Bending the Curves: Wages and Inflation

by Dany Brouillette, Madigan Dockrill, Helen Lao and Laurence Savoie-Chabot
Acknowledgements

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Abstract

As economic slack continues to be absorbed and the labour market tightens, wage growth and inflation could increase faster than expected, which would suggest convexity in their Phillips curves. This note investigates whether there is convexity in the Phillips curves for Canadian wage growth and inflation by testing different empirical approaches over the post-inflation-targeting period. We conclude that while there is some evidence that the relationship between wage growth—as indicated by the Bank of Canada wage-common measure—and the output gap is non-linear in Canada, there is no clear evidence of a convex relationship between inflation and the output gap. The presence of downward nominal wage rigidity could likely explain these results. The risk of a sharp increase in CPI inflation therefore appears to be small over the projection horizon (2018–20) since the Canadian economy is expected to experience only modest excess demand.

Bank topics: Inflation and prices; Labour markets
JEL codes: E, E24, E3, E31, J and J3

Résumé

À mesure que la capacité excédentaire de l’économie continue à se résorber et que le marché du travail se resserre, il est possible que la croissance des salaires et l’inflation augmente plus rapidement que prévu, ce qui signifierait que leurs courbes de Phillips présentent une forme convexe. Dans la présente note, nous tentons de vérifier, à l’aide de différentes méthodes empiriques, l’existence de pareilles courbes convexes pour décrire la croissance des salaires et l’inflation au Canada pendant la période postérieure à l’adoption de cibles d’inflation. Nous concluons que, malgré certains indices suggérant une relation non linéaire entre la croissance des salaires (telle qu’elle est mesurée par l’indicateur salaires-comm de la Banque du Canada) et l’écart de production au Canada, il n’existe pas d’indice incontestable d’une relation convexe entre inflation et écart de production. Ces résultats s’expliqueraient vraisemblablement par une rigidité à la baisse des salaires nominaux. Dans ces conditions, il y a peu de risque que l’inflation mesurée par l’IPC enregistre une hausse prononcée au cours de la période de projection (2018-2020), l’économie canadienne ne devant connaître qu’un léger excédent de la demande.

Sujets : Inflation et prix; Marchés du travail
Codes JEL : E, E24, E3, E31, J et J3
1. Context

As economic slack continues to be absorbed, wage growth and inflation could increase faster than expected. The lack of significant wage pressures may currently seem puzzling given the apparently tight state of the labour market. Some cyclical and structural forces, including competitive pressures from ongoing automation and offshoring as well as labour market scarring, may account for this inertia.\textsuperscript{1} Alternatively, wage growth may only increase once there is material excess demand in the economy.\textsuperscript{2} Episodes of higher wage growth, such as in the early 2000s and from 2005 to 2007, were coincident with some excess demand (Chart 1). But does this mean there is a “kink” in the wage growth curve? And would such a kink imply non-linearity in the inflation curve? This note investigates the presence of non-linearity in both the wage growth and inflation Phillips curves (WPC and IPC, respectively).

**Chart 1:** Wage growth is usually strong when the Canadian economy is in excess demand

Quarterly data

Empirical work for Canada on the convexity of the WPC is virtually non-existent and, at best, scarce for the IPC. Research in the 1990s and early 2000s focused on the IPC, with the majority finding convexity. However, these studies partly cover the period before the

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\textsuperscript{1} Box 3 in the April 2018 Monetary Policy Report offers more detail about labour market conditions and wage growth.

\textsuperscript{2} This was described in previous analysis conducted by Bank staff. See Chart A1.3 in Brouillette et al. (2017).
introduction of inflation targeting. Most evidence for the United States and some European countries support the presence of convexity for both wage growth and inflation curves, as summarized in St-Cyr (2018). US research has recently focused on the wage growth curve, for which there is strong evidence of convexity.

Section 2 describes the different methodologies used to test for non-linearity in the WPC and IPC. Section 3 presents the results for the WPC and Section 4 describes the results for the IPC. In Section 5 we compare the WPC and IPC results and discuss possible policy implications.

