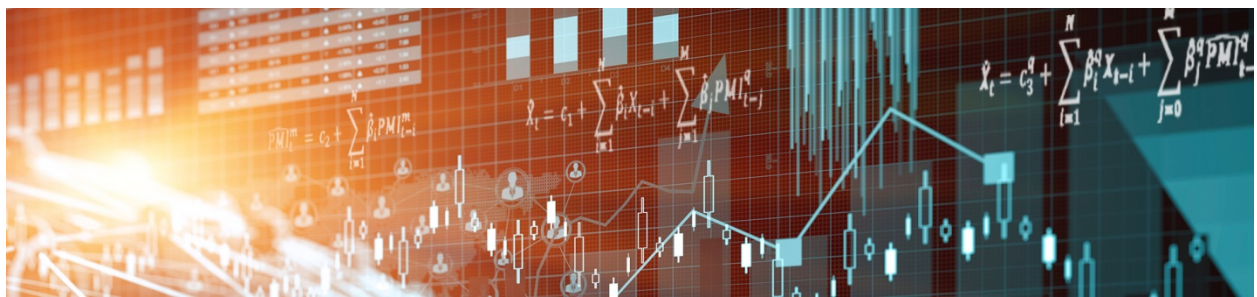


Staff Discussion Paper/Document d'analyse du personnel 2018-14

# An Alternative Estimate of Canadian Potential Output: The Multivariate State-Space Framework



by Lise Pichette, Maria Bernier and Marie-Noëlle Robitaille

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# **An Alternative Estimate of Canadian Potential Output: The Multivariate State-Space Framework**

by

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We would like to thank Pierre St-Amant for his constructive comments and suggestions. We also benefited from comments provided by Russell Barnett, Dany Brouillette and Robert Amano as well as participants at Bank of Canada internal presentations, the 2016 conference of the Société canadienne de science économique and the 2016 Canadian Economics Association conference.

## Abstract

In this paper, we extend the state-space methodology proposed by Blagrove et al. (2015) and decompose Canadian potential output into trend labour productivity and trend labour input. As in Blagrove et al. (2015), we include output growth and inflation expectations from consensus forecasts to help refine our estimates. Our alternative model, which we call the multivariate state-space framework (MSSF), adds to the Bank's existing set of tools for estimating potential output and the output gap in Canada. We find that while the MSSF shares similar dynamics to the main approaches used by the Bank, it also indicates that the economy experienced greater excess supply in both the 1990 and 2008 recessions than the Bank's other tools would suggest. Finally, the MSSF estimates that the Canadian economy has been operating close to capacity since the end of 2017.

*Bank topics: Economic models; Potential output*

*JEL codes: C5, E0, E5*

## Résumé

Dans cette étude, nous élargissons le modèle espace d'états proposé par Blagrove et autres (2015) en décomposant la production potentielle du Canada en deux facteurs : la productivité tendancielle du travail et le facteur travail tendanciel. Comme dans l'étude citée, nous incluons des prévisions de Consensus Economics concernant la croissance de la production et l'inflation en vue d'affiner nos estimations. Notre modèle alternatif, à savoir le Cadre espace d'états multivarié, s'ajoute à la gamme d'outils qu'utilise la Banque pour estimer la production potentielle et l'écart de production au Canada. Nous constatons qu'en dépit d'une certaine dynamique qu'il partage avec les principales méthodes employées à la Banque, notre modèle indique que l'économie a connu, pendant les récessions de 1990 et de 2008, une offre excédentaire plus élevée que ne le donnaient à penser les autres outils de l'institution. Enfin, toujours selon notre modèle, l'économie canadienne tournerait près des limites de sa capacité depuis la fin de 2017.

*Sujets : Modèles économiques; Production potentielle*

*Codes JEL : C5, E0, E5*

## 1. Introduction

Estimating potential output and the output gap—the difference between actual output and its potential—plays a key role in monetary policy. Unfortunately, potential output is unobservable, making its measurement difficult and uncertain. As highlighted in Pichette et al. (2015), the perfect approach to measuring potential output does not exist. Given the high level of uncertainty, it is prudent to consider multiple methods when assessing the output gap. It is therefore important that we continually expand and improve upon existing models, and innovate by testing new approaches and incorporating them into our analyses. Currently, the Bank of Canada uses two main approaches to estimate past and present potential output: the integrated framework (IF, also called the structural approach in the *Monetary Policy Report* [MPR]), and the extended multivariate filter (EMVF, also called the statistical approach in the MPR).<sup>1</sup> The estimate of potential output also relies on a few secondary methods, including the basic multivariate filter (BMVF) developed by Blagrove et al. (2015) and other sources of information, such as survey indicators and judgment.<sup>2</sup>

In this paper, we develop a useful complement to the existing set of tools used at the Bank of Canada. Our approach—the multivariate state-space framework (MSSF)—extends the state-space framework (the BMVF) proposed by Blagrove et al. (2015). More specifically, we decompose potential output into trend labour productivity and trend labour input, thus providing greater interpretability. The identification of supply and demand shocks is similar to the BMVF.

