulated equilibrium, internalizing its beneficial effect on market liquidity. For $\lambda_L^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the planner liquifies the market for uninsured loans. On the subset $\lambda_L^P < \lambda \leq \min\left\{\lambda_B^P, \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}\right\}$, the regulator increases allocative efficiency only in the quantity dimension, while on the subset $\lambda_B^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ the planner increases both the quantity and price dimension of allocative efficiency. For $\lambda \leq \lambda_L^P$ (shaded), the planner chooses the illiquid equilibrium. Liquifying the market for uninsured loans is feasible but would too severely reduce screening incentives and productive efficiency.

At $\lambda = \lambda_L^P$, the planner is indifferent between the liquid and illiquid equilibria. Screening incentives and productive efficiency is higher in the illiquid equilibrium, while the social gains from trade (allocative efficiency) are higher in the liquid equilibrium. These forces transparently show up in the definition of λ_L^P:

$$\overbrace{\nu(\lambda_L^P - 1)\left[p_U + (p_I - p_U)(1 - F(\eta^L))m^L - p_I(1 - F(\eta^{NL}))\right]}^{\text{Higher gains from trade in liquid equilibrium}} \equiv \overbrace{(\psi - \mu)A\left[F(\eta^{NL}) - F(\eta^L)\right] - \int_{\eta^L}^{\eta^{NL}} \tilde{\eta} dF}^{\text{Higher net benefits of screening in illiquid equilibrium}}, \quad (14)$$

where η^{NL} and η^L are the screening thresholds in the illiquid and liquid equilibrium.

For $\lambda \leq \lambda_L^P$, the social gains from trade have a lower impact on welfare, so the planer prefers the illiquid equilibrium with higher productive efficiency but lower gains from trades. Since these gains are only realized in the insured market in the illiquid equilibrium, the planner chooses $m^P = 1$ (as in the unregulated economy). For $\lambda > \lambda^P$, the higher social gains from trade in the liquid equilibrium exceed the benefits of more screening in the illiquid one, so the liquid equilibrium is preferred.

For $\lambda_L^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the planner insures enough loans m^P to create the liquid equilibrium, $p_U^P \geq {}^{A}/_{\lambda}$.[23] The planner exploits the benefit of loan insurance for the price of uninsured loans and sustains the liquid equilibrium for parameters for which it does not exist in the unregulated economy. For $\lambda_L^P < \lambda \leq \lambda_B^P \leq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, the regulator increases allocative efficiency on the quantity dimension only, $p_U^R = {}^{A}/_{\lambda}$. For $\lambda_B^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, however, the regulator improves allocative efficiency on both the quantity and the price dimension, $p_U^R > {}^{A}/_{\lambda}$.[24]

For $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, the planner internalizes the beneficial effect of loan insurance for market liquidity and insures more loans, $m^P > m^* > 0$ for $\mu > \tilde{\mu}_I$. The planner also insures loans for a larger set of parameters, $m^P > 0 = m^*$ for $\mu_I^P < \mu \leq \tilde{\mu}_I$, where μ_I^P is defined in Appendix B.5.[25] Thus, the planner improves

[23]For $\lambda \geq \frac{\nu+(1-\nu)(1-\psi)}{\nu\psi} \equiv \underset{\sim}{\lambda_L}$, there exists an $m \leq 1$ to support a liquid equilibrium, $p_U \geq {}^{A}/_{\lambda}$. For $\lambda < \underset{\sim}{\lambda_L}$, however, even $m = 1$ cannot keep the market liquid, where $\underset{\sim}{\lambda_L}$ arises from equation (2) with insurance of all high-cost lenders, $p_U(m = 1) = \frac{\nu\psi}{\nu+(1-\nu)(1-\psi)}A \equiv \frac{A}{\lambda}$. The bound $\underset{\sim}{\lambda_L}$ decreases in ψ and $\lambda \geq \underset{\sim}{\lambda_L}$ holds whenever screening is productive enough, $\psi\kappa \geq 1$, which we assume henceforth.

[24]Appendix B.5 defines the bound λ_B^P; there exists a $\widehat{\mu} < 1$ such that $\lambda_B^P \leq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\} \Leftrightarrow \mu \geq \widehat{\mu}$.

[25]This condition of a high probability of repayment, $\mu > \mu_I^P$, can also be expressed as a low loan payoff, $A < A_I^P$, or for a sufficiently expensive screening technology, $F(\cdot)$, or a high enough size of

the allocative efficiency on the price dimension, which reduces screening incentives.

For intuition about the planner's choice of loan insurance, we state its effects on welfare:

$$\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} + \frac{\partial W}{\partial \eta^*}\frac{d\eta^*}{dm}. \tag{15}$$

In the unregulated equilibrium, insurance and screening are chosen privately optimally, where $\frac{\partial W}{\partial m} \mid_{m=m^*}= 0$ and $\frac{\partial W}{\partial \eta} \mid_{m=m^*}= 0$ are the indifference condition for loan insurance and the screening threshold.[26] Since loan insurance improves the price of uninsured loans, welfare at $m = m^*$ increases in insurance, $\frac{dW}{dm} \mid_{m=m^*}= \frac{\partial W}{\partial p_U}\frac{dp_U}{dm} > 0$. Once insurance increases above its unregulated level, $m > m^*$, the positive welfare effect of higher social gains from trade, $\frac{\partial W}{\partial p_U} > 0$, are mitigated by the negative effect of lower screening, $\frac{\partial W}{\partial \eta} \mid_{m>m^*}> 0$. That is, lenders who in the unregulated economy do not insure (and some of whom screen) are required to insure and, therefore, do not screen. These lenders are individually worse off, while other lenders are better off due to higher gains from trade upon a liquidity shock and overall welfare increases.

4.2 Regulation

We consider a regulator R subject to a balanced-budget constraint and with the same information as outside financiers. As a result, a direct implementation of the planner's allocation by choosing insurance for each lender is infeasible. Only high-cost lenders should insure but lender screening costs are unobserved by the regulator.

Instead we consider two regulatory tools: (i) a subsidy $b_I \geq 0$ to lenders who insure their loan at $t = 0$ and (ii) a minimum price guarantee $p_{min} \geq 0$ in the market for uninsured loans at $t = 1$. This guarantee can be credibly implemented via a subsidy to sellers of uninsured loans (as originally envisioned by TARP in the US):

$$b_U \equiv \max\{p_{min} - p_U, 0\}. \tag{16}$$

the liquidity shock $\lambda_I^P < \lambda$. The condition $\nu \leq \frac{2\mu}{1+2\mu}$ is again sufficient for a bound on λ to exist.

[26] The expression of welfare used is the expected lender payoffs in equation (39) in Appendix B.5.

The regulator has commitment, announces regulation (b_I, p_{min}) at the beginning of $t = 0$, and lenders make their privately optimal choices of loan insurance, screening, and loan sales. We refer to this arrangement as the regulated economy.

These policies are funded by a lump-sum tax T on all lenders at $t = 1$. This tax is levied after loan sales and before consumption occurs. To ensure that lenders can always pay the tax (and to avoid unnecessary technical complications associated with limited liability), we introduce an additional non-pledgeable and perishable endowment n received when taxes are due, so these resources can be used to pay taxes or for consumption at $t = 1$.[27] To make both policies more comparable, we assume that the loan insurance subsidy is received at $t = 1$ as well. Figure 7 shows the timeline.

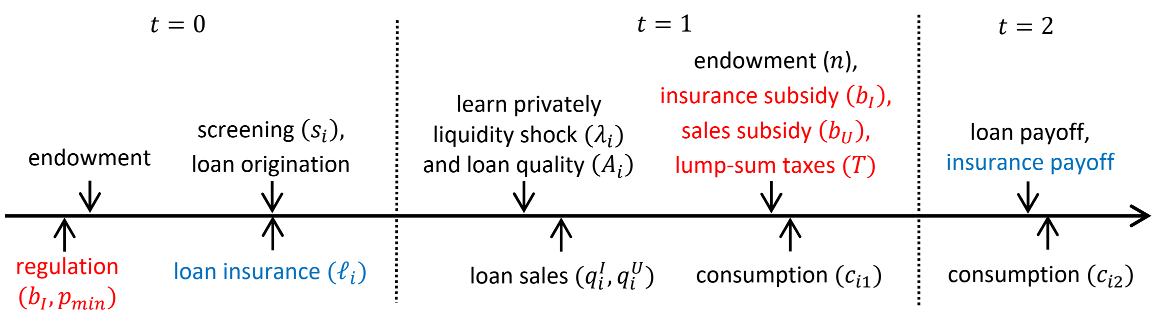


Figure 7: Timeline with loan insurance subsidy b_I and uninsured loan sale subsidy b_U.

Definition 2. *A regulated equilibrium comprises screening $\{s_i\}$, insurance $\{\ell_i\}$, insured and uninsured loan sales $\{q_i^I, q_i^U\}$, an insurance subsidy b_I, a minimum price guarantee p_{min}, lump-sum taxes T, prices p_I and p_U, and a fee k such that:*

1. *At $t = 1$, for each λ_i and A_i, each lender i optimally chooses sales q_i^I and q_i^U.*

2. *At $t = 0$, each lender i chooses screening s_i and loan insurance ℓ_i to solve*

$$\max_{s_i, \ell_i, c_{i1}, c_{i2}} \quad \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \text{subject to}$$
$$c_{i1} = q_i^I p_I + \ell_i b_I + q_i^U (p_U + b_U) + n - T,$$
$$c_{i2} = (\ell_i - q_i^I)\pi + (1 - \ell_i - q_i^U)A_i, \quad \Pr\{A_i = A\} = \psi s_i + \mu(1 - s_i).$$

[27] An endowment $n = (1-\mu)A$ covers any meaningful regulation. If $b_I = (1-\mu)A$, all lenders insure $m = 1$, $\eta = 0$, and $T = (1-\mu)A$. Similarly for the other tool, if $p_{min} = A$, then all high-quality loans are sold irrespective of the liquidity shock and $\eta = 0$, which implies that the fundamental value of loans sold is μA and the required subsidy is $b_U = (1-\mu)A = T$.

3. *The insurance fee k at $t = 0$ and the prices of loans p_I and p_U at $t = 1$ are set for outside financiers to break even in expectation.*

4. *At $t = 0$, the regulator chooses the insurance subsidy b_I and price guarantee p_{min} to maximize welfare subject to a balanced budget, $T = b_U \int q_i^U di + b_I \int \ell_i di$.*

Generalizing condition (8), loan insurance in the regulated economy satisfies

$$m \left[\nu\lambda \left(p_I - (p_U + b_U) \right) + \kappa b_I - (1-\nu)(1-\mu)(p_U + b_U) \right] = 0 \tag{17}$$

with complementary slackness. Thus, loan insurance is used, $m > 0$, whenever high-cost lenders are indifferent about insurance.[28] In this case, the uninsured sale subsidy b_U increases the value of sold uninsured loans, $p_U + b_U$, lowering the incentives to insure. By contrast, the insurance subsidy b_I increases the incentives to insure and the share of insured loans m, which indirectly increases the price of uninsured loans p_U. When high-cost lenders are not indifferent, $m = 0$, the insurance subsidy does not affect the price p_U. Generalizing the screening threshold in equation (5) yields:

$$\eta = (1-\nu)(\psi - \mu)\left[A - (p_U + b_U) \right], \tag{18}$$

where either subsidy reduces the incentives to screen (directly or via the price p_U).

The regulator solves $\max\{W_R^L, W_R^{NL}\}$, where welfare in the illiquid equilibrium is defined in Appendix B.6 and welfare in the liquid equilibrium solves:[29]

[28] Lenders prefer not to insure or are indifferent about insurance in the unregulated equilibrium, $m^* < 1$, because $\psi > \underline{\psi}$. While the regulator insures more, $m^* \leq m^R \leq 1$, it is never optimal to induce a strict preference for insurance that would reduce screening and welfare. See Appendix A.5.

[29] Without loss of generality, we focus on the interval $b_I \leq (1-\mu)A$ and $b_U \leq (1-\mu)A$, respectively. Higher subsidies have no effect on welfare, as the payoff of insured loans $\mu A + b_I$ (sold loans $p_U + b_U$) would exceed the payoff from high-quality loans, so all lenders insure (sell all high quality loans irrespective of liquidity shock) and do not screen, resulting in constant welfare $W = \kappa(\mu A + n)$.

$$W_R^L \equiv \max_{b_I, p_{min}} \overbrace{\nu(\lambda-1)\left[p_U + b_U + (p_I + b_I - (p_U + b_U))(1-F(\eta))\, m\right]}^{\text{Gains from trade}}$$
$$+ \underbrace{\left[\psi F(\eta) + \mu(1-F(\eta))\right] A}_{\text{Fundamental value}} - \underbrace{\int_0^\eta \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} + \kappa(n-T) \tag{19}$$
$$\text{s.t. } (9),\ (16),\ (17),\ (18),\ p_I = \mu A, \text{ and } \lambda p_U \geq A,$$

Our main result on regulation and loan insurance subsidies follows.

Proposition 5. ***Regulation achieves the welfare benchmark.***

1. *For $\lambda \leq \lambda_L^P$, the regulator implements the illiquid equilibrium, $p_{min}^R = 0 = b_U^R$, with $m^R = 1$ such that no loan insurance subsidies are used, $b_I^R = 0$.*

2. *For $\lambda > \lambda_L^P$, the regulator guarantees a minimum price, $p_{min}^R = A/\lambda$ to eliminate the inferior illiquid equilibrium. In the resulting (unique) liquid equilibrium, $p_U^R \geq p_{min}^R$ and thus $b_U^R = 0$.*

   (a) For $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ and $\mu \leq \mu_I^P$, insurance is not subsidized, $b_I^R = 0$.

