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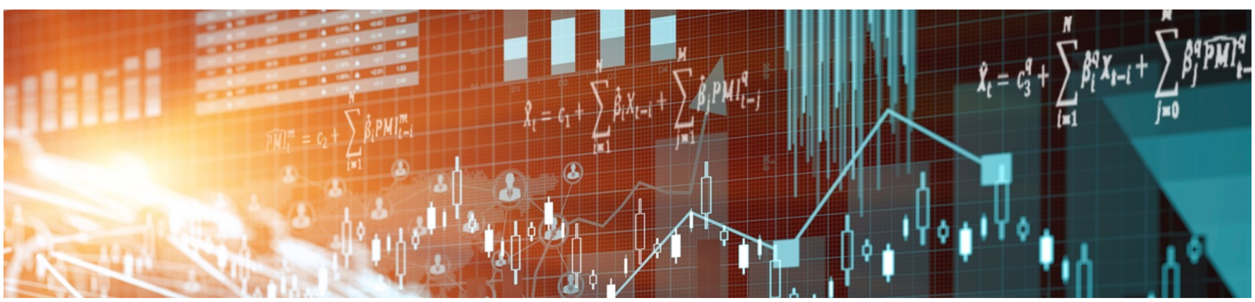

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by

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Abstract

In this paper, we assess several methods that have been used to measure the Canadian trend unemployment rate (TUR). We also consider improvements and extensions to some existing methods. The assessment is based on four criteria: (i) the extent to which methods provide explanations for changes in trend unemployment; (ii) whether revisions to unemployment gap (UGAP, the difference between the actual unemployment rate and TUR) estimates are well behaved; (iii) if UGAPs provide information about future inflation; and (iv) if UGAPs help explain historical data about wages and consumer price inflation. In our assessment of conformity to the second and third criteria, we use real-time data, i.e., the data available to policymakers at the time of making decisions. We find that while all methods we consider have both strengths and weaknesses, those based on variables thought to determine TUR provide better interpretation and tend to do at least as well as others against the other criteria. These are most promising for future work. Nevertheless, there is considerable uncertainty about the value of TUR, which suggests it would be prudent to use a range of models in research or policy work. While estimates of TUR have declined since the mid-1990s, it is assessed to range between 5.6 and 6.7 per cent in 2018Q4.

Bank topics: Business fluctuations and cycles; Economic models; Inflation and prices; Labour markets
JEL codes: C52, C53, E24, E27

Résumé

Dans cette étude, nous analysons diverses méthodes ayant servi à mesurer le taux de chômage tendanciel au Canada. Nous prenons également en considération quelques améliorations et variantes de certaines méthodes existantes. L’analyse est fondée sur quatre critères, qui se traduisent par les questions suivantes : 1. Dans quelle mesure les méthodes expliquent-elles les variations du chômage tendanciel? 2. Les révisions des estimations de l’écart de chômage (la différence entre le taux de chômage observé et le taux de chômage tendanciel) sont-elles fiables? 3. Les écarts de chômage donnent-ils des indications quant à la trajectoire future de l'inflation? 4. Ces écarts contribuent-ils à expliquer les données historiques sur le rythme d'augmentation des salaires et des prix à la consommation? Pour les deuxième et troisième critères, nous avons utilisé des données en temps réel, soit les données dont disposent les responsables des politiques économiques au moment de prendre leurs décisions. Bien que chaque méthode étudiée ait des forces et des faiblesses, nous observons que celles reposant sur des variables réputées déterminer le taux de chômage tendanciel permettent une meilleure interprétation des données et semblent répondre aux critères restants aussi bien, sinon mieux, que les autres. Ces méthodes sont les plus prometteuses pour les travaux à venir. Néanmoins, l'incertitude considérable qui entoure le niveau du taux de chômage tendanciel porte à croire qu'il serait prudent d'avoir recours à plus d'un modèle à la fois pour la recherche et l'élaboration des politiques. À la baisse depuis le milieu des années 1990, les estimations du taux de chômage tendanciel oscilleraient entre 5,6 et 6,7 % au quatrième trimestre de 2018 selon les évaluations.

Sujets : Cycles et fluctuations économiques; Modèles économiques; Inflation et prix; Marchés du travail
Codes JEL : C52, C53, E24, E27

Non-Technical Summary

We estimate the Canadian trend unemployment rate (TUR) with methods based on three types of approaches. A first type consists of estimating TUR as an unobservable variable in state-space models, often using a Phillips curve as conditioning information. A second type is based on models with variables, such as tax or demographic variables, thought to determine TUR. A third type consists of simple, or augmented, mechanical filters designed to extract the trend of time series. We evaluate each method against four criteria: the extent to which they provide an explanation for TUR movements; the behaviour of revisions to estimated unemployment gaps (UGAP is the difference between the actual unemployment rate and TUR); the information provided by UGAPs about future consumer price index (CPI) and wage inflation; and the goodness of fit of UGAPs in estimated Phillips curves.

A contribution of our work is that we use real-time data in assessing TUR measures against the second and third criteria. This is important, given that real-time data are the data available at the time policymakers make their decisions. Real-time data are the relevant data to be used in assessing TUR measures. While the unemployment rate is not significantly revised, some of the data used in estimating it can be substantially revised. Assessment of TUR measures made with revised data could therefore be very different from assessment with real-time data.

Our results suggest that TUR has trended down since the mid-1990s, with lower payroll taxes and demographic developments as likely contributors. Overall, we find that models with *structural* determinants are superior because they help interpret TUR movements and they do at least as well as others against the other criteria. These models appear to be the most promising avenue for future work. Nevertheless, there remains considerable uncertainty surrounding the actual level of TUR, which suggests that it is prudent to use a range of methods in policy work. As of 2018Q4, our best assessment (based on the top

four measures according to our criteria) places the TUR in a range between 5.6 and 6.7 per cent.

1. Introduction

In the aftermath of the 2008–09 recession and the collapse of commodity prices in 2014, the Canadian economy entered an extended period of excess supply, in part reflected in slack labour market conditions. However, in recent quarters the Canadian economy has been operating close to capacity (Bank of Canada, 2019), renewing interest in how fast the labour gap, defined as the difference between total hours worked in the economy and its trend, has been closing. This requires having a view on whether some labour market indicators, such as the employment and unemployment rates, are at their respective trends.

We define the trend unemployment rate (TUR) as the level of unemployment rate that is not a source of inflationary or disinflationary pressures—a useful definition for central banks with an inflation target. The concept of a TUR is close to that of a non-accelerating inflation rate of unemployment (NAIRU). However, the latter suggests that the rate of inflation is continuously accelerating (decelerating) when the actual unemployment rate is below (above) the NAIRU. This may not be the case in an economy where inflation expectations are anchored. We therefore prefer to work with the more general TUR concept. The TUR concept is also related to that of a natural rate of unemployment. The latter has been defined differently by various authors. For instance, Jacob and Wong (2018) see it as a longer-run, steady-state concept, corresponding to the unemployment rate that would prevail "after transitory shocks have fully worked through labour and product markets." Wilkins (2019) instead defines it as the "rate that puts neither upward nor downward pressure on inflation," a definition that is essentially the same as our TUR definition.

An important issue with TUR is that it is unobserved and needs to be estimated. Various approaches have been proposed for this. A frequently chosen approach consists of estimating TUR as an unobservable variable in a state-space model, often using a Phillips curve as conditioning information (see, for example, Gordon, 2013). A different approach (e.g. Côté and Hostland, 1996) is to develop models with structural variables that could cause TUR changes, such as payroll taxes, employment insurance generosity, level of minimum wages, and rate of unionization. A third approach is simply to use detrending techniques, such as the mechanical filter proposed by Hodrick and Prescott (1997), to separate the cyclical from the trend component of the unemployment rate.

In this paper, we estimate TUR using methods representative of these three approaches and assess their relative performances. Our objective is to determine whether a TUR, or a set of TURs, could be useful for conducting monetary policy in Canada. Our assessment of TUR measures is based on four criteria.

The first criterion is derived from the simple idea that a method will be more useful if it explains the level and dynamics of TUR. Central banks need to communicate their views about labour market developments and about their implications for inflation and monetary policy. It is therefore advantageous that a method not only provides a TUR estimate, but that it also provides an explanation for that estimate.

The second criterion is that revisions to unemployment gap—UGAP, the difference between the actual unemployment rate and TUR—estimates should be well behaved. Since the unemployment rate is hardly revised, revisions to UGAP reflect revisions to TUR. Estimates subject to revisions that are large and biased could complicate both the conduct and the communication of policy. However, this idea is debatable. One can argue that revisions are a desirable outcome, even if they are large and biased, if they bring TUR estimates closer to its true value. We further discuss this point in Section 3. We also investigate the implications for our conclusions of putting a smaller weight, or no weight, on this criterion. We calculate revisions using real-time data because UGAP estimates can be revised as new data are published and historical data get revised. Therefore, revision for a given period can be assessed by comparing real-time UGAP with UGAP based on later data vintages.

The third criterion is that UGAPs should help forecast inflation. This is critical for monetary authorities with an inflation-control target, as in Canada. Phillips curve simulations with real-time data allow us to determine which UGAP measures help forecast wage growth and consumer price index (CPI) inflation out of sample. Assuming no major structural break going forward, this should provide useful information about the likelihood that a given TUR-estimation method will help forecast future inflation.

The last criterion is the fit and theoretical plausibility of the TUR within an inflation Phillips curve. Using the last vintage of data, models are assessed based on their ability to explain past inflation movements. We also verify whether the relationship between UGAP and inflation is consistent with theory by looking at the sign and significance of the estimated coefficients. While this criterion ignores data revisions, it is a complement to a real-time forecasting exercise: if two methods lead to similar real-time inflation forecasts, the one with the best fit over history may provide a better estimate of the (unobserved) TUR.

Overall, based on the four criteria, we find that models with structural determinants generally tend to be superior because they provide an interpretation for the dynamics of TUR while performing at least as well as other methods against the other criteria. Our results show that the TUR has trended down since the mid-1990s for all models estimated. Demographic development and lower payroll taxes are likely contributors to this decline. Based on our four criteria, a likely range for the TUR, using estimates from the top four best-performing models, places the TUR between 5.6 and 6.7 per cent in 2018Q4.

Our paper contributes to the literature in several ways. First, it presents an overview, and some extensions, of different models that have been used to estimate TUR in Canada (sometimes in internal, unpublished Bank of Canada work). Second, it proposes a coherent framework, based on four criteria, to assess the usefulness for monetary policy of TUR-estimation methods. Third, real-time datasets that have recently been developed for Canada allow us to assess in real-time revisions and inflation forecasts of various TUR-estimation methods. To our knowledge this is the first paper to use real-time data to assess many TUR-estimation methods affected by substantial data revisions.[1] Fourth, we study the links between our TUR measures and a wide range of inflation measures. This includes CPI inflation, the variable upon which the inflation-control target is set in Canada, but also the three preferred measures of core inflation recently adopted to help guide Canadian monetary policy (Bank of Canada, 2016).[2] Since UGAP may be more immediately linked with wage inflation, we also assess the link between labour market gaps and wage inflation—which may eventually feed CPI inflation.

Section 2 presents the TUR-estimation methods considered in this paper. Section 3 explains our empirical methodology for assessing the methods against the following criteria: revisions to UGAP estimates; informational content about future inflationary pressures; and fit of the model. Section 4 presents our data. Our results are presented in Section 5. We conclude in Section 6.

2. Overview of the methods

2.1 Multivariate state-space models

The state-space framework has frequently been used to identify TUR. In most of these models, TUR is specified as an unobserved component where estimation is conditioned on a Phillips curve. The generic form of these models can be expressed as follows:

$$\boldsymbol{\pi_t = f(\pi_{t-1}) + g(UGAP_t) + h(Z_{t-1}) + \epsilon_t} \quad [1]$$

$$\boldsymbol{U_t^T = U_{t-1}^T + \eta_t} \quad [2]$$

$$\boldsymbol{UGAP_t = f(UGAP_{t-1}) + \varepsilon_t} \quad [3]$$

[1] However, there have been studies of methods with real-time data to estimate potential output (e.g. Orphanides and van Norden, 2002, and Pichette et al., 2019).

[2] https://www.bankofcanada.ca/rates/indicators/key-variables/inflation-indicators/.

where $\boldsymbol{\pi_t}$ denotes inflation at time $\boldsymbol{t}$, $\boldsymbol{U_t^T}$ is the TUR at time $\boldsymbol{t}$ assumed to follow a random walk (equation 2), and $\boldsymbol{UGAP_t} \equiv \boldsymbol{U_t} - \boldsymbol{U_t^T}$ is the unemployment gap assumed to be an autoregressive function of its past values (equation 3). Vector $\boldsymbol{Z}$ contains (mostly) lagged control variables.[3] The estimation is done using a Kalman filter.

