Monetary Policy, Bounded Rationality and Incomplete Markets

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Motivation

- How is monetary policy affected by
  - **Bounded Rationality?**
  - **Incomplete Markets?**
  - **Combination?**

Paper: complementarities!

Motivation

- Helps fix "bugs" of standard NK model
  - indeterminacy given interest rate paths (Taylor principle)
  - Neo-Fisherian controversies
  - effectiveness of monetary policy
  - dependence on horizon ("forward guidance puzzle")
  - effects of fiscal policy at ZLB ("fiscal multipliers puzzle")
  - explosive nature of long-lasting liquidity traps
  - …

Bounded Rationality

- Expectations management major (main) channel of policy transmission in NK model under RE
- Realistic?
  - incomplete information or inattention to policy announcement?
  - less than full understanding of its future effects?

Bounded Rationality

- "**Inductive**"
  - learning: extrapolate from past data rationally or irrationally (Sargent; Evans Honkapohja; Shleifer)
  - incomplete info and inattention: ignore, underweight, cost to process info (Sims; Mankiw-Reis; Maćkowiak-Wiederholt; Gabaix; Angeletos-Lian)
- "**Eductive**"
  - robustness (Hansen-Sargent)
  - **level-k thinking:** think through reaction of others (Stahl-Wilson; Nagel; Crawford-Costa-Gomes-Iriberri; Evans-Ramey; Woodford; García-Schmidt-Woodford)
- **Level-k thinking**
  - credible and clear announcement policy change
  - with little past experience
  - agents think through consequences, with bounded rationality

Incomplete Markets

- Standard NK model: representative agent or complete markets
- Incomplete markets alternative (Bewley-Huggett-Aiyagari)
  - lack of insurance to idiosyncratic shocks
  - borrowing constraints
- Key for effects and channels of monetary policy
  - high Marginal Propensity to Consume (MPC)
  - low intertemporal substitution
- Large and active area in macro (Guerrieri-Lorenzoni, Farhi-Werning, Chamley, Beaudry-Galizia-Portier, Ravn-Sterk, Sheedy, McKay-Nakamura-Steinsson, Auclert, Werning, Kaplan-Moll-Violante etc.)

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark	?
Bounded Rationality	?	???

Outline

- General concept of level-k
- Representative agent with level-k
- Incomplete markets without level-k
- Incomplete markets with level-k
- Start: rigid prices or effects of real interest rates
- End: sticky prices and inflation

Rational Expectations

$$C_t = C^*(\{R_{t+s}\}, Y_t, \{Y^e_{t+1+s}\})$$

$$C_t = Y_t$$

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$$Y^e_{t+s} = Y_{t+s}$$

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Level-k Thinking

Level-1 thinking:

$$\hat{C}_t^1 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^1, \{Y_{t+1+s}\})$$

$$\hat{C}_t^1 = \hat{Y}_t^1$$

status quo REE

Level-k Thinking

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status quo REE

(almost PE effect! continuous time…)

Level-k Thinking

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status quo REE

(almost PE effect! continuous time…)

Level-2 thinking:

$$\hat{C}_t^2 = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t^2, \{\hat{Y}_{t+1+s}^1\})$$

$$\hat{C}_t^2 = \hat{Y}_t^2$$

level-1 thinking

Level-k Thinking

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Level-2 thinking:

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level-1 thinking

Level-k thinking: $\{\hat{Y}_t^{k+1}\} = \Gamma(\{\hat{Y}_t^k\})$

Note: REE is a fixed point!

Level-k Thinking

- Coincides with PE for $k = 1$
- Mitigates GE, less and less as k increases
- Converges to RE as $k \to \infty$
- Determinate for any k, without Taylor rule
- Can generalize to aggregate consumption functions depending on state variable Ψ for incomplete markets (wealth distribution)

Effects of Monetary Policy

- Elasticities of output to interest rates
  - at different horizons
  - PE, GE, level-k

$$\epsilon_{t,\tau} = \lim_{\Delta R_\tau \to 0} -\frac{R_\tau}{Y_t}\frac{\Delta Y_t}{\Delta R_\tau} \qquad \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^{k} = \lim_{\Delta R_\tau \to 0} -\frac{R_\tau}{Y_t}\frac{\Delta Y_t^k}{\Delta R_\tau} \qquad \epsilon_{t,\tau}^{k} = \epsilon_{t,\tau}^{k,PE} + \epsilon_{t,\tau}^{k,GE}$$

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Bounded Rationality	?	???

