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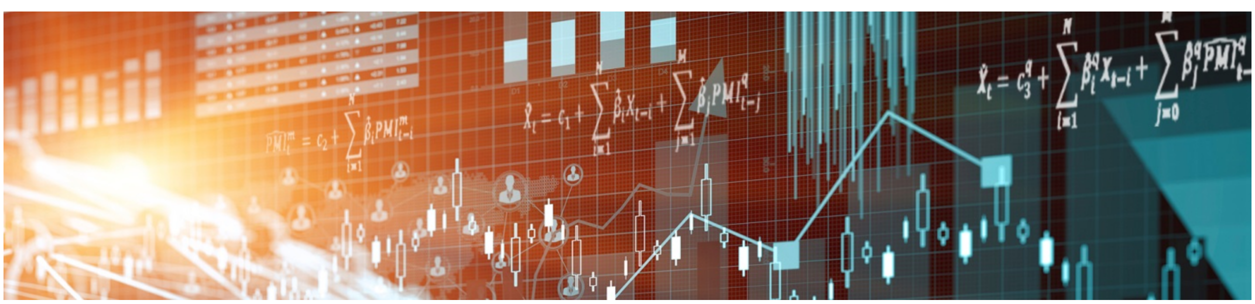

by Lorenzo Pozzi and Barbara Sadaba

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Detecting Scapegoat Effects in the Relationship Between Exchange Rates and Macroeconomic Fundamentals

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Lorenzo Pozzi[1] and Barbara Sadaba[2]

[1]Department of Economics
Erasmus University Rotterdam and Tinbergen Institute
Rotterdam, The Netherlands
pozzi@ese.eur.nl

[2]International Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
bsadaba@bankofcanada.ca

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Abstract

This paper presents a new testing method for the scapegoat model of exchange rates that aims to tighten the link between the theory on scapegoats and its empirical implementation. This new testing method consists of a number of steps. First, the exchange rate risk premium, the unobserved time-varying structural impact of the macro fundamentals on the exchange rate and the unobserved fundamental of the model are estimated. Next, the scapegoat terms in the model's exchange rate equation are estimated under the restrictions implied by these first-step estimates. The scapegoat terms consist of macro fundamentals, i.e., potential scapegoats, interacted with parameter expectations, where the latter are proxied using survey data. We use a Bayesian Gibbs sampling approach to estimate the different steps of the methodology for eight countries (five developed, three emerging) versus the US over the period 2002Q1–2014Q4. The macro fundamentals we consider are real GDP growth, the inflation rate, the long-run nominal interest rate and the current account to GDP ratio. We calculate the posterior probabilities that these macro fundamentals are scapegoats. For the inflation rate, these probabilities are considerably higher than the imposed prior probabilities of 0.5 in five out of eight countries (including the Anglo-Saxon economies). We find little evidence to suggest that the other macro fundamentals we consider are scapegoats.

Bank topics: Exchange rates; Econometric and statistical methods; International financial markets
JEL codes: G15, C32, F31

Résumé

Nous présentons dans cette étude un nouveau test pour le modèle de taux de change fondé sur la théorie du bouc émissaire (*scapegoat model of exchange rates)*. Le test vise à renforcer le lien entre cette théorie et sa traduction empirique. Ce nouveau test comporte plusieurs étapes. En premier, la prime de risque de change et l'incidence structurelle inobservée et changeante dans le temps des variables macroéconomiques fondamentales sur le taux de change sont estimées ainsi qu'une composante inobservée du modèle. Ensuite, les termes représentant le bouc émissaire dans l'équation du taux de change du modèle sont estimés en tenant compte des restrictions déduites des estimations obtenues à la première étape. Ces termes sont des variables macroéconomiques fondamentales (des boucs émissaires potentiels) en interaction avec les paramètres des anticipations, lesquelles sont approximées au moyen de données d'enquête. Nous procédons à une estimation en différentes étapes à l'aide d'une méthode d'échantillonnage bayésienne de

Gibbs pour huit pays (cinq développés et trois émergents) qui sont comparés aux États-Unis sur la période 2002T1-2014T4. Les variables fondamentales macroéconomiques examinées sont le taux de croissance du PIB réel, le taux d'inflation, le taux d'intérêt nominal de long terme et le ratio balance courante/PIB. Nous calculons les probabilités a posteriori concernant les chances de voir ces variables fondamentales être considérées comme des boucs émissaires. Pour le taux d'inflation, ces probabilités sont beaucoup plus élevées que les probabilités a priori de 0,5 imposées pour cinq des huit pays (notamment les pays anglo-saxons). Nous trouvons peu d'éléments pour conforter l'idée que les autres variables macroéconomiques sont des boucs émissaires.

Sujets : Taux de change; Méthodes économétriques et statistiques; Marchés financiers internationaux
Codes JEL : G15, C32, F31

Non-Technical Summary

One of the major puzzles in international macroeconomics is the difficulty of linking exchange rates to macroeconomic fundamentals such as money supplies, interest rates and outputs, i.e., the so-called "disconnect puzzle" of exchange rates (see Obstfeld and Rogoff, 2000). The "disconnect puzzle" manifests itself in a variety of ways, among which the lack of out-of-sample predictability of exchange rates and the instability of the ex-post relationship between exchange rates and fundamentals are probably the most striking. The latter manifestation of the "disconnect puzzle" constitutes the focus of this paper. With respect to the reasons for instability, Cheung and Chinn (2001) argue, based on a survey of US foreign exchange traders, that the instability caused by the impact of macro fundamentals on exchange rates is driven by the fact that traders regularly change the weight that they attach to macro fundamentals.

The scapegoat theory of exchange rates (see Bacchetta and van Wincoop, 2004, 2009, 2012, 2013) provides a formal theoretical framework with a potential explanation for the weak link between macro fundamentals and exchange rates. In a scapegoat model, economic agents form rational expectations but are assumed to have incomplete information. Economic agents do not observe some economic variables in the economy (e.g., money demand shifts, real exchange rate shocks) and do not know the structural parameters on the macro fundamentals that drive the exchange rate. Therefore, they form expectations about these structural parameters based on an observed "signal", which typically depends on the observed level of the exchange rate, the observed interest rate differential and the discount factor of the agents (i.e., the discount rate used by investors to discount future observed and unobserved variables). Because of imperfect information, economic agents can be rationally confused and can, as a result, rationally attribute changes in the exchange rate to changes in the observed macro fundamentals, while in fact these changes are caused by the unobserved variables. Therefore, they may erroneously give too much weight to certain observed macro fundamentals in the determination of the exchange rate. In this model, it is economic agents' expectations of the structural parameters on macro fundamentals that are driving exchange rates rather than the structural parameters themselves. As agents are assumed to frequently update their expectations about the impact of macro fundamentals on exchange rates, the theory can potentially explain the highly unstable observed relationship between macroeconomic fundamentals and exchange rates.

In this paper, we propose an alternative empirical testing strategy for the scapegoat theory of exchange rates. More specifically, the contribution of the paper is to test for scapegoat effects using the exact structural exchange rate equation implied by a scapegoat model instead of an ad hoc empirical specification. This implies that we test for scapegoat effects under the restrictions imposed on the data by the model. This approach should tighten the link between the theory on scapegoats and the empirical testing of this theory.

The results suggest, first, that there is a persistent but stationary exchange risk premium or time-varying deviation from the uncovered interest rate parity (UIRP) condition in all countries considered. Second, we identify a persistent but stationary unobserved component from the "signal" in the model, which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. Third, we find that, over the sample period, the structural parameters on the macro fundamentals are constant and often close to zero. Fourth, as far as the scapegoat terms in the exchange rate equation are concerned, we calculate posterior probabilities that these macro fundamentals are scapegoats, and find, for the inflation rate in five out of eight countries, probabilities that are considerably higher than the imposed prior probabilities of 0.5. These countries are the three Anglo-Saxon economies (Australia, Canada and the UK) and South Korea and South Africa. We find little evidence to suggest that the other considered macro fundamentals are scapegoats, however.

1 Introduction

One of the major puzzles in international macroeconomics is the difficulty linking exchange rates to macroeconomic fundamentals such as money supplies, interest rates and outputs, i.e., the so-called "disconnect puzzle" of exchange rates (see Obstfeld and Rogoff, 2000). The "disconnect puzzle" manifests itself in a variety of ways, among which the lack of out-of-sample predictability of exchange rates and the instability of the ex-post relationship between exchange rates and fundamentals are probably the most striking. The latter manifestation of the "disconnect puzzle" constitutes the focus of this paper. The instability of structural parameters has been linked to the poor performance of exchange rate models both in and out of sample (see Meese and Rogoff, 1983a,b, 1988; Bacchetta et al., 2009; Rossi, 2006, 2013).[1] With respect to the reasons for instability, Cheung and Chinn (2001) argue, based on a survey of US foreign exchange traders, that the instability caused by the impact of macro fundamentals on exchange rates is driven by the fact that traders regularly change the weight that they attach to macro fundamentals. Sarno and Valente (2009) conduct an exchange rate model selection procedure that allows them to select the best model in every period out of all possible combinations of fundamentals. They report frequent changes in the optimal model, implying frequent shifts in parameters.

The scapegoat theory of exchange rates (see Bacchetta and van Wincoop, 2004, 2009, 2012, 2013) provides a formal theoretical framework for many of these ideas and hence provides a potential explanation for the weak link between macro fundamentals and exchange rates. In a scapegoat model, economic agents form rational expectations but are assumed to have incomplete information. Economic agents do not observe some economic variables in the economy (e.g., money demand shifts, real exchange rate shocks) and do not know the structural parameters on the macro fundamentals that drive the exchange rate. Therefore, they form expectations about these structural parameters based on an observed "signal", which typically depends on the observed level of the exchange rate, the observed interest rate differential and the discount factor of the agents (i.e., the discount rate used by investors to discount future observed and unobserved variables). Because of imperfect information, they can be rationally confused and can, as a result, rationally attribute changes in the exchange rate to changes in the observed macro fundamentals, while in fact these changes are caused by the unobserved variables. Therefore, economic agents may

[1] We note that Engel and West (2005) and Engel et al. (2007) nuance this poor performance, however, by arguing that the low predictability of exchange rates using macro fundamentals is actually implied by standard present-value models of the exchange rate. When macro fundamentals are non-stationary and the discount factor used to discount expected future fundamentals is high (i.e., close to 1), then the exchange rate will be close to a random walk. In this case, expectations about future fundamentals drive the exchange rate, while current and lagged values are relatively unimportant. A testable implication then is whether exchange rates can predict future fundamentals rather than the other way around (see, e.g., Engel and West, 2005; Sarno and Schmeling, 2014).

erroneously give too much weight to certain observed macro fundamentals in the determination of the exchange rate. In this model, it is economic agents' expectations about the structural parameters on macro fundamentals that are driving exchange rates rather than the structural parameters themselves. As agents are assumed to frequently update their expectations about the impact of macro fundamentals on exchange rates, the theory can potentially explain the highly unstable observed relationship between macroeconomic fundamentals and exchange rates. The model has recently been tested empirically by Fratzscher et al. (2015) using survey scores reported biannually by Consensus Economics. These reflect the weight investors attach to certain macro fundamentals in the determination of the exchange rate in a given period. They regress changes in the exchange rate of 12 currencies on fundamentals and on fundamentals interacted with these scores. They find that the interaction terms have a significant impact on exchange rates, hence providing evidence in favor of the scapegoat model.

In this paper, we propose an alternative empirical testing strategy for the scapegoat theory of exchange rates. More specifically, the contribution of the paper is to test for scapegoat effects using the exact structural exchange rate equation implied by a scapegoat model instead of an ad hoc empirical specification. This implies that we test for scapegoat effects under the restrictions imposed on the data by the model. This approach should tighten the link between the theory on scapegoats and the empirical testing of this theory.

The scapegoat model that we consider largely follows the model presented by Bacchetta and van Wincoop (2013), except for the assumption of time-varying structural parameters (i.e., random walks) on the macro fundamentals and the explicit incorporation of a time-varying deviation from the uncovered interest rate parity (UIRP) condition in the derived exchange rate equation.[2] The model leads to an exchange rate equation that consists of four terms. First, a term that captures the standard impact of the macro fundamentals on the exchange rate. This term consists of the macro fundamentals interacted with time-varying structural parameters. Second, a term that captures the impact of macro fundamentals as scapegoats. This term consists of the macro fundamentals interacted with the expectations about the time-varying structural parameters. Third, a term related to the unobserved component which reflects unobserved relative money demand shocks and/or real exchange rate shocks. Fourth, a term related to the exchange rate risk premium or time-varying deviation from the UIRP condition.

The exchange rate equation is sufficiently complex that an estimation approach in different steps is required. First, the exchange rate risk premium or time-varying deviation from the UIRP condition is estimated using a state-space approach applied to the observed difference between the change in the exchange rate and the interest rate differential. Second, the unobserved time-varying structural

[2] A risk premium is included in the calibration exercise of Bacchetta and van Wincoop (2013), however.

parameters on the macro fundamentals and the unobserved component of the model are estimated using a state-space system applied to the observed "signal" in the model, which depends on the level of the exchange rate, the interest rate differential and the discount factor. Third, the scapegoat terms in the model's exchange rate equation, i.e., the expectations of the structural parameters interacted with the macro fundamentals, are estimated using a regression analysis where the estimation is conditional on the exchange rate risk premium, the structural parameters on the macro fundamentals and the unobserved component estimated in the previous steps. Following Fratzscher et al. (2015), we use survey data to proxy the parameter expectations that enter the scapegoat terms.

