Monetary Policy, Bounded Rationality and Incomplete Markets

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Motivation

How is monetary policy affected by
- Bounded Rationality?
- Incomplete Markets?
- Combination?

Paper: complementarities!
Motivation

- Helps fix “bugs” of standard NK model
  - indeterminacy given interest rate paths (Taylor principle)
  - Neo-Fisherian controversies
  - effectiveness of monetary policy
  - dependence on horizon (“forward guidance puzzle”)
  - effects of fiscal policy at ZLB (“fiscal multipliers puzzle”)
  - explosive nature of long-lasting liquidity traps
  - ...

Bounded Rationality

- Expectations management major (main) channel of policy transmission in NK model under RE

- Realistic?
  - incomplete information or inattention to policy announcement?
  - less than full understanding of its future effects?
Bounded Rationality

• “Inductive”
  - learning: extrapolate from past data rationally or irrationally (Sargent; Evans-Honkapohja; Shleifer)
  - incomplete info and inattention: ignore, underweight, cost to process info (Sims; Mankiw-Reis; Maćkowiak-Wiederholt; Gabaix; Angeletos-Lian)

• “Eductive”
  - robustness (Hansen-Sargent)
  - **level-k thinking**: think through reaction of others (Stahl-Wilson; Nagel; Crawford-Costa-Gomes-Iriberri; Evans-Ramey; Woodford; García-Schmidt-Woodford)

• **Level-k thinking**
  - credible and clear announcement policy change
  - with little past experience
  - agents think through consequences, with bounded rationality
Incomplete Markets

- Standard NK model: representative agent or complete markets

- Incomplete markets alternative (Bewley-Huggett-Aiyagari)
  - lack of insurance to idiosyncratic shocks
  - borrowing constraints

- Key for effects and channels of monetary policy
  - high Marginal Propensity to Consume (MPC)
  - low intertemporal substitution

- Large and active area in macro (Guerrieri-Lorenzoni, Farhi-Werning, Chamley, Beaudry-Galizia-Portier, Ravn-Sterk, Sheedy, McKay-Nakamura-Steinsson, Auclert, Werning, Kaplan-Moll-Violante etc.)
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Outline

• General concept of level-k
• Representative agent with level-k
• Incomplete markets without level-k
• Incomplete markets with level-k

• Start: rigid prices or effects of real interest rates
• End: sticky prices and inflation
Rational Expectations

\[ C_t = C^*(\{R_{t+s}\}, \gamma_t, \{\gamma_{t+1+s}^e\}) \]
\[ C_t = \gamma_t \]
Rational Expectations

\[ C_t = C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}^e\}) \]
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R.E. Equilibria. Solution for \( \{C_t, Y_t\} \) with
\[ Y_{t+s}^e = Y_{t+s} \]
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Comparative static \( \{R_{t+s}\} \rightarrow \{\hat{R}_{t+s}\} \)
Rational Expectations

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\[ \hat{C}_t - C_t = C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \]
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PE
Rational Expectations

\[ C_t = C^*(\{R_{t+s}\}, Y_t, \{Y^e_{t+1+s}\}) \]

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**R.E. Equilibria. Solution for \( \{C_t, Y_t\} \) with**

\[ Y^e_{t+s} = Y_{t+s} \]

Comparative static \( \{R_{t+s}\} \rightarrow \{\hat{R}_{t+s}\} \)

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\[ = C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) - C^*(\{R_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \]

\[ \text{PE} \]

\[ +C^*(\{\hat{R}_{t+s}\}, \hat{Y}_t, \{\hat{Y}_{t+1+s}\}) - C^*(\{\hat{R}_{t+s}\}, Y_t, \{Y_{t+1+s}\}) \]

\[ \text{GE} \]
Level-k Thinking

Level-1 thinking: \[
\hat{C}_t^1 = C^*([\hat{R}_t+s], \hat{Y}_t^1, \{Y_{t+1+s}\})
\]
\[
\hat{C}_t^1 = \hat{Y}_t^1
\]
Level-1 thinking:

$$\hat{C}^1_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}^1_t, \{Y_{t+1+s}\})$$

$$\hat{C}^1_t = \hat{Y}^1_t$$

(status quo REE)

(almost PE effect! continuous time...)}
Level-k Thinking

Level-1 thinking:
\[
\hat{C}^1_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}^1_t, \{Y_{t+1+s}\})
\]
\[
\hat{C}^1_t = \hat{Y}^1_t
\]

(status quo REE)

(almost PE effect! continuous time...)

Level-2 thinking:
\[
\hat{C}^2_t = C^* (\{\hat{R}_{t+s}\}, \hat{Y}^2_t, \{\hat{Y}^1_{t+1+s}\})
\]
\[
\hat{C}^2_t = \hat{Y}^2_t
\]

(level-1 thinking)
Level-k Thinking

Level-1 thinking:
\[
\hat{C}_t^1 = C^* (\{ \hat{R}_{t+s} \}, \hat{Y}_t^1, \{ Y_{t+1+s} \})
\]
\[
\hat{C}_t^1 = \hat{Y}_t^1
\]

status quo REE
(almost PE effect! continuous time...)