The MSSF also borrows the flexible framework of the BMVF, and as such can be easily modified and enhanced to provide additional structure or to test alternative hypotheses. This flexibility allows for many other potential improvements. Pichette et al. (2018) show that while some output gap measures appear to provide information that reduces forecast errors in some models for inflation, forecast improvements are rarely statistically significant. In future work, we could take advantage of the flexibility of the new proposed approach for measuring potential output to assess whether any enhancements could strengthen the relationship between the output gap and inflation.

We find that while the MSSF shares similar dynamics with the main approaches used by the Bank for estimating the output gap, it also indicates that the economy experienced greater excess supply in both the 1990 and 2008 recessions than the Bank's other tools would suggest. The MSSF also estimates that the Canadian economy has been operating close to capacity since the end of 2017.

The rest of this paper is organized as follows. The next section presents a detailed description of the alternative model we propose. Section 3 presents the output gap estimates obtained through the model and compares them with the Bank's existing approaches. Section 4 provides some concluding remarks and possible areas for future study.

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<sup>1</sup> See Pichette et al. (2015) for a more detailed assessment of the IF and EMVF.

<sup>2</sup> For more details on Bank staff output gap estimates, see Champagne, Poulin-Bellisle and Sekkel (2018).

## 2. Method

The MSSF is adapted from the BMVF—a state-space model that requires information on only a few variables to estimate potential output—real gross domestic product (GDP) growth, consumer price index (CPI) inflation, unemployment rate, and output and inflation expectations from consensus forecasts (Blagrove et al. 2015). It defines the output gap as the deviation of real GDP from its potential level. Real GDP is subject to three types of shocks: a shock to the level of potential, a shock to potential growth and a demand shock to the output gap. A Phillips curve, Okun’s law and consensus forecasts for output growth and inflation are added to help identify potential. Consensus forecasts are included to improve the precision of end-of-sample estimates. A forecast where output growth is stronger than actual growth could indicate that current output is below potential. A weak inflation forecast would have the same implication.

Since the BMVF requires information on only a few observable variables, it is ideal for situations where data limitations may preclude the use of production-function approaches or complex multivariate filters. However, this simplicity means that the BMVF does not incorporate some of the main determinants of potential output; even though developments in potential output are easy to interpret, they cannot be attributed to specific components since the method is applied directly to real GDP. The MSSF addresses this limitation by decomposing potential output as the product of trend productivity and trend labour input.

In the MSSF, the output gap ( $y$ ) is defined as the deviation of real GDP, in log terms ( $Y$ ), from its potential level ( $\bar{Y}$ ):

$$y_t = Y_t - \bar{Y}_t \quad (1)$$

Real GDP is decomposed as follows:

$$Y_t = \text{PROD}_t + \text{AHW}_t + \text{EMP}_t \quad (2)$$

where  $\text{PROD}$  is labour productivity,  $\text{AHW}$  is average hours worked, and  $\text{EMP}$  is the level of employment, all in log terms. Potential is given by the trend of these same variables ( $\overline{\text{PROD}}$ ,  $\overline{\text{AHW}}$ ,  $\overline{\text{EMP}}$ ), while the output gap is the sum of the deviations of each component from its trend ( $prod$ ,  $ahw$ ,  $emp$ ). An advantage of this approach is that it allows us to identify whether pressures on capacity are coming from the labour productivity gap or the labour input gap (i.e.,  $ahw + emp$ ).

