Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment

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Abstract

One frequently raised concern about a central bank digital currency (CBDC) is that it is likely to compete with bank deposits as a means of payment and therefore increase private banks’ funding costs and induce disintermediation. We develop a micro-founded general equilibrium model with money and banking to evaluate this concern both theoretically and quantitatively. We find that when banks have market powers in the deposit market, introducing a CBDC that competes with bank deposits as a means of payment can compel banks to raise the deposit rate and expand bank intermediation and output. The model calibrated to the U.S. economy suggests that a CBDC with a proper interest rate can raise bank lending by 3.55% and output by 0.50%. We also use the framework to evaluate other dimensions of the CBDC design, including acceptability, eligibility as reserves and the rule of supply, and assess the role of a CBDC as the economy becomes increasingly cashless.

Bank topics: Digital currencies and fintech; Monetary policy; Monetary policy framework; Market structure and pricing

JEL codes: E50, E58
1 Introduction

Central banks in several countries, including China, Canada and Sweden, are considering issuing central bank digital currencies (CBDCs), a digital form of central bank money that can be used for retail payments. One frequently raised concern about a CBDC is that, since a CBDC is likely to compete with bank deposits as a payment instrument, it may increase commercial banks’ funding costs and reduce bank deposits and loans, leading to bank disintermediation. For example, the International Monetary Fund (IMF) staff discussion note by Mancini-Griffoli et al. (2018) argues that “as some depositors leave banks in favor of CBDC, banks could increase deposit interest rates to make them more attractive. The higher deposit rates would reduce banks’ interest margins. As a result, banks would attempt to increase lending rates, though at the cost of loan demand.”

The objective of this study is to develop a micro-founded general equilibrium model with money and banking to formally assess this concern, both theoretically and quantitatively. In our model, households allocate funds among three assets or payment instruments, cash, bank deposits and a CBDC, that differ in terms of the types of exchange they can facilitate. Entrepreneurs have investment opportunities but have no resources to self-fund them. Banks act as intermediaries, accepting deposits from households and issuing loans to entrepreneurs subject to a reserve requirement. One critical feature of our model is that there is imperfect competition in the banking sector, especially in the deposit market (See, for example, Dreschler et al., 2017, and Wang et al., 2018, for empirical evidence). In our baseline model, banks engage in Cournot competition in the deposit market and perfect competition in the loan market.

Using this framework, we first investigate the effects of a baseline CBDC, which is a perfect substitute for bank deposits as a means of payment, bears a fixed interest rate and cannot be used as reserves. In particular, we examine the effect of the CBDC rate on the rates and quantities of bank deposits and loans, and on the output of the economy. We then explore the effect of a CBDC with alternative design choices in terms of acceptability, eligibility as reserves and the rule of supply (for example, in fixed quantity or rate). Finally, we discuss the role of a CBDC when the payment system continues to evolve and the economy becomes increasingly cashless.

Our model predicts that when banks have market power in the deposit market, a

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1Three countries have launched a CBDC. Ecuador’s dinero electronico failed after three years (2015-2018) of operations, largely due to the lack of trust in the government’s ability to issue claims to US dollars (Ecuador has been dollarized since 2000) that it might become unable or unwilling to repay. Uruguay ran a six-month pilot with e-Pesos starting in November 2017. In December 2019, the Central Bank of Bahamas started its central bank digital currency (sand dollar) pilot. In these three countries, bank disintemediation is less of a concern; in fact, a CBDC is cited as a means to improve financial inclusion and access to financial services in these countries.

2See also the BIS report by the Committee on Payments and Market Infrastructures (2018).
CBDC could surprisingly expand bank intermediation. While calibrating our model to the U.S. economy, we find that introducing the baseline CBDC expands bank intermediation if the interest rate on the CBDC is set in an appropriate range (between 0.05% and 1.79% in the calibrated model), and at the maximum, a CBDC can increase loans and deposits by 3.55% and the total output by about 0.5%.

The mechanism through which a CBDC “crowds in” bank intermediation works as follows. In an imperfectly competitive deposit market, banks restrain deposit supply and keep interest rates too low (or equivalently, the price of deposits too high) relative to the case with perfect competition. A CBDC offers an outside option to depositors and sets an interest rate floor for bank deposits. The CBDC forces banks to raise the deposit rate. A higher deposit rate attracts more deposits, lowers the loan rate and expands lending when the reserve requirement is binding. Interestingly, a CBDC has positive effects on deposits, loans and output even if it has zero market share. The mere existence of a CBDC as an outside option forces non-competitive banks to match the CBDC rate to retain their deposits. We also find that even a non-interest-bearing CBDC can discipline banks’ market power and enhance efficiency as the economy becomes increasingly cashless (without a CBDC, banks could start imposing negative deposit rates).

The positive effect of a CBDC on deposit and loan creation reverses if the CBDC rate crosses a threshold (around 1.79% while calibrated to the U.S. economy). Beyond this threshold, banks are forced to raise the lending rate to compensate for the interest paid on deposits, which reduces the equilibrium quantity of loans and deposits.

Our study is closely related to two concurrent papers that discuss the effects of a CBDC in a micro-founded model with money and banking. Keister and Sanches...
(2019) focus on the welfare implications of an interest-bearing CBDC when the banking sector (modelled as a bank-firm combination) is fully competitive. The bank is subject to credit constraints due to limited pledgeability of the returns on their projects, so the level of investment can be inefficiently low. In this environment, a CBDC always crowds out bank intermediation, which is welfare-reducing when investment frictions are large. However, introducing a CBDC could still be beneficial because it promotes efficient exchange. This benefit can dominate the cost associated with disintermediation if investment frictions are sufficiently small.

Andolfatto (2018) studies the effect of a CBDC on bank intermediation in an economy with heterogeneous households and monopolistic banking. He uses the overlapping generations (OG) framework where young households save for old age in cash, deposits or a CBDC. The latter two require costly access to the banking system, so poorer households save only in cash. He shows that the CBDC could compel the monopoly bank to increase the deposit rate, which in turn increases financial inclusion and bank deposits. On the lending side, he assumes that the central bank lends to the commercial bank at a fixed rate, which fully determines the level of loans and disconnects the bank’s loans from its deposits.

While the two papers are restricted to qualitative analysis, we develop a flexible and rich framework more suitable for quantifying the effect of a CBDC with various design choices as the payment landscape evolves. In particular, our model captures a continuous spectrum of competitiveness as the number of banks slides from one (which captures monopolistic banking in Andolfatto 2018) to infinity (which captures perfectly competitive banking in Keister and Sanches 2019). Since the degree of the competitiveness of the banking sector is likely to vary across countries and is more realistically represented by imperfect competition, we let the data guide us on the degree of the competitiveness in our calibration exercise. In addition, we explicitly model cash, deposits and a CBDC as different payment instruments to facilitate different types of transactions. This allows us to discuss the design of the CBDC in terms of its acceptability and the evolution of the payment landscape that changes the relative acceptability of different payment instruments. Modelling the payment choices also makes it easier to use method-of-payment survey data to discipline our calibration exercise. Finally, modelling the reserve requirement allows us to discuss another issue regarding the design of a CBDC: whether it can be held as bank reserves.

Other papers formalize the discussion of a CBDC. Barrdear and Kumhof (2016) develop a rich DSGE model and estimate that issuing a CBDC could increase GDP by up to 3% through reducing real interest rates. Brunnermeier and Niepelt (2019) derive conditions under which the issuance of inside money and outside money are equivalent, even if inside money and outside money have liquidity or return differences. Their results imply that introducing a CBDC does not necessarily change macroeconomic outcomes. Davoodalhosseini (2018) studies a model where a CBDC
allows balance-contingent transfers as opposed to cash, but the CBDC is more costly for agents to use than cash (because cash offers anonymity). He shows that the coexistence of cash and the CBDC may not be optimal, because cash can serve as an outside option for agents, restricting the central bank’s power in implementing monetary policy.\footnote{Williamson (2019) argues that a CBDC can raise issues regarding independence of central bank and scarcity of assets eligible to back the CBDC. Using a model of a CBDC and banking, Dong and Xiao (2019) show that certain types of CBDC can be useful in implementing a negative policy rate. In a model where banks choose the level of project monitoring, Monnet et al (2019) show that a CBDC can lead banks to take less risk, resulting in a higher output and generally higher welfare. For policy discussion papers on CBDC, see Agur, Ari, and Dell’Ariccia (2019); Mancini-Griffoli et al. (2018); Chapman and Wilkins (2019); Davoodalhosseini and Rivadeneyra (2018); Engert and Fung (2017); Fung and Halaburda (2016); Kahn, Rivadeneyra, and Wong (2018).}

Our paper is related to the literature on transmission channels of monetary policy through the banking system. In their seminal work, Bernanke and Blinder (1988) propose a bank reserve channel. A higher interest rate increases the opportunity cost of holding reserves, leading banks to reduce their lending. This channel operates in our model and is quantitatively important for our results. Dreschler, Savov, and Schnabl (2016) propose a transmission channel based on banks’ market power in deposit markets. As the nominal interest rate increases, cash becomes more expensive, leading to an increase in demand for bank deposits. Because of market power, the banks increase the price of deposits (i.e., the spread between the nominal interest rate and interest on deposits). The mechanism through which a higher interest on a CBDC leads to a lower price and higher quantity of deposits in our model is similar to the mechanism through which an expansionary policy works in their paper, because these policies reduce the banks’ market power in the deposit market. The advantage of our model is that we model the payments arrangements explicitly, enabling us to calibrate the model directly using the payments data and to conduct various counterfactual analyses regarding the effects of CBDCs with different design choices.\footnote{Using a variation of Dreschler et al. model, Kurlat (2018) shows that banks’ market power raises the cost of inflation. Scharfstein and Sunderam (2016) propose another transmission channel based on banks’ market power in loan markets. As the nominal interest rate increases, the banks reduce their markup due to lower demand for loans. Quantitatively, Wang et. al (2019) estimate a structural banking model and show that the effect of banks’ market power in monetary policy transmission is sizable and comparable to that of bank capital regulations. More specifically, when the interest rate is low, the deposit channel is more important. Given that we focus on environments with low interest rates in our counterfactual exercise, their finding supports our assumption of banks’ market power in the deposit market.}

Finally, our paper contributes to the New Monetarist literature with financial intermediation. Berentsen, Camera, and Waller (2008) first incorporate banking into the Lagos and Wright (2005) model. Our model differs from Berentsen, Camera, and Waller (2008) in two dimensions. First, banks in our model engage in imperfect competition. Second, our banks create inside money by taking deposits. We provide
conditions under which a monetary equilibrium exists and is unique. We also show that the model can have multiple equilibria under certain parameters, implying that the banking sector can introduce instability into the economy. This point is also made in Gu et al. (2018). Dong et al. (2016) study a model of oligopolistic banks that face mismatch in timing of payments, and show that both bank profit and welfare are non-monotone in the number of banks.

This rest of the paper is organized as follows. Section 2 introduces the baseline model, where there is no CBDC. Section 3 derives the equilibrium of the baseline model. Section 4 considers a CBDC with two different designs and studies its implications on the equilibrium under each design. Section 5 calibrates the model and assesses the quantitative implications. Section 6 discusses two alternative designs of a CBDC and Section 7 discusses different extensions. Section 8 concludes. The omitted proofs are collected in Appendix A. Extensions and further discussions come in other appendices.

2 Environment

We adopt the modelling framework developed by Lagos and Wright (2005). Time is discrete and continues forever from 0 to infinity. There are four types of agents: a continuum of households with measure 2, a continuum of entrepreneurs with measure 1, a finite number of $N$ bankers, and the government. The discount factor from current to the next period is $0 < \beta < 1$. In each date $t$, agents interact sequentially in two stages: a frictionless centralized market (CM), and a frictional decentralized market (DM). There are two perishable goods: good $x$ in the CM and good $y$ in the DM.

Households are divided into two permanent types, buyers and sellers, each with measure 1. In the CM, both types work and consume $x$. Their labor $h$ is transformed into $x$ one-for-one. In the DM, buyers and sellers meet bilaterally and trade good $y$. Buyers want to consume $y$, which can be produced on the spot by the sellers. The utility from consumption is $u(y)$ with $u' > 0$, $u'(0) = \infty$, and $u'' < 0$. The disutility from production is normalized to $y$. Let $y^*$ be the socially efficient consumption, which solves $u'(y^*) = 1$. To summarize, buyers and sellers have period utilities given respectively by

$$U^B(x, y, h) = U(x) - h + u(y),$$
$$U^S(x, y, h) = U(x) - h - y.$$

Young entrepreneurs are born in the CM and will become old and die in the next CM. Entrepreneurs cannot work in the CM and care about only consumption when old. Young entrepreneurs are endowed with an investment opportunity that transforms $x$ current CM goods to $f(x)$ CM goods in the next period, where $f''(0) = \infty$. 

\[ f'(\infty) = 0, \quad f' > 0, \quad \text{and} \quad f'' < 0. \]

Given the preferences and endowment patterns, there are gains from trade between buyers and sellers, and between entrepreneurs and households. Specifically, buyers would like to consume DM goods produced by sellers, and entrepreneurs would like to borrow from households to invest in their investment opportunities. However, we assume that households and entrepreneurs lack commitment and cannot enforce debt repayment, so credit arrangement among them is not viable.

Like entrepreneurs, young bankers are born in the CM and will become old and die in the next CM. They cannot work in the CM and they consume when old. Unlike households and entrepreneurs, bankers can commit to repay and enforce payment (refer to Gu et al., 2018, for the discussion of the endogenous emergence of banks). As a result, banks can act as intermediaries between households and entrepreneurs to finance the investment projects. The bank has the option to issue illiquid or liquid deposits. Illiquid (time) deposits cannot be used to make payments, while liquid (checkable) deposits can.

The government issues fiat money, an intrinsically useless durable token that can be used as a means of payment too. Money supply \( M_t \) grows at a constant gross rate \( \mu > \beta \). The change in money supply is implemented as lump-sum transfers to (if \( \mu > 1 \)) or taxes on (if \( \mu < 1 \)) households. The government also stipulates a reserve requirement that \( \chi \) fraction of bank deposits must be held in cash.

As in Zhu and Hendry (2019), there are three types of meetings in the DM, depending on which of the two means of payment, cash and deposits, can be used for transactions. From a buyer’s perspective, with \( \alpha_1 > 0 \) probability, a buyer enters into a type 1 meeting, where only fiat money can be used. With \( \alpha_2 > 0 \) probability, a buyer enters into a type 2 meeting, where only bank deposits can be used. With \( \alpha_3 \geq 0 \) probability, a buyer enters into a type 3 meeting, where both can be used. The three types of meetings can be interpreted as follows. Type 1 meetings are transactions in local stores that do not have access to debit cards; type 2 meetings are online transactions where the buyers and sellers are spatially separated and can only use debit cards or bank transfers for payment; and type 3 meetings occur at local stores with point-of-sale (POS) machines, and hence both payment methods are accepted.

