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by Toni Ahnert and Benjamin Nelson

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Opaque Assets and Rollover Risk

by

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Abstract

We model the asset-opacity choice of an intermediary subject to rollover risk in wholesale funding markets. Greater opacity means investors form more dispersed beliefs about an intermediary's profitability. The endogenous benefit of opacity is lower fragility when profitability is expected to be high. However, the endogenous cost of opacity is a "partial run," whereby some investors receive bad private signals about profitability and run, even though the intermediary is solvent. We find that intermediaries choose to be transparent (opaque) when expected profitability is low (high). Intermediaries with less-volatile profitability are also more likely to choose to be opaque.

JEL classification: G01, G2
Bank classification: Financial institutions; Financial stability

Résumé

Nous modélisons le choix qu'un intermédiaire exposé au risque de refinancement sur les marchés du financement de gros opère par rapport à l'opacité des actifs. En cas d'accroissement de l'opacité, les opinions des investisseurs sur la rentabilité d'un intermédiaire sont plus diversifiées. L'opacité a pour avantage endogène de réduire la fragilité quand la rentabilité attendue est élevée. Elle a toutefois pour coût endogène d'entraîner un mouvement partiel de ventes massives puisque certains investisseurs recevant, de source privée, des informations défavorables sur la rentabilité procèdent à des ventes massives quand bien même l'intermédiaire est solvable. Il ressort de notre analyse que les intermédiaires choisissent d'être transparents (ou opaques) lorsque la rentabilité attendue est faible (ou élevée). Les intermédiaires dont la rentabilité est moins instable ont également une plus grande propension à faire le choix de l'opacité.

Classification JEL : G01, G2
Classification de la Banque : Institutions financières; Stabilité financière

Non-Technical Summary

The crisis that struck the global financial system in 2008 arose, in part, because of opacity. Creating securitized financial claims appeared to have achieved diversification and risk sharing. When the collateral value backing those claims began to fall, however, investors awoke to the complexity and obscurity of bank assets. Having been compressed, the cost of default protection on bank debt rose precipitously, and banks rapidly became illiquid. For some time, bank equity traded below book value as investors expressed doubt about the true values of their assets. Concerns over the opacity of bank assets persist today, and a range of countervailing measures have been proposed by regulators, including recommendations to enhance disclosure.

Why did banks choose to become so opaque? We offer a stylized theory of bank opacity. A banker is subject to rollover risk in wholesale funding markets and chooses assets with either opaque or transparent returns. Since premature liquidation is costly, both a wholesale debt run and no crisis may be equilibria. Using global games, we pin down the rollover behavior uniquely. We derive the banker's optimal choice of opacity and link it to the expected return on and volatility of its assets.

Our simple framework suggests that a period of high expected returns encourages banks to choose opaque portfolios. By doing this, a bank encourages investors to 'overweight' the prior information about returns, reducing the likelihood of a bank run, even at the expense of partial runs, which occur because some investors receive bad private signals and are correspondingly skeptical about returns. A corollary is that when returns are low, banks are more likely to be transparent. When returns are low, public information about asset returns tends to encourage investors to run; in this case, banks strive to convince investors that the asset returns are actually sound by raising the precision of the private information their asset portfolios generate.

1 Introduction

The crisis that struck the global financial system in 2008 arose, in part, because of opacity. Creating securitized financial claims appeared to have achieved diversification and risk sharing. When the collateral value backing those claims began to fall, however, investors awoke to the complexity and obscurity of bank assets. Having been compressed, the cost of default protection on bank debt rose precipitously (Figure 1), and banks rapidly became illiquid. For some time, bank equity traded below book value as investors expressed doubt about the values of their assets (Figure 2). Concerns over bank asset opacity persist today (Partnoy and Eisinger, 2014; Jones et al., 2012; Sowerbutts et al., 2014). A range of countervailing regulatory measures have been proposed, including recommendations to enhance disclosure (FSB, 2012).

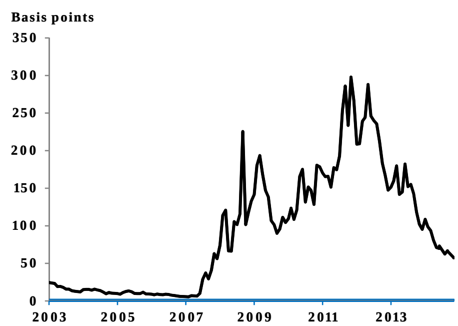


Figure 1: Senior CDS premia for major UK banks. Source: Bank of England

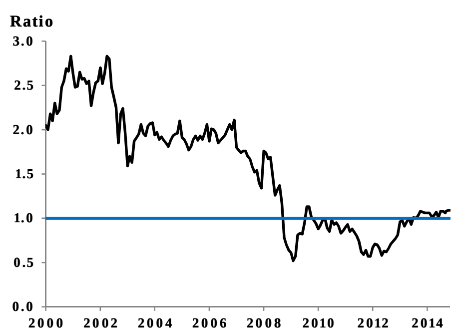


Figure 2: Price-to-book ratio for major UK banks. Source: Bank of England

To study why banks chose to become so opaque, we offer a simple and stylized theory of bank opacity. In our model, wholesale investors have two sources of information about the bank's asset returns, a public signal and a private signal. We measure transparency by the precision of the private information available to investors. This captures, in a simple way, the difference in how precisely investors can evaluate the likely cash flows from loans versus tradable securities, or securitized claims versus the underlying conventional securities (e.g., mortgages). It may also capture the level of organizational complexity of a bank (Cetorelli and Goldberg, 2014; Goldberg, 2016).

In our model, a banker chooses assets with returns that are either opaque or transparent to investors. The banker faces rollover risk as in Rochet and Vives (2004). To finance profitable but risky investment opportunities, the banker attracts wholesale funding by offering demandable debt claims.[1] Since liquidation is costly, the rollover decisions of fund managers constitute a coordination problem that leads to multiple equilibria (Diamond and Dybvig, 1983). The individual incentive of a manager to roll over increases in the proportion of managers who also roll over (strategic complementarity). We use global games methods to pin down the rollover behavior uniquely (Carlsson and van Damme, 1993; Morris and Shin, 2003). Each manager rolls over funding as long as the private signal about the return on bank assets is sufficiently high, so a wholesale debt run occurs when the realized asset return is sufficiently low.

We begin by studying the effects of asset opacity on bank fragility, defined as the likelihood of a debt run. Greater opacity reduces fragility if expected returns are high. As opacity increases, the private information available to investors becomes less precise, so investors place greater weight on public information in forming their beliefs about returns. When returns are expected to be high, investors are reassured and revise up their posterior belief, which makes them less likely to run. The opposite

[1]We take the demandable-debt contract as given. Demandable debt can be explained by demand for liquidity (Diamond and Dybvig, 1983), agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), and demand for safety (Gennaioli et al., 2013; Ahnert and Perotti, 2015).

is true when expected returns are low. In this case, the bank would rather investors down-weighted public information in their decisions; more-precise private information causes investors to revise up their beliefs, making them less likely to run.

The precision of *public* information about the bank's asset return may reflect the degree of diversification in the bank's portfolio. In other words, when the volatility of the bank's assets is low, investors' prior beliefs about returns are relatively precise. In this context, 'diversification' has effects on fragility that are of opposite sign to those of opacity. In particular, investors in a more diversified bank are more likely to disregard (possibly unfavorable) private information, reducing fragility as long as expected returns are high. The opposite holds when expected returns are low, whereby more precise public information increases fragility. An alternative interpretation of the precision of public information may be policies that mandate the disclosure of certain information about the asset portfolio (Morris and Shin, 2002).

We next endogenize the asset opacity of the bank. The global games approach allows us to study how the bank's ex ante asset opacity choice affects the ex post rollover behavior of investors. The banker internalizes this effect when choosing opacity to maximize expected equity value. Under some conditions that we make clear, there is a trade-off between an endogenous benefit of opacity and an endogenous cost.

The endogenous benefit of opacity is the possibility that it lowers fragility. As stated above, however, this depends on expected returns being high. When expected returns are low, there is no benefit of opacity. The endogenous cost of opacity arises because of 'partial runs' on fundamentally solvent banks. That is, even for a high *realized* asset return, some fund managers receive bad private signals and so do not roll over funding. This results in costly liquidation, reducing the residual equity value to the banker. This cost of opacity is independent of the expected asset return.

Therefore, for low expected returns, opacity both increases fragility *and* leads to partial runs when the bank is solvent. In this case, opacity is a dominated choice and transparency is optimal. In contrast, for sufficiently high expected returns, the fragility-reducing benefit of opacity can dominate the cost of partial runs, and so the banker chooses opacity. Thus, our model may explain how high-return environments, like that of the pre-crisis years, can generate asset opacity, while in low-return environments, like the aftermath of the crisis, banks instead strive for transparency.[2]

We also use our framework to study how the bank's opacity choice varies with the precision of public information. If public information is uninformative, investors base their decisions solely on private information. In this case, there is only a cost to opacity – that arising from partial runs – and no countervailing benefit, so the banker chooses to generate precise private information for investors. Next, suppose that public information becomes more precise and expected returns are high. Then the banker would rather investors base their actions on this favorable public information, rather than on potentially unfavorable private information, which reduces both fragility and the incidence of partial runs. In this case, the bank becomes opaque. Thus private and public information are effectively substitutes. Our model implies that encouraging greater asset diversification (less-volatile returns) would endogenously leave investors with less precise private information about bank profitability.[3]

The paper proceeds as follows. Section 2 contains the literature review. In Section 3, we describe the model. In Section 4, we solve for the rollover behavior in wholesale funding markets. In Section 5, we derive the banker's choice between opacity and transparency. Section 6 concludes.

[2]These results are consistent with empirical evidence in Giannetti and Laeven (2012) who document a 'flight home' effect. During the crisis, lenders rebalanced their portfolios towards domestic borrowers, about which information is more precise. The flight home effect is also shown to increase as funding sources become less stable.

[3]This result is consistent with empirical evidence in Flannery et al. (2004), who find that analysts are able to predict the earnings of smaller banks more precisely than the earnings of larger banks. A plausible reason is the lower volatility of small banks' equity returns, documented by Flannery et al. (2004), indicating relatively more precise public information about the asset returns of small banks.

2 Literature

Opacity and complexity have been important features of the recent financial crisis. Cetorelli et al. (2014) document the organizational complexity of US bank holding companies. Sato (2015) studies the implications of opacity in financial markets for investor behavior, asset prices, and welfare. In his model, investors do not observe an opaque fund's portfolio or an opaque asset's payoff. In Wagner (2007), opaque activities constitute an inefficient response by banks to the higher transparency of their traditional activities. By contrast, opacity in our setup is the low precision of private information about asset returns available in wholesale funding markets.

