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Abstract

We present a simple model of the risk sensitivity of bank capital regulation. A banker funds a project with uninsured deposits and costly capital, where capital resolves a moral hazard problem in the choice of the probability of default. Investors are uninformed about the project’s high or low loss given default, but a regulator receives a noisy signal and imposes minimum capital requirements. We show that the sensitivity of capital regulation to measured risk is non-monotonic. For an inaccurate signal, the regulator pools banker types via risk-insensitive capital requirements. For an accurate signal, the regulator separates types via risk-sensitive capital requirements. For an even more accurate signal, the risk sensitivity of bank capital requirements falls.

Bank topics: Financial institutions; Financial system regulation and policies
JEL codes: G21, G28

Résumé

Nous exposons, à l’aide d’un modèle simple, la sensibilité au risque de la réglementation des fonds propres bancaires. Un banquier finance un projet avec des dépôts non assurés et des fonds propres coûteux à mobiliser, les fonds propres régler un problème d’aléa moral dans le choix de la probabilité de défaillance. Les investisseurs ne sont pas informés sur la perte élevée ou faible en cas de défaillance associée au projet, mais l’organe de réglementation reçoit un signal bruyant et impose des exigences minimales de fonds propres. Nous montrons que la sensibilité de la réglementation des fonds propres au risque mesuré n’est pas monotone. Dans le cas d’un signal imprécis, l’organe de réglementation regroupe les différents types de banquiers en imposant des exigences de fonds propres insensibles au risque. À l’inverse, lorsque le signal est précis, l’organe de réglementation distingue les différents types en appliquant des exigences de fonds propres sensibles au risque. Enfin, si le signal est encore plus précis, la sensibilité au risque des exigences de fonds propres chute.

Sujets : Institutions financières; Réglementation et politiques relatives au système financier
Codes JEL : G21, G28
1 Introduction

Bank capital regulation gained formal international acceptance with the Basel I framework in 1988 and remains at the core of banking regulation today. The appropriate degree of risk sensitivity of capital regulation has been a central part of the debate and reforms over the last few decades. On the one hand, increased risk sensitivity improves the efficient use of scarce capital, supports the appropriate pricing of risk, and provides incentives for holding low-risk assets. On the other hand, these advantages of risk sensitivity may be hampered due to imperfect information about the risk of bank assets, increased complexity of regulation, and less comparability across banks (Basel Committee, 2013b). The latter concerns have led some to call for increased reliance on the leverage ratio, which is invariant to bank asset risks (Haldane, 2012).

A point missing from this debate is that banks—as any economic actors facing uncertainty—make decisions based on expected outcomes. The policy discussion on risk sensitivity has focused on capital regulation conditional on correctly determining bank risk. Bank regulators have imperfect information about bank risk, however, so the level of uncertainty in measuring bank risk affects the expected bank capital requirements. This idea is well known in the literature on the economics of crime and punishment (Becker, 1968). A lower probability of being caught for a crime requires punishment be more severe, since the expected punishment is composed of the probability of being caught and the punishment conditional on being caught.

In this paper, we construct a parsimonious model to understand the risk sensitivity of bank capital regulation when the bank regulator is uncertain about the risk of bank assets. We examine optimal risk sensitivity when the bank regulator has imperfect information and banks face expected capital requirements. For the model to shed light on this issue, it requires (i) a role for capital, (ii) heterogeneous bank
assets, (iii) a role for regulation, and (iv) an imperfectly informed regulator.

First, a banker can invest in a risky project using her own funds (equity capital) or borrowed funds (uninsured deposits). Capital is assumed to be costly, so the banker wishes to borrow as much as possible from competitive investors.¹ But a minimum level of capital is essential for resolving a moral hazard problem of the banker’s choice of risk (Holmström and Tirole, 1997). When the banker chooses the probability of default (PD), enough skin in the game provides her with incentives to control risk.

Second, project quality is uncertain. Its scrap value defines the loss given default (LGD) – a key determinant of Basel capital requirements. The banker observes the scrap value – her type – but investors do not. Third, a role for regulation arises from an information advantage of the regulator, perhaps due to its role as supervisor. The regulator receives a signal about the LGD and imposes minimum capital requirements to maximize utilitarian welfare. Our rationale for bank regulation is consistent with the representation hypothesis by Dewatripont and Tirole (1994), whereby the regulator represents depositors in collecting information about the bank.

Using our model, we study the risk sensitivity of capital regulation for three information structures: two benchmarks (observed and unobserved types) and the main analysis for a noisy but informative signal. To relate our results to the policy and practitioner literature, we use the term “risk sensitivity.” To be clear, the sensitivity of bank capital regulation in the model is with respect to the different realizations of the scrap value (that is, with respect to LGD), not the probability of default (PD).

In the first information structure, the regulator observes the banker’s type. The

¹The assumption of costly capital is critical to our analysis. Precedents for this assumption in banking theory include Hellmann et al. (2000), Repullo (2004), and Allen et al. (2011). A possible micro-foundation of this assumption is a general equilibrium setting with limited participation of investors in equity markets, which is in the spirit of Allen and Gale (1994) and Carletti et al. (2019). See also Diamond and Rajan (2000) for a microfoundation when the banker faces an agency problem.
regulator sets minimum capital requirements to disseminate the project’s scrap value to investors. Risk-sensitive capital regulation achieves the second-best allocation of risk and intermediation volume. A low banker type—whose project has negative net present value—is excluded from raising deposits. A high banker type has a project with positive NPV and can raise deposits up to the point where the equity stake still provides incentives for monitoring effort. As a comparative static, a higher scrap value of a high banker type reduces the cost of deposits, so a lower capital requirement suffices to ensure effort. One interpretation of such risk-sensitive capital requirements in practice is different risk weights of assets with different quality.

In the second information structure, the regulator cannot observe banker types and can, therefore, only impose a leverage ratio invariant to the project’s loss given default. This results in a pooling equilibrium in which the cost of deposits is risk insensitive. The leverage ratio still provides incentives for the banker to exert monitoring effort. However, a two-sided inefficiency arises on the extensive margin of intermediation. Both banker types attract deposits for a low cost of capital (excessive intermediation, relative to second-best), while financial intermediation breaks down for a high cost of capital (insufficient intermediation). In the latter case, a high banker type with a positive net present value opportunity cannot raise deposits.