Agents in our model economy engage in the following activities. In every CM, young bankers issue deposits for two purposes. First, banks issue deposits to households in exchange for fiat money which can be kept as bank reserves. Second, banks offer loans to entrepreneurs in the form of deposits, which entrepreneurs use to buy \( x \) from buyers for investment. In the DM, buyers use a combination of cash and checkable deposits to purchase goods \( y \) from sellers. In the following CM, deposits and loans are settled. Entrepreneurs sell some of the investment output for cash or deposits to settle bank loans and retain the remaining output for their own consumption.
Having collected the loan repayments, bankers redeem the deposits held by the households and retain the remaining profit for their own consumption.

For the analysis in the main text, we assume that banks engage in a Cournot competition in the deposit market, but the lending market is perfectly competitive. We choose to focus on this structure because there is stronger evidence of imperfect competition in the deposit market than in the loan market (Dreschler et al., 2017; Wang et al., 2018). In Appendix B, we extend the model to the case where the lending market also features imperfect competition.

3 Equilibrium without a CBDC

In this section, we characterize the equilibrium of the economy without a CBDC. The strategy that we follow to solve the model is as follows. First, we characterize the household’s problem to derive the demand for cash and bank deposits. Second, we lay out the problem faced by the bank, incorporating the household demand for deposits, to derive the aggregate supply curve for loans. Third, we derive the demand curve for loans. Finally, we combine the supply and demand curves for loans to derive the market clearing loan rate, and combine the solutions to all agents’ problems to characterize the equilibrium deposit rate, real cash balances (by households and banks as reserves), and the quantities of deposits and loans.

3.1 Households

We first examine the buyer’s maximization problem, and then the seller’s problem. Let $W$ and $V$ be the value function of households in the CM and DM, respectively. In the following, we suppress the time subscript and use prime to denote variables in the next period.

In the CM, the buyer chooses consumption $x$, labor $h$, and the real cash, checkable and time deposit balances in the next period, $z'$, $d'$ and $b'$, to solve

$$W^B(z, d, b) = \max_{x, h, z', d', b'} \left\{ U(x) - h + \beta V^B(z', d', b') \right\}$$

st. $x = h + z + d + b + T - \frac{\phi}{\phi'} z' - \psi d' - \psi b'$,

where $\phi$ is the value of cash in terms of CM good, and $\psi d'$ and $\psi b'$ are the real value of checkable and time deposits today, respectively. The real return on cash balances is $\phi'/\phi - 1$, the real interest rate on checkable deposits is $1/\psi - 1$, and the real interest rate on time deposits is $1/\psi_b - 1$. Substitute out $h$ using the budget

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8The type of the DM meeting is not revealed until the start of the DM. Therefore, buyers hold a portfolio of fiat money and bank deposits.
equation and rewrite the buyer’s CM problem as

\[
W^B(z, d, b) = z + d + b + T + \max_x [U(x) - x] + \max_{z', d', b'} \left\{ -\frac{\phi}{\phi'} z' - \psi d' - \psi b' + \beta V^B(z', d', b') \right\}.
\]

Note that \(W^B(z, d, b)\) is linear in \(z, d,\) and \(b\). The first-order conditions (FOCs) are

\[
x : U'(x) = 1,
\]

\[
z' : \frac{\phi}{\phi'} \geq \beta V^B_1(z', d', b'), \text{ with equality if } z' > 0,
\]

\[
d' : \psi \geq \beta V^B_2(z', d', b'), \text{ with equality if } d' > 0,
\]

\[
b' : \psi b \geq \beta V^B_3(z', d', b'), \text{ with equality if } b' > 0,
\]

where the subscripts indicate the derivative with respect to corresponding arguments. Two standard results are that all buyers will choose the same portfolio \((z', d', b')\), and \(W^B(z, d, b)\) is linear in \((z, d, b)\) with \(W^B_1(z, d, b) = W^B_2(z, d, b) = W^B_3(z, d, b) = 1\).

The buyer’s DM problem is

\[
V^B(z, d, b) = \alpha_1 [u \circ Y(z) - P(z)] + \alpha_2 [u \circ Y(d) - P(d)] + \alpha_3 [u \circ Y(z + d) - P(z + d)] + W^B(z, d, b),
\]

where \(Y(\cdot)\) and \(P(\cdot)\) are the terms of trade (TOT) and represent the amount of good \(y\) being traded and the amount of payment, respectively. Note that the TOT depends on the buyer’s usable liquidity, which varies according to the type of meetings. We will discuss the determination of the TOT later.

Now we characterize the seller’s problem. A standard result in the literature is that the seller will choose to enter the DM with zero liquidity balances, or \(z' = d' = 0\), because he/she does not need (costly) liquidity in the DM. Using this result, we can formulate the seller’s CM problem as

\[
W^S(z, d, b) = \max_{x, h} \{U(x) - h + \beta V^S(0, 0, b')\}
\]

\[
\text{st. } x + \psi b' = h + z + d + b + T.
\]

Similar to the buyer’s maximization problem, we have \(U'(x) = 1\), and \(W^S\) is linear in \(z, d\) and \(b\). The seller’s DM problem is

\[
V^S(0, 0, b) = \alpha_1 [P(\bar{z}) - Y(\bar{z})] + \alpha_2 \left[ P(\bar{d}) - Y(\bar{d}) \right] + \alpha_3 \left[ P(\bar{z} + \bar{d}) - Y(\bar{z} + \bar{d}) \right] + W^S(0, 0, b),
\]
where \( \tilde{d} \) and \( \tilde{z} \) are the cash and deposit holdings of his trading partner.

The TOT are determined by buyers making take-it-or-leave-it offers. Let \( \mathcal{L} \) be the buyer’s total available liquidity, which is equal to \( z \) in type 1 meetings, \( d \) in type 2 meetings, and \( d + z \) in type 3 meetings. The buyer offers output-payment pair \((y, p)\) to

\[
\max_{y, p} [u(y) - p] \quad \text{s.t.} \quad p \geq y \text{ and } p \leq \mathcal{L},
\]

where the first constraint is the seller’s participation constraint and the second the liquidity constraint. The TOT as a function of the buyer’s total available liquidity \( \mathcal{L} \) are

\[
Y(\mathcal{L}) = P(\mathcal{L}) = \min(y^*, \mathcal{L}). \tag{2}
\]

In words, if the real value of available payment balances is enough to purchase the optimal amount, then the optimal amount is traded; otherwise, the buyer spends all available payment balances.

Combining the FOCs of buyers with respect to \( z' \) and \( d' \) in the CM and equations (1) and (2), we obtain the buyer’s demand for payment balances,

\[
\frac{\phi}{\beta \phi'} = \alpha_1 \lambda(z') + \alpha_3 \lambda(z' + d') + 1, \tag{3}
\]

\[
\frac{\psi}{\beta} = \alpha_2 \lambda(d') + \alpha_3 \lambda(z' + d') + 1, \tag{4}
\]

where \( \lambda(\mathcal{L}) = \max[u'(\mathcal{L}) - 1, 0] \) is the liquidity premium. Note that the demand for cash and deposits is positive under both the assumptions that \( u'(0) = \infty, \alpha_1 > 0 \) and \( \alpha_2 > 0 \). At the steady state, \( \phi/\phi' = \mu \), and the demand for liquid balances \((z, d)\) is given by

\[
\mu = \alpha_1 \lambda(z) + \alpha_3 \lambda(z + d), \tag{5}
\]

\[
\frac{\psi}{\beta} - 1 = \alpha_2 \lambda(d) + \alpha_3 \lambda(z + d), \tag{6}
\]

where \( \mu = \mu/\beta - 1 \) is the nominal interest rate using the Fisher’s equation. These two equations are intuitive. The first one states that the marginal cost of holding one unit of cash (the left-hand side) should be equal to its marginal benefit (the right-hand side), which comes from the fact that the buyer can use the marginal unit of cash in type 1 and type 3 meetings to derive \( \lambda(z) \) and \( \lambda(z + d) \) additional units of utility, respectively, from consumption. The second equation has a similar interpretation for checkable deposits.

Here, (5) defines the aggregate demand for cash balances \( z \) as a function of \( d \). Given this, (6) defines the aggregate inverse demand function for checkable deposits, \( \psi = \Psi(d) \). The inverse demand function has the following properties: \( \Psi(0) = \infty \),
\[ \Psi(d) = \beta \text{ for } d \geq y^*, \quad \Psi'(d) < 0 \text{ for } d < y^*, \quad \text{and } \Psi'(d) = 0 \text{ for } d \geq y^* . \]

Finally, note that the demand for time deposits is separate from the demand for liquid assets and is given by

\[ \psi_b = \beta. \quad (7) \]

In words, since time deposits have no liquidity value, the rate of return of time deposits must compensate for discounting across time.

### 3.2 Bankers

Banks issue two types of deposits, checkable deposits \((d)\) and time deposits \((b)\), to households, and invest in two assets, cash \((z)\) and loans \((\ell)\). The \(N\) bankers engage in Cournot competition in the deposit market and are fully competitive in the loan market (where they take the loan rate, \(\rho\), as given). Bankers face a reserve requirement. \(^{10}\) At the end of each CM, the real value of a banker’s cash holding must be at least \(\chi\) fractions of the total checkable deposits, where \(\chi\) is a policy parameter set by the government.

Formally, each banker \(j\) solves the following maximization problem, taking the price of time deposits \((\psi_b = \beta)\), the market loan rate \((\rho)\) and other banks’ checkable deposit quantities \((D_{-j} = \sum_{i \neq j} d_i)\) as given:\(^{11}\)

\[
\max_{z_j, d_j, b_j} \left\{ (1 + \rho) \ell_j + \frac{z_j}{\mu} - d_j - b_j \right\} \quad (8)
\]

\[ \text{s.t. } \ell_j + z_j = \Psi(D_{-j} + d_j) d_j + \beta b_j, \]

\[ z_j \geq \chi \Psi(D_{-j} + d_j) d_j. \]

The banker maximizes consumption in the second period of life. The banker receives the repayment of loans (principal plus interest) from the entrepreneurs, \((1 + \rho) \ell_j\), the inflation-adjusted value of money holdings, and redeems the deposits \(d_j\) and \(b_j\).

The maximization problem is subject to two constraints. The first constraint is the balance sheet identity of the bank at the end of CM. The right-hand side is the liability, the real value of checkable and time deposits. The left-hand side is the asset, which includes money and loans. The second constraint reflects the reserve requirement on checkable deposits.

\(^{9}\) From equations (5) and (6), \(\Psi'(d) = \alpha_2 \beta \lambda'(d) + \frac{\alpha_3 \beta \lambda'(z + d)}{\alpha_1 \lambda(z) + \alpha_3 \lambda(z + d)} \leq 0\), and \(< 0\) if \(d < y^*\).

\(^{10}\) For simplicity, we consider cash reserves in the benchmark model. As discussed in Section 7, allowing interest-bearing reserves will not change the main findings of the model.

\(^{11}\) Note that banks cannot affect the price of time deposits, which is fixed at \(\psi_b = \beta\) from the household’s problem. To ease notation, we suppress the dependence of \(\Psi\) on \(\iota\).
In the following, we analyze the result from Cournot competition contingent on the competitive loan rate \( \rho \). We will focus on the symmetric Cournot equilibrium where each makes the same choice. Denote the quantities of checkable deposits, time deposits, cash holdings and loans supplied by each bank as \( d(\rho), b(\rho), z(\rho) \) and \( \ell(\rho) \), respectively. Note that if \( 1 + \rho > 1/\beta \), then the bank can make unlimited profits by issuing time deposits and invest in loans. As a result, in equilibrium, \( 1 + \rho \leq 1/\beta \).

We can carry out the analysis in four cases depending on the magnitude of \( 1 + \rho \) relative to the return on cash \( 1/\mu \) and the return on time deposits \( 1/\beta \).

**Case 1: cash gives a higher return than loans.** If \( 1 + \rho < 1/\mu \), then the bank does not invest in loans \( (\ell = 0) \) because its return is dominated by cash. The bank’s asset side consists of only cash, and the reserve requirement does not bind. In this case, the bank’s problem can be rewritten as

\[
\max_{z_j, d_j, b_j} \left\{ \frac{z_j}{\mu} - d_j - b_j \right\}
\]

s.t. \( z_j = \Psi(D_{-j} + d_j) + \beta b_j \).

Using the balance sheet identity to eliminate \( z_j \), one can rewrite the problem as

\[
\max_{d_j, b_j} \left\{ \frac{\Psi(D_{-j} + d_j)d_j + \beta b_j}{\mu} - d_j - b_j \right\}
\]

The first-order condition for \( d_j \) is

\[
d_j : \Psi'(D_{-j} + d_j)d_j + \Psi(D_{-j} + d_j) = \mu.
\]

The derivative with respect to \( b_j \) is \( \beta/\mu - 1 < 0 \), implying \( b_j = 0 \); i.e., the bank does not issue time deposits because the return on cash, \( 1/\mu \) is less than the deposit rate required by the household, \( 1/\beta \). In a symmetric equilibrium, each bank issues \( \mu \) units of checkable deposits and invests only in cash \( (z = d_\mu) \), where \( d_\mu \) solves

\[
\Psi'(Nd_\mu)d_\mu + \Psi(Nd_\mu) = \mu. \tag{9}
\]

**Case 2: cash and loans have the same return.** If \( 1 + \rho = 1/\mu \), then the bank is indifferent between investing in cash and reserves as long as the reserve requirement does not bind. The supply of (checkable and time) deposits remains the same as in case 1. The supply of loans for each bank lies on the interval \([0, (1 - \chi)\Psi(Nd_\mu)d_\mu]\).

**Case 3: cash is dominated in return by loans.** If \( 1 + \rho > 1/\mu \), then the reserve requirement binds, and we can rewrite the bank’s problem by plugging the constraints into the objective function in the banker’s problem (8) as

\[
\max_{d_j, b_j} \left\{ (1 + \rho)[(1 - \chi)\Psi(d_{-j} + d_j)d_j + \beta b_j] + \frac{\chi\Psi(d_{-j} + d_j)d_j}{\mu} - d_j - b_j \right\}
\]
The first-order condition for \( d_j \) is

\[
d_j : \Psi'(d_{-j} + d_j)d_j + \Psi(d_{-j} + d_j) = \frac{1}{(1 + \rho)(1 - \chi) - \chi/\mu}.
\]

In a symmetric equilibrium, the supply of checkable deposits for each bank, \( d \), solves

\[
\Psi'(Nd)d + \Psi(Nd) = \frac{1}{(1 + \rho)(1 - \chi) - \chi/\mu}, \tag{10}
\]

where the denominator is the rate of return on the bank’s assets, which is a weighted average of the return on cash and loans. In terms of time deposits, the first-order derivative with respect to \( b_j \) is \((1 + \rho)\beta - 1\). We can further divide case 3 into two sub-cases depending on the relative magnitudes of \( 1 + \rho \) and \( \beta \).

