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by Patrick D. Alexander

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# Producer Heterogeneity, Value-Added, and International Trade

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All mistakes are my own, and the views expressed in this paper are my own and do not necessarily reflect those of the Bank of Canada.

# Abstract

Standard new trade models depict producers as heterogeneous in total factor productivity. In this paper, I adapt the Eaton and Kortum (2002) model of international trade to incorporate tradable intermediate goods and producer heterogeneity in *value-added* productivity. In equilibrium, this yields a positive relationship between the international trade elasticity and the share of intermediate goods in production. This relationship is absent from the standard model and is driven by the extensive margin of trade. I then use cross-country sectoral data from 1995 to 2010 and estimate the trade elasticity, finding empirical support for this relationship and for the importance of the extensive margin. This model yields results that are similar to those of the standard model suggests that gains from trade are higher in sectors that use intermediate goods, I find that this is no longer true under the value-added heterogeneity model.

Bank topic(s): Trade integration; Economic models; International topics; Productivity

JEL code(s): F11, F12, F14

# Résumé

Les nouveaux modèles types de commerce extérieur représentent l'hétérogénéité des producteurs sur le plan de la productivité totale des facteurs. Dans cet article, nous adaptons le modèle de commerce international d'Eaton et Kortum (2002) pour prendre en compte les biens intermédiaires échangeables et l'hétérogénéité des producteurs dans la productivité *à valeur ajoutée*. En situation d'équilibre, il en résulte une relation positive entre l'élasticité des échanges internationaux et la part des biens intermédiaires dans la production. Absente du modèle type, cette relation est déterminée par la marge extensive du commerce. Nous nous servons ensuite de données sectorielles se rapportant à divers pays et couvrant la période de 1995 à 2010, et estimons l'élasticité des échanges. Nous constatons ainsi que des fondements empiriques confirment cette relation et l'importance de la marge extensive. Ce modèle génère des résultats semblables à ceux du modèle type relativement à l'ampleur globale des gains provenant des échanges. Fait important cependant, là où le modèle type indique que les gains provenant des échanges sont supérieurs dans les secteurs qui utilisent des biens intermédiaires, nous découvrons qu'il n'en va pas de même dans le modèle tenant compte de l'hétérogénéité de la productivité à valeur ajoutée.

Sujet(s) : Intégration des échanges; Modèles économiques; Questions internationales; Productivité: Code(s) JEL : F11, F12, F14

# **Non-Technical Summary**

This paper adapts a popular international trade model with heterogeneous producers to account for several important features in the data.

The paper begins by briefly discussing the evidence on the growing importance of globally fragmented production, also known as global value chain (GVC) trade. In contrast to more basic trade in final goods, GVC trade accounts for trade in intermediate components. These components can come from numerous countries and are, themselves, often composed of other intermediate goods that might be imported. In the end, the development of a single final good often includes many different intermediate stages with value-added from several different countries. One interesting finding has been that GVC trade tends to take place over shorter distances than other types of trade. Together, the relatively high pace of GVC growth in recent decades and the regionalized nature of this type of trade suggest that GVC trade is fundamentally more sensitive to differences in trade costs than other types of trade.

To account for this higher sensitivity, I consider a model of international trade where producers are heterogeneous with respect to value-added productivity. As in standard models, this heterogeneity delivers a basis for comparative advantage and international market power for relatively efficient producers. However, once intermediate goods are integrated in the model and the value-added share in production falls, the impact of producer heterogeneity declines and trade becomes more competitive globally. In equilibrium, the model delivers a predicted positive relationship between the share of intermediate goods in production and the international trade elasticity with respect to trade costs for a given industry. This relationship can qualitatively account for the growing share over time, as well as the regionalized pattern, of GVC trade.

To test this relationship, I estimate the international trade elasticity using cross-country sectorlevel trade and production data from 1995 to 2010. I find evidence of a positive and statistically significant relationship between the international trade elasticity and the share of intermediate goods in production as predicted by the theoretical model. Using baseline estimates, the use of intermediate goods raises the trade elasticity for the average sector by approximately 60 per cent, according to the model.

In terms of welfare, the model suggests similar magnitudes for the economic gains from international trade to those of other models that include traded intermediate goods. However, whereas other models typically suggest that economic gains from trade are higher in sectors that use intermediate goods, this model delivers no such relationship. As a result, if intermediate goods continue to grow in importance in the future, the suggested welfare gains from trade will be more modest, according to this model, compared with those of existing international trade models.

# 1 Introduction

The growing international fragmentation of production is a well-documented phenomenon. For example, Hummels, Ishii and Yi (2001) provide evidence that vertical specialization – the use of imported intermediates in producing goods that are exported – has grown significantly as a share of total trade over time. Importantly, this phenomenon is not simply due to the rising share of imported intermediate goods in production, but is largely due to the fact that sectors that have become more import-oriented over time, in terms of intermediate goods, have also become especially export-oriented (see Figures 1 and 2).<sup>1</sup> Meanwhile, vertical specialization trade also tends to be traded over shorter distances and, hence, is more "regionalized," than other types of trade.<sup>2</sup> Together, these patterns suggest that vertical specialization trade is particularly sensitive to variations in trade costs. With the gradual decline in trade costs over time, export growth has been particularly high in sectors that have experienced import growth on the intermediate side. Meanwhile, the trade costs imposed by bilateral distance are particularly burdensome for sectors that engage in vertical specialization, leading to a localized pattern in this type of trade. Notably, this extra sensitivity is not reflected in standard micro-founded general equilibrium trade models with intermediate goods.<sup>3</sup>

In this paper, I develop an adapted Eaton and Kortum (2002) model that generates this extra sensitivity. As in many versions of this framework, industries combine intermediate goods and value-added to produce output. However, where standard versions of the model feature heterogeneity in total factor productivity (TFP) across products, this model features cross-product heterogeneity in *value-added* productivity (VAP). In equilibrium, this adjustment generates a positive relationship between the sensitivity of exports to variations in trade costs (also known as the "trade elasticity") and the share of intermediate goods used in production. Importantly, this relationship is driven by the extensive margin of trade and does not emerge under a representative-producer model with a similar production structure.

To test this relationship empirically, I estimate the trade elasticity using cross-country sector-level data from 1995 to 2010. I find evidence of a positive and statistically significant relationship between the trade elasticity and the intermediate goods share as

<sup>&</sup>lt;sup>1</sup>Figure 1 compares the imported intermediate content of total exports (ICE) to the imported intermediate content of total output (ICO) across a broad set of major economies for the year 2005. The fact that the ICE is consistently higher than the ICO across these countries indicates that sectors that use imported intermediate goods tend to be the same sectors that export. Figure 2 shows that ICE grew faster than ICO over time for each of these countries.

<sup>&</sup>lt;sup>2</sup>See Johnson and Noguera (2012c) for evidence of this pattern.

 $<sup>^{3}</sup>$ A notable recent example that demonstrates this point is the model from Caliendo and Parro (2015), where export sensitivity to trade costs is unrelated to intermediate goods. See also Yi (2003) for more on this point.

predicted by the model. This result holds across several different measures of trade costs, including bilateral distance.<sup>4</sup> To identify the extensive margin empirically, I compute a sector-level bilateral measure of the number of products exported. I find evidence that the relationship between the trade elasticity and the intermediate goods share is particularly significant when this computed measure is used as the dependent variable. Overall, the empirical evidence provides support for the relationship derived from the model.

Despite this heightened sensitivity, the economic gains from trade are quantitatively similar to the gains derived under the standard Eaton and Kortum (2002) setting with intermediate goods. Crucially, however, while the standard setting features a significant positive relationship between the gains from trade and the share of intermediate goods used in production, this model features no such relationship. In other words, even stronger than the message from the seminal paper by Arkolakis et al. (2012), this model features the "same old gains" as the standard models in addition to the "same old gains" for sectors that use intermediate goods in production.

Overall, my findings contribute to the literature in several ways. Numerous others have aimed to distinguish between intermediate inputs and value-added in exports using input-output data. For examples, see Hummels, Ishii and Yi (2001), Antras et al. (2012), Johnson and Noguera (2012a, 2012b, 2012c), Koopman et al. (2012, 2014) and Timmer et al. (2014). These papers have generally drawn a particular country-level distinction between imported intermediates and domestic value-added in exports. In contrast, the present analysis emphasizes the distinction between intermediate inputs (domestic- or foreign-produced) and value-added at the producer level. My results broaden the findings of Johnson and Noguera (2012c), which stresses the localized pattern of exports for industries that use imported intermediate goods, to suggest that this pattern emerges for industries that use any intermediate goods, whether sourced domestically or imported.

My theoretical framework is based on the Eaton and Kortum (2002) international trade model, combined with a production setting similar to Yi (2003, 2010). Yi (2003) aims to explain the growth in vertical specialization trade with a two-country model that features industry heterogeneity and endogenous growth in both the trade elasticity and vertical specialization trade. In contrast, my model can accommodate many countries, and yields a gravity-type equation in equilibrium.<sup>5</sup> Moreover, the trade elasticity is exogenous, but varies across sectors and is higher in sectors that use intermediate goods. As a result, falling trade costs lead to endogenous growth in vertical specialization trade.

 $<sup>^4\</sup>mathrm{As}$  indicators of trade costs, I include bilateral distance, bilateral tariffs and a dummy variable for regional trade agreements.

<sup>&</sup>lt;sup>5</sup>My model adds to a long list of trade models that feature gravity model properties. See Tinbergen (1962) for the original exposition of a gravity model.

The welfare analysis in this paper adds to the literature on quantifying the gains from trade based on trade models with producer heterogeneity. For other contributions, see Arkolakis et al. (2012), Caliendo and Parro (2015), Ossa (2015), Levchenko and Zhang (2014), Costinot and Rodriguez-Clare (2014), and Melitz and Redding (2014, 2015). In contrast to these papers, I demonstrate that under the setting with heterogeneity in value-added productivity across producers, gains from trade are not higher in sectors that use intermediate goods. This finding is in particularly stark contrast to the results from Melitz and Redding (2014), which emphasize that trade in intermediate goods generates welfare gains that are significantly above the gains generated from trading final goods alone.

This paper also contributes to an existing literature that aims to correct for biases in the link between empirical estimates of trade elasticities and model-based structural parameters. Other examples include Ruhl (2008), di Giovanni et al. (2011), Simonovska and Waugh (2014), and Imbs and Mejean (2015). As in these papers, I use theory to relate empirical trade elasticity estimates to the parameters of my model. I also show, as these other papers do, that failure to do this leads to highly distorted conclusions from the model.

Finally, my findings emphasize the importance of the extensive margin of trade, which is also emphasized by other recent papers. These include Chaney (2008), Helpman, Melitz and Rubenstein (2008), Hillberry and Hummels (2008), and Crozet and Koenig (2010).

The remainder of the paper is organized as follows. Section 2 describes the theoretical framework. Section 3 describes the data. Section 4 provides empirical results for the gains from trade. Section 5 provides empirical results for trade elasticities. Section 6 concludes. An Appendix follows.

# 2 Theory

#### 2.1 The model

The following is a static multi-sectoral Eaton-Kortum (2002) model of trade with intermediate goods similar to the model derived in Caliendo and Parro (2015).<sup>6</sup>

<sup>&</sup>lt;sup>6</sup>The basic Eaton and Kortum (2002) model does not incorporate intermediate goods, although the authors provide an extension with intermediates in the second half of their original paper. Other multi-sectoral versions of the Eaton and Kortum (2002) model include Costinot et al. (2012), Shikher (2012), Levchenko and Zhang (2014), and Caliendo and Parro (2015).

