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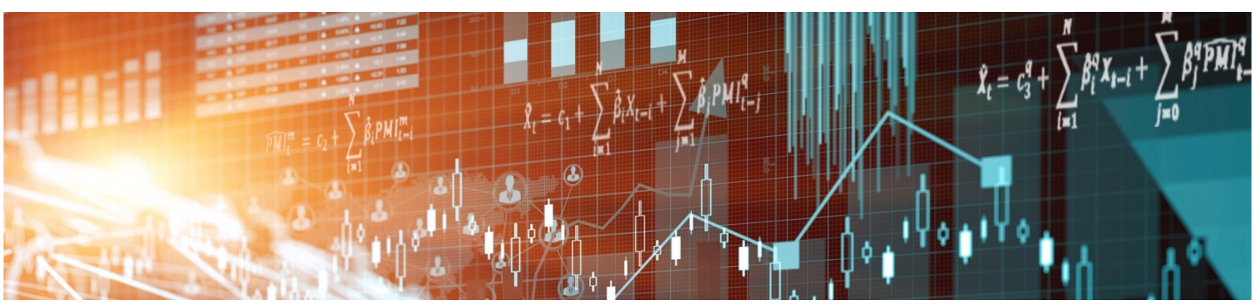

by Tom D. Holden, Paul Levine and Jonathan M. Swarbrick

Bank of Canada Staff Working Paper 2018-26

June 2018

Reconciling Jaimovich-Rebelo Preferences, Habit in Consumption and Labor

by

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ISSN 1701-9397

Acknowledgements

The authors acknowledge constructive comments from an anonymous referee.

Abstract

This note studies a form of a utility function of consumption with habit and leisure that (a) is compatible with long-run balanced growth, (b) hits a steady-state observed target for hours worked and (c) is consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch elasticity of labor supply. We employ Jaimovich-Rebello preferences, and our results highlight a constraint on the preference parameter needed to target the steady-state Frisch elasticity. This leads to a lower bound for the latter that cannot be reconciled empirically with external habit, but the introduction of a labor wedge solves the problem. We also propose a dynamic Frisch inverse elasticity measure and examine its business cycle properties.

Bank topic(s): Economic models; Business fluctuations and cycles; Labour markets
JEL code(s): E21, E24

Résumé

Dans cette note, nous étudions une fonction d'utilité qui dépend du temps de loisir et de la consommation, en tenant compte de la persistance des habitudes, et dont la forme a) est compatible avec une croissance équilibrée à long terme, b) permet d'atteindre les nombres d'heures travaillées constatés dans les données à l'état stationnaire et c) est conforme aux données microéconométriques relatives à l'élasticité de substitution intertemporelle et à l'élasticité de l'offre de travail de Frisch. Nous appliquons les préférences proposées par Jaimovich et Rebelo et nos résultats mettent en évidence une contrainte sur le paramètre de préférence nécessaire pour faire concorder l'élasticité de Frisch à l'état stationnaire avec les données. Cette contrainte entraîne une limite inférieure de l'élasticité de Frisch qui ne peut être conciliée avec les données empiriques en présence d'habitudes externes, mais l'introduction d'un écart entre le taux marginal de substitution consommation-loisir et la productivité marginale du travail résout le problème. Nous proposons également une mesure dynamique pour l'inverse de l'élasticité de Frisch et examinons ses propriétés cycliques.

Sujets : Modèles économiques; Cycles et fluctuations économiques; Marchés du travail
Codes JEL : E21, E24

Non-technical summary

Motivation and question

Macroeconomic models are developed based on assumptions about the behavior and preferences of households and firms. On the household side, two important assumptions are currently included in many models: habits in consumption and 'Jaimovich-Rebelo' household preferences. The first assumption means that households care not only about how much they consume goods and services, but also about their relative consumption. In other words, households gain additional pleasure either from consuming more today than they did yesterday (internal habits) or from consuming more than those around them (external habits). The second assumption is about the strength of the wealth-effect; that is, how much a household's wealth will affect its response to changing economic conditions. These preferences allow the modeler to control the importance of the wealth effect. Given the importance of these two assumptions, we investigate the implications for macroeconomic modeling of employing both these features together.

Methodology

First, we conduct a steady-state analysis to determine how the strength of the wealth effect constrains the values of the Frisch inverse labor supply elasticity; that is, the percentage change in working hours following a one percent change in the wage rate, holding the value of extra consumption constant. Second, we allow for a time-varying labor supply elasticity in a real business cycle (RBC) model and study the cyclical properties of this elasticity.

