Asset Encumbrance, Bank Funding and Fragility*

Toni Ahnert Kartik Anand Prasanna Gai James Chapman

Revised July 2018

Abstract

We model asset encumbrance by banks subject to rollover risk and study the consequences for fragility, funding costs, and prudential regulation. A bank's privately optimal encumbrance choice balances the benefit of expanding profitable yet illiquid investment, funded by cheap long-term senior secured debt, against the cost of greater fragility from runs on unsecured debt. We derive testable implications about encumbrance ratios. The introduction of deposit insurance or wholesale funding guarantees induces excessive encumbrance and fragility. Ex-ante limits on asset encumbrance or ex-post Pigovian taxes eliminate such risk-shifting incentives. Our results shed light on prudential policies currently pursued in several jurisdictions.

Keywords: asset encumbrance, bank runs, wholesale funding, secured debt, unsecured debt, encumbrance limits, encumbrance surcharges.

JEL classifications: G01, G21, G28.

*We are grateful to the editor (Philip Strahan) and three anonymous referees for helpful comments that significantly improved the paper. We also wish to thank Jason Allen, Scott Frame, Itay Goldstein, Philipp König, Agnese Leonello, Jean-Charles Rochet, Javier Suarez, Jing Zeng, audiences at AFA 2017, Alberta School of Business, Amsterdam, Atlanta Fed, Bank of Canada, Bank of England, Carlos III, Carleton, Danmarks Nationalbank, DGF 2016, ECB, EEA 2016, EFA 2017, FDIC 2015, Frankfurt School, HEC Montreal, HU Berlin, IESE, McGill, NFA 2015, Queen's, and RIDGE 2015 for comments. Matthew Cormier provided excellent research assistance. Gai is grateful to the Warden and Fellows of All Souls College, Oxford for their hospitality and acknowledges financial assistance from the University of Auckland (FRDF 3000875) and INET (INO140004). These views and all errors are our own. Ahnert and Chapman: Bank of Canada, 234 Wellington St, Ottawa, ON K1A 0G9, Canada. Anand: Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt, Germany. Gai: University of Auckland, 12 Grafton Rd, Auckland 1010, New Zealand.

1 Introduction

Banks attract secured funding by pledging assets on their balance sheet as collateral and ring-fencing, or "encumbering," them so that they are unavailable to, and take priority over the claims of, unsecured creditors in the event of a default. Following the global financial crisis and the European sovereign debt crisis, investors have sought safe financial assets, including instruments that are excluded from writedowns in many resolution frameworks (Haldane, 2012; CGFS, 2013). As a result, many banks in the euro area and the United States have become reliant on long-term senior secured debt for their funding. The share of covered bonds in total gross bond issuance by euro area banks rose from 26% in 2007 to 42% in 2012 (Rixtel and Gasperini, 2013). And the share of advances to non-deposit liabilities extended by the Federal Home Loan Bank (FHLB) system to US banks more than doubled from 12% in 2011 to 26% in 2016 (Adams-Kane and Wilhelmus, 2017).[1]

Greater asset encumbrance levels pose several important positive and normative questions. What is the relationship between asset encumbrance and rollover risk in unsecured debt markets? What determines a bank's encumbrance choice and how is that choice affected by policies aimed at reducing fragility, such as deposit insurance and guarantees? To the extent that such guarantees distort encumbrance choices, how should corrective measures be designed? And how do measures introduced in several jurisdictions to limit asset encumbrance fare against a normative benchmark?

[1]These collateralized debt instruments are distinct from securitization. In securitizations, the creditor is paid directly from the cash flow of the pledged asset and has no recourse to the borrower's other assets in default. By contrast, with covered bonds and FHLB advances, the issuing bank is liable to secured creditors who retain recourse to the bank's unencumbered assets. Moreover, the bank is required to replenish impaired encumbered assets. A further distinction is that, with covered bonds and advances, encumbered assets remain on balance sheet for accounting purposes. In securitizations, by contrast, assets are transferred off-balance sheet to facilitate credit risk transfer and economize on capital requirements (Schwarcz, 2011, 2013; FHLB, 2018). Covered bonds and advances are also extremely safe – German covered bonds have not been defaulted on since 1901 and no FHLB has ever incurred a loss on an advance.

Our paper proposes a model of asset encumbrance to study these issues. Building on Rochet and Vives (2004), a bank is subject to rollover risk of unsecured debt.[2] The bank has initial equity and seeks funding for profitable investment that is costly to liquidate. Funding markets are segmented – unsecured debt is issued to risk-neutral investors, while secured debt is issued to infinitely risk-averse investors by encumbering assets on the bank's balance sheet in a bankruptcy-remote entity. If the value of encumbered assets is insufficient, secured debtholders have recourse to the bank's unencumbered assets. The consequences of a shock to the bank's balance sheet are borne entirely by unsecured debtholders. Secured debtholders, by contrast, are unaffected since the encumbered assets are insulated from the shock and from any claims by unsecured debtholders in the event of bank failure.

Drawing on global games techniques to pin down a unique equilibrium (Carlsson and van Damme, 1993; Morris and Shin, 2003), a run on unsecured debt occurs whenever the balance sheet shock exceeds a threshold value. Since unencumbered assets are depleted after a run, the demandability of unsecured debt renders secured debtholders' recourse to unencumbered assets worthless. We link the incidence of runs to the bank's encumbrance choice and price secured and unsecured debt claims.

Asset encumbrance alters run dynamics by driving a wedge between the conditions for illiquidity and insolvency. If the bank prematurely liquidates assets to satisfy unsecured debt withdrawals, it can only use unencumbered assets since encumbered assets are pledged to secured debtholders. But if unsecured debt is rolled over, the bank can pay unsecured debtholders using residual encumbered assets once secured debtholders have been paid. This is possible because of the overcollateralization of

[2]Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability arises endogenously with liquidity needs (Diamond and Dybvig, 1983), as a commitment device to overcome an agency conflict (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), or to commit to providing a safe claim when investors seek safety (Ahnert and Perotti, 2018).

the asset pool backing secured debt, which ensures that value of encumbered assets exceeds the face value of secured debt. While illiquidity only depends on unencumbered assets, insolvency depends on all assets. We show that the illiquidity condition is more binding than the insolvency condition if unsecured debt is cheap. Asset encumbrance can, therefore, make solvent banks illiquid and prone to runs. This result contrasts with Rochet and Vives (2004), where an illiquid bank is always insolvent.

Greater encumbrance ratios induce two opposing effect on fragility. On the one hand, it allows a greater amount of cheap secured debt to be raised in order to finance profitable investment. On the other hand, too few unencumbered assets are available to meet unsecured debt withdrawals, exacerbating fragility.[3] The latter effect dominates when the cost of recovering encumbered assets upon bank failure is large, which is often the case in practice. For example, secured debtholders may have to assert their claims against unsecured debtholders in expensive and protracted legal proceedings (Ayotte and Gaon, 2011; Duffie and Skeel, 2012; Fleming and Sarkar, 2014). Access to critical infrastructure such as shared IT and risk management systems may also be impeded upon bank failure (Bolton and Oehmke, 2018). Thus, more encumbrance heightens fragility.[4]

The model yields a rich set of testable implications that are consistent with empirical evidence and which can inform future empirical work. The privately optimal encumbrance ratio for the bank balances marginal costs (greater fragility and a lower probability of surviving a run) against the marginal benefits (greater profitability conditional on survival). Accordingly, greater encumbrance ratios arise when (a) investment is more profitable; (b) funding costs are lower; (c) the distribution of

[3] In a corporate finance model with incomplete contracts, Bolton and Oehmke (2015) also show that a firm's increased reliance on derivatives privileged in bankruptcy can increase its credit risk.

[4] Consistent with this result, Bennett et al. (2005) and Garcia et al. (2017) show that more FHLB advances increase the default probability of US commercial banks, and greater covered bond issuance increases the default risk of European banks, respectively.

balance sheet shocks is more favourable; (d) unsecured debt is rolled over frequently; (e) recovery costs on encumbered assets are lower; (f) liquidation values of investment are higher; (g) banks hold more liquid reserves, and (h) the risk premium charged by risk-neutral investors is lower. In addition, we find that the impact of bank capital on encumbrance ratios is ambiguous.

Our framework also provides important normative insights. In many advanced countries, public guarantee schemes for unsecured debtholders are an integral part of the financial system. While such privileges usually apply to retail depositors, they often extend to unsecured wholesale depositors during financial crises. In our model, the bank does not internalize the social cost of servicing the guarantee and so has an incentive to excessively encumber assets, which exacerbates financial fragility.[5] The privately optimal encumbrance choice with guarantees is constrained inefficient.

We show how prudential safeguards can ameliorate the tendency for such excessive asset encumbrance. If the financial regulator observes asset encumbrance ex ante, limits on encumbrance ratios can achieve the social optimum. If encumbrance can only be observed ex post, however, risk-neutral Pigovian taxes on encumbrance ratios can also ensure the social optimum. A linear tax on encumbrance corrects the incentive of the bank to shift risk to the guarantee scheme, while rebating the revenue in lump-sum fashion ensures that there are sufficient resources to avoid excessive fragility. For taxes contingent on the face value of unsecured debt, we derive a closed-form solution for the optimal tax rate.

Our analysis sheds light on the regulatory approach to asset encumbrance taken in several jurisdictions. Some countries (e.g. Australia) have adopted a cap on encumbered assets similar to the analysis in this paper. In the United States, a cap is

[5] We abstract from deposit insurance premiums. In practice, these premiums are usually insensitive to encumbrance ratios, so banks have incentives to excessively encumber assets.

applied to the share of covered bonds to total liabilities. Both measures are equivalent in our model. In Italy, for example, encumbrance caps for banks are contingent on their capital ratios, with no limits for highly capitalized banks. Our results suggest that while such an approach may curb the incentive for lowly capitalized banks to encumber excessively, it does not reduce the incentives for highly capitalized institutions. Deposit insurance premia for systemic Canadian banks partly reflect the extent to which their assets are encumbered (CDIC, 2017). This encumbrance surcharge can be viewed as a form of the Pigovian taxation examined in this paper.

Our paper contributes to the literature on bank runs and global games (Morris and Shin, 2001; Goldstein and Pauzner, 2005). In the unique equilibrium, a run is the consequence of a coordination failure as bank fundamentals deteriorate. In particular, we build on Rochet and Vives (2004) where unsecured debtholders delegate their rollover decisions to professional fund managers, so the decisions to roll over are global strategic complements.[6] Our contribution is to introduce secured funding and to identify how asset encumbrance affects run risk and the pricing of unsecured debt.

An early contribution on bank funding choices is Greenbaum and Thakor (1987). In an asymmetric-information setting, banks with high-quality assets securitize these, while banks with low-quality assets use deposit funding. In our model, by contrast, a bank uses a mix of unsecured and secured funding. This interaction of funding sources allows us to examine how asset encumbrance affects fragility and the role of prudential regulation. These issues are absent in Greenbaum and Thakor (1987).

Our paper adds to a nascent literature on the interaction between secured and

[6] Goldstein and Pauzner (2005) study one-sided strategic complementarity due to the sequential service constraint of banks (Diamond and Dybvig, 1983). Matta and Perotti (2017) contrast the sequential service constraint with the mandatory stay of illiquid assets and study its impact on run risk. Eisenbach (2017) shows how rollover risk from demandable debt effectively disciplines banks for idiosyncratic shocks, while a two-sided inefficiency arises for aggregate shocks.

unsecured debt. In a corporate finance setting, Auh and Sundaresan (2015) study how short-term secured debt interacts with long-term unsecured debt. Ranaldo et al. (2017) studies short-term secured and unsecured debt in money markets, where shocks to asset values lead to mutually reinforcing liquidity spirals. Our focus, by contrast, is the interaction between long-term secured and demandable unsecured debt.

Our paper also contributes to the policy debate on the financial stability implications of asset encumbrance. Gai et al. (2013) and Eisenbach et al. (2014) develop partial equilibrium models exploring the interplay between secured and unsecured funding. In a global-game setup, Gai et al. (2013) show that interim liquidity risk and encumbrance intertwine and can generate a 'scramble for collateral' by short-term secured creditors. Eisenbach et al. (2014) study a range of wholesale funding arrangements with exogenous creditor decisions. Their model suggests that asset encumbrance increases insolvency risk when the encumbrance ratio is sufficiently high.

2 Model

Our model builds on Rochet and Vives (2004). There are three dates $t = 0, 1, 2$, a single good for consumption and investment, and a large mass of investors. Each investor has a unit endowment at $t = 0$ and can store it until $t = 2$ at a gross return $r > 0$. Although investors are indifferent between consuming at $t = 1$ and $t = 2$, they differ in their risk preferences. A first clientele is risk-neutral, while a second is infinitely risk-averse. The latter group can be thought of large institutional investors mandated to hold high-quality safe assets, e.g. pension funds (IMF, 2012).

A representative risk-neutral bank has access to illiquid investment at $t = 0$ that mature at $t = 2$ with return $R > r$. Liquidation at $t = 1$ yields a fraction

$\psi \in (0, 1)$ of the return. In order to consume at $t = 2$, the bank invests own funds, $E \geq 0$, and obtains funds from the segmented investor base by issuing unsecured demandable debt to risk-neutral investors and secured debt to risk-averse investors.

Unsecured debt, $U \equiv 1$, is withdrawn at $t = 1$ or rolled over until $t = 2$. Investors delegate the rollover decision to professional fund managers, $i \in [0, 1]$, who are rewarded for making the right decision. If the bank does not fail, a manager's payoff difference between withdrawing and rolling over is a cost $c > 0$. If the bank fails, the differential payoff is a benefit $b > 0$.[7] The conservatism ratio, $\gamma \equiv \frac{c}{b+c} \in (0, 1)$, summarizes these payoffs, with more conservative managers being less likely to roll over.[8] The face value of unsecured debt, D_U, is independent of the withdrawal date.

