

The Intertemporal Keynesian Cross

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- Q:** How does the macroeconomy propagate shocks?
- what micro moments are important?
 - Recent literature: **MPCs** are crucial for **PE** effects
 - Idea: for **PE impact response to shocks**, want models to be consistent with evidence on C response to Y
 - Applications: fiscal policy [Kaplan-Violante], monetary policy [Auclert], house prices [Berger et al], ...
 - In GE, C now and in future affects everyone's Y
 - Here: “**intertemporal MPCs**” (**iMPCs**) are crucial for the **GE impulse response**

Application: When is the fiscal multiplier large?

- Lots of theory + empirical work. Two workhorse models:

1. **Representative-agent (RA)** models

- **response of monetary policy** is key
- large when at ZLB

[Eggertsson 2004; Christiano-Eichenbaum-Rebelo 2011]

2. **Two-agent (TA)** models

- aggregate **MPC** is key
- large when deficit financed, effects not persistent

[Galí-López-Salido-Vallés 2007; Coenen et al 2012; Farhi-Werning 2017]

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New: Heterogeneous-agent (HA) models

- **iMPCs** are key, can be used for calibration
- large and persistent Y effect when deficit financed

Our contribution: Interaction of iMPCs and deficit-financing

1. **Benchmark model**, allows for RA, TA, HA
 - without capital & constant-real-rate monetary policy
 - multiplier = function of **iMPCs** and **deficits** *only*
 - = **1** if zero deficits or flat iMPCs (RA) [Woodford 2011]
 - > **1** if **deficit-financed** and **realistic iMPCs** (HA, TA?)

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 - large & persistent Y effects, despite these extra elements
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2. **Quantitative model** with capital & Taylor rule
 - large & persistent Y effects, despite these extra elements
 - iMPCs still crucial for Y response
3. Role of **iMPCs** for the **GE effects of other shocks**
 - Today (not in paper): monetary policy

- 1 The intertemporal Keynesian Cross
- 2 iMPCs in models vs. data
- 3 Fiscal policy in the benchmark model
- 4 Fiscal policy in the quantitative model
- 5 Other shocks

The intertemporal Keynesian Cross

- GE, discrete time $t = 0 \dots \infty$, no aggregate risk (MIT shocks)
- Mass 1 of households:
 - idiosyncratic shocks to skills e_{it} , various market structures
 - real pre-tax income $y_{it} \equiv W_t/P_t e_{it} n_{it}$
 - **after tax income** $z_{it} \equiv y_{it} - T_t(y_{it}) \equiv \tau_t y_{it}^{1-\lambda}$ [Bénabou, HSV]

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 - **tax revenues** $T_t = \int (y_{it} - z_{it}) di$
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 - monetary policy: fixed real rate $= r$

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- Supply side:
 - linear production function $Y_t = N_t$
 - flexible prices $\Rightarrow P_t = W_t$
 - sticky $W_t \Rightarrow \pi_t^w = \kappa^w \int N_t (v'(n_{it}) - \frac{\partial z_{it}}{\partial n_{it}} u'(c_{it})) di + \beta \pi_{t+1}^w$

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rationing

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rationing

relax later

Nesting four types of households

Household i solves

$$\max \mathbb{E} \left[\sum \beta^t \{u(c_{it}) - v(n_{it})\} \right]$$

$$c_{it} + a_{it} = (1 + r) a_{it-1} + z_{it}$$

- **RA**: no risk in e (or complete markets)
- **TA**: share μ of agents with $c_{it} = z_{it}$
- **HA-std**: one asset model, with constraint $a_{it} \geq 0$
- **HA-illiq**: simplified two asset model
 - **illiquid** account $a^{illiq} = \mathbf{fixed}$ no. of bonds (+ capital)
 - **liquid** account $a_{it} = \mathbf{all remaining}$ bonds + ra^{illiq}

- Given $\{a_{i0}\}$ and r , **aggregate consumption function** is

$$C_t = \int c_{it} di = C_t(\{Z_s\}_{s=0}^{\infty})$$

[Farhi Werning 2017, Kaplan Moll Violante 2017, ...]