   (b) For $\lambda_L^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ or $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ and $\mu > \mu_I^P$, the regulator implements the welfare benchmark by subsidizing loan insurance:

$$\kappa b_I = \underbrace{(1-\nu)(1-\mu)p_U}_{\textit{Private cost of insurance}} - \underbrace{\nu\lambda(p_I - p_U)}_{\textit{Private benefit of insurance}} \tag{20}$$

Proof. See Appendix B.6. ■

When the regulator implements the illiquid equilibrium, $p_U^* = 0$ irrespective of insurance and screening. Hence, the illiquid equilibrium of the unregulated economy equals the welfare benchmark, so the regulator does not use either tool, $p_{min} = 0 = b_I$.

Eliminating the illiquid equilibrium when it is inferior can only be achieved via guaranteeing a minimum price for uninsured loans, $p_{min} = A/\lambda$. To credibly promise this minimum price, the regulator must have the option of a subsidy to the sellers of uninsured loans at its disposal. This eliminates any mutually reinforcing self-fulfilling beliefs about a zero price of uninsured loans and no sale of high-quality loans.

In the liquid equilibrium, both an uninsured loan sale subsidy b_U and an insurance subsidy b_I can keep the market liquid. However, the insurance subsidy is superior to the sale subsidy because of its beneficial impact on market liquidity. The insurance subsidy induces high-cost lenders to forgo the option of selling lemons based on future private information about loan quality and, thus, reduces adverse selection. In contrast, the sale subsidy does not take advantage of this benefit and is therefore more expensive. Moreover, the uninsured loan sale subsidy can eliminate beneficial insurance (by making high-cost lenders prefer not to insure).[30] Figure 8 compares welfare in the liquid equilibrium when the same target price $p_U^T < A$ is achieved with an insurance subsidy, $p_U^T = p_U(b_I)$, or with an uninsured loan sale subsidy, $p_U^T = p_U + b_U$.

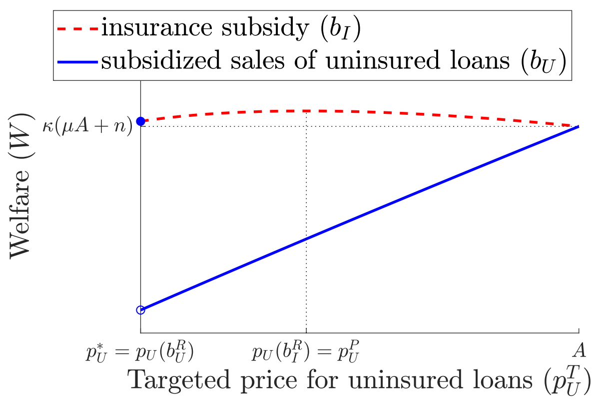


Figure 8: Insurance subsidy achieve the welfare benchmark, $p_U^R = p_U(b_I^R) = p_U^P > p_U^*$, and dominates uninsured loan sale subsidy. Sale subsidies eliminate insurance used in the unregulated equilibrium (when $\lambda \geq \tilde{\lambda}_L$ and $\mu > \tilde{\mu}_I$), resulting in a discrete drop in welfare at $p_U^T = p_U^*$. At $p_U^T = A$, all lenders receive a subsidy under both policy options and, therefore, the overall welfare levels are equalized: $W \mid_{p_U^T = A} = \kappa(\mu A + n)$. Plotted for $\psi \to 1$.

The optimal insurance subsidy implements the planner's choice of more insurance. Its size incentivizes lenders who do not insure in the unregulated equilibrium because their private costs of insurance exceed the private benefits (equation 20). The size of the subsidy captures the net social benefits of insurance and

[30]Under the optimal subsidy, the regulator does not increase welfare by redistribution between lenders, resulting in zero redistributive term, $\kappa(b_I(1 - F(\eta))m - T) = 0$. While the insurance subsidy redistributes from all lenders (taxpayers) to insured lenders, all of them have the same expected utility of consumption, κ. Thus, our welfare effects arise only from the impact on the incentives to insure m and their impact on the uninsured loan price p_U and screening incentives η.

can be expressed from the FOC of the regulator objective (19) with respect to b_I, $\left(\frac{dW_R^L}{dm} + \gamma \frac{dp_U}{dm}\right)\frac{dm}{db_I} = 0$, where γ is the Lagrange multiplier on the constraint $\lambda\, p_U \geq A$ that captures the effect of the subsidy on sustaining the liquid equilibrium:

$$\kappa\, b_I^R \underbrace{\left[1 - F(\eta^R)\right]}_{\text{Share of high-cost lenders}} = \underbrace{\left(\frac{\partial W_R^L}{\partial p_U} + \gamma\right)\frac{dp_U}{dm}}_{\text{Marginal social benefit}} - \underbrace{\frac{\partial W_R^L}{\partial \eta}\left(-\frac{d\eta}{dm}\right)}_{\text{Marginal social cost}}. \tag{21}$$

The regulator implements a subsidy that balances the marginal social benefits of higher allocative efficiency with the marginal social costs of lower productive efficiency.

The subsidy depends on the dimension of allocative efficiency improved by regulation. For low social gains from trade, $\lambda_L^P < \lambda \leq \lambda_B^P \leq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, the regulator only liquifies the market for uninsured loans, $p_U = {}^{A}\!/\!_{\lambda}$ and $\gamma > 0$, but any further improvement of allocative efficiency reduces productive efficiency too much. Hence, insurance subsidies target this minimum price to sustain a liquid equilibrium, so the subsidy decreases in λ. For higher social gains from trade, both dimensions of allocative efficiency are improved, $\lambda_B^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, or only the price dimension, $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ and $\mu > \mu_I^P$. In these cases, the constraint is slack, $\gamma = 0$, and the insurance subsidy increases in the size of the liquidity shock λ. Figure 9 illustrates.

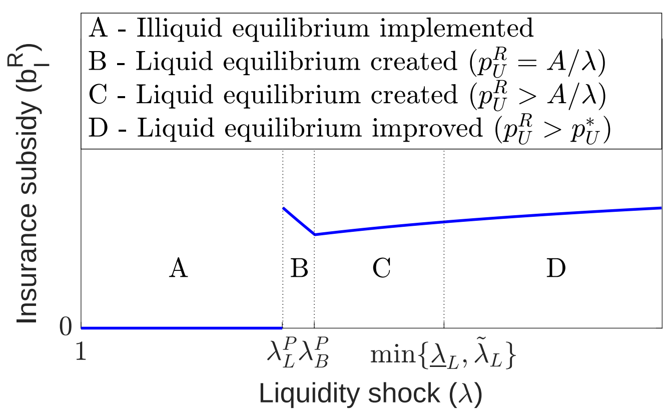


Figure 9: Insurance subsidy b_I^R is non-monotonic in the size of the liquidity shock λ (plotted for $\mu > \widehat{\mu}$ and $\psi \to 1$). The illiquid equilibrium is implemented for $\lambda \leq \lambda_L^P$, so the subsidy is zero. For $\lambda > \lambda_L^P$, the liquid equilibrium is implemented. For $\lambda_L^P < \lambda \leq \lambda_B^P$, the regulator implements a price consistent with a liquid market, $p_U^R = {}^{A}\!/\!_{\lambda}$ (improving the quantity dimension of allocative efficiency). For $\lambda_B^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the regulator improves both the quantity and the price dimension of allocative efficiency, $p_U^R > {}^{A}\!/\!_{\lambda}$. For $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the regulator improves the price dimension of allocative efficiency, $p_U^R > p_U^* \geq {}^{A}\!/\!_{\lambda}$.

5 Implications for mortgage guarantees

The beneficial effect of loan insurance on market liquidity implies insufficient insurance in the unregulated economy and motivates insurance subsidies in the following situations. First, insurance subsidies should cover loans with a low default risk (high μ), such as borrowers with high credit scores or loans in regions with low predictable default risk. Indeed, conditioning mortgage guarantees on high enough credit scores is consistent with the practices of FHA and GSEs in the US and CMHC in Canada. However, government support for mortgage guarantees does not vary across regions despite large regional variation in predictable default risk (Hurst et al., 2016).

Second, insurance subsidies should arise when loans are less profitable, borrowers have a lot of bargaining power, or lending markets are more competitive (low A). This implies that the benefits of loan insurance are higher in countries with a less concentrated lending market and lower profit margin of lenders (e.g., the United States as opposed to Canada). Similarly, our model suggests that the recent competition from Fintech (e.g., specialized online lenders) has made loan insurance more beneficial.

Third, loan insurance subsidies arise when lenders may face larger liquidity needs (high λ). This would apply in countries with high systemic vulnerabilities in the financial sector and when lenders are highly levered or have large liquidity and maturity mismatches on their balance sheets. Finally, less insurance is desirable when screening costs are lower (a shift in F). Recent technological advances and extensive data analysis of borrowers, such as big data or machine learning innovations, including by FinTechs (e.g., Fuster et al., 2019; Buchak et al., 2018), would reduce the benefits of insurance. Taken the impact of FinTechs on screening costs and lending market competition together, its impact on subsidies to mortgage guarantees is ambiguous.

Finally, we derive results about the size of the optimal insurance subsidy. First, the size of the subsidy increases in loan profitability. This suggests that the subsidy should be higher in less competitive lending markets. Similarly, higher competition brought by Fintech lenders in recent years suggests that a smaller subsidy is necessary. Second, the size of the subsidy is non-monotonic in the size of the liquidity shock λ

(Figure 9), which can proxy for countries with or during times of high systemic risk and when lenders are vulnerable due to high leverage or balance sheet mismatches.

6 Adverse selection in loan insurance market

In this extension, we study the possibility of adverse selection in the loan insurance market. To do so, we modify the screening technology: lenders who screen privately learn loan quality A_i already upon origination at $t = 0$. Lenders who do not screen still privately learn A_i at $t = 1$ as in the main model. A share $1 - \psi$ of lenders who screen may insure lemons, resulting in adverse selection in loan insurance at $t = 0$.

Proposition 6. ***Adverse selection in loan insurance market.*** *In the modified model with asymmetric information at $t = 0$, additional multiple equilibria arise:*

1. *An equilibrium with an illiquid insurance market, $k = A$ and $p_I = 0$, always exists. Lemma 1 and 2 from the benchmark model without loan insurance apply.*
2. *For $\lambda \geq \tilde{\lambda}_L^{AS}$ and $\mu > \tilde{\mu}_I^{AS}$, where $\tilde{\lambda}_L^{AS} > \tilde{\lambda}_L$ and $\tilde{\mu}_I^{AS} > \tilde{\mu}_I$, there exists an equilibrium in which the markets for insurance and uninsured loans are liquid:*
   - *(a) All high-cost lenders are indifferent about insurance and an interior share of them insures, $0 < m^* < 1$. The prices satisfy $0 < p_U^* < p_I^* < \mu A$ and all low-cost lenders with a lemon choose to insure it at $t = 0$.*
   - *(b) Compared to the liquid equilibrium in Proposition 2, the prices of both insured and uninsured loans are lower, the screening threshold is higher, and overall welfare is lower.*
   - *(c) There are potentially multiple equilibria, with different shares m^*. Among these, the equilibrium with the highest share of insured loans has the highest price of uninsured loans, the lowest screening, and the highest welfare.*

The planner insures more loans because the social benefits of insurance are not fully internalized in the unregulated economy. The regulator subsidizes loan insurance *to eliminate the welfare-dominated equilibrium with an illiquid market for insurance. If the liquid equilibrium is implemented, the optimal loan insurance subsidy is*

$$\kappa b_I^R = \underbrace{(1-\nu)\left[(1-\mu)p_U + (\mu A - p_I)\right]}_{\textit{Private costs of insurance}} - \underbrace{\nu\lambda(p_I - p_U)}_{\textit{Private benefit of insurance}} . \tag{22}$$

Proof. See Appendix B.7, where $\tilde{\lambda}_L^{AS}$ and $\tilde{\mu}_I^{AS}$ are defined. ■

Due to private learning of A_i at $t = 0$, low-cost lenders can selectively insure lemons. Thus, the additional defining feature of equilibrium is whether high-cost lenders insure and make the insurance market liquid because not only lemons are insured, $k < A$ and $p_I > 0$. The insurance markets can always be illiquid since $p_I = 0$ and low-cost lenders selectively insuring lemons are mutually consistent. The equilibrium with a liquid insurance market has a higher price for uninsured loans, lower screening, and higher welfare than the equilibrium with an illiquid insurance market—similar to the effect of loan insurance on the liquid equilibrium in Section 3.

We also compare the equilibrium with a liquid insurance market to the main model. Asymmetric information at $t = 0$ reduces the benefits of insurance because adverse selection reduces the price for insured loans, $p_I^* < \mu A$. Hence, insurance occurs for a smaller parameter range, $\tilde{\mu}_I^{AS} > \tilde{\mu}_I$, and fewer loans are insured. While lemons by low-cost lenders are removed from the market for uninsured loans, the effect of lower insurance on the price dominates, reducing the price of uninsured loans overall. Screening incentives are higher in the modified model for two reasons. First, screening has an additional benefit of learning loan quality at $t = 0$ and selectively insuring lemons (at an advantageously low fee). Second, the lower price of uninsured loans lowers the payoff from not screening, which involves selling lemons at $t = 1$.