Our main contribution arises from the third information structure of a noisy but informative signal. The signal is either good or bad and a high (low) banker type is more likely to generate a good (bad) signal. The regulator can implement a separating equilibrium, in which only the high banker type participates, via a risk-sensitive capital requirements—one for each signal. After the good signal, the regulator requires the same capital level as when the high type is observed. The capital requirement after a bad signal, in turn, is set to ensure the non-participation of the low banker type. Excluding the low type is costly with a noisy signal, since
the high type occasionally receives a bad signal and faces a high capital requirement.

The regulator can also implement a pooling equilibrium in which both types participate. The capital requirement then has the single duty of providing monitoring incentives. Since the average project quality is lower in the pooling equilibrium, uninsured deposits are more expensive. The banker then keeps less when investment succeeds, so the minimum capital requirement is higher to maintain incentives.

Our main result is a non-monotonic risk-sensitivity of bank capital regulation. The risk sensitivity is the difference between the requirement after the bad signal and after the good signal and, therefore, depends on risk as measured by the regulator. For an inaccurate signal, the regulator implements the pooling equilibrium and risk-sensitivity is zero. The intuition is that it is too costly to separate types, since the high type is misidentified as a low type too frequently. For a sufficiently accurate signal, the regulator implements a separating equilibrium with positive risk-sensitivity. That is, capital requirements after the bad signal must be higher in order to exclude the low banker type. As the signal becomes more accurate, the low type is more likely to be correctly identified, so a lower capital requirement after a bad signal is required to exclude the low type. Thus, the risk-sensitivity of capital regulation decreases in signal accuracy in the separating equilibrium. This result contrasts with some policy debate that argues for more risk sensitivity as information becomes more accurate.\(^2\)

Our paper is closely related to Morrison and White (2005), who also study bank capital regulation in the presence of moral hazard and adverse selection. MW allow for an endogenous banking sector, the regulator has different screening technology, and an additional policy tool to close banks, while our model has a unique equilibrium. A common theme is that capital regulation may be used to simply combat moral

\(^2\)See, for example, Basel Committee (2013a,b) and Zelmer (2013) and the references therein.
hazard (as in our pooling equilibrium) or to also combat adverse selection (as in our separating equilibrium). MW consider banker types heterogeneous in the probability of default (PD) and the extensive margin of regulator skill (the probability of a regulator being perfectly informed versus perfectly uninformed). In contrast, we consider banker types heterogeneous in the loss given default (LGD) and the intensive margin of regulator skill (the accuracy of an informative but noisy signal).

Our paper is also related to Repullo (2013), who offers a parsimonious model to rationalize the cyclical adjustment of risk-sensitive bank capital requirements. Both papers motivate capital with a moral hazard problem in the banker’s choice of risk. There are two main differences. First, the motivation for regulation arises from a social cost of bank failure in his model, while we consider an informational advantage of the regulator. Second, the banker type (LGD) is observed throughout his paper, while we derive rich implications from different information about the type.

Other theoretical work on the risk sensitivity of bank capital regulation includes Colliard (2019), who studies the strategic use of internal models where a bank trades off the benefit of lower capital requirements from reporting lower risk with the cost of fines after an audit by its supervisor. Work on the choice between Basel’s standardized approach (SA) and internal ratings based (IRB) approach includes Repullo and Suarez (2004), Hakenes and Schnabel (2011), and Feess and Hege (2012).

The remainder of the paper is structured as follows. Section 1.1 briefly reviews the history of Basel capital regulation. Section 2 describes the model. Section 3 characterizes the benchmarks with observed and unobserved types. We study the bank regulator with a noisy signal in section 4. Section 5 contains an extension and section 6 concludes. All derivations and proofs are relegated to the Appendix.
1.1 History of risk-sensitive capital regulation

The first capital adequacy framework by the Basel Committee in 1988 focused on credit risk and recognized the benefits of differentiating regulatory capital according to asset risk. However, the mechanisms were relatively simple and did not significantly differentiate across banks. For instance, risks associated with trading activities (e.g., market risks) and operations (e.g., legal risk) were not considered. Even for credit risk, assets were grouped into only five categories (0%, 10%, 20%, 50%, and 100%) based on negotiated levels rather than on a formal assessment of risks. Capital was set at 8% of assets and adjusted by a loan’s credit risk weight. Hence, required capital did not necessarily reflect differences in bank risk and business models (Basel Committee, 1988).

Subsequent reforms greatly increased the scope for increased risk sensitivity, starting with a modification in 1996 that opened the door to the use of banks’ internal models to determine the amount of capital required for market risk. The rationale for allowing internal models was that standardized models could not capture the complexity of trading and derivatives activities (Basel Committee, 2013b). The use of internal models was extended to the measurement of credit risk in 2004 (under Basel II). A major motivation of this new risk-based capital regime was to improve incentives for banks to better measure and manage risks and to reduce the incentives for regulatory arbitrage. Basel II also strengthened disclosure requirements to support market discipline and thereby reinforce banks’ incentives to manage their risks.

This risk sensitivity remains at the core of the new Basel III capital framework, with a continued role for internal models and extension of the framework to better manage a wider range of risks, such as liquidity risk, risks related to off-balance-sheet activities, and too-big-to-fail concerns (Gomes et al., 2017). The Basel Committee
nonetheless chose to recognize the limits to risk sensitivity and imposed a leverage requirement. The purpose of the risk insensitive leverage ratio was a credible supplementary measure to the risk-based capital requirements (Basel Committee, 2014).