Our paper is part of the global games literature (Carlsson and van Damme, 1993; Morris and Shin, 2003). In particular, we build on Rochet and Vives (2004) who propose a model of rollover risk in wholesale funding markets. The Rochet-Vives environment assumes that rollover decisions are delegated to professional managers, which gives rise to global strategic complementarity. The tractability of this environment allows us to embed a model of the opacity choice of bank assets. Goldstein and Pauzner (2005) refrain from this delegation assumption and study a bank-run setup with one-sided strategic complementarity, as in Diamond and Dybvig (1983).

Transparency versus opacity has been studied in the global games literature. Transparency can refer to either the more-precise public information about economic fundamentals (Morris and Shin, 1998, 2002; Bouvard et al., 2015) or the more-precise private information of investors (Heinemann and Illing, 2002). We follow the latter approach. The literature has focused on different economic settings, such as currency attacks (Morris and Shin, 1998; Heinemann and Illing, 2002; Metz, 2002) and rollover risk (Bouvard et al., 2015). We study rollover risk. Some literature has focused on the incentives of investors to acquire private information (Hellwig and Veldkamp, 2009; Szkup and Trevino, 2015; Ahnert and Kakhbod, 2014). In contrast, we endogenize the precision of private information from the perspective of an equity-maximizing banker.

Metz (2002), Heinemann and Illing (2002), and Iachan and Nenov (2015) are related papers that study how the precision of private information affects fragility. These papers are closely related to the benefit of asset opacity in our paper, and we explain the link in greater detail below. There are two main differences to our paper. First, the precision of private information in our paper is endogenous and chosen by a banker who maximizes the expected value of equity. Second, we also describe an endogenous cost of opacity, which is given by partial runs on a solvent bank.

In the currency attack model of Morris and Shin (1998), Metz (2002) studies the impact of higher precision of public or private information about the fundamental. Her comparative statics results depend crucially on the level of the public signal (equal to the expected asset return in our model). We confirm these results in the context of rollover risk, where more opacity reduces fragility for a high expected asset return.

Heinemann and Illing (2002) generalize the results of Morris and Shin (1998) by studying a broader class of probability distributions and allowing for sunspots. They also analyze the impact of transparency, defined as more-precise private information, on the probability of a currency crisis. They find that greater transparency reduces the probability of a crisis, which is the opposite of our result. This difference arises for two reasons. First, the payoff from attacking successfully is sensitive to the economic fundamental, which is absent in our model. Transparency would have no impact on the probability of a crisis without payoff sensitivity (see page 444). Second, there is no public signal in their model. Our model would yield that transparency would not affect the probability of a crisis if the public signal were completely imprecise.

Iachan and Nenov (2015) study information quality, defined as more-precise private information, in a generalized global game of regime change. They allow the net payoffs to depend directly on the fundamental, both in case of a regime change and when the status quo is maintained. Their specification encompasses many common applications, including currency crises and debt runs. They link the impact of higher information quality on fragility to the relative sensitivity of net payoffs to the fundamental. In an extension with an informative public signal, the level of the public signal is shown to determine the impact of higher information quality on fragility. A special case of this result is our benefit from opacity for a high expected return.

Bouvard et al. (2015) study the public disclosure of bank-specific information by a regulator when banks are subject to rollover risk. In their baseline model without signaling, the regulator chooses to disclose whenever average fundamentals are bad. Transparency leads to runs on some vulnerable banks, as opposed to a run on the entire system under opacity. When average fundamentals are good, by contrast, the regulator chooses not to disclose, which leads to no runs. While we share the idea that opacity can enhance financial stability, the mechanism is quite different. First, their focus is on a systemic bank run as opposed to an individual bank run. Second, disclosure or transparency refers to public information, not to private information. Third, they consider infinitely precise private information, so transparency does not affect the relative precision of private information important for our mechanism.

Parlatore-Siritto (2015) also studies how transparency affects bank fragility. Incorporating risky investment in Diamond and Dybvig (1983), investors learn the same noisy signal about its profitability. Transparency, defined as greater informativeness of this signal, can lead to more inefficient runs, heightening fragility ex post. As a result, greater fragility affects the optimal portfolio choice and deposit contract design ex ante. Less risk sharing occurs among investors subject to liquidity shocks, lowering welfare. There are several differences to our mechanism. First, we endogenize the transparency of investment. Second, our global games model uniquely pins down the rollover behavior of investors. Third, private information is dispersed in our model, where different investors receive different signals. Finally, we obtain that greater transparency can increase or reduce fragility, depending on the expected return.

3 Model

The model builds on Rochet and Vives (2004). There are three dates, $t = 0, 1, 2$, and no discounting. There is a banker and a unit mass of uninsured wholesale investors, all of whom are risk neutral. Investors have a unit endowment at the initial date and are indifferent about the date of consumption. Investors deposit their endowment with the banker at the initial date and can withdraw at either the interim or final date. The face value of wholesale debt $D \geq 1$ is independent of the withdrawal date.

The banker invests these funds in a risky asset with stochastic final-date return $r \in \mathbb{R}$ specified below. Liquidation at the interim date yields a fraction $\psi \in (0, 1)$ of the final-date return, perhaps because of costly fire sales (Shleifer and Vishny, 1992) or the loss of relationship-specific knowledge when asset ownership changes (Diamond and Rajan, 2001).[4]

Investors are assumed to delegate the decision to roll over funds at the interim date to a group of professional fund managers indexed by $j \in [0, 1]$. If a proportion of managers $\ell \in [0, 1]$ refuses to roll over, the banker liquidates the amount $y = \frac{\ell D}{\psi r}$ of the asset to serve withdrawals. Early closure of the bank occurs at the interim date if withdrawals exceed the maximum liquidation value, $\ell D > \psi r$. Otherwise, partial liquidation of the asset allows the banker to serve all interim-date withdrawal. At the final date, the value of the remaining asset is $r(1 - y) = r - \frac{\ell D}{\psi}$ and the banker must serve withdrawals of $(1 - \ell)D$, so bankruptcy occurs if and only if

[4] Since ψ captures the market liquidity of the assets, it may be interpreted as the bank's holding of liquid assets in reduced form, whereby a higher liquidation value would correspond to more liquid assets. Ahnert (2016) explicitly models the liquidity holdings of financial intermediaries funded with wholesale debt, considering both the ex ante cost of liquidity holdings via a forgone higher asset return and the ex post benefits of liquidity via avoiding or reducing costly liquidation.

$$r - \frac{\ell D}{\psi} < (1 - \ell)D. \tag{1}$$

The simultaneous rollover decisions of fund managers are governed by their compensation. If the bank is bankrupt or closed early, a manager's relative compensation from rolling over is negative, $-C < 0$. Otherwise, the relative compensation from rolling over is positive, $B > 0$. This specification ensures global strategic complementarity in rollover decisions. To ease the exposition, we assume that $B = C$.[5]

Figure 3 illustrates the dominance regions if the asset return r were common knowledge. If all wholesale funding is rolled over, $\ell = 0$, bankruptcy occurs when the asset return is smaller than the face value of wholesale debt, which defines a lower bound $r_L \equiv D$. It is a dominant strategy for managers not to roll over funding whenever $r < r_L$. Likewise, if no wholesale funding is rolled over, $\ell = 1$, bankruptcy is avoided whenever the asset return exceeds an upper bound, $r_H \equiv \frac{D}{\psi} > r_L$. In this case, it is a dominant strategy for managers to roll over funding whenever $r > r_H$.

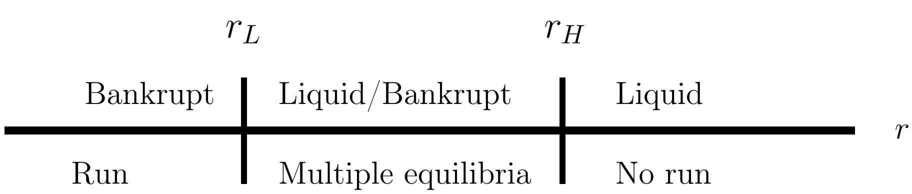


Figure 3: Tripartite classification of the asset return (complete information)

To ensure a unique equilibrium, we follow the global games literature by assuming incomplete information about the asset return r. First, the distribution of the asset return is commonly known at the initial date to follow a normal distribution with mean $\bar{r} > D$ and precision $\alpha \in (0, \infty)$.[6] Second, each fund manager j receives a noisy private signal at the interim date (Morris and Shin, 2003):

[5]This assumption implies that the cutoff level of the expected asset return, $\bar{r}_0$, is independent of the level of opacity. See Proposition 1.

[6]The common prior may be induced by a public signal, $\bar{r} = r + \eta$, where the aggregate noise $\eta \sim \mathcal{N}\left(0, \alpha^{-1}\right)$ is independent of the return r and of each of the idiosyncratic noise terms ϵ_j.

$$r \sim \mathcal{N}\left(\bar{r}, \alpha^{-1}\right) \tag{2}$$

$$x_j = r + \varepsilon_j, \qquad \varepsilon_j \sim \mathcal{N}\left(0, \beta^{-1}\right), \tag{3}$$

where the idiosyncratic noise ε_j is normally distributed with zero mean and endogenous precision, $\beta > 0$, described below. Idiosyncratic noise is independent of the asset return and independently and identically distributed across fund managers.

We consider two interpretations of the precision of the common prior. First, it may reflect policies that mandate the disclosure of certain information about the asset portfolio (see also Morris and Shin, 2002). Second, the precision may reflect the degree of diversification. Suppose the banker could invest in multiple assets with independent returns, each of which follows the distribution specified in (2). Diversification would not affect the expected return but would reduce the variance of the portfolio return. In this sense, lower volatility of the asset return may correspond to more diversification.

At the initial date, the banker chooses the precision of private information about the asset return β. The banker is protected by limited liability and maximizes her expected equity value at the final date. The banker can choose between a low or high level of information precision, $\beta \in \{\beta_L, \beta_H\}$, where $\beta_L < \beta_H$ and $\beta_H \to \infty$. Opacity refers to β_L and Transparency to β_H. Table 1 summarizes the timeline of events.

Initial date ($t = 0$)	**Interim date** ($t = 1$)	**Final date** ($t = 2$)
1. Raise wholesale debt	1. Private information x_j	1. Investment yields r
2. Asset choice β	2. Debt withdrawals ℓ	2. Debt repayment
	3. Consumption	3. Consumption

Table 1: Timeline.