$$\bar{Y}_t = \overline{\text{PROD}}_t + \overline{\text{AHW}}_t + \overline{\text{EMP}}_t \quad (3)$$

$$y_t = prod_t + ahw_t + emp_t \quad (4)$$

Following Blagrove et al. (2015), each component of GDP is modelled according to a stochastic process with three types of shocks. Equations 5.1 to 8.1 describe the process for productivity.<sup>3</sup> The level of trend productivity ( $\overline{PROD}_t$ ) evolves according to trend productivity growth ( $g\overline{PROD}_t$ ) and a level-shock term ( $\varepsilon_t^{\overline{PROD}}$ ). Trend productivity growth is also subject to shocks ( $\varepsilon_t^{g\overline{PROD}}$ ), with their persistence given by the parameter  $\theta^{PROD}$ . Finally, the gap between trend and actual productivity closes at a speed given by parameter  $\phi^{prod}$ , but is subject to demand shocks ( $\varepsilon_t^{prod}$ ).<sup>4</sup>

$$PROD_t = \overline{PROD}_t + prod_t \quad (5.1)$$

$$\overline{PROD}_t = \overline{PROD}_{t-1} + g\overline{PROD}_t + \varepsilon_t^{\overline{PROD}} \quad (6.1)$$

$$g\overline{PROD}_t = \theta^{PROD} g\overline{PROD}^{SS} + (1 - \theta^{PROD}) g\overline{PROD}_{t-1} + \varepsilon_t^{g\overline{PROD}} \quad (7.1)$$

$$prod_t = \phi^{prod} prod_{t-1} + \varepsilon_t^{prod} \quad (8.1)$$

The role of each shock is expressed graphically in **Figure 1**. In the absence of shocks, trend productivity would evolve according to its steady-state path, depicted by the solid blue line with slope  $g\overline{PROD}^{SS}$ . However, shocks to the level, growth rate or gap can cause trend productivity to deviate from its initial steady-state path (which is determined outside the model). A level shock causes trend productivity to be permanently higher (or lower) than its steady state (dashed blue line), while a shock to the growth rate causes productivity growth to be temporarily higher (lower) before returning to its steady-state growth rate (dashed red line). These shocks can be interpreted as supply shocks since they have permanent effects on the level of potential output. Conversely, shocks to the productivity gap can be seen as demand shocks since they cause only temporary deviations (green line). Similar stochastic processes are used for the evolution of each of the two elements of labour input.

For each component of GDP, the three shock terms give the calibration for the relative incidence of demand and supply shocks over the business cycle. Building on the BMVF, we assign priors to the standard deviation of shock terms so that shocks will be one-third supply and two-thirds demand.<sup>5</sup> Distributional assumptions for the shocks are also assumed to be the same as in the BMVF (see Blagrove et al. 2015).

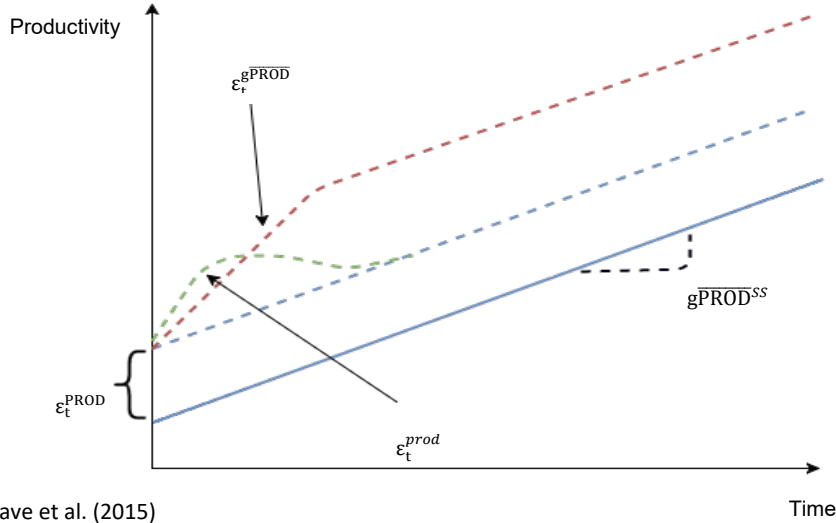
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<sup>3</sup> The stochastic processes used to model the two other components of GDP are reported in equations 5.2–8.3 in Appendix A.

<sup>4</sup> Following the assumption from Blagrove et al. (2015) regarding the output gap, the productivity gap is estimated using a first-order autoregressive process (AR(1)). As argued by Guay and St-Amant (2005), an AR(2) may be a better approximation. In the case of the MSSF, using an AR(1) or an AR(2) provides very similar results.

<sup>5</sup> Reassuringly, the posteriors of the standard deviations of the shocks are almost identical to their priors (see **Table B-1** in Appendix B).