Five of the multivariate state-space models we consider are based on a Phillips curve. Representative of this type of model are the triangle and OECD models.[4] As its name suggests, the Phillips curve of the **triangle** model (Gordon, 2013) is characterized by three main features: inertia, capturing the persistence of the inflationary process by using long lags of inflation; demand factors, accounted for by the contemporaneous and lagged unemployment gap; and supply factors, namely food and energy prices, and relative import prices effects.[5]

The Phillips curve specification in the **OECD backward-looking** model (Rusticelli, Turner and Cavalleri, 2015) is similar to that of the triangle model, the main differences being that the lagged structure of inflation is more parsimonious and that the change in inflation is used instead of inflation. Supply factors are captured by lagged real import prices and lagged oil prices. The contemporaneous UGAP is used to control for demand factor.[6] As for the backward-looking model for Canada, we follow the specification in Rusticelli, Turner and Cavalleri (2015) for the **OECD anchored** model: it includes lagged level and the change of inflation, lagged unemployment gap and real import prices. Inflation is assumed to converge to the Bank of Canada 2 per cent target.

While the **basic multivariate filter** (**BMVF**; Blagrave et al., 2015) and the **multivariate state-space framework** (**MSSF**; Pichette, Bernier and Robitaille, 2018) have a more

[3] Appendix A lists all the variables and their sources used in the models presented in this section.

[4] More detailed information about the structure of these models, and the structure of other models examined in this study, is available upon request. Generally, for models having been estimated for Canada, we carry over the same assumptions, such as priors on parameters distributions.

[5] The triangle model we estimate has a slightly different specification than the one in Gordon (2013) for the US economy. For instance, lagged labour productivity is not included due to the lack of a sufficiently long quarterly series. Also, we do not include a binary variable to control for the Nixon-era price controls.

[6] See also Guichard and Rusticelli (2011) for details about the backward-looking version.

complex structure—they are models developed to estimate potential output—the way the TUR is identified is broadly consistent with the generic form presented at the beginning of this section. Both models include a Phillips curve linking inflation to the evolution of the output gap. Inflation and the unemployment gap are indirectly related through an Okun's law linking the unemployment gap to the output gap.

The last model (**hysteresis**) is adapted from Jaeger and Parkinson (1994) and was used in internal Bank of Canada work. In this model, labour market hysteresis is accounted for in the estimation of the TUR. The model does not rely on inflation dynamics to identify the TUR but, rather, decomposes the unemployment rate in a trend (the TUR) and a cyclical part (UGAP). Labour market hysteresis is introduced by assuming that the TUR depends on past value of the UGAP. As with the BMVF and the MSSF, an Okun's law, linking the output gap and the UGAP, is used to facilitate the identification of the TUR. As for the other models in this class, this state-space model is estimated using a Kalman filter.

2.2 Models with structural determinants

The framework used by Côté and Hostland (1996) departs from the previous class of models because it does not rely on filter-based techniques to identify the TUR but, rather, on a cointegration relationship between the unemployment rate and structural factors. Choice of these structural factors is also based on the economic literature. In more recent internal work, Bank of Canada staff developed a model similar to Côté and Hostland's and found that payroll taxes and the employment insurance disincentive index (Sargent, 1995) are relevant structural factors to explain TUR.[7] The TUR is given by the fitted value of a linear regression (fully modified least squares) of the unemployment rate on structural factors. We call this model **cointegration 1**. As an extension, we consider other possible variables, such as the ratio of minimum wage to average wage and the share of prime-

[7] In their original specification, Côté and Hostland (1996) used the payroll taxes and the degree of unionization. This model, however, underperforms the other cointegration models presented in this sub-section and we therefore do not report its results.

age workers over working-age population. The most promising alternative is a cointegration model in which the minimum wage ratio is added. We label this model **cointegration 2**. Interestingly, the minimum wage ratio was considered by Côté and Hostland (1996) but was not retained in their preferred specifications.

We then estimate a model inspired by Fougère (2012). This specification assumes that changes in unit labour cost (ULC) depend on inflation expectations and UGAP (equation 4), and thus TUR. We consider several factors but settle on using the same variables as in cointegration 1. Moreover, we assume adaptive inflation expectations (equation 5).[8] We call it the **structural factors wage Phillips curve** (**SFWPC**). Cointegration between the unemployment rate and these structural factors is implicitly assumed, as in Fougère (2012). Estimation is performed by substituting equations 5 and 6 into equation 4. While simple, this model has the interesting feature that wage dynamics, rather than inflation dynamics, are used to identify a TUR.

$$\Delta \boldsymbol{w_t} = \boldsymbol{\pi_t^e} + \boldsymbol{\beta}\left(\boldsymbol{U_{t-1}} - \boldsymbol{U_{t-1}^T}\right) + \boldsymbol{\mu_t} \tag{4}$$

$$\boldsymbol{\pi_t^e} = \boldsymbol{\delta_1 \pi_{t-1}} + (\boldsymbol{1} - \boldsymbol{\delta_1})\boldsymbol{\pi_{t-2}} \tag{5}$$

$$\boldsymbol{U_t^T} = \boldsymbol{\gamma_0} + \boldsymbol{\gamma_1 PT_t} + \boldsymbol{\gamma_2 EI_t} \tag{6}$$

The last model in this class is a **simplified version of the integrated framework** (**SIF**), one of the tools used by Bank of Canada staff to estimate potential output (Pichette et al., 2015).[9] The SIF provides an estimate of trend employment rate (TER) using cohort-based regressions for men and women (Barnett, 2007). While such a model has been used by the Bank to assess potential output for some time, this is the first time

[8] We initially tried to estimate the same specifications as in Fougère (2012). We found that they generally underperform relative to the simpler specification presented here.

[9] For instance, we exclude the job offer rate and the measure of aggregate wealth due to the difficulty in getting consistent real-time data over a long period. We verified, with recent vintages, that these exclusions have very little effect on estimates.

it has been used to estimate a TUR. Assuming the same structure and specifications, an estimate of trend participation rate (TPR) can be obtained and the TUR is then derived as:[10]

$$\boldsymbol{U_t^T = 1 - \frac{TER}{TPR}} \quad [\mathbf{7}]$$

2.3 Basic and augmented mechanical filters

We consider two simple mechanical filters as benchmarks: the **HP filter** (Hodrick and Prescott, 1997) with a smoothing parameter set at 1,600 and the **band-pass filter** (**BP filter**) proposed by Christiano and Fitzgerald (2003). These filters aim at separating the cyclical from the trend component of the unemployment rate. The trend component could then be called the TUR, and the cyclical component the UGAP.

We also include in this class a method combining a mechanical filter with conditioning economic information and end-of-sample constraints, namely the **extended multivariate filter** (**EMVF**).[11]

3. Empirical methodology

This section describes the methodology we use to analyze revisions to UGAP estimates (our second assessment criterion); to assess UGAP estimates' information content about future inflation (third criterion); and to assess the fit and theoretical plausibility (fourth criterion). Results are presented in Section 5.

[10] Using actual data rather than trend, equation 7 is an identity. We assume that this identity also holds for trends.

[11] First put forward by Butler (1996). The revised version used here is discussed in Pichette et al. (2015). The EMVF has mainly been used to estimate potential output, but it also produces a TUR estimate.

3.1 Are revisions to UGAP estimates "well behaved"?

We use real-time data to estimate UGAP and assess UGAP revisions for each method described in Section 2.[12] The revision for quarter $\boldsymbol{T-1}$ is obtained by comparing the real-time UGAP estimate using the vintage $\boldsymbol{T}$ and the estimate for the same quarter $\boldsymbol{T-1}$ made after eight revisions. Keep in mind that $\boldsymbol{T-1}$ is the last historical data point of vintage $\boldsymbol{T}$ and the first release of quarter $\boldsymbol{T-1}$ data.[13] The eight-quarter hypothesis, which is based on the idea that the data must be close to final after eight revisions, has been used in the literature (e.g. Jacobs and Sturm, 2004). Technically, the revision to UGAP for a given quarter can be expressed as follows:

$$\boldsymbol{REV\_UGAP_{T-1} = UGAP_{T-1}^{T+8} - UGAP_{T-1}^{T}} \qquad [\mathbf{8}]$$

where the subscript denotes the quarter for which the revision is calculated and the superscript is the vintage used. Revisions are calculated using all vintages from 2006Q1 to 2018Q4.[14]

We consider simple statistics used in the real-time data literature (e.g. Orphanides and van Norden, 2002; Cayen and van Norden, 2005; Pichette et al., 2019). UGAPs with revisions that are biased (mean far from zero), large and volatile (as measured by the absolute mean and standard deviation) will tend to be less useful to policymakers. Moreover, noisy real-time estimates that differ significantly from revised estimates (high noise-to-signal ratio, low correlation between real-time UGAP and UGAP after eight

[12] We focus on UGAP instead of TUR because UGAP should be stationary if the trend is well defined, which facilitates some of the analysis of revisions (for instance, the calculation of the correlation between real-time and revised UGAP).

[13] Thus, for all vintages, quarters $\boldsymbol{T}$ and $\boldsymbol{T+1}$ are monitoring data. See Section 4 for more details.

[14] Given that the last historical data point available is 2018Q3 (with the last vintage 2018Q4), the last possible data point for which an eight-quarter revision can be calculated is 2016Q3. Therefore, the revision for 2016Q3 is the last one used in calculating the revision statistics presented in Section 5.1 despite the last vintage being 2018Q4.

revisions and high frequency of changing sign) may lead to forecast and policy errors or misleading communication.[15] This could happen if, for instance, revisions lead to a change to the sign of a large UGAP.

Within our framework, there are two sources of revision to UGAP. First, each subsequent vintage is adding one-quarter of data. UGAP can be revised because this expands the information set available to estimate the UGAP-identifying relationship of each model. Second, historical data can be revised, sometimes quite substantially. So even if the estimation window was kept constant, UGAP would be revised as time passes because of historical revisions. To assess the effects of abstracting from the latter source of revisions, we re-estimate the UGAP models in every quarter using the last vintage of data. A comparison of UGAP revisions with and without data revisions informs us about the contribution of data revisions to total UGAP revisions.

A caveat to the assumption that fewer revisions are a desirable outcome is that revisions may better capture the actual state of the economy and may therefore provide better—in the sense of closer to its true value—estimates of UGAP. Moreover, variables that are poorly captured in real-time data should contribute poorly to real-time forecasts, so that our forecasting criterion should partly encompass the revision criterion in terms of the information provided. Given these objections, Sections 5 and 6 discuss results and conclusions putting zero weight on the revision criterion. But overall, all else being equal, we assume that smaller, less biased and less noisy revisions are preferable, and we therefore include this criterion in our base case analysis.

This revision criterion, however, is not enough to determine whether UGAP could be informative for monetary policy. A good estimate of UGAP should be reflecting tightness in the labour market and, by extension, be informative about future inflationary pressures. This is discussed in Section 3.2.

[15] The noise-to-signal ratio is defined as the standard deviation of revision divided by the standard deviation of UGAP after eight revisions. A low value means that UGAP revisions are less volatile than the UGAP estimates after eight revisions.

3.2 Are UGAPs informative about future inflationary pressures?

Our approach to assessing this criterion builds on the one proposed by Orphanides and van Norden (2005) (and used by Pichette et al., 2019) for assessing output gap measures. It investigates whether UGAPs add useful information to simple forecasting models based on past inflation. If UGAPs cannot pass such a minimal criterion, they are unlikely to help predict inflation.

The base case model posits that inflation forecasts are dependent only on past inflation:

$$\boldsymbol{\pi_{T+h}^{h} = \alpha + \sum_{i=1}^{m} \beta_i \cdot \pi_{T-i}^{1} + e_{T+h}} \qquad [9]$$

where the number of lags $\boldsymbol{m}$ is determined with the Schwarz information criterion as estimated using the first available vintage (2006Q1). While $\boldsymbol{m}$ is specific to the inflation variable, it remains the same for all vintages of a given model. Equation 9, like all other equations discussed in this section, is estimated over a sample starting in 1992Q4, i.e. from the beginning of the inflation-targeting period in Canada (Bank of Canada, 1991).[16] Inflation is defined over $\boldsymbol{h}$ quarters (as shown in equation 10), from quarter $\boldsymbol{T-h}$ to quarter $\boldsymbol{T}$, with $\boldsymbol{P}$ being a CPI or a measure of cost pressure in the labour market.

$$\boldsymbol{\pi_{T}^{h} = \ln(P_T) - \ln(P_{T-h})} \qquad [10]$$

We forecast inflation over one quarter ($\boldsymbol{h=1}$, quarter-over-quarter inflation) and four quarters ($\boldsymbol{h=4}$, year-over-year inflation). Using equation 9, real-time out-of-sample forecasts in $\boldsymbol{T+1}$ (quarter-over-quarter) and $\boldsymbol{T+4}$ (year-over-year) are produced. We do not consider a longer horizon because forecasting inflation over the short term alleviates the need to control for the Bank of Canada’s reaction function; it has been

[16] There is evidence that the inflation process changed at that time. For instance, Demers (2003) found that the inflation process (mean, persistence and volatility) changed in Canada with the adoption of inflation targeting.

estimated that the impact of monetary policy decisions on inflation in Canada takes from six to eight quarters to materialize (Bank of Canada, 2012). We calculate the forecasting root-mean-squared error (RMSE) using the first release of data.[17] Estimations are performed for all vintages from 2006Q1 to 2018Q4.