Another new feature of this liquid equilibrium is a strategic complementarity in the lender's choice to insure. The benefit of insurance increases in the proportion of insuring high-cost lenders m ($dp_I/dm > 0$). Since all low-cost lenders insure lemons, a higher share of high-cost lenders who insure spreads the costs of cross-subsidizing insured lemons among more lenders. This lowers the insurance fee and improves the price of insured loans p_I. This strategic complementarity can result in multiple equilibria even when the market for both insurance and uninsured loans are liquid.

In an equilibrium, insurance again improves the average quality of uninsured loans traded. As in the main model, insured lenders have on average lower quality than uninsured lenders and high-cost lenders who insure give up the option to exploit future private information. Moreover, insurance removes all lemons owned by low-cost lenders from the uninsured market because informed low-cost lenders sell lemons for the highest price, which is in the market for insured loans, $p_I^* > p_U^*$. However, adverse selection in insurance lowers the private incentives to insure. Therefore, the equilibrium insurance benefits are lower in the unregulated equilibrium with liquid insurance and eliminated in the equilibrium with illiquid insurance market.

Adverse selection in the loan insurance market further strengthens the case for loan insurance subsidies compared to the main model. It creates a new and additional incentive to liquify the insurance market and improve allocative efficiency, with an independent welfare benefit. Indeed, the regulator can use insurance subsidies to eliminate the welfare-dominated equilibrium with illiquid insurance market.

Similarly to the main model, high-cost lenders also insure too few loans due to the positive externality of loan insurance on the quality of uninsured loans traded in the liquid equilibrium. Hence, there is again scope for the regulator to improve allocative efficiency and welfare. The optimal subsidy in equation (22) incentivizes high-cost lenders to insure and reflects the lower insurance incentives due to adverse selection ($p_I^* < \mu A$) compared to equation (20). The optimal subsidy balances the social benefits, which includes a higher price p_I, and the costs of insurance:

$$\kappa b_I^R \left[1 - F(\eta)\right] = \underbrace{\left(\frac{\partial W}{\partial p_U} + \gamma\right) \frac{dp_U}{dm} + \frac{\partial W}{\partial p_I} \frac{dp_I}{dm}}_{\text{Marginal social benefit}} - \underbrace{\frac{\partial W}{\partial \eta} \left(-\frac{d\eta}{dm}\right)}_{\text{Marginal social costs}}, \tag{23}$$

where the modified welfare function W is characterized in Appendix B.7.

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A Generalizations and extensions

Unless stated otherwise, we focus on the simplification $\psi \to 1$ throughout these extensions.

A.1 Loan sales upon origination

We study the option for lenders to sell loans to outside financiers upon origination at $t = 0$. This approach requires a couple of small changes to the model. First, financiers are endowed at $t = 0$ as well. Second, for lenders to consume at $t = 1$ (when their expected marginal utility of consumption is high), we introduce a storage technology for the proceeds of loan sales until $t = 1$. Lenders learn the loan payoff A_i at $t = 1$, e.g. due to relationship lending.

Proposition 7. ***Loan sales upon origination.*** *In the extended model, loan sales upon origination and loan insurance are equivalent. Hence, all of our positive and normative implications carry over from the main text to the case of loan sales upon origination.*

Loan sales upon origination reduce adverse selection in the uninsured loan market and raises its price—exactly as loan insurance does due a formal equivalence of both forms. To see this, note that both loan sales and loan insurance yield the same payoffs for lenders—κp_0 for the sale and κp_I for insurance, where $p_0 = p_I = \mu A$ in equilibrium. Since both forms commit a lender to not acting on future private information about loan quality A_i at $t = 1$, the same beneficial impact on the price of uninsured loans at $t = 1$ arises:

$$p_1 = \nu A \frac{F + \mu(1-F)(1-m^I-m^S)}{\nu\left[F + (1-F)(1-m^I-m^S)\right] + (1-\nu)(1-\mu)(1-F)(1-m^I-m^S)}, \tag{24}$$

where m^I (m^S) is the share of high-cost lenders insuring (selling upon origination). Hence, all positive and normative result from the main text extend to loan sales upon origination.

A.2 Partial insurance

Suppose lenders can choose the share ω of default costs covered by the insurance. Such an insurance contract is equivalent to guaranteeing the non-default payment A with a deductible $(1-\omega)A$, where the owner of the loan pays the insurance fee at $t = 2$. Since only high-cost lenders insure, the competitive insurance fee is actuarially fair, $k = \omega(1-\mu)A$.

Proposition 8. *Full insurance, $\omega^* = 1$, is both privately and socially optimal.*

Proof. See Appendix B.8. ■

With partial insurance, $\omega < 1$, the value of an insured loan of low quality is $\omega A - k = \omega\mu A$, which is below the value of an insured loan of high quality, $A - k = A[1-(1-\mu)\omega]$, which contrasts with the main text. There is adverse selection in the market for partially insured loans since lenders without a liquidity shock sell only low-quality loans. Adverse selection redistributes wealth from lenders with a liquidity shock (who always sell) to lenders without liquidity shock (who sell only lemons). If this redistribution is severe, insured loans of high quality are not traded and social gains from trade decrease further. Since lenders have a higher utility in states with liquidity shock, they choose full coverage, $\omega^* = 1$, to avoid the costs of adverse selection. As for social optimality, a higher insurance coverage has a positive externality on the price of uninsured loans, so a planner chooses full coverage.

An alternative interpretation of partial insurance is insurer default. We have assumed so far that the insurer has deep pockets, perhaps because of (implicit) government backing. In contrast, suppose the insurer defaults on its liabilities after the fee is paid at $t = 2$ with exogenous probability $1 - \omega$. The expected value of an insured loan is $\omega A - k$ upon loan default ($-k$ when insurer defaults and $A - k$ otherwise) and $A - k$ upon loan repayment (irrespective of insurer default). The insurance fee is $k = \omega(1 - \mu)A$. Since the expected payoffs are equal to those for partial insurance, the problem with insurer default is identical. Accordingly, Proposition 8 implies that welfare decreases in insurer default risk.

A.3 Upfront insurance fee

We consider an insurance fee k that must be paid at $t = 0$. Thus, a lender who insures can fund only a share $1-k$ of the loan, reducing the lending volume. We show that the positive implications are qualitatively the same. Since credit risk transfer upon origination via loan insurance still has a positive impact on the price of uninsured loans not internalized in the unregulated economy, our normative results are also qualitatively the same.

Proposition 9. ***Upfront insurance fee.*** *The fee paid at* $t = 0$ *is* $k^* = \frac{A(1-\mu)}{1+A(1-\mu)}$.

1. *Insurance increases the uninsured loan price, lowers screening, and increases welfare.*
2. *For* $\mu > \tilde{\mu}'_I$ *and* $\lambda \geq \tilde{\lambda}'_L$*, the screening threshold is* $\eta^{*\prime} \equiv \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}(1+\delta)$*, the price of uninsured loans is* $p^{*\prime}_U \equiv \frac{\nu\lambda\mu A-\kappa(1-\mu)A\delta}{\nu\lambda+(1-\nu)(1-\mu)}$*, and some loans are insured,* $m^{*\prime} = 1 - \frac{\kappa F(\eta^{*\prime})(1-\delta)}{(1-F(\eta^{*\prime}))\left[\mu(\lambda-1)(1-\nu)-\kappa\delta\frac{\nu+(1-\nu)(1-\mu)}{\nu}\right]} \in (0,1)$*, where* $\delta \equiv \frac{\mu A-1}{1+A(1-\mu)}$.
3. *The planner insures more loans than in the unregulated economy,* $m^{P\prime} \geq m^{*\prime}$.

Proof. See Appendix B.9 (which also defines the bounds $\tilde{\mu}'_I$ and $\tilde{\lambda}'_L$). ■

A.4 Partial loan sales

We allow for partial sales of uninsured loans, where $q_i^U \in [0, 1-\ell_i]$ is a continuous choice of lenders and retaining default risk $1 - \ell_i - q_i^U$ may signal loan quality to financiers. That is, financiers use $1 - \ell_i - q_i^U$ to update their beliefs about loan quality. A continuum of perfect Bayesian equilibria (PBE) may exist but our results are qualitatively unchanged.

Proposition 10. ***Partial loan sales.***

All PBE are pooling equilibria characterized by $q^{U*} \in (0,1]$ and screening $\eta^*(q^{U*})$, sustained by out-of-equilibrium beliefs interpreting $q^U \neq q^{U*}$ as a signal of low quality. The quality of uninsured loans remains private information. Except for the corner case of perfect screening by all lenders($\psi \to 1$, $\bar{\eta} < (1-\mu)A$, $q^{U*} < \bar{q}^U$), adverse selection remains and:

1. *For $\mu > \tilde{\mu}_I(q^U)$, some loans are insured in the liquid equilibrium, $m^* > 0$.*
2. *The planner insures more loans in the liquid equilibrium, $m^P > m^* > 0$ for $\mu > \tilde{\mu}_I(q^U)$, and for more parameters, $m^P > m^* = 0$ for $\mu_I^P(q^U) < \mu \leq \tilde{\mu}_I(q^U)$.*

Proof. See Appendix B.10 (which defines the equilibrium and bounds $\tilde{\mu}_I(q^U)$ and $\bar{q}^U$). ■

Since lenders have limited liability, any loan sale q^U would be mimicked by sellers of low-quality loans (similar to Parlour and Plantin 2008). Thus, insurance has a positive effect on the reduction of adverse selection and our positive and normative results are qualitatively unchanged. The only exception is the case where all lenders screen ($\eta^*(q^{U*}) > \bar{\eta}$) and screening technology is perfect ($\psi \to 1$), and thus it eliminates adverse selection. This arises for $\bar{\eta} < (1-\mu)A$ and $q^{U*} < \bar{q}^U$. In this case, the upper bound on screening costs is low enough so that sufficient default risk retention incentivizes all lenders to screen, so for $\psi \to 1$ all loans are of high-quality. When screening is imperfect, however, low-quality loans are originated and at $t = 1$ their holders mimic the risk retention of lenders with high-quality loans, resulting in adverse selection in the market for uninsured loans. So our results from the main text extend to partial loan sales, whereby loan insurance reduces such adverse selection and the planner insures more loans.

A.5 Low screening productivity

Proposition 11. ***Low screening productivity.*** *For $\psi \leq \underline{\psi}$ and $\lambda \geq \underset{\sim}{\lambda_L}$, there exists a liquid equilibrium with price $p_U^* = \frac{\nu\psi}{\nu+(1-\nu)(1-\psi)}A$ and screening threshold $\eta^* = \nu\lambda + (1-\nu)[\psi A + (1-\psi)p_U] - \kappa\mu A$. The planner's allocation coincides with the unregulated equilibrium: $m^P = 1 = m^*$, $\eta^P = \eta^*$, and $p_U^P = p_U^*$. There are no loan insurance subsidies.*

Proof. See Appendix B.11. ■

For $\psi > \underline{\psi}$ in the main text, some loans are uninsured, $m^* < 1$, so more insurance by the planner increases the price of uninsured loans. For $\psi \leq \underline{\psi}$, however, this positive effect is exhausted in the corner solution of insurance by all high-cost lenders, $m^* = 1$. Hence, the planner / regulator cannot improve upon the unregulated equilibrium with insurance subsidies or taxes. Indeed, insurance subsidies would only lower screening because the indifference about insurance no longer holds as all high-cost lenders strictly prefer to insure.

B Proofs

B.1 Proof of Lemma 1

Definition 3. *An equilibrium comprises screening choices $\{s_i\}$, loan sales $\{q_i^U\}$, and a price of loans p_U such that:*

1. *At $t = 1$, for each λ_i and A_i, each lender i optimally chooses loan sales, q_i^U.*
2. *At $t = 1$, the price p_U is set for outside financiers to break even in expectation.*
3. *At $t = 0$, each lender i chooses screening s_i to maximize expected utility:*

$$\max_{s_i, c_{i1}, c_{i2}} \quad \mathbb{E}[\lambda_i c_{i1} + c_{i2} - \eta_i s_i] \quad \textit{subject to}$$
$$c_{i1} = q_i^U p_U, \quad c_{i2} = (1 - q_i^U) A_i, \quad \Pr\{A_i = A\} = \psi s_i + \mu(1 - s_i).$$

In the liquid equilibrium, screening yields the expected payoff $\nu\lambda p_U + (1-\nu)[\psi A + (1-\psi)p_U] - \eta$ and not screening yields $\nu\lambda p_U + (1-\nu)[\mu A + (1-\mu)p_U]$, so the cost threshold stated in (5) follows. Inserting equation (4) in (5) yields η^* determined by equation (6).

Within the class of liquid equilibria, does a unique equilibrium exist? Regarding uniqueness, the left-hand side (LHS) of the equation (6) increases in η and its right-hand side (RHS) decreases in it, so at most one intersection exists. Regarding existence, we evaluate both sides at the bounds, using $F(0) = 0 < 1 = F(\bar{\eta})$. Note that $LHS(0) < RHS(0)$ always holds and $LHS(\bar{\eta}) > RHS(\bar{\eta})$ if $\bar{\eta} > \frac{(1-\nu)(\psi-\mu)(1-\psi)}{\nu+(1-\nu)(1-\psi)} A \equiv \bar{\eta}_L$. For $\psi \to 1$, this condition always holds. For $\psi < 1$, we assume that the screening cost is heterogeneous enough. Hence, there exists a unique and interior screening threshold $\eta^* \in (0, \bar{\eta})$. The price of loans sold at $t = 1$ is:

$$p_U^* = \nu A \frac{\psi F(\eta^*) + \mu(1 - F(\eta^*))}{\nu + (1-\nu)\left[(1-\psi) F(\eta^*) + (1-\mu)(1 - F(\eta^*))\right]}, \tag{25}$$

where η^* is given in equation (5). To verify the supposed liquid equilibrium, we combine conditions (25) and (2). Thus, the condition for the liquid equilibrium is $\lambda \geq \underline{\lambda}_L \equiv \frac{\nu+(1-\nu)[(1-\psi)F(\eta^*)+(1-\mu)(1-F(\eta^*))]}{\nu(\psi F(\eta^*)+\mu(1-F(\eta^*)))}$, where its RHS is independent of λ.

