

# The optimal inflation target and the natural rate of interest

Philippe Andrade<sup>1</sup>   Jordi Galí<sup>2</sup>   Hervé Le Bihan<sup>1</sup>  
Julien Matheron<sup>1</sup>

<sup>1</sup>Banque de France

<sup>2</sup>CREI, UPF and Barcelona GSE

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# Motivation

- ✓ Evidence of a persistent decline in the natural rate of interest
  - Holston et al (2017), Eggertsson et al. (2017) among many others
  - Implication for monetary policy ⇒ more frequent ZLB
- ✓ Calls for a higher inflation target (e.g. Ball, Blanchard, Williams)
  - Benefit: Offset increase in frequency of ZLB events
- ✓ However recommendation is controversial (e.g. Bernanke)
  - Cost: Pay higher inflation every day to limit rare ZLB events

## A trade-off

Optimal reaction of  $\pi^*$  to a 1% drop in  $r^*$ ?

- ✓  $Pr(i < ZLB) = Pr(r^* + \pi^* < ZLB)$
- ✓ Increase  $\pi^*$  by 1% to keep  $Pr(i < ZLB)$  constant?
  - Does not take into account the costs induced by  $\pi^*$
  - Moreover, changes in  $r^*$  affect the dynamic properties of the economy, so does not ensure  $Pr(i < ZLB)$  remains constant
- ✓ Keep  $\pi^*$  unchanged to avoid the day-to-day costs of higher  $\pi$ ?
  - Does not take into account the costs induced by  $Pr(i < ZLB)$

## This paper

- ✓ Quantitative welfare-based evaluation of the above trade-off
- ✓ Describe the relation between optimal inflation ( $\pi^*$ ) as a function of steady-state real interest rate ( $r^*$ )
- ✓ Based on an estimated NK model (US & euro area)
- ✓ And (intensive) simulations of the estimated model under a ZLB constraint

## Main Findings

Local slope of  $(r^*, \pi^*)$  relation  $\approx -0.9$

- ✓ A 1% drop in  $r^*$  calls for an increase in  $\pi^*$  that is close to (but below) 1%

Robust to considering

- ✓ different factors underlying the drop in  $r^*$
- ✓ the US or the EA
- ✓ parameter uncertainty
- ✓ larger shocks
- ✓ negative ELB
- ✓ different markups

## Relation to the Literature

NK models with trend inflation

Ascari (2004), Cogley & Sbordone (2008), Ascari & Sbordone (2014)...

Quantitative analysis of optimal inflation target w/o ZLB

Khan et al. (2003), Amano et al. (2009), Schmitt-Grohé and Uribe (2010), Bilbiie et al. (2014), Ascari et al. (2015), Carlsson and Westermark (2016), Adam and Weber (2017), Lepetit (2017)...

Quantitative analysis of optimal inflation target with ZLB

Coibion et al (2012), Blanco (2016), Dordal-i-Carreras et al. (2016), Kiley and Roberts (2017), ...

This paper: Analysis of the relation between  $r^*$  and  $\pi^*$

# Roadmap

The model and computing the optimal inflation target

The  $(r^*, \pi^*)$  relation

Accounting for parameters uncertainty

Other robustness exercises

Conclusion

Appendix

## The Model

Simple NK framework with trend inflation ( $\pi$ ) and

- ✓ Sticky prices & wages à la Calvo
- ✓ Less than perfect price & wage indexation
- ✓ Wages imperfectly indexed to productivity growth

⇒ Costs to a positive inflation target  $\pi$

- ✓ ZLB

⇒ Benefit of a positive  $\pi$

- ✓ Steady-state real rate  $r^* = \mu_z + \rho$  with  $\mu_z$  trend productivity growth,  $\rho$  discount rate

⇒ Two sources of variations in  $r^*$

## The Model: Households

Representative HH with preferences

$$E_t \sum_{s=0}^{\infty} \beta^s \left\{ e^{\zeta_{c,t+s}} \log(C_{t+s} - \eta C_{t+s-1}) - \frac{\chi}{1+\nu} \int_0^1 (N_{t+s}(h))^{1+\nu} dh \right\},$$

and sequence of budget constraints

$$P_t C_t + e^{\zeta_{q,t}} Q_t B_t \leq \int_0^1 W_t(h) H_t(h) dh + B_{t-1} - T_t + D_t$$

where  $\zeta_{q,t}$  is a “risk-premium” shock & discount factor  $\beta$  obeys

$$\beta \equiv \frac{1}{1+\rho}, \quad Q_t = e^{-i_t}$$

# The Model: Firms

Final good

$$Y_t = \left( \int_0^1 Y_t(f)^{(\theta_p-1)/\theta_p} df \right)^{\theta_p/(\theta_p-1)}, \quad C_t = Y_t$$

Intermediate Good

$$Y_t(f) = Z_t L_t(f)^{1/\phi}, \quad Z_t = Z_{t-1} e^{\mu_z + \zeta_{z,t}}$$

Aggregate labor

$$N_t = \left( \int_0^1 N_t(h)^{(\theta_w-1)/\theta_w} dh \right)^{\theta_w/(\theta_w-1)}, \quad N_t = \int_0^1 L_t(f) df$$

# The Model: Price Setting

Calvo probability of not resetting price:  $\alpha_p$ .

Partial indexation so that non-updating firms set

$$P_t(f) = \Pi_{t-1}^{\gamma_p} P_{t-1}(f) \text{ indexation degree } \gamma_p < 1$$

Re-optimizing firm  $f$  chooses  $P_t^*$  in order to maximize

$$E_t \sum_{s=0}^{\infty} (\beta \alpha_p)^s \lambda_{t+s} \left\{ (1 + \tau_p) e^{-\zeta_{u,t+s}} \frac{V_{t,t+s}^p P_t^*}{P_{t+s}} Y_{t,t+s} - \frac{W_{t+s}}{P_{t+s}} \left( \frac{Y_{t,t+s}}{Z_{t+s}} \right)^\phi \right\},$$

s.t. demand function

$$Y_{t,t+s} = \left( \frac{V_{t,t+s}^p P_t^*}{P_{t+s}} \right)^{-\theta_p} Y_{t+s} \text{ where } V_{t,t+s}^p = \prod_{j=t}^{t+s-1} \Pi_j^{\gamma_p}$$

and  $\zeta_{u,t}$  cost push shock, e.g. exog. variations in markup/sales taxes

# The Model: Wage Setting

Calvo probability of not resetting wage:  $\alpha_w$ .

Partial indexation so that non-updating unions set

$$W_t(h) = e^{\gamma_z \mu_z} \Pi_{t-1}^{\gamma_p} W_{t-1}(h) \text{ indexation degree } \gamma_z, \gamma_w < 1$$

If drawn to re-optimize, union  $h$  chooses  $W_t^*$  to maximize

$$E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s \left\{ (1 + \tau_w) \Lambda_{t+s} \frac{V_{t,t+s}^w W_t^*}{P_{t+s}} N_{t,t+s} - \frac{\chi}{1 + \nu} N_{t,t+s}^{1+\nu} \right\},$$

s.t. demand function

$$N_{t,t+s} = \left( \frac{V_{t,t+s}^w W_t^*}{W_{t+s}} \right)^{-\theta_w} N_{t+s} \text{ where } V_{t,t+s}^w = e^{\gamma_z \mu_z (t+s)} \prod_{j=t}^{t+s-1} \Pi_j^{\gamma_w}$$

# The Model: Monetary Policy

Monetary policy rule

$$\hat{i}_t = \rho_{TR} \hat{i}_{t-1} + (1 - \rho_{TR}) [a_\pi \hat{\pi}_t + a_y x_t] + \zeta_{R,t}$$

Effective lower bound (ELB) constraint

$$\hat{i}_t > -i + ELB$$

here

✓  $x_t \equiv \log(Y_t / Y_t^n)$ ,  $Y_t^n$  is natural output

✓  $\hat{\pi}_t \equiv \pi_t - \pi$  ,  $\pi_t \equiv \log(\Pi_t)$

$\pi$  the inflation target (may be suboptimal)