We then add the UGAP in the forecasting models:

$$\boldsymbol{\pi^h_{T+h} = \delta + \sum_{i=1}^{m} \eta_i \cdot \pi^1_{T-i} + \sum_{j=1}^{n} \gamma_j \cdot UGAP_{T-j} + \varepsilon_{T+h}} \qquad [\mathbf{11}]$$

The number of lags of inflation, $\boldsymbol{m}$, is the same as in equation 9 and the number of lags of UGAP ($\boldsymbol{n}$) is also determined by the Schwarz criterion as calculated with the first vintage (2006Q1). Equation 11 is estimated in real time in every quarter from 2006Q1 to the end of our sample and inflation forecasts are performed. The UGAPs are introduced like inflation: no matter the projection horizon $\boldsymbol{h}$, only historical values of UGAPs are used. RMSE is again calculated using the first data released.

To assess whether adding UGAPs leads to better forecasts, relative to a model that simply includes past realized inflation, we calculate the ratio of RMSE from models with UGAP (equation 11) over the benchmark (equation 9). A ratio below one indicates that adding UGAP does reduce the forecasting errors. A Clark and McCracken (2009) test is used to compare the forecasting performance of nested models in presence of data revisions. This forecasting exercise is done for all TUR models presented in Section 2.

[17] We assess forecasts against the first release of data because this is how they are usually assessed, in practice, in central banks. For instance, the 2016Q2 vintage encompasses the information available to staff in June 2016 to inform the Bank of Canada's Governing Council about the Canadian economy outlook, ahead of the publication of the July 2016 *Monetary Policy Report*. At that time, the last historical data point was 2016Q1. Thus, the first data release for the quarterly forecast for quarter 2016Q3 ($\boldsymbol{T+1}$) was with the 2016Q4 vintage. For the year-over-year forecast done for quarter 2017Q2 ($\boldsymbol{T+4}$), the first release was with the 2017Q3 vintage. See Murray (2013) for a discussion about the Bank's monetary policy decision-making process.

We use four different measures of inflation: CPI inflation (the Bank of Canada target); and the three preferred measures of core inflation (CPI-trim, CPI-median and CPI-common). For wage inflation, we use wages and ULC growth from the Productivity Accounts.[18] One advantage of ULC is that it controls for labour productivity growth, a determinant of wage growth (e.g. Brouillette, Lachaine and Vincent, 2018). Since UGAP is a measure of slack in the labour market, it may therefore be easier to identify a statistically significant relationship between UGAP and wage or ULC growth.

Our conclusions about the usefulness of UGAPs to forecast inflation could potentially be different if we considered other variables that may affect Canadian inflation. As a robustness check, we therefore estimate versions of equations 9 and 11 for inflation measures where we add changes in the exchange rate and in commodity prices.[19] These variables are used to analyze CPI inflation fluctuations in the Bank of Canada *Monetary Policy Report*. For wage and ULC growth, one lag of inflation is added to equation 11 and one lag of labour productivity growth is added for the former only.[20] Correspondingly, the benchmark equation 9 is also augmented with the same control variables for inflation and wages and the Clark and McCracken (2009) test is applied.

One remaining issue is the treatment of the output gap. In theory, the output gap is a broader measure of economic slack that could incorporate the information contained in the UGAP. And indeed, the correlations between the Bank of Canada staff output gap and the UGAPs estimated by all but two TUR-estimation methods are high, as they range from -0.77 to -0.99.[21] For this reason, we do not include the output gap in the augmented

[18] Variables from the Productivity Accounts are used because they are readily available in real time over a sufficiently long period. See Brouillette, Lachaine and Vincent (2018) for a description of other wage growth measures for Canada.

[19] We added one to four lags of the change in the exchange rate based on the Schwartz criterion for CPI and core inflation, while we consider only one lag of the change in commodity prices for CPI inflation. See Bank of Canada (2019) for discussions of factors affecting CPI inflation in Canada.

[20] This is broadly consistent with other work done at the Bank; see, for instance, Brouillette, Lachaine and Vincent (2018).

[21] The correlations for the triangle model, the BP filter and cointegration 2 are lower (in absolute value) at -0.70, -0.65 and -0.72, respectively.

equation 11, as it is most likely that UGAP would not add much explanatory power over the other gaps. Nevertheless, an interesting avenue of research would be to compare the relative performance of non-nested models. For example, equation 11 could be estimated twice, first with the output gap and then by replacing it with the UGAP. We leave these alternative specifications for future work.

Usefulness to forecast wage or CPI inflation is an important criterion, particularly for inflation-targeting central banks. However, the sample for our real-time out-of-sample forecasts is only about 10 years of quarterly data, which is a limitation of that exercise. We therefore complement it, in Section 3.3, with an examination of how well UGAPs can explain past inflation dynamics.

3.3 How well do UGAPs explain past inflation dynamics?

Our last criterion looks at how well different estimates of UGAP can be mapped into inflation dynamics over history. Equation 11 (augmented with the additional control variables described in Section 3.2) is thus estimated (not using real-time data), for all inflation and wage growth measures, over our entire sample of historical data used to assess forecasting performance (1992Q4 to 2018Q3). The fit, as measured by the adjusted R-squared, allows the identification of models that explain inflation well over history.

Of course, measures of fit also have disadvantages. First, they abstract from the issue of data revisions (unfortunately, we do not have enough vintages to assess the fit in real time over the entire sample). Second, a model able to fit historical data may not forecast or explain the more recent data well (e.g. in the presence of structural breaks).

We also verify whether the relationship between UGAP and inflation is consistent with theory, by looking at the sign and the significance of the coefficients of lag UGAPs. A negative relationship is expected as lower inflation is expected to be associated with high cyclical unemployment (positive UGAP).

4. The data

A main contribution of this paper is that it uses real-time data for many variables. We have 52 vintages, from 2006Q1 to 2018Q4. We define a vintage as follows. Taking, for instance, the 2018Q4 vintage, the last historical data point is 2018Q3 ($\boldsymbol{T-1}$) and two quarters of Bank of Canada staff monitoring are added, 2018Q4 ($\boldsymbol{T}$) and 2019Q1 ($\boldsymbol{T+1}$).[22] This definition is consistent with the release schedule of historical data used for the conduct of monetary policy in Canada (see footnote 18).

Our database builds on the one used in Pichette et al. (2019), but adds some variables specific to this project, such as ULC, wage growth and other labour market data. The data vintages were collected from a variety of sources, including Bank of Canada staff projection databases, real-time data gathered automatically in Bank of Canada data systems since 2013, and various past projects that involved collecting real-time data.[23] Also, some vintages were obtained directly from Statistics Canada. Appendix A provides detailed information about the real-time data used in the models.

A main advantage of working with real-time data is that it makes it possible to disentangle UGAP revisions due to data revisions from UGAP revisions due to the availability of new information (analysis of this is presented in Section 5.2). It is interesting to note, however, that the variables used in models of Section 2 are not revised to the same extent. For instance, the unemployment rate (Chart B1)—the main object of our study—and CPI inflation (Chart B2)—one of the main variables we forecast—are series that are not revised much.[24] However, two variables we forecast, wages (Chart B3) and ULC growth (Chart B4), are subject to substantial revisions. Some of the variables used in estimating TURs, such as GDP growth, can also be revised (Chart B5). It was therefore essential to

[22] Some series used in the estimation of TURs are not monitored by Bank of Canada staff. In those cases, a simple historical growth rate rule is used.

[23] Champagne, Poulin-Bellisle and Sekkel (2018) developed and analyzed a similar database of real-time data based on Bank of Canada staff projection databases.

[24] The CPI not seasonally adjusted is subject to a non-revision policy (Statistics Canada, 2015). However, in this paper we use the seasonally adjusted CPI and as such there are some revisions.

develop this real-time database given that we want to assess the models with the data they would have used in real time. Data revisions contribute to revisions to TUR and UGAP estimates, but also affect the real-time information content about future inflation of our UGAPs.

Unfortunately, we have only a few vintages of core inflation measures—the series were introduced with the 2016 renewal of the inflation-control target (Bank of Canada, 2016). Consequently, for core inflation we assess only the real-time forecasts with the latest available vintage of these variables.

While all TUR-estimation models have the same number of vintages, the TUR-estimation period is not necessarily the same for all models. We use the longest period available rather than restricting all estimation periods to be the same. Starting dates are as follows:

- Multivariate state-space models
    - Multivariate state-space framework: 1990 (annual)
    - Basic multivariate filter: 1990 (annual)
    - Gordon triangle model: 1982Q2
    - OECD backward-looking model: 1984Q1
    - OECD anchored model: 1990Q1—a later starting date, relative to the OECD backward-looking model, is needed because the model imposes that inflation expectations are anchored and inflation targeting in Canada started in the early 1990s
    - Hysteresis model: 1981Q1
- Models with structural determinants
    - Cointegration 1 and cointegration 2: 1981Q2
    - Structural factors wage Phillips curve: 1981Q2
    - Simplified integrated framework: 1976 (annual)
- Basic and augmented mechanical filters
    - Hodrick and Prescott and band-pass filters: 1976Q1
    - Extended multivariate filter: 1981Q1

5. Assessment

5.1 Interpretability

Our TUR-measurement methods can be categorized into three groups, depending on whether they provide no (Group 1), little (Group 2) or some (Group 3) interpretability of TUR movements.

Group 1: BP filter, HP filter, EMVF, triangle, OECD backward-looking and OECD anchored

Simple mechanical filters (HP filter and BP filter) can be a pragmatic approach because they assign a certain proportion of unemployment fluctuations to the TUR and a certain proportion to UGAP. Yet, they can neither explain why the TUR stands at a given level nor provide a narrative for its dynamics. Moreover, their shortcomings at extracting trends from time series are well documented. HP and BP filters were indeed shown to be subject to end-of-sample limitations (Baxter and King, 1999; St-Amant and Norden, 1997). And even when abstracting from this issue, they tend to perform poorly in extracting trends from persistent time series, such as the level of unemployment and real GDP (Guay and St-Amant, 2005; Hamilton, 2017). Also, while different types of shocks could have different effects in theory, this cannot be reflected in mechanical filters.

The EMVF is not much more useful in providing interpretation. Indeed, EMVF results are difficult to interpret, as they are influenced by a complicated mix of economic factors, end-of-sample conditions, and mechanical filtering (see Pichette et al., 2015, for more discussion on this point). Methods based on Phillips curves (triangle model, OECD backward-looking and anchored models) include several variables linking inflation to its determinants. These variables, and the models' specifications, are usually justified with theoretical arguments. However, the ability of these models to explain TUR estimates is very limited, as inflation movements are used to identify TUR changes. Variables other than inflation are included as controls, but they are believed to explain inflation, not necessarily TUR.

Group 2: BMVF, MSSF and hysteresis

The BMVF and MSSF provide some interpretation of the dynamics of the TUR because they include an Okun's law relationship linking shocks to output to the output gap and to the UGAP, and implicitly, the TUR. While the MSSF can provide insights into the origin of the movements in the output gap—in contrast to the BMVF, the MSSF decomposes potential output into various components—its explanatory power remains limited. Likewise, the hysteresis model provides some interpretability, because it links the output gap with TUR changes. However, none of these models can directly inform on the other factors affecting TUR dynamic.

Group 3: Cointegration 1, cointegration 2, SFWPC and SIF

Both cointegration models provide some interpretation of TUR developments because the variables included in the cointegration equations are interpreted as determinants of TUR. The estimating method of the SFWPC is similar to the cointegration approach and therefore provides a similar amount of interpretability. The main difference between the cointegration models and SFWPC is that, in the latter, movements in ULC are used to help identify TUR. Of course, these models include only a small number of variables, which could be a limitation if important factors are excluded. Also, the direction of the causality relationships may not always be entirely clear, which affects interpretability.

The components of the SIF used to estimate TUR account for few factors, but an important one is the structure of the working-age population. The SIF can thus provide an explanation of the impact of population ageing on TUR estimates. While more work is needed to improve the interpretability of Group 3 models, they nevertheless perform better than models in the two other groups in that respect.

5.2 Revisions

Table 1 summarizes some key properties of revisions to the various UGAP measures. As explained in Section 3.1, these statistics are based on the comparison of real-time estimates with the estimates after eight revisions. Charts in Appendix C show that comparing the real-time estimates with the latest vintage of data would yield similar results, supporting our eight-quarter revisions assumption.

UGAP estimates do not appear to be strongly biased.[25] The largest biases are for the hysteresis, the SIF and the triangle methods (the latter has a positive bias). This implies that, for instance, after eight quarters, UGAPs estimated by the hysteresis model are revised down by 0.17 percentage points, on average.

Overall, three models perform relatively poorly in terms of revisions: the triangle, the OECD anchored and the SIF (shown by numbers in red in Table 1). Their UGAPs have the largest mean absolute revisions and volatility of revisions, while the correlation between real-time estimates and revised estimates is among the lowest. The triangle and OECD anchored models have the largest noise-to-signal ratio—the only instance with a ratio above one—and their UGAPs change sign more than one time out of four after eight revisions.