**Case 3a.** If \( 1/\mu < 1 + \rho < 1/\beta \), then banks will not issue time deposits because the required return on time deposits exceeds the return on loans. The bank splits its checkable deposits between cash reserves in the amount of \( z = \chi \Psi(Nd)d \) and loans in the amount of \( \ell = (1 - \chi)\Psi(Nd)d \).

**Case 3b.** If \( 1 + \rho = 1/\beta \), then banks will start issuing time deposits. The amount of checkable deposits is given by \( d_\beta \), which solves equation (10) at \( \rho = 1/\beta - 1 \):

\[
\Psi'(Nd_\beta)d_\beta + \Psi(Nd_\beta) = \frac{1}{(1 - \chi)/\beta - \chi/\mu}, \tag{11}
\]

The amount of cash reserves is \( z_\beta = \chi \Psi(Nd_\beta)d_\beta \). The amount of loans supplied by each bank is \( \ell_\beta = [(1 - \chi)\Psi(Nd_\beta)d_\beta, \infty) \). To finance the loans, the bank uses both checkable deposits \( d_\beta \) and time deposits \( b = [\ell_\beta - (1 - \chi)\Psi(Nd_\beta)d_\beta]/\beta \).

Proposition 1 summarizes the result on checkable deposits from Cournot competition conditional on the loan rate \( \rho \). Throughout the paper, we assume the following holds.

**Assumption 1**

a) Given any \( d_{-j} \in [0, y^*) \) and \( \kappa > \beta \), either there exists a unique \( d_j > 0 \) such that \( \Psi'(D_{-j} + d) d + \Psi(D_{-j} + d) \geq \kappa \) if \( d \leq d_j \), or \( \Psi'(D_{-j} + d) d + \Psi(D_{-j} + d) < \kappa \) for all \( d \geq 0 \). b) In addition, \( \Psi'(Nd)d + \Psi(Nd) \) decreases with \( d \) on \([0, y^*/N)\).

Part (a) of this assumption states that \( \Psi'(D_{-j} + d) d + \Psi(D_{-j} + d) - \kappa \) as a function of \( d \) crosses the horizontal axis from the above and at most once. We need part (a) to ensure that the best response of banker \( j \) to any amount of deposits (less than \( y^* \)) created by other banks is unique. Part (b) ensures that there is at most one symmetric Nash equilibrium of the Cournot game.\(^{12}\)

\(^{12}\)One can show that Assumption 1 holds if \( u(y) = y^{\frac{1 - \sigma}{1 - \sigma}} \), with \( \sigma < 1 \) and \( \alpha_3 = 0 \). By continuity, this would hold if \( \alpha_3 \) is sufficiently small.
**Proposition 1** Under Assumption 1, there is a unique symmetric pure strategy equilibrium in the Cournot game for $\rho \leq 1/\beta - 1$, where each bank’s supply of checkable deposits $d(\rho)$ solves:

$$
\Psi'(Nd)d + \Psi(Nd) = \min \left\{ \mu, \frac{1}{(1+\rho)(1-\chi) + \chi/\mu} \right\}. \tag{12}
$$

**Proof.** See Appendix A. ■

Conditional on $\rho$, the loan supply $\ell(\rho)$ and cash holding $z(\rho)$ are given by

$$
\ell(\rho) = \begin{cases} 
0 & \text{if } 1 + \rho < 1/\mu, \\
[0, (1-\chi)d(\rho)\Psi(Nd(\rho))] & \text{if } 1 + \rho = 1/\mu, \\
(1-\chi)d(\rho)\Psi(Nd(\rho)) & \text{if } 1/\mu < 1 + \rho < 1/\beta, \\
[(1-\chi)d_\beta\Psi(Nd_\beta), \infty] & \text{if } 1 + \rho = 1/\beta,
\end{cases}
$$

and

$$
z(\rho) = \begin{cases} 
d(\rho)\Psi(Nd(\rho)) & \text{if } 1 + \rho < 1/\mu, \\
[\chi d(\rho)\Psi(Nd(\rho)), d(\rho)\Psi(Nd(\rho))] & \text{if } 1 + \rho = 1/\mu, \\
\chi d(\rho)\Psi(Nd(\rho)) & \text{if } 1 + \rho > 1/\mu,
\end{cases}
$$

and the implied price and rate for checkable deposits are

$$
\psi(\rho) = \Psi(Nd(\rho)),
$$

$$
r(\rho) = 1/\Psi(Nd(\rho)) - 1.
$$

The supply of time deposits $b(\rho)$ is described by

$$
b(\rho) = \begin{cases} 
0 & \text{if } 1 + \rho < 1/\beta, \\
[0, \infty] & \text{if } 1 + \rho = 1/\beta.
\end{cases}
$$

The aggregate supply of checkable deposits and loans are defined as $D(\rho) = Nd(\rho)$ and $L(\rho) = N\ell(\rho)$.

### 3.3 Entrepreneurs

The entrepreneurs are price takers and hence decide their demand for loans given the loan rate $\rho$. Their problem is

$$
\max_{\ell} \left\{ f(\ell) - (1 + \rho)\ell \right\}. \tag{13}
$$
This implies that the inverse loan demand for an entrepreneur is $f'(\ell) = 1 + \rho$, which defines the aggregate inverse loan demand function,

$$L^d(\rho) = f'^{-1}(1 + \rho).$$

Obviously $L^d(\cdot)$ is a decreasing function, i.e., the demand for loan decreases with the loan rate. Note also that the demand for loans is always positive because $f'(0) = \infty$ but approaches zero as $\rho$ goes to $\infty$.

### 3.4 Equilibrium

Now we combine the problems of households, banks and entrepreneurs to characterize the equilibrium of the economy. In particular, we will combine the loan demand curve derived from the entrepreneur’s problem and the loan supply curve derived from the bank’s problem to determine the equilibrium market loan rate, $\rho$. After determining $\rho$, we can use the equilibrium loan rate to derive other variables.

In figure 1, we plot the loan supply and loan demand curves in the $[(1 + \rho), L]$ space. The loan demand curve is strictly decreasing as shown by the solid blue line. The loan supply curve is illustrated by the solid black line. If $1 + \rho < 1/\mu$, then the bank issues only checkable deposits and invests only in cash so the loan supply is zero. If $1 + \rho = 1/\mu$, then banks are indifferent between lending and holding cash as long as the reserve requirement does not bind. The loan supply curve is vertical between 0 and $(1 - \chi)Nd\mu\Psi(Nd\mu)$. The loan supply curve is strictly increasing on $(1/\mu, 1/\beta)$. When $1 + \rho = 1/\beta$, it is vertical again. Note that when $1 + \rho < 1/\beta$, the loans are financed only by checkable deposits, and when $1 + \rho = 1/\beta$, supply beyond the amount of checkable deposits is supported by time deposits.

The demand and the supply have one and only one intersection, which implies the existence and uniqueness of the steady state equilibrium (we will use * to denote equilibrium values). After acquiring the equilibrium loan rate $\rho^*$, we can plug it into the Cournot solution in Proposition 1 to get equilibrium price of demand deposits $\psi^*$, the equilibrium quantity of checkable deposits $d^*$ and loans $\ell^*$. Notice that when entrepreneurs’ productivity (or the loan demand curve) is low, then the equilibrium loan rate is equal to the return on cash, i.e, $\rho^* = 1/\mu - 1$. In this case, the reserve requirement is loose and banks hold excess reserves.

**Proposition 2** Under Assumption 1, there exists at least one monetary equilibrium. If, in addition, $D\Psi(D)$ is increasing in $D$, then the equilibrium is unique.$^{13}$

**Proof.** See Appendix A.

$^{13}$See Appendix B for a discussion on equilibrium multiplicity.
4 Effects of a CBDC

This section analyzes the effects of a CBDC on bank intermediation. We will first consider a baseline design: it bears interest, is a perfect substitute for checkable deposits as a means of payment (in type 2 and type 3 meetings), and is not accessible to banks. The supply of CBDC grows at a gross rate \( \mu_e \) and pays a nominal interest \( i_e \). Both \( \mu_e \) and \( i_e \) are policy instruments of the central bank. In subsection 4.4 we modify the baseline design by allowing the bank to use CBDC as reserves. In Section 6, we will consider other alternative designs of CBDC.\(^{14}\)

With a CBDC, the firm’s problem remains the same as before. The household’s and bank’s problems will be modified. In the following, we will discuss the household’s and bank’s problems and then the effects on equilibrium prices and quantities of deposits and loans.

4.1 Household’s Problem

With a CBDC, the household’s problem changes to

\[
W^B(z, z_e, d, b) = z + z_e + d + b + T + \max_x [U(x) - x] \\
+ \max_{z', z'_e, d', b'} \left\{ -\frac{\phi'}{\phi} z' - \frac{\phi_e'}{\phi_e} \frac{1}{1 + i_e} z'_e - \psi d' - \psi b' + \beta V(z', z'_e, d', b') \right\},
\]

\(^{14}\)There are several design dimensions that we can consider using our model: whether a CBDC is interest bearing, in what types of meetings a CBDC can be used, and whether banks can hold CBDC to meet reserve requirements.
where $z_e$ is the real balance of the CBDC, and $\phi_e$ is the price of the CBDC in terms of the numeraire good. Note that $\phi_e$ can be different from $\phi$ in the equilibrium because the CBDC may pay interest or have a different growth rate.

Following steps similar to the case without a CBDC, one can obtain the steady-state household demand for all three payment instruments as functions of the price of deposit $\psi$ and policy rates $(i_e, \mu_e)$:

$$i = \frac{\mu}{\beta} - 1 \geq \alpha_1 \lambda(z) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } z > 0,$$

$$\frac{\psi}{\beta} - 1 \geq \alpha_2 \lambda(d + z_e) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } d > 0,$$

$$\frac{\mu_e}{\beta(1 + i_e)} - 1 \geq \alpha_2 \lambda(d + z_e) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } z_e > 0.$$  

From the last two equations, if $\psi > \frac{\mu_e}{1 + i_e}$, then the demand for bank deposits is 0. If $\psi < \frac{\mu_m}{1 + i_m}$, then the demand for the CBDC is 0: since the CBDC and checkable deposits are perfect substitutes as a means of payment, the household holds only the instrument that gives the higher rate of return. From (14) to (16), we can drive the inverse demand function for checkable deposits with a CBDC as follows (and denote it as $\hat{\Psi}$ to distinguish it from the demand for checkable deposits without a CBDC, $\Psi$).

$$\hat{\Psi}(D) = \begin{cases} 
\left[\frac{\mu_e}{1 + i_e}, \infty\right) & D = 0, \\
\frac{\mu_e}{1 + i_e} & D \in \left[0, \Psi^{-1}\left(\frac{\mu_e}{1 + i_e}\right)\right], \\
\Psi(D) & D \geq \Psi^{-1}\left(\frac{\mu_e}{1 + i_e}\right).
\end{cases}$$

The induced change in the demand for checkable deposits is illustrated in Figure 2.

### 4.2 Bank’s Problem

With a CBDC, the formulation of the bank’s problem remains largely the same as the case without a CBDC: we simply replace $\Psi(D)$ with $\hat{\Psi}(D)$ in (8). Similar to the analysis in the previous section, we will examine the solution to the Cournot competition for all possible values of $1 + \rho \in [0, 1/\beta]$. First note that for a given $\rho$, if $(1 + i_e)/\mu_e \leq 1 + r(\rho)$, then the rate of return on the CBDC is too low (lower than the deposit rate offered by the bank when there is no CBDC) to affect the demand for checkable deposits, and introducing the CBDC does not affect the economy. The threat from the CBDC to checkable deposits comes into effect only when $(1 + i_e)/\mu_e > 1 + r(\rho)$. As $r(\rho)$ increases in $\rho$, a CBDC with a certain return
Figure 2: Inverse Demand for Checkable Deposits

Notes. $\Psi(D)$ is the inverse demand for checkable deposits without CBDC, and $\hat{\Psi}(D)$ is the inverse demand for checkable deposits with CBDC.

$(1 + i_e)/\mu_e$ may affect the result only for a subset of $\rho$ values.\footnote{From equation (12), under Assumption 1, $\psi$ is weakly decreasing in $\rho$ and strictly decreasing in $\rho$ for $\rho > 1/\mu - 1$. The response of the implied return on checkable deposits $r(\rho)$ is the opposite. Intuitively, under the Cournot competition, a higher return on assets is passed on to the deposit rate, albeit incompletely.}

In the following, we analyze the conditional deposit and loan supply curves with a CBDC in four separate cases depending on the return on the CBDC, $(1 + i_e)/\mu_e$, relative to four threshold values: $1 + r_\mu$, where $r_\mu$ is $r(\rho)$ evaluated at $\rho = 1/\mu - 1$; $1 + r_\beta$, where $r_\beta$ is $r(\rho)$ evaluated at $\rho = 1/\beta - 1$; $1/\mu$; and $(1 - \chi)/\beta + \chi/\mu$. The first two define the range of rate on checkable deposits without the CBDC, and the second two define the range of return on assets funded by checkable deposits. Among the four values, it is straightforward that $1 + r_\mu$ is the lowest, and $1/\mu < (1 - \chi)/\beta + \chi/\mu$ is the highest. However, the rank between $1/\mu$ and $1 + r_\beta$ depends on the parameters of the economy. We will focus on the case where $1/\mu < 1 + r_\beta$ and discuss the effect of the CBDC in five regions marked by the four threshold values.

**Case 1.** If $(1 + i_e)/\mu_e < 1 + r_\mu$, then the CBDC does not affect the economy for any $\rho \in (0, 1/\beta]$, and the conditional deposit and loan supply curves remain the same as in the case without a CBDC. Note that without a CBDC, the lowest rate for checkable deposits is given by $r_\mu$.\footnote{When $1 + \rho < 1/\mu$, banks invest only in cash, so the return of the bank’s asset is bounded below by $1/\mu$, and the deposit rate without a CBDC remains the same as when $1 + \rho = 1/\mu$.} If the CBDC rate is below this value, then it will not affect the economy irrespective of the value of $\rho$.

