SHADOW INTEREST RATES, MACROECONOMIC TRENDS, & TIME-VARYING UNCERTAINTY

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Federal Reserve Board

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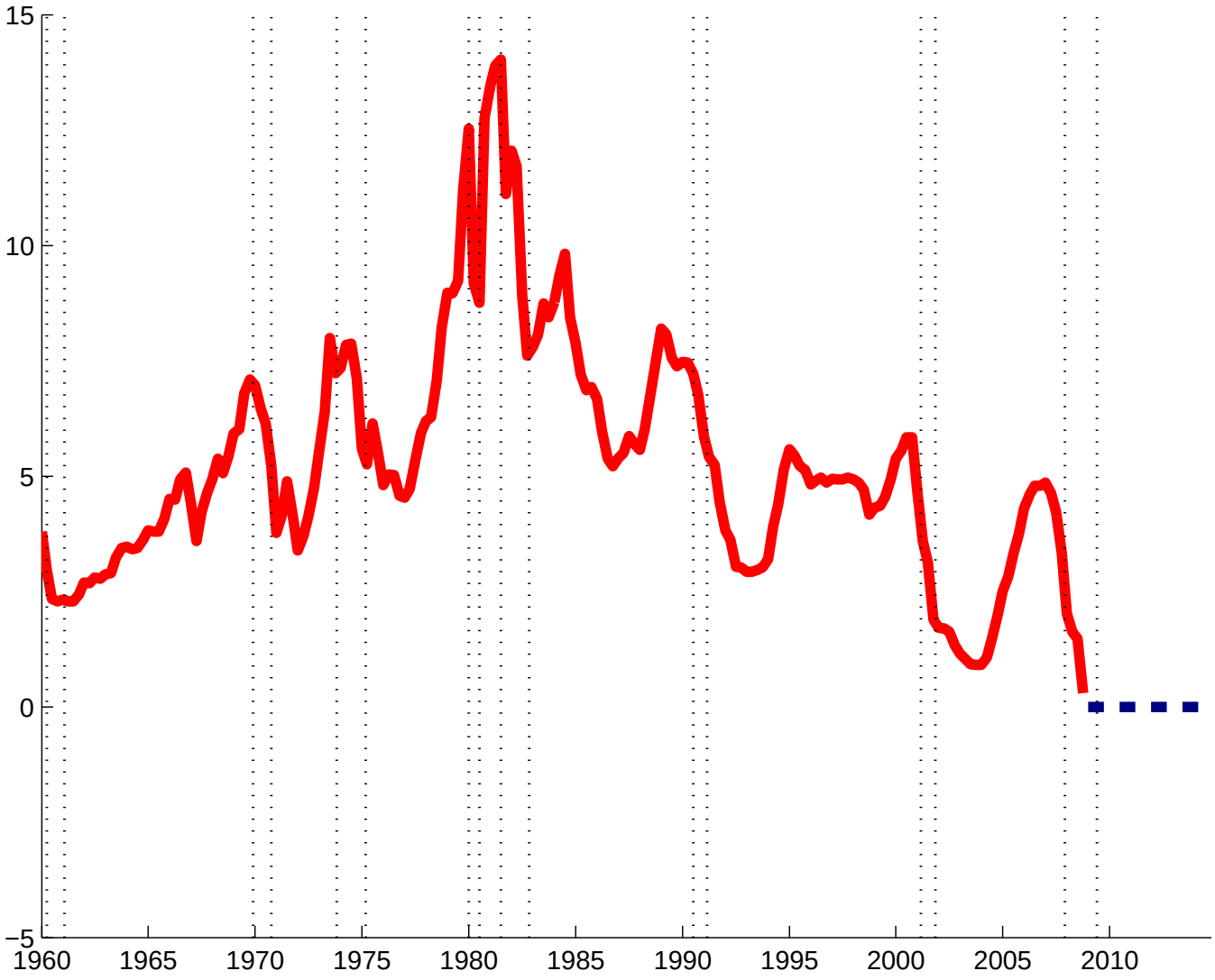
NOMINAL INTEREST RATE

Three-month T-Bill (APR)



RESEARCH AGENDA

Nominal short rate near zero since late 2008

- Typical time-series tools unusable
  (VARs, unobserved component models, time-varying parameter macro)
- How to measure variations in the stance of policy?

Shadow rate approach

- Hypothetical nominal rate, unconstrained by lower bound
- Our focus: Pure time series approach

SOME QUESTIONS

1. **What is the long-run level of the real rate, how does it relate to growth?**
2. **How did the recent recession affect trends in real and nominal variables?**
3. **How does the term structure change with short rates near the lower bound?**

RELATED LITERATURE

Macro-Time Series at the ZLB

Iwata & Wu (2006), Nakajima (2011),
Chan & Strachan (2014)

Dynamic Term-Structure Models

Kim & Wright (2005), Wright (2011),
Kim & Singleton (2011), Krippner (2013), Wu & Xia (2014),
Bauer & Rudebusch (2014)

Unobserved Component Models of the Macroeconomy

Gordon (1997), Gerlach & Smets (1997),
Staiger, Stock & Watson (1997), Laubach & Williams (2003),
Clark & Kozicki (2005), Stock & Watson (2007),
Stella & Stock (2013), Watson (2014), Mertens (2014)

AGENDA

1. **Shadow-Rate Concept**
2. **Shadow-Rate Sampling**
3. **Estimates from an Empirical Macro Model**
4. **Term-Premium Estimates**

Shadow Rate i_t

Nominal interest rate that would prevail in the absence of lower bound constraint

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Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate

$$i_t^* = \max(0, i_t)$$

THE SHADOW RATE OF INTEREST

FISCHER BLACK (1995)