≡ value used to define “inflation gap” entering Taylor rule

## The Model: Estimation

- ✓ Detrending by  $Z_t$
- ✓ Log-linearization around deterministic steady state
- ✓ Calibrated parameters:  $1/\phi = 0.7; \theta_p = 6; \theta_w = 3$
- ✓ Remaining parameters estimated, full-system Bayesian approach
- ✓ Gaussian priors for  $(\rho, \mu_z, \pi)$  with means consistent with average inflation, GDP growth and real rate in each economy
- ✓ Sample period: 1985Q1-2008Q3 (pre-ZLB)
- ✓ Observable variables

$$\mathbf{x}_t = [\Delta \log(\text{GDP}_t), \Delta \log(\text{GDP Deflator}_t), \\ \Delta \log(\text{Wages}_t), \text{Short Term Interest Rate}_t]'$$

# Estimation Results

Parameter	Posterior Mean	
	US	EA
$\rho$	0.191	0.211
$\mu_z$	0.429	0.475
$\pi$	0.617	0.787
$\alpha_p$	0.669	0.616
$\alpha_w$	0.502	0.585
$\gamma_p$	0.198	0.117
$\gamma_w$	0.445	0.341
$\gamma_z$	0.500	0.506
$a_\pi$	2.134	2.022
$a_y$	0.501	0.498
$\rho_{TR}$	0.852	0.871

▶ detailed US results

▶ detailed EA results

# Estimation Results

## Comments

- ✓ Estimated parameters in the ball park of available results
- ✓ Some specificities
  - Average growth & disc't rate larger in the EA than in the US  
 $\Rightarrow r_{EA} > r_{US}$   $\Rightarrow$  larger  $\pi$  cushion needed in the US
  - Wage & price rigidities smaller than SW (2007), more in line with micro evidence
  - Indexation parameters also smaller
  - Stronger indexation in the US  $\Rightarrow$  More inflation tolerance
  - MP more inertial in the EA than in the US

# Computing Optimal Inflation Target

- ✓ Freeze  $\theta$  to a fixed value
- ✓ 2d-order approximation to HH expected utility  $\mathcal{W}(\pi; \theta)$  [▶ Details](#)
- ✓ Simulate large sample ( $T = 100,000$ ) and compute  $\mathcal{W}(\pi; \theta)$
- ✓ Solution under ZLB: Bodenstein et al. (2009) algo. [▶ Details](#)
- ✓ Optimal (welfare-maximizing) inflation target is:

$$\pi^*(\theta) \equiv \arg \max_{\pi} \mathcal{W}(\pi; \theta)$$

## Pre-crisis benchmark

$$\Rightarrow \pi_{US}^* \in [1.85\%, 2.20\%]$$

$$\Rightarrow \pi_{EA}^* \in [1.30\%, 1.60\%]$$

[▶ US example](#)

[▶ EA example](#)

The model and computing the optimal inflation target

The  $(r^*, \pi^*)$  relation

Accounting for parameters uncertainty

Other robustness exercises

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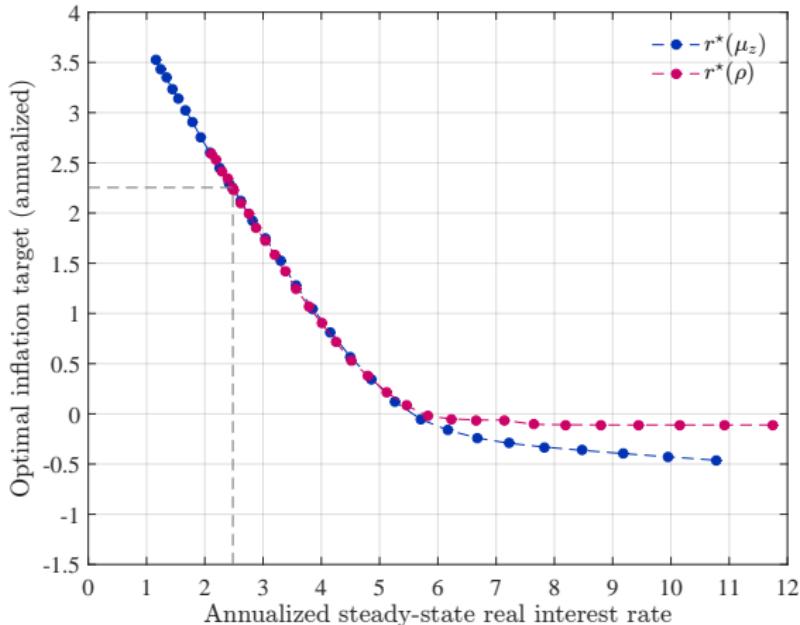
Appendix

## The $(r^*, \pi^*)$ relation

- ✓ Fix structural parameter vector at posterior mean
- ✓ Vary  $\mu_z$  or  $\rho$  on a grid of values  
(from 0.4% to 10% annualized, each)
- ✓ For each value, compute associated  $\pi^*$  and associated  $r^*$
- ✓ Draw the implied  $(r^*, \pi^*)$  relation

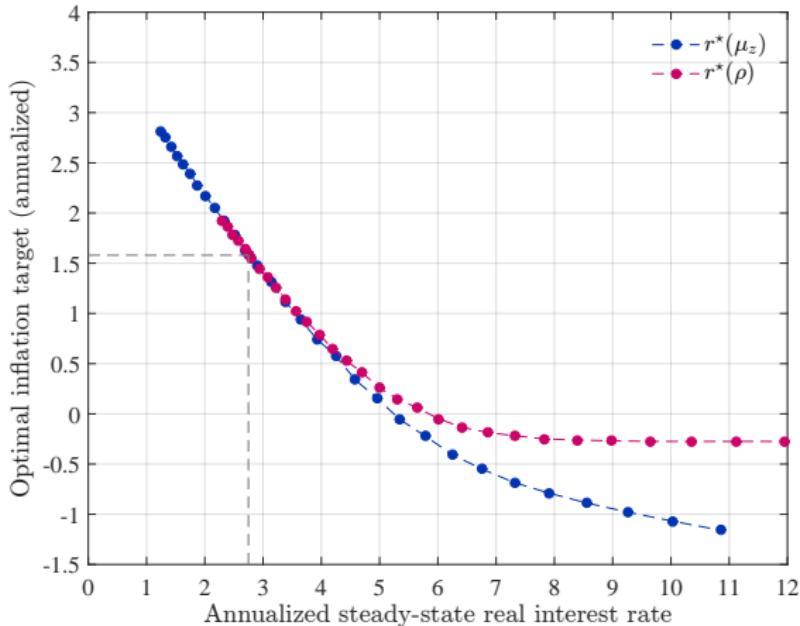
# The $(r^*, \pi^*)$ relation – US

Figure :  $(r^*, \pi^*)$  locus (at the posterior mean)



# The $(r^*, \pi^*)$ relation – EA

Figure :  $(r^*, \pi^*)$  locus (at the posterior mean)



# The $(r^*, \pi^*)$ relation

## Wrap up

- ✓ Relation is decreasing
- ✓ Slope is not one for one
  - Small ( $\approx 0$ ) for high  $r^*$
  - Close to but below 1 for low  $r^*$
- ✓ For large  $r^*$ ,  $\pi^*$  can be negative (reflecting real wage growth)
- ✓ For low values, source of change in  $r^*$  ( $\mu_z$  or  $\rho$ ) does not matter much