#### 2.1.1 Environment

Consider a world with N countries and J sectors. Labor is freely mobile across sectors in a given country, and immobile across countries. All markets are perfectly competitive. Country n has a measure  $L_n$  of representative households. Households in n purchase  $C_n^j$ units of composite final goods from each sector j to maximize the following Cobb-Douglas utility function:

$$U_n = \prod_{j=1}^J C_n^{j \, \alpha_n^j},\tag{1}$$

where  $\sum_{j=1}^{J} \alpha_n^j = 1$ . Labor is the only factor of production and the sole source of household income. As a result, the budget constraint for consumers in n is given by:

$$\sum_{j=1}^{J} P_n^j C_n^j = w_n L_n,$$
(2)

where  $w_n$  denotes the wage rate and  $P_n^j$  denotes the composite price index in sector j of country n (described in detail below).

Each sector consists of a continuum of tradable intermediate products indexed by  $\omega \in [0, 1]$ . Potential producers of product  $\omega$  in sector j of country n receive a productivity draw  $z_n^j(\omega)$  from a Fréchet distribution of the following form:

$$F_n^j(z_n^j) = exp\left\{-T_n^j z_n^{j-\theta^j}\right\}.$$
(3)

This distribution varies across both countries and sectors. A higher  $T_n^j$  implies higher average productivity for the country-sector pair, while a higher  $\theta^j$  implies lower dispersion of productivity draws within sector j. All producers in n have access to this technology for a given product  $\omega$ .<sup>7</sup>

Producers use two types of inputs in production: labor and composite intermediate goods from each of the J sectors. The corresponding production function for product  $\omega$  is:

$$q_n^j(\omega) = \left[z_n^j(\omega)l_n^j(\omega)\right]^{1-\beta^j} \left[\prod_{k=1}^J M_n^{k,j}(\omega)^{\gamma_n^{k,j}}\right]^{\beta^j},\tag{4}$$

where  $z_n^j$ ,  $l_n^j$  and  $M_n^{k,j}$  denote labor productivity, labor input and intermediate input of the composite intermediate good from sector k, respectively. The parameter  $\gamma_n^{k,j}$ 

<sup>&</sup>lt;sup>7</sup>The original Eaton and Kortum (2002) model has a single sector, so  $T_n$  depicts a parameter of country-level average productivity, while  $\theta$  provides dispersion across productivity draws and, hence, a basis for gains from trade. In the present model, variance in  $T_n^j$  across sectors provides an additional basis for gains from trade owing to comparative advantage in the traditional Ricardian sense. For more on this insight, see Levchenko and Zhang (2014).

denotes the share of the composite goods from sector k used by producers in sector j of country n, with  $\sum_{k=1}^{J} \gamma_n^{k,j} = 1$ . Equation (4) includes an important departure from standard versions of the Eaton and Kortum (2002) model. The parameter  $z_n^j(\omega)$  does not enter here as total factor productivity (TFP) but as *value-added* productivity (VAP). As shown below, this difference is not trivial: it provides for an additional relationship between intermediate goods and the trade elasticity.

Non-traded composite goods  $Q_n^j$  are produced using intermediate products as inputs. Producers of composite goods minimize costs by sourcing intermediate products from the lowest cost suppliers, whether they are located at home or abroad. These products are then assembled according to the following CES production function:

$$Q_n^j = \left(\int_0^1 q_n^j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},\tag{5}$$

where  $\sigma > 1$  denotes the elasticity of substitution across intermediate products. The composite goods from j are demanded by both consumers as final goods  $C_n^j$  and by producers as intermediate goods  $M_n^{j,k}$  across all k sectors.

Given the CES production function in (5), the composite goods producers in sector j of n have the following demand for expenditures on product  $\omega$  exported from i:

$$x_{ni}^{j}(\omega) = \left[\frac{p_{ni}^{k}(\omega)}{P_{n}^{j}}\right]^{1-\sigma} X_{n}^{j},\tag{6}$$

where  $X_n^j$  denotes total expenditures in n on goods from sector j, and

$$P_n^j = \left[\int_0^1 p_n^j(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$
(7)

denotes the composite price index for sector j in country n.

As mentioned, total expenditures  $X_n^j$  consists of spending by both consumers and producers. Given (1) and (4), this can be expressed as the following:

$$X_n^j = \alpha_n^j w_n L_n + \sum_{k=1}^J \gamma_n^{j,k} \beta^k Y_n^k, \tag{8}$$

where  $Y_n^k$  denotes the value of total production in sector k of country n. To clear the product market for this sector, total production value  $Y_n^k$  in country n must be equal to

total expenditures by all other countries on products produced by n. That is:

$$Y_{n}^{j} = \sum_{i=1}^{n} X_{in}^{j}.$$
(9)

Substituting this into total expenditures yields the following:

$$X_n^j = \alpha_n^j w_n L_n + \sum_{k=1}^J \gamma_n^{j,k} \beta^k \left( \sum_{i=1}^N X_{in}^k \right).$$
<sup>(10)</sup>

#### 2.1.2 Prices

Composite goods producers in n buy intermediate products from the lowest-cost producer. Producers are perfectly competitive, setting prices equal to marginal cost. Exports from i to n are subject to an additional iceberg trade cost of the form  $\kappa_{ni} > \kappa_{ii} = 1$ , where  $\kappa_{ni}$  units of a given product need to be exported from i for each unit that arrives in n. As a result, the price of product  $\omega$  exported from i to n takes the following form:

$$p_{ni}^j(z_i^j(\omega))^{VAP} = \frac{c_i^j \kappa_{ni}^j}{(z_i^j(\omega))^{1-\beta^j}},\tag{11}$$

where

$$c_i^j = \Psi_i^j w_i^{1-\beta^j} \left[ \prod_{k=1}^J P_i^{k\gamma_n^{k,j}} \right]^{\beta^j}$$
(12)

denotes the unit cost of production and  $\Psi_i^j$  is a constant.<sup>8</sup>

Note that (11) is different here than in the standard Eaton and Kortum (2002) model with TFP heterogeneity. In that setting, the analogous expression is the following:

$$p_{ni}^j (z_i^j(\omega))^{TFP} = \frac{c_i^j \kappa_{ni}^j}{z_i^j(\omega)}.$$
(13)

Expression (7) can be simplified, given our choice of the Fréchet distribution, to yield the following expression:

$$P_n^j = A^j \left[ \sum_{i=1}^N T_i^j \left[ c_i^j \kappa_{ni}^j \right]^{\frac{-\theta^j}{1-\beta^j}} \right]^{\frac{1-\beta^j}{-\theta^j}} = A^j \left[ \phi_n^j \right]^{\frac{1-\beta^j}{-\theta^j}}, \tag{14}$$

<sup>8</sup>Specifically,  $\Psi_i^j = \prod_{k=1}^J (\gamma_n^{k,j})^{-\gamma_n^{k,j} (1-\beta^j)} (\beta^j)^{-\beta^j} (1-\beta^j)^{\beta^j-1}.$ 

where  $A^{j}$  is a constant.<sup>9</sup> See the Appendix for a proof of (14).

#### 2.1.3 Expenditures and trade balance

We denote the share of total expenditures in n on products exported from i in sector j as  $\pi_{ni}^{j} = X_{ni}^{j}/X_{n}^{j}$ . Again, using some useful properties of the Fréchet distribution, this share can be represented by the following:

$$\pi_{ni}^{j} = \frac{X_{ni}^{j}}{X_{n}^{j}} = \frac{T_{i}^{j} \left[c_{i}^{k} \kappa_{ni}^{j}\right]^{\frac{-\theta^{j}}{1-\beta^{j}}}}{\phi_{n}^{j}}.$$
(15)

See the Appendix for a proof of equation (15).

Finally, the total trade surplus for country n can be defined as  $D_n = \sum_{j=1}^J D_n^j$ , where  $D_n^j = \sum_{i=1}^N X_{ni}^j - \sum_{i=1}^N X_{in}^j$  denotes the total trade surplus in sector j. Trade is balanced for all countries when  $D_n = 0$  for all n, which is equivalent to the following:

$$D_n = \sum_{j=1}^J \left( \sum_{i=1}^N X_{ni}^j - \sum_{i=1}^N X_{in}^j \right) = 0$$
(16)

for all n.

#### 2.1.4 Equilibrium

Following Alvarez and Lucas (2007) and Caliendo and Parro (2015), I define an equilibrium as a set of wages and prices that satisfy (10), (12) (14), (15), and (16) for all nand j.<sup>10</sup>

#### Total bilateral exports: A gravity equation

Rearranging (15) in terms of  $X_{ni}^{j}$  and substituting this into the market-clearing equation in (9) yields:

$$Y_{i}^{j} = T_{i}^{j} \left(c_{i}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}} \sum_{n=1}^{N} \frac{\left(\kappa_{ni}^{j} X_{n}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}}}{\phi_{n}^{j}}.$$
 (17)

Solving this expression for  $T_i^j(c_i^k)^{\frac{-\theta^j}{1-\beta^j}}$  and substituting into (15) yields the following

<sup>9</sup>In particular,  $A^{j} = \Gamma\left(\frac{\theta^{j} + (1-\sigma)(1-\beta^{j})}{\theta^{j}}\right)^{\frac{1}{(1-\sigma)}}$  and  $\Gamma$  is the Gamma function.

<sup>&</sup>lt;sup>10</sup>Other versions of the Eaton and Kortum (2002) model often allow for trade deficits at the country level. This element could easily be included in this model as well. However, for simplicity, I assume that trade is balanced for each country.

gravity equation:

$$X_{ni}^{j \ VAP} = X_{n}^{j} Y_{i}^{j} \frac{\left(\kappa_{ni}^{j}/P_{n}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}}}{\sum_{n=1}^{N} \left(\kappa_{ni}^{j}/P_{n}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}}}.$$
(18)

Equation (18) is different from the standard multi-sectoral Eaton and Kortum gravity equation (e.g., Caliendo and Parro (2015)). In the standard setting with TFP heterogeneity, the gravity equation is the following:

$$X_{ni}^{j\ TFP} = X_{n}^{j}Y_{i}^{j}\frac{\left(\kappa_{ni}^{j}/P_{n}^{j}\right)^{-\theta^{j}}}{\sum_{n=1}^{N}\left(\kappa_{ni}^{j}/P_{n}^{j}\right)^{-\theta^{j}}}.$$
(19)

Clearly, the main difference between these expressions relates to the  $1 - \beta^j$  term in the exponent of  $\kappa_{ni}^j$  in (18). Denoting the trade elasticity with respect to variable trade costs as  $\eta_{X,\kappa}^j$ , we can derive the following simple expression (controlling for  $X_n^j$ ,  $Y_n^j$  and  $P_n^j$ ):

$$\eta_{X,\kappa}^{j \ VAP} = \frac{-\theta^j}{1-\beta^j}.$$
(20)

In contrast, the trade elasticity according to (19) is:

$$\eta_{X,\kappa}^{j \ TFP} = -\theta^j. \tag{21}$$

In the model with heterogeneity in value-added productivity, sectors that use a higher share of intermediate goods have a higher elasticity of trade with respect to trade costs. In the model with TFP, this mechanism is absent.

#### 2.2 Extensive and intensive margins

To illustrate the role of the extensive and intensive margins separately, I reproduce the following product-level bilateral exports equation from (6):

$$x_{ni}^j(\omega) = \left[\frac{c_i^j \kappa_{ni}^j (z_i^j)^{\beta^j - 1}}{P_n^j}\right]^{1 - \sigma} X_n^j.$$

$$\tag{22}$$

Note that I have substituted in the price equation from (11). Clearly, the elasticity of trade with respect to trade costs for a given product (i.e., the intensive margin) is  $1 - \sigma$ .

Although the Eaton and Kortum (2002) setting does not yield a closed-form definition of the extensive margin, Chaney (2008) develops a gravity model from a Melitz (2003) framework with Pareto-distributed firm heterogeneity that yields separate closed-form expressions for the intensive, extensive and compositional margins.<sup>11</sup> In adapting the Chaney (2008) model to incorporate value-added productivity heterogeneity, we can recover an extensive margin elasticity that is appropriate for our setting. In doing this, we find that the extensive margin trade elasticity is equivalent to the total trade elasticity expression found in (20). In other words, the extensive margin describes the entire trade elasticity, as well as the entire equilibrium relationship between the trade elasticity and the intermediate goods share under this setting.