Representative Agent

- Representative agent (= complete markets)
- Continuous time
  - not crucial, but…
  - …partial equilibrium = level-1 thinking

$$\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} {c_t}^{1-\sigma} dt \qquad \text{s.t.} \qquad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt$$

$$p_t = e^{-\int_0^t r_s ds}$$

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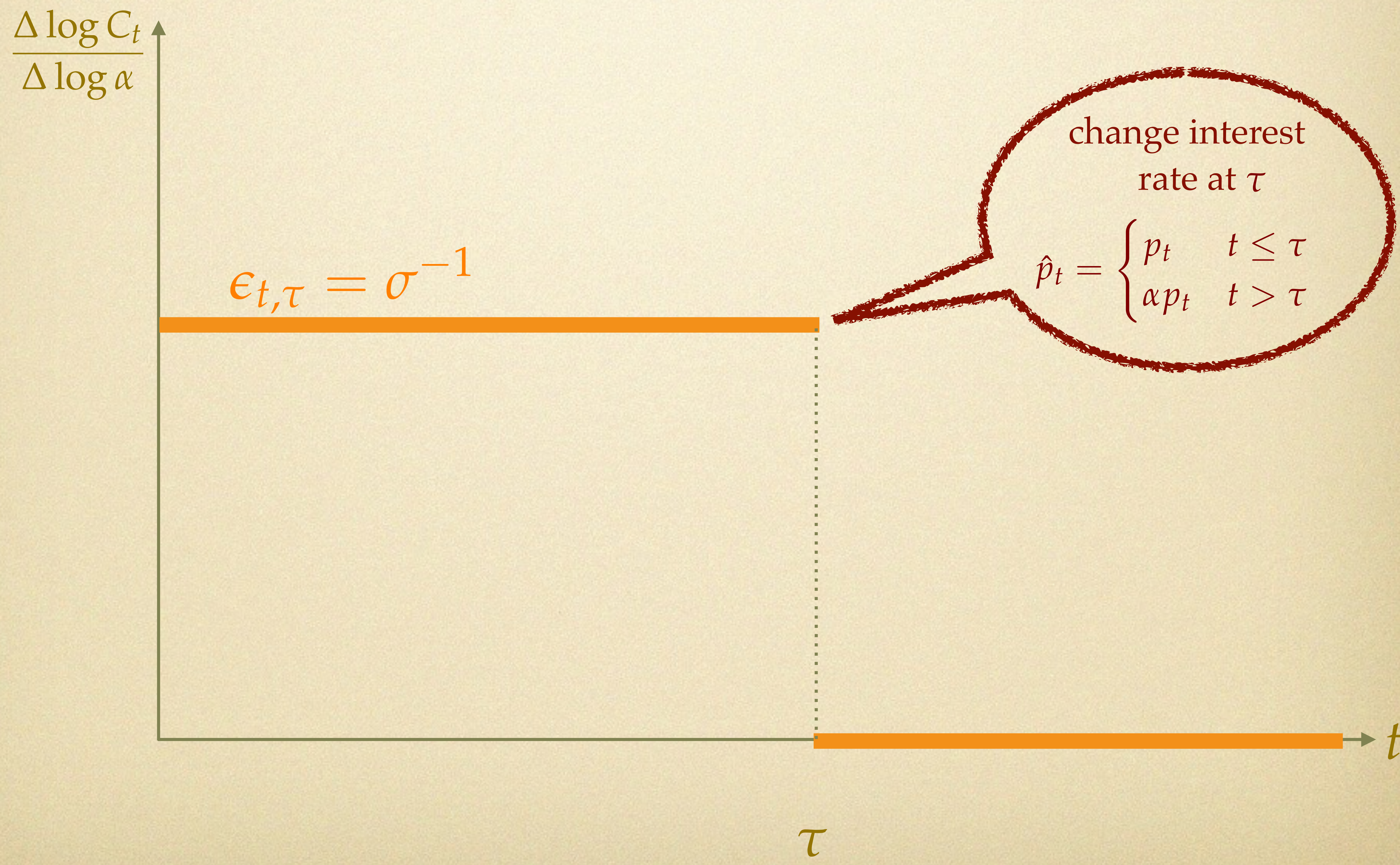