► Robustness US  $\mu_z$

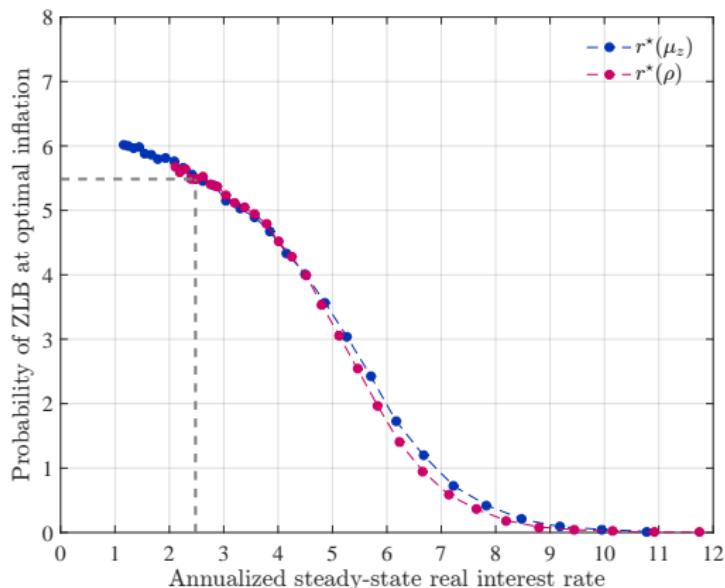
► Robustness US  $\rho$

► Robustness EA  $\mu_z$

► Robustness EA  $\rho$

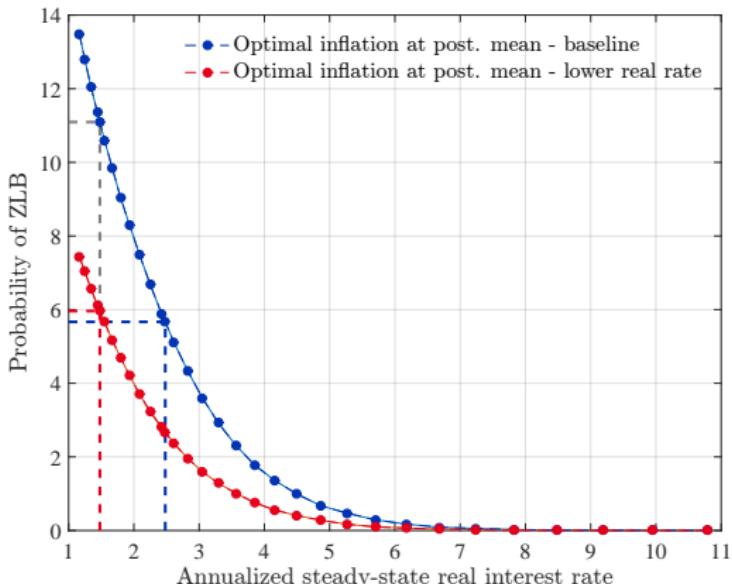
# Relation between $Pr[ZLB|\text{optimal inflation}]$ and $r^*$

US - posterior mean



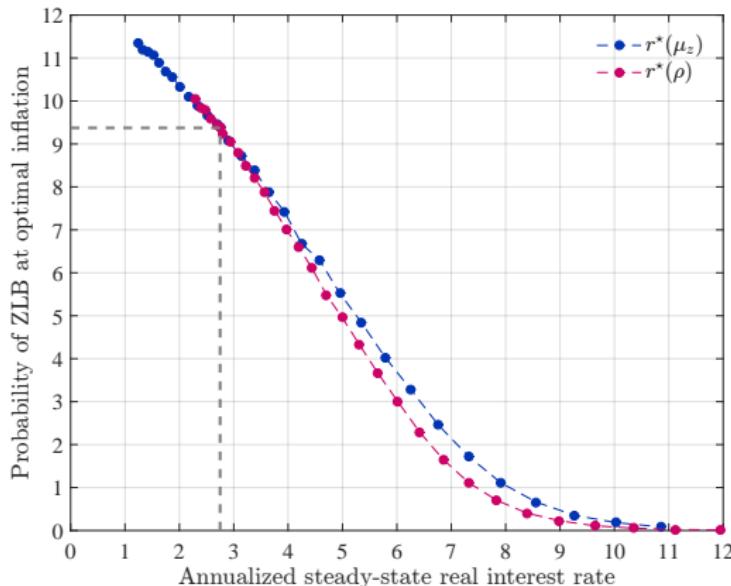
# Relation between $Pr[ZLB]$ and $r^*$ at fixed $\pi^*$

US - posterior mean



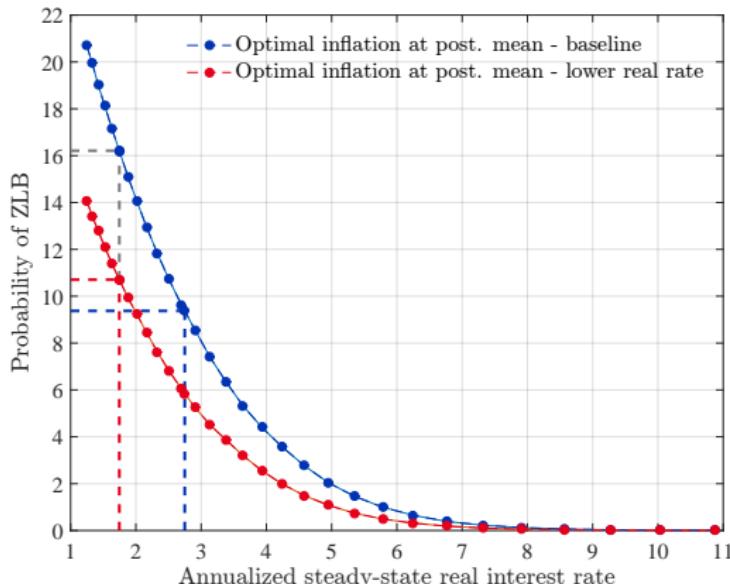
# Relation between $Pr[ZLB|\text{optimal inflation}]$ and $r^*$

Euro Area - posterior mean



# Relation between $Pr[ZLB]$ and $r^*$ at fixed $\pi^*$

Euro Area - posterior mean



The model and computing the optimal inflation target

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# Accounting for parameter uncertainty

## Computing Optimal Inflation Target

Bayesian-theoretic optimal inflation

- ✓ Draw  $\theta$  from posterior distribution
- ✓ Simulate large sample ( $T = 100,000$ ) and compute  $\mathcal{W}(\pi; \theta)$
- ✓ Solution under ZLB through Bodenstein et al. (2009) algorithm
  
- ✓ Repeat these steps  $N$  times ( $N = 500$ ) so as to get a distribution of  $\mathcal{W}(\pi; \theta)$
- ✓ Then compute

$$\pi^{**} \equiv \arg \max_{\pi} \int_{\theta} \mathcal{W}(\pi; \theta) p(\theta | X_T) d\theta$$

Pre-crisis benchmark

$$\Rightarrow \pi_{US}^{**} = 2.40\%$$

$$\Rightarrow \pi_{EA}^{**} = 2.20\%$$

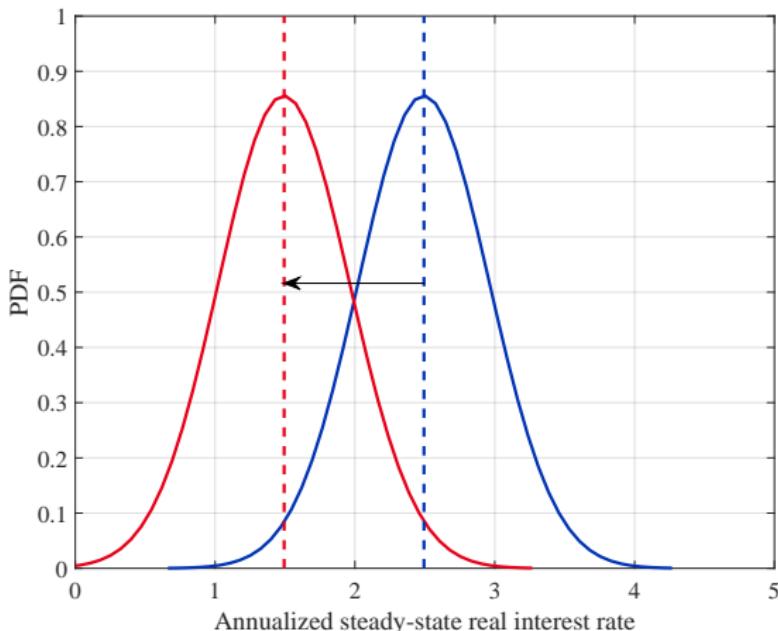
# Accounting for parameter uncertainty

Shift in central tendency of  $r^*$  distribution

- ✓ Draw a parameter vector  $\theta$  from the posterior distribution
- ✓ Recover  $r^*(\theta) = \rho(\theta) + \mu_z(\theta)$
- ✓ For that draw, shift  $\mu_z(\theta)$  downward by 1 pp (annualized)
- ✓ Compute  $\mathcal{W}(\pi; \theta)$  for this perturbed draw
- ✓ Repeat these steps  $N$  times so as to get
  - A posterior distribution of  $\mathcal{W}(\pi; \theta)$
  - A new  $\pi^{**}$  associated to the shifted posterior of  $\mathcal{W}(\pi; \theta)$