This result should not be surprising to those who are familiar with the Chaney (2008) framework. As discussed in Head and Mayer (2014), the combination of CES preferences with Pareto-distributed firm heterogeneity leads to a result in which the product-specific intensive margin of trade  $(1 - \sigma)$  is exactly counteracted by the compositional margin of trade  $(\sigma - 1)$ . In the end, the extensive margin describes the entire trade elasticity in equilibrium.

#### 2.3 Discussion

To recap, the model described above yields a positive relationship between the international trade elasticity and the intermediate goods share in production. As discussed, this mechanism is not present in standard versions of the Eaton and Kortum (2002) model with intermediate goods.

The explanation for this mechanism is fairly intuitive. In the heterogeneous producer environment, sectors are dispersed in terms of productivity. In sectors with high productivity dispersion (low  $\theta$ ), production is more concentrated, the extensive margin reacts little when trade costs change, and the international trade elasticity is low. When intermediate goods are used in production, all firms purchase these goods from the same suppliers at the same price (excluding freight), and the impact, in terms of productivity, is neutral across firms. Thus, as the share of value-added in production is replaced with intermediate goods, the distribution of market share across firms becomes more even, the extensive margin becomes more responsive to changes in trade costs, and the international trade elasticity rises.<sup>12</sup>

In models where firm productivity enhances all factors equally, such as in most versions of the Eaton and Kortum (2002) model, more productive firms are equally more productive in value-added and intermediate goods, so including intermediate goods does not flatten the productivity distribution in any way. Under the value-added productivity

<sup>&</sup>lt;sup>11</sup>The compositional margin, as defined in Head and Mayer (2014), captures the impact that distributional changes across existing (and new) products have on the trade elasticity.

<sup>&</sup>lt;sup>12</sup>The VAP heterogeneity setting yields a similar result as models with producer heterogeneity that incorporate decreasing returns to scale. For an example, see Adamopoulos and Restuccia (2014).

setting, however, this is no longer the case.

The Fréchet and Pareto distributions are popular choices in this literature mainly because they deliver clean analytical solutions. Another significant factor, however, is evidence that the size distribution of firms (and especially exporting firms) in advanced economies often closely resembles a type II extreme value distribution, such as Fréchet or Pareto. For evidence, see Axtell (2001), Luttmer (2007), and di Giovanni, Levchenko and Rancière (2011).

Importantly, this feature remains true under the value-added productivity setting described here. In fact, for both the firm size distribution and calculations of the gains from trade (see Section 2.4), the economic role of the international trade elasticity is essentially unchanged. What does change is the definition of the trade elasticity which, under the value-added setting, becomes positively related to the intermediate goods share.

#### 2.4 Gains from trade

To illustrate the welfare impact of international trade in this framework, I consider a simplified model where  $\gamma^{j,j} = 1$  and  $\gamma_n^{j,k} = 0$  for all  $k \neq j$ . That is, sector j uses only intermediate goods from its own sector in production.<sup>13</sup> Welfare per capita in country nfor this case is equal to that country's real wage, depicted as the following:

$$W_n = \frac{w_n}{P_n^c},\tag{23}$$

where  $P_n^c = \chi_n \prod_{j=1}^J P_n^{j \alpha_n^j}$  denotes the composite price index for consumers in n, and  $\chi_n$  is a constant.<sup>14</sup>

We can rearrange (15) to find the following expression for  $P_n^j$ :

$$P_n^j = \left(\frac{T_n^j}{\pi_{nn}^j}\right)^{\frac{1-\beta^j}{-\theta^j}} c_n^j,\tag{24}$$

where  $\kappa_{nn}^{j}$  is assumed to be 1. Note that, given the simplified input-output assumption, the unit cost from equation (12) reduces to  $c_{n}^{j} = \Psi_{n}^{j} w_{n}^{1-\beta^{j}} P_{n}^{j\beta^{j}}$ . Substituting this

<sup>&</sup>lt;sup>13</sup>When  $\gamma^{j,j} = 1$ , the gains from trade reduce to a simpler analytical solution under the TFP framework. This is convenient for the purposes of illustrating the mechanisms of this model. Under the VAP heterogeneity framework, gains from trade are equivalent, both with and without the  $\gamma^{j,j} = 1$  assumption. By contrast, Levchenko and Zhang (2014) find that  $\gamma^{j,j} = 1$  provides an upper bound for the gains from trade under a TFP heterogeneity.

 $<sup>^{14}\</sup>chi_n$  is equal to  $\prod_{j=1}^J (\alpha_n^j)^{-\alpha_n^j}$ .

expression into (24) and solving for the price index  $P_n^j$  yields the following:

$$P_n^j = \left(\frac{T_n^j}{\pi_{nn}^j}\right)^{\frac{1}{-\theta^j}} \Psi_n^j w_n.$$
(25)

Finally, substituting this expression into (23) yields the following expression for welfare per capita in n:

$$W_n^{VAP} = \prod_{j=1}^J \left(\frac{\lambda_{nW}^j}{\pi_{nn}^j}\right)^{\frac{\alpha_n^j}{\theta^j}},\tag{26}$$

where  $\lambda_{nW}^{j} = (T_{n}^{j})^{\frac{1}{\theta^{j}}} \alpha_{n}^{j - \alpha_{n}^{j}} \Psi_{n}^{j}$  is a constant.

To find the gains from trade, we take take the logarithm of (26) and consider the comparative static of going from autarky, where  $\pi_{nn}^{j}{}^{A} = 1$  for all j, to the status quo, where  $\pi_{nn}^{j} \leq 1$  for all j. Gains can be denoted as:

$$GFT_n^{VAP} = d\ln(W_n^{VAP}) = -\sum_{j=1}^J \frac{\alpha_n^j}{\theta^j} d\ln(\pi_{nn}^j).$$

$$\tag{27}$$

To calculate gains from trade in n, all that one needs is data on three variables: sectoral spending on final goods  $(\alpha_n^j)$  for all j in n, the share of sectoral home consumption  $(\pi_{nn}^j)$  for all j in n, and sectoral dispersion parameters  $(\theta^j)$ .

Equation (26) is different here than it would be for the case with heterogeneity in TFP. In that environment, welfare simplifies to the following:

$$W_n^{TFP} = \prod_{j=1}^J \left(\frac{\lambda_{nT}^j}{\pi_{nn}^j}\right)^{\frac{\alpha_n^j}{\theta^j(1-\beta^j)}},\tag{28}$$

where  $\lambda_{nT}^j = (T_n^j)^{\frac{1}{\theta^j}} \alpha_n^{j-\alpha_n^j} \Psi_{nT}^j$  and  $\Psi_{nT}^j = \prod_{k=1}^J (\gamma_n^{k,j})^{-\gamma_n^{k,j}} (\beta^j)^{-\beta^j}$ .

Gains from trade with TFP heterogeneity are the following:

$$GFT_n^{TFP} = d\ln(W_n^{TFP}) = -\sum_{j=1}^J \frac{\alpha_n^j}{\theta^j \left(1 - \beta^j\right)} d\ln(\pi_{nn}^j).$$
(29)

Since  $\beta^j \in (0, 1)$  for all j, it is clear that gains from trade are higher in (29) than (27) for a common set of  $\pi^j_{nn}$ ,  $\alpha^j_i$  and  $\theta^j$  across the two models.

In both models, intermediate goods are used to produce both intermediate and final products. In the TFP model, this input-output loop leads to an amplification effect in the gains from trade. As a result, the larger the share of intermediate goods, the higher the gains from trade. In contrast, when productivity enhances value-added, as in our model, this amplification effect disappears. This reveals that the amplification is not due to the use of intermediate goods, but depends on the form of the productivity parameter in the production function. In the standard TFP heterogeneity model, firm productivity enhances both value-added and intermediate goods by the same factor, creating a compounding effect for productivity through the input-output loop. This mechanism is absent from the VAP heterogeneity framework.<sup>15</sup>

This is not to say, however, that estimates of gains from trade will necessarily be higher using the TFP heterogeneity model. Equations (27) and (29) each depend on dispersion parameters  $\theta^{j}$ , which should be estimated with the model in mind. As I demonstrate in Sections 3 and 4, when these parameters are estimated using an empirical gravity equation, the estimates depend on the trade elasticity, which differs across these two models.

# 3 Data

To compute gains from trade under the specification in (27), data are needed for sectoral dispersion parameters  $(\theta^j)$ , sectoral home consumption  $(\pi_{nnt}^j)$  and sectoral consumption shares  $(\alpha_{nt}^j)$ . To compare with gains from trade under the standard TFP heterogeneity model according to (29), data for sectoral intermediate goods shares in production  $(\beta_{it}^j)$  are also needed. Since I have data from several time periods, I add a t subscript for variables that change over time.

#### 3.1 Sectoral dispersion

To find values for  $\theta^{j}$ , it is common in the literature to estimate a gravity equation based on the theoretical model. In our model with VAP heterogeneity, this equation is represented by (18). In the standard TFP heterogeneity framework, this equation is represented by (19).

Caliendo and Parro (2015) provide a prominent recent example of sectoral estimates for  $\theta^{j}$  under the TFP specification. The authors develop a multi-sectoral Eaton and Kortum (2002) model similar to the model in Section 2.1. They derive the following

<sup>&</sup>lt;sup>15</sup>Melitz and Redding (2014) reveal that gains from trade can become arbitrarily large in a framework with sequential production and TFP heterogeneity. This point can be equally demonstrated by setting  $\beta^{j}$  close to zero in the TFP model illustrated here.

trade-share equation for exports from i to n in sector j:

$$\pi_{ni}^{j} = \frac{X_{ni}^{j}}{X_{n}^{j}} = \frac{T_{i}^{j} \left[c_{i}^{k} \kappa_{ni}^{j}\right]^{-\theta^{j}}}{\sum_{i=1}^{N} T_{i}^{j} \left[c_{i}^{j} \kappa_{ni}^{j}\right]^{-\theta^{j}}}.$$
(30)

This equation is analogous to (15) in the VAP heterogeneity model. To estimate  $-\theta^{j}$ , they consider the following tetradic ratio for trade between n, i and a reference country, h, in sector j, based on (30):

$$\frac{X_{ni}^{j}X_{ih}^{j}X_{hn}^{j}}{X_{in}^{j}X_{hi}^{j}X_{nh}^{j}} = \left(\frac{\kappa_{ni}^{j}\kappa_{ih}^{j}\kappa_{hn}^{j}}{\kappa_{in}^{j}\kappa_{hi}^{j}\kappa_{nh}^{j}}\right)^{-\theta_{TFP}^{j}}.$$
(31)

This ratio conveniently eliminates everything in (30) except for bilateral trade costs and the dispersion parameter to be estimated. Note that any symmetric components of trade costs also cancel out in this expression. In fact, any country fixed effects cancel as well. To estimate (31), the authors gather asymmetric tariff data from UNCTAD-TRAINS from 1989 to 1993 across 16 economies and 20 sectors (18 manufacturing and 2 nonmanufacturing).<sup>16</sup> Denoting bilateral tariffs imposed by country n on imports from i in sector j as  $\tau_{ni}^{j}$ , they specify the following estimation equation based on the logarithm of (31):

$$ln\left(\frac{X_{ni}^{j}X_{ih}^{j}X_{hn}^{j}}{X_{in}^{j}X_{hi}^{j}X_{nh}^{j}}\right) = -\theta_{TFP}^{j}ln\left(\frac{\tau_{ni}^{j}\tau_{ih}^{j}\tau_{hn}^{j}}{\tau_{in}^{j}\tau_{hi}^{j}\tau_{nh}^{j}}\right) + \epsilon^{j},\tag{32}$$

where  $\epsilon^{j}$  denotes an i.i.d. error term. Caliendo and Parro (2015) estimate (32) using OLS with heteroskedasticity-robust standard errors, dropping observations with zero flows. In the first two columns of Table 8, I report the estimates and standard errors from their baseline full-sample estimation.<sup>17</sup>

According to the model with VAP heterogeneity described in Section 2.1.1, I derive the following analog to (31) based on (15):

$$\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} = \left(\frac{\kappa_{ni}^j \kappa_{ih}^j \kappa_{hn}^j}{\kappa_{in}^j \kappa_{hi}^j \kappa_{nh}^j}\right)^{\frac{-\theta_{VAP}^j}{1-\beta^j}}.$$
(33)

Note that the right-hand side of (33) is equal to Caliendo and Parro's expression, to the exponent of  $1/(1 - \beta^j)$ . That is,  $\theta_{VAP} = \theta_{TFP} \times (1 - \beta^j)$  can be backed out from

<sup>&</sup>lt;sup>16</sup>The economies included are Argentina, Australia, Brazil, Canada, Chile, China, the European Union, India, Indonesia, Japan, South Korea, New Zealand, Norway, Switzerland, Thailand and the United States.

