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Optimal Interbank Regulation

by Thomas J. Carter

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Abstract

Recent years have seen renewed interest in the regulation of interbank markets. A review of the literature in this area identifies two gaps: first, the literature has tended to make ad hoc assumptions about the interbank contract space, which makes it difficult to generate convincing policy prescriptions; second, the literature has tended to focus on ex-post interventions that kick in only after an interbank disruption has come underway (e.g., open-market operations, lender-of-last-resort interventions, bail-outs), rather than ex-ante prudential policies. In this paper, I take steps toward addressing both these gaps, namely by building a simple model for the interbank market in which banks optimally choose the form of their interbank contracts. I show that the model delivers episodes that qualitatively resemble the interbank disruptions witnessed during the financial crisis. Some important implications for policy then emerge. In particular, I show that optimal policy requires careful coordination between ex-post and ex-ante interventions, with the ex-ante component surprisingly doing most of the heavy lifting. This suggests that previous literature has underemphasized the role that ex-ante interventions have to play in optimal interbank regulation.

Bank topics: Financial stability; Financial system regulation and policies
JEL codes: G01, G20

Résumé

Les dernières années ont vu renaître un intérêt pour la réglementation des marchés interbancaires. Une revue de la littérature sur ce sujet souligne deux lacunes. D'abord, la plupart des auteurs se basent sur des hypothèses ad hoc quant à l'univers des contrats du marché interbancaire. Par conséquent, il est difficile de tirer des conclusions convaincantes pour la formulation des politiques. De plus, la littérature tend à se concentrer sur des interventions ex-post, qui ne sont mises en place que lorsque le marché interbancaire a subi une perturbation (par exemple, les opérations d'open-market, les interventions de prêteurs de dernier ressort, les sauvetages), plutôt que sur des politiques prudentielles ex-ante. Dans cette étude, nous visons à combler ces lacunes en construisant un modèle simple du marché interbancaire dans lequel les banques choisissent de façon optimale la forme de leurs contrats interbancaires. Nous montrons que le modèle peut générer des épisodes qui s'apparentent de façon qualitative aux perturbations des marchés interbancaires observées lors de la crise financière. Les simulations du modèle révèlent des éléments importants pour l'élaboration de politiques. Plus spécifiquement, nous montrons que la politique optimale requiert une coordination minutieuse entre interventions ex-ante et ex-post et, fait surprenant, que ce sont les interventions ex-ante qui tiennent les premiers rôles. Cela semble indiquer que les travaux existants sous-estiment le rôle des interventions ex-ante dans la réglementation interbancaire.

Sujets : Stabilité financière; Réglementation et politiques relatives au système financier
Codes JEL : G01, G20

Non-Technical Summary

Recent years have seen renewed interest in interbank regulation. However, a review of the literature in this area identifies two potentially important gaps. The first is that previous studies have tended to take the form of the interbank contract as given, rather than allowing banks to choose this contract optimally; since the famous Lucas critique, it's well known that this can make it difficult to generate convincing policy prescriptions. The second gap is that the literature to date has focused mostly on ex-post policies, which kick in after an interbank disruption is under way. In contrast, relatively little attention has been paid to policies that kick in at an ex-ante stage, including the new prudential policies now coming on line as part of the Basel III regulatory framework.

In this paper, I take steps toward addressing these gaps. More specifically, I build a simple model for the interbank market, which endogenizes the form of the interbank contract as part of an optimal contracting problem. The model's bare bones come from a canonical model of liquidity developed by Bengt Holmstrom and Jean Tirole (1998). There are three periods: $t = 0,1,2$. At $t = 0$, banks collect deposits and allocate funds between a riskless storage technology and a risky investment technology. At $t = 1$, they experience aggregate and bank-specific shocks, conditional on which they choose between maintaining or liquidating their investments. Maintained investments then mature at $t = 2$, though payouts are subject to a moral-hazard problem.

Several key results emerge from this framework. On the positive front, I show that the model delivers episodes that qualitatively resemble the interbank disruptions witnessed during the financial crisis. More specifically, if banks' initial balance sheets at $t = 0$ are sufficiently levered and illiquid, then the interaction between the information asymmetry and moral-hazard problem gives rise to episodes during which the distribution of resources at $t = 1$ is distorted along two margins. The first is that interbank debtors are unable to maintain their investments at full scale, despite the fact that some storage is left sitting idle on other banks' balance sheets; this helps to rationalize banks' accumulation of excess reserves and other liquid assets during the crisis. The second margin is that the episodes in question also have the property that the set of interbank debtors is polluted by a range of very low-productivity types, which helps to rationalize evidence of heightened counterparty fears during the crisis.

On the normative front, I expand the model to allow for the possibility of socially wasteful fire sales and explore the role for policy which then arises. More specifically, I show that banks tend to liquidate too heavily at $t = 1$, relative to the social optimum. I also show that the initial balance sheets on which banks settle at $t = 0$ are too "fragile" in a precise sense. Correcting these two tendencies requires careful coordination between an ex-post intervention at $t = 1$ and an ex-ante intervention at $t = 0$. I show that the former intervention admits a natural interpretation as a monetary stimulus, while the latter can be interpreted as a liquidity coverage ratio, coupled with a limit on the leverage that banks are able to take on in the deposit market. Crucially, it turns out that the ex-ante component does most of the heavy lifting—in fact, ex-ante interventions are always necessary and *sometimes even sufficient* for implementing the social optimum. Ex-ante interventions thus emerge as a qualitatively more important part of the overall policy mix, suggesting that previous literature may have underestimated the role that they play in optimal interbank regulation.

1 Introduction

Recent years have seen renewed interest in the regulation of interbank markets. This interest stems mainly from the events of the 2007–09 financial crisis, during which many interbank markets experienced especially severe contractions.[1] However, it's more generally understood that interbank markets play a key role in liquidity allocation, so it's natural to worry that disruptions in these markets could have spillovers into the larger economy. Indeed, some of the linkages between interbank markets and real economic outcomes have now been documented in Iyer et al. (2014).

A review of the current literature on interbank markets and their optimal regulation identifies two potentially important gaps. The first is that previous studies have tended to take the form of the interbank contract as given, rather than allowing banks to choose this contract optimally. Since the famous Lucas critique (1976), it's well known that this can make it difficult to generate convincing policy prescriptions. In fact, the problem is especially acute in the case of interbank markets, since banks' relative sophistication makes it all the more likely that they will adjust the institutional arrangements surrounding these markets in response to policy changes. As a result, progress in interbank regulation hinges in part on our developing models in which the form of the interbank contract emerges endogenously.

The second gap is that the literature to date has focused mostly on ex-post policies that kick in after an interbank disruption has come under way (e.g., open-market operations, lender-of-last-resort interventions, bailouts). In contrast, relatively little attention has been paid to policies that kick in at an ex-ante stage, including the new prudential policies now coming online as part of Basel III.[2] Since interbank markets represent a major source of liquidity for banks, this omission is especially problematic in the case of prudential policies that constrain banks' liquidity-management practices, such as the Basel III liquidity coverage ratio, which places a lower bound on banks' liquid asset holdings. A full understanding of how these policies should be designed and implemented thus requires some anticipation of their potential impact on interbank markets.

In this paper, I take steps toward addressing these gaps. More specifically,

[1] Among others, see section 2.3 in Bech and Monnet (2013) for details on the drop in overnight interbank volumes in the US, UK, Europe, and Australia.

[2] For an overview of Basel III, see Basel Committee on Banking Supervision (2010, 2013).

I build a simple model for the interbank market that endogenizes the form of the interbank contract as part of an optimal contracting problem. I then show that the model can account for interbank disruptions like those witnessed during the financial crisis and explore its predictions on the circumstances under which the economy becomes vulnerable to these disruptions, along with its implications for optimal policy design.

To be more specific about my model, its bare bones come from a canonical model of liquidity, namely Holmström and Tirole (1998), which I herein denote "HT". There are three periods: $t \in \{0, 1, 2\}$. At $t = 0$, banks collect deposits and allocate funds between a riskless storage technology and a risky investment technology. At $t = 1$, they experience aggregate and idiosyncratic shocks, conditional on which they choose between maintaining or liquidating their investments. Maintained investments then mature at $t = 2$, though payouts are subject to limited pledgeability.

I depart from HT in two respects. The first has to do with my treatment of depositors. In HT and the literature following it, banks are able to negotiate state-contingent contracts with their depositors. It's thus implicitly assumed that banks are able to revisit their depositors at $t = 1$ if shocks necessitate their raising additional funds. Though fruitful from a modelling perspective, this assumption is difficult to reconcile with one of the stronger stylized facts emerging from the empirical banking literature, namely that banks' deposit liabilities are highly inertial.[3] I therefore introduce a form of limited participation in the spirit of Allen and Gale (1994), which precludes depositors' acting as a source of funds at $t = 1$. In particular, I assume that depositors lack the sophistication needed to sign and enforce state-contingent contracts, and instead suppose that these contracts are only available when banks negotiate with other banks. As a result, banks' maintenance plans at $t = 1$ must be financed using some combination of storage, liquidations, and interbank lending.

My second departure from HT is that I introduce some information asymmetry into the economy. More specifically, I assume that the idiosyncratic shocks arriving at $t = 1$ convey private information about the productivity

[3] The sluggishness in banks' deposit liabilities has been noted by Feldman and Schmidt (2001), Dinger and Craig (2014), and Hahm et al. (2012), among others. This is due in part to deposit insurance and geographic limits, but also due to switching costs, as documented by Sharpe (1997), Kim et al. (2003), and Hannah and Roberts (2011). See also Billett and Garfinkel (2004), Song and Thakor (2007), and Huang and Ratnovski (2011), among others.

of banks' investments. This assumption is naturally motivated by experience during the financial crisis, when uncertainty on the location of sub-prime risk inside the financial system was a first-order issue.

Together, these ingredients give rise to several key results. On the positive front, I show that the model delivers episodes that qualitatively resemble the interbank disruptions witnessed during the financial crisis (proposition 3.1 and the discussion thereafter). More specifically, if the balance sheets that banks select at $t = 0$ exhibit sufficiently high deposits and sufficiently low storage, then the interaction between the information asymmetry and limited pledgeability gives rise to episodes during which the distribution of resources at $t = 1$ is distorted along two distinct margins. The first is that interbank debtors are unable to maintain their investments at full scale, despite the fact that some storage is left sitting idle on other banks' balance sheets. This helps to rationalize banks' accumulation of excess reserves and other liquid assets during the crisis, as documented by Heider et al. (2009), Ashcraft et al. (2011), Ennis and Wolman (2012), and Acharya and Merrouche (2012), among others; it also resonates with statements from policymakers suggesting that they did not "trust" interbank markets to allocate liquidity to its most urgent uses.[4] The second margin is that the episodes in question also have the property that the set of interbank debtors is polluted by a range of very low-productivity types, which helps to rationalize evidence of heightened counterparty fears during the crisis, including a widening of the LIBOR-OIS spread,[5] along with complementary evidence from Afonso et al. (2011) and Benos et al. (2014).[6,7] Going forward, I refer to these margins as the *intensive*

[4] For example, during hearings before the UK House of Commons Treasury Committee on September 20, 2007, the Chancellor of the Exchequer eschewed the use of broad open market operations as a potential solution for the Northern Rock crisis, namely in favour of a more targeted liquidity injection: "[Northern Rock] told the [Monetary Policy] Committee they have had to borrow about £13 or £14 billion from the Bank. To get that sort of money into the hands of one institution you would have to put many more billions of pounds into the market generally." In a post-mortem, the Treasury Committee concluded that "[i]t is most unlikely that any such lending operation in September...could have been of a sufficient scale to ensure that Northern Rock could have received the liquidity it then required." See House of Commons Treasury Committee (2008).

[5] See Taylor and Williams (2009) for details on the widening of the LIBOR-OIS spread.

[6] Broadly speaking, Afonso et al. (2011) find evidence that the amount and terms of lending in the federal funds market became more sensitive to borrowing banks' risk profiles following the Lehman Brothers bankruptcy.

[7] The Benos et al. (2014) data come from CHAPS, the UK's large-value payment sys-

and *extensive* margins, respectively.

My next positive result concerns comparative statics at $t = 0$. Since the episodes described in my previous paragraph arise only when banks select relatively fragile balance sheets at $t = 0$, it's natural to ask about the circumstances under which banks would be willing to make such choices ex ante. On this front, I first show that banks are more likely to select balance sheets of this sort when expected productivity at $t = 0$ is high, making the risk of interbank disruptions procyclical (proposition 4.1). This finding complements a growing literature highlighting the tendency for endogenous risk to accumulate inside the financial system during good times (e.g., Borio and Drehmann, 2009; Schularick and Taylor, 2012). I also show that interbank disruptions are more likely to occur when banks are poorly capitalized (proposition 4.2), which complements an extensive literature on the potentially stabilizing effects of bank equity (e.g., He and Krishnamurthy, 2012; Bigio, 2014).

On the normative front, I first report a strong "no-go" result. More specifically, I find that the simple model described above actually admits no role for policy: when interbank disruptions occur, they are constrained-efficient (proposition 4.3). Broadly speaking, this constrained efficiency arises because the wider interbank contract space creates greater potential for coordination among banks, relative to previous literature. Though the model is too stylized to interpret this finding as a literal prescription that policymakers should abstain from intervention in the interbank market, it does serve as a warning that interbank disruptions of the sort witnessed during the financial crisis do not necessarily count as evidence of a pathology; policies that aim to eliminate them from the economy may constitute a bridge too far.

That said, the aforementioned "no-go" result is in part a consequence of a simplifying assumption that banks' return from liquidating their investments is exogenous. This raises natural questions about the role for policy that might emerge if we expanded the model to allow for fire-sale externalities of the sort highlighted in, e.g., Lorenzoni (2008), Stein (2012), Korinek (2012), and Gersbach and Rochet (2012a,b). The remainder of the paper focuses on answering this question.

In the context of this expanded model, I first show that banks tend to

tem. Following the Lehman Brothers bankruptcy, they show that banks were more likely to delay payments due to higher-risk counterparties and argue that this behaviour was motivated by a desire for insulation against intraday defaults.

liquidate too heavily at $t = 1$, relative to the allocation preferred by a utilitarian planner, and also that the initial balance sheets on which banks settle at $t = 0$ are too "fragile" in a sense that I make precise in the sequel. Correcting these two tendencies jointly requires careful coordination between an ex-post intervention at $t = 1$ and an ex-ante intervention at $t = 0$. More specifically, I find that the appropriate ex-post intervention admits a natural interpretation as a monetary stimulus aiming to reduce the effective rate at which banks store goods between $t = 1$ and $t = 2$, thus discouraging them from liquidating their investments in favour of storage (proposition 6.1). In contrast, two instruments are needed at $t = 0$, since this period presents banks with distinct choices on the size and composition of their initial balance sheets. I show that an appropriately specified liquidity coverage ratio will do the trick if supplemented with a limit on the leverage that banks are able to take on in the deposit market (propositions 7.1 and 7.2). Crucially, it turns out that the ex-ante component does most of the heavy lifting. More specifically, I show that the aforementioned ex-ante interventions are always necessary and sometimes even sufficient for implementation of a utilitarian planner's solution (also propositions 7.1 and 7.2).

The mechanism underlying this surprising sufficiency has to do with the extensive and intensive margins mentioned earlier. In an unregulated equilibrium, I find that these two margins obey a kind of pecking order: the interbank market initially responds to the distortions described above by equilibrating along the extensive margin, allowing more and more low-productivity types into the set of interbank debtors; however, if the productivity of the marginal debtor falls beneath some threshold, then the market begins equilibrating along the intensive margin, reducing the scale at which interbank debtors are allowed to keep their investments running. When solving the planner's problem, I find that he obeys a similar pecking order. However, the threshold type around which he switches is strictly lower, since he understands that letting more types into the set of interbank debtors has the added benefit of reducing the volume of liquidations in the secondary market, thus taking some pressure off the price set therein. As a result, ex-post interventions are only needed in the case of interbank disruptions so severe that both margins are active, since the planner must then take steps to regulate the trade-off between these two margins. However, in the case of a less severe disruption during which the intensive margin remains dormant, banks' behaviour at $t = 1$ mechanically coincides with the planner's. The latter case thus has the property that the role for policy is confined to $t = 0$, while the former case—if anything—leaves the need for ex-ante intervention enhanced by moral-hazard concerns. For these reasons, ex-ante interventions emerge as a qualitatively more important part of the overall policy mix.

Related literature. In terms of related literature, the literature on interbank markets and their optimal regulation is too vast to do it much justice here. Some recent contributions include Acharya et al. (2012), who consider an environment in which interbank inefficiencies stem from an assumption that banks have some market power, which leaves liquidity-rich banks with an incentive to engage in predatory hoarding so as to force liquidity-poor banks into fire sales. They show that an appropriate lender-of-last-resort policy can eliminate this incentive by providing liquidity-poor banks with a better bargaining position. On the other hand, Allen et al. (2009) show how constraints on the interbank contract space can give rise to excessive volatility in the interbank interest rate, which the central bank can correct using state-contingent open-market operations. Similar results hold in related work by Freixas et al. (2011). Bruche and Suarez (2010) show how deposit insurance can give rise to distortions in an interbank market prone to counterparty risk. They argue that guarantees or subsidies for interbank lending can be used to correct these distortions, while Heider et al. (2009) identify a similar intervention as a potential solution for inefficiencies stemming from asymmetric information between banks, along with a few alternative policies.

