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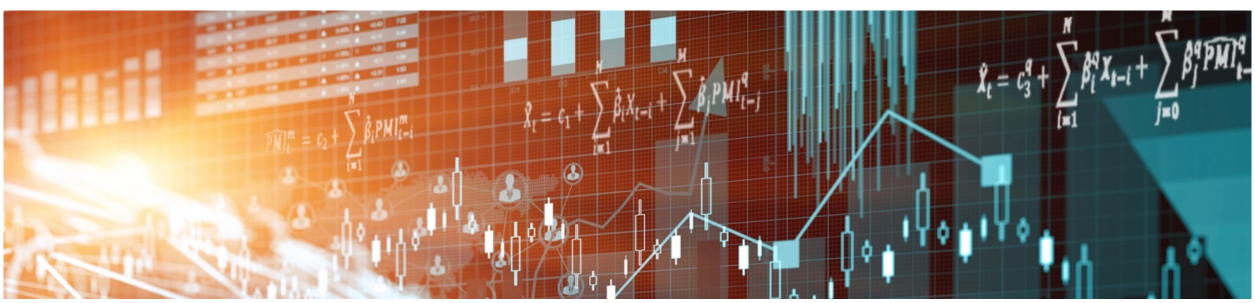

by Thierry Mayer and Walter Steingress

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Estimating the Effect of Exchange Rate Changes on Total Exports

by

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Abstract

This paper shows that real effective exchange rate (REER) regressions, the standard approach for estimating the response of aggregate exports to exchange rate changes, imply biased estimates of the underlying elasticities. We provide a new aggregate regression specification that is consistent with bilateral trade flows micro-founded by the gravity equation. This theory-consistent aggregation leads to unbiased estimates when prices are set in an international currency as postulated by the dominant currency paradigm. We use Monte-Carlo simulations to compare elasticity estimates based on this new "ideal-REER" regression against typical regression specifications found in the REER literature. The results show that the biases are small (around 1 percent) for the exchange rate and large (around 10 percent) for the demand elasticity. We find empirical support for this prediction from annual trade flow data. The difference between elasticities estimated on the bilateral and aggregate levels reduces significantly when applying an ideal-REER regression rather than a standard REER approach.

Bank topics: Econometric and statistical methods; Exchange rates; International topics
JEL codes : F11, F12, F31, F32

Résumé

La présente étude montre que l'approche couramment employée pour estimer l'incidence des variations du taux de change sur les exportations totales, qui se base sur des régressions du taux de change effectif réel (TCER), implique des estimations biaisées des élasticités sous-jacentes. Nous proposons une nouvelle spécification reposant sur des régressions agrégées qui est compatible avec une dérivation des flux commerciaux bilatéraux par le modèle de gravité. Conforme à la théorie, cette approche débouche sur des estimations non biaisées lorsque les prix sont établis dans une monnaie internationale, soit la monnaie de l'économie dominante dans les échanges. Les auteurs utilisent la méthode de Monte-Carlo pour comparer les estimations des élasticités obtenues au moyen de ces nouvelles régressions d'un TCER dit « idéal » avec celles obtenues au moyen des spécifications communément utilisées dans les études impliquant le TCER. Les résultats indiquent que le biais est faible (autour de 1 %) pour le taux de change, et prononcé (autour de 10 %) pour l'élasticité de la demande. Ces observations sont confortées de manière empirique par les données sur les flux commerciaux annuels. L'écart entre les estimations des élasticités aux niveaux bilatéral et agrégé est bien moins important avec notre spécification qu'avec l'approche standard.

Sujets : Méthodes économétriques et statistiques; Taux de change; Questions internationales
Codes JEL : F11, F12, F31, F32

Non-technical summary

Regressions that explain total exports using real effective exchange rates (REER) are ubiquitous in applied policy work, but they are often based on incorrect aggregation of theory-consistent bilateral trade equations and lead to biased estimates of trade elasticities. Models calibrated with these elasticities will have predictions that are inaccurate by exaggerating the response of exports and, by extension, output following an exchange rate shock.

This paper shows that standard REER regressions produce biased results, documents the source of the bias and proposes a new alternative regression specification. Our specification is consistent with bilateral trade flows and micro-founded by the gravity equation. Total exports are a function of the real exchange rate and foreign demand deflated by the destination-specific price index, all denominated in the dominant international currency (i.e., the US dollar). Denominating prices according to the dominant currency paradigm (DCP) has the key implication that pass-through depends on the exchange rate variation relative to the international currency and not on the bilateral exchange rate. This allows us to split the change in the relative price between the exporter and the importer into two components each denominated in the international currency. This separation provides for theory-consistent aggregation and unbiased estimation of the trade and demand elasticities.

Simulations of the theoretical model imply that standard REER regressions lead to significant aggregation bias in the underlying elasticities. The bias in the trade elasticity is rather small (close to 1 percent), while the bias in the demand elasticity is more pronounced (close to 10 percent), and both increase with measurement errors.

We show that calibrating macroeconomic models with estimates obtained by using our new regression specification improves their fit and results in predictions consistent with microeconomic behavior in bilateral trade equations.

1 Introduction

Regressions that explain total exports with real effective exchange rates (REERs) are ubiquitous in applied policy work,[1] but their analytical structures are only loosely based on trade theory. These open-economy "macro-level" regressions rely on *monadic* export equations where a country's change in total exports is the dependent variable with the REER as the main variable of interest and an additional control is considered for demand changes faced by the country. Despite considerable progress in the econometric aspects, the fundamental specification of McGuirk (1986), which derives its regression specification from Armington (1969), has remained virtually unchanged over the past few decades.[2] During the same period, considerable progress has been made in modeling the "micro-level" foundations of *dyadic (bilateral)* trade equations, which are called gravity equations. A seminal paper in the gravity literature is Anderson and Van Wincoop (2003), which is also based on Armington (1969). In the gravity literature, little attention has been given to estimate the impact of changes in the exchange rate (see Anderson et al. (2016) as a notable exception).[3] In this paper, we reconnect the two strands of the literature.

We start by assessing whether the standard approach of monadic macro regression is compatible with the micro-founded gravity equation that predicts a very precise functional form for an estimation explaining bilateral flows. Similar to Imbs and Méjean (2015), our paper is methodological in that we seek to derive the appropriate monadic aggregation of bilateral flows and compare our new "ideal-REER" approach with typical regressions from the standard REER literature. Differences arise due to the fact that the ideal-REER regression is based on proportional changes,[4] while McGuirk (1986)'s REER is based on partial derivatives (log changes). In the standard REER regression total exports are a log linear function of log changes of the *real effective exchange rate* and foreign demand. On the other hand, the aggregate export equation that follows a structural gravity approach implies a log-linear function of log changes in the *real exchange rate (RER)* and differences in foreign demand deflated by the destination-specific price index. The standard REER approach constitutes an approximation, which holds for small changes, and causes an aggregation bias.

The aggregation of bilateral trade flows inherently depends on the currency denomination. In our baseline gravity model, prices are set in a common international currency following the domi-

[1]Global projection models of central banks and international policy institutions (for example, Jakab et al. (2015)), the literature on global VAR models (Di Mauro and Pesaran (2013)) as well as on global imbalances (see, among others, Obstfeld and Rogoff (2005) and Lane and Milesi-Ferretti (2012)) rely on REER equations and their elasticities to summarize the trade linkages between the different regions in their model.

[2]Debate and progress in econometric aspects of such work is still ongoing, see Chinn (2006) for a primer on alternative measures of REER. More recently, Bennett and Zarnic (2009) introduce product heterogeneity and services exports. Bayoumi et al. (2013) and Bems and Johnson (2017) construct REER indexes based on value-added trade to account for global value chains. Patel et al. (2017) build on Bayoumi et al. (2013) and Bems and Johnson (2017) to study how input linkages affect REER measurement in a multi-sector economy.

[3]Most of the interest has been geared towards measuring the impact of bilateral trade costs, such as regional trade agreements (RTAs), tariffs and membership at the WTO. See Head and Mayer (2014) for an overview.

[4]Often referred to as exact hat algebra. See Arkolakis et al. (2012).

nant currency paradigm (DCP) as in Casas et al. (2017). DCP implies that the exchange rate variation relative to the international (dominant) currency and not the bilateral exchange rate determines pass-through. As a result, we can split the relative price between the exporter and the importer into the sum of an exporter-specific (the RER) and an importer-specific component (foreign demand deflated by the importer's price index) each denominated in the international currency. This separation provides for theory-consistent aggregation and unbiased estimation of the trade and demand elasticities. If prices are set in the producer's currency (producer currency pricing (PCP)) or in the local currency (local currency pricing (LCP)), the pass-through depends on the bilateral exchange rate and estimation of the aggregate elasticities without bias is only possible if pass-through is complete. Extending the model to allow markups to vary across trading partners reduces the response of prices to the exchange rate change and lowers the bias in both the demand and the trade elasticities.

To reveal the importance of theory-consistent aggregation for parameter inference, we resort to theoretical model simulations and quantify how departures from the model's strict form can generate biases in trade and demand elasticity estimates. To that purpose, we use the Dekle et al. (2007) version of the structural gravity model (compatible with most of the commonly used theoretical models of international trade). Monte Carlo simulations show that while there is discordance between the ideal-REER and standard REER regressions, the magnitude of the biases in elasticities are dependent on the precise regression specification as well as the underlying assumptions on the exchange rate shock. Overall, the bias in the trade (exchange rate) elasticity is rather small (close to 1 percent), while the bias in the demand elasticity is more pronounced (close to 10 percent) and increases with measurement errors. If, instead, we re-estimate the same specifications and denominate trade in the producer currency, the aggregation bias increases significantly, the overall fit of the regression is lower, the point estimates of the elasticities are smaller and the standard errors are larger in all specifications.

Next, we build on the theoretical basis of our paper and compare the empirical performance of our ideal-REER with the standard REER approach. We use annual bilateral trade and price data for 25 countries that are part of the Bank for International Settlements (BIS) narrow effective exchange rate indexes (see Klau and Fung (2006)) for the period 1964 to 2014. As a first step, we follow Boz et al. (2017) and run exchange rate pass-through regressions to test (1) whether pass-through is complete, and (2) whether the pass-through depends on the currency denomination of export and import prices. Our results show that, overall, pass-through is incomplete (with an average coefficient of 0.16) and both exporter and importer prices vary mainly with the US dollar supporting the dominant currency paradigm. In the second step, we aggregate bilateral trade flows denominated in US dollars and estimate the trade and demand elasticities for both the ideal-REER and the standard REER regression specifications following the DCP. Evidence based on the pooled sample across all countries, in addition to country-specific regressions, corroborate the simulation results. The difference between the elasticities estimated on the bilateral and the aggregate levels reduces significantly when applying an ideal-REER regression rather than the standard REER approach.

Taken together, these results highlight two additional advantages of the dominant currency paradigm, which currently is the yardstick used in evaluating pricing and exchange rate pass-through (see Boz et al. (2017) and Casas et al. (2017)). First, DCP allows for consistent aggregation of micro-level founded bilateral trade equations with a minimum set of assumptions on the pricing behavior and, second, the estimation performance of aggregate regressions improves significantly when denominating the variables in US dollars.

Our paper contributes to the macroeconomic literature on exchange rate indexes as a measure of price competitiveness. The REER is an important statistical indicator, produced by international agencies such as the IMF (see Bayoumi et al. (2005)), the BIS (see Klau and Fung (2006)) and many central banks (see Schmitz et al. (2011) for the ECB and Barnett et al. (2016) for the Bank of Canada references). We see our analysis as complementary to the recent papers that study the theoretical foundations for these indexes. These papers' focus lies more in introducing global linkages into the measurement of competitiveness (see Bayoumi et al. (2013), Patel et al. (2017) and Bems and Johnson (2017)). We show that using the REER to estimate the aggregate response of exports leads to biased estimates of the underlying elasticities. The bias exists due to functional form assumptions, currency denomination and inconsistent weighting of trade flows in the aggregation of bilateral trade flows.