with $Z_t \equiv$ aggregate after-tax labor income

$$Z_t \equiv \int z_{it} di = Y_t - T_t$$

- C summarizes the heterogeneity and market structure
- Equilibrium defined as usual

Intertemporal MPCs

- An output path $\{Y_t\}_{t=0}^{\infty}$ is part of equilibrium \Leftrightarrow

$$Y_t = G_t + C_t(\{Y_s - T_s\}) \quad \forall t \geq 0$$

- Impulse response to shock $\{dG_t, dT_t\}$

$$dY_t = dG_t + \sum_{s=0}^{\infty} \underbrace{\frac{\partial C_t}{\partial Z_s}}_{\equiv M_{t,s}} \cdot (dY_s - dT_s) \quad (1)$$

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→ Response $\{dY_t\}$ entirely characterized by $\{M_{t,s}\}$!

- *partial equilibrium* derivatives, **“intertemporal MPCs”**
- how much of income change at date s is spent at date t
- $\sum_{t=0}^{\infty} (1+r)^{s-t} M_{t,s} = 1$

The intertemporal Keynesian cross

- Stack objects: $\mathbf{M} = \{M_{t,s}\} = \left\{ \frac{\partial C_t}{\partial Z_s} \right\}$, $d\mathbf{Y} = \{dY_t\}$, etc
- Rewrite equation (1) as

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- This is an **intertemporal Keynesian cross**
 - entire complexity of model is in \mathbf{M}
 - with \mathbf{M} from data, could get $d\mathbf{Y}$ without model simulations!

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- This is an **intertemporal Keynesian cross**
 - entire complexity of model is in \mathbf{M}
 - with \mathbf{M} from data, could get $d\mathbf{Y}$ without model simulations!
- When unique, solution is

$$d\mathbf{Y} = \mathcal{M} \cdot (d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

where \mathcal{M} is (essentially) $(I - \mathbf{M})^{-1}$

iMPCs in models vs. data

Measuring aggregate iMPCs using individual iMPCs

- Object of interest: **(aggregate) iMPCs**

$$M_{t,s} = \frac{\partial C_t}{\partial Z_s}$$

where $C_t = \int c_{it} di$ and $Z_s = \int z_{is} di$

- Direct evidence on $M_{t,s}$ is hard to come by for general s
- More work on column $s = 0$ (unanticipated income shock)
- Can write

$$M_{t,0} = \int \underbrace{\frac{z_{i0}}{Z_0}}_{\text{income weight}} \cdot \underbrace{\frac{\partial c_{it}}{\partial z_{i0}}}_{\text{individual iMPC}} di$$

→ aggregate iMPCs are **weighted individual iMPCs**

Obtain date-0 iMPCs from cross-sectional microdata

- Two sources of evidence on $\frac{\partial c_{it}}{\partial z_{i0}}$:

1. Fagereng Holm Natvik (2018) measure in Norwegian data

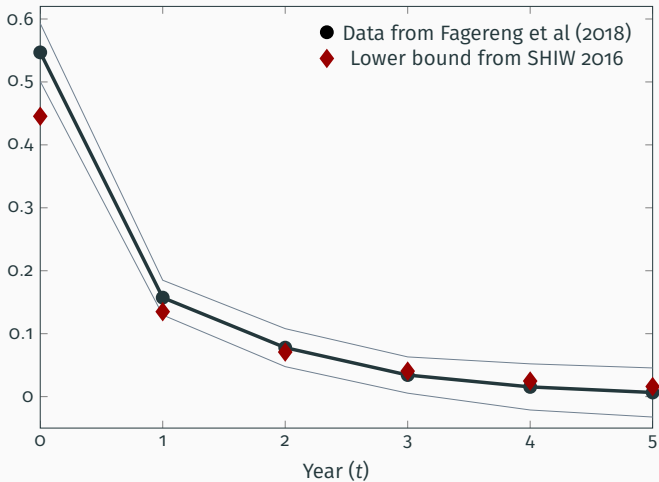
$$c_{it} = \alpha_i + \tau_t + \sum_{k=0}^5 \gamma_k \text{lottery}_{i,t-k} + \theta x_{it} + \epsilon_{it}$$