Even with this new backstop, interest in limiting risk sensitivity grew further, particularly related to the use of internal models. In finalizing Basel III, the Basel Committee commissioned a study about whether the framework has gone too far in terms of risk sensitivity, with too much given up in terms of simplicity and comparability (Basel Committee, 2013b). The rationale for limits to risk sensitivity were largely related to imperfect information, including the accuracy and completeness of the data, limitations of risk models, and the nature of risk itself. These concerns were reinforced by evidence about large divergences in measures of risk-weighted assets across banks (for instance, for credit risk in the banking book) unexplained by differences in underlying risks of the exposure (Basel Committee, 2013a). Hence, the Basel Committee adjusted the risk sensitivity of capital requirements in a number of dimensions. For instance, the standardized approach to credit risk was made more risk sensitive, while internal models were subjected to capital floors (Basel Committee, 2017). Haldane (2012) argues that capital regulation should rely primarily on the leverage ratio rather than using it as a backstop, because information about risk is too noisy and taking risk into account introduces too much complexity.

2 Model

There are two dates $t = 0, 1$, a single good for consumption and investment, and three types of risk-neutral agents: a banker, a regulator, and many competitive investors. At $t = 0$, the banker has access to a risky project that requires one unit of investment.
The investment return at $t = 1$ is $R > 1$ with endogenous probability $p$ or a random scrap value. This scrap value is interpreted as loss given default (LGD) and defines the banker’s type. It is realized at $t = 0$ and takes two values: either $S \in (0, 1)$ with probability $\gamma \in (0, 1)$ or 0. The high banker type refers to realized scrap value $S$ and the low type to value 0. The banker knows her type but investors learn it at $t = 1$.\(^3\)

The banker can exert effort by monitoring investment (Holmström and Tirole, 1997). In particular, the banker chooses the success probability $p \in \{p_L, p_H\}$ where $0 < p_L < p_H < 1$. The non-pecuniary cost of exerting effort, $p = p_H$, is $B > 0$. The effort choice and the private benefit are unobservable. The effort choice takes place after the banker raised funding at $t = 0$.

Investors have a unit endowment at $t = 0$ and are indifferent between consuming at either date, $u^I(c_0, c_1) = c_0 + c_1$, where $c_t$ is consumption at date $t$. Thus, investors are willing to fund the banker as long as they receive a unit expected return. The banker has a unit endowment at $t = 0$ and prefers early over late consumption, $u^B(c_0, c_1) = z c_o + c_1$, where $z > 1$ measures banker impatience. Hence, it is costly to use own funds (equity) compared to borrowed funds (debt).\(^4\) The banker offers investors uninsured deposits (risky debt) with face value $D$ (per unit of deposits).

This contract is independent of the effort choice but may depend on the regulator’s information about bank type. Endowed with a supervision technology, the regulator receives a private signal $x$ about the loss given default. The regulator maximizes utilitarian welfare and can commit to minimum capital requirements $k_x$ that stipulate the amount of equity the banker must invest in the project.

We make several assumptions about the parameters characterizing investment.

\(^3\)We discuss the case of uncertainty of the banker about her type in section 5.
\(^4\)The assumption of costly bank capital is common in the banking literature. For a similar assumption, see Hellmann et al. (2000), Repullo (2004), and Allen et al. (2011), among others.
First, effort is efficient irrespective of LGD. Since the incentive compatibility con-
straint to ensure effort is strictest for the high type, we obtain an upper bound on
the information rent for the unobserved effort choice:

\[
\frac{B}{\Delta p} < R - S, \tag{1}
\]

where \( \Delta p \equiv p_H - p_L \). Second, the project has negative net present value without
banker effort. This condition is toughest to satisfy for a high type, so we impose:

\[
p_L R + (1 - p_L) S < 1. \tag{2}
\]

Third, the heterogeneity in LGD is material. When the banker exerts effort, the
project with low LGD has positive net present value, while the project with high LGD
has negative net present value. Accounting for the effort cost, these assumptions are

\[
p_H R < 1 + B < p_H R + (1 - p_H) S, \tag{3}
\]

where \( NPV \equiv p_H R + (1 - p_H) S - 1 - B > 0 \) is the present value of investment with
low LGD (high type) and monitoring effort net of the costs of investment and effort.
We assume that investment at \( t = 0 \) and its return at \( t = 1 \) are verifiable, so the
banker cannot run away with funds at \( t = 0 \) or the investment proceeds at \( t = 1 \).

Table 1 shows the timeline. We assume that the banker invests own funds
initially, submits the project for supervisory review, and receives a capital requirement
from the regulator. Based on it, the banker raises funding from investors (similar to
loan sales or securitization). We assume that the banker must fund the project
irrespective of the supervisory review. Hence, the banker may have to put up equity
capital when the project is unprofitable after a negative review.
Table 1: Timeline.

The first-best allocation arises for observable banker type and effort choice. Assumption (1) ensures effort is exerted upon investment. Assumption (3) implies that investment occurs if and only if the banker type is high. Finally, costly capital implies that the banker consumes her endowment and the project is fully funded with uninsured deposits from investors. In what follows, we assume that the effort choice is unobservable (or non-contractible). Thus, we require incentive compatibility constraints for banker effort. The second-best allocation (unobservable monitoring action but observable type) is the relevant benchmark throughout the paper.

3 Benchmarks: observed and unobserved types

3.1 Observed types

Consider first a regulator who perfectly observes the type, \(x \in \{0, S\}\), and imposes a minimum capital requirement, \(k_x \in [0, 1]\). In equilibrium, investors infer the type from the strongly monotone capital requirement (which we verify below). Investors supply \(1 - k_x\) of uninsured deposits and require a face value \(D_x\) (for each measure of deposits). The case of observed types yields the second-best allocation.
For a low banker type, the net present value of the project is negative. Since investors cannot break even, the regulator sets the capital requirement high enough (e.g., \( k_0^* = 1 \)). Since the project is not worthwhile pursuing at the discount rate of investors, it is also not worthwhile pursuing at the discount rate of the banker. Anticipating the high capital requirement, which does not allow the banker to raise uninsured deposits, the low type prefers not to invest in the first place and consumes her endowment at \( t = 0 \). No financial intermediation occurs for the low type.

For the high type, the regulator sets a capital requirement \( k_S \). Since capital is costly \( (z > 1) \), the banker raises \( 1 - k_S \) in deposits, has \( k_S \) of own funds invested, and consumes \( 1 - k_S \) at \( t = 0 \). Since there is limited liability, uninsured deposits are risky as only the proceeds from investment can be used to repay investors at \( t = 1 \).

