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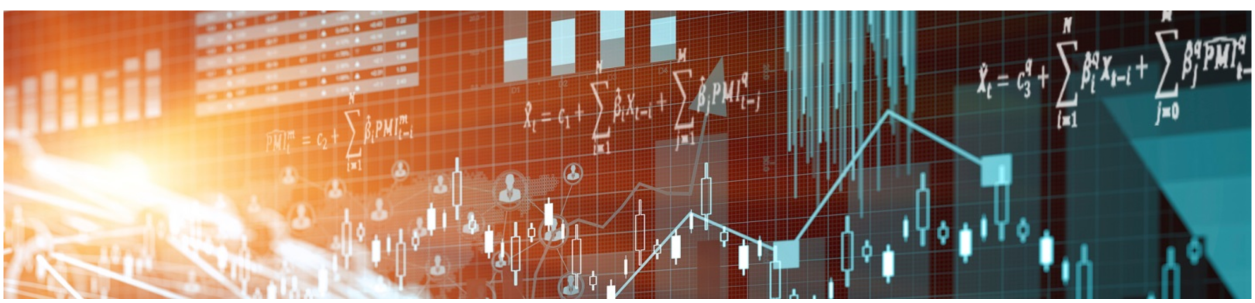
Financial Frictions, Durable Goods and Monetary Policy



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Financial Frictions, Durable Goods and Monetary Policy

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Abstract

This paper examines the effect of financial frictions on the consumption of durables and non-durables in a two-sector dynamic stochastic general equilibrium (DSGE) model with sticky prices and heterogeneous agents. The financial frictions are a combination of loan-to-value (LTV) and payment-to-income (PTI) constraints faced by borrowers. In this setting, a monetary contraction drastically reduces the maximum amount consumers can borrow to purchase durable goods. As a result, the model predicts that the consumption of durables falls, along with non-durables, even when durable prices are fully flexible. Also, output falls and the nominal interest rate increases following monetary tightening. Thus, our model's predictions better match the data than models in existing literature.

Bank topics: Monetary policy; Financial system regulation and policies
JEL codes: E44, E52

1 Introduction

When households purchase durable goods such as homes, automobiles and other big ticket items, most often they pay for them with a partial cash down payment and the remainder with loans through borrowing. Lenders on the other hand try to secure their loans with collateral requirements and, equally important, employment and income information in order to make sure that borrowers will be able to meet the flow of their periodic debt and interest payments. Such requirements are important parts of credit or financial frictions in the recent macro-finance literature. Although this literature has given extensive consideration to collateral frictions, it has paid less attention to income constraints on durable purchases.

In this paper we re-examine the reaction of durable and non-durable consumption and related macroeconomics variables to monetary shocks in the context of the two-sector sticky price model with financial frictions. In contrast to the majority of the existing studies, we consider a broader set of financial frictions consisting of both loan-to-value (LTV) limits and payment-to-income (PTI) limits on borrowing. The LTV limits are standard and constrain the debt issued to borrowers to be a net-of-down payment fraction of the expected future value of the durable goods, whereas the PTI limits constrain interest payments on the debt to be a specific fraction of the borrowers' labor income. Our study shows that the extension to this type of credit frictions is important in reconciling the predictions of the model with the empirical facts. In particular, the PTI constraint is crucial in resolving the well-known co-movement problem or the negative correlation between durables and non-durables in response to monetary shocks, even in the case of fully flexible durable prices. This is a new result which does not appear in the existing literature. This result also helps to avoid other counterfactual predictions regarding the response of output and the nominal interest rate that have plagued the sticky price model. The implication of our results is that, relative to sticky prices, the effects of considering financial frictions on aggregate demand are quite important in determining the predictive properties of the model.

In the recent macroeconomics literature, monetary policy has been considered within dynamic stochastic general equilibrium models with sticky prices and durable goods. Notable contributions in this line of research include, among others, Barsky, House and Kimball (2003, 2007), Iacoviello (2005), and Erceg and Levin (2006). It is well known that with perfect financial markets, the standard sticky price model with durable goods generates some counterfactual predictions. For low levels of durable price stickiness, the model predicts a negative correlation between durable and non-durable expenditures in response to a monetary shock. Specifically, a monetary contraction causes a fall in non-durable consumption and a rise in durable consumption, a prediction that contradicts the empirical evidence, which shows that both types of consumption fall and durables fall more than non-durables; see Bernanke and Gertler (1995), Barsky et al. (2003), Erceg and Levin (2006). Further, if the predicted rise in durables is large enough to offset the fall in non-durables, aggregate output will be unchanged; and with increasing price stickiness the nominal interest rate falls despite the monetary tightening.

Barsky et al. (2003, 2007) noted that these counterfactual results arise from the near constancy of the shadow value of durables, and suggested that financial frictions may improve the predictive power of sticky price models. Monacelli (2009) introduces collateral or LTV constraints on borrowing in the two-sector sticky price model with borrowers and lenders. Household debt in the model reflects trade between households with different rates of time preference. Collateral constraints break the near constancy of the shadow value of durables, and this helps to resolve the co-movement problem and the perverse reaction of the interest rate for a limited range of durable price stickiness, around the 2-quarter mark. For durable price stickiness below this range, or for full durable price flexibility, the co-movement problem remained unresolved. The implication of Monacelli's main findings is that financial frictions on borrowing reduce the importance of price stickiness in determining the model's behavior.

Sterk (2010) disentangled the relative importance of price stickiness and financial frictions by comparing the predictions of Monacelli's model with and without financial frictions. He showed that collateral constraints, in fact, make it more difficult to resolve the co-movement problem. This striking result is due to the general equilibrium implications of the bond or debt market. Specifically, the presence of collateral constraints reduces the ability of lenders to lend more to borrowers. As a result, they use their extra savings to smooth their consumption by buying more durable goods. If the increase in lenders' durable purchases is large enough, it is then possible that total durable consumption is higher with credit frictions than in the model without frictions. Also, with flexible durable prices, in the model with credit frictions aggregate output may even expand following a monetary contraction, contrary to the empirical evidence. Sterk's results reinstated the importance of price stickiness and downplayed the importance of credit frictions on demand in sticky price models. He went on to conclude that supply frictions, such as sticky nominal wages considered by Carlstrom and Fuerst (2006), provide a more satisfactory approach to resolving the co-movement problem.

Although collateral requirements alone may not be enough to align the predictions of the sticky price model with the empirical evidence, demand-side financial frictions can still be very useful in improving the predictions of the model. As we show below, when credit frictions are extended to include both collateral and income requirements, they provide an effective and realistic way to eliminate the counterfactual predictions of the sticky price model. Our framework is the heterogeneous agent two-sector sticky price model with financial frictions consisting of random combinations of LTV and PTI borrowing constraints, similar to Greenwald (2018). In this setting, the amount of collateralized debt is determined randomly by shocks to the borrowers' labor income. If the borrowers' income is above a threshold value, the LTV constraint binds; and if it is below the threshold, the PTI constraint binds. The actual collateralized debt obtained by borrowers is the minimum of the two debt levels associated with the two constraints. A monetary tightening in this context causes the consumption of both durable and nondurable goods to fall even when durable prices are fully flexible. Aggregate output also decreases and the nominal interest rate increases, as would be expected with a monetary contraction. Further, household debt declines more sharply in this model than in the model with collateral constraints alone.

To see how these predictions arise, it is important to understand how the interaction of the collateral and income constraints changes the dynamics of a monetary tightening in the model with collateral constraints alone. One reason for the change is the link between the monetary policy shock and the opportunity cost of leisure, and hence the labor supply of the borrowers. Another reason is the response of the shadow value of borrowing, which varies with the monetary shock.

Regarding the former, the opportunity cost of leisure for the borrowers depends on the interest rate through the PTI constraint. A monetary tightening that raises the interest rate decreases the opportunity cost of leisure and tends to reduce the borrowers' labor supply, and thus their consumption of durables and non-durables. This helps resolve the co-movement problem, even with flexible durable prices.

Regarding the latter, the monetary tightening also affects the shadow value of borrowing and the shadow value of durables. Both variables increase, but the rise in each is less with both constraints than with the collateral constraint alone. The reason is that the increase in the interest rate tightens credit conditions more through the PTI constraint. The borrower can relax this constraint by working more, but the increase in the interest rate dictates working less by reducing the current opportunity cost of leisure. Since the tighter credit conditions reduce the amount of the debt and the relative price of the durable good, the only way a borrower can purchase a more valuable durable is to pay dollar-for-dollar beyond the price that satisfies both constraints. This is not the case in an economy with only a collateral constraint, as in that case purchases of additional units of the durable relax the LTV constraint, which allows the borrower to accumulate more debt. This option is not available when the PTI constraint binds. As a result, the rise in the shadow value of borrowing and the shadow value of durables is less in the latter case than in the former. Equivalently, the user cost of durables rises by more in the two frictions case than in the case with only a collateral friction. The implication is that there is less incentive in the present model with two frictions to substitute durables for non-durables, which would otherwise tend to exacerbate the co-movement problem relative to a frictionless economy, as noted by Sterk (2010).