- Weighting by income in year of lottery receipt $\Rightarrow M_{t,0}$

2. Italian survey data (SHIW 2016) on $\frac{\partial c_{i0}}{\partial z_{i0}}$

- Lower bound for $M_{t,0}$ using distribution of MPCs
- Example: income-weighted average of $(1 - MPC_i)MPC_i \Rightarrow$ lower bound for $M_{1,0}$

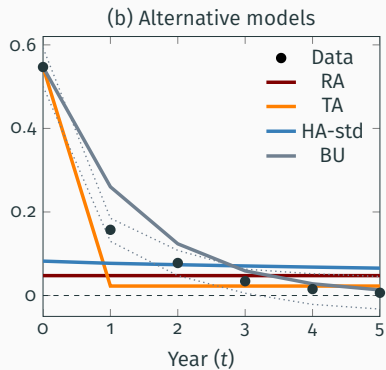
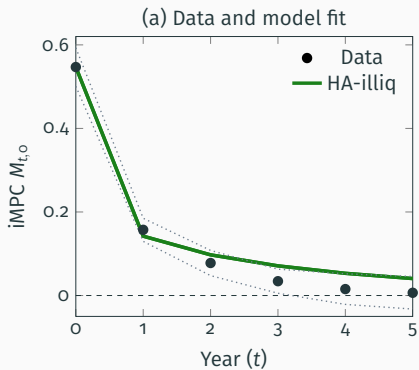
iMPCs in the data



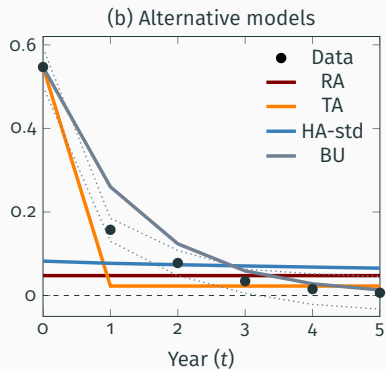
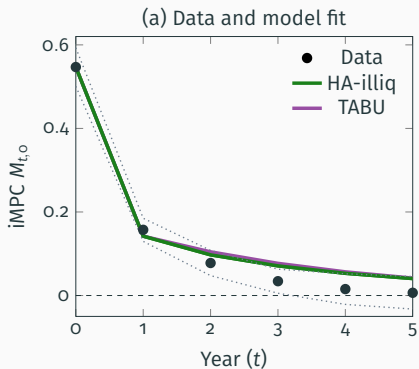
- Annual $M_{0,0}$ consistent with evidence from other sources

- **RA**
- **TA**: share of hand-to-mouth calibrated to match $M_{0,0}$
- **HA-std**: one-asset HA, standard calibration
- **HA-illiq**: two-asset HA calibrated to match $M_{0,0}$
- ... and for fun:
 - **BU**: bonds-in-utility model, calibrated to match $M_{0,0}$
[Michaillat Saez 2018; Hagedorn 2018; Kaplan Violante 2018]

iMPCs across models



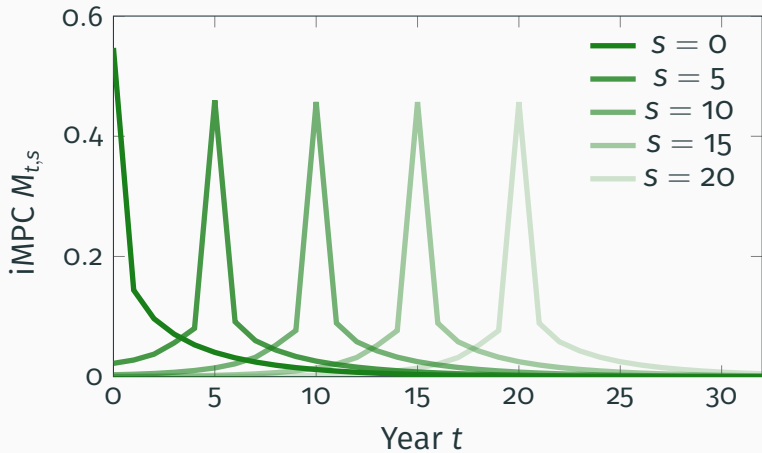
iMPCs across models including TABU



What about non-date-o iMPCs?