We turn to comparative statics summarized in Corollary 1. The size of the liquidity shock, λ, affects the existence of liquid equilibrium. Once it exists, $\lambda > \underline{\lambda}_L$, however, the shock size has no further impact on the quantity and quality of loans traded. A FOSD reduction in the screening cost distribution, $\tilde{F} \geq F$, makes screening cheaper and increases the share of low-cost lenders. Hence, the price p_U^* increases and it is easier to support a liquid equilibrium ($\underline{\lambda}_L$ falls). Higher loan profitability A (or lower bargaining power of borrowers or lower lending market competition) increases the screening benefit and thus η^*. As a result of the better pool of loans traded, the price p_U^* increases by more than the initial increase in loan profitability, making it easier to sustain the liquid equilibrium. A higher repayment probability μ (e.g., a higher credit score) improves the average quality of non-screened loans. A higher probability of liquidity shock ν implies that lenders are more likely to sell a high-quality loan. Both parameter changes lower the benefits of screening and, thus, lower η^*. The overall effect on the price p_U^* and the bound $\underline{\lambda}_L$ can be ambiguous, however. First, lower screening tends to depresses the price. Second, higher μ lowers the default probability conditional on no screening and higher ν increases the relative share of liquidity sellers. Both effects tend to increase the average quality of loans traded.

Corollary 1. *The threshold η^* increases in A and decreases in μ, ν, and after a first-order stochastic dominance (FOSD) reduction in F. The price p_U^* increases in A and after a FOSD reduction. The price can also be non-monotonic in μ and ν. Similarly, the bound $\underline{\lambda}_L$ decreases in A and after a FOSD reduction and can be non-monotonic in μ and ν.*

Proof. For the effect on the screening threshold, we use equation (6) to define

$$H \equiv \eta - \frac{(1-\nu)(\psi-\mu)\left[1-\mu-(\psi-\mu)F(\eta)\right]A}{\nu+(1-\nu)\left[1-\mu-(\psi-\mu)F(\eta)\right]} \equiv \eta - \frac{N}{D}, \tag{26}$$

with $H(\eta^*) \equiv 0$ and N and D being the numerator and denominator, respectively. To use the implicit function theorem, we obtain the following partial derivatives of H:

$$\begin{aligned}
H_\eta &= 1 + D^{-2}(1-\nu)(\psi-\mu)^2\nu A f > 0, \qquad H_\nu = D^{-2}(\psi-\mu)\left[1-\mu-F(\psi-\mu)\right]A > 0,\\
H_\mu &= D^{-2}(1-\nu)A\Big\{\left[(1-\psi)F+(1+\psi-2\mu)(1-F)\right]\nu + \left[(1-\psi)F+(1-\mu)(1-F)\right]^2(1-\nu)\Big\} > 0,\\
H_\lambda &= 0, \qquad H_A = -D^{-1}(1-\nu)(\psi-\mu)\left[1-\mu-(\psi-\mu)F(\eta)\right] < 0.
\end{aligned} \tag{27}$$

These partial derivatives imply the following comparative statics:

$$\frac{d\eta^*}{d\nu} = -\frac{H_\nu}{H_\eta} < 0, \quad \frac{d\eta^*}{d\mu} = -\frac{H_\mu}{H_\eta} < 0, \quad \frac{d\eta^*}{d\lambda} = -\frac{H_\lambda}{H_\eta} = 0, \quad \frac{d\eta^*}{dA} = -\frac{H_A}{H_\eta} > 0. \tag{28}$$

For the effect on the price, we use equation (25) and obtain these partial derivatives:

$$\frac{\partial p_U^*}{\partial \lambda} = 0, \quad \frac{\partial p_U^*}{\partial A} = \frac{p_U^*}{A} > 0, \quad \frac{dp_U^*}{d\eta^*} = \frac{(\psi-\mu)A\nu f}{D^2} > 0, \quad \frac{\partial p_U^*}{\partial \mu} = \frac{\nu(1-F)A}{D^2} > 0, \tag{29}$$

$$\frac{\partial p_U^*}{\partial \nu} = D^{-2}[\mu+(\psi-\mu)F(\eta^*)]\left[(1-\psi)F(\eta^*)+(1-\mu)(1-F(\eta^*))\right]A > 0. \tag{30}$$

The total derivatives are

$$\begin{aligned}
\frac{dp_U^*}{dA} &= \frac{\partial p_U^*}{\partial A} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dA} > 0, \qquad \frac{dp_U^*}{d\lambda} = \frac{\partial p_U^*}{\partial \lambda} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\lambda} = 0,\\
\frac{dp_U^*}{d\nu} &= \frac{\partial p_U^*}{\partial \nu} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\nu} \lesseqgtr 0, \qquad \frac{dp_U^*}{d\mu} = \frac{\partial p_U^*}{\partial \mu} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\mu} \lesseqgtr 0.
\end{aligned} \tag{31}$$

Higher ν and μ increase the price directly but decrease it indirectly via a lower screening threshold. A set of sufficient conditions for the non-monotonicity of p_U^* in μ is $\frac{dp_U^*}{d\mu}\mid_{\mu\to 1} > 0$ and $\frac{dp_U^*}{d\mu}\mid_{\mu\to 1} < 0$. Substituting into (31) from conditions (29) and (28), we evaluate derivatives for the two limits, $\frac{dp_U^*}{d\mu}\mid_{\mu\to 1} = \frac{A}{\nu} > 0$ and $\frac{dp_U^*}{d\mu}\mid_{\mu\to 0} = \frac{\nu A}{D^2}\Big((1-F) - \frac{A(1-\nu)f\psi\left(\nu[(1-\psi)F+(1+\psi)(1-F)]+(1-\nu)[(1-\psi)F+(1-F)]^2\right)}{D^2+(1-\nu)\nu A f}\Big)$. The latter derivative is negative for $\psi \to 1$ if $\eta_\mu^* \frac{f(\eta_\mu^*)}{1-F(\eta_\mu^*)} > 1$, where $\eta_\mu^* = \eta^*\mid_{\mu\to 0}$. Next, we turn to a FOSD reduction in the cost distribution, $\widetilde{F} \geq F$ (lower screening costs become more likely). Since $\frac{dp_U}{dF(\eta)} = \frac{(\psi-\mu)\nu A}{D^2} > 0$, the price increases, $\widetilde{p_U^*} > p_U^*$. Thus, the threshold decreases, $\widetilde{\eta^*} < \eta^*$. For the comparative statics of the bound $\underline{\lambda}_L$, we use the derivatives in (28) and (31) and $\underline{\lambda}_L = \frac{A}{p_U^*}$ to get:

$$\frac{d\underline{\lambda}_L}{dA} = -\frac{(\psi-\mu)f}{\nu(\psi F(\eta^*) + (1-\mu)(1-F(\eta^*))^2} \frac{d\eta^*}{dA} < 0, \frac{d\underline{\lambda}_L}{d\nu} = -\frac{A}{p_U^2} \frac{dp_U^*}{d\nu} \gtrless 0, \frac{d\underline{\lambda}_L}{d\mu} = -\frac{A}{p_U^2} \frac{dp_U^*}{d\mu} \gtrless 0.$$

The threshold $\underline{\lambda}_L$ is monotonic in A but can be non-monotonic in μ and ν. Moreover, $\underline{\lambda}_L$ decreases after a FOSD reduction in the screening cost distribution, $\widetilde{F} \geq F$, because $\frac{\underline{\lambda}_L}{dF(\eta)} = -\frac{A}{p_U^2} \frac{dp_U^*}{dF(\eta)} < 0$ (the second term is positive). Hence, $\widetilde{\underline{\lambda}_L} < \underline{\lambda}_L$. □

B.2 Proof of Proposition 1

The payoff from an insured loan is independent of the screening choice (as financiers cannot observe screening), so a lender i who insures has a payoff $\nu\lambda p_I + (1-\nu)\pi = \kappa p_I$ when not screening and a payoff $\kappa p_I - \eta_i$ when screening and, thus, generically prefers not to screen.

For productive enough screening, $\psi > \underline{\psi}$, high-cost lenders are indifferent about insurance and $m^* < 1$. For $\psi \leq \underline{\psi}$, however, insurance is weakly preferred by high-cost lenders,

$$\kappa p_I \geq \nu\lambda p_U + (1-\nu)\left[\mu A + (1-\mu)p_U\right], \tag{32}$$

so $m^* = 1$. To obtain the threshold $\underline{\psi}$, we substitute $p_I = \mu A$ and $p_U(m=1) = \frac{\nu\psi A}{\nu+(1-\nu)(1-\psi)}$ into equation (32), so $\psi \leq \frac{\lambda\mu}{\lambda\mu+\kappa(1-\mu)} \equiv \underline{\psi} \in (\mu, 1)$. The condition $\psi \leq \underline{\psi}$ can also be expressed as $\mu > \bar{\mu} \equiv \frac{\psi\kappa}{\psi\kappa+\lambda(1-\psi)}$ or $\nu < \bar{\nu} \equiv \frac{\lambda\mu(1-\psi)-\psi(1-\mu)}{\psi(1-\mu)(\lambda-1)}$. We focus on $\psi > \underline{\psi}$ henceforth.

B.3 Proof of Proposition 2

The indifference condition for loan insurance (8) pins down the price of uninsured loans, $p_U^* = \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)}$. Since high-cost lenders are indifferent about insurance, the screening threshold can be obtained by equalizing payoff of screening with payoff of not screening and not insuring in equation (5). Substituting p_U^* from above into (5), the screening cost threshold stated in the proposition follows. To ensure a liquid equilibrium, the price p_U^* must satisfy condition (2). Thus, a liquid equilibrium in which insurance is used exists if $\mu\nu\lambda^2 - \nu\lambda - (1-\mu)(1-\nu) \geq 0$. Since only the larger root is positive, this condition reduces to $\lambda \geq \tilde{\lambda}_L$. An equivalent expression is $\mu \geq \tilde{\mu}_L \equiv \frac{\kappa}{\kappa+\nu\lambda(\lambda-1)}$. When insurance is available, the liquid equilibrium exists if $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$. The comparative statics of $\tilde{\lambda}_L$ are $\frac{d\tilde{\lambda}_L}{d\mu} = -\frac{1}{2\mu^2} - \frac{1}{2}\chi\left(\frac{1}{2\mu^3} + \frac{1-\nu}{\nu\mu^2}\right) < 0$ and $\frac{d\tilde{\lambda}_L}{d\mu} = -\frac{1}{2}\chi\frac{1-\mu}{\mu\nu^2} < 0$, where $\chi \equiv \left(\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}\right)^{-\frac{1}{2}} > 0$. The competitive price of uninsured loans is

$$p_U = \nu A \frac{\psi F(\eta) + \mu\left(1-F(\eta)\right)(1-m)}{\nu\left[F + (1-F)(1-m)\right] + (1-\nu)\left[(1-\psi)F + (1-\mu)\left(1-F\right)(1-m)\right]}. \tag{33}$$

Combining (33) with the expressions for p_U^* (see above) yields for p_U/A:

$$\nu \frac{\psi F(\eta) + \mu\,(1 - F(\eta))\,(1-m)}{\nu\big[F + (1-F)(1-m)\big] + (1-\nu)\big[(1-\psi)F + (1-\mu)\,(1-F)\,(1-m)\,\big]} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)\,(1-\mu)}, \tag{34}$$

which yields the share of loans insured by high-cost lenders, m^*, stated in Proposition 2. Since the LHS of (34) increases in m, insurance is used when $\frac{p_U}{A}\,|_{m=0} < \frac{\nu\lambda\mu}{\nu\lambda+(1-\nu)(1-\mu)} \Leftrightarrow$

$$\nu \frac{\psi F(\eta(\mu,\lambda)) + \mu\,(1 - F(\eta(\mu,\lambda)))}{\nu + (1-\nu)\big[(1-\psi)F(\eta(\mu,\lambda)) + (1-\mu)\,(1-F(\eta(\mu,\lambda)))\,\big]} < \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)\,(1-\mu)}. \tag{35}$$

The LHS of (35) increases in A and after a first-order stochastic dominance shift in $F(\cdot)$ (cheaper screening), and decreases in λ. The RHS is independent of both A and $F(\cdot)$ and increases in λ. Hence, the condition for loan insurance to occur can be expressed as $A < \tilde{A}_I$, $\lambda > \tilde{\lambda}_I$, or high enough screening costs $F(\cdot)$. The parameter thresholds $\{\tilde{A}_I, \tilde{\mu}_I, \tilde{\lambda}_I\}$ are defined by $\frac{p_U}{A}\,|_{m=0} = \frac{\nu\lambda\mu}{\nu\lambda+(1-\nu)(1-\mu)}$ and the threshold $\tilde{A}_I$ can be expressed in closed form:

$$\tilde{A}_I \equiv \frac{\nu\lambda + (1-\nu)(1-\mu)}{(1-\nu)(1-\mu)(\psi-\mu)\kappa} F^{-1}\left(\frac{\mu(\lambda-1)(1-\nu)(1-\mu)}{\kappa(1-\mu)\psi - (1-\psi)\lambda\mu + \mu(\lambda-1)(1-\nu)(1-\mu)}\right).$$

Equivalently, the threshold $\tilde{\lambda}_I$ is implicitly but uniquely defined by:

$$\nu \frac{\psi F(\eta(\tilde{\lambda}_I)) + \mu\left(1 - F(\eta(\tilde{\lambda}_I)\right)}{\nu + (1-\nu)\big[(1-\psi)F(\eta(\tilde{\lambda}_I)) + (1-\mu)\left(1 - F(\eta(\tilde{\lambda}_I))\right)\big]} = \frac{\nu\tilde{\lambda}_I\mu}{\nu\tilde{\lambda}_I + (1-\nu)\,(1-\mu)}, \tag{36}$$

provided the RHS of (36) exceeds its LHS for $\lambda \to \infty$, for which $\nu \leq \frac{2\mu}{1+2\mu}$ is sufficient. Similarly, the threshold $\tilde{\mu}_I$ is implicitly defined by $m^* = 0$ given in Proposition 2:

$$\tilde{\mu}_I \equiv \frac{\kappa\psi - (1-\psi)\lambda\frac{\tilde{\mu}_I}{1-\tilde{\mu}_I}}{(\lambda-1)(1-\nu)} \frac{F\left(\eta^*(\tilde{\mu}_I)\right)}{1 - F\left(\eta^*(\tilde{\mu}_I)\right)} \in (0,1). \tag{37}$$

The bound $\tilde{\mu}_I$ is unique since the LHS of (37) increases in μ and the RHS decreases in μ. It is also interior because the following limits do not satisfy equation (37): first, $\lim_{\mu\to 0} LHS = 0$ while $\lim_{\mu\to 0} RHS > 0$ (insurance costs outweigh benefits for $\mu \to 0$); second, $\lim_{\mu\to\psi} LHS = \psi$ while $\lim_{\mu\to\psi} RHS = 0$ (insurance benefits outweigh costs for $\mu \to \psi$ because no lender screens, $F = 0$). The existence of $\tilde{\mu}_I$ follows from continuity in μ.