The rest of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents a simple example in order to provide intuition for the interaction of the PTI and LTV constraints. Section 4 presents the empirical evidence and stylized facts related to the co-movement problem. Section 5 presents the theoretical model and discusses the optimality conditions for borrowers and lenders. Section 6 calibrates the model with different credit frictions and compares the numerical results. Section 7 discusses the results of sensitivity analysis. Section 8 contains some concluding remarks.

2 Related Literature

This paper complements existing strands of the literature on sticky price models and durable goods. Ohanian, Stockman and Killian (1995) discuss the co-movement problem in a general equilibrium two-sector model without financial frictions, in which one good has a flexible price and the other has a sticky price. They show that a monetary policy shock may induce a negative co-movement of sectoral outputs. Barsky et al. (2003) note that the standard two-sector sticky price model generates counterfactual co-movements between durable and non-durable consumption and suggested two different mechanisms that could help match the model's predictions with the empirical evidence: on the supply-side, wage stickiness; and on the demand side, credit frictions. As noted above, Monacelli (2009) and Sterk (2010) study the interaction between price stickiness and collateral credit frictions, while Carlstrom and Fuerst (2006) examine the role of both credit constraints and nominal wage stickiness in resolving the co-movement problem.

Another strand of the literature attempts to tackle the co-movement problem by focusing on factors affecting the supply side of the economy. Carlstrom and Fuerst (2010) interpret durable goods as residential housing in a model with sticky nominal wages, adjustment costs in housing construction and habits over non-durable consumption. They show that the interaction of these three factors solves the co-movement problem and delivers reasonable responses of non-durable consumption and housing investment following a monetary contraction. Nonetheless, the improved predictions of this model rely on the key assumption of nominal wage stickiness, which imparts a great deal of durable (housing) price stickiness. Bouakez, Cardia and Ruge-Murcia (2011) extend the Barsky et al. (2007) sticky price model by introducing input-output intersectoral linkages and limited labor mobility across sectors. They show that these two features help to generate a positive co-movement between non-durable and durable spending after a monetary shock, thus overcoming the limits of the standard two-sector models that relies on varying degrees of price stickiness across sectors. Similarly, Sudo (2012) exploits intersectoral input-output linkages to resolve the co-movement problem.

Kim and Katiyama (2013) modify the two-sector sticky price model to allow for non-separability and complementarity between non-durable consumption and labor in household preferences. With flexibly priced durables, a monetary contraction decreases the consumption and production of non-durables and the labor supply. In turn, this raises the real wage and decreases the consumption and production of durable goods, thus resolving the co-movement problem. Alternatively, Chen and Liao (2014) resolve the co-movement problem by adding capital as another input in the production process along with a collateral constraint for borrowers. In this model, with flexible durable prices, a monetary contraction induces capital accumulation, which reduces savers' disposable income in the short run. As a result, savers reduce their expenditures on durables along with expenditures on non-durables, which have now become relatively more expensive. Nonetheless, capital accumulation leads to a counterfactual fall in the nominal interest rate in their model.

There is also an emerging literature that incorporates PTI constraints in dynamic macroeconomic models with financial frictions.[1] Corbae and Quintin (2015) incorporate a PTI constraint in their model and use it to explain the housing boom in the US, and its relationship to default risk and credit growth. Greenwald (2018) presents empirical evidence for the importance of PTI limits for the recent boom-bust cycle of the US housing market, and studies their interaction with LTV limits in a general equilibrium model with prepayable mortgage debt. He shows that PTI limits greatly amplify the effects of nominal interest rate shocks on mortgage debt and the macroeconomy.

In the present paper we introduce PTI and LTV limits in our analysis similar to Greenwald. Given the practical importance of the two constraints in actual credit markets, we exploit their interaction in order to provide a realistic alternative resolution to the co-movement problem between durable and non-durable spending. Our analysis is different from the existing literature in that we rely solely on demand-side financial frictions to resolve the co-movement problem even with fully flexible durable prices, which cannot be done in models with collateral constraints alone. An added advantage of our model is that it predicts the correct response of output and the nominal interest rate to monetary shocks.

3 The Interaction of the PTI and LTV Constraints

In this section we present a numerical example in order to illustrate the significance of the PTI and LTV constraints and show how their interaction affects the borrowing capacity of the borrowers. The section also provides intuition on how monetary policy can alter the dynamics of the model relative to the case of an LTV constraint alone.

In practice, both constraints are important. In order to extend loans to borrowers, lenders evaluate the credit worthiness and the ability of borrowers to pay back the principal and interest. The PTI ratio is used to measure the borrower's ability to manage monthly payments and debt repayment. It measures how much of monthly income goes toward paying the credit card bills, mortgage, car payments and other long-term and significant short-term monthly debts. For many lenders, the PTI limits is set at between 28% and 36% of a borrower's average monthly income. On

[1] In contrast, following the seminal papers by Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), the literature of heterogeneous agent models with collateral constraints has grown very large, partly in response to importance of the US housing sector during the great moderation and the 2008 global financial crisis; see among others Iacoviello (2005), Iacoviello and Neri (2010), Calza, Monacelli and Stracca (2013) and Justiniano, Primicerib and Tambalottie (2015).

the other hand, the LTV ratio is the ratio of the debt to the value of the collateral. Most lenders set the LTV ratio at 80% or less. The aim of the PTI limit is to prevent default, and the purpose of the LTV limit is to reduce the probability of debt repudiation by borrowers. When both constraints are considered, the amount of the debt issued to a borrower is the minimum of the debt amounts implied by the PTI and LTV limits. This is illustrated in the following example.

Consider a borrower with average monthly income of \$6000, facing a PTI ratio of 28% and an LTV ratio of 80% or equivalently a 20% down payment. Then the maximum monthly payment consistent with the PTI limit is \$1680 per month. If the borrower is quoted an effective interest rate of 4% on a loan with an amortization period of 20 years, the annualized rate will be 4.074% and the maximum loan amount based on PTI limit will be $\$275,457.30$. Because the borrower has to satisfy the LTV constraint as well, this amount has to be exactly equal to $0.80P_{\max}$, implying $P_{\max} = \$344,321.63$, where $P_{\max}$ is the maximum price of the durable good that satisfies both constraints. Further, for prices in the range $(0,\ P_{\max})$, the LTV constraint binds, and for prices above $P_{\max}$, the PTI constraint binds. In the former case, the borrower can increase purchases of the durable good by paying 20% cash and borrowing 80% for the balance. In the latter case, purchasing a higher-priced durable entails paying dollar-for-dollar with cash for a price beyond $P_{\max}$. Next, consider a monetary policy shock that raises the interest rate from 4% to 5%. Now, the maximum amount of debt allowed by the PTI constraint is $\$252,104.18$, and the corresponding maximum price is $P^{'}_{\max} = \$315,130.23$. These results are summarized in Table 1.

Table 1: Maximum Loan due to the PTI Limit

Interest Rate	Max PTI Loan	Max Durable Price
4%	\$275,457.30	\$344,321.63
5%	\$252,104.18	\$315,130.23

Notice the interest rate hike has decreased the collateralized debt and the price of the durable by 9.26%. More importantly, because $P^{'}_{\max} < P_{\max}$ and the PTI constraint binds, the monetary contraction has reduced the shadow value of borrowing, as now the borrower has to pay fully with cash in order to purchase a more valuable durable than $P^{'}_{\max}$. Clearly, that is not the case when the LTV constraint binds, because in this case the borrower can purchase a more valuable durable and use it as collateral to relax the LTV constraint and accumulate more debt to finance 80% of the additional cost. This collateral constraint effect, which is present when the LTV constraint binds, is shut-off when the PTI constraint binds. In the latter case, the borrower can relax the PTI constraint by increasing her labor income, that is, by working more. Because borrowers are more impatient than lenders, and as a result they wish to tilt consumption in the present, the shadow value of borrowing is positive in both cases. However, following the increase in the interest rate, the shadow value of borrowing should be expected to rise by less in economies with both financial frictions than in economies with only an LTV friction.

As we show below, this insight has important implications for the dynamics of the two-sector general equilibrium sticky price model with PTI and LTV frictions. As noted by Sterk (2010), models with LTV frictions alone make it harder to resolve the co-movement problem between durables and non-durables relative to models without financial frictions, because the monetary contraction increases the shadow value of borrowing and the shadow value of durables, and thus results in a substitution of durables for non-durable purchases. This substitution effect is reduced when both frictions are present and the PTI constraint binds, as then the rise in the shadow value of borrowing is relatively small. This is an important reason why the presence of the two frictions helps to resolve the co-movement problem even with fully flexible durable prices. Another important reason is that the PTI constraint directly affects the opportunity cost of leisure and hence the labor supply of the borrowers. In particular, the increase in the interest rate reduces the opportunity cost of leisure and the borrowers' labor supply, which tends to tighten their PTI constraint. As a result their consumption of both durables and non-durables falls. We show this explicitly in Section 5 below, after we discuss the empirical evidence based on US data in the next section.