Since all these examples make ad hoc assumptions on the interbank contract space and focus mostly on ex-post interventions in the interbank market, this paper's main contributions have to do with its emphasis on optimal contracts and ex-ante interventions. To some extent, Bhattacharya and Gale (1986) and Bhattacharya and Fulghieri (1994) share the former emphasis, since they take a mechanism-design approach to the interbank market, though their models fail to deliver episodes of the sort described above. On the other hand, the aforementioned work by Freixas et al. (2011) shares some of my emphasis on ex-ante intervention, since their normative analysis includes a role for the policy rate between periods $t = 0$ and $t = 1$. The same can be said of related work by Kharroubi and Vidon (2009). However, these frameworks have the property that an appropriate choice on the rate between periods $t = 0$ and $t = 1$ need not be supplemented with any kind of prudential policies, leaving open the questions raised above concerning the potential usefulness of these policies and the task of coordinating them with the other parts of the policy framework. Moreover, the particular inefficiency on which Kharroubi and Vidon (2009) focus has to do with a coordination failure that leads banks to abstain from storage at $t = 0$, implying that there's no liquidity for the interbank market to allocate at $t = 1$. In contrast, I focus on episodes during which liquidity exists inside the banking system but fails to reach its most productive use.

In addition to the literature on interbank regulation, my findings also connect with three parts of the wider post-crisis policy literature. First of all, my finding that the policy rate has a key role to play in ensuring financial stability reinforces an emerging view that financial stability and price stability cannot cleanly be separated as policy objectives (e.g., Stein, 2012, 2013; Brunnermeier and Sannikov, 2012, 2013, 2015). Secondly, my characterization of the optimal coordination between ex-post and ex-ante interventions extends a growing sub-literature on the relative merits of these two forms of intervention (e.g., Keister, 2010; Fahri and Tirole, 2012; Jeanne and Korinek, 2013; Stravrakeva, 2013; Chari and Kehoe, 2013). In particular, since this sub-literature has tended to view the main task for ex-ante intervention as the correction of the moral hazards arising from expectations of ex-post intervention, this paper provides a potentially important counterexample in which ex-ante interventions sometimes suffice to ensure constrained efficiency. Finally, my findings also connect with the current debate on introducing liquidity-based rules into the prudential toolkit, an area where regulatory practice is currently far out ahead of theory. More specifically, since a liquidity coverage ratio emerges as a key part of my implementation of the planner's solution, the framework developed herein can help to rationalize policies of this sort while providing some insight into how they might fit into the overall policy infrastructure.

Road map. The remainder of the paper is organized as follows. Section 2 describes the model, which I solve in sections 3 and 4. Section 5 then expands the model to include a fire-sale externality, and sections 6 and 7 explore the role for policy to which this externality gives rise. Section 8 concludes.

2 Model

2.1 Environment

There are three periods, $t \in \{0, 1, 2\}$, and a single homogeneous good. The economy contains a unit measure of islands, each populated by a single bank and a single household. Banks receive an endowment E^b at $t = 0$ and aim to maximize expected consumption at $t = 2$. Households receive an endowment E^h at $t = 0$ and aim to maximize

$$\mu(c_0) + \mathbb{E}(c_2),$$

where c_t denotes consumption at time t, with $\mu'(\cdot) > 0 > \mu''(\cdot)$, and $\mu'(E^h) = 1$.

Technologies. Banks have access to two technologies: a storage technology and an investment technology. The storage technology allows them to store goods at a one-to-one rate between periods. On the other hand, the investment technology is similar to that in HT. It's outlined in Figure 1.

The details on the investment technology are as follows. At $t = 0$, banks make some initial investment $I_0 \geq 0$. This investment will eventually mature at $t = 2$, when banks either *succeed*, meaning that they receive a positive payout, or *fail*, meaning that their payout is zero. However, some of the uncertainty regarding terminal payouts is resolved at $t = 1$. More specifically, at the beginning of $t = 1$, nature reveals two pieces of information: a bank-specific fundamental θ, and an aggregate state ω. The bank-specific fundamental θ is private and gives the probability with which a particular bank will succeed at $t = 2$. It's distributed over $[0, 1]$ on an i.i.d. basis, namely with some cumulative F admitting some positive density f.[8] On the other hand, the aggregate state is public and can be interpreted as an aggregate productivity shock. More specifically, it determines the payout that successful banks receive at $t = 2$. It can take one of two values, $\omega \in \{B, G\}$, where $\omega = G$ denotes a *good* state in which payouts are relatively high, while $\omega = B$ denotes a *bad* state in which payouts are relatively low. The former occurs with probability $\alpha_G \in (1/2, 1)$, while the latter occurs with probability $\alpha_B = 1 - \alpha_G$.

[8] That is, $f(\theta) > 0 \ \forall \theta \in [0, 1]$. It would also suffice if $f(\theta) > 0 \ \forall \theta \in (0, 1)$, so long as $\lim_{\theta \searrow 0} \{F(\theta)/f(\theta)\} = 0$.

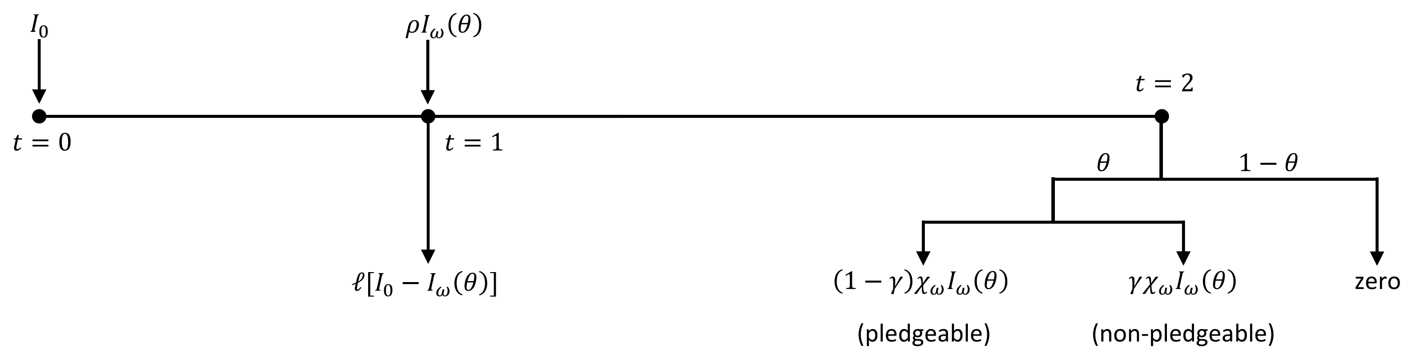

Figure 1: Banks' investment technology

After observing the pair (θ, ω), banks must choose how much of their initial investments they wish to keep running: $I_\omega(\theta) \in [0, I_0]$. As in HT, continuation entails some maintenance cost ρ—more specifically, banks must reinvest ρ goods for each unit of investment that they wish to keep on line. Any units of investment on which they opt to forego maintenance must be liquidated. Liquidation in state ω generates some small per-unit payout $\ell \in [0, 1)$.

After banks make their maintenance decisions, we move on to $t = 2$, when banks learn if they've succeeded or failed. These outcomes are independently distributed across banks, and the identity of the economy's successful banks is public. Successful banks receive some payout χ_ω per unit of investment maintained at $t = 1$, with $\chi_G > \chi_B$. However, as in HT, this payout cannot be pledged in its entirety to outsiders—rather, contracting frictions oblige successful banks to retain some fraction $\gamma \in (0, 1)$. The particular frictions underlying this requirement are not important for the sequel, so I remain agnostic over the various microfoundations that the literature offers, including shirking, absconding, and so forth. See Holmström and Tirole (2010) for a few examples.

Markets. It's natural to allow for two markets in this economy: a market for deposits and a market for interbank claims.

The details on the market for deposits are as follows. At the beginning of $t = 0$, banks post contracts under which local households can make deposits, namely on a take-it-or-leave-it basis. However, households suffer from a form of limited participation in spirit of Allen and Gale (1994), and this constrains the form of the deposit contracts that they can accept. Broadly speaking, the idea is that households are relatively unsophisticated and thus unable to use state-contingent contracts. More specifically, I assume that contracts cannot be made contingent on information revealed at $t \in \{1, 2\}$. As a result, deposit contracts can be summarized by a pair (D, R). The interpretation is that households make an initial deposit D at $t = 0$ and then receive interest and principal DR at $t = 2$ on a non-contingent basis. Though stark, this approach precludes banks' quickly adjusting their deposits in response to new information and is thus consistent with the inertia observed in banks' real-world deposit liabilities, as discussed in my introduction.

On the other hand, the details on the interbank market are as follows. At the beginning of $t = 0$, banks have an opportunity to meet with one another so as to negotiate mutual insurance contracts in anticipation of the idiosyncratic shock θ. In contrast with my approach to the market for deposits, I eschew any restriction on the form that these contracts might take and instead allow for state-contingent contracts; the idea is that banks' relative sophistication makes it possible for them to use these contracts when dealing with other banks. The contracting problem that then arises is the subject of my next subsection.

2.2 Banks' optimal contracting problem

With state-contingent contracts in the interbank market, we can think of an interbank contract as an agreement within a group of banks about how its members will behave going forward, much as in Bhattacharya and Gale (1986) and Bhattacharya and Fulghieri (1994). More precisely, we can think of an interbank contract as an object of the form

$$\mathcal{C} := \left[(D, R), I_0, \{S_\omega(\theta), I_\omega(\theta), T_{\omega f}(\theta), \Delta T_\omega(\theta)\}_{(\theta,\omega)\in[0,1]\times\{B,G\}}\right],$$

where the interpretation is as follows:

- at $t = 0$, the contract specifies the offer (D, R) that members should make to local households, along with the way that their funds should be distributed between initial investment I_0 and storage $E + D - I_0$;
- at $t = 1$, the contract specifies the portfolio of storage and investment $[S_\omega(\theta), I_\omega(\theta)]$ to which members should adjust, conditional on the pair (θ, ω);
- finally, at $t = 2$, the contract specifies the transfer $T_{\omega f}(\theta)$ that failed banks should send back to their interbank creditors, along with the additional transfer $\Delta T_\omega(\theta)$ that should be extracted from successful banks.

See Figure 2 for a visual summary.

Now, given that banks are homogeneous at the time that negotiations take place, these negotiations should settle on a contract that maximizes the average bank's expected profits—that is, banks choose $\mathcal{C}$ so as to maximize

$$\sum\nolimits_{\omega\in\{B,G\}} \alpha_\omega \int_0^1 [S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)]\, dF(\theta) - RD.$$

Of course, several constraints attend this optimization. The first is that contracts must induce banks to reveal their draws on the fundamental θ—that is,

$$S_\omega(\theta) - T_{\omega f}(\theta) - RD + \theta\left[\chi_\omega I_\omega(\theta) - \Delta T_\omega(\theta)\right]$$

$$\geq S_\omega(\theta') - T_{\omega f}(\theta') - RD + \theta\left[\chi_\omega I_\omega(\theta') - \Delta T_\omega(\theta')\right],$$

$$\forall (\theta, \theta', \omega) \in [0,1]^2 \times \{B, G\}, \quad \text{(TT)}$$

where the label (TT) stands for "truth telling". The second constraint has to do with limited pledgeability:

$$\Delta T_\omega(\theta) \leq (1-\gamma)\chi_\omega I_\omega(\theta), \;\; \forall(\theta,\omega) \in [0,1] \times \{B, G\}. \qquad \text{(LP)}$$

The next constraint is the individual rationality constraint for depositors:

$$RD = \mu(E^h) - \mu(E^h - D) =: \Delta\mu(D). \qquad \text{(IR)}$$

The remaining constraints then have to do with physical feasibility. For example:

$$\left(E^b + D - I_0\right) + \ell I_0 = \int_0^1 \left[S_\omega(\theta) + (\rho + \ell)\, I_\omega(\theta)\right] dF(\theta), \;\; \forall \omega \in \{B, G\} \qquad \text{(F1a)}$$

$$0 = \int_0^1 \left[T_{\omega f}(\theta) + \theta \Delta T_\omega(\theta)\right] dF(\theta), \;\; \forall \omega \in \{B, G\} \quad \text{(F2a)}$$

Here (F1a) says that the banks entering into the contract $\mathcal{C}$ must carry enough liquidity into $t = 1$ to cover the adjustments to which they've committed, while (F2a) says that the transfers that they make among themselves at $t = 2$

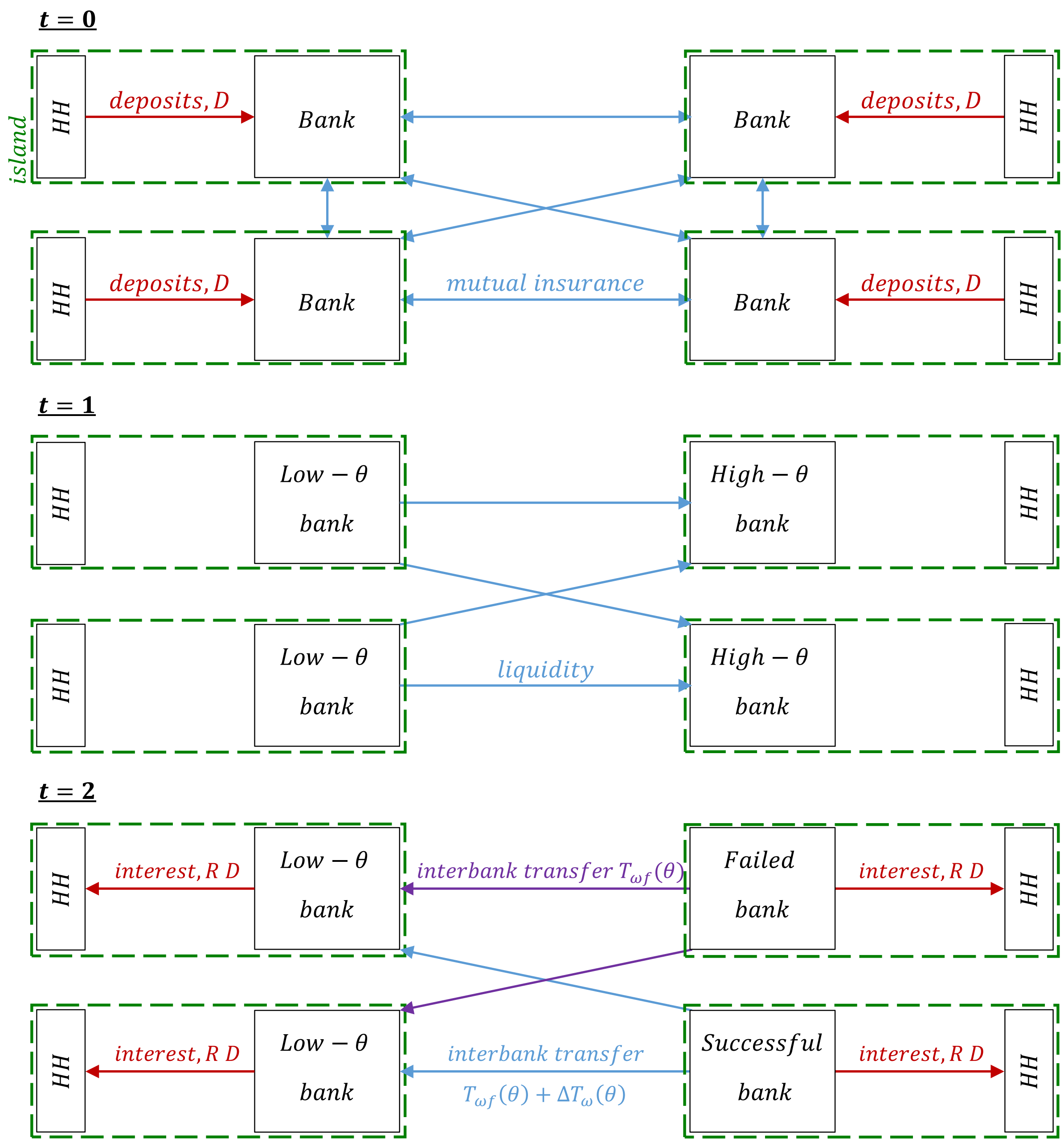

Figure 2: Interbank contract

must net to zero. Finally:

$$(D, I_0) \in \left[0, E^h\right] \times \left[0, E^b + D\right] \tag{F0}$$

$$S_\omega(\theta) \geq 0, \;\; \forall(\theta, \omega) \in [0,1] \times \{B, G\} \tag{F1b}$$

$$S_\omega(\theta) \geq T_{\omega f}(\theta) + RD, \;\; \forall(\theta, \omega) \in [0,1] \times \{B, G\} \tag{F2b}$$

$$S_\omega(\theta) + \chi_\omega I_\omega(\theta) \geq T_{\omega f}(\theta) + \Delta T_\omega(\theta) + RD, \;\; \forall(\theta, \omega) \in [0,1] \times \{B, G\} \tag{F2c}$$

$$I_\omega(\theta) \in [0, I_0], \;\; \forall(\theta, \omega) \in [0,1] \times \{B, G\} \tag{F1c}$$

$$I_\omega(\theta) = 0 \implies \Delta T_\omega(\theta) = 0, \;\; \forall(\theta, \omega) \in [0,1] \times \{B, G\} \tag{F2d}$$

Here (F0) through (F2c) are non-negativity constraints, while (F1c) reminds us that banks' investments can't be scaled up at $t = 1$. (F2d) reminds us that we can't condition on success or failure in the case of banks whose investments are fully liquidated at $t = 1$.

2.3 Parametric assumptions

I close this section with my parametric assumptions. The first three read as follows:

Assumption 2.1. $\sum_{\omega \in \{B,G\}} \alpha_\omega \mathbb{E}\left[\max\left\{\ell, \theta\chi_\omega - \rho\right\}\right] > 1$.