<sup>&</sup>lt;sup>17</sup>Caliendo and Parro estimate parameters for 20 ISIC Revision 3 industries. The values in Table 8 are converted into ISIC Revision 2 classification.

Caliendo and Parro's estimates of  $\theta_{TFP}$  coupled with data on  $\beta^{j}$ . If we then substitute  $\theta_{VAP}$  into equation (27) to find gains from trade under the value-added specification, it is clear that this formula becomes identical to (29), which corresponds to gains from trade under the TFP specification.

In other words, for cases where values for  $\theta^{j}$  and gains from trade are found using the same data, gains from trade are equivalent under the VAP and TFP specifications.

In the present case, however, and in much of the literature, values for  $\theta^{j}$  and gains from trade are found using different data. To quantify the impact of this, I adjust Caliendo and Parro's tetradic tariff ratio data to be consistent with the specification in (33). That is, I adjust the regressors from Caliendo and Parro's data to the following:

$$\theta_{VAP} = \theta_{TFP} \times \left(1 - \overline{\beta}^j\right),\tag{34}$$

where  $\overline{\beta}^{j}$  denotes the observed mean intermediate goods share across countries.<sup>18</sup>

In the third column of Table 8, I report calculated values for  $\theta_{VAP}^{j}$  based on this exercise. Not surprisingly, values for  $\theta_{VAP}^{j}$  are significantly lower than  $\theta_{TFP}^{j}$  owing to the impact of the intermediate goods adjustment. This translates into a higher degree of dispersion within sectors.

The gains from trade are higher when the sectoral dispersion parameters are low. This is true for both the VAP and TFP frameworks, as indicated by equations (27) and (29). Note, however, that the TFP specification in (29) has a  $(1 - \beta^j)$  term that equation (27) is missing. This raises the gains from trade under TFP heterogeneity. In the end, the lower estimates of  $\theta^j$  from the value-added specification counterbalance the welfarereducing impact of the missing  $(1 - \beta^j)$ , resulting in an ambiguous but modest overall difference in the gains from trade between the two theoretical models.

As mentioned above, when  $\theta^{j}$  and gains from trade are calculated using the same data, gains from trade are equal across these specifications. In the present case, however, since  $\theta_{VAP}^{j}$  and gains from trade are estimated using different data, then gains from trade across specifications will differ, albeit in a fairly modest way. **o** 

#### **3.2** Intermediate goods shares

For sectoral intermediate goods shares  $(\beta_{it}^j)$ , I use data from the World Input-Output Database (WIOD). Because these shares vary across countries, I allow  $\beta_{it}^j$  to vary across

<sup>&</sup>lt;sup>18</sup>To correspond with the 16 countries from Caliendo and Parro (2015), I calculate  $\overline{\beta}^{j}$  using the data from the World Input-Output Database (WIOD) for 1995. Unfortunately, the WIOD does not perfectly overlap with the set of countries used in Caliendo and Parro's tariff calculations. However, there is significant overlap, so I expect that observed values for  $\overline{\beta}^{j}$  are not severely biased.

*i* for the remainder of this analysis. Previous literature specifically emphasizes the relationship between the trade elasticity and the imported intermediate share (Johnson and Noguera (2012c)). However, the model presented in Section 2 suggests that trade elasticity should have a negative relationship with the intermediate goods share, regardless of whether the inputs are produced domestically or abroad.

For each country, data for 14 manufacturing ISIC Revision 2 sectors are available from the WIOD for 1995 to 2011. I use data for 1995, 2000, 2005 and 2010. I exclude one of the sectors, Leather and Footwear, which reduces the number of sectors to 13. Although the WIOD provides data for 40 countries, I restrict the analysis to 33 countries.<sup>19</sup> A list of the sectors included is provided in Table 8, and a list of the countries included is provided in Table 10.

Table 9 reports average shares of domestic-sourced  $(\beta_{ith}^j)$  and foreign-sourced  $(\beta_{itf}^j)$ intermediate goods by sector and over time across these 33 countries. As we see,  $\beta_{ith}^j$  is consistently higher than  $\beta_{itf}^j$  across all sectors except for Coke and Refined Petroleum. However, from 1995 to 2010, the domestic share generally decreased while the foreign share increased for most sectors. On average, the total share of intermediate goods used in production rose a few percentage points over the 1995 to 2010 period across these countries.

#### 3.3 Exports

For exports  $(X_{nit}^j)$ , I use the BACI export database provided by CEPII.<sup>20</sup> BACI is constructed using HS6 bilateral export values based on UN Comtrade data, which includes over 5000 potential product groups. I group the data to correspond with each of the 13 ISIC Revision 2 sectors described above using correspondence tables downloaded from the United Nations Statistical Division website, and then consider aggregate bilateral exports in addition to a count of the number of product groups  $(F_{nit}^j)$  exported bilaterally in each sector. As a result, I can define the following expression:

$$X_{nit}^j = F_{nit}^j \times \overline{X}_{nit}^j$$

, where  $\overline{X}_{nit}^{j}$  denotes the average bilateral exports per product group between *i* and *n* in sector *j* at time *t*. This provides for distinction between the total exports  $(X_{nit}^{j})$  and the extensive margin  $(F_{nit}^{j})$  in the empirical analysis. For both dependent variables, I include

<sup>&</sup>lt;sup>19</sup>The decision to restrict the analysis to these 33 countries and 13 manufacturing sectors follows Costinot and Rodriguez-Clare (2014). The main reason for these restrictions appears to be data limitations.

<sup>&</sup>lt;sup>20</sup>Centre d'Etudes Prospectives et d'Informations Internationales.

data from the 33 exporting countries to 206 import-receiving countries.<sup>21</sup>

#### **3.4** Trade costs

To estimate the trade elasticity, I consider several different measures of trade costs  $(\kappa_{nit})$ . These include bilateral distance  $(d_{ni})$ , bilateral sector-level average *ad valorem* tariffs  $(\tau_{nit}^j)$  and a dummy variable for bilateral regional trade agreement  $(rta_{nit})$ .<sup>22</sup> All of these data come from CEPII.<sup>23</sup>

#### 3.5 Other input-output parameters

To calculate the gains from trade, we need measures of home consumption shares  $(\pi_{nnt}^j)$  and sectoral consumption shares  $(\alpha_{nt}^j)$  across countries and sectors. Measures of both variables can be derived using the WIOD. Although the WIOD reports trade data for non-manufacturing sectors, I focus specifically on manufacturing trade for this analysis.

Summary statistics for data used in the gains from trade and trade elasticity exercises are provided in Table 1.

# 4 Gains from Trade: Empirical Results

I calculate the gains from manufacturing trade using data for sectoral spending on final goods  $(\alpha_{nt}^j)$ , the share of sectoral home consumption  $(\pi_{nnt}^j)$ , and sectoral dispersion parameter estimates  $(\theta^j)$ , as described in Section 3.

These are calculated according to the VAP heterogeneity specification based on the following equation (derived in Section 2.5):

$$GFT_{nt}^{VAP} = d\ln(W_{nt}^{VAP}) = -\sum_{j=1}^{J} \frac{\alpha_{nt}^{j}}{\theta_{VAP}^{j}} d\ln(\pi_{nnt}^{j})$$
(35)

I also calculate gains from trade under the TFP heterogeneity specification. This is calculated using Caliendo and Parro's estimates of  $\theta_{TFP}^{j}$  according to the following

 $<sup>^{21}\</sup>mathrm{A}$  list of recipient countries is provided in Table 11.

<sup>&</sup>lt;sup>22</sup>I also explored using common language, colonial linkage, common border and currency union dummy variables. These measures, however, provided estimates that were not positively significant in standard gravity regressions so I omitted them.

<sup>&</sup>lt;sup>23</sup>For bilateral distance, I use the population-weighted measure of agglomeration-by-agglomeration distance created in Head and Mayer (2002) that is provided by CEPII. This is calculated as:  $d_{ni} = \left(\sum_{k \in n} (pop_k/pop_n) \sum_{l \in i} (pop_l/pop_i) d_{kl}\right)$  where  $pop_k$  denotes population in agglomeration k inside country n. For ad valorem tariffs, I use measures constructed by CEPII at the HS6 level of aggregation based on tariff data from UNCTAD-WTO. See www.cepii.fr for details and links to the data.

standard gains from trade (GFT) equation:

$$GFT_{nt}^{TFP} = d\ln(W_{nt}^{TFP}) = -\sum_{j=1}^{J} \frac{\alpha_{nt}^{j}}{\theta_{TFP}^{j} \left(1 - \beta_{it}^{j}\right)} d\ln(\pi_{nnt}^{j}).$$
(36)

I report the results under the VAP specification in columns 1 and 2 of Table 10 for all 33 countries for 1995 and 2010. The results based on the TFP specification are reported in columns 3 and  $4^{24}$ 

The gains from manufacturing trade are, on average, larger by 0.8 percentage points in 1995 and smaller by 2.1 percentage points in 2010 under the VAP specification. In 1995, gains are larger under the VAP specification for 20 of the 33 countries; in 2010, gains are larger for only 10 of the countries.

These results are consistent with a central point made in Section 2.4 – that the difference in gains from trade across these two specifications is ambiguous yet modest. In cases where gains are higher under the VAP specification, the positive effect of lower  $\theta^{j}$  estimates outweighs the missing direct effect of  $\beta_{it}^{j}$  in the GFT equation. In cases where the gains are higher under the TFP specification, the opposite is true.

Figure 3 depicts growth in the gains from trade from 1995 to 2010 for each country. Growth is generally higher under the TFP specification. This result is driven by the share of intermediate goods in production, which grew over time for most sectors (see Table 9). This growth positively influences gains from trade under TFP but not under the VAP specification. Note that, for some countries, gains from trade actually decline over time. Moreover, for most cases, this decline is larger under the VAP specification. For example, in Canada, gains from trade declined by 20 and 14 percentage points under the VAP and TFP specifications, respectively. This decline is mostly concentrated in the Transport Equipment sector where Canadian imports declined significantly over the period.<sup>25</sup> Meanwhile, the use of intermediate inputs in this sector has grown over time, so the decline in Canadian import share is counterbalanced under the TFP specification. Under VAP, since intermediate goods do not enter the gains from trade formula, the fall in imports had relatively more influence on the measured gains from trade.

Overall, the gains from trade, while generally slightly higher under the VAP specification, are fairly similar in both models. Again, this is not surprising, given that the quantitative difference in welfare across the two specifications is theoretically modest and ambiguous.

<sup>&</sup>lt;sup>24</sup>The WIOD reports exports for non-manufacturing sectors as well. Since this project is focused on manufacturing sectors, the figures in Table 10 are calculated with non-manufacturing trade set to zero across all countries.