The bank attracts secured funding by encumbering a proportion $\alpha \in [0, 1]$ of assets and placing them in a bankruptcy-remote entity. Bankruptcy-remoteness ensures that secured debtholders retain an exclusive claim to encumbered assets even upon bank failure. This is achieved by legally separating encumbered assets from the bank, thereby fully insulating these assets from unsecured debt claims. If the value of encumbered assets is insufficient to meet the claims of secured debtholders upon bank failure, they also have recourse to unencumbered assets.[9] Let $S \geq 0$ be the amount of long-term secured funding and D_S its face value at $t = 2$. Table 1 shows the balance sheet at $t = 0$, after investment $I \equiv E + S + U$ and asset encumbrance.

The bank is protected by limited liability and its balance sheet is subject to a shock, A, at $t = 2$. This shock may improve the balance sheet, $A < 0$. But the crystallization of operational, market, credit or legal risks may require writedowns, $A > 0$.

[7] As an example, assume the cost of withdrawal is c; the benefit from getting the money back or withdrawing when the bank fails is $b + c$; the payoff for rolling over when the bank fails is zero.

[8] Reviewing debt markets during the financial crisis, Krishnamurthy (2010) argues that investor conservatism was an important determinant of short-term lending behavior. See also Vives (2014).

[9] Secured debtholders have equal seniority to the bank's unencumbered assets as unsecured debtholders for covered bonds and are senior to unsecured debtholders for FHLB advances.

Assets		Liabilities
(encumbered assets)	αI	S
(unencumbered assets)	$(1-\alpha)I$	U
		E

Table 1: Balance sheet at $t = 0$ after funding, investment, and asset encumbrance.

The shock has a continuous density $f(A)$ and distribution $F(A)$, with decreasing reverse hazard rate, $\frac{d}{dA}\frac{f(A)}{F(A)} < 0$, to ensure a unique equilibrium. Since encumbered assets are ring-fenced, the shock only affects unencumbered assets.[10] Table 2 shows the balance sheet at $t = 2$ for a small shock and when all unsecured debt is rolled over. Bank equity at $t = 2$ is worth $E_2(A) \equiv \max\{0, RI - A - UD_U - SD_S\}$.

Assets		Liabilities
(encumbered assets)	$R\alpha I$	SD_S
(unencumbered assets)	$R(1-\alpha)I - A$	UD_U
		$E_2(A)$

Table 2: Balance sheet at $t = 2$ after a small shock and rollover of all unsecured debt.

If a proportion $\ell \in [0, 1]$ of unsecured debt is not rolled over at $t = 1$, the bank liquidates an amount $\ell\frac{UD_U}{\psi R}$ to meet withdrawals. A bank fails due to *illiquidity* and is closed early if the liquidation value of unencumbered assets is insufficient:

$$R(1-\alpha)I - A < \frac{\ell UD_U}{\psi}, \tag{1}$$

so the illiquidity threshold of the shock is $A_{IL}(\ell) \equiv R(1-\alpha)I - \frac{\ell UD_U}{\psi}$. A larger proportion of withdrawing fund managers reduces the illiquidity threshold, $\frac{dA_{IL}}{d\ell} < 0$.

Upon early closure, secured debtholders can recover encumbered assets. But recovery may be partial, reflecting legal difficulties in seizing collateral assets (Duffie and Skeel, 2012; Ayotte and Gaon, 2011), the inability of secured debtholders to

[10] Our modelling approach is consistent with the notion of collateral replenishment, whereby non-performing encumbered assets are replaced by performing assets from the unencumbered part of the balance sheet. Replenishment concentrates credit and market risks on unsecured creditors.

properly redeploy these assets (Diamond and Rajan, 2001), or the loss of economies of scale and scope due to shared services and common IT or risk management systems (Bolton and Oehmke, 2018). Accordingly, the return for secured debtholders is $\lambda R\alpha$, where $\lambda \in [\psi, 1)$ and recovering encumbered assets is cheaper than liquidation. Unsecured creditors are assumed to face a zero recovery rate upon bank failure.

A liquid bank has assets worth $RI - \ell\frac{UD_U}{\psi} - A$ at $t = 2$. Upon repaying secured debt, the bank can use any residual encumbered assets (due to over-collateralization) to repay remaining unsecured debt. The bank fails due to *insolvency* at $t = 2$ if

$$RI - A - \frac{\ell UD_U}{\psi} < SD_S + (1-\ell)UD_U, \tag{2}$$

so the insolvency threshold of the shock is $A_{IS}(\ell) \equiv RI - SD_S - UD_U \left[1 + \ell\left(\frac{1}{\psi} - 1\right)\right]$.

At $t = 1$, each fund manager bases the rollover decision on a noisy private signal about the shock

$$x_i \equiv A + \epsilon_i, \tag{3}$$

where ϵ_i is idiosyncratic noise drawn from a continuous distribution H with support $[-\epsilon, \epsilon]$ for $\epsilon > 0$. Idiosyncratic noise is independent of the shock and i.i.d. across fund managers. Table 3 summarizes the timeline of events.

$t = 0$	$t = 1$	$t = 2$
1. Issuance of secured and unsecured debt	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
3. Asset encumbrance	3. Unsecured debt withdrawals	3. Debt repayments
	4. Consumption	4. Consumption

Table 3: Timeline of events.

3 Equilibrium

Our focus is the symmetric pure-strategy perfect Bayesian equilibrium and threshold strategies for the rollover of unsecured debt. Each fund manager rolls over unsecured debt whenever their private signal indicates a healthy balance sheet, $x_i \leq x^*$.[11]

Definition 1. *The symmetric pure-strategy perfect Bayesian equilibrium comprises an encumbrance ratio* (α^*)*, an amount of secured debt* (S^*)*, face values of unsecured and secured debt* (D_U^*, D_S^*)*, and signal and shock thresholds* (x^*, A^*) *such that:*

a. *at* $t = 1$*, the rollover decisions of all fund managers,* x^**, are optimal and the run threshold* A^* *induces bank failure for any shock* $A > A^*$*, given the encumbrance ratio and secured debt* (α^*, S^*) *and face values of debt* (D_U^*, D_S^*)*;*

b. *at* $t = 0$*, the bank optimally chooses* (α^*, S^*) *given the face values of debt* (D_U^*, D_S^*)*, the participation of secured debtholders, and the thresholds* (x^*, A^*)*;*

c. *at* $t = 0$*, secured and unsecured debt are priced by binding participation constraints, given the choices* (α^*, S^*) *and the thresholds* (x^*, A^*)*.*

We construct the equilibrium in four steps. First, we price secured debt and derive the amount of secured debt issued. Second, we describe the optimal rollover decision of fund managers. Third, we characterize the optimal asset encumbrance choice of the bank. In a final step, we price unsecured debt.

[11] Since we assume that private information is sufficiently precise, the equilibrium is unique (Morris and Shin, 2003). It is also an extremal equilibrium that is in monotone strategies (Vives, 2005). Since the rollover decision is binary, our focus on threshold strategies is without loss of generality.

3.1 Pricing secured debt

The bank promises to repay secured debtholders $D_S \geq r$. After an adverse shock, however, the bank fails at $t = 1$ due to a run, which fully depletes the value of unencumbered assets. Consequently, secured debtholders' recourse to unencumbered assets is worthless. But encumbered assets remain bankruptcy remote, i.e. legally separated from the failed bank and insulated from the shock and the claims of unsecured debtholders. Each secured debtholder, thus, recovers an equal share of the encumbered assets, $\lambda \frac{R\alpha I}{S}$. Since secured debtholders are infinitely risk-averse, they value their claim at $t = 0$ at the worst outcome, i.e. when the bank fails at $t = 1$ due to a run and is closed early, which yields $\min\left\{D_S, \lambda \frac{R\alpha I}{S}\right\}$. Under competitive pricing, this value equals the risk-free return, r.

Lemma 1. Asset encumbrance and secured debt. *Secured debt is risk-free, $D_S^* = r$, and its price is independent of the amount of secured debt issued. The maximum issuance of secured debt tolerated by risk-averse investors is $S \leq S^*(\alpha) = \lambda R\alpha \frac{U+E}{r-\lambda R\alpha}$, which increases in the asset encumbrance ratio.*

In equilibrium, the bank issues the maximum amount of secured debt for a given asset encumbrance ratio, $S^* = S^*(\alpha)$. Hence, the volume of investment is

$$I^*(\alpha) \equiv S^*(\alpha) + U + E = \frac{U + E}{1 - \frac{\lambda R}{r}\alpha}. \tag{4}$$

Encumbering more assets increases the volume of secured debt issued and, therefore, the volume of investment, $\frac{dS^*}{d\alpha} = \frac{dI^*}{d\alpha} = \frac{\lambda R r(U+E)}{(r-\lambda R\alpha)^2} > 0$.

The result in Lemma 1 applies more broadly. If senior secured debt were demandable, risk-averse investors would always be willing to roll over their risk-free claims on the bank. Also, if an investors' type were unobserved, incentive compatibility constraints would hold: a risk-averse investor strictly prefers safe secured debt, while a risk-neutral investor weakly prefers risky unsecured debt.

3.2 Rollover risk of unsecured debt

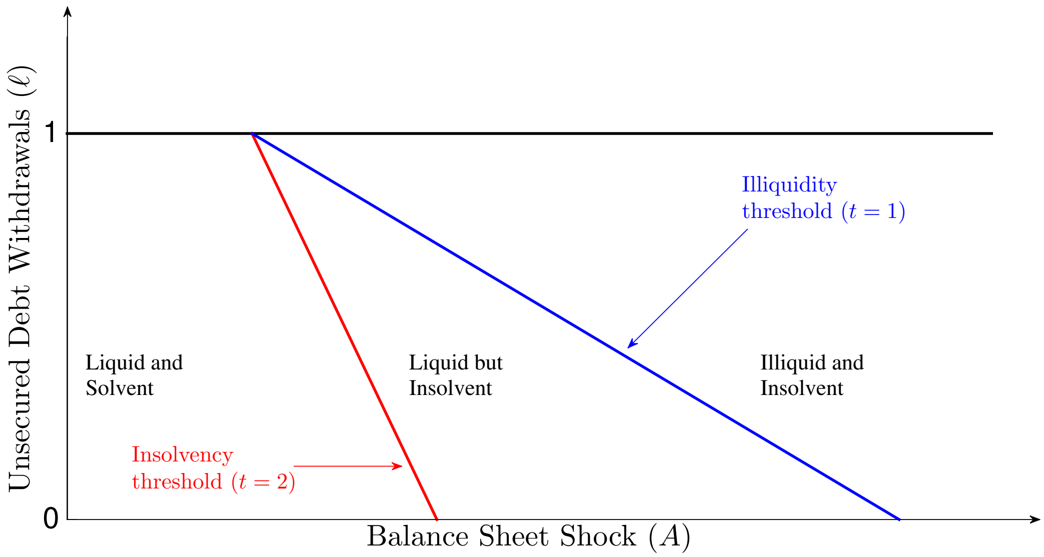
Asset encumbrance and secured debt issuance fundamentally affect run dynamics. Figure 1a shows the illiquidity and insolvency thresholds, $A_{IL}(\ell)$ and $A_{IS}(\ell)$, without encumbrance and secured debt, $\alpha = S = 0$. In this case, we recover the dynamics in Rochet and Vives (2004) without liquid reserves. An illiquid bank at $t = 1$ is always insolvent at $t = 2$, so the insolvency threshold is the relevant condition for analysis.

Figure 1b shows the illiquidity and insolvency thresholds with encumbrance and secured debt. Over-collateralization means that the thresholds do not coincide at $\ell = 1$. Additional assets worth $R\alpha I^*(\alpha) - rS^*(\alpha) = (1-\lambda)R\alpha I^*(\alpha) > 0$ are available to serve unsecured debt withdrawals at $t = 2$, which are not available at $t = 1$ because of encumbrance. As a result, a bank that is illiquid at $t = 1$ can, nevertheless, be solvent at $t = 2$, i.e. $(1-\lambda)R\alpha I^*(\alpha) \geq (1-\ell)\,UD_U$. An upper bound on the face value of unsecured debt, $D_U \leq \hat{D}_U \equiv (1-\lambda)R\alpha I^*(\alpha)$, ensures that the illiquidity threshold is the relevant condition for analysis. We suppose that this condition holds and later verify that it does in equilibrium.

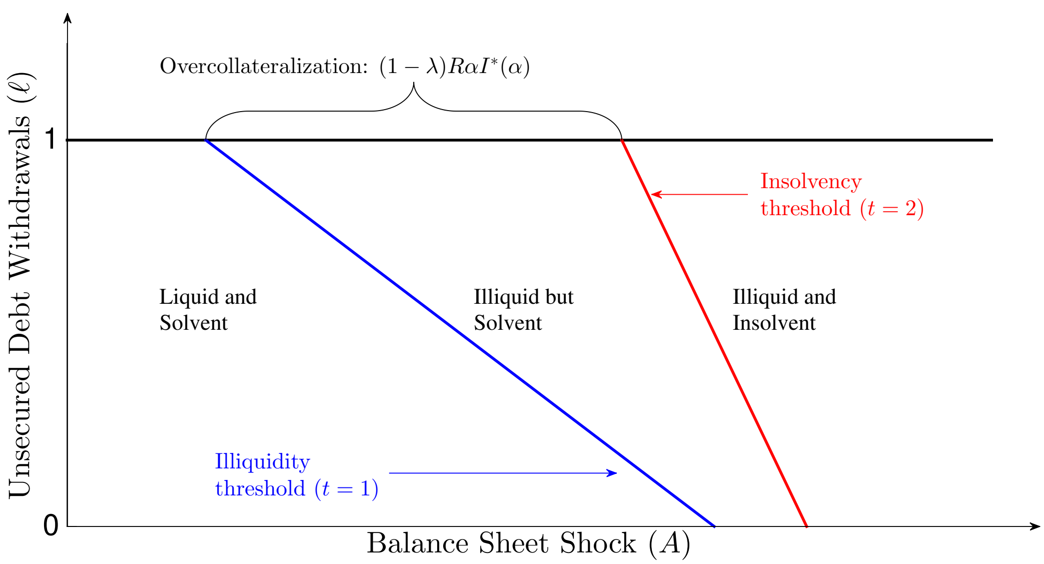
We focus on vanishing private noise about the balance sheet shock, $\epsilon \to 0$, so the rollover threshold converges to the run threshold, $x^* \to A^*$.