The participation constraint of investors allows us to price uninsured deposits. When investment succeeds, investors of mass \( 1 - k_S \) are fully repaid and receive \( D_S \) each, so the banker receives the residual, \( R - D_S(1 - k_S) \). Upon partial default of debt, each investor receives an equal share of the scrap value, \( \frac{s}{1 - k_S} \), and the banker receives zero due to limited liability because the scrap value is exhausted. For investors to break even conditional on effort, the expected payments to all investors must equal the total deposits, \( p_H D_S(1 - k_S) + (1 - p_H)S = 1 - k_S \). Rewriting yields the face value:

\[
D_S(k_S) \equiv \frac{1 - (1 - p_H) \frac{s}{1 - k_S}}{p_H}.
\]

Equity at stake provides the banker with incentives for exerting effort, \( p^* = p_H \). Since the project has negative net present value without effort, investors can break even—and financial intermediation can be sustained—only if effort is induced by the minimum capital requirement. The expected payoff with effort must exceed the expected payoff without effort, \( p_H[R - D_S(1 - k_S)] - B \geq p_L[R - D_S(1 - k_S)] \), or
\[ k \geq k_S(D_S) \equiv 1 - \frac{R - \frac{B}{\Delta p}}{D_S}, \] (5)

which is the incentive compatibility constraint of the banker.

Combining (4) and (5) yields the equilibrium values of the face value \( D_S^* \) stated in Proposition 1 and the capital requirement \( k_S^* = p_L \frac{B}{\Delta p} - NPV \). Since \( k_S^* < 1 = k_0^* \), the capital requirement is indeed strongly monotone and reveals the type to investors. Capital regulation is required, \( k_S^* > 0 \), whenever the expected banker profit net of the effort cost exceeds the net present value of investment of the high type with effort:

\[ p_L \frac{B}{\Delta p} > NPV, \] (6)

which we assume henceforth. It is easy to verify that the banker fully repays when investment succeeds, \( R > D_S^*(1 - k_S^*) = R - \frac{B}{\Delta p} \), and partially defaults when investment fails, \( S < D_S^*(1 - k_S^*) \), because of assumption 1. The latter inequality proves that investors receive a higher payoff upon repayment than after partial default, \( \frac{S}{1 - k_S} < D_S^* \).

Regarding the participation constraint, the banker receives an information rent \( R - D_S^*(1 - k_S^*) = \frac{B}{\Delta p} \) when investment succeeds and zero otherwise by limited liability. Expected profits net of the effort cost are thus \( p_H \frac{B}{\Delta p} - B = p_L \frac{B}{\Delta p} \), which is independent of the capital level. Since the alternative use of funds (consumption) yields \( z k_S^* \), participation requires a sufficiently low cost of capital (shown in Figure 1):

\[ z \leq \tilde{z} \equiv \frac{p_L \frac{B}{\Delta p}}{p_L \frac{B}{\Delta p} - NPV}. \] (7)

We assume throughout that condition (7) holds. Proposition 1 summarizes the results.

**Proposition 1.** Observed types. If the regulator observes banker types, the second-
best allocation of partial intermediation is attained via risk-sensitive capital regulation and uninsured deposits. The low type is excluded from raising deposits and deterred from investment. The high type invests, attracts deposits \(1 - k^*_S = p_H \left( R - \frac{R}{\Delta p} \right) + (1 - p_H)S\) at face value \(D^*_S = \frac{R - \frac{R}{\Delta p}}{p_H(R - \frac{R}{\Delta p}) + (1 - p_H)S}\), and exerts effort, \(p^* = p_H\).

With observed types, capital regulation is risk-sensitive. On the extensive margin, the low banker type is deterred, while the high type invests and is allowed to raise funding. On the intensive margin, a higher scrap value allows the high type to attract cheaper funding, \(\frac{dD^*_S}{dS} < 0\). Ceteris paribus, a high type keeps more when investment is successful, resulting in better incentives to exert effort. So the regulator allows the high type to reduce her equity stake in the project and lever up with deposits, \(\frac{d(1 - k^*_S)}{dS} > 0\). Since lower equity is required, a higher scrap value allows the high type to participate for a larger range of costs of capital, \(\frac{dz}{dS} > 0\) (Figure 1).

Figure 1: Observed types and partial intermediation: The low type faces a high capital requirement and is deterred from investment. The high type invests and keeps a high enough equity stake in the project to ensure her monitoring effort. For a low cost of capital, \(z \leq \bar{z}\), the expected profit from investment exceeds the opportunity cost of capital and effort and intermediation occurs. For \(z > \bar{z}\), the minimum capital required for incentives exceeds the maximum capital required for participation.

### 3.2 Unobserved types

Next, we turn to an uninformative signal and maintain our focus on uninsured deposits and a minimum capital requirement imposed by the regulator. Since the regulator
lacks any information, the capital requirement is indiscriminate at level $k_U$, a risk-insensitive leverage ratio. The face value of debt is $D_U$ (per unit of deposits).

A first result is that either both banker types participate or none. Banker types only differ in the LGD. The banker receives nothing upon partial default, such that the participation of the banker depends only on the payoffs when investment is successful. Since the capital requirement and the face value of debt are the same for both types, the participation choice of the banker is independent of the LGD and thus the same for both types (pooling equilibrium), $zk_U \leq p_H(R - (1 - k_U)D_U) - B$.\footnote{This feature is different from Morrison and White (2005), where separation is feasible when types are heterogeneous in the probability of default (PD).}

Proposition 2 summarizes the results with unobserved types.\footnote{We abstract from potential signaling by high banker types.}

**Proposition 2. Unobserved types.** A leverage ratio achieves efficient effort, $p^* = p_H$, but an inefficient level of intermediation (compared to second-best). For a low cost of capital, $z \leq z$, intermediation is excessive: both types receive funding, $1 - k^*_U = p_H \left( R - \frac{B}{\Delta p} \right) + (1 - p_H)\gamma S$; at face value $D^*_U = \frac{R - \frac{B}{\Delta p}}{p_H(R - \frac{B}{\Delta p}) + (1 - p_H)\gamma S}$. For a high cost of capital, $z \in (z, \bar{z}]$, intermediation is insufficient, as the high type cannot raise funding.