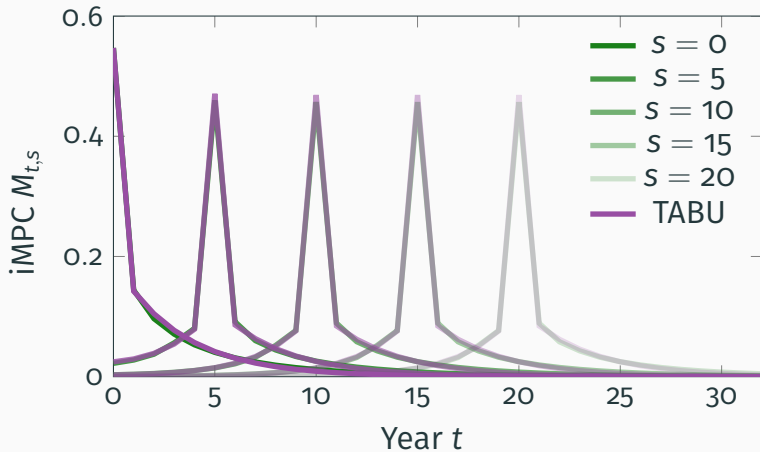
- Existing evidence useful for response to date-o income shocks, $\{M_{t,o}\}$
 - What about response to future shocks?
- rely on calibrated **HA-illiq** model to fill in the blanks!

Response of HA-illiq to other income shocks



Not entirely arbitrary \rightarrow TABU is very similar!

► Durables



Fiscal policy in the benchmark model

- Recall **intertemporal Keynesian cross**:

$$dY = dG - M \cdot dT + M \cdot dY$$

- dY entirely determined by iMPCs M and fiscal policy (dG, dT)
- Next: Characterize role of iMPCs for
 1. balanced budget policies, $dG = dT$
 2. deficit-financed policies

The balanced-budget unit multiplier

- With **balanced budget**, $dG = dT \Rightarrow$ **multiplier of 1:**

$$dY = dG$$

- Similar reasoning already in Haavelmo (1945)
- Generalizes Woodford's **RA** results
 - heterogeneity irrelevant for balanced budget fiscal policy
 - similar to Werning (2015)'s result for monetary policy
- Proof: $dY = dG$ is unique solution to

$$dY = (I - M) \cdot dG + M \cdot dY$$

Deficit-financed fiscal policy

- With deficit financing $d\mathbf{G} \neq d\mathbf{T}$ we have

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

Consumption $d\mathbf{C}$ depends on **primary deficits** $d\mathbf{G} - d\mathbf{T}$

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- Example: **TA model** with deficit financing

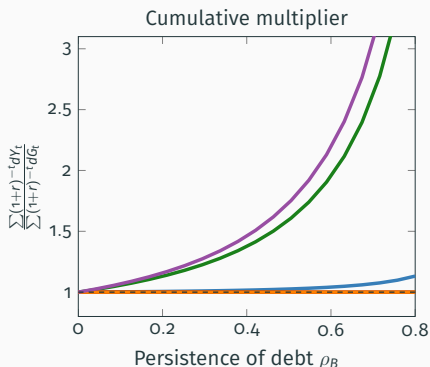
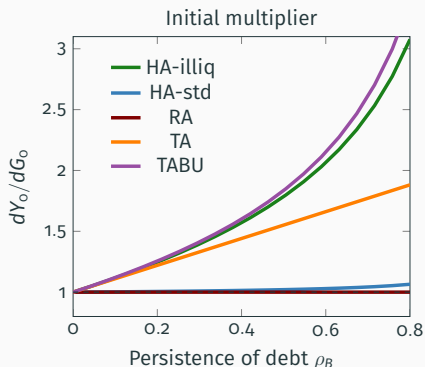
$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1 - \mu} (d\mathbf{G} - d\mathbf{T})$$