Our analysis also speaks to the old macroeconomic literature on estimating trade elasticities started by Orcutt (1950), Houthakker and Magee (1969) and Goldstein et al. (1985) and recent contributions by Spilimbergo and Vamvakidis (2003), Freund and Pierola (2012), Bussière et al. (2014a) and Ahmed et al. (2016). All of these papers rely on effective exchange rates to estimate demand and trade elasticities. Our results warrant caution in the use of these estimates for counterfactual policies, particularly in the case of the demand elasticity, where bias can be significant. As an alternative, we suggest our ideal-REER regression framework to mitigate these concerns. These conclusions also apply to studies that use aggregate time-series data to estimate exchange rate pass-through coefficients such as Campa and Goldberg (2005), Vigfusson et al. (2009) and Bussière et al. (2014b).

The remainder of the paper is structured as follows. Section 2 derives the gravity-compatible aggregate export equation in the partial as well as in the general equilibrium. Section 3 describes the standard empirical approaches used in the applied literature and discusses the functional form differences with respect to the gravity approach. Section 4 quantifies the biases of the different specifications using Monte-Carlo simulations of partial as well as general equilibrium counterfactuals of exchange rate shocks. Section 5 discusses the importance of the underlying pricing behavior and currency denomination. Section 6 introduces the data and provides empirical evidence on the key predictions of our theoretical framework. Section 7 concludes.

2 The gravity approach

We start with a description of the structural gravity equation based on Head and Mayer (2014) derived from the Armington model. The central feature of the Armington model is that it is not only at the heart of the gravity equation literature but also forms the basis of the real exchange rate regressions as in Artus and McGuirk (1981), McGuirk (1986) and Spilimbergo and Vamvakidis (2003).

2.1 Structural gravity

Consider a world economy consisting of N countries. Each country is endowed with Q_n units of a distinct good. In each country, the representative agent has Constant Elasticity of Substitution (CES) preferences. Trade between countries is costly and takes the form of iceberg trade costs. In order to sell one unit of good i in country n, exporters from country i have to ship $\tau_{ni} \geq 1$ units. Trade within the country is costless, i.e., $\tau_{ii} = 1$. To prevent arbitrage opportunities, we assume that trading bilaterally between i and n is always cheaper than trading via an intermediate country k ($\tau_{nk}\tau_{ki} \geq \tau_{ni}$ for $\forall i, j, k$). Note that given country i's endowment Q_i and total income Y_i, we can express supply as $S_i = Y_i/Q_i$, the corresponding producer price of good i as $P_{ii} = S_i$ and the price charged to consumers in country n to be $P_{ni} = \tau_{ni}S_i$. Given our assumptions, the gravity equation is a product of the share of expenditure that importer n allocates on goods from exporter i, π_{ni}, and the overall expenditure, X_n. Following Head and Mayer (2014), π_{ni} can be expressed in the following multiplicative separable form:

$$\pi_{ni} = \frac{S_i\phi_{ni}}{\Phi_n} \qquad \text{where} \qquad \Phi_n = \sum_{\ell=1}^{N} (S_\ell \phi_{n\ell}) \qquad \text{and} \qquad \Omega_i = \sum_{\ell=1}^{N} \frac{\phi_{\ell i} X_\ell}{\Phi_\ell}, \tag{1}$$

where ϕ_{ni} captures the bilateral accessibility of n to exporter i and is a function of iceberg trade costs, Φ_n represents the multilateral resistance term and Ω_i represents the weighted sum of the exporter capabilities.

In order to quantify potential estimation biases and to produce counterfactual simulation estimates of the structural gravity model, we need to specify the underlying supply structure. In this paper, we opt for the Ricardian Comparative Advantage specification with intermediate inputs from Dekle et al. (2007) based on Eaton and Kortum (2002) and Alvarez and Lucas (2007). In this model, each country produces a large number of tradable intermediate goods that are homogeneous across countries. Productivity z differs across goods and is assumed to follow a Fréchet distribution with a cumulative distribution function of $\exp(-T_i z^{-\theta})$, where T_i is a technology parameter and θ determines the amount of heterogeneity in the productivity distribution. For each intermediate good z, buyers choose the lowest cost supplier in the world. Aggregating across all goods, assuming that factor prices are denominated in the domestic currency as well as assuming complete pass-through, the expenditure share that buyers in n spend on goods from i can be written as

$$\pi_{ni} = \frac{T_i \left(r_i w_i^{\beta} p_i^{1-\beta} \tau_{ni} \right)^{-\theta}}{\left(r_n P_n \right)^{-\theta}} \qquad \text{where} \qquad P_n^{-\theta} = \sum_{\ell=1}^{N} T_l \left(\frac{r_l}{r_n} w_l^{\beta} p_l^{1-\beta} \tau_{n\ell} \right)^{-\theta}, \tag{2}$$

where w_i is the wage paid to workers in i, p_i is the price of tradable intermediate goods, r_i is the nominal exchange rate that converts domestic prices into a common international currency (the US dollar, for instance) and β is the share of intermediate goods in the production of manufacturing goods. Comparing equation (2) to equation (1) shows that the three structural gravity terms are given by $S_i = T_i \left(r_i w_i^{\beta} p_i^{1-\beta} \right)^{-\theta}$, $\phi_{ni} = \tau_{ni}^{-\theta}$ with θ as the trade elasticity and the multilateral resistance term in international currency $\Phi_n = \left(r_n P_n \right)^{-\theta}$.

Asserting a market clearing condition implies that total output in i, Y_i, corresponds to total spending, X_i. With labor as the sole factor, the value of production in local currency in the country of origin is given by $Y_i = w_i L_i$. Following Dekle et al. (2007), the market clearing conditions for the manufacturing sector in international currency become

$$r_i w_i L_i = \sum_n \pi_{ni} \left(w_n L_n + r_n D_n \right),$$

where D_i is the overall trade deficit in local currency.[5]

At this point, it is important to note that while we chose the Eaton-Kortum variant of the gravity model as the underlying theoretical foundation, all other theoretical models that adhere to structural gravity are compatible with the following analysis on the estimation bias. The main difference will be the underlying implications on the micro-structure and its corresponding interpretations.

2.2 Counterfactual analysis

To produce counterfactual analysis, we use the exact hat notation, see Dekle et al. (2007) and Arkolakis et al. (2012), to derive the aggregate export equation. More precisely, hats denote the ratio of post-shock to pre-shock values. We start by assessing the change in bilateral exports and then aggregate to obtain the change in total exports. For bilateral exports of country i to country n in international currency, we have

$$\hat{X}_{ni} \equiv \frac{X'_{ni}}{X_{ni}},$$

where X'_{ni} is bilateral trade after the change and X_{ni} is the value before any change. The resulting change in total exports after the policy change is simply the sum over all import countries with the exception of country i's home market. Furthermore, we can decompose the change in total exports ($\hat{E}_i$) as the sum of changes in the expenditure shares and the aggregate expenditure in international

[5]For exposition purposes we do not present the derivations of the market clearing condition and refer to Dekle et al. (2007) for the details.

currency across all importing countries weighted by the initial share of exports going to country n ($\omega_{ni} = X_{ni}/E_i$):

$$\hat{E}_i = \frac{1}{E_i} \sum_{n \neq i} \pi^{'}_{ni} X^{'}_n = \sum_{n \neq i} \omega_{ni} \widehat{\pi}_{ni} \hat{X}_n \tag{3}$$

This equation is extremely general and simply states that the change in a country's total exports corresponds to the weighted sum of all proportional changes in bilateral exports. Weights are the initial share of each bilateral flow in total exports. The change in each bilateral flow can have two origins: i) an increase in total expenditure in destination n, or ii) a change in competitiveness of country i relative to all other trading partners of n.

As a policy shock, we consider an exogenous change in the nominal exchange rate of country i, r_i, which is independent of the exchange rate in all other countries. The shock to r_i acts like a shifter of delivered prices to the consumer (which assumes complete pass-through at this stage).[6] Without migration or other affects to the population, we have changes in output given by changes in the nominal exchange rate and wages, $\hat{r}_i \hat{w}_i = \hat{Y}_i$. Assuming that technology and trade costs are unaffected by the shock, the implied change in the bilateral trade shares resulting from a change in the nominal exchange rate takes the following form:

$$\widehat{\pi}_{ni} = \frac{\left(\hat{r}_i \hat{w}_i^{\beta} \hat{p}_i^{1-\beta}\right)^{-\theta}}{\left(\hat{r}_n \hat{P}_n\right)^{-\theta}}, \tag{4}$$

where the change in the price index of country n is equal to the change in country i's exchange rate times the weight of country i in country n's import basket,

$$\widehat{P}_n^{-\theta} = \sum_{k=1}^{N} \pi_{nk} \left(\frac{\hat{r}_k}{\hat{r}_n} \hat{w}_k^{\beta} \hat{p}_k^{1-\beta} \right)^{-\theta}. \tag{5}$$

Plugging the calculated trade shares back into the market clearing condition, one can solve for the changes in production of each country of origin. The resulting changes in equilibrium wages (w_i) paid in local currency are

$$\hat{r}_i \hat{w}_i Y_i = \sum_n \frac{\pi_{ni} \left(\hat{r}_i \hat{w}_i^{\beta} \hat{p}_i^{1-\beta}\right)^{-\theta}}{\sum_k \pi_{nk} \left(\hat{r}_k \hat{w}_k^{\beta} \hat{p}_k^{1-\beta}\right)^{-\theta}} \left(\hat{r}_n \hat{w}_n Y_n + D_n\right). \tag{6}$$

Note that, in this model, a nominal exchange rate shock will affect relative prices and lead to changes in trade flows. The reason is that we fix the trade deficit in international currency, i.e., in US dollars. Given this assumption, the wage adjustment will not be one-to-one with exchange rate change and depends on the dollar value of the current account deficit.[7] As such, this scenario will be

[6] In this model the exchange rate shock is a nominal shock that changes the unit of account of prices.

[7] Alternatively, if one assumes that the ratio of the current account deficit to GDP is constant or trade is balanced

instructive to understand potential aggregation biases when constructing an effective exchange rate in general equilibrium.[8]

To obtain the export equation, we can simply apply equation 4 into equation 3. Taking logs and assuming that prices are set in the international currency,[9] we can write the log change in exports as a function of the log change in the real exchange rate (RER_i) and foreign demand as

$$\ln \hat{E}_i = -\theta \ln R\hat{E}R_i + \ln \sum_{n \neq i} \omega_{ni} \left(\frac{\hat{X}_n}{\left(\hat{r}_n \hat{P}_n\right)^{-\theta}} \right). \tag{7}$$

$\hat{X}_n = \hat{r}_n \hat{w}_n Y_n + D_n$ represents changes in absorption and the real exchange rate is defined as the change in the nominal exchange rate and the change in the producer price in the exporting country in local currency, i.e., changes in intermediate input prices and wages, $R\hat{E}R_i = \hat{r}_i \hat{P}_{ii} = \hat{r}_i \hat{w}_i^{\beta} \hat{p}_i^{1-\beta}$.

Equation 7 implies that, under the assumption of a constant elasticity of substitution, there is no aggregation bias in aggregate macro exchange rate regressions. In order to obtain these unbiased estimates, one needs to regress the log of the change in nominal exports on the log of the change in the real exchange rate and the foreign demand deflated by changes in the importer's price index:

$$\ln \hat{E}_i = \beta_{RER} \ln R\hat{E}R_i + \beta_X \ln \sum_{n \neq i} \omega_{ni} \left(\frac{\hat{X}_n}{\left(\hat{r}_n \hat{P}_n\right)^{\beta_{RER}}} \right) + \epsilon_i \tag{8}$$

β_{RER} will be the estimated exchange rate elasticity, β_X the corresponding demand elasticity and ϵ_i the error term. However, to our knowledge, there is no paper running regressions based on equation 8. Instead, the literature focuses on trade-weighted real (or nominal) effective exchange rate regressions, which aggregate bilateral exchange rates to a single country-specific indicator. Next, we discuss the theoretical underpinnings of the effective exchange rate regressions used in applied work and relate them to the structural gravity framework.