2. Methodology

A convex relationship between wage growth and the output gap could exist for several reasons. For example, downward nominal wage rigidity (DNWR) could bend this relationship by preventing wages from falling as much as they would in a period of excess supply—a constraint that is not playing a role in a period of excess demand when wages tend to rise (Akerlof, Dickens and Perry 1996). Capacity constraints could also lead to higher wage pressure when the economy runs hot if the labour supply did not adjust fast enough in response to a positive demand shock (Clark, Laxton and Rose 1995; Macklem 1997). In the short term, wages would have to increase to draw more workers into the labour market.

Similar methodologies are used to test for the presence of convexity in the WPC and IPC. The generic form of the equation is as follows:

\[ Y_t = \beta X_t + f(gap_t) + \epsilon_t, \]

where \( Y_t \) is either a measure of wage growth or inflation, and \( X_t \) includes explanatory variables that can vary depending on how \( Y_t \) is defined (see Appendix 1 for details on the specifications and variables included in \( X_t \)). We test three alternatives for the non-linear function \( f(\cdot) \) of the measure of economic slack \( gap_t \): \(^3\)

- **Piecewise linear function:** the sensitivity of \( Y_t \) to \( gap_t \) changes when a certain threshold is reached. For instance, if \( gap_t \) is measured by the output gap, the kink is at zero excess demand: \( f(gap_t) = \beta_1 gap_t + \beta_2 gap_t \mathbb{I}(gap_t > 0) \), where \( \mathbb{I}(gap_t > 0) \) equals 1 if \( gap_t > 0 \), 0 otherwise.

- **Quadratic function:** as for the piecewise linear, the sensitivity of \( Y_t \) to \( gap_t \) increases after reaching a certain threshold, but at an increasing speed. Thus

\(^3\) Fillion and Léonard (1997) and Kumar and Orrenius (2015) use the first option, while Laxton, Rose and Tetlow (1993) use the first and second.
\[ f(gap_t) = \gamma_1 gap_t + \gamma_2 gap_t^2 I(gap_t > 0), \]
where \( I(gap_t > 0) \) equals 1 if \( gap_t > 0 \), 0 otherwise.

- **Semi-parametric:** The relationship between \( Y_t \) and \( gap_t \) is estimated using a non-parametric approach to avoid imposing any form of convexity *ex ante*. See Appendix 1 for a summary of the method.

The preferred measure for wage growth is the wage-common, but alternative specifications are also tested with the individual components of the wage-common: hourly wages from the Labour Force Survey (LFS) and the Survey of Employment, Payrolls and Hours (SEPH); and hourly compensation from the Productivity Accounts (PA) and National Accounts (NAC).\(^4\) In the inflation equations, CPI inflation and the three preferred measures of core inflation, CPI-Trim, CPI-median and CPI-common, are used. Our preferred measure of slack is the output gap, but alternative specifications are also estimated using the unemployment rate and the labour market indicator (LMI).\(^5\)\(^,\)\(^6\)

### 3. The convexity of the wage growth Phillips curve

**A simple WPC**

Our results support the presence of non-linearity in the WPC that is robust across different types of non-linearities and measures of slack (**Table 1**). **Chart 2** shows the predicted wage growth from the semi-parametric approach. When the output gap is less than -1 per cent, average wage growth is expected to be close to 2 per cent. But with an increase in the output gap from -1 per cent to +1 per cent, wage growth increases to about 3 per cent. Adding another percentage point to the output gap (i.e., when slack reaches 2 per cent) is expected to boost wage growth above 4 per cent. Interestingly, when the output gap is 0, the model predicts that wage growth is around 2.6 per cent, close to the actual historical average for the wage-common (2.7 per cent).\(^7\)

To assess the robustness of our conclusions with the wage-common, we test for the convexity relationship using the three other measures of slack as shown in **Table 1**. We also re-estimate different non-linear models with the individual components of the wage-common (LFS, SEPH, PA and NAC). These alternative results strengthen our conclusion

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\(^4\) As highlighted in Brouillette, Lachaine and Vincent (2018), the wage-common effectively captures underlying wage pressures, reflecting a common trend across the available data sources.