The estimation in different steps is carried out through a Bayesian Gibbs sampling approach for eight countries versus the US over the period 2002Q1-2014Q4. We consider five developed economies (Australia, Canada, the euro area, Japan and the UK) and three emerging countries (Singapore, South Korea and South Africa). Our choice of macro fundamentals is based on the availability of corresponding survey data for these fundamentals. More specifically, we incorporate four macro fundamentals in the estimations that can potentially be scapegoats, i.e., the real GDP growth rate (relative to the US), the inflation rate (relative to the US), the long-run nominal interest rate (relative to the US) and the current account balance to GDP ratio. The applied Gibbs approach is advantageous because the full posterior distributions of parameters and states are calculated in every step and are conditioned upon in the next steps so that both parameter and state uncertainty can fully be taken into account in the estimation of scapegoat effects. Additionally, the Bayesian approach allows for model selection when considering which fundamentals are scapegoats. In particular, we assign binary indicators to each of the potential scapegoat terms in the exchange rate regression (see, e.g., George and McCulloch, 1993; Frühwirth-Schnatter and Wagner, 2010). These are equal to one if a particular fundamental can be considered a scapegoat and equal to zero if the fundamental does not enter the regression equation as a scapegoat. We sample these binary indicators together with the other parameters using the Gibbs sampler. From the sampled indicators, we compute the posterior probabilities that the included fundamentals are scapegoats.

The results suggest, first, that there is a persistent but stationary exchange risk premium or time-varying deviation from the UIRP condition in all countries considered. Second, we identify a persistent but stationary unobserved component from the "signal" in the model, which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. Third, we find that over the sample period the structural parameters on the macro fundamentals are constant and often close to zero. Fourth, as far as the scapegoat terms in the exchange rate equation are concerned, we calculate posterior probabilities that these macro fundamentals are scapegoats, and we find, for the inflation rate in five out of eight countries, probabilities that are considerably higher than the imposed prior probabilities of 0.5.

These countries are the three Anglo-Saxon economies (Australia, Canada and the UK) and South Korea and South Africa. We find little evidence to suggest that the other macro fundamentals we consider are scapegoats, however.

The paper is structured as follows. Section 2 presents the scapegoat model and derives a testable exchange rate equation from the model. Section 3 shows how to implement the estimation of this equation in a number of steps. Section 4 discusses the choice of macro fundamentals, the data used and the Bayesian estimation method (i.e., the outline of the Gibbs sampler and the imposed parameter priors). Section 5 presents and discusses the estimation results of the various steps. Section 6 concludes.

2 Theory

We consider an interest rate parity condition between a local and a benchmark economy and an equation that contains determinants of the interest rate differential between these economies:

$$E_t(s_{t+1}) - s_t = \bar{i}_t + z_t \tag{1}$$

$$\bar{i}_t = \mu[s_t - f_t\beta_t - x_t] \tag{2}$$

where s_t is the log nominal exchange rate (expressed as the amount of local currency per unit of benchmark currency), E_t denotes the rational expectations operator conditional on time t information, $\bar{i}_t$ is the short-term nominal interest rate differential between the local country and the benchmark country, z_t is the exchange risk premium or deviation from UIRP, f_t is a $1 \times K$ vector of observed macroeconomic fundamentals with β_t the $K \times 1$ vector of corresponding time-varying parameters, and x_t is an unobserved fundamental or component. As noted by Engel and West (2005), eq.(2) may represent the interest rate differential as obtained from the differential in Taylor rules between the local and the benchmark economies. Alternatively, it may represent the interest rate differential as obtained from the reduced-form monetary model of exchange rates, i.e., obtained by combining a purchasing power parity condition with money market equilibrium in both countries. In the Taylor rule model, the unobserved component represents a relative shock to the Taylor rules and/or potentially omitted Taylor rule terms. In the monetary model, the unobserved component represents an unobserved relative money demand shock, possibly augmented with a real exchange rate shock. We depart from the standard exchange rate framework of Engel and West (2005) by assuming that some parameters in the model are unknown (see, e.g. Bacchetta and van Wincoop, 2009, 2013). In particular, we assume that the parameter vector β_t is unknown; i.e., we have $E_t(\beta_{kt}) \neq \beta_{kt}$ for $k = 1, ..., K$. With respect to the parameter μ, we follow Bacchetta and van Wincoop (2009) and Bacchetta and van Wincoop (2013) and assume that μ is known and constant.

We define the signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$, which agents know since they observe s_t, $\bar{i}_t$ and μ. From eq.(2) we then have

$$y_t = f_t\beta_t + x_t \tag{3}$$

If β_t were known, agents could infer the value of the unobserved component x_t. Given that β_t is not known, y_t gives an imperfect signal of β_t because of the unobserved component x_t.

Combining eqs.(1), (2) and (3) and solving forward then gives

$$s_t = (1-\lambda)\left[y_t + \sum_{j=1}^{\infty}\lambda^j E_t(y_{t+j})\right] - \lambda\left[z_t + \sum_{j=1}^{\infty}\lambda^j E_t(z_{t+j})\right]$$

or

$$s_t = (1-\lambda)\left[f_t\beta_t + x_t + \sum_{j=1}^{\infty}\lambda^j E_t(f_{t+j}\beta_{t+j} + x_{t+j})\right] - \lambda\left[z_t + \sum_{j=1}^{\infty}\lambda^j E_t(z_{t+j})\right] \tag{4}$$

where the result is obtained by imposing the transversality condition $\lambda^{\infty}E_t(s_{t+\infty}) = 0$ and where we define the discount factor λ as $\lambda \equiv \frac{1}{1+\mu}$.

We assume that the unobserved variables x_t and z_t follow AR(1) processes and that the observed macroeconomic fundamentals f_t and corresponding time-varying unknown parameters β_t follow random walk processes, i.e.,

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x \tag{5}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{6}$$

$$f_{kt} = f_{k,t-1} + \varepsilon_{kt}^f \qquad k = 1, \ldots, K \tag{7}$$

$$\beta_{kt} = \beta_{k,t-1} + \varepsilon_{kt}^\beta \qquad k = 1, \ldots, K \tag{8}$$

where all processes are assumed to be mutually independent. Using these processes in eq.(4) then gives

$$s_t = (1-\Psi)f_t\beta_t + \Psi f_t E_t(\beta_t) + (1-\Psi)x_t + \Phi z_t \tag{9}$$

where $\Psi \equiv \frac{\lambda(1-\rho_x)}{1-\rho_x\lambda}$ and $\Phi \equiv -\frac{\lambda}{1-\rho_z\lambda}$. We refer to Appendix A for the derivation. The first term, $(1-\Psi)f_t\beta_t$, captures the standard impact of macro fundamentals on the exchange rate s_t via the time-varying structural parameters β_t. The second term, $\Psi f_t E_t(\beta_t)$, captures the impact of macro fundamentals on s_t through the occurrence of scapegoat effects as captured by the expectations about the unknown parameters, i.e., through $E_t(\beta_t)$. The third term, $(1-\Psi)x_t$, captures the role of the unobserved component x_t. The fourth term, Φz_t, captures the impact of UIRP deviations or exchange rate risk premiums z_t. It should be noted that for the scapegoat effects to enter the model, three conditions must be fulfilled. First, the discount factor λ must be nonzero as otherwise $\Psi = 0$ and the scapegoat term drops out of the model. Second, the unobserved variable x_t must be stationary; otherwise, if $\rho_x = 1$, we have $\Psi = 0$ and the scapegoat term drops out of the model.[3] Third, the parameters in β_t must be unknown; i.e., β_t must be different from $E_t(\beta_t)$. The first condition is met as the literature finds that λ is positive and typically close to 1 (see, e.g. Engel and West, 2005; Sarno and Sojli, 2009). Our results show that the second condition is also met as the estimates that we obtain for ρ_x are well below 1. The third condition constitutes the focus of this paper. In the empirical section we test whether proxies used for the expected parameters on macro fundamentals $E_t(\beta_t)$ have an impact on the exchange rate.

The estimation of eq.(9) is conducted in four steps, which are discussed one by one in the next section.

3 Empirical implementation

This section explains how the estimation of eq.(9) is implemented. In Section 3.1, the estimation of the UIRP deviation z_t using a state-space approach is discussed. In Section 3.2, we explain how to estimate the time-varying structural parameters β_t as well as the unobserved component x_t from the signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ also using a state-space approach. In Section 3.3, we discuss how the parameters Ψ and Φ are estimated. Finally, in Section 3.4, we use the estimates obtained for z_t, x_t, β_t, Ψ and Φ in eq.(9) and then estimate the scapegoat term $\Psi f_t E_t(\beta_t)$ using survey data to proxy $E_t(\beta_t)$. We note that the Gibbs sampler approach discussed in Section 4 below incorporates the parameter uncertainty of the first three steps into the estimation of eq.(9) as the scapegoat effects are calculated conditional on the full posterior distributions obtained for z_t, x_t, β_t, Ψ and Φ.

3.1 Estimating the exchange risk premium z_t

To calculate the time-varying deviation from UIRP or exchange rate risk premium z_t, we estimate a state-space system consisting of the following equations:

$$\Delta s_{t+1} - \bar{i}_t = z_t + \varepsilon^s_{t+1} \qquad \varepsilon^s_{t+1} \sim iid\left(0, \sigma^2_s\right) \tag{10}$$

$$z_{t+1} = \rho_z z_t + \varepsilon^z_{t+1} \qquad \varepsilon^z_{t+1} \sim iid\left(0, \sigma^2_z\right), z_1 \sim iid\left(0, \frac{\sigma^2_z}{1-\rho^2_z}\right) \tag{11}$$

The observation equation, eq.(10), equals the interest parity condition, eq.(1), in the model. This can be seen by taking expectations in period t from both sides of eq.(10) and noting that $E_t(\varepsilon^s_{t+1}) = 0$. It relates the observed variable $\Delta s_{t+1} - \bar{i}_t$ to the unobserved variable z_t. The state equation, eq.(11), is eq.(6) rewritten for period $t+1$. We refer to Appendix C for the exact specification of the state-space

[3]More specifically, as shown by Bacchetta and van Wincoop (2013), the observed fundamentals f_t and the unobserved component x_t can both follow AR(1) processes (with an AR parameter potentially equal to 1) but the AR parameters of both processes should be different.

model. Estimation of this system provides estimates of z_t. Given z_t, we can then calculate ρ_z and σ_z^2 from a simple AR(1) regression on z_t, and we can calculate σ_s^2, which is the variance of the error term $\varepsilon_{t+1}^s = \Delta s_{t+1} - \bar{i}_t - z_t$. The estimates of z_t and ρ_z are then used in the estimation of eq.(9). Note that we implicitly assume $0 < \rho_z < 1$. If $\rho_z = 0$, no distinct identification of z_t versus ε_{t+1}^s is possible. If $\rho_z = 1$, estimation is possible after adjusting the initialization for z_1. However, this is not necessary as the literature reports that UIRP deviations tend to be stationary (see, e.g., Carriero, 2006; Byrne and Nagayasu, 2012). When estimating eqs. (10) and (11), we do indeed find that $0 < \rho_z < 1$, as we report below.

3.2 Estimating the time-varying structural parameters $\boldsymbol{\beta_t}$ and the unobserved component $\boldsymbol{x_t}$

We estimate the time-varying structural parameters β_t using eq.(3), where the signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ can be calculated from s_t, $\bar{i}_t$ and the known parameter μ. Since from the model discussed in Section 2 we have $\lambda \equiv \frac{1}{1+\mu}$, we can use a value for μ obtained from estimates reported in the literature for λ. Sarno and Sojli (2009) report an average monthly discount factor of 0.989, which then amounts to setting $\lambda = 0.967$ in quarterly data.[4] This value for λ implies setting $\mu = 0.034$.

When y_t is calculated, we estimate the following state-space model to obtain estimates for the time-varying structural parameters β_{kt} where $k = 1, \ldots, K$:

$$(1-\rho_x L)y_t = (1-\rho_x L)f_t\beta_t + \varepsilon_t^x \qquad \varepsilon_t^x \sim iid\left(0, \sigma_x^2\right) \tag{12}$$

$$\beta_{k,t+1} = \beta_{kt} + \varepsilon_{k,t+1}^{\beta} \qquad \varepsilon_{k,t+1}^{\beta} \sim iid\left(0, \sigma_{\beta_k}^2\right), \beta_{k1} \sim iid\left(0, 10^6\right) \tag{13}$$

where eq.(12) is the observation equation, which relates the observed signal y_t to the unobserved states β_t. Eq.(12) equals eq.(3) premultiplied by $(1 - \rho_x L)$ (with L the lag operator), a transformation that guarantees that the observation equation has a noise shock ε_t^x as, from eq.(5), we have $(1 - \rho_x L)x_t = \varepsilon_t^x$. The state equation, eq.(13), is eq.(8) rewritten for period $t + 1$. Since the state β_{kt} follows a random walk, its initialization is diffuse. We refer to Appendix C for the exact specification of the state-space model. Estimation of this system provides estimates of β_t. Given these, we can calculate estimates for the variances $\sigma_{\beta_k}^2$. Estimates for the unobserved component x_t are then obtained by noting that $x_t = y_t - f_t\beta_t$. Given estimates for x_t, we can then calculate ρ_x and σ_x^2 from an AR(1) regression on x_t. Note that estimates for ρ_x should be smaller than 1 because, as noted in Section 2, a non-stationary x_t implies that scapegoat effects drop out of the model. As reported below, we do indeed find that $0 < \rho_x < 1$. The obtained estimates of β_t and x_t are then used in the estimation of eq.(9).

[4] This is obtained from $\lambda = (0.989)^{\frac{12}{4}}$.

