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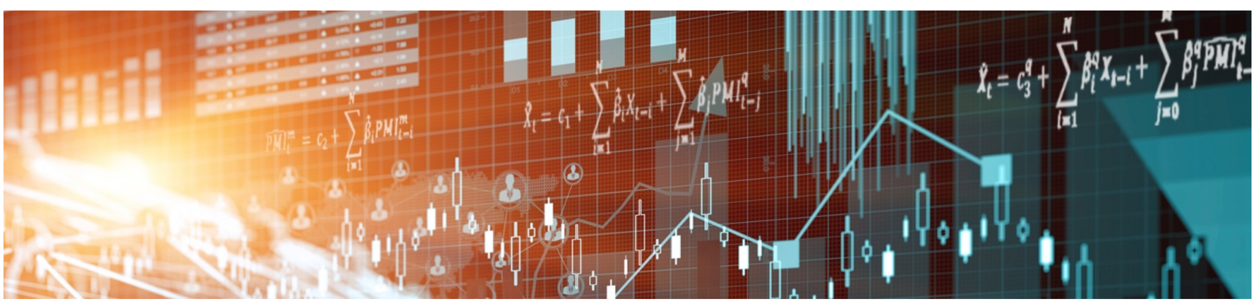

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Frictional Capital Reallocation I: Ex Ante Heterogeneity

by

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Abstract

This paper studies dynamic general equilibrium models where firms trade capital in frictional markets. Gains from trade arise due to ex ante heterogeneity: some firms are better at investment, so they build capital in the primary market; others acquire it in the secondary market. Cases are considered with random search and bargaining, or directed search and posting. For each, we provide results on existence, uniqueness, efficiency and comparative statics. Monetary and fiscal policy are discussed at length. We also discuss how productivity dispersion can be countercyclical while capital reallocation and its price are procyclical.

Bank topics: Monetary policy
JEL codes: E22, E44

Résumé

Dans cette étude, nous analysons des modèles dynamiques d’équilibre général où les entreprises font des opérations en capital sur des marchés soumis à des frictions. Les gains liés à ces opérations tiennent à l’hétérogénéité ex ante : certaines entreprises ont de meilleures stratégies d’investissement et accumulent des capitaux sur le marché primaire, tandis que d’autres le font sur le marché secondaire. Nous étudions différents scénarios en utilisant des modèles basés sur la recherche aléatoire et la négociation, ou sur la recherche dirigée et l’affichage des modalités. Dans chaque cas, nous fournissons des résultats en ce qui concerne les notions d’existence, d’unicité, d’efficience et de statique comparative. Les politiques monétaire et budgétaire font l’objet d’une discussion approfondie. Nous expliquons également comment la dispersion de la productivité peut être contracyclique, alors que la réaffectation du capital et son prix sont de nature procyclique.

Sujets : Politique monétaire
Codes JEL : E22, E44

Non-technical Summary

This paper studies a model where firms trade used capital partly with money. We show how monetary policy can affect the economy through relaxing or tightening the liquidity constraint of firms in capital reallocation. In particular, a higher inflation target tightens the liquidity constraint of firms, which makes resale of capital more difficult. This not only leads to inefficient reallocation of used capital, but also reduces the incentives of firms to invest. The interaction of these two channels amplifies the negative impact of inflation on output and welfare. Results obtained are robust to market structure/trading mechanisms for the used capital. We also discuss how fiscal policy affects the economy and output.

1 Introduction

Components necessary for efficient output and growth include (i) getting the right amount of aggregate investment; and (ii) getting the capital available at a point in time into the hands of those best able to use it. In traditional macroeconomics, the former received more emphasis, but reducing *capital mismatch* via reallocation has recently received a lot of attention (the literature is reviewed below). Components (i) and (ii) are obviously related, since the ease with which used capital can be reallocated should affect incentives for investment in new capital, just like attributes of secondary markets for houses, cars and other assets influence demand and supply in primary markets. This paper develops a dynamic general equilibrium theory of investment featuring frictional markets for existing capital with two alternative micro-market structures: first we consider random search and bargaining; then we consider directed search and price posting.[1]

Eisfeldt and Rampini (2006), and more recently Cao and Shi (2016), find the reallocation of existing capital is around 25% of total investment. However, these data exclude mergers, and ignore firms that are relatively small or not publicly traded, all of which can make 25% an underestimate. Cui (2013, 2017) and Dong et al. (2016) suggest the right number may be more like 30%. Also, these data are only for purchases, not rentals of capital, which may be at least as important. In any case, evidence suggests secondary capital markets may be qualitatively and quantitatively relevant. Moreover, again, the functioning of secondary markets

[1] We emphasize our interest here is reallocation across firms, but one can also consider reallocating capital within firms (e.g., Giroud and Mueller 2015), across sectors (e.g., Ramey and Shapiro 1998; Eberly and Wang 2009), across countries (e.g., Caselli and Feyrer 2007), etc. Also, we focus mainly on trading unbundled physical capital. As Jovanovic and Rousseau (2002) say, "There are two distinct used-capital markets. Used equipment and structures sometimes trade unbundled in that firm 1 buys a machine or building from firm 2, but firm 2 continues to exist. At other times, firm 1 buys firm 2 and thereby gets to own all of firm 2's capital. In both markets, the traded capital gets a new owner." We emphasize the first type, but make a few remarks on mergers and acquisitions (see, e.g., Harford 2005 for more).

influences investment in primary markets. Hence it seems interesting to develop models where the impact of fiscal and monetary policy on these variables can be tractably analyzed.[2]

We pursue the idea that secondary capital markets are neither perfectly competitive nor frictionless, although of course one could always model them that way (e.g., Holmes and Schmitz 1990; Jovanovic and Rousseau 2002). That capital reallocation is not frictionless is argued by, e.g., Kurmann and Petrosky-Nadeau (2007), Gavazza (2010, 2011*a,* 2011*b*), Cao and Shi (2016), Kurmann (2014), Ottonello (2015) and Kurmann and Rabinovich (2016). Imperfections include information issues related to adverse selection, financial constraints due to limited commitment, and holdup problems due to bargaining. We downplay adverse selection and related information issues (on these see, e.g., Li and Whited 2014, and references therein). This allows us to concentrate on other issues related to search, bargaining and liquidity. Thus, our secondary capital markets feature bilateral trade, as in equilibrium search theory, and the use of assets in exchange, as in modern monetary theory.

Any analysis of reallocation builds on gains from trade, with capital flowing from lower- to higher-productivity firms in the model, as in the data (e.g., see Maksimovic and Phillips 2001, Andrade, Mitchell and Stafford 2001 or Schoar 2002). The formulation here is based on ex ante heterogeneity: firms in the secondary market have different capital stocks due to differences in their invest-

[2] On fiscal policy, taxation has been shown to have big effects on capital investment, output, and welfare by, e.g., Chari et al. (1994), Cooley and Hansen (1992), McGrattan et al. (1997) and McGrattan (2012). On monetary policy, there is much precedent for studying the impact of inflation on investment, going back to Tobin (1965), Sidrauski (1967), Stockman (1981) and Cooley and Hansen (1989). These papers take reduced-form approaches, using money-in-utility-function or cash-in-advance models. In contrast, we are solidly in the New Monetarist camp that avoids such shortcuts. Work using microfoundations based on Lagos and Wright (2005), like our model, includes Aruoba and Wright (2003), Aruoba et al. (2011) and Andolfatto et al. (2016). Other related work includes Shi (1998, 1999*a*, 1999*b*), Shi and Wang (2006), Menner (2007) and Berentsen, Rojas Breu and Shi (2011), based on Shi (1997), and Molico and Zhang (2006), based on Molico (2006). Rocheteau and Nosal (2017) provide a textbook treatment.

ment ability in the primary market. Hence, there are gains to reallocating capital from those with more to those with less, since the latter have a higher marginal product, even if they have the same technology conditional on their capital stock (see also Xiao 2017). In a companion paper, Wright, Xiao and Zhu (2017), we alternatively study a formulation with ex post heterogeneity, where firms in the secondary market have similar capital stocks but different productivities due to idiosyncratic shocks. We think both cases are interesting. In any case, we assume decreasing returns to scale in production, because otherwise, given any two firms, the efficient outcome is for the more productive firm to get all the capital. With decreasing returns, the more productive firm may get some but not necessarily all of the other's capital.[3]

For random search and bargaining or directed search and posting, we prove existence and uniqueness of steady state equilibrium. Under certain conditions both specifications can be reduced to two equations, one for capital and one for money, that determine investment and reallocation, or, supply and demand in the secondary market. This allows us to easily show how investment, reallocation and other endogenous variables depend on monetary, fiscal and other exogenous variables. The analysis also provides insights into observations deemed interesting in the literature. One such observation is that reallocation is procyclical even though capital mismatch appears countercyclical (e.g., Eisfeldt and Rampini 2006, Cui 2013, Cao and Shi 2016 and Lanteri 2016). Our model is consistent with this stylized fact because in good times there may well be less incentive to reallocate capital, due to lower dispersion in productivity, but there is also more capital, so actual reallocation can be greater.

Capital investment and reallocation are not generally efficient in equilibrium.

[3]When one firm gets *all* the capital of the other, it looks like a merger or acquisition. This can happen even with decreasing returns, in general, but not with standard Inada conditions.

In fact, conditional on investment, reallocation is efficient if and only if monetary policy runs the Friedman rule, which is the limit $\iota \to 0$, where ι is the nominal interest rate. But that is conditional on investment, which is not generally efficient. In the bargaining version of the model, there is no bargaining power that delivers efficiency due to a double holdup problem. If random search and bargaining are replaced by directed search and posting, however, efficiency obtains in equilibrium at $\iota = 0$. Alternatively, we discuss how taxes or subsidies on capital income can deliver efficiency. Generally, higher nominal interest (or inflation or money supply growth) rates reduce capital reallocation, and they reduce investment under reasonable conditions. This is consistent with conventional Keynesian wisdom, but the logic is unconventional, as discussed below. Perhaps less surprisingly, but no less relevant, higher capital income taxation also reduces investment and reallocation.