Level-2 thinking:
\[
\hat{C}_t^2 = C^* (\{ \hat{R}_{t+s} \}, \hat{Y}_t^2, \{ \hat{Y}_{t+1+s}^1 \})
\]
\[
\hat{C}_t^2 = \hat{Y}_t^2
\]

level-1 thinking

Level-k thinking:
\[
\{ \hat{Y}_t^{k+1} \} = \Gamma (\{ \hat{Y}_t^k \})
\]

Note: REE is a fixed point!
Level-k Thinking

- Coincides with PE for $k = 1$

- Mitigates GE, less and less as $k$ increases

- Converges to RE as $k \to \infty$

- Determinate for any $k$, without Taylor rule

- Can generalize to aggregate consumption functions depending on state variable $\Psi$ for incomplete markets (wealth distribution)
**Effects of Monetary Policy**

- Elasticities of output to interest rates
  - at different horizons
  - PE, GE, level-k

\[
\begin{align*}
\epsilon_{t,\tau} &= \lim_{\Delta R_\tau \to 0} - \frac{R_\tau}{Y_t} \frac{\Delta Y_t}{\Delta R_\tau} \\
\epsilon_{k, t, \tau} &= \lim_{\Delta R_\tau \to 0} - \frac{R_\tau}{Y_t} \frac{\Delta Y_t^k}{\Delta R_\tau}
\end{align*}
\]

\[
\epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE}
\]

\[
\epsilon_{k, t, \tau} = \epsilon_{k, t, \tau}^{PE} + \epsilon_{k, t, \tau}^{GE}
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Representative Agent

- Representative agent (= complete markets)

- Continuous time
  - not crucial, but...
  - ...partial equilibrium = level-1 thinking
\[
\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt
\]

\[p_t = e^{-\int_0^t r_s ds}\]
\[
\max \left\{ c_t \right\} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt
\]

\[ p_t = e^{-\int_0^t r_s ds} \]

\[
\frac{\Delta \log C_t}{\Delta \log \alpha}
\]

\[ t \]

start at steady state
\[
\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt
\]

\[p_t = e^{-\int_0^t r_s ds}\]

\[\Delta \log C_t \quad \Delta \log \alpha\]

\[\epsilon_{t,\tau} = \sigma^{-1}\]

\[\hat{p}_t = \begin{cases} p_t & t \leq \tau \\ \alpha p_t & t > \tau \end{cases}\]

change interest rate at \(\tau\)
\[
\max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t dt = \int_0^\infty p_t y_t dt
\]

\[p_t = e^{-\int_0^t r_s ds}\]

\[\epsilon_{t,\tau} = \sigma^{-1}\]

\[\epsilon_{t,\tau}^{PE} = \sigma^{-1} e^{-r(t-\tau)}\]

\[\hat{p}_t = \begin{cases} 
  p_t & t \leq \tau \\
  \alpha p_t & t > \tau 
\end{cases}\]

change interest rate at \(\tau\)
Bottom line: weak mitigation and horizon effects from level-k thinking.

\[ \max_{\{c_t\}} \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} c_t^{1-\sigma} \, dt \quad \text{s.t.} \quad \int_0^\infty p_t c_t \, dt = \int_0^\infty p_t y_t \, dt \]

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Incomplete Markets

- See e.g. Werning (2015)

- Benchmark neutrality result: “as if” rep. agent

- Subtle dependence on cyclicality of
  - income risk
  - liquidity
Keynesian Cross

- Liquidity constrained cannot substitute, so...

- Q: How can incomplete markets not affect aggregate response?

- A: General Equilibrium vs. Partial Equilibrium
  - some do substitute and increase their spending...
  - ...increases income all around...
  - ...raises spending of liquidity constrained more...
  - ... increases income.... etc.

\[ \downarrow \text{PE} + \uparrow \text{GE} = \text{constant} \]
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Perpetual Youth + Aiyagari Simulations
Perpetual Youth Model

- Tractable model to easily visit all 4 squares!
- Continuum measure 1 of agents
- OLG with Poisson death and arrival $\pi \geq 0$
- Preferences
  $$\int_0^\infty e^{-(\rho + \pi)s} \log(c_{t+s}^i) ds$$
- Income
  - labor income: $(1 - \delta)Y_t$
  - Lucas tree dividend: $\delta Y_t$
- Budget with annuities
  $$\frac{d a_t^i}{dt} = (r_t + \pi) a_t^i + Y_t - c_t^i$$
Perpetual Youth Model

- Alternative interpretation
  - agents do not die
  - life separated by stochastic “periods”
  - heavy discount across periods:
    - wish to borrow against future periods
    - but cannot do so!

