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Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment*

by

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Abstract

This paper develops a micro-founded general equilibrium model of payments to study the impact of a central bank digital currency (CBDC) on intermediation of private banks. If banks have market power in the deposit market, a CBDC can enhance competition, raising the deposit rate, expanding intermediation, and increasing output. A calibration to the United States economy suggests that a CBDC can raise bank lending by 1.96% and output by 0.21%. These “crowding-in” effects remain robust, albeit with smaller magnitudes, after taking into account endogenous bank entry. We also assess the role of a non-interest bearing CBDC as the use of cash declines.

Topics: Digital currencies and fintech; Monetary policy; Monetary policy framework; Market structure and pricing

JEL codes: E50, E58
1 Introduction

Many central banks are considering issuing central bank digital currencies (CBDCs), a digital form of central bank money that can be used for retail payments. The Bank for International Settlements surveyed 65 central banks in 2020, covering 72% of the world population and 91% of the world output. Of these central banks, 86% are engaging in work regarding a CBDC; 60% have started experiments or proofs-of-concept for a CBDC; and 14% have moved forward to development and pilot arrangements (see Boar and Wehrli 2021).

In the debate around the impact of introducing a CBDC, one frequently raised concern is that, by competing with bank deposits as a payment instrument, a CBDC could increase commercial banks’ funding costs and reduce bank deposits and loans, leading to bank disintermediation. For example, Mancini-Griffoli et al. (2018) caution that a CBDC would force banks to increase their deposit interest rates, and banks would respond by increasing lending rates at the cost of loan demand. The 2018 report by the Committee on Payments and Market Infrastructures of the Bank for International Settlements raises the same concern.

This paper develops a general equilibrium model of banking and payments to assess this disintermediation concern, both theoretically and quantitatively. In this model, banks act as intermediaries, issuing loans to entrepreneurs and creating deposits, which households can use as a means of payment to trade consumption good. Besides deposits, households have access to two other payment instruments: cash and CBDC. Cash and deposits differ in the types of exchange they can facilitate. For example, cash cannot be used in online transactions while deposits can be used via debit/credit cards or electronic transfers. A CBDC, however, is a perfect substitute for deposits in terms of payment functions and bears an interest set by the central bank.

Our main finding is that introducing a CBDC does not necessarily lead to disintermediation if banks have market power in the deposit market. In this case, the impact of a CBDC is non-monotonic in its interest rate. It expands bank intermediation if its interest rate lies in an intermediate range and causes disintermediation only if its interest rate is set too high. The main mechanism through which a CBDC “crowds in” bank intermediation works as follows. In an imperfectly competitive deposit market, banks restrain the deposit supply to keep the deposit interest rate below the level under perfect competition. A CBDC offers an outside option to depositors and sets an interest rate floor for bank deposits. This floor limits the reduction in the deposit rate and reduces commercial banks’ incentive to restrain the deposit supply. If the CBDC rate is not too high, banks supply more deposits, reduce the loan rate, and expand lending.
Interestingly, a CBDC can have a positive effect on deposits, loans, and output even if it has zero market share. The mere existence of a CBDC as an outside option forces banks to match the CBDC rate and create more deposits and loans.  

A policy implication is that one should assess the effectiveness of a CBDC based on its equilibrium effect on deposits or the deposit rate instead of its usage.

Calibrating our model to the United States economy, we find that a CBDC expands bank intermediation if its interest rate is between 0.30% and 1.49%. At the maximum, it can increase loans and deposits by 1.96% and the total output by 0.21%. The CBDC leads to disintermediation, however, if its rate exceeds 1.49%. To break even, banks are forced to raise the lending rate to compensate for the interest paid on deposits. As a result, both loans and deposits decrease.  

Finally, a non-interest-bearing CBDC can still restrict banks’ market power and improve intermediation if the use of cash declines. Without a CBDC, banks would limit intermediation and pay negative deposit rates. We have also extended the model to incorporate an imperfectly competitive loan market and endogenous bank entry. A CBDC can still promote bank intermediation, albeit by a smaller magnitude relative to that in the benchmark model.

Our study highlights the role of banks’ market power in determining the effects of a CBDC on bank intermediation. The study is closely related to two concurrent papers. Keister and Sanches (2019) focus on the welfare implications of an interest-bearing CBDC when the banking sector is perfectly competitive. They find that, while the CBDC always crowds out bank intermediation, social welfare can still increase when the efficiency in exchange significantly improves, especially when financial frictions are not very severe.  

In contrast, Andolfatto (2020) studies the effect of a CBDC on banking when there is a monopolistic bank. Using an overlapping generations model, he shows that a CBDC could compel the bank to increase the deposit rate, leading to an increase in bank deposits and financial inclusion. Under the assumption that the central bank offers a lending facility and a deposit facility at the same policy rate, the bank’s deposit and loan decisions are made separately.

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1This insight is closely related to that of Lagos and Zhang (2019, 2020), who show that monetary policies discipline the equilibrium outcome by setting the value of the outside option and can be effective even if the use of money approaches zero. Rocheteau et al. (2018) show a related message that money holdings can limit the bank’s market power on the lending side.

2As suggested by Meaning et al. (2018), an important research question regarding a CBDC is “... at which point do the benefits of a new competitive force for the banking sector get outweighed by the negative consequences of the central bank disintermediating a large part of banks business models?” Our calibration exercise allows us to pin down the interest of a CBDC at which its effect on bank intermediation reverses from positive to negative.

3Using a related model, Williamson (2020a) shows that introducing a CBDC to compete with bank deposits can raise welfare by freeing up scarce collateral for banks that are subject to limited commitment.
The loan rate and quantity are fully determined by the policy rate and are not affected by the CBDC.

Compared to these papers, our framework is more suitable for quantifying the effects of a CBDC and accommodates various design choices as the payment landscape evolves. First, our model captures a complete spectrum of competitiveness. If the number of banks is one, the banking sector is monopolistic, as in Andolfatto (2020). If this number tends to infinity, the banking sector is perfectly competitive as in Keister and Sanches (2019). We use data to discipline the level of competitiveness, which is crucial for quantifying the effects of a CBDC. Second, we explicitly model cash, deposits and a CBDC as three imperfectly substitutable payment instruments that facilitate different types of transactions. This allows us to discuss the design of a CBDC in terms of its acceptability and its effect when the payment landscape evolves, for example, when the use of cash declines.

The economic literature on CBDCs is just emerging, with several lines of research complementary to our work. A number of studies focuses on the role of CBDCs as a monetary policy tool: Barrdear and Kehmho (2016) evaluate the macroeconomic consequences of a CBDC in a dynamic stochastic general equilibrium model. Davoodalhosseini (2018) explores the usage of a CBDC for balance-contingent transfers; Dong and Xiao (2019) examine its role in implementing negative policy rates; Brunnermeier and Niepelt (2019) and Niepelt (2020) derive conditions under which introducing a CBDC has no effects on macroeconomic outcomes, including bank intermediation. Jiang and Zhu (2021) discuss how the interest on a CBDC and the interest on reserves interact as two separate policy tools. Another line of research studies the financial stability implications of a CBDC such as the risk-taking behavior of banks and bank runs. Recent works by Chiu et. al (2020), Fernández-Villaverde et al. (2020), Schilling et al. (2020), Keister and Monnet (2020), Monnet et. al (2020), and Williamson (2020) have made some important progress. Our paper abstracts from these issues and focuses on the effects of a CBDC on bank intermediation in terms of deposit and loan quantities. For research related to the design of a CBDC, see Agur et al. (2020) and Wang (2020). For policy discussions on CBDCs, see Fung and Halaburda (2016); Engert and Fung (2017); Mancini-Grieffoli et al. (2018); Chapman and Wilkins (2019); Davoodalhosseini and Rivadenyra (2020); Davoodalhosseini et al. (2020); and Kahn et al. (2020).

More broadly, our paper contributes to the monetary theory literature by developing a

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4Our paper is also related to the literature on private digital currencies and currency competition; see Chiu and Koepl (2019); Fernández-Villaverde and Sanches (2019); Schilling and Uhlig (2019); Zhu and Hendry (2019); Benigno et al. (2020); Choi and Rocheteau (2020); and Zhou (2020). For a complete introduction to the issues in digital currencies, see Schar and Berentsen (2020).
tractable model with imperfect competition in inside money creation.\textsuperscript{5} It is also connected to the literature on how a bank’s market power affects monetary policy transmission. Dreschler et al. (2017) provide empirical support for banks’ market power in deposit markets and propose a transmission channel accordingly: since a lower nominal interest rate makes cash cheaper to use relative to deposits, banks are compelled to lower the spread between the nominal interest rate and the deposit rate. The effect of a lower nominal interest rate plays a similar role as a higher interest on a CBDC: both policies reduce banks’ market power in the deposit market.\textsuperscript{6}

The rest of the paper is organized as follows. Section 2 describes the physical environment. Section 3 characterizes the equilibrium. Section 4 calibrates the model and assesses its quantitative implications. Section 5 discusses motivations and implementation of a CBDC. Section 6 concludes and provides some directions for future research. Appendix A provides omitted proofs. Extensions and further discussions are collected in the Online Appendix.

## 2 Environment

Our model is based on the framework of Lagos and Wright (2005). Time is discrete and continues from zero to infinity. There are four types of agents: a continuum of households with measure 2, a continuum of entrepreneurs with measure 1, a finite number of $N$ bankers (each running a bank), and the government. The discount factor from the current period to the next is $\beta \in (0,1)$. In each period $t$, agents interact sequentially in two stages: a frictional decentralized market (DM) and a Walrasian centralized market (CM). There are two perishable goods: $y$ in the DM and $x$ in the CM.

Households are divided into two permanent types, buyers and sellers, each with measure 1. In the DM, a buyer randomly meets a seller. The meeting probability is $\Omega \in (0,1]$ for both buyers and sellers. The buyer wants to consume $y$, which is produced by the seller. The buyer’s utility from consumption is $u(y)$ with $u'(0) = \infty$, $u' > 0$, and $u'' < 0$. The seller’s disutility from production is normalized to $y$. Let $y^*$ be the socially efficient DM


\textsuperscript{6} Using a variation of the model of Dreschler et al. (2017), Kurlat (2019) shows that banks’ market power raises the cost of inflation. Scharfstein and Sunderam (2016) propose a transmission channel based on banks’ market power in the loan market. As the nominal interest rate increases, banks reduce their markup due to lower demand for loans. Wang et. al (2020) estimate a structural banking model and show that the effect of banks’ market power in monetary policy transmission is sizable and comparable to that of bank capital regulations.
consumption, which solves \( u'(y^*) = 1 \). Households lack commitment and cannot enforce debt payment. As a result, the DM trade must be quid pro quo and buyers must use a means of payment to exchange for \( y \). We will discuss available means of payment later. The terms of trade are determined by buyers making take-it-or-leave-it offers. In the CM, both buyers and sellers work and consume \( x \). Their labor \( h \) is transformed into \( x \) one-for-one. The utility from consumption is \( U(x) \) with \( U'(0) = \infty \), \( U' > 0 \), and \( U'' < 0 \). Buyers’ and sellers’ preferences can be summarized respectively by the period utilities

\[
U_B(x, y, h) = u(y) + U(x) - h,
\]

\[
U_S(x, y, h) = -y + U(x) - h.
\]

Young entrepreneurs are born in the CM and become old and die in the next CM. Entrepreneurs cannot work in the CM and consume only when old. Young entrepreneurs are endowed with an investment opportunity that transforms \( x \) current CM goods to \( f(x) \) CM goods in the next period, where \( f'(0) = \infty \), \( f'(\infty) = 0 \), \( f' > 0 \), and \( f'' < 0 \). Entrepreneurs would like to borrow from households to invest. However, entrepreneurs and households lack commitment and cannot enforce debt repayment, so no credit arrangement among them is viable.

Like entrepreneurs, young bankers are born in the CM, become old and die in the next CM, cannot work in the CM, and consume only when old. Unlike households and entrepreneurs, bankers can commit to repay their liabilities and enforce the repayment of debt from entrepreneurs (for the discussion of the endogenous emergence of banks, see Gu et al., 2018). Therefore, banks can act as intermediaries between households and entrepreneurs to finance investment projects. A bank can finance its loans by issuing two liabilities: liquid checkable deposits and illiquid time deposits. Checkable deposits can be used as a medium of exchange to facilitate trading between buyers and sellers in the DM. Banks are subject to the reserve requirement that a bank’s reserve holdings must cover at least a fraction \( \chi \geq 0 \) of its checkable deposits.

The government is a combination of monetary and fiscal authorities. The monetary authority, or the central bank, issues three forms of liabilities: physical currency (or cash), central bank reserves, and a CBDC. Currency is a physical token, pays a zero interest rate, and can be used as a means of payment. The reserves are electronic balances that pay a net nominal

\[7\text{Infinitely-lived banks complicate expositions, but have little impact on the results. In this model, banks do not have incentives to retain profits for investment because deposit financing is cheaper. Therefore, they behave as if they live for one period.}\]
interest rate $i_r \geq 0$; they can be held only by banks and cannot be used for retail payments. The CBDC is a digital token or electronic entry that can be used for retail payments. It pays a net nominal interest $i_e$. We focus on stationary monetary policies, where the total liabilities of the central bank (currency, CBDC, and reserves) grow at a constant gross rate $\mu > \beta$ and the central bank stands ready to exchange its three forms of liabilities at par in the CM. We abstract from government purchases. The government collects revenues from the issuance of new liabilities to pay interest on the CBDC and reserves, and the difference finances lump-sum transfers ($T$) to buyers (a negative $T$ represents lump-sum taxes).

In the following, we describe how payments flow in the economy. In the DM, buyers use cash, CBDC, and checkable deposits to purchase good $y$ from sellers. We assume that the two electronic payment methods, CBDC and deposits, are perfect substitutes in terms of payment functions. Sellers are distinguished in three types by the payment methods they accept (Lester et al., 2012; Zhu and Hendry, 2019). Type 1 sellers (of measure $\omega_1 > 0$) accept only cash and can be interpreted as local cash-only stores that do not accept electronic payments. Type 2 sellers (of measure $\omega_2 > 0$) accept deposits and CBDC, and can be interpreted as online stores. Type 3 sellers (of measure $\omega_3 \geq 0$) accept all three payment methods and can be interpreted as local stores with point-of-sale machines that accept both cash and electronic payment methods.

Since the CM is Walrasian, the equilibrium allocation can be supported by different patterns of payment flows. Some possible types of transactions are as follows. Buyers trade $x$ to rebalance their payment portfolio, spending the deposits issued by old banks, acquiring deposits issued by new banks, and adjusting their cash and CBDC balances. Sellers use their earnings in the previous DM (in cash, CBDC, and old deposits) to buy $x$. Young entrepreneurs acquire loans from young banks in the form of new deposits to purchase $x$ for investment. During the process, new deposits are transferred from young entrepreneurs to buyers and old entrepreneurs. Old entrepreneurs sell $x$ in exchange for new and old bank deposits to repay their loans from old banks. Between the banks, young banks use their newly issued deposits to exchange for reserves from old banks. Old banks use acquired new deposits to repay remaining liabilities and to purchase $x$. Note that, as in reality, banks engage in purely financial transactions, except when they use their profits to buy $x$. Figure 1 summarizes the activities and timeline for all private agents.

In the benchmark model analyzed in the next section, we assume that banks cannot hold the CBDC. We also assume that $N$ is fixed, banks engage in Cournot competition in the deposit market, and the lending market is perfectly competitive. This simple environment
Figure 1: Timeline
transparency illustrates our main mechanism. In Online Appendix B.2, we consider the case where banks can use the CBDC as reserves. In Section 4.4 and Online Appendices C and D, we study a model where $N$ is endogenous and the lending market also features imperfect competition. Online Appendix F shows a model with price competition in the deposit market following Burdett and Judd (1983) and Head et al. (2012). Online Appendix G shows an extension of the model that incorporates risk-taking considerations. Our main findings are robust in all of these extensions.