Proposition 1. Run threshold. *There exists a unique run threshold*

$$A^*(\alpha) \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma\,UD_U}{\psi}, \tag{5}$$



(a) Without asset encumbrance ($\alpha = S = 0$)



(b) With asset encumbrance

Figure 1: Run dynamics without and with asset encumbrance. Panel (a) replicates the results of Rochet and Vives (2004) without liquid reserves, where the relevant condition is the insolvency threshold. Panel (b) shows that over-collateralization due to encumbrance, which is strictly positive for $\lambda < 1$, shifts the insolvency threshold to the right. As long as $D_U \leq \hat{D}_U$, the relevant condition is the illiquidity threshold.

where $I^*(\alpha) = \frac{U+E}{1-\frac{\lambda R}{r}\alpha}$ *is investment. All fund managers withdraw and the bank closes early if and only if* $A > A^*$*.*

Proof. See Appendix A.1. ■

Proposition 1 pins down the unique incidence of an unsecured debt run using global games methods. More conservative fund managers decrease the run threshold, $\frac{\partial A^*}{\partial \gamma} < 0$. A higher return on investment increases the value of unencumbered assets and the amount of secured debt raised for a given encumbrance ratio. Both effects act to reduce run risk, so $\frac{\partial A^*}{\partial R} > 0$. Higher liquidation values decrease the extent of strategic complementarity among fund managers and increases the issuance of secured debt for given encumbrance. Both these effects also lower run risk, $\frac{\partial A^*}{\partial \psi} > 0$. Similarly, a decrease in the cost of recovering encumbered assets after early closure of the bank (higher λ) enables more secured funding for given encumbrance. So the stock of unencumbered assets increases and run risk decreases, $\frac{\partial A^*}{\partial \lambda} > 0$. A higher cost of funding decreases the amount of secured debt for given encumbrance and thereby lowers the value of unencumbered assets, $\frac{\partial A^*}{\partial r} < 0$. Increased bank capital reduces run risk via its effect on increased investment and unencumbered assets, $\frac{\partial A^*}{\partial E} > 0$.

Lemma 2 links bank fragility to secured debt issuance and the recovery of encumbered assets after early closure.

Lemma 2. Encumbrance and fragility. *Asset encumbrance affects bank fragility according to*

$$\frac{dA^*}{d\alpha}\left(\frac{\lambda R}{r} - 1\right) \geq 0, \tag{6}$$

with strict inequality whenever $\lambda R \neq r$*. More encumbrance increases fragility,* $\frac{dA^*}{d\alpha} < 0$*, when the cost of recovering encumbered assets is high,* $\lambda R < r$*.*

Proof. See Appendix A.1. ■

Greater encumbrance affects the run threshold in two opposing ways. First, for given investment, greater encumbrance reduces the stock of unencumbered assets and increases fragility. Second, greater encumbrance allows the bank to issue more secured debt, which increases investment. As a result, the stock of unencumbered assets increases and run risk is reduced. Whether the second effect dominates depends on the cost of recovering encumbered assets, $1 - \lambda$. If the cost is high, the bank must encumber more than one unit of assets for each unit of secured funding raised. In this case, the stock effect dominates and greater encumbrance increases fragility. Conversely, if recovery is cheap, the bank is able to encumber less than a unit of assets per unit of secured funding – so greater encumbrance reduces fragility.

We assume that the cost of recovering encumbered assets is sufficiently high,

$$\lambda R < r, \tag{7}$$

i.e. the recovery of secured assets upon maturity is below the risk-free return. This assumption implies that greater asset encumbrance increases bank fragility. Empirical evidence consistent with this implication includes Bennett et al. (2005), who show that FHLB advances increase the default probability of U.S. banks, and Garcia et al. (2017), who show that covered bond issuance increases the default risk of European banks.[12] The inability to fully recover encumbered assets can also reflect legal reasons – secured senior debtholders often face costly and protracted proceedings in court to recover collateral assets upon bank failure (Duffie and Skeel, 2012).[13]

[12] Proxies for the recovery value are readily available for FHLB advances. According to FHLB (2018), the discount applied to a single-family mortgage loan – the most common form of collateral – ranges from 4% - 99% (table 45), and depends on detailed loan information, including interest rate and FICO score. The implied average recovery value is $\lambda = 0.26$. Therefore, our assumption, $\lambda R < r$, is likely to hold for banks with high-interest loans and borrowers with low FICO scores.

[13] Fleming and Sarkar (2014) highlight how even special-status secured financial instruments (qualifying for so-called 'safe harbor' provisions) are not immune from legal logjams. They document the protracted legal settlements following the failure of Lehman. Although safe harbor provisions al-

3.3 Optimal asset encumbrance and secured debt issuance

The bank encumbers assets to maximize expected equity value, taking as given the face value of unsecured debt D_U, the run threshold $A^*(\alpha)$, and the (maximum) amount of secured debt, $S \leq S^*(\alpha)$. Since more secured debt for a given encumbrance ratio increases expected equity value and lowers fragility, $S^* = S^*(\alpha)$ and the bank solves

$$\max_{\alpha} \pi \equiv \int_{-\infty}^{\infty} E_2(A) dF(A) = \int_{-\infty}^{A^*(\alpha)} \left[RI^*(\alpha) - UD_U - S^*(\alpha)\, r - A \right] dF(A). \quad (8)$$

Figure 2 shows the relationship between encumbrance and expected equity value, highlighting the unique interior encumbrance ratio for a given face value of debt.[14]

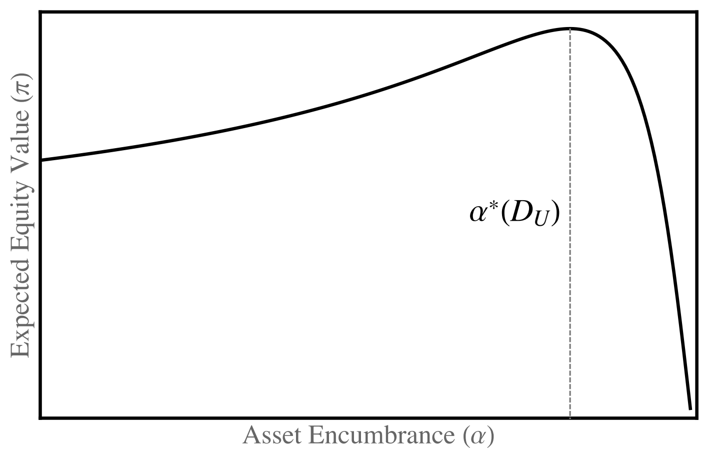


Figure 2: The encumbrance schedule is unique.

Proposition 2. Asset encumbrance schedule. *There is a unique encumbrance schedule, $\alpha^*(D_U)$. If fund managers are sufficiently conservative, $\gamma > \psi$, then the schedule decreases in the face value of unsecured debt, $\frac{d\alpha^*}{dD_U} \leq 0$, and an interior*

lowed retail counterparties to terminate contracts upon bankruptcy, the settlement of their claims on encumbered assets remained incomplete for several years.

[14] Throughout we use the numerical example of $R = 1.5$, $r = 1.1$, $E = 0.5$, $\psi = 0.6$, $\lambda = 0.66$, $\gamma = 0.8$, $D_U = 3.3$ (if exogenous), and a normally distributed shock with mean -3 and unit variance.

solution for $\underline{D}_U < D_U < \overline{D}_U$ is implicitly given by:

$$\frac{F(A^*(\alpha^*))}{f(A^*(\alpha^*))} = \frac{\left(1 - \frac{\lambda R}{r}\right)}{\lambda\left(\frac{R}{r} - 1\right)} \left[(1-\lambda)R\alpha^* I^*(\alpha^*) + \left(\frac{\gamma}{\psi} - 1\right) U D_U\right]. \tag{9}$$

Proof. See Appendix A.2. ■

The bank balances the marginal benefits and costs of asset encumbrance when choosing the privately optimal encumbrance ratio. The marginal benefit is an increase in the amount of secured funding raised. Since secured debt is cheap and investment is profitable, the equity value of the bank – in the absence of an unsecured debt run – is higher. The marginal cost is greater fragility, i.e. a higher probability of an unsecured debt run. So a higher face value of unsecured debt exacerbates rollover risk and lowers the run threshold, inducing the bank to encumber fewer assets.

3.4 Pricing of unsecured debt

Repayment of unsecured debt depends on the balance sheet shock. Absent a run, $A < A^*$, unsecured debtholders receive face value D_U, while for larger shocks, $A > A^*$, they receive zero in bankruptcy. A binding participation constraint for risk-neutral investors equalizes the value of an unsecured debt claim with the required return for any encumbrance ratio,

$$r = D_U^* F(A^*(\alpha, D_U^*)). \tag{10}$$

Proposition 3. Private optimum. *If bank capital is scarce, $E < \bar{E}$, and managers are conservative, $\gamma \geq \underset{\sim}{\gamma}$, then there exists a unique face value of unsecured debt, $D_U^* > r$. If funding is costly, $r > \underset{\sim}{r}$, asset encumbrance is interior, $\alpha^{**} \equiv \alpha^*(D_U^*) \in (0, 1)$.*

Proof. See Appendix A.3. ■

Figure 3 shows the privately optimal allocation and its construction. The condition $\gamma \geq \underset{\sim}{\gamma}$ ensures that the schedule $D_U^*(\alpha)$, derived from the pricing of unsecured debt, is upward-sloping in the vicinity of the encumbrance schedule. Consistent with this result, Garcia et al. (2017) document that spreads on unsecured debt increase in covered bond issuance of European banks. The upward-sloping schedule, in turn, leads to a unique characterization of the joint equilibrium for the encumbrance ratio, α^{**}, and the face value of unsecured debt, D_U^{**}. Next, scarce bank capitalization $E < \bar{E}$ ensures that $D_U^* < \hat{D}_U(\alpha^*)$, so the illiquidity condition is more binding that the insolvency condition, as supposed. Finally, the lower bound on the cost of funding ensures a face value of unsecured debt high enough for an interior encumbrance ratio.

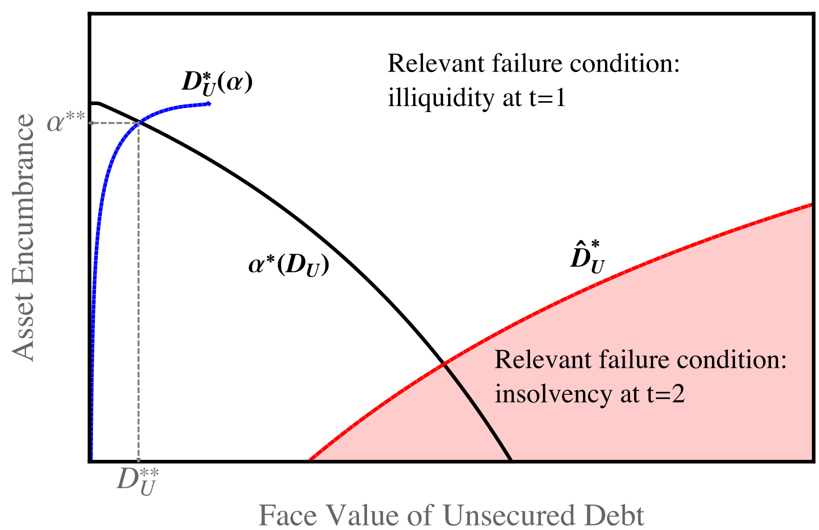


Figure 3: Privately optimal asset encumbrance and face value of unsecured debt. The curve labeled $\hat{D}_U^*$ demarcates the boundary at which the relevant failure condition switches from illiquidity at $t = 1$ to insolvency at $t = 2$ (shaded area).

The allocation in Proposition 3 is constrained efficient. A social planner who takes as given the incomplete information structure of the game (the private information of fund managers about balance sheet adjustment) would choose the same allocation. Constrained efficiency arises since the pricing of debt is competitive.

4 Testable Implications

Our model yields a rich set of comparative static results and testable implications. Parameter changes affect the unique interior equilibrium in two ways. For a given face value of unsecured debt, the bank trades off heightened fragility against more profitable investment funded with cheap secured debt. The face value of unsecured debt required for investor participation also changes with underlying parameter values.

Proposition 4. Comparative Statics. *The privately optimal encumbrance ratio* α^{**} *decreases in funding costs* r*, conservatism of fund managers* γ*, and the costs of recovering encumbered assets,* $1-\lambda$*. Encumbrance increases in investment profitability* R*, the liquidation value* ψ*, and improvements in the shock distribution* F *according to reverse hazard rate dominance. The effect of higher bank capital* E *is ambiguous.*

Proof. See Appendix A.4. ∎

Lower funding costs increases the benefits of asset encumbrance because more secured debt can be issued for a given asset encumbrance ratio. Since the required face value of unsecured debt is also lowered, the two effects combine to increase encumbrance. Empirical evidence consistent with this result includes Meuli et al. (2016) who find that issuance of Swiss covered bonds between 1932–2014 was lower during periods of higher interest rates.[15]

Improvements in the shock distribution reduce the likelihood of runs. Owing to this reduced fragility, the face value of unsecured debt is also lower. This, in turn, boosts the bank's incentives to encumber more assets and issue more secured debt. Consistent with this implication, Ashley et al. (1998) document that banks with larger

[15] Low funding costs may also reflect looser monetary conditions. Juks (2012) and Bank of England (2012) document an increasing trend in the encumbrance ratios of Swedish and UK banks following the extraordinary monetary policy measures in the aftermath of the global financial crisis.

deferred loan loss reserve ratio use fewer FHLB advances (intensive margin).