4 Empirical Evidence

In this section we examine empirically the reaction of real durable and non-durable spending, real output, and real household debt in response to a contractionary monetary policy shock. Our results, based on the most recent US data, replicate the stylized facts: gross domestic product and consumption spending on both durable and non-durable goods decline, durable spending declines more than non-durable spending, and real household debt also declines.

4.1 Empirical Evidence from Vector Auto-regression

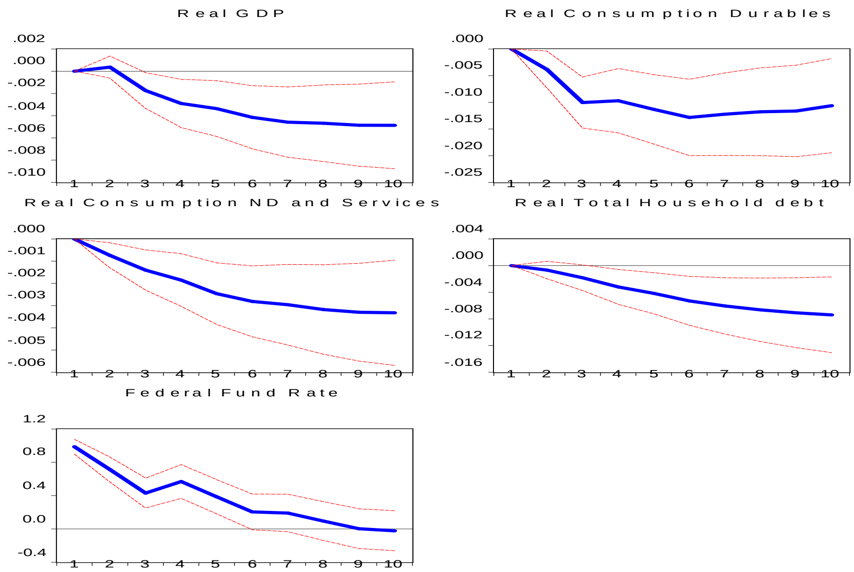
Our empirical analysis complements the vector autoregressive (VAR) framework of Monacelli (2009) by including more recent data for the US over the period 1954:1 – 2017:3. We fit the data to the following six-dimensional VAR model:

$$Y_t = \alpha_o + \alpha_1 t + \sum_{j=1}^{L} A_j Y_{t-j} + B\varepsilon_t. \tag{1}$$

The VAR system includes a constant and a time trend, and the vector Y_t comprises six variables: (i) real GDP, (ii) real durable consumption, (iii) real non-durable consumption and services, (iv) the GDP deflator, (v) total real household debt and (vi) the federal funds rate. The error term is $\varepsilon_t^{'} = [\varepsilon_t^{g}\ \varepsilon_t^{d}\ \varepsilon_t^{nd}\ \varepsilon_t^{gd}\ \varepsilon_t^{hd}\ \varepsilon_t^{ms}\ ]$, consisting of shocks to GDP, durable and non-durable goods, GDP deflator, households debt, and the federal funds rate, respectively. All variables except the federal funds rate are measured in logs, and all variables except the federal funds rate and the GDP deflator are deflated by the GDP deflator. We want to measure the dynamic responses of these variables to one standard deviation increase in ε_t^{ms}. To identify ε_t^{ms}, we use the standard recursive identification scheme based on the Cholesky decomposition (Christiano, Eichenbaum and Evans, 1999). The lag order value $L = 4$ was chosen by the minimized values of the Final Prediction Error (FPE), Hannan-Quinn (HQ), Akaike (AIC) and Swartz Bayesian (BIC) information criteria.

Figure 1 shows the empirical impulse response functions, following a one-standard-deviation increase in the federal funds rate. The dashed red lines represent two standard error bands. As shown, real GDP and both durable and non-durable consumptions fall in response to the monetary tightening. The decline in durable consumption is higher than the decline in non-durable consumption and is about four to five times larger than the decline in non-durables. Real household debt also declines.

Figure 1 Impulse response functions



Note: Empirical impulse response functions to one-standard-deviation increase in the federal funds rate: real GDP, real consumer durables, real consumption of non-durables and services, real household debt, and the federal funds rate.

In the next section we consider a heterogeneous agent sticky-price dynamic stochastic general equilibrium (DSGE) model with durables and non-durable goods and PTI and LTV financial frictions. This model generates predictions that are consistent with these empirical findings, even when durable prices are fully flexible. Considering PTI frictions along with LTV frictions is crucial in obtaining these predictions.

5 The Model

We extend the two-sector heterogeneous agent sticky-price model with durable and non-durable goods, whereby borrowers are subject to both an LTV constraint and a PTI constraint. In this framework, we examine the effects of a monetary contraction on durable and non-durable consumption, output, real household debt and the nominal interest rate.

5.1 Households

The economy has two types of representative households, impatient borrowers and patient savers/lenders, of measure ω and $1-\omega$, respectively. Each individual household's time endowment is normalized to 1. All households derive utility from the consumption of a non-durable final good and from the services of a durable final good. Debt accumulation reflects intertemporal trading between the borrowers and the savers. The borrowers are subject to two constraints: a collateral constraint, where the borrowing limit is tied to the expected future value of the stock of durables, and an income constraint, such that the amount of the loan secured by the borrowers is limited by their random employment income.

5.2 The Borrowers

Following the literature, in our model a typical borrower consumes a composite of a non-durable good and the services from a durable good indexed by

$$X_t = ((1-\mu)^{\frac{1}{\eta}}(C_t)^{\frac{\eta-1}{\eta}} + (\mu)^{\frac{1}{\eta}}(D_t)^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} \tag{2}$$

where C_t denotes consumption of the final non-durable good and D_t denotes services from the stock of the final durable good at the end of period t; $\mu > 0$ is the share of durable goods in the composite consumption index; and $\eta \geq 0$ is the elasticity of substitution between services of durable and non-durable consumption. The instantaneous utility function of the borrowers is given by

$$U(X_t, N_t) = \log(X_t) - \frac{\nu N_t^{1+\varphi}}{1+\varphi}, \tag{3}$$

where N_t is the labor supply, φ is the inverse elasticity of labor supply and ν is a parameter that indexes the preference for hours worked for each borrower.

A new loan $B_{i,t}$ by borrower i must satisfy both the LTV and PTI constraints, defined by

$$R_t B_{i,t}^{ltv} \leq \kappa^{ltv}(1-\delta)E_t\{D_{i,t}P_{d,t+1}\} \tag{4}$$

$$(R_t - 1 + \tau)B_{i,t}^{pti} \leq \kappa^{pti} W_t N_{i,t} e_{i,t}, \tag{5}$$

where $P_{d,t}$ is the price of durables; R_t is the gross nominal interest rate; δ is the rate of depreciation of durables; κ^{ltv} and κ^{pti} are the exogenous LTV and PTI ratios; τ accounts for taxes, insurance, and other borrowing costs associated with debt issuance and payment; and $e_{i,t}$ is a random shock to the borrower's labor income that is log-normally distributed with mean 1 and c.d.f F_e. The LTV limit is standard and constrains the debt issued to each borrower to be a net-of-down payment fraction of their expected future value of the durable goods. On the other hand, the PTI limit constrains interest payments on the debt to be a specific fraction of each borrower's random labor income. Since each borrower must satisfy both constraints, $B_{i,t}$ must be the minimum of the two debt limits implied by the LTV and PTI constraints: $B_{i,t} = \min(B_{i,t}^{pti}, B_{i,t}^{ltv})$.