3 Equilibrium Characterization

We focus on stationary monetary policies and stationary equilibria where real allocations are constant over time. It takes four steps to solve for the equilibrium. First, characterize the household’s problem to derive the demand for cash, CBDC, and bank deposits as functions of the deposit rate. Second, solve the Cournot game for banks, incorporating the household demand for deposits, to derive the aggregate deposit supply and loan supply as functions of the competitive loan rate. Third, derive the aggregate demand for loans from entrepreneurs. Finally, we equate the supply and demand for loans to derive the equilibrium loan rate and loan quantity and plug them into the solutions to private agents’ problems to obtain other equilibrium objects, such as the rate and quantity of deposits.

3.1 Households

We first present the buyer’s problem, and then the seller’s problem. Let $W$ and $V$ be the household’s value functions in the CM and DM, respectively. We suppress the time subscript and use prime to denote variables in the next period. Define $\vec{a} = (z, e, d, b)$ as the vector of the real value of cash, CBDC, checkable deposits, and time deposits held by an agent. Let $\vec{i} = (i_z, i_e, i_d, i_b)$ be the vector of net nominal returns, and $\vec{R} = (R_z, R_e, R_d, R_b) = (1 + \vec{i})/\mu$ be the vector of real gross returns. For example, the net nominal interest on cash is $i_z = 0$, and its real gross return is $R_z = 1/\mu$. For brevity, we often refer to $R_e$ as the CBDC rate, $R_d$ the (checkable) deposit rate, and $R_\ell$ the loan rate.

In the CM, a buyer chooses consumption $x$, labor $h$, and the real asset portfolio $\vec{a}'$ carried to the next DM and measured at the current price. The value function for a buyer holding an asset portfolio $\vec{a}$ is
\[ W^B(\vec{a}) = \max_{x,h,\vec{a}'} \{ U(x) - h + \beta V^B(\vec{a}') \} \]

subject to \( x + \vec{1} \cdot \vec{a}' = T + h + \vec{R} \cdot \vec{a}, \)

where \( \vec{1} \) is the unit vector \((1, 1, 1, 1)\) and \( \vec{1} \cdot \vec{a}' \) denotes the inner product of two vectors. The first-order condition with respect to asset portfolio \( \vec{a}' \) is

\[ \beta \frac{\partial}{\partial a} V^B(\vec{a}') \leq 1, \text{with equality if } a' > 0 \text{ for } a = z,e,d,b. \] (1)

Note that, since the type of the DM meeting is not revealed until the start of the DM, buyers carry a portfolio of cash, CBDC, and bank deposits to the DM. Three standard results of the Lagos-Wright model are \( U'(x) = 1, \) all buyers choose the same portfolio \( \vec{a}', \) and \( \partial W^B(\vec{a}) / \partial a = R_a \) for \( a = z,e,d,b. \)

The buyer’s DM value function is

\[ V^B(\vec{a}) = \sum_{j=1}^{3} \alpha_j [u(Y(\mathcal{L}_j)) - P(\mathcal{L}_j)] + W^B(\vec{a}), \] (2)

where \( \alpha_j = \omega_j \Omega \) is the (unconditional) probability of meeting a seller of type \( j, \) and \( Y(\mathcal{L}) \) and \( P(\mathcal{L}) \) are the terms of trade and represent the amount of good \( y \) being traded and the amount of payment, respectively. The terms of trade in a type \( j \) meeting depend on the buyer’s usable liquidity \( \mathcal{L}_j, \) which incorporates the expected return of the asset. Specifically,

\[ \mathcal{L}_1 = R_z z, \] (3)
\[ \mathcal{L}_2 = R_e e + R_d d, \] (4)
\[ \mathcal{L}_3 = R_z z + R_e e + R_d d. \] (5)

Next we turn to the seller’s problem. Without loss of generality, we assume that the seller does not take any asset into the DM, or \( \vec{a}' = \vec{0}. \) Therefore, a type \( j \) seller’s CM problem is

\[ \text{It can be shown that, if the liquidity premium, defined below, on a liquid asset is positive, then the seller does not take that asset into the DM. The seller is indifferent between holding zero or a positive amount of illiquid time deposits when } R_b = 1/\beta, \text{ which holds in equilibrium as shown below. For simplicity, we assume the seller does not hold time deposits either. Note that a seller enters the CM with positive asset balances } (\vec{a} > 0) \text{ after trading in the previous DM.} \]
\[ W^S_j(\vec{a}) = \max_{x,h} \{ U(x) - h + \beta V^S_j(\vec{0}) \} \]

subject to \( x = h + T + \vec{R} \cdot \vec{a} \).

The type \( j \) seller’s DM value function is

\[ V^S_j(\vec{0}) = \Omega[-Y(\hat{\mathcal{L}}_j) + P(\hat{\mathcal{L}}_j)] + W^S(\vec{0}), \]

where \( \hat{\mathcal{L}} \) is usable liquidity held by the seller’s trading partner.

The terms of trade in the DM are determined by buyers making take-it-or-leave-it offers and solve

\[ \max_{y,p} [u(y) - p] \text{ subject to } p \geq y \text{ and } p \leq \mathcal{L}, \]

where the first constraint is the seller’s participation constraint and the second is the liquidity constraint. The solution is

\[ Y(\mathcal{L}) = P(\mathcal{L}) = \min(y^*, \mathcal{L}). \tag{6} \]

In words, if the buyer has enough payment balances to purchase the optimal amount, then the optimal amount is traded; otherwise, the buyer’s liquidity constraint binds and the buyer spends all available payment balances.

Combining (1) to (6), we can characterize the household’s solution as follows. First, the demand for time deposits is separable from the demand for liquid assets and is given by \( R_b = 1/\beta \). Since time deposits have no liquidity value, their return must compensate for discounting across time. Second, the buyer’s demand for payment balances \((z, e, d)\) is determined by

\[ \frac{1}{\beta R_z} - 1 = \alpha_1 \lambda(\mathcal{L}_1) + \alpha_3 \lambda(\mathcal{L}_3), \tag{7} \]

\[ \frac{1}{\beta R_a} - 1 \geq \alpha_2 \lambda(\mathcal{L}_2) + \alpha_3 \lambda(\mathcal{L}_3) \text{ with equality iff } a > 0, \text{ for } a = e, d, \tag{8} \]

where \( \mathcal{L}_j \) is defined by (3) to (5), and \( \lambda(\mathcal{L}) = \max\{u'(\mathcal{L}) - 1, 0\} \) is the liquidity premium.

Equation (7) states that the marginal cost of holding cash (left-hand side) equals its marginal benefit (right-hand side). The former is because the buyer must delay consumption and bear the inflation cost to accumulate cash. The latter is because more cash allows the buyer to consume more in type 1 and type 3 meetings. Equation (8) is for the CBDC and checkable
deposits and has a similar interpretation.

Under the assumption $u' (0) = \infty$ and $\alpha_1 > 0$, the demand for cash is positive, so (7) holds as an equality. Similarly, if $\alpha_2 > 0$, the demand for total electronic (CBDC plus checkable deposits) balances is also positive. However, because the CBDC and checkable deposits are perfect substitutes, buyers hold only the instrument with the higher rate of return. From (8), if $R_d < R_e$, then the demand for checkable deposits is zero. If $R_d > R_e$, then the demand for the CBDC is zero. If $R_d = R_e$, then the buyer is indifferent between the CBDC and checkable deposits and cares only about the total electronic payment balances.

Equations (7) and (8) define $R_d$ as a function of $d$, which is the inverse demand function for checkable deposits, denoted as $R_{d}(d)$. To derive $R_{d}(d)$, it is useful to first obtain the inverse deposit demand without a CBDC. We denote it as $\hat{R}_d(d)$ (from now on, we will use the accent “-” to denote variables or functions if there is no CBDC). We can solve $\hat{R}_d(d)$ from (7) and

$$\frac{1}{\beta R_d} - 1 = \alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_3),$$

(9)
after imposing $e = 0$. For certain values of $d$, there may exist multiple values of $R_d$ that solve (7) and (9). This is because although (7) and (9) uniquely determine $d$ given $R_d$, $d$ may not be monotone in $R_d$. Intuitively, as $R_d$ increases, there are two opposing effects: the substitution effect implies a higher $d$ and the wealth effect implies a lower $d$. Throughout this paper, we assume that the substitution effect dominates and $d$ is monotonically increasing in $R_d$. Then, $\hat{R}_d(d)$ is well-defined and increasing in $d$, with $\hat{R}_d(0) = 0$ and $\hat{R}_d(d) = 1/\beta$ for $d \geq \beta y^*$. With a CBDC, households hold only the electronic payment instrument that bears a higher rate of return. Therefore,

$$R_d(d) = \begin{cases} [0, R_e) & \text{if } d = 0, \\ R_e & \text{if } d \in (0, \hat{R}_d^{-1}(R_e)], \\ \hat{R}_d(d) & \text{if } d > \hat{R}_d^{-1}(R_e). \end{cases}$$

Figure 2 illustrates the inverse demand for checkable deposits. The solid line represents the demand with a CBDC, and the dashed line represents the demand without a CBDC. The two functions overlap if $R_d > R_e$. Once $R_d$ is below $R_e$, the demand for checkable deposits drops to zero.

\footnote{Without this assumption, an equilibrium of the model still exists but may not be unique. A sufficient condition for this assumption is that $-xu''(x)/u'(x) \leq 1$.}
Figure 2: Inverse Demand for Checkable Deposits

Notes. The solid line is the inverse demand for checkable deposits with a CBDC, $R_d(d)$; and the dashed line is the inverse demand for checkable deposits without a CBDC, $\hat{R}_d(d)$. The two lines coincide with each other if $R_d \geq R_e$.

3.2 Banks

Banks issue two types of deposits, checkable deposits ($d$) and time deposits ($b$), and invest in two assets, reserves ($r$), and loans ($\ell$). They do not invest in cash under the assumption $i_r \geq 0$. Bankers maximize consumption in the second period of life, which equals the return from loans and reserves, minus interest payments on deposits. They engage in Cournot competition in the deposit market and perfect competition in the loan market. Formally, banker $j$ chooses $\{r_j, \ell_j, d_j, b_j\}$ to maximize its profit, taking as given the gross real rates for time deposits ($R_b = 1/\beta$), reserves ($R_r$) and loans ($R_\ell$), the inverse demand function for checkable deposits ($R_d(\cdot)$), and other banks’ checkable deposit quantities ($D_{-j} = \sum_{i \neq j} d_i$):

$$\max_{r_j, \ell_j, d_j, b_j} \left\{ R_\ell \ell_j + R_r r_j - R_d(D_{-j} + d_j)d_j - b_j/\beta \right\} \quad (10)$$

subject to $\ell_j + r_j = d_j + b_j$, $r_j \geq \chi d_j$.

This problem has two constraints. The first is a balance sheet identity at the end of the banker’s first CM. The right-hand side is liabilities, which include checkable and time deposits. The left-hand side is assets, which include reserves and loans. The second is the reserve requirement constraint. We also implicitly impose that $d_j$, $b_j$, and $\ell_j$ are non-negative throughout the paper.

If $R_\ell > 1/\beta$, then the bank can make unlimited profits by issuing time deposits and investing
in loans. As a result, \( R_\ell \leq 1/\beta \) in equilibrium. From now on, we restrict our attention to \( R_\ell \in [0, 1/\beta] \). We also assume \( R_r < 1/\beta \). We can separate the bank’s problem into two steps. In the first step, the bank chooses funding sources \((d_j, b_j)\):

\[
\max_{d_j, b_j} \left\{ [\xi - R_d(D_j + d_j)]d_j + (\xi_b - 1/\beta)b_j \right\},
\]

where

\[
\xi \equiv \max\{R_r, \chi R_r + (1 - \chi)R_\ell\}
\]

is the gross return on the bank’s checkable deposits, and \( \xi_b \equiv \max\{R_r, R_\ell\} \) is the gross return on time deposits. The first term in (11) is the profit from issuing checkable deposits, while the second term is the profit from issuing time deposits. Banks can hold their assets in loans or reserves, and therefore the return on assets is the higher of the two. Note that the return on checkable deposits accounts for the cost of satisfying the reserve requirement, while this consideration is absent for time deposits. Additionally, \( b_j = 0 \) if \( R_\ell < 1/\beta \) and \( b_j \in [0, \infty) \) if \( R_\ell = 1/\beta \): the bank issues time deposits only if the return on loans is sufficient to cover the return of \( 1/\beta \) required by households.

In the second step, conditional on the choice in the first step, \((d_j, b_j)\), the bank solves an asset allocation problem. If \( R_\ell < 1/\beta \), then the bank issues only checkable deposits. It invests only in reserves if loans have a lower return than reserves, and invests only a fraction \( \chi \) of assets in reserves to satisfy the reserve requirement if loans have a higher return. If the two assets have the same return, then the bank is indifferent between any allocations that satisfy the reserve requirement. If \( R_\ell = 1/\beta \), then the bank starts to issue time deposits and \( \ell_j \) can take any value in \([(1 - \chi)d_j, \infty)\).

We focus on a symmetric pure strategy equilibrium in which every bank makes the same choice \((d, b, \ell)\). Denote the equilibrium checkable deposits of the Cournot game as \( d(R_\ell) \) to indicate its dependence on the loan rate \( R_\ell \). Following the discussion in the above paragraph, conditional on \( d(R_\ell) \), we can express the equilibrium loan supply function \( \ell(R_\ell) \) as

\[
\ell(R_\ell) = \begin{cases} 
0 & \text{if } R_\ell < R_r, \\
[0, (1 - \chi)d(R_\ell)] & \text{if } R_\ell = R_r, \\
(1 - \chi)d(R_\ell) & \text{if } R_r < R_\ell < 1/\beta, \\
[(1 - \chi)d(1/\beta), \infty) & \text{if } R_\ell = 1/\beta.
\end{cases}
\]

(12)

To establish the existence and uniqueness of the equilibrium in the Cournot game, the
assumption below, Assumption 1, is maintained throughout the paper. As discussed above, \( b_j \) is indeterminate if \( R_\ell = 1/\beta \), and \( \ell_j \) is indeterminate for certain values of \( R_\ell \). We say that the Cournot game has a unique symmetric equilibrium if the symmetric checkable deposit supply is unique.

**Assumption 1** a) For any \( D \in [0, \beta y^* - D) \) and \( \zeta \leq 1/\beta \), there exists a unique \( d_j \in [0, \beta y^* - D) \) such that \( \hat{R}_d'(D + d) + \hat{R}_d(D + d) \leq \zeta \) if \( d \leq d_j \) and \( d \in [0, \beta y^* - D) \). b) \( \hat{R}_d'(Nd)d + \hat{R}_d(Nd) \) increases with \( d \) on \( [0, \beta y^*/N) \) and is less than \( R_r \) if \( d \) is sufficiently small.

In the following, we first characterize the Cournot equilibrium if \( R_e = 0 \), which is equivalent to the case without a CBDC. It serves as a basis for analyzing the general case where \( R_e > 0 \). We characterize the Cournot equilibrium by taking the first-order condition of the bank’s deposit-issuing problem (11) and imposing symmetry.

**Proposition 1** In the absence of a CBDC, the Cournot game has a generically unique symmetric pure strategy equilibrium, where each bank supplies \( \hat{d}(R_\ell) \in [0, \beta y^*/N) \) checkable deposits. In addition, \( \hat{d}(R_\ell) \) increases with \( R_\ell \) and solves the following equation in \( d \):

\[
\hat{R}_d'(Nd)d + \hat{R}_d(Nd) = \zeta.
\] (13)

**Proof.** See Appendix A. 

In Figure 3, we plot the aggregate checkable deposit supply curve \( \hat{D}^*(R_\ell) = N\hat{d}(R_\ell) \) (black curve in the left panel) and the loan supply curve \( \hat{L}^*(R_\ell) = N\hat{\ell}(R_\ell) \) (black curve in the right panel) in the absence of a CBDC. Without a CBDC, banks always issue checkable deposits, and the loan supply is positive if \( R_\ell \geq R_r \). If \( R_\ell < R_r \), banks hold only reserves as assets, the checkable deposit supply is flat and the loan supply is zero. If \( R_\ell = R_r \), the loan supply is vertical. Banks are indifferent between loans and reserves as long as the reserve requirement is satisfied. Both checkable deposits and loans strictly increase with \( R_\ell \) if \( R_r < R_\ell < 1/\beta \). If \( R_\ell = 1/\beta \), then banks start to issue time deposits to finance loans. They are willing to supply any amount of loans that is no less than \( (1 - \chi)N\hat{d}(1/\beta) \).