Key contributions

First, we formally detail the restriction on the Frisch inverse labor supply elasticity and propose amendments that can help reconcile the value of this elasticity with empirical evidence. Second, we derive a time-varying measure of the Frisch elasticity and discuss its cyclical properties in an RBC model.

Findings

We find that standard macroeconomic models with external habits exhibit a set of values of the Frisch inverse elasticity that cannot be reconciled empirically with either very weak or very strong wealth effects. However, we find that introducing a labor wedge to the labor supply condition, equivalent to a labor tax, can help alleviate this problem. We also find that our proposed measure of Frisch inverse labor supply elasticity in the RBC model is pro-cyclical when the wealth effect is strong but becomes counter-cyclical as it weakens.

Future work

The findings of this research suggest that the introduction of labor market frictions in models with external habits will be necessary to produce dynamics consistent with the data. In addition, it would be worthwhile studying further the behavior of our proposed measure of Frisch inverse labor supply elasticity in alternative models.

1 Introduction

Whether it is in the context of the equity-premium puzzle (see, for example, Abel, 1999), the savings-growth relation (Carroll and Weil, 2000) or monetary policy–business cycle analysis (Christiano *et al.*, 2005), researchers have used the concept of relative preferences to advance their various agendas. In particular, real business cycle and dynamic stochastic general equilibrium (RBC-DSGE) models in which a consumer's utility level depends not only on her consumption level but also on how that level compares to a standard set either by her own past consumption levels (internal habit-formation) or the levels of those in her peerage (catching up with the Joneses or external habit) are now ubiquitous in the literature.

At the same time, to achieve co-movement of output, hours, consumption and investment, modelers turn to preferences proposed by Jaimovich and Rebelo (2008) (henceforth JR) that control short-run wealth effects. This note discusses this form of this utility function, $U(C, L)$, where C is consumption modified by habit and $L = 1-H$ is leisure, as the proportion of the day, H being hours. The objective is to choose a form that (a) is compatible with long-run balanced growth, (b) hits a steady-state observed target for H and (c) is consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch inverse elasticity of labor supply.

2 The Household Problem

We write the JR utility function as:

$$U_t = U(C_t, H_t, X_{t-1}) = \frac{(C_t - \varrho H_t^{1+\psi} C_t^{\gamma} X_{t-1}^{1-\gamma})^{1-\sigma}}{1-\sigma}\,; \tag{1}$$

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}\,; \quad \gamma \in [0,1],\, \psi > 0. \tag{2}$$

We suppose that the household's problem at time t is to choose paths for consumption (C_t), labor supply ($H_t = 1 - L_t$, where L_t is leisure), capital (K_t), investment (I_t) and bond holdings (B_t) to maximize:

$$V_t = V_t(B_{t-1}, K_{t-1}, X_{t-1}) = \mathbb{E}_t\left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}, H_{t+s}, X_{t+s-1})\right],$$

subject to the budget constraint:

$$B_t = R_{t-1}B_{t-1} + r_t^K K_{t-1} + W_t H_t - C_t - I_t - T_t,$$

and the law of motion for capital:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

where felicity U is given by the JR preferences in (1) and (2), r_t^K is the rental rate of capital, W_t is the wage rate, R_t is the gross interest rate and T_t are lump-sum taxes. All variables are real throughout. We further assume that the investment adjustment costs $S\left(\frac{I_t}{I_{t-1}}\right)$ satisfy S', $S'' \geq 0\,;\ S(1) = S'(1) = 0$.

2.1 Solution of the Household Problem

To solve the household problem, we form a Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t\Big[&\sum_{s=0}^{\infty}\beta^s\Big(U(C_{t+s}, H_{t+s}, X_{t+s-1})\\&+\lambda_{t+s}[R_{t+s-1}B_{t+s-1} + W_{t+s}H_{t+s} + r^K_{t+s}K_{t+s-1}\\&-C_{t+s} - I_{t+s} - T_{t+s} - B_{t+s}]\\&+\lambda_{t+s}Q_{t+s}[(1-\delta)K_{t+s-1} + (1 - S\left(I_{t+s}/I_{t+s-1}\right))\,I_{t+s} - K_{t+s}]\\&+\mu_{t+s}[X_{t+s} - C^{\gamma}_{t+s}X^{1-\gamma}_{t+s-1}]\Big)\Big].\end{aligned}$$