**Case 2.** If $1 + r_\mu < (1 + i_e)/\mu_e < 1/\mu$, then the CBDC affects the economy if and
only if (iff) $\rho < \bar{\rho}$, where $\bar{\rho}$ solves

$$1 + r(\bar{\rho}) = \frac{1 + i_e}{\mu_e}. \tag{17}$$

In this case, the conditional checkable demand deposit and loan supply curves will adjust for low values of $\rho$ while coinciding with the curves without a CBDC for higher values of $\rho$ (remember that $r(\rho)$ increases in $\rho$). In particular, if $\rho > \bar{\rho}$, then the rate for checkable deposits without a CBDC exceeds the CBDC rate and the CBDC does not pose a threat to checkable deposits. If $\rho < \bar{\rho}$, then the CBDC rate is higher than the rate for checkable deposits without a CBDC and banks must adjust their decisions. Banks must match the CBDC rate and offer $(1 + i_e)/\mu_e - 1$ on their checkable deposits, and they lose the ability to affect the price and rate of checkable deposits. Given that $(1 + i_e)/\mu_e < 1/\mu$, the marginal net benefit of an additional unit of checkable deposit is positive and banks are willing to supply checkable deposits to fully satisfy the household demand for electronic liquidity. The equilibrium aggregate quantity of checkable deposits is fully determined by the CBDC rate and given by

$$D_e = \Psi^{-1} \left( \frac{\mu_e}{1 + i_e} \right). \tag{18}$$

In a symmetric Cournot equilibrium, each bank supplies $d_e = D_e/N$ checkable deposits.

**Case 3.** If $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$, then, similar to case 2, the CBDC affects the economy iff $\rho < \bar{\rho}$. The difference is that there are two possible adjustments. If $\rho < \rho < \bar{\rho}$, where $\bar{\rho}$ is the solution to \footnote{At $\rho = \bar{\rho}$, the rate paid by the bank on checkable deposits (which is the CBDC rate) is equal to the return from its assets, and therefore making zero profit from checkable deposits. Note that $\bar{\rho} < \bar{\rho}$.}

$$(1 - \chi)(1 + \rho) + \chi \frac{1}{\mu} = \frac{1 + i_e}{\mu_e}, \tag{19}$$

then the marginal net benefit of an additional unit of checkable deposits is positive and each bank supplies $d_e = D_e/N$ checkable deposits. If $\rho < \bar{\rho}$, then the return from assets is insufficient to cover the promised deposit rate, and banks will shut down their checkable deposits and loan businesses.

If the CBDC rate continues to increase such that $(1 + i_e)/\mu_e > 1 + r_\beta$, then the CBDC rate affects the economy for all $\rho$ values. We can further divide the discussion in two cases.
Case 4. $1 + r_\beta < (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu$. Similar to case 3, banks can respond in two ways. If $\rho < \bar{\rho}$, then the required checkable deposit rate is higher than the return on the bank’s assets, and the bank does not offer checkable deposits and loans. If $\rho > \bar{\rho}$, then the bank will match the CBDC rate and supply checkable deposits $d_e$.

Case 5. $(1 + i_e)/\mu_e > (1 - \chi)/\beta + \chi/\mu$. In this case, the required rate for checkable deposits is higher than the highest possible return on the bank’s assets. As a result, the bank does not offer any checkable deposits for all $\rho \in (0, /\beta - 1]$. For $\rho = 1/\beta - 1$, the bank could still offer time deposits.

Proposition 3 summarizes how the result of the Cournot competition responds to a CBDC.

Proposition 3 Suppose Assumption 1 holds and $1 + r_\beta > 1/\mu$. With a CBDC, conditional on the loan rate $\rho$, each bank’s supply of checkable deposits $\hat{d}(\rho)$ from the Cournot competition is given as follows:

1. If $(1 + i_e)/\mu_e < 1 + r_\mu$, then the CBDC does not affect the economy for any $\rho \in (0, 1/\beta]$, and $\hat{d}(\rho) = d(\rho)$.

2. If $1 + r_\mu < (1 + i_e)/\mu_e < 1/\mu$, then the CBDC affects the economy iff $\rho < \bar{\rho}$, and

$$\hat{d}(\rho) = \begin{cases} d_e > d(\rho) & \text{if } \rho < \bar{\rho}, \\ d(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases}$$

3. If $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$, then the CBDC affects the economy iff $\rho < \bar{\rho}$, and

$$\hat{d}(\rho) = \begin{cases} 0 & \text{if } \rho < \bar{\rho}, \\ [0, d_e] & \text{if } \rho = \bar{\rho}, \\ d_e > d(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases}$$

4. If $1 + r_\beta < (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu$, then the CBDC rate affects the economy for all $\rho \in (0, 1/\beta - 1]$, and

$$\hat{d}(\rho) = \begin{cases} 0 & \text{if } \rho < \bar{\rho}, \\ [0, d_e] & \text{if } \rho = \bar{\rho}, \\ d_e > d(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases}$$

5. If $(1 + i_e)/\mu_e > (1 - \chi)/\beta + \chi/\mu$, then the CBDC rate affects the economy for
all \( \rho \in (0, 1/\beta - 1] \), and
\[
\hat{d}(\rho) = 0 \text{ for all } \rho \in (0, 1/\beta - 1].
\]

**Proof.** See Appendix A. ■

The effect of a CBDC on the supply of loans is as follows. First, if \((1 + i_e)/\mu_e < 1 + r_\mu\), then the CBDC does not affect the loan supply and \(\hat{\ell}(\rho) = \ell(\rho)\). If \(1 + r_\mu < (1 + i_e)/\mu_e < 1/\mu\), then
\[
\hat{\ell}(\rho) = \begin{cases}
0 & \text{if } \rho < 1/\mu, \\
\left[0, (1 - \chi) \frac{\mu_e}{1 + i_e} d_e\right] & \text{if } \rho = 1/\mu, \\
(1 - \chi) \frac{\mu_e}{1 + i_e} d_e > \ell(\rho) & \text{if } \rho \in (1/\mu, \bar{\rho}], \\
(1 - \chi) \psi(\rho)d(\rho) = \ell(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1), \\
[(1 - \chi) \psi(\rho)d(\rho), \infty) = \ell(\rho) & \text{if } \rho = 1/\beta - 1.
\end{cases}
\]

If \(1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta\), then
\[
\hat{\ell}(\rho) = \begin{cases}
0 & \text{if } \rho < \bar{\rho}, \\
\left[0, (1 - \chi) \frac{\mu_e}{1 + i_e} d_e\right] & \text{if } \rho = \bar{\rho}, \\
(1 - \chi) \frac{\mu_e}{1 + i_e} d_e > \ell(\rho) & \text{if } \rho \in (\bar{\rho}, \bar{\rho}], \\
(1 - \chi) \psi(\rho)d(\rho) = \ell(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1), \\
[(1 - \chi) \psi(\rho)d(\rho), \infty) = \ell(\rho) & \text{if } \rho = 1/\beta - 1.
\end{cases}
\]

If \(1 + r_\beta < (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu\), then
\[
\hat{\ell}(\rho) = \begin{cases}
0 & \text{if } \rho < \bar{\rho}, \\
\left[0, (1 - \chi) \frac{\mu_e}{1 + i_e} d_e\right] & \text{if } \rho = \bar{\rho}, \\
(1 - \chi) \frac{\mu_e}{1 + i_e} d_e > \ell(\rho) & \text{if } \rho \in (\bar{\rho}, 1/\beta - 1), \\
[(1 - \chi) \frac{\mu_e}{1 + i_e} d_e, \infty) & \text{if } \rho = 1/\beta - 1.
\end{cases}
\]

If \((1 + i_e)/\mu_e > (1 - \chi)/\beta + \chi/\mu\), then
\[
\hat{\ell}(\rho) = \begin{cases}
0 & \text{for all } \rho \in (0, 1/\beta - 1), \\
[0, \infty) & \text{if } \rho = 1/\beta - 1.
\end{cases}
\]

### 4.3 Equilibrium

The intersection of the bank’s new loan supply curve \(\hat{d}(\rho)\) and the firm’s loan demand curve (which remains the same as without a CBDC) will determine the equilibrium
loan rate, and the equilibrium loan rate can then be plugged into the functions $\hat{d}(\rho)$ and $\hat{\ell}(\rho)$ to determine the general equilibrium quantities.

Figure 3 plots the loan demand and loan supply curves before and after introducing a CBDC. We will focus on the case where $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$. As in previous sections, we focus on the case where $D\Psi(D)$ is increasing. The blue curve is the loan demand, the black curve is the loan supply without the CBDC, and the red curve is the loan supply with the CBDC. The blue and black curves intersect at point a, which represents the equilibrium without the CBDC. After the CBDC is introduced, the loan demand curve is not affected, whereas the loan supply curve is changed to the red line.

As stated by Proposition 3, the new loan supply curve has the following properties. If $\rho < \bar{\rho}$, the loan supply is 0. This is because the loan rate is too low and is insufficient to cover the checkable deposit rate at $(1 + i_e)/\mu_e$, and it is not profitable for banks to issue checkable deposits or loans. If $\rho = \bar{\rho}$, the loan rate is just high enough to cover the cost of issuing deposits, and banks are indifferent between a range of deposits $\hat{d} \in [0, d_e]$. If $\rho \in (\bar{\rho}, \rho)$, then issuing deposits is profitable and the bank issues enough deposits to meet the demand for electronic liquidity at the CBDC rate ($\hat{d} = d_e$). With the binding reserve requirement, the loan supply is at $\hat{\ell} = \ell_e = (1 - \chi)\mu_e / (1 + i_e)d_e$. If $\rho > \bar{\rho}$, then the deposit rate offered by banks in the absence of a CBDC is higher than the CBDC rate, and the CBDC does not affect the economy, so the loan supply curves with and without a CBDC coincide.

As the CBDC rate increases, the economy goes through four regimes. As shown in figure 3(a), regime 1 happens when $i_e$ is low so that the return on the CBDC, $(1 + i_e)/\mu_e - 1$, falls below the equilibrium rate for checkable deposits, $r^*$ (or equivalently, $\bar{\rho} < \rho^*$). As the CBDC rate increases, both $\rho$ and $\bar{\rho}$ increase. However, as long as $i_e$ is lower than $i_{e1}$, which solves $(1 + i_e)/\mu_e - 1 = r^*$, the economy remains in regime 1 and changes in the CBDC rate do not affect the economy.

As $i_e$ increases to the point $i_{e1}$ (or equivalently, $\bar{\rho} = \rho^*$), a further increase in $i_e$ pushes the economy into regime 2, as shown figure 3(b). In this regime, compared with the case without a CBDC, the CBDC forces up the deposit rate and induces a higher demand for electronic payment balances. Given that the marginal benefit from one additional unit of deposits is positive, the bank is willing to fully absorb the higher demand from households and the CBDC is not used in equilibrium. The higher level of checkable deposits also translates into higher supply of loans and a lower equilibrium loan rate. In regime 2, the amount of checkable deposits is given by $\hat{d}^* = d_e$, which solves $\Psi(ND_e) = (1 + i_e)/\mu_e$. As the CBDC rate rises, the equilibrium

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We focus on this case because it covers all three ways that the bank responds to a CBDC: no change of actions, setting the deposit rate to match the CBDC rate, or shutting down the checkable and loan businesses. In this case, $\rho$ and $\bar{\rho}$ are well defined and lie within the interval $[1/\mu - 1, 1/\beta - 1]$. 

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Figure 3: Effects of a CBDC

Notes. (1) The blue curve is the loan demand, the black curve is the loan supply without a CBDC, and the red curve is the loan supply with a CBDC. Note that the red curve joins the black curve for $\rho > \bar{\rho}$. (2) The figure illustrates the effect of a CBDC when $1/\mu < (1 + i_e)/\mu_e < 1 + \beta$. 

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rate and quantity of checkable deposits continue to rise. On the asset side, the bank
invests \((1 − χ)\) fraction of deposits on loans so \(\hat{\ell} = \ell_e = \frac{(1 − χ)\mu_e}{1 + i_e} d_e\). As the
loan supply increases, the loan rate decreases (the equilibrium loan rate is given by
the marginal product of firm investment \(\hat{ρ}^* = f'(N\hat{ℓ}^*) - 1\). The increasing deposit
rate and falling loan rate reduce the bank’s profit margin. At some point \(i_{e2}\), which
solves \((1 − χ)f'(N\ell_e) + χ/\mu = (1 + i_e)/\mu_e\) (or equivalently, \(\rho = f'(N\ell_e) - 1\)), the
bank makes zero profit.

As \(i_e\) increases beyond \(i_{e2}\), the economy enters into regime 3, illustrated in figure 3(c).
In this regime, any increase in \(i_e\) induces the deposit and loan rates to go up, and the
quantities of deposits and loans to decrease. As \(i_e\) increases beyond \(i_{e2}\), the bank can
only match the higher required deposit rate by scaling down its lending and deposit
activities. Note that the household’s electronic liquidity balance increases with
the CBDC rate. With the bank scaling down deposit taking, households increase
their CBDC holdings. As long as the CBDC rate is lower than \(i_{e3}\), which solves
\((1 − χ)\hat{ρ}^* + χ/\mu = (1 + i_e)/\mu_e\) (or equivalently, \(\bar{ρ} = \hat{ρ}^*\)), the equilibrium with a CBDC
still features higher loans and deposits than the equilibrium without a CBDC.

Finally, as \(i_e\) increases beyond \(i_{e3}\), the return on the CBDC exceeds the equilibrium
return on the bank’s assets without a CBDC, and the economy enters into regime 4 (see Figure 3(d)). In this regime, the CBDC causes disintermediation relative to
the case without a CBDC. As the CBDC rate continues to rise, the response of the
economy is similar to regime 3: deposits and loans rates will increase, deposits and
loans shrink, and households hold higher electronic liquidity and increasingly use
the CBDC for transactions.\(^{19}\)

The following proposition summarizes these discussions.

**Proposition 4** Suppose bankers cannot hold a CBDC. Under Assumption 1, sup-
pose \(D\Psi(D)\) is increasing, then there exists a unique monetary equilibrium. As
\((1 + i_e)/\mu_e - 1\) increases from \(1/\mu\) to \(r_β\), the effect of the CBDC is as follows:

1. if \(i_e \leq i_{e1}\), or \(\bar{ρ} \leq ρ^*\), then the CBDC does not affect the economy relative to
   the case without a CBDC;

2. if \(i_e \in (i_{e1}, i_{e2})\), or equivalently, \(\bar{ρ} > ρ^*\) and \(\rho < f'(N\ell_e) - 1\), then the CBDC
   increases lending relative to the case without a CBDC, and a higher \(i_e\) induces
   more lending;

3. if \(i_e \in (i_{e2}, i_{e3})\), or equivalently, \(\bar{ρ} > ρ^*\) and \(\rho > f'(N\ell_e) - 1\), then the CBDC
   increases lending relative to the case without a CBDC, and a higher \(i_e\) induces
   less lending;

\(^{19}\)This is the case where introducing a CBDC crowds out bank lending and private investment
(Keister and Sanches, 2018).
4. if $i_e > i_{e3}$, or equivalently, $\rho^* < \bar{\rho}$, then the CBDC decreases lending relative to the case without a CBDC, and a higher $i_e$ induces less lending.