3.3 Estimating the parameters Ψ and Φ

The parameters Ψ and Φ in eq.(9) are given by $\Psi \equiv \frac{\lambda(1-\rho_x)}{1-\rho_x\lambda}$ and $\Phi \equiv -\frac{\lambda}{1-\rho_z\lambda}$, respectively. Hence, to estimate Ψ and Φ we need estimates for ρ_z, ρ_x and λ. Estimates for ρ_z and ρ_x are obtained when estimating z_t and x_t, respectively, as detailed in Sections 3.1 and 3.2. For λ, as noted in Section 3.2, we use the estimates reported by Sarno and Sojli (2009) and set $\lambda = 0.967$. When calculating the posterior distributions of Ψ and Φ, we keep λ fixed so that the posteriors of Ψ and Φ incorporate only the dispersion contained in the posterior distributions of ρ_x and ρ_z, respectively. We find, however, that our results are robust to imposing slightly different values for the discount factor λ.[5]

3.4 Estimating the scapegoat effects $E_t(\beta_t)$

Using the estimates obtained for z_t, x_t, β_t, Ψ and Φ in Sections 3.1, 3.2 and 3.3, we rewrite eq.(9) as

$$\tilde{s}_t = \tilde{f}_t E_t(\beta_t) \tag{14}$$

where $\tilde{s}_t \equiv s_t - (1-\hat{\Psi})f_t\hat{\beta}_t - (1-\hat{\Psi})\hat{x}_t - \hat{\Phi}\hat{z}_t$ and $\tilde{f}_t \equiv \hat{\Psi} f_t$. Upon noting that $\tilde{f}_t$ and $E_t(\beta_t)'$ are $1 \times K$ vectors, we can write

$$\tilde{s}_t = \sum_{k=1}^{K} E_t(\beta_{kt})\tilde{f}_{kt} \tag{15}$$

where $k = 1, ..., K$. Following Fratzscher et al. (2015), we proxy the scapegoat effects $E_t(\beta_{kt})$ by setting $E_t(\beta_{kt}) = \phi_k \tau_{kt}$ for $k = 1, ..., K$, where τ_{kt} is a survey outcome denoting the weight attached to fundamental k by investors in period t and where ϕ_k captures the impact of τ_{kt} on the exchange rate.[6] Assuming that τ_{kt} is a good proxy for the scapegoat effect $E_t(\beta_{kt})$, if the macro fundamental f_{kt} functions as a scapegoat in the exchange rate determination, we should find a nonzero ϕ_k for this fundamental. Hence, eq.(15) becomes

$$\tilde{s}_t = \sum_{k=1}^{K} \phi_k \tau_{kt} \tilde{f}_{kt} \tag{16}$$

We then add an intercept and an error term to the equation, which gives

$$\tilde{s}_t = c + \sum_{k=1}^{K} \phi_k \tau_{kt} \tilde{f}_{kt} + \varepsilon_t \tag{17}$$

where c is a constant and ε_t is a zero-mean error term. Next, as a model selection device, we add binary indicators δ_k to each of the K scapegoat terms (see George and McCulloch, 1993; Frühwirth-Schnatter and Wagner, 2010). If fundamental k can be considered a scapegoat, then $\delta_k = 1$. Otherwise, $\delta_k = 0$.

[5] These results are unreported but are available from the authors upon request.

[6] A constant added to the specification for $E_t(\beta_{kt})$ was generally found to be equal to zero.

From the sampled indicator δ_k (for $k = 1, ..., K$) we can then calculate the posterior probability that fundamental k is a scapegoat. Our estimable test equation is now given by

$$\tilde{s}_t = c + \sum_{k=1}^{K} \delta_k \phi_k \tau_{kt} \tilde{f}_{kt} + \varepsilon_t \tag{18}$$

Finally, if the model fits the data well, the error term ε_t should be independent and identically distributed (*iid*). Our results suggest, however, that there is residual autocorrelation. This is not surprising since we include only a limited number of fundamentals in the equation, i.e., fundamentals k for which we have survey data τ_{kt} available to proxy $E_t(\beta_{kt})$. Calculated autocorrelation and partial autocorrelation functions applied to estimates for ε_t obtained under the assumption that ε_t is *iid* suggest that, for all currencies, there is substantial first-order autocorrelation of the autoregressive form (see Appendix B). To deal with this when estimating eq.(18), we explicitly model this autocorrelation so that we have,

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \varepsilon_t^* \qquad \varepsilon_t^* \sim iid\left(0, \sigma_\varepsilon^2\right) \tag{19}$$

Upon multiplying both sides of eq.(18) by $(1 - \rho_\varepsilon L)$, we obtain

$$(1 - \rho_\varepsilon L)\tilde{s}_t = c^* + \sum_{k=1}^{K} \delta_k \phi_k (1 - \rho_\varepsilon L) \tau_{kt} \tilde{f}_{kt} + \varepsilon_t^* \tag{20}$$

where $c^* = (1 - \rho_\varepsilon L)c = (1 - \rho_\varepsilon)c$ and where the regression error term ε_t^* is now an *iid* shock so that Bayesian Ordinary Least Squares (OLS) can be applied to estimate this equation. Technical details are provided in Appendix C.

4 Estimation method

In this section we discuss the macroeconomic fundamentals included in f_t that could potentially be scapegoats. Then we discuss the data used and its sources. Finally, we elaborate on the Bayesian estimation method; i.e., we discuss the Gibbs sampler and the assumed parameter priors.

4.1 Choice of macroeconomic fundamentals f_t that can be scapegoats

We include macroeconomic fundamentals in f_t that can be expected to have an impact on both the interest rate differential as given by eq.(2) and the exchange rate as given by eq.(9). Additionally, as these variables are the ones that can become scapegoats according to the model—i.e., the parameters β_t on f_t are unknown, and therefore we can have $E_t(\beta_t) \neq \beta_t$—we choose to include macroeconomic fundamentals in f_t for which proxies are available for $E_t(\beta_t)$.

Following Fratzscher et al. (2015), we proxy $E_t(\beta_t)$ by survey data from Consensus Economics (see

Section 4.2).[7] As such, we use the following fundamentals in f_t for which survey data from Consensus Economics are available. First, the real GDP growth rate differential between the local and the benchmark country, $\bar{g}_t$. Second, the inflation rate differential between the local and the benchmark country, $\bar{\pi}_t$. Third, the long-term nominal interest rate differential between the local and the benchmark country, $\bar{i}_t^L$.[8] Fourth, the current account balance to GDP ratio of the local country, ca_t (where the latter is not considered in deviation from the benchmark country).[9]

We have also conducted estimations where f_t contains variables for which we have no survey data to proxy $E_t(\beta_t)$—i.e., an intercept, the money supply differential between the local and benchmark economies and the one-period lagged short-term nominal interest rate differential (see Bacchetta and van Wincoop, 2009, 2013). For these variables, by necessity, we have to assume that $E_t(\beta_t) = \beta_t$ (for all t). The results obtained when these additional fundamentals are included in the regression equation do not differ much from our reported results. Hence, we do not report them, but they are available from the authors upon request.

4.2 Data

We use quarterly data over the period 2002Q1-2014Q4 so that the sample size equals $T = 52$. The availability of the surveys τ_{kt} determines the sample period chosen (see below). We conduct estimations using data for five developed or industrialized economies (Australia, Canada, the euro area, Japan and the United Kingdom) and for three emerging economies (Singapore, South Africa and South Korea). The data for exchange rates and all macro fundamentals used in the baseline estimations, with the exception of the long-term interest rate, are taken from Oxford Economics via Datastream. Data for the long-run interest rates are taken from national sources.[10]

The currencies considered are the Australian dollar, the Canadian dollar, the euro, the Japanese yen, the UK pound, the Singapore dollar, the South African rand and the Korean won. All exchange rate

[7] An alternative proxy could be $E_t(\beta_t)$ as estimated from the Kalman filter applied when estimating the state-space system given by eqs. (12) and (13). However, the Kalman filter output cannot be considered a good proxy for $E_t(\beta_t)$ as it tends to converge to the "smoother" $E_T(\beta_t)$ when filtering nears the end of the sample period, where $E_T(\beta_t)$ is basically what is used to estimate the time-varying structural parameter β_t. While the scapegoat model *does* predict that $E_t(\beta_t)$ tends to β_t when the scapegoat effects wear off (see Fratzscher et al., 2015), there is of course no reason for this convergence to occur only and precisely at the end of the sample period.

[8] We do not add the short-term interest rate differential $\bar{i}_t$ in f_t since it appears on the left hand side (LHS) of eq.(2).

[9] Note that the variables $\bar{g}_t$, $\bar{\pi}_t$ and $\bar{i}_t^L$ are, given that $\bar{i}_t^L$ can be considered a proxy for inflation expectations, in accordance with variables one would include in eq.(2) under a Taylor rule differential interpretation of eq.(2). The variables $\bar{g}_t$, $\bar{\pi}_t$ and ca_t are also in accordance with the variables one would use in a reduced-form monetary model of exchange rates (see, e.g. Meese and Rogoff, 1983a; Cheung et al., 2005). We note that the expected signs of the coefficients β_t on the fundamentals f_t can vary according to the assumed underlying model.

[10] I.e., from the Reserve Bank of Australia, the Bank of Canada, Eurostat, the Ministry of Finance of Japan, the Bank of England, the US Treasury, the Bank of Korea, the South African Reserve Bank and the Monetary Authority of Singapore.

data are expressed versus the US dollar. In particular, the (log) nominal exchange rate s_t is expressed as (the log of) the amount of local currency that one US dollar is worth.

The other variables are calculated as follows. For $\bar{i}_t$ we use the short-term nominal interest rate (relative to the US), for $\bar{g}_t$ we use real GDP growth (relative to the US), for the inflation rate $\bar{\pi}_t$ we use the quarterly change in the consumer price index (relative to the US), for $\bar{i}_t^L$ we use the yield on 10-year government bonds relative to the US as a proxy for the long-term interest rate differential, and for ca_t we use the ratio of the current account balance to GDP where a positive value of ca_t indicates a surplus.

Data for the survey outcomes τ_{kt}—i.e., the weights attached by exchange market participants to macro fundamentals in the determination of the exchange rate—are taken from Consensus Economics. In their survey, 40 to 60 exchange market participants are asked to rank the current importance of macro fundamentals for the exchange rate of a country (versus the US dollar) where each fundamental receives a score from 0 (no influence) to 10 (very strong influence). The survey scores are available over the sample period for all currencies in our sample for the macro fundamentals included in f_t, i.e., for $\bar{g}_t$, $\bar{\pi}_t$, $\bar{i}_t^L$ and ca_t. Since they are only available on a biannual basis, we use linear interpolation to obtain a quarterly series over the period 2002Q1-2014Q4.[11]

While some papers in the literature model exchange rates at the monthly frequency (see, e.g., Bacchetta and van Wincoop, 2013; Fratzscher et al., 2015), we prefer to use quarterly data for three reasons. First, this avoids the need to interpolate quarterly macro data to obtain macro data at the monthly frequency. Second, since the survey data that are used as proxies for the parameter expectations $E_t(\beta_t)$ are only available on a biannual basis, it makes more sense to consider the model at the quarterly frequency rather than at the monthly frequency. Third, we can use real GDP—which is available at the quarterly but not at the monthly frequency—to construct $\bar{g}_t$ instead of having to use industrial production.

4.3 Bayesian estimation

Bayesian methods are used to estimate eq.(9). In particular, we use a Gibbs sampling approach, which is a Markov chain Monte Carlo method used to simulate draws from the intractable joint and marginal posterior distributions of the parameters and unobserved states using only tractable conditional distributions. As described in Section 3, estimation is conducted in different steps, with steps 1 and 2 (Sections 3.1 and 3.2) being fully independent, step 3 (Section 3.3) using the results of steps 1 and 2, and step 4 (Section 3.4) using the results obtained in steps 1, 2 and 3. A Bayesian method is advantageous when compared with classical methods like maximum likelihood because the full posterior distributions of pa-

[11] We find very similar results when instead we construct a quarterly series by assigning the last-available survey score to the quarter in which the survey is not conducted. These results are not reported but are available upon request.

rameters and states are calculated in every step and can be used in the next steps. Hence, the parameter uncertainty of the first two steps can be incorporated into step 3, and the parameter and state uncertainty of the first three steps can be incorporated into the estimation of eq.(9) in step 4. Additionally, our Bayesian approach allows us to do model selection, i.e., compute the posterior probabilities that the macro fundamentals included in eq.(9) are scapegoats. Finally, a Bayesian approach can be conducted without making specific assumptions about the orders of integration of the variables used in the analysis. As a Bayesian analysis relies on sampling posterior distributions rather than on using asymptotic approximations, statistical inference in the presence of non-stationarity variables is less complicated compared to inference conducted in a classical setting.

The general outline of the Gibbs sampler is presented in Section 4.3.1, while technical details about the implementation of the Gibbs sampler are relegated to Appendix C. The Bayesian parameter priors are then discussed in Section 4.3.2.

4.3.1 Gibbs sampler

The Gibbs sampling scheme is as follows:

1. Sample the exchange rate risk premium z_t and parameters from the state-space model eqs. (10) and (11). First, sample the state z_t conditional on the data and the parameters in the system, namely σ_s^2, ρ_z and σ_z^2. To this end, the Bayesian state-space approach with multimove sampling of Carter and Kohn (1994) and Kim and Nelson (1999) is implemented (i.e., the forward-filtering, backward-sampling approach). Second, sample the parameter σ_s^2 conditional on the data and the state z_t using a Bayesian OLS regression approach applied to eq.(10) (see, e.g. Bauwens et al., 2000). Third, sample ρ_z and σ_z^2 conditional on the data and the state z_t using a Bayesian OLS regression approach applied to eq.(11).