An increase in investment profitability has two effects. First, for each unit of the asset encumbered, the bank can raise more secured funding. Second, since the value of unencumbered assets is greater, the likelihood of a run is reduced, which reduces fragility and the face value of unsecured debt. The bank, in turn, responds by encumbering more assets so as to raise more secured debt and increase investment. In line with this implication, Stojanovic et al. (2008) show that U.S. commercial banks with higher returns on assets were more likely to become FHLB members over the period 1992–2005. Membership proxies for the use of advances on the extensive margin. The evidence on the intensive margin is inconclusive as Ashley et al. (1998) report no significant effect of return on assets on FHLB advances during 1985–1991.

The effect of higher bank capital on asset encumbrance is ambiguous. More capital allows the bank to withstand larger shocks, lowering fragility. While this 'loss absorption' effect induces greater encumbrance, it also means that the bank risks losing more of its own funds in bankruptcy. The effect of such 'greater skin in the game' is to lower encumbrance. The net result is a non-monotonic relationship between bank capital and asset encumbrance.

Existing evidence on FHLB advances and their the relationship with bank capital is indeed mixed. On the extensive margin, Stojanovic et al. (2008) find that higher equity ratios reduce the likelihood of FHLB membership – a negative association between capital and encumbrance. Along the intensive margin, Ashley et al. (1998) associate lower capital ratios with greater FHLB advances. But Ashcraft et al. (2010) find that funding ratios are negatively associated with FHLB advances. Since the capital ratio and the funding ratio should add up to one, their result suggests a positive association between changes in advances and the equity ratio.

5 Guarantees and Excessive Encumbrance

We next proceed to examine the relationship between asset encumbrance and guarantee schemes. Such schemes, which typically apply to retail depositors, often extend to wholesale unsecured debtholders during times of crisis. While such measures aim to reduce bank fragility ex post, they distort behavior ex ante. In our model, the bank fails to internalize the costs of the guarantee upon its failure and, therefore, has incentives to excessively encumber assets that, in turn, creates excessive fragility.

Let $0 < m \leq \widehat{m}$ be the fraction of unsecured debt that is fully and credibly guaranteed.[16] Guaranteed debt is always rolled over.[17] It has face value D_G and ℓ is the withdrawal proportion of non-guaranteed unsecured debt. The bank is illiquid at $t = 1$ if $R(1-\alpha)I - A \leq \frac{\ell(1-m)UD_U}{\psi}$ and insolvent at $t = 2$ if $RI - A - \frac{\ell(1-m)UD_U}{\psi} \leq SD_S + (1-\ell)(1-m)UD_U + mUD_G$. Guaranteed debt is safe and, thus, cheap, $D_G^* = r$. From Lemma 1, $D_S^* = r$, $S^* = S^*(\alpha)$, and $I^* = I^*(\alpha)$.[18] And the illiquidity and insolvency thresholds in the presence of guarantees are:

$$A_{IL}(\ell) = R(1-\alpha)I^*(\alpha) - \ell\frac{(1-m)UD_U}{\psi}, \tag{11}$$

$$A_{IS}(\ell) = R(1-\alpha\lambda)I^*(\alpha) - (1-m)\left(1+\ell\left[\frac{1}{\psi}-1\right]\right)UD_U. \tag{12}$$

Figure 4 depicts run dynamics with guarantees. A run on unsecured non-guaranteed debt occurs whenever $A > A_m^* \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma(1-m)UD_U}{\psi}$. The direct effect of the guarantee is to achieve its intended purpose of increasing the run threshold, $\frac{\partial A_m^*}{\partial m} = \frac{\gamma UD_U}{\psi} > 0$. But the guarantee impacts the bank's incentive to encumber

[16] The bound $\widehat{m} < 1$ reflects the fiscal capacity of the guarantor (König et al., 2014) or the minimum amount of demandable debt required to control bank moral hazard (Rochet and Vives, 2004).

[17] We follow Allen et al. (2015) and consider guarantees that eliminate both inefficient and efficient runs. Such guarantee schemes better resemble real-world deposit guarantees.

[18] The upper bound on the face value of unsecured debt is less restrictive with guarantees. So the conditions previously imposed continue to suffice for the illiquidity threshold to be more binding.

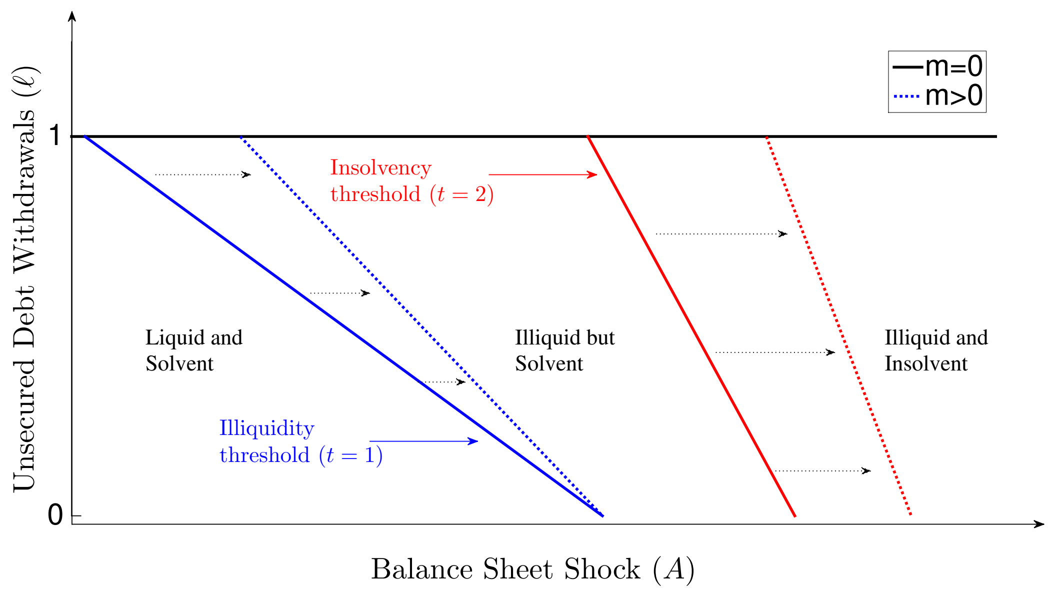


Figure 4: Run dynamics with guarantees: thresholds as functions of the withdrawal proportion without (solid) and with (dashed) guarantees. Guarantees reduce the responsiveness of both thresholds to the withdrawal proportion. The insolvency threshold shifts outwards, while the illiquidity threshold pivots outwards.

assets as well as the face value of unsecured debt. The bank's problem is now:

$$\alpha_m^* \equiv \max_{\alpha} \pi_m(\alpha) = \int_{-\infty}^{A_m^*(\alpha)} \left[RI^*(\alpha) - S^*(\alpha)\, r - (1-m)UD_U - mUr - A\right] dF(A). \quad (13)$$

Proposition 5. Privately optimal encumbrance with guarantees. *The equilibrium with guarantees is unique and has a higher encumbrance ratio,* $\alpha_m^{**} > \alpha^{**}$.

Proof. See Appendix A.5. ■

Guarantees reduce the stock of non-guaranteed debt that may be withdrawn and, therefore, run risk. This has two effects on encumbrance. First, the bank has greater incentives to encumber assets – an outward shift of the encumbrance schedule $\alpha_m^*(D_U)$. Second, unsecured debt is repaid more often. This reduces the face value of unsecured debt and shifts the participation constraint of risk-neutral investors $D_U^*(\alpha)$ inwards. In sum, guarantees unambiguously increase the encumbrance ratio, α_m^{**}.

Since the bank ignores the social cost of providing the guarantee, its incentives to encumber assets are distorted. In contrast, a social planner accounts for these costs, mUr, incurred upon bank failure with probability $1 - F(A^*_m)$. The planner takes as given the incomplete information structure of the model and chooses an encumbrance schedule for a given face value of non-guaranteed unsecured debt:[19]

$$\alpha^*_P(D_U) \equiv \max_{\alpha} \quad \pi_m(\alpha) - \left[1 - F(A^*_m(\alpha))\right] mUr. \tag{14}$$

Even though the encumbrance schedules of the bank and planner differ, $\alpha^*_P(D_U) \leq \alpha^*_m(D_U)$, the participation constraint of unsecured debtholders is the same, $D^*_U(\alpha)$. Any difference in allocations is, therefore, due to differences in encumbrance schedules. The face value of unsecured debt chosen by the planner D^{**}_P solves the fixed point problem $D^{**}_P \equiv D^*_U(\alpha^*_P(D^*_U))$. Figure 5 shows the private and social optimums.

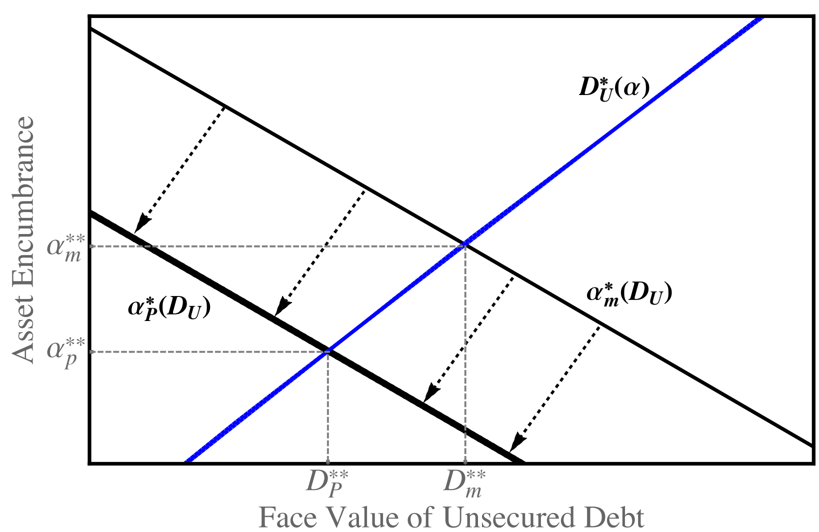


Figure 5: Constrained inefficiency of equilibrium with guarantees.

[19]The payoffs to fund managers do not enter the planner's objective function. This approach is consistent with taking the limits of $b \to 0$ and $c \to 0$ subject to a constant ratio $\gamma = \frac{c}{b+c}$. Moreover, including the payoffs would add $bF(A^*)$ as each fund manager refuses to roll over unsecured debt exactly when the bank fails (due to vanishing noise). As a result, the gap between the privately and socially optimal encumbrance ratios would actually increase, strengthening our results on regulation.

Proposition 6. Constrained inefficiency. *The privately optimal encumbrance ratio and the face value of unsecured debt are excessive,* $\alpha_m^{**} > \alpha_P^{**}$ *and* $D_m^{**} > D_P^{**}$*.*

Proof. See Appendix A.6. ■

Since guarantees are a structural feature of many financial systems, it is worth exploring policy tools that seek to curb excessive encumbrance and fragility within such an environment. Prudential safeguards include explicit limits on encumbrance ($\alpha \leq \bar{\alpha}$) and revenue-neutral linear Pigovian taxes on encumbrance ($\tau \geq 0$). Let $\alpha_R^*(D_U)$ denote the bank's optimal encumbrance schedule subject to regulation.

Proposition 7. Prudential regulation. *For given a guarantee* m*, a planner achieves the social optimum* $(\alpha_P^{**}, D_P^{**})$ *by imposing:*

a. a limit on asset encumbrance at α_P^{**}*;*

b. a contingent linear tax on asset encumbrance imposed at $t = 2$*, combined with a lump-sum rebate of the generated revenue,* $T = \alpha\tau$*. The optimal rate is*

$$\tau^*(D_U) = \frac{\left(1 - \frac{\lambda R}{r}\right) RI^*(\alpha_P^*)Umrf(A_m^*(\alpha_P^*))}{\left(1 - \frac{\lambda R}{r}\alpha_P^*\right) F(A_m^*(\alpha_P^*))}, \tag{15}$$

which depends on the face value of unsecured non-guaranteed debt.

c. a linear tax on asset encumbrance at $t = 2$ *that is not contingent on the face value of debt, combined with a lump-sum rebate of the generated revenue: there exists a unique rate* $\tau^* > 0$ *such that* $\alpha_R^{**}(\tau^*) = \alpha_P^{**}$ *and* $D_R^{**}(\tau^*) = D_P^{**}$*.*

Proof. See Appendix A.7. ■

Imposing an encumbrance limit, $\alpha \leq \alpha_P^{**}$, the bank's constrained encumbrance schedule for a given face value of debt is

$$\alpha_R^*(D_U) \equiv \begin{cases} \alpha_m^*(D_U) & \alpha_m^*(D_U) < \alpha_P^{**} \\ & \text{if} \\ \alpha_P^{**} & \alpha_m^*(D_U) \geq \alpha_P^{**}. \end{cases} \tag{16}$$

The bank chooses the socially optimal encumbrance ratio and, therefore, unsecured debt is also priced at the socially optimal level. When the planner can directly control encumbrance at $t = 0$, an encumbrance limit is effective.

This encumbrance limit can also be implemented via bank capital regulation. In the model, the bank's capital ratio at $t = 0$, $e \equiv e(\alpha) = \frac{E}{I^*(\alpha)}$, is sensitive to changes in the encumbrance ratio, that is $\frac{de}{d\alpha} < 0$. A minimum capital ratio, $e \geq \underline{e}$, translates to a limit on encumbrance, so the social optimum can be achieved with the capital requirement $e \geq \underline{e}(\alpha_P^{**})$. The argument generalizes to risk-based capital requirements if encumbered and unencumbered assets are assigned different risk-weights.

A limitation of caps on encumbrance is that they require the planner to observe the bank's encumbrance at $t = 0$. As an alternative, we consider a revenue-neutral policy that imposes a linear tax τ on encumbrance at $t = 2$, combined with a lump-sum rebate $T = \tau\alpha$. Higher taxes reduce the privately optimal encumbrance ratio, since, in the absence of a run, the bank's equity value decreases in encumbrance. When the tax rate can be made contingent on the face value of non-guaranteed debt, the optimal tax rate $\tau^*(D_U)$ ensures that the privately and socially optimal encumbrance schedules are aligned. This policy achieves the social optimum.