Now we analyze how a CBDC affects the checkable deposit and loan supply. Since the CBDC is a perfect substitute for checkable deposits regarding payment functions, it alters the

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\(^{10}\)Part (a) of Assumption 1 guarantees that the symmetric Cournot equilibrium is generically unique. Part (b) guarantees that the equilibrium deposit supply is increasing in \( R_\ell \) and banks issue checkable deposits for any \( R_\ell \). If \( u(y) = y^{1-\sigma}/(1-\sigma) \), Assumption 1 holds if \( \sigma < 1 \) and \( R_\ell \) is not too small.

\(^{11}\)The equilibrium is unique unless \( R_\ell = 1/\beta \) and \( \chi = 0 \). In this case, there is one equilibrium where banks make positive profits, and a continuum of equilibria with \( d \geq N\beta y^*/(N-1) \) in which banks make zero profits. We select the single positive-profit equilibrium.
checkable deposit and loan supply only if the CBDC rate, $R_e$, exceeds checkable deposit rate in the Cournot equilibrium without a CBDC, which is denoted by $\hat{R}_d^*(R_e) \equiv \hat{R}_d(N\hat{d}(R_e))$. From (13) and Assumption 1, $\hat{R}_d^*(R_e)$ is constant if $R_\ell \leq R_r$ and strictly increases in $R_\ell$ if $R_\ell > R_r$. Intuitively, under Cournot competition, a higher return on assets is partly passed on to the checkable deposit rate. Therefore, for a given CBDC rate $R_e$, the CBDC tends to alter the checkable deposit and loan supply only for low values of $R_\ell$. In the following, we discuss in detail how a CBDC alters the checkable deposit and loan supply. To ease presentation, we focus on the case in which $R_e \in (R_r, \hat{R}_d^*(1/\beta))$. Other cases are analyzed in Online Appendix B.1.

If $R_\ell \geq \bar{R}_\ell$, where $\bar{R}_\ell$ solves $\hat{R}_d^*(\bar{R}_\ell) = R_e$, the CBDC rate is lower than the deposit rate in the Cournot equilibrium without a CBDC, and a CBDC does not affect the deposit and loan supply.\(^{12}\) If $R_\ell < \bar{R}_\ell$, where $\bar{R}_\ell$ solves $(1 - \chi)\bar{R}_\ell + \chi R_r = R_e$, then a bank’s return on assets is insufficient to cover the cost of serving deposits, and it stops operating.

If $R_r < R_\ell < \bar{R}_\ell$, then a bank matches the CBDC rate and supplies $d_e = D_e/N$ checkable deposits, where

$$D_e = \hat{R}_d^{-1}(R_e).$$

Intuitively, if a bank reduces its supply of checkable deposits below $d_e$, then the checkable deposit rate remains equal to the CBDC rate, because the latter sets a floor for the former.

\(^{12}\) The highest deposit rate in the Cournot equilibrium without a CBDC is $\hat{R}_d^*(1/\beta)$. If $R_e < \hat{R}_d^*(1/\beta)$, then $\bar{R}_\ell < 1/\beta$. 

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**Figure 3**: Effects of a CBDC on the Supply of Checkable Deposits and Loans

Notes. (1) $D_r = N\hat{d}(R_r)$. (2) The red line is the case with a CBDC, and the black line represents the case without a CBDC. The two curves coincide with each other when $R_\ell > \bar{R}_\ell$. 

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The deviating bank has a strictly lower profit because the marginal return of checkable deposits is higher than the marginal cost, that is, $(1 - \chi)R_\ell + \chi R_r > R_e$. Therefore, no bank wants to reduce checkable deposits, because that raises the deposit rate and lowers profits. Notice that a CBDC raises deposit quantity compared to the case without a CBDC; i.e., $d_e > \hat{d}(R_\ell)$. Without a CBDC, banks restrict deposit supply and pay a deposit rate lower than $R_e$. With a CBDC, this is no longer possible, because $R_e$ becomes a lower bound for the deposit rate. This reduces banks’ incentives to restrict the deposit supply and leads to more deposits.

Finally, if $R_\ell = R_\ell$, the bank is indifferent between operating and not, and the deposit supply lies in the interval $[0, d_e]$. Proposition 2 summarizes a bank’s checkable deposit supply in the Cournot equilibrium with a CBDC.

**Proposition 2** If $R_e \in (R_r, \hat{R}_d(1/\beta))$, a bank’s supply of checkable deposits in the symmetric pure strategy equilibrium of the Cournot game is given by

$$d(R_\ell) = \begin{cases} 
0 & \text{if } R_\ell < \bar{R}_\ell, \\
[0, d_e] & \text{if } R_\ell = \bar{R}_\ell, \\
d_e > \hat{d}(R_\ell) & \text{if } \bar{R}_\ell < R_\ell < \bar{R}_\ell, \\
\hat{d}(R_\ell) & \text{if } \bar{R}_\ell \leq R_\ell \leq 1/\beta.
\end{cases} \quad (14)$$

**Proof.** See Appendix A. ■

Figure 3 illustrates how a CBDC affects the aggregate checkable deposit and loan supply, $D^*(R_\ell) = N \hat{d}(R_\ell)$ and $L^*(R_\ell) = N \ell(R_\ell)$, graphed in red. If $R_\ell \geq \bar{R}_\ell$, then the deposit rate offered by banks in the absence of a CBDC is higher than the CBDC rate, and the CBDC does not affect the economy. Therefore, the checkable deposit and loan supply curves with and without a CBDC coincide. If $\bar{R}_\ell < R_\ell < \bar{R}_\ell$, then the supply of checkable deposits and loans is dictated by the CBDC rate $R_e$. The supply of checkable deposits stays at $D_e$ and the supply of loans stays at $(1 - \chi)D_e$. This corresponds to the positive horizontal part of the solid red line. In this interval, the red curve is above the black curve, reflecting that a CBDC can increase the deposit and loan supply. If $R_\ell = R_e$, banks break even, and the supply of checkable deposits can take any value between zero and $D_e$, and the supply of loans lies between zero and $(1 - \chi)D_e$. This corresponds to the vertical part of the solid red line. If $R_\ell < R_\ell$, banks cannot compete with the CBDC and do not operate.$^{13}$

$^{13}$Online Appendix B.1 shows that the introduction of a CBDC with $R_e \in (\hat{R}_d^*(R_r), (1 - \chi)/\beta + \chi R_r)$ expands the supply of checkable deposits and loans for some values of $R_\ell$ (the second and/or third branch of (14) applies).
3.3 Entrepreneurs

Entrepreneurs take the gross loan rate \( R_\ell \) as given and solve

\[
\max_\ell \{ f(\ell) - R_\ell \ell \}.
\]

The inverse loan demand for an entrepreneur is \( f'(\ell) = R_\ell \), which defines the aggregate loan demand function,

\[
L_d(R_\ell) = f'^{-1}(R_\ell).
\]

Obviously \( L_d(\cdot) \) is a decreasing function. It is always positive and approaches zero (infinity) as \( R_\ell \) approaches infinity (zero). The loan demand function is not affected by a CBDC.

3.4 Effects of a CBDC

We now combine the aggregate loan supply curve, \( L^s(R_\ell) \), with the aggregate loan demand curve, \( L^d(R_\ell) \), to determine the equilibrium loan quantity and rate. The loan market equilibrium is unique because the aggregate loan supply curve is non-decreasing. We can then use the equilibrium loan rate to derive the equilibrium quantity and rate of checkable deposits and loans. In the steady state equilibrium, the government budget constraint is

\[
z + e + r = R_z z + R_e e + R_r r + R_z T,
\]

which determines the equilibrium transfer to buyers.

Note that the transfer \( T \) does not affect the household’s demand for real payment balances, so it does not affect the analysis that determines the equilibrium rates and the quantities of deposits and loans.

Figure 4 shows the equilibrium with and without a CBDC. The blue curve is the aggregate loan demand and the black curve is the aggregate loan supply without a CBDC. They intersect at point \( a \), which corresponds to the equilibrium without a CBDC. Let \( \hat{R}_\ell^* \) and \( \hat{R}_d^* \) be the rates of loan and checkable deposits, respectively, in this equilibrium.

The red curves illustrate the aggregate loan supply under different values of \( R_e \). They intersect with the loan demand at point \( b \), which corresponds to the equilibrium with a CBDC. We focus on the case in which \( \hat{R}_\ell^* > R_r \). If \( R_e \leq \hat{R}_d^* \), a CBDC does not affect the equilibrium; otherwise, the effect of a CBDC can be distinguished by three regimes as \( R_e \) increases from \( \hat{R}_d^* \). These regimes are distinguished along two dimensions: the effect of introducing a CBDC (relative to the case without a CBDC), and the comparative statics of

\[14\]

If the loan demand is sufficiently low, then the intersection \( a \) in Figure 4 would lie on the first vertical part of the loan supply curve and \( \hat{R}_\ell^* = R_r \). The reserve requirement constraint would be slack and the equilibrium loan rate would be determined by the interest on reserves \( R_r \). In this case, a CBDC can increase deposits without affecting lending, as in Andolfatto (2020).
Regime 1: \( R_e \in (\hat{R}_e^*, R_{e1}) \)

Regime 2: \( R_e \in (R_{e1}, R_{e2}) \)

Regime 3: \( R_e \in (R_{e2}, 1/\beta) \)

Figure 4: Effects of a CBDC

Notes. The blue curve is the aggregate loan demand, the black curve is the aggregate loan supply without a CBDC, and the red curve is the aggregate loan supply with a CBDC. The red curve joins the black curve for \( R_\ell > \bar{R}_\ell \).

Deposits and loans with respect to \( R_e \).

Regime 1 is shown in Figure 4(a). Compared with the case without a CBDC, a CBDC raises the deposit rate and the demand for electronic payment balances. If the CBDC had not been introduced, banks would have restricted their supply of checkable deposits and offered a lower deposit rate. However, the CBDC sets a floor for the rate of checkable deposits. Losing the ability to further reduce deposit rate, banks supply \( D_e \) checkable deposits to meet all the demand for electronic payment balances at the CBDC rate, because the marginal profit from checkable deposits is positive. In this regime, the CBDC is not used. A bank invests a fraction \( 1 - \chi \) of its checkable deposits on loans, so the aggregate loan quantity is \( L_e = (1 - \chi)D_e \). Now we analyze how the economy responds as the CBDC rate increases.

As \( R_e \) rises, \( R_\ell \) and \( \bar{R}_\ell \) move to the right and the horizontal part of the loan supply curve rises, because \( D_e \) increases. Therefore, the rate and quantity of checkable deposits increase. The loan quantity increases and the loan rate decreases. Banks then have a lower profit margin because of the higher deposit rate and the lower loan rate. If \( R_e = R_{e1} \), which solves \( R_e = (1 - \chi)f'(L_e) + \chi R_r \) as an equation in \( R_e \), the profit margin reaches zero and all banks make a zero profit.

As \( R_e \) increases beyond \( R_{e1} \), the economy enters into regime 2, illustrated in Figure 4(b). In this regime, a higher \( R_e \) increases the rates of checkable deposits and loans. The marginal profit from checkable deposits is zero and banks behave as if they are perfectly competitive. To stay break-even, banks must increase the loan rate to compensate for a higher deposit rate. This lowers the equilibrium loan quantity. Banks then create fewer checkable deposits.
to finance loans. However, households increase their electronic payment balances by holding more CBDC. If \( R_e < R_{e2} \equiv (1 - \chi)\hat{R}_d^* + \chi R_r \) (or equivalently, \( R_d^* = \hat{R}_d^* \)), a CBDC still leads to more loans and deposits relative to the case without it.

Finally, as \( R_e \) increases beyond \( R_{e2} \), regime 3 occurs. It is the same as regime 2, except that the CBDC rate is too high and the quantities of checkable deposits and loans drop below the level without a CBDC. In other words, introducing the CBDC causes disintermediation if and only if \( R_e > R_{e2} \). The following proposition summarizes these discussions.

**Proposition 3** There exists a unique steady-state monetary equilibrium with a CBDC. If \( R_e \leq \tilde{R}_d^* \), then a CBDC does not affect the economy. If \( R_e > \tilde{R}_d^* \), the effects of a CBDC are as follows.

1. **Effects of introducing a CBDC:** Introducing a CBDC promotes lending relative to the case without a CBDC if \( R_e \in (\tilde{R}_d^*, R_{e2}) \), and reduces lending if \( R_e \in [R_{e2}, 1/\beta] \).

2. **Comparative statics with respect to the CBDC rate:** Raising \( R_e \) promotes lending if \( R_e \in (\tilde{R}_d^*, R_{e1}) \), and reduces lending if \( R_e \in [R_{e1}, 1/\beta] \).

Our analysis delivers two important messages. First, introducing a CBDC does not necessarily cause disintermediation or reduce bank loans and deposits. Indeed, the CBDC expands bank intermediation by introducing more competition to the banking sector if its rate falls between \( \tilde{R}_d^* \) and \( R_{e2} \). Second, one should not judge the effectiveness of the CBDC based on its usage, but, rather, on how much it affects the deposit and lending rates or quantities. Throughout regime 1, the CBDC is not used, but it increases both deposits and loans. In fact, it maximizes lending at \( R_e = R_{e1} \), which is the upper bound of regime 1. Here, the CBDC works as a potential entrant. It disciplines the off-equilibrium outcome: if banks reduce their deposit rates below the CBDC rate, then buyers would switch to the CBDC.

We end this section by comparing a CBDC and an interest-rate-floor policy that sets a minimum deposit rate that banks can legally offer. These two policies are similar but, in general, can deliver different outcomes. If the CBDC rate \( R_e \) is below \( R_{e1} \), then the CBDC is not used and its effect is identical to a policy that mandates banks to pay a real rate no less than \( R_e \). However, the two policies lead to different outcomes if \( R_e \) is larger than \( R_{e1} \). With a CBDC, households use both the CBDC and deposits for transactions, and their total electronic balance increases with \( R_e \) despite shrinking deposits. With an interest-rate-floor policy, households end up with lower electronic payment balances and therefore consume less in type 2 and type 3 meetings. The effects of the interest-rate-floor policy (or the CBDC with \( R_e < R_{e1} \)) are also related to the effects of a minimum wage on employment in the
labor literature. It has been shown that a minimum wage can increase employment if firms have monopsony power (Burdett and Mortensen, 1998; Flinn, 2006; and Ashenfelter et al., 2010).

4 Quantitative Analysis

Theoretically, a CBDC can increase bank lending if its interest rate lies in a certain range. Empirical questions remain as to how large a range this is and how big the effect of a CBDC can be. To answer these questions, we calibrate our model without a CBDC to the United States economy, and conduct a counterfactual analysis to assess the effect of introducing a CBDC. We also use the calibrated model to study the effects of a non–interest-bearing CBDC when the economy trends toward cashless.

4.1 Calibration

We introduce two modifications to the model. First, we assume that banks incur a management cost $c$ per unit of deposits. This simply adds the term $-c(d_j + b_j)$ to the profit function in (10). In our model, this cost is equivalent to a variable asset management cost. Second, we allow sellers in the DM to have some market power. Specifically, the DM terms of trade are determined by the Kalai bargaining, with bargaining power $\theta$ to the buyer. These modifications do not affect the qualitative results but capture two features in the data: banks have operational costs and sellers have substantial markups. Both features can be quantitatively important.