## Posterior Distribution of $r^*$ – US

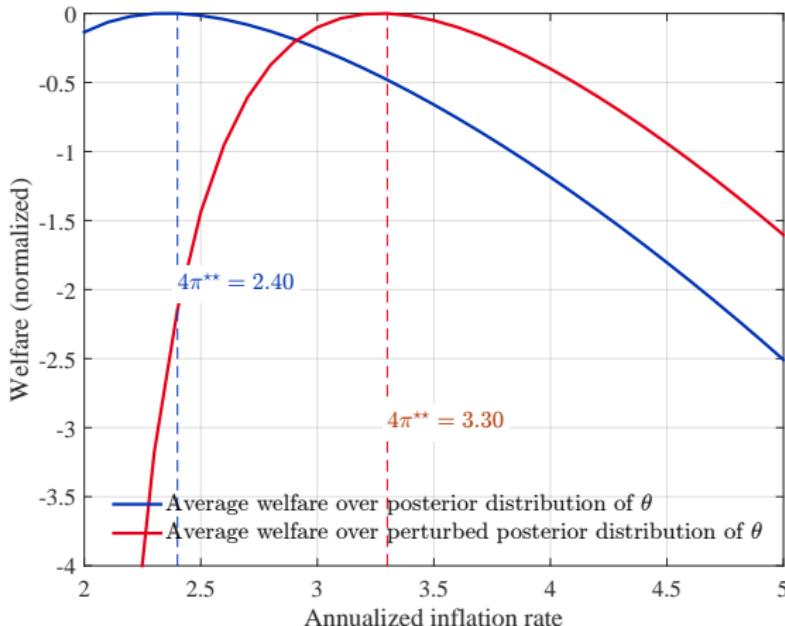
Figure : Posterior Distributions of  $r^*$  and counterfactual  $r^*$



Plain curve: PDF of  $r^*$ ; Dashed vertical line: Mean value, i.e.  $E_\theta(\pi^*(\theta))$ . Remark: distribution of  $r^*$  roughly symmetric; does not explain the asymmetry in distribution of  $\pi^*$ .

# Bayesian Robust Welfare – US

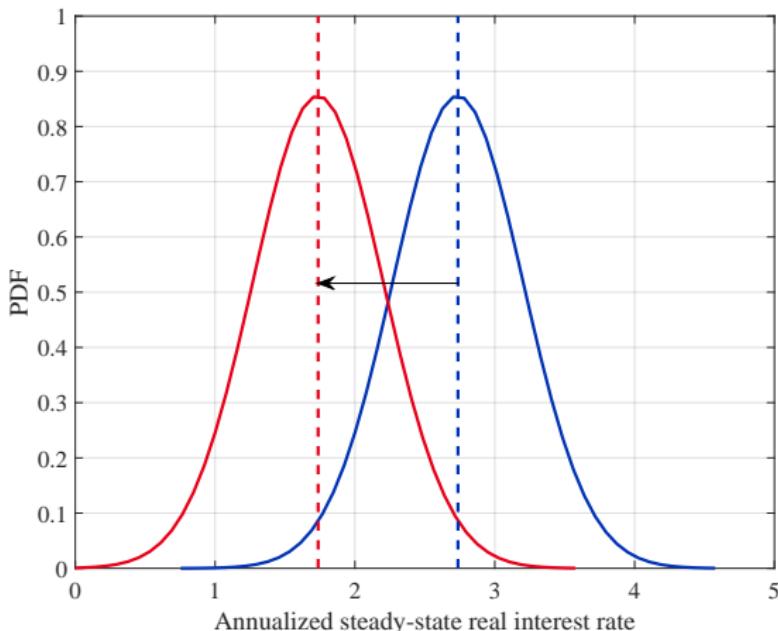
Figure :  $E_\theta(\mathcal{W}(\pi, \theta))$



Blue curve:  $E_\theta(\mathcal{W}(\pi, \theta))$ ; Red curve:  $E_\theta(\mathcal{W}(\pi, \theta))$  with lower  $r^*$

# Posterior Distribution of $r^*$ – EA

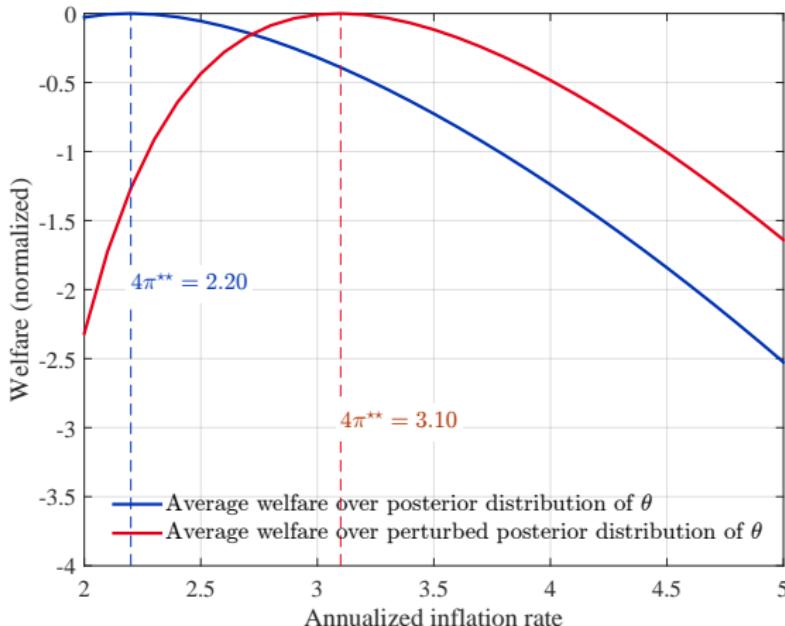
Figure : Posterior Distributions of  $r^*$  and counterfactual  $r^*$



Plain curve: PDF of  $r^*$ ; Dashed vertical line: Mean value, i.e.  $E_\theta(r^*(\theta))$ . Remark: distribution of  $r^*$  roughly symmetric; does not explain the asymmetry in distribution of  $\pi^*$ .

# Bayesian Robust Welfare – EA

Figure :  $E_\theta(\mathcal{W}(\pi, \theta))$



Blue curve:  $E_\theta(\mathcal{W}(\pi, \theta))$ ; Red curve:  $E_\theta(\mathcal{W}(\pi, \theta))$  with lower  $r^*$

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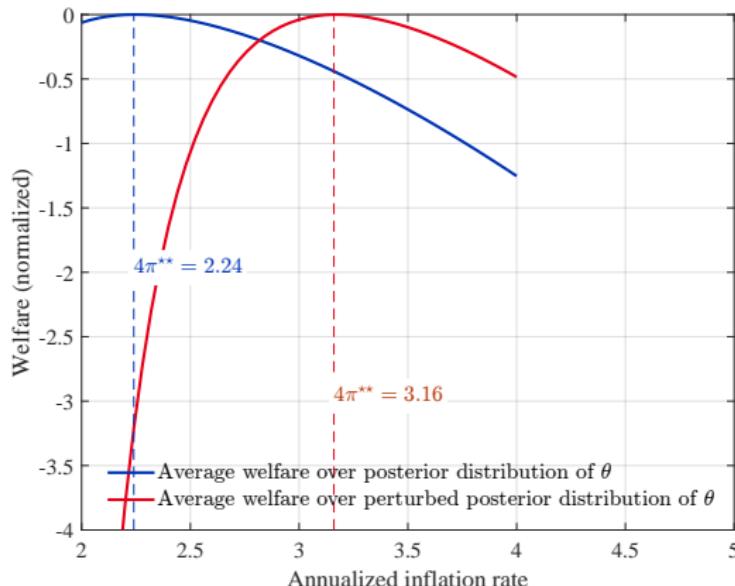
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# No uncertainty on reaction function

Holding MP parameters at posterior mean – US

Figure :  $E_\theta(\mathcal{W}(\pi, \theta))$

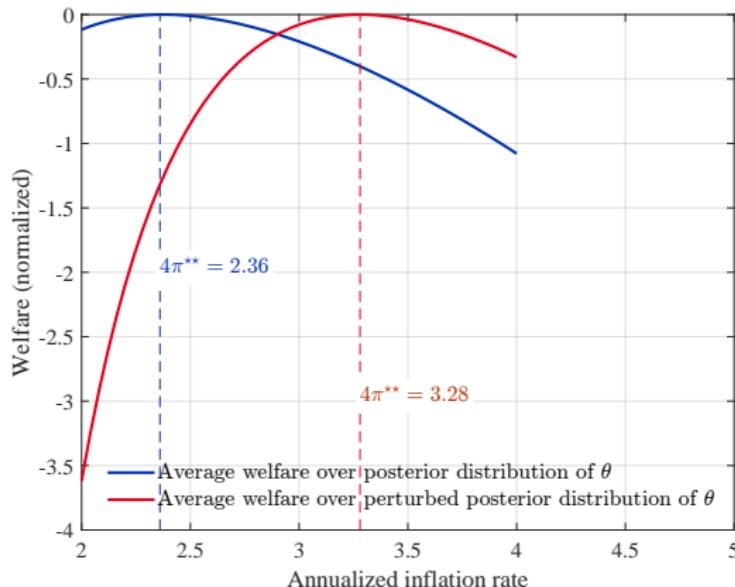


Blue curve:  $E_\theta(\mathcal{W}(\pi, \theta))$ ; Red curve:  $E_\theta(\mathcal{W}(\pi, \theta))$  with lower  $r^*$ . In each case,  $\rho_{TR}$ ,  $a_\pi$ , and  $a_y$  are frozen at their posterior mean values.