($D_n = 0, \forall n$), then changes in the exchange rate will be one-to-one offset by changes in the local wage and have no real effects. In this case the equilibrium condition looks as follows:

$$\hat{r}_i \hat{w}_i Y_i = \sum_n \frac{\pi_{ni} \left(\hat{r}_i \hat{w}_i^{\beta} \hat{p}_i^{1-\beta}\right)^{-\theta}}{\sum_{\ell} \pi_{n\ell} \left(\hat{r}_l \hat{w}_l^{\beta} \hat{p}_l^{1-\beta}\right)^{-\theta}} \left(\hat{r}_n \hat{w}_n \left(Y_n + D_n\right)\right)$$

The appendix discusses this point in more detail and provides a proof that a nominal exchange rate shock has real effects when fixing the current account deficit in international dollars.

[8] Anderson et al. (2016) consider an alternative approach that leads to changes in relative prices across countries after a nominal exchange rate shock. They assume an exogenous degree of incomplete exchange rate pass-through into import prices ($\hat{r}_i = \hat{r}_i^{\rho_n}$ where ρ_n is the exogenous degree of import pass-through). However, in this case aggregation of the bilateral flows to an effective exchange rate without bias will not be possible. The change in the exporter's price will depend on the importing country n and cannot be separated from the demand effect as in equation 7.

[9] We discuss the importance of this assumption together with alternative pricing assumptions on the exchange rate pass-through in section 5.

3 The real effective exchange rate approach

McGuirk (1986) is the standard reference that describes the methods used to construct an aggregate price competitiveness indicator, or the real effective exchange rate. This indicator attempts to measure a country's price change in the tradable sector relative to those of other countries after converting each of them into a common currency. In this section we show that aggregation based on the effective exchange rate relies on a functional form approximation and that using this indicator for parameter inference will lead to bias in the estimated coefficients.

The theoretical foundations of the McGuirk (1986) approach rests on the Armington (1969) assumption of imperfect substitutability between goods and a constant elasticity of substitution (CES), which is consistent with structural gravity. The starting point is writing the bilateral import demand function of country n for goods from country i , X_{ni}, as a function of total expenditure X_n and the bilateral shares defined in equation 2. Taking the log and considering the change from the initial equilibrium to the new equilibrium $\ln \hat{X}_{ni} = \triangle \ln X_{ni} = (\ln X'_{ni} - \ln X_{ni})$, we get

$$\ln \hat{X}_{ni} = -\theta \ln \left(\frac{\hat{r}_i}{\hat{r}_n} \frac{\hat{P}_{ni}}{\hat{P}_n} \right) + \ln \hat{X}_n, \tag{9}$$

where $\ln \hat{P}_{ni}$ denotes the change in the price that exporting country i charges to consumers in country n and $\ln \hat{P}_n$ is the change in the price index of the importing country n, both denoted in local currency.[10] Concerning the changes in the importing country's price changes, McGuirk (1986) approximates the log change of the importer's price as a weighted average of the log changes in the production prices:

$$\ln \hat{r}_n \hat{P}_n = \sum_{k=1}^{K} \pi_{nk} \ln \left(\hat{r}_k \hat{P}_{kk} \right), \tag{10}$$

where $\hat{P}_{kk}$ represents the change in producer price index in country k and π_{nk} is the expenditure shares of country n. Using further simplifications outlined in the appendix, we can substitute the import price index in equation 9 with equation 10 and aggregate across all importing countries. To do so, we sum the bilateral trade flows over all markets (excluding the home market) by weighting the flows by the share of i's output sold in each market:

$$\sum_{n \neq i} \omega_{ni} \ln \hat{X}_{ni} = -\theta \sum_{n \neq i} \sum_{k \neq i} \omega_{ni} \pi_{nk} \ln \left(\frac{\hat{r}_i}{\hat{r}_k} \frac{\hat{P}_{ii}}{\hat{P}_{kk}} \right) + \sum_{n \neq i} \omega_{ni} \ln \hat{X}_n,$$

where the weight ω_{ni} is defined as the share of export revenues of country i that comes from sales in destination n with respect to total export sales. Next, we define the double-weighted real effective exchange rate (REER) as follows:

[10] Based on the model in section 2 and the assumption that trade costs are fixed, the change in the foreign price equals the change in producer prices, which equals the change in marginal costs $\ln \hat{P}_{ni} = \ln \hat{P}_{ii} = \ln \left(\hat{w}_i^{\beta} \hat{p}_i^{1-\beta} \right)$.

$$\ln RE\hat{E}R_i = \sum_{n\neq i}\sum_{k\neq i}\omega_{ni}\pi_{nk}\ln\left(\frac{\hat{r}_i}{\hat{r}_k}\frac{\hat{P}_{ii}}{\hat{P}_{kk}}\right) \tag{11}$$

and write the standard REER regression used in the literature as

$$\ln \hat{E}_i = \beta_{REER}\ln RE\hat{E}R_i + \beta_X\sum_{n\neq i}\omega_{ni}\ln\hat{X}_n + \epsilon_i, \tag{12}$$

where β_{REER} is the estimated exchange rate elasticity of specification and β_X the demand elasticity. ϵ_i represents the estimation error. The estimation produces unbiased estimates if the exchange rate elasticity ($\hat{\beta}_{REER}$) is equal to θ and if the foreign demand elasticity ($\hat{\beta}_X$) is equal to one.

3.1 Aggregation bias

The essential difference between the REER regression in equation 12 and the ideal-REER regression implied by gravity in equation 7 is that the change in total exports is approximately equal to the weighted sum of the bilateral export changes, i.e., $\ln\hat{E}_i \approx \sum_{n\neq i}\omega_{ni}\ln\hat{X}_{ni}$. To see the difference between the two approaches explicitly, we rewrite the REER in equation 9 as a weighted average of the bilateral real exchange rate without substituting for the importer's price index using equation 10. The resulting change in the REER is simply the log difference between the changes in the real exchange rate (RER) of exporter i and a weighted average of the price index changes of every trading partner:

$$\ln RE\hat{E}R_i = \ln\left(\hat{P}_{ii}\hat{r}_i\right) - \sum_{n\neq i}\omega_{ni}\ln\left(\hat{P}_n\hat{r}_n\right). \tag{13}$$

Substituting the implied REER in equation 13 back into the standard REER regression defined in equation 12, we can compare the ideal REER regression in equation 7 with the standard REER regression:

$$-\theta\ln\left(\hat{P}_{ii}\hat{r}_i\right) + \ln\sum_{n\neq i}\omega_{ni}\left(\frac{\widehat{X_n}}{\left(\hat{P}_n\hat{r}_n\right)^{-\theta}}\right) \approx -\theta\left(\ln\left(\hat{P}_{ii}\hat{r}_i\right) - \sum_{n\neq i}\omega_{ni}\ln\left(\hat{P}_n\hat{r}_n\right)\right) + \sum_{n\neq i}\omega_{ni}\ln\hat{X}_n$$

Canceling the RER_i on both sides and rewriting the destination-specific terms, we can apply Jensen's inequality and arrive at the following inequality:[11]

$$\ln\sum_{n\neq i}\omega_{ni}\left(\frac{\hat{X}_n}{\left(\hat{P}_n\hat{r}_n\right)^{-\theta}}\right) \geq \sum_{n\neq i}\omega_{ni}\ln\left(\frac{\hat{X}_n}{\left(\hat{P}_n\hat{r}_n\right)^{-\theta}}\right) \tag{14}$$

[11] The result follows from Jensen's inequality and parallels the proof of the inequality of arithmetic and geometric means, more formally, $\ln\sum_{n\neq i}\omega_{ni}x_n \geq \ln\prod_{n\neq i}\left(x_n^{w_{ni}}\right)$.

Equation 14 states that the log of total exports (the LHS) is larger than the weighted average of the bilateral trade flows (the RHS). The difference between the two expressions is related to the Theil inequality index[12], which measures the heterogeneity in the destination-specific changes. Intuitively, the Theil index and the resulting bias will be larger the higher the variance is from destination-specific shocks. In the extreme case, if prices and demand change only in one destination, the Theil index is at a maximum value and the estimation biases in the demand and exchange rate elasticities of the REER approach will be at their largest.[13] On the other hand, when the price and demand changes in all destinations are the same, the Theil index will be zero and both regression specifications will lead to the same elasticities. In the simulation section, we quantify the estimation biases as a function of the exchange rate shocks and the trade elasticity. Before, we discuss other real effective exchange rate regression specifications found in the literature starting with an alternative weighting scheme.

3.2 Weighting scheme

The effective exchange rate calculated by policy institutions such as the IMF (Bayoumi et al. (2005)), the European Central Bank (ECB) (Schmitz et al. (2011)) and the Bank for International Settlements (BIS) (Klau and Fung (2006)) follows the double weighting approach approach of McGuirk (1986) but uses slightly different weights. Their calculations include the sales of the exporter country in its domestic market. The REER is defined as follows:

$$\ln R\hat{E}ER_i^{IMF} = \sum_{k \neq i} TW_{ki} \ln \left(\frac{\hat{r}_i}{\hat{r}_k} \frac{\hat{P}_{ii}}{\hat{P}_{kk}} \right),$$

and the implied weights are given by

$$TW_{ki} = \sum_{n=1}^{N} \psi_{ni} \pi_{nk}, \tag{15}$$

where ψ_{ik} is the export weight including the home market shares. ψ_{ik} relates to our export weights in the following way:

$$\psi_{ni} = \frac{X_{ni}}{\sum_{n=1}^{N} X_{ni}} = \omega_{ni} \left(\frac{\sum_{n \neq i} X_{ni}}{\sum_{n=1}^{N} X_{ni}} \right)$$

However, as Bayoumi et al. (2005) note, the weights implied by equation 15 do not sum to 1. As a result, the weights are normalized leading to the following definition:[14]

[12] The Theil L entropy index is defined as $T_L = \sum_{i=1}^{N} w_i \ln \left(\frac{\mu}{x_i} \right)$, where w_i is the importance weight of observation i's characteristic x_i. The sum of w_i over all observations i equals one. μ is the corresponding arithmetic mean.

[13] New Zealand is a good example. An exchange rate shock to the New Zealand dollar will change prices and demand in Australia but is likely to have little impact on other parts of the world.

[14] The appendix provides details on the derivations.

$$TW_{ki} = \frac{\sum_{n=1}^{N} \psi_{ni} \pi_{nk}}{\sum_{n=1}^{N} \psi_{ni} \left(1 - \pi_{ni}\right)} \tag{16}$$

The denominator of equation 16 ensures that TW_{ki} sums to 1. The resulting estimation equation is then given by

$$\ln \hat{E}_i = \beta_{REER} \ln R\hat{E}ER_i^{IMF} + \beta_X \sum_{k \neq i} \omega_{ni} \ln \hat{X}_n + \epsilon_i, \tag{17}$$

where the weights for the foreign demand continue to be the export shares ω_{ni}. We do not have any prior on the direction of the bias in the demand (β_X) and the exchange rate elasticity (β_{RER}) caused by the weighting scheme and refer to section 4 for the quantitative simulation results.

3.3 Multilateral resistance term

The value of exports depends not only on the country's export price and foreign demand, but also on the willingness of foreigners to substitute domestic for foreign goods. The multilateral resistance term Φ_n defined in equation 1 picks this idea up. This subsection discusses the estimation bias in the demand and trade elasticities provoked by the absence of the multilateral terms in the ideal-REER and the standard REER regression approach.