4 Wholesale debt runs

We analyze pure-strategy perfect Bayesian equilibrium. Solving for the equilibrium by working backwards, we start with the wholesale debt rollover subgame between fund managers at the interim date. A subgame is defined by the opacity choice of the banker at the initial date. Because of the global strategic complementarity in rollover decisions among fund managers, we can apply standard global games techniques. We focus on symmetric equilibria in threshold strategies without loss of generality. Morris and Shin (2000, 2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies; see also Frankel et al. (2003).

Proposition 1. ***Debt rollover subgame.*** *If private information is sufficiently precise, $\beta > \underline{\beta} \equiv \left(D\left(\frac{1}{\psi}-1\right)\frac{\alpha}{\sqrt{2\pi}}\right)^2 \in (0,\infty)$, there exists a unique Bayesian equilibrium in each subgame. This equilibrium is characterized by a bankruptcy threshold r^* and a signal threshold x^*. Fund manager j rolls over funding whenever $x_j > x^*$, and bankruptcy occurs whenever $r < r^*$. The bankruptcy threshold is implicitly defined by:*

$$r^* = D\left[1+\left(\frac{1}{\psi}-1\right)\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[r^*-\bar{r}\right]\right)\right] \in \left[r_L, r_H\right], \tag{4}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standardized Gaussian random variable. More-expensive wholesale funding raises the bankruptcy threshold, $\frac{dr^*}{dD} > 0$. In contrast, both a higher liquidation value and a higher expected asset return reduce the bankruptcy threshold, $\frac{dr^*}{d\psi} < 0$ and $\frac{dr^*}{d\bar{r}} < 0$.

The effects of more diversification and greater opacity on the bankruptcy threshold depend on the expected asset return: $\frac{dr^*}{d\alpha}\left(\bar{r}-\bar{r}_0\right) \leq 0$ and $\frac{dr^*}{d\beta}\left(\bar{r}-\bar{r}_0\right) \geq 0$, each with strict inequality if $\bar{r} \neq \bar{r}_0 \equiv \frac{D}{2}\frac{1+\psi}{\psi}$.

Proof. See Appendix A. ■

As the cost of wholesale funding rises, a higher asset return must be realized for the banker to repay all debt. As a result, the range of asset returns that induce a run increases (greater bank fragility). Next, when the liquidation value of assets increases, strategic complementarity between fund managers falls: if other fund managers withdraw funding, the damage caused by premature liquidation is smaller and reduces the incentive to join the run. As a result, both the bankruptcy threshold and fragility are lower. A higher expected asset return leads to more favorable beliefs about the asset return and therefore induces fund managers to roll over for a larger range of private signals, which reduces both the bankruptcy threshold and fragility.

The impact of asset opacity and diversification depends on the expected asset return. If the expected return exceeds the threshold $\bar{r}_0$, both greater diversification (lower return volatility) and more opacity (less-precise private signal) reduce bank fragility. Either greater diversification or greater opacity reduces the precision of the private signal in (3) relative to the precision of the prior about the asset return in (2), so managers base their rollover decision more on the prior. As a result, fragility is reduced whenever the expected asset return is high, that is, when managers are more likely to disregard (potentially unfavorable) private information.

Next, we describe how the volume of actual withdrawals $\ell^*(r)$ depends on the realized asset return and other variables, as summarized in Proposition 2.

Proposition 2. ***Actual withdrawal volume.*** *For a given realized return r, withdrawals are $\ell^*(r) = \Phi(z)$, where $z \equiv \sqrt{\beta}\left[\frac{\alpha}{\beta}(r^* - \bar{r}) + r^* - r\right]$. The withdrawal volume changes according to:*

$$\frac{\partial \ell^*}{\partial r} \equiv -\sqrt{\beta}\,\phi\left(z\right) < 0 \tag{5}$$

$$\frac{\partial \ell^*}{\partial r^*} \equiv \frac{\alpha+\beta}{\sqrt{\beta}}\,\phi\left(z\right) > 0 \tag{6}$$

$$\frac{\partial \ell^*}{\partial \bar{r}} \equiv -\frac{\alpha}{\sqrt{\beta}}\,\phi\left(z\right) < 0 \tag{7}$$

$$\frac{\partial \ell^*}{\partial \alpha} \equiv \frac{r^*-\bar{r}}{\sqrt{\beta}}\,\phi\left(z\right) \tag{8}$$

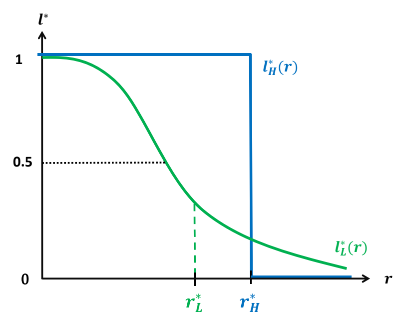
$$\frac{\partial \ell^*}{\partial \beta} \equiv \frac{\left(1-\frac{\alpha}{\beta}\right)r^* - r + \frac{\alpha}{\beta}\bar{r}}{2\sqrt{\beta}}\,\phi\left(z\right), \tag{9}$$

where $\phi(x) \equiv \frac{1}{\sqrt{2\pi}}e^{\{-\frac{x^2}{2}\}}$ *is the probability density function of the standardized Gaussian random variable. Furthermore,* $\frac{\partial \ell^*(r)}{\partial \alpha}\left(\bar{r}-\bar{r}_0\right) \leq 0$*, with strict inequality if* $\bar{r} \neq \bar{r}_0$*. Moreover,* $\frac{\partial \ell^*(r)}{\partial \beta}\left(r-\tilde{r}\right) \leq 0$*, with strict inequality if* $r \neq \tilde{r} \equiv r^* - \frac{\alpha}{\beta}(r^*-\bar{r})$*.*

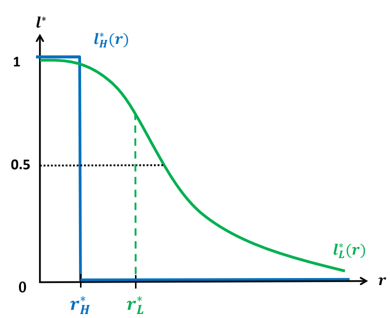
Proof. See Appendix A. ■

The intuition for these results follows. A higher realized return r implies that fund managers receive more-favorable private signals on average, so fewer withdrawals occur. Either a higher bankruptcy threshold r^* or a lower expected return $\bar{r}$ implies a higher signal threshold, $x^* \equiv \left(1+\frac{\alpha}{\beta}\right)r^* - \frac{\alpha}{\beta}\bar{r}$, below which fund managers refuse to roll over debt, thereby increasing the withdrawal volume. The effects of diversification and opacity are in general ambiguous. If the expected asset return is low, $\bar{r} < \bar{r}_0$, more diversification increases the withdrawal volume for any given realized asset return r, $\frac{\partial \ell^*(r)}{\partial \alpha} > 0$, since fund managers put more weight on the common prior and tend to disregard (potentially favorable) private information. Finally, the effect of opacity depends on the realized asset return. More opacity reduces the withdrawal volume, $\frac{\partial \ell^*(r)}{\partial \beta} > 0$, if and only if the realized return is sufficiently low, $r < \tilde{r}$. That is, if the realized return is low, greater opacity implies that fund managers place less weight on their private signal and therefore are less likely to withdraw, reducing fragility.

Figure 4 shows how asset transparency or opacity affects the bankruptcy threshold r^* and the actual withdrawal volume ℓ^* for the cases of high and low expected asset returns. For a high expected asset return ($\bar{r} > \bar{r}_0$), shown in Figure 4a, asset opacity ($\beta = \beta_L$) reduces fragility, $r_L^* \equiv r^*(\beta_L) < r^*(\beta_H) \equiv r_H^*$. However, asset opacity also induces partial runs for high realized returns, $\ell_L^*(r) > 0$ for $r > r_H^*$. For a low expected asset return ($\bar{r} \leq \bar{r}_0$), shown in Figure 4b, asset opacity increases fragility, $r_L^* \geq r_H^*$, which is the opposite of the previous result. The second effect of asset opacity is unchanged and partial runs occur for high realized returns, $\ell_L^*(r) > 0$ for $r > r_H^*$. Taken together, asset opacity is never beneficial if expected returns are low.



(a) $\bar{r} > \bar{r}_0$



(b) $\bar{r} \leq \bar{r}_0$

Figure 4: The bankruptcy threshold r^* and actual withdrawal volumes ℓ^* for asset opacity (β_L) and transparency (β_H). Panel (a) shows the case of high expected returns and panel (b) shows the case of low expected returns. Asset opacity may be beneficial for high expected returns but is unambiguously costly for low expected asset returns.

5 Transparent versus opaque assets

Having characterized the optimal rollover behavior in the wholesale funding market at the interim date, we now turn to the optimal asset choice by the banker at the initial date. We restrict the two choices of opacity, $\beta \in \{\beta_L, \beta_H\}$, to satisfying the following conditions. First, to guarantee uniqueness in all rollover subgames, we impose a lower bound on transparency, $\beta_L \geq \underline{\beta}$. Second, as we will see, full transparency is sometimes optimal, so we focus on the limiting case in which $\beta_H \to \infty$. These parameterizations ensure that the asset choice of the banker yields a unique equilibrium in all subgames.

First, we derive the equity value of the banker. For low realized returns, bankruptcy occurs and the banker is protected by limited liability. For high realized returns, the banker is profitable and receives the asset return net of funding costs. Because of incomplete information, some withdrawals occur at $t = 1$ even if the banker is solvent, as some fund managers receive a low private signal and withdraw – a *partial run*. The withdrawal volume at $t = 1$ is $\ell^*(r)$ and cost $\frac{\ell^*(r)}{\psi}D$ due to liquidation, while the withdrawal volume at $t = 2$ is $1 - \ell^*(r)$ and cost $[1 - \ell^*(r)]D$. In sum, the equity value for a given realized asset return r is

$$E(r) \equiv \max\left\{0, r - D\left[1 + \left(\frac{1}{\psi} - 1\right)\ell^*(r)\right]\right\}. \tag{10}$$

The equity value is zero at the bankruptcy threshold, $E(r^*) = 0$. Therefore, the bankruptcy threshold r^* is the relevant lower bound on returns for evaluating the expected equity value, as it exceeds the realized return for which early closure occurs.[7]

Next, we derive the *expected* equity value of the banker V. This is the equity value $E(r)$ integrated over all asset returns above the bankruptcy threshold, $r \geq r^*$:

[7]Since early closure at $t = 1$ occurs if the realized asset return of the banker cannot serve withdrawals by liquidating the asset, $r < r_{EC}$, where the early closure threshold return r_{EC} is implicitly defined by $r_{EC} \equiv \frac{D}{\psi}\ell^*(r_{EC})$. It follows that $r^* > r_{EC}$. See also Rochet and Vives (2004).

$$V \equiv \int_{-\infty}^{\infty} E(r)\, f(r)\, dr = \int_{r^*}^{\infty} \left(r - D\left[1 + \left(\frac{1}{\psi} - 1\right) \ell^*(r)\right]\right) f(r)\, dr, \quad (11)$$

where $f(r) \equiv \sqrt{\alpha}\phi(\sqrt{\alpha}[r - \bar{r}])$ is the probability density function of the common prior about the asset return and $F(r) \equiv \Phi(\sqrt{\alpha}[r - \bar{r}])$ is its cumulative distribution function.