# No uncertainty on reaction function

Holding MP parameters at posterior mean– EA

Figure :  $E_\theta(\mathcal{W}(\pi, \theta))$

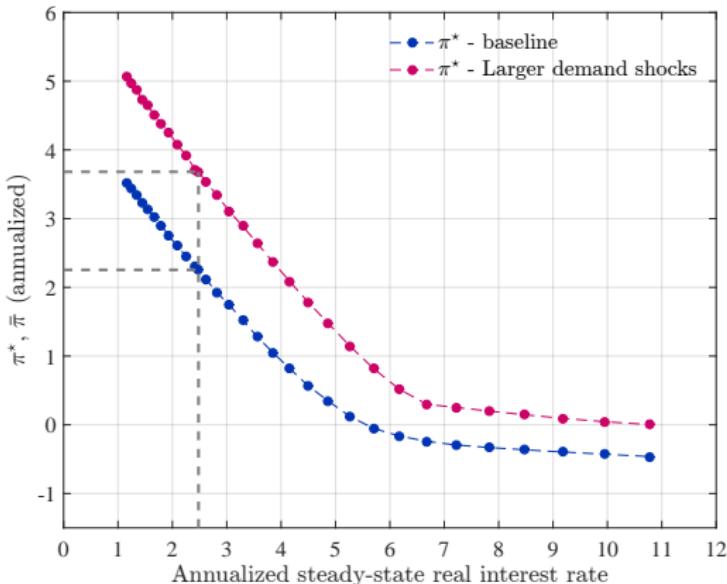


Blue curve:  $E_\theta(\mathcal{W}(\pi, \theta))$ ; Red curve:  $E_\theta(\mathcal{W}(\pi, \theta))$  with lower  $r^*$ . In each case,  $\rho_{TR}$ ,  $a_\pi$ , and  $a_y$  are frozen at their posterior mean values.

# What if shocks are larger?

Set standard deviation of demand shocks to 1.3 their baseline value

Figure :  $(r^*, \pi^*)$  relation with larger demand shocks - US



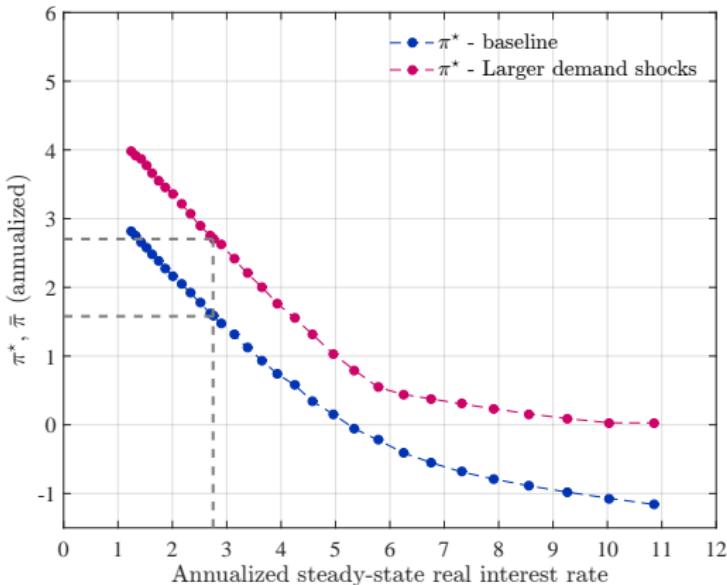
Note: blue dots  $\equiv$  baseline scenario :all the structural parameters set at posterior mean  $\bar{\theta}$ .

The red dots counterfactual simulation with  $\sigma_q$  &  $\sigma_g$  set to 30% higher than their baseline value.

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Set standard deviation of demand shocks to 1.3 their baseline value

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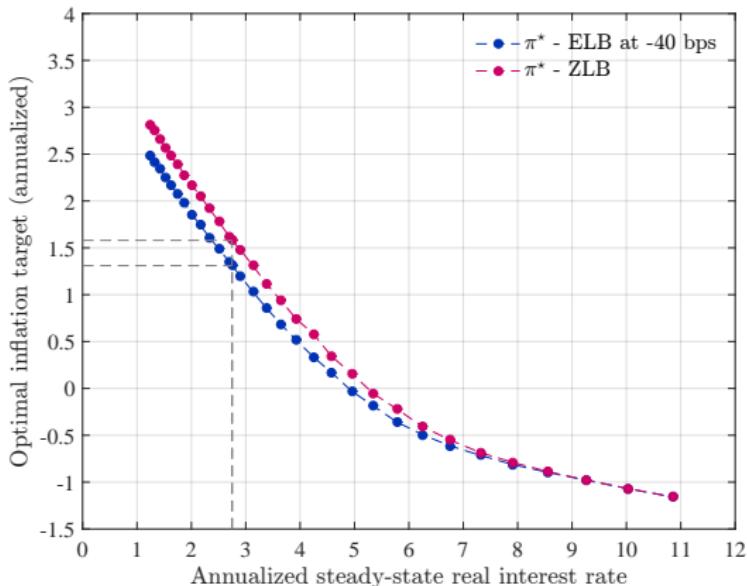
The red dots counterfactual simulation with  $\sigma_q$  &  $\sigma_g$  set to 30% higher than their baseline value.

# A Negative Effective Lower Bound

ELB: the nominal rate  $i_t$ , such that  $i_t \geq ELB$

Here set  $ELB$  for EA to  $-40$  basis points instead of zero.

Matches the ECB Deposit Facility Rate level attained in March 2016.