When the tax rate cannot be contingent on the face value of debt, the privately and socially optimal encumbrance schedules do not align. But since a higher tax rate lowers encumbrance, there exists a unique rate at which the social optimum is achieved. Graphically, the private encumbrance schedule shifts inward following the marginal tax on encumbrance so as to intersect with the social optimum.

Our results on prudential regulation are relevant to the policy debate on asset encumbrance. Increasingly, policymakers are expressing concern that the increased collateralization of bank balance sheets may heighten fragility (Haldane, 2012; CGFS, 2013). In many jurisdictions, concerns about excessive encumbrance has resulted in explicit restrictions that apply either (a) through limits on asset that can be pledged when secured debt is issued; or (b) via limits on bond issuance. Table 4 summarizes.

Country	Policies	
	Assets	Liabilities
Australia	8%	
Belgium	8%	
Canada	4%	
Italy	$\begin{cases} 25\% \text{ of assets if } 6\% \leq CET1 < 7\% \\ 60\% \text{ of assets if } 7\% \leq CET1 < 8\% \\ \text{No Limit} \quad \text{if } CET1 \geq 8\% \end{cases}$	
Netherlands	Determined on a case-by-case basis so ratio is 'healthy'	
United States		4%

Table 4: Caps on asset encumbrance across countries

While most countries have opted for limits on encumbered assets, the United States limits the share of secured debt to total liabilities. In our model, these asset and liability-side restrictions are equivalent. In Italy, the encumbrance limit depends on a bank's capital ratio (Core Equity Tier 1), with less capitalized banks facing stricter encumbrance limits. While this approach curbs the incentive to excessively encumber for banks with low capital, it does not reduce the incentives for highly capitalized banks (Proposition 6). Our results suggest that banks of all capital levels should be subject to some limits on encumbrance.

In our model, the optimal level α_P^{**} is sensitive to economic and financial conditions (Proposition 6), so policymakers have to set asset encumbrance limits that vary over time. In the Netherlands, for example, the cap is set on a case-by-case basis for individual banks, taking into account the financial position, solvency risk of the issuing bank, its risk profile, and the riskiness inherent in its assets (DNB, 2015).

A form of Pigovian taxation is also used in Canada to supplement asset encumbrance limits. The deposit insurance premiums levied by the Canadian Deposit Insurance Corporation on systemically important domestic banks reflects the extent to which balance sheets are encumbered. Specifically, 5% of the score used to calculate the premium reflects encumbrance considerations (CDIC, 2017). The surcharge is not revenue neutral and can be viewed as a non-contingent tax on encumbrance.

Next, we consider the case of a regulator who chooses guarantee coverage to minimize fragility, that is to maximize the run threshold A^*.

Proposition 8. Optimal guarantee coverage. *If the recovery value of encumbered assets is sufficiently low, $\lambda \leq \bar{\lambda} \equiv \frac{r}{2R-r}$, the fragility-minimizing coverage is $m^* = \widehat{m}$.*

Proof. See Appendix A.8. ■

Higher guarantee coverage has a positive total effect on the run threshold, $\frac{dA_m^*}{dm} > 0$. It can be decomposed into a direct effect and two indirect effects. The direct effect is positive and stems from a lower share of non-guaranteed debt. The indirect effects include changes to the encumbrance schedule and the face value of (non-guaranteed) unsecured debt. The former effect is negative but small for low recovery values, while the latter is positive. For a low recovery value, the bank can raise only a small amount of secured debt per unit of encumbered assets, providing few incentives to encumber. A low recovery value also implies greater fragility, further reducing the incentives to encumber. Following a marginal increase in coverage, the bank trades off a low benefit from raising secured debt against a high cost through greater fragility. Overall, encumbrance increases by a small amount and fragility is lower.

6 Additional Implications

In this section, we derive the implications of (i) liquid reserves held by banks; (ii) a risk premium; (iii) the precision of private information about the balance sheet shock.

6.1 Liquid reserves

The model sheds light on how a bank's liquid reserves shape run dynamics and encumbrance choice. As in Rochet and Vives (2004), we allow the bank to have exogenous liquid reserves $L \geq 0$ to serve interim withdrawals, which lowers the amount of liquidated unencumbered assets. The return on liquid reserves is normalized to one and is below the return on investment, $R > 1$. Because of this return differential at $t = 2$, the bank can attract less secured debt when encumbering liquid reserves. Moreover, liquid reserves are better than investment at reducing illiquidity at $t = 1$. Taken together, it is never optimal for the bank to encumber liquid reserves. Table 5 shows the bank's balance sheet at $t = 0$, where now $I + L = S + U + E$.

Assets		Liabilities
(encumbered assets)	αI	S
(unencumbered assets)	$(1-\alpha)I$	U
(liquid reserves)	L	E

Table 5: Balance sheet at $t = 0$ with liquid reserves.

There are three cases. If $\ell U D_U > L + \psi[(1-\alpha)I - A]$, the bank is illiquid and closes early. If $L < \ell UD_U \leq L + \psi[(1-\alpha)I - A]$, the bank is liquid at $t = 1$ and liquidates the amount $\frac{\ell UD_U - L}{\psi}$ to serve withdrawals. The bank is insolvent at $t = 2$ if $RI - A - \frac{\ell UD_U - L}{\psi} < SD_S + (1-\ell)UD_U$. If $L \geq \ell UD_U$, the bank is 'super-liquid' and has enough reserves to serve all interim withdrawals. A super-liquid bank is insolvent at $t = 2$ if

$$RI + L - A < UD_U + SD_S. \tag{17}$$

Similar to König (2015), an increase in liquid reserves reduces the bank's investment. Since liquid reserves have a lower return, this leads to an increase in the bank's insolvency risk at $t = 2$. Figure 6 shows the run dynamics with liquid reserves and depicts the thresholds as functions of the balance sheet shock A and the withdrawal proportion ℓ. Unlike Figure 1b, there is now a 'super-liquidity' region. If the bank is super-liquid, the relevant critical mass condition is insolvency at $t = 2$. Otherwise, the relevant condition is illiquidity at $t = 1$ (as in the benchmark model).

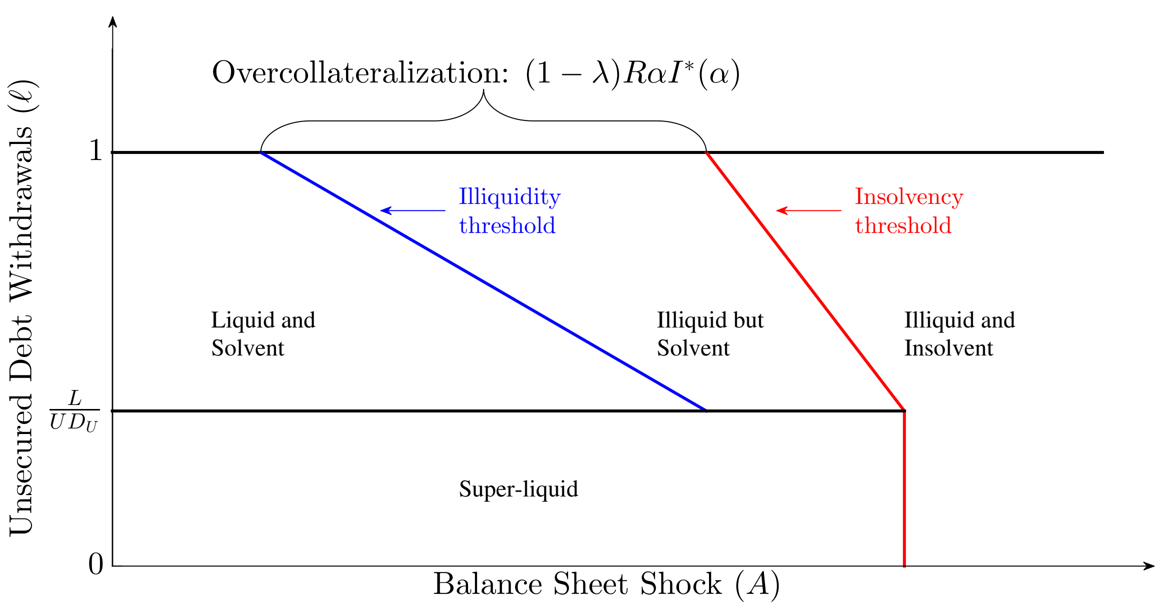


Figure 6: Run dynamics with liquid reserves: a super-liquid region in which the bank never fails at $t = 1$ exists.

Proposition 9. Runs with liquid reserves. *There exists a unique run threshold:*

$$A^* = \begin{cases} A_1^* \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma U D_U - L}{\psi} & \gamma > \frac{L}{UD_U} \\ & \text{if} \qquad\qquad , \\ A_2^* \equiv RI^*(\alpha) + L - UD_U - S^*(\alpha)r & \gamma \leq \frac{L}{UD_U} \end{cases} \tag{18}$$

where $I^*(\alpha) = \frac{U+E-L}{1-\frac{\lambda R}{r}\alpha}$ *and* $S^* = \frac{\lambda R}{r}\alpha I^*$. *More liquid reserves increase fragility of a super-liquid bank,* $\frac{dA_2^*}{dL} < 0$. *If* $R < \frac{1}{\psi}$, *they reduce fragility of an illiquid bank,* $\frac{dA_1^*}{dL} > 0$.

Proof. See Appendix A.9. ■

Greater liquid reserves have the opportunity cost of a higher investment return at the final date, R, but can also reduce the required amount of liquidation at the interim date, earning an implicit return $\frac{1}{\psi}$. For a bank with abundant reserves, more reserves always increase fragility since the marginal benefit is zero. By contrast, for a bank with few liquid reserves, more reserves are not idle and reduce fragility when the liquidation cost savings exceed the opportunity cost.

Restricting attention to a bank with scarce liquidity in order to facilitate comparison with the benchmark model yields a further result on encumbrance choice:

Proposition 10. Asset encumbrance choice with liquid reserves. *For scarce bank liquidity,* $L \leq \gamma U r$, *the unique encumbrance schedule is interior and given by:*

$$\frac{F(A_1^*(\alpha^*))}{f(A_1^*(\alpha^*))} = \frac{1-\frac{\lambda R}{r}}{\lambda(\frac{R}{r}-1)} \left[(1-\lambda)R\alpha^* I^*(\alpha^*) - L\left(\frac{1}{\psi}-1\right) + \left(\frac{\gamma}{\psi}-1\right)UD_U\right]. \tag{19}$$

The privately optimal level of asset encumbrance increases in liquid reserves, $\frac{d\alpha^*}{dL} > 0$.

Proof. See Appendix A.9 ■

In other words, greater liquid reserves lower fragility and induce greater encumbrance. Ashcraft et al. (2010) document evidence consistent with this implication.

6.2 Risk premium on unsecured debt

We also establish a testable implication about the risk premium on unsecured debt and the encumbrance ratio. Assume that risk-neutral investors have access to a menu of risk-free and risky storage. The return on risk-free storage continues to be r but the expected return on risky storage is $\tilde{r} > r$, implying a risk premium of $p \equiv \tilde{r} - r$. Since the risk premium leaves the pricing of secured debt, the illiquidity condition, and the encumbrance schedule unaffected, $D_S^* = r$, $S^*(\alpha)$, $A^*(\alpha)$, and $\alpha^*(D_U)$ are unchanged. But an increase in the risk premium impacts the pricing of unsecured debt, shifting the schedule $D_U^*(\alpha; p)$ outwards. Proposition 11 summarizes.

Proposition 11. Risk premium. *A higher risk premium p lowers the privately optimal encumbrance ratio, α^{**}, and increases the face value of unsecured debt, D_U^{**}.*

6.3 Limited precision of information and liquidity support

In the limit of infinitely precise private information about the balance sheet shock, $\epsilon \to 0$, the mass of fund managers who withdraw is a step function, $\ell^*(A, x) = \mathbf{1}_{\{\mathbf{A} > \mathbf{A}^*\}}$. If the bank's insolvency line, $A_{IS}(\ell)$, and illiquidity line, $A_{IL}(\ell)$, are sufficiently close, $\ell^*(A, x)$ crosses both curves for the same A^*, so the illiquidity and insolvency thresholds coincide. For finite precision, by contrast, the mass of fund managers who withdraw is $\ell^*(A, x^*) = H(x^* - A)$ and the illiquidity and insolvency thresholds, A_{IL}^* and A_{IS}^*, respectively, differ, as shown in Figure 7.

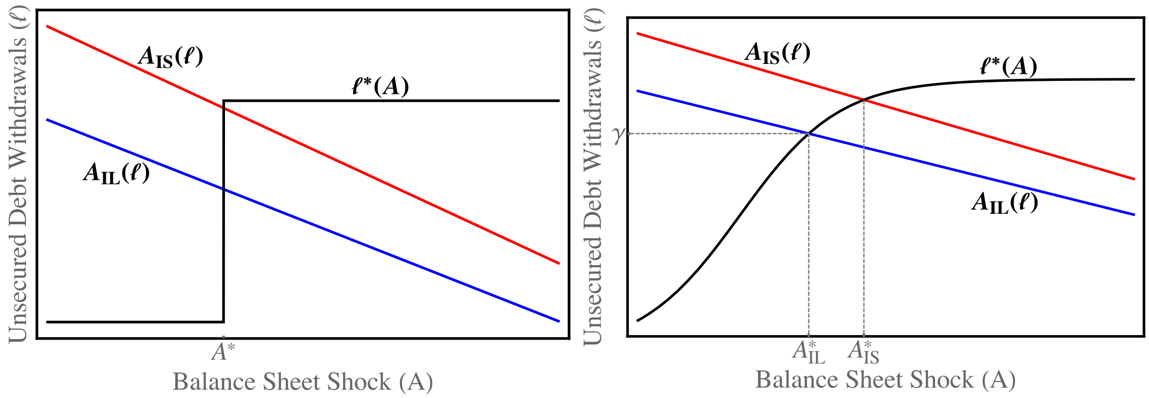


Figure 7: The precision of private information and the range of balance sheet shocks for which a bank is illiquid but solvent. The left panel shows the case of infinitely precise private information. The right panel shows the case of limited precision of private information, so the illiquidity and insolvency thresholds are A^*_{IL} and A^*_{IS}.