The banker's asset opacity choice affects the expected equity value of the banker through a number of channels. First, although asset transparency or opacity affects the run threshold (the lower limit of the integral), the equity value at this limit is zero, $E(r^*) = 0$. Therefore, opacity affects the expected equity value only via a second set of effects: it changes the actual withdrawal volume $\ell^*(r)$ governing the scale of partial runs. These are both directly affected by opacity, via $\frac{\partial \ell^*(r)}{\partial \beta}$, and indirectly via the bankruptcy threshold, $\frac{\partial \ell^*(r)}{\partial r^*}\frac{\partial r^*}{\partial \beta}$. Lemma 1 states the total effect of changes in asset opacity choice on the bank's expected equity value.

Lemma 1. ***Asset choice and expected equity value.*** *The total effect of greater asset transparency on the expected value of banker equity is given by:*

$$\frac{dV}{d\beta} = D\left(\frac{1}{\psi} - 1\right)\frac{S}{2\beta}\left[\frac{1}{\sqrt{2\pi}\sqrt{\alpha+\beta}} + (r^* - \bar{r})\frac{\alpha + \beta\Delta}{2\beta(1-\Delta)}\right], \quad (12)$$

where $S \equiv \sqrt{\frac{\alpha\beta}{2\pi(\alpha+\beta)}} e^{\{-\frac{\alpha(\alpha+\beta)}{2\beta}[r^* - \bar{r}]^2\}} > 0$, $\Delta \equiv D\left(\frac{1}{\psi} - 1\right)\phi\left(\frac{\alpha}{\sqrt{\beta}}\,[r^* - \bar{r}]\right)\frac{\alpha}{\sqrt{\beta}} \in (0, 1)$.

Proof. See Appendix B. ■

We next state our first result on the banker's optimal choice of asset opacity versus transparency at the initial date.

Proposition 3. ***Asset transparency.*** *If the expected return is low, $\bar{r} \leq \bar{r}_0$, then $r^* \leq \bar{r}$ and the banker chooses asset transparency, $\beta^* = \beta_H \to \infty$. The bankruptcy threshold is $r_H^* \equiv r^*(\beta_H) \to \bar{r}_0$, the equity value is $E_H(r) \to r - D$, and the expected equity value is $V_H \equiv V(\beta_H) = \int_{r_H^*}^{\infty} E_H(r)\, f(r)dr \to \frac{f(r_H^*)}{\alpha} + (\bar{r} - D)\big[1 - F(r_H^*)\big]$.*

The expected equity value under asset transparency increases in the expected asset return, $\frac{dV_H}{d\bar{r}} > 0$, and in the liquidation value, $\frac{dV_H}{d\psi} > 0$. In contrast, it decreases in the level of diversification, $\frac{dV_H}{d\alpha} < 0$, and in the face value of wholesale funding, $\frac{dV_H}{dD} < 0$.

Proof. See Appendix C. ■

As shown in Figure 4b, when expected asset returns are low, asset opacity is associated with two costs and no benefit. First, asset opacity always implies a partial run on a solvent bank, as some fund managers receive low private signals and withdraw their funds even if the realized asset return is high. This cost of opacity, via partial runs, is independent of the expected asset return. Second, and only when expected returns are low, asset opacity increases the bankruptcy threshold. Asset opacity implies that fund managers put a larger weight on their common prior about the asset return. Since the prior is unfavorable for a low expected return, asset opacity leads to a higher bankruptcy threshold.

In sum, for a low expected asset return, asset transparency dominates asset opacity. This is shown in Figure 5, where the equity value under opacity, $E_L(r)$ always lies below the equity value under transparency, $E_H(r)$, over the relevant domain of returns. Clearly in this case, opacity is always dominated by transparency.

Figure 6 offers intuition for the comparative statics results of the expected equity value under transparency. It shows the components of the expected equity value, $V_H = \int_{r_H^*}^{\infty} E_H(r)\, f(r)dr$. A higher liquidation value ψ reduces the bankruptcy threshold and increases the range of returns over which the equity value is positive. A higher face value of debt D has two effects, each of which reduces the expected equity value:

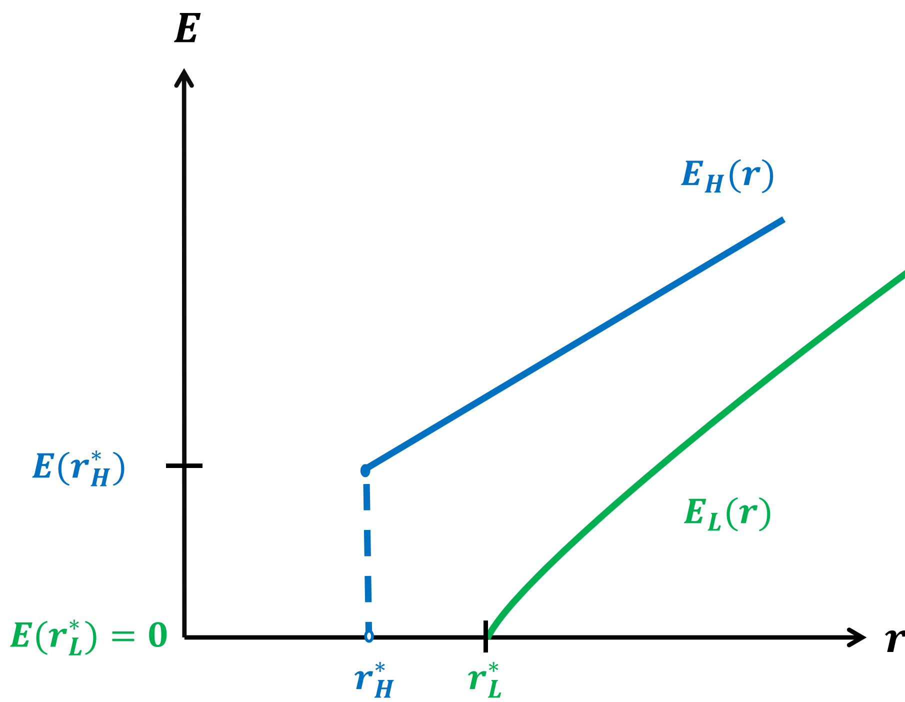


Figure 5: Bank equity as a function of the realized asset return, $E(r)$, for a low expected asset return, $\bar{r} \leq \bar{r}_0$. We compare the cases of opacity (β_L) and transparency (β_H). First, opacity is always associated with partial runs, $E_H(r) > E_L(r)$ for $r \geq r_L^*$. Second, for low expected returns, opacity results in higher fragility, $r_H^* \leq r_L^*$. In sum, transparency dominates opacity for low expected asset returns, $E_H(r) \geq E_L(r)$.

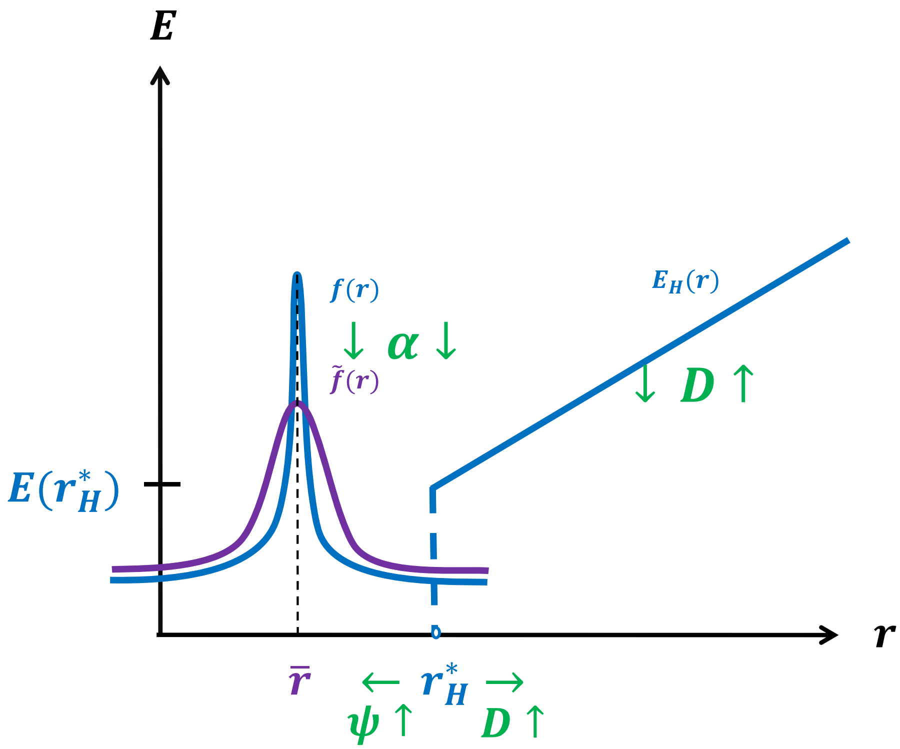


Figure 6: Comparative statics of expected bank equity for transparency, $V_H = \int_{r_H^*}^{\infty} E_H(r)\, f(r) dr$. A higher liquidation value ψ reduces the bankruptcy threshold r_H^* and thus increases the range of asset returns over which the equity value is positive. A higher face value of wholesale debt D has two effects, each of which reduces the expected equity value. First, it reduces the equity value for a given asset return. Second, it increases the bankruptcy threshold r_H^*. Finally, both a higher expected asset return $\bar{r}$ and a lower level of diversification α increase the expected equity value, since those asset returns for which the equity value is positive are more probable.

it reduces the equity value for a given asset return and it increases the bankruptcy threshold. Finally, both a higher expected asset return $\bar{r}$ and a lower level of diversification α increase the expected equity value. Since $\bar{r} \leq r_H^*$, either of these changes increases the probability mass of the returns for which the equity value is positive.

We label the result that $dV_H/d\alpha < 0$ the *equity value effect*. This arises because greater diversification is a mean-preserving contraction of the distribution of returns. Since the equity value is truncated at r_H^*, lower volatility reduces the expected value of equity, which is a call option on the bank's assets.