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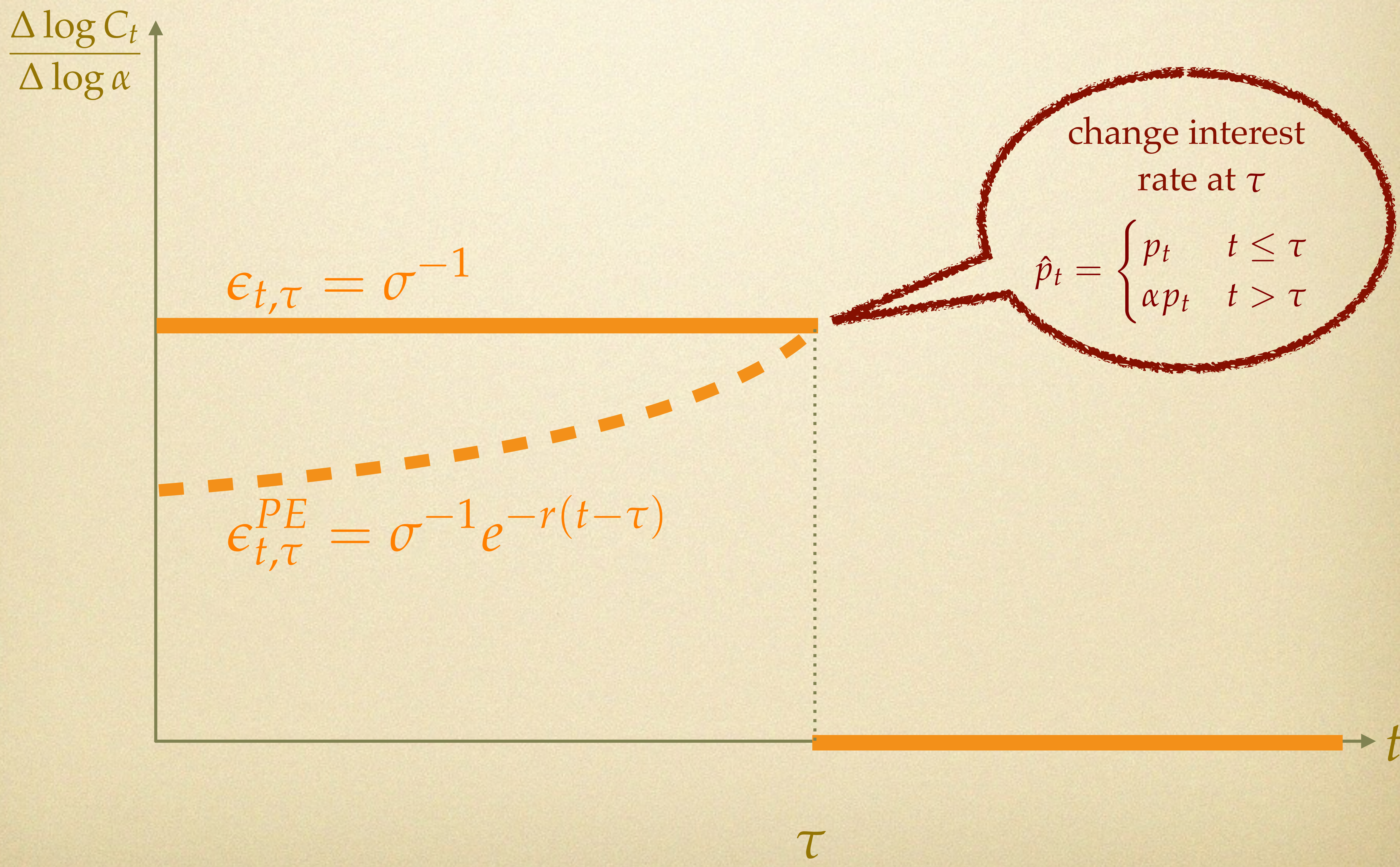