If the shock falls in the range $[A^*_{IL}, A^*_{IS}]$, the bank's failure is driven by illiquidity and not insolvency concerns. As a result, there is a role for a lender of last-resort, as considered by Rochet and Vives (2004). In particular, if a regulator observes the shock without noise (perhaps due to its supervisory function) it can offer loans at an interest rate $\rho \in [0, \frac{1}{\psi} - 1)$. This policy shifts out the illiquidity threshold for given encumbrance. Since the bank is solvent, no taxpayer's money is at risk. Interim liquidity support also affects the initial incentives to encumber assets.

7 Conclusion

Our paper studies asset encumbrance by banks and its implications for rollover risk, funding costs, and prudential regulation. A bank's privately optimal encumbrance choice balances profitable yet illiquid investment, funded by cheap long-term senior secured debt, against a greater probability of runs on unsecured debt. But asset encumbrance and bank fragility can become excessive in presence of deposit insurance or wholesale funding guarantees. We show how caps on asset encumbrance and revenue-neutral Pigovian taxation ameliorate these risk-shifting incentives. Our results thus contribute to the policy debate on the risks that rising asset encumbrance levels pose for banks. The model also yields testable implications that are consistent with existing evidence, and which might help guide future empirical research.

Our model assumes that the stock of unsecured debt is fixed and that the bank holds all the bargaining power in funding markets. Relaxing the first assumption would allow the bank to scale up its investment by marginally increasing the stock of unsecured debt. But this exacerbates the coordination failure between fund managers and heightens fragility. The bank's choice of unsecured debt issuance decreases in the level of asset encumbrance, implying that secured and unsecured debt are substitutes in equilibrium. Relaxing the second assumption does not qualitatively alter the results. The amount of secured funding raised would be lower under other market structures, reducing investment, equity value, and the incentives to encumber assets. Future work might explore these issues, and how a lender-of-last-resort shapes ex ante encumbrance choice, in greater detail.

References

Adams-Kane, J. and J. Wilhelmus (2017). The real story behind the surge in FHLB advances: Macroprudential policy changed how banks borrow. Milken Institute.

Ahnert, T. and E. Perotti (2018). Seeking safety. *Mimeo, Bank of Canada*.

Allen, F., E. Carletti, I. Goldstein, and A. Leonello (2015). Government guarantees and financial stability. CEPR Discussion Paper 10560.

Ashcraft, A., M. Bech, and S. Frame (2010). The Federal Home Loan Bank System: The lender of next-to-Last resort? *Journal of Money, Credit and Banking 42*(4), 551–83.

Ashley, L., E. Brewer, and N. Vincent (1998). Access to FHLBank advances and the performance of thrift institutions. *FRB Chicago Economic Perspectives*, 33–52.

Auh, J. and S. Sundaresan (2015). Repo rollover risk and the bankruptcy code. Mimeo, Columbia Business School.

Ayotte, K. and S. Gaon (2011). Asset-backed securities: Costs and benefits of "bankruptcy remoteness". *Review of Financial Studies 24*, 1299–1336.

Bank of England (2012). Three medium-term risks to financial stability. *Financial Stability Report 31*, 30–46.

Bennett, R., M. Vaughan, and T. Yeager (2005). Should the FDIC worry about the FHLB? The impact of Federal Home Loan Bank advances on the bank insurance fund. *FRB Richmond Working Paper No. 05-05*.

Bolton, P. and M. Oehmke (2018). Bank resolution and the structure of global banks. Mimeo, Columbia Business School.

Calomiris, C. and C. Kahn (1991). The Role of Demandable Debt in Structuring Optimal Banking Arrangements. *American Economic Review 81*(3), 497–513.

Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica 61*(5), 989–1018.

CDIC (2017). Differential premiums by-law manual. *http://www.cdic.ca/en/financial-community/Pages/differential-premiums.aspx*.

CGFS (2013). Asset encumbrance, financial reform and the demand for collateral assets. *Committee on the Global Financial System Publications No. 49, BIS, Basel*.

Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy 91*, 401–419.

Diamond, D. and R. Rajan (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy 109*(2), 287–327.

DNB (2015). Testing of a healthy ratio of covered bonds to disposable assets. http://www.toezicht.dnb.nl/en/3/51-203144.jsp.

Duffie, D. and D. Skeel (2012). *Bankruptcy Not Bailout: A Special Chapter 14*, Chapter A Dialogue on the Costs and Benefits of Automatic Stays for Derivatives and Repurchase Agreements. Hoover Press.

Eisenbach, T. (2017). Rollover risk as market discipline: A two-sided inefficiency. *Journal of Financial Economics 126*.

Eisenbach, T., T. Keister, J. McAndrews, and T. Yorulmazer (2014). Stability of funding models: an analytical framework. *FRBNY Economic Policy Review*.

Fleming, M. and A. Sarkar (2014). The failure resolution of Lehman Brothers. *Federal Reserve Bank of New York Economic Policy Review 20*, 175–206.

Gai, P., A. Haldane, S. Kapadia, and B. Nelson (2013). Bank funding and financial stability. In A. Heath, M. Lilley, and M. Manning (Eds.), *Liquidity and Funding Markets: Proceedings of the Reserve Bank of Australia Annual Conference*, pp. 237–52.

Garcia, E., S. Gatti, and G. Nocera (2017). Covered Bonds, Asset Encumbrance and Bank Risk: Evidence from the European Banking Industry. *Mimeo, University of Zurich*.

Goldstein, I. and A. Pauzner (2005). Demand deposit contracts and the probability of bank runs. *Journal of Finance 60*(3), 1293–1327.

Greenbaum, S. and A. Thakor (1987). Bank funding modes: Securitization versus deposits. *Journal of Banking and Finance 11*(3), 379–401.

Haldane, A. (2012). Financial arms races. Speech at the Institute for New Economic Thinking, April 14, Centre for International Governance Innovation and Mercator Research Institute on Global Commons and Climate Change Third Annual Plenary Conference 'Paradigm Lost: Rethinking Economics and Politics', Berlin.

IMF (2012). Chapter III: Safe Assets – Financial System Cornerstone? *Global Financial Stability Report*, 3547–92.

Juks, R. (2012). Asset encumbrance and its relevance for financial stability. *Sveriges Riksbank Economic Review 3*, 67–89.

König, P. (2015). Liquidity requirements: A double-edged sword. *International Journal of Central Banking 11*(4), 129–68.

König, P., K. Anand, and F. Heinemann (2014). Guarantees, transparency and the interdependency between sovereign and bank default risk. *Journal of Banking and Finance 45*, 321–37.

Krishnamurthy, A. (2010). How debt markets have malfunctioned in the crisis. *Journal of Economic Perspectives 24*, 3–28.

Matta, R. and E. Perotti (2017). Insecure debt and liquidity runs. *Mimeo, UvA Amsterdam*.

FHLB (2018). Combined Financial Report for the year ended December 31, 2017. Federal Home Loan Banks Office of Finance.

Meuli, J., T. Nellen, and T. Nitschka (2016). Securitisation, loan growth and bank funding: the Swiss experience since 1932. Working Papers 2016-18, Swiss National Bank.

Morris, S. and H. Shin (2001). *NBER Macroeconomics Annual 2000*, Volume 15, Chapter Rethinking Multiple Equilibria in Macroeconomic Modeling, pp. 139–182. MIT Press.

Morris, S. and H. Shin (2003). Global games: Theory and Applications. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics (Proceedings of the 8th World Congress of the Econometric Society)*. Cambridge University Press.

Ranaldo, A., M. Rupprecht, and J. Wrampelmeyer (2017). Fragility of money markets. Mimeo, University of St. Gallen.

Rixtel, A. V. and G. Gasperini (2013). Financial crises and bank funding: recent experience in the euro area. *BIS Working Papers No 406*.

Rochet, J.-C. and X. Vives (2004). Coordination failures and the Lender of Last Resort: was Bagehot right after all? *Journal of the European Economic Association 2*(6), 1116–47.

Schwarcz, S. L. (2011). The conundrum of covered bonds. *Business Lawyer 561*(3), 561–586.

Schwarcz, S. L. (2013). *Handbook of key global financial markets, institutions, and infrastructure*, Chapter Securitization and structured finance. Elsevier.

Stojanovic, D., M. Vaughan, and T. Yeager (2008). Do Federal Home Loan Bank membership and advances increase bank risk-taking? *Journal of Banking and Finance 32*, 680–98.

Vives, X. (2005). Complementarities and Games: New Developments. *Journal of Economic Literature 43*, 437–479.

Vives, X. (2014). Strategic complementarity, fragility, and regulation. *Review of Financial Studies 27*(12), 3547–92.

A Proofs

A.1 Proof of Proposition 1

The proof is in two steps. First, we show that the dominance regions at the rollover stage, based on the illiquidity threshold, are well defined for any bank funding structure. If the balance sheet shock were common knowledge, the rollover behavior of fund managers would exhibit multiple equilibria (Figure 8). If no unsecured debt is rolled over, $\ell = 1$, the bank is liquid whenever the shock is below $\underline{A} \equiv R(1-\alpha)I - \frac{UD_U}{\psi}$. For $A < \underline{A}$, it is a dominant strategy for fund managers to roll over. If $\ell = 0$, the bank is illiquid whenever the shock is above $\overline{A} \equiv R(1-\alpha)I$. For $A > \overline{A}$, it is a dominant strategy for managers not to roll over.

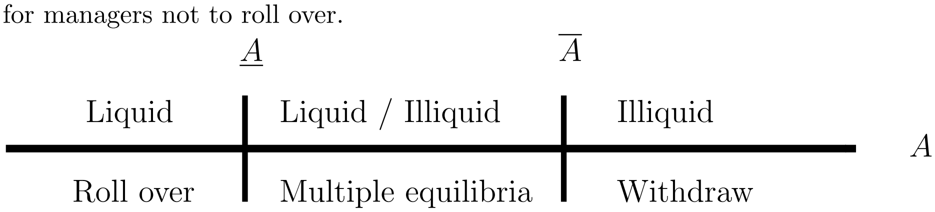


Figure 8: Tripartite classification of the balance sheet shock

Second, we characterize the threshold equilibrium. For a given realization $A \in [\underline{A}, \overline{A}]$, the proportion of fund managers who do not roll over unsecured debt is:

$$\ell\Big(A, x^*\Big) = \text{Prob}\left(x_i > x^* \big| A\right) = \text{Prob}\left(\epsilon_i > x^* - A\right) = 1 - H\Big(x^* - A\Big). \tag{20}$$

A **critical mass condition** states that illiquidity occurs when the shock equals A^*, where the proportion of managers not rolling over is evaluated at A^*:

$$R(1-\alpha)I - A^* \equiv \ell\big(A^*, x^*\big)\frac{UD_U}{\psi}. \tag{21}$$

The posterior distribution of the shock conditional on the private signal is derived using Bayes' rule. An **indifference condition** states that a manager who receives the threshold signal $x_i = x^*$ is indifferent between rolling and not rolling over, $\gamma = \Pr\left(A < A^* | x_i = x^*\right)$. Using the definition $x_j = A + \epsilon_j$, the conditional probability is $1 - \gamma = \Pr\left(A > A^* | x_i = x^* = A + \epsilon_j\right) = H\Big(x^* - A^*\Big)$. The indifference condition implies $x^* - A^* = H^{-1}\big(1-\gamma\big)$. Inserting it into $\ell\big(A^*, x^*\big)$, the withdrawal proportion at the threshold shock A^* is $\ell\Big(A^*, x_i = x^*\Big) = 1 - H\Big(x^* - A^*\Big) = 1 - H\Big(H^{-1}\Big(1-\gamma\Big)\Big) = \gamma$. Let $z \equiv \frac{R}{r}$. The run threshold A^* follows, where $\frac{dA^*}{dD_U} = -\frac{\gamma U}{\psi} < 0$ and $\frac{dA^*}{d\alpha} = R\left(\lambda z - 1\right)\frac{I^*(\alpha)}{1-\alpha\lambda z}$.

A.2 Proof of Proposition 2

We begin by specifying the full constrained problem faced by the bank and derive the results of Lemma 1. The bank's problem is given by

$$\begin{aligned} \max_{\{\alpha,S,D_S\}} \pi \quad &\equiv \quad \int_{\infty}^{A^*} \big[R\big(S+U+E\big) - UD_U - SD_S - A\big]\, dF(A) \\ \text{s.t} \quad & r = \min\left\{D_S, \lambda\, \frac{R\alpha(S+U+E)}{S}\right\}, \end{aligned}$$

where the failure threshold is $A^*(\alpha, S) = R(1-\alpha)(S+U+E) - \frac{\gamma U D_U}{\psi}$. It follows that $\frac{\partial \pi}{\partial S} > 0$ and $\frac{\partial \pi}{\partial D_S} < 0$. We prove $S^* = S^*(\alpha) \equiv \alpha\lambda R \frac{U+E}{r-\alpha\lambda R}$ and $D_S^* = r$ by contraction. First, suppose $D_S^* > \alpha\lambda R\frac{S^*+U+E}{S^*}$. Infinitely risk-averse investors value their secured debt claim at $\alpha\lambda R\frac{S^*+U+E}{S^*}$ since bank failure occurs with positive probability. This violates the optimality for D_S^* since the bank's expected equity value decreases in the face value of secured debt – a contradiction that implies that $D_S^* \leq \alpha\lambda R\frac{S^*+U+E}{S^*}$. Next, suppose $D_S^* < \alpha\lambda R\frac{S^*+U+E}{S^*}$. It implies that the infinitely risk-averse investors value secured debt at D_S^* that, however, violates the optimality of S^* since expected equity value increases in S. This contradiction implies $D_S^* \geq \alpha\lambda R\frac{S^*+U+E}{S^*}$. In sum, we have $D_S^* = r$ and $S^* = S^*(\alpha) \equiv \alpha\lambda R\frac{U+E}{r-\alpha\lambda R}$.