\(^5\) While the labour gap may seem to be a better choice for the WPC, the output gap was used for consistency with the analysis of inflation.

\(^6\) For more details on the LMI, see Zmitrowicz and Khan (2014).

\(^7\) When the labour gap is zero, the predicted growth of the wage-common is about 3 per cent.
about the existence of a convex relationship between wage growth and measures of slack in Canada.

### Table 1: There is evidence of a convex relationship between wage growth and measures of slack

<table>
<thead>
<tr>
<th>Wage growth</th>
<th>Measures of slack</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Output gap</td>
</tr>
<tr>
<td>Piecewise linear</td>
<td>Yes</td>
</tr>
<tr>
<td>Quadratic</td>
<td>Yes</td>
</tr>
<tr>
<td>Semi-parametric</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: Yes means that we find statistical evidence of a non-linear relationship between wage growth from the wage-common and the measure of slack.

### Chart 2: There is evidence of a non-linear relationship between wage growth and the output gap

Overall, our results provide some support for the hypothesis that the relationship of wage growth to economic slack is non-linear. There is nevertheless one major caveat. Historically, the output gap has almost never fallen outside the range of -2 per cent to 1 per cent. This makes identification of the relationship at high positive values of the output gap more tenuous, which is where higher wage growth, relative to periods of
excess supply, should be more evident. Nevertheless, we find evidence that the relationship could be non-linear over the -2 per cent to 1 per cent range.

Implications of a non-linear WPC

The piecewise linear function is used to illustrate the increase in wage growth as the economy moves into excess demand. Chart 3 shows wage growth as measured by the wage-common under the base case (the output gap remains at zero after closing, red line) and under two simulations where the output gap gradually widens to 3 per cent (blue line) and narrows to -3 per cent (green line) by the end of 2020. A likely interpretation of the results shown in Chart 3, and in Chart 2, is that DNWR is keeping wages higher than otherwise in periods of excess supply (i.e., the sensitivity of wage growth is weaker with large excess supply, green line). However, convexity does not necessarily imply that wage growth would be much higher in periods of excess demand or that this acceleration would last and be associated with persistent high inflation: in such cases, monetary policy would react to keep inflation on target, which would reduce excess demand and curb wage growth as well.

Chart 3: Wage growth is higher when the economy is in excess demand

4. The convexity of the inflation Phillips curve

This section evaluates whether the evidence of a kink in the WPC also implies a kink for Canadian CPI inflation and the preferred measures of core inflation.

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8 Although a positive output gap of 3 per cent has never been observed, we use a range of -3 per cent to +3 per cent to show an extreme case. The quadratic empirical specifications yield similar results.
Using the output gap, we find that estimates for both CPI inflation and core inflation do not support the hypothesis of convexity in the IPC (Table 2). Moreover, this result is robust across most approaches we tested (see Chart 4 for the semi-parametric results and Chart 2A to Chart 2C in Appendix 2 for the measures of core inflation).

Table 2: There is no clear evidence of a convex relationship between measures of inflation and the output gap

<table>
<thead>
<tr>
<th></th>
<th>Output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI inflation</td>
<td></td>
</tr>
<tr>
<td>Piecewise linear</td>
<td>No</td>
</tr>
<tr>
<td>Quadratic</td>
<td>No</td>
</tr>
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<td>Semi-parametric</td>
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</tr>
<tr>
<td>Core inflation measures</td>
<td></td>
</tr>
<tr>
<td>Piecewise linear</td>
<td>No</td>
</tr>
<tr>
<td>Quadratic</td>
<td>No</td>
</tr>
<tr>
<td>Semi-parametric</td>
<td>No</td>
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</tbody>
</table>

Note: Yes means that we find statistical evidence of a non-linear relationship between the measure of inflation and the output gap.

Chart 4: The output gap does not appear to have a non-linear relationship with CPI inflation

Source: Bank of Canada calculations and estimates
These results are in line with findings in the most recent research on the subject in Canada, which noted evidence of non-linearity only in the period before the adoption of inflation targeting (Demers 2003).