In contrast, four methods do relatively well: the MSSF, the EMVF and both cointegration models. Their revisions to real-time estimates tend to be relatively small and unbiased. Revised estimates also tend to be highly correlated with real-time estimates (correlation coefficient of at least 0.96) and to have the same sign. In addition, the MSSF and cointegration 1 yield the lowest noise-to-signal ratios. The BMVF also tends to do relatively well in various dimensions. These results are consistent with the charts in Appendix C, showing that, for these five methods, real-time UGAP estimates are close to revised ones.

[25] Pichette et al. (2019) find much more accentuated and one-sided biases in revisions to output gap estimates.

Table 1: Properties of UGAP revisions after eight quarters—Sample: 2005Q4 to 2016Q3

MODEL	Mean	Mean absolute	Standard deviation	Correlation	Noise-to-signal	Freq. of Opposite Signs (%)
MSSF	**0.01**	**0.12**	**0.16**	**0.98**	**0.19**	**2.27**
Triangle	**0.14**	**0.45**	**0.57**	**0.84**	**1.07**	**25.00**
OECD anchored	0.04	**0.30**	**0.37**	**0.75**	**1.01**	**31.82**
OECD backward-looking	0.04	0.26	0.33	0.87	0.57	**25.00**
Cointegration 1	-0.02	**0.10**	**0.13**	**0.98**	**0.23**	**4.55**
BP filter	**0.02**	0.18	0.26	0.85	0.54	13.64
HP filter	**0.00**	0.25	0.34	**0.76**	**0.65**	11.36
BMVF	**-0.12**	0.22	0.30	**0.94**	**0.37**	**0.00**
SIF	**-0.14**	**0.36**	**0.57**	**0.65**	**0.81**	18.18
Hysteresis	**-0.17**	**0.32**	0.34	0.88	0.48	4.55
SFWPC	-0.07	0.27	**0.37**	0.89	0.50	**29.55**
EMVF	**0.01**	**0.15**	**0.20**	0.94	**0.34**	13.64
Cointegration 2	0.05	**0.14**	**0.17**	**0.96**	0.41	**0.00**

Numbers in green denote the top four best-performing models for each statistic. In red, the bottom four performers.

As discussed in Section 3, UGAP revisions can be seen as coming from two sources. They could come from data revisions or they could reflect changes to estimations caused by the fact that more data become available for the estimations. To distinguish between these two sources, we calculate revisions when the models are estimated with the latest vintage of the data (but extending the estimation window). This is shown in Table 2.

Comparing Table 2 with Table 1 suggests that data revisions can be a notable source of UGAP revisions. The performance of the hysteresis model and that of the SFWPC particularly improve when abstracting from data revisions. For instance, the mean absolute revision of the hysteresis model falls from 0.32 to 0.01 and that of SFWPC falls from 0.27 to 0.14. While the noise-to-signal ratios fall for all models, the largest falls are again registered by the hysteresis model and SFWPC. In contrast, data revisions are a less important source of revisions to UGAP estimates for the triangle model and the EMVF.

Indeed, their statistics, as reported in Table 1 and Table 2, are similar whether or not the data revisions are accounted for.

Table 2: Properties of UGAP revisions after eight quarters—Sample: 2005Q4 to 2016Q3 No revisions to input data (2018Q4 vintage)

MODEL	Mean	Mean absolute	Standard deviation	Correlation	Noise-to-signal	Freq. of Opposite Signs (%)
MSSF	0.00	0.10	0.12	0.99	0.15	0.00
Triangle	0.14	0.45	0.56	0.85	0.94	31.82
OECD anchored	0.02	0.37	0.46	0.58	0.94	34.09
OECD backward-looking	0.02	0.25	0.35	0.87	0.58	13.64
Cointegration 1	-0.01	0.09	0.12	0.98	0.20	6.82
BP filter	0.00	0.17	0.24	0.88	0.50	11.36
HP filter	-0.01	0.25	0.33	0.78	0.63	9.09
BMVF	-0.12	0.18	0.26	0.96	0.31	0.00
SIF	-0.31	0.34	0.48	0.77	0.67	13.64
Hysteresis	0.00	0.01	0.01	1.00	0.01	0.00
SFWPC	-0.05	0.14	0.18	0.96	0.28	6.82
EMVF	0.01	0.15	0.20	0.94	0.33	9.09
Cointegration 2	0.11	0.14	0.14	0.96	0.34	4.55

In some cases, however, abstracting from data revisions did increase the extent of the revisions. For example, data revisions help to reduce the mean revisions of the SIF and cointegration 2: average revisions are twice as small when data revisions are factored in (Table 1) relative to the case when data revisions are ignored (Table 2).

5.3 Real-time forecasts

This section discusses whether, and which, UGAPs improve forecasts of CPI or wage inflation one and four quarters ahead. Table 3 compares real-time out-of-sample forecasts of CPI inflation and CPI-common, based on models excluding and including the

UGAP measures. As described in Section 3.2, a simple model, with only lags of inflation—equation 9—is taken as the benchmark. It is compared with models augmented with only lagged UGAP estimates—equation 11. Statistical significance for the difference between the RMSE of equation 9 and that of equation 11 is assessed using an approach developed by Clark and McCracken (2009).[26]

It is immediately obvious from Table 3 that UGAPs have difficulties providing useful information about future CPI inflation. Most RMSE ratios (ratios of RMSEs with UGAPs to ratios without UGAPs) are above one, meaning that models with UGAPs do worse than models without them. In addition, differences between the forecasts of models with and without UGAPs are often not statistically significant. UGAPs have more useful information about future CPI-common. All RMSE ratios are below one, except the BP filter at horizon $\boldsymbol{T+4}$, and some results are statistically significant, particularly at horizon $\boldsymbol{T+1}$. Results are less often significant at horizon $\boldsymbol{T+4}$. Table D1 in Appendix D shows the results for CPI-median and CPI-trim. CPI-common is the core inflation measure for which the forecast improvements are the most promising.

[26] Using an eight-quarter window (as for the revisions) to calculate the RMSE, as opposed to the first release, did not materially change our conclusions. We also considered an alternative specification where projected UGAPs were used to forecast inflation—the UGAP terms in equation 11 thus become $\boldsymbol{UGAP_{T+h-j}}$—but this gave mixed results. While we notice that this addition improves the forecasting ability of some models, it degrades it for some others. Overall, our conclusions are not too sensitive to the use of this alternative model.

Table 3: Comparison of CPI inflation and CPI-common forecasts at various horizons
Ratio of RMSEs between models with and without UGAPs[a, b]

	CPI inflation		CPI-common	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	1.01	1.17	0.90	0.84
Triangle	1.01	1.19	0.90	0.86
OECD anchored	0.95	1.20	0.86	0.88
OECD backward-looking	1.00	1.15	0.87	0.81
Cointegration 1	1.02	1.18	0.88***	0.83***
BP filter	1.00	1.01	0.97***	1.01
HP filter	1.01	1.07	0.94***	0.97
BMVF	1.01	1.15	0.89	0.83
SIF	1.00	1.09	0.91	0.87
Hysteresis	0.99***	1.05	0.85	0.81
SFWPC	1.04	1.21	0.88***	0.81
EMVF	1.02	1.16	0.88***	0.86**
Cointegration 2	1.01	1.09	0.95***	0.94**

[a] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.
[b] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

Results for wages are mixed (Table 4). RMSE ratios are sometimes above and sometimes below one, meaning that UGAPs do not always improve forecasts. BMVF, MSSF, SIF and cointegration 1 have a ratio below one at all horizons, although only the first one shows a statistically significant forecasting improvement for the $\boldsymbol{T+1}$ horizon. Results for ULC are more promising. RMSEs are often significantly smaller when UGAPs are included. Interestingly, the improvement is often most pronounced at horizon $\boldsymbol{T+4}$. Triangle, OECD backward-looking and SFWPC UGAPs do particularly well in bringing statistically significant improvements.

Table 4: Comparison of wages and ULC forecasts at various horizons
Ratio of RMSEs between models with and without UGAPs[a, b]

	Wages		ULC	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	0.96	0.94	0.98	0.99
Triangle	1.01	1.04	0.93***	0.76***
OECD anchored	1.03	0.97	1.04	0.99
OECD backward-looking	1.00	0.97	0.94***	0.79***
Cointegration 1	0.97	0.93	0.99	1.01
BP filter	1.10	1.08	0.99	0.96
HP filter	1.01	1.00	0.98***	0.97**
BMVF	0.96***	0.92	0.96***	0.91***
SIF	0.96	0.94	0.96***	0.94*
Hysteresis	0.96***	1.01	0.96	0.90
SFWPC	1.01	1.09	0.92***	0.79**
EMVF	1.00	0.95	0.97*	0.95
Cointegration 2	0.99	1.03	0.98***	0.99**

[a] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.
[b] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

As a robustness check, we also estimate versions of equations 9 and 11 for inflation measures where we add changes in the exchange rate and commodity prices, as discussed in Section 3.2. For wage and ULC growth, inflation is added to equation 11 and labour productivity growth is added for the former only. Results are presented in tables D2 to D4 in Appendix D. They are generally similar to the ones already discussed in this section.

5.4 UGAP and past inflation dynamics

This section discusses whether, and which, UGAPs contribute to the fit of the equations simulated in the previous section, more specifically the equations augmented with other control variables, such as changes in the exchange rate for inflation and inflation and growth in productivity for wages.[27] The fit, as measured by the adjusted R-squared, allows the identification of models that explain inflation well over history. We also verify whether the relationship between UGAPs and inflation is consistent with theory, by looking at the sign and significance of the coefficients. Our fit measures, however, have disadvantages. First, they abstract from the issue of data revisions. Second, a model able to fit historical data may not forecast or explain the more recent data well, e.g. in the presence of structural breaks.

As expected, the signs on the sum of the coefficients of UGAPs are negative for all models when CPI inflation is the forecasted variable (Table 5), except for the BP filter with CPI four-quarters-ahead forecasts. The lags of UGAPs are also significant for all models, apart from the HP filter, BP filter and EMVF no matter the forecast horizon, and cointegration 2 with one-quarter-ahead CPI forecasts. While the fit, as measured by the adjusted R-squared, is quite low for CPI inflation, it is much better for the CPI-common. While there is little dispersion among the models, the SIF and BMVF offer the best fit for all horizons. At the other end stand the BP filter and cointegration 2. Table D5 in Appendix D presents the results for CPI-median and CPI-trim. The conclusions on the sign and statistical significance are similar, but the fit tends to be lower than for the CPI-common and higher than for CPI inflation.

For wage growth and ULC, the sign of the sum of the coefficients of UGAPs are all negative and statistically significant except for the BP filter with horizon $\boldsymbol{T+1}$ (Table 6). Regarding the fit for ULC, hysteresis, SIF, SFWPC, BMVF and the triangle models are in the lead. The adjusted R-squared tend to be higher for four-quarters-ahead forecasts than for one-quarter-ahead forecasts. This is likely because year-over-year growth rates are less volatile and thus easier to forecast than quarter-over-quarter growth rates. It is also the case that the adjusted R-squared are lower for ULC than for CPI-common, most likely reflecting the higher volatility of the former.

[27] Removing control variables does not generally change the consistency with economic theory but does generally lead to some smaller adjusted R^2.

Table 5: Fit of the equations for CPI inflation and CPI-common and consistency with theory[a]

	CPI inflation						CPI-common					
Models	**T+1**			**T+4**			**T+1**			**T+4**		
	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]
MSSF	-	**	0.07	-	***	0.14	-	***	0.59	-	***	0.67
Triangle	-	**	0.07	-	***	0.13	-	***	0.58	-	***	0.62
OECD anchored	-	***	0.16	-	***	0.21	-	***	0.61	-	***	0.67
OECD backward-looking	-	*	0.06	-	**	0.10	-	***	0.60	-	***	0.66
Cointegration 1	-	*	0.05	-	**	0.11	-	***	0.56	-	***	0.66
BP filter	-		0.03	+		0.05	-	***	0.50	-	***	0.46
HP filter	-		0.03	-		0.05	-	***	0.57	-	***	0.58
BMVF	-	**	0.06	-	***	0.13	-	***	0.61	-	***	0.69
SIF	-	**	0.06	-	***	0.13	-	***	0.61	-	***	0.69
Hysteresis	-	*	0.06	-	***	0.15	-	***	0.59	-	***	0.66
SFWPC	-	*	0.05	-	***	0.11	-	***	0.58	-	***	0.68
EMVF	-		0.05	-		0.07	-	***	0.59	-	***	0.64
Cointegration 2	-		0.05	-	**	0.09	-	***	0.53	-	***	0.59

[a] Sample over which the equations are estimated: 1992Q4–2018Q3. The equations are augmented with control variables.

[b] Sign of the sum of coefficients on the lags of UGAPs.

[c] P-value of the sum of coefficients on the lags of UGAPs. *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.

[d] Adjusted R-squared of the models with UGAPs.