Consider an annual model and the functional forms $U(x) = B \log x$, $u(y) = [(y + \epsilon)^{1-\sigma} - \epsilon^{1-\sigma}] / (1 - \sigma)$, and $f(k) = Ak^{\eta}$. The parameter $\epsilon$ is set to 0.001. This guarantees $u(0) = 0$ so that the Kalai bargaining is well-defined for all $\sigma$. It has little effect on our counterfactual analysis. There are 15 parameters to calibrate: $(A, B, N, \Omega, \omega_1, \omega_2, \omega_3, \sigma, c, i_r, \theta, \beta, \eta, \chi, \mu)$. Nine parameters, $i_r, c, \beta, \eta, \mu, \chi$, and $\omega_i$ ($i = 1, 2, 3$), are set directly. The rest are calibrated internally. We calibrate $\omega_1, \omega_2, \omega_3, c, A, N, i_r, \chi$ and $\mu$ using data from 2014 to 2019. The calibration of $(\Omega, B, \sigma, \eta)$ follows the standard approach of matching the money and loan demand curves, which requires the use of longer time series data.

We use four data sets in our calibration exercise: (1) data from the Survey of Consumer Payment Choice (SCPC) and the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; (2) call report data from the Federal Financial Institutions Examination Council; (3) new M1 series from Lucas and Nicolini (2015); and (4) several
time series on macro variables and reserves from Federal Reserve Economic Data (FRED). In what follows, we briefly discuss the calibration of several key parameters. For more details, see Online Appendix H.

We obtain the payment acceptance parameters, the $\omega$s, from the SCPC (Greene and Stavins 2018) and the DCPC (Premo 2018). The SCPC contains information on the fraction of online transactions, and the DCPC contains information on the perceived fraction of point-of-sale transactions that do not accept cash or debit/credit cards. We use data from the 2016 wave, and the numbers are similar in 2015 and 2017. The SCPC documents that an average household makes 67.8 transactions per month. This includes 6.6 automatic bill payments, 5.9 online bill payments, and 4.7 online or electronic non-bill payments. We count these as online transactions and they represent 25.37% of all transactions. We assume that all online transactions accept only deposits. At the point of sale, the DCPC reports that 15% of transactions do not accept debit/credit cards and 2% of transactions do not accept cash. Then, cash-only transactions are those at points of sale that do not accept cards. This implies $\omega_1 = 15\%(1 - 25.37\%) = 11.19\%$. Deposit-only transactions include online transactions and point-of-sale transactions that do not accept cash. Hence, $\omega_2 = 25.37\% + 2\%(1 - 25.37\%) = 26.86\%$, and $\omega_3 = 1 - \omega_1 - \omega_2 = 61.94\%$.

Next, we calibrate the DM trading probability ($\Omega$), the parameters of the utility functions ($\sigma, B$), and the bargaining power ($\theta$) jointly to match the money demand curve and a 20% retail markup. For the monetary aggregate, we use the new M1 series from Lucas and Nicolini (2015), which include cash, checkable deposits, and some interest-bearing liquid accounts. To calculate the money demand in the model, we also need the deposit rates. The call report data, which was also used in Drechsler et al. (2017, 2018), contain the interest expenses and balances on the transaction accounts of the United States banks. We take the ratio of these two variables to obtain an average interest rate on transaction deposits. For this calibration, we use data from 1987 to 2008 for the following reasons. First, interest expenses on transaction accounts are not available before 1987. Second, after 2008, demand for M1 rises sharply likely due to non-transactional demand, such as store-of-value or foreign demand. Because our model focuses on the transactional demand, it is more reasonable to exclude the post-crisis data. In our calibration exercise, the $\omega$s are set to their values in 2016. It is possible that the $\omega$s changed during 1987–2008 and differ from their values in 2016. However, since we calibrate the parameters to match the aggregate money demand curve, our approach remains valid if changes in the $\omega$s affect mainly the composition of M1 but not its aggregate. This is likely, because the money demand curve is stable during 1987–2008.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>Value</th>
<th>Calibration Targets</th>
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<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
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<td>Standard in literature</td>
</tr>
<tr>
<td>Curvature of production</td>
<td>$\eta$</td>
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<td>Elasticity of commercial loans</td>
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<tr>
<td>Reserve requirement</td>
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<td>5.60%</td>
<td>2014–19 avg. required reserves/transaction balances</td>
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<tr>
<td>Interest rate on reserves</td>
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<td>2014–19 avg. IORR</td>
</tr>
<tr>
<td>Cost of handling deposits</td>
<td>$c$</td>
<td>0.02</td>
<td>Avg. operating cost per dollar asset 2.02%</td>
</tr>
<tr>
<td>Gross money growth rate</td>
<td>$\mu$</td>
<td>1.0152</td>
<td>2014–19 avg. annual inflation 1.52%</td>
</tr>
<tr>
<td>Frac. of type 1 trades</td>
<td>$\omega_1$</td>
<td>11.19%</td>
<td>SCPC 2016</td>
</tr>
<tr>
<td>Frac. of type 2 trades</td>
<td>$\omega_2$</td>
<td>26.86%</td>
<td>SCPC 2016</td>
</tr>
<tr>
<td>Frac. of type 3 trades</td>
<td>$\omega_3$</td>
<td>61.94%</td>
<td>SCPC 2016</td>
</tr>
</tbody>
</table>

Calibrated internally
- Prob. of DM trading: $\Omega = 0.22$ (Money demand 1987-2008)
- Coeff. on CM consumption: $B = 2.33$ (Money demand 1987-2008)
- Curv. of DM consumption: $\sigma = 1.66$ (Money demand 1987-2008)
- Total factor productivity: $A = 1.44$ (Rate on transaction accounts 0.3049%)
- Number of banks: $N = 19$ (Spread b/w transaction accounts and loans 3.39%)
- Buyer’s bargaining power: $\theta = 0.9988$ (Retailer markup 20%)

Table 1: Calibration Results

We set $\eta$, the parameter that governs the curvature of the entrepreneur’s production function, to match the elasticity of commercial loans with respect to the prime rate, using the time series from FRED. Choose $\chi$ to match the average ratio of required reserves to the total balance in transaction accounts in 2014–2019. Set $i_r = 1.02\%$ to match the average interest rate on required reserves. To calibrate $A$, $c$, and $N$, we use several statistics on the banking sector in 2014-2019 calculated from the call report data. We choose $c$ to match average non-interest expenditures, excluding expenditures on premises or rent, per dollar of assets. Given $\eta$, $\chi$, $i_r$, and $c$, pick $A$ and $N$ jointly to match the average interest rate on transaction accounts and the spread between the loan rate and the rate on transaction accounts.

Table 1 summarizes all the parameter values along with their calibration targets. Figure 5(a) shows the model-predicted money demand curve against the data between 1987 and 2008. Figure 5(b) shows the loan supply and loan demand without a CBDC under the calibrated parameters.

The money demand is rather flat during 1987-2008, so $\sigma$ is larger than one. This result implies that the DM utility has a high curvature and a $\theta$ close to one is needed to match the markup. We have also done a calibration using data from 1987 and 2019. This alternative calibration suggests $\sigma = 0.45$ and $\theta = 0.80$, and the effect of a CBDC is larger than the benchmark calibration.
4.2 Effects of an Interest-Bearing CBDC

Now we conduct counterfactual analysis and introduce an interest-bearing CBDC as a perfect substitute for checkable deposits. We are particularly interested in how the CBDC affects lending and output with different interest rates. Figure 6 shows the results. In all figures, the horizontal axis is the net nominal interest on CBDC $i_e$. The first row shows the net nominal deposit and loan interest rates and their difference, that is, the spread. All interest rates are in percentages. The second row displays the percentage changes of deposits, loans and total output relative to the equilibrium without a CBDC.

First note that if the interest rate on the CBDC ($i_e$) is below 0.30%, which is the deposit rate without a CBDC in our calibration, then the CBDC does not affect the economy; this corresponds to the flat parts of the figures.

Once $i_e$ exceeds 0.30%, the CBDC rate becomes an effective floor of the deposit rate, and from that point on the deposit rate follows the 45° line. As $i_e$ and the deposit rate increase, total checkable deposits increase as long as banks make positive profits. At $i_e = 0.85\%$, a bank’s profit becomes zero, and the checkable deposits reach their maximum. After that, a further increase in $i_e$ leads to a reduction in checkable deposits. To break even with a higher deposit rate, banks must raise their lending rates, which reduces the loan demand and hence the deposit supply. The effect on loan quantity is the same as the effect on deposits. Because a higher loan supply reduces the loan rate, the loan rate first decreases and then increases.
with $i_e$. From Figure 6, if the CBDC rate is between 0.30% and 1.49%, then the CBDC increases both deposits and loans compared to the equilibrium without a CBDC. At the maximum, the CBDC increases checkable deposits and loans by 1.96% and reduces the loan rate to around 3.00% from about 3.70%.

We next focus on the spread. The CBDC competes with checkable deposits, and a higher CBDC rate reduces the spread as long as banks make positive profits. If $i_e$ is sufficiently high, then banks earn zero profits and the spread starts to increase. Intuitively, as the CBDC rate increases, the interest rate on deposits increases. Because of the reserve requirement, a bank can lend only a fraction of its deposits. Therefore, the loan rate must increase even more to compensate for the increase in the deposit rate, explaining the increasing segment of the spread curve.

Lastly, we move to output. The pattern is qualitatively similar to that of loans: as $i_e$ increases, the total output first increases and then decreases. Quantitatively, the expansionary effect on output is more modest relative to lending, because of the diminishing return in production. Introducing a CBDC increases the total output (relative to the case without a CBDC) if $i_e \in (0.30\%, 1.26\%)$. The highest increase in output is 0.21%, which is achieved at $i_e = 0.85\%$.

In addition to the baseline calibration, we have conducted some robustness checks. First,
we extend the model to allow banks to hold the CBDC as reserves. The results are almost identical. Second, we assess the sensitivity of our results with respect to values of $\chi$ and $i_r$. We range $\chi$ from zero to 10% and $i_r$ from zero to 1.02%. The results are very close in magnitude. For example, if $\chi = 0$ and $i_r = 1.02\%$, the CBDC increases deposits and loans if its rate is between 0.30% and 1.63%. It increases output if the rate is between 0.30% and 1.37%. At the maximum, deposits and loans increase by 2.13% and output increases by 0.24%.

4.3 Non–Interest-Bearing CBDC in a Cashless Economy

We have so far focused on an interest-bearing CBDC. However, central banks may be wary of paying interest on a CBDC, at least in initial stages of its introduction. If the CBDC does not pay interest, can it still have any effect on intermediation? This section assesses the effect of a zero-interest CBDC as the payment landscape evolves, captured by changes in the $\omega$s.

In particular, we consider the trend of declining cash usage experienced in many countries. Several central banks consider this trend an important motivation for issuing a CBDC. We capture this trend by converting $\Delta\%$ of type 3 sellers to type 2 sellers; that is, the fraction of type 3 sellers changes to $\omega_3 - \Delta\% \times \omega_3$ and that of type 2 sellers changes to $\omega_2 + \Delta\% \times \omega_3$. One interpretation is that some brick-and-mortar sellers have closed their physical stores and sell exclusively online. Therefore, more stores accept only electronic payment methods. However, such a change can occur for other reasons. For example, some consumers (merchants) have recently stopped using (accepting) cash for fear of transmitting and contracting the COVID-19 virus. We evaluate how an economy with and without a CBDC differs as $\Delta$ increases.

The solid blue line in Figure 7 illustrates the results without a CBDC. As $\Delta$ increases, checkable deposits become a better payment instrument. Banks gain more market power and reduce the deposit rate. Buyers hold more deposits despite the reduced deposit rate, because deposits can be used in more transactions. Banks issue more deposits and make more loans, and the loan rate decreases. More loans lead to a higher output. The spread increases as the reduction in deposit rate exceeds the reduction in the loan rate.

The dashed red curve shows the economy with a zero-interest CBDC. If $\Delta$ is low, then the CBDC rate is lower than the deposit rate. Therefore, it does not affect the equilibrium and the dashed red curve overlaps with the solid blue curve. As $\Delta$ increases, the CBDC

\[16\text{The Bank of Canada’s contingency planning for a CBDC involves a cash-like CBDC that does not pay interest. China’s Digital Currency Electronic Payment does not pay interest either.}\]
Figure 7: Effects of a Non–Interest-Bearing CBDC as Economy Becomes Cashless

Notes. The horizontal axis represents the % of type 3 meetings that change into type 2 meetings.

prevents the deposit rate from becoming negative, that is, zero becomes a hard floor of the deposit rate. As a result, banks find it optimal to create more deposits and make more loans. Compared to the case without a CBDC, the deposit rate and output are higher, while the loan rate and spread are lower. As Δ increases, banks gain more market power and the CBDC has larger effects. A zero-interest CBDC starts to affect the economy if 3.40% of type 3 sellers stop accepting cash. Therefore, the Unites States could reach a situation in which a zero-interest CBDC affects the economy with a modest change in the payment landscape.

4.4 Endogenous Bank Entry

We have so far taken the banks’ market power as given and assumed that the number of banks is fixed. In this subsection we endogenize it by modelling bank entry through a fixed operating cost.\footnote{There is indeed evidence that banks have significant fixed operating costs. According to the call report data, expenses on business premises and fixed assets were, on average, 5.9% of a bank’s income between 1987 and 2010. Corbae and D’Erasmo (2020) estimate that the fixed cost for all banks in the United States is 0.77% (scaled by loans). Liu (2019) finds substantial fixed costs associated with complying with regulations. In addition, Online Appendix E shows that banks have decreasing average costs, which is consistent with fixed operating costs.} We study how endogenous entry affects our quantitative results.

With endogenous entry, banks play a two-stage game. In the entry stage, they decide whether
to incur the fixed operating cost, denoted by κ, to become active. In the second stage, the 
N active banks play the Cournot game analyzed in Section 3. We solve the model backward 
in two steps. First, we solve the N-bank Cournot game and obtain each active bank’s profit 
π(N), which decreases with N. Second, we solve the entry game. The equilibrium number 
of banks, denoted by N*, satisfies

$$\pi(N^*) \geq \kappa \text{ and } \pi(N^* + 1) < \kappa. \quad (15)$$

There exists a unique equilibrium with active banks (N* ≥ 1) if κ is sufficiently small.

It turns out that if the loan market is competitive, then endogenizing bank entry does not 
modify the basic results from the model with a fixed N. The effects of a CBDC on the rate 
and quantity of deposits and loans remain the same as in the benchmark model as long as Re 
is not too big so that at least one bank is active. Intuitively, the banks’ market power on the 
deposit side is disciplined by the CBDC rate, which fully determines the aggregate deposit 
supply. On the loan side, the market rate is competitive and independent of the number of 
active banks. As a result, the reduction in bank profits and the subsequent exit of banks do 
not affect either side. The remaining active banks satisfy the demand for electronic payment 
balances at the CBDC rate and lend up to the reserve requirement.

Now suppose banks have market power and engage in Cournot competition on the loan side 
too. In a two-sided Cournot game, an individual bank j considers its price impact on both 
the deposit and the loan markets and solves

$$\max_{\ell_j, r_j, d_j, b_j} \left\{ R_\ell(L_{-j} + \ell_j)\ell_j - R_d(D_{-j} + d_j)d_j + R_r r_j - b_j/\beta \right\}$$

$$\text{st } \ell_j + r_j = d_j + b_j \text{ and } \chi d_j \leq r_j,$$

where $R_\ell(\cdot) = f'(\cdot)$ is the inverse loan demand function and $L_{-j}$ is the aggregate loan supply 
of banks other than j. After we solve the N-bank two-sided Cournot equilibrium (see Online 
Appendix D for detailed analysis), we can compute the profit of each bank, π(N), and solve 
N* from (15).