- consumption $d\mathbf{C}$ depends only on **current** deficits
- **initial multiplier** can be large $\in \left[1, \frac{1}{1-\mu}\right] \dots$
- but **cumulative multiplier** is = 1!

$$\frac{\sum (1+r)^{-t} dY_t}{\sum (1+r)^{-t} dG_t} = 1$$

- Parametrize: $dG_t = \rho_G dG_{t-1}$ and $dB_t = \rho_B (dB_{t-1} + dG_t)$
 - vary **degree of deficit-financing** ρ_B

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Fiscal policy in the quantitative model

- **Government:**

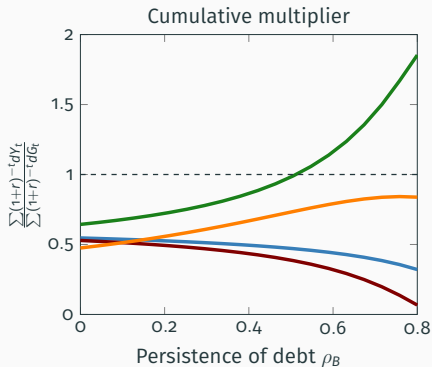
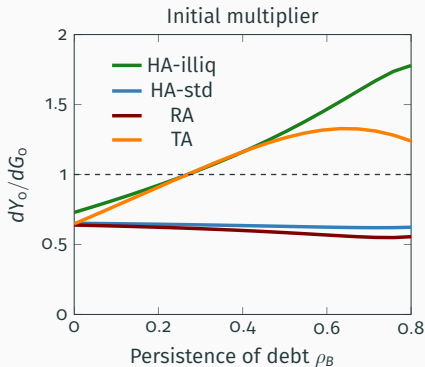
- gov spending shock, $dG_t = \rho_G dG_{t-1}$
- fiscal rule, $dB_t = \rho_B (dB_{t-1} + dG_t)$
- Taylor rule, $i_t = r_{ss} + \phi \pi_t, \phi > 1$

- **Supply side:**

- Cobb-Douglas production, $Y_t = K_t^\alpha N_t^{1-\alpha}$
- K_t subject to quadratic capital adjustment costs
- sticky prices à la Calvo, $\pi_t = \kappa^P mc_t + \frac{1}{1+r_t} \pi_{t+1}$

- **Two reasons for lower multipliers:**

- distortionary taxation & crowding-out of investment



Calibration: $\rho_G = 0.7$, $\kappa^W = \kappa^P = 0.1$, $\phi = 1.5$

Other shocks

What can we learn for other shocks? – back to benchmark

- Aggregate consumption may depend on **other shocks** θ ,

$$C_t = C_t(\{Z_s\}, \theta)$$

[e.g. deleveraging, inequality, preferences, **mon. policy**]

- Can generalize intertemporal Keynesian cross as

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \underbrace{C_\theta d\theta}_{\equiv \partial \mathbf{C}} + \mathbf{M}d\mathbf{Y}$$

→ **iMPCs also determine propagation of other shocks**

$$d\mathbf{Y} = d\mathbf{G} + \mathcal{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T}) + \mathcal{M}\partial \mathbf{C}$$

Monetary policy experiment

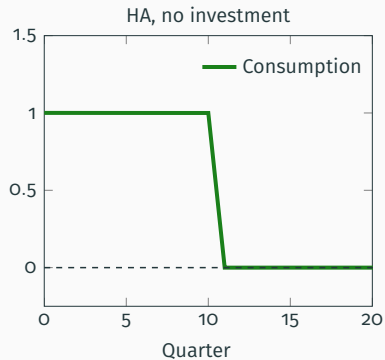
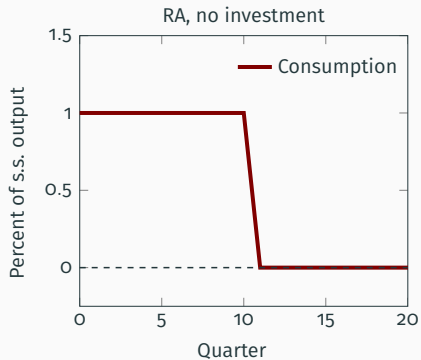
- Economy starts in steady state
- Monetary policy sets $\{r_t\}$ according to