Table 6: Fit of the equations for wages and ULC and consistency with theory[a]

	Wages						ULC					
Models	T+1			T+4			T+1			T+4		
	Sign[b]	Pval[c]	$R^{2\ d}$	Sign[b]	Pval[c]	$R^{2\ d}$	Sign[b]	Pval[c]	$R^{2\ d}$	Sign[b]	Pval[c]	$R^{2\ d}$
MSSF	-	***	0.10	-	***	0.17	-	***	0.12	-	***	0.28
Triangle	-	***	0.10	-	***	0.16	-	***	0.13	-	***	0.32
OECD anchored	-	***	0.10	-	***	0.13	-	***	0.12	-	***	0.25
OECD backward-looking	-	***	0.10	-	***	0.15	-	***	0.13	-	***	0.29
Cointegration 1	-	***	0.10	-	***	0.16	-	***	0.09	-	***	0.25
BP filter	-		0.01	-	***	0.13	-	*	0.03	-	***	0.22
HP filter	-	**	0.06	-	*	0.04	-	***	0.08	-	***	0.15
BMVF	-	***	0.11	-	***	0.18	-	***	0.14	-	***	0.33
SIF	-	***	0.13	-	***	0.21	-	***	0.15	-	***	0.34
Hysteresis	-	***	0.09	-	***	0.22	-	***	0.17	-	***	0.40
SFWPC	-	***	0.10	-	***	0.15	-	***	0.13	-	***	0.33
EMVF	-	***	0.08	-	***	0.08	-	***	0.10	-	***	0.20
Cointegration 2	-	***	0.08	-	***	0.14	-	***	0.06	-	***	0.20

[a] Sample over which the equations are estimated: 1992Q4–2018Q3. The equations are augmented with control variables.
[b] Sign of the sum of coefficients on the lags of UGAPs.
[c] P-value of the sum of coefficients on the lags of UGAPs. *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.
[d] Adjusted R-squared of the models with UGAPs.

5.5 Overall assessment against our criteria

Our criteria can be used to rank the models we consider. We present two examples below. Results for CPI, CPI-common, CPI-trim, CPI-median, hourly compensation and ULC are all considered in ranking models against the interpretability, revisions, forecast and fit criteria (Table 7).

Table 7: Ranking of the models for estimating TUR for each criterion

Interpretability[a]	Revisions[b]	Forecast[c]	Fit[d]	Overall ranking[e]	Alternative ranking[f]
1-SIF	1-MSSF	1-OECD backward	1-Hysteresis	1-SFWPC	1-SIF
1-Cointegration 1	2-Cointegration 1	2-SFWPC	2- BMVF	2-Cointegration 1	2-SFWPC
1-SFWPC	3-Cointegration 2	3-SIF	3- SIF	3-BMVF	3-BMVF
1-Cointegration 2	4-EMVF	4-BMVF	4-SFWPC	4-SIF	4-Cointegration 1
5-MSSF	5-BMVF	5-Triangle	5-OECD anchored	5-MSSF	5-Hysteresis
5-BMVF	6-BP filter	6-Cointegration 1	6-MSSF	6-Cointegration 2	6-OECD backward
5-Hysteresis	7-HP filter	7-EMVF	7-Triangle	7-Hysteresis	7-MSSF
8-OECD backward	8-OECD backward	8-MSSF	8-OECD backward	8-OECD backward	8-Cointegration 2
8-OECD anchored	9-Hysteresis	9-HP filter	8-Cointegration 1	9-EMVF	9-Triangle
8-Triangle	10-SFWPC	10-Cointegration 2	10-Cointegration 2	10-Triangle	10-OECD anchored
8-EMVF	11-OECD anchored	11-Hysteresis	11-EMVF	11-OECD anchored	11-EMVF
8-HP filter	12-Triangle	12-OECD anchored	12-HP filter	12-HP filter	12-HP filter
8-BP filter	13-SIF	13-BP filter	13-BP filter	13-BP filter	13-BP filter

[a] The ranking of the models in terms of their interpretability is based on the discussion in Section 5.1.
[b] The ranking for the revisions criterion uses the information provided by the six statistics presented in Table 1. These statistics are given equal weight in ranking the models.
[c] The ranking related to the ability to forecast is based on the extent to which the models improve the forecast and on whether the improvements are statistically significant in the models where control variables were added.
[d] The ranking related to the fit of the equations is based on the discussion in Section 5.4 and gives equal weight to the three variables of interest the sign of the coefficient in front of the sum of UGAPs, the p-value and the adjusted R2.
[e] Equal weight is given to the criteria.
[f] In this alternative ranking, no weight is given to the revision criterion. The three remaining criteria are equally weighted.

Clearly, models with structural determinants do better in terms of interpretability, which helps them do well in the overall ranking. Three of the top four models are structural models. The BMVF, MSSF and hysteresis models top the ranking of multivariate state-space models. Basic and augmented mechanical filters do badly on interpretability and do not do particularly well vis-à-vis the other criteria. These results should not be surprising, given the limitations of these filters as mentioned in Section 5.1. We conclude that they can safely be dismissed.

All in all, the top four best-performing models appear to be the SFWPC, cointegration 1, BMVF and SIF. These models differ in terms of their input data and structure, which means that aggregating them to estimate the TUR allows for a large range of information to be incorporated in assessments.

Of course, policymakers could decide to use our criteria differently. For instance, they could choose to put more weight on a criterion they find particularly important or they could choose to exclude one of our criteria. We verified that the models we identify as best performing also tend to do well when the weight assigned to the criteria is changed somewhat or when the alternative specifications of the different criteria are used. For instance, putting no weight on revisions (last column in Table 7), a criterion that has limitations (see Section 3.1), does not change the top four of our ranking—although the ordering of the models changes. But of course, some substantial changes to the weight could change the conclusions.[28]

6. Conclusions

In this paper, we assess various measures of the Canadian trend unemployment rate (TUR) against four criteria: interpretability, behaviour of revisions, information content about future inflation and information about past inflation.

The TUR-measurement methods we examine are not explicitly derived from structural models. Therefore, they are all limited in terms of the interpretability they can provide.

[28] Rankings were constructed for various alternative specifications of the criteria (e.g. revisions calculated on the last vintage instead of after eight revisions, or forecasting errors based on Phillips curves without control variables). In general, the top four were still doing well and the worst-performing models remained the same as in Table 7. However, the hysteresis model or the MSSF were sometimes among the top four, depending on the specifications used.

However, some methods do better in that respect. The cointegration, the SFWPC and the SIF methods are indeed attempts at linking TUR with its determinants and can therefore provide some interpretation. Other methods are more limited against this criterion.

TUR-estimation methods are also quite different in terms of the behaviour of revisions to UGAP estimates. While some methods give real-time UGAPs that are noisy estimates of revised UGAPs (SIF, triangle, OECD anchored), others are less noisy (MSSF and the two cointegration methods, EMVF and BMVF). In some cases, data revisions are an important driver of revisions to UGAP estimates (e.g. hysteresis model). However, in many cases revisions to UGAP estimates mostly reflect the fact that we are using an extending window for UGAP estimations.

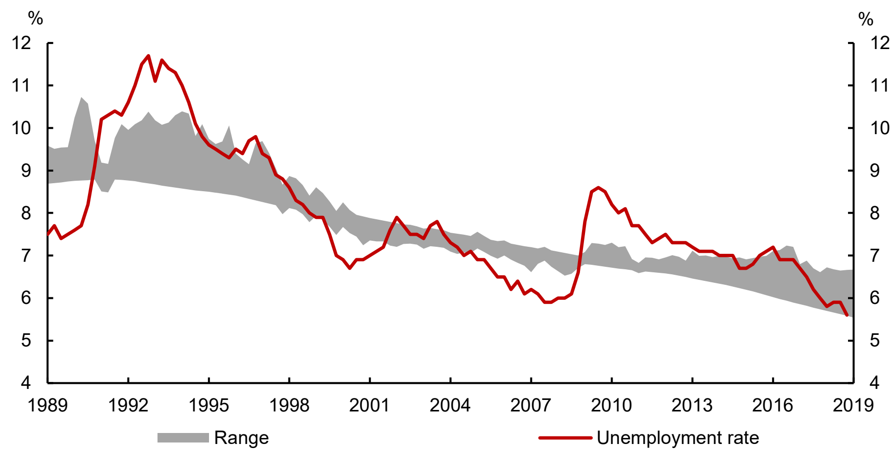
In general, UGAPs appear very limited in their ability to help forecast CPI inflation. They do better at reducing core CPI inflation and wage growth forecast errors, but these improvements are often not statistically significant. Results for ULC are more encouraging, with improvements over simple autoregressive models of ULC growth that are often statistically significant. The following methods do particularly well at forecasting in general: OECD backward-looking, SFWPC, SIF, triangle and BMVF.

Lastly, coefficients in front of UGAPs in core inflation, wage growth and ULC growth equations are statistically significant and of the right sign. Results vary more in the case of CPI inflation equations, with some UGAPs not statistically significant and one (BP filter) of the wrong sign. In terms of fit, the adjusted R^2 are usually higher with the CPI-common and very low with CPI inflation. Hysteresis, BMVF and SIF perform relatively well for this criterion.

The weight to be put on these TUR measures in policymaking clearly depends on the relative importance policymakers assign to our four criteria. With a simple approach giving equal weight to the four criteria, we find that SFWPC, cointegration 1, SIF and BMVF perform best. Using these four methods to estimate a range of TURs may be an approach worth considering by policymakers. This approach suggests that Canada's TUR was between 5.6 and 6.7 per cent in 2018Q4. The actual (measured by Statistics Canada) rate of unemployment was 5.6 per cent in that quarter. Chart 1 shows the actual unemployment rate with the historical range of estimates based on our four preferred methods (the 2018Q4 vintage is used for this estimation).

Chart 1: Range for trend unemployment rate and unemployment rate (2018Q4 vintage)

Quarterly data



Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2019Q1

Chart 1 clearly indicates that Canada's TUR has been declining since the early 1990s. SFWPC and cointegration 1 imply that this is partly explained by a trend decline in the payroll tax rate since the early 1990s. SIF highlights the role of demographic developments, particularly the increased weight in the working-age population of age groups that tend to have lower unemployment rates. Nevertheless, developing a better understanding of the structural factors accounting for TUR developments should be a priority in future research. There may well be other factors at play.

In future work it may also be worth exploring alternative models, such as search and match models, and models specifically allowing for non-linearities in the relationship between UGAPs and inflation. Finally, methods for combining various TUR estimates may be worth considering.

References

Bank of Canada. 1991. “Press Release: Targets for Reducing Inflation.” *Bank of Canada Review*. March. https://www.bankofcanada.ca/wp-content/uploads/2011/12/bocreview-mar1991.pdf.

Bank of Canada. 2012. “How Monetary Policy Works: The Transmission of Monetary Policy.” Backgrounders, https://www.bankofcanada.ca/wp-content/uploads/2010/11/how_monetary_policy_works.pdf.

Bank of Canada. 2016. “Renewal of the Inflation-Control Target: Background Information.” October. http://www.bankofcanada.ca/wp-content/uploads/2016/10/background_nov11.pdf.

Bank of Canada. 2019. *Monetary Policy Report*. January. https://www.bankofcanada.ca/wp-content/uploads/2019/01/mpr-2019-01-09.pdf.

Barnett, R. 2007. “Trend Labour Supply in Canada: Implications of Demographic Shifts and the Increasing Labour Force Attachment of Women.” *Bank of Canada Review*. Summer. https://www.bankofcanada.ca/wp-content/uploads/2010/06/barnett.pdf.

Baxter, M. and R. G. King. 1999. “Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series.” *Review of Economics and Statistics* 8(4): 575–593.

Blagrave, P., R. Garcia-Saltos, D. Laxton and F. Zhang. 2015. “A Simple Multivariate Filter for Estimating Potential Output.” International Monetary Fund, IMF Working Papers 15/79.

Brouillette, D., J. Lachaine and B. Vincent. 2018. “Wages: Measurement and Key Drivers.” Bank of Canada Staff Analytical Note 2018-2. https://www.bankofcanada.ca/wp-content/uploads/2018/01/san2018-2.pdf.

Butler, L. 1996. “The Bank of Canada’s New Quarterly Projection Model, Part 4. A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter.” Bank of Canada Technical Report No. 77. https://www.bankofcanada.ca/wp-content/uploads/2010/01/tr77.pdf.

Cayen, J.-P. and S. van Norden. 2005. “The Reliability of Canadian Output-Gap Estimates.” *The North American Journal of Economics and Finance* 16(3): 373–393.

Champagne, J., G. Poulin-Bellisle and R. Sekkel. 2018. "Evaluating the Bank of Canada Staff Economic Projections Using a New Database of Real-Time Data and Forecasts." Bank of Canada Staff Working Paper 2018-52.

Christiano, L. J. and T. J. Fitzgerald. 2003. "The Band Pass Filter." *International Economic Review* 44(2): 435–465.

Clark, T. and M. McCracken. 2009. "Tests of Equal Predictive Ability with Real-Time Data." *Journal of Business & Economic Statistics* 27(4): 441–454.

Côté, D. and D. Hostland. 1996. "An Econometric Examination of the Trend Unemployment Rate in Canada." Bank of Canada Staff Working Paper 1996-7. https://www.bankofcanada.ca/wp-content/uploads/2010/05/wp96-7.pdf.