Further in terms of the literature, Cao and Shi (2016) also use a search-based model, but it is quite different; in particular, they focus on market tightness as determined by entry, while we abstract from that to highlight other margins. Ottonello (2015) also models capital reallocation through search to explain recovery from a financial crisis, but there are again many differences – e.g., he has households selling capital to entrepreneurs, while we have firms trading with each other. Kurmann and Rabinovich (2016) and Dong et al. (2016) are also complementary with key differences – e.g., they have capital trade intermediated by dealers with access to a frictionless interdealer market, while again we have firms trading bilaterally. Moreover, none of the above papers consider monetary economies. Rocheteau, Wright and Zhang (2017) study a monetary economy with some common features, but it has 100% depreciation, linear utility and various other special restrictions that we relax, although it also has some things we omit, like banks. Perhaps most importantly, that model has firms buying capital from competitive suppliers, not trading with each other. To recap a key difference between our setup and previous papers, we have firms trading with each other, and moreover our firms make these trades using cash (internal finance).

Also related is a body of empirical work studying how differences in TFP (total factor productivity) and related variables depend on allocative efficiency.[4] Their findings are generally consistent with the model presented below. In particular, Buera, Kabaosky and Shin (2011) find that "financial frictions explain a substantial part of the [empirical] regularities. Essentially, financial frictions distort the allocation of capital across heterogeneous production units ... lowering aggregate and sector-level TFP. While self-financing can alleviate the resulting misallocation, it is inherently more difficult to do so in sectors with larger scale and larger financing needs. Thus, sectors with larger scale (e.g., manufacturing) are affected disproportionately more by financial frictions." Our model has explicit self-financing decisions that depend directly on monetary policy, plus investment decisions that depend directly on fiscal policy. While it is important to try to match the data quantitatively, that is beyond the scope of this paper, where we focus on deriving analytical results and developing economic intuition.

To be clear, our goal is to develop a microfounded model of capital investment and reallocation across heterogeneous firms in markets with frictions related to search, bargaining and liquidity. To this end the rest of the paper is organized as follows. Section 2 describes the basic environment. Section 3 specializes a few assumptions in the interests of tractability. These specifications use random search and bargaining. Section 4 studies a version with directed search and price

[4] Hsieh and Klenow (2009) study how misallocation lowers aggregate TFP using microdata from China and India. Midrigan and Xu (2014) evaluate the impact of financial frictions on TFP and the returns to capital using producer data. Cooper and Schott (2016) study the effects of capital reallocation on aggregate productivity. Other work on financial considerations includes Buera, Kaboski and Shin (2011), Ai, Li and Yang (2016) and David and Venkateswaran (2017).

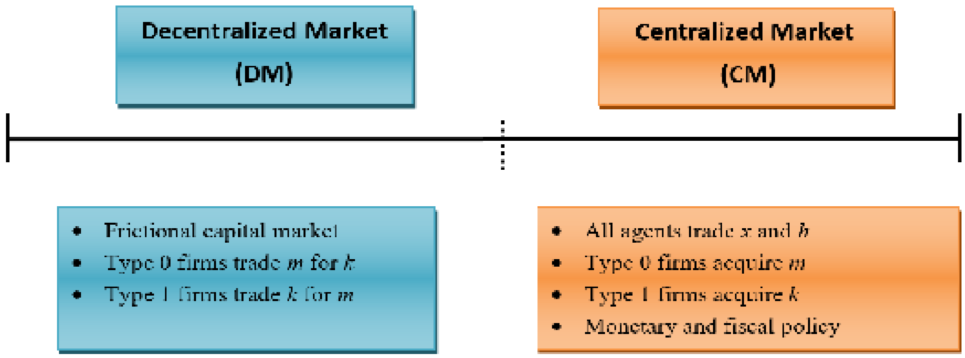


Figure 1: Timeline

posting. Section 5 concludes.[5]

2 The Model

Time is discrete and continues forever. Following one branch of the standard New Monetarist framework, as in Lagos and Wright (2005), in each period t two markets convene sequentially: a decentralized market, or DM, characterized by frictions detailed below; and a frictionless centralized market, or CM. This is illustrated in Figure 1. In the CM agents trade a numeraire consumption good x_t, labor hours h_t, money m_t, and capital k_t. A novelty compared to most previous papers using the framework is this: rather than having households trading consumption goods in the DM, we have firms trading k_t, although sometimes for convenience we refer to these agents as the households that own the firms, rather than firms per se. The price of k_t in terms of numeraire in the CM is 1 since, as usual, k_t and x_t are the same physical object; the price in the DM need not be 1, as discussed below.

[5] We mention that the frictions we highlight are potentially important not only in the market for capital, but also for other inputs, including the market for ideas (technologies) studied using similar methods by Silveira and Wright (2010), Chiu and Meh (2011) and Chiu, Meh and Wright (2017). In particular, Chiu, Meh and Wright (2017) study a model with endogenous growth, which would be interesting to pursue in future work, even if we stick to steady states here.

Heterogeneity in k_t is one way to generate DM gains from trade, and this results here from firms having different costs to carrying k (as mentioned, in Wright et al. 2017, we alternatively use productivity shocks in the DM to generate gains from trade). The simplest case has two types of firms, $j = \{0, 1\}$, with a measure of type j given by n_j, where type 1 has no cost of carrying k_t while type 0 has a prohibitive cost. Hence, if type 0 want any capital for the next CM they must acquire it in the DM. If there were perfect credit then, in exchange for some capital in the DM, a type 0 firm could promise a payment to type 1 in the next CM. This version is not without interest, but we want to include payment frictions to engender a role for liquidity. As is well known (e.g., see Kocherlakota 1998), this requires a lack of commitment to preclude simple credit, plus some version of anonymity to preclude credit supported by punishments for default (e.g., as in Kehoe and Levine 1993).

These frictions imply a role for assets as payment instruments. Here the only such asset is fiat money, although one can replace or augment this with other assets.[6] Hence, type 0 firms (or their owners) bring money to the DM to pay for capital, which we interpret as internal finance, since firms' cash comes from their retained earnings (or, depending on parameters, possibly also from their owners' labor income). This form of liquidity is generally costly, however, because of inflation.[7]

[6] As just two examples, Geromichalos, Licari and Suárez-Lledó (2007) show how to add dividend-bearing Lucas trees, while Lagos and Rocheteau (2008) show how to add neoclassical capital, both of which can compete with money as media of exchange. As long as the exogenous supply of trees is low, in the first case, or the efficient level of capital is endogenously low, in the second case, the inside liquidity provided by these assets will be scarce, and outside liquidity – fiat money – still has an essential role. See Venkateswaran and Wright (2013) for updated versions of those models; see Lagos, Rocheteu and Wright (2017) for a general survey of the related literature.

[7] Why can't firms pay with claims to profit income Π in the next CM? For the same reason they can't pay with claims to labor or any other income accruing in the next CM: anonymity and a lack of commitment. A lack of commitment underlies most models of liquidity, including those following Kiyotaki and Moore (1997) or Holmstrom and Tirole (2011), who often frame the issue in terms of pledgeability. However, as is well known in monetary theory, something

In the DM, $\alpha_j \in [0,1]$ is the probability that type j meets $j' \neq j$. As usual, this comes from a CRS (constant returns to scale) meeting technology that yields $\chi(n_0, n_1)$ meetings between the types, when n_j is the measure of type j in the market, and $\alpha_j = \chi(n_0, n_1)/n_j$. By CRS, α_j depends only on market tightness, n_0/n_1, and when the measures n_j are fixed we can take α_j as exogenous. In any case, when types 0 and 1 meet, they trade (p_t, q_t) where q_t is the capital transferred and p_t the payment, so that $P_t = p_t/q_t$ is the unit price of k_t in the DM. It is convenient to have the deal take the form of a rental agreement: type 0 gets to use q_t in the CM that period, then returns $(1-\delta)\, q_t$ to type 1, where $\delta \in (0,1]$ is depreciation. This arrangement, as opposed to an outright purchase, is natural given that type 0 cannot store capital across periods, although it is equivalent to have them buy q_t and then sell it off in the frictionless CM.

While it might be interesting in extensions, here we do not let types 0 and 1 enter into long-term relationships, meeting in subsequent DMs to trade again. This can be interpreted in terms of different kinds of capital – say, fixed at different locations – and assuming type 0 needs a different kind each period.[8] As a special case we can set $\alpha_0 = 1$, so type 0 can always find a supplier, although that may not be quite as good as a long-term relationship. Still, there is no reason to impose $\alpha_0 = 1$ at this point, because it does not really simplify things, and we find it interesting to consider changes in meeting probabilities. In any case, it may help to imagine type 0 firms as smaller enterprises that do not own their own property, plant or equipment, consistent with the idea that it is harder for them to obtain external finance, and making them more reliant on earnings held as liquid assets.

like anonymity is necessary to preclude credit based on punishments.

[8] See Corbae and Ritter (2004), e.g., for a discussion of repeated relationships in the context of consumer credit, where buyers sever relations with suppliers whenever they need a new kind of good. As they emphasize, one advantage of repeated relationships is that they may allow bilateral credit, something from which we abstract in this study.

All agents discount at rate $\beta \in (0,1)$. Period utility is $U(x_t, h_t) = u(x_t) - Ah_t$, where $u(\cdot)$ satisfies standard assumptions.[9] A type j firm (or its owner) can produce $F^j(k_t, h_t)$ units of x_t, using capital brought in from the previous DM and labor hired competitively in the CM. We assume $F^j(\cdot)$ exhibits decreasing returns. Were one to assume constant returns, given that labor is hired in the frictionless CM, there would be no reason to transfer capital from type 1 to type 0 if they are equally productive, and if one is more productive it would be efficient to allocate all the capital to them, which may be interesting because it looks like a merger or acquisition. However, the baseline model has decreasing returns, so that type 0 agents want some but not necessarily all of type 1's capital in DM meetings. In case it is not obvious, we highlight that, ceteris paribus, the efficient q equates capital's marginal product for the counterparties.