Assumption 2.2. $\chi_B > \rho + \ell$.

Here assumption 2.1 ensures that it's profitable for banks to engage in some investment at $t = 0$, while assumption 2.2 ensures that continuation is profitable for some types in both states.

Next, it will be useful to impose the following restriction on the form of the distribution from which banks draw their types:

Assumption 2.3. The cumulative function F is strictly log-concave.[9]

[9] That is, $\frac{d^2 \log F(\theta)}{d\theta^2} < 0$, $\forall\theta \in [0, 1]$.

This assumption is relatively common in models with mechanism-design components and holds for a wide range of distributions over $[0, 1]$, including the uniform, truncated (log-)normal, and truncated exponential cases.[10,11] It will also be useful to impose a lower bound on the productivity differential across states:

Assumption 2.4. The bad state is "sufficiently bad," namely in the sense that the payout χ_B satisfies an upper bound given in a technical appendix.[12] Conversely, the good state is "sufficiently good," namely in the sense that the payout χ_G satisfies an analogous lower bound.

Finally, I follow HT in assuming that households are deep-pocketed:

Assumption 2.5. The household endowment E^h is relatively large in comparison with the endowment E^b received by banks—specifically, $E^b + E^h - \Delta\mu(E^h) < 0$.

3 Solution at $t = 1$

I'll now begin solving the model and characterizing its key properties. Due to the model's relative simplicity, this amounts to solving the optimal contracting problem outlined in subsection 2.2. On this front, the game plan is as follows. The present section focuses on banks' behaviour at $t = 1$, taking as given the initial balance sheets selected at $t = 0$, as summarized by the initial investment I_0 and the deposit contract (D, R). More specifically, subsection 3.1 shows that banks' optimal contracting problem admits an intuitive reformulation, which subsection 3.2 then exploits to solve for banks'

[10] See Mohtashami Borzadaran and Mohtashami Borzadaran (2011) for a relatively comprehensive list of examples.

[11] At the risk of being a bit pedantic, I note that there is a distinction between log-concavity of a given *cumulative function* and log-concavity of the underlying *distribution*, since the latter refers to a situation in which the *density function* is log-concave. The latter assumption is a bit more common in mechanism design, but also constitutes a stronger condition, since any log-concave density function is known to admit a log-concave cumulative function, though the converse is untrue. (See theorem one in Bagnoli and Bergstrom, 2005.) An important example of the distinction in question would be the (truncated) log-normal distribution, which is not log-concave but still admits a log-concave cumulative function.

[12] Available at `http://tjcarter.weebly.com/technical-appendix.html`.

behaviour in the bad state. Subsection 3.2 repeats for the good state. In section 4, I'll then step back to $t = 0$ and focus on endogenizing banks' initial balance-sheet choices.

3.1 Reformulation of banks' contracting problem

Lemma 3.1. *Fix some initial balance sheet (D, I_0, R), and define a* subcontract

$$\mathcal{C}_\omega := \{S_\omega(\theta), I_\omega(\theta), T_{\omega f}(\theta), \Delta T_\omega(\theta)\}_{\theta\in[0,1]},$$

which collects all those terms in the interbank contract obtaining in state ω. Banks' optimal choice on this subcontract can be summarized by two numbers: a threshold fundamental $\theta_\omega \in [0,1]$, and an investment scale $I_\omega \in [0, I_0]$. The interpretation is that (almost) all types in $[\theta_\omega, 1]$ maintain exactly I_ω units of investments, whereas all types in $[0, \theta_\omega)$ liquidate completely—that is,

$$I_\omega(\theta) = \begin{cases} I_\omega & \text{for (almost) all } \theta \in [\theta_\omega, 1] \\ 0 & \text{for all } \theta \in [0, \theta_\omega). \end{cases}$$

More specifically, banks choose $(\theta_\omega, I_\omega)$ so as to maximize the surplus generated by their maintained investments,

$$I_\omega \int_{\theta_\omega}^{1} (\theta\chi_\omega - \rho - \ell)\, dF(\theta) =: I_\omega \Pi_\omega(\theta_\omega),$$

subject to two constraints. The first is a physical constraint *stipulating that banks must carry enough liquidity into $t = 1$ to cover the maintenance plans to which they've committed:*

$$(E^b + D - I_0) + \ell I_0 \geq \int_{\theta_\omega}^{1} (\rho + \ell)\, I_\omega dF(\theta) =: I_\omega \Psi_\omega(\theta_\omega). \tag{PC$_\omega$}$$

The second is a financial constraint *on which I'll elaborate in a moment:*

$$(E^b + D - I_0) + \ell I_0 \geq RD - I_\omega \left[\Pi_\omega(\theta_\omega) - \theta_\omega \gamma \chi_\omega F(\theta_\omega) - \int_{\theta_\omega}^{1} \theta\gamma\chi_\omega dF(\theta)\right]. \tag{FC$_\omega$}$$

Proof. All proofs are contained in the technical appendix. ■

Here the financial constraint (FC_ω) captures the combined effects of the economy's information and pledgeability frictions. Its interpretation hinges on our recognizing the term

$$I_\omega \theta_\omega \gamma \chi_\omega F(\theta_\omega)$$

as the total present value that must be promised to the types in $[0, \theta_\omega)$, whom we can think of as *interbank creditors*; were this promise any smaller, creditors would be tempted to mimic the types in $[\theta_\omega, 1]$, whom we can think of as *interbank debtors*. Similarly, the term

$$I_\omega \int_{\theta_\omega}^{1} \theta \gamma \chi_\omega dF(\theta)$$

gives the total present value that must be retained by interbank debtors due to limited pledgeability. Therefore, the term

$$I_\omega \left[\Pi_\omega(\theta_\omega) - \theta_\omega \gamma \chi_\omega F(\theta_\omega) - \int_{\theta_\omega}^{1} \theta \gamma \chi_\omega dF(\theta) \right] =: I_\omega \Delta_\omega(\theta_\omega)$$

gives the portion of total surplus that can be pledged to depositors, which I'll herein refer to as *pledgeable surplus*. The financial constraint can then be interpreted as a statement about the way that liquidity can help to "grease the wheels" at $t = 1$. In particular, a buffer of liquidity is needed to fill any gap between the promise to depositors, RD, and pledgeable surplus, $I_\omega \Delta_\omega(\theta_\omega)$—that is,

$$(\text{FC}_\omega) \iff (E^b + D - I_0) + \ell I_0 \geq RD - I_\omega \Delta_\omega(\theta_\omega).$$

In light of these findings, we now see that banks' optimal contracting problem can be reformulated as a choice over

$$\left[(D, R), I_0, \{\theta_\omega, I_\omega\}_{\omega \in \{B,G\}} \right],$$

where the goal is to maximize

$$\sum\nolimits_{\omega \in \{B,G\}} \alpha_\omega \left[(E^b + D - I_0) + \ell I_0 + I_\omega \Pi_\omega(\theta_\omega) - RD \right],$$

subject only to the physical and financial constraints described above, along with (IR), (F0), and the requirement that $(\theta_\omega, I_\omega) \in [0,1] \times [0, I_0]$ $\forall \omega \in \{B, G\}$. Denote this program ($\mathbb{P}$).

Now, it's natural to make certain conjectures on the form of the solution that this program takes. In particular, it's natural to conjecture that the physical constraint is lax in the bad state, though the financial constraint associated with this state might bind. The intuition is that the bad state is one in which relatively few types draw fundamentals that justify maintenance, so the physical constraint should be relatively easy to satisfy. At the same time, all those types whose fundamentals warrant liquidation must be bribed to reveal themselves, so satisfying the financial constraint may be difficult. Similar reasoning leads to a conjecture that the financial constraint should be lax in the good state, though the physical constraint associated with this state might bind. I also conjecture that the non-negativity constraint (F0) should be lax. Let ($\mathbb{P}$-rex) denote the relaxed program implied by these three conjectures. In the sequel, I'll focus on solving this relaxed program before verifying my conjectures in section 4.

3.2 Details on the bad state

Under program ($\mathbb{P}$-rex), the constraint that banks have to worry about in the bad state is the financial constraint. To get a sense for how this constraint might distort their behaviour, I've used Figure 3 to plot the per-unit surplus function $\Pi_B(\theta_B)$, along with the per-unit pledgeable surplus function $\Delta_B(\theta_B)$. The former is in dashed red, while the latter is in solid blue.

These functions have several key properties, all of which can easily be verified. The first is that the per-unit surplus function naturally peaks around the type for whom maintenance is NPV-neutral, $\theta_B^\Pi := (\rho + \ell)/\chi_B$. This means that banks would prefer a subcontract of the form $(\theta_B, I_B) = \big(\theta_B^\Pi, I_0\big)$, all else equal, namely because this subcontract keeps all NPV-positive types operating at full scale.

On the other hand, the per-unit pledgeable surplus function peaks around a lower type, $\theta_B^\Delta < \theta_B^\Pi$. The intuition for this lower type is that a reduction in the threshold θ_B lowers the bribe that must be paid to inframarginal creditors so as to discourage their mimicking debtors. As a result, allowing some NPV-negative types into the set of interbank debtors may nonetheless increase pledgeable surplus so long as their expected losses are offset by the corresponding savings on the aforementioned bribe—indeed, θ_B^Δ has the property that these two effects offset each other exactly:

$$\underbrace{\left(\rho+\ell-\chi_B\theta_B^\Delta\right)}_{\substack{\text{expected losses}\\ \text{for type } \theta_B^\Delta}} f(\theta_B^\Delta) = \underbrace{\gamma\chi_B F(\theta_B^\Delta)}_{\substack{\text{savings on}\\ \text{the bribe to}\\ \text{inframarginal}\\ \text{creditors}}}$$

It will also be useful to note that the per-unit pledgeable surplus function is strictly negative over $[0,1]$, namely due to the scarcity of pledgeable income in the bad state (assumption 2.4). On the other hand, per-unit surplus either exhibits single-crossing from below over the interval $[0,1)$, namely at some type $\underline{\theta}_B^\Pi < \theta_B^\Pi$, or otherwise is positive over all of this interval, in which case I adopt a convention that $\underline{\theta}_B^\Pi = 0$.

With these points in mind, we have a few cases to consider, depending on banks' initial balance sheets. Suppose first that initial balance sheets are relatively liquid, namely in the sense

$$(E^b + D - I_0) + \ell I_0 \geq RD - I_0\Delta_B(\theta_B^\Pi). \tag{1}$$

In this case, banks will be able to implement the unconstrained optimum $(\theta_B, I_B) = \left(\theta_B^\Pi, I_0\right)$, as described above. Next, suppose that initial balance sheets instead exhibit relatively low liquidity and high leverage, namely in the sense that

$$(E^b + D - I_0) + \ell I_0 < RD. \tag{2}$$

Since the per-unit pledgeable surplus function is always negative, this situation has the property that there's no choice on the subcontract (θ_B, I_B) that balances the financial constraint. As a result, banks would be forced to default on their deposits. Since banks are obliged to keep their deposits non-contingent, we can conclude that initial balance sheets in this range cannot be selected at $t=0$. In effect, the problem is that debt-overhang vis-à-vis depositors prohibits banks' maintaining any investments at $t=1$; at the same time, the proceeds from liquidating these investments are too low to cover the promise to depositors.

Of course, the bounds in (1) and (2) are not mutually exclusive. The most interesting case arises when initial balance sheets exhibit moderate liquidity and moderate leverage, namely in the sense that

$$(E^b + D - I_0) + \ell I_0 \in \left[RD, RD - I_0\Delta_B(\theta_B^\Pi)\right).$$

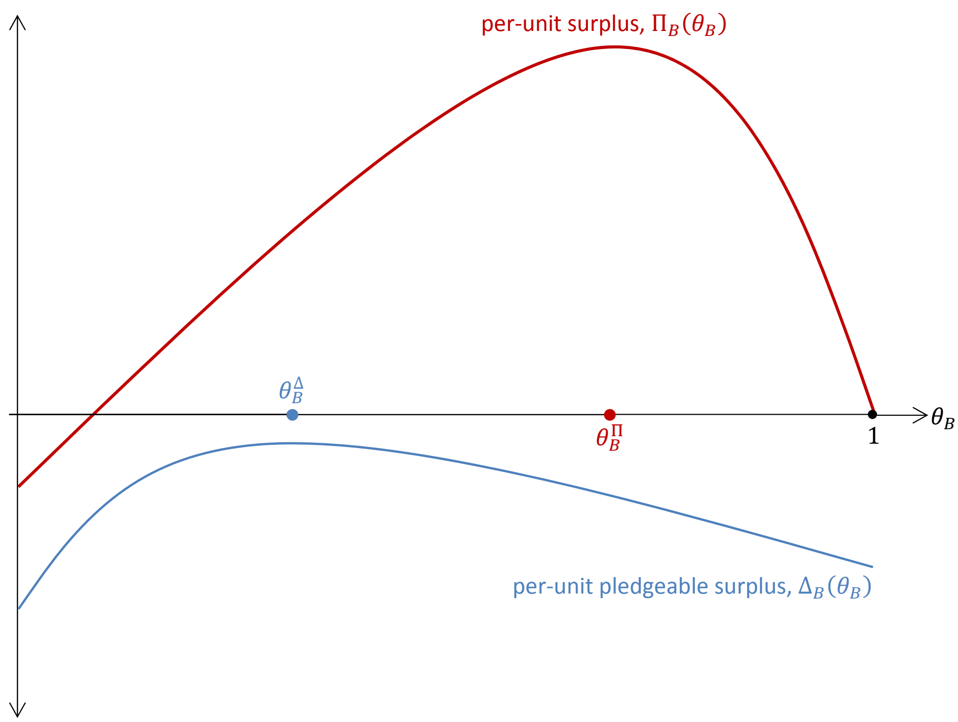

Figure 3: Per-unit surplus functions

For initial balance sheets in this range, banks can't implement their unconstrained optimum and instead adjust the subcontract (θ_B, I_B) so as to rebalance the financial constraint. Now, from Figure 3, we see that there are two options for how banks might go about rebalancing the financial constraint. The first is to reduce the threshold θ_B to some point in $\left[\theta_B^\Delta, \theta_B^\Pi\right)$. The alternative is to reduce the investment scale I_B. For brevity, I'll respectively refer to these two options as the *extensive* and *intensive margins*. Both margins have their drawbacks: reliance on the intensive margin distorts the distribution of liquidity across the set of interbank debtors—in particular, types near the top of the interval $[\theta_B, 1]$ receive relatively too little liquidity, whereas types near the bottom receive relatively too much; on the other hand, reliance on the extensive margin introduces relatively unprofitable types into the set of debtors.

When banks trade off between these margins, it turns out that they obey a strict pecking order. More specifically, it can be shown that the interval $\left(\max\left\{\underline{\theta}_B^\Pi, \theta_B^\Delta\right\}, \theta_B^\Pi\right)$ admits a critical type θ_B^Ξ, with the special property that banks prefer to rely on the extensive margin until they reach a point at which further reliance would require that they let θ_B^Ξ into the set of interbank debtors. At this point, banks prefer to revert to the intensive margin rather than allow any more inferior types into the set. The intuition for this pecking order should be relatively clear from Figure 3: types near θ_B^Π are very close to breaking even in NPV terms, so the costs associated with the extensive margin are second-order in this neighbourhood; however, as we lean on the extensive margin, each successive type that we let into the set of debtors generates less pledgeable surplus at the cost of greater expected losses.

To get a bit more precise about this pecking order, I note that the rate of transformation along the financial constraint is given by

$$(dI_B/d\theta_B)\,\Delta_B(\theta_B) + I_B\Delta_B'(\theta_B) = 0 \iff dI_B/d\theta_B = (-1)I_B\Delta_B'(\theta_B)/\Delta_B(\theta_B), \tag{3}$$

so banks prefer the extensive margin whenever

$$(dI_B/d\theta_B)\,\Pi_B(\theta_B) + I_B\Pi_B'(\theta_B) = I_B \underbrace{\left[\Pi_B'(\theta_B) - \frac{\Pi_B(\theta_B)\Delta_B'(\theta_B)}{\Delta_B(\theta_B)}\right]}_{*} \leq 0.$$

Now, it should be clear that the starred term is strictly negative at $\theta_B = \theta_B^\Pi$ but strictly positive at $\theta_B = \max\left\{\underline{\theta}_B^\Pi, \theta_B^\Delta\right\}$. A sufficient condition for the pecking order in question is then that the starred term also exhibit single-crossing, which can easily be verified.