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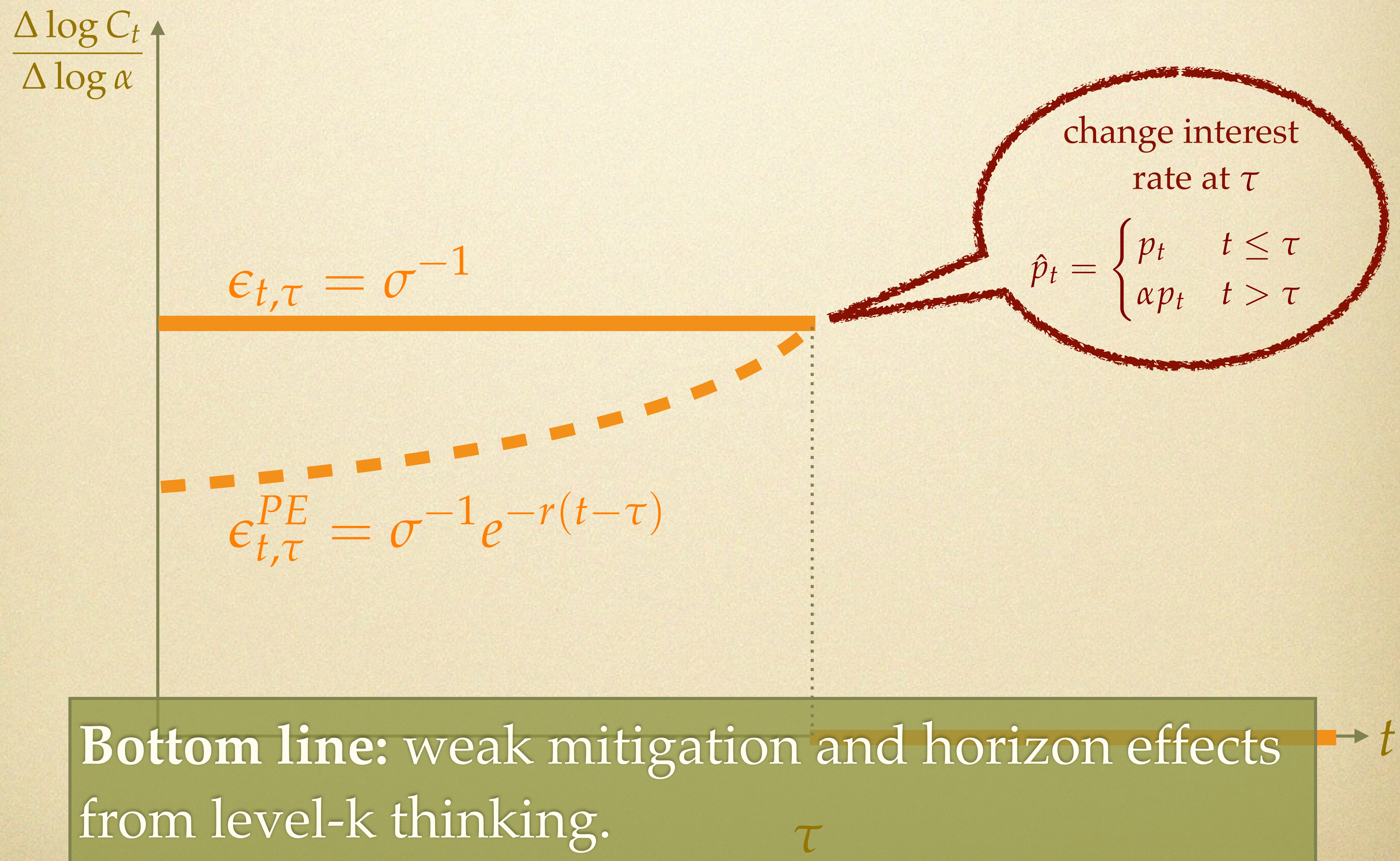

Bottom line: weak mitigation and horizon effects from level-k thinking.

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Incomplete Markets

- See e.g. Werning (2015)
- Benchmark neutrality result: “as if” rep. agent
- Subtle dependence on cyclicality of
  - income risk
  - liquidity

Keynesian Cross

- Liquidity constrained cannot substitute, so…
- Q: How can incomplete markets not affect aggregate response?
- A: General Equilibrium vs. Partial Equilibrium
  - some do substitute and increase their spending…
  - …increases income all around…
  - …raises spending of liquidity constrained more…
  - … increases income…. etc.

$$\downarrow \text{PE} \quad + \uparrow \text{GE} \quad = \quad \text{constant}$$

	Complete Markets	Incomplete Markets
Rational Expectation	benchmark →	small effects
Bounded Rationality	↓ small effects	???

Perpetual Youth
+
Aiyagari Simulations

Perpetual Youth Model

- Tractable model to easily visit all 4 squares!
- Continuum measure 1 of agents
- OLG with Poisson death and arrival $\pi \geq 0$
- Preferences

$$\int_0^\infty e^{-(\rho+\pi)s} \log(c^i_{t+s}) ds$$

- Income
  - labor income: $(1-\delta)Y_t$
  - Lucas tree dividend: δY_t
- Budget with annuities

$$\frac{da^i_t}{dt} = (r_t + \pi)a^i_t + Y_t - c^i_t$$

Perpetual Youth Model

- Alternative interpretation
  - agents do not die
  - life separated by stochastic "periods"
  - heavy discount across periods:
    - wish to borrow against future periods
    - but cannot do so!