This proposition delivers an important message, i.e., introducing a CBDC does not necessarily cause disintermediation and reduce bank loans and deposits. Furthermore, with a properly chosen CBDC rate, a CBDC may expand bank intermediation by introducing more competition to the banking sector. In particular, there is a range of $i_e \in (i_{e1}, i_{e3})$ at which introducing a CBDC increases lending relative to the case without a CBDC.

It is also worth noticing that in Figure 3(b), although the CBDC has a real effect on the economy, it is not used and buyers continue to hold only checkable deposits for transactions in type 2 and 3 meetings. The existence of the CBDC disciplines the off-equilibrium outcome: banks know that if they reduce their deposit rates below the CBDC rate, then buyers would switch to the CBDC. In this case, the CBDC acts like a potential entrant. An important implication is that the effectiveness of the CBDC should not be judged on its actual usage, but rather on how much it affects the deposit rate. Indeed, the CBDC increases lending most if $i_e$ is set such that at the equilibrium $\rho^* = \bar{\rho}$; in this case, the CBDC is still not used as a means of payment.

If $i_e$ is set such that the loan demand curve intersects the loan supply curve with the CBDC on its vertical region (as shown in Figures 3(c) and 3(d)), then both CBDC and checkable deposits are used as means of payment. To support the high deposit rate, the bank must scale back its lending and checkable deposits. At the same time, buyers’ demand for electronic liquidity increases, as liquid balances earn a higher return. The difference between total electronic liquidity and checkable deposits is met with the CBDC.

4.4 CBDC as Reserves

In the above subsections, we discuss the effect of a CBDC with a baseline design: it bears interest, serves as a perfect substitute for checkable deposits as a payment method and is not accessible by banks. Now we modify the the baseline design along one dimension and consider the case where banks can hold the CBDC as reserves (and earn the interest paid on the CBDC).\textsuperscript{20} The household’s and firm’s problems remain the same as with the baseline CBDC design. In the following, we first discuss

\textsuperscript{20}If banks can invest in the CBDC but cannot use it as reserves, then the effect of the CBDC remains the same as the baseline design. The intuition is as follows. If the return on the CBDC is less or equal to cash, then banks will not hold the CBDC as assets so the CBDC does not affect the economy. If the return on the CBDC exceeds cash, then the marginal benefit of investing in the CBDC is negative. To invest in the CBDC, the bank needs to issue deposits and pay at least the CBDC return on its deposits. In addition, the bank must hold cash reserves and invest only a fraction of deposits on the CBDC, which implies that the total cost of investing in the CBDC exceeds the return.
how the CBDC with the new design changes the bank’s problem, and then its effect on the equilibrium outcome.\footnote{In the analysis in this subsection, we continue to assume that Assumption 1 holds and $D\Psi(D)$ is decreasing.}

### 4.4.1 The Bank’s Problem

When a CBDC can be used as reserves, the bank’s problem is modified to

$$
\max_{\ell_j, z_j, z^e_j, d_j} \left\{ (1 + \rho) \ell_j + \frac{z_j}{\mu} (1 + i_e) \frac{z^e_j}{\mu} - d_j \right\}
$$

s.t.

$$
\ell_j + z_j + z^e_j = \tilde{\Psi} (d_{-j} + d_j) d_j,
$$

$$
z_j^e + z_j \geq \chi \tilde{\Psi} (d_{-j} + d_j) d_j,
$$

where $z^e_j$ is bank $j$’s CBDC balance.

First note that if the CBDC is dominated in return by cash, or $(1 + i_e)/\mu_e \leq 1/\mu$, then the bank does not hold the CBDC, so the analysis remains the same as in the case with the baseline CBDC design.\footnote{When $(1 + i_e)/\mu_e \leq 1/\mu$, the bank’s demand and loan supply conditional on the loan rate are described by the first two cases in subsection 4.2.}

In the following, we focus on the case where $(1 + i_e)/\mu_e > 1/\mu$ and banks hold the CBDC instead of cash as reserves. To solve for the Cournot competition conditional on the loan rate with the CBDC that can serve as reserves, we take two steps similar to the strategy used to study the effect of the CBDC with the baseline design. In the first step, we solve for the Cournot competition result conditional on the loan rate, assuming that the households cannot use the CBDC for payments (but banks can hold the CBDC as reserves). Denote the supply of checkable deposits and loans as $d^R(\rho)$ and $\ell^R(\rho)$, respectively, where the superscript “R” stands for reserves. The implied price and rate of checkable deposits and deposit rate are $\psi^R(\rho)$ and $r^R(\rho)$, respectively. In the second step, we compare the return on the CBDC $(1 + i_e)/\mu_e$ with $r^R(\rho)$ to analyze how the possibility of a CBDC as a payment method affects the household demand for checkable deposits, and the bank’s supply of checkable deposits, $\psi^R(\rho)$, and the supply of loans, $\ell^R(\rho)$.

The analysis of the first step remains largely the same as in Section 3, with $\mu$ in the bank’s problem replaced by $\mu_e/(1 + i_e)$. Specifically, when banks hold the CBDC as reserves and households cannot use the CBDC as a means of payment, the bank’s supply for checkable deposits $d^R(\rho)$ under the Cournot competition solves

$$
\psi'(Nd^R(\rho))d^R(\rho) + \Psi(Nd^R(\rho)) = \min \left\{ \frac{\mu_e}{1 + i_e}, \frac{1}{(1 + \rho)(1 - \chi) + \chi(1 + i_e)/\mu_e} \right\}. \tag{20}
$$
The loan supply $\ell^R(\rho)$ is given by
\[
\ell^R(\rho) = \begin{cases} 
0 & \text{if } 1 + \rho < (1 + i_e)/\mu_e, \\
[0, (1 - \chi)d^R(\rho)\Psi(Nd^R(\rho))] & \text{if } 1 + \rho = (1 + i_e)/\mu_e, \\
(1 - \chi)d^R(\rho)\Psi(Nd^R(\rho)) & \text{if } (1 + i_e)/\mu_e < 1 + \rho < 1/\beta, \\
[(1 - \chi)d^R(\rho)\Psi(Nd^R(\rho)), \infty] & \text{if } 1 + \rho = 1/\beta.
\end{cases}
\]

The implied price of checkable deposits is
\[
\psi^R(\rho) = \Psi(Nd^R(\rho)),
\]
and the implied rate of return on checkable deposits is
\[
r^R(\rho) = 1/\Psi(Nd^R(\rho)) - 1.
\]

In step 2, the solution is also similar to the case with the baseline CBDC design with the variables replaced by their counterparts when the bank uses the CBDC instead of cash as reserves. Define $r^R_\beta$ to be $r^R(\rho)$ evaluated at $\rho = 1/\beta$, which is the highest deposit rate offered by the bank when households cannot use the CBDC for payments. When the CBDC can serve as a means of payment, the bank’s supply of checkable deposits and loans are described as follows. If $(1 + i_e)/\mu_e < 1 + r^R_\beta$, then
\[
\hat{d}^R(\rho) = \begin{cases} 
[0, d_e] & \text{if } \rho \leq (1 + i_e)/\mu_e - 1, \\
d_e > d^R(\rho) & \text{if } \rho \in ((1 + i_e)/\mu_e - 1, \bar{\rho}^R], \\
d^R(\rho) & \text{if } \rho \in (\bar{\rho}^R, 1/\beta - 1].
\end{cases}
\]

\[
\hat{\ell}^R(\rho) = \begin{cases} 
0 & \text{if } \rho < (1 + i_e)/\mu_e - 1, \\
\left[0, (1 - \chi)\frac{\mu_e}{1 + i_e}d_e\right] & \text{if } \rho = (1 + i_e)/\mu_e - 1 \\
(1 - \chi)\frac{\mu_e}{1 + i_e}d_e > \hat{\ell}^R(\rho) & \text{if } \rho \in ((1 + i_e)/\mu_e - 1, \bar{\rho}^R], \\
(1 - \chi)\psi^R(\hat{\ell}^R(\rho))d^R(\rho) = \ell^R(\rho) & \text{if } \rho \in (\bar{\rho}^R, 1/\beta - 1), \\
[(1 - \chi)\psi^R(\rho)d^R(\rho), \infty) = \ell^R(\rho) & \text{if } \rho = 1/\beta - 1, \\
\end{cases}
\]

where $\bar{\rho}^R$ solves
\[
1 + r^R(\bar{\rho}^R) = (1 + i_e)/\mu_e.
\]

If $(1 + i_e)/\mu_e > 1 + r^R_\beta$, then
\[
\hat{d}^R(\rho) = d_e > d^R(\rho) \text{ for } \rho \in (0, 1/\beta - 1],
\]
\[ \hat{\ell}(\rho) = \begin{cases} 0 & \text{if } \rho < (1 + i_e)/\mu_e - 1, \\ [0, (1 - \chi) \frac{\mu_e}{1+i_e} d_e] & \text{if } \rho = (1 + i_e)/\mu_e - 1, \\ (1 - \chi) \frac{\mu_e}{1+i_e} d_e > \hat{\ell}(\rho) & \text{if } \rho \in ((1 + i_e)/\mu_e - 1, 1/\beta - 1), \\ [1 - \chi) \frac{\mu_e}{1+i_e} d_e, \infty) & \text{if } \rho = 1/\beta - 1. \end{cases} \]

The intuition of the equations for \( \hat{(d)}(\rho) \) and \( \hat{\ell}(\rho) \) is as follows. We will illustrate with the case where \((1 + i_e)/\mu_e < 1 + r^R_c\). At \( \rho = \bar{\rho}^R \), the deposit rate offered by the bank when the CBDC cannot be used for payments is equal to the return on the CBDC. When \( \rho > \bar{\rho}^R \), introducing a CBDC as a means of payment does not affect the economy (relative to the case where the CBDC is used as reserves but cannot be used as a means of payment).

For \( \rho < \bar{\rho}^R \), the rate for checkable deposits must match the return on the CBDC. For \( \rho < (1 + i_e)/\mu_e - 1 \), the supply for checkable deposits lies between zero and \( d_e \). The bank accepts deposits and offers the CBDC return on the deposits. On the asset side, it invests only in the CBDC. The bank earns zero profit in the process and is indifferent between operating or not.\(^{23}\) When \( \rho \in ((1 + i_e)/\mu_e, \bar{\rho}^R] \), the bank’s profit margin is positive, and the bank is willing to fully satisfy the household demand for checkable deposits at the CBDC rate. On the asset side, the bank holds just enough CBDC to satisfy the reserve requirement, and the supply of loans is \((1 - \chi)\) fraction of (the current value of) checkable deposits. In this case, the amount of checkable deposits and loans is fixed and determined by the return on the CBDC rate.

For \( \rho \in ((1 + i_e)/\mu_e - 1, 1/\beta - 1) \), the CBDC does not affect the supply of deposits and loans relative to the case where banks use the CBDC as reserves but households cannot use it for payments. When \( \rho = 1/\beta \), the bank starts issuing time deposits and the supply of loans becomes vertical and goes to infinity.

### 4.4.2 Equilibrium

After we solve for the Cournot competition with the new CBDC design, we can use the new loan supply curve to analyze the effect of a CBDC on the general equilibrium.

The results are illustrated in Figure 4. The blue line is the loan demand curve, the black line is the loan supply curve before introducing the CBDC. Following the analysis above, we can acquire the supply curve with the CBDC in two steps. In the

\(^{23}\) In the current setup, when the CBDC serves as reserves, the bank’s profit margin is at least zero for all values of \( \rho \). If the bank incurs a proportional management fee for its deposits, then the profit margin will become negative for low values of \( \rho \). In particular, suppose the per unit cost is \( c \), then when \( 1 + \rho < (1 + i_e)/\mu_e - c/(1 - \chi) \), the bank’s profit margin for issuing checkable deposits will become negative and the bank stops issuing deposits and loans.
Figure 4: CBDC as Reserves

Notes. (1) The blue curve is the loan demand, the black curve is the loan supply without a CBDC, the dashed black line is the loan supply when a CBDC is used as reserves but cannot be used for payments, and the red curve is the final new loan supply with a CBDC that can be used for both reserves and payments. The dashed line coincides with the red curve except for $\rho \in \left((1 + i_e)/\mu_e, \bar{\rho}^R\right]$. All three curves join each other at $1 + \rho = 1/\beta$. (2) The figure illustrates the effect of a CBDC when $1/\mu < (1 + i_e)/\mu_e < 1 + r^R_\beta$. 

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first step, we acquire the curve where the CBDC is used for reserves but cannot be
used for payments, and in the second step, we allow the CBDC to be used for both
reserves and payments. The final new supply curve is depicted in red; the first-step
curve follows the dashed line when $\rho \in \left(\frac{(1 + i_e)}{\mu_e}, \tilde{\rho}^R\right]$ and coincides with the red
curve in other parts.

As in the case with the baseline CBDC design (Figure 3), the loan supply curve with
CBDC coincides with the horizontal axis for low values of $\rho$. For intermediate values
of $\rho$, the curve is flat, the deposit rate matches the CBDC return, and the quantity of
loans is fully determined by the CBDC rate. For $\rho > \tilde{\rho}^R$, the return on the CBDC is
dominated by the deposit rate offered by the bank, and the supply curve is upward
sloping. Compared with the case with the baseline design, besides introducing a
potential alternative payment method to checkable deposits (call this the payment
competition effect), the CBDC that can be used as reserves also increases the return
on the bank’s asset bundle or reduces the bank’s cost of holding reserves (call this the
cost-saving effect). The change of the loan supply curve with the CBDC relative to
the case without the CBDC combines the two effects. The movement from the black
solid curve to the dashed curve captures the cost-saving effect, and the movement
from the dashed curve to the red curve captures the payment competition effect.

Similar to the CBDC with the baseline design, the effect of a CBDC that can be
used as reserves can be classified into four regimes. Let $r^*_R$ be the equilibrium rate
for checkable deposits when the CBDC is used as reserves but cannot be used for
payments.

When $i_e < i^R_1$, which solves $(1 + i_e)/\mu_e - 1 = r^*_R$ (shown in Figure 4(a)), the loan
demand curve intersects the new loan supply (red) curve with the CBDC in its
increasing region and the economy is in regime 1. In this regime, buyers strictly
prefer bank deposits over the CBDC and the payment competition effect is not
operative. However, the cost-saving effect is still present. When the CBDC offers a
higher return than cash, its presence improves the bank’s profitability. As a result,
the bank supplies more deposits and loans (point $b$), and the equilibrium loan rate is
lower compared with the equilibrium without the CBDC (point $a$). Furthermore, in
this regime, as $i_e$ increases, the bank expands deposits and loans, and the equilibrium
deposit rate increases and the loan rate decreases.