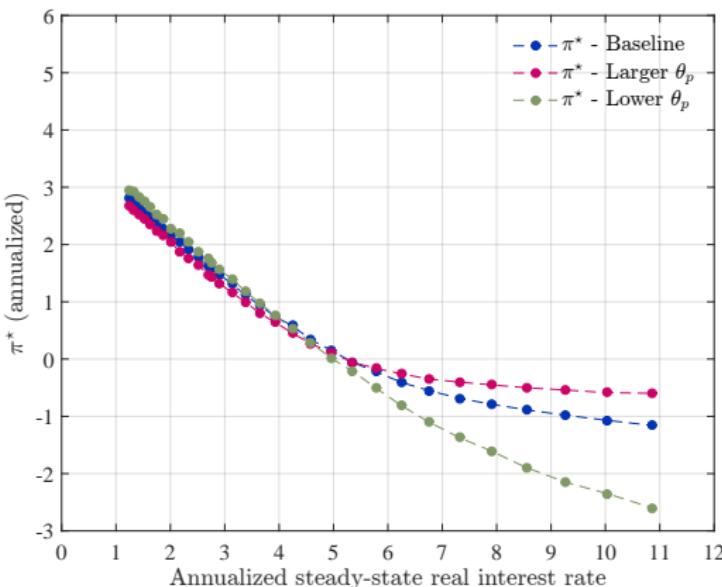


# Lower (or larger) markups

## On products market

Baseline  $\theta_p = 6$  / low  $\theta_p = 3$  / high  $\theta_p = 10$

Figure :  $(r^*, \pi^*)$

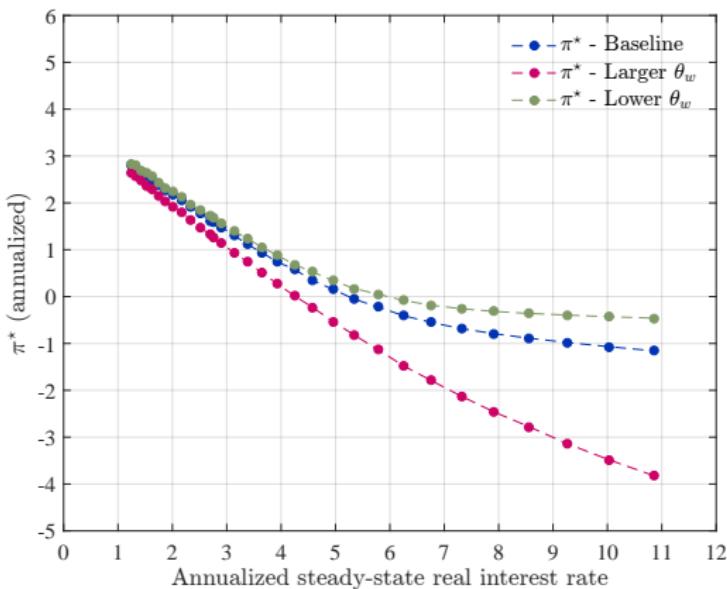


# Lower (or larger) markups

## On labor market

Baseline  $\theta_w = 3$  / low  $\theta_w = 1.5$  / high  $\theta_p = 8$

Figure :  $(r^*, \pi^*)$



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# Conclusion

## Analysis of the $(r^*, \pi^*)$ relation

- ✓ Robust finding: starting from a pre-crisis benchmark, a 1% decline in  $r^*$  calls for an increase of about 0.9% in  $\pi^*$

## Alternatives to an increase in $\pi^*$

- ✓ Unconventional MP
- ✓ Countercyclical fiscal policies
- ✓ Alternative MP strategies (price level targeting)

## Transition to new $\pi^*$ and credibility issues

# Appendix

# The Welfare Cost of Inflation

A second-order approximation to welfare :

$$U_0 = -\frac{1}{2} \frac{1-\beta\eta}{1-\eta} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_y [x_t - \delta x_{t-1} + (1-\delta)\bar{x}]^2 \right.$$

$$\left. + \lambda_p [(1-\gamma_p)\pi + \hat{\pi}_t - \gamma_p \iota_p \hat{\pi}_{t-1}]^2 \right\}$$

$$+ \lambda_w [(1-\gamma_z)\mu_z + (1-\gamma_w)\pi + \hat{\pi}_{w,t} - \gamma_w \iota_w \hat{\pi}_{t-1}]^2 \right\} + \text{t.i.p.} + \mathcal{O}(\|\zeta, \pi\|^3)$$

where gaps are defined as:  $x_t \equiv \hat{y}_t - \hat{y}_t^n$ ,  $\bar{x} \equiv \log \left( \frac{Y_z}{Y_z^n} \right)$

Strictly positive inflation

- is harmful due to induced dispersion in prices and quantities
- but limits variability of  $\hat{\pi}_t$  that results from ZLB

▶ Back

## The Model: Solution under ZLB

Log-linearized version of the model

Simulation under ZLB via a “OccBin” algorithm following Bodenstein et al. (2009) or Guerrieri-Iacoviello (2015)

### General idea

At each date  $t$ , given shocks  $\epsilon_t$

- ✓ Postulate ZLB entry date  $T_e$  and ZLB exit date  $T_x$
- ✓ Solve by backward induction for time-varying state-space representation
- ✓ Check whether postulated dates are correct; else shift leftward or rightward as appropriate
- ✓ Iterate upon convergence

# Estimation Results - [Back](#)

**Table : Estimation Results - US**

Parameter	Prior Shape	Prior Mean	Priod std	Post. Mean	Post. std	Low	High
$\rho$	Normal	0.20	0.05	0.19	0.05	0.11	0.27
$\mu_z$	Normal	0.44	0.05	0.43	0.04	0.36	0.50
$\pi^*$	Normal	0.61	0.05	0.62	0.05	0.54	0.69
$\alpha_p$	Beta	0.66	0.05	0.67	0.03	0.61	0.73
$\alpha_w$	Beta	0.66	0.05	0.50	0.05	0.43	0.58
$\gamma_p$	Beta	0.50	0.15	0.20	0.07	0.08	0.32
$\gamma_w$	Beta	0.50	0.15	0.44	0.16	0.21	0.68
$\gamma_z$	Beta	0.50	0.15	0.50	0.18	0.26	0.75
$\eta$	Beta	0.70	0.15	0.80	0.03	0.75	0.85
$\nu$	Gamma	1.00	0.20	0.73	0.15	0.47	0.97
$a_\pi$	Gamma	2.00	0.15	2.13	0.15	1.89	2.38
$a_y$	Gamma	0.50	0.05	0.50	0.05	0.42	0.58
$\rho_{TR}$	Beta	0.85	0.10	0.85	0.02	0.82	0.89
$\sigma_z$	Inverse Gamma	0.25	1.00	1.06	0.22	0.74	1.38
$\sigma_R$	Inverse Gamma	0.25	1.00	0.10	0.01	0.09	0.11
$\sigma_q$	Inverse Gamma	0.25	1.00	0.39	0.11	0.16	0.61
$\sigma_g$	Inverse Gamma	0.25	1.00	0.23	0.04	0.16	0.29
$\sigma_u$	Inverse Gamma	0.25	1.00	0.24	0.05	0.06	0.46
$\rho_R$	Beta	0.25	0.10	0.51	0.06	0.41	0.61
$\rho_z$	Beta	0.25	0.10	0.27	0.13	0.09	0.45
$\rho_g$	Beta	0.85	0.10	0.98	0.01	0.97	1.00
$\rho_q$	Beta	0.85	0.10	0.88	0.04	0.80	0.95
$\rho_u$	Beta	0.80	0.10	0.80	0.10	0.65	0.96

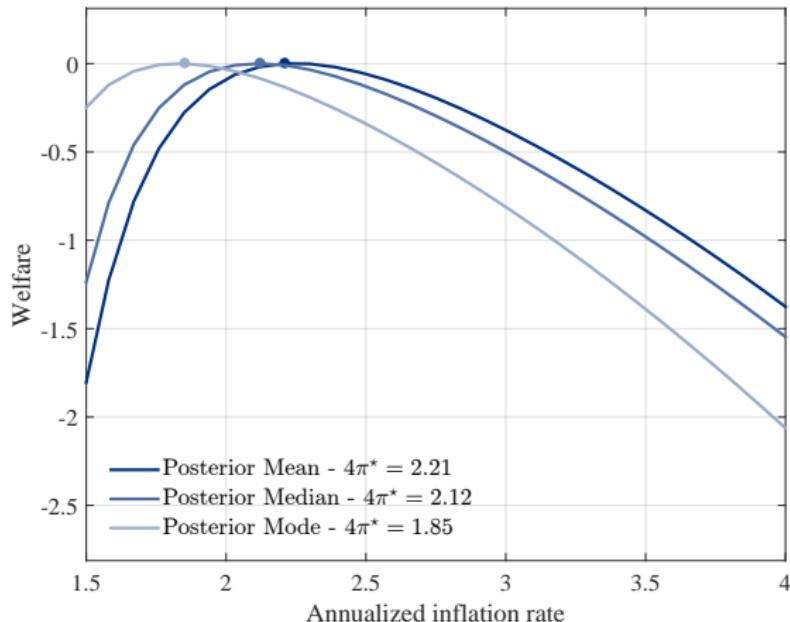
# Estimation Results - [Back](#)