Shadow Rate i_t

Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate

$$i_t^* = \max(0, i_t)$$

Key idea

Model i_t with conventional tools and handle $\max$ operator

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AR(1) EXAMPLE

Observer

$$i_t^* = \max(0, i_t)$$

Shadow Rate as Latent State

$$(i_t - \bar{i}) = \rho\,(i_{t-1} - \bar{i}) \qquad + \sigma\, e_t$$

AR(1) EXAMPLE

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Shadow Rate as Latent State

$$(i_t - \bar{i}_t) = \rho_t(i_{t-1} - \bar{i}_{t-1}) + \sigma_t e_t$$

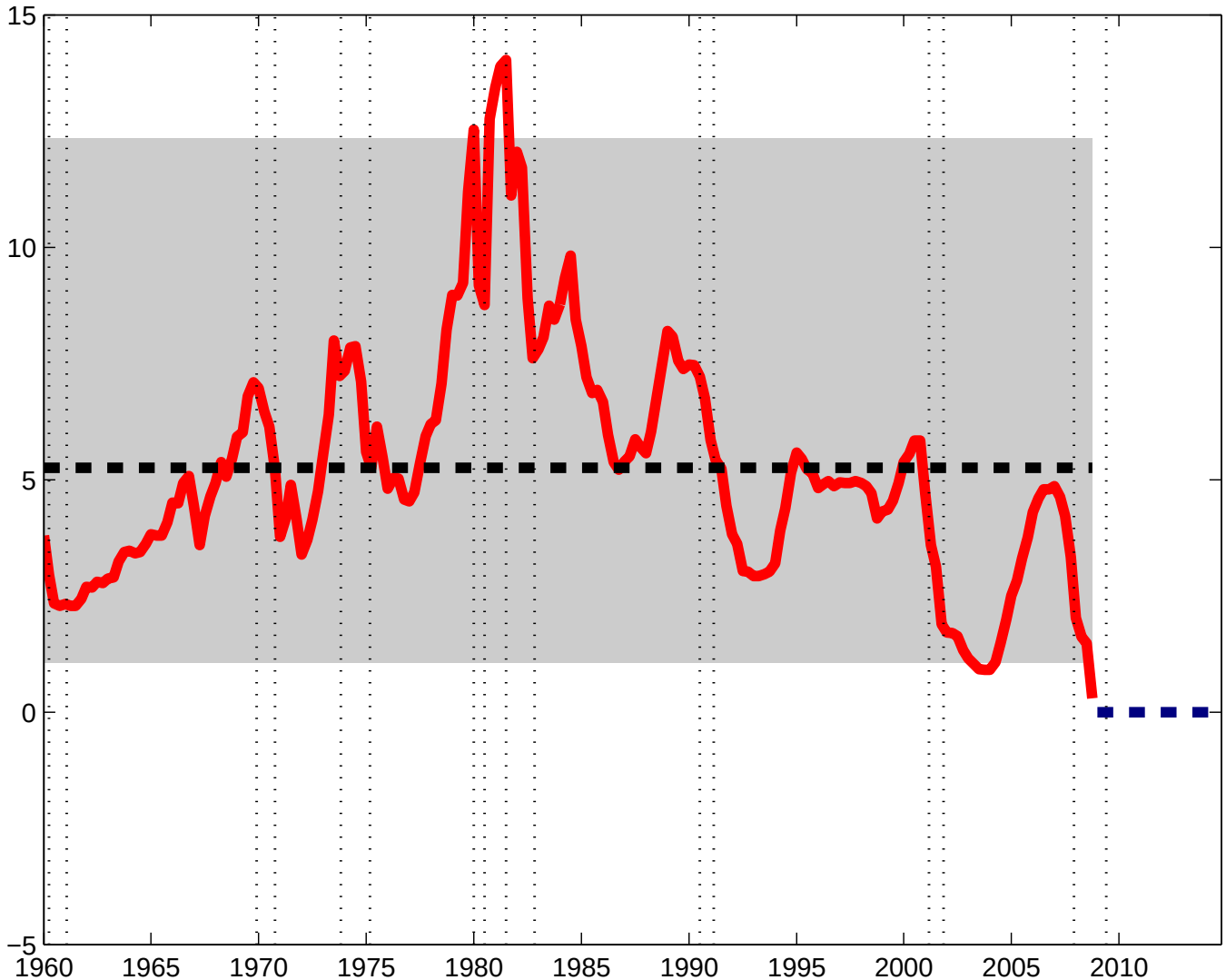
SHADOW RATE SAMPLING

- Let $\boldsymbol{Y^*} = \begin{bmatrix} \boldsymbol{i_1^*} \\ \boldsymbol{i_2^*} \\ \vdots \\ \boldsymbol{i_T^*} \end{bmatrix} = \boldsymbol{\max(0, Y)}$
- Denote non-zero values of $\boldsymbol{Y^*}$ by $\boldsymbol{X}$
- Kalman smoother implies $\boldsymbol{Y}|\boldsymbol{X} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{V})$
- Thus $\boldsymbol{Y^*}|\boldsymbol{X} \sim \text{trunc}\boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{V})$
- Model only needs to be conditionally linear

NOMINAL INTEREST RATE

Three-month T-Bill (APR), 90 percent quantile range

U.S. DATA



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TREND AND CYCLE IN NOMINAL INTEREST RATE

Trend-cycle decomposition for the shadow rate

$$i_t = \bar{i}_t + \tilde{i}_t \qquad \bar{i}_t = E_t i_{t+\infty} \qquad \tilde{i}_t \sim I(0)$$

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Taylor-type reaction function for the gap

$$\tilde{i}_t = \rho\, \tilde{i}_{t-1} + d_\pi(\pi_t - \bar{\pi}_t) + d_y(y_t - \bar{y}_t) + \varepsilon_t^i$$

TREND-CYCLE MODEL FOR MACRO VARIABLES

Observables

Real GDP:	$y_t = \bar{y}_t + \tilde{y}_t$
Unemployment rate:	$u_t = \bar{u}_t + \tilde{u}_t$
Inflation:	$\pi_t = \bar{\pi}_t + \tilde{\pi}_t$
Nominal Rate:	$i_t^* = \max(0, i_t)\ i_t = \bar{r}_t + \bar{\pi}_t + \tilde{i}_t$

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Nominal Rate: $i_t^* = \max(0, i_t)\ i_t = \bar{r}_t + \bar{\pi}_t + \tilde{i}_t$

Trends

$$\bar{y}_t = \mu_{t-1} + \bar{y}_{t-1} + \bar{\varepsilon}_t^y \qquad \mu_t = \mu_{t-1} + \eta_t^\mu$$

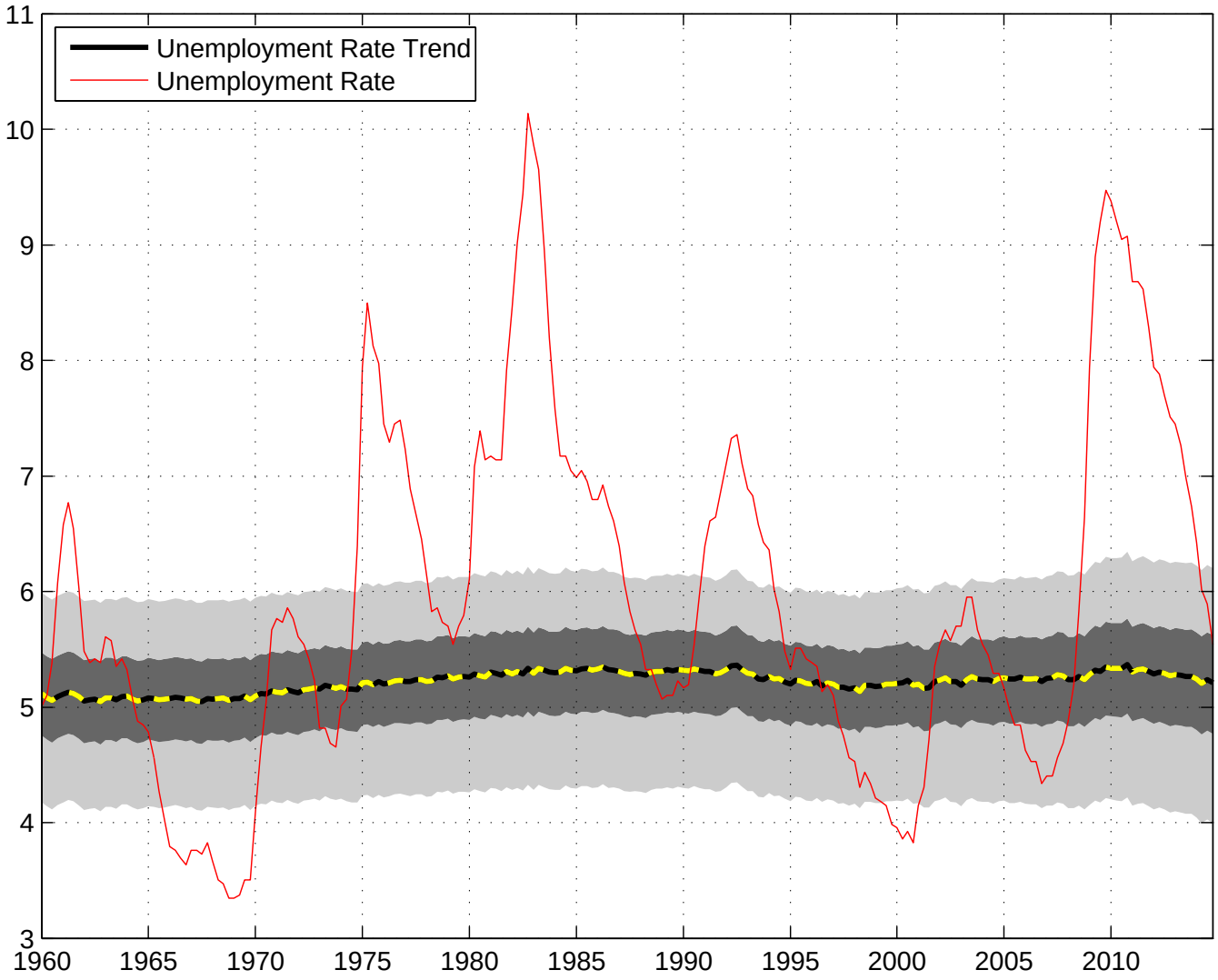
$$\bar{u}_t = \bar{u}_{t-1} + \bar{\varepsilon}_t^u$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \bar{\varepsilon}_t^\pi$$