In the model with endogenous entry and Cournot competition on both sides, the impact of a 
CBDC on intermediation is more complicated. Since a CBDC reduces bank profits, it reduces 
the number of active banks. On the one hand, a CBDC can raise the competitiveness in the 
deposit market and lead to more deposits and loans. On the other hand, a smaller number of 
active banks lowers the competitiveness in the loan market and results in fewer deposits and
loans. Therefore, the overall effect of a CBDC on intermediation is a quantitative question.

We calibrate the model with two-sided Cournot competition and endogenous entry. The calibration follows two steps. We first calibrate a model with two-sided Cournot competition and a fixed number of banks. From this calibration, we obtain the number of banks and compute \( \pi(N) \), the profit of active banks. We then set \( \kappa = \pi(N) \) and solve the model with endogenous entry to evaluate the effect of a CBDC.

We find that a CBDC can still increase bank intermediation and output, but the magnitude is about a quarter of that in the benchmark model. The CBDC increases deposits and loans if \( i_e \) is between 0.30% and 0.56%. It increases output if \( i_e \) is between 0.30% and 0.51%. The maximum increase in loans is 0.42% and in output is 4.53 basis points. The smaller effects are due to two reasons. First, banks have less market power in the deposit market in this calibration, because part of the market power is attributed to loans. Second, a reduction in the number of banks counteracts the effects of the CBDC in the deposit market.\(^{18}\) If the CBDC does not pay interest, it can have a positive effect on bank lending and output if more than 4.00% of type 3 sellers stop accepting cash.

Before we conclude this section, we briefly discuss the welfare implication of a CBDC in our model. First, different types of agents are affected differently by the CBDC. As \( i_e \) increases, buyers bring more electronic payment balances. Based on the calibration, buyers are not constrained in type 3 meetings. More electronic payment balances increase consumption in type 2 meetings but do not affect consumption in other meetings. As a result, the welfare of buyers and type 2 sellers increases with \( i_e \), but the welfare of type 1 and type 3 sellers is not affected. Entrepreneurs benefit from the CBDC only if it reduces the loan rate. Banks lose because the CBDC reduces their market power and hence profits.

Second, the government can potentially set \( i_e \) to maximize the overall welfare and use a transfer scheme to generate a Pareto improvement. As pointed out by Keister and Sanches (2019), the optimal CBDC rate must consider payment efficiency and investment efficiency. A higher CBDC rate raises electronic payment balances and the DM consumption, and improves payment efficiency if \( R_e < 1/\beta - c. \)\(^{19}\) The effect on investment efficiency is more complex. A higher CBDC rate may increase or decrease lending. Moreover, higher lending

\(^{18}\) If the loan market features Cournot competition and the number of banks is exogenous, a CBDC increases deposits and loans if \( i_e \in (0.30\%, 0.78\%) \) and output if \( i_e \in (0.30\%, 0.69\%) \). The maximum increase in loans is 0.78% and in output is 8.35 basis points. Therefore, bank exits significantly dampens the expansionary effect of a CBDC.

\(^{19}\) While calculating the overall welfare, we assume that the central bank incurs the same marginal cost \( c \) as commercial banks for providing electronic payment balances.
may increase or decrease investment efficiency, because there may be over-investment induced by cheap deposit funding or under-investment induced by imperfect competition in the loan market (relative to the efficient level of investment characterized by $f'(\ell) = 1/\beta$). If bank entry is endogenous, the optimal policy also needs to economize on the fixed operating cost. In general the optimal CBDC rate depends on model parameters and a simple prescription is not possible. According to our calibration, the optimal CBDC rate is 3.85% in the benchmark model and 3.58% with endogenous bank entry and an imperfectly competitive loan market.

5 Discussion

In this section, we discuss some important issues related to CBDCs, including the motivations for issuing a CBDC and the implementation scheme.

5.1 Motivations for Issuing a CBDC

According to the Bank for International Settlements survey on CBDC (Boar and Wehrli, 2021), domestic payments efficiency is a key motivation for issuing a retail CBDC in both advanced and emerging market economies. Related to this motivation, our findings suggest that a CBDC can discipline the bank’s market power in providing transaction deposit balances and improves payment efficiency. In the past, cash has served this disciplinary role. As the economy enters an increasingly “cashless” digital world, the role of cash weakens, and a CBDC offers the general public an outside option for conducting electronic payments.

While introducing a CBDC could promote bank competition, one may ask why the payment market requires special treatment—after all, the government does not enter the supply side of each market that is subject to imperfect competition. In our view, a couple of features make the payment market somewhat special relative to others. Central banks are already and have been intervening in the payment market for a long time by providing payment balances in the form of cash. Given this historical involvement, the public sector has accumulated some tangible and intangible capital (e.g., payment infrastructures and social trust) that are valuable for its entry into the digital payments industry. Furthermore, the government’s taxation power implies that the public sector has an advantage, relative to the private sector, in providing safe, liquid balances (Holmström and Tirole, 1998). \(^{20}\)

Another motivation for issuing a CBDC is related to the implementation of monetary policy.

\(^{20}\)The flight to safe government securities and bank notes in a financial crisis is evidence of the public sector’s relative advantage in providing safe assets of which the nominal redemption value is certain.
It is commonly argued that, if interest rates are close to zero, monetary policy becomes less effective, because individuals and intermediaries can hold cash to avoid negative interest rates. In our model, cash is an outside option for households as a means of payment, and for banks as reserve balances. The existence of cash limits the ability of a central bank to reduce the interest on reserves and deposits. If cash is replaced with a CBDC, which can bear a negative nominal interest rate, then the limit on the interest rates on reserves and deposits can be relaxed. In practice, completely eliminating cash is unlikely in the near future, but whether cash is around or not, our model suggests that the interest on the CBDC becomes a new policy tool. Combining it with traditional monetary policy tools such as the interest rate on reserves, the central bank can implement a larger set of equilibrium allocations. It is straightforward to adapt our framework to discuss such issues; see, for example, Jiang and Zhu (2021).

Finally, there are other motivations, including safety and resiliency of the payment system, financial inclusion, monetary policy sovereignty and data privacy.\textsuperscript{21} Regardless of the motivation, our paper helps central banks understand the potential impact of a CBDC on the financial industry.

5.2 CBDC Implementation

We now discuss some issues related to the implementation of a CBDC. In our model, the central bank sets the interest rate on the CBDC. An alternative arrangement is to control its quantity, with the interest rate on the CBDC and deposits endogenously determined by the market (Kumhof and Noone, 2018). Under this quantity rule, competition from the central bank will still induce private banks to raise the deposit rate. However, a key difference is that the CBDC always causes bank disintermediation. Specifically, the CBDC takes a positive market share away from banks, leading to fewer deposits and loans relative to the case without a CBDC.

Another implementation issue is related to the architecture of the CBDC system. Our results on bank intermediation require two conditions: the CBDC pays an interest set by the central bank, and the CBDC is a close substitute for bank deposits in terms of payment functionality. In addition, our theoretical analysis requires that the central bank can offer payment services

\textsuperscript{21}Related to the safety motivation, Chiu et al. (2020) suggest that private agents might not fully internalize the benefit of circulating safe payment balances. Andolfatto (2020) shows that a CBDC can expand deposit funding through greater financial inclusion and desired saving. Our result that a CBDC can expand intermediation suggests a similar impact. Also, Garratt and van Oordt (2021) argue that introducing a CBDC helps to promote privacy which is a public good.
as efficiently as banks. In practice, what architecture can meet these requirements without returning the market power to commercial banks? One option is that the central bank runs an independent CBDC system by itself. The central bank may be able to utilize new payment technologies to reduce the costs of operating an independent payment system. For example, the Riksbank’s e-krona pilot considers running an independent payment system based on the Distributed Ledger Technology. However, in such a system, it may still be challenging to provide comprehensive customer service and to satisfy anti-money-laundering/know-your-customer requirements.

Alternatively, the central bank can delegate payment and customer services to third parties and take measures to avoid returning market power to banks. One choice is to delegate the operation of the CBDC system to new non-bank players such as private fintech companies. For example, the Central Bank of the Bahamas has built the infrastructure, technology and regulatory framework for its CBDC (the Sand Dollar), and authorises non-banks as Sand Dollar agents to enrol customers. The central bank can also rely on traditional banks to provide payment and customer services and impose sufficient regulations. For example, the central bank could request banks to offer segregated CBDC accounts and impose the following regulations. Banks should not be allowed to charge excessive account fees or delay CBDC transactions, and should pay the CBDC interest in full (for further discussion, see Kahn et al., 2018).

6 Conclusion and Future Research

This paper develops a model with imperfect competition in the deposit market to analyze whether introducing a CBDC would cause disintermediation in the banking sector. We show that, contrary to the common wisdom, a CBDC can promote bank intermediation. Intuitively, if banks have market power, they restrict the deposit supply to lower the deposit rate. An interest-bearing CBDC introduces more competition, which leads to more deposits and lending, and a lower loan rate. However, greater intermediation arises only if the interest rate on the CBDC lies in some intermediate range. If the CBDC rate is too low, then the CBDC does not affect the equilibrium. If the CBDC rate is too high, disintermediation occurs.

Our model is useful for analyzing the effects of CBDCs with various design choices: interest bearing or not, cash like or deposit like, serving as reserves or not, with a fixed quantity or a fixed rate, and so forth. It can also be used to study the role of a CBDC in an increasingly cashless world and the interaction between CBDC-related policies and existing monetary
policy instruments, such as interest on reserves.

Our model abstracts from the financial stability issue related to a CBDC. For instance, an interest-bearing CBDC would increase banks’ funding costs. As a result of their lower charter value, on the asset side, banks could invest in riskier projects, increasing the total risk in the financial system. On the liability side, banks could switch to less stable funding sources, such as wholesale funding, increasing the likelihood of runs in the wholesale market. Hence, introducing a CBDC could lead to financial stability concerns. Other policy tools (e.g., capital requirements and emergency lending facilities) could help limit banks’ risk taking and alleviate the risk of runs in the banking system. Combining these policies with a well-designed CBDC could potentially promote bank intermediation without increasing risk in the economy. We take some initial steps to explore these issues in Online Appendix G, and leave a full analysis of this important issue for future research.22

Finally, our paper does not investigate central bank lending policy. As CBDCs become an alternative to deposit accounts, the central bank may also reconsider its lending policy.23 Brunnermeier and Niepelt (2019) show that central bank lending to private banks can insulate private lending and investment against the reduction in bank deposits caused by a CBDC.24 Our analysis suggests that a CBDC can promote bank intermediation without central bank lending. However, central bank lending can still serve as an additional (or complementary) policy tool to affect aggregate investment. For example, with endogenous bank entry and non-competitive loan market, the introduction of a CBDC weakens competition in the loan market, and central bank lending can counteract this negative effect. We leave a full analysis of how to coordinate central bank lending and CBDC policies to a separate paper.

22In Online Appendix G, we incorporate the risk-taking channel based on Boyd and De Nicolo (2005), and show that our main result on the level of bank intermediation remains robust, while the introduction of a CBDC can induce more or less risk taking, depending on modeling assumptions.

23In the short run, large-scale, direct lending to firms seems difficult due to legal and political considerations, and operational challenges such as managing credit relationships, and screening and monitoring borrowers. Central bank lending through private banks seems to be a more viable option. However, technological developments (e.g., mobile banking) could reduce the operational costs and make direct lending more likely in the long run.

24Niepelt (2020) generalizes the equivalence result in Brunnermeier and Niepelt (2019) to a model with central bank reserves. Fernández-Villaverde et al. (2020) study the equivalence regarding maturity transformation between private and central bank intermediation.
References


Appendix

A Proofs

Proof of Proposition 1. The bank’s choice of checkable deposits solves:

$$\max_{d_j} \left[ \xi - \hat{R}_d(D_{-j} + d_j) \right] d_j.$$ 

First, suppose $\xi < 1/\beta$, which occurs if $\chi > 1$ or $R_\ell < 1/\beta$. Focus the case where $D_{-j} + d_j < \beta y^*$ because banks make negative profit otherwise. By Assumption 1(a), this problem has a unique solution. It satisfies $\hat{R}'_d(D_{-j} + d_j) d_j + \hat{R}_d(D_{-j} + d_j) = \xi$. Then the symmetric pure strategy Nash equilibrium $d$ must satisfy (13). Because $\hat{R}'_d$ is positive and $\hat{R}_d(\beta y^*) = 1/\beta > \xi$, $\hat{R}'_d(Nd) d + \hat{R}_d(Nd) > \xi$ if $d$ is slightly smaller than $\beta y^*/N$. By Assumption 1, equation (13) has a unique solution, which is increasing in $\xi$ and hence increasing in $R_\ell$.

Next, we show that there is a solution to (13) on $[0, \beta y^*/N]$ if $\xi = 1/\beta$. Let $\xi_n$ be an increasing sequence that converges to $1/\beta$ and $d_n$ be the solution to (13) if $\xi = \xi_n$. Then $d_n$ is the Cournot equilibrium supply of checkable deposits if $\xi = \xi_n$. Let $\tilde{d} = \lim_n d_n \leq \beta y^*/N$. We show that $\tilde{d} < \beta y^*/N$ and therefore solves (13) under $\xi = 1/\beta$ by continuity. Suppose towards contradiction $\tilde{d} = \beta y^*/N$. Then a bank’s profit under $\xi_n$ is $[\xi_n - \hat{R}_d(Nd_n)]d_n$, which converges to 0 because $d_n$ converges to $\beta y^*/N$ and $\xi_n$ converges to $1/\beta$. But if a bank unilaterally deviate to $d_n/2$, its profit is $[\xi_n - \hat{R}((N - 1/2)d_n)]d_n$, which converges to $[1/\beta - \hat{R}((N - 1/2)\beta y^*/N)]y^*/2N > 0$. This implies that for $n$ sufficiently large, a bank can choose $d_n/2$ and gets a higher profit. Therefore, $d_n$ cannot be an equilibrium. This leads to a contradiction. As a result, $\tilde{d} < \beta y^*/N$ and solves (13) if $\xi = 1/\beta$.∎

Proof of Proposition 2. We only prove the third branch of equation (14), which says $d(R_\ell) = d_e > \hat{d}(R_\ell)$ if $R_\ell < R_\ell < \bar{R}_\ell$. The other branches are obvious. First, if the total supply of checkable deposits $D$ is lower than $Nd_e = D_e$, then increasing $d_j$ does not change the real gross rate of deposits, which is fixed at $R_e$. The first-order derivative of equation (11) with respect to $d_j$ is $\xi - R_e$, which is positive if $R_\ell > R_\ell$ by the definition of $R_\ell$. Therefore, bank $j$ can always increase its profit by increasing $d_j$. Second, by the definition of $\bar{R}_\ell$, if $R_\ell = \bar{R}_\ell$, then $\hat{R}_d(D_e) + \hat{R}'_d(D_e) \frac{D}{N} = \xi$. Therefore, by Assumption 1, the marginal profit of a bank $\xi - \hat{R}_d(D) - \hat{R}'_d(D) \frac{D}{N} < 0$ for all $D > D_e$ and $R_\ell < \bar{R}_\ell$. It is profitable for a bank to reduce its supply of deposit if $D > D_e$. Combining both arguments, banks supply $D_e$ checkable deposits in total and $d(R_\ell) = d_e$ by symmetry.∎
Appendices for Online Publication

B Supplementary Analysis

B.1 Detailed Analysis of Checkable Deposit Supply

In the main text, the discussion assumes $R_e \in (R_r, \hat{R}_d^*(1/\beta))$, where the checkable deposit curve experiences all four branches in (14). For $R_e$ located out of this range, some branches in (14) disappear. We now describe $d(R_e)$ for all $R_e \in [0, 1/\beta]$ in more detail. One main take away is that for $R_e \in (\hat{R}_d^*(R_r), (1 - \chi)/\beta + \chi R_r)$, the introduction of a CBDC could expand the supply of checkable deposits and loans for some values of $R_\ell$ (the second and/or the third branch of (14) apply).