**Proof.** See Appendix A, which also defines $z$. \hfill \blacksquare

An increase in the average quality of investment, $\gamma$, has intuitive effects. It reduces the cost of raising uninsured deposits when types are unobserved, $\frac{dD^*_U}{d\gamma} < 0$, and reduces the capital requirement, $\frac{dk^*_U}{d\gamma} < 0$ (the banker keeps more upon successful investment and has to hold less capital for incentives). As a result, the opportunity cost of investment (forgone consumption value $zk^*_U$) is lower, so intermediation occurs for a larger range of costs of capital, $\frac{dz}{d\gamma} > 0$. In other words, a higher average quality of investment moves the allocation closer the second-best benchmark.
Figure 2: Unobserved types and deviations from the second-best level of financial intermediation: For a low cost of funding, $z \leq \bar{z}$, both banker types receive funding and invest, so financial intermediation is excessively high as the low type participates. For an intermediate cost of capital, $\bar{z} < z \leq \bar{z}$, no banker type receives funding and financial intermediation breaks down. Financial intermediation is excessively low because the high type, with a positive net present value project, cannot raise funding.

A final consideration is whether the high type who is shut out from deposit funding would fund investment completely with equity. To exclude this option, we assume that capital is too costly and some (cheap) uninsured deposit funding is required for investment. We augment assumption 3 with the following upper bound:

$$p_H R + (1 - p_H) S < z + B.$$  \hspace{1cm} (8)

4 Noisy information about types

We turn to a noisy signal about the loss given default. The regulator receives a signal $x \in \{0, S\}$, where $q \in (\frac{1}{2}, 1)$ is the accuracy of the signal for each type, $\Pr\{x = S | S\} = \Pr\{x = 0 | 0\} \equiv q$. We study how the accuracy $q$ affects the allocations implemented by the regulator and the implied risk-sensitivity of capital regulation.

The regulator can implement three allocations. In autarky, the participation constraints of each banker type are violated (e.g. by setting $k_x = 1$). No intermediation occurs and welfare is $W_{Aut} = 1 + z$. In a separating equilibrium, only the high type participates, while the low type is excluded. In a pooling equilibrium, both types participate. We study separation and pooling below and then compare them.
4.1 Separation

In the separating equilibrium, only the high type participates. The low type has a negative NPV project and is separated out from the high types (via capital requirements) such that the low type does not participate in the separating equilibrium. As a result, the Bayesian posterior of investors about the share of the high type is one for each signal. Competitive pricing of uninsured deposits is thus based on the high type’s LGD, so

\[ D_x(k_x) \equiv \frac{1-(1-p_H)\frac{k_x}{S}}{p_H}, \]

where we supposed effort. The corresponding incentive compatibility constraints simplify to

\[ k \geq pL\frac{x}{\Delta p} - NPV \equiv k = k^*_S \]

for both signals, where \( k^*_S \) is the capital requirement of the high type when types are observed. To avoid confusion with the labels for observed types, we label \( x = S \) \((x = 0)\) as the good (bad) signal and the corresponding capital requirement as \( k_G \) \((k_B)\) henceforth. Note that the capital requirements only depend on the regulator’s signal, that is measured risk. We define the degree of risk sensitivity of capital regulation by

\[ \rho \equiv k_B - k_G. \quad (9) \]

The participation constraint of the high banker type is \((z-1)[qk_G+(1-q)k_B] \leq NPV.\)

Intuitively, the participation requires that the opportunity cost difference, \( z - 1 \), of the expected capital requirement for a high banker type, \( E_x[k_x|S] \equiv qk_G + (1-q)k_B \), is below the surplus from intermediation generated by a high type, \( NPV \). Similarly, the non-participation constraint of the low type is

\[ p_H D_x(1-k_x) = 1-k_x-(1-p_H)S. \]

\[ \text{Not investing allows for consuming the entire endowment, yielding } z. \text{ For the high banker type, investing requires to contribute own funds } k_G \text{ with probability } q \text{ and } k_B \text{ with probability } 1-q. \text{ Exerting effort costs } B. \text{ At } t = 1, \text{ the good state occurs with probability } p_H, \text{ in which the banker receives } R - D_G(1-k_G) \text{ with probability } q \text{ and } R - D_B(1-k_B) \text{ with probability } 1-q. \text{ Thus, the participation constraint is } z < [q(1-k_G) + (1-q)(1-k_B)] + p_H[R-qD_G(1-k_G)-(1-q)D_B(1-k_B)] - B \text{ and simplifies to the stated condition, where we used } p_H D_x(1-k_x) = 1-k_x-(1-p_H)S. \]
\[(z - 1)[qk_B + (1 - q)k_G] > NPV, \quad (10)\]

where the expected capital requirement of a low type is \(E_x[k_x|0] \equiv qk_B + (1 - q)k_G\).

Since the signal has some accuracy, \(q > \frac{1}{2}\), a high type is more likely to generate a good signal than a bad signal (and the low type is more likely to generate a bad signal). Hence, the high type’s expected capital requirement is below the low type’s expected requirement as \(k_G < k_B\). Proposition 3 characterizes the separating equilibrium.

**Proposition 3. Noisy signal about types and separating equilibrium.** The regulator can implement a separating equilibrium—only the high type participates—by setting capital requirements \(k^*_G = k^*_S\) and \(k^*_B = k^*_G + \frac{1}{q} \left(\frac{z}{z-1}NPV - pL_B \Delta p\right)\). The degree of risk sensitivity is thus \(\rho^* = \frac{1}{q} \left(\frac{z}{z-1}NPV - pL_B \Delta p\right) > 0\). Welfare in the separating equilibrium exceeds its autarky level and increases in signal accuracy, \(\frac{dW^*_{sep}}{dq} > 0\), where

\[
W^*_{sep} = 1 + z + \gamma \frac{2q - 1}{q} \left( zNPV - (z - 1)pL_B \frac{B}{\Delta p} \right) \geq 1 + z. \quad (11)
\]

The degree of risk sensitivity of capital regulation decreases in signal accuracy, \(\frac{d\rho^*}{dq} < 0\).