We have shown that asset transparency is optimal for a low expected asset return. If the expected return is high, however, the banker may choose asset opacity. This depends on the trade-off between the costs and benefits of opacity as expected returns rise. Figure 7 illustrates for the case where $\bar{r} > \bar{r}_0$. The cost of asset opacity is unchanged, namely the presence of partial runs even when the bank is solvent, $r > r_H^*$. This cost arises since some fund managers receive low private signals and withdraw.

However, the benefit of asset opacity in the case of a high expected asset return is lower fragility, $r_L^* < r_H^*$. If assets are opaque, fund managers place a smaller weight on their private signal and a larger weight on the prior. Since the prior is favorable, this reduces the bankruptcy threshold. Therefore, for a high expected asset return, opacity increases the range of asset returns for which the equity value is positive – the benefit of opacity. Since this requires high expected returns, a *necessary* condition for the banker to choose opacity is that returns are expected to be high.

Under asset opacity, $\beta = \beta_L$, the bankruptcy threshold is pinned down by $r_L^* \equiv D\left[1 + \left(\frac{1}{\psi} - 1\right)\Phi\left(\frac{\alpha}{\sqrt{\beta_L}}\left[r_L^* - \bar{r}\right]\right)\right] < r_H^*$. This inequality highlights the benefit of opacity in the form of lower bank fragility when the expected asset return is high. However, the endogenous cost of opacity is that the withdrawal volume is now less sensitive to the realized asset return, so liquidation occurs even if the bank is solvent,

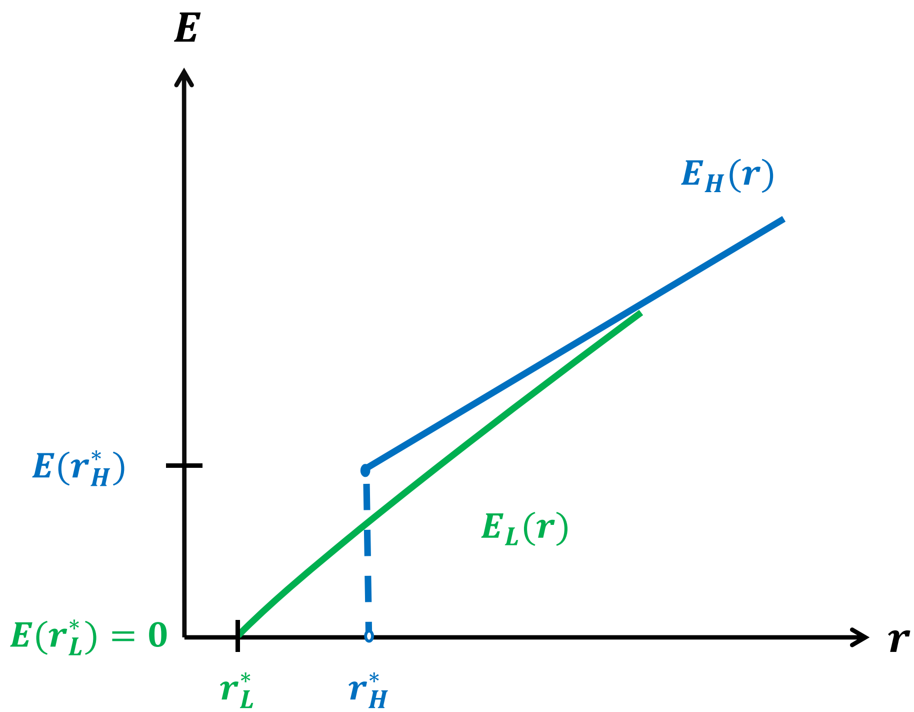


Figure 7: Bank equity as a function of the realized asset return, $E(r)$, for a high expected asset return, $\bar{r} > \bar{r}_0$. $E_H(r)$ denotes equity value under transparency and $E_L(r)$ denotes equity value under opacity. The endogenous benefit of asset opacity is lower bank fragility, $r_L^* < r_H^*$, since fund managers place a lower weight on their private signal and a larger weight on the (favorable) prior about the asset return. The endogenous cost of opacity is the partial run on the bank even for high realized asset returns, $r > r_H^*$, since some fund managers receive a low private signal and withdraw.

$\ell_L^*(r) \equiv \Phi\left(\sqrt{\beta_L}\left[\frac{\alpha}{\beta_L}(r_L^* - \bar{r}) + r_L^* - r\right]\right) > 0$. Let the equity value be denoted by $E_L(r) \equiv r - D\left[1 + \left(\frac{1}{\psi} - 1\right)\ell_L^*(r)\right]$. The expected equity value is $V_L = V(\beta_L) = \int_{r_L^*}^{\infty} E_L(r)\, f(r) dr$.

Proposition 4. ***Asset opacity.*** *If the expected return is high, $\bar{r} > \bar{r}_0$, the banker chooses asset opacity, $\beta^* = \beta_L$, if and only if the benefit of opacity exceeds its cost:*

$$\underbrace{\int_{r_L^*}^{r_H^*} E_L(r)\, f(r) dr}_{\text{Benefit of opacity: lower fragility}} \geq \underbrace{\int_{r_H^*}^{\infty} \Big[E_H(r) - E_L(r)\Big] f(r) dr}_{\text{Cost of opacity: partial runs}}. \tag{13}$$

If $\bar{r} \geq \bar{r}_2 \equiv \sqrt{\frac{2}{\pi}} \frac{\beta_L(1-\Delta(\beta_L))}{(\alpha+\beta_L\Delta(\beta_L))\sqrt{\alpha+\beta_L}} + D\left[\frac{1}{\psi} + \left(\frac{1}{\psi} - 1\right)\Phi\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{\beta_L}{\alpha+\beta_L}} \frac{\alpha(1-\Delta(\beta_L))}{\alpha+\beta_L\Delta(\beta_L)}\right)\right] > \bar{r}_0$, the banker always chooses asset opacity. Moreover, if $\alpha \geq \beta_L$ and $\bar{r} \leq \bar{r}_1 \equiv \sqrt{\frac{\beta_L}{\alpha(\alpha+\beta_L)}} + D\left[\frac{1}{\psi} + \left(\frac{1}{\psi} - 1\right)\Phi\left(\sqrt{\frac{\alpha}{\alpha+\beta_L}}\right)\right] > \bar{r}_0$, there exists a unique threshold $\tilde{\alpha}$ such that the banker chooses asset opacity if and only if $\alpha > \tilde{\alpha}$.

Proof. See Appendix D. ■

This establishes a *sufficient* condition for the bank to choose opacity. In effect, sufficiently high expected returns ensure that the benefit of opacity – less fragility – outweighs the cost – more partial runs. The intuition is that with high expected returns, the banker wants to encourage investors to place a high weight on their priors, so reducing fragility. For sufficiently high returns that is strong enough to outweigh the effects of partial runs resulting from imprecise private information.

To obtain some intuition for the second part of the proposition, which says that opacity dominates when the common prior is sufficiently good over some range of expected returns, consider first the case of an uninformative prior, $\alpha \to 0$. Then, the mean of the prior is irrelevant for the bankruptcy threshold. As a result, there is no benefit from opacity. However, there remains the cost of opacity via partial runs, since some fund managers receive bad private signals and withdraw from the bank for good realizations of the asset return. Therefore, the banker never chooses opacity because transparency allows the bank to minimize costly partial runs.

As the prior becomes more informative and α increases, two effects occur. First, a wedge opens up between the bankruptcy threshold under opacity and under transparency, such that the bank is proportionately less fragile under opacity. This is because more diversification (lower volatility of the prior) reassures fund managers when expected returns are high. Thus, the bank would rather managers weighed these signals in their withdrawal decisions relatively heavily, so that runs are ex ante less likely and fragility is low. Second, greater diversification reduces the incidence of partial runs, because the prior now provide managers with a greater 'anchor' for their beliefs, and this anchor is reassuring. Therefore, as the precision of the prior improves, the bank would rather investors base their actions on this information rather than on (possibly unfavorable) private signals. As a result, the banker chooses opacity for a sufficiently precise prior. In effect, public and private information are substitutes.

Finally, we study how parameters affect the expected equity value of the banker under opacity. Proposition 5 summarizes these comparative statics results.

Proposition 5. ***Comparative statics in the case of asset opacity.*** *Under opacity, $\beta^* = \beta_L$, the expected equity value increases in the expected return, $\frac{dV_L}{d\bar{r}} > 0$, and in the liquidation value, $\frac{dV_L}{d\psi} > 0$. In contrast, it decreases in the face value of debt, $\frac{dV_L}{dD} < 0$. The effect of changes in diversification is in general ambiguous, $\frac{dV_L}{d\alpha} \lesseqgtr 0$.*

Proof. See Appendix E. ■

Figure 8 offers some intuition for these comparative statics results. It shows the components of the expected equity value under opacity, $V_L = \int_{r_L^*}^{\infty} E_L(r)\, f(r) dr$. The liquidation value ψ and the face value of debt D have two effects. Both a higher liquidation value and a lower face value of debt reduce the bankruptcy threshold r_L^*, which increases the range of asset returns over which the equity value is positive. A higher liquidation value and a lower face value of debt also increase the equity value for a given asset return. The expected asset return $\bar{r}$ and diversification α have three effects. Both a higher expected asset return and greater diversification reduce the bankruptcy as well as increase the equity value for a given asset return. Moreover, they affect the prior distribution $f(r)$. Since greater diversification reduces the weight on very positive realizations of the asset return, for which the equity value is very positive, the overall effect of greater diversification is ambiguous.