2. Sample the time-varying structural parameters β_t from the state-space model of eqs. (12) and (13). First, sample the K states β_t conditional on the data and the parameters in the system, namely ρ_x, σ_x^2 and $\sigma_{\beta_k}^2$, using the Bayesian state-space approach with multimove sampling of Carter and Kohn (1994) and Kim and Nelson (1999). Second, obtain estimates for the unobserved component x_t from $x_t = y_t - f_t\beta_t$. Third, sample the parameter $\sigma_{\beta_k}^2$ conditional on the state β_{kt} (for $k = 1, ..., K$) using a Bayesian OLS regression approach applied to eq.(13). Fourth, sample the parameters ρ_x and σ_x^2 conditional on x_t using a Bayesian OLS regression approach applied to eq.(5).

3. Calculate the parameters $\Psi = \frac{\lambda(1-\rho_x)}{1-\rho_x\lambda}$ and $\Phi = -\frac{\lambda}{1-\rho_z\lambda}$ using the sampled values for ρ_z and ρ_x and the imposed value for λ, i.e., $\lambda = 0.967$.

4. (a) Using estimates for z_t, x_t, β_t, Ψ and Φ, calculate $\tilde{s}_t \equiv s_t - (1-\hat{\Psi})f_t\hat{\beta}_t - (1-\hat{\Psi})\hat{x}_t - \hat{\Phi}\hat{z}_t$ and $\tilde{f}_t \equiv \hat{\Psi} f_t$.

   (b) For $k = 1, ..., K$, sample the binary indicator δ_k using eq.(20) while marginalizing over the parameter vector ϕ_k over which model selection is carried out. The approach follows the stochastic variable selection procedure for regressions by George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010).

   (c) Jointly sample c^*, σ_ε^2 and the slope coefficients ϕ_k, for which the corresponding binary indicators δ_k are equal to 1 via a Bayesian OLS regression approach applied to eq.(20). Set the slope coefficients ϕ_k to 0 if the corresponding binary indicators δ_k are equal to 0.

   (d) Calculate the estimates of the residuals ε_t from $\varepsilon_t = \tilde{s}_t - c - \sum_{k=1}^{K} \delta_k \phi_k \tau_{kt} \tilde{f}_{kt}$. Sample the AR coefficient ρ_ε for given variance σ_ε^2 using Bayesian OLS applied to eq.(19).

Sampling from these steps is iterated D times, and, after a sufficiently large number of burn-in draws B, the sequence of draws $(B+1, ..., D)$ approximates a sample from the posterior distributions of the sampled quantities. The results reported below are based on $D = 20,000$ iterations with the first $B = 10,000$ draws discarded as a burn-in sequence; i.e., the reported results are based on posterior distributions constructed from $D - B = 10,000$ draws.

4.3.2 Parameter priors

For the regression parameters—i.e., ρ_z, ρ_x, c^*, ϕ and ρ_ε—we use a Gaussian prior $\mathcal{N}(b_0, V_0)$ defined by setting the prior mean b_0 and prior variance V_0. For the variance parameters—i.e., σ_s^2, σ_z^2, σ_x^2, $\sigma_{\beta_k}^2$ (for $k = 1, ..., K$) and σ_ε^2—we use the Inverse Gamma prior $\mathcal{IG}(c_0, C_0)$ where the shape $c_0 = \nu_0 T$ and scale $C_0 = c_0\sigma_0^2$ parameters are calculated from the prior belief σ_0^2 about the variance parameter and the prior strength ν_0, which is expressed as a fraction of the sample size T.[12] More details are provided in Appendix C. For the binary indicators δ_k (with $k = 1, ..., K$) in eq.(20) that determine which fundamentals are scapegoats, we choose Bernoulli prior distributions where every indicator δ_k has a prior probability p_0 of being equal to 1, i.e., $p(\delta_k = 1) = p_0$ (for $k = 1, ..., K$).

With respect to the parameter priors used when estimating the state-space system given by eqs. (10) and (11), we set $b_0 = 0$ and $V_0 = 1$ for the AR(1) parameter ρ_z so that the prior distribution covers the full range of possible values for this parameter. For both variance parameters σ_s^2 and σ_z^2, we set the prior belief equal to half the unconditional variance of the data series $\Delta s_{t+1} - \bar{i}_t$; i.e., for σ_s^2 and σ_z^2 we set

[12] Since this prior is conjugate, $\nu_0 T$ can be interpreted as the number of fictional observations used to construct the prior belief σ_0^2.

$\sigma_0^2 = 0.5 \times V(\Delta s_{t+1} - \bar{i}_t)$. The strength is set equal to $\nu_0 = 0.01$ for both variances, which amounts to imposing a very loose prior.

With respect to the parameter priors used when estimating the state-space system given by eqs. (12) and (13), we set $b_0 = 0$ and $V_0 = 1$ for the AR(1) parameter ρ_x so that the prior distribution covers the full range of possible values for this parameter. For the variance parameter σ_x^2, we set the prior belief equal to half the unconditional variance of the variable $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$, which follows eq.(3); i.e., for σ_x^2 we set $\sigma_0^2 = 0.5 \times V(y_t)$. For the variances $\sigma_{\beta_k}^2$ (for $k = 1, ..., K$), we set belief $\sigma_0^2 = 0.01$, which is not too low and not too high to allow for slow structural movement in β_k without imposing that β_k is constant. The strength is set equal to $\nu_0 = 0.01$ for all variances so that the priors are given little weight in the estimation results.

With respect to the parameters of regression eq.(20), we set $p_0 = 0.5$ for every binary indicator δ_k (with $k = 1, ..., K$), which amounts to assuming that there is an a priori 50% chance that fundamental k is a scapegoat. For the intercept c^* and the regression slope parameters ϕ_k that are included in the regression (i.e., those for which $\delta_k = 1$), we set $b_0 = 0$ and $V_0 = 10$, which allows for a wide range of possible estimates for c^* and ϕ_k. For the regression error variance σ_ε^2, we set belief $\sigma_0^2 = 0.01$ and strength $\nu_0 = 0.01$, which, again, implies a loose prior imposed on a variance, in this case on σ_ε^2.

With respect to the AR parameter of regression eq.(19), we set $b_0 = 0$ and $V_0 = 1$ for ρ_ε so that the prior distribution covers the full range of possible values for this parameter. Note that we sample ρ_ε given the sampled value of σ_ε^2 obtained when estimating eq.(20).

5 Results

5.1 Estimates of the exchange rate risk premium

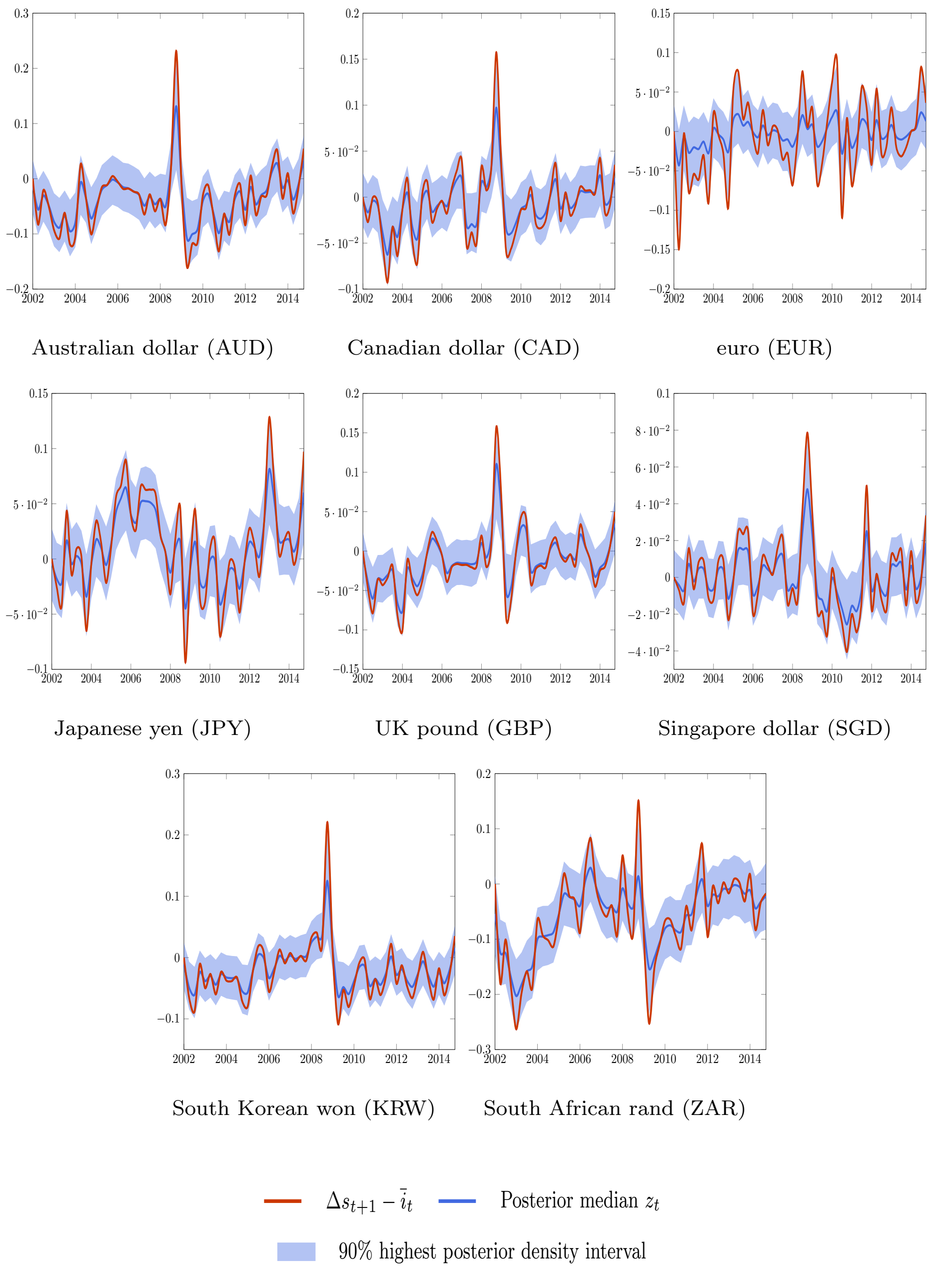
Table 1 presents the parameter estimates of the state-space system in eqs. (10) and (11), while Figure 1 presents the signal $\Delta s_{t+1} - \bar{i}_t$ and the estimated exchange risk premium z_t for all eight currencies in the sample. From the table we note that for all currencies the AR(1) parameter lies between 0.13 and 0.84, suggesting that there is a persistent though stationary deviation of the UIRP condition in the model. This result is in line with results reported in the literature (see, e.g., Carriero, 2006; Byrne and Nagayasu, 2012). As the scapegoat model discussed in Section 2 shows that z_t enters the exchange rate equation derived from the model, i.e., eq.(9), we condition the estimation of this equation on the estimates obtained for z_t (i.e., on the full posterior distribution of z_t).

Table 1: Posterior distributions of the parameters of the state-space system eqs.(10) and (11)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
ρ_z	0.6232	0.4208	0.1347	0.5971	0.4957	0.3240	0.5817	0.8426
	[0.340; 0.880]	[0.109; 0.689]	[-0.420; 0.622]	[0.262; 0.879]	[0.219; 0.749]	[-0.079; 0.625]	[0.262; 0.865]	[0.643; 0.975]
σ_z^2	0.0023	0.0008	0.0007	0.0008	0.0010	0.0002	0.0012	0.0021
	[0.001; 0.004]	[0.000; 0.001]	[0.000; 0.002]	[0.000; 0.002]	[0.000; 0.002]	[0.000; 0.001]	[0.000; 0.002]	[0.001; 0.005]
σ_s^2	0.0014	0.0005	0.0018	0.0007	0.0005	0.0002	0.001	0.0026
	[0.000; 0.003]	[0.000; 0.001]	[0.000; 0.003]	[0.000; 0.001]	[0.000; 0.001]	[0.000; 0.001]	[0.000; 0.002]	[0.001; 0.005]

Note: Reported are the medians and the 90% highest posterior density intervals of the posterior distributions of the AR parameter and variance parameters of state-space system in eqs.(10) and (11).

Figure 1: The signal $\Delta s_{t+1} - \bar{i}_t$ and the estimated exchange risk premium z_t



5.2 Structural parameters of macro fundamentals and unobserved component

In this section we discuss the results of the estimation of the state-space system eqs.(12) and (13). This estimation provides estimates for the potentially time-varying structural parameters β_t on the macro fundamentals $\bar{g}$, $\bar{\pi}$, $\bar{i}^L$ and ca. These are presented in Figure 3, while the posterior distributions of the estimates of the variances $\sigma^2_{\beta_k}$ (for $k = 1, ..., K$) of the shocks to the random walks β_t are reported in Table 2. Figure 3 shows that—while there is some time-variation in the structural parameters, since the variances reported in Table 2 are positive—the highest posterior density intervals (HPD) around the

β_t's are rather wide, and these parameters can de facto all be considered constant. This means that our theoretical model essentially collapses to the model considered by Bacchetta and van Wincoop (2013) with constant structural parameters. While this result may be specific to the considered sample period (which contains relatively few observations), the result nonetheless seems to suggest that the instability between exchange rates and macro fundamentals and the "disconnect puzzle" cannot be explained simply by imposing time-varying structural parameters in the model.