The labor income shock induces borrower heterogeneity and determines endogenously the fraction of borrowers with a binding PTI limit in equilibrium. Let

$$\bar{B}_t^{ltv} = \frac{\kappa^{ltv}(1-\delta)E_t\{D_t P_{d,t+1}\}}{R_t} \tag{6}$$

$$\bar{B}_t^{pti} = \frac{\kappa^{pti} W_t N_t}{(R_t - 1 + \tau)} \tag{7}$$

be the maximum average LTV and PTI debt limits, respectively, when $E(e_{it}) = 1$. Also let $\bar{e}_t = \bar{B}_t^{ltv}/\bar{B}_t^{pti}$ be the threshold value of the income shock $e_{i,t}$ such that when $e_{i,t} < \bar{e}_t$ the PTI constraint binds, and when $e_{i,t} > \bar{e}_t$ the LTV constraint binds. Then, the combined average borrower debt $\bar{B}_t$ is given by

$$
\begin{aligned}
\bar{B}_t &= \int \min(\bar{B}_t^{pti} e_i, \bar{B}_t^{ltv}) dF_e(e_i) && (8)\\
&= \int_0^{\bar{e}_t} \bar{B}_t^{pti} e_i dF_e(e_i) + \int_{\bar{e}_t}^{\infty} \bar{B}_t^{ltv} dF_e(e_i) && (9)\\
&= \underbrace{\bar{B}_t^{pti} \int_0^{\bar{e}_t} e_i dF_e(e_i)}_{\text{PTI Constrained}} + \underbrace{\bar{B}_t^{ltv}(1 - F_e(\bar{e}_t))}_{\text{LTV Constrained}}, && (10)
\end{aligned}
$$

where the first and second terms in equation (10) represent the borrowing capacity of PTI and LTV constrained households, respectively. The properties of the lognormal distribution imply the closed form expression,

$$
\begin{aligned}
G_e(\bar{e}_t) = \int_0^{\bar{e}_t} e_i dF_e(e_i) &= \Phi\left(\frac{\log \bar{e}_t - \frac{\sigma_e^2}{2}}{\sigma_e}\right) && (11)\\
F_e(\bar{e}_t) &= \Phi\left(\frac{\log \bar{e}_t + \frac{\sigma_e^2}{2}}{\sigma_e}\right), && (12)
\end{aligned}
$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution. This is convenient since the borrower's problem aggregates into a single representative borrower: Greenwald (2018).

In this setting, the representative borrower maximizes expected discounted lifetime utility based on information available at time 0:

$$E_0\{\sum_{t=0}^{\infty} \beta^t (\log(X_t) - \frac{\nu N_t^{1+\varphi}}{1+\varphi})\}, \tag{13}$$

subject to the debt constraint,

$$B_t \leq \bar{B}_t, \tag{14}$$

and the average budget constraint,

$$P_{c,t}C_t + P_{d,t}I_t + R_{t-1}B_{t-1} = B_t + W_t N_t + T_t, \tag{15}$$

where, in addition, $P_{c,t}$ is the price of non-durables, $I_t = D_t - (1-\delta)D_{t-1}$ are real purchases of new durables, B_t is nominal debt at the end of period t, W_t is the nominal wage rate, and T_t is lump-sum money transfers from the government. Since labor income is random, the term $W_t N_t$ in equation (15) represents average labor income.

For optimization in units of non-durable consumption, the budget constraint (15) is given by

$$C_t + q_t I_t + R_{t-1}\frac{b_{t-1}}{\pi_{c,t}} = b_t + \frac{W_t}{P_{c,t}} N_t + \frac{T_t}{P_{c,t}}, \tag{16}$$

where $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$ is the relative price of the durable goods, $\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}$ is non-durable good inflation, and $b_t = \frac{B_t}{P_{c,t}}$ is real debt. Similarly, letting $\bar{b}_t = \frac{\bar{B}_t}{P_{c,t}}$ and

$$\bar{b}_t^{ltv} = \frac{B_t^{ltv}}{P_{c,t}} = \frac{\kappa^{ltv}(1-\delta)E_t\{D_t P_{d,t+1}\}}{R_t P_{c,t}} \tag{17}$$

$$\bar{b}_t^{pti} = \frac{B_t^{pti}}{P_{c,t}} = \frac{\kappa^{pti} W_t N_t}{(R_t - 1 + \tau) P_{c,t}}, \tag{18}$$

$$\bar{b}_t = \bar{b}_t^{pti} \int_0^{\bar{e}_t} e_i dF_e(e_i) + \bar{b}_t^{ltv}(1 - F_e(\bar{e}_t)), \tag{19}$$

we can write the real debt constraint as

$$b_t \leq \bar{b}_t. \tag{20}$$

The representative borrower chooses the real quantities $\{C_t, N_t, D_t, b_t\}$ to maximize expected lifetime utility. The Lagrangian for this problem is

$$\begin{aligned} L = \sum_{t=0}^{\infty} [\beta^t [(\log(X_t) - \frac{\nu N_t^{1+\varphi}}{1+\varphi}) \\ + \lambda_t (b_t + \frac{W_t}{P_{c,t}} N_t + \frac{T_t}{P_{c,t}} - C_t - q_t(D_t - (1-\delta)D_{t-1}) - R_{t-1}\frac{b_{t-1}}{\pi_{c,t}}) \\ + \lambda_t \psi_t (\bar{b}_t^{pti} G_e(\bar{e}_t) + \bar{b}_t^{ltv}(1 - F_e(\bar{e}_t)) - b_t)]], \end{aligned}$$

where λ_t and $\lambda_t \psi_t$ are the multipliers associated with the budget constraint and combined debt constraint, respectively.

The optimality conditions for the borrowers' maximization problem are

$$\frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} [1 + \psi_t \frac{\kappa^{pti}}{R_t - 1 + \tau} G_e(\bar{e}_t)] \tag{21}$$

$$\begin{aligned} U_{c,t} q_t = U_{d,t} + \beta(1-\delta) E_t[q_{t+1} U_{c,t+1}] \\ + U_{c,t} \psi_t q_t [\frac{\kappa^{ltv}(1-\delta)E_t(\pi_{d,t+1})}{R_t}(1 - F_e(\bar{e}_t))] \end{aligned} \tag{22}$$

$$\psi_t = 1 - \beta E_t [\frac{U_{c,t+1}}{U_{c,t}} \frac{R_t}{\pi_{c,t+1}}]. \tag{23}$$

Equation (21) sets the marginal rate of substitution between consumption and leisure equal to the opportunity cost of leisure. The latter is the product of the real wage in terms of non-durables times the term in the square brackets, which is positive and greater than unity when the PTI constraint binds, as in this case $\psi_t > 0$ and $G_e(\bar{e}_t) > 0$. With the LTV constraint alone, $\kappa^{pti} = 0$ and the term in the square brackets is unity, in which case equation (21) becomes the standard labor market condition. Clearly, the opportunity cost of leisure is higher in the present model with both LTV and PTI constraints than in the model with only the LTV constraint because an extra hour of work generates more income and relaxes the PTI constraint. Further, condition (21) depends explicitly on the interest rate R_t, so that the opportunity cost of leisure changes with monetary policy shocks. For instance, a monetary contraction, by raising R_t, reduces the opportunity cost of leisure, resulting in a reduction in the borrowers' labor supply. This is important for the propagation of monetary shocks in the economy and the dynamics of the model relative to models with only collateral frictions. Equation (22) sets the marginal utility of non-durable consumption equal to the shadow value of durables. The latter has three components: i) the direct marginal utility of one unit of the durable in period t, ii) the discounted expected utility from selling one unit of the durable and using the proceeds to expand future non-durable consumption, and iii) the marginal utility of relaxing the collateral constraint through the purchase of an additional unit of the durable. Notice that this is proportional to ψ_t, which measures the tightness of the collateral constraint, and to $(1 - F_e(\bar{e}_t))$, the fraction of borrowers with a binding collateral constraint. When the LTV constraint is not binding, both of these terms are equal to zero and the shadow value of one unit of durables reduces to the sum of the first two terms.

Equation (23) is the Euler equation for debt. The shadow value of borrowing, ψ_t, is the extra utility that would result from borrowing an extra dollar, using it to increase current consumption and reducing consumption next period by an appropriate amount. When the debt constraint, $b_t \leq \bar{b}_t$, is binding, $\psi_t > 0$ and at the constrained optimum; the marginal utility of current consumption is greater than the discounted expected marginal utility of future consumption. When the debt constraint is not binding, $\psi_t = 0$, and this condition reduces to the standard Euler equation.

Equation (22) can be rewritten to equate the marginal rate of substitution between durables and non-durables, $U_{d,t}/U_{c,t}$, to the user cost of durables, Z_t, given by

$$Z_t = q_t[1 - \psi_t(\frac{\kappa^{ltv}(1-\delta)E_t\{\pi_{d,t+1}\}}{R_t}(1 - F_e(\bar{e}_t)))] - \beta(1-\delta)E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}q_{t+1}\right\}. \tag{24}$$

Log linearizing equation (24) around the deterministic steady state we obtain

$$\begin{aligned} \hat{z}_t = \bar{Z}^{-1}(1-\delta)[&\Upsilon_1\hat{q}_t - \beta E_t\hat{q}_{t+1} + \gamma\hat{R}_{r,t} + (\gamma-\beta)(((1-\kappa^{ltv}) + \kappa^{ltv}F(\bar{e}))\hat{\psi}_t - \hat{\zeta}_t) \\ &- \gamma(\gamma-\beta)\kappa^{ltv}[1 - F_e(\bar{e})]\hat{R}_t], \end{aligned} \tag{25}$$

in which

$$\begin{aligned} \Upsilon_1 &= \frac{(1 - \gamma\kappa^{ltv}(1-\delta)(\gamma-\beta)[1 - F_e(\bar{e})])}{1-\delta} \\ \hat{\zeta} &= E_t(\kappa^{ltv}(1 - F_e(\bar{e}) - \pi_{c,t+1}) \\ \bar{Z} &= 1 - (1-\delta)(\beta + \gamma\kappa^{ltv}(\gamma-\beta)[1 - F_e(\bar{e})]) \\ \hat{R}_{r,t} &= \hat{R}_t - E_t[\hat{\pi}_{c,t+1}], \end{aligned}$$

where $\bar{Z}$ is the steady-state value of the user cost and $\hat{R}_{r,t}$ is the real interest rate steady-state deviation in units of non-durables.