As $i_e$ increases beyond $i^R_1$, a further increase in $i_e$ pushes the economy into regime 2,
as shown in Figure 4(b). The regime 2 of the economy with the new CBDC design is
the same as with the baseline design. Compared with the case without the CBDC, the
CBDC forces up the deposit rate and induces a higher demand for electronic

\[24\text{In the increasing part of new (red) loan supply curve } (\rho \in [\hat{\rho}^R, 1/\beta - 1]), \text{the payment competition effect is muted, and the higher loan supply relative to the old (black) supply curve is due to the cost-saving effect. In contrast, with the baseline CBDC design, the cost-saving effect is absent and the loan supply curves with and without the CBDC coincide with each for } \rho \in (\hat{\rho}, 1/\beta - 1).\]
payment balances. If the marginal benefit from one additional unit of deposits is positive, the bank is willing to fully absorb the higher demand from households and the CBDC is not used in equilibrium. The bank issues checkable deposits, invests \( \ell_e \) on loans, and the equilibrium loan rate is given by \( f'(N\ell^*) - 1 \). In terms of comparative statics, as the CBDC rate rises, the equilibrium rate and quantity for checkable deposits rises, the amount of loan supply increases and the loan rate decreases. The increasing deposit rate and falling loan rate reduces the bank’s profit margin. At some point \( i^R_{e2} \), which solves \((1 + i_e)/\mu_e = f'(N\ell_e)\), the bank makes zero profit.

If \( i_e \) increases beyond \( i^R_{e2} \), the economy enters into regime 3 as shown in Figure 4(c). In this regime, any increase in \( i_e \) induces the deposit and loan rates to increase, and the quantity of loans to decrease. As \( i_e \) increases beyond \( i^R_{e2} \), the bank can only match the higher required deposit rate by scaling down its lending; otherwise, the return on the loan would be lower than the required return on deposits and the bank’s profit would be negative. The supply of checkable deposits is indeterminate. The household’s electronic liquidity balance increases with the CBDC rate. The bank is indifferent between issuing just enough deposits to support its lending, and meeting the total demand for liquidity with checkable deposits and investing the extra deposits in the CBDC. This indeterminacy disappears when there is a positive proportional fee to manage the deposits (as the cost of issuing deposits will be less than the return on the CBDC investment). Finally, note that although the equilibrium loan quantity decreases with \( i_e \) in this regime, as long as it is lower than \( i^R_{e3} = (1 + \rho^*)\mu_e - 1 \) (or equivalently, \((1 + i_e)/\mu_e < 1 + \rho^*)\), the equilibrium with a CBDC still features higher loans than the equilibrium without a CBDC.

As \( i_e \) increases beyond \( i^R_{e3} \), the economy enters into regime 4 (see Figure 4(d)). In this regime, the CBDC causes disintermediation relative to the case without a CBDC. As the CBDC rate continues to rise, the response of the economy is similar to regime 3: deposits and loans rates will increase, loans shrink, and households hold higher electronic liquidity.

To summarize, as \((1 + i_e)/\mu_e \) increases from \(1/\mu\), the effect of the CBDC that can be used as reserves is as follows: (1) If \( i_e \leq i^R_{e1} \), then the CBDC does not threaten checkable deposits as a means of payment. However, the interest on reserves reduces the cost of reserves and induces higher lending relative to the case without a CBDC. As \( i_e \) increases, lending increases. (2) If \( i_e \in (i^R_{e1}, i^R_{e2}) \), then the CBDC increases lending relative to the case without a CBDC, and a higher \( i_e \) induces more lending. (3) If \( i_e \in (i^R_{e2}, i^R_{e3}) \), then the CBDC increases lending relative to the case without a CBDC, and a higher \( i_e \) induces less lending. (4) If \( i_e > i^R_{e3} \), then the CBDC decreases lending relative to the case without the CBDC, and a higher \( i_e \) induces less lending.
5 Quantitative Analysis

In the previous section, we establish theoretically that an interest-bearing CBDC can increase bank lending if the interest rate on the CBDC lies in a certain range. It remains an empirical question how large this range is. This is crucial for policy decisions. To answer this question, we calibrate our model to the U.S. data and then conduct a counterfactual analysis to evaluate (1) how large the relevant range of interest is, (2) how much additional lending can be induced by the introduction of CBDC, and (3) what the effects on output and welfare are.

5.1 Calibration

We consider an annual model. We use the functional forms \( U(x) = B \log x \), \( u(y) = [(y + \epsilon)^{1-\sigma} - 1^{1-\sigma}]/(1 - \sigma) \), and \( f(k) = Ak^\eta \). We set \( \epsilon = 0.001 \) to guarantee that \( u(0) = 0 \) if \( \sigma \geq 1 \). The choice of \( \epsilon \) has little effect on the results as long as it is very small. For our calibration, we introduce two modifications to the theoretical model in sections 2 and 4. First, we assume that banks incur a management cost \( c \) per unit of deposits. Second, we allow sellers in the DM to have some market power. Specifically, in the DM, the terms of trade are determined by Kalai bargaining, and the buyer’s bargaining power is \( \theta \). These two modifications do not affect the qualitative analysis of the model but may improve the model’s ability to quantitatively match the data. We also transform the probabilities of the three types of meetings to facilitate matching with available data. Specifically, define \( \Omega = \alpha_1 + \alpha_2 + \alpha_3 \) as the probability of a buyer meeting a seller and therefore consuming in the DM. Define \( \hat{\alpha}_i = \alpha_i/\Omega, i = 1, 2, 3 \), as the probabilities of being in a type \( i \) meeting conditional on being matched with a seller. We can then replace \( \alpha_i \) with \( \Omega \hat{\alpha}_i \).

There are 14 parameters in our calibration: \( (A, B, N, \Omega, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \sigma, c, \theta, \beta, \eta, \chi, \mu) \). Eight parameters, \( \beta, \eta, \mu, \chi, \hat{\alpha}_i (i = 1, 2, 3) \), and \( c \), are set directly using data. The rest of the parameters are calibrated internally to hit calibration targets.

We set \( \beta = 0.96 \) to generate an annual real rate of 4%. We set \( \eta = 0.66 \) to match the elasticity of commercial loans with respect to the prime rate and \( \mu = 1.02 \) to match a 2% annual inflation. The reserve requirement ratio \( \chi \) is set to be 0.1 to match the regulation in the US.

We acquire the \( \hat{\alpha}_i \)s from the 2016 Survey of Consumer Payment Choice (SCPC) from the Federal Reserve Bank of Atlanta. This survey contains information on the fraction of online transactions and on the perceived fraction of point-of-sale transactions.

---

\(^{25}\)The bank’s profit will have a minus term \( c\psi b \) compared with the expressions in the previous sections.

\(^{26}\)Since the buyer may not meet a seller in the DM in every period, \( \Omega \) can be less than 1.

\(^{27}\)For more details on data and calibration, see Appendix D.
that do not accept cash or debit/credit cards. According to the 2016 survey, around 25.37% transactions happen online. At the point of sale, respondents report on average 15% transactions do not accept debit/credit card and 2% transactions do not accept cash. Assuming that all online transactions accept only deposits, cash-only trades are those transactions at the points of sale that do not accept cards. This implies that \( \hat{\alpha}_1 = 15\%(1 - 25.37\%) = 11.19\% \). Deposit-only transactions include online transactions and point-of-sale transactions that do not accept cash. Hence, \( \hat{\alpha}_2 = 25.37\% + 2\%(1 - 25.37\%) = 26.86\% \). Then \( \hat{\alpha}_3 = 1 - \hat{\alpha}_1 - \hat{\alpha}_2 = 61.94\% \).

To calculate \( c \), we use the bank balance sheet dataset from Drechsler et al. (2017) and Drechsler et al. (2018), which is an unbalanced panel spanning 1976 to 2013 for all U.S. banks. We first calculate a bank-specific cost for each bank by dividing the annual non-interest expenses by total liabilities \( (c_{it}) \). We then take the average of \( c_{it} \) over the time period to get a bank-specific average cost, \( c_i \). Then we take the median of \( c_i \) across the banks, which gives us 0.89%.

We pick \( A \) to match the deposit rate on checking accounts 0.05%, \( \theta \) to match the 20% markup for retailers, \( N \) to match the net interest margin for banks, and \( (B, \Omega, \sigma) \) to match the money demand curve. We use the “new M1” data in Lucas and Nicollini (2015), which cover the period from 1915 to 2012. Because historical data on bank deposit interest are not available for the whole period, while matching the money demand curve, we use only data from 1935 to 1983 when regulation Q was in place, which prevented banks from paying interest on deposits. We assume that the nominal deposit rates are 0 during this period.\(^{28} \)

Table 1 summarizes all the parameter values. Figure 5(a) shows the scatter plot of M1 to GDP ratio versus the 3-month T-Bill rate. Different colors are used to indicate different time periods. One can see that money demand is more or less stable over the whole sample period. However, the money demand between 1935 and 1983 is steeper than other periods, which could be partly attributed to regulation Q implemented in this period. Regulation Q restricts interest payment on demand deposits, so banks cannot raise the deposit rate in response to rising nominal rates to retain deposits in this period. Therefore, M1 drops more. Figure 5(b) shows the model-predicted money demand curve against the data between 1935 and 1983. The model fits the data well. Notice that there is a kink in the fitted money demand curve. Intuitively, when the nominal interest is sufficiently high, households become liquidity-constrained in type 3 meetings.\(^{29} \) As a result, money demand becomes less elastic.

\(^{28}\)Notice that we do not assume \( \hat{\alpha}_s \) in our sample period are the same as those in the year 2016. We instead rely on the assumption that \( \hat{\alpha}_s \) do not affect the demand for M1. This assumption is consistent with the data. The demand for M1 is very stable over a long horizon, during which \( \hat{\alpha}_s \) may have changed a lot with the introduction of credit/debit cards.

\(^{29}\)The liquidity constraint always binds in type 1 and type 2 meetings.
### Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
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<tr>
<td>Curvature of production</td>
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<td>Elasticity of commercial loans</td>
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<td>Reserve requirement</td>
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<td>Gross money growth rate</td>
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<td>2% Annual inflation</td>
</tr>
<tr>
<td>Frac. of type 1 trades</td>
<td>$\hat{\alpha}_1$</td>
<td>11.19%</td>
<td>SCPC 2016</td>
</tr>
<tr>
<td>Frac. of type 2 trades</td>
<td>$\hat{\alpha}_2$</td>
<td>26.86%</td>
<td>SCPC 2016</td>
</tr>
<tr>
<td>Frac. of type 3 trades</td>
<td>$\hat{\alpha}_3$</td>
<td>61.94%</td>
<td>SCPC 2016</td>
</tr>
<tr>
<td>Calibrated externally</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob. of DM trading</td>
<td>$\Omega$</td>
<td>0.7</td>
<td>Money demand 1935-83</td>
</tr>
<tr>
<td>Coeff. on CM consumption</td>
<td>$B$</td>
<td>1.7881</td>
<td>Money demand 1935-83</td>
</tr>
<tr>
<td>Curv. of DM consumption</td>
<td>$\sigma$</td>
<td>0.4814</td>
<td>Money demand 1935-83</td>
</tr>
<tr>
<td>TFP</td>
<td>$A$</td>
<td>1.3071</td>
<td>Deposit rate 0.05%</td>
</tr>
<tr>
<td>Number of banks</td>
<td>$N$</td>
<td>9</td>
<td>NIM 2.98% (Dempsey 2018)</td>
</tr>
<tr>
<td>Buyer’s bargaining power</td>
<td>$\theta$</td>
<td>0.8395</td>
<td>Retailer markup 20%</td>
</tr>
</tbody>
</table>

#### Table 1: Calibration Results

(a) Data in Lucas-Nicolini (2015)

(b) Model Fit

Figure 5: Money Demand Curve
5.2 Effects of a CBDC on Banking

Now we consider introducing a CBDC that is a perfect substitute for checkable deposits. We consider both the case where the CBDC does not serve as reserves and the case where the CBDC serves as reserves. The growth rate of the total supply of CBDC is set to be the same as that of fiat money. We then focus on how changes in the interest rate on the CBDC affect the real economy as well as the deposit and lending rates. In particular, we are interested in whether the CBDC increases lending and output.

Figure 6 shows the results. The first row displays real variables divided by corresponding equilibrium values without a CBDC as the CBDC rate changes. The second row shows the changes in deposit and loan rates and their difference, i.e., the spread. The blue curve is constructed assuming that the CBDC does not serve as reserves and the red curve is under the scenario where the CBDC serves as reserves. Notice all the rates are nominal rates and are in percentages.

Let us first examine the quantities of checkable deposits and loans. If $i_e$ is lower than 0, the real rate of the CBDC is below that of fiat money and the rate for checkable deposits. Therefore, it is not used and the economy behaves as if there were no CBDC. In this region, the blue curve and the red curve overlap and are flat.

Although it is not clear in the graph, the red curve starts to increase once $i_e$ becomes positive, while the blue curve stays flat until $i_e$ exceeds a small positive value. In
this region, the deposit rate without a CBDC is still above the CBDC rate, so the
CBDC does not compete with checkable deposits as a payment method. If the
CBDC does not serve as reserves, then it does not change the equilibrium (as shown
by the blue curve). If the CBDC serves as reserves, then interest on the CBDC
reduces the lending cost through the reduced cost of holding reserves and increases
lending (as show by the red curve). Quantitatively, this cost-saving effect is very
small and therefore not obvious in the graph.

As $i_e$ further increases, the CBDC becomes an effective competitor for bank deposits
and forces the banks to increase their deposit rates, which attracts more deposits.
Losing the ability to affect the deposit rate, the bank is willing to fully meet the
higher demand for deposits as long as their profit margin is positive (and in equilib-
rium, the CBDC is not used for payments). Notice that when both curves increase,
the equilibrium is fully determined by the CBDC rate and the two curves overlap
with each other.

If $i_e$ is sufficiently high, then deposits and loan supply start to decrease. In the
decreasing segment, banks act as if they were in a perfectly competitive deposit
market, and their profit is driven to 0. To compensate for the higher deposit rate,
they must charge a higher lending rate, which reduces lending. Since the total
electronic payment balance increases, households hold increasingly more CBDC for
payment purposes. Another observation is that the red curves enter the decreasing
segment at a higher $i_e$, and stays above the blue curves after the blue curve starts
to decrease; this is due to the cost-saving channel.

Relative to the economy without CBDC, introducing a CBDC increases lending if
its rate is between 0.05% and 1.79% if it does not serve as reserves. When it serves
as reserves, this range expands to between 0% and 1.98%. With a properly chosen
rate, the CBDC can increase lending relative to the case without a CBDC by as
much as 3.55% when it cannot be used as reserves, and 3.69% when it can be used
as reserves.