<sup>&</sup>lt;sup>25</sup>This decline is likely due to the reconfiguration of the North American automotive supply chain away from Canada and towards Mexico that occurred over the 1995 to 2010 period.

Perhaps the most important difference across these models relates to trade policy. The TFP gains from trade formula might lead one to conclude that sectors that use intermediate goods should be targeted for trade promotion since they have a greater relative impact on welfare. In contrast, the VAP framework suggests no such policy. In addition, as emphasized by Melitz and Redding (2014), the TFP formula suggests that future gains from trade might be inevitable, owing to the rising share of intermediate goods in production. In contrast, according to the VAP specification, future gains from trade must come about through greater openness, which is not necessarily expected to consistently rise over time.

# 5 The Trade Elasticity: Empirical Results

#### 5.1 Empirical specification

The gravity model in (18) has two distinct features that are different from previous gravity equations in the literature. The first is that the elasticity of trade with respect to trade costs (the "trade elasticity") is positively related to the share of intermediate goods used in production  $(\beta_{it}^j)^{26}$  The second is that this relationship is driven by the extensive margin: the number of products exported from i to n is more sensitive to changes in trade costs when intermediates goods are used in production.

To examine these relationships in a reduced form, one must control for every item in expression (18) except for the bilateral trade cost expression  $(\kappa_{nit}^{j})^{\frac{-\theta^{j}}{1-\beta^{j}}}$ ). One way to achieve this would be to divide these expressions by exporter and importer home consumption  $(\pi_{nnt}^{j})^{27}$  This approach, however, requires input-output tables for all importing and exporting countries. Since we have these tables for only 33 countries, the sample would be significantly restricted. Note that I also do not want to use countrysector fixed effects, since this would eliminate much of the variation in  $\beta_{it}^{j}$  that I wish to exploit. Instead, I follow a tetradic ratio approach employed by Romalis (2007) and Head et al. (2010). Considering sectoral exports between n, i, a reference exporter l and a reference importer k in sector j at time t, we can derive the following tetradic ratio that accords with (18):

$$\frac{X_{nit}^j X_{klt}^j}{X_{nlt}^j X_{kit}^j} = \left(\frac{\kappa_{nit}^j \kappa_{klt}^j}{\kappa_{nlt}^j \kappa_{kit}^j}\right)^{\frac{-\theta^j}{1-\beta_{it}^j}}.$$
(37)

This ratio conveniently cancels out any exporter and importer sectoral fixed effects

<sup>&</sup>lt;sup>26</sup>As in Section 4, I now add time subscripts (t) for variables that change across time periods in the data. I also allow intermediate goods  $(\beta_{it}^j)$  to vary across countries.

<sup>&</sup>lt;sup>27</sup>This approach is referred to in the literature as the Head-Ries Index.

that are found in the theoretical gravity equation. Unlike (31) from Caliendo and Parro (2015), however, this ratio does not cancel out symmetric bilateral trade costs. Taking the logarithm of (37), I define the following log-linearized theoretically consistent empirical gravity specification for bilateral trade costs:

$$ln\left(\widetilde{X}_{nit}^{j}\right) = \left(\frac{-\theta^{j}}{1 - \overline{\beta}_{ilt}^{j}}\right) ln\left(\widetilde{\kappa}_{nit}^{j}\right) + \epsilon_{nit}^{j},\tag{38}$$

where  $\widetilde{X}_{nit}^k = \frac{X_{nit}^j X_{klt}^j}{X_{nlt}^j X_{ki}^j}$ ,  $\widetilde{\kappa}_{nit}^k = \frac{\kappa_{nit}^j \kappa_{klt}^j}{\kappa_{nlt}^j \kappa_{kit}^j}$  and  $\epsilon_{nit}^j$  is an error term assumed to be i.i.d.

Data on  $\beta_{it}^{j}$  vary across countries empirically. Allowing for this variation, I define  $\overline{\beta}_{ilt}^{j}$  as the mean of intermediate goods shares between exporter i and the reference exporter l in sector j at time t.

In theory,  $\tilde{\kappa}_{nit}^{j}$  consists of both observed and unobserved bilateral trade costs. To capture these, I examine data on the log of bilateral distance  $(d_{ni})$ , sector-level bilateral tariffs  $(\tau_{nit})$  and a dummy variable for bilateral regional trade agreement  $(rta_{nit})$ . I assume that any unobserved determinants of intermediate goods shares and trade costs that are excluded are orthogonal to the error term  $\epsilon_{nit}^{j}$ .

The tetradic reference country method raises the difficulty of choosing reference countries. Including reference countries inevitably restricts the sample of observations. Ideally, both countries should have large economies that are relatively open to imports in order to provide as many observations as possible. In line with these considerations, I have chosen Germany and France as the reference exporter and importer, respectively.<sup>28</sup>

I analyze the relationship between  $X_{nit}^{j}$ ,  $\kappa_{nit}^{j}$  and  $\overline{\beta}_{ilt}^{j}$  in (38) using two methodologies. In the first, I estimate the following equation using NLS based on the theoretical trade elasticity:

$$ln\left(\widetilde{X}_{nit}^{j}{}^{VAP}\right) = \lambda_o + \lambda_1 \left(\frac{1}{1 - \lambda_2 \overline{\beta}_{ilt}^j}\right) ln\left(\widetilde{\kappa}_{nit}^j\right) + \epsilon_{nit}^j.$$
(39)

This equation is analogous to a typical gravity equation with fixed effects, which is usually specified as the following:

$$ln\left(\widetilde{X}_{nit}^{j\ TFP}\right) = \lambda_o + \lambda_1 ln\left(\widetilde{\kappa}_{nit}^j\right) + \epsilon_{nit}^j.$$
(40)

Note that our specification differs from the standard approach owing to the structure of the trade elasticity associated with the trade cost variable in (39). I am interested in whether or not  $\lambda_2$  is positive, as well as its magnitude. In our theoretical framework,

<sup>&</sup>lt;sup>28</sup>I also considered other reference countries, including Great Britain, the Netherlands and the United States. The results presented here are generally robust across these alternatives.

 $\lambda_2 = 1.$ 

As a robustness check, I also estimate the following more reduced-form equation using OLS based on the theoretical trade elasticity:

$$ln\left(\widetilde{X}_{nit}^{j}\right) = \lambda_{o} + \lambda_{1}ln\left(\widetilde{\kappa}_{nit}^{j}\right) + \lambda_{2}ln\left(\overline{\beta}_{ilt}^{j}\right) + \lambda_{3}ln\left(\widetilde{\kappa}_{nit}^{j}\right) \times ln\left(\overline{\beta}_{ilt}^{j}\right) + \epsilon_{nit}^{j}.$$
 (41)

For this case, I am interested in whether or not the coefficient on the interaction term,  $\lambda_3$ , is significant and has the same sign as the  $\lambda_1$  coefficient, which should be the case according to our theoretical gravity model.

To explore the extensive margin and average exports per product, I also estimate (39), (40) and (41) replacing  $X_{nit}$  by  $F_{nit}$  and  $\overline{X}_{nit}$  in each equation. I expect, based on the model, that these relationships will be present in the extensive margin regressions.

All regressions include year dummy variables, and include data from 1995, 2000, 2005 and 2010. For estimates from equations (39) and (40), errors are adjusted for clustering by sector using the standard cluster robust variance estimator (CRVE) procedure.<sup>29</sup>

#### 5.2 Results

Results from estimating (39) and (40) using distance  $\tilde{d}_{ni}$  as a proxy for trade costs are reported in Table 2.<sup>30</sup> Columns 1 and 2 report results from estimating (40) and (39) respectively, with total bilateral exports as the dependent variable. The estimate of  $\lambda_1$ in column 1 is significant at the 1% level and fairly close to 1 in magnitude, which is consistent with most estimates in the literature.

In column 2, we see that the estimate of  $\lambda_1$  is lower than in column 1 and that  $\lambda_2$  is positive and significant at the 5% level. Since this estimate is positive and above 0.5, I take the result to be fairly consistent with the predictions of the model where  $\lambda_2 = 1$ . Note that in the standard framework with TFP heterogeneity,  $\lambda_2$  is set equal to 0, which is clearly rejected by the data.

Table 3 reports similar results using sectoral bilateral tariffs  $\tilde{\tau}_{nit}^{j}$  as a measure of trade

<sup>&</sup>lt;sup>29</sup>The choice to adjust errors for clustering by sector follows the spirit of Romalis (2007), who focuses on data at the commodity level and adjusts for clustering by commodity. I also clustering by importer, exporter, and country-pair, but these adjustments did not affect the standard errors in a significant way. Note that the data have only 14 sectors/clusters, which raises concerns over the appropriateness of using the CRVE for inference. Cameron, Gelbach and Miller (2008) suggest an alternative wild cluster bootstrap-t procedure for cases when the number of clusters is small. Unfortunately, the bootstrap methods employed in this procedure are not valid for non-linear estimation. In the Appendix, I consider robustness using reduced-form linear equations and adjust for clustering using the wild cluster bootstrapt procedure. Results from this exercise are not significantly different from results using the standard CRVE.

<sup>&</sup>lt;sup>30</sup>Results from estimating equation (41) are provided in the Appendix. They are generally consistent with the results provided here.

cost. Again, columns 1 and 2 report estimates for equations (40) and (39), respectively. From column 1, the impact of bilateral tariffs is negative and significant at the 1% level, which is consistent with previous literature. From column 2, as in Table 1, the estimate of  $\lambda_2$  is positive and statistically significant at the 5% level. Again, this coefficient is close to 0.5.

Table 4 reports results using a regional trade agreement dummy variable  $\widetilde{RTA}_{nit}$  as a measure of trade cost.<sup>31</sup> In this case,  $\lambda_1$  is positive in column 1, which is consistent with previous research.  $\lambda_2$  in column 2 is positive and significant at the 1% level, and between 0.5 and 1, which is consistent with the predictions of the theoretical model.

Overall, the findings based on total exports are strongly consistent with the model. Again, the standard model with TFP hetereogeneity assumes that  $\lambda_2 = 0$ , which is not supported by the data.

#### 5.2.1 Extensive margin

The theory predicts that the positive relationship between the intermediate goods share and the trade elasticity is driven by the extensive margin rather than the intensive/compositional margin.

To test this prediction, I replace total exports with the number of products exported as the dependent variable in specification (39). Columns 3 and 4 in Table 2 report results for equations (40) and (39) respectively with bilateral distance as a measure of trade costs. From column 3, we observe that the trade elasticity at the extensive margin is lower in magnitude than its equivalent value for total exports (reported in column 1).<sup>32</sup> From column 4, we observe that the estimate of  $\lambda_2$  is positive, relatively large in magnitude, and more statistically significant than in column 2 where total exports is the dependent variable. By contrast, results from the intensive/compositional margin, reported in column 6, suggest that the impact of intermediate goods is statistically insignificant at this margin. This is consistent with the theoretical model.

Columns 3 and 4 in Tables 3 and 4 report similar findings at the extensive margin using  $\tau_{nit}^{j}$  and  $RTA_{nit}$  as measures of trade costs. Again, the relationship between the trade costs and the share of intermediate goods (indicated by  $\lambda_2$  in column 4) at the extensive margin is larger in magnitude and more significant than the equivalent relationship at the total or intensive/compositional margins (indicated by  $\lambda_2$  in columns 2 and 6). For tables 3 and 4, the estimate of  $\lambda_2$  at the intensive/compositional margin is insignificant,

<sup>&</sup>lt;sup>31</sup>I also considered estimating equations (39) and (40) with  $d_{ni}$ ,  $\tau_{nit}^{j}$  and  $RTA_{nit}$  in the same regression. With this specification, the impact  $d_{ni}$  remains significant, while that of  $\tau_{nit}^{j}$  and  $RTA_{nit}$  becomes insignificant.