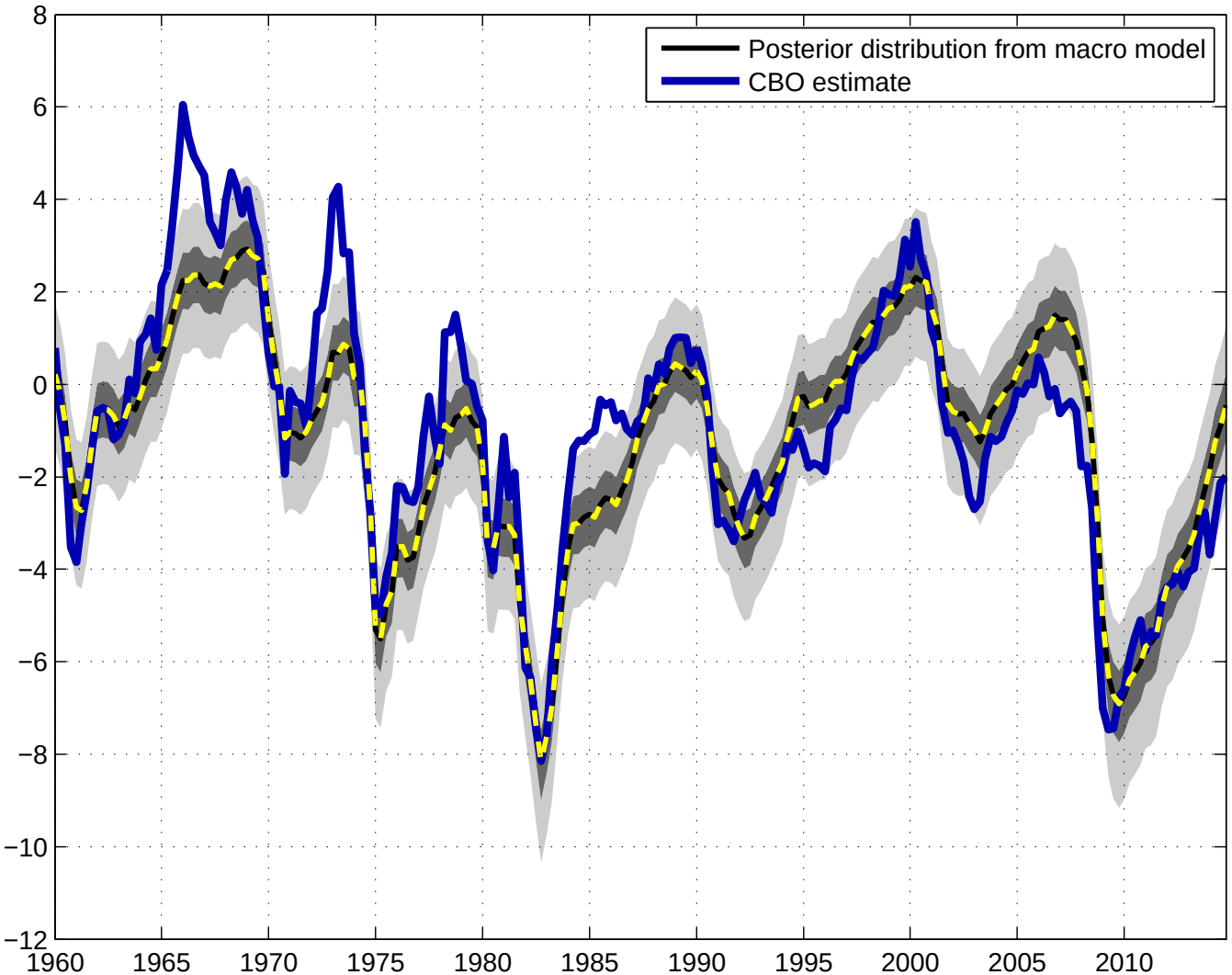
Dynamic factor model for the gaps

$$a(L)\tilde{y}_t = \tilde{\varepsilon}_t \quad \tilde{u}_t = b_u(L)\tilde{y}_t + \varepsilon_t^u \quad \tilde{\pi}_t = b_\pi(L)\tilde{y}_t + \varepsilon_t^\pi$$

TREND UNEMPLOYMENT

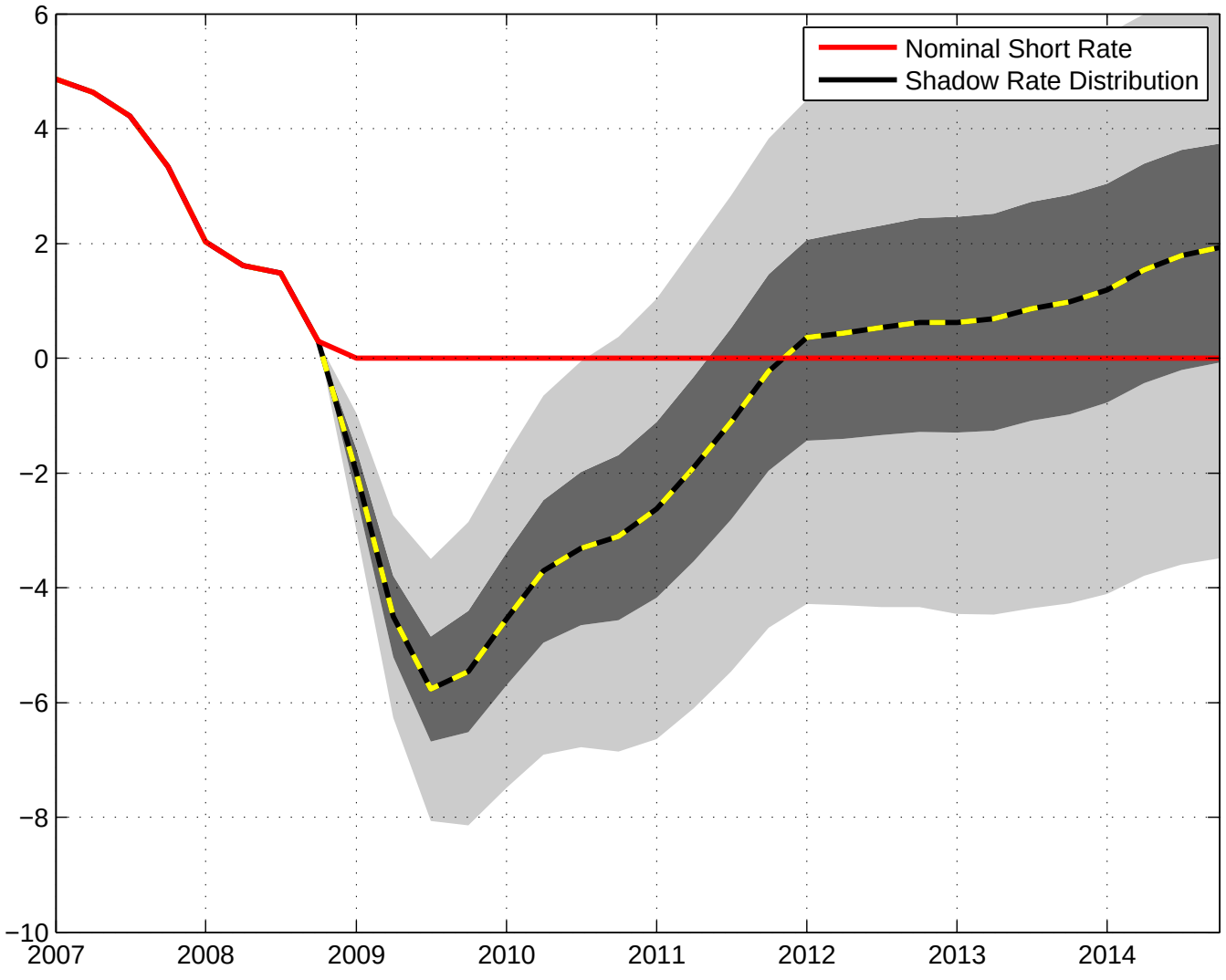


OUTPUT GAP


Posterior distribution from macro model
CBO estimate
8
6
4
2
0
−2
−4
−6
−8
−10
−12
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010