To further assess the robustness of our conclusions, we tested convexity with the unemployment rate and LMI as measures of economic slack. Most of these alternative estimations led to similar results (see the summary of the results in Table 2A and Table 2B in Appendix 2).

Overall, convexity between measures of inflation and the output gap was rejected regardless of the approach used.

5. Policy implications and concluding remarks

Overall, our results suggest there is limited evidence of convexity in the Canadian IPC. As such, if the Canadian economy begins to experience modest excess demand over the next few years, as expected in the April MPR, CPI inflation is unlikely to increase more than expected (even if wage growth does). First, nearly all estimates rejected the possibility of convexity in the IPC. Second, as noted in the April MPR, excess demand is expected to be modest on average in 2019 and 2020 (grey bars in Chart 13 of the April 2018 MPR). This suggests that even if such convexity is currently present in Canada, its effect would probably be small over the projection horizon, assuming that inflation expectations remain well anchored. Third, while we find some evidence of non-linearity for wage growth, the expected increase in wage growth should also be modest given the modest amount of excess demand expected, unless one of the upside risks highlighted in the April MPR materialize.

It is important to also investigate why we find some evidence of convexity in the WPC but not in the IPC. Although testing for possible factors behind this divergence is left for further research, we briefly discuss some potential explanations.

First, it is possible that the non-linearity in wage growth comes not from wages rising faster during expansions, but from wages not falling during recessions because of some form of DNWR (see Chart 3). If nominal wages are downwardly rigid, they will not adjust during periods of excess supply. And if these episodes are long enough and some form of “pent up wage deflation” is present, a significant amount of excess demand may be needed for wage growth to start increasing (Yellen 2014).

Second, if competitive pressures intensify during periods of excess demand, retail firms in Canada may curtail their margins (i.e., firms will not fully pass increases in labour costs to consumer prices). Chart 5 presents margins in the retail sector together with the integrated framework output gap. At the beginning of the 2000s, when the Canadian
economy was operating well above potential, retailers’ margins were below their historical average and declining. This could explain why upward pressures on wage growth due to the presence of convexity would not have translated into inflationary pressures. However, before the 2008–09 recession, when the Canadian economy was also operating above capacity, margins were above their historical average and increasing. As such, it is hard to draw any definitive conclusions regarding the possible impact of competition, through compressed margins, on the disconnect between wages and inflation when the economy experiences excess demand.\textsuperscript{9, 10}

Finally, it is possible that this disconnect between the sensitivity of wage growth and inflation to the output gap when the economy experiences excess demand is partly related to the success of inflation targeting in Canada. Simply, inflation expectations have remained well anchored. Since the 1990s, CPI inflation and the core inflation measures have never been outside the target band (1–3 per cent) for a persistent period. As such, it might not be surprising that there is only limited evidence of convexity in the Phillips curve for Canadian inflation over that period.

\textbf{Chart 5: In the early 2000s, margins were declining as excess demand was increasing, in contrast to before 2009}  

Quarterly data

Red line: Retail trade profit margins (left scale)  
Blue line: IF output gap (right scale)

Note: The dotted line is the historical average of retail trade profit margins (2000-15).  
Sources: Statistics Canada and Bank of Canada estimates  
Last observation: 2017Q4

\textsuperscript{9} A strong Canadian dollar (between 0.80 and 0.95 USD) could also explain why retailers’ margins did not decrease during the period of excess demand experienced between 2005 and 2008, while they seem to have at the beginning of the 2000s (when the dollar was below 0.70 USD).

\textsuperscript{10} There are potential data limitations and measurement issues with the profit margins presented in Chart 5. They do not control for a shift in composition (e.g., a shift toward more efficient, i.e., higher-margin, retailers). Mergers and acquisitions can also lead to volatility in data on profit margins.
References


Appendix 1

The general form of the wage growth Phillips curve (WPC) and inflation Phillips curve (IPC) is given by

\[ Y_t = \beta X_t + f(gap_t) + \varepsilon_t, \quad [1A] \]

where \( f(gap_t) \) represents one of the three alternatives for the non-linear function of the slack measure presented in Section 2. Variables included in \( X_t \) depend on the dependent variables and are listed below.