<sup>&</sup>lt;sup>32</sup>This finding is consistent with results from Santos Silva, Tenreyro and Wei (2014).

whereas this estimate at the extensive margin is positive and significant at the 1% level.

Overall, this evidence suggests that when the extensive margin is isolated, the relationship between the intermediate goods share and the trade elasticity remains strong. This is consistent with the theoretical model, which predicts that this relationship is driven by the extensive margin of trade.

# 6 Conclusion

This paper makes several contributions. First, I extend the Eaton and Kortum (2002) model of international trade to a setting with intermediate goods and producer heterogeneity in *value-added* productivity. This adjustment generates a positive relationship between the international trade elasticity and the share of intermediate goods in production, which is absent from the standard model. According to the theory, this positive relationship is driven by the extensive margin of trade, or the number of products traded internationally.

Second, I estimate the trade elasticity in accordance with the theoretical relationship derived from the model. I find evidence that the trade elasticity is positively related to the share of intermediate goods in production. I also find evidence that this relationship is particularly strong at the extensive margin of trade.

These results help explain the rapid growth, and regional concentration, of vertical specialization trade since the 1970s. Since sectors that use intermediate goods are particularly sensitive to trade costs, falling trade costs have ad a larger impact on exports and imports for these goods. Meanwhile, distance-related trade costs also have a larger impact on these sectors, leading to a "regionalized" pattern in this type of trade.

Remarkably, despite this heightened elasticity, the country-level gains from international trade are quantitatively similar to the gains under standard international trade models with intermediate goods. However, at the sector level, whereas standard models suggest higher gains for sectors that use intermediate goods, the value-added productivity specification generates no such relationship. In other words, this model extends the message from Arkolakis et al. (2012) by generating the "same old gains" as the standard models in addition to the "same old gains" for sectors that use intermediate goods in production.

# References

- ADAMOPOULOS, T. AND D. RESTUCCIA (2014): "The Size Distribution of Farms and International Productivity Differences," *American Economic Review*, 104, 1667–97.
- ALVAREZ, F. AND R. J. LUCAS (2007): "General equilibrium analysis of the Eaton-Kortum model of international trade," *Journal of Monetary Economics*, 54, 1726–1768.
- ANTRAS, P., D. CHOR, T. FALLY, AND R. HILLBERRY (2012): "Measuring the Upstreamness of Production and Trade Flows," *American Economic Review*, 102, 412–16.
- ARKOLAKIS, C., A. COSTINOT, AND A. RODRÍGUEZ-CLÁRE (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.
- AXTELL, R. L. (2001): "Zipf Distribution of U.S. Firm Size," Science, 293, 1818–1820.
- CALIENDO, L. AND F. PARRO (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 82, 1–44.
- CAMERON, A. C., J. B. GELBACH, AND D. L. MILLER (2008): "Bootstrap-Based Improvements for Inference with Clustered Errors," *The Review of Economics and Statistics*, 90, 414–427.
- CHANEY, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98, 1707–21.
- COSTINOT, A., D. DONALDSON, AND I. KOMUNJER (2012): "What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas," *Review of Economic Studies*, 79, 581–608.
- COSTINOT, A. AND A. RODRÍGUEZ-CLÁRE (2014): "Trade Theory with Numbers: Quantifying the Consequences of Globalization," *Handbook of International Economics*, 4, 197–261.
- CROZET, M. AND P. KOENIG (2010): "Structural gravity equations with intensive and extensive margins," *Canadian Journal of Economics*, 43, 41–62.
- DI GIOVANNI, J., A. A. LEVCHENKO, AND R. RANCIÈRE (2011): "Power laws in firm size and openness to trade: Measurement and implications," *Journal of International Economics*, 85, 42–52.
- EATON, J. AND S. KORTUM (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

- HEAD, K. AND T. MAYER (2002): "Illusory Border Effects: Distance Mismeasurement Inflates Estimates of Home Bias in Trade," Working Papers 2002-01, CEPII research center.
- (2014): Gravity Equations: Workhorse, Toolkit, and Cookbook, Elsevier, vol. 4 of Handbook of International Economics, chap. 0, 131–195.
- HEAD, K., T. MAYER, AND J. RIES (2010): "The erosion of colonial trade linkages after independence," *Journal of International Economics*, 81, 1–14.
- HELPMAN, E., M. MELITZ, AND Y. RUBINSTEIN (2008): "Estimating Trade Flows: Trading Partners and Trading Volumes," *The Quarterly Journal of Economics*, 123, 441–487.
- HILLBERRY, R. AND D. HUMMELS (2008): "Trade responses to geographic frictions: A decomposition using micro-data," *European Economic Review*, 52, 527–550.
- HUMMELS, D. (2007): "Transportation Costs and International Trade in the Second Era of Globalization," *Journal of Economic Perspectives*, 21, 131–154.
- HUMMELS, D., J. ISHII, AND K.-M. YI (2001): "The nature and growth of vertical specialization in world trade," *Journal of International Economics*, 54, 75–96.
- IMBS, J. AND I. MEJEAN (2015): "Elasticity Optimism," American Economic Journal: Macroeconomics, 7, 43–83.
- JOHNSON, R. C. AND G. NOGUERA (2012a): "Accounting for intermediates: Production sharing and trade in value added," *Journal of International Economics*, 86, 224–236.
- (2012b): "Fragmentation and Trade in Value Added over Four Decades," NBER Working Papers 18186, National Bureau of Economic Research, Inc.
- (2012c): "Proximity and Production Fragmentation," American Economic Review, 102, 407–11.
- KOOPMAN, R., Z. WANG, AND S.-J. WEI (2012): "Estimating domestic content in exports when processing trade is pervasive," *Journal of Development Economics*, 99, 178–189.
- ——— (2014): "Tracing Value-Added and Double Counting in Gross Exports," *American Economic Review*, 104, 459–94.

- LEVCHENKO, A. A. AND J. ZHANG (2014): "Ricardian productivity differences and the gains from trade," *European Economic Review*, 65, 45–65.
- LUTTMER, E. G. J. (2007): "Selection, Growth, and the Size Distribution of Firms," The Quarterly Journal of Economics, 122, 1103–1144.
- MELITZ, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.
- MELITZ, M. J. AND S. J. REDDING (2014): "Missing Gains from Trade?" American Economic Review, 104, 317–21.
- (2015): "New Trade Models, New Welfare Implications," *American Economic Review*, 105, 1105–46.
- Ossa, R. (2015): "Why Trade Matters After All," *Journal of International Economics*, 97, 266–277.
- ROMALIS, J. (2007): "NAFTA's and CUSFTA's Impact on International Trade," *The Review of Economics and Statistics*, 89, 416–435.
- RUHL, K. J. (2008): "The International Elasticity Puzzle," Working Papers 08-30, New York University, Leonard N. Stern School of Business, Department of Economics.
- SHIKHER, S. (2012): "Putting industries into the Eaton–Kortum model," *The Journal* of International Trade & Economic Development, 21, 807–837.
- SILVA, J. M. C. S. AND S. TENREYRO (2006): "The Log of Gravity," *The Review of Economics and Statistics*, 88, 641–658.
- (2014): "Estimating the extensive margin of trade," Journal of International Economics, 93, 67–75.
- SIMONOVSKA, I. AND M. E. WAUGH (2014): "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 92, 34–50.
- TIMMER, M. P., A. A. ERUMBAN, B. LOS, R. STEHRER, AND G. J. DE VRIES (2014): "Slicing Up Global Value Chains," *Journal of Economic Perspectives*, 28, 99–118.
- TINBERGEN, J. (1962): "An Analysis of World Trade Flows," New York, NY Twentieth Century Fund, Shaping the World Economy.
- YI, K.-M. (2003): "Can Vertical Specialization Explain the Growth of World Trade?" Journal of Political Economy, 111, 52–102.

(2010): "Can Multistage Production Explain the Home Bias in Trade?" *American Economic Review*, 100, 364–93.

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# A Appendix

#### A.1 Proofs

#### A.1.1 Proof of equation (14)

Let  $F_{ni}^{j}(p)$  denote the probability that the price at which country *i* can supply a given variety in sector *j* to country *n* is lower than or equal to *p*. Rearranging (11) in terms of  $z_{i}^{j}$  and using the distribution expression in (3), we find that:<sup>33</sup>

$$F_{ni}^{j}(p)^{VAP} = 1 - F_{i}^{j}\left(z_{i}^{j}(\omega)\right) = 1 - F_{i}^{j}\left(\left(\frac{c_{i}^{j}\kappa_{ni}^{j}}{p}\right)^{\frac{1}{1-\beta^{j}}}\right)$$
  
=  $1 - exp\left\{-T_{i}^{j}\left(c_{i}^{j}\kappa_{ni}^{j}\right)^{-\frac{\theta^{j}}{1-\beta^{j}}}p^{\frac{\theta^{j}}{1-\beta^{j}}}\right\}.$  (42)

It follows that the probability of receiving a price in n below p for a given variety from any country is equal to  $F_n^j(p) = \prod_{i=1}^N F_{ni}^j(p)$ . Solving for this expression yields:

$$F_{n}^{j}(p) = 1 - \prod_{i=1}^{N} exp\left\{-T_{i}^{j}\left(c_{i}^{j}\kappa_{ni}^{j}\right)^{-\frac{\theta^{j}}{1-\beta^{j}}}p^{\frac{\theta^{j}}{1-\beta^{j}}}\right\} = 1 - exp\left\{\sum_{i=1}^{N} -\phi_{n}^{j}p^{\frac{\theta^{j}}{1-\beta^{j}}}\right\},\qquad(43)$$

where

$$\phi_n^j = \sum_{i=1}^N T_i^j \left[ c_i^j \kappa_{ni}^j \right]^{\frac{-\theta^j}{1-\beta^j}}.$$
(44)

Expression (7) can now be rearranged to  $(P_n^j)^{1-\sigma} = \int_0^1 p_n^j(\omega)^{1-\sigma} d\omega = \int_0^\infty p^{1-\sigma} dF_n^j(p)$ . Expanding  $dF_n^j(p)$  using (43) and substituting this into the price index yields the following:

$$\left(P_n^j\right)^{1-\sigma} = \int_0^\infty p^{1-\sigma} \phi_n^j \left(\frac{\theta^j}{1-\beta^j}\right) p^{\frac{\theta^j}{1-\beta^j}-1} exp\left\{-\phi_n^j p^{\frac{\theta^j}{1-\beta^j}}\right\} dp.$$
(45)

From here, one can employ integration by substitution. Letting  $x = \phi_n^j p^{\frac{\theta^j}{1-\beta^j}}$ , it follows that  $dx = \phi_n^j \left(\frac{\theta^j}{1-\beta^j}\right) p^{\frac{\theta^j}{1-\beta^j}-1} dp$  and  $p = (x/\phi_n^j)^{\frac{1-\beta^j}{\theta^j}}$ . Substituting these expressions into

 $^{33}$ This probability is different from the standard Eaton and Kortum (2002) model, where the analogous expression is the following:

$$F_{ni}^{j}(p)^{TFP} = 1 - F_{i}^{j}\left(z_{i}^{j}(\omega)\right) = 1 - F_{i}^{j}\left(\frac{c_{i}^{j}\kappa_{ni}^{j}}{p}\right).$$