Demers, F. 2003. "The Canadian Phillips Curve and Regime Shifting." Bank of Canada Staff Working Paper 2003-32. https://www.bankofcanada.ca/wp-content/uploads/2010/02/wp03-32.pdf.

Fougère, M. 2012. "How Close to Equilibrium Is Canada's Rate of Unemployment?" Human Resources and Skills Development Canada. https://www.researchgate.net/publication/235770371_How_close_to_equilibrium_is_Canada's_rate_of_unemployment.

Gordon, R. J. 2013. "The Phillips Curve Is Alive and Well: Inflation and the NAIRU During the Slow Recovery." National Bureau of Economic Research, Working Paper 19390.

Guay, A. and P. St-Amant. 2005. "Do the Hodrick-Prescott and Baxter-King Filters Provide a Good Approximation of Business Cycles?" *Annals of Economics and Statistics* 77: 133–155.

Guichard, S. and E. Rusticelli. 2011. "Reassessing the NAIRUs after the Crisis." OECD Economics Department, Working Paper, No. 918, OECD Publishing, Paris.

Hamilton, J. 2017. "Why you Should Never Use the Hodrick-Prescott Filter." National Bureau of Economic Research, Working Paper 23429.

Hodrick, R. J. and E. C. Prescott. 1997. "Postwar US Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking* 29(1): 1–16.

Jacob, P. and M. Wong. 2018. "Estimating the NAIRU and the Natural Rate of Unemployment for New Zealand." Reserve Bank of New Zealand Analytical Notes, AN 2018/4.

Jacobs, J.P.A.M., and J.-E. Sturm. 2004. "Do IFO Indicators Help Explain Revisions in German Industrial Production?" CESifo Working Paper No. 1205.

Jaeger, A. and M. Parkinson. 1994. "Some Evidence on Hysteresis in Unemployment Rates." *European Economic Review* 38: 329–342.

Murray, J. 2013. "Monetary Policy Decision Making at the Bank of Canada." *Bank of Canada Review*, Autumn 2013: 1–9. https://www.bankofcanada.ca/wp-content/uploads/2013/11/boc-review-autumn13-murray.pdf.

Orphanides, A. and S. van Norden. 2005. "The Reliability of Inflation Forecasts Based on Output Gap Estimates in Real Time." *Journal of Money, Credit and Banking*. Blackwell Publishing, 37(3): 583–601.

Orphanides, A. and S. van Norden. 2002. "The Unreliability of Output-Gap Estimates in Real Time." *The Review of Economics and Statistics* 84(4): 569–583.

Pichette, L., P. St-Amant, B. Tomlin and K. Anoma. 2015. "Measuring Potential Output at the Bank of Canada: The Extended Multivariate Filter and the Integrated Framework." Bank of Canada Staff Discussion Paper 2015-1. https://www.bankofcanada.ca/wp-content/uploads/2015/01/dp2015-1.pdf.

Pichette, L., M.-N. Robitaille, M. Salameh and P. St-Amant. 2019. "Dismiss the Output Gaps? To Use with Caution Given their Limitations." *Economic Modelling* 76C: 199–215.

Pichette, L., M. Bernier and M.-N. Robitaille. 2018. "An Alternative Estimate of Canadian Potential Output: The Multivariate State-Space Framework." Bank of Canada Staff Discussion Paper 2018-14. https://www.bankofcanada.ca/wp-content/uploads/2018/11/SDP2018-14.pdf.

Rusticelli, E., D. Turner and M. C. Cavalleri. 2015. "Incorporating Anchored Inflation Expectations in the Phillips Curve and in the Derivation of OECD Measures of Equilibrium Unemployment." OECD Economics Department Working Papers, No. 1231, OECD Publishing, Paris.

Sargent, T. C. 1995. “An Index of Unemployment Insurance Disincentives.” Department of Finance, Working Paper No. 95-10.

St-Amant, P. and S. van Norden. 1997. “Measurement of the Output Gap: A Discussion of Recent Research at the Bank of Canada.” Bank of Canada Technical Report No. 79. https://www.bankofcanada.ca/wp-content/uploads/2010/01/tr79.pdf.

Statistics Canada. 2015. “The Canadian Consumer Price Index Reference Paper.” Available online: https://www150.statcan.gc.ca/n1/pub/62-553-x/62-553-x2015001-eng.htm.

Wilkins, C. 2019. “A Look Under the Hood of Canada’s Job Market.” Remarks to the Toronto Region Board of Trade. January 31. Available online: https://www.bankofcanada.ca/2019/01/look-under-hood-canadas-job-market/.

Appendix A: Data sources

• VINT_BC: Databases with vintages collected by the Bank of Canada accessible to Bank staff. Most of the real-time data used in the paper come from this source.[29]

• MON: Bank of Canada short-term forecast databases. These are vintages of data used by Bank of Canada staff in monitoring the Canadian economy.

• STAT CAN: Data obtained from Statistics Canada through special requests.

• CF: Consensus forecasts from Consensus Economics.

• NR: The data are not revised.

Multivariate state-space models

Triangle, OECD backward-looking and OECD anchored

Series	Definition	Source	Real-time data source
Series used in the three models			
CPI inflation	All-items consumer price index; seasonally adjusted; quarter-over-quarter percentage change	Statistics Canada – Consumer Price Index	MON
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force		
Labour force	Labour force (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Import price inflation	Implicit price index; imports of goods and services; seasonally adjusted; quarter-over-quarter percentage change	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON

[29] Prior to 2013, data vintages were collected in individual projects and initiatives. We want to mention the excellent contribution of Paul Gilbert and Hope Pioro, who collected weekly vintages from 2006 until the Bank of Canada put in place a process to do this automatically.

Series specific to the triangle model			
CPI excluding food and energy (CPIXFE) inflation[30]	All-items excluding food and energy consumer price index; seasonally adjusted; quarter-over-quarter percentage change	Statistics Canada – Consumer Price Index	NR[31]
Series specific to the OECD backward-looking and anchored models			
Import penetration	Imports divided by GDP less net exports (all nominal)		
Nominal gross domestic product	Gross domestic product at market prices ($ x 1,000,000); expenditure based; current prices; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Nominal exports	Exports of goods and services ($ x 1,000,000); expenditure based; current prices; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Nominal imports	Imports of goods and services ($ x 1,000,000); expenditure based; current prices; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Series specific to the OECD backward-looking model			
Oil price inflation	Nominal West Texas Intermediate (WTI) oil price per barrel; in USD; quarter-over-quarter percentage change	Haver Analytics	NR
Oil intensity	Total domestic oil supply divided by real domestic output		
Total domestic oil supply	Total Canadian oil supply; thousands of barrels per day; quarterly	International Energy Agency – Monthly Oil Data Service	NR
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure based; chained (2007) $; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON

[30] Seasonally adjusted data for CPIXFE were only available since 1992. From 1976 to 1992, seasonally adjusted data were obtained by applying the Eviews x12 command to non-seasonally adjusted CPIXFE data.

[31] Real-time CPIXFE data were not used because revisions are negligible and the real-time data were only available starting in 2011Q4.

MSSF and BMVF

Series	Definition	Source	Real-time data source
Series used in the two models			
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force		
Labour force	Labour force (x 1,000); both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure based; chained (2007) $	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Gross domestic product (consensus forecast)	Consensus forecasts of gross domestic product (the next five years)	Consensus Economics	CF
Inflation	All-items consumer price index; year-over-year percentage change convert to annual	Statistics Canada – Consumer Price Index	MON
Inflation (consensus forecast)	Consensus forecasts of total consumer price index inflation (the next year)	Consensus Economics	CF
Series specific to the MSSF			
Labour productivity	Real gross domestic product divided by total hours worked		
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure based; chained (2007) $	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Total hours worked	Total actual hours worked for all jobs	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Average hours worked	Total hours worked divided by employment		
Employment	Employment (x 1,000); both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC & MON

Total hours worked	Total actual hours worked for all jobs	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC & MON

Hysteresis model

Series	Definition	Source	Real-time data source
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force		
Labour force	Labour force (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Output gap	Difference between real GDP and potential output as a percentage of real GDP	Bank of Canada (internal)	VINT_BC

Models with structural determinants

Cointegration 1, cointegration 2 and SFWPC

Series	Definition	Source	Real-time data source
Series used in the three models			
Unemployment rate	Unemployment rate; both genders; 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	MON
Employment insurance disincentives index	Employment insurance disincentive index assuming constant unemployment rate of 7.5% for all regions	Department of Finance Canada	NR
Payroll tax rate (excluding CPP)	Payroll taxes (excluding CPP) as a share of wages, salaries, and supplementary income		
Employer and employee contributions to employment insurance	General governments; revenue; federal government; contributions to social insurance plans; of which: employer and employee	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC

	contributions to employment insurance ($ x 1,000,000); seasonally adjusted at annual rates		
Employers' contribution to workers' compensation	Provincial administration, education and health; revenue; provincial and territorial administration; contribution to social insurance plans; employers' contribution to workers' compensation ($ x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Provincial and territorial payroll taxes	Provincial administration, education and health; revenue; provincial and territorial administration; payroll taxes ($ x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Employer and employee contributions to industrial employees' vacations	Provincial administration, education and health; revenue; provincial and territorial administration; contribution to social insurance plans; employer and employee contributions to industrial employees' vacations ($ x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Contributions to social insurance plans, other	Provincial administration, education and health; revenue; provincial and territorial administration; provincial contributions to social insurance plans, other ($ x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Compensation of employees	Compensation of employees paid by resident entities ($ x 1,000,000); seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Series specific to cointegration 2			

Minimum wage ratio[32]	Ratio of minimum wage in Canada to average hourly wage		
Minimum wage by provinces	Minimum wage rates by provinces	Minimum wage database, Government of Canada	NR
Employment by provinces	Employment; both sexes; 15 years of age and over (x 1,000); seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC
Employment	Employment; both sexes; 15 years of age and over (x 1,000); seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC
Wage	Compensation of employees Canada; wages and salaries; seasonally adjusted at annual rates	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	VINT_BC
Total hours worked	Total actual hours worked for all jobs	Statistics Canada – Labour force survey estimates	VINT_BC
Series specific to the SFWPC			
CPI inflation	All-items consumer price index; seasonally adjusted; quarter-over-quarter percentage change	Statistics Canada – Consumer Price Index	MON
Unit labour cost	Unit labour cost, total economy; index 2007 = 100; seasonally adjusted; quarter-over-quarter percentage change	Statistics Canada – Labour Productivity Measures	VINT_BC & MON

SIF

Series	Definition	Source	Real-time data source
Education	Ratio of university degree holders to working-age population		
University degree holders	University degree holders	Statistics Canada – Labour force survey estimates	VINT_BC

[32] Minimum wage ratio is constructed. Minimum wage for Canada is based on the weighted sum of minimum wage in each province using employment shares. Average hourly wage is constructed using the wages and salaries in the national accounts divided by the total hours worked. The Canadian minimum wage is then divided by this average hourly wage series.

Working-age population	Number of persons of working age, 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	VINT_BC
Employment disincentive index	Employment insurance disincentive index assuming constant unemployment rate of 7.5% for all regions	Department of Finance Canada	NR
Enrollment rate	Full-time students; aged 15-24 years; divided by total population aged 15–24 (students + non-students)		
Full-time students (15-24)	Full-time students; aged 15–24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT_BC
Total non-students (15-24)	Non-students; aged 15–24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT_BC
Total students (15-24)	Students; aged 15–24 years; unadjusted for seasonality	Statistics Canada – Labour force survey estimates	VINT_BC
Nominal interest rate	Bank of Canada Bank Rate	Bank of Canada	MON
Trend inflation	The trend in the consumer price index inflation rate	Internal	MON
Trend nominal interest rate	The trend in the nominal interest rate	Internal	MON
Share of service-producing sector	Ratio of employment in the service-producing sector to total employment		
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Service-producing sector employment	Employment (x 1,000); service-producing sector; both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC
Working-age population	Number of persons of working age; 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	VINT_BC
Employment rate, single age groups	Employment rate; males, females, and both genders; single age group	Statistics Canada – Labour force survey estimates (special request)	STAT CAN
Participation rate, single age groups	Participation rate; males, females, and both genders; single age group	Statistics Canada – Labour force survey estimates (special request)	STAT CAN

Working-age group, single age groups	Number of persons of working age; males, females, and both genders; single age group (x 1,000)	Statistics Canada – Labour force survey estimates (special request)	STAT CAN