The CM price of money in terms of x_t is ϕ_t, making real balances $z_t = \phi_t M_t$, where M_t is the money supply. Assume $M_{t+1} = (1 + \mu_t) M_t$. Changes in M_t satisfy the government CM budget constraint, $G_t = T_t + \phi_t(M_{t+1} - M_t)$, where G_t is spending on x_t and T_t is lump-sum taxes minus transfers; it does not matter for present purposes if cash injections involve higher G_t or lower T_t. In general, the inflation rate is $\pi_t = \phi_t/\phi_{t+1} - 1$. In steady state, where all real variables are constant, including $z = \phi_t M_t$, we have $\pi = \mu$. Also, in steady state, the Fisher equation $1 + \iota = (1 + \pi)(1 + r)$ gives ι as the net return on a nominal bond that is illiquid in the sense that it cannot be traded in the DM, and $1 + r = 1/\beta$ gives r as the return on an illiquid real bond.[10] Therefore, in steady state, it is

[9] As a special case, it is fine to assume $u(x) = x$ and $A = 0$, which means agents simply maximize profit. Or, we can generalize utility to any $U(x_t, h_t)$ that is homogeneous of degree 1 and get very similar results (see Wong 2016). Another way to generalize utility is discussed in fn. 14. We also mentiont that, given any such preferences, it is unimportant if a household owns one firm or many, and if a firm is owned by one household or many.

[10] As usual, we can price these bonds whether or not they trade in equilibrium. To facilitate understanding, it may help to think of $1 + \iota$ as simply the amount of cash in the next CM that makes one willing to give up a dollar today, and $1 + r$ as the amount of numeraire in the next CM that makes one willing to give up a unit today. One can also introduce bonds that

equivalent to describe monetary policy as the choice of μ, ι, or π. We impose $\mu > \beta - 1$, which is equivalent in steady state to $\iota > 0$, but also consider the limit $\iota \to 0$, which is the Friedman rule.

Let the CM and DM value functions for type j at t be $W_t^j(m_t, k_t, \tilde{k}_t)$ and $V_t^j(m_t, k_t)$. Here k_t is capital held in the beginning of the DM while $\tilde{k}_t$ is capital available after DM trade; note that type 0 (type 1) may have $\tilde{k}_t$ greater (less) than k_t if they traded in the DM. The CM problem for type j is

$$\begin{aligned} W_t^j(m_t, k_t, \tilde{k}_t) &= \max_{x_t, h_t, \hat{m}_t, \hat{k}_t} \{u(x_t) - Ah_t^j + \beta V_{t+1}^j(\hat{m}_t, \hat{k}_t)\} \\ \text{st } x_t + \phi_t \hat{m}_t + \hat{k}_t &= w_t h_t^j + \phi_t m_t + (1-\delta)k_t + \Pi^j(\tilde{k}_t) - T_t \\ \Pi^j(\tilde{k}_t) &= \max_{\tilde{h}_t} \{F^j(\tilde{k}_t, \tilde{h}_t) - w_t \tilde{h}_t\}, \end{aligned} \tag{1}$$

where h_t^j is labor supply, $\hat{m}_t$ and $\hat{k}_t$ are money and capital demand, and $\Pi^j(\tilde{k}_t)$ is profit, which also depends on w_t, but we suppress that in the notation. Labor demand for a type j firm with $\tilde{k}_t$ is

$$\tilde{h}^j(\tilde{k}_t) = \arg\max_{\tilde{h}_t} \{F^j(\tilde{k}_t, \tilde{h}_t) - w_t \tilde{h}_t\}, \tag{2}$$

where $\tilde{h}^j(\tilde{k}_t)$ also depends on w_t, but that is also suppressed in the notation.

Several features can be emphasized in (1). First, notice that $(1-\delta)k_t$ appears on the RHS of the budget equation, rather than $(1-\delta)\tilde{k}_t$, reflecting our capital rental arrangement. Also note that the functions $\tilde{h}^j(\cdot)$ and $\Pi^j(\cdot)$ do not depend on t, although their values can vary over time with $\tilde{k}_t$ and w_t. Moreover, although these functions are defined for any $\tilde{k}_t$, in equilibrium there are four relevant cases: $\tilde{k}_t = 0$ for type 0 that did not trade in the DM; $\tilde{k}_t = q_t$ for type 0 that did; $\tilde{k}_t = k_t$ for type 1 that did not trade in the DM; and $\tilde{k}_t = k_t - q_t$ for type 1 that did. If type j firms have $\tilde{k}_t^j = 0$, they may produce in the CM using $\tilde{h}_t^j > 0$, or may go

might trade in the DM, although perhaps not as readily as cash due to recognizability problems (information frictions); see Rocheteau, Wright and Xiao (2017) and references therein.

to the corner $\tilde{h}_t^j = 0$; we do not restrict this. Also note that $\tilde{h}_t^j$ is not the same as h_t^j, since labor demand by a firm is generally different from the labor supply of its owner. Indeed, since hours are traded in the CM, theory does not pin down who works for whom, although it would be interesting to also incorporate frictional labor markets in future work (e.g., Berentsen, Menzio and Wright 2011; Dong and Xiao 2017).

Using the budget constraint and the obvious result that $k_t = \hat{k}_t = 0$ for type 0, we can reduce their problem to

$$\begin{aligned} W_t^0(m_t, 0, \tilde{k}_t) &= \frac{A}{w_t}[\phi_t m_t + \Pi^0(\tilde{k}_t) - T_t] + \max_{x_t}\{u(x_t) - \frac{A}{w_t}x_t\} \\ &\quad + \max_{\hat{m}_t}\{-\frac{A}{w_t}\phi_t \hat{m}_t + \beta V_{t+1}^0(\hat{m}_t, 0)\}. \end{aligned}$$

From this it is immediate that the relevant envelope conditions are $\partial W_t^0/\partial m_t = A\phi_t/w_t$ and $\partial W_t^0/\partial \tilde{k}_t = AF_k^0[\tilde{k}_t, h^0(\tilde{k}_t)]/w_t$, while the first-order conditions for interior solutions are

$$x_t \; : \; u'(x_t) = \frac{A}{w_t} \tag{3}$$

$$\hat{m}_t \; : \; \frac{A\phi_t}{w_t} = \beta \frac{\partial V_{t+1}^0(\hat{m}_t, 0)}{\partial \hat{m}_t}. \tag{4}$$

As is standard in models following Lagos and Wright (2005), (3)-(4) imply x_t and $\hat{m}_t$ do not depend on wealth at the start of the CM, and hence the distribution of $\hat{m}_t$ across type 0 agents in the DM is degenerate.

For type 1, $\hat{m}_t = 0$ because they do not need liquidity in the DM, but they might have $m_t > 0$ in the CM from trading in the previous DM. Their envelope conditions for m_t and $\tilde{k}_t$ are the same as those for type 0, but since type 1 can own (and not just rent) capital, we also have $\partial W_t^1/\partial k_t = A(1-\delta)/w_t$. Their first-order conditions are (3) and

$$\hat{k}_t : \frac{A}{w_t} = \beta \frac{\partial V_{t+1}^1(0, \hat{k}_t)}{\partial \hat{k}_t}. \tag{5}$$

Similar to the result that $\hat{m}_t$ is degenerate across type 0 agents, (5) implies $\hat{k}_t$ is degenerate across type 1 agents.

In a DM meeting at $t+1$ between type 0 with $\hat{m}_t$ and type 1 with $\hat{k}_t$, brought in from the previous CM, the former rents q_{t+1} units of capital from the latter and pays p_{t+1} in cash, subject to $q_{t+1} \leq \hat{k}_t$ and $p_{t+1} \leq \hat{m}_t$. It is standard that $p_{t+1} \leq \hat{m}_t$ is binding, since it is costly to carry cash, but $q_{t+1} \leq \hat{k}_t$ is slack if we assume Inada conditions. Given this, it is notationally convenient to let $\hat{q}_t = q_{t+1}$ and $\hat{z}_t = \phi_{t+1}\hat{m}_t$, so that for type 0

$$\begin{aligned} V^0_{t+1}(\hat{m}_t, 0) &= W^0_{t+1}(\hat{m}_t, 0, 0) + \alpha_0[W^0_{t+1}(0, 0, \hat{q}_t) - W^0_{t+1}(\hat{m}_t, 0, 0)] \\ &= W^0_{t+1}(\hat{m}_t, 0, 0) + \frac{\alpha_0 A}{w_{t+1}}[\Pi^0(\hat{q}_t) - \Pi^0(0) - \hat{z}_t]. \end{aligned} \tag{6}$$

The interpretation is straightforward: a type 0 agent might not trade, in which case he goes to the next CM with his cash but no capital, as reflected in the first term; but, if he does trade, he gets a surplus from renting some capital net of the payment, as reflected in the second term. Similarly, for type 1,

$$\begin{aligned} V^1_{t+1}(0, \hat{k}_t) &= W^1_{t+1}(0, \hat{k}_t, \hat{k}_t) + \alpha_1[W^1_{t+1}(\hat{m}_t, \hat{k}_t, \hat{k}_t - \hat{q}_t) - W^1_{t+1}(0, \hat{k}_t, \hat{k}_t)] \\ &= W^1_{t+1}(0, \hat{k}_t, \hat{k}_t) + \frac{\alpha_1 A}{w_{t+1}}[\hat{z}_t + \Pi^1(\hat{k}_t - \hat{q}_t) - \Pi^1(\hat{k}_t)]. \end{aligned} \tag{7}$$

For now the terms of trade in the DM are determined by bargaining (later we alternatively consider posting with directed search). For tractability we use Kalai's (1977) proportional bargaining solution.[11] If θ is the bargaining power of type 0, the Kalai solution in our context reduces to

$$\hat{z}_t = (1-\theta)[\Pi^0(\hat{q}_t) - \Pi^0(0)] + \theta[\Pi^1(\hat{k}_t) - \Pi^1(\hat{k}_t - \hat{q}_t)], \tag{8}$$

making the payment a weighted average of type 0's gain and type 1's cost. Using (6)-(8) in (4)-(5), it is now routine to derive the Euler equations for money and

[11] Kalai bargaining has become quite popular in models of liquidity since Aruoba et al. (2007), for several reasons, including the fact that it is algebraically much easier than Nash bargaining. Still, we mention below how some results change if we use Nash.