To summarize, the situation is as follows:

Proposition 3.1 (banks' behaviour in the bad state)**.** *Under program* ($\mathbb{P}$-rex)*, the subcontract* (θ_B, I_B) *exhibits the following dependence on banks' initial balance sheets:*

- *if initial balance sheets exhibit low leverage and high liquidity, namely in the sense that*

$$(E^b + D - I_0) + \ell I_0 \geq RD - I_0 \Delta_B(\theta_B^\Pi),$$

  then $(\theta_B, I_B) = \big(\theta_B^\Pi, I_0\big)$*. Since all NPV-positive types thus receive enough liquidity to keep operating at full scale, I'll refer to this situation as one in which banks experience a* liquidity surplus*;*

- *if initial balance sheets exhibit moderate leverage and moderate liquidity, namely in the sense that*

$$(E^b + D - I_0) + \ell I_0 \in \big[RD - I_0 \Delta_B(\theta_B^\Xi), RD - I_0 \Delta_B(\theta_B^\Pi)\big) ,$$

  then banks rely only on the extensive margin—that is, $I_B = I_0$*, with* θ_B *set s.t. the financial constraint holds with equality. I'll thus refer to this situation as one in which banks experience an* extensive distortion*;*

- *if initial balance sheets exhibit high leverage and low liquidity, namely in the sense that*

$$(E^b + D - I_0) + \ell I_0 \in \big[RD, RD - I_0 \Delta_B(\theta_B^\Xi)\big) ,$$

  then banks rely on both margins—specifically, $\theta_B = \theta_B^\Xi$*, with* I_B *set s.t. the financial constraint holds with equality. I'll thus refer to this situation as one in which banks experience a* dual distortion*;*

- *finally, if initial balance sheets exhibit very high leverage and very low liquidity, namely in the sense that*

$$(E^b + D - I_0) + \ell I_0 < RD,$$

  then the subcontract (θ_B, I_B) *cannot be chosen to satisfy the financial constraint.*

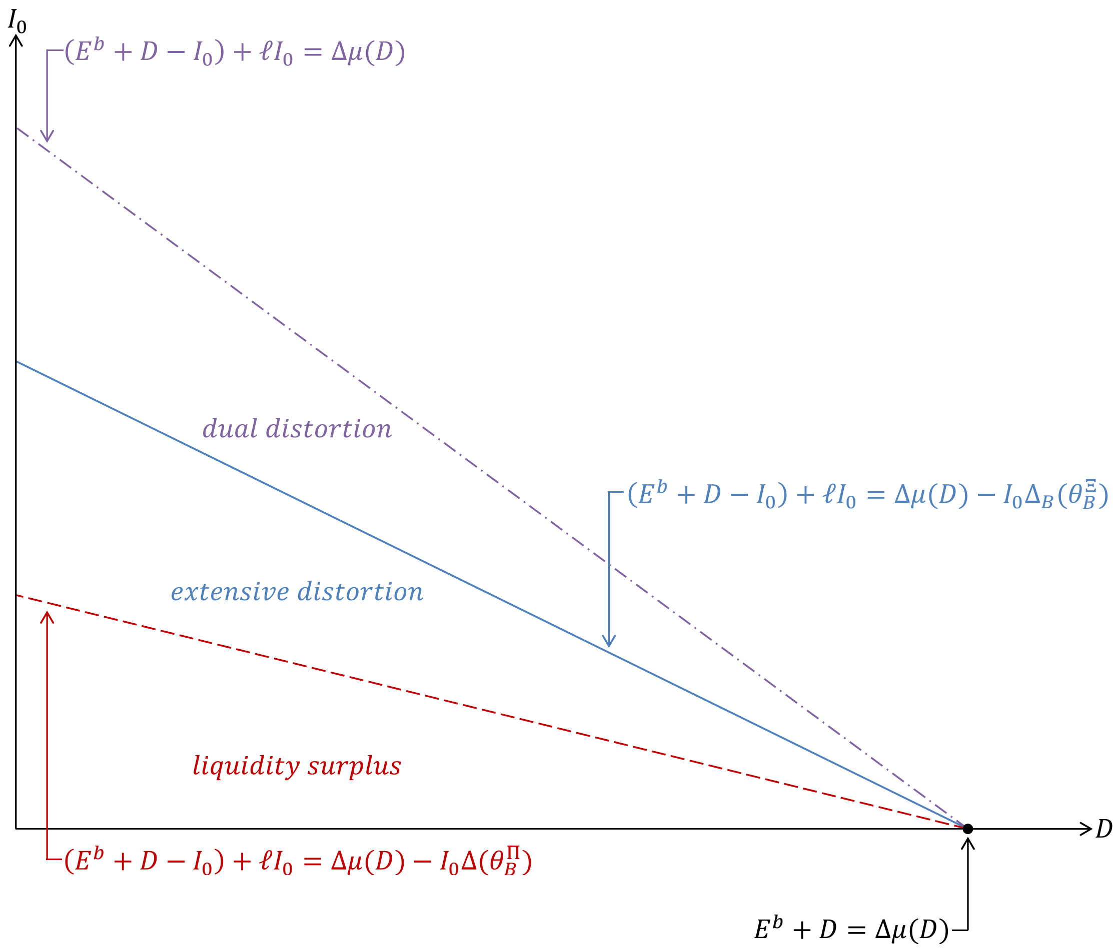

Figure 4: Banks' behaviour in the bad state as a function of their initial balance-sheet choices

See Figure 4 for an illustration. When constructing this figure, I've used the individual rationality constraint for depositors, (IR), to eliminate the interest rate R as choice variable, so the pair (D, I_0) will herein suffice as a summary of banks' initial balance-sheet choices.

Proposition 3.1 constitutes one of my main results, namely because the dual distortion regime exhibits three key properties suggestive of an interbank disruption like those witnessed during the financial crisis:

- the first such property is that an active intensive margin causes liquidity to be misallocated *across the set of interbank debtors*. In particular, as mentioned earlier, types near the top of the interval $[\theta_B, 1]$ receive relatively too little liquidity, while types near the bottom receive relatively too much;

- the second property is that liquidity is also being misallocated *between interbank debtors and interbank creditors*. More specifically, with the intensive margin active but the physical constraint lax, the dual distortion regime has the property that banks with strong fundamentals are unable to keep operating at full scale, despite the fact that some liquidity is still sitting idle inside the banking system. As mentioned in my introduction, this helps to rationalize banks' accumulation of excess reserves and other liquid assets during the crisis, as documented by Heider et al. (2009), Ashcraft et al. (2011), Ennis and Wolman (2012), Acharya and Merrouche (2012), and others;

- finally, because the extensive margin is active, the dual distortion regime also has the property that the set of interbank debtors is polluted by a subset of negative-NPV types. As mentioned in my introduction, this rationalizes reports of heightened counterparty fears during the crisis, as documented by Taylor and Williams (2009), Afonso et al. (2011), Benos et al. (2014), and others.

Now, before closing this subsection, it will be useful to record banks' value functions under each of the regimes above, namely as a first step toward eventually pinning down their initial balance-sheet choices:

- under the liquidity surplus regime, banks' payout is given by

$$v_B^{LS}(D, I_0) := (E^b + D - I_0) + \ell I_0 + I_0 \Pi_B\left(\theta_B^{\Pi}\right) - \Delta\mu(D);$$

- on the other hand, their payout under the extensive distortion regime is given by

$$v_B^{ED}(D, I_0) := (E^b + D - I_0) + \ell I_0 + I_0 \Pi_B \left[\theta_B^{ED}(D, I_0)\right] - \Delta\mu(D),$$

  where the threshold $\theta_B^{ED}(D, I_0)$ is chosen to satisfy the financial constraint—i.e.,

$$(E^b + D - I_0) + \ell I_0 = \Delta\mu(D) - I_0 \Delta_B \left[\theta_B^{ED}(D, I_0)\right]; \tag{4}$$

- finally, their payout under the dual distortion regime is given by

$$v_B^{DD}(D, I_0) := (E^b + D - I_0) + \ell I_0 + I_B^{DD}(D, I_0) \Pi_B \left(\theta_B^{\Xi}\right) - \Delta\mu(D),$$

  where the scale $I_B^{DD}(D, I_0)$ is chosen to satisfy the financial constraint—i.e.,

$$(E^b + D - I_0) + \ell I_0 = \Delta\mu(D) - I_B^{DD}(D, I_0) \Delta_B \left(\theta_B^{\Xi}\right). \tag{5}$$

3.3 Details on the good state

I now turn my attention to the good state, in which ($\mathbb{P}$-rex) has the property that the constraint banks have to worry about is now the physical constraint, rather than the financial constraint. As a result, their behaviour in this state is relatively mechanical. Suppose, for example, that initial balance sheets exhibit high liquidity, namely in the sense that

$$(E^b + D - I_0) + \ell I_0 \geq I_0 \Psi_G \left(\theta_G^{\Pi}\right).$$

In this case, banks are able to implement the unconstrained optimum described in my previous subsection, $(\theta_G, I_G) = \left(\theta_G^{\Pi}, I_0\right)$. I'll thus refer to this situation as one in which banks experience another *liquidity surplus*. If instead

$$(E^b + D - I_0) + \ell I_0 \in \left[0, I_0 \Psi_G \left(\theta_G^{\Pi}\right)\right),$$

then banks are forced to ration liquidity. With the financial constraint lax, they're able to do so in a way that sends each unit of liquidity to its best possible use, namely by setting $I_G = I_0$ while increasing θ_G until the physical constraint binds. I'll thus refer to this situation as one in which banks experience *liquidity rationing*. Finally, if both of the conditions above fail, then it should be clear that the subcontract (θ_G, I_G) cannot be chosen to balance the physical constraint. As a result, initial balance sheets in this last range cannot be selected at $t = 0$.

Now, much as in my previous subsection, it will be useful to record banks' value functions under each of the regimes above before finally shifting our attention back to $t = 0$:

- under the liquidity surplus regime, banks' payout is given by

$$v_G^{LS}(D, I_0) := (E^b + D - I_0) + \ell I_0 + I_0 \Pi_G \left(\theta_G^{\Pi}\right) - \Delta\mu(D);$$

- on the other hand, their payout under the liquidity rationing regime is given by

$$v_G^{LR}(D, I_0) := (E^b + D - I_0) + \ell I_0 + I_0 \Pi_G \left[\theta_G^{LR}(D, I_0)\right] - \Delta\mu(D),$$

  where the threshold $\theta_G^{LR}(D, I_0)$ is chosen to satisfy the physical constraint—i.e.,

$$(E^b + D - I_0) + \ell I_0 = I_0 \Psi_G \left[\theta_G^{LR}(D, I_0)\right]. \tag{6}$$

4 Solution at $t = 0$

We're now ready to solve for banks' initial balance-sheet choices under program ($\mathbb{P}$-rex). Based on the analysis in my last section, we know that there's a total of six cases for us to consider, since there are three possible regimes associated with the bad state, $r_B \in \{LS, ED, DD\}$, and two possible regimes associated with the good state, $r_G \in \{LS, LR\}$. See Figure 5 for a visual summary.

Fortunately, some cases can be ruled out almost immediately. Suppose, for example, that $(r_G, r_B) = (LS, DD)$. In this case, the marginal deposit generates return[13]

$$\alpha_G(v_G^{LS})_D(D,I_0)+\alpha_B(v_B^{DD})_D(D,I_0) = [1-\Delta\mu'(D)]-\alpha_B[1-\Delta\mu'(D)]\frac{\overbrace{\Pi_B(\theta_B^{\Xi})}^{>0}}{\underbrace{\Delta_B(\theta_B^{\Xi})}_{<0}} < 0.$$

The interpretation is relatively straightforward once we recognize that increasing D while holding I_0 constant amounts to taking the marginal deposit, raised at cost $\mu > 1$, and placing it in storage, where it only generates a unit return. In the good state, this deposit ends up sitting idle, since all NPV-positive projects are already operating at full scale. On the other hand, it contributes to a tighter financial constraint in the bad state. As a result, it's strictly optimal for banks to select out of this case in favour of one involving lower leverage. A similar argument can be used to rule out the case where $(r_G, r_B) = (LS, ED)$, under which

$$\alpha_G(v_G^{LS})_D(D,I_0)+\alpha_B(v_B^{ED})_D(D,I_0) = [1-\Delta\mu'(D)]-\alpha_B[1-\Delta\mu'(D)]\frac{\overbrace{\Pi_B'[\theta_B^{ED}(D,I_0)]}^{>0}}{\underbrace{\Delta_B'[\theta_B^{ED}(D,I_0)]}_{<0}} < 0.$$

The case where $(r_G, r_B) = (LS, LS)$ can also be ruled out, though the argument is a bit different. Under this case, we have

$$\alpha_G(v_G^{LS})_I(D,I_0) + \alpha_B(v_B^{LS})_I(D,I_0) = \sum\nolimits_{\omega\in\{B,G\}} \alpha_\omega[\ell_\omega + \Pi(\theta_\omega^{\Pi})] - 1 > 0,$$

where the inequality follows from the fact that investment is relatively profitable from an ex-ante perspective (assumption 2.1). In effect, with the financial constraint lax in both states, all of the surplus generated by the marginal

[13] To be clear, the derivation is as follows: (5) $\implies$ $(I_B^{DD})_D(D,I_0) = (-1)[1 - \Delta\mu'(D)]/\Delta_B(\theta_B^{\Xi})$, so

$$\alpha_G(v_G^{LS})_D(D,I_0) + \alpha_B(v_B^{DD})_D(D,I_0) = \alpha_G[1-\Delta\mu'(D)] + \alpha_B[1-\Delta\mu'(D) + (I_B^{DD})_D(D,I_0)\Pi_B(\theta_B^{\Xi})]$$

$$= [1-\Delta\mu'(D)] - \alpha_B[1-\Delta\mu'(D)]\frac{\Pi_B(\theta_B^{\Xi})}{\Delta_B(\theta_B^{\Xi})}.$$

All of this section's marginal returns can be derived in similar ways.

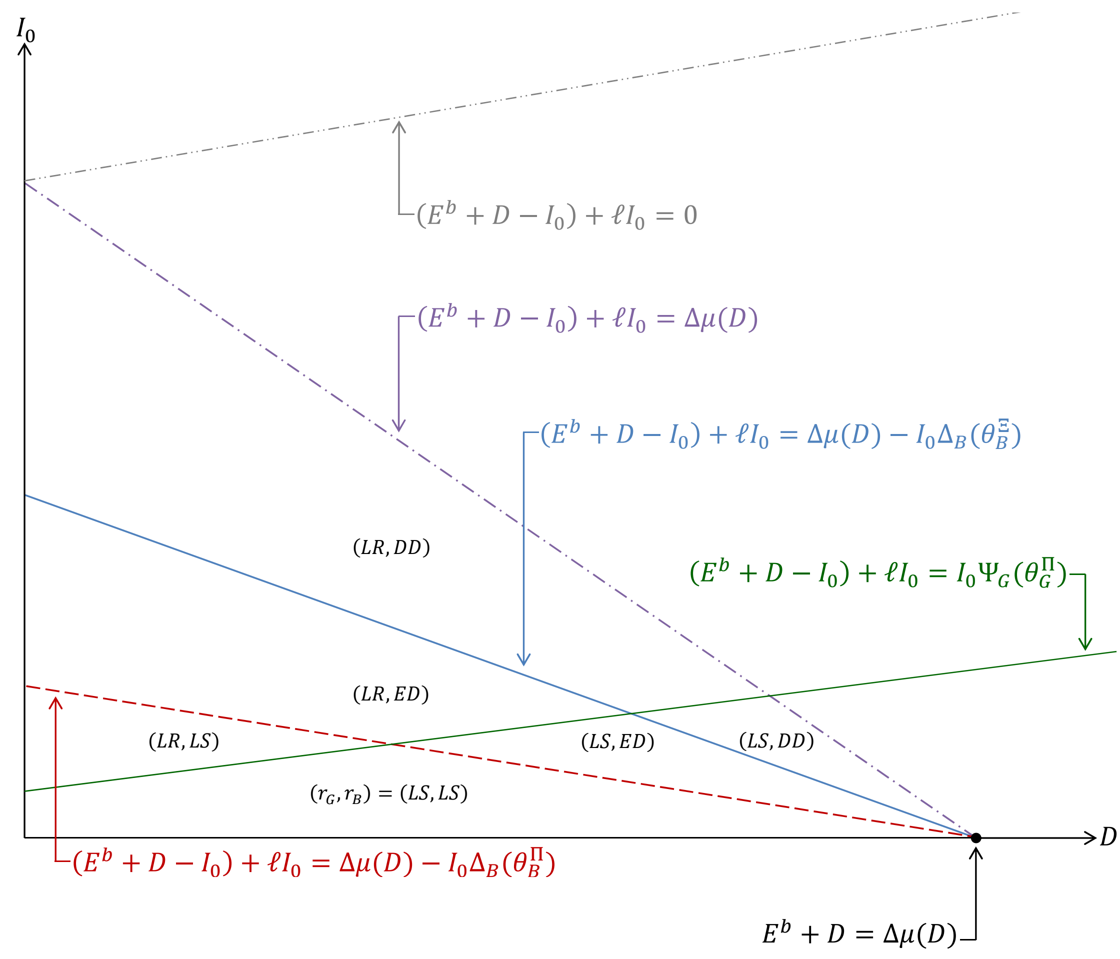

Figure 5: Banks' behaviour in both states as a function of their initial balance-sheet choices

unit of investment accrues directly to banks, rather than depositors, leaving banks with a strict incentive to select out this case in favour of one involving higher investment.