Basic and augmented mechanical filters

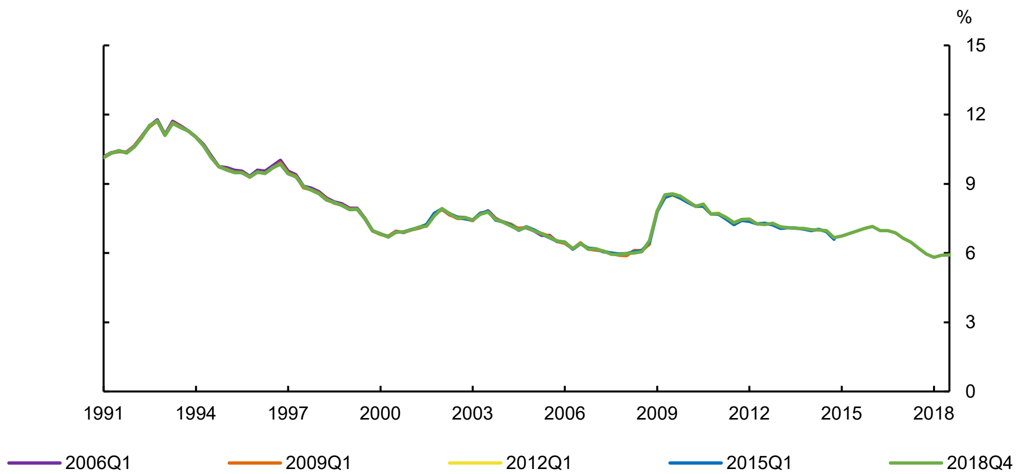
HP filter, BP filter and EMVF

Series	Definition	Source	Real-time data source
Series used in the three models			
Unemployment rate	Number of people unemployed (labour force minus employed) divided by labour force		
Labour force	Labour force (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over; seasonally adjusted	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Series specific to the EMVF			
Average hours worked	Total hours worked divided by employment		
Total hours worked	Total actual hours worked for all jobs	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Employment	Employment (x 1,000); both genders; aged 15 years and over	Statistics Canada – Labour force survey estimates	VINT_BC & MON
Gross domestic product	Gross domestic product at market prices (x 1,000,000); expenditure based; chained (2007) $	Statistics Canada – National Gross Domestic Product by Income and by Expenditure Accounts	MON
Trend unemployment rate	Series is the one estimated with the cointegration model 1		
Working-age population	Number of persons of working age; 15 years of age and over (x 1,000)	Statistics Canada – Labour force survey estimates	VINT_BC & MON

Appendix B: Data revisions

Chart B1: Real-time unemployment rate

Per cent, quarterly data

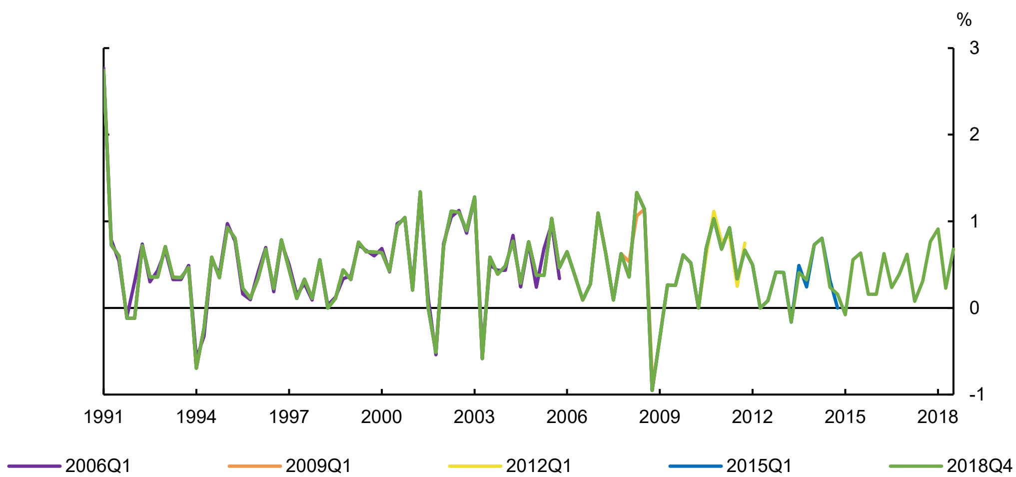


Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2018Q3

Chart B2: Real-time CPI inflation

Quarter-over-quarter percentage change, quarterly data

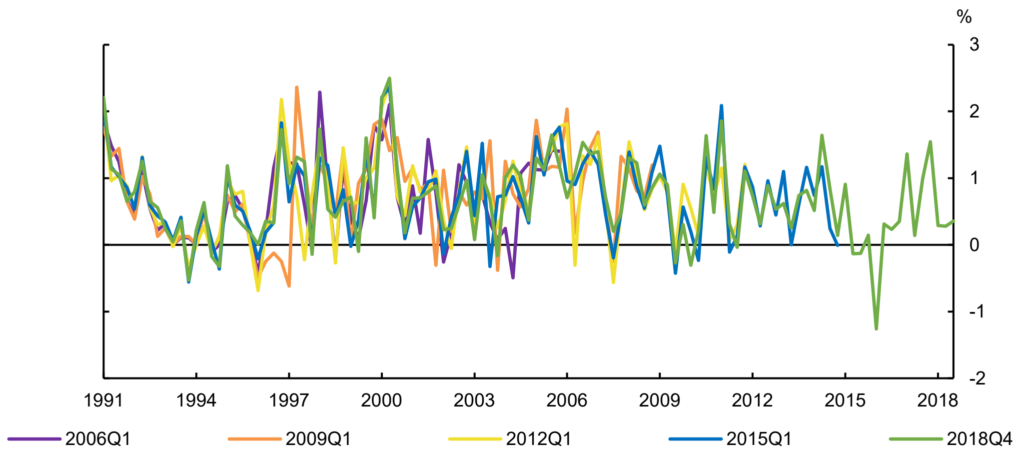


Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2018Q3

Chart B3: **Real-time wage growth**

Hourly compensation from the Productivity Accounts, quarter-over-quarter percentage change, quarterly data

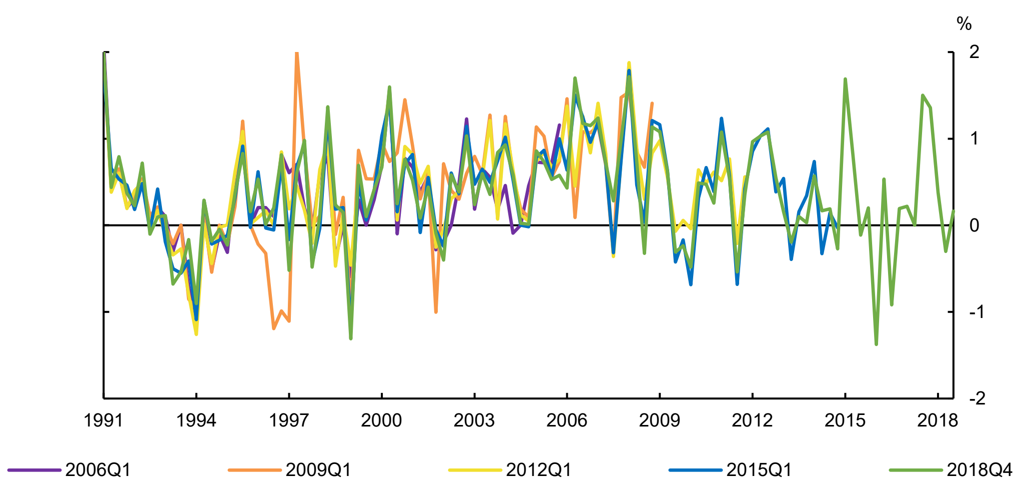


Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2018Q3

Chart B4: **Real-time unit labour cost growth**

Quarter-over-quarter percentage change, quarterly data

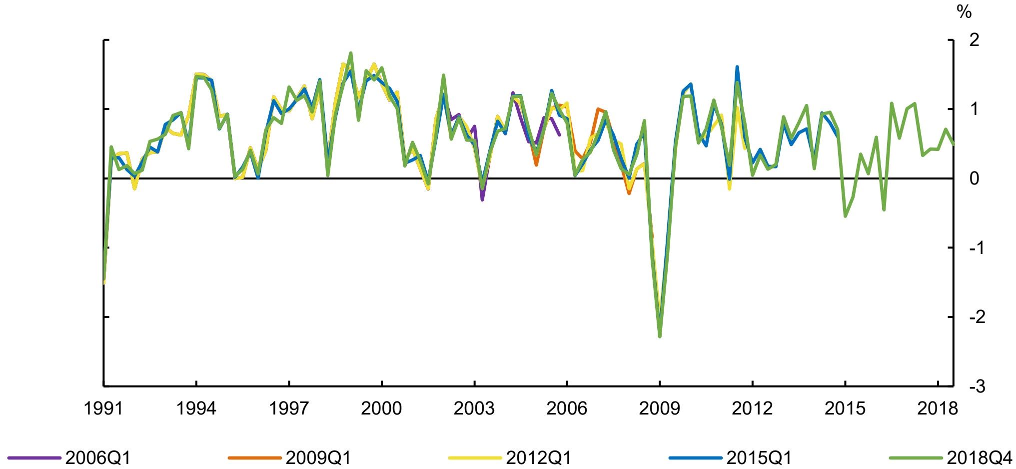


Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2018Q3

Chart B5: **Real-time real GDP growth**

Quarter-over-quarter percentage change, quarterly data

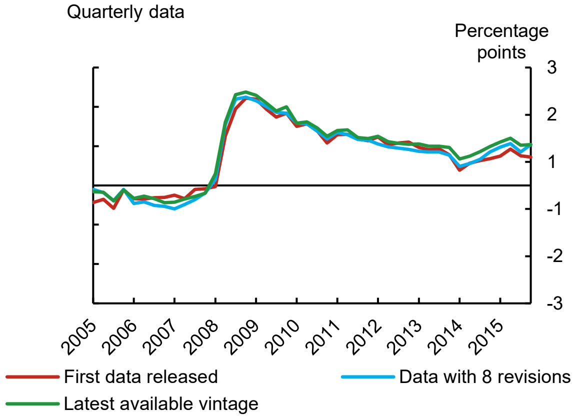


Sources: Statistics Canada and Bank of Canada calculations

Last observation: 2018Q3

Appendix C: Unemployment gap estimates

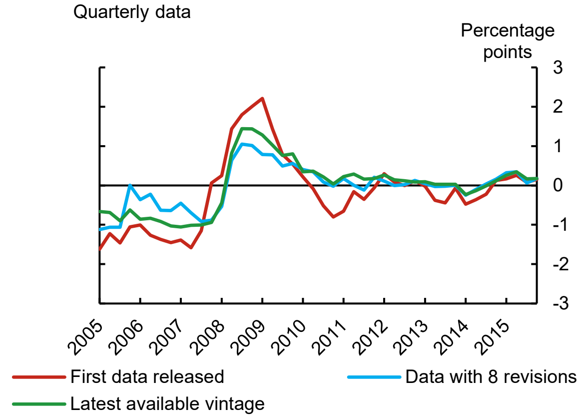
Chart C1: MSSF
Quarterly data



Source: Bank of Canada calculations

Last observation: 2016Q3

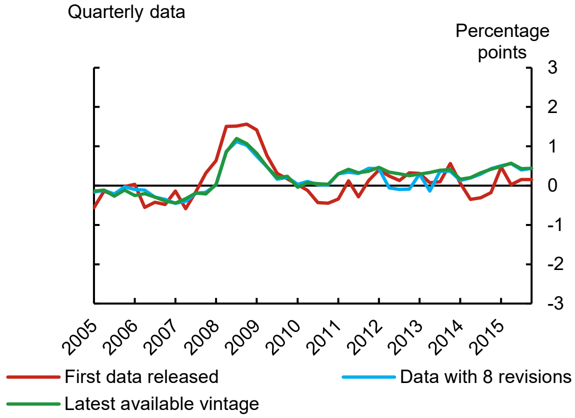
Chart C2: Triangle
Quarterly data



Source: Bank of Canada calculations

Last observation: 2016Q3

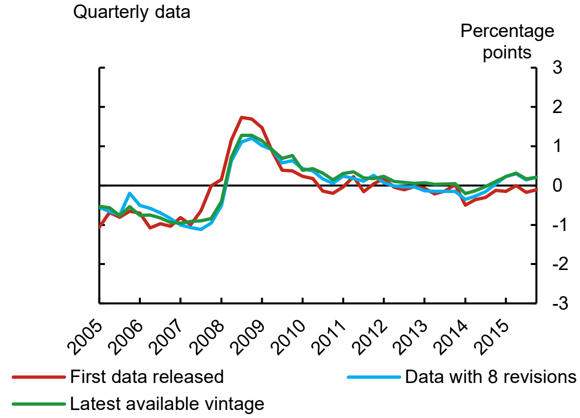
Chart C3: OECD anchored
Quarterly data



Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C4: OECD backward-looking
Quarterly data