capital,

$$\begin{aligned}
\frac{\phi_t w_{t+1}}{\beta \phi_{t+1} w_t} &= 1 + \alpha_0 \theta \Phi(\hat{k}_t, \hat{q}_t) && (9) \\
\frac{w_{t+1}}{\beta w_t} &= 1 - \delta + F_k^1[\hat{k}_t, \tilde{h}^1(\hat{k}_t)] + \alpha_1 (1-\theta) F_k^0[\hat{q}_t, \tilde{h}^0(\hat{q}_t)] \Psi(\hat{k}_t, \hat{q}_t), && (10)
\end{aligned}$$

where to keep the expressions manageable we introduce

$$\begin{aligned}
\Phi(k,q) &= \frac{F_k^0[q, \tilde{h}^0(q)] - F_k^1[k-q, \tilde{h}^1(k-q)]}{\Delta} && (11) \\
\Psi(k,q) &= \frac{F_k^1[k-q, \tilde{h}^1(k-q)] - F_k^1[k, \tilde{h}^1(q)]}{\Delta} && (12) \\
\Delta(k,q) &= (1-\theta) F_k^0[q, \tilde{h}^0(q)] + \theta F_k^1[k-q, \tilde{h}^1(k-q)]. && (13)
\end{aligned}$$

By virtue of (3), the LHS of (10) is $w_{t+1}/\beta w_t = u'(x_t)/\beta u'(x_{t+1}) = 1 + r_t$, where r_t is the real interest rate on illiquid savings, and by the Fisher equation, the LHS of (9) is $1 + \iota_t$, where ι_t is the nominal interest rate on illiquid savings.[12] Using this notation, we can rewrite (9)-(10) as

$$\begin{aligned}
\iota &= \alpha_0 \theta \Phi(k,q) && (14) \\
r + \delta &= F_k^1[k, \tilde{h}^1(k)] + \alpha_1 (1-\theta) F_k^0[q, \tilde{h}^0(q)] \Psi(k,q). && (15)
\end{aligned}$$

Notice we take off our hats and time subscripts in (14)-(15) – e.g., writing q instead of $\hat{q}_t$ – because all variables are evaluated at the same date. However, this does not make the model static: r and ι generally change over time with k during transitions; and, since this is a monetary economy, they can potentially also change over time due to beliefs.

Heuristically, (15) equates $r + \delta$, the time-plus-depreciation cost of carrying capital, to the marginal product F_k^1, as in standard growth theory, plus a non-standard term given by type 1's probability of DM trade α_1 times his share $1 - \theta$

[12] As mentioned above, $1 + r_t$ is the amount of numeraire and $1 + \iota_t$ is the amount of money due in the CM at $t + 1$ that makes one willing to give up a unit in the CM at t, although here we are not restricting attention to steady state.

of the additional surplus generated by bigger k. Similarly, (14) equates ι, the marginal cost of carrying cash, to type 0's probability of trade α_0 times his share θ of the additional surplus generated by bigger m. We say more about this later, but for now, consider at one extreme $\theta = 1$, so type 0 get the entire DM surplus. Then (15) becomes $r + \delta = F_k^1[k, \tilde{h}(k)]$, as in standard growth theory, while (14) becomes

$$\iota = \alpha_0 \frac{F_k^0[q, \tilde{h}(q)] - F_k^1[k - q, \tilde{h}(k - q)]}{F_k^1[k - q, \tilde{h}(k - q)]},$$

equating the cost of liquidity to the expected gain from reallocation. At the other extreme, $\theta = 0$ implies type 1 get the entire surplus, but then (14) becomes $\iota = 0$, which means that no one holds $m > 0$ for any $\iota > 0$.[13]

While some insights emerge directly from the Euler equations, to close the model we need market clearing conditions. For money, this means $n_0 \hat{m} = (1 + \mu) M$. For labor, it means

$$h = n_0 \alpha_0 \tilde{h}^0(q) + n_0 (1 - \alpha_0) \tilde{h}^0(0) + n_1 \alpha_1 \tilde{h}^1(k - q) + n_1 (1 - \alpha_1) \tilde{h}^1(k), \quad (16)$$

where the LHS aggregates supply across workers, while the RHS aggregates demand across firms. Notice (16) depends on w_t, but that is subsumed in the notation. In fact, the labor market clears automatically if the goods markets clears (Walras' Law), so we focus on the latter. The aggregate supply of output is given by

$$\begin{aligned} y \;=\; & \alpha_0 n_0 F^0[q, \tilde{h}^0(q)] + (1 - \alpha_0) n_0 F^0[0, \tilde{h}^0(0)] \quad (17) \\ & + \alpha_1 n_1 F^1[k - q, \tilde{h}^1(k - q)] + (1 - \alpha_1) n_1 F^1[k, \tilde{h}^1(k)]. \end{aligned}$$

This also depends on w_t, again subsumed in the notation. Goods market clearing

[13] We show below a stronger result: $\theta > \hat{\theta}$ is necessary for money to be valued, where $\hat{\theta} > 0$. This is consistent with other monetary models using Kalai bargaining; with Nash, in those models, money can be valued for any $\theta > 0$ but $q \to 0$ and $z \to 0$ as $\theta \to 0$.

obtains when w_t adjusts to satisfy

$$y = (n_0 + n_1)\, x + n_1 \left[\hat{k} - (1 - \delta)k\right] + G. \tag{18}$$

We are now in a position to define equilibrium. First, as in any model of fiat currency, there is a nonmonetary equilibrium, which reduces here to a standard growth model.[14] From now on we focus on monetary equilibria, with $z_t > 0$. For initial conditions, we start at $t = 0$ in the DM with all type 0 holding m_0 and all type 1 holding k_0. To conserve notation, we do not carry around $h^j(\cdot)$ in our definition, since it is simply given by (2). Similarly, we include z but not ϕ, as that can be recovered from $\phi = M/z$; so ϕ_t/ϕ_{t+1} in (9) should be read as $(1 + \mu_t)z_t/z_{t+1}$. Moreover, we specify monetary policy in terms of μ, the growth rate of M, but one can alternatively target π or ι, and again it does not matter in steady state. With this in hand, we have the following:[15]

Definition 1 *Given time paths for policy, $\{\mu, G, T\}$, an equilibrium is a list of nonnegative and bounded paths for $\{k, q, x, w, z\}$ that at every date satisfy: the Euler equations (9)-(10); the bargaining condition (8); the first-order condition (3); market clearing (18); and the initial conditions.*

From what is known about monetary models, in general, there can be many dynamic equilibria based solely on beliefs, including, for some parameters, cyclic, chaotic and stochastic equilibria (e.g., Rocheteau and Wright 2013). This is an

[14] If we add shocks, the nonmonetary equilibrium replicates exactly Hansen's (1985) real business cycle model. The only detail is that he does not start with $u(x) - Ah$; he derives it from general utility $U(x, h)$ by imposing indivisible labor, $h \in \{0, 1\}$, and incorporating employment lotteries à la Rogerson (1988). But the same trick works here: with indivisible labor and lotteries, agents with any $U(c, h)$ act as if their utility functions were $u(x) - Ah$, which leads to all same simplifications, including the result that $(\hat{m}, \hat{k})$ is independent of (m, k) (the proof follows closely the argument in Rocheteau et al. 2008).

[15] By boundedness of the equilibrium paths in this definition, we mean $\lim_{t\to\infty}\beta^t z_t < \infty$, as implied by transversality conditions for this kind of monetary model (e.g., see Rocheteau and Wright 2013), plus the analogous conditions for k_t from standard growth theory.

inescapable implication of taking liquidity seriously in dynamic general equilibrium theory, but although such outcomes may be worth studying in future work, in what follows we mainly focus on a simpler notion:

Definition 2 *Given constant policy,* $\{\mu, G, T\}$*, a steady state is a time-invariant list* $\{k, q, x, w, z\}$ *that is an equilibrium ignoring the initial conditions.*

3 A Convenient Parameterization

While generality may be desirable in quantitative work, tractable models are also useful for developing and communicating salient economic ideas. With this in mind, consider a special case of Section 2 with the quasi-linear technology $F^j(k,h) = f^j(k) + h$, where we assume $f^j(0) = 0$, $f^j_k(k) > 0$, $f^j_{kk}(k) < 0$ and standard Inada conditions. This implies $w = 1$, making it easier to solve for the rest of the equilibrium and to distill key results.[16] Also, we now include a capital income tax τ to discuss fiscal policy, which requires modifying the government budget constraint in the obvious way.

The CM problem for type j is now

$$W^j_t(m,k,\tilde{k}) = \max_{x,h,\hat{m},\hat{k}} \{u(x) - Ah + \beta V^j_{t+1}(\hat{m},\hat{k})\} \tag{19}$$
$$\text{st } x + \phi\hat{m} + \hat{k} = h + \phi m + (1-\delta)k + (1-\tau)f^j(\tilde{k}) - T,$$

[16] Having technology and preferences both linear in h has some disadvantages, e.g., it may make transitional dynamics less interesting. However, transitional dynamics are still interesting due to the constraint $h_t \in [0,1]$, where 1 is the normalized time endowment. Consider the case with $n_0 = 0$ so there is no role for the DM – i.e., consider the standard growth model with utility and production functions linear in h. There is a unique steady state $\bar{k}$ ($k = 0$ is not a steady state since output can be produced with h even if $k = 0$). At $\bar{k}$, x solves $u'(\bar{x}) = A$ and h solves $\bar{h} = \bar{x} + \delta\bar{k} - f(\bar{k})$ as long as $\bar{h} \leq 1$, which we suppose is true. Also, suppose first that the initial k_0 is below $\bar{k}$, but not too far below. Then the economy jumps to steady state immediately by setting $h_0 = \bar{x} + \bar{k} - f(k_0) - (1-\delta)k_0$. Now suppose k_0 is sufficiently low that $\bar{x} + \bar{k} - f(k_0) - (1-\delta)k_0 > 1$, so the hours constraint initially binds. Then the transition has $h_t = 1$, with x_t and k_t determined as in the simplest growth model, until we reach a k such that $h = \bar{x} + \bar{k} - f(k) - (1-\delta)k \leq 1$, whence we jump to steady state.

which is similar to (1) except we use $w = 1$, insert $\Pi^j(\cdot) = f^j(\cdot)$ and include τ. It is easy to derive type 0's first-order condition for $\hat{m}$ and type 1's first-order condition for $\hat{k}$, as well as the DM value functions

$$\begin{aligned} V^0_{t+1}(\hat{m}, 0) &= W^0_{t+1}(\hat{m}, 0, 0) + \alpha_0 A[(1-\tau)f^0(\hat{q}) - \hat{z}] \\ V^1_{t+1}(0, \hat{k}) &= W^1_{t+1}(0, \hat{k}, \hat{k}) + \alpha_1 A[\hat{z} + (1-\tau)f^1(\hat{k} - \hat{q}) - (1-\tau)f^1(\hat{k})] \end{aligned}$$

which are simpler here due to $w = 1$.