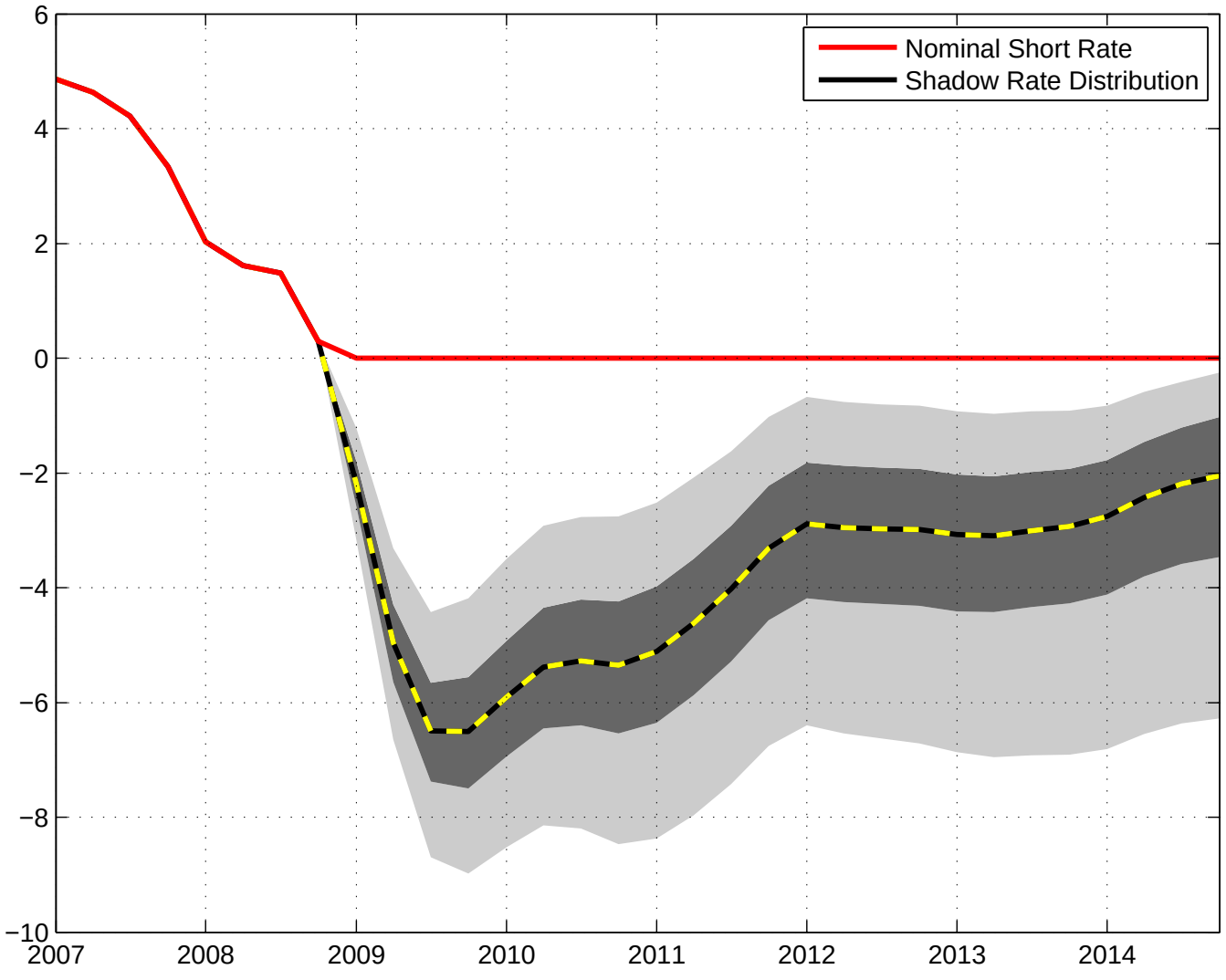
SHADOW RATE ESTIMATES

$i_t^* = 0$ treated as missing data



SHADOW RATE ESTIMATES

Censored sampling: $i_t < 0$ when $i_t^* = 0$



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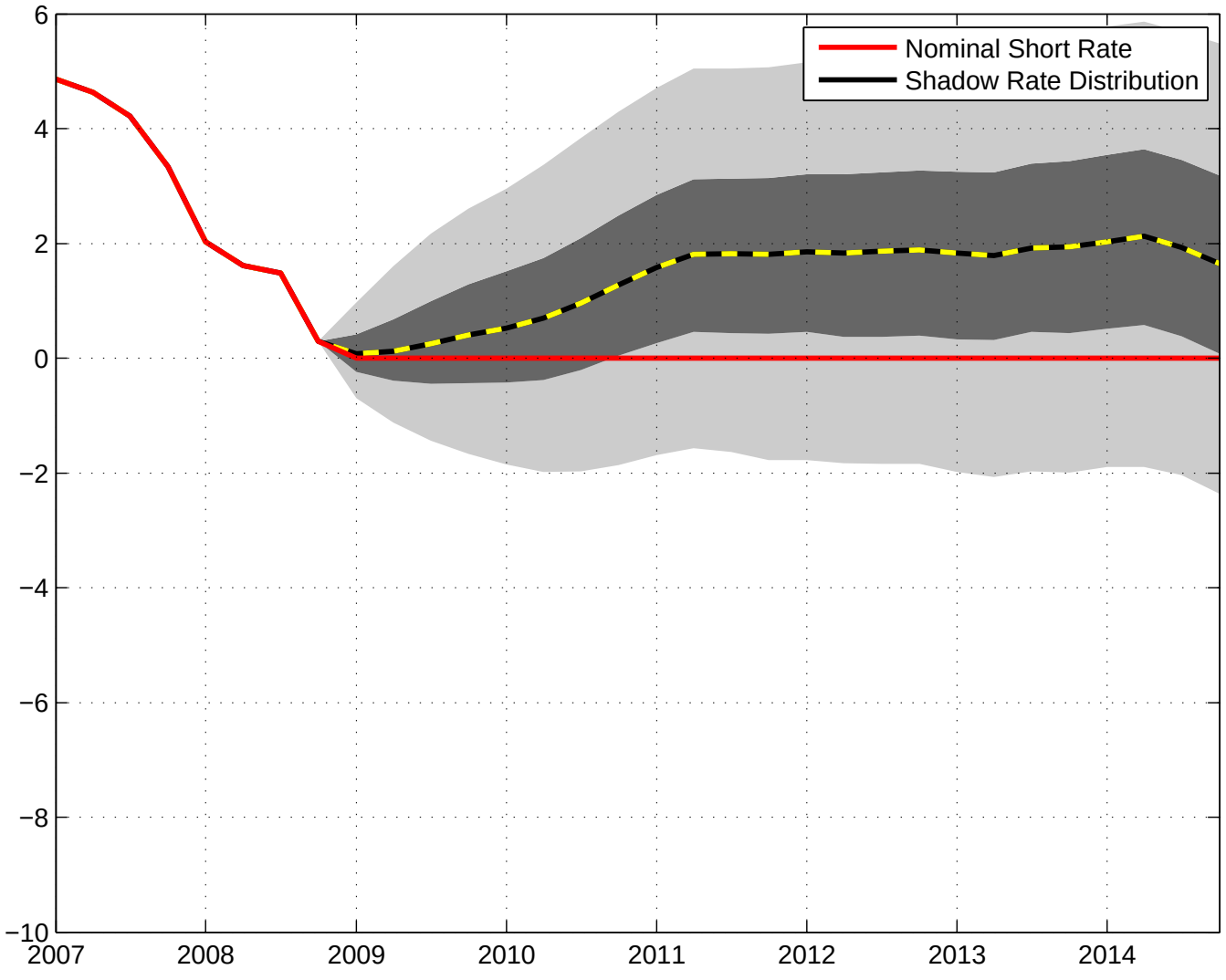
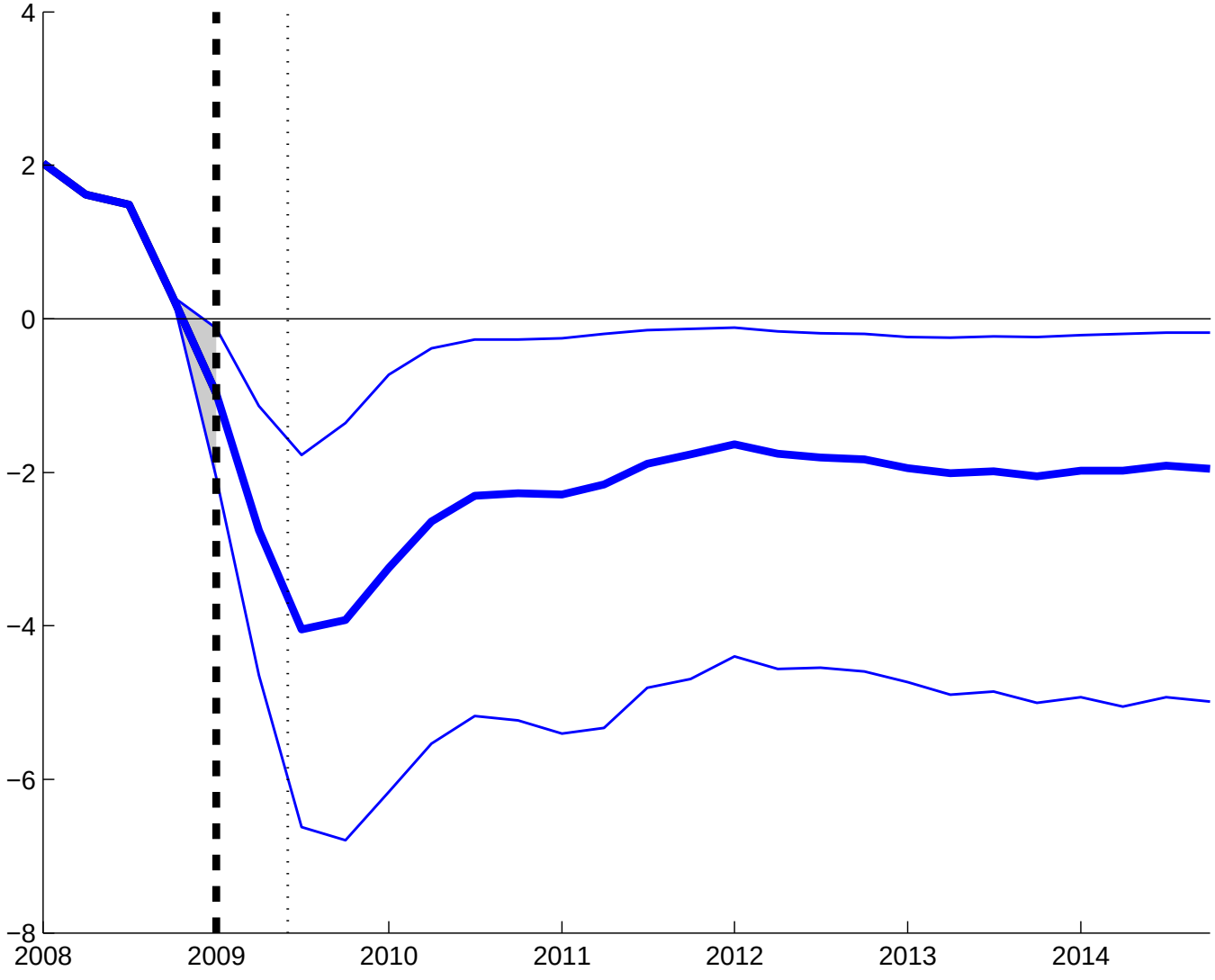