**Figure 1: Shocks to the level of trend productivity, its growth rate and gap**



Source: Blagrove et al. (2015)

Equations 1 to 8 presented above constitute the core model. To help better identify shocks, the core model is extended by adding a Phillips Curve, linking the evolution of the output gap to observable data on inflation ( $\pi_t$ ).

$$\pi_t = \lambda\pi_{t+1} + (1 - \lambda)\pi_{t-1} + \beta y_t + \varepsilon_t^\pi \quad (9)$$

Next, equations describing the evolution of unemployment are included to provide further information for the estimation of the output gap.

$$\bar{U}_t = (\tau_4 \bar{U}^{SS} + (1 - \tau_4)\bar{U}_{t-1}) + g\bar{U}_t + \varepsilon_t^{\bar{U}} \quad (10)$$

$$g\bar{U}_t = (1 - \tau_3)g\bar{U}_{t-1} + \varepsilon_t^{g\bar{U}} \quad (11)$$

$$u_t = \tau_2 u_{t-1} + \tau_1 y_t + \varepsilon_t^u \quad (12)$$

$$u_t = \bar{U}_t - U_t \quad (13)$$

In equations 10 to 13,  $\bar{U}_t$  is the equilibrium value of the unemployment rate (the non-accelerating inflation rate of unemployment [NAIRU]), which is time-varying and subject to shocks ( $\varepsilon_t^{\bar{U}}$ ) as well as variation in the trend ( $g\bar{U}_t$ ). The latter is itself also subject to shocks ( $\varepsilon_t^{g\bar{U}}$ ). This specification allows for persistent deviations of the NAIRU from its steady-state value. Equation 12 specifies an Okun's law relationship wherein the gap between the actual unemployment rate ( $U_t$ ) and its equilibrium process (given by  $u_t$ ) is a function of the amount of slack in the economy ( $y_t$ ). For the shock terms in equations 10 to 13, we use the same distributional assumptions as in the BMVF, as well as the same priors for their standard deviation (see **Appendix B**).

Finally, the model is augmented with forward-looking data on output growth and inflation expectations to improve the accuracy of estimates at the end-of-sample period and to further help identify shocks.



$$\pi_{t+j}^C = \pi_{t+j} + \varepsilon_{t+j}^{\pi^C} \quad j = 0 - 1 \quad (14)$$

$$\text{GROWTH}_{t+j}^C = \text{GROWTH}_{t+j} + \varepsilon_{t+j}^{\text{GROWTH}^C} \quad j = 0 - 5 \quad (15)$$

Following Blagrove et al. (2015), for each observation, forecasts from Consensus Economics are added for time  $t$  and five subsequent years for real GDP growth ( $\text{GROWTH}_t^C$ ), and for time  $t$  and one year ahead in the case of inflation ( $\pi_t^C$ ). These equations relate the model's expectations for growth ( $\text{GROWTH}_{t+j}$ ) and inflation ( $\pi_{t+j}$ ) to what Consensus forecasters expect over various time horizons (up to five years ahead) throughout the estimation period. Allowing forecasts to influence but not completely override the model's expectations can be thought of as adding forward-looking information at the end of the sample to alleviate well-documented issues associated with real-time output gap estimates (see Orphanides and van Norden 2002). Indeed, Blagrove et al. (2015) still find an end-of-sample problem in the BMVF, but their approach helps address the issue to some extent, relative to a simple HP filter.

The system of equations is estimated simultaneously using Bayesian estimation techniques on an annual basis over the 1990–2017 period.<sup>6,7</sup> A Kalman filter is used to obtain unobserved variables (Hamilton 1994). Steady states for the growth in GDP components are exogenously assigned using the average over the 1990–2017 period (see **Table B-2** in Appendix B). Priors of estimated parameters are chosen according to Blagrove et al. (2015).<sup>8</sup> They are shown, as well as their associated posteriors, in **Table B-3** of Appendix B. For the three GDP components, posteriors for the speed at which the gap between trend and actual data closes ( $\phi$ ) are similar to those associated with GDP in the BMVF. Shocks to trend growth ( $\theta$ ) seem to be slightly more persistent for average hours worked than GDP as estimated in the BMVF. As in Blagrove et al. (2015), estimated posteriors are generally close to their priors. The main exception is  $\beta$ , which is estimated at its lower bound (i.e., 0.05), challenging the linear relationship between inflation and the output gap. This is in line with work by Simon, Matheson and Sandri (2013) who suggest that the slope of the Phillips curve has flattened over the past several decades. Quarterly estimates of potential output and its components are obtained using a cubic interpolation.

### 3. Results

To shed light on the role of the different components of the MSSF, we present output gap estimates obtained in the step-by-step construction of the model. First, we consider the core model (excluding equations 9 to 15)—the specification arising solely from the decomposition of potential output into trend labour input and trend labour productivity (**Chart 1**, green line).