Let G be the difference between the RHS and the LHS of (35). Then, $G = 0$ defines the boundary of the extensive margin of insurance. The results derived above can be expressed as $\frac{dG}{dA} < 0$, $\frac{dG}{d\lambda} > 0$, $\frac{dG}{d\mu} > 0$ and G decreases after a FOSD reduction in F. Hence, $\frac{d\tilde{\mu}_I}{dA} = -\frac{dG}{dA}/\frac{dG}{d\mu} > 0$, $\frac{d\tilde{\mu}_I}{d\lambda} = -\frac{dG}{d\lambda}/\frac{dG}{d\mu} < 0$, and $\tilde{\mu}_I$ increases after a FOSD reduction in F.

Comparative statics: screening threshold and price of uninsured loans. Using (33), the total derivative of the price of uninsured loans w.r.t. loan insurance is:

$$\frac{dp_U^*}{dm^*} = \frac{\partial p_U^*}{\partial m^*} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*} = \frac{\frac{\partial p_U^*}{\partial m^*}}{1 - \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}} > 0, \tag{38}$$

since $\frac{\partial p_U^*}{\partial m^*} = \nu(\psi-\mu)F(\eta^*)(1-F(\eta^*))A\big[\nu\left(1-\left(1-F\left(\eta^*\right)\right)m\right)+(1-\nu)\big[(1-\psi)F\left(\eta^*\right)+(1-\mu)\left(1-F\left(\eta^*\right)\right)\left(1-m^*\right)\big]\big]^{-2}>0$, $\frac{dp_U^*}{d\eta^*}>0$, and $\frac{d\eta^*}{dp_U^*}=-(1-\nu)(\psi-\mu)<0$. Since the price increases in loan insurance, the screening threshold falls, $\frac{d\eta^*}{dm^*}=\frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*}<0$. Since the threshold $\tilde{\lambda}_L$ decreases in the price p_U^*, it decreases in m^*: $\frac{d\tilde{\lambda}_L}{dm^*}=\frac{d\tilde{\lambda}_L}{dp_U^*}\frac{dp_U^*}{dm^*}<0$. As a result, when insurance is used, $m^*>0$, the threshold for the existence of a liquid equilibrium is lower compared to the case when insurance is unavailable, $\tilde{\lambda}_L<\underline{\lambda}_L$.

We use the screening threshold stated in Proposition 2 and $D'\equiv\nu\lambda+(1-\nu)\left(1-\mu\right)$:

$$
\begin{aligned}
\frac{d\eta^*}{dA} &= \frac{(1-\nu)\left(1-\mu\right)\left(\psi-\mu\right)\kappa}{D'}>0, \quad \frac{d\eta^*}{d\lambda}=-\frac{\nu(1-\nu)^2\mu(1-\mu)(\psi-\mu)A}{D'^2}<0,\\
\frac{d\eta^*}{d\mu} &= -\frac{(1-\nu)\,\kappa A\left((1+\psi-2\mu)\nu\lambda+(1-\nu)(1-\mu)^2\right)}{D'^2}<0,\\
\frac{d\eta^*}{d\nu} &= -\frac{(1-\mu)\left(\psi-\mu\right)A\left[\kappa^2+\mu(1-\nu)^2(\lambda-1)\right]}{D'^2}<0, \quad \frac{d\eta^*}{d\psi}=\frac{(1-\nu)(1-\mu)\kappa A}{D'}>0.
\end{aligned}
$$

For the effect on the price, we use p_U^* given in Proposition 2 to obtain:

$$
\begin{aligned}
\frac{dp_U^*}{dA} &= \frac{\nu\lambda\mu}{D'}>0, \quad \frac{dp_U^*}{d\mu}=\frac{\nu\lambda A\kappa}{D'^2}>0, \quad \frac{dp_U^*}{d\psi}=0,\\
\frac{dp_U^*}{d\lambda} &= \frac{\nu(1-\nu)\mu(1-\mu)A}{D'^2}>0, \quad \frac{dp_U^*}{d\nu}=\frac{\lambda\mu(1-\mu)A}{D'^2}>0.
\end{aligned}
$$

Comparative statics: share of high-cost lenders who insure their loan. Since m^* is given in Proposition 2 as a function of η^*, the total effect of parameters $\alpha\in\{\nu,\lambda,\mu\}$ on m^* consists of a direct and indirect effect via screening, $\frac{dm^*}{d\alpha}=\frac{\partial m^*}{\partial\alpha}+\frac{dm^*}{d\eta^*}\frac{d\eta^*}{d\alpha}$:

$$
\begin{aligned}
\frac{dm^*}{d\eta^*} &= -\frac{[\kappa(1-\mu)\psi-(1-\psi)\lambda\mu]f}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2}<0, \quad \frac{\partial m^*}{\partial\lambda}=\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(1-\nu)(\lambda-1)^2(1-F)}F>0,\\
\frac{\partial m^*}{\mu} &= \frac{\kappa\psi(1-\mu)^2+(1-\psi)\lambda\mu^2}{\mu^2(1-\mu)^2(\lambda-1)(1-\nu)(1-F(\eta^*))}F(\eta^*)>0, \qquad \frac{\partial m^*}{\partial A}=0,\\
\frac{\partial m^*}{\partial\nu} &= -\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)^2(1-F(\eta^*))}\lambda F(\eta^*)<0,\\
\frac{\partial m^*}{\partial F} &= -\frac{\kappa(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2}<0, \quad \frac{\partial m^*}{\partial\psi}=-\frac{\kappa(1-\mu)+\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)}F<0.
\end{aligned}
$$

The following total derivatives are unambiguous, $\frac{dm^*}{d\mu}>0$, $\frac{dm^*}{dA}<0$, $\frac{dm^*}{d\psi}<0$, and the FOSD shift. The total effect of ν on m^* can be ambiguous since the direct effect is negative and the indirect one is positive. A sufficient condition for non-monotonicity is the opposite sign of derivatives at both limits, $\nu\to\{0,1\}$, where $\lim_{\nu\to1}\frac{dm^*}{d\nu}=-\infty$ and

$$
\lim_{\nu\to0}\frac{dm^*}{d\nu}=-\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-F)}\lambda F+A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-F)^2}f.
$$

The sufficient condition for non-monotonicity is $\frac{F(1-F)}{f}\mid_{\nu\to0}<A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{(1-\mu)\lambda\psi-(1-\psi)\lambda\mu}$.

B.4 Proof of Proposition 3

The illiquid equilibrium always exists. If the price of uninsured loans is zero, only lemons are sold in this market, which justifies $p_U^* = 0$. The screening threshold is given by the indifference of the marginal lender who compares payoffs from screening, $\psi A - \eta$, and not screening but insuring, $\kappa\mu A$. Equating those yields the stated threshold $(\psi - \kappa\mu)A$. This threshold is is below the threshold in the illiquid equilibrium without insurance, $(\psi - \mu)A$.

B.5 Proof of Proposition 4

Deriving welfare in liquid equilibrium. Utilitarian welfare is the sum of expected payoffs to lenders and financiers. Up to a constant for financiers who expect to break even, welfare W is the expected payoffs to lenders. In a liquid equilibrium, low-cost lenders, $\eta_i \leq \eta^*$, of mass $F(\eta^*)$ receive $\nu\lambda p_U^* + (1-\nu)[\psi A + (1-\psi)p_U^*] - \eta_i$, uninsured high-cost lenders of mass $(1 - F(\eta^*))(1 - m^*)$ receive $\nu\lambda p_U^* + (1-\nu)[\mu A + (1-\mu)p_U^*]$, and insured high-cost lenders of mass $(1 - F(\eta^*))m^*$ receive κp_I^*. Integrating over all lenders i yields

$$W = \overbrace{\nu\lambda\Big\{p_U^* \left[F + (1-F)(1-m^*)\right] + p_I^*(1-F)m^*\Big\}}^{\text{Liquidity Shock}} + \overbrace{(1-\nu)p_I^*(1-F)m^*}^{\text{No shock, insured}} \tag{39}$$
$$+ \overbrace{(1-\nu)\Big\{\big[\psi A + (1-\psi)p_U^*\big]F + [\mu A + (1-\mu)p_U^*](1-F)(1-m^*)\Big\}}^{\text{No shock, uninsured}} - \overbrace{\int_0^{\eta^*} \eta dF(\eta)}^{\text{Screening costs}},$$

where we used the short-hand $F = F(\eta^*)$ unless stated otherwise. Substituting the price of uninsured loans, $p_U\big\{\nu\big[F+(1-F)(1-m)\big]+(1-\nu)\big[(1-\psi)F+(1-\mu)(1-F)(1-m)\big]\big\} = \nu A\big[\psi F + \mu(1-F)(1-m)\big]$, results in the simplified expression of welfare in equation (11).

Planner's choice in liquid equilibrium. We prove existence of m^P and that the planner insures more loans by showing that welfare increases in m on the interval $m \in [0, m^*]$. Welfare is continuous and defined everywhere, so the planner's choice m^P exceeds the unregulated level m^*, except in the corner of $m^* = 1 = m^P$. To see this, the total derivative of welfare in (39), $\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} + \frac{\partial W}{\partial \eta^*}\frac{d\eta^*}{dm}$, is evaluated using

$$\begin{aligned} \frac{\partial W}{\partial m} &= (1-F)\big(\kappa p_I^* - \nu\lambda p_U^* - (1-\nu)[\mu A + (1-\mu)p_U^*]\big) \geq 0, \\ \frac{\partial W}{\partial p_U^*} &= \nu\lambda[F + (1-F)(1-m)] + (1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)(1-m)\right] > 0, \\ \frac{\partial W}{\partial \eta^*} &= f\left((1-\nu)(\psi-\mu)(A - p_U^*) - \eta^* + m^*\left\{\nu\lambda p_U^* + (1-\nu)[\mu A + (1-\mu)p_U^*] - \kappa p_I^*\right\}\right) = 0. \end{aligned} \tag{40}$$

At the level of the unregulated equilibrium $m = m^*$, the total derivative $\frac{dW}{dm}$ is positive due to the positive pecuniary externality, $\frac{dW}{dm}\mid_{m=m^*} = \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} > 0$, where $\frac{dp_U^*}{dm} > 0$ reflects the improvement in the average quality of uninsured loans traded because of loan insurance (e.g. Figure 4). By an envelope-type-argument, the direct effect of insurance (except in the corner of $m = 1$) and screening on welfare is zero in the unregulated economy, $\frac{\partial W}{\partial m} = 0 = \frac{\partial W}{\partial \eta^*}$, as lenders choose insurance and screening privately optimally. Moreover, for any $m < m^*$, the total derivative is also positive because $\frac{\partial W}{\partial m}|_m > 0$, $\frac{\partial W}{\partial p}|_m > 0$, $\frac{\partial W}{\partial \eta^*}|_m < 0$, and $\frac{d\eta^*}{dm} < 0$, whereby loan insurance reduces screening incentives.

Therefore, we conclude that $\frac{dW}{dm} > 0$ for any $m \le m^*$, resulting in $m^P > m^*$ (except for the corner case). It follows immediately from the proof of Proposition 1 (see equation 38) that the price of uninsured loans is higher and screening lower, $p_U^P > p_U^*$ and $\eta^P < \eta^*$.