SHADOW RATE ESTIMATES IN PSEUDO-REAL TIME



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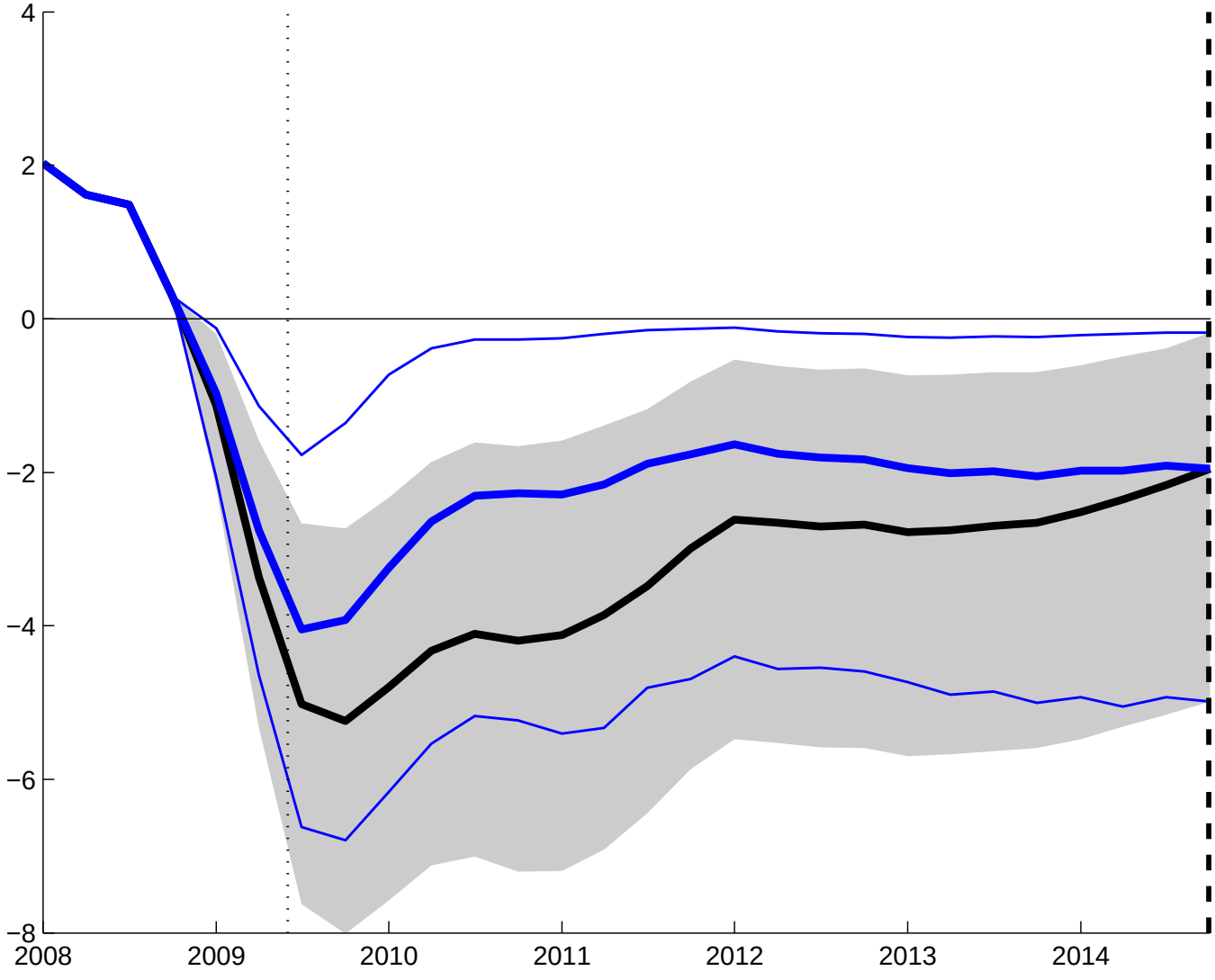


REAL (SHADOW) RATE TREND

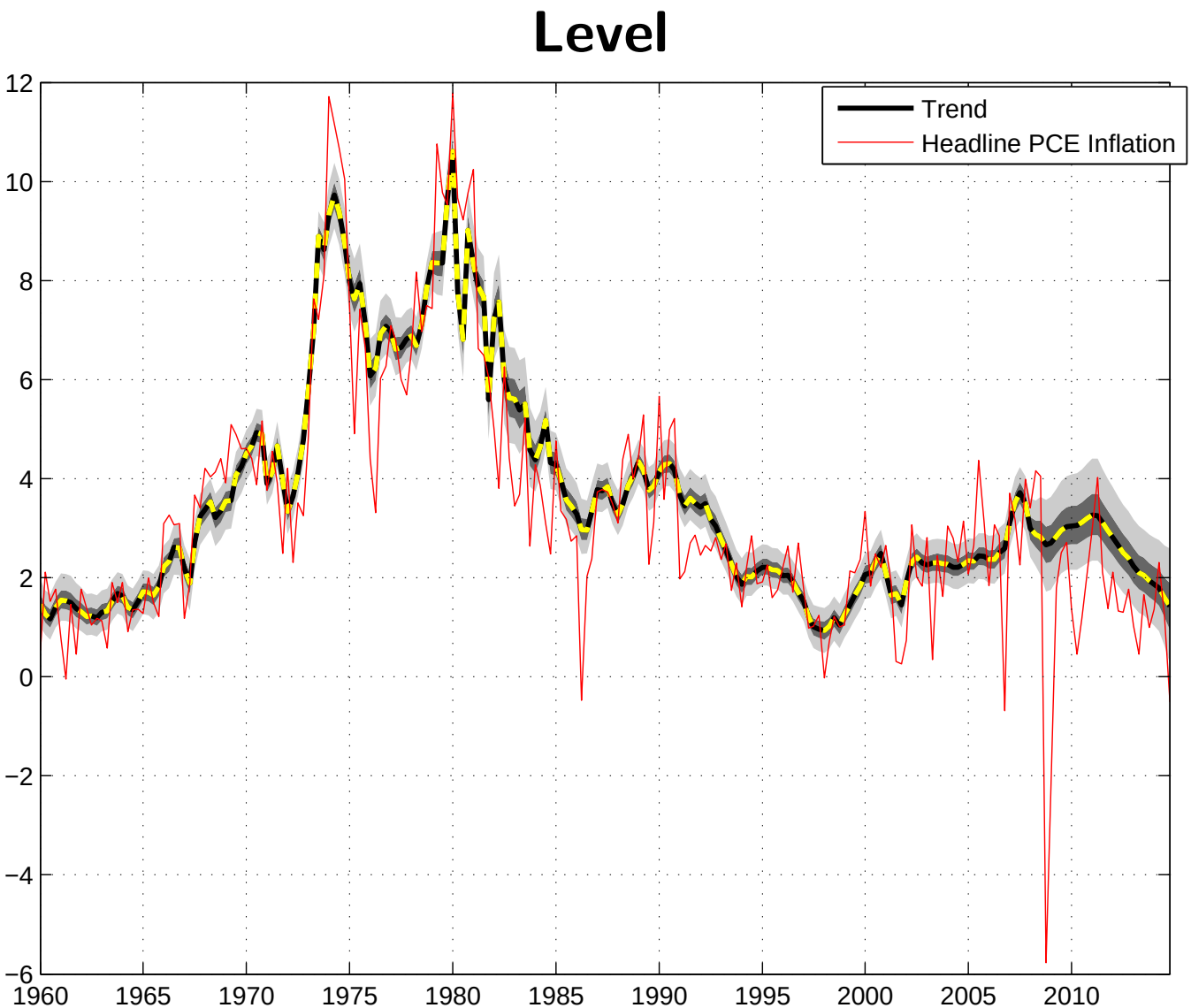
Real shadow-rate trend $\bar{r}_t$

$$\begin{aligned}\bar{i}_t &= \bar{r}_t + \bar{\pi}_t \\ \bar{r}_t &= \bar{r}_{t-1} + e_t \\ &= \beta \cdot \mu_t + r_t^{\perp}\end{aligned}$$