- **OLG ~ borrowing constraints**
  - short or interrupted time horizons
  - no precautionary savings
  - linear consumption function and aggregation

Perpetual Youth Model

$$V_t = \int_t^\infty e^{-\int_t^s r_u du} \delta Y_s^e ds$$

$$H_t = \int_t^\infty e^{-\int_t^s (r_u + \pi) du} (1-\delta) Y_s^e ds$$

individual consumption function → $c_t^i = (\rho + \pi)(a_t^i + H_t)$

$$\int_0^1 a_t^i di = V_t \qquad \textbf{equilibrium} \qquad \int_0^1 c_t^i di = Y_t$$

$$C_t = (\rho + \pi)(V_t + H_t)$$
$$C_t = Y_t$$

← **aggregate consumption function**

Steady State

- Steady state

$$Y_t = Y$$

$$1 = (1-\delta)\frac{\rho+\pi}{r+\pi} + \delta\frac{\rho+\pi}{r}$$

- Comparative static ("MIT shock")
  - new path for interest rate
  - compute
    - rational expectations equilibrium
    - k-level thinking

Mitigation and Horizon

$$1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}$$

$$\epsilon_{t,\tau}^{PE} = (1-\delta)\frac{\rho+\pi}{r+\pi}e^{-(r+\pi)(\tau-t)} + \delta\frac{\rho+\pi}{r}e^{-r(\tau-t)}$$

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$$\frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0 \qquad \frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} < 0$$

$$\frac{\partial^2 \epsilon_{t,\tau}}{\partial \pi \partial \tau} = 0 \qquad \frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} < 0$$

Result. Complementarity between incomplete markets and bounded rationality.

Speed of Convergence

- Recall, level-1 = PE, level- ∞ = RE
- Level-k

$$\epsilon^k_{t,\tau} = (1-\delta)e^{-(\rho+\pi)(\tau-t)}\left[\sum_{\ell=1}^{k}\frac{(\rho+\pi)^{\ell-1}(\tau-t)^{\ell-1}}{(\ell-1)!}\right]$$

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Complementarity: Asymptotic convergence to RE slower for higher π.

Bewley-Aiagari-Hugget

- Assumptions:
  - idiosyncratic income uncertainty
  - no insurance
  - borrowing constraints

- Results:
  - occasionally binding borrowing constraints
  - precautionary savings
  - concave consumption functions (varying MPC)

- Monetary policy and bounded rationality?
  - general theoretical characterization
  - numerical simulations

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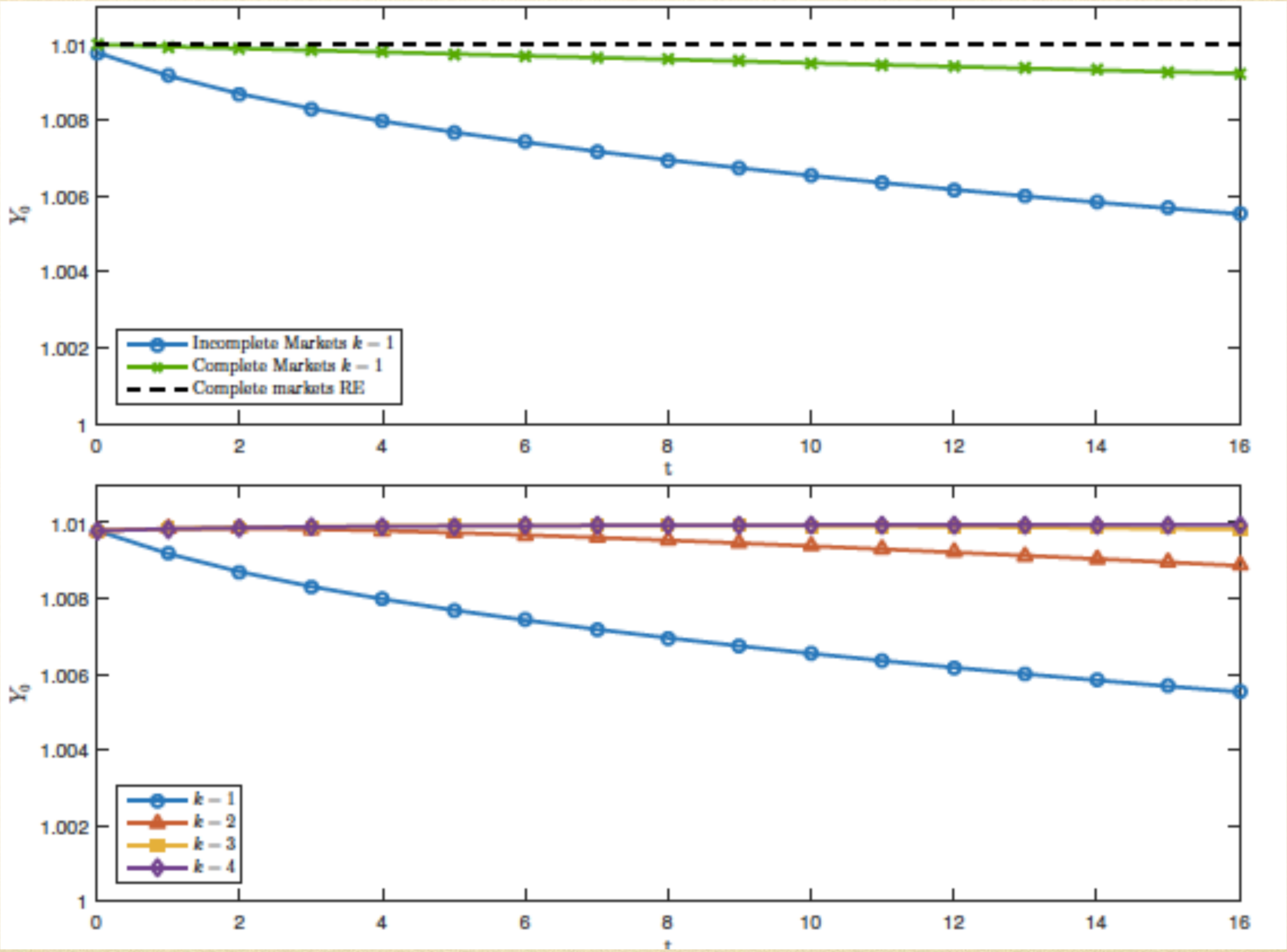
Bewley-Aiyagari-Huggett Model