Defining the stochastic discount factor as $\Lambda_{t,t+1} \equiv \beta\frac{\lambda_{t+1}}{\lambda_t}$, the first-order conditions (FOC) are:

$$\text{Euler Consumption } : 1 = R_t\mathbb{E}_t\left[\Lambda_{t,t+1}\right], \tag{3}$$

$$\text{Labor Supply} : -U_{H,t} = \lambda_t W_t, \tag{4}$$

$$\begin{aligned}\text{Investment FOC} : 1 = {}& Q_t(1 - S(I_t/I_{t-1}) - (I_t/I_{t-1})S'(I_t/I_{t-1}))\\&+\mathbb{E}_t\left[\Lambda_{t,t+1}\,Q_{t+1}S'(I_{t+1}/I_t)(I_{t+1}/I_t)^2\right],\end{aligned}$$

$$\text{Capital Supply} : 1 = \mathbb{E}_t\left[\Lambda_{t,t+1}R^K_{t+1}\right],$$

where $\lambda_t = U_{C,t} - \gamma\mu_t C_t^{\gamma-1}X_{t-1}^{1-\gamma}$, $\mu_t = \beta\mathbb{E}_t[(1-\gamma)\frac{X_{t+1}}{X_t}\mu_{t+1} - U_{X,t+1}]$, and R_t^K, the gross return on capital, is given by $R_t^K = \frac{\left[r_t^K + (1-\delta)Q_t\right]}{Q_{t-1}}$.

The zero-growth steady state of the above first-order conditions is:

$$\begin{aligned}&R = R^K = \frac{1}{\beta}\,; \quad X = C\,; \quad \Lambda = \beta\\&\lambda = U_C - \gamma\mu\,; \quad \mu = -\frac{\beta}{1-\beta(1-\gamma)}U_X\,; \quad Q = 1\\&W = -\frac{U_H}{\lambda}\,; \quad r^K = \frac{1}{\beta} - 1 + \delta.\end{aligned}$$

2.2 Deriving Labor Supply Parameter Bounds

If we define $\kappa_t \equiv (C_t - \varrho H_t^{1+\psi} C_t^{\gamma} X_{t-1}^{1-\gamma})^{-\sigma}$, we have $U_{C,t} = (1 - \gamma\varrho H_t^{1+\psi} C_t^{\gamma-1} X_{t-1}^{1-\gamma})\kappa_t$, $U_{H,t} = -\varrho(1+\psi) H_t^{\psi} C_t^{\gamma} X_{t-1}^{1-\gamma}\kappa_t$ and $U_{X,t} = -(1-\gamma)\varrho H_t^{1+\psi} C_t^{\gamma} X_{t-1}^{-\gamma}\kappa_t$. The steady state then becomes:

$$
\begin{aligned}
\mu &= \frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}\varrho H^{1+\psi}\kappa, \\
\lambda &= \left(1 - \frac{\gamma\varrho H^{1+\psi}}{1-\beta(1-\gamma)}\right)\kappa, \\
W &= \frac{\varrho(1+\psi)H^{\psi}C}{1 - \frac{\gamma\varrho H^{1+\psi}}{1-\beta(1-\gamma)}}.
\end{aligned}
$$

With these preferences and the steady-state capital share $\alpha = 1 - \frac{WH}{Y}$, we arrive at

$$
\varrho H^{1+\psi} = \frac{(1-\alpha)}{(1+\psi)c_y + \frac{\gamma}{1-\beta(1-\gamma)}(1-\alpha)}, \tag{5}
$$

where $c_y \equiv \frac{C}{Y}$. For a given c_y and H (determined in a general equilibrium with a supply side), this pins down ϱ, given the remaining parameters.

3 The Frisch Elasticity of Labor Supply

We now derive a steady-state Frisch inverse elasticity. Log-linearizing around the steady state, we have:

$$
\begin{aligned}
\hat{u}_{H,t} \equiv \log\frac{U_{H,t}}{U_H} &= \frac{U_{HC}C}{U_H}\hat{c}_t + \frac{U_{HH}H}{U_H}\hat{h}_t + \frac{U_{HX}X}{U_H}\hat{x}_{t-1} \\
\hat{\lambda}_t \equiv \log\frac{\lambda_t}{\lambda} &= \frac{\lambda_C C}{\lambda}\hat{c}_t + \frac{\lambda_H H}{\lambda}\hat{h}_t + \frac{\lambda_\mu \mu}{\lambda}\hat{\mu}_t + \frac{\lambda_X X}{\lambda}\hat{x}_{t-1} \\
\hat{w}_t &= -\hat{u}_{H,t} - \hat{\lambda}_t.
\end{aligned}
$$