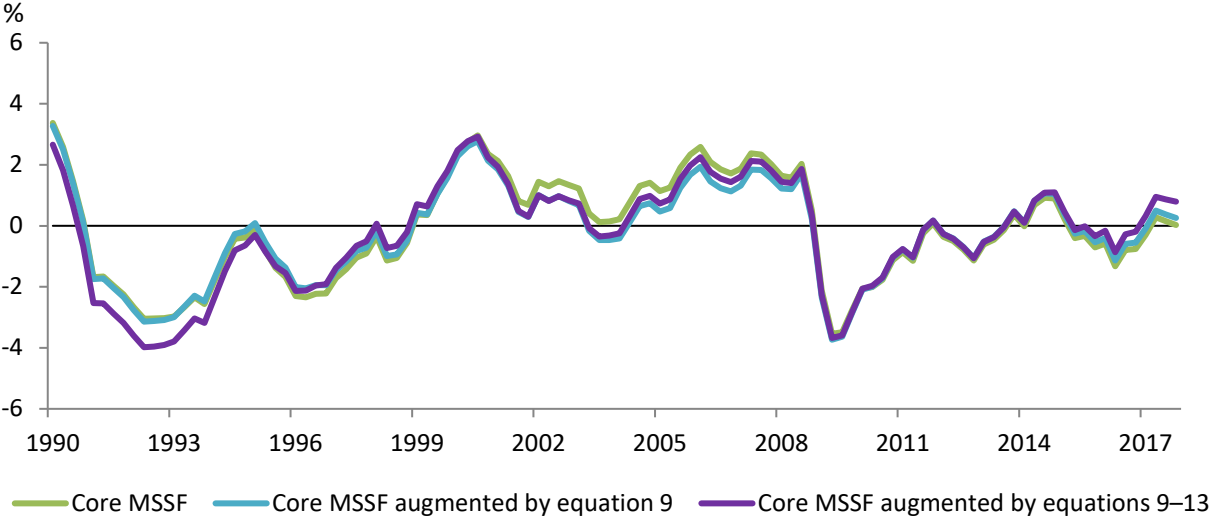
<sup>6</sup> More specifically, we use regularized maximum likelihood, as in Blagrove et al. (2015).

<sup>7</sup> While all inputs are available on a quarterly basis, we choose to keep the annual frequency for simplicity, as the original model was calibrated for annual data. In future work, we plan to convert to quarterly frequency, as this will be more useful for conducting monetary policy.

<sup>8</sup> Results are not very sensitive to the assigned priors as long as they remain reasonable.

Next, we examine the impact of adding a Phillips curve (equation 9) and the block on unemployment including Okun’s law (equations 10 to 13) to the model (**Chart 1**, blue and purple lines, respectively). The addition of equation 9 has little impact on the output gap estimate. This result is not surprising, since other studies have shown that Phillips curves that use measures of the output gap to forecast inflation perform poorly (e.g., Orphanides and van Norden [2005] for the United States; Champagne, Poulin-Bellisle and Sekkel [2018] and Pichette et al. [2018] for Canada). Also, considering the low estimate for  $\beta$  in the Phillips curve, the relationship between inflation and the output gap is weak, thus it does not help to refine the potential output estimate. The addition of equations 10 to 13 also has limited impact on the output gap estimates.