The first point to note is that the change in the multilateral resistance term summarizes all the price changes due to variations in the competitiveness of exporter n as well as changes in country i's own competitiveness in its domestic market:

$$\hat{\Phi}_n = \left(\hat{r}_n \hat{P}_n\right)^{-\theta} = \sum_{k=1}^{N} \pi_{kn} \left(\hat{r}_k \hat{P}_{kk}\right)^{-\theta} \tag{18}$$

In our framework, the price change in exporter's competitiveness $\hat{P}_{kk}$ is simply given by the change in the exporter's per unit costs of production (wages and intermediate input prices) and the exchange rate. The parameter θ describes consumers' willingness to substitute domestic for foreign goods following a price change. Omitting the multilateral resistance term in the export equation, we run the following regression:

$$\ln \hat{E}_i = \beta_{RER} \ln \underbrace{\left(\hat{P}_{ii} \hat{r}_i\right)}_{R\hat{E}R_i} + \beta_X \ln \sum_{n \neq i} \omega_{ni} \hat{X}_n + u_i \tag{19}$$

Equation 19 implies that the multilateral resistance term enters the error $u_i = -\ln \sum_{n \neq i} \omega_{ni} \left(\hat{r}_n \hat{P}_n\right)^{-\theta} + \epsilon_i$. The omitted variable bias in the estimate of θ depends on the co-variance between the change in the exporter's price and the multilateral resistance term:

$$\hat{\beta}_{RER} = -\theta + \frac{Cov\left(\ln \hat{r}_i \hat{P}_{ii}, \ln \sum_{n \neq i} \omega_{ni} \left(\hat{r}_n \hat{P}_n\right)^{-\theta}\right)}{Var\left(\ln \hat{r}_i \hat{P}_{ii}\right)} \tag{20}$$

In general, the multilateral resistance term will be positively correlated with changes in the real exchange rate. The following example with the US as the exporter and Canada as the importer illustrates this correlation. Suppose that the US dollar depreciates $(\hat{r}_i \uparrow)$, which increases competitiveness of US exports in all destinations. Depending on the Canadian expenditure share on goods from the US, this change lowers the Canadian price index directly by making US goods cheaper and indirectly by reducing Canadian producer prices through lower costs for intermediate inputs. More generally, countries with the highest expenditure share on US goods (Canada and Mexico) will experience the largest decline in prices. Given that the multilateral resistance term in equation 18 depends negatively on the price index, the correlation with the exchange rate shock is positive and the trade elasticity will be biased towards zero.

Similarly, the bias in the demand elasticity depends on the co-variance between the change in the foreign demand and the multilateral resistance term in the error:

$$\hat{\beta}_X = 1 - \frac{Cov\left(\ln \sum_{n \neq i} \omega_{ni} \widehat{X_n}, \ln \sum_{n \neq i} \omega_{ni} \left(\hat{r}_n \hat{P}_n\right)^{-\theta}\right)}{Var\left(\ln \sum_{n \neq i} \omega_{ni} \widehat{X_n}\right)} \tag{21}$$

In this case we expect a downward bias in the demand elasticity due to a positive correlation between the multilateral resistance term and foreign demand. The reasoning parallels the correlation with the real exchange rate. An exchange rate depreciation of country i, r_i, reduces the price index in destination n and increases the multilateral resistance term. The lower prices increase demand and imply a positive correlation between the multilateral resistance term and foreign demand.

So far, we have shown that if the assumptions of structural gravity hold, one can consistently aggregate bilateral exchange rate changes and estimate the trade and demand elasticities using the aggregate export equation. However, the empirical literature follows an alternative approach based on real effective exchange rates that essentially approximates the aggregate change of exports using a weighted average of changes in bilateral exports. As we have shown, this approximation will lead to an aggregation bias in the estimated elasticities if the underlying exogenous shock causes a heterogeneous response in bilateral trade. We have also discussed alternative approaches in the applied literature that are based on differences in the weighting scheme as well as highlighted the importance of accounting for the multilateral resistance term. In the following section, we simulate a standard structural gravity model in order to quantify the magnitude of the estimation biases when using real effective exchange rate regressions for parameter inference.

4 Simulation results

Having described the underlying micro-structure, we quantify potential estimation bias in various exchange rate regression specifications using the data set provided by Dekle et al. (2007). These data comprise bilateral trade flows in manufactures, GDP as well as balance of payments information for 39 countries (plus the Rest of the World) in the year 2004.[15]

1. Ideal-REER

The first specification captures the real exchange rate model implied by the exact hat algebra in equation 8. This ideal-REER regression is the benchmark specification and will produce unbiased estimates of the trade elasticity ($\beta^1_{RER} = -\theta$) and the foreign demand elasticity ($\beta^1_X = 1$) with no error. Note that the regression equation 8 is non-linear because the trade elasticity is unknown. We estimate the relevant elasticities using SILS (structurally iterated least squares) as described in Head and Mayer (2014).

2. Multilateral resistance ("Gold medal mistake")

An important problem when estimating REER regressions is the presence of the multilateral resistance term. This term captures the implied price changes in the rest of the world due to policy shocks. We assess the importance of the omitted variable bias due to the absence of the multilateral terms by estimating equation 19.

3. Real effective exchange rate *à la* McGuirk

The third specification quantifies the aggregation bias when estimating the double-weighted REER regression in equation 12.

4. Approximation via log changes

One difference between the double-weighted REER regression *à la* McGuirk and the ideal-REER regression is that the multilateral resistance term deflates export prices rather than foreign demand. To assess the importance of this functional form assumption, we approximate the baseline regression with log changes and keep the non-linear form of equation 8:

$$\ln \hat{E}_i = \beta_{RER} \ln R\hat{E}R_i + \beta_X \sum_{n \neq i} \omega_{ni} \left(\frac{\ln \hat{X}_n}{\ln \left(\hat{r}_n \hat{P}_n\right)^{\beta_{RER}}} \right) + \epsilon_i \tag{22}$$

Similar to the baseline specification, we estimate equation 22 non-linearly using SILS.

[15]The bilateral trade share matrix is a sufficient statistic for a set of parameters including trade costs. For further details, please refer directly to Dekle et al. (2007).

5. Real effective exchange rate with different weights

The fifth specification quantifies the estimation bias when running a double-weighted REER regression and using an alternative weighting scheme, i.e., IMF weights, as defined in equation 17.

For each of the 5 specifications, we consider two types of policy experiments. The first one is the partial equilibrium approach (modular trade impact (MTI) counterfactual), where we consider an exogenous change in the nominal exchange rate and assume that wages do not adjust (however, intermediate input prices do adjust). In the second experiment, the general equilibrium trade integration (GETI) counterfactual, wages and prices will fully adjust following a shock to the nominal exchange rate.

4.1 Modular Trade Impact (MTI) counterfactual

The MTI counterfactual considers changes in prices and leaves wages constant by allowing the trade deficit to change. This case is identical to the one considered by McGuirk (1986). This approach allows us to abstract from income changes in the interpretation of the bias in the exchange rate elasticity. The counterfactual consists of a random nominal exchange rate shock ($\hat{r}_i$) specific to country i (France, for example), keeping the exchange rates of all other countries fixed. Each shock is drawn from a normal distribution with a variance of 0.1 (equivalent to a 10 percent appreciation or depreciation). For every country, we run 100 different exchange rate shocks to obtain a sample of 39x39x100 data points. Next, we run a cross-section OLS regression for each replication and record the demand and exchange rate coefficients. Table 1 contains the results.

The top value in Table 1 reports the average exchange rate and demand elasticity for the 3900 estimated coefficients. The second row corresponds to the implied bias of the estimate, i.e., the difference between the sample mean and the true value of the coefficients. The third row shows the standard deviation of the estimates. Lastly, the fourth row gives the square root of the estimated mean squared error (MSE), our selection criteria for the specification with the lowest aggregation bias. The first column shows the results for the baseline specification. The estimates are equal to the true values of the elasticities (-4 and 1, respectively) and therefore unbiased.[16] All other specifications lead to a bias. In general, the bias is more pronounced for the demand than for the exchange rate elasticities. The MSE for the demand elasticities is significantly larger than the MSE for the exchange rate elasticities. The omission of the multilateral resistance term in Specification 2 (the "Gold medal mistake") shown in column (2) produces the largest biases for the exchange rate elasticity (-3.872). The specifications with the smallest bias are the double-weighted REER approach *à la* McGuirk in column (3) and the log approximation of the baseline specification in column (4). The point estimate

[16] In the baseline simulation we calibrate $\theta = 4$, which is a common value found in the literature (see Simonovska and Waugh (2014)). However, the magnitude of the bias does not depend on the level of this elasticity. Table 9 in the appendix contains simulation results for when the elasticity is $\theta = 0.8$.

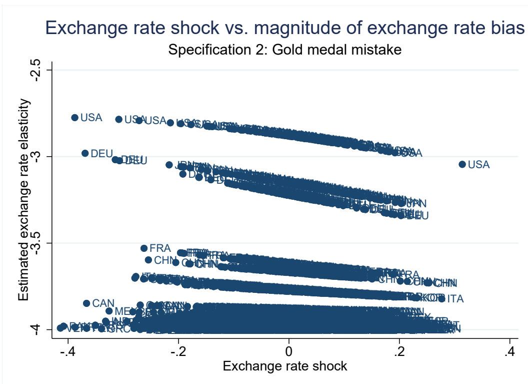
Table 1: MTI simulated elasticities of country-specific random exchange rate shocks

	Baseline ("ideal-REER") (1)	GM mistake (2)	REER McGuirk ("real-REER") (3)	d ln approx. (4)	Alternative weights (5)
Exchange rate					
Mean	-4	-3.872	-4	-4	-3.957
Bias	0	.128	0	0	.043
Std dev.	0	.247	.004	.003	.08
MSE $^{1/2}$	0	.278	.004	.003	.091
Foreign demand					
Mean	1	0	0	1.002	0
Bias	0	0	0	.002	0
Std dev.	0	0	0	.039	0
MSE $^{1/2}$	0	0	0	.039	0

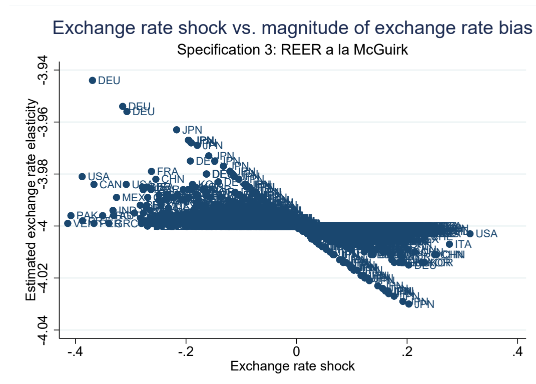
Notes: The sample consists of 39 countries and each country receives 100 random exchange rate shocks. The total number of estimated exchange rate and demand elasticities is 3900 for each of the 5 different specifications. The top cell labeled "Mean" reports the sample mean of the respective estimates for each type of elasticity. The rows labeled "Bias" give the estimated bias, where est. bias = sample mean - true value. Rows labeled "Std Dev." give the sample standard deviation of the 3900 point estimates. Rows labeled "MSE$^{1/2}$" give the square root of the estimated mean squared error (MSE), where estimated MSE = (est. bias)^2 + (std dev)^2.

of the exchange rate elasticity is -4 with a standard deviation smaller than 0.001. The coefficient of the demand elasticity for the log approximation is slightly larger than 1 with 1.002. Finally, the last column (5) shows the bias in the case of alternative weights, which overestimates the exchange rate elasticity (-3.957).

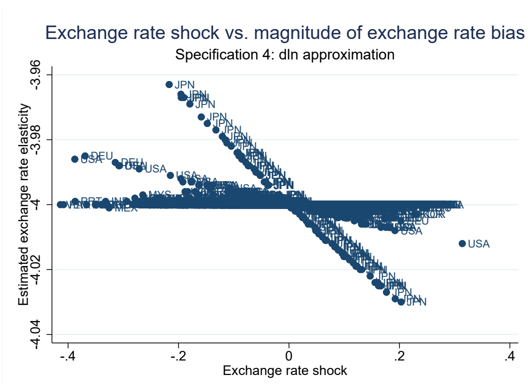
In addition to the average elasticities reported in table 1, figure 1 plots the country-specific bias in the exchange rate elasticity as a function of the exchange rate shock. The magnitude of the bias increases in the size of the exchange rate shock in all specifications. We also observe that in the case of the "McGuirk" (sub-figure b) and the "log-approximation" (sub-figure c) that the direction of the bias is the opposite of the exchange rate shock (a depreciation leads to a downward bias while an appreciation leads to an upward bias). The other two scenarios show the importance of using the adequate functional form. Omitting the variation in the weighted average of the competitor's price index (scenario "Gold medal mistake") or having the wrong weighting scheme (scenario "Alternative weights") leads to larger biases for countries that have more important implications in the international trading system. For example, the United States has the largest bias because it is, on average, the most important trading partner for the countries in our sample and, as a result, an exchange rate shock to the US dollar induces the largest variation in the partner countries' price indexes.