It is clear from equation (25) that the user cost of durables rises with changes in the relative price of durables, q_t, the shadow value of borrowing, ψ_t, and the nominal interest rate, R_t, and falls with expected future changes in the relative price of durables. With a perfect financial market, $\psi_t = 0$ and $\beta = \gamma$, and the user cost of durables reduces to the well-known result,

$$\hat{z}_t = (1 - \beta(1-\delta))^{-1}(1-\delta)(\frac{\hat{q}_t}{1-\delta} - \beta E_t\hat{q}_{t+1} + \beta\hat{R}_{r,t}). \tag{26}$$

5.3 The Savers

The savers are owners of the monopolistic competitive firms that produce the intermediate goods, and in each period, they collect the monopolistic profits. Like borrowers, they consume, work and earn wages, and receive transfers from government. The representative saver is infinitely-lived and seeks to maximize expected discounted lifetime utility,

$$E_0\{\sum_{t=0}^{\infty}\gamma^t(log(\tilde{X}_t) - \frac{\nu\tilde{N}_t^{1+\varphi}}{1+\varphi})\}, \tag{27}$$

subject to

$$\tilde{C}_t + q_t\tilde{I}_t + R_{t-1}\frac{\tilde{b}_{t-1}}{\pi_{c,t}} = \tilde{b}_t + \frac{\tilde{W}_t}{P_{c,t}}\tilde{N}_t + \frac{\tilde{T}_t}{P_{c,t}} + \frac{\tilde{\Gamma}_{c,t}}{1-\omega} + \frac{q_t\tilde{\Gamma}_{d,t}}{1-\omega}, \tag{28}$$

where a ($\tilde{}$) over a variable denotes saver quantities, γ is the saver's discount factor such that $\gamma > \beta$, $\tilde{b}_t$ is end-of-period t real debt (credit), and $\tilde{\Gamma}_{j,t}$, $j=(c,\ d)$ are aggregate nominal profits from the monopolistic competitive firms.

The saver chooses $\{\tilde{N}_t, \tilde{C}_t, \tilde{b}_t, \tilde{D}_t\}$ to maximizes equation (27) subject to equation (28). The optimality conditions for this problem are

$$\frac{-\tilde{U}_{n,t}}{\tilde{U}_{c,t}} = \frac{\tilde{W}_t}{P_{c,t}} \tag{29}$$

$$\tilde{U}_{c,t} = \gamma E\{\frac{\tilde{U}_{c,t+1}}{\tilde{\pi}_{c,t+1}}R_t\} \tag{30}$$

$$q_t = \frac{\tilde{U}_{d,t}}{\tilde{U}_{c,t}} + \gamma(1-\delta)E\{\frac{\tilde{U}_{c,t+1}}{\tilde{U}_{c,t}}q_{t+1}\}. \tag{31}$$

These are conventional optimality conditions for perfect financial markets, as savers are not subject to any financial frictions.

5.4 Firms

There are intermediate and final goods producers in both the durable and non-durable sectors.

5.4.1 Final Goods Producers

In each sector, perfectly competitive firms produce the final good $Y_{j,t}$ $\ j = c, d$. The production function in the j-th sector is given by

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon_j - 1}{\varepsilon_j}}(i)di\right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}, \tag{32}$$

where $Y_{j,t}(i)$ is the intermediate good produced by firm $i \ \epsilon\ [0,1]$ in sector j, and $\varepsilon_j > 1$ is the elasticity of substitution between intermediate goods in sector j.

5.4.2 Production and Pricing of Intermediate Goods

The production of intermediate goods is undertaken in both sectors of the economy, each populated by a continuum of monopolistic competitive firms, each of which has monopolistic power over the production of its own variety and price. In setting its price, each firm faces quadratic adjustment costs proportional to output, as in Monacelli (2009). We assume the following linear production function for firm i:

$$Y_{j,t}(i) = N_{j,t}(i) - F_j, \tag{33}$$

where $N_{j,t}(i)$ the demand for labor by firm i in sector j, and $F_j \geq 0$ is a fixed cost of production reflecting cost of adjusting the labor input, as in Christiano et al. (1999) and Notarpierto and Siviero (2015). The problem of each firm i in sector j is to choose the sequence of employment and prices, $\{\, N_{j,t}(i), P_{j,t}(i)\}$, in order to maximize expected discounted profits. In a symmetric equilibrium, all firms choose the same price and employ the same amount of labor in each sector. In this case, the optimality condition of the intermediate producer in sector $j = c, d$ is the same as in Sterk (2010):

$$((1-\epsilon_j) + \epsilon_j mc_{j,t}) = \vartheta_j(\pi_{j,t}-1)\pi_{j,t} - \gamma\vartheta_j E\{\frac{\Lambda_{j,t+1}}{\Lambda_{j,t}}\frac{P_{j,t+1}}{P_{j,t}}\frac{Y_{j,t+1}}{Y_{j,t}}(\pi_{j,t+1}-1)\pi_{j,t+1}\}, \tag{34}$$

where $\Lambda_{j,t} = \gamma\tilde{\lambda}_{t+1}/\tilde{\lambda}_t$ is the savers' stochastic discount factor, in which $\tilde{\lambda}_t$ is the savers' marginal utility of nominal income, and the parameter ϑ_j measures the degree of sectoral nominal price stickiness. In particular, when $\vartheta_j = 0$, prices are fully flexible in sector j. Also $\pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}}$ is the gross inflation rate in sector j, and

$$mc_{j,t} = \frac{W_t}{P_{j,t}}$$

is the real marginal cost of production in sector j. Finally, sectoral inflation and relative prices are related by

$$\frac{\pi_{d,t}}{\pi_{c,t}} = \frac{q_t}{q_{t-1}}. \tag{35}$$

5.5 Equilibrium

A competitive equilibrium for this model is a sequence of variables, $N_t, N_{j,t}, \tilde{N}_t, \tilde{N}_{j,t}, b_t, \tilde{b}_t\; D_t, C_t, \tilde{C}_t, \tilde{D}_t$, for $j = c$, d and prices $P_{c,t}, P_{d,t}, \frac{W_t}{P_{c,t}}, \frac{\tilde{W}_t}{P_{c,t}}$, R_t, $q_t, \pi_{c,t}, \pi_{d,t}$, such that

1. Given prices, $\{N_t, b_t, D_t, C_t\}$ solves the borrowers' problem.
2. Given prices, $\{\tilde{N}_t, \tilde{b}_t, \tilde{D}_t, \tilde{C}_t\}$ solves the savers' problem.
3. Given inflation, R_t satisfies the monetary policy rule.
4. The goods, labor and credit markets clear.

5.6 Market Clearing Conditions

Equilibrium in the goods market requires that the production of the final goods be allocated to total households' expenditures and to resource costs originating from the adjustment of prices,

$$Y_{c,t} = \omega C_t + (1-\omega)\tilde{C}_t + \frac{\vartheta_c}{2}(\pi_{c,t}-1)^2 Y_{c,t} \tag{36}$$

$$Y_{d,t} = \omega I_t + (1-\omega)\tilde{I}_t + \frac{\vartheta_d}{2}(\pi_{d,t}-1)^2 Y_{d,t}. \tag{37}$$

Equilibrium in the credit and labor markets requires, respectively,

$$b_t + \tilde{b}_t = 0 \text{ and} \tag{38}$$

$$N_{c,t} + N_{d,t} = \omega N_t + (1-\omega)\tilde{N}_t. \tag{39}$$

5.7 Monetary Policy

Monetary policy follows the simple Taylor-type rule,

$$\frac{R_t}{R} = (\frac{\tilde{\pi}_t}{\tilde{\pi}})^{\phi_\pi} \exp(\varepsilon_t) \qquad \phi_\pi > 0 \tag{40}$$

$$\tilde{\pi}_t = \pi_{c,t}^{1-\mu}\pi_{d,t}^{\mu} \tag{41}$$

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t, \tag{42}$$

where $\tilde{\pi}_t = \pi_{c,t}^{1-\mu}\pi_{d,t}^{\mu}$ is a composite inflation index, R and $\tilde{\pi}$ are the steady-state levels of the gross nominal interest rate and inflation index, respectively, and ε_t is an exogenous monetary shock.