**Chart 1: Effect of the Phillips curve and Okun’s law on MSSF estimates of the output gap**

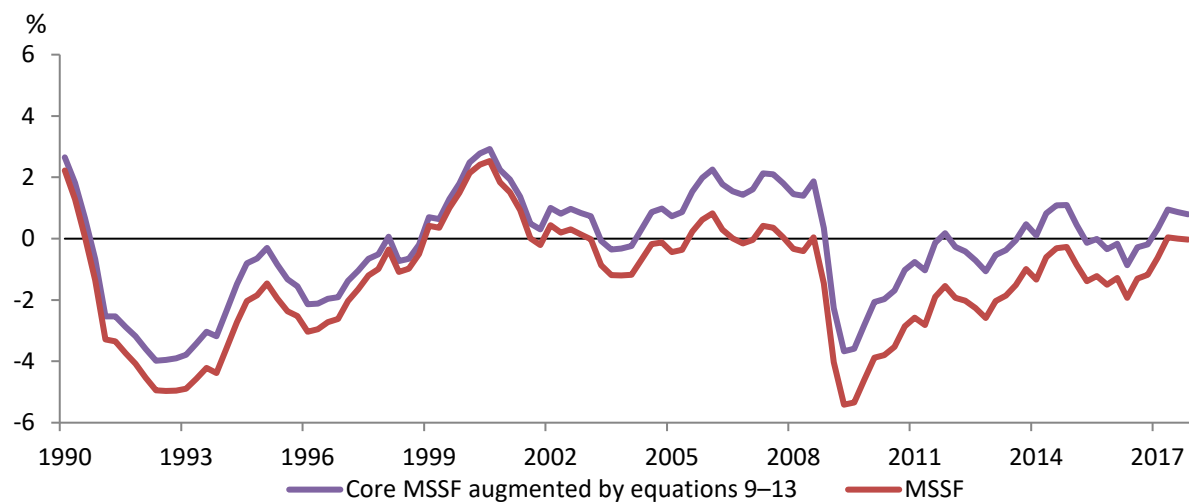


Note: MSSF is multivariate state-space framework.

Last observation: 2017Q4

Finally, adding consensus forecasts to the MSSF, as suggested by Blagrove et al. (2015), results in a sizable downshift of the model’s estimates of the output gap, particularly in recent years (**Chart 2**). This is likely due to persistently strong growth forecasts relative to actual data, which the model interprets as evidence that the observed decline in growth is unlikely to be long-lasting. It attributes the weakness to cyclical factors rather than long-term trends reflected in the potential.

**Chart 2: Output gap estimates for the MSSF with and without forecasts**



Note: MSSF is multivariate state-space framework.

Last observation: 2017Q4

A quasi real-time analysis shows that the estimates of the output gap using the full MSSF, including the consensus forecasts on GDP growth and inflation, are subject to smaller revisions than the core MSSF model (**Table 1**).<sup>9</sup> As described in Stock and Watson (2003), this exercise uses only the latest vintage of data over the entire period, so that revisions to the output gap originate from the model's endpoint properties rather than data revisions.<sup>10</sup>

The mean, absolute mean, standard deviation, root mean squared error (RMSE) and noise-to-signal ratio (NSR) of the revisions, shown in **Table 1**, all indicate that the revisions of the core MSSF are on average larger and more volatile than those of the full MSSF, suggesting that the inclusion of consensus forecasts in the MSSF plays a major role in reducing the size of its revisions.

<sup>9</sup> These estimates are constructed by sequentially estimating the output gap in each year, using only the data available as of that date. For example, the quasi real-time estimates of the output gap in 2003 would have used data from the beginning of the sample (i.e., 1990) through 2003 only. The estimates are *quasi* real-time in the sense that actual vintage data are not used for this exercise (but rather only the latest available data, which have been revised over time). Revisions are then calculated as the difference between output gaps estimated with all available data (i.e., final output gaps) and the endpoint of an estimation (i.e., output gaps in 2003 for estimations ending in 2003). See Appendix A in Pichette et al. 2015 for more details on a similar exercise.

<sup>10</sup> As in Pichette et al. (2015), these statistics are calculated using a truncated sample period; the latest observation used for these calculations is 2015Q4.

**Table 1: Properties of revisions for the MSSF (2003Q1–2015Q4)**

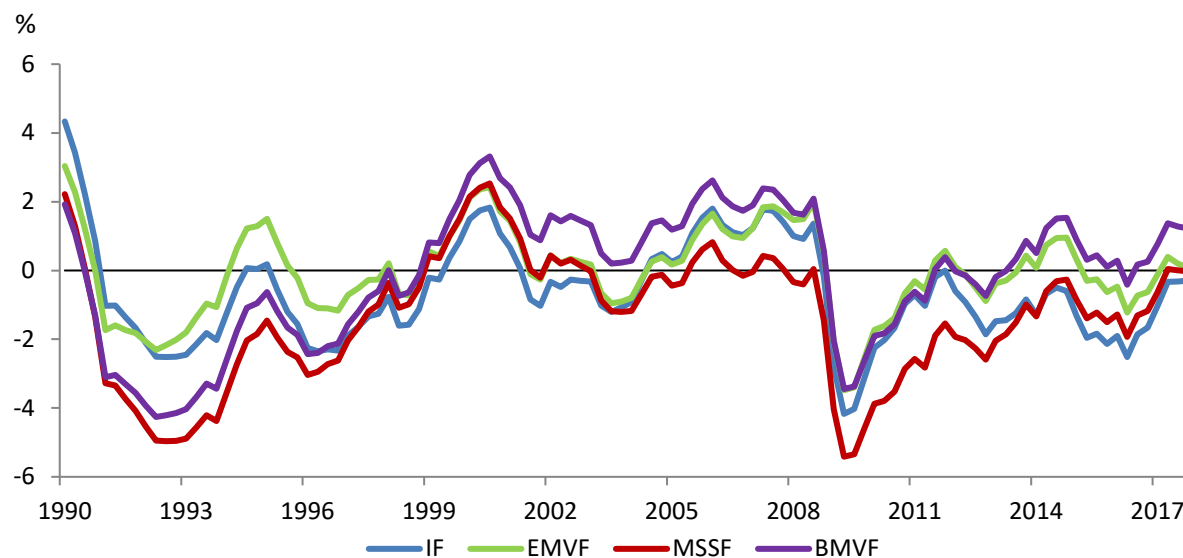
	Mean	Absolute mean	Standard deviation	RMSE	NSR
Core MSSF	0.68	0.82	0.96	1.17	0.63
MSSF	0.18	0.24	0.27	0.32	0.18