The bargaining solution is similar to before, except the tax shows up,

$$\frac{\hat{z}}{1-\tau} = (1-\theta)f^0(\hat{q}) + \theta[f^1(\hat{k}) - f^1(\hat{k} - \hat{q})]. \tag{20}$$

These observations lead to simplified versions of (14)-(15),

$$\iota = \alpha_0 \theta \frac{f^0_k(q) - f^1_k(k-q)}{(1-\theta)f^0_k(q) + \theta f^1_k(k-q)} \tag{21}$$

$$\frac{r+\delta}{1-\tau} = (1-\alpha_1) f^1_k(k) + \alpha_1 f^1_k(k-q)\Gamma(k,q), \tag{22}$$

where

$$\Gamma(k,q) = \frac{(1-\theta)f^0_k(q) + \theta f^1_k(k)}{(1-\theta)f^0_k(q) + \theta f^1_k(k-q)}. \tag{23}$$

Notice $\theta > 0$ implies $\Gamma(k,q) < 1$, lowering the marginal value of capital to type 1 firms, and capturing the strategic consideration that bringing additional k to the DM worsens their terms of trade.

Equilibrium can now be conveniently described recursively: first, (21)-(22) pin down (k,q); then given (k,q), (20) determines z; and finally, given x from $u'(x) = 1$, total employment h is

$$h = x + \delta k + G - n_0\alpha_0 f^0(q) - n_1\alpha_1 f^1(k-q) - n_1(1-\alpha_1) f(k). \tag{24}$$

In particular, independent of the other endogenous variables we can discuss capital investment and capital reallocation, k and q. This especially nice for the analysis of monetary and fiscal policy, because ι appears in (21) but not (22), while τ appears in (22) but not (21).

Conditional on k, in the limit as $\iota \to 0$, it is immediate from (21) that q efficiently equates marginal products: $f_k^0(q) = f_k^1(k-q)$. Intuitively, at the Friedman rule $\iota = 0$ liquidity is not scarce, so type 0 bring enough to get the efficient q given k. Note that this is independent of θ, as long as it is big enough to support a monetary equilibrium (see below). Away from the Friedman rule there is a wedge $f_k^0(q) > f_k^1(k-q)$, depending on search and bargaining frictions, α_0 and θ, as well as the liquidity friction represented by $\iota > 0$.[17]

As with any framework that is not completely standard, the first order of business is to discuss existence and uniqueness:

Proposition 1 *Given $F^j(k,h) = f^j(k) + h$, in the model with bargaining, a monetary steady state exists iff $\iota < \hat{\imath} = \alpha_0\theta/(1-\theta)$. If it exists, it is unique.*

Proof: First, define the RB (for real balances) curve as the implicit solution of (21) for $q \in (0,k)$ as a function of k. One can check that the RHS of (21) is $\hat{\imath} = \alpha_0\theta/(1-\theta) > 0$ at $q = 0$, negative at $q = k$, and strictly decreasing in q. Hence, $\iota < \hat{\imath}$ implies there is a unique $q \in (0,k)$ solving (21) for each k. As ι increases, RB rotates clockwise, until $\iota = \hat{\imath}$, at which point RB coincides with the horizontal axis. Hence, $\iota \geq \hat{\imath}$ implies there is no steady state with $q > 0$. Given $\iota < \hat{\imath}$, since the RB curve is strictly increasing, let us invert it to write $k = K_1(q)$. It can be checked that $q \to 0$ as $k \to 0$ and $q \to \infty$ as $k \to \infty$ along RB. Hence, it must look like the one shown in Figure 2 below.

[17] The efficiency of q for a given k at $\iota = 0$ is also true in the general case, not only for a quasi-linear technology, and in dynamic equilibrium, not only in steady state. To see this, rewrite (9) as $\phi_t w_{t+1}/\beta\phi_{t+1}w_t - 1 = \alpha_0\theta\Phi(\hat{k}_t, \hat{q}_t)$. The LHS is the nominal rate ι_t in or out of steady state. If $\iota_t = 0$ then $\Phi(\hat{k}_t, \hat{q}_t) = 0$, which from (11) tells us that marginal products are equated. Having said that, the result is not completely robust: if we were to replace Kalai by Nash bargaining then q is too low, so that $f_k^0(q) > f_k^1(k-q)$, even when $\iota \to 0$, unless we have $\theta = 1$ (the proof follows closely the methods in Lagos and Wright 2005).

Next, define the CI (for capital investment) curve by solving (22) for k as a function of q, say $k = K_2(q)$. One can check $K_2(q) \geq q$, and $K_2(0) = k_0 > 0$ where k_0 is defined by $f_k^1(k_0) = (\delta + r) / (1 - \tau)$. Also, we claim that $K_2(q)$ is not defined for $q > \bar{q}$, where $\bar{q}$ satisfies

$$\frac{\delta + r}{1 - \tau} = f_k^1(\bar{q}) + \alpha_1 \frac{(1-\theta) f_k^0(\bar{q})}{\theta}.$$

To see this, notice $q > \bar{q}$ implies the RHS of (22) is below $(\delta + r) / (1 - \tau)$ at $k = q$ and is decreasing in k, so there is no $k > q$ satisfying (22). In addition, $k \to \bar{q}$ as $q \to \bar{q}$. We know CI is increasing near $k = k_0$, but not if it is globally increasing. So it can look like the left or right panel of Figure 2.

After some algebra, one can derive

$$\mathbf{J} \begin{bmatrix} dq \\ dk \end{bmatrix} = \begin{bmatrix} d\iota \\ d\left(\frac{r+\delta}{1-\tau}\right) \end{bmatrix}, \tag{25}$$

which is useful for establishing uniqueness, where

$$\mathbf{J} = \frac{1}{\Delta^2} \begin{bmatrix} \alpha_0 \theta \left[f_k^0(q) f_{kk}^1(k-q) + f_k^1(k-q) f_{kk}^0(q) \right] & -\alpha_0 \theta f_k^0(q) f_{kk}^1(k-q) \\ \Upsilon_1 & \Upsilon_2 \end{bmatrix}$$

with the notation

$$\begin{aligned} \Upsilon_1 &= \alpha_1(1-\theta)\theta \left[f_k^1(k-q) f_{kk}^0(q) + f_{kk}^1(k-q) f_k^0(q) \right] \left[f_k^1(k-q) - f_k^1(k) \right] \qquad (26) \\ &\quad -\alpha_1(1-\theta) f_k^0(q) f_{kk}^1(k-q) \left[(1-\theta) f_k^0(q) + \theta f_k^1(k-q) \right] \\ \Upsilon_2 &= (1-\alpha_1)(1-\theta)^2 f_k^0(q)^2 f_{kk}^1(k) + \theta^2 f_k^1(k-q)^2 f_{kk}^1(k) \qquad (27) \\ &\quad + (2-\alpha_1)\theta(1-\theta) f_k^0(q) f_k^1(k-q) f_{kk}^1(k) \\ &\quad + \alpha_1(1-\theta) f_k^0(q) f_{kk}^1(k-q) \left[(1-\theta) f_k^0(q) + \theta f_k^1(k) \right], \end{aligned}$$

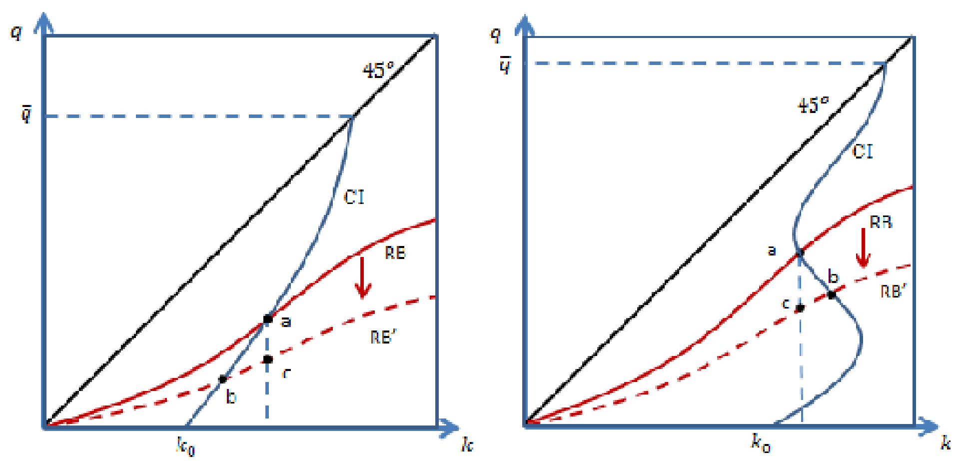
while $\Delta = (1-\theta) f_k^0(q) + \theta f_k^1(k-q)$ is a special case of (13). It can be verified that $\Upsilon_2 < 0$ and $\det \mathbf{J} > 0$, but Υ_1 may be positive or negative.

Given $\iota < \hat{\iota}$, so that $K_1(q)$ and $K_2(q)$ are well defined, any $q > 0$ that solves $K_1(q) = K_2(q)$ is a monetary steady state. Assuming a solution exists,

$$K_1'(q) - K_2'(q) = \frac{\det \mathbf{J}}{\Upsilon_2 f_k^0(q) f_{kk}^1(k-q)} > 0,$$

which means there cannot be more than one solution, and thus we establish uniqueness. Finally, one can check $K_1(0) - K_2(0) = -k_0 < 0$ and $K_1(\bar{q}) - K_2(\bar{q}) = K_1(\bar{q}) - \bar{q} > 0$, and thus we establish existence. ■

In the above proof, the solution for q as a function of k in (21) is called the RB curve, and the solution for k as a function of q in (22) is called the CI curve. For different configurations, these are depicted in Figure 2 in (k, q) space, which is natural because k and q represent investment and reallocation, or, heuristically, the supply and demand for used capital. Both curves lie below the 45^o line. As shown above, the RB curve starts at $(0, 0)$ and is monotone increasing. As also shown, while it may not be globally increasing, the CI curve starts at $(0, k_0)$, is increasing near $(0, k_0)$, and is increasing near $(\bar{k}, \bar{q})$ at least under the mild condition $\lim_{k\to 0} f_k^1(k) / f_{kk}^1(k) = -\infty$.