**Proof.** See Appendix B, where the objective function is derived and Figure 4 shows the construction of the separating equilibrium. 

The main result is that, in the separating equilibrium, the degree of risk sensitivity decreases in signal accuracy, \(\frac{d\rho^*}{dq} = \frac{d(k_B^* - k_G^*)}{dq} = \frac{dk_B^*}{dq} < 0\), so more precise regulatory information reduces the spread of capital requirements. The capital requirement after a good signal equals that for perfectly observed types, \(k^*_G = k^*_S\), and is thus independent of signal accuracy \(q\). The capital requirement after a bad signal, however, is higher than after a good signal and ensures the non-participation of the low type. As
the signal becomes more accurate, the low type is more likely to generate a bad signal, so a lower capital requirement is needed to ensure her non-participation, $\frac{dk_h}{dy} < 0$.

We can express this result in terms of the economics of crime language of Becker (1968). When the signal of the bank regulator is more accurate, then the probability for a bad type with negative NPV of being caught increases. Therefore, punishment in terms of higher capital requirement can be lowered. Since the high type is less often misidentified and, thus, faces lower expected capital requirements, welfare increases.

### 4.2 Pooling

In the pooling equilibrium, both types participate and the share of the high type is thus $\gamma$ for each signal, and the regulator ignores the signal when setting capital requirements. The competitive pricing of uninsured deposits reflects the average LGD, so $D_x(k_x) = \frac{1-(1-p_H)\frac{\gamma S}{k_x}}{p_H}$. The supposed effort is ensured by the usual incentive compatibility constraints, $k \geq k_x \equiv 1 - \frac{R - \beta}{D_x}$. Combining both equations yields a the minimum capital required for monitoring incentives (irrespective of the signal):

$$k \geq p_e \frac{B}{A_p} - \left[ NPV - (1-p_H)(1-\gamma)S \right] \equiv k_\gamma. \quad (12)$$

The minimum capital requirement implied by investor participation and banker incentives in the pooling equilibrium is tighter than in the separating equilibrium, $k_\gamma > k$.

The lower average quality under pooling increases the face value of debt required by investors to break even, so the banker keeps less when investment succeeds. To incentivize monitoring, the banker must thus have a higher equity stake in the project.

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8Moreover, the risk sensitivity decreases in the cost of capital, $\frac{d\rho^*}{dS} < 0$, and the monitoring cost, $\frac{d\rho^*}{dS} < 0$. It increases in the attractiveness of investment, $\frac{d\rho^*}{dR} > 0$, $\frac{d\rho^*}{dS} > 0$, and $\frac{d\rho^*}{dp_H} > 0$. 

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Proposition 4. Noisy signal about types and pooling equilibrium. If \( z \leq \bar{z} \), the regulator can implement a pooling equilibrium—both types participate—by setting risk-insensitive capital requirements \( k^*_G = k^*_B = k_\gamma \), a leverage ratio. Welfare in the pooling equilibrium exceeds its autarky level and is independent of signal accuracy:

\[
W^*_\text{Pool} = 1 + z + \left( zNPV^- - (z-1)p_L\frac{B}{\Delta p} \right) \geq 1 + z, \tag{13}
\]

where the expected net present value of the average project is \( NPV^- \equiv p_H R + (1-p_H)\gamma S - 1 - B = NPV - (1-p_H)(1-\gamma)S \).

**Proof.** See Appendix C for derivations and Figure 5 that shows the construction of the pooling equilibrium. □

### 4.3 Capital regulation choice of regulator

Finally, we turn to the regulator’s choice to implement either the separating or the pooling equilibrium since autarky is never optimal. We characterize under what conditions the regulator implements separation and relate these to signal accuracy.

Consider the extreme cases of signal accuracy. When the signal becomes perfect, one can show that the welfare under separation exceeds the welfare under pooling, \( \lim_{q \to 1} W^*_\text{Sep} > W^*_\text{Pool} \). When the signal becomes uninformative, separation yields autarky welfare and pooling is preferable, \( \lim_{q \to 1/2} W^*_\text{Sep} = 1 + z \leq W^*_\text{Pool} \). By Proposition 3, the welfare under separation increases in signal accuracy, so there is a unique threshold signal accuracy \( q^* \) such that pooling and separation yield the same welfare. Figure 3 shows the regulator’s choice of allocations and Proposition 5 summarizes.

Proposition 5. Implementation. If \( \underline{z} < z \leq \bar{z} \), the pooling equilibrium is infeasible
Figure 3: Equilibrium and risk-sensitivity for $z \leq z^*$. The regulator implements a separating equilibrium with positive but decreasing risk-sensitivity for an accurate signal, $q \geq q^*$, and a pooling equilibrium with zero risk-sensitivity otherwise, $q < q^*$.

and the regulator implements the separating equilibrium. If $z \leq z^*$, there exists a

$$q^* \equiv \gamma \frac{\frac{z}{z-1}NPV - p_L \frac{B}{\Delta p}}{(1-2\gamma) \left[ p_L \frac{B}{\Delta p} - \frac{z}{z-1}NPV \right] + z(1-p_H)(1-\gamma)S} \in \left( \frac{1}{2}, 1 \right) \quad (14)$$

such that the regulator implements the separating equilibrium if and only if $q \geq q^*$.

Figure 3 shows that the risk-sensitivity of capital regulation is non-monotonic. For a fairly inaccurate signal, $q < q^*$, the regulator imposes a pooling equilibrium with capital regulation insensitive to risk. When the signal is not accurate enough, it is too costly to separate types because the high type would face too often a bad signal and, thus, a high capital requirement. When the signal is sufficiently accurate, however, the regulator separates types via capital requirements that are sensitive to measured risk (regulator’s signal). As the signal gets more accurate, the capital requirement after a bad signal can be relaxed. In line with the intuition of Becker (1968), bank capital regulation is less risk-sensitive as the regulator measures risk more accurately.
5 Discussion: uncertain banker type

In this section, we discuss an alternative environment with coarser information by assuming that the banker is uncertain about her own type. As a result, both high and low banker types are active in equilibrium.