Note: The root mean squared error (RMSE) is calculated as the root mean square of the revisions, while the noise-to-signal ratio (NSR) is calculated as the ratio of the standard deviation of the revision to that of the final estimates of the output gap. MSSF is the multivariate state-space framework.

**Chart 3** shows the output gap estimates for the MSSF, the BMVF and the two main approaches used at the Bank of Canada—the EMVF and the IF—estimated with the data available in 2018Q1.<sup>11</sup> Although the MSSF estimates tend to be lower than those of the other methods between 2004 and 2014, they roughly align with those of the IF from 2014 onward. Using data available in the first quarter of 2018, the IF, EMVF and MSSF estimate that current output has remained below potential in 2015 and 2016. For 2017, the MSSF suggests a closed output gap—an estimate similar to the IF and EMVF but slightly below the BMVF, which points to some excess demand.

The MSSF produces gaps that are generally more persistent than in the other models. For instance, the MSSF suggests more persistent economic slack in the early 1990s, an assessment that seems consistent with a period of important structural adjustment, where restrictive monetary and fiscal policies contributed to the depth and duration of the recession (Thiessen 2001).

**Chart 3: Output gap estimates for the IF, EMVF, MSSF and BMVF**



Note: IF is integrated framework, EMVF is extended multivariate filter, MSSF is multivariate state-space framework and BMVF is basic multivariate filter. Last observation: 2017Q4

<sup>11</sup> The EMVF and IF output gap estimates shown in **Chart 3** may not be the same as those currently published on the Bank of Canada website, as they were produced using the 2018Q1 vintage of data and may have been revised.

The episode before the 2008–09 recession might appear more striking, as the MSSF output gap results in a roughly balanced position while other models indicate excess demand. Yet, recall that the output gap is estimated with a high degree of uncertainty, which can potentially lead to large confidence intervals. Unfortunately, the approaches used here do not allow for easy calculation of those intervals, but it is reasonable to think that they are all included in each others' confidence bands, and that the true value of the output gap may be somewhere in between those different estimates.

#### 4. Discussion

In this paper, we propose an alternative measure of potential output and the output gap, building on the state-space methodology proposed by Blagrove et al. (2015)—the BMVF—and drawing on previous work at the Bank of Canada to estimate potential output (notably Pichette et al. 2015). Given the uncertainty surrounding estimates of potential output, it is important to continually expand and improve upon existing models, testing new approaches and incorporating them into our analysis of potential output and the output gap. Our proposed approach defines the output gap as the sum of the labour productivity gap and the labour input gap; we incorporate some economic relationships and supply and demand shocks, which provide improved interpretability although it remains a reduced-form approach. Moreover, like Blagrove et al. (2015), we find that forward-looking data on output and inflation expectations help refine our estimates of the output gap—an important feature for conducting monetary policy. Pichette et al. (2018) also find the MSSF to be subject to fewer revisions than other measures of output gaps when using real-time data. Finally, the MSSF provides flexibility: the structure can be modified and enhanced to incorporate additional features or test alternative hypotheses, thus opening the door for many other potential improvements.

For instance, we might consider adding some factors underlying the employment and average hours worked components to account for demographic changes. Continuing population aging, along with a slowdown in growth of the working-age population, are expected to reduce growth in trend labour input from 0.7 per cent in 2017 to 0.5 per cent in 2021 (Agospwicz et al. 2018). A small offset should come from higher levels of both immigration and educational attainment of workers. Although such structural factors are not yet embedded in the MSSF, the proposed framework is flexible enough to allow for each block to be enhanced with factors driving trends of productivity, employment and hours worked. This would greatly improve the economic interpretation of any change in potential output.