So, we can now restrict attention to initial balance sheets under which the good state is associated with liquidity rationing—that is,

$$(E^b + D - I_0) + \ell I_0 < I_0 \Psi_G(\theta_G^\Pi),$$

but

$$(E^b + D - I_0) + \ell I_0 \geq \Delta\mu(D). \tag{7}$$

For balance sheets in this range, marginal returns are given by

$$\alpha_G(v_G^{LR})_D(D, I_0) + \alpha_B(v_B^{r_B})_D(D, I_0)$$

$$= \alpha_G \left[1 - \Delta\mu'(D) + \frac{\Pi'_G[\theta_G^{LR}(D, I_0)]}{\Psi'_G[\theta_G^{LR}(D, I_0)]}\right]$$

$$+ \alpha_B \left\{ \begin{array}{ll} 1 - \Delta\mu'(D) & \text{if } r_B = LS \\ [1 - \Delta\mu'(D)] \left[1 - \dfrac{\Pi'_B[\theta_B^{ED}(D, I_0)]}{\Delta'_B[\theta_B^{ED}(D, I_0)]}\right] & \\ & \text{if } r_B = ED \\ [1 - \Delta\mu'(D)] \left[1 - \dfrac{\Pi_B(\theta_B^\Xi)}{\Delta_B(\theta_B^\Xi)}\right] & \text{if } r_B = DD, \end{array} \right\}$$

and

$$\alpha_G(v_G^{LR})_I(D, I_0) + \alpha_B(v_B^{r_B})_I(D, I_0)$$

$$= \alpha_G \left[\begin{array}{l} \ell + \Pi_G[\theta_G^{LR}(D, I_0)] - 1 \\ \quad - \left[1 - \ell + \Psi_G[\theta_G^{LR}(D, I_0)]\right] \dfrac{\Pi'_G\left[\theta_G^{LR}(D, I_0)\right]}{\Psi'_G\left[\theta_G^{LR}(D, I_0)\right]} \end{array} \right]$$

$$+\,\alpha_B \left\{ \begin{array}{ll} \ell + \Pi_B(\theta_B^{\Pi}) - 1 & \text{if } r_B = LS \\[2ex] \ell + \Pi_B[\theta_B^{ED}(D, I_0)] - 1 & \\[2ex] \quad + \left[1 - \ell - \Delta_B[\theta^{ED}(D, I_0)]\right] \dfrac{\Pi_B'[\theta^{ED}(D, I_0)]}{\Delta_B'[\theta^{ED}(D, I_0)]} & \\[2ex] & \text{if } r_B = ED \\[2ex] (-1)(1-\ell)\left[1 - \dfrac{\Pi_B(\theta_B^{\Xi})}{\Delta_B(\theta_B^{\Xi})}\right] & \text{if } r_B = DD. \end{array} \right\}$$

Now, the envelope theorem ensures that these expressions are continuous in (D, I_0), even around the boundaries separating the various regimes associated with the bad state of the world. As a result, solutions for program ($\mathbb{P}$-rex) must take one of three distinct forms. The first is an interior solution under which both of the expressions above reach zero. The second in principle would be a corner solution under which $D = 0$, but this possibility is precluded by the fact that depositors' outside options satisfy $\mu'(E^h) = 1$, meaning that they lack a good use to which they can put the last unit of their endowments.

The third and final possibility is a corner solution under which the "no-default" constraint on line (7) binds—that is, banks select initial balance sheets so levered and illiquid that they're barely able to pay their depositors in the bad state. Under a solution of this form, banks anticipate that they won't collect any profits in the bad state and instead focus exclusively on the good state, namely by choosing (D, I_0) to satisfy the first-order condition

$$(v_G^{LR})_I(D, I_0) + \underbrace{\left[\frac{1-\ell}{1-\Delta\mu'(D)}\right]}_{*}(v_G^{LR})_D(D, I_0) = 0, \tag{8}$$

where the starred term gives the rate of transformation along the "no-default" constraint. Since this solution has the property that no interbank transfers occur, I'll refer to it as a situation in which the interbank market *collapses*.

To summarize:

Lemma 4.1. *Solutions for* ($\mathbb{P}$*-rex*) *must have the property that banks experience liquidity rationing in the good state. As for their behaviour in the bad* *state, there are two possible cases: one under which the "no-default" constraint is lax, with*

$$\alpha_G(v_G^{LR})_x(D, I_0) + \alpha_B(v_B^{r_B})_x(D, I_0) = 0, \;\; \forall x \in \{D, I_0\},$$

and another under which the "no-default" constraint binds, with (8) *holding.*

Moreover:

Lemma 4.2. *A solution of the aforementioned form exists, is unique, and generalizes to the more constrained program* ($\mathbb{P}$)*.*

In light of these two lemmata, it's now natural to ask about the conditions under which the model's solution takes a corner form, along with the regime that this solution associates with the bad state of the world in non-corner cases. For now, I focus on how the answers to these questions vary over the business cycle, namely by taking comparative statics with respect to two parameters governing the productivity of banks' investment technology: the payout that successful projects generate in the good state, χ_G, and the probability on this state, α_G. Now, intuitively speaking, we should expect that banks are only comfortable exposing themselves to some risk of distortion in the bad state if expected productivity at $t = 0$ is relatively high. Indeed:

Proposition 4.1 (procyclical risk in the interbank market)**.** *Fix all parameters save for the payout* χ_G *and let* $\underline{\chi}_G$ *denote the lower bound on this payout at which my parametric assumptions begin to fail. The range of potential values for this payout then admits a partition* $\underline{\chi}_G \leq \chi_G^{LS} \leq \chi_G^{ED} \leq \chi_G^{DD} \leq \infty$ *such that the solution for program* ($\mathbb{P}$) *exhibits the following properties:*

- *if* $\chi_G \in (\underline{\chi}_G, \chi_G^{LS}]$*, then banks experience a liquidity surplus in the bad state;*
- *if* $\chi_G \in (\chi_G^{LS}, \chi_G^{ED}]$*, then banks experience an extensive distortion in the bad state;*
- *if* $\chi_G \in (\chi_G^{ED}, \chi_G^{DD}]$*, then banks experience a dual distortion in the bad state, but the "no-default" constraint remains lax;*
- *if* $\chi_G \in (\chi_G^{DD}, \infty)$*, then an interbank collapse occurs in the bad state.*

Similar results obtain if the parameter being varied is instead the probability on the good state, α_G*.*

See Figure 6 for an illustration.

Here proposition 4.1 implies that the risk of interbank distortions is procyclical. It also associates higher levels of expected productivity at $t = 0$ with qualitatively more severe distortions, namely as these distortions spread from the extensive margin to the intensive margin and eventually give rise to a full interbank collapse. These findings complement a growing literature on endogenous risk inside the financial system, and more specifically reinforce the view that financial crises represent "booms gone bad," as argued by Borio and Drehmann (2009), Schularick and Taylor (2012), and others.

Derivations very similar to those underlying proposition 4.1 also allow us to characterize the economy's response to changes in banks' net worth:

Proposition 4.2 (stabilizing effect of bank equity)**.** *Fix all parameters save for the endowment E^b received by banks, and let $\overline{E}$ denote the upper bound on this endowment implied by the requirement that households be deep-pocketed (assumption 2.5). The interval $[0, \overline{E}]$ then admits a partition $0 \leq E^{DD} \leq E^{ED} \leq E^{LS} \leq \overline{E}$ such that the solution for program* ($\mathbb{P}$) *exhibits the following properties:*

- *if $E^b \in [E^{LS}, \overline{E})$, then banks experience a liquidity surplus in the bad state;*
- *if $E^b \in [E^{ED}, E^{LS})$, then banks experience an extensive distortion in the bad state;*
- *if $E^b \in [E^{DD}, E^{ED})$, then banks experience a dual distortion in the bad state, but the "no-default" constraint remains lax;*
- *if $E^b \in (0, E^{DD})$, then an interbank collapse occurs in the bad state.*

See Figure 6 for an illustration. That low capitalization in the banking sector thus leaves the economy more vulnerable to interbank distortions complements an extensive literature on the potentially stabilizing benefits of bank equity, including recent work by He and Krishnamurthy (2012), Bigio (2014), and many others. Moreover, in the context of a more ambitious model that endogenized banks' endowments, lemma 4.2 could provide a rationale for capital injections of the sort witnessed during the financial crisis, though I leave this issue as a topic for future research.

Apart from these mainly positive findings, propositions 4.1 and 4.2 also have some important normative implications. On this front, I note that the

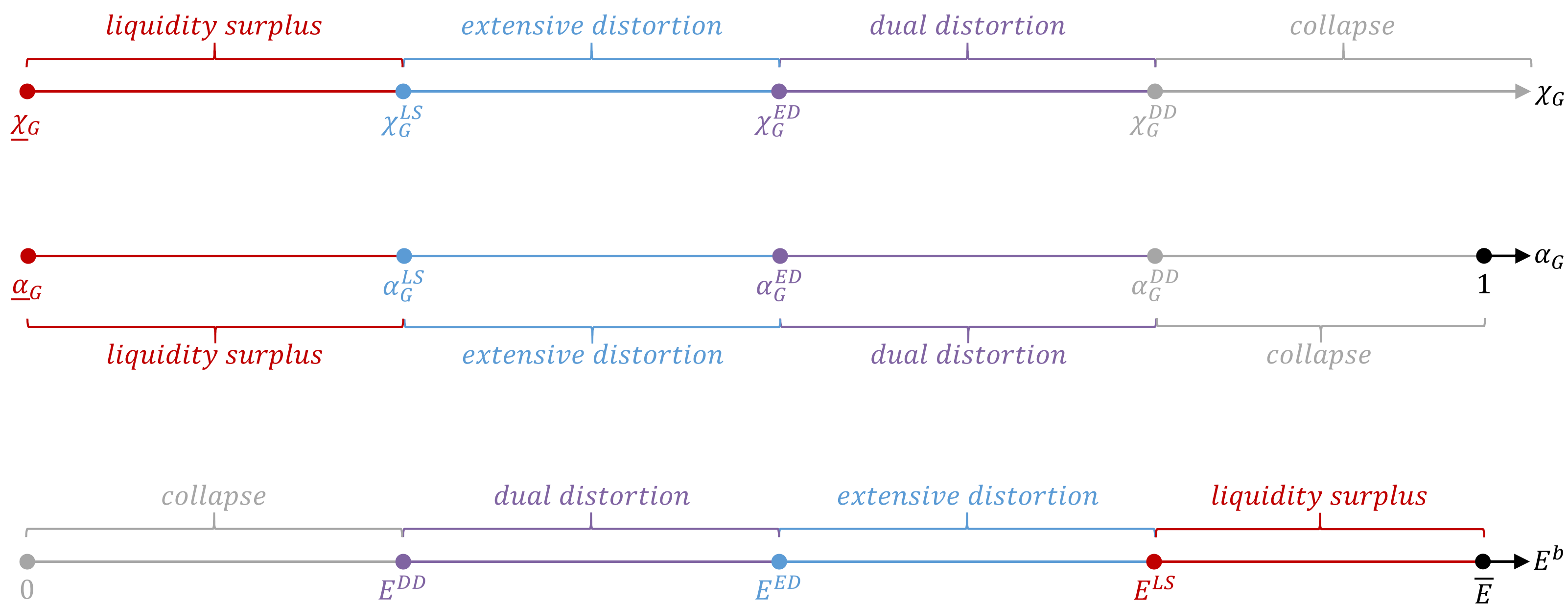

Figure 6: Banks' behaviour at $t = 0$ as a function of parameters

NB: $\underline{\alpha}_G$ denotes the lower bound on α_G implied by my parametric assumptions.

relatively simple model at hand has been constructed in a way which precludes constrained inefficiencies. In fact, it can easily be shown that banks' solution for program ($\mathbb{P}$) coincides with the contract on which a utilitarian planner would settle if he faced the same information and pledgeability frictions as do private agents. That is:

Proposition 4.3 ("no-go" for policy)**.** *The interbank contract $\mathcal{C}$ selected by banks also maximizes utilitarian welfare,*

$$\sum\nolimits_{\omega\in\{B,G\}} \alpha_\omega \int_0^1 [S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)]\, dF(\theta) + \mu(E^h - D),$$

subject to the truth-telling constraint, (TT)*; limited pledgeability constraint,* (LP)*; individual rationality constraint for depositors,* (IR)*; and feasibility constraints* (F0) *through* (F2d)*.*

Though propositions 4.1 and 4.2 imply that the parameter space includes regions in which the economy is vulnerable to interbank distortions, we can thus conclude that these distortions do not give rise to a need for some kind of policy intervention—neither at $t = 0$, nor at $t = 1$. Intuitively speaking, this constrained efficiency is a consequence of banks' having access to a larger contract space, which creates greater potential for coordination among banks. In light of this strong "no-go" result, it's now natural to ask about the role for policy that might emerge if we expanded the model to include a potential source of constrained inefficiency. I take up this question in my next section, namely by introducing a fire-sale externality into the economy.

5 An expanded model with fire sales

5.1 Changes to the economic environment

The structure of the expanded model is unchanged relative to the baseline model above, with the exception that the liquidation value ℓ that banks are able to extract from their unmaintained investments is now endogenous and state-specific. To endogenize this payout, I take an approach similar to that in Lorenzoni (2008). More specifically, I assume that unmaintained investments are useless to banks, but may still be useful to households, each of whom owns a firm operating in a "traditional sector," as distinct from the sector in which banks deploy their funds. At $t = 1$, these firms have access to a salvaging technology with which L_ω units of unmaintained investments can be converted back into $\Lambda(L_\omega)$ units of consumption, where $\Lambda'(\cdot) > 0 > \Lambda''(\cdot)$, with $\Lambda'(0) \leq 1$, and $\lim_{L \nearrow \infty}\{\Lambda'(L)\} = 0$. The proceeds are then remitted to households as a dividend. Under the assumption that unmaintained investments trade hands in a competitive secondary market, liquidation values are thus pinned down by the first-order condition

$$\ell_\omega = \Lambda'(L_\omega). \tag{9}$$

See Figure 7 for an illustration.

5.2 Definitions

In this environment, it will be useful for us to follow Freixas et al. (2011) in adopting a notion of "ex-ante" versus "ex-post" equilibrium, where the latter isolates banks' behaviour at $t = 1$ from their behaviour at $t = 0$. This will allow us to address the questions raised in my introduction concerning the relative merits of intervention at $t = 0$ versus $t = 1$.

Intuitively speaking, the relevant definitions are as follows. Given some state ω, along with some initial balance sheet (D, I_0, R), an ex-post equilibrium is a subcontract $\mathcal{C}_\omega$ and secondary-market price ℓ_ω such that (i) banks find it optimal to use the subcontract $\mathcal{C}_\omega$ in state ω, taking the price ℓ_ω as given; (ii) the subcontract $\mathcal{C}_\omega$ dictates liquidation decisions that cause the secondary market to clear at price ℓ_ω in state ω. More formally:

Definition. Given some state ω and an initial balance sheet (D, I_0, R) satisfying the participation constraint for depositors, (IR), an *ex-post equilibrium* is a subcontract $\mathcal{C}_\omega$ and a secondary-market price ℓ_ω such that the following two conditions hold:

- the subcontract $\mathcal{C}_\omega$ maximizes

$$\int_0^1 [S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)]\, dF(\theta),$$

taking ℓ_ω as given, subject to the following constraints:

$$S_\omega(\theta) - T_{\omega f}(\theta) - RD + \theta\left[\chi_\omega I_\omega(\theta) - \Delta T_\omega(\theta)\right]$$

$$\geq S_\omega(\theta') - T_{\omega f}(\theta') - RD + \theta\left[\chi_\omega I_\omega(\theta') - \Delta T_\omega(\theta')\right], \ \ \forall(\theta, \theta') \in [0, 1]^2 \tag{TT$_\omega$}$$

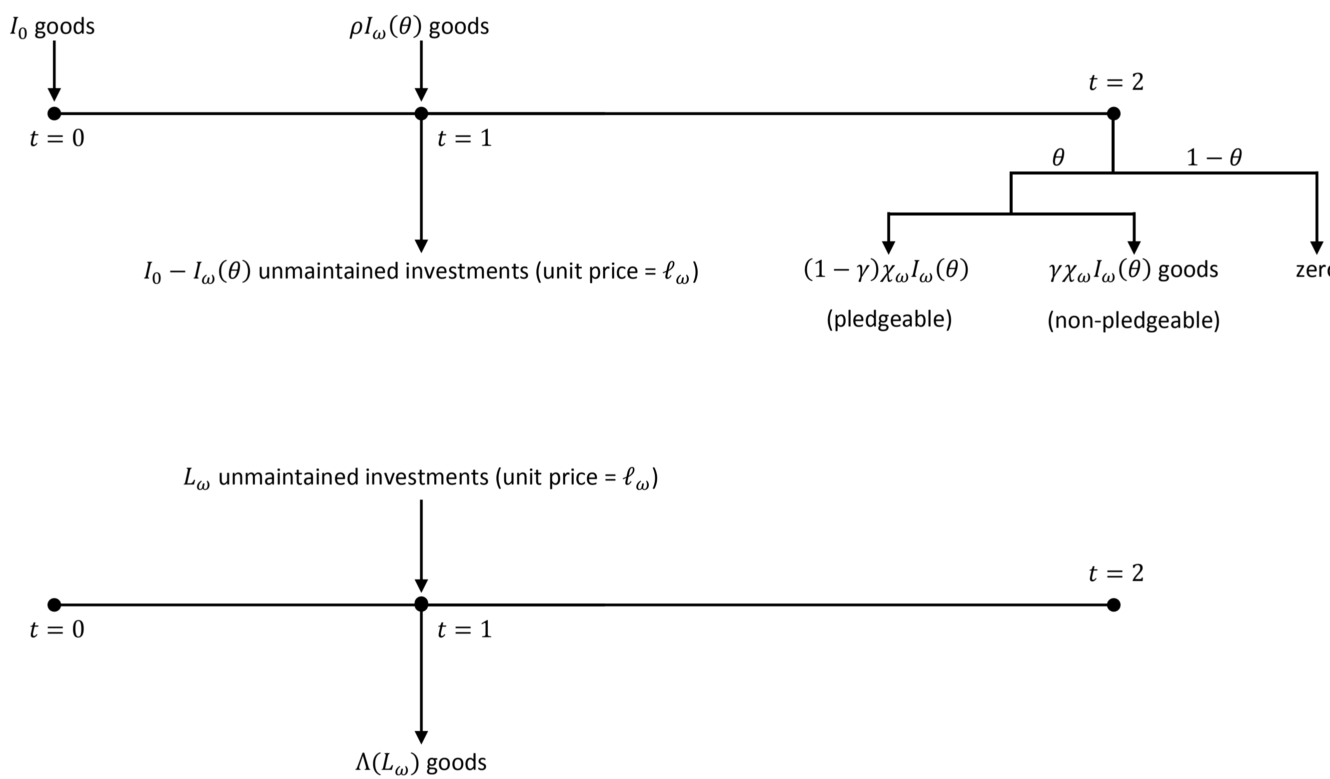

Figure 7: Banks' investment technology (top) and firms' salvaging technology (bottom) under the expanded model

$$\Delta T_\omega(\theta) \leq (1-\gamma)\chi_\omega I_\omega(\theta), \ \ \forall\theta \in [0,1] \tag{LP$_\omega$}$$

$$\left(E^b + D - I_0\right) + \ell_\omega I_0 = \int_0^1 \left[S_\omega(\theta) + \left(\rho + \ell_\omega\right) I_\omega(\theta)\right] dF(\theta) \tag{F1a$_\omega$}$$

$$0 = \int_0^1 \left[T_{\omega f}(\theta) + \theta\Delta T_\omega(\theta)\right] dF(\theta) \tag{F2a$_\omega$}$$

$$S_\omega(\theta) \geq 0, \ \ \forall\theta \in [0,1] \tag{F1b$_\omega$}$$

$$S_\omega(\theta) \geq T_{\omega f}(\theta) + RD, \ \ \forall\theta \in [0,1] \tag{F2b$_\omega$}$$

$$S_\omega(\theta) + \chi_\omega I_\omega(\theta) \geq T_{\omega f}(\theta) + \Delta T_\omega(\theta) + RD, \ \ \forall\theta \in [0,1] \tag{F2c$_\omega$}$$

$$I_\omega(\theta) \in \left[0, I_0\right], \ \ \forall\theta \in [0,1] \tag{F1c$_\omega$}$$

$$I_\omega(\theta) = 0 \implies \Delta T_\omega(\theta) = 0, \ \ \forall\theta \in [0,1] \tag{F2d$_\omega$}$$

- the secondary market clears—i.e., $\ell_\omega = \Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]]$.