- Bewley-Aiyagari-Huggett economy
- Discrete periods (quarters)
- Calibration
  - income process $\log y_t = \rho \log y_{t-1} + \epsilon_t$
    $\rho = 0.966 \quad \sigma_\epsilon = 0.017$
  - steady state interest rates at 2%
  - choose δ to match outside liquidity to output 1.44 (fraction of borrowing constrained agents 15%), as in McKay et al. (2016)

Simulations



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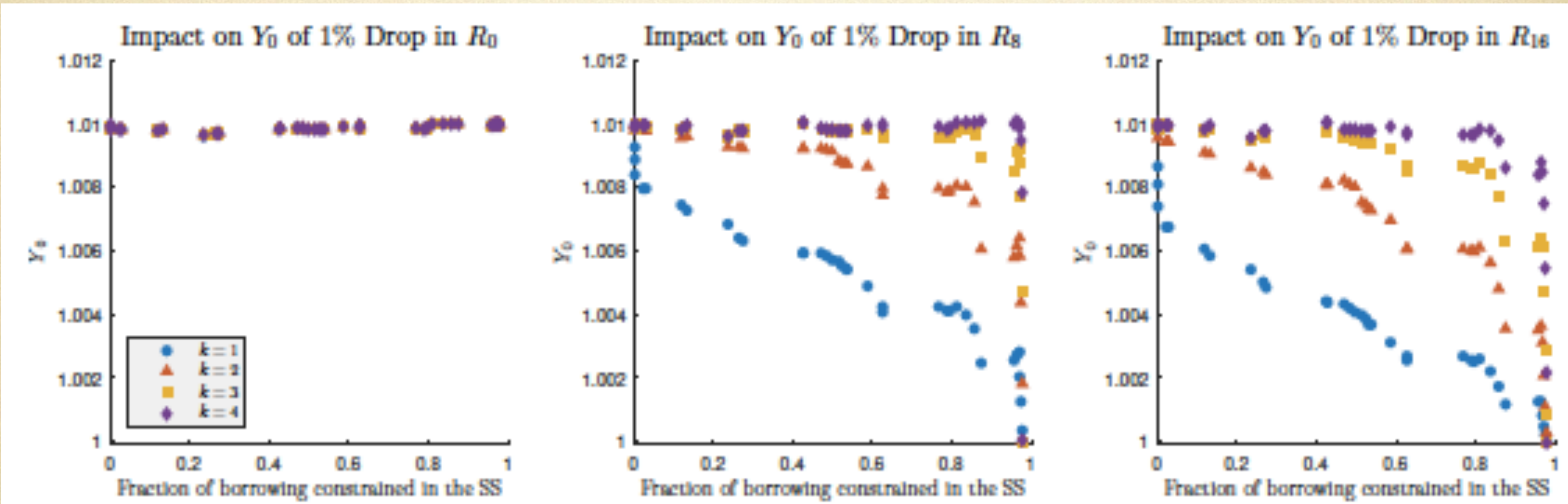