6 Calibration

In this section we calibrate the model to quarterly frequency, and unless otherwise specified, most parameters were drawn from Monacelli (2009). The parameter values are summarized in Table 2. We set the savers' discount factor γ to 0.995 and the borrowers' discount factor β is set at 0.985.[2] The steady-state gross quarterly interest rate, R, is pinned down by the savers' discount factor at 1.01. The quarterly depreciation rate for durable goods, δ, was set at 0.01. The elasticity of substitution between varieties, ε_j, was set to 6 in both sectors, $j = c$, d, which yields a steady-state mark-up of 20%. The parameters for nominal price stickiness were set according to $\vartheta_j = \frac{\theta(\varepsilon_j - 1)}{(1-\theta)(1-\gamma\theta)}$, where θ is the probability of a firm not resetting prices in the standard Calvo-Yun model. The non-durable price stickiness parameter ϑ_c was pinned down by setting $\theta = 0.75$, which implies an average frequency of non-durable price adjustment, $1/(1-\theta)$, of 4 quarters. In the main experiment we wanted to measure the reaction of the endogenous variables to a monetary tightening when durable prices are fully flexible. Accordingly, we set $\vartheta_d = 0$, that is, 1 quarter durable price stickiness. For the sensitivity analysis the value of ϑ_d was changed to allow for 1.5, 2 and 3 quarters durable price stickiness. The elasticity of substitution between durables and non-durables η was set at 1, and the share of durables in total consumption μ was set at 0.20. The preference parameter ν was set such that the borrowers' and savers' labor supplies are equal to 1/3 of their total normalized time. We

[2]The steady-state value of ψ_t was determined from the borrowers' pseudo-Euler equation, as $\psi = (\gamma - \beta)/\gamma$.

Table 2: Parameter Values

Parameter	Description	Both Constraints	LTV Constraint
β	Borrowers' discount factor	.985	.985
γ	Savers' discount factor	0.995	0.995
δ	Depreciation rate of durables	0.01	0.01
$\varepsilon_c, \varepsilon_d$	El. of sub. between varieties	6	6
η	El. of sub. b/w dur. and non-dur.	1	1
μ	Share of durable consumption	0.20	0.20
α	Durable inflation share	0.20	0.20
ω	Fraction of borrowers	0.54	0.54
ϕ_π	Coeff. of inflation in Taylor rule	1.5	1.5
ρ	Persistence of monetary shock	0.48	0.48
φ	Inv. El. of labor supply	1	1
κ^{ltv}	LTV ratio	0.75	0.75
κ^{pti}	PTI limit	0.28	-
τ	Debt cost parameter	0.024	-
σ_e	Income dispersion	0.411	-
ε_{bc}, ε_{sc}	Habit parameters: non-durables	0.50, 0.30	0.50, 0.30
ε_{bd}, ε_{sd}	Habit parameters: durables	0.60, 0.40	0.60, 0.40

set the fraction of borrowers $\omega = 0.54$, an intermediate value between Monacelli's value of 0.50 and the value of 0.61 estimated by Justiniano et al. (2015) using data from the Survey of Consumer Finances. The inverse elasticity of the labor supply φ was set at 1. The coefficient on inflation in the Taylor rule ϕ_π was set at 1.5, and the persistence parameter ρ of the monetary shock was set at 0.5. Following Greenwald (2018), we set the LTV and PTI ratios κ^{ltv} and κ^{pti} at 0.75 and 0.28, respectively, and the debt cost parameter τ at 0.024. Similarly, the values of the borrowers' labor income shock $e_{i,t}$ were simulated from the log-normal distribution,

$$\log(e_{it}) \sim N(\frac{-\sigma_e^2}{2}, \sigma_e), \tag{43}$$

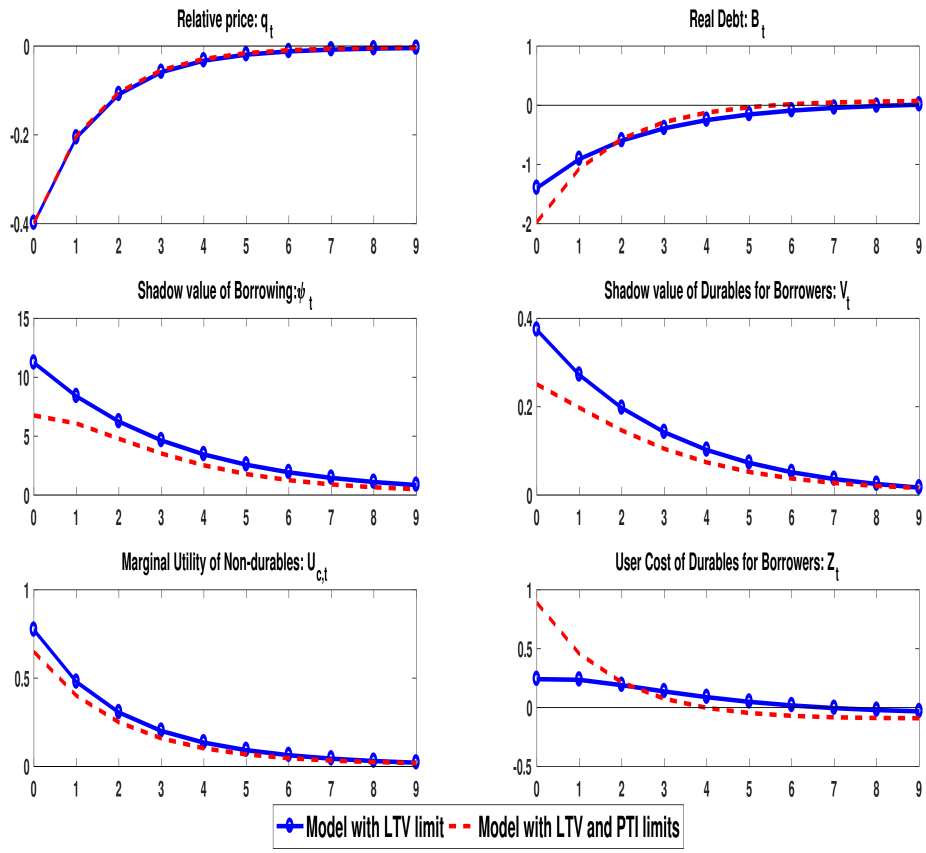
by setting the borrowers' income dispersion, σ_e, to 0.411. Finally, we carried out an additional simulation by assuming habits in the consumption of both non-durables and durables. In that case, discussed below, the habit persistence parameters for borrowers, ε_{bc}, ε_{bd}, and for savers, ε_{sc}, ε_{sd}, were chosen such that the the impact response of durable spending to the monetary shock was about four to five times the response of non-durable spending.

Given these parameter values, we simulate a monetary contraction by a positive shock to ε_t in the monetary policy rule, which raises the nominal interest rate by 25 basis points. As noted above, our main focus in the simulations is the case with fully flexible durable prices by setting $\vartheta_d = 0$.

6.1 Results

First, we present some useful results that clarify the way in which the present model with two credit frictions resolves the co-movement problem relative to the model with only an LTV friction. Figure 2 plots the impulse response functions (IRFs) for each model of the relative price of durables, the real debt, the shadow value of borrowing, the shadow value of durables, the marginal utility from durable consumption, and the borrowers' user cost of durable consumption.

Figure 2 Comparison of the models with two frictions and one friction



Note: Impulse responses are percentage deviations from the steady state in the relative price of durables, q_t; real debt, b_t; shadow value of borrowing, ψ_t; shadow value of durables for borrowers, V_t; marginal utility of non-durables, $U_{c,t}$; and the user cost of durables for borrowers, Z_t.

These results are consistent with the two models' predictions. Following the increase in the nominal interest rate, the relative price of durables falls by the same percentage in the two models, thus causing similar tightening in the two LTV constraints. However, when the interest rate increases, the PTI constraint binds more often, and as a result the percentage fall in the borrowers' real debt is greater in the model with the two financial frictions than in the model with an LTV friction alone. Since the borrowers are constrained by their labor income, they can buy more valuable durables than those allowed by their PTI limit by paying dollar-for-dollar for the difference. This implies that the shadow value of borrowing in the economy with both financial frictions, ψ_t^2, rises by less on impact than in the economy with only the LTV friction, ψ_t^1: $\psi_t^2 < \psi_t^1$, as shown in Figure 2. In turn, this means that $V_t^2 < V_t^1$ for the corresponding shadow values of durables. To establish this formally, let $V_t^2 \equiv U_{c,t} q_t$ denote the shadow value of durables in the model with the two frictions, and use the optimality condition (22) to obtain

$$V_t^2 = \frac{U_{d,t} + \beta(1-\delta)E_t V_{t+1}^2}{1 - \psi_t^2 (\frac{\kappa^{ltv}(1-\delta)E_t \pi_{d,t+1}}{R_t}(1 - F_e(\bar{e}_t))} \ . \tag{44}$$

The corresponding V_t^1 for the model with one friction is

$$V_t^1 = \frac{U_{d,t} + \beta(1-\delta)E_t V_{t+1}^1}{1 - \psi_t^1 (\frac{\kappa^{ltv}(1-\delta)E_t \pi_{d,t+1}}{R_t})}. \tag{45}$$

Since $\psi_t^2 < \psi_t^1$ and $(1 - F(\bar{e}_t)) < 1$, it follows that $V_t^2 < V_t^1$. Equivalently, borrowers tend to consume more non-durables than durables in the model with two frictions than the model with one friction. Consequently, the marginal utility of non-durable consumption $U_{c,t}$ is larger in the former case than in the later. This result is useful because, as noted by Sterk (2010), a high value for V_t^1 makes the co-movement problem harder to resolve with collateral frictions than no frictions at all.