Hence, the unconstrained problem is given in (8). The total derivative $\frac{d\pi}{d\alpha}$, which takes indirect effects via $A^*(\alpha)$ and $S^*(\alpha)$ into account, yields $\frac{d\pi}{d\alpha} = \frac{RI^*(\alpha)}{1-\alpha\lambda z} f(A^*)G(\alpha)$ where

$$G(\alpha) = \frac{F(A^*)}{f(A^*)}\lambda(z-1) - (1-\lambda z)\left[RI^*(\alpha)\alpha(1-\lambda) + UD_U\left(\frac{\gamma}{\psi} - 1\right)\right]. \tag{22}$$

If an interior solution $0 < \alpha^* < 1$ exists, it is given by $G(\alpha^*) = 0$. It is a local maximum:

$$\frac{dG}{d\alpha} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\alpha}\lambda(z-1) - (1-\lambda z)\frac{R(1-\lambda)I^*(\alpha)}{1-\alpha\lambda z} < 0, \tag{23}$$

where the sign arises from the decreasing reverse hazard rate of $f(A)$ and $\lambda z < 1$. Using the implicit function theorem, we obtain $\frac{d\alpha^*}{dD_U} < 0$ for the interior solution since

$$\frac{dG}{dD_U} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dD_U}\lambda(z-1) - (1-\lambda z)U\left(\frac{\gamma}{\psi} - 1\right) < 0, \tag{24}$$

because of $\gamma > \psi$, so $\frac{d\alpha^*}{dD_U} \leq 0$ follows for the entire encumbrance schedule.

An interior solution to $G(\alpha^*) = 0$ requires two conditions, $G(\alpha = 0) > 0$ and $G(\alpha = 1) < 0$. First, evaluating the implicit function when there is no encumbrance, $\alpha = 0$, yields

$$\frac{F(A^*(0))}{f(A^*(0))}\lambda(z-1) - (1-\lambda z)\left(\frac{\gamma}{\psi} - 1\right) U D_U \,, \tag{25}$$

which strictly decreases in D_U. To ensure $\alpha^* > 0$, the face value of unsecured debt must satisfy $D_U < \bar{D}_U$, where $\bar{D}_U$ is uniquely and implicitly defined by

$$\frac{F(R(U+E) - \frac{\gamma}{\psi}U\bar{D}_U)}{f(R(U+E) - \frac{\gamma}{\psi}U\bar{D}_U)}\lambda(z-1) - (1-\lambda z)\left(\frac{\gamma}{\psi} - 1\right) U\bar{D}_U = 0. \tag{26}$$

Second, evaluating the implicit function when all assets are encumbered, $\alpha = 1$, yields

$$\frac{F(A^*(1))}{f(A^*(1))}\lambda(z-1) - (1-\lambda z)\left[\frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left(\frac{\gamma}{\psi} - 1\right) U D_U\right] \,, \tag{27}$$

which also decreases in D_U. To ensure $\alpha^* < 1$, the expression in equation (27) must be strictly negative. Hence, the face value of unsecured debt must be bounded from below, $D_U > \underline{D}_U$, where $\underline{D}_U < \bar{D}_U$ is uniquely and implicitly defined by

$$\frac{F(-\frac{\gamma}{\psi}U\underline{D}_U)}{f(-\frac{\gamma}{\psi}U\underline{D}_U)}\lambda(z-1) - (1-\lambda z)\left[\frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left(\frac{\gamma}{\psi} - 1\right) U\underline{D}_U\right] = 0\,. \tag{28}$$

A.3 Proof of Proposition 3

The proof is in four steps. First, we ensure an interior encumbrance ratio, $\alpha^{**} \in (0,1)$. Since $\alpha = 0$ implies $\hat{D}_U = 0$, there is no equilibrium consistent with the supposition $D_U \leq \hat{D}_U$, so α^{**} must be positive (verified below). If $\alpha = 1$, the run threshold and value of an unsecured debt claim are $A^*(1) = -\frac{\gamma U D_U}{\psi}$ and $V(1, D_U) = D_U F(A^*(1))$. The value of unsecured debt, $V(\alpha, D_U) \equiv D_U\, F(A^*)$, attains a maximum at $D_U = D_{max}$ uniquely and implicitly defined by $\frac{F\left(-\frac{\gamma}{\psi}UD_{max}\right)}{f\left(-\frac{\gamma}{\psi}UD_{max}\right)} - \frac{\gamma}{\psi}UD_{max} = 0$. $V(\alpha, D_U)$ decreases in α, so any solution for the encumbrance ratio, if it exists, is interior if the outside option satisfies $r > \underset{\sim}{r} \equiv V(1, D_{max})$.

Second, we show that the face value of unsecured debt satisfies $D_U^* > r$. While $\frac{\partial V}{\partial D_U}$ has an ambiguous sign in general, the derivative evaluated at the encumbrance schedule is

$$\left.\frac{\partial V}{\partial D_U}\right|_{\alpha^*(D_U)} \quad = \quad f(A^*)\left[\frac{1-\lambda z}{\lambda(z-1)}R(1-\lambda)\alpha^* I^*(\alpha^*)+\beta_0 U D_U\right], \tag{29}$$

which is non-negative whenever $\beta_0 \equiv \frac{1-\lambda z}{\lambda(z-1)}\left(\frac{\gamma}{\psi}-1\right)-\frac{\gamma}{\psi} \geq 0 \Leftrightarrow \gamma \geq \underset{\sim}{\gamma} \equiv \frac{1-\lambda z}{1-\lambda z-\lambda(z-1)}\psi$. Having established conditions under which V increases in D_U, at least in the vicinity of the encumbrance schedule, it follows that $D_U = r$ always violates the participation constraint, $V(D_U = r) = rF(A^*(D_U = r)) < r$. Thus, $D_U^* > r$.

Third, we establish that the intersection between the encumbrance schedule and participation constraint of unsecured debtholders yields a unique joint equilibrium. Proposition 2 states that $\alpha^*(D_U)$ decreases in D_U. Also, from the second step of this proof, we have sufficient conditions that ensure, in the vicinity of the encumbrance schedule, the market-implied face value of unsecured debt, $D_U^*(\alpha)$, increases in α. Hence, there is at most only one intersection of these two curves, establishing uniqueness.

We also show that the equilibrium specified above exists. Define $T(D_U) = r/F(A^*(\alpha^*(D_U), D_U))$ as a mapping from the set $\mathcal{U}$ of face values of unsecured debt into itself. If $\mathcal{U}$ is a closed and compact set then, by Brouwer's fixed-point theorem, there exists at least one fixed-point for the mapping. The lower bound on D_U is r. For the upper bound, note that if the bank could pledge all assets to unsecured investors, then $D_U \leq RI^*(\alpha) - A$. Truncating the shock distribution at some arbitrary $-A_L < 0$ yields a well-defined upper bound on D_U.

Fourth, we verify the supposition $D_U \leq \hat{D}_U$. Denoting the run threshold evaluated at $D_U = \hat{D}_U$ by $\hat{A}^*(\alpha) \equiv A^*(\hat{D}_U(\alpha))$, we have $\hat{A}^*(\alpha) = RI^*(\alpha)\left[1-\alpha\left(1+\frac{\gamma}{\psi}U(1-\lambda)\right)\right]$ with $\frac{d\hat{A}^*}{d\alpha} = -\frac{RI^*(\alpha)}{1-\alpha\lambda z}\left[1-\lambda z+\frac{\gamma}{\psi}U(1-\lambda)\right] < 0$. Next, define $\hat{\alpha}^*$ as the equilibrium encumbrance ratio evaluated at $D_U = \hat{D_U}(\hat{\alpha}^*)$, which is implicitly and uniquely defined by

$$\frac{F(\hat{A}^*(\hat{\alpha}^*))}{f(\hat{A}^*(\hat{\alpha}^*))} = \frac{1-\lambda z}{\lambda(z-1)}\frac{\gamma}{\psi}U(1-\lambda)R\hat{\alpha}^* I^*(\hat{\alpha}^*), \tag{30}$$

because the left-hand side decreases in α, while the right-hand side increases in it. We next translate the condition $D_U \leq \hat{D}_U$ into a condition for unsecured debt pricing, $r \leq V(\hat{\alpha}^*, \hat{D}_U(\hat{\alpha}^*))$. A stricter sufficient condition is to require $r \leq V(1, \hat{D}_U(1))$. Using the condition for $\hat{\alpha}^*$:

$$r \leq \frac{\psi\lambda(z-1)}{\gamma U(1-\lambda z)} \frac{F\left(-\frac{R(U+E)}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)^2}{f\left(-\frac{R(U+E)}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)}, \tag{31}$$

where right-hand side decreases in the bank's own funds E. This suggests that imposing an upper bound, $E \leq \bar{E}$, on bank capital ensures that, in equilibrium, $D_U^* \leq \hat{D}_U(\alpha^*)$. The upper bound on capital is implicitly defined as $\frac{\psi\lambda(z-1)}{\gamma U(1-\lambda z)} \frac{F\left(-\frac{R(U+\bar{E})}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)^2}{f\left(-\frac{R(U+\bar{E})}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)} = r$.

A.4 Proof of Proposition 4

The proof is in two steps. First, we show the effect of a parameter on the encumbrance schedule $\alpha^*(D_U)$. Second, we show that this direct effect is reinforced by an indirect effect via the equilibrium cost of unsecured debt D_U^*. For the direct effect via $\alpha^*(D_U)$, we take D_U as given and use the implicit function theorem, whereby $\frac{d\alpha^*}{dy} = -\frac{\frac{dG}{dy}}{\frac{dG}{d\alpha}}$ for $y \in \{\gamma, r, R, \psi, E, F(\cdot)\}$:

$$\frac{dG}{d\gamma} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\gamma}\lambda(z-1) - (1-\lambda z)U\frac{D_U}{\psi} < 0, \tag{32}$$

$$\begin{aligned}
\frac{dG}{dr} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dr}\lambda(z-1) - \frac{F(A^*)}{f(A^*)}\lambda\frac{z}{r} - \lambda\frac{z}{r}\left[\frac{R(1-\lambda)\alpha(1-\alpha)I^*(\alpha)}{1-\alpha\lambda z} + D_U\left(\frac{\gamma}{\psi}-1\right)\right] < 0 \\
\frac{dG}{dR} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dR}\lambda(z-1) + \frac{F(A^*)}{f(A^*)}\frac{\lambda}{r} + \frac{\lambda}{r}D_U\left(\frac{\gamma}{\psi}-1\right) + (1-\lambda)\alpha I^*(\alpha)\left\{\lambda z - \frac{(1-\lambda z)}{1-\alpha\lambda z}\right\} > 0,
\end{aligned}$$

where we could sign the expression in equation (33) by evaluating it at α^* and substituting $F(A^*)/f(A^*)$ from the first-order condition in equation (9). Moreover, we have:

$$\frac{dG}{d\psi} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\psi}\lambda(z-1) + (1-\lambda z)UD_U\frac{\gamma}{\psi^2} > 0 \tag{33}$$

$$\begin{aligned}
\frac{dG}{d\lambda} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\lambda}\lambda(z-1) + \frac{F(A^*)}{f(A^*)}(z-1) + zRI^*(\alpha)\alpha(1-\lambda) \\
&+ zUD_U\left(\frac{\gamma}{\psi}-1\right) + \frac{(1-\lambda z)(1-\alpha z)}{1-\alpha\lambda z}R\alpha I^*(\alpha) > 0,
\end{aligned} \tag{34}$$

$$\frac{dG}{dE} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dE}\lambda(z-1) - \frac{1-\lambda z}{1-\alpha\lambda z}R(1-\lambda)\alpha \lesseqgtr 0. \tag{35}$$

Finally, consider a balance sheet shock distribution $\widetilde{F}$ that first-order stochastically dominates the distribution F according to the reverse hazard rate criterion: $\frac{\widetilde{f}}{\widetilde{F}} \geq \frac{f}{F}$, or, equivalently, $F/f \geq \widetilde{F}/\widetilde{f}$. Let $\widetilde{G}(\widetilde{\alpha}^*) = 0$ denote the implicit function defining the privately optimal encumbrance ratio, $\widetilde{\alpha}^*$, under $\widetilde{F}$. Thus, $\widetilde{G}(\alpha) \leq G(\alpha)$ for all ratios of encumbrance. Since $d\widetilde{G}/d\alpha < 0$ and $dG/d\alpha < 0$, the privately optimal encumbrance ratio satisfies $\widetilde{\alpha}^* \geq \alpha^*$.

The indirect effects arise from the face value of unsecured debt. For any given encumbrance ratio, they are given by the implicit function $J(\alpha, D_U^*) = 0$ where $J \equiv -r + D_U F(A^*(\alpha, D_U))$. Using the implicit function again, and noting that $\frac{\partial J}{\partial D_U}\Big|_{\alpha^*} > 0$, we obtain reinforcing effects:

$$\begin{aligned} \frac{\partial J}{\partial R} &= D_U f(A^*)\frac{dA^*}{dR} > 0, \qquad \frac{\partial J}{\partial \psi} = D_U f(A^*)\frac{dA^*}{d\psi} > 0, \qquad \frac{\partial J}{\partial \lambda} = D_U f(A^*)\frac{dA^*}{d\lambda} > 0 \\ \frac{\partial J}{\partial \gamma} &= D_U f(A^*)\frac{dA^*}{d\gamma} < 0, \qquad \frac{\partial J}{\partial E} = D_U f(A^*)\frac{dA^*}{dE} > 0, \qquad \frac{\partial J}{\partial r} = -1 + D_U f(A^*)\frac{dA^*}{dr} < 0. \end{aligned} \tag{36}$$

Finally, an improvement in the distribution of the balance sheet shock also increases J.

A.5 Proof of Proposition 5

The derivation for the private optimum with guarantees follows closely that in Appendix A.3. For brevity, we only state the key conditions for the existence of a unique equilibrium.