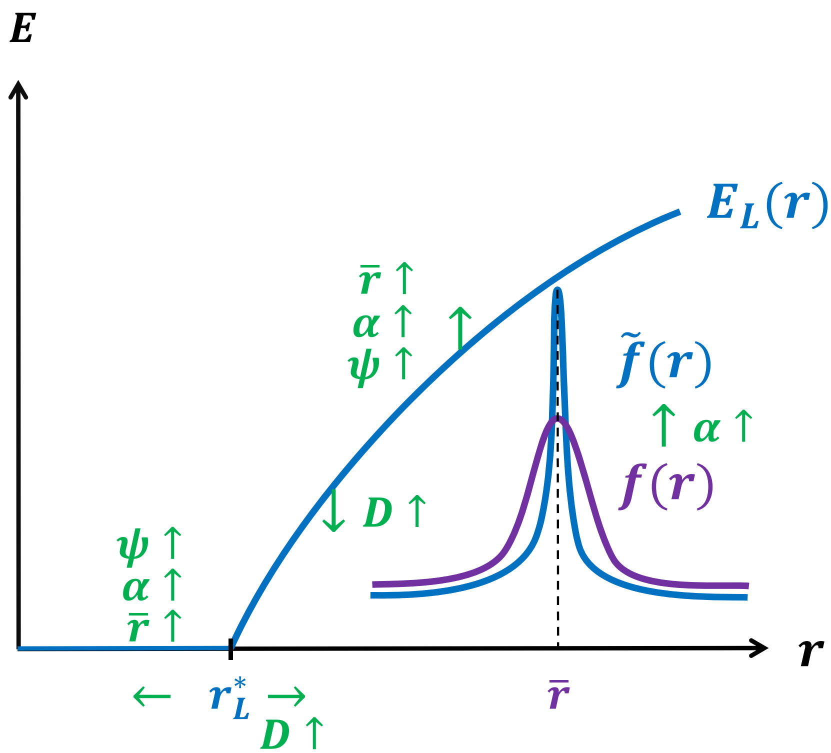


Figure 8: Comparative statics of the expected equity value of the banker in the case of opacity, $V_L = \int_{r_L^*}^{\infty} E_L(r)\, f(r) dr$, when the expected asset return is high, $\bar{r} > \bar{r}_0$. The liquidation value ψ and the face value of debt D have two effects. Both a higher liquidation value and a lower face value of debt reduce the bankruptcy threshold r_L^*, increasing the range of asset returns over which the equity value is positive. A higher liquidation value and a lower face value of debt also increase the equity value for a given asset return. The expected asset return $\bar{r}$ and the level of diversification α have three effects. Both a higher expected asset return and a greater diversification reduce the bankruptcy as well as increase the equity value for a given asset return. Moreover, they affect the prior distribution $f(r)$. Since greater diversification reduces the weight on very positive realizations of the asset return, the overall effect is ambiguous.

6 Conclusion

One notable feature of the financial crisis was the opacity of bank assets. Why did banks choose to be so opaque? Our simple framework suggests that a period of high expected returns encourages banks to choose opaque portfolios. By doing this, a bank encourages investors to 'overweight' the priors about returns, reducing the likelihood of a bank run, even at the expense of partial runs, which occur because *some* investors receive bad private signals and are correspondingly skeptical about returns. A corollary is that when returns are low, banks are more likely to be transparent. When returns are low, public information about asset returns tends to encourage investors to run; in this case, banks strive to convince investors that their assets are actually sound by raising the precision of the private information their asset portfolios generate.

Our framework formalizes these intuitions. The key insight of our model is that the bank chooses asset opacity to trade off the endogenous costs of creating opaque claims, via partial runs on a solvent bank in good times, with the endogenous benefits of opacity, which take the form of a greater use of public information by investors in assessing portfolio quality. When returns are expected to be high, asset opacity reduces bank fragility. We link the banker's optimal opacity choice to the expected return on its assets and the volatility of asset returns, interpreted as diversification.

A number of avenues remain to be explored in future work. These include the impact of the bank's opacity choice on its cost of funding and the effect of bank capitalization on the fragility-opacity nexus. It would also be worthwhile to study the impact of a single large investor in wholesale funding markets on coordination, including its potential for signaling. Finally, it would be interesting to extend the model to a system context – studying the interplay between bank opacity and the channels of contagion across banks.

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A Proof of Propositions 1 and 2

This proof builds on Morris and Shin (2000, 2003). Each fund manager j uses the commonly known distribution of the asset return given in (2) and the private signal in (3) to update the belief about the asset return. **Bayesian updating** yields the posterior about the return of manager j: $r|x_j \sim \mathcal{N}\left(\frac{\alpha\bar{r}+\beta x_j}{\alpha+\beta}, \frac{1}{\alpha+\beta}\right)$. Next, an **indifference condition** states that the fund manager who receives the threshold signal, $x_j = x^*$, is indifferent between rolling over and withdrawing funds. Using the posterior about the asset return and $B = C$, we obtain:

$$\begin{aligned} C \ \Pr\{r < r^*|x^*\} &= B \ \Pr\{r > r^*|x^*\} \\ \Rightarrow x^* &= r^* + \frac{\alpha}{\beta}[r^* - \bar{r}]. \end{aligned} \tag{14}$$

A **critical mass condition** states that bankruptcy occurs at the threshold r^*:

$$r^* = D\left[1 + \left(\frac{1}{\psi} - 1\right)\ell^*(r^*)\right], \tag{15}$$

where the actual volume of withdrawals is (by a law of large numbers):

$$\ell^*(r) = \Pr\{x_j < x^*|r\} = \Pr\{\epsilon_j + r < x^*\} = \Pr\{\epsilon_j < x^* - r\} \tag{16}$$

$$= \Phi\big(\sqrt{\beta}[x^* - r]\big) = \Phi\left(\frac{\alpha+\beta}{\sqrt{\beta}}r^* - \frac{\alpha}{\sqrt{\beta}}\bar{r} - \sqrt{\beta}r\right) \equiv \Phi(z), \tag{17}$$

where we used the distribution of the private signal ϵ_j, and $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and cumulative distribution function of the standard Gaussian random variable, respectively.

Evaluating the withdrawal volume at $r = r^*$, inserting x^* from the indifference condition, and combining the result with the critical mass condition yields the bankruptcy threshold stated in Proposition 1.

In line with the standard uniqueness argument, the right-hand side of equation (4) is bounded by the interval $[r_L, r_H]$, while the left-hand side has full support. Both sides increase in the asset return. Thus, a unique solution r^* exists if the slope of the left-hand side exceeds the slope of the right-hand side:

$$1 > \Delta \equiv D\left(\frac{1}{\psi} - 1\right)\phi\Big(\frac{\alpha}{\sqrt{\beta}}\,[r^* - \bar{r}]\Big)\frac{\alpha}{\sqrt{\beta}} > 0. \tag{18}$$

For this condition to hold, a lower bound on the precision of private information suffices. Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$, an upper bound of the right-hand side of condition (18) is $RHS \leq D\left(\frac{1}{\psi} - 1\right)\frac{\alpha}{\sqrt{2\pi}\sqrt{\beta}}$. This upper bound decreases in β. As a result, there exists a unique value $\underline{\beta} \equiv \left(D\left(\frac{1}{\psi} - 1\right)\frac{\alpha}{\sqrt{2\pi}}\right)^2 \in (0, \infty)$ such that condition (18) holds for all $\beta > \underline{\beta}$ (sufficient condition). Given sufficiently precise private information, there exists a unique threshold of the asset return that is implicitly given by equation (4). The unique signal threshold follows from the indifference condition in equation (14).

Next, we study how the bankruptcy threshold r^* is affected by other variables of interest. First, consider the impact of wholesale funding costs on the bankruptcy threshold. More expensive wholesale funding raises the threshold:

$$\frac{dr^*}{dD} = \frac{1}{1-\Delta}\,\frac{r^*}{D} > 0. \tag{19}$$

Moreover, a higher liquidation value reduces the bankruptcy threshold:

$$\frac{dr^*}{d\psi} = -\frac{D}{\psi^2(1-\Delta)}\Phi\Big(\frac{\alpha}{\sqrt{\beta}}\,[r^* - \bar{r}]\Big) = -\frac{r^* - D}{(1-\Delta)\psi(1-\psi)} < 0. \tag{20}$$

Next, a higher expected asset return reduces the equilibrium bankruptcy threshold:

$$\frac{dr^*}{d\bar{r}} = -\frac{\alpha}{\sqrt{\beta}}\phi\Big(\frac{\alpha}{\sqrt{\beta}}\,[r^* - \bar{r}]\Big)\frac{D\left(\frac{1}{\psi} - 1\right)}{1-\Delta} = -\frac{\Delta}{1-\Delta} < 0. \tag{21}$$

Asset opacity affects the bankruptcy threshold according to:

$$\frac{dr^*}{d\beta} = -\frac{\Delta}{1-\Delta}\frac{r^* - \bar{r}}{2\beta}. \tag{22}$$

Greater asset opacity, a lower β, reduces the bankruptcy threshold, $\frac{dr^*}{d\beta} > 0$, whenever $r^* < \bar{r}$. Using the bankruptcy threshold in equation (4), the inequality $r^* < \bar{r}$ arises if and only if the expected asset return is high:

$$\bar{r} > \bar{r}_0 \equiv \frac{D}{2}\frac{1+\psi}{\psi}, \tag{23}$$

where this threshold of the expected asset return increases in the cost of wholesale funding, $\frac{d\bar{r}_0}{dD} > 0$, and decreases in the liquidation value, $\frac{d\bar{r}_0}{d\psi} < 0$. The same lower bound on the expected asset return is also necessary and sufficient for more diversification to reduce the threshold, $\frac{dr^*}{d\alpha} < 0$, since

$$\frac{dr^*}{d\alpha} = \frac{\Delta}{1-\Delta}\frac{r^* - \bar{r}}{\alpha}. \tag{24}$$

Likewise, $\frac{dr^*}{d\beta} < 0$ and $\frac{dr^*}{d\alpha} > 0$ if $r^* > \bar{r}$, or $\bar{r} < \bar{r}_0$. Finally, if $\bar{r} = \bar{r}_0$, then $r^* = \bar{r}$ and $\frac{dr^*}{d\beta} = 0 = \frac{dr^*}{d\alpha}$. Taken together, this yields the expression stated in Proposition 1.

Regarding Proposition 2, the actual volume of withdrawals is derived above. Its partial derivatives are immediate. Next, we have that, for all values of the realized asset return, $\frac{\partial \ell^*(r)}{\partial \alpha} > 0 \Leftrightarrow r^* > \bar{r} \Leftrightarrow \bar{r} < \bar{r}_0$, as was shown above. Moreover, $\tilde{r}$ follows directly from $\frac{\partial \ell^*(\tilde{r})}{\partial \beta} \equiv 0$.