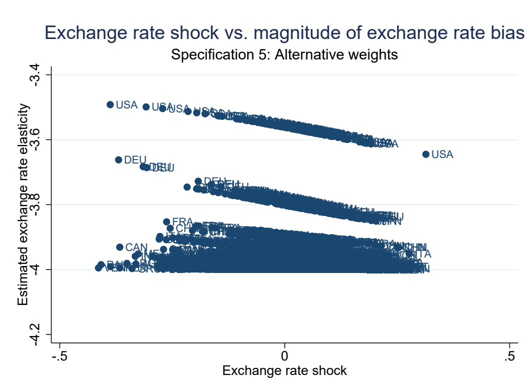
(a) Gold medal mistake



(b) McGuirk



(c) Log-approximation



(d) Alternative weights

Figure 1: MTI: Relationship between an exchange rate shock and the bias of the exchange rate elasticity for $\theta = 4$.

Table 2: GETI simulated elasticities of country-specific random exchange rate shocks

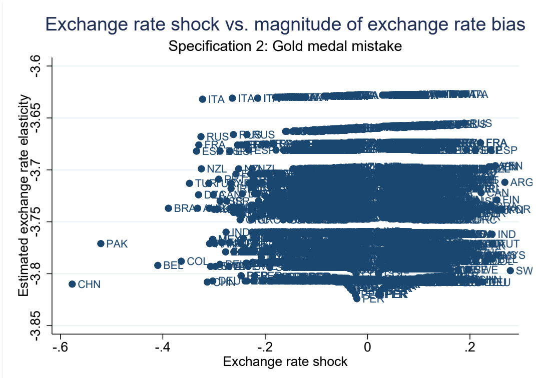
	Baseline ("ideal-REER") (1)	GM mistake (2)	REER McGuirk ("real-REER") (3)	d ln approx. (4)	Alternative weights (5)
Exchange rate					
Mean	-4	-3.74	-4	-4.001	-3.918
Bias	0	.26	0	0.001	.082
Std dev.	0	.045	.019	.032	.022
MSE $^{1/2}$	0	.264	.019	.032	.085
Foreign demand					
Mean	1	.988	1.004	1.004	1.002
Bias	0	.012	.004	.004	.002
Std dev.	0	.023	.053	.053	.053
MSE $^{1/2}$	0	.026	.053	.053	.053

Notes: The sample consists of 39 countries and each country receives 100 random exchange rate shocks. The total number of estimated exchange rate and demand elasticities is 3900 for each of the 5 different specifications. The top cell labeled "Mean" reports the sample mean of the respective estimates for each type of elasticity. The rows labeled "Bias" give the estimated bias, where est. bias = sample mean - true value. Rows labeled "Std Dev." give the sample standard deviation of the 3900 point estimates. Rows labeled "MSE$^{1/2}$" give the square root of the estimated mean squared error (MSE), where estimated MSE = (est. bias)^2 + (std dev)^2.

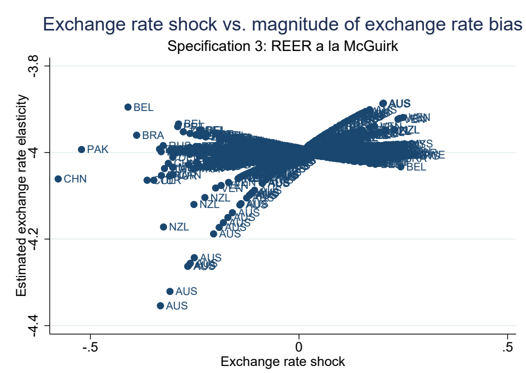
4.2 General Equilibrium Trade Integration (GETI) counterfactual

In the GETI counterfactual, we allow wages to adjust following an exchange rate shock. In particular, we impose that wages have to adjust in order to keep the current account imbalance constant in international currency. To engineer an exchange rate shock in the general equilibrium, we use the same exchange rate shocks as in the MTI part. When a country receives an idiosyncratic exchange rate shock, the exchange rate does not change for other countries. However, exports change for all countries because prices and wages adjust after the shock. Table 2 presents the general equilibrium results using a trade elasticity of 4.

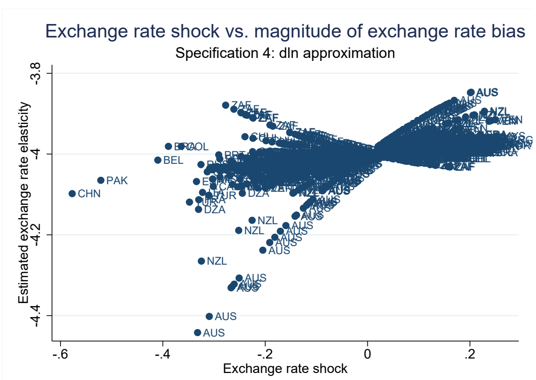
The direction and the magnitude of the underlying biases in the exchange rate elasticities are similar to the MTI version. Wrong functional form assumptions and incorrect weighting schemes in the scenarios “Gold medal mistake” and “Alternative weights” bias the exchange rate elasticity towards zero. The key difference is the increase in the estimation error captured by the MSE. Figures 2 and 3 illustrate this change by plotting the bias in the exchange rate and the demand elasticity as a function of the exchange rate shock for the four different scenarios. In the GETI counterfactual an exchange rate shock leads to a much larger estimation bias than in the MTI. The reason why the average biases are small is that they move in the opposite direction of the exchange rate shock (appreciation leads to upward bias and depreciation leads to downward bias) and cancel each other out when averaging.



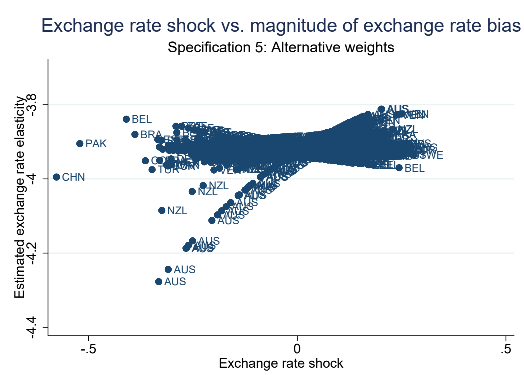
(a) Gold medal mistake



(b) McGuirk

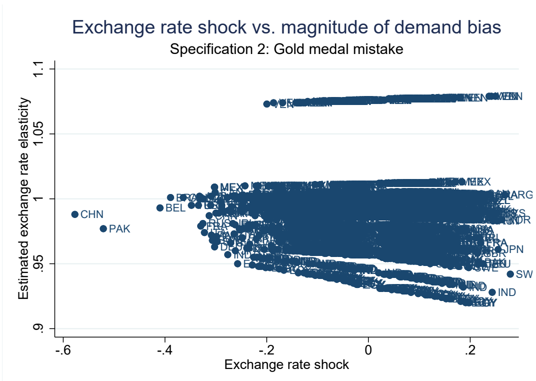


(c) Log-approximation

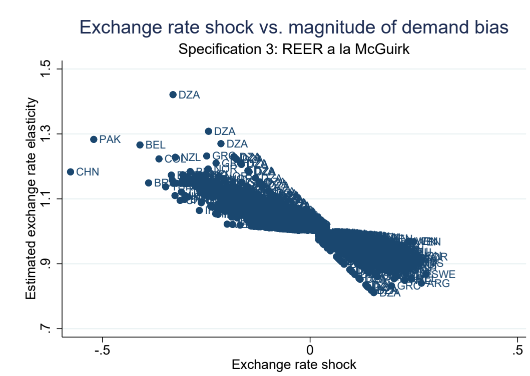


(d) Alternative weights

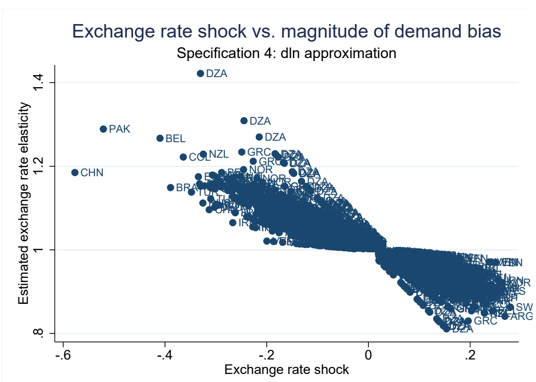
Figure 2: GETI: Relationship between exchange rate shock and the bias in the exchange rate elasticity.



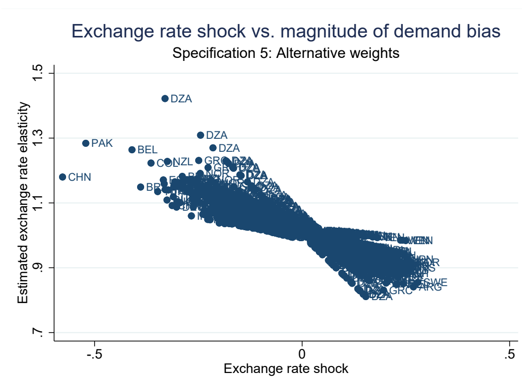
(a) Gold medal mistake



(b) McGuirk



(c) Log-approximation



(d) Alternative weights

Figure 3: GETI: Relationship between exchange rate shock and the bias in the demand elasticity.

Table 3: GETI simulated elasticities of country-specific random exchange rate shocks, stochastic trade costs and $\theta = -4$.

	Baseline ("ideal-REER") (1)	GM mistake (2)	REER McGuirk ("real-REER") (3)	d ln approx. (4)	Alternative weights (5)
Exchange rate					
Mean	-4	-3.951	-4.022	-4.041	-3.997
Bias	0	.049	.022	.041	.003
Std dev.	0	.141	.171	.166	.178
MSE $^{1/2}$	0	.149	.173	.171	.178
Foreign demand					
Mean	1	1.28	1.111	1.092	1.192
Bias	0	.28	.111	.092	.192
Std dev.	0	.28	.247	.262	.335
MSE $^{1/2}$	0	.396	.270	.278	.386

Notes: The sample consists of 39 countries and each country receives 100 random exchange rate and trade cost shocks. The total number of estimated exchange rate and demand elasticities is 3900 for each of the 5 different specifications. The top cell labeled "Mean" reports the sample mean of the respective estimates for each type of elasticity. The rows labeled "Bias" give the estimated bias, where est. bias = sample mean - true value. Rows labeled "Std Dev." give the sample standard deviation of the 3900 point estimates. Rows labeled "MSE$^{1/2}$" give the square root of the estimated mean squared error (MSE), where estimated MSE = (est. bias)^2 + (std dev)^2.

4.3 Monte-Carlo with stochastic trade costs

Up until now, our simulations are based on the assumption that the structural gravity equation is the true data-generating process for bilateral trade flows. To allow for the fact that the gravity equation does not perfectly explain bilateral trade flows, we run a Monte Carlo exercise and assume, as in Head and Mayer (2014), that bilateral trade flows change due to a stochastic term η_{ni} in the trade cost function:

$$\phi_{ni} = \exp(-\ln \text{Dist}_{ni})\eta_{ni}, \tag{23}$$

where Dist_{ni} is the distance between importer n and exporter i. η_{ni} is a log-normal random term and the only stochastic term in the simulation since GDP and distance are all set by actual data. We calibrate the variance of $\ln(\eta_{ni})$ to replicate the root mean squared error of the least squares dummy variables (LSDV) regression on real data. We use the method of Dekle et al. (2007) and solve the model in changes. Based on actual data in trade flows and incomes, we first simulate changes in the trade costs with a random shock $\hat{\eta}_{ni}$, i.e. $\hat{\tau}_{ni} = \hat{\eta}_{ni}$, and then calculate the changes in wages, the prices for intermediate goods and the aggregate price indexes resulting from the random exchange rate shock.

Table 3 reports the results. Introducing stochastic trade costs increases the bias and the estimation error. The MSE significantly increases for the point estimates of the exchange rate and the demand elasticities. Interestingly, the bias in the demand elasticity becomes more pronounced than in the exchange rate elasticity. Across specifications, the average magnitude of the bias is close to 20 percent for the demand elasticity, whereas it is close to 1 percent for the exchange rate elasticity. The specification with the lowest MSE is the log approximation in column (4), suggesting that deflating the changes in foreign demand by changes in the aggregate price index is important to reduce the bias in the demand elasticity. These simulation results suggest that with the ideal-REER approach it is possible to estimate the response of aggregate exports to exchange rate changes without bias even if the data-generating process does not fully adhere to structural gravity.