Case 1 Case 2

Figure 2: Monetary Policy under Bargaining, Increasing ι

While we cannot establish that CI is increasing, in general, it is under additional restrictions. Two options for such restrictions are

- **Condition 1**: *$f^j(k) = \varepsilon_j f(k)$ where $f(k) = k^\eta$, with $\eta \in (0,1)$.*

- **Condition 2**: *$f^j(k) = f(k)$ where $f(k)$ is any production function, θ is close to $1/2$, and ι is close to 0.*

The first (second) of these guarantees CI is increasing globally (at steady state). As the functional form $f(k) = k^\eta$ is standard, Condition 1 is not overly harsh. Condition 2 does not parameterize the $f(k)$, but makes it the same for both types, and assumes that bargaining is not too asymmetric and the inflation distortion is not too bad.[18]

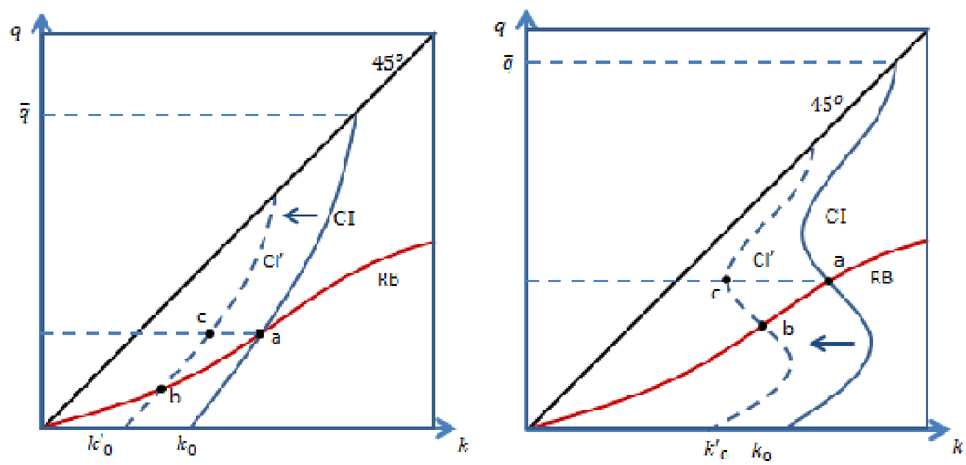
Proposition 1 says RB and CI cross at $q > 0$ iff $\iota < \hat{\iota}$. In the graphs, an increase in ι rotates RB clockwise, and when ι hits $\hat{\iota}$ the curve coincides with the horizontal axis. At that point $k = k_0$, which is its value in the nonmonetary equilibrium. It also says RB and CI cannot cross more than once even if they both slope upward. Heuristically, RB captures this: if type 1 bring more k then type 0 bring more z to get more q. Similarly, CI captures this: if type 0 bring more z then type 1 bring more k at least under Condition 1 or 2. This means there are complementarities and hence multipliers at work, but in this specification they are not strong enough to generate multiple monetary steady states.

As regards monetary policy, an increase in ι rotates RB but does not affect CI, and so steady state moves from point a to b in Figure 2. Hence, q must decrease, while k decreases or increases as the CI slopes up or down. In other words, lower nominal interest (or inflation or money supply growth) rates increase capital reallocation, and increase investment under reasonable conditions, consistent with conventional Keynesian wisdom. But the logic is unconventional: in this model, lower ι reduces the cost of liquidity and thus facilitates trade in secondary mar-

[18] The slope of the CI curve is $-\Upsilon_2/\Upsilon_1$, where Υ_1 and Υ_2 are defined in (26) and (27). We know $\Upsilon_2 < 0$, but not the sign of Υ_1, in general; one can check $\Upsilon_1 > 0$ globally under Condition 1 and $\Upsilon_1 > 0$ at steady state under Condition 2.

kets, which raises the *option value* of investing in the primary market. To see the multipliers at work, observe that the increase in ι would move us from a to c if k were fixed, but since k in fact reacts we move instead to b, which accentuates the fall in q in the left panel of Figure 2 and attenuates it in the right panel.

As regards fiscal policy, an increase in τ shifts the CI curve to the left but does not affect RB, and so q and k both decrease, as shown in Figure 3. To see the fiscal multipliers at work, observe that the increase in τ would move steady state from point a to c if q were fixed, but since q in fact reacts we move instead to b, which accentuates the fall in k in the left panel of Figure 3 and attenuates it in the right panel.



Case 1 Case 2

Figure 3: Fiscal Policy under Bargaining, Increasing τ

This CI-RB curve shifting may remind one of the IS-LM approach to undergrad macro. Well, our CI and RB curves actually *are* the IS and LM curves, even if we take a different approach to microfoundations than most macro courses, and depict the outcomes in (k, q) rather than (y, r) space. In any case, we find it attractive that the general equilibrium effects of fiscal and monetary policy can be illustrated using simple diagrams, despite the theory featuring intricacies like search, bargaining and monetary exchange.[19]

The next result describes the impact of changes in parameters more generally (details are in an Online Appendix). For this we set $F^j(k,h) = B\varepsilon_j f(k) + Ch$, where B and ε_j capture aggregate and type-specific capital productivity, while C captures labor productivity. The effects on output y are omitted because, except for changes in A or C, they are the same as the effects on k; to see this, note that in steady state $x + \delta k = y$, where $u'(x) = A/C$ by (3). Hence, we identify higher k with higher output and better economic times.

Proposition 2 *Given $F^j(k,h) = B\varepsilon_j f(k) + Ch$ and bargaining, the effects of parameter changes are shown in Table 1, where * means the result holds if the CI curve is upward sloping at steady state, which is true at least under Condition 1 or 2, and ** means the result holds at least under Condition 1.*

	ι	τ	α_0	α_1	ε_0	ε_1	A	B	C
k	$-^*$	$-$	$+^*$	$+$	$+$	$+$	0	$+$	0
q	$-$	$-$	$+$	$+$	$+$	$+^{**}$	0	$+$	0
x	0	0	0	0	0	0	$-$	0	$+$
w	0	0	0	0	0	0	0	0	$+$
z	$-$	$-^{**}$	$+$	$+$	$+$	?	0	$+^{**}$	0

Table 1: Parameter Changes under Bargaining.

The results in Table 1 accord well with intuition. Consider first productivity. Naturally the k chosen by type 1 goes up with ε_1, but perhaps more subtly it also goes up with ε_0: intuitively, higher ε_0 makes the secondary market more lucrative for type 1, so they invest more in the primary market. Higher ε_0 increases q, too, and higher ε_1 increases it at least under Condition 1: intuitively, higher ε_1 means

[19] A similar diagrammatic approach is used in a similarly microfounded model in Berentsen, Menzio and Wright (2011), except that their framework has no capital, and has unemployment v as in Pissarides (2000), so the outcomes are shown in (v,q) rather than (k,q) space.

type 1 want more k in the CM, but q can still rise because there is more k in the DM. As an application, recall the fact that reallocation is procyclical and capital mismatch is countercyclical. The model accounts for that if ε_0 and ε_1 both go up in good times, but ε_0 goes up more, as is true in the data.[20] Then good times have higher q even though ε_0 and ε_1 are closer together. Similarly, while z and q can both go up in good times, the former can go up more, making the DM price of capital, $P = z/q$, procyclical.

On search frictions, increasing α_1 raises k and q: when their probability of trade is higher, type 1 acquire more capital in the CM and trade more in the DM. This also raises z, as type 0 use more liquidity to get more q. Similarly, increasing α_0 raises z and q, since type 0 use more liquidity when their probability of trade is higher, and this raises k at least under Condition 1 or 2.[21]

These results shed new light on the connections between money and capital, between investment and reallocation, and between fiscal and monetary policy. Of course Table 1 is based on quasi-linear technology, so w is invariant to changes in parameters other than C. While more general technologies entail more general equilibrium effects that complicate these predictions, the forces in Table 1 still seem important. Also, with quasi-linear utility, changes in parameters other than A and C do not affect consumption, which is pinned down by $u'(x) = A/C$, but do affect leisure. With alternative specifications such as those mentioned in

[20] Kehrig (2015), e.g., establishes these results: "First, crosssectional productivity dispersion is countercyclical; the distribution of total factor productivity levels across establishments is about 12% more spread-out in a recession than in a boom. Second, the bottom quantiles of the productivity distribution are more cyclical than the top quantiles. In other words, the countercyclicality of productivity dispersion is mostly due to a higher share of relatively unproductive establishments during downturns."

[21] Observe that increasing α_0 is like decreasing ι, since only ι/α_0 matters; to understand this, note that ι is the cost of holding cash per period, and $1/\alpha_0$ is the average number of periods it is held, so ι/α_0 is the average cost of being liquid in the DM. Also, these experiments involve increasing α_1 holding α_0 constant, or vice versa, say by changing the efficiency of the meeting technology $\chi(n_0, n_1)$ and n_0/n_1. One can also increase the efficiency of $\chi(n_0, n_1)$ with n_0/n_1 fixed, so α_0 and α_1 both increase, which raises q and k.

footnotes 9 and 14, x as well as h are affected by parameters.

To say more about efficiency, consider a planner that maximizes welfare subject to the search frictions,

$$\begin{aligned} W^*(k_0) &= \max_{\{h_t, x_t, q_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[u(x_t) - Ah_t\right] \\ \text{st } x_t &= y_t + (1-\delta) n_1 k_t - n_1 k_{t+1} - G_t \\ y_t &= n_1 \alpha_1 f^1(k_t - q_t) + n_1(1-\alpha_1) f^1(k_t) + n_0 \alpha_0 f^0(q_t) + h_t, \end{aligned} \tag{28}$$

where $n_0\alpha_0 = n_1\alpha_1$. Assuming an interior solution, the first-order conditions are given by

$$x_t \ : \ u'(x_t) = A \tag{29}$$

$$q_t \ : \ f_k^1(k_t - q_t) = f_k^0(q_t) \tag{30}$$

$$k_{t+1} \ : \ r + \delta = \alpha_1 f_k^1(k_{t+1} - q_{t+1}) + (1-\alpha_1) f_k^1(k_{t+1}), \tag{31}$$

with h_t then determined by the constraint.