B Proof of Lemma 1

Since $E(r^*) = 0$, the direct and indirect effects of asset transparency add up to:

$$\begin{aligned}
\frac{dV}{d\beta} &= -D\left(\frac{1}{\psi}-1\right)\int_{r^*}^{\infty}\frac{d\ell^*(r)}{d\beta}\,f(r)\,dr \\
&= -D\left(\frac{1}{\psi}-1\right)\int_{r^*}^{\infty}\left[\frac{\partial\ell^*(r)}{\partial\beta}+\frac{\partial\ell^*(r)}{\partial r^*}\frac{\partial r^*}{\partial\beta}\right]f(r)\,dr \qquad (25)\\
&= \frac{D}{2\sqrt{\beta}}\left(\frac{1}{\psi}-1\right)\int_{r^*}^{\infty}\phi(z)f(r)\left[r-r^*+\frac{r^*-\bar{r}}{\beta}\left(\alpha+\frac{(\alpha+\beta)\Delta}{1-\Delta}\right)\right]dr,
\end{aligned}$$

where Δ and the partial derivatives are defined in Appendix A. Using the symmetry $\phi(-z) = \phi(z)$, let $g(r) \equiv \sqrt{\beta}\phi(z)$, where g is the probability density function (pdf) of a Gaussian random variable with mean $r^* + \frac{\alpha}{\beta}(r^* - \bar{r})$ and precision β. We use the fact that the product of two Gaussian pdfs is proportional to a Gaussian pdf, with a scaling factor that is a Gaussian pdf itself (DeGroot, 1970). Specifically, if $f : r \sim \mathcal{N}(\bar{r}, \alpha^{-1})$ and $g : r \sim \mathcal{N}\left(r^* + \frac{\alpha}{\beta}(r^* - \bar{r}), \beta^{-1}\right)$, then the scaling factor becomes S, as stated in Lemma 1, and the Gaussian pdf becomes $h(r)$:

$$h : r \sim \mathcal{N}\left(r^*, (\alpha+\beta)^{-1}\right),$$

since the precision of the product is the sum of the individual precisions, $\alpha + \beta$, and the mean of the product is the weighted average means with the precision as weights, $\frac{\alpha}{\alpha+\beta}\bar{r} + \frac{\beta}{\alpha+\beta}\left[r^* + \frac{\alpha}{\beta}(r^* - \bar{r})\right] = r^*$. Therefore, we can replace $\phi(z)f(r)$ with $\frac{S}{\sqrt{\beta}}h(r)$:

$$\frac{dV}{d\beta} = \frac{D}{2\beta}\left(\frac{1}{\psi}-1\right)S\int_{r^*}^{\infty}h(r)\left[r-r^*+\frac{\alpha}{\beta}(r^*-\bar{r})+(\alpha+\beta)\frac{\Delta}{1-\Delta}\frac{r^*-\bar{r}}{\beta}\right]dr.$$

Finally, we use the facts: $\int_{r^*}^{\infty} h(r)dr = \frac{1}{2}$ and $\int_{r^*}^{\infty}(r - r^*)h(r)dr = \frac{1}{\sqrt{2\pi}\sqrt{\alpha+\beta}}$. The first result follows from the symmetry of the Gaussian pdf. The second result can be obtained with the help of the elementary integral $\int x\phi(bx)dx = -\frac{\phi(bx)}{b^2} + C$, where $x \equiv r - r^*$. (A simple proof is to differentiate the right-hand side.) Taken together:

$$\frac{dV}{d\beta} = \frac{D}{2\beta}\left(\frac{1}{\psi} - 1\right) S \left[\frac{1}{\sqrt{\alpha+\beta}\sqrt{2\pi}} + \left(\frac{\alpha}{2\beta} + \frac{\alpha+\beta}{2\beta}\frac{\Delta}{1-\Delta}\right)(r^* - \bar{r})\right]. \tag{26}$$

C Proof of Proposition 3

Since $\bar{r} \leq \bar{r}_0$ is assumed, it follows that $r^* \geq \bar{r}$ and $\frac{dV}{d\beta} > 0$ for all β. Thus $\beta^* = \beta_H \to \infty$. As a result, $r^* \to r_H^* \equiv \bar{r}_0$ and $\ell^*(r)|_{\beta^*} \to \mathbf{1}_{r<\bar{r}_0}$, where all fund managers withdraw if the realized return is below the bankruptcy threshold and no manager withdraws if it is above. The equity values converges to $E_H \to r - D$, so

$$\begin{aligned}
V_H &\equiv V(\beta_H) \to \int_{r_H^*}^{\infty} (r - D)\, f(r)\, dr && (27)\\
&= \sqrt{\alpha}\int_{r_H^*}^{\infty} \big(r - \bar{r}\big)\, \phi(\sqrt{\alpha}[r-\bar{r}])dr + \sqrt{\alpha}\big(\bar{r} - D\big)\int_{r_H^*}^{\infty} \phi(\sqrt{\alpha}[r-\bar{r}])dr && (28)\\
&= \frac{f(r_H^*)}{\alpha} + (\bar{r} - D)(1 - F(r_H^*)), && (29)
\end{aligned}$$

where we used $x \equiv r - \bar{r}$, the indefinite integrals stated above, and $\int \phi(\sqrt{\alpha}x)dx = \frac{\Phi(\sqrt{\alpha}x)}{\sqrt{\alpha}} + C$. Also note that $\int x^2\phi(\sqrt{\alpha}x)dx = \sqrt{\alpha}^{-3}\Phi(\sqrt{\alpha}x) - \alpha^{-1}x\phi(\sqrt{\alpha}x) + C$. Taken together, we can establish some useful properties of the Gaussian pdf and cdf:

$$\begin{aligned}
\frac{df(r)}{d\bar{r}} &\equiv \alpha(r - \bar{r})f(r) && (30)\\
\frac{df(r)}{dr} &\equiv -\alpha(r - \bar{r})f(r) && (31)\\
\frac{df(r)}{d\alpha} &\equiv \frac{f(r)}{2}\left[\frac{1}{\alpha} - (r - \bar{r})^2\right] && (32)\\
\frac{dF(r)}{d\bar{r}} &\equiv -f(r) < 0 && (33)\\
\frac{dF(r)}{dr} &\equiv f(r) > 0 && (34)\\
\frac{dF(r)}{d\alpha} &\equiv \frac{r - \bar{r}}{2\alpha} f(r) && (35)
\end{aligned}$$

Using these results, we obtain the following comparative static results of the expected equity value under transparency:

$$\frac{dV_H}{d\psi} \equiv -(r_H^* - D)\,\frac{dr_H^*}{d\psi}\,f(r_H^*) > 0 \tag{36}$$

$$\frac{dV_H}{dD} \equiv -\left[1 - F(r_H^*) + (r_H^* - D)\,\frac{dr_H^*}{dD}\,f(r_H^*)\right] > 0 \tag{37}$$

$$\frac{dV_H}{d\bar{r}} \equiv 1 - F(r_H^*) + (r_H^* - D)\,f(r_H^*) > 0 \tag{38}$$

$$\frac{dV_H}{d\alpha} \equiv -\frac{f(r_H^*)}{2\alpha}\left[\frac{1}{\alpha} + (r_H^* - \bar{r})\,(r_H^* - D)\right] < 0, \tag{39}$$

where we used $\frac{dr_H^*}{dD} \equiv \frac{1+\psi}{2\psi} > 0$ and $\frac{dr_H^*}{d\psi} \equiv -\frac{D}{2\psi^2} < 0$.

Note that the sign of the last comparative static arises because $\frac{dV_H}{d\alpha} < 0 \Leftrightarrow \frac{1}{\alpha} + (r_H^* - \bar{r})\,(r_H^* - D) > 0 \Leftrightarrow \bar{r} \leq \bar{r}_0 + \frac{2\psi}{\alpha D(1-\psi)} > \bar{r}_0$. This always holds because $\bar{r} \leq \bar{r}_0$.

D Proof of Proposition 4

The expected equity values are V_L (under opacity) and V_H (under transparency). Therefore, opacity is optimal if and only if $\delta \equiv V_L - V_H \geq 0$, where

$$\delta = \int_{r_L^*}^{r_H^*} \left(r - D\left[1 + \left(\frac{1}{\psi} - 1\right)\ell_L^*(r)\right]\right) dF(r) - D\left(\frac{1}{\psi} - 1\right)\int_{r_H^*}^{\infty} \ell_L^*(r)\,dF(r). \tag{40}$$

The first term reflects the endogenous benefit of asset opacity via reduced bank fragility ($r_L^* < r_H^*$). The second term reflects the endogenous cost of opacity via partial runs on a solvent bank ($\ell^*(r) > 0$ for $r > r_H^*$).

Next, we study how δ varies with α. Differentiation yields $\frac{d\delta}{d\alpha} = \lambda_1 + \lambda_2 + \lambda_3$, where we look at each of these terms in greater detail below.

We start with λ_1.

$$\begin{aligned}
\lambda_1 &\equiv -D\left(\frac{1}{\psi}-1\right)\int_{r_L^*}^{\infty}\frac{d\ell_L^*(r)}{d\alpha}f(r)dr \\
&= -D\left(\frac{1}{\psi}-1\right)\left[\frac{r_L^*-\bar{r}}{\beta_L}+\frac{\alpha+\beta_L}{\beta_L}\frac{r_L^*-\bar{r}}{\alpha}\frac{\Delta}{1-\Delta}\right]\int_{r_L^*}^{\infty}g(r)f(r)dr \qquad (41)\\
&= D\left(\frac{1}{\psi}-1\right)\frac{S}{2}\frac{\bar{r}-r_L^*}{\beta_L}\frac{\alpha+\beta_L\Delta}{\alpha(1-\Delta)}>0,
\end{aligned}$$

where we used $\frac{d\ell_L^*(r)}{d\alpha}=\frac{\partial\ell_L^*(r)}{\partial\alpha}+\frac{\partial\ell_L^*(r)}{\partial r_L^*}\frac{\partial r_L^*}{\partial\alpha}$ as we moved from the first to the second line and used the corresponding expressions from Appendix A. We also used $f(r)g(r)=Sh(r)$ and related results from Appendix B. Since $r_L^*<\bar{r}$ for $\bar{r}>\bar{r}_0$, the first term is unambiguously positive, $\lambda_1>0$.

Next, we turn to λ_2.

$$\lambda_2\equiv\int_{r_L^*}^{r_H^*}E_L(r)\frac{df(r)}{d\alpha}dr=\int_{r_L^*}^{r_H^*}E_L(r)\frac{f(r)}{2}\left[\frac{1}{\alpha}-(r-\bar{r})^2\right]dr. \qquad (42)$$

Note that $0=E_L(r_L^*)\le E_L(r)\le E_L(r_H^*)$ for all $r\in[r_L^*,r_H^*]$ since $E_L(r)$ monotonically increases in r. Therefore, we can bound the absolute value of the integral by taking out $E_L(r_H^*)$. It remains to solve $\int_{r_L^*}^{r_H^*}\frac{f(r)}{2}\left[\frac{1}{\alpha}-(r-\bar{r})^2\right]dr$. Using $x=r-\bar{r}$, we first solve the indefinite integral, using the previously stated facts about Gaussian pdfs:

$$\begin{aligned}
&\int\frac{f(r)}{2}\left[\frac{1}{\alpha}-(r-\bar{r})^2\right]dr=\int\frac{\sqrt{\alpha}\phi(\sqrt{\alpha}\,x)}{2}\left[\frac{1}{\alpha}-x^2\right]dx\\
=&\ \frac{1}{2\sqrt{\alpha}}\int\phi(\sqrt{\alpha}\,x)dx-\frac{\sqrt{\alpha}}{2}\int x^2\phi(\sqrt{\alpha}\,x)dx=\frac{r-\bar{r}}{2\alpha}f(r). \qquad (43)
\end{aligned}$$

As a result, $\int_{r_L^*}^{r_H^*}\frac{f(r)}{2}\left[\frac{1}{\alpha}-(r-\bar{r})^2\right]dr=(\bar{r}-r_L^*)f(r_L^*)-(\bar{r}-r_H^*)f(r_H^*)\equiv y_L\phi(y_L)-y_H\phi(y_H)$, where $y_L\equiv\sqrt{\alpha}[\bar{r}-r_L^*]$ and $y_H\equiv\sqrt{\alpha}[\bar{r}-r_H^*]$.