Wage growth Phillips curve specification

For all measures of wage growth, the WPC includes labour productivity growth (\( LP \)), a four-quarter moving average of CPI inflation and a constant:

\[ \Delta W_t = c + \beta_1 f(gap_t) + \beta_2 \Delta LP_t + \beta_3 \sum_{i=1}^{4} \frac{1}{4} \Delta CPI_{t-i} + \varepsilon_t, \]

where \( \Delta \) denotes year-over-year growth. Data are quarterly, and estimations are done over the 1998Q1–2017Q4 period.

Inflation Phillips curve specification

For the CPI-inflation IPC specification, the variables included in \( X_t \) are a constant, the contemporaneous change in the Bank of Canada commodity price index (\( BCPI \)) and changes in the exchange rate (\( xrate \)) from \( t \) to \( t-4 \):

\[ \Delta CPI_t = c + \beta_1 f(gap_t) + \beta_2 \Delta BCPI_t + \sum_{i=0}^{4} \beta_{i+3} \Delta xrate_{t-i} + \varepsilon_t, \]

where \( \Delta \) denotes quarter-over-quarter growth. Data are quarterly, and estimations are done over the 1995Q1–2017Q4 period, except for the equations that use the labour market indicator (LMI) given that the series starts in 2003.

For core inflation measures, \( X_t \) includes the inflation target (\( \pi^*_t \)) and one lag of the dependent variable. Changes in the exchange rate and the BCPI were also included from \( t \) to \( t-4 \):

\[ \Delta Core_t = \beta_1 f(gap_t) + \beta_2 \pi^*_t + (1 - \beta_2) \Delta Core_{t-1} + \sum_{i=0}^{4} \beta_{3+i} \Delta BCPI_{t-i} + \sum_{j=0}^{4} \beta_{8+j} \Delta xrate_{t-j} + \varepsilon_t, \]

where \( \Delta \) denotes quarter-over-quarter growth. Data are quarterly, and estimations are done over the 1995Q1–2017Q4 period, except for the equations that use the LMI.
The semi-parametric model

The semi-parametric model uses the Robinson (1988) partially linear model. For all measures of wage growth and inflation, $X_t$ remains the same as for the parametric models. The difference is how the function $f(gap_t)$ is identified. The methodology is as follows:

1. Estimate the conditional expectations $E(Y_t|gap_t)$ and $E(X_t|gap_t)$ using a non-parametric method.
2. Identify $\beta$ using the ordinary least squares method on
   \[ Y_t - E(Y_t|gap_t) = (X_t - E(X|gap_t)) \beta + \epsilon_t. \]
   This has the advantage of consistently estimating $\beta$ without having to make any assumption on $f(gap_t)$.
3. The identification of $\beta$ allows us to construct the residuals $(Y_t - \hat{\beta}X_t)$ from equation [1A].
4. The shape of $f(gap_t)$ can then be estimated non-parametrically by regressing $(Y_t - \hat{\beta}X_t)$ on $gap_t$. 

Appendix 2

Results on the convexity of the Phillips curve for Canadian inflation

Chart 2A: Partially linear CPI-trim curve

Source: Bank of Canada calculations and estimates

Chart 2B: Partially linear CPI-median curve

Source: Bank of Canada calculations and estimates
Chart 2C: Partially linear CPI-common curve

Source: Bank of Canada calculations and estimates
Table 2A: Dependent variable—CPI inflation\textsuperscript{a, b}

<table>
<thead>
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<th>Output gap</th>
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<th>Labour market indicator</th>
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<td></td>
<td>Unemployment rate</td>
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<tr>
<td>Semi-parametric</td>
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\textsuperscript{a} Using alternative measures of inflation, such as CPI inflation excluding mortgage interest cost, and different specifications does not change the results significantly.

\textsuperscript{b} In all tables, Yes means that we find statistical evidence of a non-linear relationship between inflation and the measure of slack.

Table 2B: Dependant variable—CPI-trim

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Table 2B: Dependant variable—CPI-median

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Table 2B: Dependant variable—CPI-common

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