Table 2: Posterior distributions of the parameters of the state-space system eqs.(12) and (13)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
ρ_x	0.5166	0.6919	0.6650	0.7102	0.4855	0.5031	0.7014	0.6481
	[0.149; 0.807]	[0.306; 0.908]	[0.140; 0.956]	[0.199; 0.944]	[0.149; 0.773]	[0.160; 0.780]	[0.240; 0.941]	[0.393; 0.899]
σ_x^2	0.0476	0.0551	0.1574	1.1264	0.0866	0.0294	6.7532	0.2933
	[0.022; 0.260]	[0.016; 0.729]	[0.031; 6.075]	[0.283; 18.05]	[0.042; 0.263]	[0.015; 0.087]	[2.094; 64.24]	[0.107; 5.283]
$\sigma_\beta^2(\bar{g})$	0.0227	0.0231	0.0195	0.0163	0.0172	0.0205	0.0325	0.0225
	[0.002; 2.586]	[0.003; 1.088]	[0.003; 0.886]	[0.002; 1.079]	[0.002; 0.476]	[0.003; 0.495]	[0.003; 1.894]	[0.003; 4.772]
$\sigma_\beta^2(\bar{\pi})$	0.0168	0.0206	0.0229	0.0185	0.0212	0.0213	0.0265	0.0292
	[0.003; 0.547]	[0.003; 2.459]	[0.003; 4.554]	[0.002; 0.670]	[0.003; 1.393]	[0.003; 1.169]	[0.003; 1.227]	[0.003; 9.418]
$\sigma_\beta^2(\bar{i}^L)$	0.0164	0.0135	0.0280	0.0163	0.0143	0.0235	0.0252	0.0181
	[0.002; 0.650]	[0.002; 1.166]	[0.002; 2.380]	[0.002; 0.360]	[0.002; 1.244]	[0.003; 2.293]	[0.003; 4.118]	[0.002; 1.238]
$\sigma_\beta^2(ca)$	0.0178	0.0437	0.0225	0.0261	0.0452	0.0214	0.0214	0.0270
	[0.002; 0.407]	[0.003; 3.979]	[0.003; 2.251]	[0.003; 6.065]	[0.003; 7.857]	[0.003; 0.124]	[0.002; 1.256]	[0.003; 2.280]

Note: Reported are the medians and the 90% highest posterior density intervals of the posterior distributions of the AR parameter and variance parameters of state-space system in eqs.(12) and (13).

Additionally—and related to the width of the HPD intervals—most structural parameters have HPD intervals that contain the value of 0, suggesting that the impact of the macro fundamentals on the signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ is rather limited. Macro fundamentals that have a clear nonzero structural impact on the exchange rate are the inflation rate $\bar{\pi}$ for Australia and the UK, the long-run interest rate $\bar{i}^L$ for Australia, Japan, the UK and Singapore, and the current account to GDP ratio ca for Australia and the UK. The inflation differential $\bar{\pi}$ has a positive impact, suggesting that higher inflation rates in Australia and the UK versus the US depreciate the exchange rates of these countries; the long-run interest rate $\bar{i}^L$ has a negative impact, suggesting that higher long-run interest rates in Australia, Japan, the UK and Singapore versus the US appreciate those countries' exchange rates. The current account balance ca has a negative impact for Australia and a positive impact for the UK, meaning that a higher current account surplus or a lower deficit appreciates the Australian dollar and depreciates the UK pound.

The estimation of eqs.(12) and (13) then allows us to calculate estimates for the unobserved component x_t from $x_t = y_t - f_t\beta_t$. The estimated AR(1) and variance parameters of this component, i.e., ρ_x and σ_x^2, are reported in Table 2, while Figure 2 presents graphs for the signal y_t and for the posterior median of

x_t and its 90% HPD interval. From Figure 2 we note that the difference between y_t and x_t, which reflects $f_t\beta_t$, is often rather important, suggesting that even though the HPD intervals around the β's are wide and often contain the value of 0, the magnitude of the estimated β's is non-negligible. From Table 2 we note that, for all currencies, the AR(1) parameter is positive and lies between 0.48 and 0.71, suggesting that there is a persistent though stationary unobserved component in the model which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. The existence of this component is a precondition for the potential presence of scapegoat effects. We investigate the presence of scapegoat effects in the next section.

Figure 2: The signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ and the unobserved component x_t

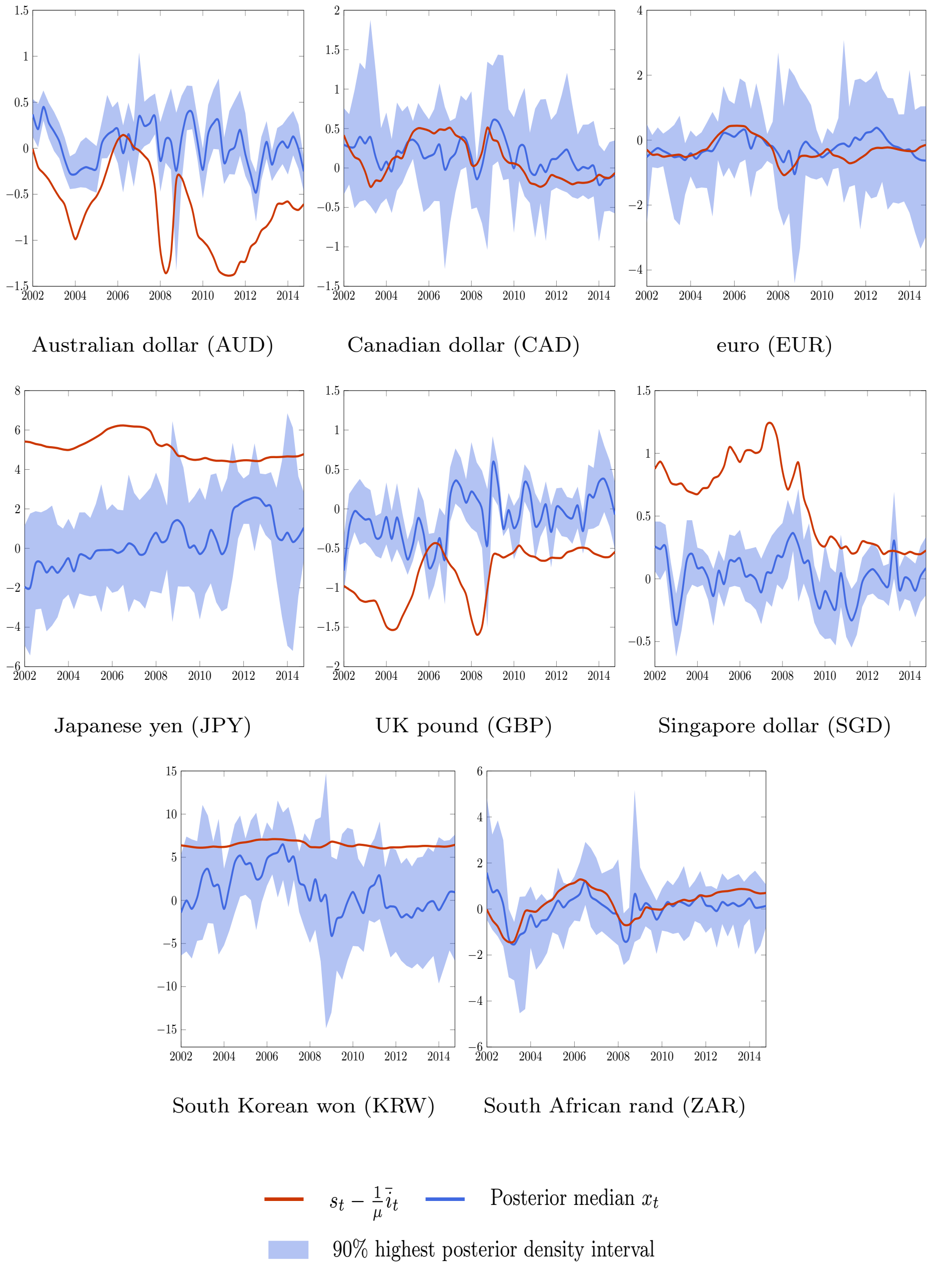
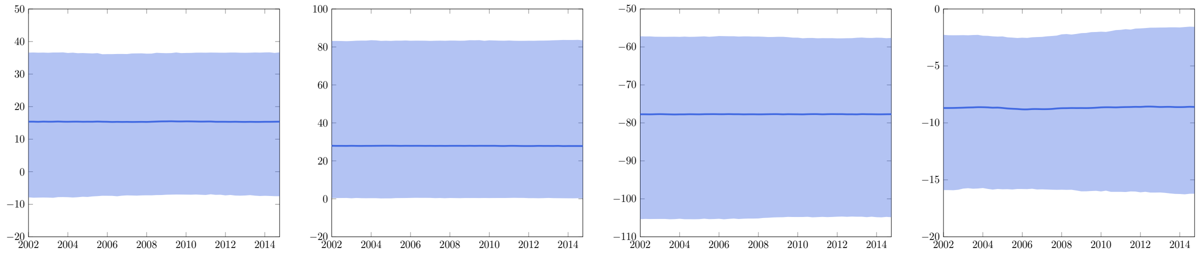
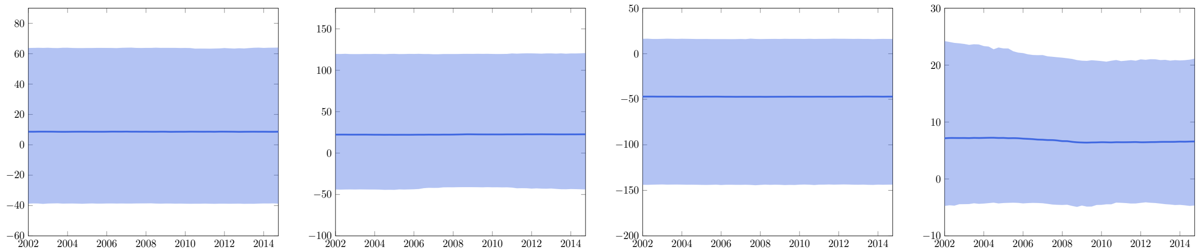


Figure 3: Structural parameters β_{kt}



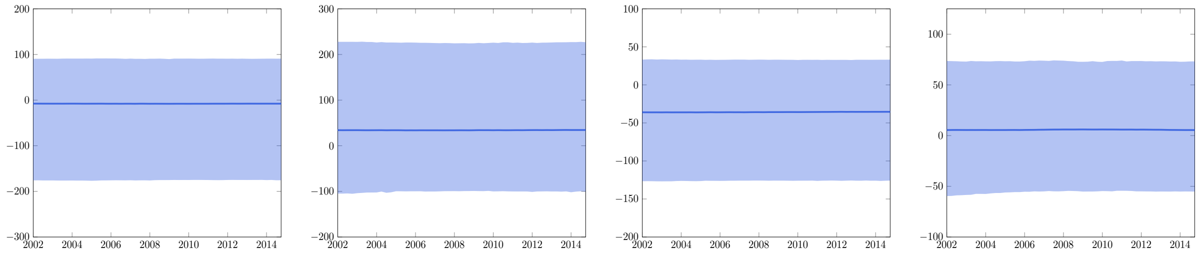
(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Australian dollar (AUD)



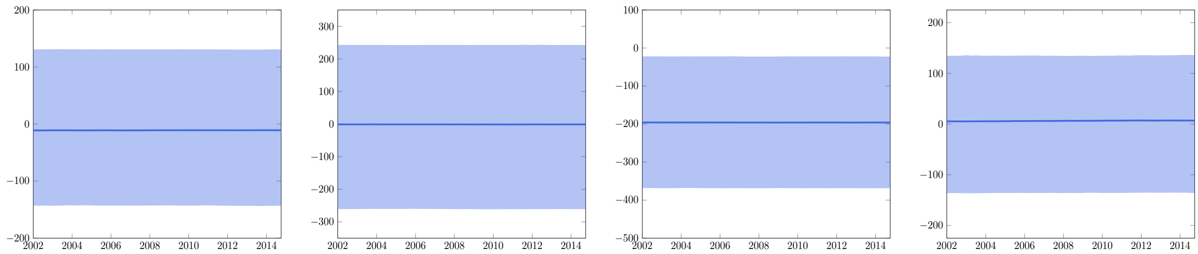
(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Canadian dollar (CAD)



(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

euro (EUR)



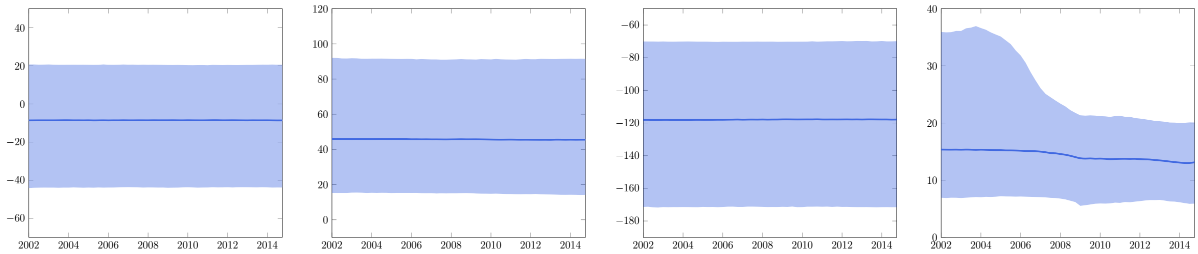
(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Japanese yen (JPY)

Posterior median β_{kt}

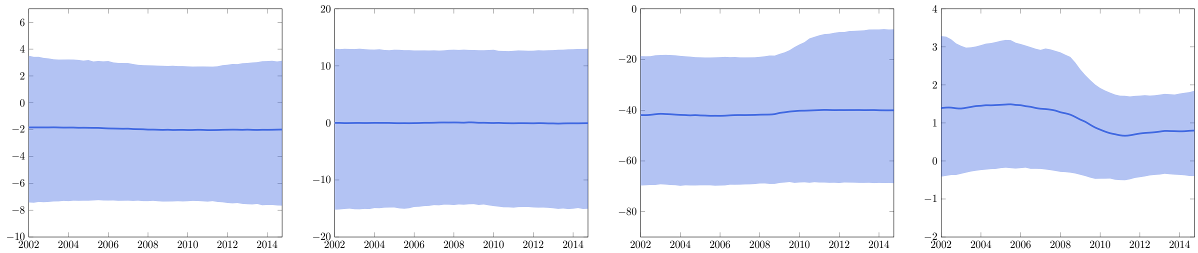
90% highest posterior density interval

Figure 3: Structural parameters β_{kt} (cont'd)