(45) yields the following:

$$(P_n^j)^{1-\sigma} = \int_0^\infty \left(\frac{x}{\phi_n^j}\right)^{\frac{(1-\sigma)\left(1-\beta^j\right)}{\theta^j}} exp\left\{-x\right\} dx$$

$$= \left(\phi_n^j\right)^{\frac{(1-\sigma)\left(1-\beta^j\right)}{-\theta^j}} \int_0^\infty x^{\frac{(1-\sigma)\left(1-\beta^j\right)}{\theta^j}} exp\left\{-x\right\} dx.$$

$$(46)$$

The second part of this expression can be simplified as

$$\int_{0}^{\infty} x \frac{(1-\sigma)(1-\beta^{j})}{\theta^{j}} exp\left\{-x\right\} dx = \Gamma\left(\frac{\theta^{j} + (1-\sigma)\left(1-\beta^{j}\right)}{\theta^{j}}\right),\tag{47}$$

where  $\Gamma$  denotes the Gamma function (a constant).<sup>34</sup> Substituting (47) into (46) and multiplying by the exponent of  $1/(1-\sigma)$  yields the expression (14):

$$P_n^j = \left(\phi_n^j\right)^{\frac{1-\beta^j}{-\theta^j}} \Gamma\left(\frac{\theta^j + (1-\sigma)\left(1-\beta^j\right)}{\theta^j}\right)^{\frac{1}{(1-\sigma)}}.$$
(48)

#### A.1.2 Proof of equation (15)

We can represent  $\pi_{ni}^{j}$  as  $\pi_{ni}^{j} = Pr\left(p_{ni}^{j}(\omega^{j}) \leq \min\left\{p_{nk}^{j}(\omega^{j}); k \neq i\right\}\right)$ . Suppose that  $p_{ni}^{j}(\omega^{j}) = p$ ; then, this probability can be represented as:

$$\prod_{k \neq i} \Pr\left(p_{nk}^{j}(\omega^{j}) \geq p\right) = \prod_{k \neq i} \left[1 - F_{nk}^{j}\left(p\right)\right] = exp\left\{\sum_{k \neq i} -T_{i}^{j}\left(c_{i}^{j}\kappa_{ni}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}}p^{\frac{\theta^{j}}{1-\beta^{j}}}\right\} = exp\left\{-\phi_{n \neq i}^{j}p^{\frac{\theta^{j}}{1-\beta^{j}}}\right\},$$
(49)

where  $\phi_{n\neq i}^{j} = \sum_{k\neq i} -T_{i}^{j} \left(c_{i}^{j} \kappa_{ni}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}}$ . To find  $\pi_{ni}^{j}$ , we multiply (49) by the density  $dF_{ni}^{j}(p)$  and integrate this product over all possible p's. The density  $dF_{ni}^{j}(p)$  is equal to:

$$dF_{ni}^{j}(p) = T_{i}^{j} \left(c_{i}^{j} \kappa_{ni}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}} \frac{\theta^{j}}{1-\beta^{j}} p^{\frac{\theta^{j}}{1-\beta^{j}}-1} exp\left\{-T_{i}^{j} \left(c_{i}^{j} \kappa_{ni}^{j}\right)^{\frac{-\theta^{j}}{1-\beta^{j}}} p^{\frac{\theta^{j}}{1-\beta^{j}}} dp\right\}.$$
 (50)

<sup>34</sup>The general formula for the Gamma function is  $\Gamma(a) = \int_{-\infty}^{\infty} x^{a-1} e^{-x} dx$ .

We can therefore solve for  $\pi_{ni}^{j}$  as:

$$\begin{aligned} \pi_{ni}^{j} &= \int_{0}^{\infty} exp \left\{ -\phi_{n\neq i}^{j} p^{\frac{\theta^{j}}{1-\beta^{j}}} dp \right\} dF_{ni}^{j}(p) \\ &= \left( \frac{T_{i}^{j} \left[ c_{i}^{j} \kappa_{ni}^{j} \right]^{\frac{-\theta^{j}}{1-\beta^{j}}}}{\phi_{n}^{j}} \right) \int_{0}^{\infty} \frac{\theta^{j}}{1-\beta^{j}} \phi_{n}^{j} exp \left\{ -\phi_{n}^{j} p^{\frac{\theta^{j}}{1-\beta^{j}}} \right\} p^{\frac{\theta^{j}}{1-\beta^{j}}-1} dp. \end{aligned}$$
(51)

The portion of the expression above that is to the right of the integral is equal to  $dF_n^j(p)dp$ . Since  $\int_0^\infty dF_n^j(p)dp = 1$ , (15) has been proven.

#### A.2 Robustness Regression Tables

Tables 5 through 7 report estimates of equation (41) using various different measures of trade costs. All errors in these tables are adjusted for clustering by sector. To address concerns over the small number of sectors/clusters, I use the wild cluster bootstrap-t procedure described by Cameron, Gelbach and Miller (2008). Resulting standard errors are not significantly different than those obtained from using the standard CRVE procedure.

Table 5 provides estimates using the  $d_{ni}$  trade cost measure. The model predicts that estimates of the interaction term should be negative and significant, indicating a positive relationship between the share of intermediate goods and the international trade elasticity. The standard TFP heterogeneity model assumes that there is no relationship here.

In column 1 of Table 5, the interaction term is negative but statistically insignificant in the regression where total exports is used as the dependent variable. From columns 2 and 3, we observe that the interaction term is negative and significant at the 10% level at the extensive margin, and insignificant at the intensive margin. Although the model predicts that this relationship should exist for both total exports and the extensive margin, the extensive margin is predicted to drive the overall relationship, so I take this evidence to be somewhat consistent with the findings of the model.

Table 6 provides similar evidence using  $\tilde{\tau}_{ni}^{j}$  as a measure of trade costs. In this case, the coefficient on the interaction term in column 1 is negative and significant at the 10% level. However, this term is more statistically significant at the intensive margin (reported in column 3) than at the extensive margin (reported in column 2). While the evidence for total exports is consistent with the predictions of the model, the differential pattern at the extensive margins goes against the prediction of the model.

Table 7 provides evidence using  $\widetilde{RTA}_{ni}$  as a measure of trade costs. In this case, the interaction term in column 1 is positive and significant at the 10% level. At the extensive margin (reported in column 2), this term remains positive and significant at

the 10% level, while at the intensive margin (reported in column 3), it is insignificant. Both of these results are consistent with the predictions of the VAP heterogeneity model.

# A.3 Tables and Figures

	Gui.	no nom maa	c Data		
Variable	Mean	Std. Dev.	Min.	Max.	Ν
$\alpha_{nt}^j$	0.02	0.02	0	0.14	858
$eta_{it}^j$	0.66	0.09	0.36	0.95	858
$\pi^{j}_{nnt}$	0.62	0.25	0	1.00	858

Table 1: Summary Statistics
Gains from Trade Data

Variable	Mean	Std. Dev.	Min.	Max.	Ν
$ln(X_{nit}^j)$	7.09	3.29	0	18.82	296,987
$ln(F_{nit}^j)$	2.77	1.72	0	6.75	$296,\!987$
$ln(\overline{X}_{ni}^{j})$	4.32	2.02	0	14.82	$296,\!987$
$\beta_{it}^j$	0.63	0.11	0.08	0.96	2,112
$ln(d_{ni})$	8.57	0.85	5.08	9.89	6,035
$ au_{nit}^{j}$	0.09	0.11	0	7.05	264,408
$RTA_{nit}$	0.16	0.37	0	1.00	2
$ heta_{VAP}^{j}$	3.24	3.42	0.09	12.47	13
$ heta_{TFP}^{j}$	9.47	12.76	0.37	51.08	13

Trade Elasticity Data

**Table 2:** Equations (39) and (40):  $\kappa_{nit} = ln(\tilde{d}_{ni})$ 

	(1) Total	(2) Total	(3) Extensive	(4) Extensive	(5) Intensive	(6) Intensive
$\lambda_1 \ \lambda_2$	-1.194 (0.00)	$\begin{array}{c} -0.736 \\ (0.005) \\ 0.568 \\ (0.044) \end{array}$	-0.412 (0.00)	$\begin{array}{c} -0.177 \\ (0.00) \\ 0.840 \\ (0.00) \end{array}$	-0.781 (0.00)	$\begin{array}{c} -0.695 \\ (0.077) \\ 0.164 \\ (0.81) \end{array}$
$\begin{array}{c} \text{Observations} \\ R^2 \\ \text{RMSE} \end{array}$	$236,467 \\ 0.258 \\ 2.289$	$236,467 \\ 0.259 \\ 2.288$	$236,467 \\ 0.122 \\ 1.267$	$236,467 \\ 0.124 \\ 1.265$	$236,467 \\ 0.212 \\ 1.712$	$236,467 \\ 0.212 \\ 1.712$

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector.

	(1) Total	(2) Total	(3) Extensive	(4) Extensive	(5) Intensive	(6) Intensive
$\lambda_1 \ \lambda_2$	-0.305 (0.00)	$\begin{array}{c} -0.201 \\ (0.002) \\ 0.501 \\ (0.048) \end{array}$	-0.094 (0.00)	$\begin{array}{c} -0.034 \\ (0.00) \\ 0.923 \\ (0.00) \end{array}$	-0.212 (0.00)	$\begin{array}{c} -0.212 \\ (0.072) \\ -0.000 \\ (1.00) \end{array}$
$\begin{array}{c} \text{Observations} \\ R^2 \\ \text{RMSE} \end{array}$	$213,535 \\ 0.109 \\ 2.437$	$213,535 \\ 0.110 \\ 2.288$	$213,535 \\ 0.043 \\ 1.267$	$213,535 \\ 0.044 \\ 1.265$	$213,535 \\ 0.100 \\ 1.712$	$213,535 \\ 0.100 \\ 1.712$

**Table 3:** Equations (39) and (40):  $\kappa_{nit} = ln(\tilde{\tau}_{nit}^{j})$ 

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector.

<b>Table 4:</b> Equations (39) and (40): $\kappa_{nit} = RTA_{nit}$							
	(1) Total	(2) Total	(3) Extensive	(4) Extensive	(5) Intensive	(6) Intensive	
$\lambda_1$	1.972 (0.00)	0.982 (0.002)	$0.659 \\ (0.00)$	0.225 (0.00)	$1.312 \\ (0.00)$	$0.941 \\ (0.057) \\ 0.421$	
$\lambda_2$		(0.743) (0.00)		(0.968)		(0.421) (0.372)	
$\begin{array}{c} \text{Observations} \\ R^2 \\ \text{RMSE} \end{array}$	$236,467 \\ 0.135 \\ 2.471$	$236,467 \\ 0.137 \\ 2.288$	$236,467 \\ 0.061 \\ 1.267$	$236,467 \\ 0.065 \\ 1.265$	$236,467 \\ 0.116 \\ 1.712$	$236,467 \\ 0.117 \\ 1.712$	

**Table 4:** Equations (39) and (40):  $\kappa_{nit} = \widetilde{RTA}_{nit}$ 

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector.

	. ,	· · · · · · · · · · · · · · · · · · ·	
	(1)	(2)	(3)
	Total	Extensive	Intensive
Distance	-1.449 $(0.002)$	-0.531 (0.002)	-0.917 (0.002)
Intermediate share	-0.405	0.421	-0.826
	(0.138)	(0.766)	(0.278)
Interaction term	-0.626	-0.297	-0.329
	(0.344)	(0.066)	(0.72)
$\frac{\text{Observations}}{R^2}$	$236,467 \\ 0.258$	$236,467 \\ 0.124$	$236,467 \\ 0.213$

**Table 5:** Equation (41):  $\kappa_{nit} = ln(\tilde{d}_{ni})$ 

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector using the wild cluster bootstrap-t procedure described by Cameron, Gelbach and Miller (2008).

	(1)	(2)	(2)
	(1) Total	(2) Extensive	(3) Intensive
Import tariff	-0.391	-0.116	-0.275
	(0.002)	(0.002)	(0.002)
Intermediate share	-0.607	0.484	-1.092
	(0.238)	(0.634)	(0.024)
Interaction term	-0.215	-0.062	-0.152
	(0.082)	(0.476)	(0.096)
Observations	213,535	213,535	213,535
$R^2$	0.110	0.045	0.103

**Table 6:** Equation (41):  $\kappa_{nit} = ln(\tilde{\tau}_{nit}^{j})$ 

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector using the wild cluster bootstrap-t procedure described by Cameron, Gelbach and Miller (2008).