Hence, in the region of the steady state, by eliminating $\hat{c}_t$, we have:

$$
\hat{w}_t = \delta_F\hat{h}_t + \text{terms in } \hat{\lambda}_t + \text{terms in } \hat{\mu}_t + \text{terms in } \hat{x}_{t-1},
$$

where δ_F is a constant Lagrange multiplier (shadow prices of wealth and habit stock) inverse elasticity of labor supply, given by:

$$
\delta_F = \frac{U_{HC}H}{U_H}\left(-\frac{\lambda_H}{\lambda_C} + \frac{U_{HH}}{U_{HC}}\right), \tag{6}
$$

where:

$$
\frac{\lambda_H}{\lambda_C} = \frac{U_{CH}}{U_{CC} + \gamma(1-\gamma)\mu C^{\gamma-2}X^{1-\gamma}} = \frac{U_{CH}}{U_{CC} + \gamma(1-\gamma)\mu C^{-1}}. \tag{7}
$$

δ_F is a generalization of the constant marginal utility of consumption Frisch elasticity proposed by Bilbiie (2011) for KPR preferences (those proposed by King *et al.*, 1988). The derivatives (derived below in Section 3.5) are now functions of γ. Evaluating these at the steady state, we arrive at the steady-state Frisch elasticity:

$$\delta_F = \delta_F(\psi, \gamma) = \left(-\gamma + \sigma \frac{A(\psi, \gamma)}{A(\psi, 1)} \right) \left(\frac{\sigma(1+\psi)B(\psi) + \psi A(\psi, 1)}{\sigma A(\psi, \gamma) - \gamma A(\psi, 1)} \right.$$
$$\left. - \frac{(1+\psi)B(\psi)(\sigma A(\psi, \gamma) - \gamma A(\psi, 1))}{\sigma A(\psi, \gamma)^2 - \gamma(1-\gamma)B(\psi)A(\psi, 1)(1 + 1/(1-\beta(1-\gamma)))} \right), \tag{8}$$

where we emphasize the dependency on the reference parameters ψ, γ and we have defined $A(\psi, \gamma) \equiv (1-\gamma\varrho H^{1+\psi})$ and $B(\psi) \equiv \varrho H^{1+\psi}$. Note that wealth effects parameterized by γ enter directly through (8) and indirectly through its impact on steady-state hours $H = H(\gamma)$ as in (5). Two special cases are worth noting:

$$\text{KPR } (\gamma = 1) : \delta_F(\psi, 1) = \psi + \frac{(1+\psi)\varrho H^{1+\psi}(2\sigma - 1)}{\sigma(1 - \varrho H^{1+\psi})},$$
$$\text{GHH } (\gamma = 0) : \delta_F(\psi, 0) = \psi,$$

where GHH preferences are those proposed by Greenwood *et al.* (1988). Note that although $\delta(\psi, 1) > \delta(\psi, 0)$ for $\sigma > \frac{1}{2}$, $\delta(\psi, \gamma)$ is not monotonically decreasing owing to the term $\gamma(1-\gamma)$ in (8), which peaks at $\gamma = \frac{1}{2}$.

3.1 The Lower Bound on the Steady-State Frisch Elasticity

A necessary condition for the utility to be well defined and an equilibrium to exist is that $\varrho H^{1+\psi} < 1$. This places the following lower bound on ψ:

$$\psi > \underline{\psi} \equiv \frac{(1-\alpha)(1-\beta)(1-\gamma)}{c_y(1-\beta(1-\gamma))} - 1 \tag{9}$$

For $\gamma = 0$, this becomes $\psi > \frac{1-\alpha}{c_y} - 1$, whereas for $\gamma = 1$ (KPR preferences), we have $\psi > -1$ and the constraint disappears. Since we restrict ourselves to $\psi > 0$, this implies a threshold for γ, γ^* say, below which the bound is relevant. This is given by:

$$\gamma^* = \frac{(1-\beta)(1-\alpha-c_y)}{(1-\alpha)(1-\beta) + \beta c_y} \tag{10}$$

For our calibration below, we find that $\gamma^* = 0.0017$. The bound therefore matters only for values of γ very close to the GHH case.

Theorem 1
In the GHH case, $\delta_F(\psi, 0)$ is bounded below at a value $\psi = \underline{\psi}$ given by (9).

A sting in the tail arises if we introduce *external habit* with C_t in the utility function replaced by $C_t - \chi C_{t-1}$. Then c_y is replaced with $c_y(1-\chi)$, pushing the constraint on ψ into an implausible range. This we now show can be mitigated by making habit *internal rather than external.*