Now we turn to the deposit and lending rates and the spread, which are shown in
the second row of Figure 6. All these rates are nominal. The deposit rate is shown
in the first panel in the second row. It is constant if $i_e$ is low and coincides with
the 45°-line once the CBDC rate exceeds the deposit rate offered by the bank in
the absence of a CBDC. This reflects that the CBDC rate serves as a floor of the
deposit rate. It is also worth noting that if $i_e$ is positive but small, the deposit rate
increases if the CBDC can be used as reserves, but the effect is too small to discern
in the graph.

The loan rate reverses the pattern of loans, as shown in the second panel. If $i_e$ is set
appropriately, then the loan rate reduces to less than 2% from more than 3% when
there is no CBDC. If $i_e$ is too high, then the loan rate can be higher compared with
the equilibrium without a CBDC.
The spread, defined as the difference between the nominal lending rate and the nominal deposit rate, is shown in the third panel in the second row. The CBDC reduces the spread by introducing more competition into the deposit market. If $i_e$ is sufficiently high (at which the deposit and loan curves start to decrease), then bankers act as if the market is perfectly competitive, and the lending rate equals the marginal cost of lending, which is the cost of maintaining deposits and holding reserves. In this region, if the CBDC does not serve as reserves, then the spread increases with the CBDC rate. Note that this increase in the spread does not reflect higher market power but simply reflects the higher cost of holding reserves as the difference between the deposit rate and return on cash reserves (which is constant) increases. This difference disappears when the CBDC serves as reserves, and the spread will become constant (as shown by the red curve).

5.3 Effects on Total Output and Welfare

Now we move to total output, which is shown in the third graph in the first row. The pattern is qualitatively similar to loans: as $i_e$ increases, total output first increases and then decreases. Quantitatively, there are two differences. First, introducing a CBDC increases total output for $i_e \in (0.05\%, 4.02\%)$ when it does not serve as reserves and for $i_e \in (0, 6.25\%)$ when it serves as reserves. These ranges are wider compared with the regions where lending increases. This is because the CBDC also increases DM trades. Second, the percentage increase in output is much smaller than loans. The highest increase in output is 0.50% if the CBDC does not serve as reserves and 0.52% if the CBDC serves as reserves. They are achieved at $i_e = 0.69\%$ and $i_e = 0.72\%$, respectively. The relatively modest expansionary effect of the CBDC on total output compared with loans is due to two reasons. First, investment by entrepreneurs is subject to diminishing returns. Second, a higher CBDC rate induces households to substitute out of cash into electronic balances and reduces consumption in type 1 meetings, which partially offsets the expansionary effect of the CBDC on CM production and DM production in type 2 and 3 meetings.

We next discuss effects of CBDC on the welfare of different agents. Figure 7 shows changes in welfare for buyers, sellers, entrepreneurs, and bankers. Welfare is measured as the percentage change in the equilibrium consumption without a CBDC that makes an agent indifferent between no CBDC and the CBDC with interest rate

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30To see this, it helps to rewrite the bank’s profit $(1 + \rho)(1 - \chi)\psi d + \chi\psi d\zeta - d - \psi dc$ as $\ell[(1 + \rho - 1/\psi) - \chi/(1 - \chi)(1/\psi - \zeta) - 1/(1 - \chi)c]$, where $\zeta$ is the return on reserves.

31The difference between the loan and deposit rates is just enough to cover the account cost $1/(1 - \chi)c$. The coefficient $1/(1 - \chi)$ before $c$ reflects the reserve requirement: the bank pays the account fee for all deposits, but can only loan $1 - \chi$ fraction out. In our calibration, $1/(1 - \chi)c = 0.089%/0.9 = 0.099\%$.

32The total output aggregates the output in the DM and the CM. See Appendix D for the formula to calculate output.
i_e. If it is positive, the CBDC increases the welfare of the agent. Otherwise, the CBDC reduces the welfare. All the y-axes are in percentages.

Buyers and sellers benefit from the introduction of a CBDC, and their surpluses increase in the range of i_e considered in Figure 7. Without hurting lending, a buyer’s surplus can be raised by around 0.2% together with a modest increase in a seller’s surplus. The kink in these two welfare graphs corresponds to the point where i_e is sufficiently high such that buyer’s liquidity constraint becomes loose in type 3 meetings. Before that point, buyers are liquidity-constrained in type 3 meetings, and an increase in i_e leads to a decrease in the value of cash and reduces transactions in cash meetings. This offsets the positive welfare effect of i_e on electronic liquidity. Hence, the slope of the welfare function is lower before the kink.

Entrepreneurs benefit highly from the CBDC because of the lower lending rate (resulting from higher deposit and loan supplies). Their maximum welfare gain is about 2.5%. A CBDC improves their welfare as long as i_e < 1.79% if the CBDC does not serve as reserves, and as long as i_e < 1.98% if it serves as reserves. Intuitively, bankers lose because the CBDC introduces more competition into the deposit market. If i_e is sufficiently high, bankers behave as if the market is perfectly competitive. Their profit and hence consumption is reduced to 0, which is a 100% reduction compared with the equilibrium without a CBDC.

5.4 Non-Interest-Bearing CBDC in a Cashless Economy

So far our analysis considers the effects of an interest-bearing CBDC. Central banks may be cautious about using the interest on a CBDC as an active tool and consider only a zero-interest CBDC at least in the initial stage of introducing a CBDC.\footnote{For example, the Bank of Canada’s contingency planning for a CBDC involves a cash-like CBDC that does not pay interest (see https://www.bankofcanada.ca/2020/02/contingency-planning-central-bank-digital-currency).} If the CBDC does not pay interest, can it still have any effect on banking? This section assesses the effect of a zero-interest CBDC as the payment landscape evolves, captured by changes in the \( \alpha \)s.

More specifically, we consider the cashless trend experienced in many countries as transactions increasingly move online. In our model, this can be captured by converting a fraction \( \Delta \) of transactions from type 3 meetings to type 2 meetings, i.e., the probability of type 3 meetings changes to \( \alpha_3 - \Delta \) and that of the type 2 meetings changes to \( \alpha_2 + \Delta \). The interpretation is that some brick and mortar sellers close their physical stores and move online to sell. The calibration results are shown in Figure 8.

First, let us investigate the effect of increasing \( \Delta \) without a CBDC, represented by the blue line. As \( \Delta \) increases, deposits become a better payment instrument
Figure 7: Welfare Change for Each Type of Agent
Figure 8: Economy Becomes Cashless

6 Alternative Designs of a CBDC

In sections 4 and 5, we consider a CBDC that serves as perfect substitute for checkable deposits as a mean of payment. The CBDC policies are captured by the growth rate $\mu_e$, its interest rate $i_e$ and whether it can be used as reserves. In this section, we consider other dimensions of the CBDC design, and discuss how they affect our
analysis of the effect of a CBDC.

6.1 Fixed Quantity, Instead of Fixed Rate, of a CBDC

The designs that we have considered above assume that the central bank fixes the rate of the CBDC, and its quantity is endogenously determined in equilibrium. Now suppose that the central bank fixes the quantity of CBDC instead and announces it publicly at the beginning of each period. The rate on the CBDC and deposits are then endogenously determined in equilibrium.

In general, bankers take that quantity as given and solve a similar Cournot maximization problem. It is easy to see that in this case, the central bank acts like a new bank entering the deposit market to compete with incumbent bankers. We can show that the amount of deposits that each bank attracts will decrease. This leads to disintermediation because the aggregate amount of deposits by bankers and, hence, the aggregate amount of loans decrease in the equilibrium with the CBDC. Note that the total amount of electronic balances (deposits plus CBDC) increases. The interest rate on deposits increases, similar to the result in the benchmark model, but the interest rate on loans increases, unlike in the benchmark model as depicted in Figure 6 in the intermediate range. Another difference from the benchmark model is that, in this case, the CBDC always has a positive market share.

6.2 CBDC as a Cash Substitute

In the benchmark model we assume that the CBDC is a deposit substitute in its payment function. Now, suppose a CBDC is still interest bearing, as in the benchmark model, but is a perfect substitute for cash, i.e., it can be used in type 1 and type 3 meetings. If the nominal interest rate that the CBDC pays is above zero, then the CBDC dominates cash for households, so they do not hold cash. In essence, introducing a cash-substitute CBDC works as if the central bank pays interest on cash.

Now, we investigate how the aggregate amounts of deposits and loans and the interest rates change. We focus on the cases in Figure 1 where \( L^d \) intersects \( L^s \) in its strictly increasing region. Paying interest on the CBDC makes the CBDC more desirable relative to deposits.\(^{34}\) For a given \( \rho \), bankers need to pay more to raise resources, so the amount of deposits that each bank raises and, consequently, the aggregate amount of deposits decrease. If the price of deposits \( \Psi \) decreases or slightly increases so that \( \Psi D \) decreases, then the total amount of loans decreases for a given \( \rho \), causing the supply curve to shift downward. As a result, \( \rho \) increases and \( L \)

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\(^{34}\)Note that \( \Psi \) (the price of bank deposits) is an increasing function of \( \iota \) (the opportunity cost of holding cash) because cash and deposits are substitutes when \( \alpha_3 > 0 \). An increase in the interest rate on cash decreases \( \iota \). Hence, \( \Psi \) decreases, and bankers raise less funds from issuing one unit of deposits.
decreases in equilibrium, and the cash-substitute CBDC leads to disintermediation. However, if the price of deposits increases such that \( \Psi D \) increases, then the supply curve shifts upward. As a result, \( \rho \) decreases and \( L \) increases in equilibrium and the CBDC leads to more intermediation. To summarize, the effects of a cash-substitute CBDC on intermediation depends on how the inverse demand for deposits changes with the opportunity cost of holding cash.

7 Discussion

This section briefly discusses the implications of changing certain assumptions of our model.

7.1 Interest-Bearing Reserves

In the benchmark model, we assume that bankers hold non-interest-bearing money as reserves. If bankers can hold interest-bearing reserves, the analysis of the benchmark model stays unchanged except that the term \( 1/\mu \) in the banker's problem is replaced by \( (1 + i_r)/\mu \), where \( i_r \) is the interest rate on central bank reserves. This is because the interest on reserves effectively decreases the cost of holding reserves. The analysis in the benchmark model can be viewed as a special case where \( i_r = 0 \).

7.2 Interest Rate Floor versus a CBDC

In the benchmark model, the introduction of a CBDC has the effect of imposing an interest rate floor in the deposit market. Here we briefly compare the difference between a CBDC and a direct interest rate floor. We argue here that the interest rate floor regulation is generally less effective than introducing a CBDC.

To elaborate, if the interest rate on the CBDC is in the intermediate range, then this regulation is equally effective as the CBDC. This is because, even with the CBDC, the CBDC is not used in equilibrium, so if the regulation sets the floor with the same interest rate as the CBDC rate, bankers have to pay the same interest rate to depositors. Hence, the funding cost is identical to that with the CBDC, and so is the interest rate on loans.

However, if the interest rate on the CBDC is sufficiently high, the results will be different. With the CBDC, the demand for means of payment is partially satisfied by bank deposits and partially by the CBDC; i.e., agents use the CBDC in equilibrium. If the regulation sets the floor with the same interest rate as the CBDC rate, since the rate is too high, bankers are not willing to satisfy all the demand they face for their deposits. (If they take all those deposits, they would receive a negative profit because there would not be enough profitable opportunities on the lending side.) Therefore, there will be rationing of deposit taking by banks. The outcome on the
lending side is identical to that with the CBDC, but depositors will be rationed and cannot obtain as much electronic balances as they want for transactions if the CBDC is not available as a substitute. Therefore, introducing a CBDC directly is more effective in this case than the floor policy.

7.3 Endogenous Bank Entry

In the benchmark model, the number of bankers is exogenously fixed. Suppose, instead, that the number of bankers in equilibrium is endogenously determined. In particular, suppose that there are many potential bankers who can pay a (per period) fixed cost to enter the market. We briefly discuss the effects of introducing a CBDC in this alternative environment.

If banks have market power only on the deposit side as in our benchmark model, the results of the paper are unaffected. A higher interest rate on the CBDC leads to a higher funding cost for bankers. Thus, the bankers’ profit is lower, and a smaller number of banks enter the market in equilibrium. This means that the interest rate on the CBDC still serves as the floor for the interest rate on deposits, so the CBDC rate still determines the deposit rate and lending rate and the results are not affected.

Suppose the deposit market is perfectly competitive but the lending side is imperfectly competitive. In this case, for a fixed number of bankers, introducing a CBDC will reduce both deposit and loan supplies. With endogenous bank entry, the number of banks will decline, which will further reduce the loan supply. This means that allowing for entry reinforces the reduction in loan supply, because the increase in deposit cost reduces the amount of deposits.

8 Conclusion

Our paper develops a model with imperfect competition in the deposit market to analyze whether introducing a CBDC would cause disintermediation in banks. We show that, contrary to common wisdom, the CBDC does not necessarily lead to disintermediation. Indeed, we find that an interest-bearing CBDC can promote bank intermediation. Intuitively, if banks have market power, they would restrict the deposit supply to lower the deposit rate. An interest-bearing CBDC reduces their market power by setting a floor for the deposit rate, which leads to the creation of more deposits and lending and a lower loan rate. However, more intermediation happens only if the interest rate on the CBDC is set properly. If the CBDC rate is too low, then the CBDC does not affect the equilibrium. If the CBDC rate is too high, then banks must raise the loan rate to support the high deposit rate, and disintermediation will occur. The CBDC only expands intermediation if its interest rate lies in some intermediate range.
We then quantify the effect of a CBDC by calibrating our model to the U.S. economy, using payments survey data from the Federal Reserve Bank of Atlanta, money demand and bank interest rate data from FRED, as well as banks’ balance sheet data from U.S. call reports. We find that introducing an interest-bearing CBDC that is a perfect substitute for bank deposits as a payment instrument expands bank intermediation when the CBDC rate lies between 0.05% and 1.79%, and at the maximum, a CBDC can increase loans and deposits by 3.6% and the total output by 0.5%.

The framework we construct is useful for exploring the effects of CBDCs with various design choices: interest-bearing or not, cash-like or deposit-like, serving as reserves or not, with a fixed quantity or rate, etc. The framework can also be used to study the role of a CBDC in an increasingly cashless world, and the interaction between CBDC-related policies and existing monetary policy instruments, such as interest on reserves.