Sticky Prices

- So far: rigid prices or equivalently real interest rates
- Now: sticky prices
- Differences:
  - additional GE effect: output-inflation feedback loop
  - baseline representative agent features anti-horizon
  - can get big difference from level-k alone

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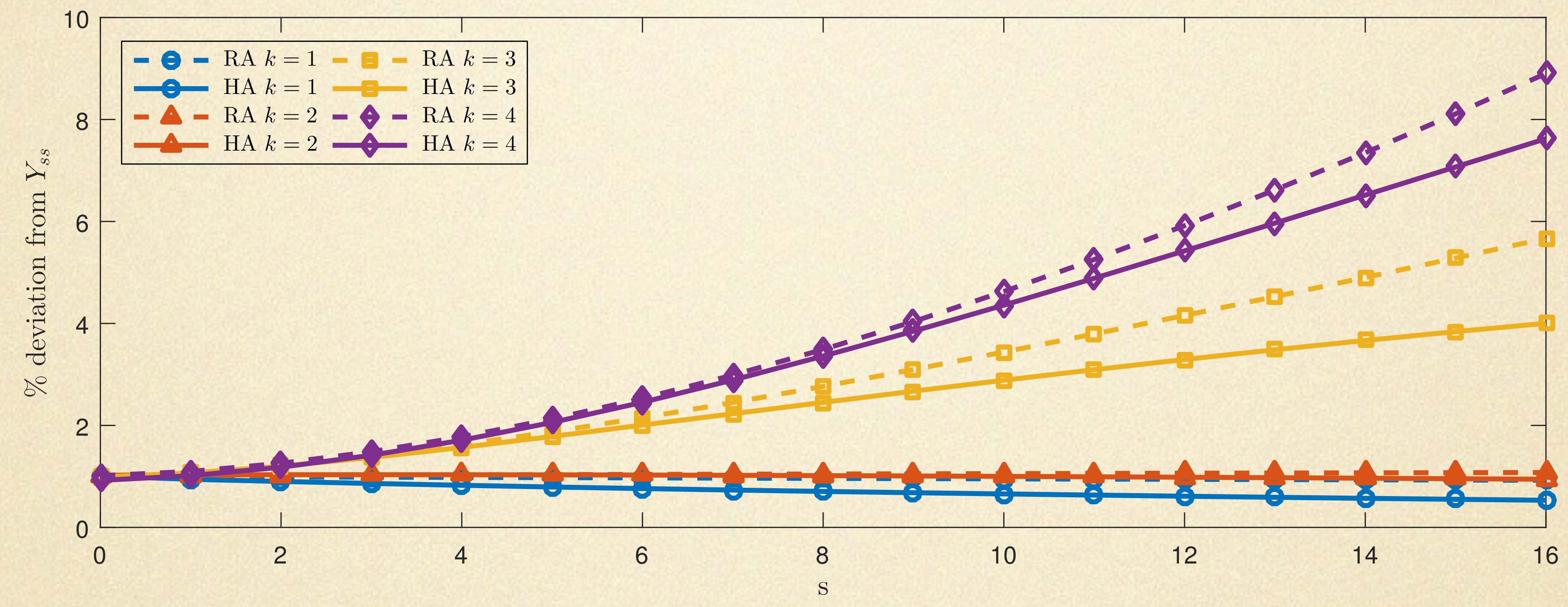