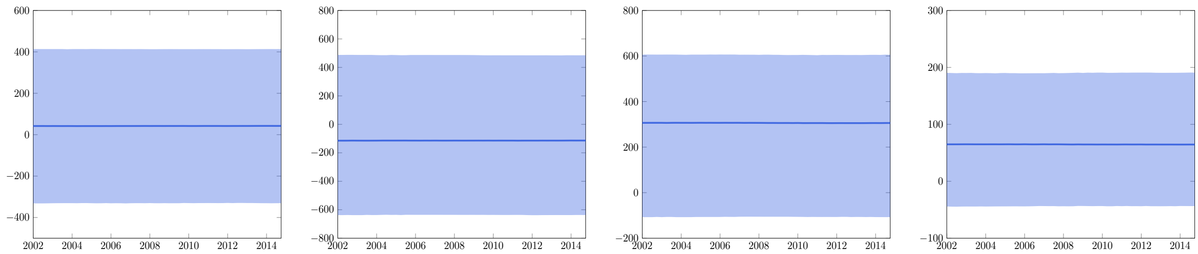
(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

UK pound (GBP)



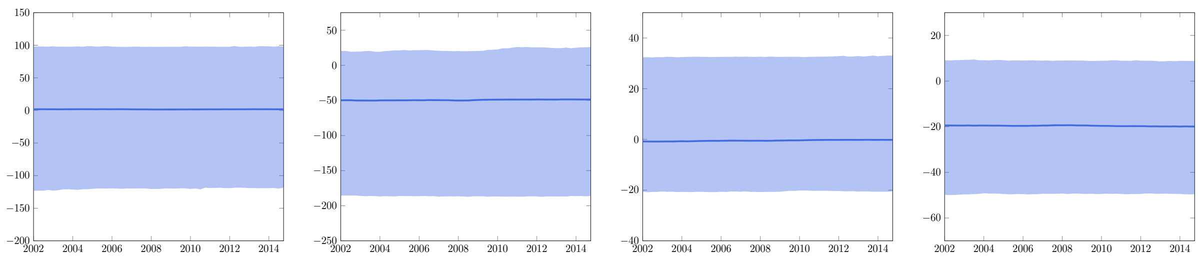
(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Singapore dollar (SGD)



(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

South Korean won (KRW)



(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

South African rand (ZAR)

Posterior median β_{kt}

90% highest posterior density interval

5.3 Scapegoat effects

The results of estimating eq.(20) using Bayesian OLS are reported in Tables 3 and 4. The posterior inclusion probabilities p that the included macro fundamentals $\bar{g}$, $\bar{\pi}$, $\bar{i}^L$ and ca are scapegoats are reported in Table 3. These probabilities are calculated as the average over the iterations of the Gibbs sampler of the binary indicators δ_k included in eq.(20). From the table, we note that only the scapegoat term for the inflation rate $\bar{\pi}$, which is the inflation rate interacted with the Consensus survey score for the inflation rate, tends to *systematically* have a posterior inclusion probability that is higher than the imposed prior probability of $p_0 = 0.5$. This is the case for the Anglo-Saxon countries (Australia, Canada, the UK), where the inclusion probabilities are all close to or even above 0.9, and for South Korea and South Africa, where the inclusion probabilities are lower but still well above 0.5.

Table 3: Posterior probabilities p that fundamentals are scapegoats

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
$\bar{g}$	0.5534	0.3444	0.2488	0.1051	0.1309	0.0347	0.1082	0.3658
$\bar{\pi}$	0.8838	0.9313	0.3713	0.2309	0.9337	0.3159	0.6405	0.7109
$\bar{i}^L$	0.8719	0.5003	0.2605	0.5021	0.3384	0.2098	0.3665	0.3869
ca	0.2858	0.2731	0.1932	0.2998	0.1359	0.0257	0.1057	0.2961

Note: Reported are the posterior probabilities p that the fundamentals $\bar{g}$, $\bar{\pi}$, $\bar{i}^L$ and ca (as defined in Section 4.1) are scapegoats. These probabilities are calculated as the average of the sampled binary indicators δ_k over the iterations of the Gibbs sampler. The prior inclusion probabilities are equal to $p_0 = 0.5$ in all cases.

The estimates for the other parameters of eq.(20) are reported in Table 4. The posterior medians and 90% HPD intervals of the coefficients ϕ_k on the macro fundamentals $\bar{g}$, $\bar{\pi}$, $\bar{i}^L$ and ca interacted with the Consensus survey scores support the results obtained for the posterior probabilities as reported Table 3. For the Anglo-Saxon economies (Australia, Canada and the UK) and for South Korea and South Africa, the impact of the inflation rate interacted with the survey weight as captured by the parameter ϕ_k for inflation is different from zero and positive. Also, while we find posterior inclusion probabilities for the inflation rate below 0.5 for the euro area and Singapore, the estimates for ϕ_k are nonetheless above zero for these countries as well (even though the value of zero is included in the HPD interval). We note further that the structural parameters β_k on the inflation rate are found to be positive in Australia and in the UK as can be seen in Figure 3, while they are essentially zero in Canada, South Korea and South Africa. Hence, the scapegoat effects ϕ_k intensify the impact of the structural parameters β_k, a result that supports the predictions of the scapegoat model. Apart from the inflation rate in the five countries mentioned above, only the long-run interest rate in Australia can be considered a scapegoat, since the long-run interest rate interacted with its survey weight has a negative impact on the exchange rate; i.e., it has a nonzero value for ϕ_k. This result is in line with the posterior probability $p = 0.87$ reported for

$\bar{i}^L$ for Australia in Table 3. Since in Figure 3 the structural parameter β_k on the long-run interest rate is also negative, this suggests that for the Australian dollar the long-run interest rate is a scapegoat. For the other countries no convincing evidence can be found that $\bar{i}^L$ is a scapegoat. Neither is there evidence that the macro fundamentals $\bar{g}$ and ca are scapegoats. While our results confirm the empirical findings of Fratzscher et al. (2015) as far as the inflation rate is concerned, we find considerably less evidence in favor of scapegoat effects when looking at the other macro fundamentals. Interestingly, the finding that the inflation rate shows up as a scapegoat in the model is not surprising, since, from looking at the survey scores of Consensus Economics, Fratzscher et al. (2015) infer that the inflation rate most frequently is selected as the main driver of exchange rates out of the six variables that they consider (for industrialized countries and, to a lesser extent, for emerging economies).[13]

Table 4: Estimation of the parameters of regression eq.(20)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
c	0.1251	0.0116	-0.0694	0.1806	-0.1692	0.0118	0.3290	0.3220
	[0.005; 0.295]	[-0.011; 0.041]	[-0.143; -0.016]	[0.024; 1.253]	[-0.312; -0.043]	[-0.007; 0.048]	[0.039; 3.160]	[-0.007; 0.768]
$\phi(\bar{g})$	1.0505	0.4828	-0.3570	0.2133	-0.0066	0.0189	-0.2692	2.1988
	[0.207; 2.155]	[-0.018; 1.156]	[-1.074; 0.154]	[-0.327; 0.761]	[-0.769; 0.647]	[-0.046; 0.086]	[-1.332; 0.392]	[-0.736; 7.127]
$\phi(\bar{\pi})$	2.5528	2.0192	0.6959	-0.5592	1.9264	0.2570	1.8762	3.1964
	[0.910; 4.885]	[0.711; 3.693]	[-0.420; 1.985]	[-2.178; 0.645]	[0.738; 3.550]	[-0.018; 0.573]	[0.309; 4.347]	[0.802; 7.464]
$\phi(\bar{i}^L)$	-3.1574	1.2036	0.1417	-1.4683	-0.5088	-0.1112	1.4568	1.9722
	[-5.895; -1.017]	[-0.834; 3.392]	[-1.794; 2.665]	[-4.799; 0.243]	[-2.615; 1.534]	[-0.700; 0.443]	[-0.620; 7.887]	[-1.097; 4.391]
$\phi(ca)$	0.7744	-0.3526	-0.0676	0.7260	-0.2282	0.0096	0.2593	-1.2904
	[-0.177; 2.138]	[-0.864; 0.163]	[-1.429; 0.861]	[-0.194; 2.116]	[-0.749; 0.243]	[-0.057; 0.065]	[-0.293; 0.935]	[-3.740; 0.956]
ρ_ε	0.6672	0.8581	0.7003	0.9512	0.6476	0.9495	0.9421	0.7467
	[0.364; 0.933]	[0.647; 0.999]	[0.414; 0.921]	[0.655; 0.999]	[0.371; 0.911]	[0.843; 0.999]	[0.462; 0.999]	[0.314; 0.999]
σ^2_ε	0.0138	0.0041	0.0046	0.0118	0.0066	0.0016	0.0179	0.0564
	[0.007; 0.047]	[0.002; 0.009]	[0.002; 0.012]	[0.004; 0.056]	[0.003; 0.014]	[0.000; 0.003]	[0.005; 0.229]	[0.023; 0.420]

Note: Reported are the medians and the 90% highest posterior density intervals of the posterior distributions of the parameters c, ϕ_k (with corresponding fundamental denoted between brackets), ρ_ε and σ^2_ε.

In Figure 4 we present the posterior medians and 90% HPD intervals of the estimated parameter expectations, i.e., $E_t(\beta_{kt}) = \phi_k \tau_{kt}$. These are calculated using the posterior distributions of ϕ_k and the survey data τ_{kt}. In line with the findings reported in both tables, we find that these are strictly larger than zero for the inflation rate in the five aforementioned countries (Australia, Canada, the UK, South Korea and South Africa) and positive (but with the value of zero contained in the HPD interval) in the euro area and Singapore. The time-variation found in these estimates allows us to explain potentially why the relationship between exchange rates and macro fundamentals—in particular, the inflation rate—can

[13] See Tables I and II in their Internet Appendix B. They also find that this holds for the short-run interest rate, which we do not include in f_t since it appears on the left hand side (LHS) of eq.(2) in the model.

be unstable.

Figure 4: Expectations of parameters $E_t(\beta_{kt})$

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

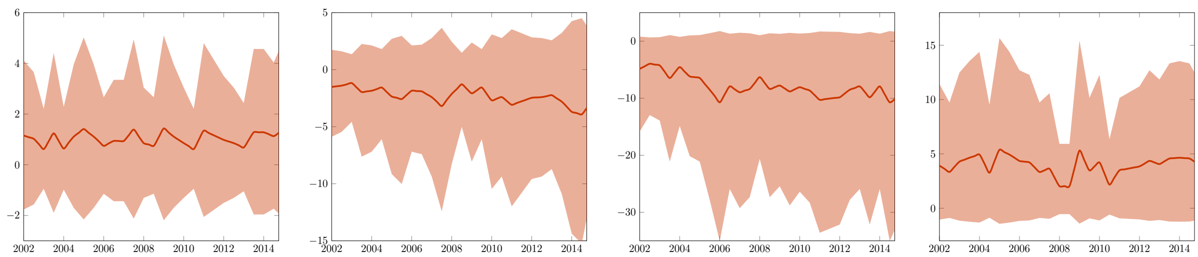
Australian dollar (AUD)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Canadian dollar (CAD)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

euro (EUR)



(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Japanese yen (JPY)

Posterior median $E_t(\beta_{kt})$

90% highest posterior density interval

Figure 4: Expectations of parameters $E_t(\beta_{kt})$ (cont'd)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

UK pound (GBP)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

Singapore dollar (SGD)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

South Korean won (KRW)

(a) $\bar{g}$ (b) $\bar{\pi}$ (c) $\bar{i}^L$ (d) ca

South African rand (ZAR)

Posterior median $E_t(\beta_{kt})$

90% highest posterior density interval

6 Conclusions

This paper proposes a new empirical testing strategy for the scapegoat theory of exchange rates that uses the exact structural exchange rate equation implied by a scapegoat model instead of an ad hoc empirical specification. The approach followed should tighten the link between the theory on scapegoats and the empirical testing of this theory.

From theory we derive an exchange rate equation that can be estimated in different steps. First, the exchange rate risk premium or time-varying deviation from the UIRP condition is estimated using a state-space approach applied to the observed difference between the change in the exchange rate and the interest rate differential. Second, the unobserved time-varying structural parameters on the macro fundamentals and the unobserved component of the model are estimated using a state-space system applied to the observed "signal" in the model, which depends on the level of the exchange rate, the interest rate differential and the discount factor. Third, the scapegoat component in the model's exchange rate equation is estimated using a regression analysis where the estimation is conditional on the estimates obtained in the previous steps.

The estimation is carried out through a Bayesian Gibbs sampling approach for eight countries versus the US over the period 2002Q1-2014Q4. We consider five developed economies (Australia, Canada, the euro area, Japan and the UK) and three emerging countries (Singapore, South Korea and South Africa), and we incorporate four macro fundamentals in the estimations that can potentially be scapegoats, i.e., the real GDP growth rate (relative to the US), the inflation rate (relative to the US), the long-run interest rate (relative to the US) and the current account balance to GDP ratio. We use survey data from Consensus Economics to proxy the parameter expectations that enter the scapegoat term.