5 Exchange rate pass-through and currency denomination

We derived our ideal-REER regression specification under two important assumptions. First, markups do not vary with the exchange rate. Second, prices are set in an international currency. In this section, we relax these assumptions and discuss alternative estimation methods based on different pricing assumptions.

One way to change the exporter's price response to exchange rates is to follow Anderson et al. (2016) and assume an exogenous pass-through coefficient ϱ_{in}. The pass-through coefficient ϱ_{in} captures the exporter's price adjustment following a variation in the exchange rate, for example, due to changes in the markup or price rigidities in the invoicing currency. The coefficient is defined on the unit interval $0 < \varrho_{in} < 1$. If the pass-through is complete, the exporter will pass on the entire bilateral exchange rate variation to the importer and $\varrho_{in} = 0$. If the export price is fixed in the destination currency and the exporter absorbs all of the exchange rate variation by changing the markup, the pass-through coefficient equals one, $\varrho_{in} = 1$. Given this definition, we can write the change in the export price denominated in importer n's currency as follows:

$$\hat{P}_i^E = \left(\frac{\hat{r}_i}{\hat{r}_n}\hat{P}_{ii}\right)^{1-\varrho_{in}}$$

However, if the pass-through coefficient varies at the bilateral level, we will not be able to identify all parameters at the aggregate level:

$$\ln \hat{E}_i = \ln \sum_{n \neq i} \omega_{ni} \left(\left(\frac{\hat{r}_i}{\hat{r}_n}\hat{P}_{ii}\right)^{1-\varrho_{in}} \hat{P}_n^{-1}\right)^{-\theta} \hat{X}_n$$

because the number of parameters to estimate is larger than the number of export equations.

Instead, we rely on the dominant currency paradigm (DCP, see Casas et al. (2017)) and assume that exporter and importer prices are fixed in the dominant (international) currency. In this case, the pass-through coefficient depends only on the exchange rate changes vis-à-vis the international currency ($\varrho_{in} = \rho_i \rho_n$). As a result, we can write the bilateral exchange rate as a product of both the exporter exchange rate and the importer exchange rate to the international currency and neglect the co-variance between the two. Reducing the dimensionality further by assuming a common pass-through coefficient, we can write the DCP export equation as follows:

$$\ln \hat{E}_i = \beta_{REER} \ln \left(\hat{r}_i \hat{P}_{ii}\right) + \beta_x \ln \sum_{n \neq i} \omega_{ni} \frac{\hat{X}_n}{\left(\hat{r}_n \hat{P}_n\right)^{\beta_{REER}}}, \tag{24}$$

where the estimated exchange rate elasticity represents the product of the trade elasticity and the exchange rate pass-through coefficient (ρ), i.e., $\beta_{REER} = \theta(1 - \rho)$, for both the exporter-specific and the importer-specific components. Introducing incomplete pass-through reduces the effect of the exchange rate but leaves the overall functional form unchanged compared with the baseline specification in equation 7. The lower exchange rate elasticity implies a reduction in the magnitude of the estimation biases in standard REER specifications discussed above. Both the qualitative results and the country-specific biases plotted in figures 1 to 3 remain the same.

An alternative pricing assumption is that exporters set their prices in the producer's currency (Producer Currency Pricing, or PCP) and absorb part of the changes in marginal costs by the common pass-through coefficient ρ. In this case, we cannot split the bilateral exchange rate into an exporter/importer component because the importer-specific component depends on the exporter's price reaction following an exchange rate shock. The resulting exporter equation is a function of the change in producer prices in the exporter's currency and a weighted average of foreign demand deflated by the price index converted into the exporter's currency:

$$\ln \hat{E}_i = \beta_{REER} \ln \left(\hat{P}_{ii}\right) + \beta_x \ln \sum_{n \neq i} \omega_{ni} \left(\left(\frac{\hat{r}_i}{\hat{r}_n} \right)^{1-\varrho} \hat{P}_n^{-1} \right)^{\beta_{REER}} \hat{X}_n \tag{25}$$

The estimated exchange rate elasticity is the same for the exporter- and importer-specific components only if exchange rate pass-through is complete, i.e., $\varrho = 0$. If exchange rate pass-through is incomplete ($\varrho > 0$), the importer-specific component depends on the exporter's price reaction following an exchange rate shock and the estimated exchange rate elasticity β_{REER} will be biased.[17]

Finally, the third common pricing assumption is that exporters set their prices in the destination's currency (Local Currency Pricing, or LCP). Under LCP, exporters absorb the bilateral exchange rate variation and the pass-through on import prices is zero. In this case, the exchange rate shock does not affect prices, demand and exports. As a result, there is no counterfactual variation.

Overall, consistent aggregation under incomplete pass-through is only possible when prices are set in an international currency as postulated by the dominant currency paradigm. If prices are set in the producer's currency (producer currency pricing (PCP)) or in the local currency (local currency pricing (LCP)), the pass-through depends on the bilateral exchange rate, and estimation of the aggregate elasticities without bias is only possible if pass-through is complete. Next, we test

[17] Our simulation results, presented in the appendix, show that under the assumption of the complete pass-through the estimation biases in standard REER specifications with PCP are qualitatively similar to the ones under DCP.

whether the ideal-REER approach also reduces the estimation bias when aggregating bilateral data empirically.

6 Empirical evidence

A key implication from the theoretical analysis is that running exchange rate regressions on the bilateral or on the aggregate level does not lead to a significant bias in the elasticities if the trade elasticity is the same for all countries. While recognizing that in practice exchange rate elasticities differ across countries (see, for example, Spilimbergo and Vamvakidis (2003) or Bussière et al. (2016)), we test the theoretical prediction of no-significant bias in exchange rate regressions when pooling across countries. Later we relax this assumption and test for significant differences between elasticities estimated on the bilateral and aggregate levels on a country-per-country basis.

Our empirical approach uses data for bilateral exchange rates from the IMF financial statistics database. Data on real effective exchange rates are coming from the Bank for International Settlements. We use the Narrow Index as it has the advantage of covering a longer sample period. The Narrow Index is a trade-weighted effective exchange rate over 25 economies (excluding the euro area). Inflation (proxied by the GDP deflator) and nominal GDP data are from national accounts and converted into international currency (USD) using the bilateral exchange rate with the USD. Bilateral trade data are retrieved from CEPII's historical trade database (see Fouquin and Hugot (2016)). We use absorption, defined as GDP minus net exports, to be a proxy for foreign demand. For changes in aggregate export prices, we use producer price indexes (PPI) from national accounts, the OECD and the IMF depending on data availability. Taken together, our sample comprises a panel of 25 countries with yearly observations from 1964 to 2014.

Given that the ideal-REER estimation equation depends on the currency denomination of trade flows, the first step is to run exchange rate pass-through regressions and shed light on the pricing behavior of exporters. To estimate the reaction of export prices to exchange rate changes, we follow Casas et al. (2017) and regress the export price of country i to country j (denominated in country i's currency,$P^E_{i,j,t}$) on country i's exchange rate with the USD $(r_{i,USD,t})$, the bilateral exchange rate between country i and country j $(r_{ij,t})$ and the producer price index of country i $(PPI_{i,t})$, a proxy for changes in marginal costs:

$$\Delta \ln P^E_{i,j,t} = \alpha_1 \Delta \ln r_{i,USD,t} + \alpha_2 \Delta \ln r_{i,j,t} + \alpha_3 \Delta \ln PPI_{i,t} + u_{i,j,t}$$

The results in Table 4 show that the export price varies mainly with the USD exchange rate. The estimated coefficient of the USD exchange rate is -0.15 compared with a bilateral exchange rate coefficient of -0.05 (see column (4)).

We repeat the exercise for the import side and estimate the reaction of the import price denominated in importer's currency to exchange rate changes with the following specification:

Table 4: Exchange Rate Pass-Through into Export Prices

Dependent variable:	Log-change (exporter price)			
	(1)	(2)	(3)	(4)
Log-change (exchange rate to USD)	0.341***	0.427**	0.177**	0.149**
	(0.0794)	(0.120)	(0.0508)	(0.0403)
Log-change (bilateral exchange rate)		0.134**		0.0517**
		(0.0423)		(0.0169)
Log-change (producer price index)			0.759***	0.744***
			(0.0617)	(0.0680)
Observations	29350	29350	29350	29350
R^2	0.305	0.331	0.625	0.629

Standard errors clustered at the exporting country in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

$$\Delta \ln P^{I}_{i,j,t} = \alpha_1 \Delta \ln r_{i,USD,t} + \alpha_2 \Delta \ln r_{i,j,t} + \alpha_3 \Delta \ln PPI_{j,t} + u_{i,j,t},$$

where $P^{I}_{i,j,t}$ denotes the import price of country i from country j (denominated in country j's currency). Similar to the export price, the variation of the import price is predominantly driven by the USD exchange rate. Table 5 column (4) shows that the pass-through coefficient for the USD import price is -0.89 while the coefficient of the bilateral exchange rate is -0.21. These findings are similar to Casas et al. (2017) and support the dominant currency paradigm.

Table 5: Exchange Rate Pass-Through into Import Prices

Dependent variable:	Log-change (importer price)			
	(1)	(2)	(3)	(4)
Log-change (exchange rate to USD)	0.445***	0.553***	0.700***	0.869***
	(0.00752)	(0.00986)	(0.00811)	(0.0124)
Log-change (bilateral exchange rate)		0.170***		0.072***
		(0.00651)		(0.0104)
Log-change (producer price index)			0.434***	0.612***
			(0.00687)	(0.0112)
Observations	29350	29350	29350	29350
R^2	0.383	0.414	0.539	0.563

Standard errors clustered at the importing country in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Next, we evaluate the empirical performance of regression specifications 1 to 5 discussed in the

Table 6: Empirical results with variables denominated in current USD (DCP)

	Bilateral	Aggregate				
		Baseline ("ideal-REER")	GM mistake	REER McGuirk ("real-REER")	$d \ln$ approx.	Alternative weights
	(1)	(2)	(3)	(4)	(5)	(6)
Exchange rate						
Point estimate	-.459	-.409	-.380	-.584	-.291	-.254
Std. error	(0.024)	(0.051)	(0.059)	(0.057)	(0.066)	(0.067)
Foreign demand						
Point estimate	1.184	1.199	.758	1.092	1.207	1.122
Std. error.	(0.027)	(0.063)	(0.048)	(0.037)	(0.083)	(0.049)
Observations	27940	1077	1077	1142	1077	1198
R2	.157	.567	.570	.612	.579	.516

Notes: The sample consists of 25 countries over the period 1964-2014.

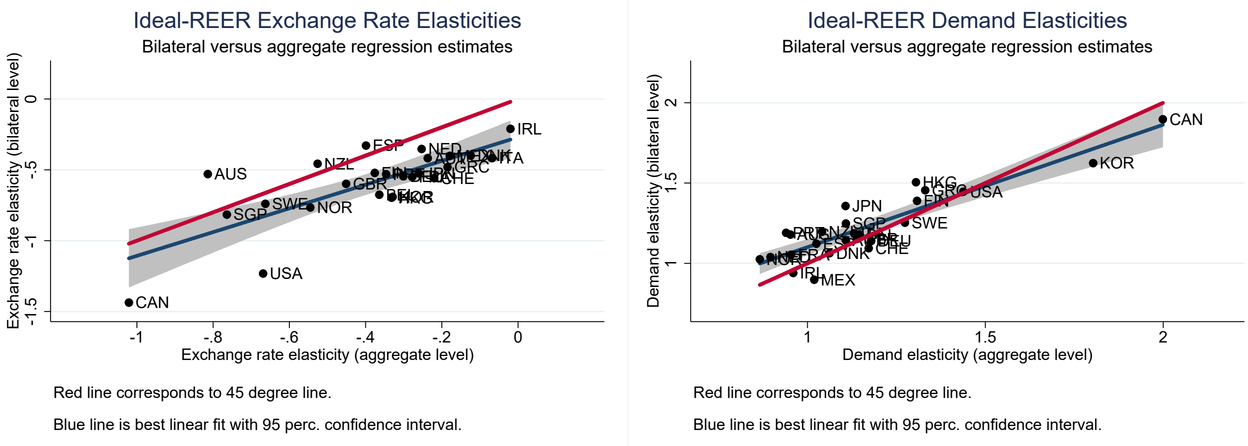
simulation section. We convert all variables into US dollars and use the following definitions for the real exchange rate and foreign demand.[18] The real exchange rate of exporter i (RER_{it}) is defined as the change in the nominal exchange rate of country i vis-à-vis the USD multiplied by the change in the PPI of country i. To calculate foreign demand, we use the current value of absorption in the importing country j denominated in the international currency (X_{jt}). Inflation proxies for changes in the importer's price index $(\hat{P}_{jt})$. For the weights ω_{nit}, we use the share of exports of country i going to country n in year t. This definition of the variables applies to all aggregate regressions with the exception of the REER in the "alternative weights" specification defined in equation 17. In this case, we use the BIS Narrow Index as the REER's empirical counterpart.