$$r_t = \begin{cases} r & t \neq T \\ r - dr & t = T \end{cases}$$

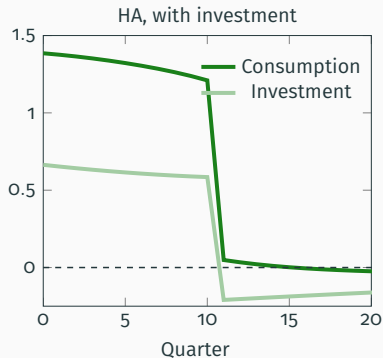
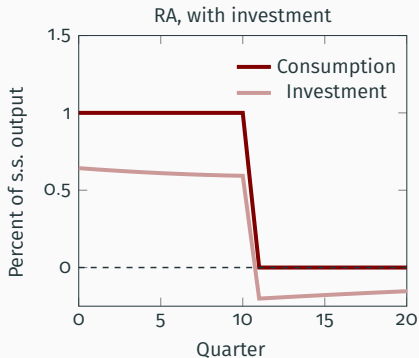
with shock at **horizon** $t = T$

- **Next:** Compare responses
 - **RA** vs **HA-illiq** (matching iMPCs)
 - **investment** vs **no investment** ($\delta = 0, \infty$ adj. costs)

No investment: $RA \sim HA$ (Werning 2015)



With investment: **HA** is amplified, \gg **RA**



→ “Forward guidance is more powerful than you think!”

M matters for **Macro** !

- crucial for GE propagation
- new insights for fiscal policy

New avenues: {
more evidence on **M**
implications for other shocks

Extra slides

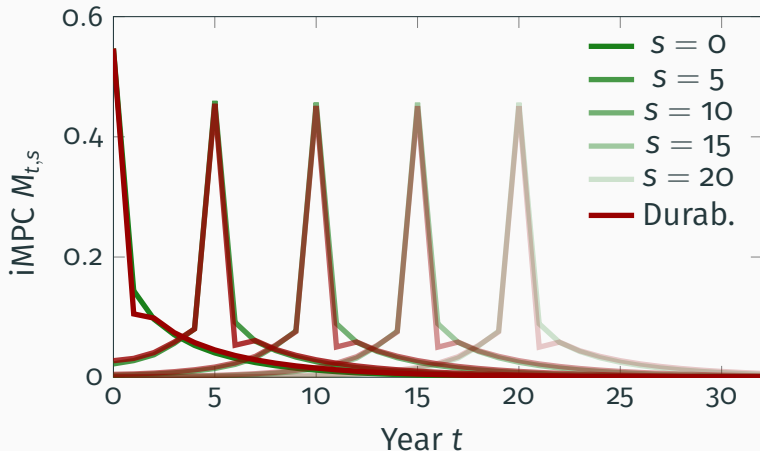
- Mass 1 of unions. Each union k
 - employs every individual, $n_i \equiv \int n_{ik} dk$
 - produces task $N_k = \int e_i n_{ik} di$ from member hours
 - pays common wage w_k per efficient unit of work e
 - requires that all individuals work $n_{ik} = N_k$
- Final good firms aggregate $N \equiv \left(\int_0^1 N_k^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$
- Union k sets w_{kt} each period to maximize

$$\max_{w_{kt}} \sum_{\tau \geq 0} \beta^\tau \left\{ \int \{u(c_{it+\tau}) - v(n_{it+\tau})\} di - \frac{\psi}{2} \left(\frac{w_{kt+\tau}}{w_{kt+\tau-1}} \right)^2 \right\}$$