In contrast:

Definition. An *ex-ante equilibrium* is a contract $\mathcal{C}$ and a schedule of secondary-market prices (ℓ_B, ℓ_G) such that

- the contract $\mathcal{C}$ maximizes

$$\sum\nolimits_{\omega \in \{B,G\}} \alpha_\omega \int_0^1 \left[S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)\right] dF(\theta) - RD,$$

  taking (ℓ_B, ℓ_G) as given, subject to (TT), (LP), (IR), (F0), (F1a$_B$), (F1a$_G$), and (F1b) through (F2d);

- the secondary market clears in both states—i.e., $\ell_\omega = \Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]]$, $\forall\omega \in \{B, G\}$.

Moreover, an ex-ante equilibrium is *monotonic* if it satisfies $\ell_G \geq \ell_B$—i.e., more liquidations occur in the bad state.

When evaluating the efficiency of a particular equilibrium, the benchmark on which I focus is one under which the social planner internalizes the price-setting process but otherwise faces the same information and contracting frictions as do banks. More specifically:

Definition. The *planner's ex-ante problem* involves choosing an interbank contract $\mathcal{C}$ and a price schedule (ℓ_B, ℓ_G) so as to maximize utilitarian welfare,

$$\sum\nolimits_{\omega\in\{B,G\}} \alpha_\omega \begin{bmatrix} \Lambda\left[I_0 - \int_0^1 I_\omega(\theta)dF(\theta)\right] - \ell_\omega\left[I_0 - \int_0^1 I_\omega(\theta)dF(\theta)\right] \\ + \int_0^1 \left[S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)\right] dF(\theta) + \mu(E^h - D) \end{bmatrix},$$

subject to the same constraints facing banks in ex-ante equilibrium, along with the market-clearing condition $\ell_\omega = \Lambda'[I_0 - I_\omega[1-F(\theta_\omega)]]$, $\forall\omega\in\{B,G\}$.

Similarly:

Definition. Given some state ω, along with some initial balance sheet (D, I_0, R) satisfying the participation constraint for depositors, (IR), the *planner's ex-post problem* involves choosing a subcontract $\mathcal{C}_\omega$ and price ℓ_ω so as to maximize

$$\Lambda\left[I_0 - \int_0^1 I_\omega(\theta)dF(\theta)\right] - \ell_\omega\left[I_0 - \int_0^1 I_\omega(\theta)dF(\theta)\right] + \int_0^1 \left[S_\omega(\theta) + \theta\chi_\omega I_\omega(\theta)\right] dF(\theta),$$

subject to the same constraints facing banks in ex-post equilibrium, along with the market-condition $\ell_\omega = \Lambda'[I_0 - I_\omega[1-F(\theta_\omega)]]$.

5.3 Parametric assumptions

I'll now close this section with my parametric assumptions. Most are inherited from the simpler model above, with adjustments ensuring that they now hold over the full range of potential liquidation values:

Assumption 5.1. Investment at $t = 0$ is profitable—i.e.,

$$\sum\nolimits_{\omega\in\{B,G\}} \alpha_\omega \mathbb{E}\left[\max\{0, \theta\chi_\omega - \rho\}\right] > 1.$$

Assumption 5.2. Continuation at $t = 1$ is always profitable for some types—i.e., $\chi_B > \rho + \Lambda'(0)$.

Assumption 5.3. The bad state is "sufficiently bad," namely in the sense that the payout χ_B satisfies an upper bound given in the technical appendix. Conversely, the good state is "sufficiently good," namely in the sense that the payout χ_G satisfies an analogous lower bound.

Assumption 5.4. Households are deep-pocketed in comparison with banks—specifically, $E^b + E^h - \Delta\mu(E^h) < 0$.

Apart from these assumptions, it will be useful to make some functional-form assumptions, beginning with the form of the salvaging technology:

Assumption 5.5. $\frac{d}{dL}\big[L\Lambda'(L)\big] > 0 > \frac{d^2}{dL^2}\big[L\Lambda'(L)\big]$, $\forall L \in \mathbb{R}_+$.

In light of firms' first-order condition, (9), we can read this assumption as a stipulation that the revenues raised in the secondary market should increase with the total volume of liquidations, but do so at a decreasing rate—e.g., $\Lambda(L) = \Lambda_0 \log(1 + L)$, with $\Lambda_0 \in (0, 1]$. It will also be useful to make a simplifying assumption on the distribution from which banks draw their types:

Assumption 5.6. The distribution F is standard uniform.[14]

6 Optimal policy at $t = 1$

In this section, I fix banks' initial balance-sheet choices and solve for the ex-post equilibria emerging in each state at $t = 1$, along with the planner's preferred ex-post allocations and the policy interventions needed to implement them. More specifically, subsection 6.1 focuses on the bad state, while subsection 6.2 repeats for the good state.

6.1 Details on the bad state

Since banks take prices as given, lemma 3.1 still holds as a description of their behaviour at $t = 1$. Their choices on the subcontract $\mathcal{C}_\omega$ can thus be

[14] To be clear, assumption 5.6 is stronger than necessary for all of my positive results and some of my normative results, most importantly including my implementation of the planner's solution. More specifically, lemmata 6.1 through 6.8 and proposition 6.1 all go through so long as the cumulative function $F(\cdot)$ is log-concave, while lemmata 7.1-7.2 and propositions 7.1-7.2 go through if we further assume that $f(1) \geq 1$, which holds for a wide range of distributions, including the uniform distribution and appropriate truncations of the (log-)normal distribution.

summarized by a pair $(\theta_\omega, I_\omega) \in [0,1] \times [0, I_0]$, with the usual interpretation that all types in $[0, \theta_\omega)$ liquidate completely, whereas (almost) all others keep operating at scale I_ω. More specifically, banks choose this pair to maximize the surplus generated by their maintained investments,

$$I_\omega \int_{\theta_\omega}^{1} (\theta\chi_\omega - \rho - \ell_\omega)\, dF(\theta) =: I_\omega \Pi_\omega(\theta_\omega, \ell_\omega),$$

subject to the usual physical and financial constraints:

$$(E^b + D - I_0) + \ell_\omega I_0 \geq \int_{\theta_\omega}^{1} (\rho + \ell_\omega)\, I_\omega dF(\theta) =: I_\omega \Psi_\omega(\theta_\omega, \ell_\omega) \qquad (\mathrm{PC}_\omega)$$

$$(E^b + D - I_0) + \ell_\omega I_0$$

$$\geq \Delta\mu(D) - I_\omega \underbrace{\left[\Pi_\omega(\theta_\omega, \ell_\omega) - \theta_\omega \gamma \chi_\omega F(\theta_\omega) - \int_{\theta_\omega}^{1} \theta\gamma\chi_\omega dF(\theta)\right]}_{=:\Delta_\omega(\theta_\omega, \ell_\omega)} \quad (\mathrm{FC}_\omega)$$

Moreover, the productivity differential across states (assumption 5.3) can be shown to ensure that the constraint banks have to worry about in the bad state is the financial constraint, rather than the physical one. As a result, when $\omega = B$, solutions for the program above must fall under one of the three regimes listed in proposition 3.1, namely *liquidity surplus* ($r_B = LS$), *extensive distortion* ($r_B = ED$), and *dual distortion* ($r_B = DD$).

So, when searching for ex-post equilibria in the bad state, we can restrict attention to candidates under which one of these three regimes obtains. I'll begin with the case of a liquidity surplus. Under an ex-post equilibrium of this type, it should be clear that banks settle on a subcontract of the form $(\theta_B, I_B) = [\theta_B^{LS}(I_0), I_0]$, where $\theta_B^{LS}(I_0)$ satisfies

$$\chi_B \theta_B^{LS}(I_0) = \rho + \Lambda' \left[I_0 F \left[\theta_B^{LS}(I_0)\right]\right] \qquad (10)$$

—that is, $\theta_B^{LS}(I_0)$ gives the type for whom continuation is NPV-neutral, after taking market-clearing into account. That this equation admits a unique solution should be obvious, so all that remains is to verify the financial constraint,

$$(E^b + D - I_0) + I_0\Lambda'\left[I_0 F\left[\theta_B^{LS}(I_0)\right]\right]$$

$$\geq \Delta\mu(D) - I_0\Delta_B\left[\theta_B^{LS}(I_0), \Lambda'\left[I_0 F\left[\theta_B^{LS}(I_0)\right]\right]\right]. \quad (11)$$

It can easily be shown that this inequality is more likely to hold when initial balance sheets are especially conservative, specifically in the sense that they exhibit high liquidity and low leverage. More precisely, the situation is as follows:

Lemma 6.1. *$\forall D \in \mathbb{R}_+$ s.t. $E^b + D - \Delta\mu(D) \geq 0$, $\exists \overline{I}_B^{LS}(D) \in \mathbb{R}_+$ s.t.* (11) *holds i.f.f. $I_0 \leq \overline{I}_B^{LS}(D)$, and in this case the bad state admits a unique ex-post equilibrium, namely of the "liquidity surplus" type. Moreover, this function is strictly decreasing, with $E^b + D - \Delta\mu(D) = 0 \implies \overline{I}_B^{LS}(D) = 0$.*

Things are a bit more complicated when the distorted regimes obtain, namely because the financial constraint binds. Now, when working with the simpler model above, I argued that banks have two ways to go about balancing a binding financial constraint: one option would be to reduce the marginal type θ_B (the *extensive margin*), and the alternative would be to reduce the investment scale I_B (the *intensive margin*). I also argued that these two margins admit a strict pecking order, with banks preferring the former until θ_B falls below some critical type. As a result, an ex-post equilibrium of the "extensive distortion" type should have the property that banks are able to balance the financial constraint without driving θ_B so low that the intensive margin comes online—that is, $(\theta_B, I_B) = [\theta_B^{ED}(D, I_0), I_0]$, with

$$(E^b + D - I_0) + I_0\Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]$$

$$= \Delta\mu(D) - I_0\Delta_B[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]],$$

and

$$(\Pi_B)_\theta[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]]$$

$$\leq \Pi_B[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]] \times \cdots$$

$$\cdots \times \frac{(\Delta_B)_\theta[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]]}{\Delta_B[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]]}. \quad (12)$$

On the other hand, an ex-post equilibrium of the "dual distortion" type would have $(\theta_B, I_B) = [\theta_B^{DD}(D, I_0), I_B^{DD}(D, I_0)]$, with

$$(E^b + D - I_0) + I_0\Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]$$

$$= \Delta\mu(D) - I_B^{DD}(D, I_0)\Delta_B[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]],$$

and

$$(\Pi_B)_\theta[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]]$$

$$= \Pi_B[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]] \times \cdots$$

$$\cdots \times \frac{(\Delta_B)_\theta[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]]}{\Delta_B[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]]}.$$

Moreover, it can be shown that the former case is most likely to obtain when initial balance sheets exhibit moderate leverage and liquidity. More specifically:

Lemma 6.2. *$\forall D \in \mathbb{R}_+$ s.t. $E^b + D - \Delta\mu(D) \geq 0$, $\exists! \overline{I}_B^{ED}(D) \in [\overline{I}_B^{LS}(D), \infty)$ s.t.* (12) *holds with equality when $I_0 = \overline{I}_B^{ED}(D)$. Moreover, this function has the property that the bad state admits a unique ex-post equilibrium, namely of the the "extensive distortion" type, whenever $I_0 \in (\overline{I}_B^{LS}(D), \overline{I}_B^{ED}(D)]$. It's also strictly decreasing, with $E^b + D - \Delta\mu(D) = 0 \implies \overline{I}_B^{ED}(D) = 0$.*

On the other hand, if initial balance sheets exhibit higher leverage and lower liquidity, then the bad state either admits an ex-post equilibrium of the "dual distortion" type or otherwise fails to admit any ex-post equilibria, namely because the financial constraint fails under all candidates:

Lemma 6.3. *$\forall D \in \mathbb{R}_+$ s.t. $E^b + D - \Delta\mu(D) \geq 0$, $\exists! \overline{I}_B^{DD}(D) \in [\overline{I}_B^{ED}(D), \infty)$ s.t.*

$$[E^b + D - \overline{I}_B^{DD}(D)] + \overline{I}_B^{DD}(D)\Lambda'[\overline{I}_B^{DD}(D)] = \Delta\mu(D).$$

Moreover, this function has the property that the bad state admits a unique ex-post equilibrium, namely of the "dual distortion" type, whenever $I_0 \in (\overline{I}_B^{ED}(D), \overline{I}_B^{DD}(D)]$. *It's also strictly decreasing, with* $E^b + D - \Delta\mu(D) = 0 \implies \overline{I}_B^{DD}(D) = 0$.

Lemma 6.4. *If instead* $E^b + D - \Delta\mu(D) < 0$, *or* $E^b + D - \Delta\mu(D) \geq 0$ *with* $I_0 > \overline{I}_B^{DD}(D)$, *then the bad state admits no ex-post equilibria of any type.*

See Figure 8 for an illustration.

Turning now to the economy's efficiency in the bad state, I note that the allocation obtaining in ex-post equilibrium mechanically solves the planner's ex-post problem whenever the financial constraint is lax, namely because the fire-sale externality remains dormant. However, once the financial constraint binds, this coincidence begins to break down. The intuition for this break-down hinges on our recognizing that the extensive and intensive margins discussed above have opposite implications for the price at which the secondary market clears and thus, by extension, the tightness of the financial constraint. In particular, since the extensive margin involves letting more types into the set of interbank debtors, it's associated with fewer liquidations, a higher price, and greater slack in the financial constraint, while the intensive margin works in the opposite direction. As a result, we should expect the planner to lean more heavily on the extensive margin, relative to banks' behaviour in ex-post equilibrium.

As a first step toward verifying this intuition, I note that the planner's ex-post problem admits a reformulation similar to that obtaining for banks. In particular, the arguments underlying lemma 3.1 can be used to show that the planner's ex-post problem in state ω amounts to a choice over the pair $(\theta_\omega, I_\omega) \in [0,1] \times [0, I_0]$ with the usual interpretation that all types in $[0, \theta_\omega)$ shut down, whereas (almost) all others operate at scale I_ω. More specifically, the planner chooses this pair to maximize the surplus generated by both the economy's technologies,[15]

$$\int_{\theta_\omega}^{1} (\theta\chi_\omega - \rho) I_\omega dF(\theta) + \Lambda[I_0 - I_\omega[1 - F(\theta_\omega)]],$$

subject to the usual physical and financial constraints, evaluated at market-

[15] The superscript "SP" stands for "social planner".