Pichette et al. (2015) present four criteria in evaluating methods used to measure potential output. Because potential output developments need to be analyzed and communicated, the first criterion is that an approach should provide economic interpretation. Consistent with the Bank's inflation-targeting mandate, the second criterion is that estimated potential output should be helpful in identifying inflationary and disinflationary pressures in the economy. The third criterion is that the assumptions and statistical relationships conditioning potential output estimates should be consistent with the data. Fourth, given that the Bank needs to estimate historical and future growth rates of potential, the tools should be helpful for both estimating and projecting potential. The MSSF meets the first and third criteria, at the very least. Its simple potential output decomposition is easy to interpret and consistent with the

data. It could be readily adapted to changing economic data. This paper does not provide sufficient information to inform on the second and fourth criteria. Pichette et al. (2018) analyze the relationship between various output gap measures, including the MSSF, and inflation. They find that estimates of the output gaps examined provide only limited information for forecasting inflation. However, taking advantage of the flexibility of the MSSF, efforts should be made in future work to assess whether enhancements to the model could improve the information content of the output gap for inflation forecasts.

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## Appendix A: Stochastic processes used to model other GDP components

$$AHW_t = \overline{AHW}_t + ahw_t \quad (5.2)$$

$$\overline{AHW}_t = \overline{AHW}_{t-1} + g\overline{AHW}_t + \varepsilon_t^{\overline{AHW}} \quad (6.2)$$

$$g\overline{AHW}_t = \theta^{AHW} g\overline{AHW}^{SS} + (1 - \theta^{AHW}) g\overline{AHW}_{t-1} + \varepsilon_t^{g\overline{AHW}} \quad (7.2)$$

$$ahw_t = \phi^{ahw} ahw_{t-1} + \varepsilon_t^{ahw} \quad (8.2)$$

$$EMP_t = \overline{EMP}_t + emp_t \quad (5.3)$$

$$\overline{EMP}_t = \overline{EMP}_{t-1} + g\overline{EMP}_t + \varepsilon_t^{\overline{EMP}} \quad (6.3)$$

$$g\overline{EMP}_t = \theta^{EMP} g\overline{EMP}^{SS} + (1 - \theta^{EMP}) g\overline{EMP}_{t-1} + \varepsilon_t^{g\overline{EMP}} \quad (7.3)$$

$$emp_t = \phi^{emp} emp_{t-1} + \varepsilon_t^{emp} \quad (8.3)$$



## Appendix B

**Table B-1: Estimated shock terms**

	Priors	Posteriors
$\varepsilon_t^{\overline{PROD}}$	0.2	0.20
$\varepsilon_t^{g\overline{PROD}}$	0.2	0.20
$\varepsilon_t^{prod}$	0.8	0.80
$\varepsilon_t^{\overline{AHW}}$	0.2	0.20
$\varepsilon_t^{g\overline{AHW}}$	0.2	0.20
$\varepsilon_t^{ahw}$	0.8	0.80
$\varepsilon_t^{\overline{EMP}}$	0.2	0.20
$\varepsilon_t^{g\overline{EMP}}$	0.2	0.20
$\varepsilon_t^{emp}$	0.8	0.80
$\varepsilon_t^{\overline{U}}$	0.1	0.10
$\varepsilon_t^{g\overline{U}}$	0.1	0.10
$\varepsilon_t^u$	0.5	0.50
$\varepsilon_t^\pi$	0.25	0.68

**Table B-2: Steady-state assumptions**

Trend productivity growth	Growth in trend average hours worked	Growth in trend employment	Trend unemployment rate
$g\overline{PROD}^{SS}$	$g\overline{AHW}^{SS}$	$g\overline{EMP}^{SS}$	$\overline{U}^{SS}$
1.184%	-0.213%	1.246%	7.95

**Table B-3: Estimated parameters**

	Priors	Posteriors
$\lambda$	0.25	0.32
$\beta$	0.25	0.05
$\phi^{prod}$	0.6	0.70
$\phi^{ahw}$	0.6	0.69
$\phi^{emp}$	0.6	0.71
$\theta^{PROD}$	0.1	0.14
$\theta^{AHW}$	0.1	0.20
$\theta^{EMP}$	0.1	0.12
$\tau_1$	0.3	0.37
$\tau_2$	0.3	0.31
$\tau_3$	0.1	0.10
$\tau_4$	0.1	0.10