To prove a positive share of insured loans for a larger set of parameters, we compare the thresholds at which insurance is zero in the unregulated equilibrium, $\{\tilde{A}_I, \tilde{\mu}_I, \tilde{\lambda}_I\}$, and in the planner's choice, $\{A_I^P, \mu_I^P, \lambda_I^P\}$, and the screening cost distributions $F(\cdot)$. First, $\{\tilde{A}_I, \tilde{\mu}_I, \tilde{\lambda}_I\}$ satisfy $m^* = 0$ and $\frac{\partial W}{\partial m} = (1-F)\{\kappa\mu A - \nu\lambda p_U^* - (1-\nu)[\mu A + (1-\mu)p_U^*]\} = 0$ (the indifference condition for insurance). Substituting p_U^* from (9) into $\frac{\partial W}{\partial m} = 0$ yields condition (35) satisfied with equality. Thus, $m^* > 0 \Leftrightarrow \frac{\partial W}{\partial m}|_{m=0} > 0 \Leftrightarrow \frac{p_U}{A}|_{m=0} < \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)}$. Appendix B.3 derives the bounds $\{\tilde{A}_I, \tilde{\mu}_I, \tilde{\lambda}_I\}$ and high enough screening costs in terms of $F(\cdot)$. Using $\frac{\partial W}{\partial m} = \left[-F\left(\kappa\psi + (1-\psi)\lambda\frac{\mu}{1-\mu}\right) + (\lambda-1)\mu(1-\nu)(1-F)\right](1-\mu)(1-F)\nu A / \nu + (1-\nu)((1-\psi)F + (1-\mu)(1-F)) = 0$, gives the lower bound $\tilde{\mu}_I$ in equation (37). Thus, insurance is positive if $\mu > \tilde{\mu}_I$. Second, $\{A_I^P, \mu_I^P, \lambda_I^P\}$ satisfy $m^P = 0$ and $\frac{dW}{dm} = 0$. We substitute p_U^* to obtain for p_U/A:

$$\frac{\nu[\psi F + \mu(1-F)]}{\nu + (1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)\right]} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)} + \frac{\frac{\partial W}{\partial p_U}\frac{dp_U}{dm}}{(1-F)[\nu\lambda + (1-\nu)(1-\mu)]A} \tag{41}$$

Since the pecuniary externality term $\frac{\partial W}{\partial p_U}\frac{dp_U}{dm}$ is positive, the LHS of (41) exceeds the LHS of (35). The LHS of (35) and (41) have the same functional form, increase in A and after a FOSD reduction in F, and decrease in λ. Hence, the planner uses insurance for larger parameter space $A_I^P > \tilde{A}_I$, $\lambda_I^P < \tilde{\lambda}_I$, and cheaper screening. A sufficient condition for λ_I^P to exist is $\nu \le \frac{2\mu}{1+2\mu}$. Rewriting $\frac{dW}{dm} = 0$ also yields an implicitly defined lower bound μ_I^P:

$$\mu_I^P \equiv \frac{\kappa\psi - (1-\psi)\lambda\frac{\mu}{1-\mu}}{(\lambda-1)(1-\nu)(1-F)}F - \frac{\nu + (1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)\right]}{(\lambda-1)(1-\nu)(1-\mu)(1-F)^2\nu A}\frac{\partial W}{\partial p_U}\frac{dp_U}{dm}. \tag{42}$$

A direct comparison of (37) and (42) implies that $\mu_I^P < \tilde{\mu}_I$.

Comparing illiquid and liquid equilibrium. Next, we define the threshold λ_L^P and show that it exists and is unique. Welfare in the liquid equilibrium can be expressed as

$$W^L = \nu(\lambda-1)\left[p_U + (\mu A - p_U)(1 - F(\eta^L))m\right] + \left[\mu + (\psi - \mu)F(\eta^L)\right]A - \int_0^{\eta^L}\eta dF, \tag{43}$$

subject to η^L in (5), p_U in (33), and $\lambda p_U \ge A$. Welfare in the illiquid equilibrium is

$$W^{NL} = \nu(\lambda-1)\mu A\left[1 - F(\eta^{NL})\right] + \left[\psi F(\eta^L) + \mu(1 - F(\eta^L))\right]A - \int_0^{\eta^{NL}}\eta dF(\eta), \tag{44}$$

where $\eta^{NL} = (\psi - \kappa\mu)A$. At some λ_L^P given in equation (14), the planner is indifferent between both equilibria, $W^{NL} \equiv W^L$. This equation implicitly and uniquely defines a $\lambda_L^P \in (1, \infty)$. For existence, the gains from trade term dominates for $\lambda \to \infty$, so $\lambda_L^P < \infty$, while this term vanishes for $\lambda \to 1$. The existence of λ_L^P follows from continuity.

For uniqueness, we show that the welfare difference $W^L - W^{NL}$ increases in λ. But first we need to characterize the liquid equilibrium at $\lambda = \lambda_L^P$, which does not exist for in the unregulated economy. The liquid equilibrium can only be sustained with a high enough share of insured loans that satisfy $\lambda p_U \geq A$. Hence, the FOC for m^P is $\frac{dW^L}{dm} + \gamma \frac{dp_U}{dm} = 0$, where γ is the Lagrange multiplier on $\lambda p_U \geq A$. When the planner values the price dimension of allocative efficiency, $p_U^P > A/\lambda$, the liquid equilibrium is preferred ($\lambda > \lambda_L^P$). At $\lambda = \lambda_L^P$, by construction the planner is indifferent between the illiquid equilibrium and the liquid equilibrium with the highest possible screening consistent with $p_U = A/\lambda$, so $\gamma > 0$. In fact, the Lagrange multiplier is positive for $\lambda_L^P \leq \lambda < \lambda_B^P$, where λ_B^P is implicitly defined by $dW^L/dm = 0$ and $p_U = A/\lambda$. Note that λ_B^P is defined on the interval $\mu \geq \widehat{\mu}$, where $\widehat{\mu} < 1$ solves $\underline{\lambda}_L(\widehat{\mu}) = \lambda_I^P(\widehat{\mu})$. That is, $\widehat{\mu}$ satisfies $dW^L/dm = 0$, $p_U = A/\lambda$, and $m = 0$. Next, $\frac{dW^L}{dm} + \gamma \frac{dp_U}{dm} = 0$ and $\frac{dp_U}{dm} > 0$ imply $\frac{dW^L}{dm} < 0$ at $\lambda = \lambda_L^P$, so the planner would insure fewer loans without the binding constraint for a liquid equilibrium.

The total derivative of the difference $W^L \mid_{p_U = A/\lambda} - W^{NL}$ with respect to λ is:

$$\underbrace{\frac{dW^L}{dm}}_{<0} \underbrace{\frac{dm \mid_{p_U = A/\lambda}}{d\lambda}}_{<0} + \underbrace{\nu \left(\frac{A}{\lambda} + \left(\mu A - \frac{A}{\lambda} \right) [1 - F(\eta^L)] m - \mu A \left[1 - F(\eta^{NL}) \right] \right)}_{\text{higher gains from trade in liquid equilibrium}(>0)} > 0. \quad (45)$$

This derivative is positive because both effects of a higher size of the liquidity shock λ are positive: the indirect effect through a lower level of insured loans and the direct effect of higher gains from trade. For $\gamma > 0$, insurance m^P targets $p_U = A/\lambda$, thus a higher λ means that less insurance is needed to achieve the reduced price necessary to liquify the market (recall that $\frac{dp_U}{dm} > 0$). Equation (14) has already established that the gains from trade in the liquid equilibrium are higher and thus the direct effect is also positive. For $\gamma = 0$, the indirect effect of higher λ on the welfare difference is zero because insurance is set such that $\frac{dW^L}{dm} = 0$, so the total effect is still positive. In sum, the welfare difference between a liquid and illiquid equilibrium monotonically increases in λ, so equation (14) defines λ_L^P uniquely.

B.6 Proof of Proposition 5

We start by showing that the regulator does not intervene conditional on the illiquid equilibrium. Next, an insurance subsidy achieves the welfare benchmark conditional on the liquid equilibrium. We also derive comparative statics about the size of the subsidy. Next, we show that an uninsured loan sale subsidy can eliminate the illiquid equilibrium. In the liquid equilibrium, however, the sale subsidy does worse than the insurance subsidy.

Illiquid equilibrium: no intervention. Conditional on the illiquid equilibrium, the regulator solves:

$$W_R^{NL} \equiv \max_{b_I, p_{min}} W + \kappa n$$
$$\text{s.t. } \eta = \psi A + (1-\psi) b_U - \max\{\kappa(\mu A + b_I), \mu A + (1-\mu) b_U \kappa\}, \ p_U = 0, \lambda p_{min} < A,$$

and subject to the privately optimal insurance choice. Lenders insure if its payoff, $\kappa(\mu A + b_I)$, exceeds the payoff from not insuring, $\mu A + \kappa(1-\mu) b_U$. In the unregulated equilibrium, all high-cost lenders insure, $m^* = 1$, and the screening threshold is $\eta^* = (\psi - \kappa\mu)A$. This allocation corresponds to the planner's choice (see Proposition 4). This level of screening also maximizes welfare: $\frac{dW}{d\eta} = [(\psi - \kappa\mu)A - \eta]\, f = 0$ yields η^*. Thus, an uninsured loan sale subsidy can affect screening and lower insurance but is undesirable. Hence, $p^R_{min} = 0 = b^R_I$.

Liquid equilibrium: insurance subsidy attains welfare benchmark. Without uninsured sale subsidies, $b_U = 0$, the objective functions of the planner in (11) and the regulator in (19) are identical (up to a constant for interim endowment), and so are the screening threshold and the uninsured loan price. To see this, we can rewrite (19) as

$$\max_{b_I} W^L_R = \max_{b_I} \overbrace{\nu(\lambda-1)\left[p_U + (p_I - p_U)(1-F(\eta))\, m\right]}^{\text{Gains from trade}} + \underbrace{[\psi F(\eta) + \mu(1-F(\eta))]\, A}_{\text{Fundamental value}} - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} + \kappa(n + \underbrace{b_I(1-F(\eta))m - T}_{\text{Policy redistribution }(=0)}).$$

Hence, the subsidy is set to achieve the uninsured loan price in welfare benchmark. Solving equation (17) and evaluating at $p_U(b_I) = p^P_U$ yields the value of b^R_I stated in the proposition.

We solve for b^R_I when condition (2) binds (so the Lagrange multiplier is positive, $\gamma > 0$), so $p_U(b^R_I) = {}^{A}\!/\!{}_{\lambda}$. Since higher λ relaxes condition (2), this case arises for $\lambda^P_L \leq \lambda < \lambda^P_B$. Substituting into (17) yields $b^R_I = A\frac{\nu\lambda(1-\lambda\mu)+(1-\nu)(1-\mu)}{\kappa\lambda}$. Hence, b^R_I linearly increases in A, decreases in μ and λ, and is independent of screening technology parameters ($F(\cdot)$, $\bar{\eta}$, ψ).

Subsidies for sales of uninsured loans. It is immediate that an illiquid equilibrium, $p^*_U = 0$, can be eliminated with a subsidy $b^R_U = {}^{A}\!/\!{}_{\lambda}$ because $p^*_U + b_U \geq {}^{A}\!/\!{}_{\lambda}$. Appendix B.5 defines λ^P_L and the liquid equilibrium is superior for $\lambda \geq \lambda^P_L$.

Next, we compare welfare when the same target price $p^T_U < A$ is achieved (i) with an insurance subsidy, $p^T_U = p_U(b_I)$, and (ii) with an uninsured loan sale subsidy, $p^T_U = p_U + b_U$. Using the insurance indifference in (17), welfare with an insurance subsidy in (19) equals

$$W^L_R(b_I) = \overbrace{\nu\lambda p^T_U + (1-\nu)\left[\psi F(\eta) + \mu\left(1-F(\eta)\right)\right] A + (1-\nu)\left[(1-\psi)F(\eta) + (1-\mu)\left(1-F(\eta)\right)\right] p^T_U + n}^{\text{Value to lenders}} - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} - \underbrace{\kappa b_I\left(1-F(\eta)\right) m}_{\text{Policy costs}},$$

where $p_U = p^T_U$ and $b_I(p^T_U), \eta(p^T_U)$, and $m(\eta(p^T_U))$ are given by (17), (5), and (33).

In contrast, welfare with subsidized sales of uninsured loans, $p^T_U > p^*_U$, is

$$W^L_R(b_U) = \overbrace{\nu\lambda p^T_U + (1-\nu)\left[\psi F(\eta) + \mu\left(1-F(\eta)\right)\right] A + (1-\nu)\left[(1-\psi)F(\eta) + (1-\mu)\left(1-F(\eta)\right)\right] p^T_U + n}^{\text{Value to lenders}} - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} - \underbrace{\kappa b_U \int q^U_i di}_{\text{Policy costs}},$$

where p_U is given by (25), $b_U = p^T_U - p_U$, $\eta = (1-\nu)(\psi-\mu)(A - p^T_U)$, and the quantity of uninsured loans sold $\int q^U_i di = \nu + (1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)\right]$.

The screening threshold is the same in both cases, so welfare only differs in the policy costs. An insurance subsidy has higher welfare than subsidized loan sales whenever $m\,(1-F)\,b_I < (p_U^T - p_U)\int q_i^U di$, which holds generically because $p_U^T < A$. To see this, substitute b_I from (17), $m(1-F) = \frac{p_U^T(\nu+(1-\nu)((1-\psi)F+(1-\mu)(1-F)))-\nu(\psi F+\mu(1-F))A}{p_U^T(1-\mu+\nu\mu)-\nu\mu A}$ from (33), and for $p_U \int q_i^U di = \nu\,(\psi F + \mu(1-F))\,A$ from (33), we can rewrite the required inequality as $\frac{1}{\kappa}\frac{[\nu\lambda+(1-\nu)(1-\mu)]p_U^T-\nu\mu A\lambda}{p_U^T(1-\mu+\nu\mu)-\nu\mu A} < 1$, which collapses to $p_U^T < A$.