The results are shown in table 6. The first observation is that the elasticities estimated from bilateral trade flows and from aggregate trade flows using the ideal-REER approach are not significantly different from each other. The point estimate of the exchange rate elasticity using bilateral flows is -0.46 (column (1)) and the estimate using the ideal-REER is -0.41 (column (2)). The demand elasticities are almost identical with the point estimates of 1.18 and 1.19, respectively. For all other specifications (columns (3) to (6)), the aggregation method results in significant differences in the estimated elasticities compared with the bilateral level in column (1). The specification with the largest gap is the "Gold medal (GM) mistake" approach in column (3) with a point estimate of -0.38 for the exchange rate and 0.76 for the demand elasticity. The standard REER approach based on McGuirk (column (4)) fits the data best with the highest R2 but produces a higher exchange rate (-0.58) and a lower demand elasticity (1.09). The specification with the lowest R2 is the "Alternative weights" approach in column (6). Overall, these results highlight the trade-off one faces when

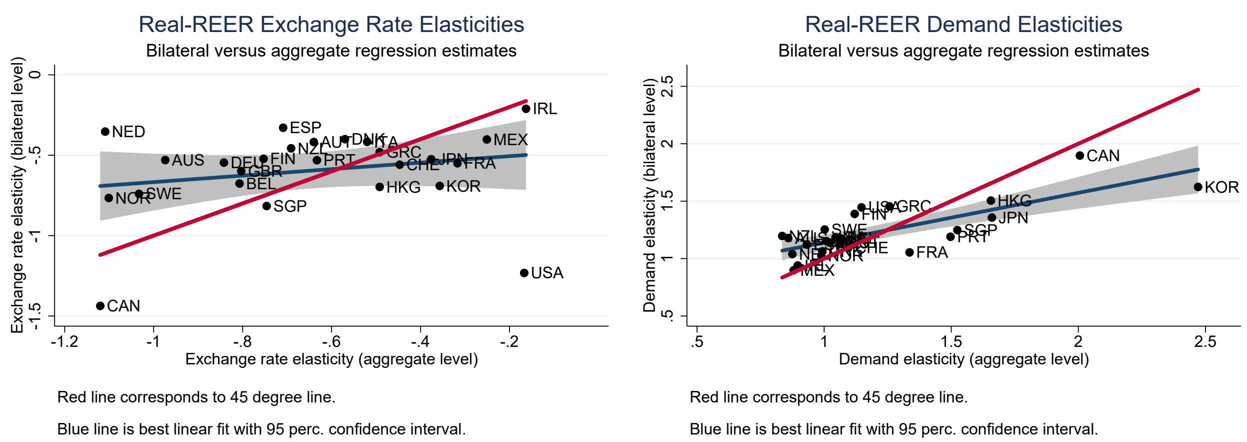
[18]The US is the reference for all countries in our sample except for the US. In this case, the reference currency is the pound sterling.

Figure 4: Country-specific estimates of aggregate vs. bilateral demand and exchange rate elasticities.

(a) Baseline ("ideal-REER")



(b) REER McGuirk ("real-REER")



deciding which elasticities to use. If forecasting is the main objective, then the model with the best fit may be the preferred one. If the aim is to use the elasticities of the standard REER approach to calibrate a macro-model, which relies on aggregate export equations to summarize trade linkages between the different regions in the model, then the counterfactual predictions are inconsistent with the “micro-level” foundations implied by structural gravity. In this case, the estimates obtained from the ideal-REER regression may be the preferred ones.

The pooled estimates in table 6 hide potential heterogeneity in the exchange rate elasticities across countries. Spilimbergo and Vamvakidis (2003) (and many others later) documented this cross-country heterogeneity. We investigate this point further by running the bilateral and the aggregate real exchange rate regressions on a country-per-country basis. We obtain country-specific exchange rate and demand elasticities to test whether they differ in a significant way.

Figure 4 plots the relationship between the elasticities obtained from bilateral and aggregate regressions based on the ideal-REER approach (panel a) as well as the standard REER approach *à la*

McGuirk (panel b). All four panels exhibit significant variation in elasticities across countries and these results are smaller for elasticities estimated on the bilateral than on the aggregate level. One possible explanation is the limited number of observations. The aggregate country-specific regressions consist of a time-series analysis with 50 years of observations. In contrast, the bilateral variation consists of up to 1200 observations (50 years multiplied by the number of trading partners). In the absence of an aggregation bias, we would expect that the difference in the elasticities does not move in a particular direction, i.e., the difference between the elasticities should be more or less proportional to the size of the country-specific elasticities. In other words, the best linear fit between the two sets of elasticities should have a slope close to 1. For this reason, we include the best linear fit (in blue) with the 95 percent confidence interval as well as the 45 degree line (in red) into figure 4. Note that in all cases the estimated slope is lower than 1, which confirms the initial observation that the cross-country variation in the elasticities estimated on the bilateral level is smaller than those obtained from the aggregate level.

With respect to a potential aggregation bias, we expect that the ideal-REER approach will result in smaller differences between the elasticities than the standard REER approach. Panels (a) and (b) in figure 4 show that this is indeed the case: the absolute difference between the point estimates of the elasticities obtained from bilateral trade flows and the ones obtained from aggregate trade flows is smaller for the ideal-REER approach compared with the standard REER approach. Moreover, the difference is less biased in a particular direction, i.e., the best linear fit (in blue) of the ideal-REER estimates is closer to the 45 degree line (in red) than the best linear fit of the standard REER estimates. Taken together, the empirical results support our theoretical findings, namely that the difference in the elasticities is related to the functional form assumption in the aggregation of bilateral trade flows. The ideal-REER approach aggregates the bilateral variation in accordance with the gravity model and reduces bias from the aggregation.

Before concluding, we consider an alternative pricing paradigm and estimate our empirical specifications in accordance with the PCP shown in equation 25. Table 7 shows the estimated coefficients. The main difference with respect to the evidence based on DCP is that denominating the variables in the producer's currency worsens the estimation performance. Compared with table 6, the R2 and the point estimates of the elasticities are lower while the standard errors of the elasticities are higher in all specifications of table 7. With respect to the differences in elasticities across different levels of aggregation, the ideal-REER elasticities are closest to the ones estimated on the bilateral level. All other specifications lead to significant differences in elasticities with respect to the bilateral level. Overall, these results suggest that currency denomination matters for the estimation performance and the quantitative results but not for the qualitative results on the aggregation bias.

Table 7: Empirical results with variables denominated in the exporter's currency (PCP)

	Bilateral	Aggregate				
		Baseline ("ideal-REER")	GM mistake	REER McGuirk ("real-REER")	$d \ln$ approx.	Alternative weights
	(1)	(2)	(3)	(4)	(5)	(6)
Exchange rate						
Point estimate	-.461	-.384	-.289	-.567	-.243	-.181
Std. error	(0.039)	(0.072)	(0.057)	(0.062)	(0.082)	(0.074)
Foreign demand						
Point estimate	1.132	1.102	.942	1.066	1.095	.979
Std. error.	(0.019)	(0.106)	(0.096)	(0.048)	(0.112)	(0.072)
Observations	27940	1077	1077	1142	1077	1198
R2	.157	.407	.497	.587	.489	.460

Notes: The sample consists of 25 countries over the period 1964-2014.

7 Conclusion

One reason why country-specific bilateral exchange rate and demand elasticities differ from aggregate elasticities is incorrect aggregation. The standard approach taken in the literature is based on functional form assumptions and an incorrect weighting scheme. A consequence of these assumptions is that both the trade (exchange rate) and the demand elasticity are biased. However, our simulations and the empirical evidence show that these biases are quantitatively small but statistically significant.

The fact that many macroeconomics models are calibrated with elasticities based on standard REER regressions has important policy implications. In particular, models calibrated with the elasticities estimated from aggregate data lead to inaccurate predictions by exaggerating the response of exports and, by extension, output following an exchange rate shock. Calibrating these models with elasticity estimates using our new ideal-REER regression specification with variables denominated in the dominant international currency (i.e., the US dollar) improve the model's fit and results in predictions consistent with microeconomic behavior in bilateral trade equations.

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8 Appendix

8.1 Simulation results for alternative values of θ

Table 8: GETI simulation elasticities of country-specific random exchange rate shocks, stochastic trade costs and $\theta = 0.8$

	Baseline ("ideal-REER") (1)	GM mistake (2)	REER McGuirk ("real-REER") (3)	d ln approx. (4)	Alternative weights (5)
Exchange rate					
Mean	-.8	-.757	-.788	-.787	-.781
Bias	0	.043	.012	.013	.019
Std dev.	0	.101	.126	.124	.132
MSE $^{1/2}$	0	.109	.127	.125	.133
Foreign demand					
Mean	1	.795	1.092	1.085	1.127
Bias	0	.205	.092	.085	.127
Std dev.	0	.283	.182	.191	.201
MSE $^{1/2}$	0	.350	.203	.209	.237

Notes: The sample consists of 39 countries and each country receives 100 random exchange rate and trade cost shocks. The total number of estimated exchange rate and demand elasticities is 3900 for each of the 5 different specifications. The top cell labeled "Mean" reports the sample mean of the respective estimates for each type of elasticity. The rows labeled "Bias" give the estimated bias, where est. bias = sample mean - true value. Rows labeled "Std Dev." give the sample standard deviation of the 3900 point estimates. Rows labeled "MSE$^{1/2}$" give the square root of the estimated mean squared error (MSE), where estimated MSE = (est. bias)^2 + (std dev)^2.

8.2 Simulation results for producer currency pricing

Table 9: GETI simulation elasticities of country-specific random exchange rate shocks in producer currency pricing (PCP) and stochastic trade costs

	Baseline ("ideal-REER") (1)	GM mistake (2)	REER McGuirk ("real-REER") (3)	d ln approx. (4)	Alternative weights (5)
Exchange rate					
Mean	-4	-3.891	-4.029	-4.032	-3.967
Bias	0	.119	.029	.032	.033
Std dev.	0	.143	.176	.185	.179
MSE $^{1/2}$	0	.186	.178	.187	.182
Foreign demand					
Mean	1	1.26	1.112	1.101	1.224
Bias	0	.26	.112	.101	.224
Std dev.	0	.291	.243	.256	.344
MSE $^{1/2}$	0	.390	.267	.275	.411

Notes: The sample consists of 39 countries and each country receives 100 random exchange rate and trade cost shocks. The total number of estimated exchange rate and demand elasticities is 3900 for each of the 5 different specifications. The top cell labeled "Mean" reports the sample mean of the respective estimates for each type of elasticity. The rows labeled "Bias" give the estimated bias, where est. bias = sample mean - true value. Rows labeled "Std Dev." give the sample standard deviation of the 3900 point estimates. Rows labeled "MSE$^{1/2}$" give the square root of the estimated mean squared error (MSE), where estimated MSE = (est. bias)^2 + (std dev)^2.