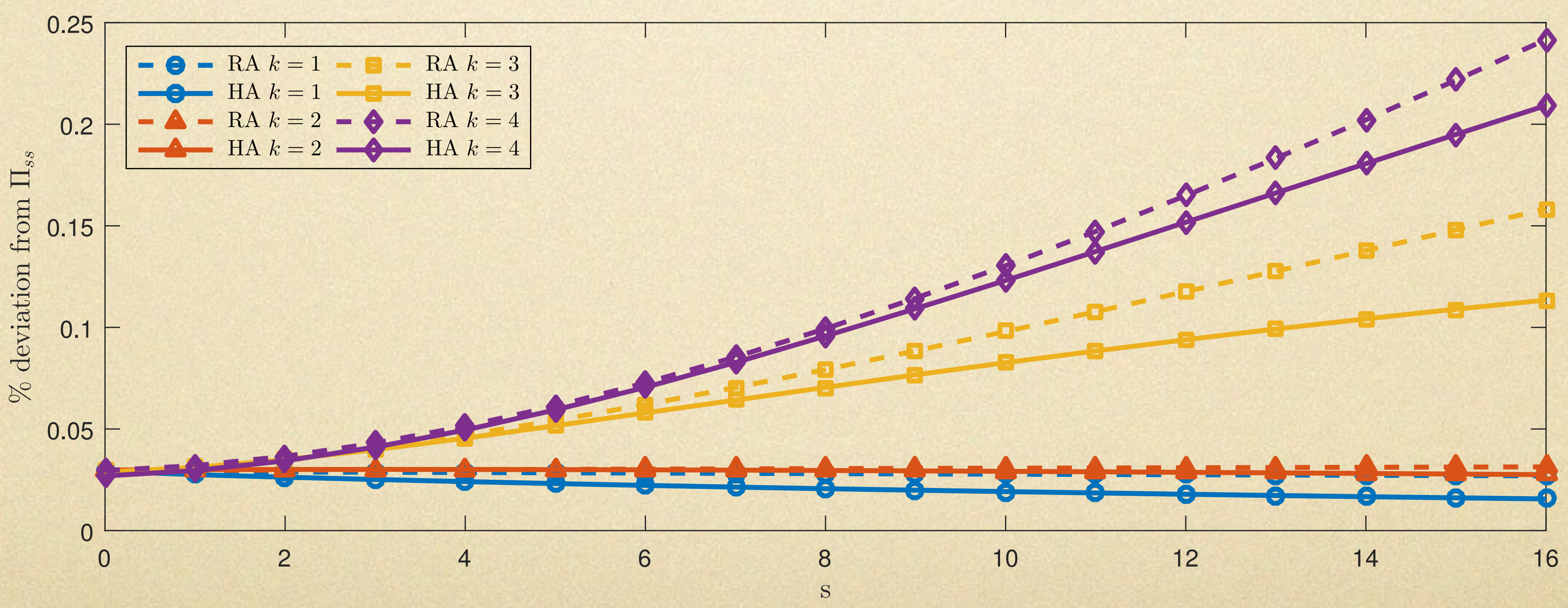

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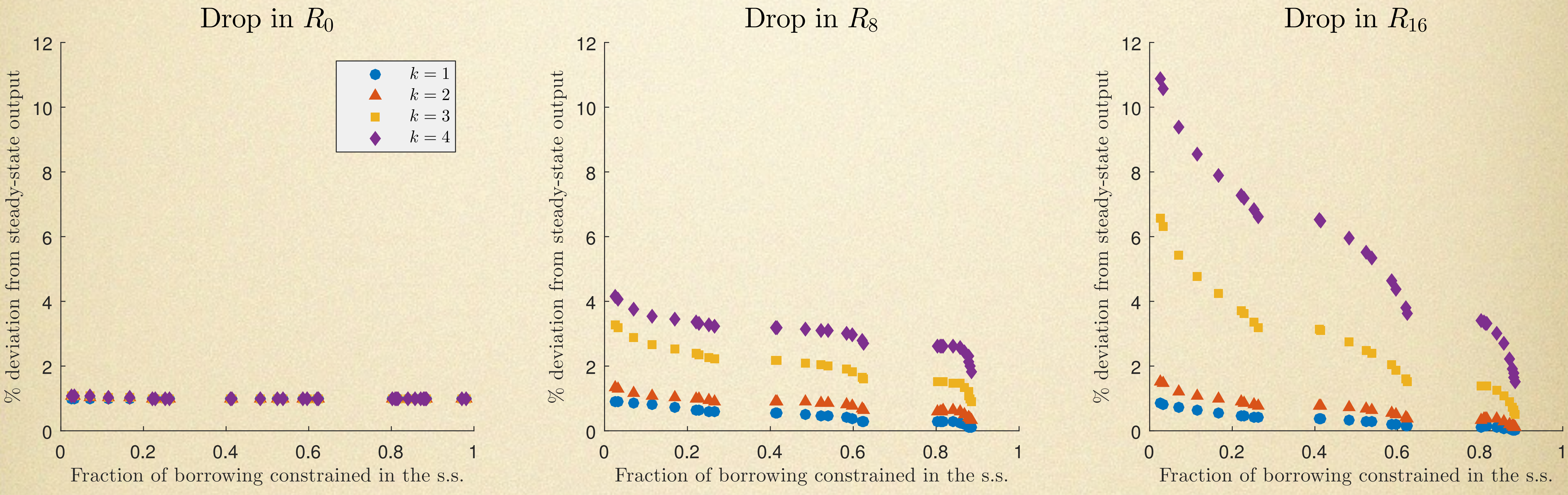

Conclusion

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Bounded Rationality	↓ small effects	large effects