Also, as a consequence of these results the user cost of durables with two credit frictions tends to be larger than the case with a collateral friction, as shown in Figure 2. Formally, writing the user cost Z_t in equation (24) explicitly in terms of ψ_t and V_{t+1} we have

$$Z_t \quad = \quad q_t \left[1 - \psi_t \left(\frac{\kappa^{ltv}(1-\delta)\mathbb{E}_t\{\pi_{d,t+1}\}}{R_t} (1 - F_e(\bar{e}_t)) \right) \right] - \beta(1-\delta)\mathbb{E}_t \left\{ \frac{V_{t+1}}{U_{c,t}} \right\}, \tag{46}$$

from which it can be easily seen that $Z_t^2 > Z_t^1$.

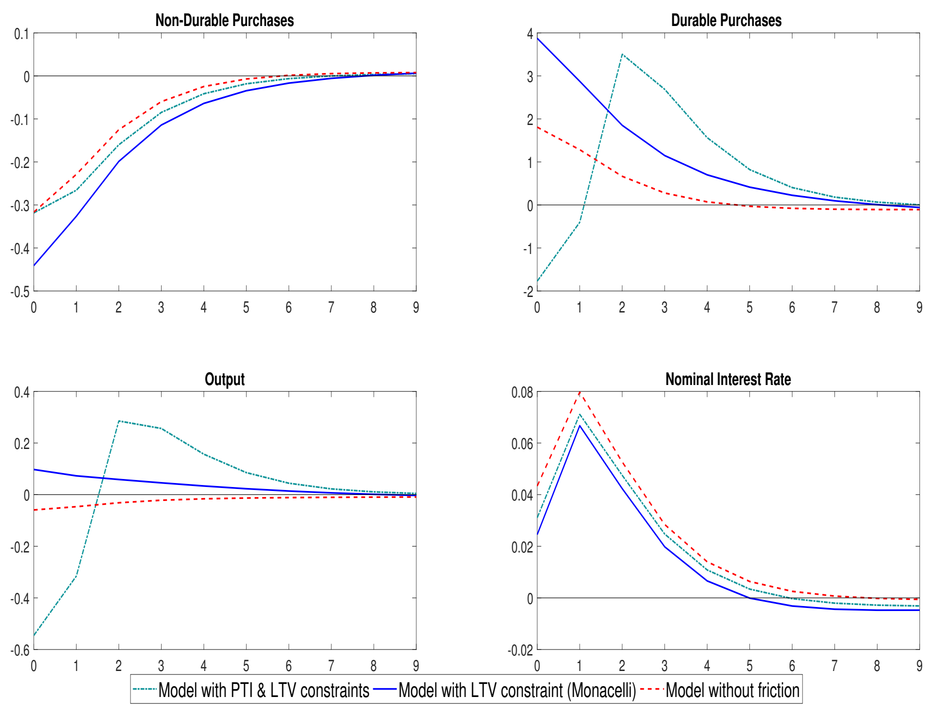
6.2 Improving the Predictions of the Model

Given the results above, we can now show that two-sector DSGE model with LTV and PTI frictions provides an adequate resolution to the co-movement problem, even when durable prices are fully flexible. Further, the monetary contraction results in a fall in aggregate output, and a rise in nominal interest, as is the case with the empirical evidence.

Consider the reaction of borrowers and lenders to the interest rate hike. Given flexible durable prices, the monetary tightening reduces the relative price of durables and results in a reduction in the consumption of non-durables for both borrowers and lenders. Thus, the aggregate consumption of non-durables falls. At the same time, the monetary contraction increases $\bar{e}_t$, the threshold value of the borrowers' income shock, and thus the fraction of borrowers with a binding PTI constraint. This creates tighter credit conditions, which reduce the amount of real debt that the borrowers can obtain. As a result, they cut back on their purchases of durables. Because $Z_t^2 > Z_t^1$, the fall in durable purchases is greater than the level implied by the LTV constraint alone. Also, because $V_t^2 < V_t^1$, borrowers have less of an incentive to substitute durables for non-durables, which prevents durable purchases from rising and non-durable purchases from declining further, and this makes it easier to resolve the co-movement problem. On the other hand, lenders are now forced to reduce their lending activity; and since they do not face any frictions, they spend their extra savings to smooth their consumption by purchasing more durables goods. However, the increase in their durables purchases is not large enough to offset the large reduction of durable purchases by borrowers. As a result, aggregate purchases of durables also decline. With both aggregate non-durables and durable purchases falling after the monetary tightening, aggregate output also declines, as is the case in the data. With flexible durable prices, the nominal interest rate also increases, consistent with conventional wisdom and the empirical evidence.

Figure 3 shows these results and compares the predictions of the present model having two credit frictions with the predictions of both the model having an LTV friction, and the model having no credit frictions. As shown, unlike the model with two credit frictions, non-durable and durable purchases move in the opposite direction in the model with the LTV constraint and the model with no credit frictions. Clearly, at fully flexible durable prices, the co-movement problem remains unresolved in these two models; and in the model with the LTV friction, the reaction of durables is much greater than in the model with no frictions. As a result, output expands counterfactually in the model with the LTV constraint despite the monetary contraction.

Figure 3 Comparison of the models: two frictions, one friction, and no frictions, at fully flexible durable prices



Note: Impulse responses are percentage deviations from the steady state in non-durable purchases, durable purchases, output and the nominal interest rate.

6.3 Adding Habits in Consumption

The empirical evidence suggests that the reduction of durable purchases to a monetary contraction is about four to five times larger than the reaction of non-durable purchases. Further, the adjustment of both durables and non-durables to the steady state is not immediate. The impulse responses in Figure 3 above show that on impact the decline in durables in the model with two credit frictions is larger than what the empirical evidence suggests. Also durable consumption adjusts sharply within one period following the monetary shock.

In this subsection, we add habits in non-durable and durable consumption in order to improve further the predictions of the model and bring them closer to the empirical evidence, including the adjustment of durable consumption after the monetary shock. Among others, Iacoviello and Neri (2010) have considered habits over non-durable consumption alone, whereas Guerrieri and Iacoviello (2017) have added habits over both types of consumption goods. In our setting, with habit over consumption the utility functions of borrowers and savers depend on both current and past consumption, through their respective composite consumption indices:

$$X_t = ((1-\mu)^{\frac{1}{\eta}}(C_t - \varepsilon_{bc}C_{t-1})^{\frac{\eta-1}{\eta}} + (\mu)^{\frac{1}{\eta}}(D_t - \varepsilon_{bd}D_{t-1})^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} \tag{47}$$

and

$$\tilde{X}_t = ((1-\mu)^{\frac{1}{\eta}}(\tilde{C}_t - \varepsilon_{sc}\tilde{C}_{t-1})^{\frac{\eta-1}{\eta}} + (\mu)^{\frac{1}{\eta}}(\tilde{D}_t - \varepsilon_{sd}\tilde{D}_{t-1})^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}}, \tag{48}$$

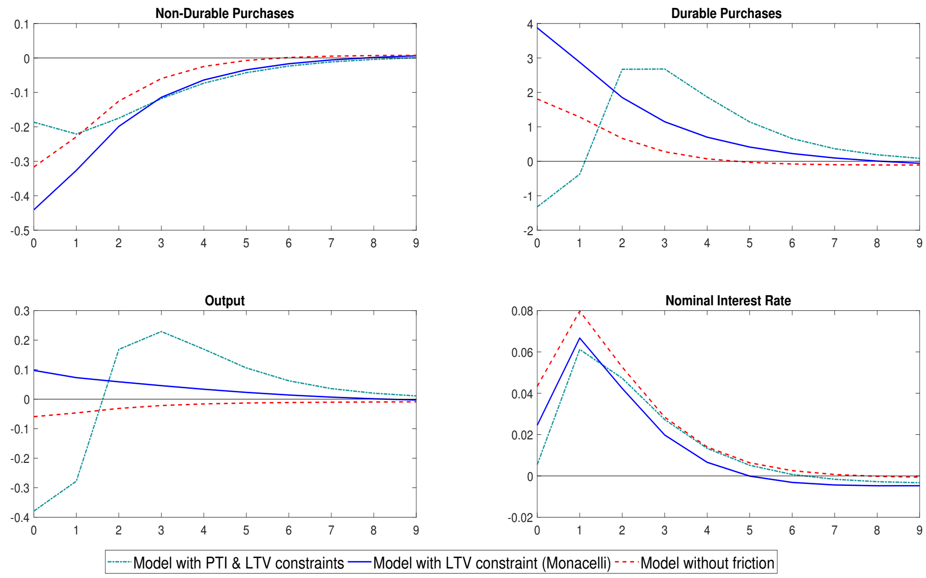
where ε_{bc} and ε_{bd} are the habit parameters of borrowers for non-durable and durable consumption, respectively, and similarly, ε_{sc} and ε_{sd} are the habit parameters of savers.