3.2 External versus Internal Habit

With external habit in consumption, household j has a single-period utility

$$
\begin{aligned}
U_t^j &= \frac{(C_t^j - \chi C_{t-1} - \varrho (H_t^j)^{1+\psi} X_t^j)^{1-\sigma}}{1-\sigma}\,; \quad \chi \in [0,1), \\
X_t^j &= (C_t^j - \chi C_{t-1})^{\gamma} (X_{t-1}^j)^{1-\gamma}\,; \quad \gamma \in [0,1]
\end{aligned}
$$

where C_{t-1} is aggregate per capita consumption, whereas with internal habit we have

$$
\begin{aligned}
U_t^j &= \frac{(C_t^j - \chi C_{t-1}^j - \varrho (H_t^j)^{1+\psi} X_t^j)^{1-\sigma}}{1-\sigma}\,; \quad \chi \in [0,1) \\
X_t^j &= (C_t^j - \chi C_{t-1}^j)^{\gamma} (X_{t-1}^j)^{1-\gamma}\,; \quad \gamma \in [0,1]
\end{aligned}
$$

Now defining $\kappa_t \equiv (C_t - \chi C_{t-1} - \varrho H_t^{1+\psi} (C_t - \chi C_{t-1})^{\gamma} X_{t-1}^{1-\gamma})^{-\sigma}$, in a symmetric equilibrium the household first-order conditions are as before with marginal utility

$$
U_{C,t} = \left(1 - \gamma \varrho H_t^{1+\psi} (C_t - \chi C_{t-1})^{\gamma-1} X_{t-1}^{1-\gamma}\right) \kappa_t,
$$

and for external habit and internal habit, respectively, we have

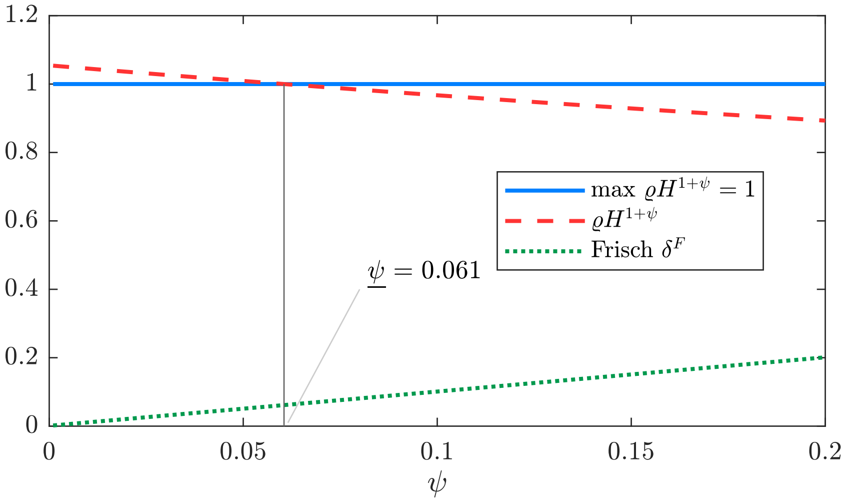
$$
\begin{aligned}
\lambda_t &= U_{C,t} - \frac{\gamma \mu_t X_t}{(C_t - \chi C_{t-1})} \\
\lambda_t &= U_{C,t} - \beta \chi \mathbb{E}_t[U_{C,t+1}] \\
&\quad - \gamma \left(\frac{\mu_t X_t}{(C_t - \chi C_{t-1})} - \beta \chi \mathbb{E}_t \left[\frac{\mu_{t+1} X_{t+1}}{(C_{t+1} - \chi C_t)} \right] \right),
\end{aligned}
$$

The zero-growth steady state then becomes $U_C = (C(1-\chi) - \varrho H^{1+\psi} X)^{-\sigma}$, $\lambda = U_C - \frac{\gamma \mu X}{(C(1-\chi))}$ for external habit and $\lambda = U_C(1-\beta\chi) - \frac{\gamma(1-\beta\chi)\mu X}{(C(1-\chi))}$ for internal habit. These results lead to:

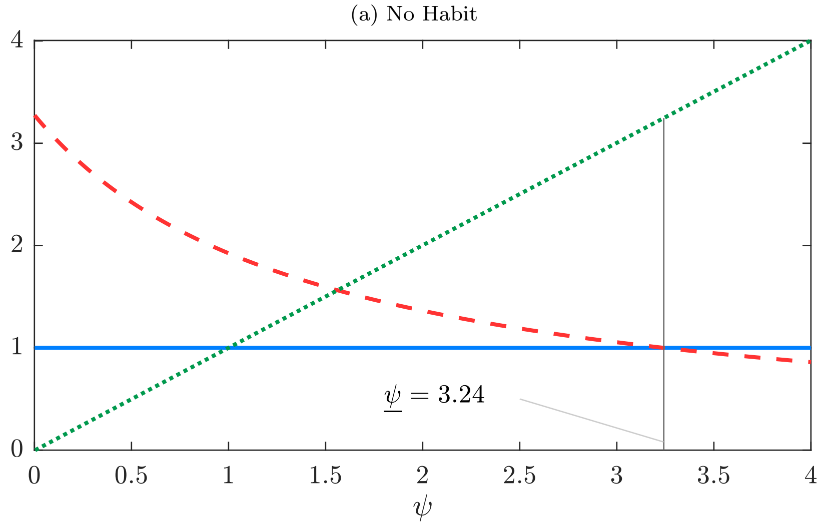
Theorem 2
The results of Theorem 1 apply to habit in consumption with c_y replaced with $c_y(1-\chi)$ for external habit and $c_y \frac{1-\chi}{1-\beta\chi}$ for internal habit.