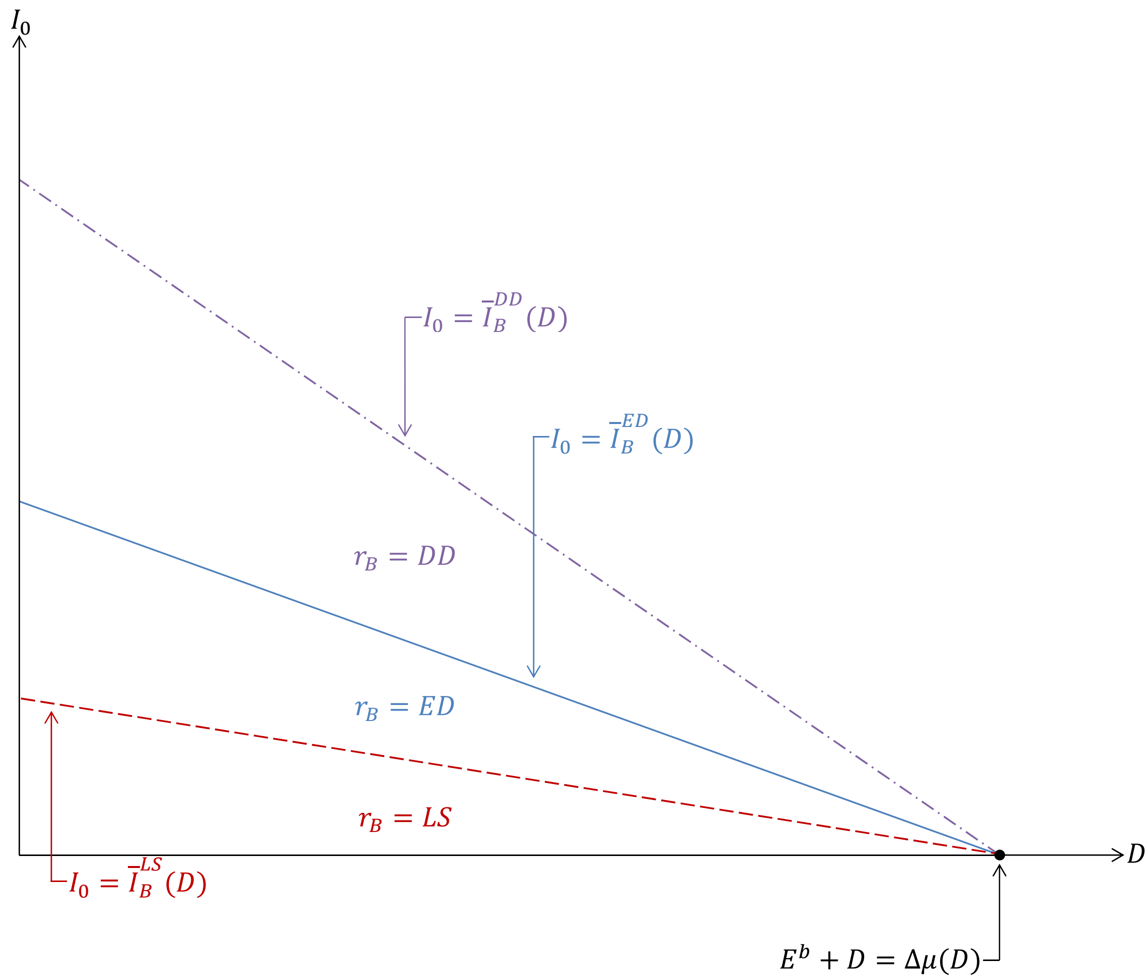

Figure 8: Banks' behaviour in the bad state as a function of their initial balance-sheet choices

clearing prices:

$$(E^b + D - I_0) + I_0\Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]] \geq I_\omega \Psi_\omega[\theta_\omega, \Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]]] \tag{PC$_\omega^{SP}$}$$

$$(E^b + D - I_0) + I_0\Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]]$$
$$\geq \Delta\mu(D) - I_\omega \Delta_\omega[\theta_\omega, \Lambda'[I_0 - I_\omega[1 - F(\theta_\omega)]]] \tag{FC$_\omega^{SP}$}$$

Moreover, if $\omega = B$, then the productivity differential across states (assumption 5.3) ensures that the constraint that the planner has to worry about is the financial constraint, rather than the physical constraint.

Now, when the financial constraint binds, the rate of transformation along this constraint is given by

$$\frac{dI_B}{d\theta_B} = \frac{-I_B\left[(\Delta_B)_\theta[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]] + f(\theta_B)\delta(\theta_B, I_B, I_0)\right]}{\Delta_B[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]] - [1 - F(\theta_B)]\delta(\theta_B, I_B, I_0)},$$

where

$$\delta(\theta_B, I_B, I_0) := [I_0 + I_B(\Delta_B)_\ell \theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]]\Lambda''[I_0 - I_B[1 - F(\theta_B)]]$$
$$= [I_0 - I_B[1 - F(\theta_B)]]\Lambda''[I_0 - I_B[1 - F(\theta_B)]] < 0$$

is a wedge distinguishing this rate from that perceived by banks—c.f. (3). The planner thus prefers the extensive margin so long as

$$\frac{dI_B}{d\theta_B}\underbrace{\left[\int_{\theta_B}^1 (\theta\chi_B - \rho)dF(\theta) - [1 - F(\theta_B)]\Lambda'[I_0 - I_B[1 - F(\theta_B)]]\right]}_{=\Pi_B[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]]}$$
$$+ I_B \overbrace{f(\theta_B)[\rho - \chi_B\theta_B + \Lambda'[I_0 - I_B[1 - F(\theta_B)]]]}^{=(\Pi_B)_\theta[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]]} \leq 0,$$

or equivalently

$$(\Pi_B)_\theta[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]]$$

$$\leq \Pi_B[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]] \times \cdots$$

$$\cdots \times \frac{(\Delta_B)_\theta[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]] + f(\theta_B)\delta(\theta_B, I_B, I_0)}{\Delta_B[\theta_B, \Lambda'[I_0 - I_B[1 - F(\theta_B)]]] - [1 - F(\theta_B)]\delta(\theta_B, I_B, I_0)}. \quad (13)$$

It can be shown that this condition is weaker than the analogous condition for banks, confirming our intuition that the planner should be inclined to rely more heavily on the extensive margin. More specifically, it can be shown that the situation is as follows:

Lemma 6.5. *$\forall D \in \mathbb{R}_+$ s.t. $E^b + D - \Delta\mu(D) \geq 0$, $\exists! \overline{I}_B^{ED|SP}(D) \in [\overline{I}_B^{ED}(D), \overline{I}_B^{DD}(D)]$ s.t.* (13) *holds with equality when $\theta_B = \theta_B^{ED|SP}(D, I_0)$, with $I_B = I_0 = \overline{I}_B^{ED|SP}(D)$. Moreover, this function has the following properties. If $I_0 \in (\overline{I}_B^{LS}(D), \overline{I}_B^{ED|SP}(D)]$, then the planner's ex-post problem in the bad state admits a unique solution, namely under which an extensive distortion occurs. On the other hand, if $I_0 \in (\overline{I}_B^{ED|SP}(D), \overline{I}_B^{DD}(D)]$, then the planner's ex-post problem still admits a unique solution, but this solution now has the property that a dual distortion occurs—specifically, $(\theta_B, I_B) = [\theta_B^{DD|SP}(D, I_0), I_B^{DD|SP}(D, I_0)]$, where this pair is pinned down by binding versions of* (13) *and (FC_B^{SP}). Also, $\overline{I}_B^{ED|SP}(D)$ is strictly decreasing, with $E^b + D - \Delta\mu(D) = 0 \implies \overline{I}_B^{ED|SP}(D) = 0$.*

Lemma 6.6. *If instead $E^b + D - \Delta\mu(D) < 0$, or $E^b + D - \Delta\mu(D) \geq 0$ with $I_0 > \overline{I}_B^{DD}(D)$, then the planner's ex-post problem in the bad state is insoluble.*

See Figure 9 for an illustration.

In light of these last two lemmata, we see that there's no need for policy intervention in the bad state when initial balance sheets are relatively liquid and unlevered. More specifically, if $I_0 \leq \overline{I}_B^{ED}(D)$, then the allocation obtaining in ex-post equilibrium coincides with the solution for the planner's ex-post problem, either because the financial constraint is lax ($I_0 \leq \overline{I}_B^{LS}(D)$), or because it binds so weakly that the planner and banks both prefer to rely exclusively on the extensive margin ($\overline{I}_B^{LS}(D) < I_0 \leq \overline{I}_B^{ED}(D)$).

On the other hand, if $I_0 \in (\overline{I}_B^{ED}(D), \overline{I}_B^{DD}(D)]$, then one of two problems arises: it's either the case that banks activate the intensive margin despite the planner's preferring to rely only on the extensive margin ($\overline{I}_B^{ED}(D) < I_0 \leq \overline{I}_B^{ED|SP}(D)$) or otherwise that both margins are active, but the planner prefers an allocation under which the extensive margin does more of the work ($\overline{I}_B^{ED|SP}(D) < I_0 \leq \overline{I}_B^{DD}(D)$). Either way, some kind of policy intervention is now needed. The appropriate intervention should have the property that it reduces the private return from liquidation and thus disincentivizes banks' reliance on the intensive margin. On this front, I follow Fahri and Tirole (2012) in assuming that the policymaker is able to levy some tax τ_B on banks' storage at $t = 1$, and then remits the proceeds as a lump sum T_B at $t = 2$. We can think of this as a simple metaphor for monetary policies aiming to drive the interest rate beneath its natural level.[16] Moreover:

Proposition 6.1 (optimal ex-post intervention)**.** $\forall (D, I_0) \in \mathbb{R}_+^2$ *s.t.* $E^b + D - D\mu'(E^h - D) \geq 0$ *and* $I_0 \leq \overline{I}_B^{DD}(D)$*, the tax* τ_B *and transfer* T_B *can jointly be chosen to implement the solution for the planner's ex-post problem in the bad state. Moreover, the appropriate choice satisfies* $\tau_B \geq 0$*, with strict inequality whenever* $I_0 > \overline{I}_B^{ED}(D)$*.*

This proposition constitutes one of this section's main results, specifically due to its implication that an instrument normally associated with price stability also has an important role to play in ensuring financial stability. As mentioned in the introduction, this reinforces a key theme in the post-crisis policy literature, namely that these two policy objectives cannot be separated cleanly.

6.2 Details on the good state

I now turn my attention to the good state. In this state, the productivity differential across states (assumption 5.3) now ensures that banks have to worry about the physical constraint, rather than the financial one. As a result, ex-post equilibria must take one of two forms. The first would be the usual *liquidity surplus* scenario ($r_G = LS$) under which banks are able to keep all NPV-positive types operating at full scale—i.e., $(I_G, \theta_G) = [I_0, \theta_G^{LS}(I_0)]$,

[16] See section I.B in Fahri and Tirole (2012) for three concrete interpretations along these lines.

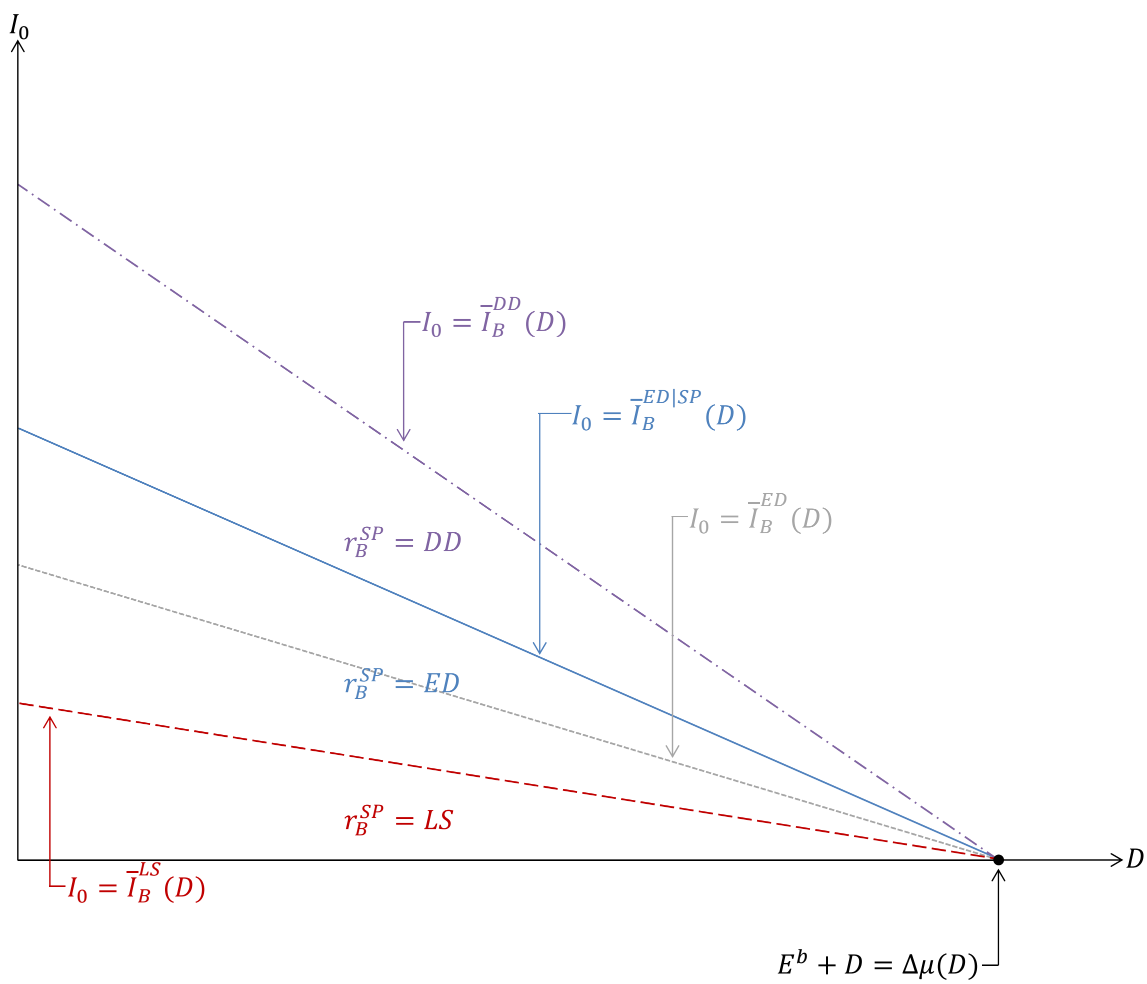

Figure 9: Planner's behaviour in the bad state as a function of his initial balance-sheet choices

with

$$(E^b + D - I_0) + I_0\Lambda'[I_0F[\theta_G^{LS}(I_0)]] \geq I_0\Psi_G[\theta_G^{LS}(I_0), \Lambda'[I_0F[\theta_G^{LS}(I_0)]]]. \quad (14)$$

The alternative would be a *liquidity rationing* scenario ($r_G = LR$) under which the physical constraint binds—specifically, $(I_G, \theta_G) = [I_0, \theta_G^{LR}(D, I_0)]$, where $\theta_G^{LR}(D, I_0)$ solves

$$(E^b + D - I_0) + I_0\Lambda'[I_0F[\theta_G^{LR}(D, I_0)]] = I_0\Psi_G[\theta_G^{LR}(D, I_0), \Lambda'[I_0F[\theta_G^{LR}(D, I_0)]]].$$

Now, it should be clear that (14) is more likely to hold the lesser is I_0 and the greater is D, so we should expect the former case to obtain when banks raise lots of deposits but allocate most of them to storage. More specifically, the situation is as follows:

Lemma 6.7. *For any initial balance sheet onto which banks would be willing to select in a monotonic equilibrium, the financial constraint associated with the good state is lax. As for the physical constraint, I note the following:* $\forall D \in \mathbb{R}_+$, $\exists! \overline{I}_G^{LS}(D) \in \mathbb{R}_+$ *s.t.* (14) *holds with equality when* $I_0 = \overline{I}_G^{LS}(D)$*. If the balance sheet in question satisfies* $I_0 \leq \overline{I}_G^{LS}(D)$*, then the good state admits a unique ex-post equilibrium, namely under which banks experience a liquidity surplus. If instead* $I_0 > \overline{I}_G^{LS}(D)$*, then the good state still admits a unique ex-post equilibrium, but this ex-post equilibrium now has the property that banks experience liquidity rationing. Moreover,* $(\overline{I}_G^{LS})'(D) > 0$*, with* $\overline{I}_G^{LS}(0) < \overline{I}_B^{LS}(0)$*.*

See Figure 10 for an illustration.