In the literature, habit parameter estimates vary across different models, data sets, countries and estimation methods, with microeconomic estimates being much smaller than macroeconomic estimates: Havranek, Rusnak and Sokolova (2017). For our simulations we experimented with different parameter values and chose those that produced the best possible results for the reaction of durable and non-durable consumption, output, and the nominal interest rate to the monetary shock. These values were $\varepsilon_{bc} = 0.50$ and $\varepsilon_{bd} = 0.60$ for borrowers, and $\varepsilon_{sc} = 0.30$ and $\varepsilon_{sd} = 0.40$ for savers. We set the habit parameters higher for borrowers than savers because the former are constrained and need stronger habits to smooth their consumption over time. Also, because durable spending is lumpy and thus more difficult to smooth, we set the habit parameters at higher values for durables than non-durables for both borrowers and savers.

The simulation results with habits are shown in Figure 4. As indicated, the impulse responses with habits in consumption provide a more realistic co-movement between durable and non-durable spending. Now, the impact decline in durable consumption is about four times larger than the decline in non-durable consumption, which is closer to the stylized facts. Also, the adjustment of durable consumption is less steep and stays longer below the steady state than in the model with no habits.[3] Also, the adjustment of output below the steady state seems more prolonged, and the nominal interest rate still reacts positively to the monetary contraction.

[3] Unlike the adjustment of non-durable consumption, it is more difficult to smooth out the adjustment of durable consumption because habits and durability work in the opposite direction: durability makes spending on durables lumpy, whereas habits smooth it: Ferson and Constantinides (1991).

Figure 4 Impulse responses with habits over consumption



Note: Impulse responses are percentage deviations from the steady state in non-durable purchases, durable purchases, output and the nominal interest rate.

7 Sensitivity Analysis

In this section we consider the sensitivity of the quantitative results to changes in the degree of durable price rigidity across the three models.

7.1 Durable Price Rigidity

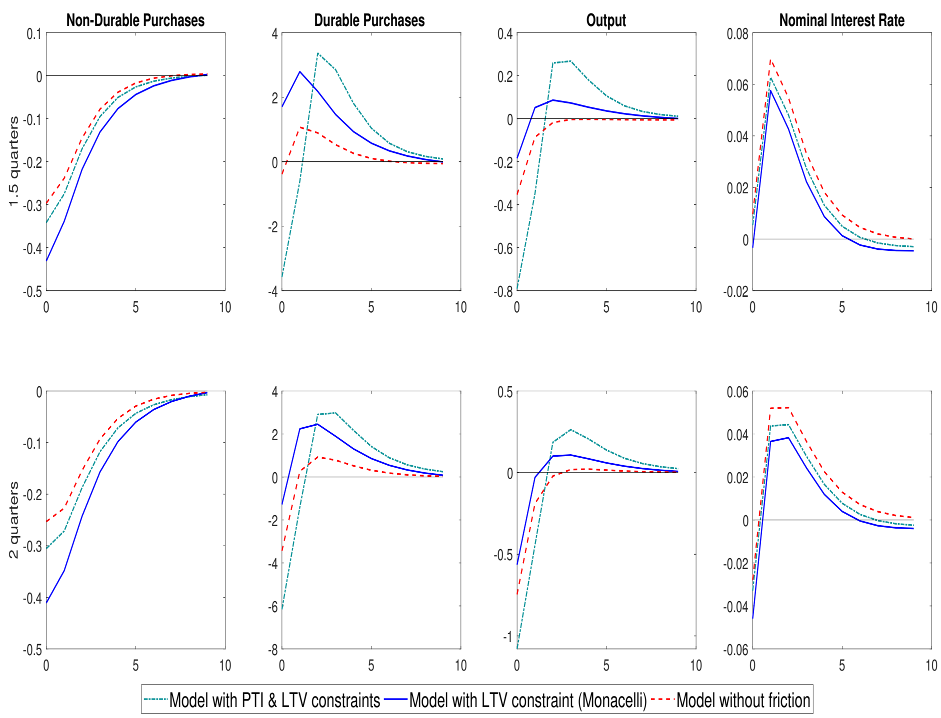
We examine the sensitivity of the simulation results to successive increases in the degree of durable price stickiness at 1.5 and 2 quarters, while keeping non-durable price stickiness at 4 quarters. We compare the numerical results for the three models: the model with the two financial frictions, the model with an LTV friction, and the frictionless model. For ease of comparison with the existing models we drop habits from the models for these simulations. More important, since the one friction and the frictionless models are nested versions of the general two frictions model, any difference in the results will be due to the additional features of the general model rather than to a different calibration.

Figure 5 shows the impulse response functions of non-durable and durable purchases, output and the nominal interest rate at different degrees of durable price stickiness, following the monetary contraction. As shown, at 1.5 quarters of durable price stickiness, the two frictions model resolves the co-movement problem whereas the the model with only the LTV constraint does not, and in this regard, the latter does worse than the model with no frictions. Also, the negative response of output is more pronounced in the model with two frictions than the other two models, where the output response is very weak. Only the interest rate response is similar across the three models.

At 2-quarters durable price stickiness, the results are consistent with those reported in Monacelli (2009) and Sterk (2010). Because durable purchases are higher in the model with the LTV friction than the one with no friction at all, the co-movement problem becomes more difficult to solve with LTV friction,the as pointed out by Sterk.

In contrast, the model with both the PTI and LTV constraints avoids the Sterk criticism. Also, it does better than the other two models in that it resolves co-movement problem for all levels of durable price stickiness, including the case of fully flexible durable prices. Only the nominal interest rate declines at 2-quarters durable price stickiness despite the monetary contraction. This result is common to sticky price models at high levels of durable price stickiness, as in this case the nominal interest moves one-to-one with expected durable price deflation.

Figure 5 Comparison of the models at different levels of durable price stickiness



Note: The rows correspond to different degrees of durable price stickiness. Impulse responses are percentage deviation from the steady state for non-durable purchases, durable purchases, output and the nominal interest rate.

8 Conclusions

In this paper we considered the interaction between monetary policy and credit frictions in the two-sector DSGE sticky price model with durable and non-durable goods, and including two credit constraints on borrowers: collateral and income constraints. The two credit constraints are randomly combined by shocks to the borrowers' labor income and determine jointly the amount of collateralized loans that borrowers can obtain. We examined the predictions of the model following a monetary contraction, when durable prices are fully flexible.

In this model, an increase in the nominal interest rate increases the threshold level of the labor shock and results in a greater fraction of borrowers with a binding PTI constraint. As a result there is a sharp decline in the amount of collateralized loans obtained by borrowers, who cut back on their purchases of durables, along with the relatively more expensive non-durables. The lenders decrease their purchases of non-durables but increase their purchases of durables by an amount less than the fall in durable purchases by borrowers. As a consequence, aggregate durable purchases fall and co-move positively with non-durables as in the data, even when durable prices are fully flexible. Aggregate output also declines and the nominal interest rate rises consistent with the empirical evidence.

Overall these results imply that financial frictions on the demand side of the economy are still important in matching the predictions of the sticky price model with the empirical evidence, even with flexible durable prices. Also, they complement to a large extent the results from other strands of literature that rely on supply side frictions or non-separable preferences.

The results in the present paper generate questions for future research. One would be to extend the model with different types of borrowers based on their initial wealth, and examine how financial frictions and monetary policy affect the consumption, employment and credit growth of each type of borrower. An empirical investigation of these issues is also feasible based on extensive information of loan-based panel data for different types of borrowers. Another extension would be to examine in detail the welfare implications of LTV and PTI frictions, and the role of these frictions in designing better macroprudential policy rules. A third extension would be to study the effects of LTV and PTI constraints in the context of other models with (1) non-separable preferences and (2) production functions with capital accumulation.

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