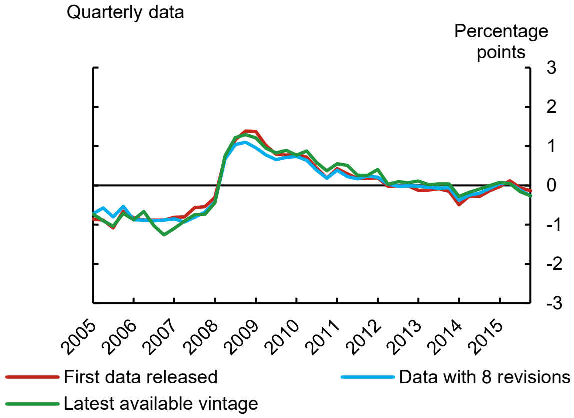


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C5: Cointegration 1

Quarterly data

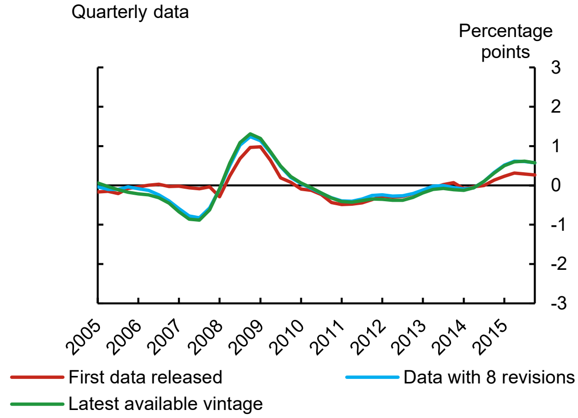


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C6: BP filter

Quarterly data

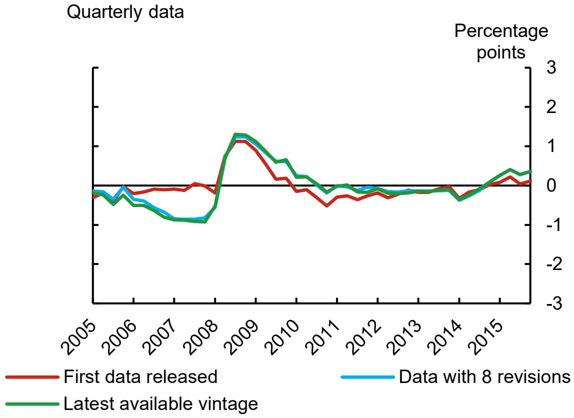


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C7: HP filter

Quarterly data

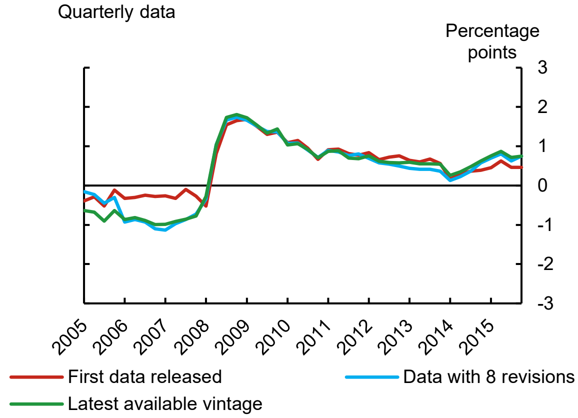


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C8: BMVF

Quarterly data

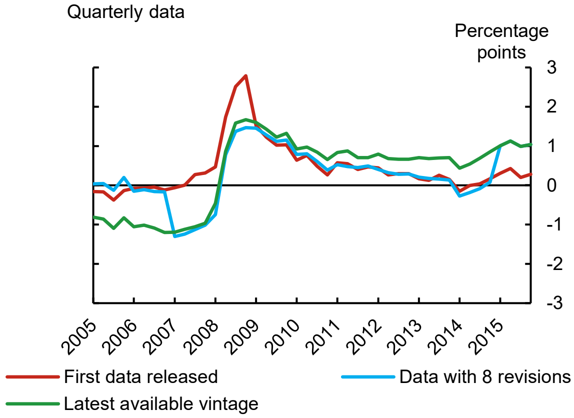


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C9: SIF

Quarterly data

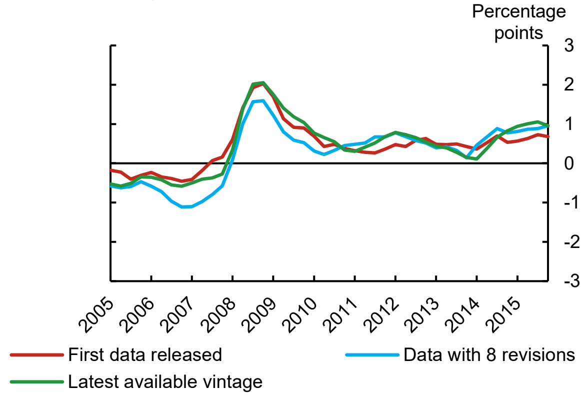


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C10: Hysteresis

Quarterly data

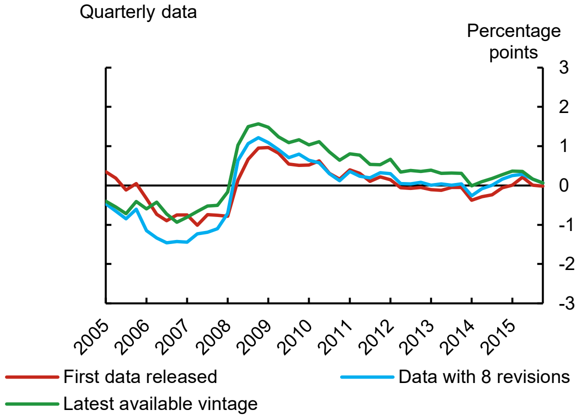


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C11: SFWPC

Quarterly data

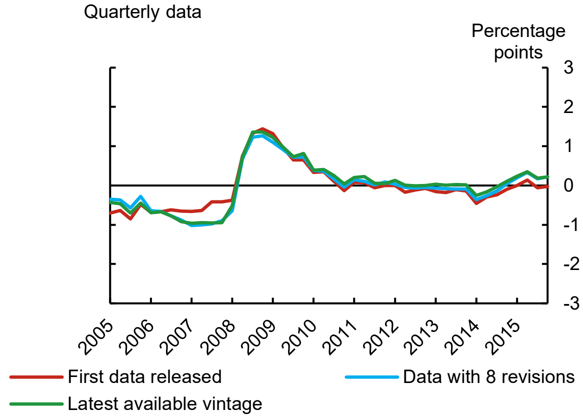


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C12: EMVF

Quarterly data

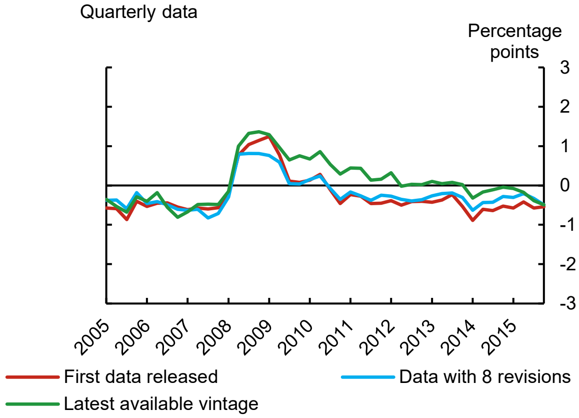


Source: Bank of Canada calculations

Last observation: 2016Q3

Chart C13: Cointegration 2

Quarterly data



Source: Bank of Canada calculations

Last observation: 2016Q3

Appendix D: Alternative results for the last two criteria

Table D1: Comparison of CPI-median and CPI-trim forecasts at various horizons
Ratio of RMSEs between models with and without UGAPs[a, b]

	CPI-median		CPI-trim	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	0.95	0.92	0.95	0.90
Triangle	0.93	0.88	0.95	0.93
OECD anchored	0.97	0.98	0.96***	0.94
OECD backward-looking	0.92	0.88	0.94	0.91
Cointegration 1	0.96	0.93	0.95	0.90
BP filter	1.00	0.99	1.01	0.95
HP filter	0.98	0.98	0.99	0.99
BMVF	0.94	0.90	0.95	0.91
SIF	0.96	0.91	0.96	0.92
Hysteresis	0.96	0.92	0.93	0.89
SFWPC	0.91	0.86	0.96	0.94
EMVF	0.96	0.95	0.95	0.93
Cointegration 2	0.99	1.00	0.99	1.00

[a] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.

[b] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

Table D2: Comparison of CPI inflation and CPI-common forecasts at various horizons with additional control variables[a]

Ratio of RMSEs between models with and without UGAPs[b, c]

	CPI inflation		CPI-common	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	1.00	1.18	0.89	0.79
Triangle	1.01	1.20	0.88	0.86
OECD anchored	0.98	1.21	0.85	0.88
OECD backward-looking	1.00	1.16	0.86	0.78
Cointegration 1	1.02	1.17	0.86*	0.74
BP filter	1.00	1.01	0.96	0.98
HP filter	1.00	1.06	0.90	0.90
BMVF	1.00	1.17	0.88	0.80
SIF	0.99**	1.09	0.89	0.82
Hysteresis	0.99	1.07	0.85	0.88
SFWPC	1.03	1.24	0.87***	0.81
EMVF	1.01	1.15	0.84	0.75
Cointegration 2	1.00***	1.07	0.92	0.88

[a] The equations for CPI inflation include lag(s) of changes in the Bank of Canada Commodity Price Index, in CPI inflation and changes in the exchange rate. The equations for CPI-common include lag(s) of CPI-common and of changes in the exchange rate.

[b] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.

[c] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

Table D3: Comparison of CPI-median and CPI-trim forecasts at various horizons with additional control variables[a]

Ratio of RMSEs between models with and without UGAPs[b, c]

	CPI-median		CPI-trim	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	0.97	0.93	0.95	0.90
Triangle	0.94	0.90	0.95	0.94
OECD anchored	0.99	0.99	0.97***	0.95
OECD backward-looking	0.94	0.89	0.94	0.91
Cointegration 1	0.97	0.92	0.94	0.88
BP filter	1.00	0.99	1.01	0.96
HP filter	0.98	0.97	0.98	0.98
BMVF	0.98	0.93	0.97	0.92
SIF	0.96	0.91	0.95	0.91
Hysteresis	1.01	0.96	0.96	0.92
SFWPC	0.96	0.90	0.98***	0.96
EMVF	0.96	0.93	0.94	0.91
Cointegration 2	0.99	1.00	0.99	0.99

[a] The equations for CPI-trim and CPI-median include lag(s) of CPI-trim or CPI-median and of changes in the exchange rate.

[b] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.

[c] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

Table D4: Comparison of wages and ULC forecasts at various horizons with additional control variables[a]

Ratio of RMSEs between models with and without UGAPs (first release)[b, c]

	Wages		ULC	
Models	**T+1**	**T+4**	**T+1**	**T+4**
MSSF	0.96***	0.96	1.00	1.01
Triangle	1.02	1.06	0.95***	0.77***
OECD anchored	1.03	0.98	1.06	1.01
OECD backward-looking	1.01	0.98	0.96***	0.80***
Cointegration 1	0.98	0.95	1.00	1.03
BP filter	1.02	1.12	1.00	1.01
HP filter	1.02	1.00	0.98***	0.96**
BMVF	0.96***	0.94	0.97***	0.92*
SIF	0.98	0.95	0.98*	0.96
Hysteresis	0.97	1.03	0.98	0.95
SFWPC	1.02	1.10	0.92***	0.79**
EMVF	1.00	0.95	0.98	0.96
Cointegration 2	1.00	1.03	0.98***	0.99

[a] The equations include lag(s) of growth in wages or ULC, one lag of labour productivity (in the case of wages) and lag(s) of inflation.

[b] *** p-value < 0.01; ** p-value < 0.05; * p-value <0.1.

[c] Sample size varies according to the forecasting horizon: T+1, 2006Q2–16Q3; T+4, 2007Q1–16Q3.

Table D5: Fit of the equations for CPI-median and CPI-trim and consistency with theory[a]

	CPI-median						CPI-trim					
Models	T+1			T+4			T+1			T+4		
	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]	Sign[b]	Pval[c]	R^{2} [d]
MSSF	-	***	0.29	-	***	0.29	-	***	0.22	-	***	0.23
Triangle	-	***	0.29	-	***	0.29	-	***	0.19	-	***	0.17
OECD anchored	-	***	0.33	-	***	0.30	-	***	0.28	-	***	0.28
OECD backward-looking	-	***	0.29	-	***	0.27	-	***	0.20	-	***	0.17
Cointegration 1	-	***	0.28	-	***	0.27	-	***	0.21	-	***	0.22
BP filter	-	**	0.25	-	***	0.23	-	**	0.18	-	***	0.19
HP filter	-	**	0.26	-	**	0.19	-	**	0.18	-	**	0.09
BMVF	-	***	0.31	-	***	0.33	-	***	0.22	-	***	0.23
SIF	-	***	0.30	-	***	0.31	-	***	0.21	-	***	0.22
Hysteresis	-	***	0.32	-	***	0.33	-	***	0.28	-	***	0.31
SFWPC	-	***	0.31	-	***	0.34	-	***	0.22	-	***	0.24
EMVF	-	***	0.28	-	***	0.22	-	***	0.21	-	***	0.15
Cointegration 2	-	*	0.25	-	***	0.20	-	**	0.17	-	***	0.13

[a] Sample over which the equations are estimated: 1992Q4–2018Q3. The equations are augmented with control variables.
[b] Sign of the sum of coefficients on the lags of UGAPs.
[c] P-value of the sum of coefficients on the lags of UGAPs. *** p-value < 0.01; ** p-value < 0.05; * p-value < 0.1.
[d] Adjusted R-squared of the models with UGAPs.