3.3 Empirical Estimates of the Frisch Elasticity

Microeconomic and macroeconomic estimates of the Frisch elasticity differ significantly, the former typically ranging from 0 to 0.5 and the latter from 2 to 4 (Peterman, 2016). Estimations of the elasticity of labor supply found using microeconomic data depend on factors such as gender, age, marital status and dependents. Keane (2011) offers a survey of labor supply, restricting the sample to men, finding a range of between 0 and 0.7 with an average of 0.31. Reichling and Whalen (2017) give a thorough review of the estimates found in the literature based on microeconomic data, finding that estimates typically range from 0 to over 1. The higher estimates correspond to married women



(a) No Habit



(b) External Habit

Figure 1: The lower bound on ψ with $\gamma = 0.001$.

with children, whereas the labor supply of men is far lower. Combining the results, Reichling and Whalen (2017) propose a range of between 0.27 and 0.53, with a central point estimate of 0.4. This corresponds to a Frisch coefficient, δ, between 1.89 and 3.7, with a point estimate of 2.5.

3.4 Numerical Illustration

Table 1 illustrates the analysis so far. Parameter values are $\alpha = 0.3$, $c_y = 0.6$, $\beta = .99$, $\sigma = 2.0$, $\chi = 0.75$ and stated values for γ.[1] We can now assess the empirical plausibility of JR preferences

[1] In fact, $\gamma > 0$ is required for balanced growth, but γ can be very small.

γ	No Habit	External Habit	Internal Habit	External Habit and Labor Wedge
1	1.750	**7.000**	1.803	**4.900**
	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)
0.75	1.079	2.307	1.103	2.008
	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)
0.5	0.580	0.911	0.589	0.853
	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)
0.25	0.239	0.320	0.241	0.309
	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)	($\psi = 0$)
0	0.167	**3.667**	0.202	2.267
	($\psi = \underline{\psi}$)	($\psi = \underline{\psi}$)	($\psi = \underline{\psi}$)	($\psi = \underline{\psi}$)

Table 1: Lower Bound $\delta_F(\underline{\psi})$ with Habit and Constraints on JR Preferences

with habit in consumption. From our discussion in 3.3, we wish to calibrate ψ to hit an inverse elasticity $\delta_F \in [1.89, 3.70]$ with a central value of 2.50. From our numerical results for the lower bound $\delta_F(\underline{\psi})$, this rules out external habit for the KPR and GHH extremes.[2]

However, we can resolve the problem by introducing a labor wedge into the household problem. Then (4) becomes $\frac{U_{H,t}}{\lambda_t} = -W_t(1-\tau)$ where $\tau \in [0.27, 0.37]$ is the wedge (as in Shimer, 2009), and $1-\alpha$ in (10) is replaced with $(1-\alpha)(1-\tau)$.

3.5 Wealth Effects and the Dynamic Frisch Elasticity

Up to now we have constructed a Frisch inverse elasticity of labor supply in the steady state. However, the wealth effect and therefore the Frisch elasticity are, in fact, time-varying in the type of models we are considering.

This subsection constructs a dynamic Frisch elasticity by decomposing the substitution and wealth effects in a standard RBC model with JR household preferences. We adopt a full general equilibrium analysis (as opposed to the partial equilibrium illustration in Jaimovich and Rebelo (2008)). The

[2]Values for the lower bound of the Frisch inverse elasticities outside or very close to the boundary of empirical estimates are highlighted in bold; see also Figure 1.

supply side of the model is that of a standard RBC model.

$$\text{Output}: Y_t = F(A_t, H_t, K_{t-1}) \tag{11}$$
$$\text{Labor Demand}: F_{H,t} = W_t \tag{12}$$
$$\text{Capital Demand}: F_{K,t} = r_t^K$$
$$\text{Equilibrium}: Y_t = C_t + G_t + I_t$$

where (11) is a production function that in the simulations we assume to be Cobb-Douglas with capital share $\alpha = 0.3$. A_t and G_t are exogenous technology and demand processes.