Next, consider $m(y)=y\phi(y)$. Geometrically, this is the area of a rectangle between $[0,y]$ on the horizontal axis and $[0,\phi(y)]$ on the vertical axis. We wish to understand how the area changes as y increases in order to simplify the above bound on λ_2. Intuitively, the area is quite small for large values of y because the normal distribution has thin tails. Formally, we have $\frac{dm}{dy} = (1-y^2)\phi(y)$, so $\frac{dm}{dy} > 0 \Leftrightarrow -1 < y < 1$. Since $y_L > y_H$, it follows that a sufficient condition for $\lambda_2 > 0$ is $y_L < 1$. This condition can be written as $r_L^* \geq r_{L1}^* \equiv \bar{r} - \frac{1}{\sqrt{\alpha}}$. We revisit this condition below.

Next, we move to λ_3.

$$\lambda_3 \equiv -D\left(\frac{1}{\psi}-1\right)\int_{r_H^*}^{\infty} \ell_L^*(r)\frac{df(r)}{d\alpha}dr. \tag{44}$$

Using the same steps as for λ_2, we wish to find a lower bound on the integral. Since $\ell_L^*(r)$ decreases monotonically in r, we take out $\ell_L^*(r_H^*)$ and evaluate the integral in order to find a lower bound:

$$\lambda_3 \geq \lambda_3^{min} \equiv -\frac{D}{2\alpha}\left(\frac{1}{\psi}-1\right)\ell_L^*(r_H^*)(\bar{r}-r_H^*)f(r_H^*) < 0. \tag{45}$$

Therefore, a sufficient condition for $\lambda_1 + \lambda_3 > 0$ is $\lambda_1 + \lambda_3^{min} \geq 0$, which can be expressed as the following inequality:

$$\begin{aligned}
\frac{D}{2\alpha}\left(\frac{1}{\psi}-1\right)\ell_L^*(r_H^*)(\bar{r}-r_H^*)f(r_H^*) &\leq D\left(\frac{1}{\psi}-1\right)\frac{S}{2}\frac{\bar{r}-r_L^*}{\beta_L}\frac{\alpha+\beta_L\Delta}{\alpha(1-\Delta)} && (46)\\
\ell_L^*(r_H^*)(\bar{r}-r_H^*)f(r_H^*) &\leq S\frac{\bar{r}-r_L^*}{\beta_L}\frac{\alpha+\beta_L\Delta}{1-\Delta} && (47)\\
\ell_L^*(r_H^*)(\bar{r}-r_H^*)e^{-\frac{\alpha}{2}[\bar{r}-r_H^*]^2} &\leq \frac{(\alpha+\beta_L\Delta)\sqrt{\alpha+\beta_L}}{(\alpha+\beta_L)(1-\Delta)\sqrt{\beta_L}}[\bar{r}-r_L^*]e^{-\frac{\alpha(\alpha+\beta_L)}{2\beta_L}[\bar{r}-r_L^*]^2},
\end{aligned}$$

where we inserted the expression for the scaling factor S. Note the structure of this inequality, which compares the product of two positive factors in either side, $ab \leq cd$. In what follows, we determine sufficient conditions to ensure that $a \leq c$ and $b \leq d$ to ensure the overall inequality.

First, we wish to ensure that $\ell_L^*(r_H^*) \leq \frac{\alpha+\beta_L\Delta}{(\alpha+\beta_L)(1-\Delta)}$. Since $\Delta > 0$ and $\ell_L^*(r_H^*) < \frac{1}{2}$

for $\bar{r} > \bar{r}_0$, a first sufficient condition arises by setting $\Delta = 0$ and $\ell_L^*(r_H^*) = \frac{1}{2}$, which yields $\alpha \geq \beta_L$. This condition is more stringent than needed but has the advantage of simplicity. Since we have to ensure that $\beta_L \geq \underline{\beta}$ for uniqueness in the subgame, there always exist values of $\beta_L \geq \alpha$ if $\psi > \underline{\psi} \equiv \frac{\sqrt{\frac{\alpha}{2\pi}}D}{1+\sqrt{\frac{\alpha}{2\pi}}D} \in (0,1)$, which we assume henceforth.

Second, $(\bar{r} - r_H^*)\sqrt{\frac{\alpha}{2\pi}}e^{-\frac{\alpha}{2}[\bar{r}-r_H^*]^2} \leq \sqrt{\frac{\alpha(\alpha+\beta_L)}{2\pi\beta_L}}[\bar{r} - r_L^*]e^{-\frac{\alpha(\alpha+\beta_L)}{2\beta_L}[\bar{r}-r_L^*]^2}$, where we added the factor $\sqrt{\frac{\alpha}{2\pi}}$ to express the inequality in terms of Gaussian pdfs. Using the argument based on $m(y)$ from above, a sufficient condition for the second inequality is $\sqrt{\frac{\alpha(\alpha+\beta_L)}{\beta_L}}[\bar{r} - r_L^*] \leq 1$ or, equivalently, $r_L^* \geq r_{L2}^* \equiv \bar{r} - \sqrt{\frac{\beta_L}{\alpha(\alpha+\beta_L)}}$. Since $r_{L2}^* > r_{L1}^*$, the constraint $r_L^* \geq r_{L2}^*$ yields a more restrictive upper bound on the expected asset return (recall that $\frac{dr^*}{d\bar{r}} < 0$), we use r_{L2}^* to determine a sufficient condition. Inserting r_{L2}^* in the bankruptcy threshold in equation (4), we obtain the value of $\bar{r}_1$ reported in Proposition 4. It follows that $\frac{d\delta}{d\alpha} > 0$, whereby the net benefit of opacity increases in diversification.

Next, as α becomes very small, $\alpha \to 0$, r_L^* converges to r_H^*, such that the benefit of opacity converges to zero. However, the cost of opacity in terms of partial runs remains positive for very small α, since $\lim_{\alpha\to 0} \ell_L^*(r) > 0$. Therefore, there exists a value $\underline{\alpha} > 0$ such that $\delta(\alpha) < 0$ for all $\alpha < \underline{\alpha}$. Likewise, as α becomes very large, r_L^* converges to D, such that the benefit of opacity remains strictly positive. However, the cost of opacity in terms of partial runs becomes very small for very large α, since $\ell_L^*(r)$ becomes very small. Therefore, there exists a value $\bar{\alpha} < \infty$ such that $\delta(\alpha) > 0$ for all $\alpha > \bar{\alpha}$. Because of continuity and strict monotonicity, there exists a unique threshold $\tilde{\alpha}$, defined by $\delta(\tilde{\alpha}) \equiv 0$, such that $\delta(\alpha) > 0$ if and only if $\alpha > \tilde{\alpha}$.

The final part of Proposition 4 can be shown by considering Lemma 1. We have that $\frac{dV}{d\beta} < 0$ if $r^* < r_2^* \equiv \bar{r} - \frac{\sqrt{2}\beta_L(1-\Delta(\beta_L))}{\sqrt{\pi}\sqrt{\alpha+\beta_L}(\alpha+\beta_L\Delta(\beta_L))}$. Using the bankruptcy threshold in equation (4), we obtain the value of $\bar{r}_2$ reported in Proposition 4.

E Proof of Proposition 5

Recall that $\bar{r} > \bar{r}_0$ and $V_L \equiv \int_{r_L^*}^{\infty} E_L(r) f(r) dr$. Using the partial derivatives of $\ell_L^*(r)$ stated in Proposition 2 and the partial derivatives of $f(r)$ stated in Appendix C, we obtain the following comparative statics results:

$$\frac{dV_L}{dD} \equiv -[1 - F(r_L^*)] - \left(\frac{1}{\psi} - 1\right)\left[\int_{r_L^*}^{\infty} \ell_L^*(r) dF(r) + \frac{\alpha + \beta_L}{2\beta_L} S \frac{r_L^*}{1 - \Delta}\right] < 0 \tag{48}$$

$$\frac{dV_L}{d\psi} \equiv \frac{D}{\psi^2} \int_{r_L^*}^{\infty} \ell_L^*(r) dF(r) + D^2 \frac{1 - \psi}{\psi} \frac{\alpha + \beta_L}{2\beta_L} S \frac{\ell_L^*(r_L^*)}{1 - \Delta} > 0 \tag{49}$$

The signs of $\frac{dV_L}{dD}$ and $\frac{dV_L}{d\psi}$ arise, since $\ell_L^*(r) > 0$ over the entire domain.

The comparative static with respect to $\bar{r}$ can be decomposed according to $\frac{dV_L}{d\bar{r}} \equiv \rho_1 + \rho_2$, where the individual components are:

$$\rho_1 \equiv D \frac{1 - \psi}{\psi} \frac{\alpha + \beta_L \Delta}{2\beta_L(1 - \Delta)} S > 0 \tag{50}$$

$$\rho_2 \equiv \int_{r_L^*}^{\infty} E_L(r) \frac{df(r)}{d\bar{r}} dr > 0, \tag{51}$$

The positive sign of ρ_1 reflects the reduction in the 'partial run effect' as the expected return increases. Also, the sign on ρ_2 is intuitive. As $\bar{r}$ increases, mass shifts away from low realizations of the asset return to high realizations of the return. Using the symmetry of the Gaussian pdf, for any $r_- < \bar{r}$ that loses weight, there exists a unique $r_+ > \bar{r}$ that has gained the same amount of weight. Since $E_L(r)$ strictly increases in r, we have $E_L(r_+) > E_L(r_-)$ for any such pair (r_+, r_-), thus yielding $\rho_2 > 0$.

The comparative static with respect to α can be decomposed according to $\frac{dV_L}{d\alpha} \equiv a_1 + a_2$, where the individual components below result in a generally ambiguous sign:

$$a_1 \equiv D\frac{1-\psi}{\psi}(\bar{r}-r_L^*)\frac{\alpha+\beta_L\Delta}{2\alpha\beta_L(1-\Delta)}S > 0 \tag{52}$$

$$a_2 \equiv \int_{r_L^*}^{\infty} E_L(r)\frac{df(r)}{d\alpha}dr \lesseqgtr 0. \tag{53}$$

The positive sign of a_1 reflects the reduction in the 'partial run effect' as α increases.