Taking the derivative of expected equity value with respect to α, the optimal encumbrance ratio, given face value of unsecured debt, is implicitly defined by $G_m(\alpha_m^*(D_U)) = 0$:

$$G_m(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - (1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U\left(\frac{\gamma}{\psi} - 1\right) - mUr\right]$$

and $\frac{dG_m}{d\alpha} < 0$, so the solution is a local maximum. As before, for an interior solution, $\alpha_m^*(D_U) \in (0,1)$, we require that $D_U \in (\underline{D}_U(m), \bar{D}_U(m))$. For the rest of the normative analysis, we assume that the encumbrance schedule yields interior solutions that are local maximums. Bounds similar to those in the previous section can be derived.

The face value of the non-guaranteed unsecured debt satisfies $r = V(\alpha, D_U, m) \equiv D_U^* F(A_m^*(\alpha, D_U^*))$. Following the lines of previous reasoning, we obtain that a joint equilibrium exists if (i) $r > \underset{\sim}{r}_m \equiv V(1, D_{max}, m)$, (ii) $\gamma \geq \underset{\sim}{\gamma} \equiv \frac{1-\lambda z}{1-\lambda z-\lambda(z-1)}\psi$, and (iii) $E \leq \bar{E}_m$.

Consider the new comparative static. As before, we derive separately the direct effect on the encumbrance schedule and the indirect effect from the face value of unsecured debt. For the direct effect, the derivative of the implicit function, $G_m(\alpha)$, with respect to coverage is

$$\frac{dG_m}{dm} = \frac{d\frac{F(A_m^*)}{f(A_m^*)}}{dA_m^*}\frac{dA_m^*}{dm}\lambda(z-1) + (1-\lambda z)U\left[D_U\left(\frac{\gamma}{\psi}-1\right)+r\right] > 0\,. \tag{37}$$

Since $dG_m/d\alpha < 0$, the implicit function theorem implies $d\alpha_m^*/dm > 0$ for given D_U, so the encumbrance schedule shifts outwards after an increase in coverage. The indirect effect concerns the face value of unsecured debt given by the implicit function $J_m(\alpha, D_U^*(m)) = 0$, where $J_m \equiv -r + D_U F(A_m^*(\alpha, D_U))$. Hence, $\left.\frac{dJ_m}{dD_U}\right|_{\alpha^*} > 0$ and $\frac{dJ_m}{dm} = D_U f(A_m^*)\frac{dA_m^*}{dm} > 0$, so the face value of unsecured debt decreases as coverage increases, reinforcing asset encumbrance.

A.6 Proof of Proposition 6

Taking the derivative of the planner's objective function with respect to the encumbrance ratio, we obtain the first-order condition $\frac{d\pi_m}{d\alpha} + f(A_m^*)\frac{dA_m^*}{d\alpha}mUr = 0$. The planner's encumbrance schedule, $\alpha_P^*(D_U)$, is given by $G_P(\alpha_P^*) = 0$, where

$$G_P(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - (1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U\left(\frac{\gamma}{\psi}-1\right)\right], \tag{38}$$

which decreases in α. We again focus on the interior solutions. Comparing equation (38) to the implicit function that provides the bank's encumbrance schedule in equation (37), we have that $G_P(\alpha_m^*) < 0$ for all permissible D_U. Hence, $\alpha_m^*(D_U) > \alpha_P^*(D_U)$ for any m.

A.7 Proof of Proposition 7

With a limit on asset encumbrance, the bank's constrained problem is given by

$$\alpha_m^* \equiv \max_{\alpha\in[0,\alpha_P^{**}]} \pi_m(\alpha) = \int^{A_m^*(\alpha)} \left[RI^*(\alpha) - (1-m)UD_U - S^*(\alpha)\,r - mUr - A\right]dF. \tag{39}$$

Since the bank profit is concave around α_m^*, the marginal profit at $\alpha = \alpha_P^{**}$ is positive. The constrained optimum stated in equation (16) follows. For the tax and transfer schemes, the planner imposes a linear tax $\tau > 0$ on encumbrance at $t = 2$ combined with a lump-sum transfer T. The privately optimal encumbrance schedule subject to this regulation is given by the implicit function $G_R(\alpha)$, where $\alpha_R^*(D_U)$ solves $G_R(\alpha_R^*(D_U)) \equiv 0$:

$$\begin{aligned} G_R(\alpha) \quad &\equiv \quad \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - \frac{F(A_m^*)}{f(A_m^*)}\frac{1-\alpha\lambda z}{RI^*(\alpha)}\tau + \cdots \\ &\quad -(1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) - \tau\alpha + T + (1-m)UD_U\left(\frac{\gamma}{\psi}-1\right) - mUr\right]. \end{aligned} \tag{40}$$

Consider the pure transfer scheme, $\tau = 0$. Comparing $G_R(\alpha)|_{\tau=0}$ with $G_P(\alpha)$ in equation (38), the two encumbrance schedules are the same whenever $T = Umr$.

Next, consider a revenue-neutral scheme, $T = \tau\alpha$ for all α. We evaluate $\alpha_R^*(D_U)$ at $T = \tau\alpha$ and solve for the optimal tax. Equalizing the socially optimal and the privately optimal schedule under regulation, $\alpha_R^* = \alpha_P^*$, we obtain τ^* stated in equation (15). This tax rate depends on the face value of unsecured and non-guaranteed debt.

Finally, we consider a linear tax on encumbrance independent of D_U. Using the implicit function theorem, we obtain that $\frac{d\alpha_R^*}{d\tau} < 0$ since $\frac{dG_R}{d\alpha} < 0$ by optimality and

$$\left.\frac{dG_R}{d\tau}\right|_{T=\tau\alpha} = -\frac{1-\alpha\lambda z}{RI^*(\alpha)}\frac{F(A_m^*)}{f(A_m^*)} < 0. \tag{41}$$

A.8 Proof of Proposition 8

The equilibrium is given by the triple, α^{**}, D_U^{**} and A^{**} that solve this system of equations:

$$\begin{aligned} g(\alpha^{**}, D_U^{**}, A^{**}) \quad &\equiv \quad \frac{F(A^{**})}{f(A^{**})}\lambda(z-1) \\ &\quad - \quad (1-\lambda z)\left[R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) + (1-m)UD_U^{**}\left(\frac{\gamma}{\psi}-1\right) - mUr\right] = 0 \end{aligned} \tag{42}$$

$$V(\alpha^{**}, D_U^{**}, A^{**}) \quad \equiv \quad D_U^{**}F(A^{**}) - r = 0 \tag{43}$$

$$T(\alpha^{**}, D_U^{**}, A^{**}) \quad \equiv \quad A^{**} - R(1-\alpha^{**})I^*(\alpha^{**}) + (1-m)\frac{\gamma}{\psi}UD_U^{**} = 0\,. \tag{44}$$

To derive the optimal guarantee, we first need to analyze how the run threshold A^{**} depends on m, both directly and via changes to the level of encumbrance and face value of unsecured debt. By the implicit function theorem and Cramer's rule, we have $\frac{dA^{**}}{dm} = \frac{|J_m^A|}{|J|}$, where $|J|$ is the determinant of the Jacobian:

$$|J_m^A| = \begin{vmatrix} g_{\alpha^{**}} & g_{D_U^{**}} & -g_m \\ V_{\alpha^{**}} & V_{D_U^{**}} & -V_m \\ T_{\alpha^{**}} & T_{D_U^{**}} & -T_m \end{vmatrix}, \quad |J| = \begin{vmatrix} g_{\alpha^{**}} & g_{D_U^{**}} & g_{A^{**}} \\ V_{\alpha^{**}} & V_{D_U^{**}} & V_{A^{**}} \\ T_{\alpha^{**}} & T_{D_U^{**}} & T_{A^{**}} \end{vmatrix}. \tag{45}$$

Since $V_{\alpha^{**}} = V_m = 0$, it follows that

$$\begin{aligned} |J_m^A| &= V_{D_U^{**}}\left[-g_{\alpha^{**}}T_m + g_m T_{\alpha^{**}}\right] \\ &= -\frac{F(A^{**})RI^*(\alpha^{**})}{1-\alpha^{**}\lambda z}(1-\lambda z)\left[\lambda(z-1)\frac{\gamma}{\psi}UD_U^{**} + (1-\lambda z)U(D_U^{**}-r)\right] < 0, \\ |J| &= g_{\alpha^{**}}\left[V_{D_U^{**}}T_{A^{**}} - V_{A^{**}}T_{D_U^{**}}\right] + g_{D_U^{**}}V_{A^{**}}T_{\alpha^{**}} - g_{A^{**}}V_{D_U^{**}}T_{\alpha^{**}} \\ &= -f(A^{**})\frac{R(1-\lambda z)I^*(\alpha^{**})}{1-\alpha^{**}\lambda z}\left[(1-\lambda)\left\{\frac{F(A^{**})}{f(A^{**})} - (1-m)\frac{\gamma}{\psi}UD_U^{**}\right\}\right. \\ &+ \left.(1-\lambda z)(1-m)\left(\frac{\gamma}{\psi}-1\right)UD_U^{**} + (1-\lambda z)\frac{F(A^{**})}{f(A^{**})}\frac{d\frac{F(A^{**})}{f(A^{**})}}{A^{**}}\right]. \end{aligned}$$

A sufficient condition for $|J| < 0$ is $\frac{F(A^{**})}{f(A^{**})} - (1-m)UD_U^{**}\left[\frac{\gamma}{\psi} - \frac{1-\lambda z}{1-\lambda}\left(\frac{\gamma}{\psi}-1\right)\right] > 0$. Replacing the inverse reverse hazard rate using $g(\alpha^{**}, D_U^{**}, A^{**}) = 0$ yields:

$$\frac{1-\lambda z}{\lambda(z-1)}\left[R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) - mUr\right] > (1-m)UD_U^{**}\left[\frac{\gamma}{\psi} - \left(\frac{\gamma}{\psi}-1\right)\left\{\frac{1-\lambda z}{1-\lambda} + \frac{1-\lambda z}{\lambda(z-1)}\right\}\right].$$

Since $UD_U < U\hat{D}_U$, we have $R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) - mUr > (1-m)UD_U^{**}$ in equilibrium. Thus, the sufficient condition for $|J| < 0$, $\lambda < \bar{\lambda} \equiv \frac{1}{2z-1}$, can be derived from this inequality:

$$\frac{\gamma}{\psi}\left[\frac{\lambda(z-1)}{1-\lambda z} - \frac{1-\lambda}{\lambda(z-1)}\right] + 1 < 0.$$

A.9 Proof of Propositions 9 and 10

These proofs parallel the proofs without liquid reserves.

We start with the dominance bounds. When all fund managers roll over ($\ell = 0$), the bank fails at $t = 2$ if $A > \bar{A} \equiv RI + L - UD_S - SD_S$. For $A > \bar{A}$, withdrawing is a dominant action for fund managers. When all fund managers withdraw ($\ell = 1$), the bank survives if $A < \underline{A} \equiv R(1-\alpha)I - \frac{UD_U - L}{\psi}$. For all $A < \underline{A}$, rolling over is a dominant action.

We turn to the global games solution. Fund managers use threshold strategies and roll over whenever $x_i < x^*$. Since the introduction of liquid reserves does not alter the signals that fund managers receive, the fraction of withdrawing fund managers remains unchanged: $\ell(A, x^*) = Pr(x > x^*|A) = 1 - H(x^* - A)$. Likewise, the indifference condition for fund managers is also unaffected, so $\gamma = 1 - H(x^* - A^*)$. Taken together, $\ell(A^*, x^*) = \gamma$. If $\gamma \leq \frac{L}{UD_U}$, the bank is super-liquid and the critical mass condition is given by the insolvency condition at $t = 2$ in equation (17), yielding the run threshold A_2^*. Otherwise ($\gamma > \frac{L}{UD_U}$), the critical mass condition is given by the usual illiquidity condition at $t = 1$, yielding A_1^*.

Taking into account the pricing of secured debt in the run threshold, we obtain:

$$\frac{dA_1^*}{dL} = \frac{1}{\psi} - \frac{R(1-\alpha)}{1-\alpha\lambda z}, \quad \frac{dA_2^*}{dL} = 1 - \frac{R(1-\alpha\lambda)}{1-\alpha\lambda z} < 0. \tag{46}$$

The cross-derivative of A_1^* with respect to L and α is positive. Also, $\left.\frac{dA_1^*}{dL}\right|_{\alpha=0} = \frac{1-R\psi}{\psi}$, so $R\psi <$ is sufficient for $\frac{dA_1^*}{dL} > 0$. The illiquidity run threshold changes with encumbrance:

$$\frac{dA_1^*}{d\alpha} = -RI^*(\alpha) + R(1-\alpha)\frac{dI^*(\alpha)}{d\alpha} = \frac{RI^*(\alpha)}{1-\alpha\lambda z}(\lambda z - 1) < 0. \tag{47}$$

The expected bank equity value is $\pi = \int_{-\infty}^{A^*} \left[RI^*(\alpha) + L - UD_U - S^*(\alpha)r - A\right] dF(A)$. For a given D_U, if $\gamma \leq \frac{L}{UD_U}$, we have $A^* = A_2^*$, else $A^* = A_1^*$ with which we proceed. Paralleling the previous steps, the encumbrance schedule is implicitly defined by $G(\alpha^*) = 0$:

$$G(\alpha) \equiv \frac{F(A_1^*(\alpha))}{f(A_1^*(\alpha))}\lambda(z-1) - (1-\lambda z)\left[RI^*(\alpha)\alpha(1-\lambda) - L\left(\frac{1}{\psi} - 1\right) + UD_U\left(\frac{\gamma}{\psi} - 1\right)\right],$$

and $G_\alpha < 0$. To evaluate how encumbrance depends on liquid reserves, we note that

$$G_L = \frac{d\frac{F(A_1^*)}{f(A_1^*)}}{dA_1^*}\frac{dA_1^*}{dL}\lambda(z-1) + (1-\lambda z)\left[\frac{1}{\psi} - 1 + \frac{R\alpha(1-\lambda)}{1-\alpha\lambda z}\right] > 0.$$

Using the implicit function theorem, we have that $\frac{d\alpha^*}{dL} = -\frac{G_L}{G_\alpha} > 0$.