Table : Estimation Results - EA

Parameter	Prior Shape	Prior Mean	Priod std	Post. Mean	Post. std	Low	High
$\rho$	Normal	0.20	0.05	0.21	0.05	0.13	0.29
$\mu_z$	Normal	0.50	0.05	0.47	0.05	0.40	0.55
$\pi^*$	Normal	0.80	0.05	0.79	0.05	0.71	0.86
$\alpha_p$	Beta	0.66	0.05	0.62	0.05	0.55	0.68
$\alpha_w$	Beta	0.66	0.05	0.59	0.04	0.52	0.65
$\gamma_p$	Beta	0.50	0.15	0.12	0.04	0.04	0.19
$\gamma_w$	Beta	0.50	0.15	0.34	0.12	0.15	0.53
$\gamma_z$	Beta	0.50	0.15	0.51	0.18	0.26	0.76
$\eta$	Beta	0.70	0.15	0.74	0.04	0.69	0.80
$\nu$	Gamma	1.00	0.20	0.96	0.18	0.65	1.25
$a_\pi$	Gamma	2.00	0.15	2.02	0.14	1.80	2.25
$a_y$	Gamma	0.50	0.05	0.50	0.05	0.42	0.58
$\rho_{TR}$	Beta	0.85	0.10	0.87	0.02	0.84	0.90
$\sigma_z$	Inverse Gamma	0.25	1.00	0.86	0.16	0.63	1.10
$\sigma_R$	Inverse Gamma	0.25	1.00	0.11	0.01	0.10	0.12
$\sigma_q$	Inverse Gamma	0.25	1.00	0.23	0.05	0.13	0.32
$\sigma_g$	Inverse Gamma	0.25	1.00	0.21	0.04	0.15	0.27
$\sigma_u$	Inverse Gamma	0.25	1.00	0.23	0.05	0.06	0.43
$\rho_R$	Beta	0.25	0.10	0.39	0.07	0.27	0.50
$\rho_z$	Beta	0.25	0.10	0.24	0.10	0.09	0.39
$\rho_g$	Beta	0.85	0.10	1.00	0.01	0.99	1.00
$\rho_q$	Beta	0.85	0.10	0.94	0.03	0.90	0.98
$\rho_u$	Beta	0.80	0.10	0.79	0.10	0.64	0.96

# Example of (Normalized) Welfare Functions - US

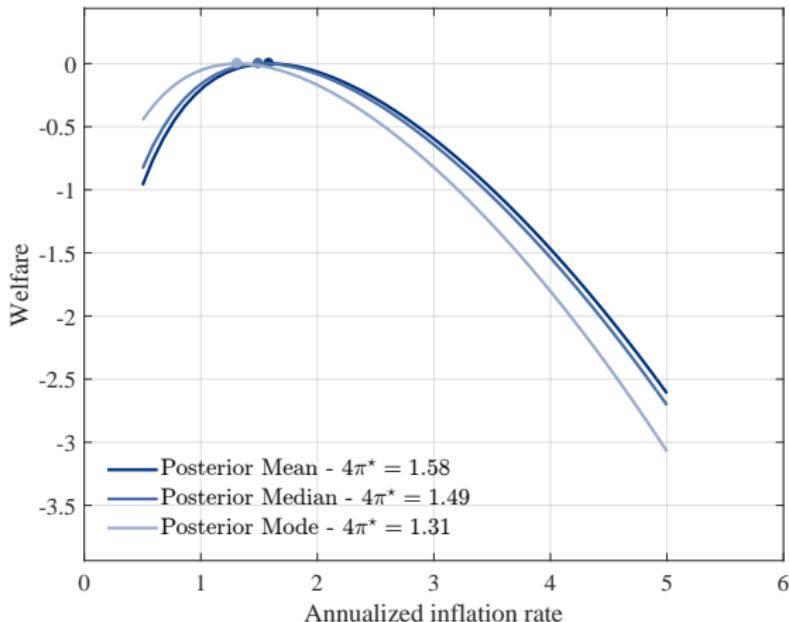
▶ Back



Blue: parameters set at posterior mean; light blue: parameters set at the posterior median;  
Lighter blue: parameters set at posterior mode.

# Example of (Normalized) Welfare Functions - EA

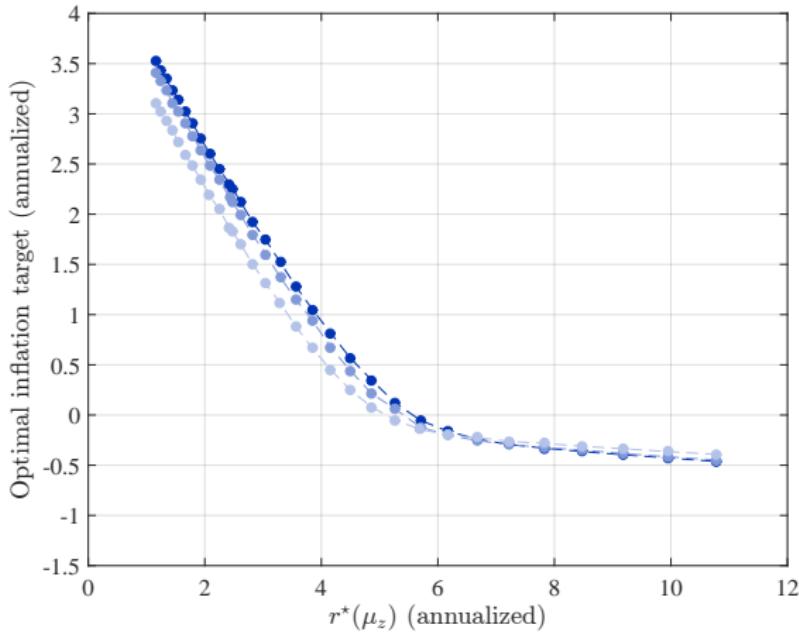
▶ Back



Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode

# The $(r^*, \pi^*)$ relation – US – [Back](#)

Figure :  $(r^*, \pi^*)$  locus when  $\mu_z$  varies

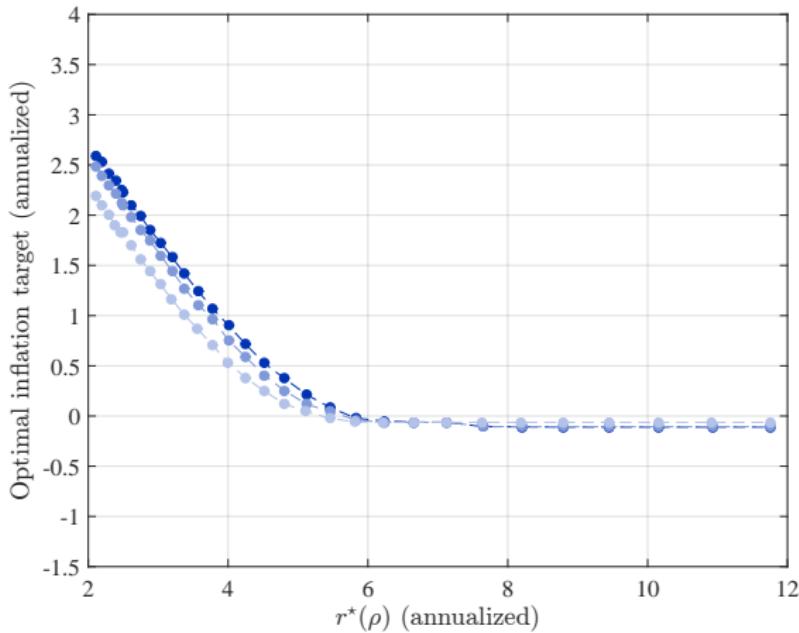


Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode

Memo:  $r^* = \rho + \mu_z$ . Range for  $\mu_z$ : 0.4% to 10% (annualized)

# The $(r^*, \pi^*)$ relation – US – [Back](#)

Figure :  $(r^*, \pi^*)$  locus when  $\rho$  varies

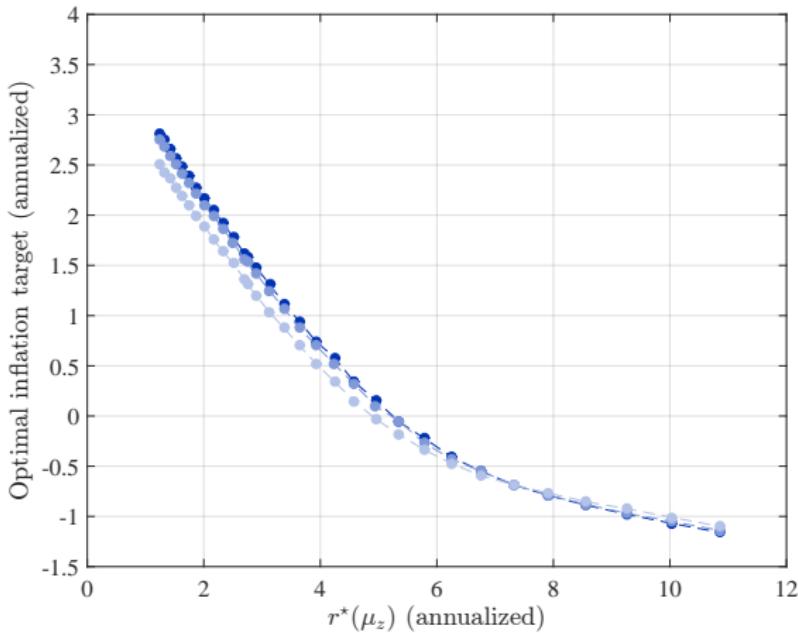


Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode

Memo:  $r^* = \rho + \mu_z$ . Range for  $\rho$ : 0.4% to 10% (annualized)