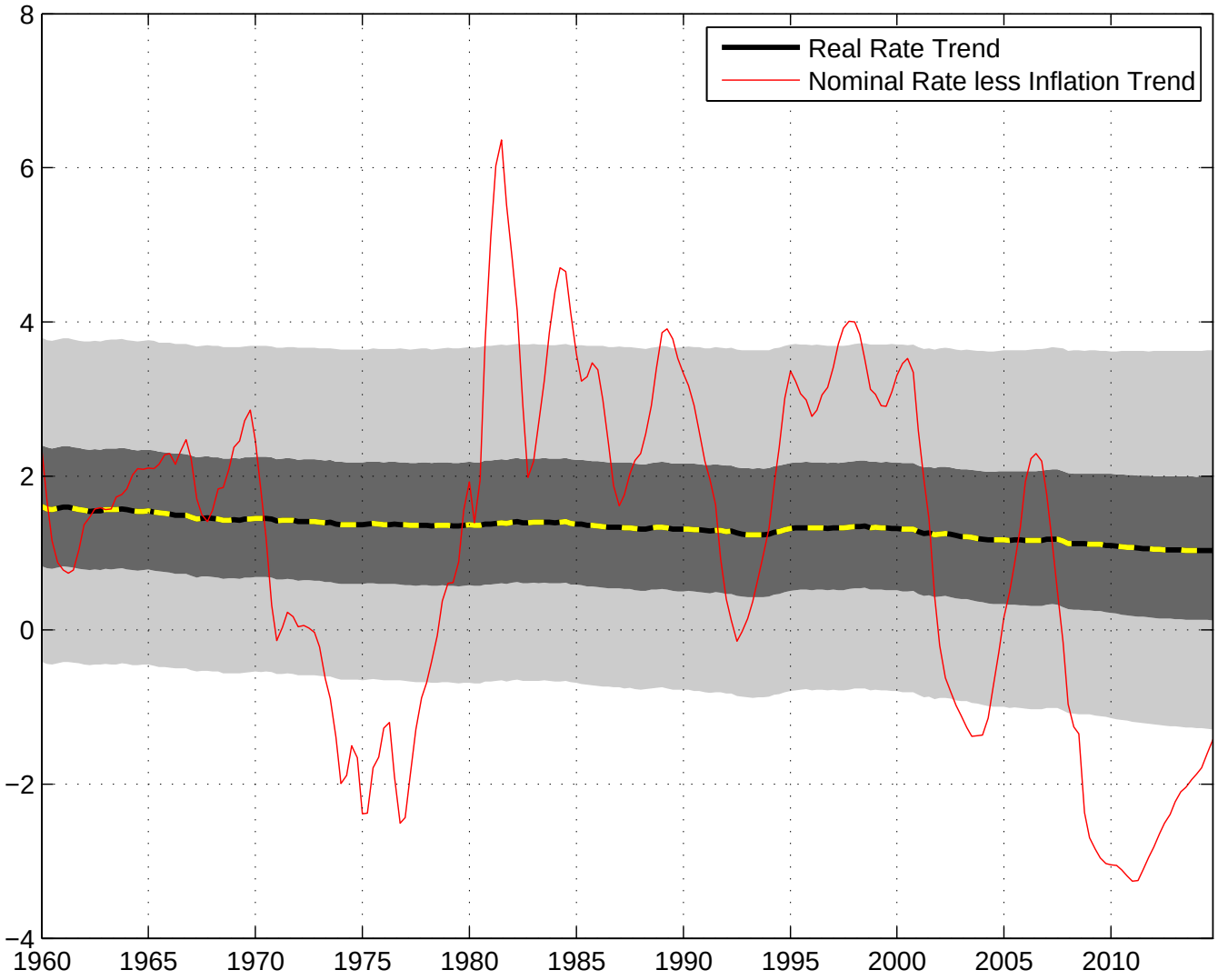
Trend growth μ_t

$$\begin{aligned}y_t &= \bar{y}_t + \tilde{y}_t \\ \bar{y}_t &= \mu_{t-1} + \bar{y}_{t-1} + \bar{\varepsilon}_t^y \\ \mu_t &= \mu_{t-1} + \eta_t^{\mu}\end{aligned}$$