We understand that our framework abstracts from some important issues related to the discussion of CBDC, such as endogenous decisions of banks on the composition of both the asset side and the liability side of their balance sheets in terms of risk and maturity, and how these endogenous decisions may have macroeconomic and financial stability implications. More specifically, introducing an interest-bearing CBDC would increase banks’ funding costs. On the asset side of their balance sheets, it may induce banks to take on more risk to make up for their lower profit margin. This can increase the total risk to the financial system, leading to a less stable system. On the liability side, it can induce banks to switch to other sources, such as wholesale funding. These funding sources are generally considered less stable than deposits. Therefore, more reliance of banks on wholesale funding may increase the likelihood of runs in the wholesale market. We leave these issues for future research.
Appendices

A Proofs

Proof of Proposition 1. Banker $j$’s problem can be rewritten as

$$\max_{d_j} \xi \Psi(D_{-j} + d_j) - d_j,$$

where $\xi = \max\{1/\mu, (1+\rho)(1-\chi) + \chi/\mu\}$ is the return on assets. By Assumption 1, there is a unique solution to this problem. This solution satisfies $\Psi'((D_{-j} + d_j) + d_j + D_{-j} + d_j) = 1/\xi$. Then the symmetric pure strategy Nash equilibrium $d$ must satisfy $\Psi'((Nd)d + \Psi(Nd) = 1/\xi$. Recall that $\Psi(0) = \infty$ and $\Psi(y^*) = \beta < 1/\xi$. Therefore, $\Psi'(Nd)d + \Psi(Nd) < 1/\xi$ for all $d$ that are smaller than but sufficiently close to $y^*/N$. Because $\Psi'(Nd)d + \Psi(Nd)$ is continuously decreasing on $[0, y^*/N)$, there exists a unique solution $d(\rho)$.

Proof of Proposition 2. The loan supply curve $L(\rho)$ is continuous and is increasing in the interval $[1/\mu - 1, 1/\beta - 1)$ under the assumption that $\Psi(D)D$ is increasing. In addition, $L(0) = 0$ for sufficiently small $\rho$, ranges from $(1 - \chi)d\beta \Psi(Nd\beta)$ to $\infty$ when $\rho = 1/\beta - 1$, and $= \infty$ when $\rho > 1/\beta - 1$. On the other hand, $L^d(\rho)$ is continuous and decreasing for any $\rho > -1$, with $L^d(-1) = \infty$ and $L^d(\infty) = 0$. As a result, $L^d > L$ for sufficiently small $\rho$ and $L^d < L$ for sufficiently big $\rho$. By the intermediate value theorem, there exists a unique equilibrium with the loan rate on $(0, 1/\beta - 1)$.

Proof of Proposition 3. We only prove that $\hat{d}(\rho) = d_e$ if $\rho \in (\rho, \bar{\rho}]$. The other parts are obvious. First, suppose the total supply of checkable deposits is lower than $Nd_e$, then increasing $d_j$ does not change the price of the deposit, which is fixed at $\mu_m/(1 + i_m)$. The first-order derivative with respect to $d_j$ is

$$[(1 + \rho)(1 - \chi) + \chi/\mu] \frac{\mu_e}{1 + i_e} - 1 > 0,$$

for $\rho < \rho$ under Assumption 1. Therefore, bank $j$ can always increase its profit by increasing $d_j$.

Now suppose the aggregate supply of checkable deposits $D$ is higher than $Nd_e$ so that the rate of the CBDC falls below the return on deposits without a CBDC, then a banker’s marginal profit is

$$[(1 + \rho)(1 - \chi) + \chi/\mu] [\Psi(D) + \Psi'(D)D/N] - 1 < 0$$

for $\rho < \bar{\rho}$. This shows that it is profitable for a banker to reduce its supply of deposit if $D > Nd_e$. ■
Now we show that this model can have multiple equilibria even under perfect competition in both the deposit and loan markets. Set $\alpha_3 = 0$, which implies the demand for cash and checkable deposits dichotomize. Assuming that $\iota$ is sufficiently small, in the equilibrium, real cash balances are determined by

$$\iota = \alpha_1 \lambda(Z).$$

(21)

Given $\psi$, the demand for checkable deposits is determined by

$$\psi = \alpha_2 \beta \lambda(D) + \beta.$$  

(22)

Notice that $\psi$ can never go below $\beta$; if $\psi = \beta$, then the demand for $D$ can be any value lying between $D^*$ and $\infty$; if $\psi < \beta$, then $D = \infty$.

Suppose there is a continuum of banks with measure 1 and they are price takers. Given $\psi$ and $\rho$, they solve

$$\max_{\ell, d} \left\{ \left( 1 + \rho - \frac{1}{\mu} \right) \ell_j - \left( 1 - \frac{\psi}{\mu} \right) d_j \right\}$$

(23)

subject to

$$\ell_j \leq (1 - \chi) \psi d_j.$$  

(24)

As long as $\psi < \mu$, the constraint is binding and the problem reduces to

$$\max_{d} \left\{ \left[ (1 + \rho) (1 - \chi) + \frac{\chi}{\mu} \right] \psi d_j - d_j \right\}$$

(25)

Because $\rho = \varrho(L) \equiv f'(L) - 1$, the optimization problem along with the the constraint imply

$$\{1 + \varrho[(1 - \chi) \psi D)] (1 - \chi) + \frac{\chi}{\mu} = 1/\psi.$$  

(26)

Given $\psi$, this equation defines $D$ as a function of $\psi$: $D = \Delta(\psi)$, which can be non-monotone depending on the curvature of the production function. If $\psi \geq \mu$, then the constraint is not binding, which means $\rho = 1/\mu - 1$ and $L = \varrho^{-1}(1/\mu - 1)$. If $\psi > \mu$, then $D = \infty$. If $\psi = \mu$, then banks are indifferent between any amount of capital. To be consistent with market clearing, $D \geq \varrho^{-1}(1/\mu - 1) (1 - \chi)$. One can show that $\varrho^{-1}(1/\mu - 1) (1 - \chi) = \Delta(\psi)$ if $\psi = \mu$. To summarize, the supply for deposit is the following:

$$D = \begin{cases} 
\Delta(\psi) & \psi < \mu, \\
\infty & \psi > \mu, \\
[\Delta(\psi), \infty) & \psi = \mu.
\end{cases}$$

(27)

Any intersection between (22) and (27) determines an equilibrium.
Proposition 5  Monetary equilibrium exists iff \( \iota < \alpha_1 \lambda(0) \) and \( \beta \leq \mu \).

Proof. From (21), one can see that \( Z > 0 \) iff \( \iota < \alpha_1 \lambda(0) \). If \( \psi \) is sufficiently small, (22) defines \( D = \infty \) and if \( \psi \) is sufficiently large, that \( D \) is sufficiently small. On the other hand, (27) implies that \( D \) is finite if \( \psi \) is low and \( D = \infty \) for \( \psi \) that is sufficiently large. Then by continuity, these two curves have at least one intersection. Hence, at least one equilibrium exists. ■

In general, the equilibrium is not unique. We next use numerical examples to illustrate this. To this end, parametrize \( f'(l) = A(l + \varepsilon)^{-\xi} \mathbf{1}\{l > \bar{l}\} + B l^{-\omega} \mathbf{1}\{l \leq \bar{l}\} \). Here, \( A, \varepsilon, \bar{l} > 0 \) and \( \xi > 1, 0 < \omega < 1 \) are the parameters to choose. Then \( B \) is chosen such that \( f' \) is continuous. One can integrate this function and impose \( f(0) = 0 \) to obtain \( f \). Since \( f' \) is positive and strictly decreasing, \( f \) is strictly increasing and concave.

Results are shown in \( \psi-D \) space in Figure 9. In all graphs, the blue curve is the deposit demand curve and the red curve is the deposit supply curve. The demand curve is monotonically decreasing while the supply curve can be monotonically increasing or non-monotone, depending on the curvature of \( f \). If \( f(l) = Al^\omega \) with \( \omega < 1 \), the supply curve is increasing as in Figures 9(a) and 9(b). In this case, we have a unique equilibrium. If, however, \( f(l) = A \left[ (l + \varepsilon)^{1-\xi} - \varepsilon^{1-\xi} \right] / (1 - \xi) \) with \( \xi > 1 \) and \( \varepsilon \) that is sufficiently small, the supply curve is decreasing for the most part if \( \psi < (1/\mu)^{-1} \). This is shown in 9(c). In this case, we have three equilibria. One has \( \psi = \beta \), i.e., the deposit does not carry a liquidity premium. One has \( \rho = 1/\mu - 1 \) and \( \psi = (1/\mu)^{-1} \). In this case, the price for deposit is sufficiently high that the banks are willing to take any amount of deposits and then hold them in cash reserves. Notice at this intersection, \( D > \Delta(\psi) \). Consequently, the reserve requirement constraint is not binding, i.e., banks hold voluntary reserves. There is another equilibrium where the reserve requirement constraint is binding.

If \( f \) is a combination of the previous two cases, the supply curve can be non-monotone even if \( \psi < \mu \), as shown in Figure 9(d). In this case, there are two equilibria where the reserve requirement constraint is binding.

C  Imperfect Competition in Lending Market

Now consider the extension that there is a competitive interbank market and the lending market features imperfect competition. We consider two cases: a Cournot lending market, and a search and matching market. In both cases, the deposit market has Cournot competition as in the previous section.
Figure 9: Equilibrium

(a) Case 1

(b) Case 2

(c) Case 3

(d) Case 4
C.1 Cournot Lending Market

Now let $\rho^B$ be the real rate in the interbank market. Then the loan supply function $L^s$ is the same as before except that now it depends on the interbank rate $\rho^B$. The loan makers solve

$$\max_{\ell_j} f' \left( \frac{\ell_j - \frac{\ell - j}{NE}}{NE} \right) \ell_j - (1 + \rho^B) \ell_j.$$ 

To guarantee the existence of a pure strategy equilibrium given $\rho^B$ on the loan side, we require the following condition:

**Condition 1** $f''(L) + f'''(L) L \leq 0$.

Then the equilibrium satisfies

$$f'' \left( \frac{L}{NE} \right) \frac{L}{NE} + f' \left( \frac{L}{NE} \right) = 1 + \rho^B.$$ 

This defines $L^d(\rho^B)$, which is decreasing and continuous in $\rho^B$. Now the equilibrium interbank market rate is determined by $L^s(\rho^B) = L^d(\rho^B)$. Then the loan rate is given by $\rho = f' \left( L^s(\rho^B) \right) - 1$.

**Proposition 6** If $\iota < \bar{\iota}$ and Assumption 1 and Condition 1 hold, there exist one or three monetary equilibria. If, in addition, $\Psi(D)D$ is increasing, the monetary equilibrium is unique.

Then comparative statics can be analyzed as before.

C.2 Search for Loans

Now suppose that each bank has a continuum of loan officers with measure $N_l$. They have access to a competitive interbank market and randomly search and match with firms. The matching probability is $\alpha(\lambda)$, where $\lambda = N_f/N_lN_b$. Upon a meeting, the loan officer bargains with the firm on the terms of loans given the interbank market rate $\rho$. The surplus is split with Kalai’s bargaining solution, where the firm has a bargaining power $\eta$:

$$\max_{\ell, p} f(\ell) - p \quad \text{st} \quad f(\ell) - p = \eta \left[ f(\ell) - (1 + \rho^B) \ell \right].$$

In this case, $f'(\ell) = \ell + \rho^B$ and $p = (1 - \eta) f(\ell) + (1 + \rho^B) \ell$. This implies the loan rate,

$$\rho^E = \eta \rho^B + (1 - \eta) \left[ \frac{f(\ell)}{\ell} - 1 \right].$$
which is a weighted sum of the interbank market rate $\rho^B$ and the average investment return $f(\ell) / \ell - 1$. Then the loan demand curve is

$$L^d(\rho^B) = \alpha(\lambda) N_i N_b f' - 1 \left(1 + \rho^B\right).$$

Then the equilibrium $\rho^B$ is determined in the same fashion as in the competitive loan market case. Consequently, we have the following proposition:

**Proposition 7** If $i < \bar{i}$ and Assumption 1 holds, there exist one or three monetary equilibria. If, in addition, $\Psi(D) D$ is increasing, the monetary equilibrium is unique.

Notice that in this case, the total supply of loans is efficient given the interbank market rate $\rho^B$. However, the bankers get a positive surplus from lending. We can then calculate the spread between the interbank rate and lending rate as $\rho^E - \rho^B = (1 - \eta) \left[\frac{f(\ell)}{\ell} - 1 - \rho^B\right]$.

### D Calibration Method and Data

FRED lists nominal interest on non-jumbo deposits after the year 2009. The average annual rate on checking accounts is less than 0.05% and that on saving accounts is around 0.09%. This is consistent with the Survey of Consumer Payment Choice data, where more than 80% of respondents report checking account rates below or at 0.05%. (See Consumer Payment Research Center (2018) in the reference list.) We then use 0.05% in our calibration.

One straightforward way to calibrate the model is to solve the whole equilibrium given the parameter and then fit the money demand curve and the deposit rates. This, however, can be computationally cumbersome because one needs to solve the model for each data point used for the money demand and then optimize over a six-dimensional parameter. One key insight is that the money demand can be solved independent of the banking sector. This leads to the following algorithm that greatly simplifies the calibration.

1. Fix the value of $\Omega$. Fit the money demand and cash to the checking account balance ratio by choosing $B$, $\sigma$ given the values of $A$, $\theta$. More specifically, for each interest rate, calculate the steady state equilibrium using the nominal interest rate and the deposit rate. Then solve a nonlinear least squares problem over $(B, \sigma)$. The M1 to GDP ratio in the model can be calculated by $(Z + D) / Y$, where

$$Y = \sum_{j=1}^3 \alpha_j y_j + 2B + A \left[(1 - \chi) D \Psi(D)\right]^\eta - D + (1 - \chi) D \Psi(D)$$
is the output. Here $Y$ is the sum of the consumption of households in DM and CM, the consumption of bankers and entrepreneurs, and the investment.

2. Solve the banking competition problem to find $N$ such that the solution of the Cournot competition leads to a net interest margin 2.98%.

3. Calculate the markups and the deposit rate implied by the model under the current inflation level $\mu - 1 = 2\%$. If the markup is less than 20%, decrease $\theta$, otherwise increase $\theta$. Similarly, decrease $A$ if the deposit rate is higher than 0.05% and increase $A$ if otherwise.

4. Repeat 1-4 until the markup is 20% and deposit rate is 0.05% under $\mu - 1 = 2\%$. Record the sum of squares from the nonlinear least squares problem.

5. Do 1-5 for different values of $\Omega$ and find the value that yields the smallest sum of squares. This identifies $\Omega$, and the other parameters are those obtained corresponding to this $\Omega$.

This greatly simplifies the calibration because we reduce a problem of six parameters into three problems of lower dimensions that are linked by $A$ and $\theta$. In practice, we consider $\Omega = 0.1, 0.2, 0.4, \cdots, 1$ and $\Omega = 0.7$ leads to the smallest sum of squares.
References