# The $(r^*, \pi^*)$ relation – EA – [Back](#)

Figure :  $(r^*, \pi^*)$  locus when  $\mu_z$  varies

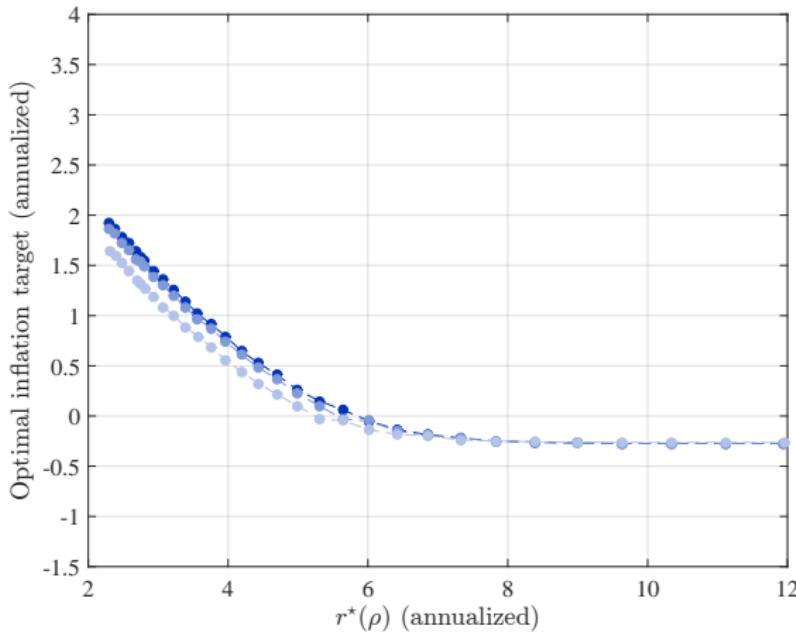


Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode

Memo item:  $r^* = \rho + \mu_z$ . Range for  $\mu_z$ : 0.4% to 10% (annualized)

# The $(r^*, \pi^*)$ relation – EA – [Back](#)

Figure :  $(r^*, \pi^*)$  locus when  $\rho$  varies



Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode

Memo item:  $r^* = \rho + \mu_z$ . Range for  $\rho$ : 0.4% to 10% (annualized)

## Summing Up

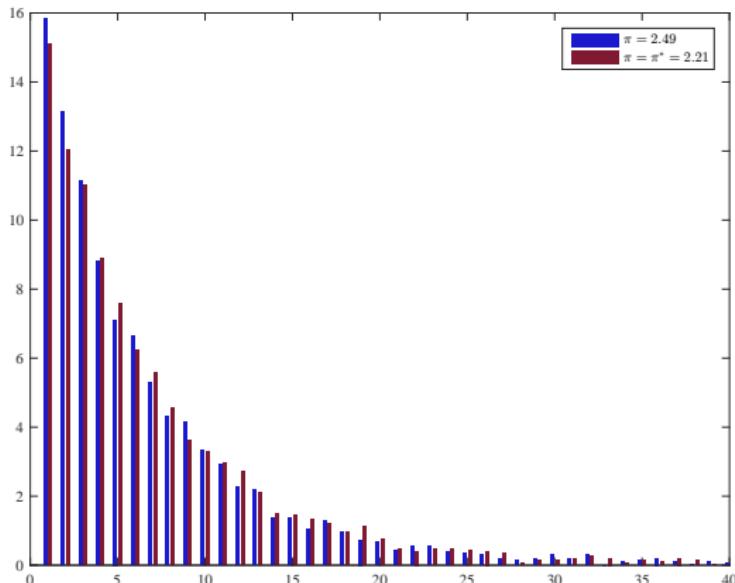
Table : Effect of a decline in  $r^*$  under alternative notions of optimal inflation

	US		EA	
	Baseline	Lower $r^*$	Baseline	Lower $r^*$
Mean of $\pi^*$	2.00	3.00	1.79	2.60
Median of $\pi^*$	1.96	2.90	1.47	2.28
$\pi^*$ at post. mean	2.21	3.20	1.58	2.39
$\pi^*$ at post. median	2.12	3.11	1.49	2.30
$\pi^{**}$	2.40	3.30	2.20	3.10
$\pi^{**}$ , frozen MP	2.24	3.16	2.36	3.28
$\pi^*$ at post. mean, ELB -40 bp	—	—	1.31	2.08
$E(\pi)$ at post. mean	2.20	3.19	1.56	2.36
$E(\pi)$ at post. mean, ELB -40 bp	—	—	1.24	1.97

Note: annualized percentage rate.

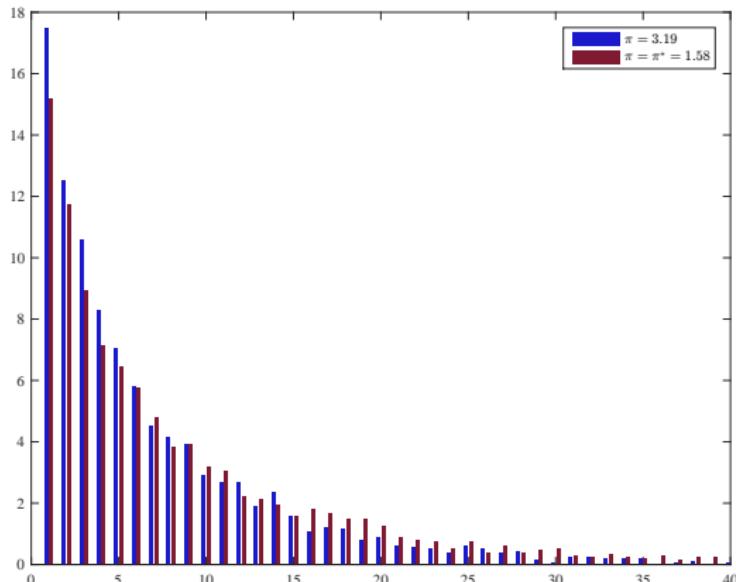
# The distribution of ZLB spells duration - US

Figure : Distribution of ZLB spells duration at the posterior mean



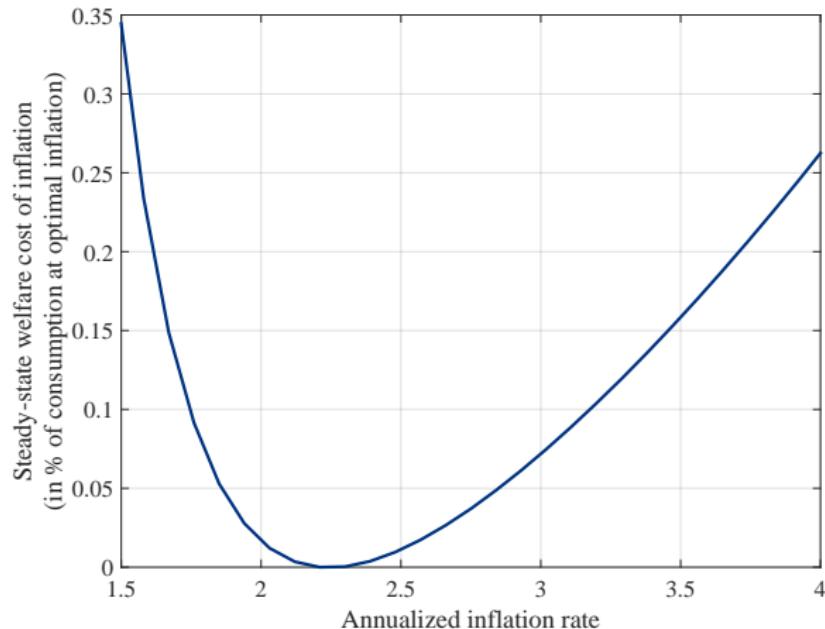
# The distribution of ZLB spells duration - EA

Figure : Distribution of ZLB spells duration at the posterior mean



# The welfare cost of inflation - US

Figure : Welfare cost of inflation at the posterior mean



# The welfare cost of inflation - EA

Figure : Welfare cost of inflation at the posterior mean