From (29), the efficient x_t is the same as the equilibrium x_t. From (30), q_t equates the marginal products across firms that meet in the DM, $f_k^1(k_0 - q_0) = f_k^0(q_0)$, which obtains in equilibrium iff $\iota = 0$. Now consider k. First, if we impose $\tau = 0$, then after a little algebra k is efficient iff $\theta = 0$. The desirability of $\theta = 0$ is a version of the conditions in Mortensen (1982) and Hosios (1990): since the gains from DM trade depend on what type 1 brings to the market, and the cost is sunk upon meeting type 0, the former must get all the gains from trade if he is to choose k efficiently. But $\theta = 0$ implies monetary equilibrium cannot exist for $\iota > 0$. This is a two-sided holdup problem: we need θ big to support efficient money demand; we need θ small to support efficient capital demand; and no $\theta \in [0,1]$ delivers both. More discussion of this is contained in Section 4.

First, let us relax $\tau = 0$ to consider corrective fiscal policy, and in particular to allow an investment subsidy $\tau < 0$. Simply by comparing the planner's conditions and the equilibrium conditions, it is straightforward to verify the following:

Proposition 3 *Given $F^j(k,h) = B\varepsilon_j f(k) + h$ and bargaining, for any θ such that a steady state monetary equilibrium exists, it is efficient if $\iota = 0$ and*

$$\tau = \tau^* \equiv 1 - \frac{r+\delta}{f_k^1(k) + \alpha_1(1-\theta)f_k^0(q)\frac{f_k^1(k-q)-f_k^1(k)}{(1-\theta)f_k^0(q)+\theta f_k^1(k-q)}} < 0, \qquad (32)$$

where k and q in (32) solve the planner's problem.

4 Competitive Search

We now consider a different way of organizing the secondary capital market, directed search and price posting, instead of random search and bargaining. As discussed in the survey by Wright et al. (2016), the combination of directed search and posting is often called *competitive search equilibrium* because those who post the terms of trade compete to attract customers. Compared to random search and bargaining, we interpret this as a different environment, where agents can now communicate and commit to the posted terms before meeting in the DM.[22] It is known from other studies (see below) that competitve search can help get around holdup problems, and we want to investigate how this plays out in our application to capital investment and reallocation.

There are different ways to formulate the competitive search equilibrium concept. One is to say that sellers post the terms of trade and buyers direct their search to those they find attractive, which does not necessarily mean those posting the lowest price, because buyers have to take into account the probability of trade (i.e., the probability of trade of meeting a seller). Alternatively, one can

[22] Even if agents lack commitment, so that posting is cheap talk, it can still affect outcomes (Menzio 2007; Kim and Kircher 2015). Therefore, to be safe, we can interpret random search and bargaining as a lack of communication rather than just a lack of commitment.

say that buyers post and sellers can direct their search to those offering attractive terms. Or, one can say that third parties called *market makers* post terms to attract both buyers and sellers to their submarkets, with the idea being that they can earn profit by charging participants fees, but free entry into market making drives the fees to 0. These three approaches give the same outcome here (although not in all environments; see Delacroix and Shi 2016).

Since it does not matter, for convenience we frame the problem in terms of market makers. A market maker designs a submarket for DM trade by posting (k, q, z, n) in the previous CM. After seeing all of the alternative submarkets, buyers and sellers make their decisions. If they choose to visit a submarket posting (k, q, z, n), type 1 must bring capital k and type 0 must bring real balances z.[23] Then the participants in each submarket engage in a bilateral random matching process just like the one used above, and if type 0 and type 1 meet they exchange z for q. There is no renegotiation in meetings – for better or worse, posting in this context means commitment. If one prefers to interpret this as, say, commitment by sellers to terms they post, that is fine; our use of the market maker fiction is then only a technical device used to characterize the equilibrium outcome.

Let the measure of type 0 be $n_0 = n$ and normalize the measure of type 1 to $n_1 = 1$, so n is aggregate market tightness in the DM. Market tightness in any open submarket will equal aggregate tightness in equilibrium, but for now let n be arbitrary in a particular submarket. This captures the idea that submarkets can compete in terms of tightness as well as the terms of trade, in principle, even if they end up being the same in equilibrium. Then the meeting probabilities are

[23] An alternative formulation has the market makers post (q, p, n), while type 1 and 0 agents unilaterally choose k and z. One can check that they bring the same k and m determined below. Relatedly, it does not matter if n is posted, or if instead agents figure it out from the other variables. Another feature of the approach that is worth mentioning, and that is emphasized in the above-mentioned survey by Wright et al. (2016), is this: one can study finite versions, with integer numbers N_0 and N_1 of types 0 and 1, then take the limit as these numbers get large holding $n = N_0/N_1$ fixed; the outcome converges to the one in the text.

$\alpha_0 = \alpha(n)$ and $\alpha_1 = \alpha(n)/n$, where the function $\alpha(\cdot)$ comes from a CRS meeting technology $\chi(n_0, n_1)$, as in Section 2, and again we assume $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$, $\alpha(n) \leq 1$ $\forall n > 0$, plus standard Inada conditions.

The market maker's problem is to maximize v_0, the payoff to type 0 in his submarket that period, subject to type 1 getting a payoff v_1, which is taken as given for now, but is determined endogenously in equilibrium below. In steady state $V_j = v_j/(1-\beta)$, but we use the short-run payoff v_j to indicate that, conceptually, there is no commitment beyond the current period. Formally, the market maker's problem is

$$v_0 = \max_{k,q,z,n} \left\{ \frac{\alpha(n)}{n} [(1-\tau) f^0(q) - z] - \iota z \right\} \tag{33}$$
$$\text{st } (1-\tau) f^1(k) + \alpha(n) \left\{ z - (1-\tau) [f^1(k) - f^1(k-q)] \right\} - (r+\delta) k = v_1.$$

Importantly, note that there is a cost ιz for buyers and a cost $(r+\delta)k$ for sellers participating in the market that is paid whether or not they trade.

Eliminating z using the constraint, we reduce the problem to

$$\max_{k,q,n} \left\{ \frac{\alpha(n)}{n} (1-\tau) f^0(q) - \left[\iota + \frac{\alpha(n)}{n}\right] (1-\tau) \left[f^1(k) - f^1(k-q)\right] \right.$$
$$\left. - \left[\iota + \frac{\alpha(n)}{n}\right] \frac{v_1 + (r+\delta) k - (1-\tau) f^1(k)}{\alpha(n)} \right\}.$$

The first-order conditions are

$$q \;:\; \iota = \frac{\alpha(n)}{n} \frac{f_k^0(q) - f_k^1(k-q)}{f_k^1(k-q)} \tag{34}$$
$$k \;:\; \frac{\delta + r}{1-\tau} = f_k^1(k) + \alpha(n)[f_k^1(k-q) - f_k^1(k)] \tag{35}$$
$$n \;:\; v_1 = \frac{[1-e(n)]\alpha(n)}{\frac{\iota n}{\alpha(n)} e(n) + 1} (1-\tau) \left[f^0(q) + f^1(k-q) - f^1(k)\right] \tag{36}$$
$$- (r+\delta) k + (1-\tau) f^1(k),$$

where $e(n) = n\alpha'(n)/\alpha(n)$ is the elasticity of $\alpha(n)$. Using these and the constraint in (33), we obtain

$$\frac{z}{1-\tau}=\frac{(1-e)\,f_k^1\,(k-q)\,f^0\,(q)+ef_k^0\,(q)\,[f^1(k)-f^1(k-q)]}{(1-e)\,f_k^1\,(k-q)+ef_k^0\,(q)}, \tag{37}$$

where in the interest of space the argument is omitted from $e=e\,(n)$. Conditions (34)-(37) determine (q,k,n,z) as a function of v_1.

Given that there is a unique solution to this problem, every active submarket in equilibrium has the same (q,k,n,z). This implies two things: by CRS in the meeting technology, it suffices to consider one representative submarket; and in equilibrium, $n=n_0/n_1$ in the representative submarket is the same as aggregate tightness.[24] Now the above conditions determine (q,k,z), and v_1 rather than n, while v_0 solves (33). Hence, competitive search equilibrium is described recursively as follows: (34) and (35) define the RB and CI curves and their intersection yields (k,q); given this, (37) determines z; given $w=1$, x solves $u'\,(x)=A$; and feasibility determines h.

We now provide an analog to Proposition 1:

Proposition 4 *Given $F^j(k,h)=f^j\,(k)+h$, with competitive search, a monetary steady state always exists and it is unique.*

Proof: First (k,q) are determined by the RB and CI curves defined by (34) and (35). One can show that (34) gives q as an increasing function of k, say $Q_1\,(k)$, where $Q_1\,(k)>0\ \forall k>0$. In addition, as $k\to 0$, $Q_1\,(k)\to 0$; and as $k\to\infty$, $Q_1\,(k)\to\infty$ and $k-Q_1\,(k)\to\infty$. Also (35) gives k as an increasing function of q, or inversely, gives q as an increasing function of k, say $q=Q_2\,(k)$. Then $Q_2\,(k_0)=0$ where $f_k^1(k_0)=(r+\delta)\,/\,(1-\tau)$. Also, as $k\to\infty$, $k-Q_2\,(k)\to$

[24] To be clear, a market maker can choose any n, in principle, but in equilibrium it will be the same as the aggregate ratio of type 0 and type 1 firms, with v_1 adjusting to equilibrate the market. An alternative approach is to allow entry by one side, say by type 1, at cost κ; in this case we get $v_1=\kappa$ with n adjusting to equilibrate the market. Entry is worth consideration, but in the interest of space, it is deferred to future work.

$c < \infty$, where $\alpha(n)f_k^1(c) = (r+\delta)/(1-\tau)$. Any k solving $Q_1(k) = Q_2(k)$ is a steady state equilibrium. Notice $Q_1(k_0) > Q_2(k_0)$ and

$$Q_1(k) - Q_2(k) = Q_1(k) - k - [Q_2(k) - k] \to -\infty \text{ as } k \to \infty.$$

Therefore, steady state equilibrium exists.