As for the economy's efficiency in this state, matters are somewhat simpler than in my previous subsection. In particular, it can be shown that the allocation described in lemma 6.7 also solves the planner's ex-post problem. More precisely:

Lemma 6.8. *For any initial balance sheet onto which the planner would be willing to select at* $t = 0$*, the financial constraint associated with the good state is lax. As for the physical constraint, one of two cases must obtain. If the balance sheet in question satisfies* $I_0 \leq \overline{I}_G^{LS}(D)$*, then the planner's ex-post problem admits a unique solution, namely under which a liquidity surplus occurs—i.e.,* $(I_G, \theta_G) = [I_0, \theta_G^{LS}(I_0)]$*. If instead* $I_0 > \overline{I}_G^{LS}(D)$*, then the planner's ex-post problem admits a unique solution, namely under which liquidity rationing occurs—i.e.,* $(I_G, \theta_G) = [I_0, \theta_G^{LR}(D, I_0)]$*.*

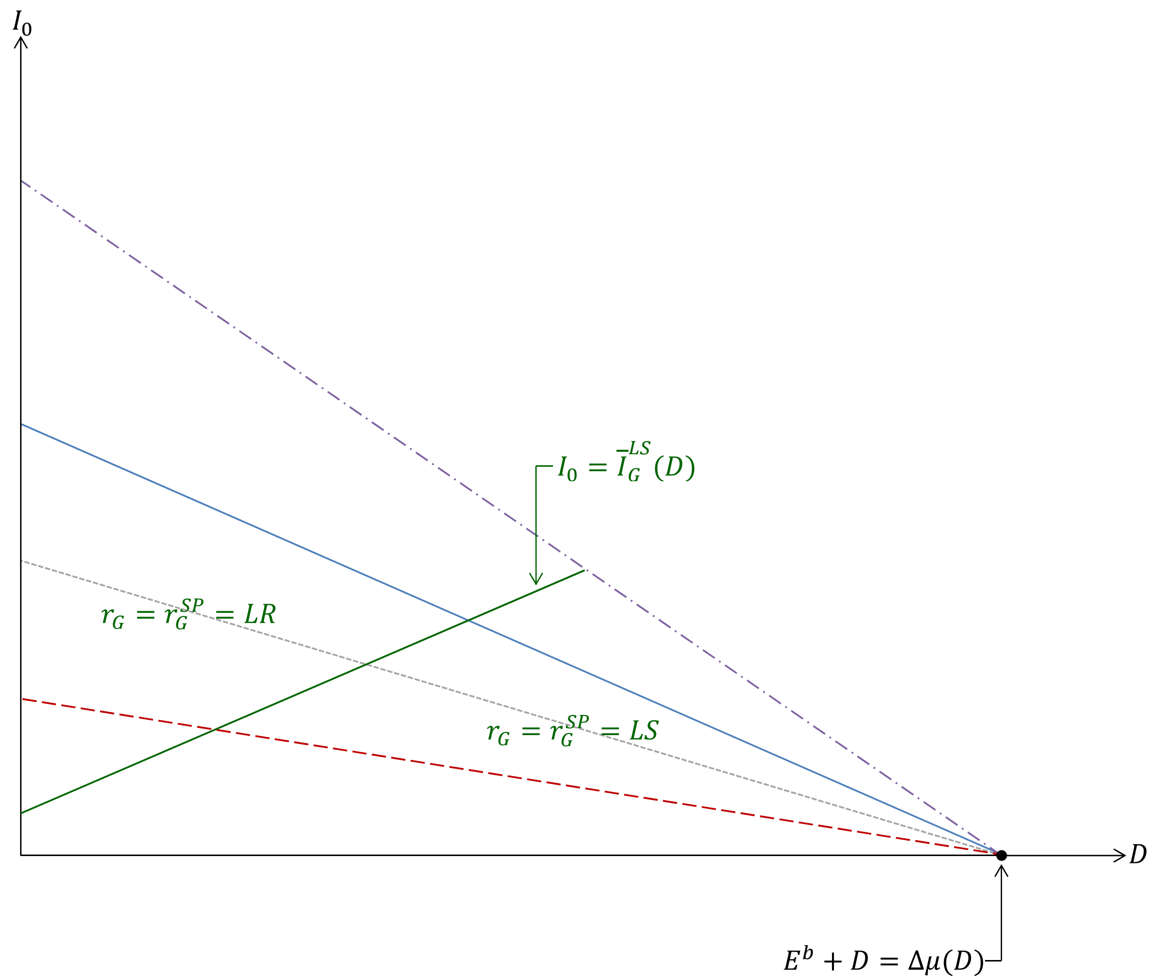

Figure 10: Banks' and planner's behaviour in the good state as a function of their initial balance-sheet choices

In the case of a liquidity surplus, this coincidence with the ex-post equilibrium allocation naturally follows from the physical constraint's being lax. On the other hand, in the case of liquidity rationing, the intuition for this coincidence hinges on our recognizing that a binding physical constraint pins down the total volume of investments being maintained and thus, by extension, the total volume of liquidations taking place. This eliminates the only potential source of disagreement between the planner and banks. As a result, the model admits no role for policy in the good state: to the extent that interventions are needed at $t = 1$, they should be confined to the bad state.

7 Optimal policy at $t = 0$

In this section, I finally shift my attention to $t = 0$, when banks settle on an equilibrium whose general form resembles that exhibited by the simpler model above. More specifically:

Lemma 7.1. *A monotonic equilibrium exists, is unique, and has the property that banks experience liquidity rationing in the good state. As for their behaviour in the bad state, one of four cases must obtain:*[17]

- *the first has a liquidity surplus occurring in the bad state, with banks' choices on D and I_0 respectively pinned down by the first-order conditions*

$$\alpha_G \left[1 - \Delta\mu'(D) + \frac{(\Pi_G)_\theta[\theta_G^{LR}(D, I_0), \Lambda'[I_0 F[\theta_G^{LR}(D, I_0)]]]}{(\Psi_G)_\theta(\cdot)}\right] + \alpha_B[1 - \Delta\mu'(D)] = 0,$$

and

$$\alpha_G \left[\Lambda'(\cdot) + \Pi_G(\cdot) - 1 - [1 - \Lambda'(\cdot) + \Psi_G(\cdot)\frac{(\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)}\right]$$

$$+ \alpha_B[\Lambda'[I_0 F[\theta_B^{LS}(I_0)]] + \Pi_B[\theta_B^{LS}(I_0), \Lambda'[I_0 F[\theta_B^{LS}(I_0)]]] - 1] = 0, \quad (15)$$

where I use $\cdot$ to suppress obvious arguments;

[17] NB: since banks take secondary-market prices as given, all of the first-order conditions given in this lemma take the same form as under the baseline model, except that we now evaluate at market-clearing prices. Compare with section 4 in particular.

- *the second case has an extensive distortion occurring in the bad state, so banks' first-order conditions instead read as*

$$\alpha_G \left[1 - \Delta\mu'(D) + \frac{(\Pi_G)_\theta[\theta_G^{LR}(D, I_0), \Lambda'[I_0 F[\theta_G^{LR}(D, I_0)]]]}{(\Psi_G)_\theta(\cdot)}\right]$$

$$+ \alpha_B[1 - \Delta\mu'(D)] \left[1 - \frac{(\Pi_B)_\theta[\theta_B^{ED}(D, I_0), \Lambda'[I_0 F[\theta_B^{ED}(D, I_0)]]]}{(\Delta_B)_\theta(\cdot)}\right] = 0,$$

and

$$\alpha_G \left[\Lambda'(\cdot) + \Pi_G(\cdot) - 1 - [1 - \Lambda'(\cdot) + \Psi_G(\cdot)] \frac{(\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)}\right]$$

$$+ \alpha_B \left[\Lambda'(\cdot) + \Pi_B(\cdot) - 1 + [1 - \Lambda'(\cdot) - \Delta_B(\cdot)] \frac{(\Pi_B)_\theta(\cdot)}{(\Delta_B)_\theta(\cdot)}\right] = 0; \quad (16)$$

- *the third case has the property that a dual distortion occurs in the bad state, but the "no-default" constraint remains lax, so banks' first-order conditions read as*

$$\alpha_G \left[1 - \Delta\mu'(D) + \frac{(\Pi_G)_\theta[\theta_G^{LR}(D, I_0), \Lambda'[I_0 F[\theta_G^{LR}(D, I_0)]]]}{(\Psi_G)_\theta(\cdot)}\right] + \alpha_B[1 - \Delta\mu'(D)] \times \cdots$$

$$\cdots \times \left[1 - \frac{\Pi_B[\theta_B^{DD}(D, I_0), \Lambda'[I_0 - I_B^{DD}(D, I_0)[1 - F[\theta_B^{DD}(D, I_0)]]]]}{\Delta_B(\cdot)}\right] = 0,$$

and

$$\alpha_G \left[\Lambda'(\cdot) + \Pi_G(\cdot) - 1 - [1 - \Lambda'(\cdot) + \Psi_G(\cdot)] \frac{(\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)}\right] = \alpha_B[1 - \Lambda'(\cdot)] \left[1 - \frac{\Pi_B(\cdot)}{\Delta_B(\cdot)}\right];$$

- *the final case is an "interbank collapse" scenario under which a dual distortion occurs in the bad state, with the "no-default" constraint now binding.*

Moreover, non-monotonic equilibria do not exist.

As for the planner's ex-ante problem, its solution takes a similar form, though he adjusts his first-order conditions to take account of the price-setting process:

Lemma 7.2. *The planner's ex-ante problem admits a unique solution, namely under which liquidity rationing occurs in the good state. As for the bad state, one of four cases must obtain:*

- *the first has a liquidity surplus occurring in the bad state, with the planner's choices on D and I_0 respectively pinned down by the first-order conditions*

$$\alpha_G\left[1-\Delta\mu'(D)+\frac{(\Pi_G)_\theta[\theta_G^{LR}(D,I_0),\Lambda'[I_0F[\theta_G^{LR}(D,I_0)]]]}{(\Psi_G)_\theta(\cdot)\underbrace{-f[\theta_G^{LR}(D,I_0)]\delta[\theta_G^{LR}(D,I_0),I_0,I_0]}_{*}}\right]+\alpha_B[1-\Delta\mu'(D)]=0,$$

and

$$\alpha_G\left[\Lambda'(\cdot)+\Pi_G(\cdot)-1-\frac{[1-\Lambda'(\cdot)+\Psi_G(\cdot)\overbrace{-F(\cdot)\delta(\cdot)}^{*}](\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)\underbrace{-f(\cdot)\delta(\cdot)}_{*}}\right]$$

$$+\alpha_B[\Lambda'[I_0F[\theta_B^{LS}(I_0)]]+\Pi_B[\theta_B^{LS}(I_0),\Lambda'[I_0F[\theta_B^{LS}(I_0)]]]-1]=0, \quad (17)$$

where I've highlighted the wedges that distinguish the planner's first-order conditions from those obtaining in equilibrium;[18]

- *the second case has an extensive distortion occurring in the bad state, with first-order conditions*

$$\alpha_G\left[1-\Delta\mu'(D)+\frac{(\Pi_G)_\theta[\theta_G^{LR}(D,I_0),\Lambda'[I_0F[\theta_G^{LR}(D,I_0)]]]}{(\Psi_G)_\theta(\cdot)\underbrace{-f[\theta_G^{LR}(D,I_0)]\delta[\theta_G^{LR}(D,I_0),I_0,I_0]}_{*}}\right]$$

$$+\alpha_B[1-\Delta\mu'(D)]\left[1-\frac{(\Pi_B)_\theta[\theta_B^{ED}(D,I_0),\Lambda'[I_0F[\theta_B^{ED}(D,I_0)]]]}{(\Delta_B)_\theta(\cdot)\underbrace{+f[\theta_B^{ED}(D,I_0)]\delta[\theta_B^{ED}(D,I_0),I_0,I_0]}_{*}}\right]=0,$$

[18] That wedges emerge in the good state may be surprising at first glance, since we know that the relevant constraint in this state has the flavour of a budget constraint, and budget constraints generally fail to effect pecuniary externalities. However, banks' inability to go short in the secondary market causes the usual logic to break down on this front.

and

$$\alpha_G\left[\Lambda'(\cdot)+\Pi_G(\cdot)-1-\frac{[1-\Lambda'(\cdot)+\Psi_G(\cdot)\overbrace{-F(\cdot)\delta(\cdot)}^{*}](\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)\underbrace{-f(\cdot)\delta(\cdot)}_{*}}\right]$$

$$+\,\alpha_B\left[\Lambda'(\cdot)+\Pi_B(\cdot)-1+\frac{[1-\Lambda'(\cdot)-\Delta_B(\cdot)\overbrace{-F(\cdot)\delta(\cdot)}^{*}](\Pi_B)_\theta(\cdot)}{(\Delta_B)_\theta(\cdot)\underbrace{+f(\cdot)\delta(\cdot)}_{*}}\right]=0; \tag{18}$$

- *the third case has the property that a dual distortion occurs in the bad state, but the "no-default" constraint remains lax, with first-order conditions*

$$\alpha_G\left[1-\Delta\mu'(D)+\frac{(\Pi_G)_\theta[\theta_G^{LR}(D,I_0),\Lambda'[I_0F[\theta_G^{LR}(D,I_0)]]]}{(\Psi_G)_\theta(\cdot)\underbrace{-f[\theta_G^{LR}(D,I_0)]\delta[\theta_G^{LR}(D,I_0),I_0,I_0]}_{*}}\right]+\alpha_B[1-\Delta\mu'(D)]\times\cdots$$

$$\cdots\times\left[1-\frac{\Pi_B[\theta_B^{DD|SP}(D,I_0),\Lambda'[I_0-I_B^{DD|SP}(D,I_0)[1-F[\theta_B^{DD|SP}(D,I_0)]]]]}{\Delta_B(\cdot)\underbrace{-[1-F[\theta_B^{DD|SP}(D,I_0)]]\delta[\theta_B^{DD|SP}(D,I_0),I_B^{DD|SP}(D,I_0),I_0]}_{*}}\right]=0,$$

and

$$\alpha_G\left[\Lambda'(\cdot)+\Pi_G(\cdot)-1-\frac{[1-\Lambda'(\cdot)+\Psi_G(\cdot)\overbrace{-F(\cdot)\delta(\cdot)}^{*}](\Pi_G)_\theta(\cdot)}{(\Psi_G)_\theta(\cdot)\underbrace{-f(\cdot)\delta(\cdot)}_{*}}\right]$$

$$=\alpha_B\left[1-\Lambda'(\cdot)-[1-\Lambda'(\cdot)\overbrace{-\delta(\cdot)}^{*}]\frac{\Pi_B(\cdot)}{\Delta_B(\cdot)\underbrace{-[1-F(\cdot)]\delta(\cdot)}_{*}}\right];$$

- *the final case is an "interbank collapse" scenario under which a dual distortion occurs in the bad state, with the "no-default" constraint now binding.*

Broadly speaking, we should expect that the wedges highlighted in this last lemma lead the planner to settle on an initial balance sheet that's in some sense more conservative than the initial balance sheet preferred by banks. One way to formalize this intuition is as follows:

Lemma 7.3. *The solution for the planner's ex-ante problem has the property that one of the distorted regimes obtains in the bad state only if this is also true in ex-ante equilibrium—that is, the parameter space in which the ex-ante equilibrium is vulnerable to the emergence of a distorted regime is a superset of the parameter space in which the planner's ex-ante solution exhibits this vulnerability.*

At this point, it's now natural to ask about the policies that can be used to close the gap between the solution for the planner's ex-ante problem and the allocation arising in ex-ante equilibrium. Based on the analysis in my last two subsections, it should be clear that the answers to these questions hinge critically on where the initial balance sheet preferred by the planner lies in relation to the locus

$$\{(D, I_0) \in \mathbb{R}_+ \text{ s.t. } I_0 = \overline{I}_B^{ED|SP}(D), \text{ with } E^b + D \geq \Delta\mu(D)\}. \tag{19}$$

Suppose, for example, that the solution for the planner's ex-ante problem places the initial balance sheet (D, I_0) beneath this locus. In this case, the results reported in subsection 6.1 (6.2) imply that the ex-post equilibrium to which this initial balance sheet gives rise in the bad (good) state will yield an allocation that also solves the corresponding ex-post problem facing the planner. As a result, the only role for policy is to discipline banks' initial balance-sheet choices at $t = 0$; conditional on banks' being directed to the right initial balance sheet, the secondary market will subsequently take care of itself. Now, with a fire-sale externality at work, a natural candidate for disciplining banks' choice on I_0 would be a liquidity coverage ratio. As explained in Basel Committee on Banking Supervision (2013), the liquidity coverage ratio requires that banks project their liquidity needs over a 30-day stress-test scenario and then hold enough liquid assets to cover a certain portion of these needs. Since in-model liquidity needs stem from the maintenance requirement ρ, we can think of a policy of this sort as a requirement of the form

$$E^b + D - I_0 \geq \underline{s}\rho I_0, \tag{20}$$

with $\underline{s} \in [0,1]$. As for disciplining banks' choice on D, it can be shown that a leverage limit of the form

$$\frac{D}{E^b} \leq \overline{d} \tag{21}$$

will do the trick:

Proposition 7.1 (optimal ex-ante intervention)**.** *If the solution for the planner's ex-ante problem places the initial balance sheet* (D, I_0) *beneath the locus on line 19, then the ex-ante intervention* $(\underline{s}, \overline{d})$ *can be chosen to implement this solution as an ex-ante equilibrium, with no subsequent need for ex-post intervention. Moreover, the appropriate choice on* $(\underline{s}, \overline{d})$ *has the property that* (20) *and* (21) *both bind.*

On the other hand, for initial balance sheets lying above the locus on line 19, the analysis in subsection 6.1 implies that an ex-post intervention (τ_B, T_B) is needed in the bad state to bring the allocation obtaining in ex-post equilibrium into alignment with the solution for the planner's ex-post problem. If anything, this fact enhances the need for ex-ante intervention, since the expectation that policymakers will reduce the effective interest rate in the bad state creates a further disincentive against storage at $t = 0$:

Proposition 7.2 (optimal ex-ante intervention, cont'd)**.** *If the solution for the planner's ex-ante problem instead places the initial balance sheet* (D, I_0) *above the locus on line 19, then the ex-ante intervention* $(\underline{s}, \overline{d})$ *and ex-post intervention* (τ_B, T_B) *can jointly be chosen to implement this solution as an ex-ante equilibrium. Moreover, the appropriate choice on* $(\underline{s}, \overline{d}, \tau)$ *has the property that* (20) *and* (21) *both bind.*

These last two lemmata constitute some of this section's main findings, mainly due to their implication that the ex-ante intervention is always necessary and sometimes even sufficient for implementation of the planner's solution. As a result, a planner who focuses exclusively on ex-post intervention will not be able to implement this solution—likewise one who views ex-ante intervention merely as a corrective for moral hazard. As explained in my introduction, this suggests that previous literature may have underestimated the role that ex-ante interventions have to play in the optimal regulation of interbank markets.

8 Conclusion

I close by briefly identifying some promising directions in which this work could be extended. The first would be to enrich the modelling of the deposit market with an eye toward connecting with the growing literature on the wholesale-versus-retail composition of banks' liabilities (e.g., Jeong (2009), Altunbas et al. (2011), Huang and Ratnovski (2011), Damar et al. (2013), Dewally and Shao (2013), Lopez-Espinosa et al. (2012), Hahm et al. (2013), and Bolgona (2015)). A complementary extension could aim to embed the model at hand into a larger macro framework, namely with an eye toward quantifying the size and cyclicality of the optimal policies emerging from my analysis, along with the welfare gains associated with their implementation.

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