			1000
	(1)	(2)	(3)
	Total	Extensive	Intensive
RTA	2.661	0.929	1.731
	(0.0)	(0.0)	(0.126)
Intermediate share	-0.312	0.493	-0.805
	(0.232)	(0.664)	(0.306)
Interaction term	1.688	0.673	1.015
	(0.074)	(0.038)	(0.01)
Observations	236,467	236,467	236,467
$R^2$	0.136	0.064	0.118

**Table 7:** Equation (41):  $\kappa_{nit} = \widetilde{RTA}_{nit}$ 

Note: Robust p-values are reported in parentheses. All errors are adjusted for clustering by sector using the wild cluster bootstrap-t procedure described by Cameron, Gelbach and Miller (2008).

ISIC Revision 2 Group	$\theta_{TFP}$	Se.	$\theta_{VAP}$	Obs
Food, Beverages and Tobacco	-2.55	(0.61)	-0.75	495
Textiles and Products	-5.56	(1.14)	-1.92	437
Wood and Products	-10.83	(2.53)	-3.72	315
Pulp, Paper and Printing	-9.07	(1.69)	-3.57	507
Coke, Ref. Petroleum	-51.08	(18.05)	-15.41	91
Chemicals and Products	-4.75	(1.77)	-1.69	430
Rubber and Plastics	-1.66	(1.41)	-0.59	376
Other Non-Metallic Min.	-2.76	(1.44)	-1.16	342
Basic Metals and Fabricated	-7.99	(2.53)	-2.75	388
Machinery, Nec	-1.52	(1.81)	-0.55	397
Electrical and Optical	-10.60	(1.38)	-3.73	343
Transport Equipment	-0.37	(1.08)	-0.11	245
Manufacturing, Nec	-5.00	(0.92)	-1.93	412
Aggregate	-4.55	(0.35)	-2.91	7212

 Table 8: Dispersion Parameters for ISIC Revision 2 Groups

**Table 9:** Average Sectoral Intermediate Goods Shares  $(\beta_{it}^j)$  across Countries and Time

ISIC Revision 2 Group	$\beta_{95h}$	$\beta_{10h}$	$\beta_{95f}$	$\beta_{10f}$	$\beta_{95}$	$\beta_{10}$
Food, Beverages and Tobacco	0.64	$\downarrow 0.59$	0.09	$\uparrow 0.11$	0.72	$\downarrow 0.71$
Textiles and Products	0.46	$\downarrow 0.43$	0.17	$\uparrow 0.20$	0.63	$\uparrow 0.63$
Wood and Products	0.53	$\downarrow 0.52$	0.12	$\uparrow 0.13$	0.65	-0.65
Pulp, Paper and Printing	0.48	-0.48	0.13	$\uparrow 0.15$	0.62	$\uparrow 0.63$
Coke, Ref. Petroleum	0.39	$\downarrow 0.36$	0.34	$\uparrow 0.42$	0.73	$\uparrow 0.79$
Chemicals and Products	0.46	-0.46	0.17	$\uparrow 0.21$	0.63	$\uparrow 0.67$
Rubber and Plastics	0.46	$\downarrow 0.45$	0.19	$\uparrow 0.22$	0.64	$\uparrow 0.66$
Other Non-Metallic Min.	0.46	$\uparrow 0.48$	0.11	$\uparrow 0.13$	0.57	$\uparrow 0.61$
Basic Metals and Fabricated	0.48	$\downarrow 0.47$	0.18	$\uparrow 0.23$	0.66	$\uparrow 0.70$
Machinery, Nec	0.45	$\downarrow 0.43$	0.17	$\uparrow 0.21$	0.62	$\uparrow 0.65$
Electrical and Optical	0.42	$\downarrow 0.41$	0.22	$\uparrow 0.27$	0.64	$\uparrow 0.68$
Transport Equipment	0.46	$\downarrow 0.45$	0.21	$\uparrow 0.27$	0.68	$\uparrow 0.72$
Manufacturing, Nec	0.47	$\downarrow 0.45$	0.14	$\uparrow 0.19$	0.61	$\uparrow 0.64$
Average	0.47	$\downarrow 0.46$	0.17	$\uparrow 0.21$	0.65	$\uparrow 0.67$

	VAP		TFP	
Country	1995	2010	1995	2010
Australia	15.2%	20.1%	12.5%	21.3%
Austria	47.3%	60.7%	41.7%	59.6%
Belgium	70.1%	71.8%	72.4%	78.0%
Brazil	7.1%	9.6%	6.7%	9.9%
Canada	49.9%	30.3%	48.0%	33.7%
China	8.1%	7.8%	9.3%	11.6%
Czech Republic	26.5%	38.3%	31.1%	45.8%
Germany	21.4%	33.5%	18.1%	34.7%
Denmark	47.6%	70.9%	37.6%	86.7%
Spain	20.2%	22.6%	21.3%	26.5%
Finland	22.4%	32.0%	17.8%	28.7%
France	19.5%	25.8%	22.7%	33.5%
Great Britain	28.4%	37.0%	24.6%	34.8%
Greece	24.0%	43.0%	16.2%	29.5%
Hungary	23.2%	50.9%	28.6%	55.5%
Indonesia	16.2%	13.1%	14.0%	11.8%
India	4.1%	9.3%	5.5%	12.8%
Ireland	44.7%	45.5%	35.9%	38.5%
Italy	18.2%	21.4%	19.8%	25.5%
Japan	2.5%	3.3%	2.4%	3.5%
Korea	12.0%	9.9%	12.8%	12.9%
Mexico	17.8%	31.1%	17.1%	28.5%
Netherlands	55.0%	56.5%	54.8%	56.6%
Poland	12.2%	40.2%	11.3%	51.3%
Portugal	43.0%	46.7%	48.8%	46.0%
Romania	12.5%	29.8%	11.1%	22.0%
Russia	18.9%	31.4%	16.9%	36.1%
Slovakia	36.6%	44.2%	38.5%	49.3%
Slovenia	58.8%	69.4%	65.1%	75.1%
Sweden	22.0%	27.7%	19.6%	32.8%
Turkey	18.5%	33.4%	12.9%	31.5%
Taiwan	23.8%	23.4%	25.6%	32.5%
United States	11.0%	13.0%	11.4%	15.6%
Average	26.0%	33.4%	25.2%	35.5%

 Table 10: Gains from Manufacturing Trade

Aruba	Dominican Republic	Lebanon	Sudan
Afghanistan	Algeria	Liberia	Senegal
Angola	Ecuador	Libyan Arab Jamahiriya	Singapore
Anguilla	Egypt	Saint Lucia	Saint Helena
Albania	Eritrea	Sri Lanka	Solomon Islands
Andorra	Spain	Lithuania	Sierra Leone
Netherland Antilles	Estonia	Latvia	El Salvador
United Arab Emirates	Ethiopia	Morocco	San Marino
Argentina	Finland	Moldova, Rep.of	Somalia
Armenia	Fiji	Madagascar	St. Pierre and Miquelon
Antigua and Barbuda	Falkland Islands	Maldives	Sao Tome and Principe
Australia	France	Mexico	Suriname
Austria	Micronesia	Marshall Islands	Slovakia
Azerbaijan	Gabon	Macedonia	Slovenia
Burundi	United Kingdom	Mali	Sweden
Belgium and Luxembourg	Georgia	Malta	Seychelles
Benin	Ghana	Myanmar/Burma	Syrian Arab Republic
Burkina Faso	Gibraltar	Mongolia	Turks and Caicos Islands
Bangladesh	Guinea	Northern Mariana	Chad
Bulgaria	Gambia	Mozambique	Togo
Bahrain	Guinea-Bissau	Mauritania	Thailand
Bahamas	Equatorial Guinea	Martinique	Tajikistan
Bosnia and Herzegovina	Greece	Malawi	Tokelau
Belarus	Grenada	Malaysia	Turkmenistan
Belize	Greenland	New Caledonia	East Timor
Bermuda	Guatemala	Niger	Tonga
Bolivia	Guyana	Norfolk Island	Trinidad and Tobago
Brazil	Hong Kong	Nigeria	Tunisia
Barbados	Honduras	Nicaragua	Turkey
Brunei Darussalam	Croatia	Niue	Tuvalu
Bhutan	Haiti	Netherlands	Taiwan
Central African Republic	Hungary	Norway	Tanzania
Canada	Indonesia	Nepal	Uganda
Switzerland	India	Nauru	Ukraine
Chile	Ireland	New Zealand	Uruguay
China	Iran	Oman	United States of America
Côte d'Ivoire	Iraq	Pakistan	Uzbekistan
Cameroon	Iceland	Panama	St. Vincent and the Gren.
Congo	Israel	Peru	Venezuela
Cook Islands	Italy	Philippines	British Virgin Islands
Colombia	Jamaica	Palau	Viet Nam
Comoros	Jordan	Papua New Guinea	Vanuatu
Cape Verde	Japan	Poland	Wallis and Futuna
Costa Rica	Kazakstan	Korea, Dem. Rep.	Samoa
$\operatorname{Cuba}$	Kenya	Portugal	Yemen
Cayman Islands	Kyrgyzstan	Paraguay	Serbia and Montenegro
Cyprus	Cambodia	French Polynesia	South Africa
Czech Republic	Kiribati	Qatar	Congo (Dem. Rep.)
Germany	Saint Kitts and Nevis	Romania	Zambia
Djibouti	Korea	Russian Federation	Zimbabwe
Dominica	Kuwait	Rwanda	
Denmark	Laos	Saudi Arabia	

# Table 11: List of Import-Receiving Countries



Figure 1: Import Content of Exports (ICE) and Output (ICO), 2005

Notes: The ICE is calculated as  $uA^M[1-A^D]^{-1}X/X^k$ , where u is a  $1 \times n$  vector of 1s,  $A^M$  is the  $1 \times n$  import coefficient matrix,  $A^D$  is the domestic coefficient matrix, X is an  $n \times 1$  vector of exports,  $X_k$  is total country exports, and n is the number of sectors. The ICO is calculated similarly as  $uA^M[1-A^D]^{-1}Y/Y^k$ , where Y is an  $n \times 1$  vector of output and  $Y_k$  is total country output. Data are taken from OECD input-output tables for 2005. Countries included are Australia, Canada, Germany, Denmark, France, the United Kingdom, Japan, the Netherlands and the United States.



Figure 2: Import Content of Exports (ICE) and Output (ICO), Growth over time

Notes: The ICE is calculated as  $uA^{M}[1-A^{D}]^{-1}X/X^{k}$ , where u is a  $1 \times n$  vector of 1s,  $A^{M}$  is the  $1 \times n$  import coefficient matrix,  $A^{D}$  is the domestic coefficient matrix, X is an  $n \times 1$  vector of exports,  $X_{k}$  is total country exports, and n is the number of sectors. The ICO is calculated similarly as  $uA^{M}[1-A^{D}]^{-1}Y/Y^{k}$ , where Y is an  $n \times 1$  vector of output and  $Y_{k}$  is total country output. Data are calculated from OECD input-output tables as growth from the late 1960s (for Australia and the United Kingdom) or the early 1970s (for Canada, Germany, Denmark, France, Japan, the Netherlands and the United States) to 2005.



Figure 3: % Growth in Gains from Trade, 1995 to 2010



Figure 3: (continued) % Growth in Gains from Trade, 1995 to 2010

Notes: Gains from trade are calculated according to equations (27) and (29) using data for  $\alpha_{nt}^{j}$ ,  $\beta_{it}^{j}$  and  $\pi_{nnt}^{j}$  constructed from the WIOD, and values of  $\theta_{TFP}^{j}$  and  $\theta_{VAP}^{j}$  estimated using data from Caliendo and Parro (2015).