In the main model, the banker perfectly knows her type. As a result, the regulator faces only the high banker type in the separating equilibrium but has to impose a high capital requirement after a bad signal to deter participation of the low type. One concern of this outcome is that the banker may sue the regulator since both the banker and the regulator are certain about the high quality of the investment.\footnote{We thank the referee for pointing out this concern.}

The purpose of this section is to introduce some noise to the banker’s information about her type. This extension allows us to show that the regulator faces both low-type and high-type bankers in the separating equilibrium. To capture this idea in a parsimonious way, suppose that the banker receives a signal \( y \in \{0, S\} \) about her type at \( t = 0 \) with some noise \( \epsilon \in (0, \frac{1}{2}) \). That is, the probability of the signal being correct conditional on the true unobserved type is \( 1 - \epsilon \) for each type.

Bayesian updating about her type implies that the banker with a positive signal assigns the probability \( \gamma_H \equiv \frac{\gamma(1-\epsilon)}{\gamma(1-\epsilon)+(1-\gamma)\epsilon} \in (\gamma, 1) \) to the high type. Next, the banker needs to form a belief about the signal \( x \) received by the regulator. Conditional on \( y = S \), the banker assigns the following probability to a good regulatory signal:

\[
\Pr\{x = S|y = S\} \equiv q_H = \gamma_H q + (1 - \gamma_H)(1 - q) < q. \tag{15}
\]

In a separating equilibrium with uncertain banker types, the banker with a good private signal, \( y = S \), participates and the banker with a bad private signal does not.
As a result, some truly low types participate and the regulatory examination charges a high capital requirement to some of them. Thus, capital requirements have a dual role in this extended model: (i) they prevent the participation of bankers with a low signal, \( y = 0 \), of which most are truly a low type; and (ii) they charge a higher capital requirement to truly low banker types who received a good private signal, \( y = S \).

6 Conclusion

Inspired by the current policy debate on the risk sensitivity of bank capital regulation, we have provided a simple model to investigate this issue for a regulator with imperfect information about bank risk. The model’s key ingredients are a moral hazard problem in the bank’s choice of the probability of default (PD) that necessitates costly bank capital for incentives, heterogeneous losses given default (LGD), and a rationale for capital regulation based on superior but noisy regulatory information.

Our main result is to identify the non-monotonic risk-sensitivity of optimal bank capital regulation. With a noisy but informative signal, the regulator can implement (i) a separating equilibrium by offering risk-sensitive capital requirements to exclude the low type; or (ii) a pooling equilibrium in which both types participate. The regulator chooses pooling when the signal is too inaccurate, so capital requirements are insensitive to risk. For a sufficiently accurate signal, the regulator chooses separation, making capital requirements sensitive to risk. The risk sensitivity of bank capital regulation then decreases in the accuracy of the signal, because lower capital requirements after a bad signal suffice to exclude the low type from participation.

Our analysis can be extended along several directions. First, the role of the regulator is, effectively, to disseminate information about the banker to investors, for
example, by direct information release, a stress test, or setting capital requirements. It could be interesting to extend the analysis to other regulatory measures, such as bank closure. Second, our emphasis is on a microprudential regulator by focusing on an individual bank without considerations of systemic risk. A macroprudential dimension could be added in an extension with multiple bankers and market-determined losses given default. Third, we have maintained commitment of the bank regulator throughout. The case of limited commitment is an interesting avenue for further work that is perhaps best studied in a dynamic model in order to allow for reputation concerns of the regulator. Finally, we have considered uninsured deposits throughout the paper but expect our results to extend qualitatively to partial deposit insurance.
References


A Proof of Proposition 2

Turning to the incentive compatibility constraint of the banker, effort again requires the banker to have sufficient skin in the game (minimum capital):

\[ k \geq k_U(D_U) \equiv 1 - \frac{R - B}{D_U}. \]  

(16)

In equilibrium, the minimum capital requirement binds because capital is costly. Imposing a lower requirement is not optimal, since the banker would shirk and the project would have negative net present value and investors would not break even. In an equilibrium with participation of both types, uninsured deposits are priced according to average investment quality, \( \gamma_S \). Conditional on effort, the face value is therefore

\[ D_U(k_U) = \frac{1 - (1 - p_H)\gamma \frac{S}{1 - k_U}}{p_H}. \]  

(17)

Using (16) and (17) yields the stated face value \( D_U^* \) and the capital requirement \( k_U^* = p_L \frac{B}{\Delta p} - [NPV - (1 - \gamma)(1 - p_H)S] \). Banker participation again requires the expected net profits, \( p_L \frac{B}{\Delta p} \), to exceed the opportunity cost, \( zk_U^* \), and yields another upper bound on the cost of capital:

\[ z \leq \bar{z} \equiv \frac{p_L \frac{B}{\Delta p}}{p_L \frac{B}{\Delta p} - [NPV - (1 - p_H)(1 - \gamma)S]} < \bar{z}. \]  

(18)

The high type cross-subsidizes the low type when financial intermediation occurs, so the relevant expected net present value is that of the average project, \( p_H R + (1 - p_H)\gamma S - 1 - B = NPV - (1 - p_H)(1 - \gamma)S = NPV^- \). Hence, uninsured deposits are more expensive, \( D_U^* > D_S^* \), the capital requirement is more restrictive, \( k_U^* > k_S^* \), and the bound on the costs of capital is lower than for observed types, \( \bar{z} < \bar{z} \), as shown in Figure 2.
B Proof of Proposition 3

For a graphical analysis, we rewrite the participation constraints, \( PC_H \) and \( PC_L \) as

\[
k_G \leq \frac{NPV}{q(z - 1)} - \frac{1 - q}{q} k_B, \quad k_G \geq \frac{NPV}{(1 - q)(z - 1)} - \frac{q}{1 - q} k_B.
\]

Both participation constraints intersect at \( \frac{NPV}{z - 1} \geq k \), where the inequality is because of \( z \leq \overline{z} \). Assumption (8) ensures that \( k = \frac{NPV}{z - 1} < 1 \). Figure 4 shows all constraints and shades the feasible set of minimum capital requirements \((k_G, k_B)\) consistent with separation.