Case 1. If $R_e \leq \hat{R}_d^*(R_r)$, then the CBDC rate cannot beat the lowest deposit rate offered by the bank regardless of the level of the lending rate, and the CBDC does not affect the deposit supply. Therefore, $d(R_\ell) = \hat{d}(R_\ell)$ for all $R_\ell \in [0, 1/\beta]$ and only the last branch in (14) remains.

Case 2. If $\hat{R}_d^*(R_r) < R_e < R_r$, then $1/\beta > \tilde{R}_\ell \geq R_r$. The bank makes a positive profit for all $R_\ell$, and only the last two branches in (14) remain: $d(R_\ell) = \hat{d}(R_\ell)$ if $R_\ell \in [\tilde{R}_\ell, 1/\beta]$ and $d(R_\ell) = d_e$ if $R_\ell < \tilde{R}_\ell$.

Case 3. If $R_e = R_r$, then $1/\beta > \tilde{R}_\ell > R_r$ and $\tilde{R}_\ell = R_r$. Note $d(R_\ell)$ remains the same for all $R_\ell \leq R_r$ (the return of the bank’s deposit is bounded below by $R_h$), $d(R_\ell) = d(R_\ell) = [0, d_e]$ if $R_\ell \leq \tilde{R}_\ell$. In this case, the bank can always break even (though may not make positive profits), and the last three branches in (14) remain: $d(R_\ell) = \hat{d}(R_\ell)$ if $R_\ell \in [\tilde{R}_\ell, 1/\beta]$; $d(R_\ell) = d_e$ if $R_\ell < \tilde{R}_\ell < R_r$; and $d(R_\ell) = [0, d_e]$ if $R_\ell \leq R_r$.

Case 4. If $R_r < R_e < \hat{R}_d^*(1/\beta)$, then we are back to the case described in the main text and all four branches in (14) apply.

Case 5. If $\tilde{R}_d^*(1/\beta) \leq R_e < (1 - \chi)/\beta + \chi R_r$, then the CBDC rate exceeds the highest deposit rate offered by the bank (without a CBDC) and the CBDC affects the economy for all values of $R_\ell \in [0, 1/\beta]$. At the same time, we have $1/\beta > \tilde{R}_\ell > R_r$. The first three branches in (14) remain: $d(R_\ell) = 0$ if $R_\ell < \tilde{R}_\ell$; $d(R_\ell) = d_e$ if $R_\ell \in (\tilde{R}_\ell, 1/\beta)$; and $d(R_\ell) = [0, d_e]$ if $R_\ell = \tilde{R}_\ell$.

Case 6. If $R_e = (1 - \chi)/\beta + \chi R_r$, then the CBDC affects the Cournot equilibrium for all values of $R_\ell \in [0, 1/\beta]$, and $\tilde{R}_\ell = 1/\beta$ (or the bank cannot make positive profits and can
break even only when $R_\ell = 1/\beta$). Only the first two branches of (14) remain.

**Case 7.** If $R_e > (1 - \chi)/\beta + \chi R_r$, then the required rate for checkable deposits is higher than the highest possible return on the bank’s assets. As a result, $d(R_\ell) = 0$ for all $R_\ell \in (0, 1/\beta]$. In this case, only the first branch in (14) remains. If $R_\ell = 1/\beta$, the bank could still offer time deposits. This case occurs only if $\chi > 0$.

### B.2 CBDC as Reserves

Now we allow banks to hold the CBDC to satisfy the reserve requirement.\(^{25}\) The CBDC then plays two roles. First, it is a means of payment that competes with checkable deposits. Second, it can lower the cost for the banks to hold reserves if it has a higher return than reserves.

The household’s and entrepreneur’s problems remain the same as in the main text. The bank’s problem changes to

$$\max_{b_j,e_j,r_j,\ell_j,d_j} \left\{ R_\ell \ell_j + R_r r_j + R_e e_j - R_d(D_{-j} + d_j)d_j - b_j/\beta \right\}$$

s.t. $\ell_j + e_j + r_j = b_j + d_j$, $e_j + r_j \geq \chi d_j$,

where $e_j$ is bank $j$’s CBDC balance. As before, we solve the Cournot game among banks for each value of $R_\ell$ and trace out the aggregate loan supply. The only difference is that now banks hold the CBDC to satisfy reserve requirement if its rate is higher than the rate on reserves.

The red curves in Figure 8 illustrate the resulting aggregate loan supply curve. Again, we focus on the case with $R_e > R_r$. Same as in Figure 4, the aggregate loan supply curve with the CBDC coincides with the horizontal axis if $R_\ell$ is low. If $R_\ell$ is intermediate, the curve is flat. The deposit rate matches the CBDC rate, and the quantity of loans is fully determined by the CBDC rate and equals $D_e$. If $R_\ell$ is above a cut-off $\bar{R}_\ell^R$, the return of the checkable deposits is higher than that of the CBDC, and the aggregate loan supply curve is upward sloping.

\(^{25}\)If banks can hold the CBDC but not as reserves, then the effect of a CBDC remains the same as in the benchmark model and banks do not hold any CBDC. The intuition is as follows. If $R_e \leq R_r$, then banks prefer reserves to the CBDC. If $R_e > R_r$, then the marginal benefit of investing in the CBDC is negative. To invest in the CBDC, banks need to issue deposits with at least the same return as the CBDC. In addition, banks must hold reserves and invest only a fraction of deposits on the CBDC, which implies that the total cost of holding the CBDC exceeds the return.
(a) Regime 1: Low $R_e$

(b) Regime 2: Medium low $R_e$

(c) Regime 3: Medium high $R_e$

(d) Regime 4: High $R_e$

Figure 8: CBDC as Reserves

Notes. (1) The blue curve is the loan demand, the black curve is the loan supply without a CBDC, the dashed black line is the loan supply when a CBDC is used as reserves but cannot be used for payments, and the red curve is the final new loan supply with a CBDC that can be used for both reserves and payments. The dashed line coincides with the red curve except for $R_\ell \in [R_e, R_\ell^R]$. All three curves join each other at $R_\ell = 1/\beta$. 

41
Compared with the baseline design, besides being an alternative payment method (payment competition effect), the CBDC can also reduce the bank’s cost to hold reserves (cost-saving effect). The two effects together shift the loan supply curve without a CBDC, shown by the solid black curve, upward to the one with the CBDC. To decompose these two effects, we also plot the aggregate loan supply curve in an auxiliary model, where we shut down the payment competition effect. It is the dashed blue curve on \((R_e, \bar{R}_L^R)\) but overlaps with the red curve otherwise. The shift from the solid black curve to the dashed curve captures the cost-saving effect, and the shift from the dashed curve to the red curve captures the payment competition effect.\(^{26}\)

We can analyze Figure 8 in the same way as Figure 4. It is easy to see that a CBDC that serves as reserves can also promote bank intermediation. Compared with the benchmark model, the CBDC has the additional cost-saving effect. This effect can be active and increase lending even if the CBDC has a lower rate of return than checkable deposits, as shown in Figure 8(a). This happens if the rate on reserves is low. Therefore, this design can increase bank intermediation more compared to the benchmark design. Moreover, it can promote bank intermediation for a wider range of \(i_e\) than in the benchmark model.

### C Imperfectly Competitive Loan Market

This section considers the setup where banks engage in Cournot competition in both deposit and loan markets. We show that the equilibrium is the same as in a setup where banks have deposit and loan departments and these departments allocate funds among themselves in a perfectly competitive market. Therefore, we only need to analyze the equilibrium in the latter setup. The latter setup is attractive because we can use the loan supply and loan demand curves in the interbank market to study the equilibrium. Moreover, the loan supply curve is similar to the one obtained in the main text. A similar analysis shows that the CBDC can increase both deposits and loans. Throughout, we impose the following assumption.

**Assumption 2** The production function satisfies \(2f''(L) + f'''(L)L < 0\).

\(^{26}\)In the increasing part of new (red) loan supply curve \((R_L \in [\bar{R}_L^R, 1/\beta])\), the payment competition effect is muted, and the higher loan supply relative to the old (black) supply curve is due to the cost-saving effect. In contrast, with the baseline CBDC design, the cost-saving effect is absent and the loan supply curves with and without the CBDC coincide if \(R_L \in (\bar{R}_L, 1/\beta))\).
First, we consider the original setup without the interbank market. Bank $j$ solves

$$\max_{\ell_j, r_j, d_j, b_j} \left\{ R_{\ell}(L_{-j} + \ell_j)\ell_j - R_d(D_{-j} + d_j)d_j + R_r r_j - b_j/\beta \right\}$$

subject to $\ell_j + r_j = b_j + d_j$ and $\chi d_j \leq r_j,$

where $R_{\ell}(\cdot) = f'(\cdot)$ is the inverse loan demand function and $L_j$ is the aggregate loan supply of banks other than $j$. Now bank $j$ internalizes its impact on the loan rate. To solve (16), form the Lagrangian

$$\max_{\ell_j, r_j, d_j, b_j} \left\{ R_{\ell}(L_{-j} + \ell_j)\ell_j - R_d(D_{-j} + d_j)d_j + R_r r_j - b_j/\beta + \Lambda_1 (b_j + d_j - \ell_j - r_j) + \Lambda_2 (r_j - \chi d_j) \right\}.$$ 

One can obtain the first-order conditions and then impose symmetry to obtain the following set of equilibrium conditions.

$$0 = R'_{\ell}(L) \frac{L}{N} + R_{\ell}(L) - \Lambda_1,$$  

$$0 \geq [(1 - \chi)\Lambda_1 + \chi R_r] - \frac{\partial^+ R_d(D)}{\partial D} \frac{D}{N} - R_d(D),$$  

$$0 \leq [(1 - \chi)\Lambda_1 + \chi R_r] - \frac{\partial^- R_d(D)}{\partial D} \frac{D}{N} - R_d(D),$$  

$$0 = R_r - \Lambda_1 + \Lambda_2,$$  

$$0 = \Lambda_2 \text{ if } L < (1 - \chi)D \text{ and } 0 < \Lambda_2 \text{ if } L = (1 - \chi)D,$$  

$$1 = \beta \Lambda_1 \text{ if } b_j > 0 \text{ and } 1 > \beta \Lambda_1 \text{ if } b_j = 0,$$

where $\frac{\partial^+ R_d(D)}{\partial D}$ and $\frac{\partial^- R_d(D)}{\partial D}$ denote the right and the left derivatives of $R_d$, respectively. They are introduced because $R_d(D)$ has a kink at $D = D_e$. The above conditions determine the equilibrium with Cournot competition in both the deposit and loan markets. Assumption 2 and the assumptions on $R_d$ imply that the above conditions are necessary and sufficient, and the equilibrium is unique.

Now consider the setup with an interbank market. Each bank has a deposit department and a loan department. Each of the department cares only about their own profit. The deposit departments create deposits and lend them in a competitive interbank market. The loan departments borrow deposits in the interbank market and lend to entrepreneurs. The equilibrium interbank market rate $R_I$ equates the demand and the supply of funds in the interbank market. Deposit departments engage in Cournot competition with each other in
deposit creation while the loan departments engage in Cournot competition with each other in lending.

Then bank \( j \)'s deposit department solves

\[
\max_{\ell_j, r_j, d_j, b_j} \left\{ R_I \ell_j - R_d(D - j + d_j) d_j + R_r r_j - b_j/\beta \right\}
\]

\[
\text{st } \ell_j + r_j = b_j + d_j \text{ and } \chi d_j \leq r_j,
\]

which is the same as the problem in our benchmark model except that now the lending rate in the interbank market appears in the problem.

The Cournot equilibrium among the deposit departments determines the aggregate loan supply in the interbank market. Without loss of generality, assume that \( R_I \geq R_r \). Then we can form the Lagrangian

\[
\max_{\ell_j, r_j, d_j, b_j} \left\{ R_I \ell_j - R_d(D - j + d_j) d_j + R_r r_j - b_j/\beta + \Lambda_1 (b_j + d_j - \ell_j - r_j) + \Lambda_2 (r_j - \chi d_j) \right\}.
\]

Again, take the first-order conditions and impose symmetry to obtain

\[
0 \geq [(1 - \chi) R_I + \chi R_r] - \frac{\partial^+}{\partial D} R_d(D) \frac{D}{N} - R_d(D),
\]

\[
0 \leq [(1 - \chi) R_I + \chi R_r] - \frac{\partial^-}{\partial D} R_d(D) \frac{D}{N} - R_d(D),
\]

\[
0 = R_r - R_I + \Lambda_2,
\]

\[
0 = \Lambda_2 \text{ if } L^* < (1 - \chi)D \text{ and } 0 < \Lambda_2 \text{ if } L^* = (1 - \chi)D,
\]

\[
1 = \beta R_I \text{ if } b_j > 0 \text{ and } 1 > \beta R_I \text{ if } b_j = 0,
\]

where \( L^* = \sum_{j=1}^N \ell_j \) is the aggregate loan supply. Notice that the loan supply curve obtained here is identical to the one in the benchmark model except it depends on \( R_I \) instead of \( R_\ell \).

The lending department \( j \) solves

\[
\max_{\ell_j} \left\{ R_\ell (L - j + \ell_j) \ell_j - R_I \ell_j \right\}.
\]

From the Cournot game among the loan departments, we can obtain the aggregate loan
demand in the interbank market. Under symmetry, the loan demand $L^d$ satisfies

$$R^d_i(L^d) \frac{L^d}{N} + R_c(L^d) = R_I.$$  

(30)

Notice that if we combine (24)-(30) and use the market clearing condition in the interbank market, $L^d = L^s = L$, we obtain the same set of equilibrium conditions as in (17)-(22) with $R_I = \Lambda_1$. In the original problem, $\Lambda_1$ is the shadow value of loanable funds. It is equal to the interbank market rate $R_I$ if there is a perfectly competitive interbank market. This proves that the model with and without a perfect competitive interbank market yield the same equilibrium.\(^\text{27}\)

### D Fixed Operating Costs and Endogenous $N$

Assuming that the number of banks is fixed in the benchmark model, we found that a CBDC can increase bank lending by reducing their market power. However, if banks have fixed operating costs, a CBDC could cause banks to exit because it reduces the profit of operating banks. This in turn can strengthen the market power of remaining banks and offsets the positive effect of the CBDC. In this section, we show theoretically that taking this into account, the CBDC can still improve bank intermediation. In the following, we first study the case with a perfectly competitive loan market and then the case with a Cournot loan market.

Each period, $\tilde{N}$ potential bankers decide whether to be active. If a potential banker decides to be active, the banker needs to pay a fixed operating cost $\kappa$. After the decisions are made, active bankers engage in Cournot competition in the deposit market and perfect competition in the loan market. This model can be solved in two steps. We first solve our benchmark model for different values of $N$ and calculate the profit of a bank without the fixed operating cost, denoted by $\pi(N)$. Then the equilibrium number of banks $N^*$ satisfies $\pi(N^*) \geq \kappa$ and $\pi(N^* + 1) < \kappa$. We can again use the loan supply and demand curve to show that the CBDC can increase bank lending. The only difference is that we need to take into account the effect on the number of banks.

Figure 9(a) shows the equilibria with and without a CBDC. Again, the blue curve is the loan demand curve and the black curve is the loan supply curve if $N = \hat{N}^*$, where $\hat{N}^*$ is the equilibrium number of banks without a CBDC and point $a$ corresponds to the equilibrium.\(^\text{27}\)

\(^{27}\)If banks are heterogeneous, the equilibrium with and without the interbank market can differ.
Now consider introducing a CBDC with \( R_e \) higher than the gross real deposit rate without a CBDC but not too high. At the new equilibrium, \( N^* < \hat{N}^* \) because the CBDC reduces bank profits. The red curve plot the loan supply if \( N = N^* \). The same as in the benchmark model, the loan supply is 0 if \( R_{\ell} < R_{\ell}^* \). If \( R_{\ell} \in (\hat{R}_{\ell}, \bar{R}_{\ell}) \), the aggregate loan supply is \((1 - \chi)D_e\), which is also the same as in the benchmark model. The intuition is also the same: in this region, banks cannot reduce the deposit rate by reducing the supply because the CBDC rate is an effective lower bound. As they are making positive profits per unit of deposits, they satisfy all the demand for electronic payment balances. As \( R_{\ell} \) moves beyond \( \bar{R}_{\ell} \), the CBDC is not effective and the loan supply equals to the one without the CBDC under \( N = N^* \). Notice that \( \bar{R}_{\ell} \) is higher than its value if the number of active banks is fixed at \( \hat{N}^* \). Because there are fewer banks and more market power, a higher lending rate is needed to make a CBDC ineffective. Moreover, the red curve lies below the black curve if \( \bar{R}_{\ell} < R_{\ell} < 1/\beta \), because the aggregate loan supply decreases with \( N \) if the CBDC is not effective.