To compute the substitution effect without wealth effects, consider notional hours supplied by households, H_t^s, as given by the system

$$\begin{aligned}
\frac{U_{H,t}}{\lambda} &= -W_t \qquad (13)\\
\lambda_t &= U_{C,t} - \gamma\mu_t \frac{X_t}{C_t}\\
\mu_t &= -U_{X,t} + \beta(1-\gamma)\mathbb{E}_t \frac{\mu_{t+1}X_{t+1}}{X_t}\\
X_t &= C_t^{\gamma} X_{t-1}^{1-\gamma},
\end{aligned}$$

where $U_{H,t} = U_{H,t}(C_t, H_t^s, X_{t-1})$ and $U_{C,t} = U_{C,t}(C_t, H_t^s, X_{t-1})$. Actual hours (with wealth plus substitution effects) and the supply side of the model are then that of a standard RBC model.

We now construct a dynamic Frisch inverse elasticity in a similar fashion to how the static one was constructed. However, rather than log-linearizing around the steady state, we now log-linearize around the current value, giving:

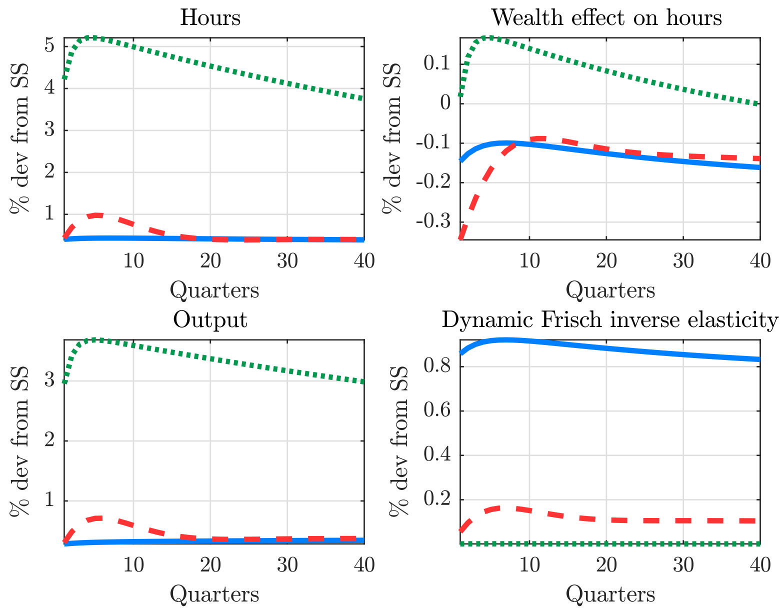
$$\begin{aligned}
\hat{u}_{H,t}^* \equiv \log \frac{U_{H,t}}{U_{H,t}} &= \frac{U_{HC,t}C_t}{U_{H,t}}\hat{c}_t^* + \frac{U_{HH,t}H_t}{U_{H,t}}\hat{h}_t^* + \frac{U_{HX,t}X_t}{U_{H,t}}\hat{x}_{t-1}^*\\
\hat{\lambda}_t^* \equiv \log \frac{\lambda_t}{\lambda_t} &= \frac{\lambda_{C,t}C_t}{\lambda_t}\hat{c}_t^* + \frac{\lambda_{H,t}H_t}{\lambda_t}\hat{h}_t^* + \frac{\lambda_{\mu,t}\mu_t}{\lambda_t}\hat{\mu}_t^* + \frac{\lambda_{X,t}X_t}{\lambda_t}\hat{x}_{t-1}^*\\
\hat{w}_t^* &= -\hat{u}_{H,t}^* - \hat{\lambda}_t^*,
\end{aligned}$$

where partial derivatives are now indexed by time to indicate they are evaluated at the current values. By construction, all of the variables with $*$ must always equal zero, but this remains a helpful representation for what-if analysis. Proceeding as before, by eliminating $\hat{c}_t^*$ we have:

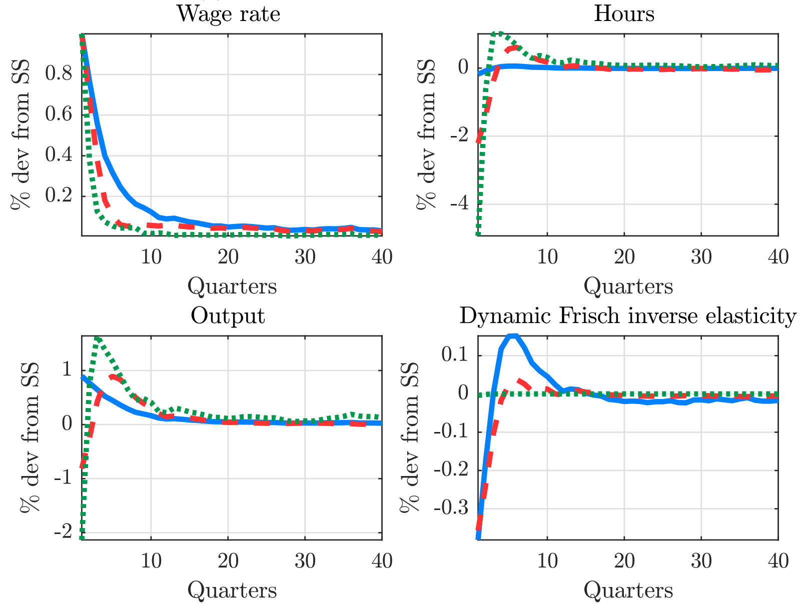
$$\hat{w}_t^* = \delta_{F,t}\hat{h}_t^*,$$

where we have removed the extra unneeded zero terms, and where $\delta_{F,t}$ is our dynamic Frisch inverse elasticity of labor supply, given by:

$$\delta_{F,t} = \frac{U_{HC,t}H_t}{U_{H,t}}\left(-\frac{\lambda_{H,t}}{\lambda_{C,t}} + \frac{U_{HH,t}}{U_{HC,t}}\right), \tag{14}$$



(a) Partial Equilibrium, Real Wage Shock



(b) General Equilibrium, Technology Shock

Figure 2: Substitution and Wealth Effects; Dynamic Frisch Elasticity. The blue solid line is with $\gamma = 1$; the red dashed line with $\gamma = 0.1$; and the green dotted line with $\gamma = 0.001$.

where:

$$\frac{\lambda_{H,t}}{\lambda_{C,t}} = \frac{U_{CH,t}}{U_{CC,t} + \gamma(1-\gamma)\mu_t C_t^{\gamma-2} X_{t-1}^{1-\gamma}}. \tag{15}$$

Figure 2 (a) first carries out a partial equilibrium exercise similar to Jaimovich and Rebelo (2008) (with the same qualitative results) to show the decomposition of hours supplied into substitution and wealth effects following a permanent exogenous wage shock. Then (12) is replaced with this exogenous process. We see from the impulse response function of figure 2 that the Frisch inverse elasticity becomes time-varying as we move away from the GHH case where it remains constant at its steady-state value.

Then we proceed in (b) to the general equilibrium case with a exogenous technology AR(1) process for A_t with persistence parameter 0.78 and standard deviation 0.67% (see Dejong and Dave (2007), page 137). G_t is held fixed at its steady state. The impulse responses for different values of γ have been scaled so that the first period impacts coincide. The dynamic Frisch inverse elasticity is then pro-cyclical for the KPR case, but becomes counter-cyclical as we close down wealth effects by moving toward the GHH case. This is confirmed by second moments computed from second-order perturbation solutions in Table 2, where throughout this subsection we have calibrated the preference parameter at ψ to hit a steady-state Frisch elasticity of $\delta_F = 2.0$ for the KPR case. But this calibration comes at the expense of an implausibly low standard deviation of output. For $\gamma = 0.001$ this feature is mended, but then the Frisch elasticity is in the low range suggested only by micro-econometric studies.

γ	δ_F	$\mathrm{sd}\,(Y_t)$ (%)	$\mathrm{sd}\,(\delta_{F,t})/\mathrm{sd}\,(Y_t)$	$\mathrm{corr}\,(\delta_{F,t}, Y_t)$
1	2.0	0.82	0.29	0.22
0.5	0.67	0.78	0.33	-0.68
0.25	0.27	0.75	0.97	-0.80
0.1	0.03	0.87	8.22	-0.63
0.001	0.17	2.05	1.04	-0.48

Table 2: Business Cycle Properties of the Dynamic Frisch Inverse Elasticity

4 Conclusions

This note has reviewed a utility function commonly used in RBC-DSGE models due to Jaimovich and Rebelo (2008) that is non-separable in habit-adjusted consumption and leisure, compatible with balanced growth and eliminates counterfactual wealth effects. Our main contributions are, first, Theorems 1 and 2, which highlight a constraint on the preference parameter ψ needed to target the steady-state Frisch inverse elasticity. This leads to a lower bound for the latter that cannot be reconciled empirically with external habit at the KPR and GHH extremes. However, the introduction of a labor wedge solves the problem for modest departures from the KPR case. Second, we propose a concept of a dynamic Frisch inverse elasticity. A numerical solution of a standard RBC model driven by a technology AR(1) shock process suggests this elasticity is pro-cyclical for the KPR case

($\gamma = 1$), but counter-cyclical as we move away from this extreme.

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