Also notice that

$$\begin{aligned}
Q_1'(k) &= \frac{f_k^0(q)f_{kk}^1(k-q)}{f_{kk}^0(q)f_k^1(k-q) + f_k^0(q)f_{kk}^1(k-q)}, \\
Q_2'(k) &= \frac{[1-\alpha(n)]\,f_{kk}^1(k) + \alpha(n)\,f_{kk}^1(k-q)}{\alpha(n)f_{kk}^1(k-q)}.
\end{aligned}$$

Then $Q_1'(k) - Q_2'(k)$ takes the same sign as

$$\begin{aligned}
&\alpha(n)f_k^0(q)f_{kk}^1(k-q)^2 - \left\{[1-\alpha(n)]\,f_{kk}^1(k) + \alpha(n)\,f_{kk}^1(k-q)\right\}\Upsilon \\
&\quad = -\alpha(n)\,f_{kk}^1(k-q)f_{kk}^0(q)f_k^1(k-q) - [1-\alpha(n)]\,f_{kk}^1(k)\,\Upsilon < 0,
\end{aligned}$$

where $\Upsilon = f_{kk}^0(q)f_k^1(k-q) + f_k^0(q)f_{kk}^1(k-q)$. Therefore, $Q_1(k) - Q_2(k)$ crosses 0 at most once, and uniqueness is established. ■

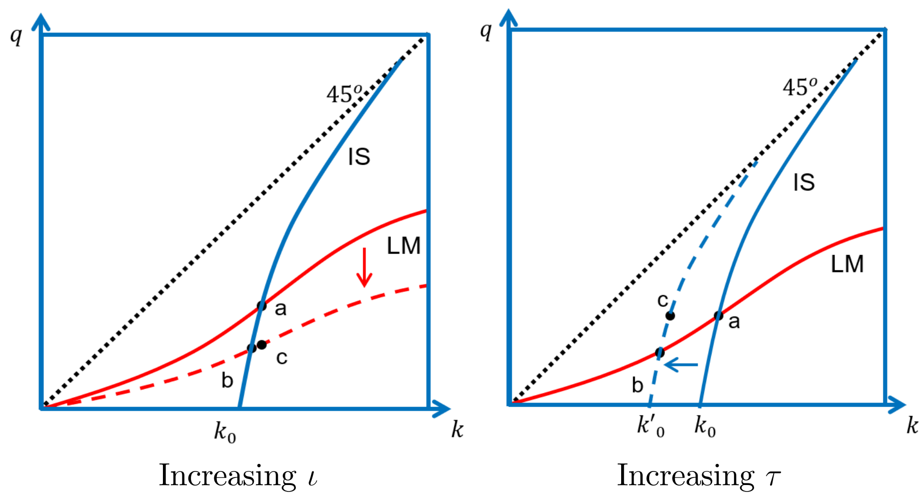


Figure 4: Monetary and Fiscal Policy under Competitive Search

Figure 4 plots equilibrium and the effects of policy changes for this formulation. Note that with competitive search CI is globally increasing without any side conditions, but Proposition 4 again guarantees it can only cross RB once, similar to Proposition 1. An interesting difference from the version with Kalai bargaining is that now we do not need a condition like $\iota < \hat{\imath}$ for existence. To understand this, first note that competitive search equilibrium behaves in some ways like a model with generalized Nash bargaining – see below for more on this, but for now simply note that (37) is exactly what one gets with generalized Nash if we replace e by bargaining power θ. This makes the result similar to other models along the lines of Lagos and Wright (2005), where monetary equilibria exist for all ι under Nash bargaining, but only for ι below a threshold under Kalai bargaining.

The next result is the analogue to Proposition 2 (again, derivations are in an Online Appendix). Since most of the effects are similar to the bargaining model, we do not go into more detail.[25]

Proposition 5 *Given $F^j(k,h) = B\varepsilon_j f(k) + Ch$ and competitive search, the effects of parameter changes are given in Table 2 where ** means the result holds under Condition 1.*

	ι	τ	μ	ε_0	ε_1	A	B	C
k	$-$	$-$	$+$	$+$	$+$	0	$+$	0
q	$-$	$-$	$+$	$+$	$+^{**}$	0	$+$	0
x	0	0	0	0	0	$-$	0	$+$
w	0	0	0	0	0	0	0	$+$
z	$-$	$-^{**}$	$+^{**}$	$+$	?	0	$+^{**}$	0

Table 2: Parameter Changes under Competitive Search.

[25] There are a few differences – e.g., now k is unambiguously decreasing in ι, while that required a side condition in Section 3. It is not uncommon for directed search theory to yield less ambiguous predictions than bargaining (e.g., see Dong 2011).

The final result concerns efficiency, the analog to Proposition 3, with the proof again following directly from comparing the conditions for equilibrium and for the planner's problem.

Proposition 6 *Given $F^j(k,h) = f^j(k) + h$, with competitive search and $\tau = 0$, equilibrium is efficient iff $\iota = 0$.*

Several aspects of Proposition 6 are noteworthy. First, as mentioned above, (37) looks like what one gets in a model with Nash bargaining when the bargaining power of type 0 is $\theta = e$. Next, (34) looks like what one gets with Nash bargaining when $\theta = 1$, which is what it takes to get efficient q at $\iota = 0$. Finally, (35) looks like what one gets with Nash bargaining when $\theta = 0$, which is what it takes to get efficient k. Thus competitive search avoids the holdup problems inherent in bargaining and makes it easier to achieve efficiency. We do not take a stand on which solution concept is more reasonable or realistic; the goal instead is to sort out logically the inefficiencies that obtain under different market structures and the implications for policy.[26]

5 Conclusion

This paper has explored capital investment and reallocation in dynamic general equilibrium models featuring frictional markets, motivated by what we see as a

[26] To compare these results to the literature, Kurmann and Rabinovich (2016) have a k holdup problem but not an m holdup problem, as they do not model liquidity. Aruoba, Waller and Wright (2011) have both, but their DM trade involves consumption, not capital, so the economics are rather different. In particular, one might say that we have a two-sided holdup problem, while that model has two one-sided holdup problems, similar to work in labor economics by Masters (1998, 2011) and Acemoglu and Shimer (1999), where firms invest in physical capital and workers in human capital. Also, the results here are different than in the model with ex post heterogeneity in Wright, Xiao and Zhu (2017), where all agents bring (m,k) to the DM. In that case there is a θ that delivers efficiency. This is related to a result in Rocheteau and Wright (2005): when buyers choose money and sellers face an entry decision, the results are quite different than when buyers choose money and face an entry decision.

consensus in the literature that this is a fruitful area of exploration.[27] The framework also featured monetary exchange, due to explicit frictions, as opposed to reduced-form restrictions. For specifications with random search and bargaining or directed search and posting, we provided strong results on existence, uniqueness, efficiency and comparative statics. The analysis was quite tractable: steady state can be reduced to two equations in k and m we called the CI and RB curves, but they are basically the IS and LM curves taught in common macro courses. Although their microfoundations are arguably more firm, our curves can just as easily be used to illustrate the effects of monetary and fiscal policy.

Some results were consistent with mainstream macro – e.g., decreasing the nominal interest rate stimulates real investment – even if the reasons were different. In particular, lower ι in our setup means that type 0 will be more liquid in the secondary market, and this encourages investment by type 1 in the primary market. As regards efficiency, the first-best outcome obtains under competitive search when $\iota = \tau = 0$, and under bargaining when $\iota = 0$ and $\tau = \tau^* < 0$, since an investment subsidy is required when there are bargaining wedges. Although we did not go into this, in terms of second-best results, when $\tau > \tau^*$ one can check that $\iota = 0$ is still optimal, even if it does not achieve full efficiency. But this is somewhat delicate: in the companion paper, with ex post heterogeneity due to firm-specific productivity shocks, we show that $\iota > 0$ can be optimal.

In terms of future research, first, it is of interest to explore in more detail

[27] In addition to what we said earlier, as motivation, consider Ottonello (2015), who compares his model to one without search frictions. He finds the latter predicts "both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature." Also, "Results indicate that investment search frictions and capital unemployment are a relevant propagation mechanism for financial shocks: While these shocks account for 33% of output fluctuations in the model with investment search frictions, they only account for 1% of output fluctuations in the benchmark real model without investment search frictions." This is a nice example of how it is useful to look at capital reallocation through the lens of search theory. Although this project focused on pure theory, rather than numbers, the goal was to provide additional examples of how this is useful.

other motives for capital reallocation, which we do to some degree in the companion paper just mentioned. Second, obviously one could study the models quantitatively, but that might best proceed after relaxing a few simplifying assumptions (e.g., allowing more than two types), combining the ex ante and ex post specifications, and adding aggregate shocks. Third, it may be interesting to study endogenous growth in these models, perhaps as in Chiu et al. (2017), where financial intermediation facilitates reallocation across producers for some reasons that are straightforward and others that are more subtle. Finally, it might be interesting to combine frictional capital and frictional labor markets. To us, these and other extensions/applications make frictional capital reallocation an exciting area for further exploration.

In closing, we thank our discussant for interesting and challenging comments. We agree with most of them, and certainly think it is important to look carefully into institutional descriptions of secondary capital markets. But, just like the use of search in labor, housing, marriage, goods and other markets, ours is *not meant to be a literal description* of how they work. While obviously a stylized and abstract description, this does not mean it conveys no insights into the actual process of capital exchange, any more than it does for the other markets. Moreover, we try to check robustness by studying random search and bargaining as well as directed search and posting. Future work should of course look into other frictions, including information problems. Search theory is well designed to make progress on that issue.

Finally, the only other issue in the discussion to which we want to respond concerns the nice "history of thought" provided on search-based models. We do not agree at all that our model is an application of Duffie, Gârleanu and Pedersen (2005). First, as nice as that paper may be, it uses a second-generation search-and-bargaining model – i.e., one with indivisible assets – while we use the more modern third-generation approach, with everything divisible. More than a technical difference, this opens up many new paths to substantive insights. Our approach is solidly in the recent New Monetarist tradition, following Lagos and Wright (2005). What we do that is novel is, first, to switch from consumers buying goods to firms buying inputs, and second, add various details meant to capture salient elements of that market at least in an abstract way. This is a natural extension, and allows us to make use of many results in the monetary literature. Moreover, this certainly could have been accomplished without ever seeing Duffie et al. (2005). Otherwise, we agree with the discussion.

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