TREND INFLATION

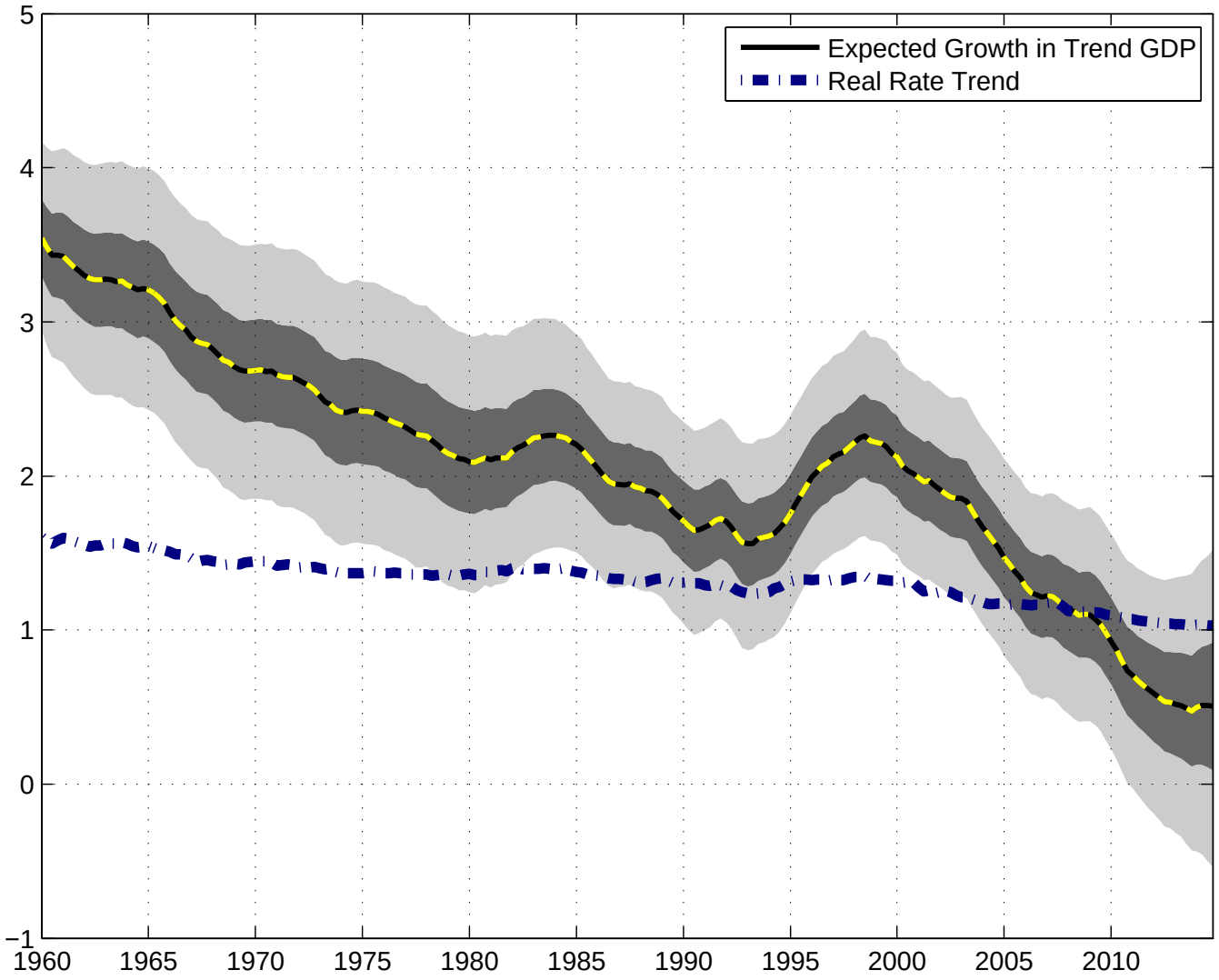


REAL RATE TREND



TREND GROWTH AND REAL RATE TREND

$\bar{r}_t = \beta \cdot \mu_t + r_t^{\perp}$



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TERM PREMIUM

A decomposition for h-period bond yield

$$i^*_{t,t+h} = \frac{1}{h}\sum_{j=0}^{h-1} E_t i^*_{t+j} + p_{t,t+h}$$

Expected future short rates

$E_t i^*_{t+j}$: mean of truncated predictive density for i_{t+j}

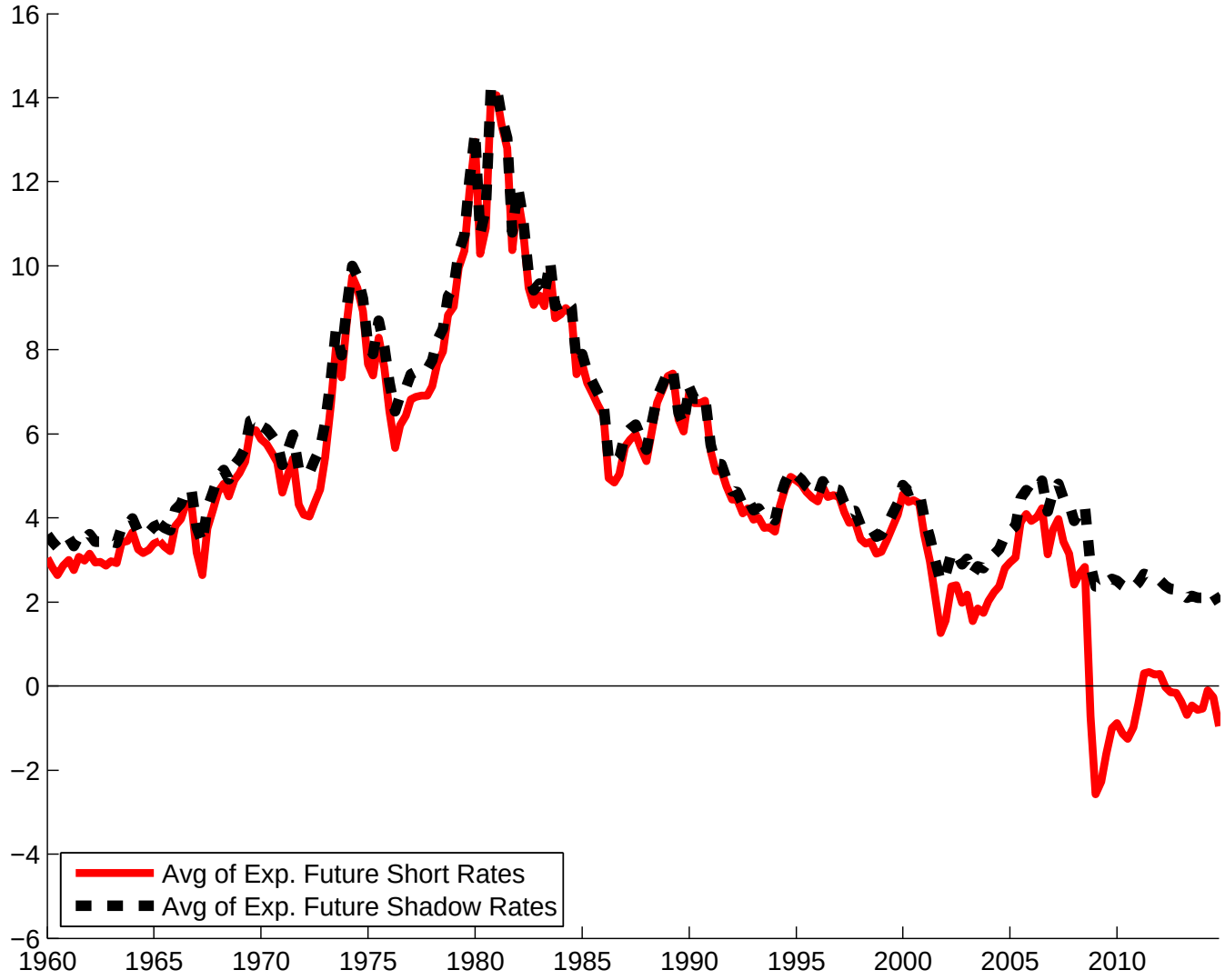
$$\frac{\partial E_t i^*_{t+j}}{\partial E_t i_{t+j}} < 1 \quad \text{the more mass on } i_{t+j} < 0$$

LONG RATES AND EXPECTED FUTURE SHORT RATES

Expected future shadow rates (10-year avg)

LONG RATES AND EXPECTED FUTURE SHORT RATES

Expected future rates (actual vs. shadow, 10-year avg)



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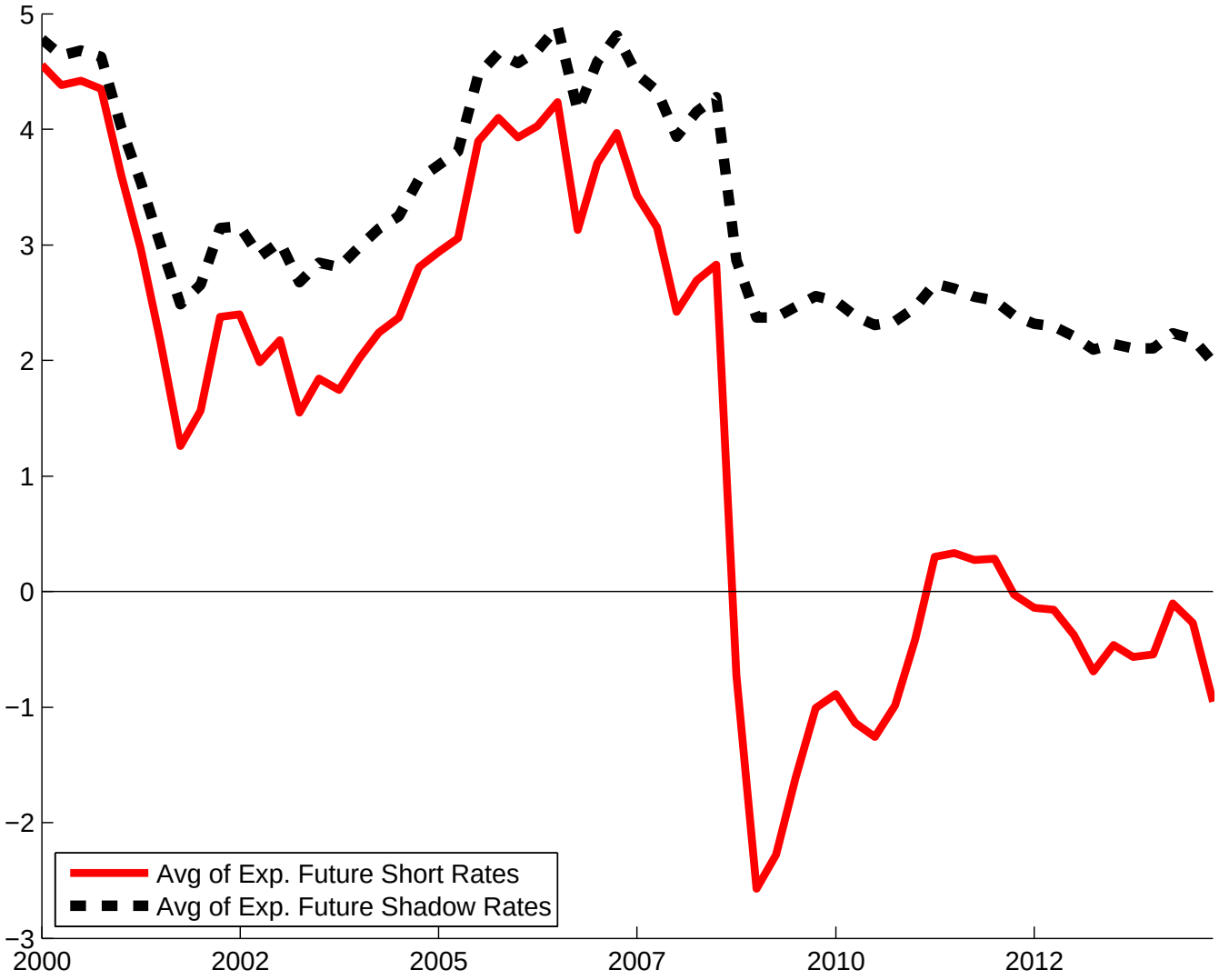