The equilibrium with the CBDC is point \( b \). It features more deposits, loans and lower lending rate compared to the equilibrium without the CBDC. Moreover, it coincides with the equilibrium with the number of banks fixed at \( N^* \). Intuitively, the CBDC sets the deposit rate and force active banks to compete, which completely off-sets the effect of fewer active banks. Indeed, as long as there is one active bank, the equilibrium remains at point \( b \).

The above result depends crucially on the assumption of perfect competition in the loan
market. If the banks also have market power in the loan market, they would reduce aggregate
loan supply if fewer banks are active. As shown in Section C, the equilibrium with a fixed
$N$ is the same as the equilibrium in a model where each bank is split into a deposit and a
loan department and they trade in a competitive interbank market. Therefore, we can focus
on the latter to simplify the analysis. In particular, we can plot the loan supply and loan
demand in the interbank market to obtain the equilibrium just as in previous analysis.

Figure 9(b) shows the effect of a CBDC if the loan market is imperfectly competitive. Same
as the above, suppose that $N = \hat{N}^*$ in the equilibrium without the CBDC. The black curve
is the loan supply in the interbank market if $N = \hat{N}^*$. The blue curve is the loan demand
curve. Their intersection, point $a$, is the equilibrium without the CBDC. With the CBDC,
$N$ decreases to $N^*$. The the red curve is the loan supply in the interbank market with the
CBDC if $N = N^*$. The loan supply is the same as in the case with a perfectly competitive
loan market, except that it depends on the lending rate in the interbank market, $R_I$. But
there is an additional effect on the loan demand. Because fewer banks are active, they have
more monopoly power in the final loan market. Active banks then demand higher lending
rate from entrepreneurs and lend out less for any rate in the interbank market. As a results,
the loan demand in the interbank market declines. This shifts the loan demand to the left
to the green curve. If the shift is not too big, the equilibrium changes to point $b$. The
amount of loans in the interbank market is higher after introducing the CBDC. Because the
lending departments lend out all the borrowed funds in the interbank market, entrepreneurs
get more loans and face a lower rate. As a result, the CBDC promotes bank intermediation.
However, if the loan demand shifts too much, it could intersect on the vertical part of the
loan supply curve, as a result, the loan quantity may decrease and a CBDC reduces bank
intermediation.

E Empirical Evidence of Fixed Operating Costs

We now show that banks have increasing return to scales, which is majorly caused by de-
creasing average costs. This provide an indirect evidence of significant fixed operating costs.
We first calculate a bank’s profit and revenue per unit of assets from the call report data
between 1987 and 2010, which we call profit rate and revenue rate, respectively. We then
regress it on the bank’s assets (in trillion dollars). To eliminate the outliers, we drop the
observations with the lowest 1% of asset or the lowest 1% profit rate.

Table 2 shows the regression results. The first three columns are results on revenue rates.
The first column is from a simple OLS. The second column adds bank fixed effects and
the last column further adds time fixed effects. The coefficients on assets are negative and significant at 1% level in all specifications. This suggests that average revenue drops with total assets. The last three columns are the same regressions but with the dependant variable being the profit rate. The coefficients on assets are positive and highly significant except one specification. This means that the profit rate is increasing in total assets. Since the revenue rate is decreasing in assets, this suggests the average cost must be decreasing in assets. This is consistent with fixed operating costs.

F Price Competition in the Deposit Market

Consider an alternative deposit market structure, where banks set the real interest rate on deposits. They have market power due to information frictions following Burdett and Judd (1983) and Head et. al (2012). There are a continuum of banks. Each of them quote rates for its checkable deposits and time deposits. Due to information frictions, a buyer does not see all the quotes. Instead, he or she sees one quote with probability $b_1$ and two quotes with probability $b_2$. For simplicity, assume that $b_1 + b_2 = 1$. If the buyer sees two quotes, he or she chooses to stay with the bank that quotes a higher rate. After choosing the bank, the buyer works and makes portfolio choices. There is heterogeneity in the portfolio choices because buyers face different interest rates on their deposits. Nevertheless, the inverse demand function for a buyer remains to be $R_d$. Throughout this section, we require that Assumption 1 holds with $N = 1$. It is convenient to work with demand function instead of inverse demand. Therefore, define $D(R_d) = R_d^{-1}(R_d)$ and $\hat{D}(R_d) = \hat{R}_d^{-1}(R_d)$. Notice that $D(R_d) = \hat{D}(R_d)$ if $R_e = 0$.

Banks engage in perfect competition in the loan market. Given the loan rate, they choose
\( R_d \) and \( R_b \) to maximize the expected profit. Same as before, banks offer time deposit only if \( R_\ell \geq 1/\beta \). If \( R_\ell = 1/\beta \), they are indifferent between any amount of time deposits and \( R_b = 1/\beta \). Therefore, we can focus only on the choice of \( R_d \). As in the main paper, we derive the loan supply as a function of \( R_\ell \) from the bank’s problem and then intersect it with the loan demand curve to determine the equilibrium. The latter remains unchanged, so we focus on the former.

Following Head et. al (2012), there is a continuum of \( R_d \) quoted in the equilibrium. They all lead to the same expected profit. Banks trade off the profit from a customer and the probability of getting a customer. Given \( R_\ell \), the distribution of \( R_d \) is \( F(\cdot; R_\ell) \). One can show that \( F(\cdot; R_\ell) \) is non-atomic and has an interval support. Given \( R_\ell \), the lowest quoted rate solves

\[
\max_{r, \ell, R_d} b_1 [R_\ell \ell + R_r r - R_d D(R_d)] \\
\text{st} \quad r + \ell = D(R_d) \quad \text{and} \quad r \geq \chi D(R_d).
\]

A bank with the lowest rate gets a customer only if the customer sees only one quote. This happens with probability \( b_1 \). Conditional on having a customer, the problem is the same as before with \( D_{-j} = 0 \), i.e., the bank is a local monopoly. This problem can be rewritten as

\[
\max_{R_d} b_1 (\xi - R_d) D(R_d). \tag{31}
\]

where \( \xi = \max \{R_\ell (1 - \chi) + \chi R_r, R_r\} \). Let \( \hat{R}_d(R_\ell) \) solve the following equation in \( R_d \):

\[
(\xi - R_d)\hat{D}'(R_d) - \xi \hat{D}(R_d) = 0.
\]

Then the solution to (31) is \( \hat{R}_d(R_\ell) = \max\{R_\ell, \hat{R}_d(R_\ell)\} \). The CBDC rate sets a floor for the lowest deposit rate. Then \( F(\cdot; R_\ell) \) satisfies the equal-profit condition

\[
b_1 [\xi - \hat{R}_d(R_\ell)] D(\hat{R}_d(R_\ell)) = \{b_1 + 2b_2 F(R_d; R_\ell)\} (\xi - R_d) D(R_d).
\]

The left hand side is the profit from quoting the lowest rate. The right hand side is the profit from quoting the higher rate \( R_d \). A bank with rate \( R_d \) gets a customer if either the customer has a quote only from this bank or the customer’s other quote has a lower rate. This equal-profit condition gives a closed-form solution

\[
F(R_d; R_\ell) = \frac{b_1}{2b_2} \left\{ \frac{[\xi - \hat{R}_d(R_\ell)] D(\hat{R}_d(R_\ell))}{[\xi - R_d] D(R_d)} - 1 \right\}.
\]
Denote the highest rate in the support of \( F \) as \( \bar{R}(\ell) \). It solves \( F(R_d; \ell) = 1 \). If \( \ell \leq 1/\beta \), then \( \bar{R}(\ell) < 1/\beta \).

The accepted quotes have the price distribution

\[
G(R_d; \ell) = b_1 F(R_d; \ell) + b_2 F(R_d; \ell)^2.
\]

Use \( \hat{F} \) and \( \hat{G} \) to denote the equilibrium distributions of quoted and accepted rates if there is no CBDC (i.e., if \( R_e = 0 \)). By Assumption 1, \( \bar{R}(\ell) \) is increasing in \( \ell \). This implies that as \( \ell \) increases, \( F, \hat{F}, G \) and \( \hat{G} \) increases in the sense of first-order stochastic dominance.

Similar as in the benchmark model, the aggregate loan supply without the CBDC is

\[
\hat{L}^s(\ell) = \begin{cases} 
0 & \text{if } \ell < R_r \\
[0, (1 - \chi) \int D(x) \, d\hat{G}(x; \ell)] & \text{if } \ell = R_r \\
(1 - \chi) \int \hat{D}(x) \, d\hat{G}(x; \ell) & \text{if } 1/\beta > \ell > R_r \\
[(1 - \chi) \int \hat{D}(x) \, d\hat{G}(x; \ell), \infty) & \text{if } \ell = 1/\beta
\end{cases}
\]

It takes the same form as the case with Cournot competition. It increases with \( \ell \) because \( \hat{D} \) is increasing with \( \ell \) and \( \hat{G} \) increases with \( \ell \) in the sense of first-order stochastic dominance.

Now introduce a CBDC with \( R_e < R_e < 1/\beta \) as in the main text. Again, define \( \ell \) as the solution to

\[
(1 - \chi) \ell + \chi R_r = R_e.
\]

Let \( \bar{\ell} \) satisfy \( \bar{R}(\bar{\ell}) = R_e \). If \( \ell < \bar{\ell} \), banks do not operate. If \( \ell \geq \bar{\ell} \), \( \bar{R}(\ell) \geq R_e \) and the CBDC does not change the distribution of \( R_d \), i.e., \( F(\cdot; \ell) = \hat{F}(\cdot; \ell) \) and \( G(\cdot; \ell) = \hat{G}(\cdot; \ell) \). If \( R_e < \ell < \bar{\ell} \), then \( R_e > \bar{R}(\ell) \). This implies that \( F(\cdot; \ell) \) and \( G(\cdot; \ell) \) first-order stochastically dominate \( \hat{F}(\cdot; \ell) \) and \( \hat{G}(\cdot; \ell) \), respectively. If \( \ell = \bar{\ell} \), then \( R_d = R_e \) is degenerate, so banks are indifferent between any amount of deposits and lend up to the reserve requirement. On the other hand, households are indifferent between the CBDC and checkable deposits. As a result, the deposit quantity can be anything between...
0 and $\hat{D}(R_e)$. The aggregate loan supply is

$$L_s(R_\ell) = \begin{cases} 
0 & \text{if } R_\ell < \bar{R}_\ell \\
[0, (1 - \chi)\hat{D}(R_e)] & \text{if } R_\ell = \bar{R}_\ell \\
(1 - \chi)\int \hat{D}(x) dG(x; R_\ell) & \text{if } \bar{R}_\ell > R_\ell > \bar{R}_\ell \\
(1 - \chi)\int \hat{D}(x) dG(x; R_\ell) & \text{if } \frac{1}{\beta} > R_\ell \geq \bar{R}_\ell \\
\left[(1 - \chi)\int \hat{D}(x) dG(x; R_\ell), \infty\right] & \text{if } R_\ell = 1/\beta 
\end{cases}$$

Figure 10 shows the loan demand and loan supply curves. The black curve is the aggregate loan supply curve without the CBDC and the red curve is the curve with the CBDC. Similar to the Cournot model, the red curve is above the black curve if $R_\ell \in (R_\ell, \bar{R}_\ell)$ and overlaps with the black curve if $R_\ell > \bar{R}_\ell$. Different from the Cournot model, it is increasing on $(R_\ell, \bar{R}_\ell)$. The blue curve is the loan demand curve. Its intersections with the loan supply curves correspond to equilibria with and without the CBDC. In this figure, we plot the case where $i_e$ is intermediate. The equilibrium with the CBDC (point $b$) has a higher loan quantity and a lower loan rate than the equilibrium without the CBDC (point $a$). Because the equilibrium is between $R_\ell$ and $\bar{R}_\ell$, households do not use the CBDC in equilibrium despite that it has a positive effect on bank intermediation. Same as in the Cournot model, a higher $i_e$ increases both $\bar{R}_\ell$ and $\bar{R}_\ell$. As $R_e$ increases from $R_r$, the CBDC first increases bank intermediation and then decreases bank intermediation.

**Proposition 4** There exists a unique steady-state monetary equilibrium. With a proper interest rate, the CBDC can increase bank intermediation.
G   Bank Profit and Risk-Taking Behaviors

So far, we show that the CBDC can increase bank lending by reducing the market power of banks. A reduction in market power lowers bank profits, which may have implications on the risk of the economy if investment is risky. Although this is not the main focus of this paper, our framework is flexible enough for studying the effect of the CBDC on the risk. It turns out the CBDC can increase or decrease risk in the economy depending on whether it is the entrepreneurs or the banks who decide the risk level of the investment projects. If entrepreneurs make the decision, the CBDC can increase loans and decrease risk taking. If banks make the decision, the CBDC can increase both loans and risk taking. In the latter case, the government can use contingent transfers to reduce the risk taking and use the CBDC to reduce banks’ market power. This opens the door to the general question of coordinations between the CBDC policy and policies targeting directly at the risk-taking behaviors. We leave a careful study of this policy coordination problem to future research.

G.1   Risk-Taking Behaviors of the entrepreneurs

Following Boyd and De Nicolo (2005), each entrepreneur has access to a risky project that requires 1 unit of investment. Each project gives $s$ CM good with probability $p(s)$ and 0 with probability $1 - p(s)$, where $p$ is strictly decreasing and concave with $p(\bar{s}) = 0$ and $1 \geq p(0) > 0$. The projects with the same risk level are perfectly correlated. Each entrepreneur has a heterogeneous cost $\kappa$ to operate the investment. The measure of entrepreneurs with $\kappa$ less than $x$ is $G(x)$.

Entrepreneurs have limited liability. They pay back the loans only if the project is successful. Notice that entrepreneurs care only about the expected profit and do not have incentives to diversify investments across projects with different risk levels. Therefore, their problem is

$$\max_s p(s)(s - R_\ell),$$

which yields a decision for risk level

$$R_\ell = s + \frac{p(s)}{p'(s)}.$$

This defines risk taking as an increasing function of $R_\ell$ denoted as $s(R_\ell)$. Higher lending rate induces entrepreneurs to take more risk and utilize their limited liabilities. Then the
profit of a entrepreneur is
\[ \pi(s(R_\ell)) = p[s(R_\ell)] [s(R_\ell) - R_\ell], \]
which is decreasing in \( R_\ell \) by the envelope condition. An entrepreneur invests if and only if \( \pi(s(R_\ell)) \geq \kappa \). Therefore, the total loan demand is \( G(\pi(s(R_\ell))) \), which is decreasing in \( R_\ell \). We can invert it to obtain the inverse loan demand function \( R_\ell \).

Now we consider the bank’s problem. Banks engage in Cournot competition in both the deposit and the loan markets. To simplify presentation, we assume that all bank deposits are insured by the government at zero cost. The government finances deposit insurance by lump-sum tax in case of a bank default. We assume that banks do not have enough assets to cover liabilities if entrepreneurs default on the loan, which requires \( \chi \) to be small. If a bank fails, it exhausts its assets to pay back deposits but do not have further obligation. Banks engage in Cournot competition in both deposit and loan markets. Therefore, bank \( j \) solves

\[
\max_{\ell_j, r_j, d_j} q(L_{-j} + \ell_j) [R_\ell(L_{-j} + \ell_j)\ell_j - R_d(D_{-j} + d_j)d_j + R_r r_j] \\
\text{st } \ell_j + r_j = d_j \text{ and } \chi d_j \leq r_j,
\]

where \( q(\cdot) = p[s(R_\ell(\cdot))] \).