- \Rightarrow nonlinear wage Phillips curve

$$(1 + \pi_t^w) \pi_t^w = \frac{\epsilon}{\psi} \int N_t \left(v'(n_{it}) - \frac{\epsilon-1}{\epsilon} \frac{\partial z_{it}}{\partial n_{it}} u'(c_{it}) \right) di + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)$$

- Given $\{G_t, T_t\}$, a **general equilibrium** is a set of prices, household decision rules and quantities s.t. at all t :
 1. firms optimize
 2. households optimize
 3. fiscal and monetary policy rules are satisfied
 4. the goods market clears



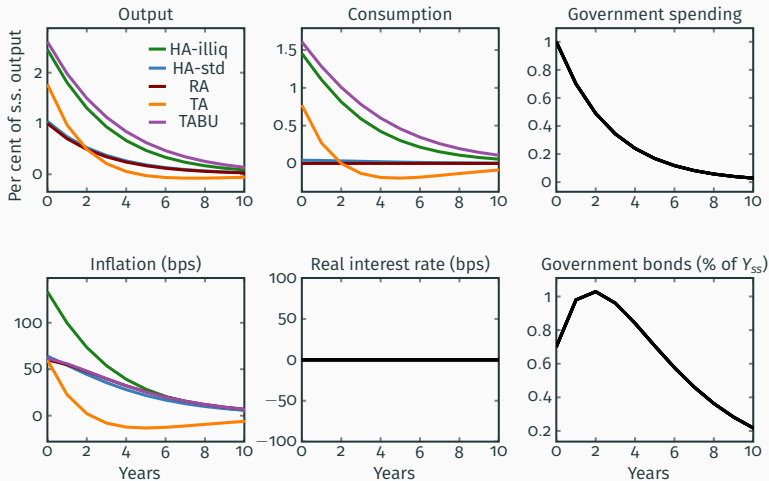
Calibration: homothetic durables model with $d_{it} = 0.1 \cdot c_{it}$ and $\delta_D = 20\%$

- Preferences: $u(c) = \frac{c^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}$, $v(n) = b \frac{n^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$
- Income process: $\log e_t = \rho_e \log e_{t-1} + \sigma \epsilon_t$

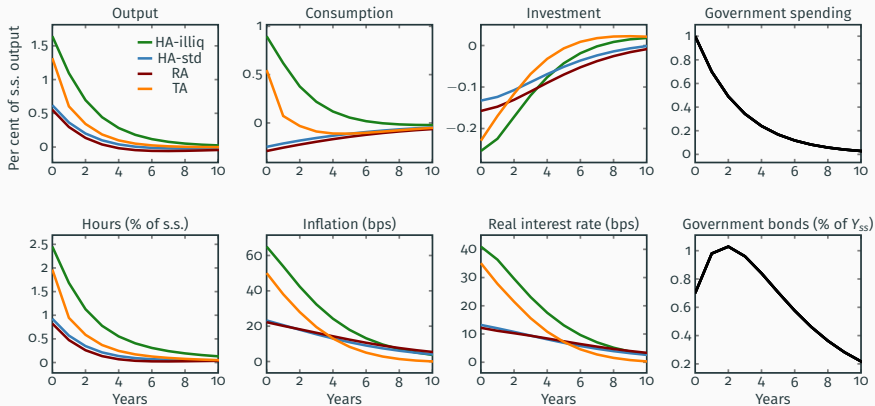
Parameter	Parameter	HA-illiq	HA-std
ν	EIS	0.5	
ϕ	Frisch	1	
ρ_e	Log e persistence	0.91	
σ	Log e st dev	0.92	
λ	Tax progressivity	0.181	
G/Y	Spending-to-GDP	0.2	
A/Z	Wealth-to-aftertax income	8.2	
B/Z	Liquid assets to aftertax income	0.15	8.2
β	Discount factor	0.80	0.92
r	Real interest rate	0.05	
κ^w	Wage flexibility	0.1	

- As in benchmark model, plus:

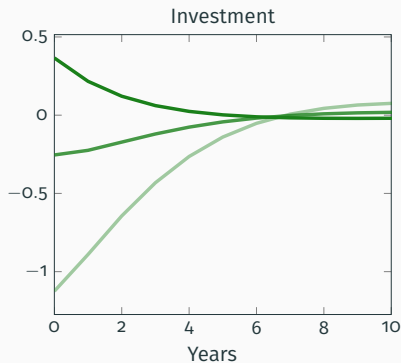
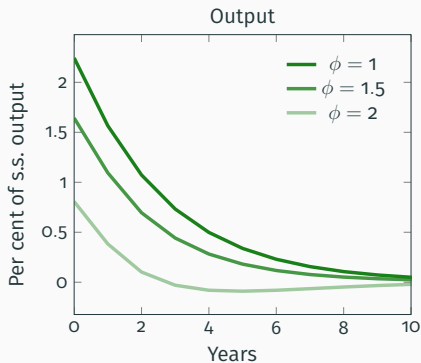
Parameter	Parameter	
α	Capital share	0.33
B/Y	Debt-to-GDP	0.7
K/Y	Capital-to-GDP	2.5
μ	SS markup	1.015
δ	Depreciation rate	0.08
ϵ_I	Invest elasticity to q	4
κ^P	Price flexibility	0.1
κ^W	Wage flexibility	0.1
ϕ	Taylor rule coefficient	1.5



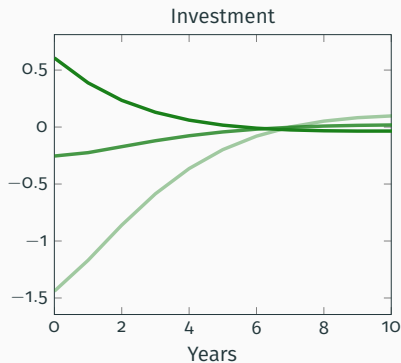
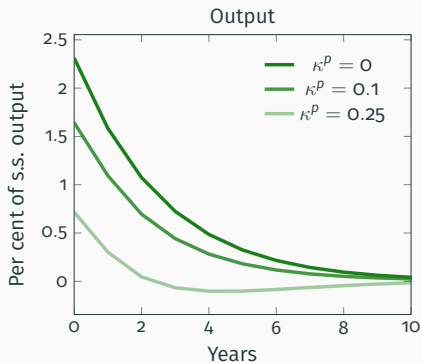
Calibration: $\rho_G = \rho_B = 0.7$



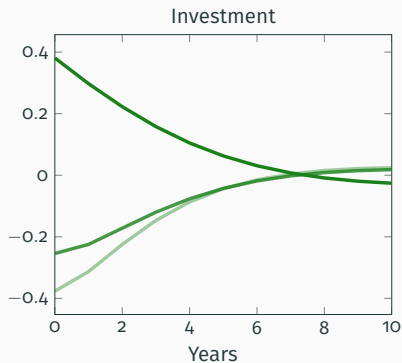
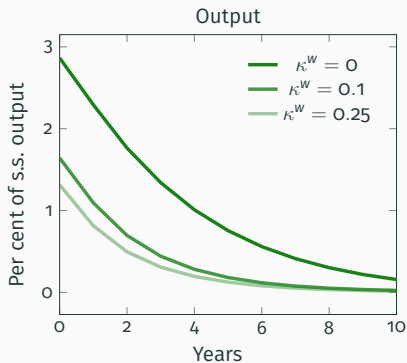
Calibration: $\rho_G = \rho_B = 0.7$, $\kappa^W = \kappa^D = 0.1$, $\phi = 1.5$



Calibration: $\rho_G = 0.7$, $\kappa^W = \kappa^P = 0.1$, $\rho_B = 0.5$, and vary ϕ in Taylor rule



Calibration: $\rho_G = 0.7$, $\kappa^W = 0.1$, $\rho_B = 0.5$, $\phi = 1.5$, and vary κ^P in price Phillips curve



Calibration: $\rho_G = 0.7$, $\kappa^P = 0.1$, $\rho_B = 0.5$, $\phi = 1.5$, and vary κ^W in wage Phillips curve