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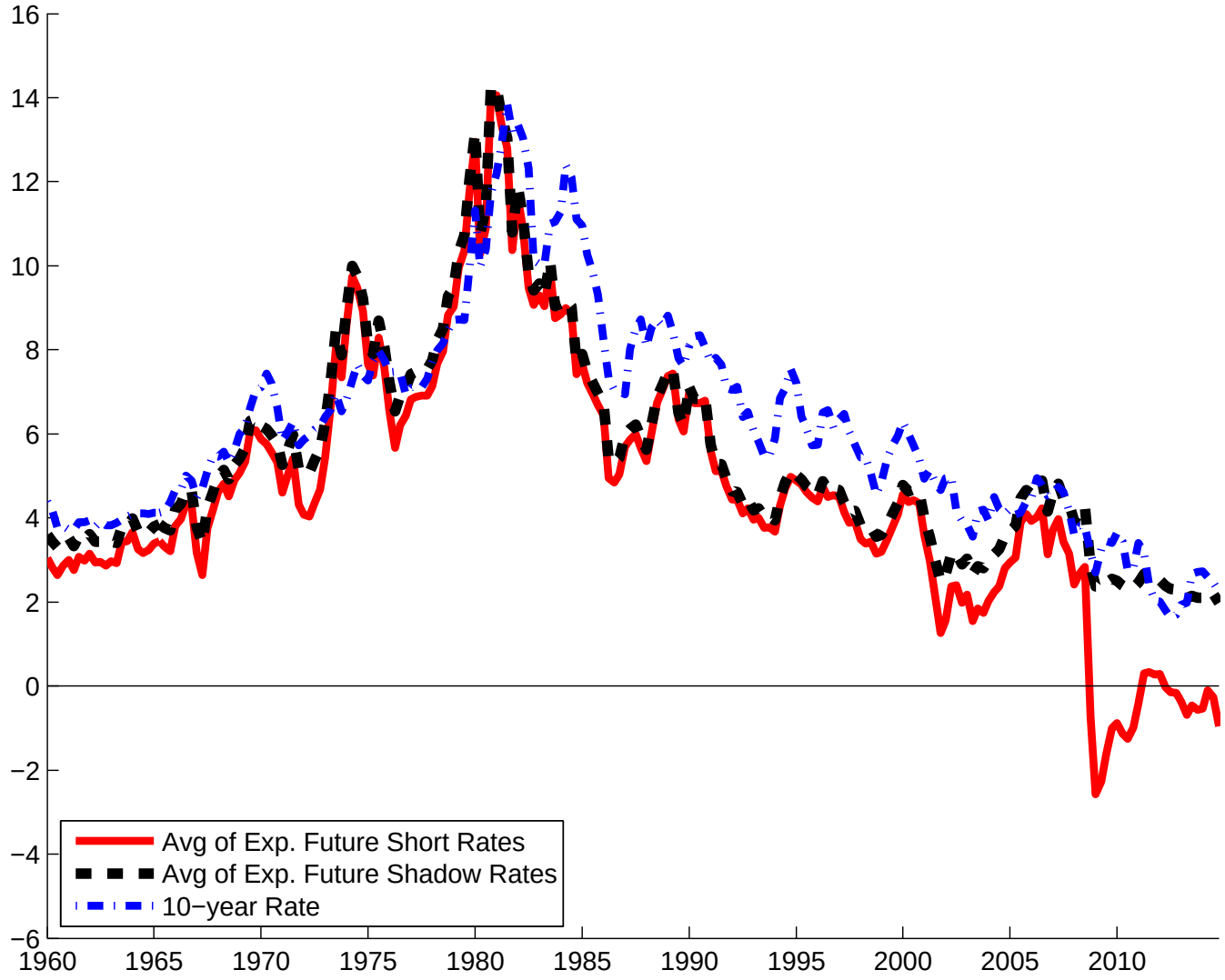
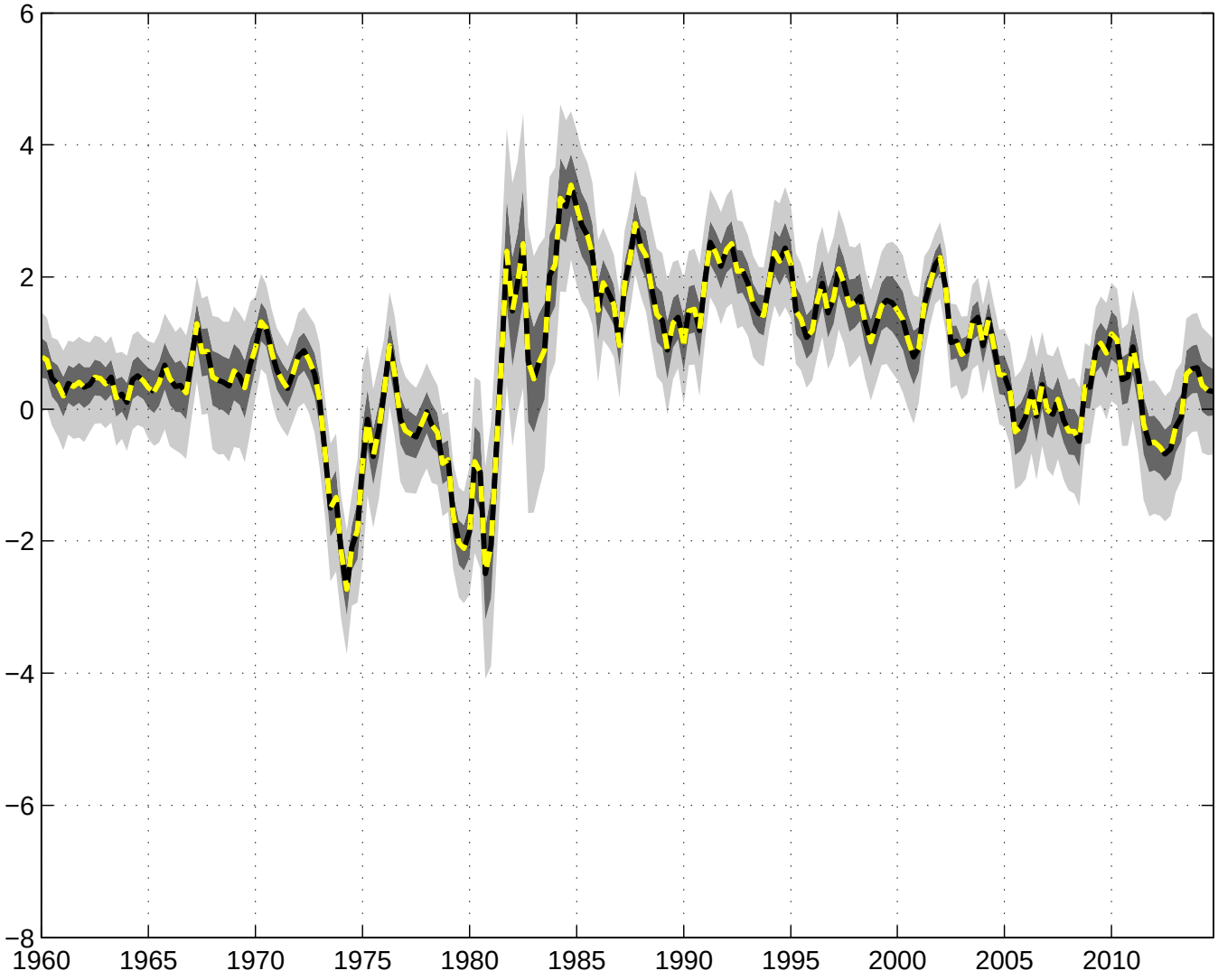
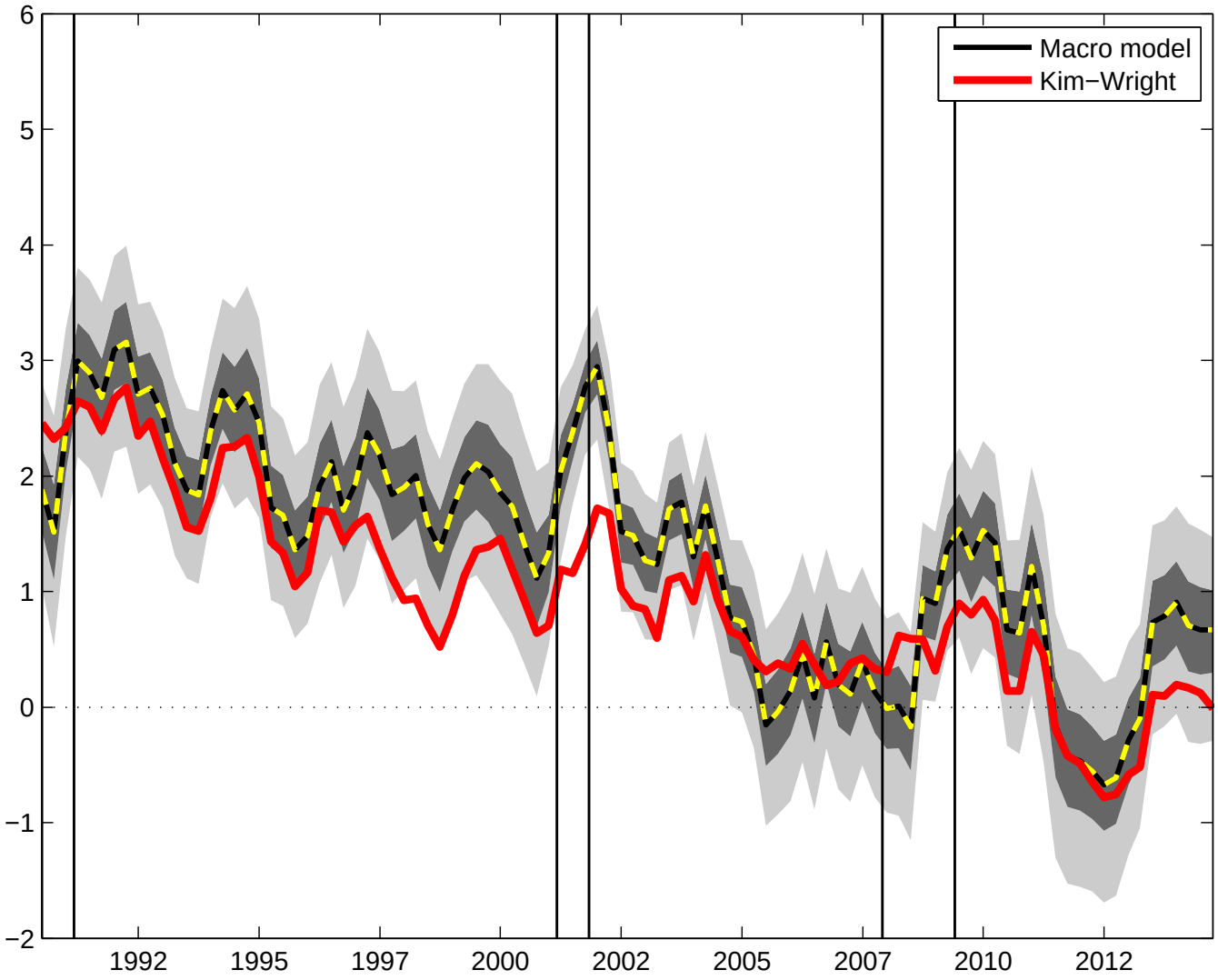


TERM PREMIUM ESTIMATES

10-year rate



TERM PREMIUM ESTIMATES
10-year rate



SOME QUESTIONS

and answers

1. **What is the long-run level of the real rate, how does it relate to growth?**

   Trend real rate depends only weakly on growth, stands currently at just about one percent

2. **How did the recent recession affect trends in real and nominal variables?**

   "Not much" as decline in trends started earlier

3. **How does the term structure change with short rates near the lower bound?**

   Expected future short rates get less sensitive to news