The results suggest, first, that there is a persistent but stationary exchange risk premium or time-varying deviation from the UIRP condition in all countries considered. Second, we identify a persistent but stationary unobserved component from the "signal" in the model, which potentially reflects unobserved quantities such as money demand shocks or real exchange rate shocks. Third, we find that, over the sample period, the structural parameters on the macro fundamentals are constant and often close to zero. Fourth, as far as the scapegoat terms in the exchange rate equation are concerned, we calculate posterior probabilities that these macro fundamentals are scapegoats, and we find, for the inflation rate in five out of eight countries, probabilities that are considerably higher than the imposed prior probabilities of 0.5. These countries are the three Anglo-Saxon economies (Australia, Canada and the UK) and South Korea and South Africa. We find little evidence to suggest that the other macro fundamentals we considered are scapegoats.

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Appendix A Derivation of eq.(9)

Note that from eq.(5) we can derive $x_{t+j} = \rho_x^j x_t + \sum_{l=0}^{j-1} \rho_x^l \varepsilon_{t+j-l}^x$ and therefore write

$$E_t(x_{t+j}) = \rho_x^j E_t(x_t) \tag{A-1}$$

Similarly, from eq.(6) we can derive $z_{t+j} = \rho_z^j z_t + \sum_{l=0}^{j-1} \rho_z^l \varepsilon_{t+j-l}^z$ and therefore write

$$E_t(z_{t+j}) = \rho_z^j E_t(z_t) \tag{A-2}$$

Given the assumption that the processes f_{kt} and β_{kt} (for $k = 1, ..., K$) are independent, we can write

$$E_t(f_{k,t+j}\beta_{k,t+j}) = E_t(f_{k,t+j})E_t(\beta_{k,t+j}) = f_{kt}E_t(\beta_{kt}) \qquad k = 1, \ldots, K, \tag{A-3}$$

where the last step follows from the random walk processes in eqs. (7) and (8) assumed for f_{kt} and β_{kt}.

We note that as the signal $y_t \equiv s_t - \frac{1}{\mu}\bar{i}_t$ is observed, we have $E_t(y_t) = y_t$ or $y_t - E_t(y_t) = 0$ (but, since the parameters β_t are unknown, $E_t(\beta_t) \neq \beta_t$ and $E_t(x_t) \neq x_t$). From eq.(3), this implies that $f_t\beta_t + x_t - f_tE_t(\beta_t) - E_t(x_t) = 0$ or

$$E_t(x_t) = x_t + f_t\beta_t - f_tE_t(\beta_t) \tag{A-4}$$

For z_t, on the other hand, we have

$$E_t(z_t) = z_t \tag{A-5}$$

This can be seen by taking expectations in period t from both sides of eq.(1) in the text to obtain $E_t(s_{t+1}) - s_t = \bar{i}_t + E_t(z_t)$, which can only be equal to eq.(1) if eq.(A-5) holds.

Using eqs.(A-1), (A-2), (A-3), (A-4) and (A-5) in eq.(4) while noting that $\sum_{j=1}^{\infty} \lambda^j = \sum_{j=0}^{\infty} \lambda^j - 1 = \frac{\lambda}{1-\lambda}$, $\sum_{j=1}^{\infty}(\lambda\rho_x)^j = \sum_{j=0}^{\infty}(\lambda\rho_x)^j - 1 = \frac{\lambda\rho_x}{1-\lambda\rho_x}$ and $\sum_{j=1}^{\infty}(\lambda\rho_z)^j = \sum_{j=0}^{\infty}(\lambda\rho_z)^j - 1 = \frac{\lambda\rho_z}{1-\lambda\rho_z}$, we obtain eq.(9) in the text.

Appendix B Autocorrelation and partial autocorrelation functions of the estimated residuals of eqs.(18) and (20)

Table B-1: Autocorrelations and partial autocorrelations of the residuals ε_t in eq.(18) under the *iid* assumption.

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
$acf(1)$	0.5120	0.4127	0.6640	0.6681	0.4632	0.6705	0.3623	0.4102
	[0.271; 0.691]	[0.191; 0.632]	[0.406; 0.816]	[0.454; 0.816]	[0.257; 0.656]	[0.469; 0.824]	[0.101; 0.653]	[0.179; 0.645]
$acf(2)$	0.3224	0.2426	0.5458	0.4728	0.2564	0.4653	0.1356	0.2077
	[0.022; 0.538]	[0.068; 0.451]	[0.279; 0.677]	[0.180; 0.676]	[0.028; 0.476]	[0.178; 0.693]	[-0.103; 0.454]	[-0.025; 0.463]
$acf(3)$	0.1587	0.1227	0.4083	0.4784	0.1609	0.4419	0.1253	0.1232
	[-0.163; 0.403]	[-0.043; 0.347]	[0.142; 0.534]	[0.240; 0.631]	[-0.076; 0.382]	[0.214; 0.638]	[-0.099; 0.376]	[-0.103; 0.350]
$acf(4)$	0.0995	0.1485	0.3022	0.1190	0.4045	0.3308	0.0341	0.0480
	[-0.192; 0.329]	[-0.027; 0.344]	[0.042; 0.433]	[0.186; 0.544]	[-0.037; 0.341]	[0.058; 0.554]	[-0.173; 0.295]	[-0.158; 0.253]
$pacf(1)$	0.5120	0.4127	0.6640	0.6681	0.4632	0.6705	0.3623	0.4102
	[0.271; 0.691]	[0.191; 0.632]	[0.406; 0.816]	[0.454; 0.816]	[0.257; 0.656]	[0.470; 0.824]	[0.101; 0.653]	[0.179; 0.645]
$pacf(2)$	0.0621	0.0738	0.1678	0.0382	0.0397	0.0166	-0.0151	0.0329
	[-0.179; 0.256]	[-0.096; 0.230]	[-0.095; 0.315]	[-0.146; 0.165]	[-0.133; 0.190]	[-0.098; 0.132]	[-0.213; 0.190]	[-0.155; 0.212]
$pacf(3)$	-0.0470	-0.0003	-0.0273	0.2560	0.0313	0.2043	0.0792	0.0168
	[-0.236; 0.126]	[-0.142; 0.149]	[-0.150; 0.121]	[0.043; 0.405]	[-0.121; 0.167]	[0.114; 0.307]	[-0.123; 0.238]	[-0.176; 0.195]
$pacf(4)$	0.0026	0.0898	-0.0122	-0.0295	0.0760	-0.1005	-0.0449	-0.0352
	[-0.173; 0.171]	[-0.085; 0.238]	[-0.166; 0.106]	[-0.181; 0.119]	[-0.065; 0.191]	[-0.215; 0.022]	[-0.220; 0.123]	[-0.204; 0.131]

Note: The *iid* assumption for the residuals in eq.(18) implies the assumption that $\rho_\varepsilon = 0$ in eq.(19). $acf(l)/pacf(l)$ denotes autocorrelation/partial autocorrelation of order l. Reported are the medians and 90% highest posterior density intervals of the posterior distributions of the autocorrelations/partial autocorrelations.

Table B-2: Autocorrelations and partial autocorrelations of the residuals ε_t^* in eq.(20)

	Australia	Canada	Euro area	Japan	UK	Singapore	South Korea	South Africa
$acf(1)$	-0.0561	-0.1449	-0.1360	-0.0398	0.0397	-0.0795	-0.0051	0.0017
	[-0.300; 0.242]	[-0.352; 0.141]	[-0.381; 0.381]	[-0.206; 0.127]	[-0.174; 0.277]	[-0.230; 0.134]	[-0.216; 0.142]	[-0.231; 0.256]
$acf(2)$	-0.0850	-0.0430	0.0640	-0.1879	-0.1152	-0.0796	-0.0671	-0.0251
	[-0.277; 0.127]	[-0.246; 0.179]	[-0.137; 0.311]	[-0.431; 0.034]	[-0.313; 0.108]	[-0.239; 0.110]	[-0.305; 0.085]	[-0.244; 0.212]
$acf(3)$	-0.0856	-0.0331	-0.0303	-0.0080	-0.0448	-0.0928	-0.0177	0.0733
	[-0.287; 0.114]	[-0.235; 0.164]	[-0.226; 0.182]	[-0.161; 0.188]	[-0.226; 0.135]	[-0.261; 0.080]	[-0.173; 0.208]	[-0.173; 0.283]
$acf(4)$	-0.0348	0.0284	-0.0602	0.1285	0.0635	-0.1157	-0.0517	0.0074
	[-0.217; 0.154]	[-0.168; 0.211]	[-0.221; 0.112]	[-0.033; 0.314]	[-0.090; 0.187]	[-0.238; -0.043]	[-0.214; 0.052]	[-0.208; 0.206]
$pacf(1)$	-0.0561	-0.1449	-0.1360	-0.0398	0.0397	-0.0795	-0.0051	0.0017
	[-0.300; 0.242]	[-0.352; 0.141]	[-0.381; 0.381]	[-0.206; 0.127]	[-0.174; 0.277]	[-0.230; 0.134]	[-0.216; 0.142]	[-0.231; 0.259]
$pacf(2)$	-0.1169	-0.0908	0.0022	-0.2030	-0.1348	-0.1009	-0.0806	-0.0460
	[-0.303; 0.089]	[-0.278; 0.136]	[-0.205; 0.230]	[-0.449; 0.013]	[-0.326; 0.066]	[-0.251; 0.090]	[-0.330; 0.073]	[-0.261; 0.177]
$pacf(3)$	-0.1101	-0.0626	-0.0482	-0.0383	-0.0408	-0.1139	-0.0242	0.0612
	[-0.301; 0.093]	[-0.246; 0.137]	[-0.233; 0.137]	[-0.178; 0.145]	[-0.244; 0.141]	[-0.273; 0.067]	[-0.175; 0.147]	[-0.179; 0.271]
$pacf(4)$	-0.0818	-0.0151	-0.0993	0.0733	0.0250	-0.1617	-0.0699	-0.0201
	[-0.281; 0.122]	[-0.212; 0.179]	[-0.279; 0.060]	[-0.080; 0.234]	[-0.162; 0.164]	[-0.308; 0.026]	[-0.265; 0.034]	[-0.231; 0.174]

Note: $acf(l)/pacf(l)$ denotes autocorrelation/partial autocorrelation of order l. Reported are the medians and 90% highest posterior density intervals of the posterior distributions of the autocorrelations/partial autocorrelations.

Appendix C Technical details of the Gibbs sampler

C.1 State-space models (steps 1 and 2)

C.1.1 General approach

The unobserved states are sampled conditional on the parameters using a state space approach. In particular, we use the forward-filtering backward-sampling approach discussed in detail in Kim and Nelson (1999) to sample the unobserved states. The general form of the state-space model is given by,

$$Y_t = Z_t S_t + V_t, \qquad V_t \sim iid\mathcal{N}\left(0, H_t\right), \tag{C-1}$$

$$S_{t+1} = T_t S_t + K_{t+1} E_{t+1}, \qquad E_{t+1} \sim iid\mathcal{N}\left(0, Q_{t+1}\right), \qquad t = 1, \ldots, T, \tag{C-2}$$

$$S_1 \sim iid\mathcal{N}\left(s_1, P_1\right), \tag{C-3}$$

where Y_t is a $T \times 1$ vector of observations and S_t an unobserved $n^s \times 1$ state vector. The matrices Z_t, T_t, K_t, H_t, Q_t and the mean s_1 and variance P_1 of the initial state vector S_1 are assumed to be known (conditioned upon) and the error terms V_t and E_t are assumed to be serially uncorrelated and independent of each other at all points in time. Note that E_t is a $n^{ss} \times 1$ matrix (where $n^{ss} \leq n^s$). As eqs. (C-1) to (C-3) constitute a linear Gaussian state-space model, the unknown state variables in S_t can be filtered using the standard Kalman filter. Sampling $S = [S_1, \ldots, S_T]$ from its conditional distribution can then be done using the multimove Gibbs sampler of Carter and Kohn (1994).

C.1.2 Step 1

In step 1 of the Gibbs sampler, we sample the state z_t conditional on the data and parameters in the state-space system eqs. (10) and (11), namely σ_s^2, ρ_z and σ_z^2. We have $n^s = 1$ and $n^{ss} = 1$. The system matrices are given by $Y_t = \Delta s_{t+1} - \bar{i}_t$, $Z_t = 1$, $S_t = z_t$, $V_t = \varepsilon_{t+1}^s$, $H_t = \sigma_s^2$, $Q_{t+1} = \sigma_z^2$, $T_t = \rho_z$, $K_{t+1} = 1$, $E_{t+1} = \varepsilon_{t+1}^z$, $s_1 = 0$ and $P_1 = \frac{\sigma_z^2}{1-\rho_z^2}$.

C.1.3 Step 2

In step 2 of the Gibbs sampler, we sample the time-varying structural parameters β_t from the state-space model eqs. (12) and (13); i.e., we sample the K states in β_t conditional on the data and parameters in the system, namely ρ_x, σ_x^2 and $\sigma_{\beta_k}^2$ (with $k = 1, ..., K$). We have $n^s = 2K$ and $n^{ss} = K$. The system matrices are given by $Y_t = \tilde{y}_t = (1 - \rho_x L)y_t$, $Z_t = \begin{bmatrix} f_t & -\rho_x f_{t-1} \end{bmatrix}$ (a $1 \times 2K$ matrix), $S_t = \begin{bmatrix} \beta_t & \beta_{t-1} \end{bmatrix}'$ (the $2K \times 1$ state vector), $V_t = \varepsilon_t^x$, $H_t = \sigma_x^2$, $T_t = \begin{bmatrix} I_K & 0_K \\ I_K & 0_K \end{bmatrix}$ (a $2K \times 2K$ matrix), $K_{t+1} = \begin{bmatrix} I_K \\ 0_K \end{bmatrix}$ (a

$2K \times K$ matrix), $E_{t+1} = \left[\begin{array}{ccc} \varepsilon^{\beta}_{1,t+1} & ... & \varepsilon^{\beta}_{K,t+1} \end{array} \right]'$ (a $K \times 1$ matrix), $Q_{t+1} = \left[\begin{array}{ccc} \sigma^2_{\beta_1} & & 0 \\ & \ddots & \\ 0 & & \sigma^2_{\beta_K} \end{array} \right]$ (a $K \times K$ matrix), $s_1 = \left[\begin{array}{ccc} 0 & ... & 0 \end{array} \right]'$ (a $2K \times 1$ vector) and $P_1 = 10^6 \times I_{2K}$ (a $2K \times 2K$ matrix).