8.3 Derivations for McGuirk (1986)

Consider the CES price index in country n:

$$P_n = \left(\sum_{k=1}^{K} P_{nk}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{26}$$

where σ is the elasticity of substitution. Note that by the CES demand, the export price of country k in destination n can be written as follows:

$$P_{nk}^{1-\sigma} = (P_{nk} X_{nk}) \frac{P_n^{-\sigma}}{X_n} \tag{27}$$

Next, we can replace the export price in the sum

$$P_n^{1-\sigma} = \left(\sum_{k=1}^{K} (P_{nk} X_{nk}) \frac{P_n^{-\sigma}}{X_n} \right)$$

and simplify to

$$P_n = \sum_{k=1}^{K} \frac{P_{nk} X_{nk}}{X_n}.$$

Taking the total derivative implies

$$\frac{dP_n}{P_n} = \sum_{k=1}^{K} \frac{P_{nk} X_{nk}}{X_n P_n} \frac{dP_{nk}}{P_{nk}} + \sum_{k=1}^{K} \frac{P_{nk} X_{nk}}{X_n P_n} \frac{d\left(\frac{X_{nk}}{X_n}\right)}{\frac{X_{nk}}{X_n}},$$

where the second term is a weighted average of percentage changes in market shares and equals zero. Noting that the term $\frac{P_{nk} X_{nk}}{X_n P_n}$ reflects the import shares π_{nk} , we get

$$\frac{dP_n}{P_n} = \sum_{k=1}^{K} \pi_{nk} \frac{dP_{nk}}{P_{nk}}.$$

If we consider a log change, we get

$$\triangle \ln P_n = \sum_{k=1}^{K} \pi_{nk} \triangle \ln P_{nk},$$

where the change in price index is a weighted geometric average of the export price changes.[19] Now, we replace the import price index in the bilateral export equation 9 by the sum of the bilateral prices

[19]Instead of the approximation of the price index, we can calculate the exact change in the price index by using the hat notation. Simply, consider equation 26 with the new prices after the policy change (P_n' and P_{kn}') and divide both sides by P_n. Using the fact that the relative prices equal the import expenditure share ($\pi_{nk} = P_{kn}^{1-\sigma}/P_n^{1-\sigma}$, see equation 27), we obtain the exact price change ($\ln \hat{P}_n^{1-\sigma} = \ln \sum_{k=1}^{K} \pi_{nk} \hat{P}_{nk}^{1-\sigma}$).

and get

$$\triangle \ln X_{ni} = \triangle \ln X_n - \theta \left(\triangle \ln P_{ni} - \sum_{k=1}^{K} \pi_{nk} \triangle \ln P_{nk} \right).$$

Note that $\sum_{k=1}^{K} \pi_{nk} = 1$, thus the equation simplifies to

$$\triangle \ln X_{ni} = \triangle \ln X_n - \theta \left((1 - \pi_{ni}) \triangle \ln P_{ni} - \sum_{k \neq i} \pi_{nk} \triangle \ln P_{nk} \right)$$

and noting again that $\sum_{k \neq i} \pi_{nk} = (1 - \pi_{ni})$, we get the following bilateral equation:

$$\triangle \ln X_{ni} = \triangle \ln X_n - \theta \sum_{k \neq i} \pi_{nk} \triangle \ln \left(\frac{P_{ni}}{P_{nk}} \right).$$

Using the fact that the change in the bilateral price index equals the change in producer prices denominated in producer currency times the exchange rate, $\triangle \ln P_{ni} = \triangle \ln r_i P_{ii}$ and $\triangle \ln P_{nk} = \triangle \ln r_k P_{kk}$, we can aggregate across all importing countries and obtain the double-weighted exchange rate:

$$\sum_{n \neq i} \omega_{ni} \triangle \ln X_{ni} = -\theta \sum_{n \neq i} \omega_{ni} \sum_{k \neq i} \pi_{nk} \triangle \ln \left(\frac{r_i}{r_k} \frac{P_{ii}}{P_{kk}} \right) + \sum_{n \neq i} \omega_{ni} \triangle \ln X_n$$

Or, as outlined in the text, in hat notation (using $\triangle \ln x = \ln \hat{x}$):

$$\sum_{n \neq i} \omega_{ni} \ln \hat{X}_{ni} = -\theta \sum_{n \neq i} \omega_{ni} \sum_{k \neq i} \pi_{nk} \ln \left(\frac{\hat{r}_i}{\hat{r}_k} \frac{\hat{P}_{ii}}{\hat{P}_{kk}} \right) + \sum_{n \neq i} \omega_{ni} \ln \hat{X}_n$$

8.4 Derivation of alternative weights

Consider the correct REER equation 11. Bayoumi et al. (2005) start from this definition but include the domestic market share of the exporter i and assumes that the change in the export price is proportional to the change in the producer index (i.e., $P_{ni} \propto P_{ii}$), i.e.,

$$\ln RE\hat{E}R_i = \sum_{n=1}^{N} \sum_{k \neq i} \psi_{ni} \pi_{nk} \ln \left(\frac{\hat{P}_{ii}}{\hat{P}_{kk}} \right). \tag{28}$$

So their weights are defined as

$$W_{ki} = \sum_{n=1}^{N} \psi_{ni} \pi_{nk}.$$

To derive the IMF weights, start from equation 28:

$$\ln R\hat{E}ER_i = \sum_{n=1}^{N} \sum_{k \neq i} \psi_{ni} \pi_{nk} \ln \left(\frac{\hat{P}_{ii}}{\hat{P}_{kk}} \right)$$

Note that we can get $\ln \hat{P}_i$ out of the sum by $\sum_{n=1}^{N} \psi_{ni} \sum_{k \neq i} \pi_{nk} = \sum_{n=1}^{N} \psi_{ni}(1 - \pi_{ni}) = 1 - \sum_{n=1}^{N} \psi_{ni} \pi_{ni}$:

$$\ln R\hat{E}ER_i = (1 - \sum_{n=1}^{N} \psi_{ni} \pi_{ni}) \ln \left(\hat{P}_{ii} \right) - \sum_{k \neq i} \sum_{n=1}^{N} \psi_{ni} \pi_{nk} \ln \left(\hat{P}_{kk} \right),$$

which can be rewritten as

$$\ln R\hat{E}ER_i = \ln \left(\hat{P}_{ii} \right) - \sum_{k=1}^{N} \sum_{n=1}^{N} \psi_{ni} \pi_{nk} \ln \left(\hat{P}_{kk} \right).$$

But note that these weights do not sum to one. To show this, start by

$$\sum_{k \neq i} W_{ki} = \sum_{n=1}^{N} \sum_{k \neq i} \psi_{ni} \pi_{nk} = \sum_{n=1}^{N} \psi_{ni} \left(1 - \pi_{ni} \right)$$

since

$$\sum_{k \neq i} \pi_{nk} = (1 - \pi_{ni})$$

and hence we get something different from one:

$$\sum_{k \neq i} W_{ki} = 1 - \sum_{n=1}^{N} \psi_{ni} \pi_{ni}$$

The term $\sum_{n=1}^{N} \psi_{ni} \left(1 - \pi_{ni} \right)$ is exactly the normalizing factor of the IMF weights. Their weights are defined as

$$W_{ki} = \frac{\sum_{n=1}^{N} \psi_{ni} \pi_{nk}}{\sum_{n=1}^{N} \psi_{ni} \left(1 - \pi_{ni} \right)}.$$

Note that they are wrong for export equations because they include the domestic market of the exporter in their weights. Also, they are re-normalized, which should not be the case.

8.5 Trade deficit

In the general equilibrium counterfactual we use a nominal exchange rate shock and the presence of trade deficits to generate variation in income. In the absence of trade deficits a nominal exchange rate shock will be fully absorbed by the nominal wage of the numeraire country and the shock will not have any real effects. When we introducing a trade deficit, the trade literature generally considers the following two scenarios:[20]

1. Assume that it is a constant fraction of national GDP $\left(c = \frac{D_i}{Y_i}\right)$ where Y_i is national GDP.

2. Assume that it is a constant fraction of world GDP $\left(c = \frac{D_i}{Y}\right)$ where Y is world GDP.

The assumption that the deficit is a constant fraction of national GDP implies that a nominal exchange rate shock does not lead to any real effects. On the other hand, if the deficit is fixed in international currency the exchange rate shock will have real effects. The proof for this statement follows.

Using the structural gravity model outlined in the paper, we can define our model as follows:

$$X_{ni} = \frac{1}{Y}\frac{Y_i X_n}{\Omega_i \Phi_n}\phi_{ni}, \qquad \text{where} \qquad \Phi_n = \sum_{\ell=1}^{N}\left(\frac{Y_i}{\Omega_i}\phi_{ni}\right) \qquad \text{and} \qquad \Omega_i = \sum_{n=1}^{N}\frac{\phi_{ni}X_n}{\Phi_n},$$

where X_{ni} denotes the bilateral trade flows, Φ_n the outward and Ω_i the inward multilateral resistance term. Introducing the bilateral exchange rate $\frac{r_i}{r_n}$ we can rewrite the equations of structural gravity as follows:

$$\pi_{ni} = \frac{S_i\phi_{ni}}{\Phi_n}, \quad \text{where} \quad \phi_{ni} = \left(\frac{r_i}{r_n}\tau_{ni}\right)^{-\theta}, \quad \Phi_n = (P_n)^{-\theta} = \sum_i S_i\phi_{ni} \quad \text{and} \quad S_i = T_i\left(w_i^{\beta}p_i^{1-\beta}\right)^{-\theta}$$

Let's start by rewriting the bilateral export equation:

$$\frac{X_{ni}Y}{Y_i X_n} = \frac{\phi_{ni}}{\Omega_i\Phi_n} \tag{29}$$

and the multilateral resistance terms as

$$\Phi_n = \sum_{i=1}^{N}\left(\frac{Y_i}{\Omega_i}\phi_{ni}\right) \qquad \text{and} \qquad \Omega_i = \sum_{n=1}^{N}\frac{\phi_{ni}X_n}{\Phi_n}.$$

Plugging in the exchange rate and trade costs for ϕ_{ni} and simplifying, we get:

$$\Phi_n r_n^{-\theta} = \sum_{i=1}^{N}\left(\frac{Y_i}{\Omega_i}(r_i\tau_{ni})^{-\theta}\right) \qquad \text{and} \qquad \Omega_i r_i^{\theta} = \sum_{n=1}^{N}\left(r_n^{-1}\tau_{ni}\right)^{-\theta}\frac{X_n}{\Phi_n}.$$

To solve the system of equations, define the shock to the bilateral exchange rate as a function of the initial values for the multilateral resistance terms:

$$\left(\frac{r_i}{r_n}\right)^{-\theta} = \left(\Omega_i^0\Phi_n^0\right)$$

[20] See the *Handbook of International Economics* chapter by Costinot and Rodríguez-Clare (2014).

Plugging the exchange rate shocks into the multilateral resistance terms shows that the RHS of equation 29 does not vary with the exchange rate shock and cancels out in the numerator and the denominator:

$$\frac{\phi_{ni}}{\Omega_i \Phi_n} = \frac{\left(\Omega_i^0 \Phi_n^0\right)\left(\tau_{ni}\right)^{-\theta}}{\frac{\Omega_i \Phi_n}{\Phi_n^0 \Omega_i^0}} = \frac{\left(\tau_{ni}\right)^{-\theta}}{\Omega_i \Phi_n}$$

To complete the proof, we have to show that the LHS of equation 29 is also invariant to exchange rate changes. We start by writing total expenditure X_n as a function of output Y_n and the trade deficit D_n:

$$\frac{X_{ni}Y}{Y_i X_n} = \frac{X_{ni}Y}{Y_i Y_n + D_n}$$

If the deficit is a constant fraction of GDP, we can write

$$\frac{r_n X_{ni} r_i Y}{r_n Y_n \left(1 + \frac{D_n}{Y_n}\right) r_i Y_i} = \frac{X_{ni}Y}{Y_i X_n},$$

which is invariant to an exchange rate change. A nominal exchange rate shock does not have any real effects. If, on the other hand, the deficit is fixed in international currency, we replace total expenditure X_n as a function of output Y_n and the trade deficit $D_n$as follows:

$$\frac{r_n X_{ni} r_i Y}{\left(r_n Y_n + \frac{D_n}{Y}\right) r_i Y_i} \neq \frac{X_{ni}Y}{Y_i X_n}$$

In this case the deficit does not change in proportion to the exchange rate shock and the LHS of equation 29 is *not* invariant to exchange rate changes. A nominal exchange rate shock will have real effects.