- OLG ~ borrowing constraints
  - short or interrupted time horizons
  - no precautionary savings
  - linear consumption function and aggregation
Perpetual Youth Model

\[ V_t = \int_t^\infty e^{-\int_t^s r_u du} \delta \gamma_s^e ds \]

\[ H_t = \int_t^\infty e^{-\int_t^s (r_u + \pi) du} (1 - \delta) \gamma_s^e ds \]

Individual consumption function

\[ c_t^i = (\rho + \pi)(a_t^i + H_t) \]

\[ \int_0^1 a_t^i di = V_t \quad \text{equilibrium} \quad \int_0^1 c_t^i di = \gamma_t \]

Aggregate consumption function

\[ C_t = (\rho + \pi)(V_t + H_t) \]

\[ C_t = \gamma_t \]
Steady State

- Steady state
  \[ Y_t = Y \]
  \[ 1 = (1 - \delta) \frac{\rho + \pi}{r + \pi} + \delta \frac{\rho + \pi}{r} \]

- Comparative static ("MIT shock")
  - new path for interest rate
  - compute
    - rational expectations equilibrium
    - k-level thinking
Mitigation and Horizon

\[ 1 = \varepsilon_{t,\tau} = \varepsilon_{t,\tau}^{PE} + \varepsilon_{t,\tau}^{GE} \]

\[ \varepsilon_{t,\tau}^{PE} = (1 - \delta) \frac{\rho + \pi}{r + \pi} e^{-(r+\pi)(\tau-t)} + \delta \frac{\rho + \pi}{r} e^{-r(\tau-t)} \]
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Mitigation and Horizon

\[ 1 = \epsilon_{t,\tau} = \epsilon_{t,\tau}^{PE} + \epsilon_{t,\tau}^{GE} \]

\[ \epsilon_{t,\tau}^{PE} = (1 - \delta) \left( r + \pi \right) e^{-(r+\pi)(\tau-t)} + \delta \left( r + \pi \right) e^{-r(\tau-t)} \]

\[ \frac{\partial \epsilon_{t,\tau}}{\partial \pi} = 0 \quad \frac{\partial \epsilon_{t,\tau}^{PE}}{\partial \pi} < 0 \]

\[ \frac{\partial^2 \epsilon_{t,\tau}}{\partial \pi \partial \tau} = 0 \quad \frac{\partial^2 \epsilon_{t,\tau}^{PE}}{\partial \pi \partial \tau} < 0 \]

Result. Complementarity between incomplete markets and bounded rationality.
Speed of Convergence

- Recall, level-1 = PE, level-∞ = RE
- Level-k

\[ e_{t,\tau}^k = (1 - \delta)e^{-(\rho+\pi)(\tau-t)} \left[ \sum_{\ell=1}^{k} \frac{(\rho + \pi)^{\ell-1}(\tau - t)^{\ell-1}}{[(\ell - 1)!]} \right] \]

\[ + \delta e^{-\rho(\tau-t)} \left[ \sum_{\ell=1}^{k} \frac{\rho^{\ell-1}(\tau - t)^{\ell-1}}{[(\ell - 1)!]} \right]. \]
Speed of Convergence

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Complementarity: Asymptotic convergence to RE slower for higher \( \pi \).
Assumptions:
- idiosyncratic income uncertainty
- no insurance
- borrowing constraints

Results:
- occasionally binding borrowing constraints
- precautionary savings
- concave consumption functions (varying MPC)

Monetary policy and bounded rationality?
- general theoretical characterization
- numerical simulations
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Monetary policy and bounded rationality?
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Result. Complementarity between incomplete markets and bounded rationality.
Bewley-Aiyagari-Huggett Model

- Bewley-Aiyagari-Huggett economy
- Discrete periods (quarters)

Calibration
- income process \( \log y_t = \rho \log y_{t-1} + \epsilon_t \)
  \[ \rho = 0.966 \quad \sigma_\epsilon = 0.017 \]
- steady state interest rates at 2%
- choose \( \delta \) to match outside liquidity to output 1.44 (fraction of borrowing constrained agents 15%), as in McKay et al. (2016)
Simulations
Simulations
Sticky Prices

- So far: rigid prices or equivalently real interest rates

- Now: sticky prices

- Differences:
  - additional GE effect: output-inflation feedback loop
  - baseline representative agent features anti-horizon
  - can get big difference from level-k alone
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Simulations

Figure 3: Proportional output response $e_k$, $t$ and inflation response $e_{P,k}$ at date 0 to a 1% interest rate cut at different horizons $t$ for the baseline incomplete-markets economy (dashed lines) and the complete-markets or representative-agent economy (solid lines). Different colors represent equilibrium output under level-$k$-thinking with different values of $k$.

Figure 4: Proportional output response $e_k$, $t$ at date 0 to a 1% interest rate cut at a horizon of $t = 0$, $t = 8$ quarters, and $t = 16$ quarters. Different colors represent equilibrium output under level-$k$-thinking with different values of $k$. Different dots of the same color correspond to economies with different fractions of borrowing-constrained agents in steady state. This variation is achieved by varying the discount factor $b$ and amount of liquidity $d$ and keeping the steady-state annual interest rate constant at 2%.
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## Conclusion

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