C.2 OLS regressions (steps 1, 2 and 4)

C.2.1 General approach

Regression parameters (intercept, slope parameters and error variance) can be sampled from a standard regression model,

$$y = w^r \gamma^r + \chi \tag{C-4}$$

where y is a $T \times 1$ vector containing T observations on the dependent variable, w^r is a $T \times M$ matrix containing T observations of M predictor variables, γ^r is the $M \times 1$ parameter vector, and χ is the $T \times 1$ vector of error terms for which $\chi \sim iid\mathcal{N}\left(0, \sigma^2_{\chi} I_T\right)$. If there are no binary indicators ι in the regression or if all binary indicators in the regression ι are equal to 1, then $w^r = w$ and $\gamma^r = \gamma$, where w and γ are the unrestricted predictor matrix and the corresponding unrestricted coefficient vector. Otherwise, the restricted parameter vector γ^r and the corresponding restricted predictor matrix w^r contain only those elements of w and γ for which the corresponding binary indicators ι are equal to 1. The prior distribution of γ^r is given by $\gamma^r \sim \mathcal{N}\left(b^r_0, B^r_0 \sigma^2_{\chi}\right)$ with b^r_0 a $M \times 1$ vector and B^r_0 a $M \times M$ matrix. The prior distribution of σ^2_{χ} is given by $\sigma^2_{\chi} \sim \mathcal{IG}\left(c_0, C_0\right)$ with scalars c_0 (shape) and C_0 (scale). The posterior distributions (conditional on y, w^r and ι) of γ^r and σ^2_{χ} are then given by $\gamma^r \sim \mathcal{N}\left(b^r, B^r \sigma^2_{\chi}\right)$ and $\sigma^2_{\chi} \sim \mathcal{IG}\left(c, C^r\right)$, with

$$\begin{aligned} B^r &= \left[(w^r)' w^r + (B^r_0)^{-1}\right]^{-1} \\ b^r &= B^r \left[(w^r)' y + (B^r_0)^{-1} b^r_0\right] \\ c &= c_0 + T/2 \\ C^r &= C_0 + \frac{1}{2}\left[y'y + (b^r_0)'(B^r_0)^{-1} b^r_0 - (b^r)'(B^r)^{-1} b^r\right] \end{aligned} \tag{C-5}$$

Following Frühwirth-Schnatter and Wagner (2010), we marginalize over the parameters γ when sampling ι and next draw γ^r conditional on ι. The posterior distribution of the binary indicators ι is obtained from Bayes' theorem as,

$$p(\iota | y, w, \sigma^2_{\chi}) \propto p(y | w, \sigma^2_{\chi}, \iota) p(\iota) \tag{C-6}$$

where $p(\iota)$ is the prior distribution of ι and $p(y | w, \sigma^2_{\chi}, \iota)$ is the marginal likelihood of regression eq.(C-4), where the effect of the parameters γ has been integrated out. We refer to Frühwirth-Schnatter and

Wagner (2010) (their eq.(25)) for the closed-form expression of the marginal likelihood for the general regression model of eq.(C-4).

C.2.2 Step 1

The regressions estimated in step 1 are unrestricted, so in eq.(C-4) we have $w^r = w$ and $\gamma^r = \gamma$.

Sampling σ_s^2 conditional on the state z_t and the data is implemented by setting $y = \Delta s - \bar{i} - z$, $w^r = w = 0$, $\gamma^r = \gamma = 0$, $\sigma_\chi^2 = \sigma_s^2$ and $\chi = \varepsilon^s$, where Δs, $\bar{i}$, z and ε^s contain the stacked values of Δs_{t+1}, $\bar{i}_t$, z_t and ε_t^s over T. Sampling σ_s^2 is from the distribution $\sigma_s^2 \sim \mathcal{IG}(c, C)$, where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2}[y'y]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = \nu_0 T = 0.01T$ and $C_0 = c_0\sigma_0^2 = 0.01T \times 0.5V(\Delta s - \bar{i})$.

Sampling ρ_z and σ_z^2 conditional on the state z_t is implemented by setting $y = z$, $w^r = w = z_{-1}$, $\gamma^r = \gamma = \rho_z$, $\sigma_\chi^2 = \sigma_z^2$ and $\chi = \varepsilon^z$, where z, z_{-1} and ε^z contain the stacked values of z_t, z_{t-1} and ε_t^z over T. Sampling ρ_z is from the distribution $\mathcal{N}\left(b, B\sigma_z^2\right)$ with, from eq.(C-5), $B = \left[w'w + (B_0)^{-1}\right]^{-1}$ and $b = B\left[w'y + (B_0)^{-1}b_0\right]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0\sigma_z^2 = 1$ so that $B_0 = \frac{1}{\sigma_z^2}$ where we use the prior belief σ_0^2 for σ_z^2. Sampling σ_z^2 is from the distribution $\sigma_z^2 \sim \mathcal{IG}(c, C)$, where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2}\left[y'y + (b_0)'(B_0)^{-1}b_0 - b'(B)^{-1}b\right]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = \nu_0 T = 0.01T$ and $C_0 = c_0\sigma_0^2 = 0.01T \times 0.5V(\Delta s - \bar{i})$. Note that first σ_z^2 is sampled from $\mathcal{IG}(c, C)$ and then, given a draw for σ_z^2, ρ_z is sampled from $\mathcal{N}\left(b, B\sigma_z^2\right)$.

C.2.3 Step 2

The regressions estimated in step 2 are unrestricted, so in eq.(C-4) we have $w^r = w$ and $\gamma^r = \gamma$.

Sampling $\sigma_{\beta_k}^2$ (for $k = 1, ..., K$) conditional on the state β_{kt} is implemented by setting $y = \beta_k - \beta_{k,-1}$, $w^r = w = 0$, $\gamma^r = \gamma = 0$, $\sigma_\chi^2 = \sigma_{\beta_k}^2$ and $\chi = \varepsilon_k^\beta$, where β_k, $\beta_{k,-1}$ and ε_k^β contain the stacked values of β_{kt}, $\beta_{k,t-1}$ and ε_{kt}^β over T. Sampling $\sigma_{\beta_k}^2$ is from the distribution $\sigma_{\beta_k}^2 \sim \mathcal{IG}(c, C)$, where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2}[y'y]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.01T$ and $C_0 = c_0\sigma_0^2 = 0.01T \times 0.01$.

Sampling ρ_x and σ_x^2 conditional on the calculated $x_t = y_t - f_t\beta_t$ is implemented by setting $y = x$, $w^r = w = x_{-1}$, $\gamma^r = \gamma = \rho_x$, $\sigma_\chi^2 = \sigma_x^2$ and $\chi = \varepsilon^x$, where x, x_{-1} and ε^x contain the stacked values of x_t, x_{t-1} and ε_t^x over T. Sampling ρ_x is from the distribution $\mathcal{N}\left(b, B\sigma_x^2\right)$ with, from eq.(C-5), $B = \left[w'w + (B_0)^{-1}\right]^{-1}$ and $b = B\left[w'y + (B_0)^{-1}b_0\right]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0\sigma_x^2 = 1$ so that $B_0 = \frac{1}{\sigma_x^2}$ where we use the prior belief σ_0^2 for σ_x^2. Sampling σ_x^2 is from the distribution $\sigma_x^2 \sim \mathcal{IG}(c, C)$, where $c = c_0 + \frac{T}{2}$ and $C = C_0 + \frac{1}{2}\left[y'y + (b_0)'(B_0)^{-1}b_0 - b'(B)^{-1}b\right]$ with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.01T$ and $C_0 = c_0\sigma_0^2 = 0.01T \times 0.5V(y_t)$. Note that first σ_x^2 is sampled from $\mathcal{IG}(c, C)$ and then, given a draw for σ_x^2, ρ_x is sampled from $\mathcal{N}\left(b, B\sigma_x^2\right)$.

C.2.4 Step 4 (parts b, c and d)

We first sample the binary indicators $\iota = \delta$ in eq.(20). In particular, we follow George and McCulloch (1993) and Frühwirth-Schnatter and Wagner (2010) and use a single-move sampler in which the binary indicators δ_k are sampled one by one for $k = 1, ..., K$. We calculate the marginal likelihoods $p(y|\delta_k = 1, \delta_{-k}, w, \sigma_\varepsilon^2)$ and $p(y|\delta_k = 0, \delta_{-k}, w, \sigma_\varepsilon^2)$ (see Frühwirth-Schnatter and Wagner, 2010, for the correct expressions based on a regression of the type of eq.(C-4) with priors as discussed below). Upon combining the marginal likelihoods with the Bernoulli prior distributions of the binary indicators $p(\delta_k = 1) = p_0$ and $p(\delta_k = 0) = 1 - p_0$, the posterior distributions $p(\delta_k = 1|y, \delta_{-k}, w, \sigma_\varepsilon^2)$ and $p(\delta = 0|y, \delta_{-k}, w, \sigma_\varepsilon^2)$ are obtained, from which the probability $prob(\delta_k = 1|y, \delta_{-k}, w, \sigma_\varepsilon^2) = \frac{p(\delta_k=1|y,\delta_{-k},w,\sigma_\varepsilon^2)}{p(\delta_k=1|y,\delta_{-k},w,\sigma_\varepsilon^2)+p(\delta_k=0|y,\delta_{-k},w,\sigma_\varepsilon^2)}$ is calculated, which is used to sample δ_k i.e., draw a random number r from a uniform distribution with support between 0 and 1 and set $\delta_k = 1$ if $r < prob(.)$ and $\delta_k = 0$ if $prob(.) > r$.

We then sample the following parameters from eq.(20): the intercept c^*, the residual variance σ_ε^2 and the slope coefficients ϕ_k for which the corresponding binary indicators δ_k are equal to 1. The dependent variable y in eq.(C-4) contains the stacked values of $(1 - \rho_\varepsilon L)\tilde{s}_t$ over T. The restricted predictor matrix w^r in eq.(C-4) contains a $T \times 1$ vector of ones for the intercept and the stacked (over T) values of the regressors $(1 - \rho_\varepsilon L)\tau_{kt}\tilde{f}_{kt}$ for those k for which the binary indicators are equal to 1. The coefficient vector γ^r in eq.(C-4) contains c^* and the coefficients ϕ_k for those k for which the binary indicators are equal to 1. The error term in eq.(C-4) is given by $\chi = \varepsilon$, where ε contains the stacked values of ε_t and $\sigma_\chi^2 = \sigma_\varepsilon^2$. Sampling γ^r is from the distribution $\mathcal{N}\left(b^r, B^r\sigma_\varepsilon^2\right)$, where B^r and b^r are defined by eq.(C-5) for which, as noted in Section 4.3.2 in the main text, we have $b_0 = 0$ (an $M \times 1$ vector of zeros) and $V_0 = B_0\sigma_\varepsilon^2 = 10I_M$ so that $B_0 = \frac{10I_M}{\sigma_\varepsilon^2}$ where we use the prior belief σ_0^2 for σ_ε^2. Sampling σ_ε^2 is from the distribution $\sigma_\varepsilon^2 \sim \mathcal{IG}(c, C^r)$, where c and C^r are given by eq.(C-5) with, as noted in Section 4.3.2 in the main text, the shape c_0 and scale C_0 of the prior distribution given by $c_0 = 0.01T$ and $C_0 = c_0\sigma_0^2 = 0.01T \times 0.01$. Note that first σ_ε^2 is sampled from $\mathcal{IG}(c, C^r)$, and then, given a draw for σ_ε^2, γ^r is sampled from $\mathcal{N}\left(b^r, B^r\sigma_\varepsilon^2\right)$.

Finally, we sample the parameter ρ_ε in eq.(19) conditional on ε_t *and* on σ_ε^2. Estimates for ε_t are obtained from $\varepsilon_t = \tilde{s}_t - c - \sum_{k=1}^{K} \phi_k\tau_{kt}\tilde{f}_{kt}$. The regression is unrestricted so that in eq.(C-4) we have $w^r = w$ and $\gamma^r = \gamma$. We set $y = \varepsilon$, $w = \varepsilon_{-1}$, $\gamma = \rho_\varepsilon$, $\sigma_\chi^2 = \sigma_\varepsilon^2$ and $\chi = \varepsilon^*$ where ε, ε_{-1} and ε^* contain the stacked values of ε_t, ε_{t-1} and ε_t^* over T. Sampling ρ_ε is from the distribution $\mathcal{N}\left(b, B\sigma_\varepsilon^2\right)$ with, from eq.(C-5), $B = \left[w'w + (B_0)^{-1}\right]^{-1}$ and $b = B\left[w'y + (B_0)^{-1}b_0\right]$ where, as noted in Section 4.3.2 in the main text, $b_0 = 0$ and $V_0 = B_0\sigma_\varepsilon^2 = 1$ so that $B_0 = \frac{1}{\sigma_\varepsilon^2}$ where we use the prior belief σ_0^2 for σ_ε^2. When sampling ρ_ε from $\mathcal{N}\left(b, B\sigma_\varepsilon^2\right)$, we use the draw for σ_ε^2 obtained when estimating eq.(20).