B.7 Proof of Proposition 6

We focus on the equilibrium in which the market for loan insurance at $t = 0$ is liquid. We exclude the equilibrium in which the market for uninsured loans is illiquid, $p_U = 0$, but the loan insurance market is liquid, $p_I > 0$, based on its instability. This equilibrium requires that high-cost lenders are indifferent about insurance, $0 < m < 1$. Insurance by all high-cost lenders, $m = 1$, is not an equilibrium: since all lemons by low-cost lenders are also insured, no lemons are traded at $t = 1$, implying $p_U = A$ and violating the supposed $p_U = 0$. Insurance by no high-cost lenders, $m = 0$, is not an equilibrium either because the price of insured loans would be $p_I = 0$, violating the supposed $p_I > 0$. Since the price of insured loans increases in m, any deviation from the equilibrium level of m leads to the equilibrium in which all markets are illiquid or to the one in which all markets are liquid.

Positive analysis. Consider the equilibrium with liquid markets for loan insurance at $t = 0$ and sales of uninsured loans at $t = 1$. For the former market to be liquid, some high-cost lenders must insure, $m > 0$, so the payoff from insuring exceeds that from not insuring:

$$\kappa p_I \geq \nu\lambda p_U + (1-\nu)\left[\mu A + (1-\mu)p_U\right]. \tag{46}$$

Some high-cost lenders insure loans worth μA and low-cost lenders may insure lemons worth 0, so $p_I \leq \mu A$. Combining this with equation (46), we find that $p_I > p_U$. Hence, all low-cost lenders insure their lemons and $p_I < \mu A$. Moreover, only some high-cost lenders insure in equilibrium, $m < 1$, because $m = 1$ would imply that a price $p_U = A > p_I$, a contradiction. Hence, equation (46) holds with equality. The screening threshold equalizes the payoff from screening, $\psi[\nu\lambda p_U + (1-\nu)A] + (1-\psi)\kappa p_I - \eta$, and from not screening, $\nu\lambda p_U + (1-\nu)[\mu A + (1-\mu)p_U]$. Using equation (46) with equality to simplify yields

$$\eta = \psi(1-\nu)(1-\mu)(A-p_U), \tag{47}$$

which is a higher schedule than in the main model because of the additional benefit of screening—the option to insure lemons. The competitive prices of loans are:

$$p_I = \frac{\mu A(1-F)m}{(1-F)m+(1-\psi)F} = \mu A - \overbrace{\mu A\frac{(1-\psi)F}{(1-F)m+(1-\psi)F}}^{\text{adverse selection discount}}, \tag{48}$$

$$p_U = \nu A\frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-\mu)(1-F)(1-m)}. \tag{49}$$

The adverse selection discount in the loan insurance market vanishes for $\psi \to 1$ as no more lemons are insured. Loan insurance by high-cost lenders m increases both prices p_U and p_I.

As in the main model in eq. (38), we have $dp_U/dm > 0$. Moreover, $\frac{dp_I}{dm} = \frac{\partial p_I}{\partial m} + \frac{dp_I}{d\eta}\frac{d\eta}{dp_U}\frac{dp_U}{dm} > 0$, since $\frac{\partial p_I}{\partial m} > 0$, $\frac{dp_I}{d\eta} < 0$, $\frac{d\eta}{dp_U} < 0$, and $\frac{dp_U}{dm} > 0$. Using eq. (46) with equality yields:

$$p_U = \mu A \frac{\nu\lambda - \overbrace{\frac{(1-\psi)F}{(1-F)m + (1-\psi)F}}^{\text{adverse selection discount}}\kappa}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{50}$$

Equations (48) and (50) state that adverse selection in the loan insurance market reduces the prices of both insured and uninsured loans relative to the main model (adverse selection discount). A lower price together with a higher screening schedule implies that the screening threshold η is higher than in the main model. Combining equations (49) and (50) gives

$$\nu \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-\mu)(1-F)(1-m)} = \mu \frac{\nu\lambda - \frac{(1-\psi)F}{(1-F)m+(1-\psi)F}\kappa}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{51}$$

where η is given by (47). The RHS of (51) is the benefit of insurance and the LHS its opportunity costs. In the limit of $\mu \to 1$, $F(\eta) = 0$ and the RHS of (51) collapses to 1 and exceeds the LHS that collapses to ν. Therefore, insurance is strictly preferred in this limit. In contrast, for $\mu = \tilde{\mu}_I$, which satisfies equation (35), the insurance costs exceed insurance benefits. The insurance benefits are lower (RHS of 51 is smaller than the RHS of 35) and costs larger (LHS of 51 is larger than the LHS of 35). By continuity, there exist a $\tilde{\mu}_I^{AS} \in (\tilde{\mu}_I, 1)$ such that insurance is used, $m > 0$, for $\mu > \tilde{\mu}_I^{AS}$.

Both RHS and LHS of (51) increase in m, so multiple equilibria with a liquid insurance market may exist. Fig. 10 shows an example. Higher λ increases p_U directly (eq. 50) and indirectly via lower screening (eq. 47) and thus fewer lemons insured by low-cost lenders. Hence, there is a threshold $\tilde{\lambda}_L^{AS}$ such that for $\lambda \geq \tilde{\lambda}_L^{AS}$ the liquid equilibrium exists (condition 2 holds) conditional on insurance used. Due to the negative effect of adverse selection (see 50), this threshold is higher than in the main model, $\tilde{\lambda}_L < \tilde{\lambda}_L^{AS}$.

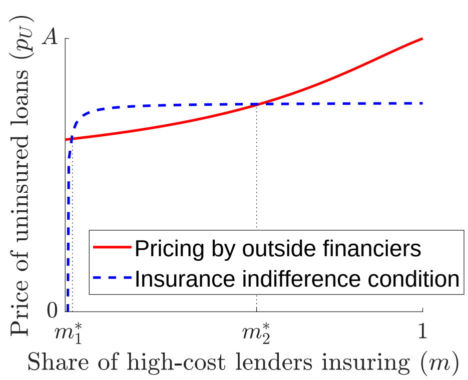

Figure 10: Example of multiple equilibria for liquid markets of both insurance and uninsured loan sales. The red solid line plots the competitive price of uninsured loans p_U and the blue dashed line shows the price p_U at which high-cost lenders are indifferent about insurance. There are two equilibria with positive insurance (m_1^*, m_2^*) but only m_2^* is stable. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$ with $\bar{\eta} = 1$, $\nu = 0.1$, $\mu = 0.9$, $\psi = 0.99$, $\lambda = 5$, $A = 3$.

Normative analysis. The equilibrium with insurance welfare-dominates equilibria with an illiquid insurance market. The screening and insurance choice are privately optimal and all externalities are pecuniary. And since the equilibrium with insurance has higher prices in secondary markets, both p_I (by definition because $p_I = 0$ in the illiquid insurance equilibrium) and p_U (see above for dp_U/dm), welfare in the equilibrium with insurance welfare-dominates the equilibria without this option. As in the main model, we express the welfare in the equilibrium with a liquid insurance market as the sum of lender payoffs, $W = F\left[\psi(\nu\lambda p_U + (1-\nu)A) + (1-\psi)\kappa p_I m^l\right] + (1-F)\left[\kappa p_I m + \left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right](1-m)\right] - \int_0^{\eta} \eta_i dF$, where p_U and η are given by generalized (49) and (47):

$$
\begin{aligned}
p_U &= \nu A \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-\mu)(1-F)(1-m) + (1-\psi)(1-F)(1-m^l)}. \\
\eta &= \psi(1-\nu)(1-\mu)(A - p_U) - (p_I - p_U)(1-m^l), \qquad (52)
\end{aligned}
$$

and m^l is the share of low-cost lenders who insure lemons ($m^l = 1$ in unregulated equilibrium). As in the unregulated equilibrium, more insurance by high-cost lenders increases welfare by raising the price of uninsured loans. Moreover, more insurance improves the price of insured loans (as in the unregulated equilibrium) given by the final term: $\frac{dW}{dm} = \underbrace{\frac{\partial W}{\partial m}}_{=0} + \underbrace{\frac{\partial W}{\partial p_U^*}}_{>0}\underbrace{\frac{dp_U^*}{dm}}_{>0} + \underbrace{\frac{\partial W}{\partial \eta^*}}_{=0}\frac{d\eta^*}{dm} + \underbrace{\frac{\partial W}{\partial p_I^*}}_{>0}\underbrace{\frac{dp_I^*}{dm}}_{>0} > 0$. To evaluate the effect of insurance by low-cost lenders on welfare dW/dm^l, it is useful to rearrange welfare using condition (48) to obtain an expression as a sum of payoffs of low-cost and high-cost lenders without direct redistribution due to adverse selection:

$$
\begin{aligned}
W' &= F\left[\psi(\nu\lambda p_U + (1-\nu)A)\right] - \int_0^{\eta} \eta_i dF \\
&\quad + (1-F)\left[\kappa\mu A m + \left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right](1-m)\right].
\end{aligned}
$$

Thus, m^l does not affect welfare directly but only via the price of uninsured loans, the screening threshold, and insurance by high-cost lenders. These are given (implicitly) by (52) and generalizations of (50) and (51): $p_U = \left[\nu\lambda - \frac{(1-\psi)Fm^l\kappa}{(1-F)m + (1-\psi)Fm^l}\right]\frac{\mu A}{\nu\lambda + (1-\nu)(1-\mu)}$ and

$$
\nu \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-\mu)(1-F)(1-m) + (1-\psi)F(1-m^l)} = \mu \frac{\nu\lambda - \frac{(1-\psi)Fm^l}{(1-F)m + (1-\psi)Fm^l}\kappa}{\nu\lambda + (1-\nu)(1-\mu)}.
$$

Hence,

$$
\frac{dW'}{dm^l} = \underbrace{\frac{\partial W'}{\partial m^l}}_{=0} + \underbrace{\frac{\partial W'}{\partial m^*}}_{>0}\underbrace{\frac{dm^*}{dm^l}}_{<0} + \underbrace{\frac{\partial W'}{\partial \eta^*}}_{<0}\underbrace{\frac{d\eta^*}{dm^l}}_{>0} + \underbrace{\frac{\partial W'}{\partial p_U^*}}_{>0}\underbrace{\frac{dp_U^*}{dm^l}}_{<0} < 0, \qquad (53)
$$

where $\frac{d\eta^*}{dm^l} = -\psi(1-\nu)(1-\mu)\frac{dp_U^*}{dm^l} + (p_I^* - p_U^*) > 0$, $\frac{dp_U^*}{dm^l} = \frac{dp_U}{dm^*}\frac{dm^*}{dm^l} - \frac{(1-\psi)F(1-F)m^*\mu A\kappa}{((1-F)m^* + (1-\psi)Fm^l)^2\left[\nu\lambda + (1-\nu)(1-\mu)\right]} < 0$, and:

$$
\begin{aligned}
\frac{\partial W'}{\partial p_U^*} &= \nu\lambda\psi F + \left[\nu\lambda + (1-\nu)(1-\mu)\right](1-F)(1-m^*) > 0, \\
\frac{\partial W'}{\partial m^*} &= (1-F)\left[\kappa\mu A - \left[\nu\lambda p_U^* + (1-\nu)(\mu A + (1-\mu)p_U^*)\right]\right] > 0,
\end{aligned}
$$

$$\begin{aligned}\frac{\partial W'}{\partial \eta^*} &= f\Big[\psi(\nu\lambda p_U^* + (1-\nu)A) - \eta^* - [\kappa\mu A m^* + [\nu\lambda p_U^* + (1-\nu)(\mu A + (1-\mu)p_U^*)]\,(1-m^*)]\Big] \\ &= -f\kappa\,[(\mu A - p_I^*)m^* + (1-\psi)p_I^*] < 0, \qquad (54)\end{aligned}$$

Insurance by low-cost lenders unambiguously lowers welfare. The direct effect of higher m^l is higher adverse selection in insurance market and lower adverse selection in the secondary market for uninsured loans. Regarding the price dimension of the allocative efficiency, the former adverse selection redistributes resources from low-cost lenders to high-cost lenders that both have the same expected marginal utility of consumption, and thus there in no direct impact on the social gains from trade. In contrast, the adverse selection in the market for uninsured loans redistributes resources from liquidity shocked lenders to lenders without liquidity shock and this reduces the social gains from trade and allocative efficiency.

The key negative effect of insurance by low-cost lenders is that it reduces insurance by high-cost lenders. Thus, the overall adverse selection in both the market for insured and the market for uninsured loans increases with a negative effect on allocative efficiency (lower p_I^* and p_U^*). The insurance by low-cost lenders also increases screening incentives, but this has negative effects on welfare $\partial W'/\partial\eta^* < 0$ because higher screening is due to two factors: the option to selectively insure lemons by low-cost lenders (marginal benefits of $\kappa(1-\psi)p_I^*$) and lower benefits of insurance for high-cost lenders due to adverse selection in insurance market (benefits lowered by $\kappa(\mu A - p_I^*)$), see equation (54). Both factors reduce welfare.

The planner who observes screening costs and makes insurance choice on behalf of lenders eliminates insurance by low cost lenders, $m^l = 0$, and chooses the welfare benchmark of the main model, $m = m^P$. Next, a regulator who does not observe screening costs cannot directly eliminate adverse selection in the loan insurance market. However, the regulator can improve welfare by subsidizing loan insurance, where the optimal subsidy in the liquid equilibrium satisfies equation (23). Moreover, an insurance subsidy eliminates the welfare-dominated equilibrium with an illiquid insurance market.