We form the Lagrangian

\[
\max_{\ell_j, r_j, d_j} \left\{ q(L_{-j} + \ell_j) [R_\ell(L_{-j} + \ell_j)\ell_j - R_d(D_{-j} + d_j)d_j + R_r r_j] \\
+ \Lambda_1(d_j - \ell_j - r_j) + \Lambda_2(r_j - \chi d_j) \right\}.
\]
Imposing symmetry, we can obtain a set of equilibrium conditions

\[ 0 = q(L) \left[ R'_e(L) \frac{L}{N} + R_e(L) \right] + q' \left( L \right) \left[ R_e(L) \frac{L}{N} - R_d(D) \frac{D}{N} + R_rr_j \right] - \Lambda_1, \]  
\[ 0 \geq [(1 - \chi) \Lambda_1 + q(L) \chi R_r] - q(L) \left[ \frac{\partial^+}{\partial D} R_d(D) \frac{D}{N} + R_d(D) \right], \]  
\[ 0 \leq [(1 - \chi) \Lambda_1 + q(L) \chi R_r] - q(L) \left[ \frac{\partial^-}{\partial D} R_d(D) \frac{D}{N} + R_d(D) \right], \]  
\[ 0 = q(L) R_r - \Lambda_1 + \Lambda_2, \]  
\[ 0 = \Lambda_2 \text{ if } L < (1 - \chi) D \text{ and } 0 < \Lambda_2 \text{ if } L = (1 - \chi) D. \]

Again $\Lambda_1$ is the marginal value of loanable funds. Different from the case without risk, the bank profit when it does not fail enters into (33). If this profit is lower, $\Lambda_1$ is lower. Intuitively, banks have an incentive to issue more loans to reduce entrepreneur’s risk, which increases their probability of being profitable. However, if the bank profit drops, they have less incentive to reduce the risk and as a result the value of loanable funds drops. This channel could reduce bank lending and increase risk if the CBDC cut bank profits.

**Proposition 5** If entrepreneurs decide the level of risk, a CBDC can increase bank lending and decrease risk if $R_e$ is slightly higher than the gross real rate of deposits without the CBDC.

**Proof.** If there is no CBDC, $R_d = \hat{R}_d$ is a smooth function. Therefore, (34) and (35) hold as equality. Denote the equilibrium quantities without the CBDC as $\hat{D}^*, \hat{L}^*, \hat{\Lambda}_1^*, \hat{\Lambda}_2^*$. Consider the case where $\hat{\Lambda}_2^* > 0$ and the reserve requirement binds. We show the if $i_e$ is set such that $R_e$ is only slightly higher than $\hat{R}_d(\hat{D}^*)$, then $D^* = D_e, L^* = (1 - \chi) D^*$ constitute an equilibrium with a CBDC. To achieve this, we only need to show that we can find $\Lambda_1^* > 0$ and $\Lambda_2^* > 0$ such that (33)-(36) hold with $(D^*, L^*, \Lambda_1^*, \Lambda_2^*)$. We first define $\Lambda_1^*$ to be the solution of (33) if we replace $(D, L)$ by $(D^*, L^*)$. Notice that $\Lambda_1^*$ is continuous in $D_e$ and hence continuous in $R_e$. Now we evaluate (35) at $(D^*, L^*, \Lambda_1^*)$. Because $\frac{\partial}{\partial D} R_d(D_e) = 0$ and $\frac{\partial^-}{\partial D} \hat{R}_d(\hat{D}^*) > 0$, the right-hand side of (35) is strictly greater than 0 if $R_e$ is slightly higher than $\hat{R}_d(\hat{D}^*)$. Similarly, the right-hand side of (34) evaluated at $(D^*, L^*, \Lambda_1^*)$ is less than or equal to 0. Otherwise, it suggests that the equilibrium without the CBDC should feature more than $D^*$ deposits and $L^*$ loans, which contradicts the assumption. Lastly, because $\Lambda_1^*$ and $L^*$ are continuous in $R_e$, (36) defines a positive $\Lambda_2^*$ if $R_e$ is not too far from $\hat{R}_d(\hat{D}^*)$. Now we have constructed $\Lambda_1^*$ and $\Lambda_2^*$ that satisfy the requirement. Therefore, $D^*$ and $L^*$ are the equilibrium quantities if $R_e$ is higher but not too much higher than $\hat{R}_d(\hat{D}^*)$. Because $L^* > \hat{L}^*$ and risk is decreasing in total loan supply, a CBDC can increase bank lending and reduce risk. ■
G.2 Risk-Taking Behaviors of Banks

We next study a variation of the model where banks choose the risky investment. This model is also studied in Boyd and Di Nicolo (2005). It is the same as the above except that now the banks decide both the risk of the investment \( s \) and the quantity of the investment \( \ell \). The investment returns \( s\ell \) with probability \( p(s) \) and 0 with probability \( 1 - p(s) \). Returns are perfectly correlated across projects.

Bank \( j \) solves

\[
\max_{s_j, \ell_j, r_j, d_j} p(s_j) [s_j\ell_j - R_d(D-j + d_j)d_j + R_r r_j] \tag{38}
\]

subject to \( \ell_j + r_j = d_j \) and \( \chi d_j \leq r_j \).

We form the Lagrangian:

\[
\max_{s_j, \ell_j, r_j, d_j} \left\{ p(s_j) [s_j\ell_j - R_d(D-j + d_j)d_j + R_r r_j] + \Lambda_1 (d_j - \ell_j - r_j) + \Lambda_2 (r_j - \chi d_j) \right\}.
\]

Impose symmetry and eliminate \( \Lambda_1 \) and \( \Lambda_2 \) to obtain

\[
0 \geq [(1 - \chi)s + \chi R_r] - \left[ \frac{\partial^+}{\partial D} R_d(D) \frac{D}{N} + R_d(D) \right] \tag{39}
\]

\[
0 \leq [(1 - \chi)s + \chi R_r] - \left[ \frac{\partial^-}{\partial D} R_d(D) \frac{D}{N} + R_d(D) \right] \tag{40}
\]

\[
0 = p(s) \frac{L}{N} + p'(s) \left[ s \frac{L}{N} - R_d(D) \frac{D}{N} + R_r \frac{D - L}{N} \right] \tag{41}
\]

\[
s = R_r \text{ if } L < (1 - \chi)D \text{ and } s > R_r \text{ if } L = (1 - \chi)D \tag{42}
\]

**Proposition 6** If banks decide the risk level directly, a CBDC can increase bank lending and risk taking if \( R_e \) is slightly higher than the gross real rate of deposits without the CBDC.

**Proof.** Let \( \hat{s}^*, \hat{D}^*, \hat{L}^* \) be the equilibrium without the CBDC. And assume that \( \hat{s}^* > R_r \). Then \( \hat{L}^* = (1 - \chi)\hat{D}^* \). Therefore, \( \hat{s}^*, \hat{D}^* \) solve

\[
0 = [(1 - \chi)s + \chi R_r] - \left[ \hat{R}'_d(D) \frac{D}{N} + \hat{R}_d(D) \right] \tag{43}
\]

\[
0 = [s + \frac{p(s)}{p'(s)}(1 - \chi) + \chi R_r - \hat{R}_d(D)]. \tag{44}
\]
Now we show that if \( R_e \) is higher than but close to \( \hat{R}_d(\hat{D}^*) \), then \( D^* = D_e \) and \( L^* = (1 - \chi)D^* \) constitute an equilibrium. Let \( s^* \) be the solution to (44) when \( D = D^* \). If \( R_e \) is close to \( \hat{R}_d(\hat{D}^*) \), \( D^* \) is close to \( \hat{D}^* \). Therefore, \( s^* \) is close to \( \hat{s}^* \) and strictly larger than \( R_r \). Moreover, because \( D^* > \hat{D}^* \) and \( s + p(s)/p'(s) \) is increasing, \( s^* > \hat{s}^* \). If \( s^* \), \( D^* \) and \( L^* \) satisfies (39)-(41), they are the equilibrium with the CBDC. First, (41) is satisfied by the definition of \( s^* \) and the fact that \( \hat{R}_d(D^*) = R_d(D^*) = R_e \). Second, (40) is satisfied by a similar argument as in the previous section. Lastly, one can show that (39) is satisfied because otherwise, the equilibrium without the CBDC should have a deposit quantity larger than \( D^* \). This shows that \( (D^*, L^*, s^*) \) is the equilibrium if \( R_e \) is slightly higher than \( \hat{R}_d(\hat{D}^*) \). Therefore, both bank lending and risk level increase after introducing the CBDC. □

The government can keep the risk level unchanged by giving banks a lump-sum transfer if they do not fail. Define

\[
\tau = \left\{ R_e - \left[ \hat{s}^* + \frac{p(\hat{s}^*)}{p'(\hat{s}^*)}(1 - \chi) - \chi R_r \right] \right\} \frac{D^*}{N} = \left[ R_e - \hat{R}_d(\hat{D}^*) \right] \frac{D^*}{N}.
\]

One can check that \( (D^*, L^*, \hat{s}^*) \) constitutes an equilibrium. Notice that \( \tau \) and the CBDC rate are two orthogonal policy tools. Although the government can choose \( \tau \) to maximize the expected productivity \( p(s)s \), the quantity of investment can be too low due to the market power of banks. The CBDC can help to raise investment quantity. Although such transfers may not be possible in practice, the message is that the government may have other policy tools that target directly at the risk-taking behaviors. These tools can be used in conjunction with the CBDC rate to reduce banks’ market power without making them more risky. An example of such a tool is the contingent transfer studied above. There are more practical tools like capital requirements. A careful study of the interaction between these policies and the CBDC rate can be very interesting and is beyond the scope of this paper. Therefore, we leave it for future research.

\section{H Calibration Method and Data}

In the calibration, we use Kalai bargaining as the DM trading mechanism. It is more flexible and allows sellers in the DM to have positive mark-ups. The bargaining power to the buyer is \( \theta \in [0, 1] \). The solution maximizes the buyer’s surplus given that he or she gets \( \theta \) fraction of the total surplus and the liquidity constraint, i.e., if the buyer has a real balance \( L \), the
Kalai solution solves
\[
\max_{y,p} \left[ u(y) - p \right] \text{ s.t. } u(y) - p = \theta \left[ u(y) - y \right] \text{ and } p \leq \mathcal{L}.
\]

All the analysis in the main text stays unchanged except that \( \lambda(\mathcal{L}) = \max\left\{ u'[Y(\mathcal{L})](1-\theta)u'[Y(\mathcal{L})] + \theta - 1, 0 \right\} \),

where \( Y(\mathcal{L}) \) satisfies \((1 - \theta)u'[Y(\mathcal{L})] + \theta Y(\mathcal{L}) = \mathcal{L}\). If \( \theta = 1 \), Kalai bargaining reduces to the buyer take-it-or-leave it offer studied in the main text.

Data

From FRED, we obtain the time series for inflation, 3-month t-bill rates, prime rates, GDP and total commercial loans. The SCPC data contain the number of transactions by type. Table 9 in Greene and Stavins (2018) contains these numbers for 2015-2017. DCPC asks consumers to record whether they think a transaction accepts cash or cards. Page 13 and 14 in Premo (2018) contain summary statistics of answers to these questions.

For calibration, we need times series of interest rates on transaction deposits and loan, and information on the operation costs of banks. We obtain them from call report data from 1987-2019. This data contain quarterly information on balance sheet and income statement of banks in the US. We obtain this data from WRDS by using the SARS code by Drechsler et al. (2017). To obtain the rates on transaction deposits, we first divide interest expenses on transaction accounts (item code: RIAD 4508) by total transaction deposits (RCON2215) to obtain the rates for each bank in a given quarter. Then we take the average across all banks weighted by their transaction deposits to obtain a quarterly industry average. Lastly, we aggregate to the annual level. Notice that RIAD4508 starts only from 1987.

To obtain the loan rates, we first divide the interest income from loans (RIAD4010) by the loan quantity (RCON3360) to obtain bank-level loan rates. These rates are very heterogeneous across banks. This could be because loans of different banks have different risk. Since we do not model risky investment, we focus only on the safe loans. We define our loan rate to be the first percentile of the loan rate distribution in the data. This leads to rates comparable to the FRED on loans of minimal risk, which is reported between 1998 and 2008. In this period, our average loan rate is about 4.4%, while the FRED data have an average rate of 4.7%. This gives a 3.69% loan rate. We have also done a calibration with the average loan rate 5.19%. The results are similar but the positive effect of the CBDC is
larger because the higher loan rate implies a higher market power in the banking sector.

Lastly, we compute the operational cost per dollar of asset between 2014 and 2019 to calibrate $c$. Unfortunately, assets data are missing for many banks during this time frame. But we observe total deposits (RCON2200+RCFN2200) for this period. We also observe both assets and deposits between 1987 and 2010. In this period, assets is about 1.505 times the total deposits. We then assume that the this ratio is stable over time. Therefore, we can divide the operational cost per dollar of deposits by 1.505 to obtain operational cost per dollar of assets. To this end, We first calculate the average operational cost for each bank by subtracting expenses on premises or rent (RIAD4217) from the non-interest expenses (RIAD4903). Then take an average across banks weighted by total deposits to get the industry average. Finally, we aggregate to annual level and set $c$ to be the time average divided by 1.505.

**Computation**

One straightforward way to calibrate the model is to solve the equilibrium given each parameter value and choose one that best fits the money demand curve, the deposit rates and the spread. This method, however, can be computationally cumbersome because one needs to solve the model for each data point used for the money demand and then optimize over a six-dimensional parameter. One key insight is that the money demand can be solved independent of the banking sector. This leads to the following algorithm that greatly simplifies the calibration.

1. Match the money demand between 1987 and 2008 to obtain $B, \sigma, \theta, \Omega$. In this step, we set $i_r = 0$ and $\chi = 2.4\%$ to match the average interest on reserves and the average ratio of required reserves excluding vault cash to transaction balances during this period.

   (a) Fix the value of $\Omega$ and $\theta$. Fit the money demand curve by choosing $B, \sigma$ to minimize the distance between the model predicted M1 to GDP ratio and the data. The M1 to GDP ratio in the model can be calculated by $(R_z Z + R_d D) / Y$, where
   
   \[ Y = \sum_{j=1}^{3} \alpha_j P(y_j) + 2B + \frac{\chi D}{1 + \pi} + AL^\eta - R_d D + L \]

   is the output and $\pi$ is the inflation rate. It is the sum of the consumption of households in DM and CM, the consumption of bankers and entrepreneurs, and...
the investment. The DM consumption is measured by the amount of payment. Because the reserve requirement binds during this period, \( L = (1 - \chi)D \). The first-order condition of the entrepreneurs implies \( A\eta L^{\eta - 1} = R_\ell \). Therefore,

\[
Y = \sum_{j=1}^{3} \alpha_j P(y_j) + 2B + \frac{\chi D}{1 + \pi} + \frac{(R_\ell + \eta)(1 - \chi)D}{\eta} - R_d D.
\]

We plug in the time series for \( R_\ell \) and \( R_d \). The above formula does not involve \( A \). This insight allows us to calibrate \( B, \sigma \) independent of the bank’s problem.

(b) Calculate the markup. If it is less than 20%, then decrease \( \theta \), otherwise increase \( \theta \). Repeat 1-2 until the markup in the model matches 20%.

(c) Calculate the model fit at different values of \( \Omega \). And find the value that gives the best fit.

2. Match banking data from 2014 to 2019 to obtain \( A, N \)

(a) Set \( N \) such that the solution of the Cournot competition leads to a spread of 3.39% between loans and transaction account.

(b) Set \( A \) to match a 0.3049% interest rate on transaction accounts.