Utilitarian welfare \( W \) sums the expected utility of the banker and investors. Investors are competitive and receive 1. In a separating equilibrium, the low banker type (with probability \( 1 - \gamma \)) does not participate and consumes her endowment, yielding \( (1 - \gamma)z \). The high type (with probability \( \gamma \)) invests, exerts effort and pays the monitoring cost, keeps an equity stake \( \mathbb{E}_x[k_x|S] = qk_G + (1 - q)k_B \), raises \( \mathbb{E}_x[1 - k_x|S] \) in uninsured deposits at \( D_x \), and receives all profits when investment succeeds, where the investors receive \( \mathbb{E}_x[(1 - k_x)D_x|S] = q(1 - k_G)D_G + (1 - q)(1 - k_B)D_B \). Taken together, welfare under separation is:

\[
W_{Sep} \equiv 1 + (1 - \gamma)z + \gamma \{ z \mathbb{E}_x[1 - k_x|S] + p_H (R - \mathbb{E}_x[(1 - k_x)D_x|S]) - B \} \\
= 1 + z + \gamma \{ NPV - (z - 1) [qk_G + (1 - q)k_B] \}. \tag{20}
\]

The iso-welfare curve has a slope of \( -\frac{1 - q}{q} \), which is the same as the participation constraint of the high banker type. The regulator shifts the iso-welfare curves as close to the origin as possible (Figure 4) because welfare is higher for lower capital

\footnote{We henceforth use a weak inequality for the participation constraint of the low type, \( PC_L \). This approach is without loss of generality if we add an amount \( \epsilon > 0 \) to the minimum requirement of a banker with a bad signal, \( k_B \), and take the limit \( \epsilon \to 0 \) to ensure non-participation of the low type.}
requirements (due to their opportunity cost). It follows that the separating equilibrium is given by the intersection of the incentive compatibility constraint and the participation constraint of the low type. Rearranging yields the capital requirements stated in Proposition 3.

Figure 4 shows the construction of the separating equilibrium. We abstract from the feasibility constraint, \( k_B^* \leq 1 \). One can show that our approach is innocuous in the following sense. If \( p_L \frac{B}{2p} < 1 + NPV \), then there exists a \( \tilde{z} \equiv \frac{1 + NPV + p_L \frac{B}{2p}}{1 + p_L \frac{B}{2p} - NPV} < \bar{z} \). If \( z \in (\tilde{z}, \bar{z}) \), then the feasibility constraint is slack, \( k_B^* < 1 \), for all \( q \in \left(\frac{1}{2}, 1\right) \).

The expression for welfare arises from inserting the separating capital requirements into expression (20). Welfare in the separating equilibrium, given in equation (11), exceeds autarky welfare, since the factor \( \frac{2q - 1}{q} \) is strictly positive \( (q > \frac{1}{2}) \) and the term in parenthesis is positive whenever \( z \leq \bar{z} \) because of condition (7), with strict inequality for \( z < \bar{z} \).
C Proof of Proposition 4

The participation constraint are derived as before, but both types participate under pooling and the average investment quality defines the net present value $NPV^-$:

$$k_G \leq \frac{NPV^-}{q(z-1)} - \frac{1-q}{q}k_B, \quad k_G \leq \frac{NPV^-}{(1-q)(z-1)} - \frac{q}{1-q}k_B. \quad (21)$$

Figure 5 shows all constraints with shaded feasible set of minimum capital requirements $(k_G, k_B)$ consistent with a pooling equilibrium. Since the incentive compatibility constraints have shifted outward and the participation constraints inward, we first establish when the set of feasible requirements is non-empty, as depicted in Figure 5. This yields a tighter upper bound on the opportunity cost of capital, $z$:

$$z \leq \bar{z} = \frac{p_L B}{\Delta p} - \frac{B}{\Delta p} NPV^- \times \left( \frac{\gamma q + (1-\gamma)q}{1-q} \right) k_G + \left( (\gamma(1-q) + (1-\gamma)q) k_B \right). \quad (22)$$

which is the same bound as in the case where types are unobserved. Quite intuitively, both in the pooling equilibrium and in the unobserved types case, both banker types participate and the cross-subsidization from the high type to the low type restricts the range of capital costs for which this arrangement can be supported in equilibrium.

We turn to the objective function. Its derivation parallels that under separation, with the difference being that both types invest, exert effort and pay the monitoring cost, and receive the net proceeds from investment when it succeeds.

$$W_{Pool} \equiv 1 + \gamma \{ z \mathbb{E}_x [1 - k_x | S] + p_H \left[ R - \mathbb{E}_x [(1 - k_x)D_x | S] \right] - B \} + \cdots$$

$$\cdots + (1-\gamma) \{ z \mathbb{E}_x [1 - k_x | 0] + p_H \left[ R - \mathbb{E}_x [(1 - k_x)D_x | 0] \right] - B \} \quad (23)$$

$$= 1 + z + NPV^- - (z-1) \left[ (\gamma q + (1-\gamma)(1-q)) k_G + (\gamma(1-q) + (1-\gamma)q) k_B \right].$$
It follows that the iso-welfare curve has a slope of \( -\frac{\gamma(1-q)+(1-\gamma)q}{\gamma q+(1-\gamma)(1-q)} \), which is between the slopes of the two participation constraints.

Figure 5 shows the construction of the pooling equilibrium, which is similar to that of the separating equilibrium.

Figure 5: Pooling equilibrium. The shaded green area is the set of feasible capital requirements. The iso-welfare curve is the red dotted line. The pooling equilibrium is at the intersection of both incentive compatibility constraints. Capital requirements are risk-insensitive, \( \rho^* = 0 \).

The iso-welfare curve has an intermediate slope between the slope of the two participation constraints. The regulator again shifts the iso-welfare curves as close to the origin as possible, so the pooling equilibrium is at the intersection of the two incentive compatibility constraints. Both participation constraints are slack in equilibrium. The expression for welfare arises from inserting the pooling capital requirements into expression (23). Welfare in the pooling equilibrium exceeds autarky welfare because the term in parenthesis is positive whenever \( z \leq \bar{z} \), with strict inequality for \( z < \bar{z} \).