B.8 Proof of Proposition 8

We derive the privately optimal insurance coverage ω^*. The price for insured loans is $p_I = \frac{\nu(\mu A+(1-\mu)\omega A-k)+(1-\nu)(1-\mu)(\omega A-k)}{\nu+(1-\nu)(1-\mu)} = \frac{\nu+(1-\nu)(1-\mu)\omega}{\nu+(1-\nu)(1-\mu)}\mu A$, which implies that p_I monotonically increases in insurance coverage, $\frac{dp_I}{d\omega} > 0$. If $\omega < \frac{(1-\nu)(1-\mu)-\nu(\mu\lambda-1)}{(1-\mu)[\nu+(1-\nu)(1+\mu(\lambda-1))]}$, then high-quality insured loans are not sold in the market as $p_I\lambda < A - k$ and the price for insured loans drops further to $p_I(\omega) = \omega\mu A$. Lenders who insure again do not screen, so they solve

$$\max_{\omega} \nu\lambda p_I + (1-\nu)\,[\mu(A-k) + (1-\mu)p_I] = \frac{\nu\kappa + (1-\nu)(1-\mu)\,[\kappa + \nu(\omega-1)(\lambda-1)]}{\nu + (1-\nu)(1-\mu)}\mu A.$$

Since this expected payoff increases in ω, the corner solution $\omega^* = 1$ is privately optimal.

Next, we consider the planner's choice of insurance coverage. The payoff of uninsured low-cost lenders, $\nu\lambda p_U + (1-\nu)A - \eta_i$, and high-cost lenders, $\nu\lambda p_U + (1-\nu)[\mu A + (1-\mu)p_U] - \eta_i$, also increases in ω because insurance coverage raises the price of uninsured loans, $\frac{dp_U}{d\omega} = \frac{dp_U}{dp_I}\frac{dp_I}{d\omega} > 0$. Hence, the planner also chooses full coverage, $\omega^{SP} = 1$:

$$\omega^{SP} = \arg\max_{\omega} \overbrace{\nu\lambda p_U \left[F + (1-F)(1-m)\right] + (1-\nu)\left[AF + (\mu A + (1-\mu)p_U)(1-F)(1-m)\right]}^{\text{Value to uninsured lenders}}$$
$$+ \underbrace{\left(\nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right)\right)(1-F)\, m}_{\text{Value to insured lenders}} - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}}$$
$$= \arg\max_{\omega} \nu\lambda p_U + (1-\nu)\left[AF + (\mu A + (1-\mu)p_U)(1-F)\right] - \int_0^{\eta} \tilde{\eta} dF(\tilde{\eta}), \tag{55}$$

subject to (5) and (33), where equation (55) is obtained after substituting the indifference condition (8). The solution, $\omega^{SP} = 1$, follows from $\frac{dW}{d\omega} = \Big(\underbrace{\frac{\partial W}{\partial p_U^*}}_{>0} + \underbrace{\frac{\partial W}{\partial \eta^*}}_{=0} \underbrace{\frac{d\eta^*}{dp_U^*}}_{<0} \Big) \underbrace{\frac{dp_U^*}{dp_I^*}}_{>0} \underbrace{\frac{dp^{I*}}{d\omega}}_{>0} > 0$.

B.9 Proof of Proposition 9

In the equilibrium with insurance, the competitive insurance fee solves $k = (1-k)A(1-\mu)$, yielding the k^* stated. High-cost lenders are indifferent between insurance, $(1-k)\kappa A = \kappa A(\mu - \delta(1-\mu))$, and no insurance, $\nu\lambda p_U' + (1-\nu)[\mu A + (1-\mu)p_U']$, which yields the stated expressions for $p_U^{*\prime}$. Combining this equation with the competitive price in equation (33) gives $m^{*\prime}$ stated in the proposition. Finally, substituting $p_U^{*\prime}$ into equation (5) gives the screening cost threshold $\eta^{*\prime}$ stated in the proposition. Since the price $p_U^{*\prime}$ must satisfy condition (2), a necessary condition for a liquid equilibrium when insurance is used is $\nu\left[\mu - \delta(1-\mu)\right]\lambda^2 - \left[\nu + \delta(1-\mu)(1-\nu)\right]\lambda - (1-\nu)(1-\mu) \geq 0$. Since only the larger root of this quadratic condition is positive, the condition collapses to $\lambda \geq \tilde{\lambda}_L' \equiv \frac{\nu+\delta(1-\mu)(1-\nu)}{2\nu[\mu-\delta(1-\mu)]} + \sqrt{\frac{[\nu+\delta(1-\mu)(1-\nu)]^2}{4\nu^2[\mu-\delta(1-\mu)]^2} + \frac{(1-\mu)(1-\nu)}{\nu[\mu-\delta(1-\mu)]}}$. Insurance takes place on the subset $A < \tilde{A}_I'$ or alternatively when $\mu > \tilde{\mu}_I'$, where thresholds $\tilde{A}_I'$ and $\tilde{\mu}_I'$ are implicitly defined by combining $p_U^{*\prime}$ and equation (33) evaluated at $m^{*\prime} = 0$:

$$\kappa(1-\delta)F(\eta) = (1 - F(\eta))\left[\mu(1-\nu)(\lambda-1) - \frac{\kappa}{\nu}\delta(\nu + (1-\nu)(1-\mu))\right]. \tag{56}$$

Regarding normative implications, it is easy to show that the planner's choice of insurance exceeds the unregulated level, following the same steps as in Appendix B.5.

B.10 Proof of Proposition 10

This proof proceeds as follows. First, we define the equilibrium in which lenders can sell a share of the loan and financiers update their beliefs based on it. Second, we show that risk retention via partial loan sales cannot result in a separating equilibrium in which loan quality is revealed at $t = 1$. The only exception is the corner solution with full screening and effectively just one type of lenders. Finally, we show that our main results carry over to the continuum of pooling equilibria sustained by particular out-of-equilibrium beliefs.

Definition 4. *An equilibrium comprises screening* $\{s_i\}$*, insurance* $\{\ell_i\}$*, loan sales* $\{q_i^I, q_i^U\}$*, beliefs about loan quality* $\phi_{i,t}$*, prices* p_I *and* p_U*, and a fee* k *such that:*

1. *At* $t = 1$*, for each* λ_i *and* A_i*, each lender* i *optimally chooses loan sales,* q_i^I *and* q_i^U*.*
2. *At* $t = 0$*, each lender* i *chooses screening* s_i *and loan insurance* ℓ_i *to solve*

$$\max_{s_i,\ell_i,c_{i1},c_{i2}} \quad \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to}$$

$$\begin{aligned} c_{i1} &= q_i^I\, p_I + q_i^U\, p_U, \\ c_{i2} &= (\ell_i - q_i^I)\pi + (1 - \ell_i - q_i^U)A_i, \qquad \Pr\{A_i = A\} = \psi s_i + \mu(1 - s_i). \end{aligned}$$

3. *At* $t = 1$*, financiers use Bayes' rule to update their beliefs* $\phi_{i,1}(q_i^U, q_i^I, \ell_i)$ *on the equilibrium path, and prices* p_I *and* p_U *are set for financiers to expect to break even.*
4. *At* $t = 0$*, financiers use Bayes' rule to update their beliefs* $\phi_{i,0}(\ell_i)$ *on the equilibrium path, and the fee* k *is set for financiers to expect to break even.*

Risk retention as signal of loan type. Suppose there is a separating equilibrium with both high-cost and low-cost lenders. In such an equilibrium, lenders with different loan qualities choose different risk retention and thus outside financiers learn the loan quality that is reflected in the price. Sellers of high-quality loans choose $q^U \in (0, 1]$ (since $\ell_i \in \{0, 1\}$), and sellers of low-quality loans choose $q^{U\prime} \neq q^U$, and thus $p_U(q^U) = A$ and $p_U(q^{U\prime}) = 0$. For this equilibrium to exist, the sellers of low-quality loans must find it optimal to signal their type and receive the zero price rather than mimic the risk-retention of sellers with high-quality loans. Since lenders cannot commit to negative consumption, high-cost lenders with lemons will always want to mimic sellers with high-quality loans since $q^U p_U(q^U) = q^U A > q^{U\prime} p_U(q^{U\prime}) = 0$. Hence, there exists no separating equilibrium with partial screening, $\eta^* < \bar{\eta}$.

However, there could exist an equilibrium with $q^U < 1$, where all lenders screen and, thus, for $\psi \to 1$ the quantity of low-quality loans originated vanishes and so does uncertainty about the quality of loans traded. This equilibrium is pooling as the vanishing amount of lenders with low-quality loans mimic the risk retention of lenders with high-quality loans, but the adverse selection in the market for uninsured loans vanishes. We derive the threshold screening cost by equating the payoff from screening, $\nu[\lambda p_U q^U + (1 - q^U)A] + (1 - \nu)A - \eta$, and payoff when not screening, $\nu[\lambda p_U q^U + (1 - q^U)\mu A] + (1 - \nu)(\mu A + (1 - \mu)p_U q^U)$:

$$\eta = (1 - \mu)[\nu(1 - q^U)A + (1 - \nu)(A - p_U q^U)] = (1 - \mu)(1 - q^U)A, \tag{57}$$

where the second equality comes from $p_U = A$ (under screening by all lenders and $\psi \to 1$). Equation (57) implies that there are no high-cost lenders, $\eta \geq \bar{\eta}$, if retention is large enough, $(1 - q^U) \geq \frac{\bar{\eta}}{(1-\mu)A}$. Thus, a sufficient condition for ruling out this equilibrium is $\bar{\eta} \geq (1 - \mu)A$.

Pooling equilibria with partial sales. The remainder of the proof focuses on pooling equilibria with adverse selection and shows that our main results are qualitatively unchanged. Let the maximum loan sales consistent with full screening, $\eta^* \geq \bar{\eta}$, be $\bar{q}^U \equiv \min\left\{0, 1 - \frac{\bar{\eta}}{(1-\mu)A}\right\}$. Then there exists a continuum of PBE with adverse selection, where $q^U \in (\bar{q}^U, 1]$ in the appropriately generalized liquid equilibrium, $\lambda > \tilde{\lambda}_I(q^U)$, and out-of-equilibrium beliefs $\phi_{i,1} = 0$ if $q_i^U \neq q^U$.

If insurance is used in this equilibrium, high-cost lenders have to be indifferent between payoff when not insuring, $\nu\lambda p_U q^U + \nu(1-q^U)\mu A + (1-\nu)[\mu A + (1-\mu)p_U q^U]$, and insurance when insuring, $\kappa\mu A$. Equating those payoffs determines the price of uninsured loans:

$$p_U^* = \frac{\nu\mu A\left[\lambda + \frac{(\lambda-1)(1-q^U)}{q^U}\right]}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{58}$$

which is a generalization of p_U^* in Proposition 2. It decreases in q^U, $dp_U/dq^U < 0$, because higher sales of uninsured loans make insurance relatively less attractive, and a lower price of uninsured loans satisfies the indifference about insurance. Using (57), the screening cost threshold is

$$\eta^* = \frac{(1-\mu)\kappa A}{\nu\lambda + (1-\nu)(1-\mu)}\left[(1-\nu)(1-\mu) + \nu(1-q^U)\right], \tag{59}$$

which is a generalization of the threshold in Proposition 2. It decreases in q^U, $d\eta/dq^U < 0$, since a higher q^U lowers the net benefits of screening from loans held to maturity upon a liquidity shock, $(1-\mu)\nu(1-q^U)A$, and increases the payoff from the sale of lemons when not screening, $(1-\nu)p_U q^U$, where $dp_U q^U/dq^U > 0$. Combining equation (58) with equation (33) yields

$$m^* = 1 - \frac{\left[\kappa q^U(1-\mu) - \mu(\lambda-1)(1-q^U)\right]F(\eta^*)}{\mu(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu](1-F(\eta^*))}, \tag{60}$$

which is a generalization of the expression for m^* stated in Proposition 2. Hence, $m^* > 0$ whenever

$$A < \tilde{A}_I(q^U) \equiv \frac{\nu\lambda + (1-\nu)(1-\mu)}{(1-\mu)\kappa[(1-\nu)(1-\mu) + (1-q^U)\nu]}F^{-1}\left(\frac{\mu(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu]}{\kappa(1-\mu)q^U + \mu(\lambda-1)(1-\nu)(q^U-\mu)}\right).$$

The equilibrium insurance condition can also be expressed as $\mu > \tilde{\mu}_I(q^U)$, where $\tilde{\mu}_I(q^U)$ is implicitly defined by (60) after substituting $m^* = 0$, similarly as in the main model. It is easy to show that the planner again insures more loans than in the unregulated economy, using the same steps as in Appendix B.5.

B.11 Proof of Proposition 11

On the positive side, the price arises from substituting $m = 1$ into equation (9). The screening threshold equalizes the payoff from screening, $\nu\lambda p_U + (1-\nu)[\psi A + (1-\psi)p_U]$, with the payoff from not screening but insuring, κp_I.

On the normative side, welfare increases in insurance for $m < m^*$ (Appendix B.5). Here, we have $m^* = 1$. Since insurance is strictly preferred by high-cost lenders for $\psi < \underline{\psi}$, a regulator that affects insurance payoff can affect only screening. It can lower screening with insurance subsidies and increase screening with insurance taxes without any effect on m or p_U up to the point where the insurance indifference condition is satisfied. Since screening has no pecuniary externality when $m = 1$, the planner's and lenders' screening choices coincide, implying zero subsidies and taxes, $b_I^R = 0$.